# A Quantitative Theory of Information and Unsecured Credit[*]

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## Abstract

Over the past three decades four striking features of aggregates in the unsecured credit market have been documented: (1) rising availability of credit along both the intensive and extensive margins, (2) rising debt accumulation, (3) rising bankruptcy rates and discharge in bankruptcy, and (4) rising dispersion in interest rates across households. We provide a quantitative theory of unsecured credit that is consistent with these facts. Specifically, we show that all four outcomes mentioned above are consistent with improvements in the ability of lenders to observe more components of individual income now than in earlier periods. A novel feature is that we allow for individualized loan pricing under asymmetric information. In addition, the paper makes a methodological contribution: an algorithm to locate equilibria with asymmetric information, a task that is complicated by the requirements that (i) lenders must use all information revealed by household choices and (ii) off-equilibrium beliefs and prices matter for equilibrium outcomes.

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# 1. Introduction

Over the past three decades there have been dramatic changes in the unsecured credit market. First, the availability of unsecured credit has increased both along the extensive and intensive margin; Narajabad (2007) documents that the fraction of US households with positive credit card limits increased by 17 percentage points between 1989 and 2004, while the average credit limit more than doubled over the same time period. In addition to the increase in availability of credit, Krueger and Perri (2006) measure that unsecured debt (utilized credit) as a fraction of disposable income has risen from 2 percent to 9 percent from 1980 to 2005. Several researchers have documented the significant rise in Chapter 7 bankruptcies over the same time period, including Livshits, MacGee, and Tertilt (2006) and Sullivan, Warren, and Westbrook (2000). Athreya (2004) notes explicitly that this increase continued over the entire 1990s; quantitatively, the filing rate per 1000 households went from 2 in 1980 to 9 in 2002. Finally, Sullivan, Warren, and Westbrook (2000) also notes that defaults are not only more common but also much larger; median non-mortgage debt-to-income ratio for households filing for bankruptcy doubled from 0.75 to 1.54 over the period 1981-1997 (see Figure 1, taken from Sullivan, Warren, and Westbrook 2000).

Recent empirical work on the functioning of consumer credit markets has also documented striking changes in the sensitivity of credit terms to borrower characteristics. A summary of this work is that credit terms, especially for unsecured loans, exhibited little variation across US households as recently as 1990, even though in the cross-section these households exhibited substantial heterogeneity in income, wealth, and default risk.[1] In the subsequent period, from 1990 to the present, a variety of financial contracts, ranging from credit card lines to auto loans to insurance, now exhibit terms that depend nontrivially on regularly updated measures of default risk, particularly a household's credit score. Three related findings stand out from the literature. First, the sensitivity of credit card loan rates to the conditional bankruptcy probability grew substantially between after the mid-1990's (Edelberg 2006). Second, credit scores themselves became more informative. Furletti (2003), for example, finds that the spread between the rates paid by highest and lowest risk classifications grew from zero in 1992 to 800 basis points by 2002. Third, the distribution of interest rates for unsecured credit was highly concentrated in 1983 and very diffuse by 2001 (Livshits, MacGee, and Tertilt 2007b and Figure 2). Each of the preceding three findings relates to the amount of information available to lenders at the time of extending and pricing credit.

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[1]A survey of these empirical findings can be found in Hunt (2005).

In a world without default risk, changes in the information available to lenders would have little or no bearing on the availability or terms for credit. However, if default is a possibility, then the changes summarized above can be expected to alter the behavior of both households and lenders.

A good deal of recent attention has been given over to the task of accounting for the rapid growth and relatively high incidence of unsecured indebtedness and bankruptcy seen among US households, including Gross and Souleles (2003), Athreya (2004), Livshits, MacGee, and Tertilt (2006), and Narajabad (2007). The candidate explanations for the rise in debts and default fall into two (non-exclusive) categories. First, there is the possibility that the personal costs incurred by defaulters have fallen substantially, either as a result of improved bankruptcy filing procedures, the learning by households from each other about navigating the bankruptcy process, or even lower psychic costs (stigma). Gross and Souleles (2003) argue households did appear to be more willing to default in the late 1990's than in earlier periods, all else equal. Unfortunately, these explanations tend to produce rising default rates combined with declining discharges on average, as households become less able to borrow and therefore default on less debt.

A second class of explanations for rising debt and default hinges on the extent to which transactions costs associated with lending are likely to have fallen as a result of improved information storage and processing technologies available to lenders. Athreya (2004) and Livshits, MacGee, and Tertilt (2006) explore this possibility; unlike changes in costs at the individual level, falling risk-free rates or transactions costs can produce both an increase in default rates and an increase in the amount of debt discharged in bankruptcy, making this mechanism a more promising candidate than lower individual default costs for the time series observations in the credit market.

A common feature of the models that underlie the preceding explanations is full information: the information available to lenders always includes the entire relevant household state vector. A central motivation for our work is that these "non-information"-based explanations do not produce the rise in the dispersion of terms that we mention above, mainly because they affect all agents identically.[2] In an elegant but highly stylized framework, Narajabad (2007) innovates along this dimension by analyzing the effects of change in the informativeness of a signal received by lenders on a borrower's long term income level, showing that such a change is qualitatively consistent with the increased indebtedness, increased default, and increased dispersion in loan terms observed. Narajabad (2007) does not address changes in the degree of asymmetric information but rather

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[2]For example, reductions in transactions costs are simply reductions in the costs of running a lending business and so affect the risk-free lending rate; as a result they tend to affect all agents in very similar ways.

changes in the quality of symmetric information; in contrast, this paper focuses on changes in the extent to which information is asymmetric between borrowers and lenders.[3]

The preceding facts suggest that information available to lenders has improved over the past several decades. Our goal in this paper is to provide a quantitative theory of how improved information changes the unsecured credit market. Our work is novel because we provide the first model (to our knowledge) that allows for the quantitatively-serious measurement of how unsecured credit markets operate under asymmetric information and how changes in information alter outcomes when loan pricing is individualized. As is well known, equilibria under asymmetric information require a specification of the precise interaction of borrowers and lenders, which we model as a signalling game.[4] We are guided in our choice of market microstructure by the requirement that households perceive a price function for loans as a function of default risk; thus, we need to solve for prices for arbitrary borrowing levels, including those not observed in equilibrium. A second complication that must be dealt with under asymmetric information is the extent to which information is revealed by household decisions. In particular, in a conjectured equilibrium, the information conveyed by a borrower's chosen debt level must not provide incentives for a lender to deviate in the terms offered, given the information available to the intermediary; in our economy, this requirement states that the beliefs used to construct default rates (on the equilibrium path) must be consistent with the stationary distribution produced by the model.

Our model features well-defined intertemporal and inter-state motives for borrowing, as well as a rich endowment structure that generates the heterogeneity essential to understand how better information may have changed credit markets. Our paper is directly related to Chatterjee, Corbae, and Ríos-Rull (2006b), which attempts to provide a theory of reputation in unsecured borrowing; relative to that paper we simplify matters by abstracting from dynamic scoring of credit terms. Our justification for this approach is that under full information credit scores are irrelevant, while our interpretation of the period of partial information as the 1980s implies that credit scores were not used, even though they were collected; in turn, a key payoff of this assumption is quantitative tractability. Our paper is also complementary to Livshits, MacGee, and Tertilt (2007b) and Drozd and Nosal (2007), who offer theories of increased differentiation of borrowers based on declining

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[3]Narajabad (2007) features only *ex post* asymmetric information; all contracts are executed under symmetric information. Strong commitment assumptions ensure that the *ex post* asymmetric information does not alter equilibrium outcomes.

[4]Hellwig (1990) makes this point clearly.

transactions costs.[5]

To understand how improvements in information affect outcomes in the credit market, we study two equilibria. First, we allow lenders to observe all relevant aspects of the state vector necessary to predict default risk. We then compare this allocation to one where lenders are no longer able to observe all of these variables. We follow the literature's preferred specification of household labor income over the life-cycle. In this formulation, households draw stochastic incomes that are the sum of four components: a permanent shock realized prior to entry into the labor market (representing formal education), a deterministic age-dependent component with a peak several years before retirement, a persistent shock, and a purely transitory shock.[6] Thus, information changes alter the observability of the components of household labor income; specifically, we prohibit the lenders from observing total income and the two stochastic components. The difference across these allocations is a quantitative measure of the effect of improved information in unsecured credit markets.

Our first set of results focuses on the full information economy, where the pricing of debt incorporates all relevant information from the model. In this model bankruptcy is largely an intertemporal-smoothing phenomenon and not an inter-state smoothing one. That is, the bulk of borrowing occurs for life cycle purposes, and the bulk of default occurs when income expectations indicate that future borrowing capacity is not very valuable. As a result, default occurs among the unlucky young, and after age 50 is very low, both of which are features of the data. Under full information, our model matches the default rate and median borrowing on the unsecured credit market, but fails to match the unconditional mean of debt discharged through bankruptcy. However, given that we abstract from shocks to net worth that generate large involuntary defaults the appropriate target for discharged debt is smaller than that measured in the data (net worth shocks play an important role in Livshits, MacGee, and Tertilt 2006 and Chatterjee *et al.* 2007).

Once we depart from the full information setting, however, we find that the equilibrium levels of debt and default fall dramatically. In other words, the model produces outcomes similar to those obtaining in a period (before 1990) that observers have characterized as one with limited information. We show first constructively that an allocation in which all households can borrow large amounts at the risk-free rate is not an equilibrium: those households with weak future income prospects (i.e. high-risk households) have an incentive to deviate by borrowing large amounts,

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[5]Similar to Narajabad (2007), both papers assume strong *ex post* commitment to contracts on the part of lenders.
[6]Some representative citations include Hubbard, Skinner, and Zeldes (1994) and Krueger and Perri (2006).

generating nontrivial default risk. In turn, as the premium for borrowing is raised, the low-risk households refuse to borrow as much, revealing the type of all those who do; the market then requires an increase in the interest rate to ensure that lenders break even, which reduces borrowing by the high-risk types until they pool again with the low-risk types. This process continues until the incentives to deviate are offset by the need to smooth consumption. As a quantitative matter, this pooling equilibrium occurs at a low enough debt level to sustain risk-free lending to almost every borrower. Thus, a natural method for modelling the reduction of information leads to an outcome qualitatively consistent with the salient aspects of unsecured credit markets prior to 1990. One anomaly remains, however: our model predicts that average rates should have been lower before 1990 than today; because there is no default under partial information, observed interest rates are very low. Augmenting the model with net worth shocks should help resolve this puzzle, provided their distribution does not change dramatically over time.[7]

It is important to point out here the current debate over the effects of better information in the credit market. It is often asserted that better information in the credit market would harm disadvantaged groups, such as racial minorities, that benefit from pooling.[8] Our model predicts that all agents are better off under full information, as every individual can borrow more at lower rates; furthermore, we show that better information will lead to both "democratization" and "intensification" of credit – that is, we obtain increases in both the extensive and intensive margins of the unsecured credit market. In terms of welfare, the intensification is quantitatively more significant: high school agents benefit less than college agents under full information. Thus, information is not redistributing credit from bad to good borrowers, it is expanding it for everyone (as in the classic lemons problem).

The remainder of the paper is organized as follows. The next section introduces the model, followed by the algorithm used to compute equilibria, and then results of the quantitative model. The final section concludes.

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[7]Livshits, MacGee, and Tertilt (2006) argue that the distributions of net worth shocks are quite similar across the time periods under consideration. It is likely that changes in transactions costs have played an important role in reducing average costs as well.

[8]Specifically, Section 215 of FACT Act directs the Federal Reserve Board, the Federal Trade Commission, and the Office of Fair Housing and Equal Opportunity (a department of HUD) to study "the consideration or lack of consideration of certain factors...could result in negative treatment of protected classes under the Equal Credit Opportunity Act." Report to Congress on Credit Scoring and Its Effects on the Availability and Affordability of Credit," Board of Governors of the Federal Reserve System, August 2007.

## 2. Model

Households in the model economy live for a maximum of $J < \infty$ periods. Each household of age $j$ has a probability $\psi_j < 1$ of surviving to age $j + 1$ and has a pure time discount factor $\beta < 1$. Households value consumption and attach a negative value $\lambda$ (in utility terms) to the stigma of filing for bankruptcy.[9] Preferences are represented by the expected utility function

$$\sum_{j=1}^{J} \beta^{j-1} \left( \prod_{i=0}^{j} \psi_{j,y} \right) \Pi \left( s^j \right) \left[ n_j u \left( \frac{c_j}{n_j} \right) - \lambda \mathbf{1} \left( m_j = 1, m_{j-1} = 0 \right) \right], \tag{2.1}$$

where $m_j = 1$ if the household is currently under a bankruptcy flag and $m_j = 0$ otherwise. $\Pi \left( s^j \right)$ is the probability of a given history of events $s^j$. We assume that households are risk averse, so that $u'' (c) < 0$. Households retire exogenously at age $j^* < J$, and $n_j$ is the number of adult-equivalent household members at age $j$. Thus, the event $\{m_j = 1, m_{j-1} = 0\}$ implies that the household is declaring bankruptcy in the current period and paying the utility cost $\lambda$ to do so.[10]

The household budget constraint during working age is given by

$$c_j + q \left( b, I \right) b + \delta \mathbf{1} \left( m_j = 1, m_{j-1} = 0 \right) \leq a + \omega_{j,y} y e \nu, \tag{2.2}$$

where $q$ is a bond price that depends on some individual characteristics $I$ and total borrowing $b$, $a$ is current net worth, $\delta$ is the pecuniary cost of filing for bankruptcy, and the last term is current income. Log income is the sum of four terms: a permanent shock $y$ realized prior to entry into the labor market, a deterministic age term $\omega_{j,y}$ that depends on the permanent shock realization, a persistent shock $e$ that evolves as an AR(1)

$$\log \left( e' \right) = \rho \log \left( e \right) + \epsilon', \tag{2.3}$$

and a purely transitory shock $\log (\nu)$. Both $\epsilon$ and $\log (\nu)$ are independent mean zero normal random variables with variances that depend on $y$. The budget constraint during retirement is

$$c_j + q \left( b, I \right) b \leq a + \theta \omega_{j^*-1} y_{j^*-1} e_{j^*-1} \nu_{j^*-1}, \tag{2.4}$$

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[9]This stigma cost is intended to be a parsimonious method for capturing all of the factors and complications associated with bad credit other than pecuniary costs and credit market terms, as in Athreya (2002).

[10]In terms of consumption, $\lambda$ is a more severe punishment for wealthy individuals, which is consistent with a stigma notion, because utility functions are concave. In contrast, the pecuniary filing cost we discuss below is more painful for poor individuals. In our calibration both costs end up approximately the same size.

where for simplicity we assume that pension benefits are a fraction $\theta \in (0,1)$ of income in the last period of working life. Because bankruptcy is not a retiree phenomenon, we deliberately keep the specification of retirees simple. There do not exist markets for insurance against income risk or survival risk and we abstract from any sources of long-run growth. Net worth tomorrow will either equal $b$ or $0$, depending on whether the household exercises the default option or not; thus, we write $a'$ as a function of current state variables and $(e', \nu')$.

The survival probabilities $\psi_{j,y}$ and the deterministic age-income terms $\omega_{j,y}$ differ according to the realization of the permanent shock. We will interpret $y$ as differentiating between college and high school education levels, as in Hubbard, Skinner, and Zeldes (1994), and the differences in these life-cycle parameters will reflect the differing incentives to borrow across types. In particular, college workers will have higher survival rates and a steeper hump in earnings. College workers also face smaller shock variances. As a result, college workers have a strong demand to borrow for purely intertemporal reasons – they want to smooth out the severe hump in their earnings – but limited demand for interstate smoothing because their shocks are (relatively) small; high school workers have the opposite motives.

## 2.1. Recursive Formulation

The recursive version of the household problem is useful for understanding the household's problem, and especially the default decision. A household of age $j$ faces the dynamic program

$$
v\left(a, y, e, \nu, j, m=0\right) = \max_{b, d(e', \nu') \in \{0,1\}} \left\{ \begin{array}{c} n_j u\left(\frac{c_j}{n_j}\right) + \beta \psi_{j,y} \sum_{e'} \sum_{\nu'} \pi_e\left(e'|e\right) \pi_\nu\left(\nu'\right) \\ \times \left[ \begin{array}{c} \left(1 - d\left(b, e', \nu'\right)\right) v\left(b, y, e', \nu', j+1, m'=0\right) + \\ d\left(b, e', \nu'\right) v^D\left(0, y, e', \nu', j+1, m'=1\right) \end{array} \right] \end{array} \right\}
$$

$$
(2.5)
$$

where

$$
v^D\left(0, y, e, \nu, j+1, m=1\right) = \left\{ \begin{array}{c} n_j u\left(\frac{c_j}{n_j}\right) - \lambda + \beta \psi_{j,y} \sum_{e'} \sum_{\nu'} \pi_e\left(e'|e\right) \pi_\nu\left(\nu'\right) \\ \times \left[ \begin{array}{c} \xi v\left(a', y, e', \nu', j+1, m'=0\right) + \\ \left(1 - \xi\right) v\left(a', y, e', \nu', j+1, m'=1\right) \end{array} \right] \end{array} \right\}
$$

$$
(2.6)
$$

and

$$v\left(a, y, e, \nu, j, m = 1\right) = \max_{a' \geq 0} \left\{ \begin{array}{c} n_j u\left(\frac{c_j}{n_j}\right) + \beta \psi_{j,y} \sum_{e'} \sum_{\nu'} \pi_e\left(e'|e\right) \pi_\nu\left(\nu'\right) \\ \times \left[ \begin{array}{c} \xi v\left(a', y, e', \nu', j+1, m'=0\right) + \\ \left(1 - \xi\right) v\left(a', y, e', \nu', j+1, m'=1\right) \end{array} \right] \end{array} \right\}. \qquad (2.7)$$

The first expression is the program for a household with good credit and the second is a household that is defaulting in the current period. The punishment for default is the nonpecuniary cost $\lambda$ and the pecuniary cost $\delta$ mentioned above, a ban on saving in the period of default, and a subsequent probabilistic prohibition from borrowing, where the parameter $\xi \in (0,1)$ governs the likelihood of being allowed to reenter the credit market to borrow. That is, a household with "bad credit" (i.e. $m = 1$) is permitted to reenter the credit market with probability $\xi \in (0,1)$; reentry is assumed to trigger the release of the bad credit marker, so that $m' = 0$. In each period the household initially makes a consumption-savings decision $(c, b)$, where $b$ is the amount of borrowing/saving. The household also makes a conditional default decision $d\left(b, e', \nu'\right)$ that equals 1 if the household declares bankruptcy in the event that next period's shocks are $(e', \nu')$ and 0 otherwise. In the event of default $a' = 0$, otherwise $a' = b$. Households with $m = 1$ cannot borrow but are permitted to save.[11]

## 2.2. Loan Pricing

We focus throughout on competitive lending. There exists a competitive market of intermediaries who offer one-period debt contracts and utilize available information to offer individualized credit pricing. Let $I$ denote the information set for a lender and $\widehat{\pi}^b$: $b_{|I} \rightarrow [0, 1]$ denote the function that assigns a probability of default to a loan of size $b$, given information $I$. $\widehat{\pi}^b$ is identically zero for positive levels of net worth and is equal to 1 for some sufficiently large debt level. The break-even pricing function must satisfy

$$q\left(b, I\right) = \left\{ \begin{array}{ll} \frac{1}{1+r} & \text{if } b \geq 0 \\ \frac{\left(1 - \widehat{\pi}^b\right)\psi_j}{1+r+\phi} & \text{if } b < 0 \end{array} \right. \qquad (2.8)$$

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[11]Exclusion is both theoretically and empirically tenuous. As argued in Chatterjee *et al.* (2007), regulators may impose a ban on lending to defaulters in order to prevent intermediaries from diluting the penalty from default, since *ex post* there is no reason to exclude borrowers. Empirically we observe only that individuals do not borrow a lot post-bankruptcy, but not that they cannot – it could be they simply do not like the terms offered. A theory of the terms of credit offered after bankruptcy is the subject of Chatterjee, Corbae, and Ríos-Rull (2006b).

given $\widehat{\pi}^b$. $r$ is the exogenous risk-free saving rate and $\phi$ is a transaction cost for lending, so that $r + \phi$ is the risk-free borrowing rate; the pricing function takes into account the automatic default by those households that die at the end of the period.[12] With full information, a variety of pricing arrangements will lead to the same price function. However, as is well known (e.g. Hellwig 1990), under asymmetric information settings outcomes often depend on the particular "microstructure" being used to model the interaction of lenders and borrowers. Under full information our approach is completely standard (see Chatterjee *et al.* 2007, Livshits, MacGee, and Tertilt 2006, and Athreya and Simpson 2006), as we seek a setting that delivers to households a function $q(b, y, e, \nu, j, m)$ : $b \rightarrow \left[0, \frac{1}{1+r}\right]$ that they can take parametrically when optimizing; the compactness of the range for $q$ implies that the household problem has a compact opportunity set and therefore possesses a solution.

We now detail explicitly the microstructure that underlies our pricing function, which we model as a three-stage game between borrowers and lenders. In the first stage, borrowers name a level of debt $b$ that they wish to issue. Second, a continuum of lenders compete in an auction where they simultaneously post a price for the desired debt issuance of the household and are committed to delivering the amount $b$ in the event their 'bid' is accepted; that is, the lenders are engaging in price competition for borrowers. Third, borrowers choose the lender who posts the lowest interest rate (highest $q$) and are committed to borrowing the amount $b$. Thus, households view the pricing functions as schedules and understand how changes in their desired borrowing will alter the terms of credit (that is, they know $D_b q(b, I)$) because they compute the locus of Nash equilibria under price competition. Exactly how the pricing function depends on the components in $I$ will be specified next. We defer a formal statement of equilibrium until after our discussion of $q$.

### 2.2.1. Full Information

In the full information case, $I$ includes all components of the household state vector: $I = (a, y, e, \nu, j, m)$. Zero profit for the intermediary requires that the probability of default used

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[12]We do not use a market-clearing condition to determine $r$. Most of the capital in the US economy is held by the wealthy (who do not hold very much unsecured debt) and the model does not have the ingredients needed to match the observed wealth distribution and thus produce the right risk-free rate. Chatterjee *et al.* (2007) utilize a neoclassical production function to determine factor prices endogenously; the computational burden of their model is tremendous, as matching the distribution of wealth requires agents with very high realizations of income. Our OLG setup would require even larger realizations and operative bequest motives, both of which would dramatically increase the computational burden.

to price debt must be consistent with that observed in the stationary equilibrium, implying that

$$\widehat{\pi}^b = \sum\nolimits_{e',\nu'} \pi_e\left(e'|e\right) \pi_\nu\left(\nu'\right) d\left(b\left(a,y,e,\nu,j,m\right),e',\nu'\right). \tag{2.9}$$

Since $d\left(b,e',\nu'\right)$ is the probability that the agent will default in state $(e',\nu')$ tomorrow at debt level $b$, integrating over all such events *tomorrow* is the relevant default risk. This expression also makes clear that knowledge of the persistent component $e$ is critical for predicting default probabilities, since the transitory component contributes little predictive power; the more persistent $e$ is, the more useful it becomes in assessing default risk.[13] With partial information we will need to integrate over current states as well as future ones, since pieces of the state vector will not be observable.

### 2.2.2. Partial Information

The main innovation of this paper is to take a first step in evaluating the consequences of changes in the information available for predicting default risk on one-period debt. Default risk, in turn, arises from a combination of indebtedness and the risk associated with future income. Under asymmetric information, we make an anonymous markets assumption: no past information about an individual (other than their current credit market status $m$) can be used to price credit. This assumption rules out the creation of a credit score that encodes past default behavior through observed debt levels; since income shocks are persistent, past borrowing would convey useful information, although it is an open question how much. Given the difficulties encountered by other researchers in dealing with dynamic credit scoring, we think it useful to consider an environment for which we can compute equilibria.[14]

Partial information in our economies will manifest itself through the observability of the stochastic components of income (including total income). We maintain the assumption throughout that age and education are observable and that total income and the transitory and persistent components of income are not.[15] In addition, we do not let lenders observe the current net worth of the borrower. Therefore, we have $I = (y, j, m)$ (with $(a, e, \nu)$ not observed).[16]

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[13]We leave as future work the case where neither borrowers nor lenders know how to decompose their income changes.

[14]See Chatterjee, Corbae, and Ríos-Rull (2006b).

[15]Again, we note that regulatory restrictions prohibit the use of age in determining credit terms, at least in the unsecured credit market, along with race and gender. We study the possibility that types are unobserved in a companion paper, which focuses on estimating the costs of such regulations.

[16]We separate $b$ from $I$ even though $b$ is observable because the borrower takes the derivative of $q$ with respect to $b$ and it is therefore more convenient to make it a separate argument.

The first concern for solving the partial information economy is that lenders must hold beliefs over the probability of an individual being in a particular state $(e, \nu)$ given *whatever* is observed, knowing also that what is observed is a function of lenders' *a priori* beliefs; that is, beliefs must satisfy a fixed point condition. Let $\Pr(e, \nu | b, y, j, m)$ denote the probability that an individual's shock vector in any period takes a given value $(e, v)$, conditional on observing the size of borrowing, the permanent shock, age, and credit status. Given this assessment, the lender can compute the likelihood of default on a loan of size $b$:

$$\widehat{\pi}^b = \sum_e \sum_\nu \left[ \sum_{e'} \sum_{\nu'} \pi_e \left( e' | e \right) \pi_\nu \left( \nu' \right) d \left( b, e', \nu' \right) \right] \Pr \left( e, \nu | b, y, j, m \right). \tag{2.10}$$

In a stable environment with a small number of creditors, or one with an efficient technology for information sharing, intermediaries must form beliefs that incorporate everything they either know or can infer from observables; competitors who exploit this information may be able to 'cream-skim' the best borrowers away from those who form beliefs in any other way.[17] In equilibrium, if this information exists it must be incorporated by all intermediaries into their belief functions; we view this arrangement as a natural analogue to the conditions that prevailed in the early 1980s, for reasons that will become clear later. Figure 3 illustrates the inference problem of the intermediary – for a given level of borrowing there may exist several different individuals who could be issuing that $b$. $\Pr(e, \nu | b, y, j, m)$ assigns a probability to each of these types.

In the partial information environment the calculation of $\Pr(e, \nu | b, y, j, m)$ is nontrivial, because it involves the distribution of endogenous variables. First, let the invariant distribution over states be denoted by $\Gamma(a, y, e, \nu, j, m)$. In a stationary equilibrium the joint conditional probability density over shock pair $(e, v)$ must be given by

$$\Pr(e, \nu | b, y, j, m) = \int_a \Gamma \left( a = f \left( b, y, e, \nu, j, m \right), y, e, \nu, j, m \right), \tag{2.11}$$

where $f$ is the inverse of $g$ with respect to the first argument wherever $\Gamma(a, y, e, \nu, j, m) > 0$; that is,

$$a = f(b, y, e, \nu, j, m)$$

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[17] This point is related to the extensive survival literature, which investigates whether agents who form beliefs that deviate from rational expectations can survive in asset markets.

and

$$b = g\left(a, y, e, \nu, j, m\right).$$

Thus, the decision rule of the household under a given pricing scheme is inverted to infer the state conditional on borrowing. Using this function the intermediary then integrates over the stationary distribution of net worth, conditional on observables, and uses this probability to formulate beliefs.

It is possible that intermediaries in the partial information world would find it profitable to offer a menu of contracts and separate types (meaning agents with different realizations of the shocks $(e, \nu)$) in this manner. We restrict attention to the pure signalling model, which is not only tractable but also consistent with the relative homogeneity of unsecured loan contracts prior to 1990.[18]

### 2.2.3. Off-Equilibrium Beliefs

In addition to ensuring that pricing reflects equilibrium information transmission, the second key complication present under asymmetric information is how to assign beliefs about a household's state for values of the state *not* observed in equilibrium. That is, how should lenders assign beliefs regarding repayment by households where $\Gamma\left(a, y, e, \nu, j, m\right) = 0$? This issue matters because a household's decision on the equilibrium path depends on its understanding of lender behavior at *all* feasible points in the state space, including those that never arise. Our theory does not restrict off-equilibrium beliefs in a clear way, since we require only zero profit on the equilibrium path, so we must specify a rule for off-equilibrium outcomes. Given the proliferation of equilibria typically present in signaling models, we want to discipline this choice as tightly as possible.[19]

The assignment of off-equilibrium beliefs turns out to be closely related to the algorithm we use to compute equilibria. Our algorithm is iterative – we guess pricing functions, compute implied default rates, recompute pricing functions based on the new default rates, and iterate to convergence. The critical choice of the algorithm is therefore the initial pricing function and the rule for updating. We assign the initial off-equilibrium beliefs in order to minimize the effects on equilibrium outcomes; specifically, we begin by guessing a pricing function $q^0$ with the following

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[18]Why the intermediaries did not use these contracts in the earlier period is a question beyond the scope of this paper. Livshits, MacGee, and Tertilt (2007b) and Drozd and Nosal (2007) make contributions to this literature using costs of offering contracts.

[19]It turns out that modelling the game as signaling rather than screening is significantly easier. In a screening game the lenders would move first, and then we would need to check deviations in the infinite-dimensional space of alternative pricing functions. Here households move first and we only need to check deviations in the space of borrowing levels, which is implicit in our use of the pricing function as a schedule confronting the borrower.

properties: it is constant at the risk-free borrowing rate $\frac{1}{1+r+\phi}$ over the range $[0, b_{\min})$, where $b_{\min}$ is a debt level such that no agent could prevent default if they borrowed that much, and then drops to 0 discontinuously. The implied beliefs for the intermediary are such that default never occurs except when it *must* in every state of the world; this assumption has the appeal that it is very weak requirement, as no equilibrium pricing function could possibly permit more borrowing. Since our algorithm will generate a monotone mapping over pricing functions, it is imperative that we begin with this function if we are to avoid limiting credit opportunities unnecessarily.[20] It is useful to compare our initial pricing function with the natural borrowing limit, the limit implied by requiring consumption to be positive with probability 1 in the absence of default. Our initial debt limit is larger than the natural borrowing limit, as agents can use default to keep consumption positive in some states of the world; we only require that they not need to do this in every state of the world.[21]

### 2.2.4. Equilibrium

We now formally define an equilibrium for the game between borrowers and lenders. We denote the state space for households by $\Omega = \mathcal{B} \times \mathcal{Y} \times \mathcal{E} \times \mathcal{V} \times \mathcal{J} \times \{0,1\} \subset \mathcal{R}^4 \times \mathcal{Z}_{++} \times \{0,1\}$ and space of information as $\mathcal{I} \subset \mathcal{Y} \times \mathcal{E} \times \mathcal{V} \times \mathcal{J} \times \{0,1\}$.

**Definition 1.** *A Perfect Bayesian Equilibrium for the model consists of (i) household strategies for borrowing $b^* : \Omega \to \mathcal{R}$ and default $d^* : \Omega \times \mathcal{E} \times \mathcal{V} \to \{0,1\}$ and intermediary strategies for lending $q^* : \mathcal{R} \times \mathcal{I} \to \left[0, \frac{1}{1+r}\right]$ and (ii) beliefs about the borrower state $\Omega$ given borrowing $\mu^* (\Omega|b)$, that satisfy*

1. **Lenders optimize**: *Given beliefs $\mu^* (\Omega|b)$, $q^*$ is the pure-strategy Nash equilibrium under price competition.*

2. **Households optimize**: *Given prices $q^* (b, I)$, $b^*$ solves the household problem.*

3. **Beliefs are consistent with Bayes' rule**: *The stationary joint density of $\Omega$ and $b$, $\Gamma_{\mu^*} (\Omega, b)$, that is induced by (i) lender beliefs and the resultant optimal pricing, (ii) household optimal borrowing strategies, and (iii) the exogenous process for earnings shocks and*

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[20]This algorithm is similar to ones used to compute endogenous borrowing limits in models of limited enforcement, such as Zhang (1997).

[21]This point is also made in Chatterjee *et al.* (2007).

mortality, is such that the associated conditional distribution of $\Omega$ given $b$, denoted $\Gamma^b_{\mu^*}(b)$, is $\mu^*(\Omega|b)$.

4. **Off-Equilibrium Beliefs**: $q^*(b, I) = 0$ $\forall b$ such that $\Gamma^b_{\mu^*}(b) = 0$.

One clarifying point needs to be made here. Since our shocks are continuous random variables, the debt levels that get zero weight in the stationary distribution are those above and below any levels that get positive weight ($\Gamma$ has a connected support). Obviously, for default decisions the upper limit is irrelevant; thus, as noted above, we are imposing a belief about the behavior of agents who borrow more than *any* agent would in equilibrium, no matter what unobserved state they are currently in. Given that $q$ is weakly-decreasing in $b$, the natural assumption is that this agent intends to default with probability one.

## 2.3. Computing Partial Information Equilibria

The imposition of conditions on beliefs off-the-equilibrium path makes the computational algorithm we employ relevant for outcomes, and in this section we therefore discuss in some detail our algorithm for computing partial information competitive equilibria. The computation of the full information equilibrium is straightforward using backward induction; since the default probabilities are determined by the value function in the next period, we can solve for the entire equilibrium, including pricing functions, with one pass. The partial information equilibrium is not as simple, since the lender beliefs regarding the state of borrowers influence decisions and are in turn determined by them; an iterative approach is therefore needed.

1. Guess the initial function $q^0(b, y, j, m)$ discussed above;

2. Solve household problem to obtain $g(a, y, e, \nu, j, m)$, $f(b, y, e, \nu, j, m)$, and $d(e', \nu'|a, y, e, \nu, j, m)$;

3. Compute $\Gamma(a, y, e, \nu, j, m)$ and $P(b, y, e, \nu, j, m) = \Gamma(f(b, y, e, \nu, j, m), y, e, \nu, j, m)$;

4. Locate $b_{\min}(y, e, \nu, j, m)$, the minimum level of debt observed conditional on the other components of the state vector;

5. Set $q^*(b \le b_{\min}, y, j, m) = 0$ (that is, borrowing that exceeds any observed triggers default with probability 1);

6. Compute
$$\pi^d(b, y, e, \nu, j, m) = \sum_{e'} \sum_{\nu'} \pi_e(e'|e) \, \pi_\nu(\nu') \, d(e', \nu'); \qquad (2.12)$$

15

7. Compute $\Pr(e, \nu | b, y, j, m)$ from $P(b, y, e, \nu, j, m)$ for each $(b, y, j, m)$, the probability that an individual is in $(e, \nu)$ given observed $(b, y, j, m)$;

8. Compute

$$\widehat{\pi}^d(b, y, j, m) = \sum_e \sum_\nu \pi^d(b, y, e, \nu, j, m) \Pr(e, \nu | b, y, j, m), \qquad (2.13)$$

the expected probability of default for an individual in observed state $(b, y, j, m)$;

9. Set

$$q^*(b, y, j, m) = \frac{\left(1 - \widehat{\pi}^d(b, y, j, m)\right) \psi_j}{1 + r + \phi} \qquad \text{for all } b \geq b_{\min}(b, y, j, m); \qquad (2.14)$$

10. Set

$$q^1(b, y, j, m) = \Xi q^0(b, y, j, m) + (1 - \Xi) q^*(b, y, j, m)$$

and repeat until the pricing function converges, where $\Xi$ is set very close to 1.

Because the household value function is continuous but not differentiable or concave, we solve the household problem on a finite grid for $a$, using linear interpolation to evaluate it at points off the grid. Similarly, we use linear interpolation to evaluate $q$ at points off the grid for $b$. To compute the optimal savings behavior we use golden section search (see Press *et al.* 1993 for details of the golden section algorithm) after bracketing with a coarse grid search; we occasionally adjust the brackets of the golden section search to avoid the local maximum generated by the nonconcave region of the value function.

Let $\mathcal{Q}$ denote the compact range of the pricing function $q$; as noted above, $\mathcal{Q}$ is a compact subset of the unit interval. Our iterative procedure maps $\mathcal{Q}$ back into itself. To ensure the existence of a unique fixed point for this mapping, we would want to establish the contraction property for this mapping; however, once the price at a particular point reaches zero it can never become positive, so the contraction property will not hold. As a result, both the initial condition and the updating scheme could matter for outcomes (since the equilibrium pricing function may not be unique). We have detailed above our approach for selecting the initial condition and the updating procedure; we then set $\Xi$ close enough to 1 that the iterative procedure defines a monotone mapping, ensuring the existence of at least one fixed point.[22] $q = 0$ is also an equilibrium under certain restrictions on lender beliefs – if no agent receives any current consumption for issuing debt no debt is issued

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[22]Our results are the same if we set $\Xi \in [0.95, 0.999]$. Due to the slow updating the program typically takes several days to converge.

and intermediaries therefore make zero profit; provided that their off-equilibrium beliefs are that any debt will be defaulted upon with probability 1 optimality of lender decisions is also satisfied – and is a fixed point of our iterative procedure.[23] The key advantage of our initial condition is that it guarantees convergence to the competitive equilibrium which supports the largest amount of borrowing.[24]

Our interest in the equilibrium which permits the most borrowing at the lowest rates arises from the fact that such an equilibrium Pareto-dominates all the others. In our economy all pricing is individualized and $r$ is exogenous, meaning that the decisions of one agent do not impose pecuniary externalities on any other. Thus, we can analyze the efficiency of an allocation individual-by-individual (in an *ex ante* sense). For any individual, the outcome under $q^0$ dominates any other, whether they exercise the default option or not, because it maximizes the amount of consumption-smoothing that an individual can achieve. Since $q$ is a monotone-decreasing function of $b$, it follows that any allocation which generates higher $q$ for each $b$ dominates one with lower $q$; that is, $q_1 \succeq q_2$ implies that allocation 1 Pareto-dominates allocation 2. Since $q = 0$ is the "worst" equilibrium in the sense that no borrowing is permitted at all, any equilibrium with positive borrowing at finite rates must yield higher *ex ante* social welfare.

## 3. Calibration

The key parameters for the model are those governing household income. The income process is taken from Hubbard, Skinner, and Zeldes (1994), which estimates separate processes for high school and college-educated workers. Figure 4 presents the path for $\omega_{j,y}$ for each type; the significant hump present for the college workers turns out to matter a lot for default. The income process is taken from Hubbard, Skinner, and Zeldes (1994), which estimates separate processes for high school and college-educated workers. The process of income for each type is

$$
\begin{aligned}
\log\left(e'\right) &= 0.95 \log\left(e\right) + \epsilon' \\
\epsilon &\sim N\left(0, 0.025\right) \\
\log\left(\nu\right) &\sim N\left(0, 0.021\right)
\end{aligned}
$$

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[23]Note that these beliefs are consistent with the ones we impose off the equilibrium path – with no borrowing any debt level is beyond that observed in equilibrium and therefore expected to generate default with probability one.

[24]Though we do not establish the conditions under which an equilibrium with positive borrowing exists, we found such an equilibrium for every case we computed.

for high school agents and

$$\log\left(e'\right) = 0.95\log\left(e\right) + \epsilon'$$

$$\epsilon \sim N\left(0, 0.016\right)$$

$$\log\left(\nu\right) \sim N\left(0, 0.014\right)$$

for college agents; we then discretize this process with 15 points for $e$ and 3 points for $\nu$.

We set the utility function to CRRA,

$$u\left(c\right) = \frac{c^{1-\sigma} - 1}{1 - \sigma},$$

with $\sigma = 2$, and choose $\beta = 0.96$. We also select $r = 0.02$ and $\phi = 0.06$, implying a 4 percent spread between risk-free lending and borrowing rates, and $\delta = 0.03$, reflecting a filing cost of $1200 in a world with average income equal to $40,000.[25] We set $\theta = 0.4$ at an exogenous retirement (model) age of 45. The age-dependent family size parameters and the age- and type-dependent mortality rates are estimated using the US Census data. Given these choices and the calibrated processes for mortality, income, and family size, we choose $\lambda$ to match a target default rate of 0.8 percent, approximately 80 percent of the empirical default rate in 2000, for the model with full information; the resulting value is $\lambda = 0.048$. We choose 80 percent because around 20 percent of filers explicitly claim medical expenditures as a reason for their default, and we exclude these shocks from the model.[26]

## 4. Results

This section is divided into five subsections. The first two examine the equilibria under full and partial information, with particular attention devoted to the facts regarding debt, bankruptcy, and discharge. The third subsection explores the robustness of the main finding. The fourth and fifth compute measures of consumption smoothing and welfare and compare them across information settings.

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[25]A 4 percent spread is larger than the 2 percent measured from the National Income and Product Accounts by Mehra, Prescott, and Piguuillem (2007) but smaller than the wedge between the prime rate for borrowers and the average deposit rate. $\beta \in \left(\frac{1}{1+r+\phi}, \frac{1}{1+r}\right)$ is chosen as a compromise between equating the time rate of preference to the risk-free borrowing rate $r + \phi$ and equating it to the risk-free lending rate $r$.

[26]See Sullivan, Warren, and Westbrook (2000).

## 4.1. Full Information

Table 1 presents aggregate default statistics for the model under full information. The model produces a reasonable fraction of borrowers in the population and the average income of a filer is about one-third of average income, consistent with observations. The model also produces a reasonable amount of credit card debt relative to the data; for example, the median credit card borrowing in 2001 was $2000, equal to 0.05 model units. However, the model underpredicts the average amount of debt discharged by bankruptcy – in the data the median ratio of discharge to income is around 1, while the model produces a number around 0.05. As noted in the introduction, the low discharge levels are partly attributable to the deliberate exclusion of large shocks to net worth; these shocks play an important role in matching the average default rate in Livshits, MacGee, and Tertilt (2006) and Chatterjee *et al.* (2007) and produce more debt in discharge because they can generate states of the world where consumption sets without default are empty.[27]

Figure 5 displays the price functions for college types at age 29; as would be expected the higher the realization of $e$ the more credit is extended (the lower the implied interest rate).[28] For low realizations the price functions look like credit lines – borrowing can occur at a fixed rate (in this case, the risk-free rate) up to some specified level, after which the implied interest rate goes to $\infty$ very rapidly. For higher realizations the increase in the interest rate is more gradual. For comparison purposes, we also plot the price functions for the high school types at age 29; they are similar but high school types can borrow strictly less than college types. Figure 6 shows the same functions but for agents at age 46; older agents can borrow more than younger ones, but typically do not choose to do so. Young agents borrow due to their upward-sloping income profiles, with more borrowing desired by the college workers. Older agents are saving for retirement, so it requires a very bad sequence of shocks to produce borrowing by the old.

Figure 7 plots the lifecycle distribution of default for the model and the data; consistent with empirical observations, default occurs mainly in the early periods of economic life, because those are the periods in which agents are borrowing. This feature is driven by the hump-shape in average earnings; if we eliminate the hump default drops nearly to zero and becomes more uniformly spread across ages. When average income is upward-sloping early in life, households are less concerned

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[27] As noted in Livshits, MacGee, and Tertilt (2006), much of the default in their model is accounted for by households that receive a large expense shock. This finding is inconsistent with the survey data in Sullivan, Warren, and Westbrook (2000).

[28] The top panel plots the price functions for the lowest 5 realizations of $e$, the middle panel the middle 5, and the bottom panel the highest 5. $\nu$ is set to the mean value.

about the exclusion cost of default because they do not intend to borrow in the future. Thus, they often default when income prospects in the future are promising but current debt is large; because shocks to $e$ are highly persistent, the first good realization after a series of bad ones triggers default (default is rare because this sequence of realizations is rare).[29] With no expectation of higher average earnings, the strength of this mechanism is much weaker, so that what little default occurs only due to repeated bad shocks.

To further explain the mechanisms that generate default in our model, Figure 8 presents statistics from the unsecured credit market as we vary the variances of $\epsilon$ and $\nu$ holding fixed the unconditional variance of labor income. We focus first on the left side of Figure 8, where the persistent component $e$ is very small relative to the transitory one $\nu$; it is important to note that the conditional variance on the left side is essentially entirely driven by $\nu$ because the variance of $\epsilon$ must be extremely small to reduce the contribution of $e$ to the unconditional variance to such a small number. Default rates are relatively high at this end of the graph. With large transitory shocks to income, households are very willing to borrow for consumption-smoothing purposes, both because the conditional volatility of income is large and because self-insurance is an effective mechanism against transitory shocks. Thus, households are willing to pay high rates to borrow and end up defaulting at relatively high rates. Thus, even though exclusion is costly here – because the credit market is effective insurance and the shocks are large, agents find the prospect of being excluded to be quite unpleasant – the magnitude of the shocks is sufficient to generate high default rates.

On the right hand side of Figure 8 (where the benchmark parametrization lies), the persistent component $e$ drives all the unconditional (and conditional) variance. With very persistent shocks, credit markets are not useful for consumption-smoothing against shocks, so that all debt is accumulated for one of two reasons: lifecycle smoothing and future defaults. If a household with debt receives a good income shock they find default attractive, since exclusion is irrelevant (they intend to save going forward).[30] Furthermore, default is an attractive option for even those receiving bad shocks, because the expected value of lifetime income has declined and without default they would have to adjust consumption; furthermore, exclusion is irrelevant for these households as well. As a result, default is relatively high when persistent shocks are dominant as well. Moving from left to right in Figure 8, we therefore see that the penalty from exclusion is declining in importance while

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[29]The intuition here is the same as in standard limited commitment models of imperfect risk sharing, such as Krueger and Perri (2006).

[30]The intuition is the same as in limited enforcement models, such as Krueger (1999).

the value of default as an insurance mechanism is also falling. This conflict triggers the U-shape – initially, the falling value of default dominates and leads to less default, but eventually the falling penalty from exclusion becomes small enough that default begins to increase.

## 4.2. Partial Information

Next, we study the economy under partial information. We assume that lenders use all observed information, including borrowing, to draw inferences about the unobserved states.[31] Table 2 shows that the default rate drops to essentially zero. The reason that default rates are so low is that the unsecured credit market has collapsed – since no one ends up borrowing very much, default is not valued and is therefore not exercised. If we look at the default rates over the lifecycle, we see that only college agents of ages 21 and 33 default at all. At age 21 agents who draw very bad initial persistent shocks default immediately. Agents whose initial persistent shocks are bad but not extremely bad do not default initially – because they want access to borrowing they delay default to age 33, when future access to credit markets is not particularly valuable.

Table 2 shows that partial information implies a decline in default, debt, and also debt discharged through bankruptcy. The first two are clear when compared with Table 2; the third one is more subtle. Average debt discharged in bankruptcy in the aggregate appears to rise under partial information, but that is due to a compositional effect only. Under full information high school types have a higher default rate and discharge slightly less debt on average than college types, but under partial information they essentially do not default at all (what default does occur is very rare and on minute amounts of debt). If we look only at those agents who continue to default in reasonable numbers – the college types – we see that average discharge also falls under partial information.

Figure 9 shows the pricing function across different values of $(e, \nu)$, compared to the one obtained under full information. One clearly sees that all agents, not merely the low risk ones, are subjected to higher interest rates and lower effective credit limits under partial information. Almost all borrowing that does occur is done at the risk-free rate; since the low levels of debt observed in equilibrium are very close to the pecuniary cost $\delta$, it is almost never cost effective to exercise the option to default.[32] Thus, at a qualitative level improvements in information appear consistent

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[31]An alternative approach would be to assume that total borrowing is not observed (that is, the $b$ that is brought to an intermediary is not informative about the total $b$ that the household is issuing in the current period). Preliminary results for this economy suggest that default increases relative to the full information economy.

[32]In an earlier version of the paper we did not include $\delta$ as a cost of filing; in that setting we found a slightly higher

with observed trends in the credit market for both debt and default – with less information in the earlier period one would observe lower debt levels, fewer defaults, less debt discharged per default, and more uniform terms for credit. Our quantitative model agrees with the more-stylized model in Narajabad (2007) – the better the information about the income prospects of borrowers the more default will occur. The key difference in our model relative to Narajabad (2007) is the extent to which lenders are committed to the contract. In his model borrowing may reveal information about a borrower's type but lenders cannot alter the contract to incorporate it; in contrast, our borrowers are repriced each period, with lenders incorporating current borrowing into their assessment of default risk. We now show that the ability of lenders to use information revealed by borrowing decisions is critical for understanding how asymmetric information unravels the credit market.

Why does the credit market collapse in the economy with partial information? Consider our function $q^0$ above – that is, a pricing function that allows risk-free borrowing out to a level of debt that would generate default in every state of the world tomorrow; Chatterjee *et al.* (2007) refer to this level of debt as the endogenous borrowing constraint driven by a lack of commitment to repay. This pricing function cannot be an equilibrium, as high risk borrowers with debt near this level would default in at least some states of the world, creating a risk premium in lending. At the new higher interest rates low risk borrowers will borrow less, meaning that the high risk borrowers will now face even higher rates as their type is revealed. As a result, the high risk borrowers reduce their borrowing as well. This process, which is exactly the insight that we use to compute equilibria, apparently continues until the market reaches very low levels of debt and default – essentially, the bad risk types chase the good risk types all the way to zero. The thing that distinguishes good and bad credit risks in our model is the amount of borrowing they desire. Bad risk borrowers would like to borrow a lot, provided they can do so cheaply, and then default; the only way to get cheap credit is to pool with the good risk borrowers. But good risk borrowers are unwilling to pay high prices to borrow, so their borrowing is low and, ultimately, so is the borrowing of the bad risks. That is, our model displays a very strong 'lemons' effect.

We demonstrate in the next subsection that our main qualitative result is robust – better information always leads to more debt, more default, more discharged debt, and more dispersion in terms. It is natural to ask what features would be needed to sustain pooling at a high level of debt; that is, what would a model need to produce in order for the partial information world to lead to more debt and default than the full information one? What the model would need to

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default rate. The qualitative features are very similar.

produce is homogeneity in the "value of default" across the agents who are being pooled together; unfortunately, in a model with unobservable income shocks that homogeneity is hard to produce. Agents with good income prospects do not want to pay a premium to borrow, meaning that they are generally unwilling to remain pooled with households that face bad income prospects who value the default option highly. It seems that the model needs more heterogeneity to pool agents at high levels of debt.[33]

We do not display the counterpart to Figure 8 under partial information because the plot for the default rate would lie essentially on top of the x-axis. Our key result about the drop in default rates is therefore seen not to be sensitive to the exact specification of persistent and transitory shocks used here – essentially any model with uninsurable idiosyncratic shocks to earnings would produce the same outcome. If we were to zoom in closely we would find a slight tendency for default rates to decline as we move from left to right (that is, as the contribution of $e$ to the unconditional variance rises). The default rate does not move upward again because the limited borrowing that is possible in the partial information world means that exclusion is not costly at all, no matter how important the persistent shocks get; therefore, the only mechanism operating left to right is the declining value of default.

### 4.3. Robustness

The prediction that the credit market would collapse in the presence of partial information is quite robust. Qualitatively, we find that it does not depend on the value of either $\lambda$ or $\delta$, the two costs of bankruptcy, nor does it depend on $\sigma$, the coefficient of relative risk aversion. However, it may depend on $\beta$, the discount factor, because $\beta$ plays an important role in determining the level of borrowing. Attempts to calibrate $\beta$ jointly with $\lambda$ to match both the default rate and the debt discharged in bankruptcy were unsuccessful – they resulted in values for $\beta$ we regard as much too low to be consistent with facts outside the context of this model, such as the wealth-income ratio in the US. It is standard in endowment economies to choose $\beta$ equal to the price of a risk-free bond, implying that the slope of the consumption profile would be zero in the absence of credit market restrictions. Unfortunately, because our risk-free lending and savings rates do not coincide, a

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[33]We have briefly investigated an alternative assumption about information, namely that total indebtedness of borrowers is not observable; in that setup lenders form beliefs using only unconditional probabilities. We found there that default rates and borrowing are higher under partial information than full information; we intend to pursue this direction more completely in the future. We omit discussion here because the game is more burdensome to describe and it would seriously lengthen the paper.

choice of $\beta = (1 + r)^{-1}$ would involve little to no borrowing while $\beta = (1 + r + \phi)^{-1}$ would involve almost no saving; our initial choice was a compromise between the two values. Given that our main focus involves the behavior of borrowers, we choose to set $\beta$ equal to the price of risk-free borrowing and recompute the model under full and partial information. We find that this change does not qualitatively affect our results either.[34]

### 4.4. Consumption Smoothing

Ultimately what matters in our model is the effect of information on the consumption process that agents end up with. In Figure 10 we plot the mean and variance of log consumption over the lifecycle for both types of agents under the two assumptions about information. Mean consumption is lower under full information than partial information due to the transactions costs of borrowing $\phi$ and defaulting $\delta$; when agents borrow and default more, they destroy more resources and leave themselves less wealth to finance lifetime consumption (since the risk-free rates are unchanged, the present value of lifetime earnings is not affected). One can thus interpret $\phi$ and $\delta$ as insurance premia paid for the right to borrow and introduce state-contingency into returns. It is interesting to note that this drop in consumption is mainly concentrated among older households.

Looking at the variance of log consumption, one sees that college agents face a riskier consumption process under partial information at every age, although the general shape is quite similar. The variance of consumption is high early in the lifecycle because households are restricted in their ability to borrow when young, inhibiting consumption-smoothing. As they age, their borrowing ability increases and so does their income on average, making consumption-smoothing relatively easy during peak earnings years. Consumption smoothing becomes less effective for most ages under partial information because borrowing is essentially impossible; thus, the young in particular experience significant consumption fluctuations.

The final observation we point out here is that consumption-smoothing for the older high school households is better under partial information than full information. The intuition for this outcome comes from Athreya (2007): households that borrow early in life must repay debts as they age, leaving them exposed to income risk later in life. Because the partial information economy does not permit borrowing, these older households are able to smooth their consumption effectively using

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[34]There are small quantitative differences across parameter values, of course. As mentioned previously, setting $\delta = 0$ generates slightly more default under partial information, particularly among the high school types. Similar results emerge from reducing $\lambda$, increasing $\sigma$, or reducing $\beta$.

buffer stocks of savings accumulated earlier in life; of course, the fact that they can smooth their consumption effectively when old does not mean that they are better off in an *ex ante* sense.

## 4.5. Welfare

We conclude this section of the paper with a simple welfare calculation – how costly is the loss of information? In Table 3, the consumption equivalent $C_{eq}$ is the percentage that consumption must be increased in each period to make newborn households indifferent between the two economies, after observing their permanent shock $y$.[35] The model suggests that the welfare costs of pooling can be significant – these costs are orders of magnitude larger than the welfare cost of business cycles, for example, even in models with incomplete markets (Krusell and Smith 2002). It is important to stress that these calculations are not the welfare gains generated by a change in policy; that calculation would require computing the transitional dynamics between steady state distributions. Given that we abstract from capital accumulation and there are no general equilibrium effects at play (other than the individual pricing functions), we do not think that paying the costs of computing a transition are worth it; fortunately, the same considerations suggest that our welfare calculation is not too inaccurate.

The welfare gains are almost twice as large for the college type than the high school type. The welfare gains are larger for the college type because their income profile has a more severe hump, leading to a stronger desire to borrow for purely intertemporal (as opposed to inter-state) reasons. Under partial information they are not able to use high expected future income to finance current consumption; the pooling over $(e, \nu)$ disrupts this process by eliminating the ability to borrow. Thus, their mean consumption profile tilts upward more than desired, leading to significant welfare losses. This increase is partially mitigated by the fact that college types face smaller earnings shocks.

## 5. Conclusion

This paper has evaluated the role played by information for the functioning of unsecured credit markets; a key technical contribution is an algorithm to compute equilibria with individualized pricing and asymmetric information. In two companion papers we make use of our model and algorithm to study two questions. First, we study how regulatory conditions that constrain infor-

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[35]The welfare cost before observing $y$ would just be the weighted average of the two costs.

mation in the credit market affect the economy; for example, the Equal Credit Opportunity Act explicitly bans the use of age, race, and gender for the determination of credit. Such bans may serve noneconomic goals, but they may also impose costs on the economy by pooling different types of borrowers together.[36] Second, we reconsider the question of Krueger and Perri (2006): does more income risk lead to better consumption smoothing? Our contribution in that paper is to use a credit market setup that is easier to map into the institutional arrangements we observe in the data; furthermore, our test is more stringent since we require that the model reproduce the unsecured market trends as well as the income risk changes.[37]

A feature of recent work on consumer default, including the present paper, is that it imposes a type of debt product that does not mimic all the features of a standard unsecured contract offered by the credit market. That is, in our model, as in all extant quantitative studies of bankruptcy and endogenous credit markets, individuals issue one-period bonds in the credit market. As a result, any bad outcome is immediately reflected in the terms of credit, making consumption smoothing in response to bad shocks difficult (credit tightens exactly during the period in which it is most needed); this arrangement would seem to artificially increase the incentive for default. We plan to construct a model of credit lines – contracts that specify a fixed interest rate and a fixed borrowing limit – to better capture the risk sharing that may be conducted via unsecured debt. Given that credit conditions are typically only adjusted by two events – default or entering the market to either purchase more credit or to retire existing lines – this representation seems more appealing. Whether it makes a significant difference for consumption smoothing is the subject of future work.[38]

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[36]Inference in a model with regulatory constraints is subject to more restrictions than those discussed here. Intermediaries want to form beliefs about the unobserved characteristics of borrowers, but if those inferences prove too accurate they can be fined by the regulator; that is, if the equilibrium reveals that a banned characteristic is informative about credit terms the intermediary is subject to penalties, even if the characteristic is not explicitly used, limiting the value of the sophisticated inference procedure considered here.

[37]Preliminary results suggest that information changes reinforce income risk changes and generate better risk sharing. More empirical support for the changing variance of shocks can be found in Gottschalk and Moffitt (1994).

[38]Matteos-Planos (2007) is a recent attempt to model credit lines with homogeneous interest rates, which would appear to be the right model for the period before 1990. An open question is how credit lines with individualized interest terms would be determined.

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**Table 1**

Aggregates under Full Information

|  | College | High School | Aggregate |
|---|---|---|---|
| Default Rate | 0.7006 | 0.8933 | 0.8162 |
| Average Income | 1.3478 | 0.9370 | 1.1013 |
| Average Borrowing | 0.0358 | 0.0299 | 0.0344 |
| Fraction of Borrowers | 0.1672 | 0.1088 | 0.1438 |
| Average Discharge | **0.0479** | **0.0388** | **0.0419** |
| Average Income of Defaulter | 0.7501 | 0.4821 | 0.5741 |

**Table 2**

Aggregates under Partial Information

|  | College | High School | Aggregate |
|---|---|---|---|
| Default Rate | 0.0107 | 0.0000 | 0.0040 |
| Average Income | 1.3478 | 0.9370 | 1.1013 |
| Average Borrowing | 0.0324 | 0.0258 | 0.0284 |
| Fraction of Borrowers | 0.1751 | 0.1161 | 0.1594 |
| Average Discharge | **0.0425** | **0.0000** | **0.0425** |
| Average Income of Defaulter | 0.5135 | 0.0000 | 0.5135 |

**Table 3**

Welfare Gains

| Information | College | High School |
|:---:|:---:|:---:|
| Full | $-55.1097$ | $-76.4768$ |
| Partial | $-55.3120$ | $-76.6440$ |
| $C_{eq}$ | $0.37\%$ | $0.22\%$ |

## Figure 1
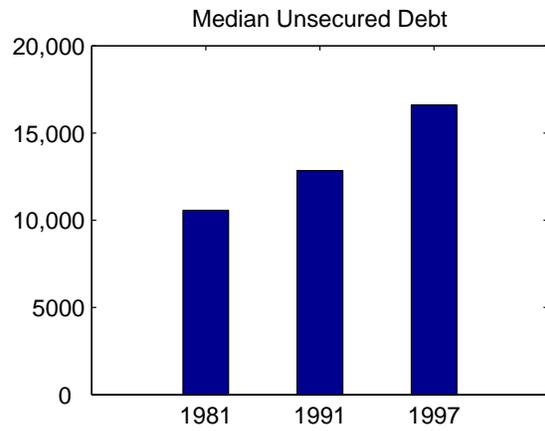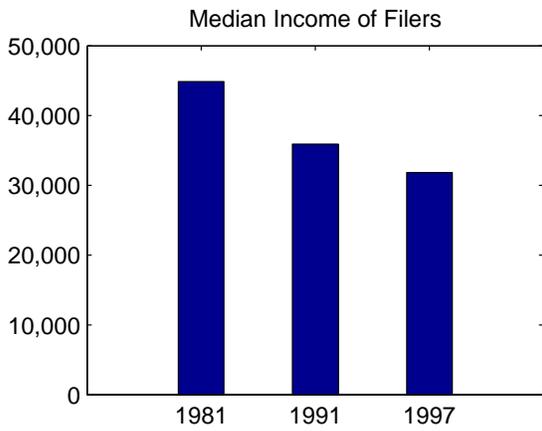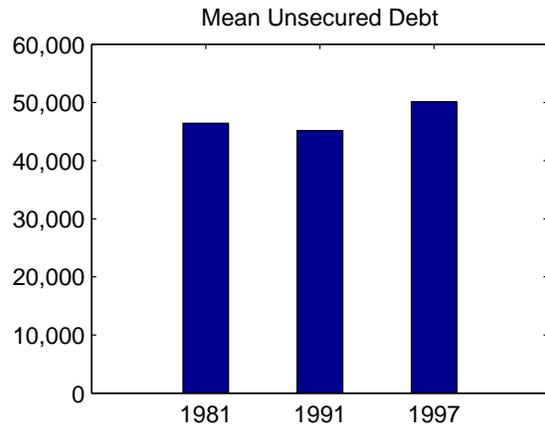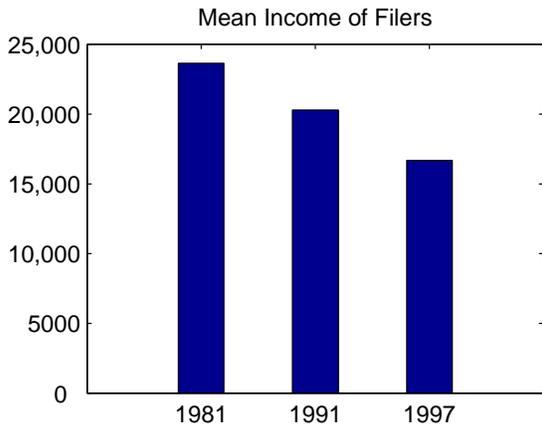
**Figure 2**

Interest Rates Paid By Positive CC Balance
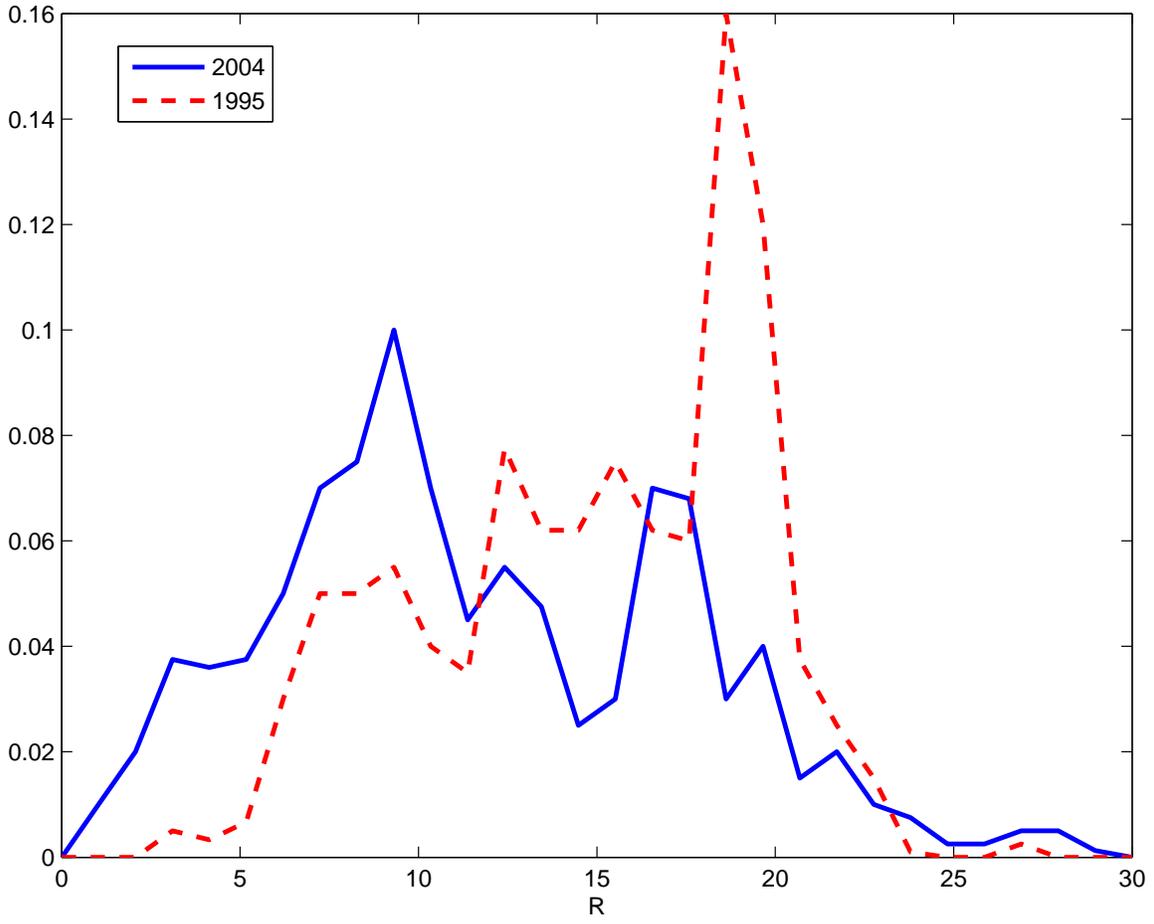
**Figure 3**

Inference Problem
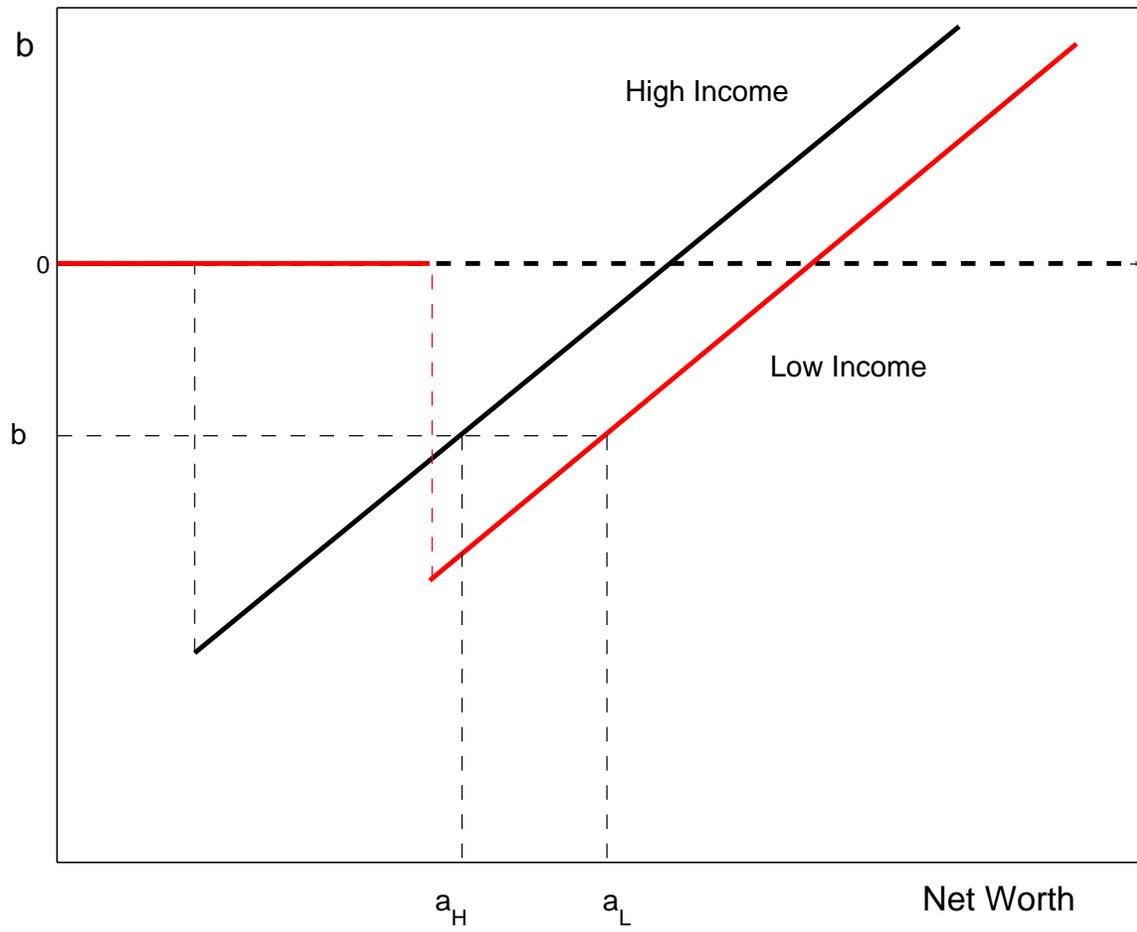
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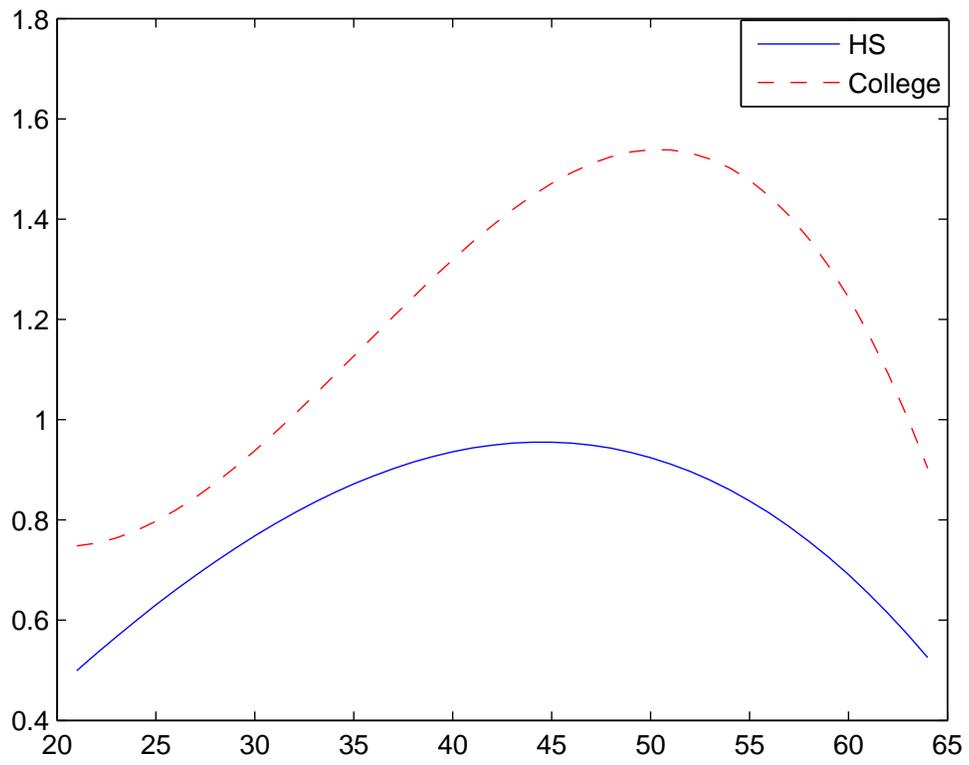
# Figure 5a

## Age 29 Coll, Interest rate
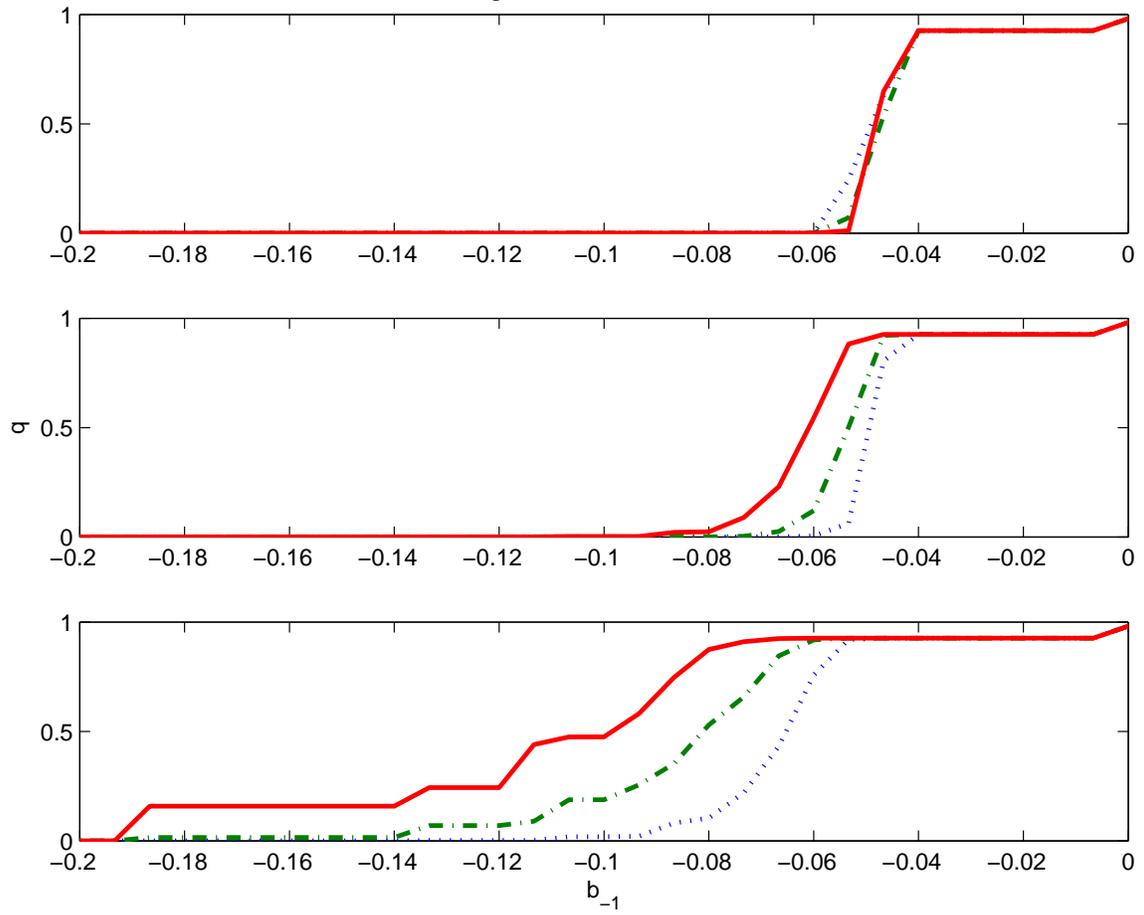
# Figure 5b

Age 29 HS, Interest rate
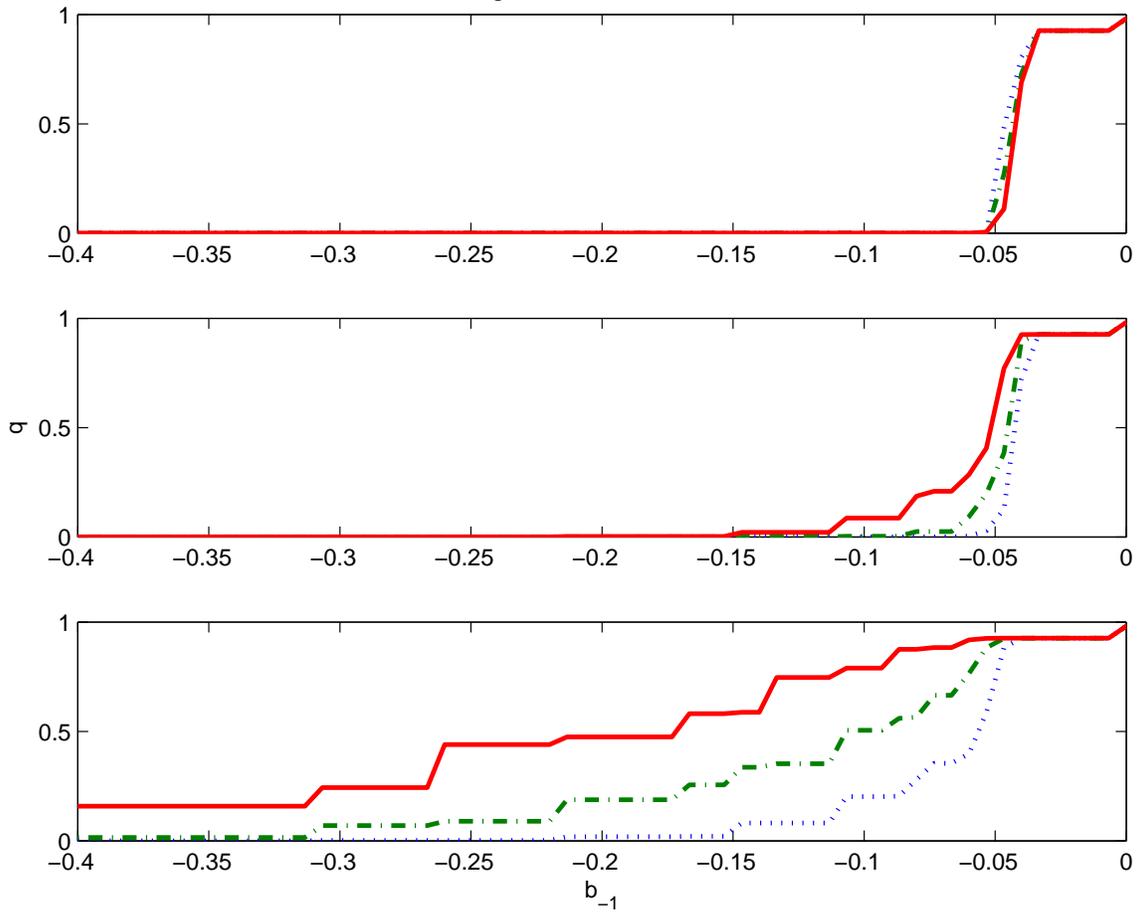
# Figure 6



Age 46 Coll, Interest rate

# Figure 7



40
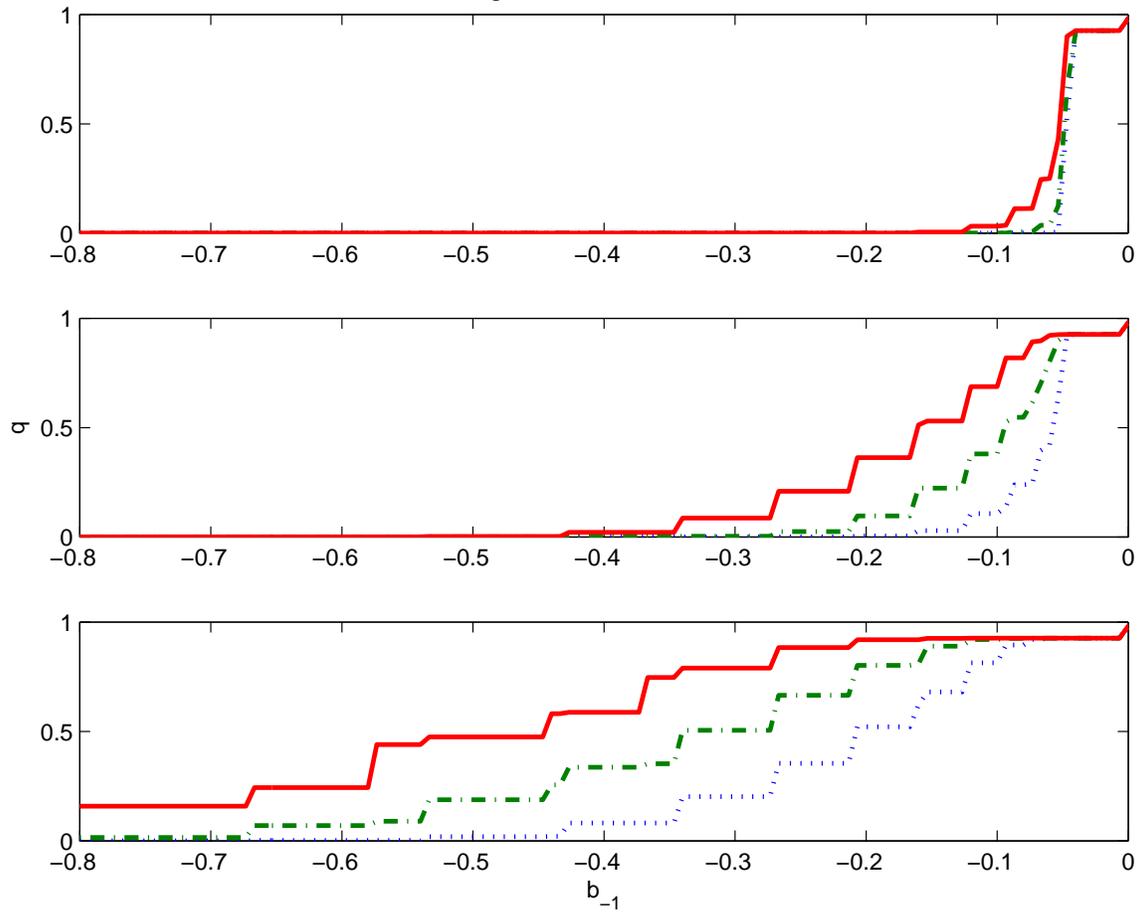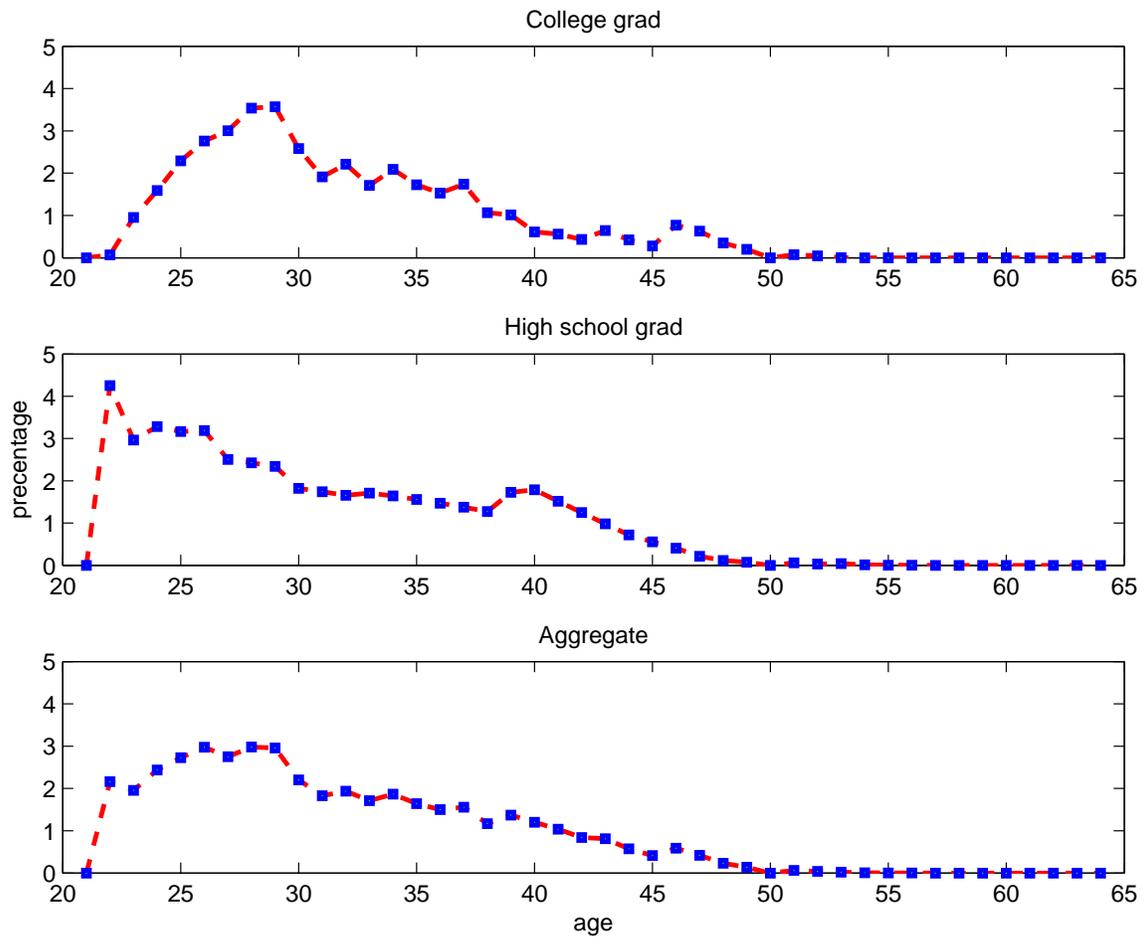
Figure 8

Default, Full Information
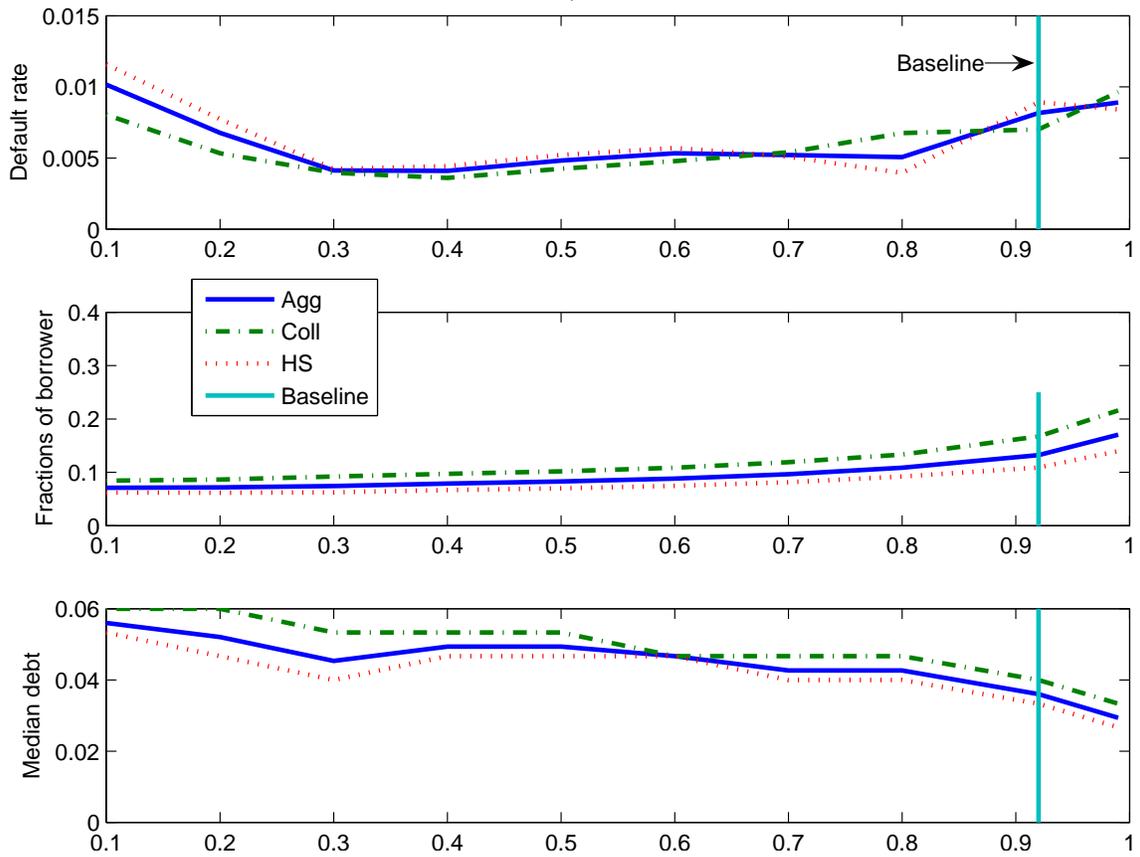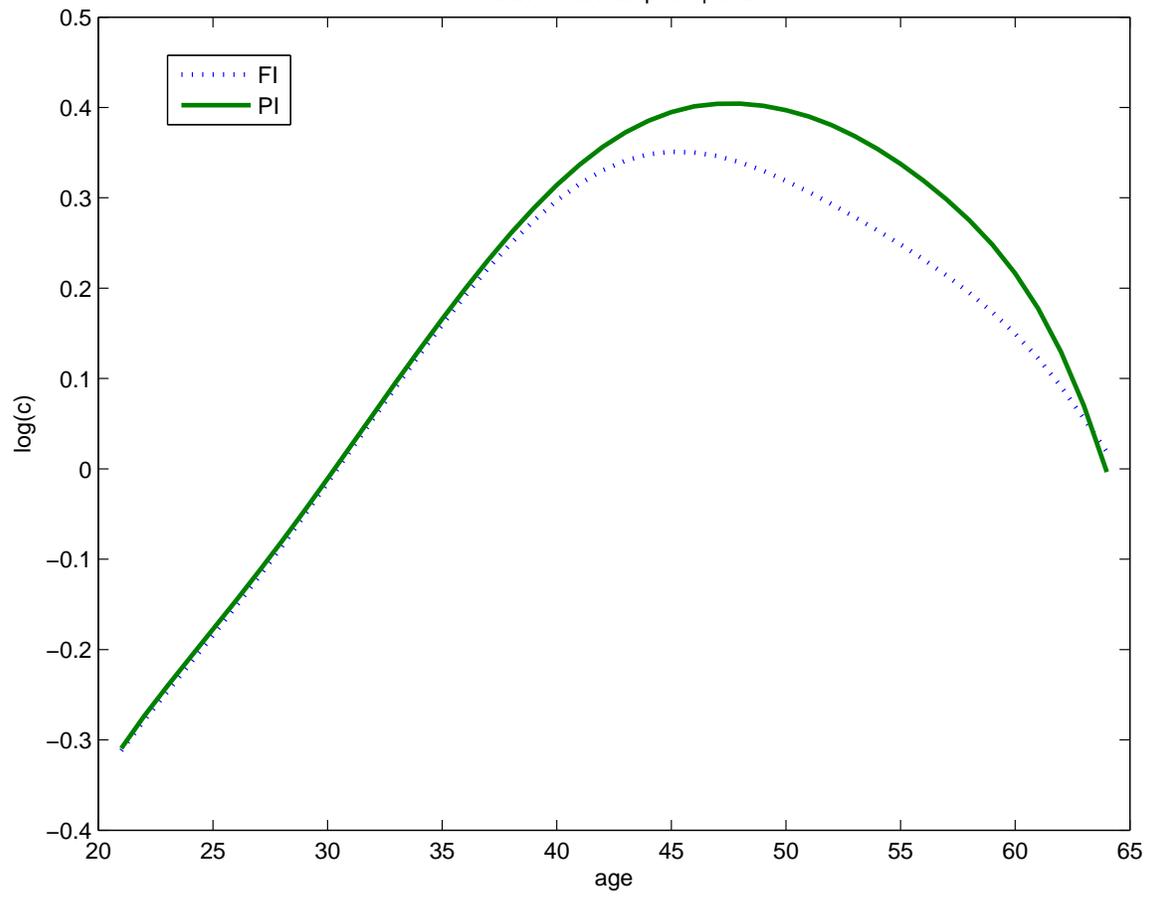
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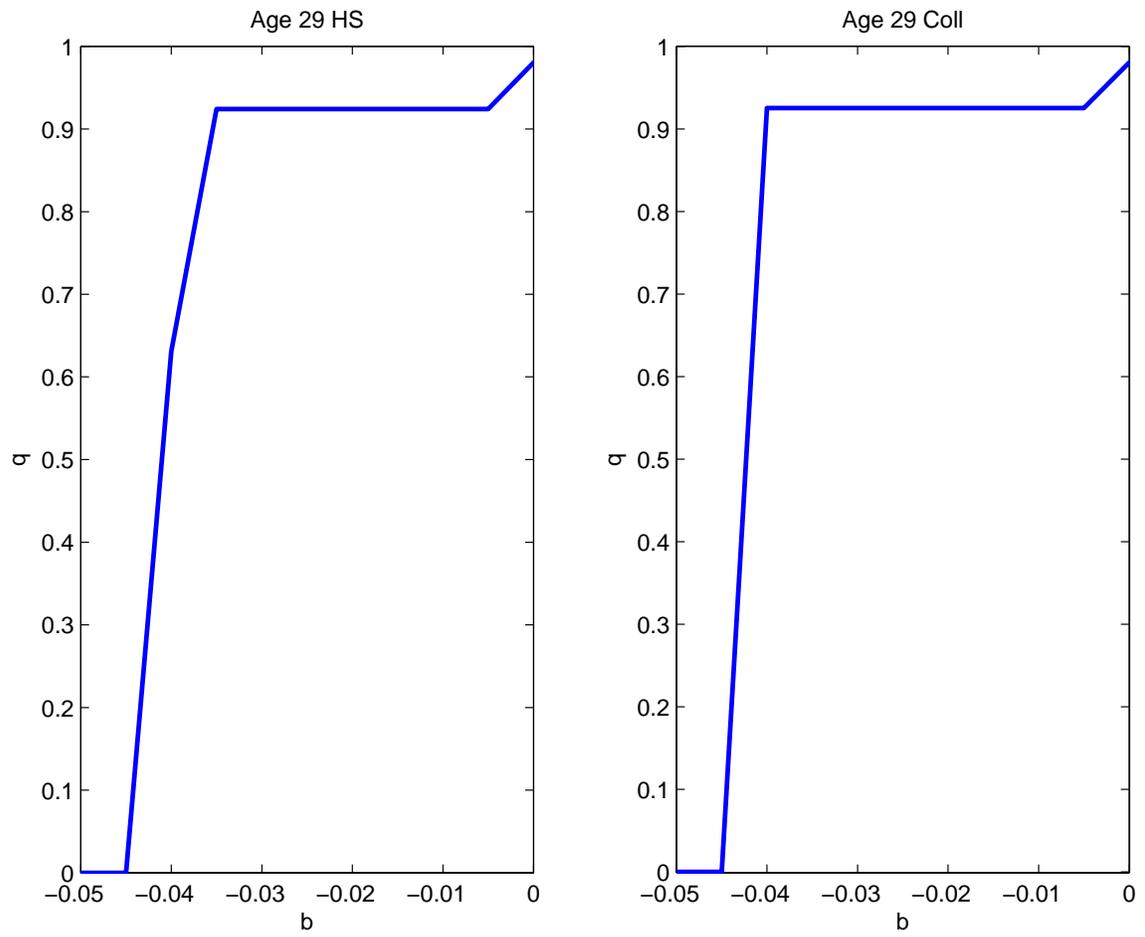
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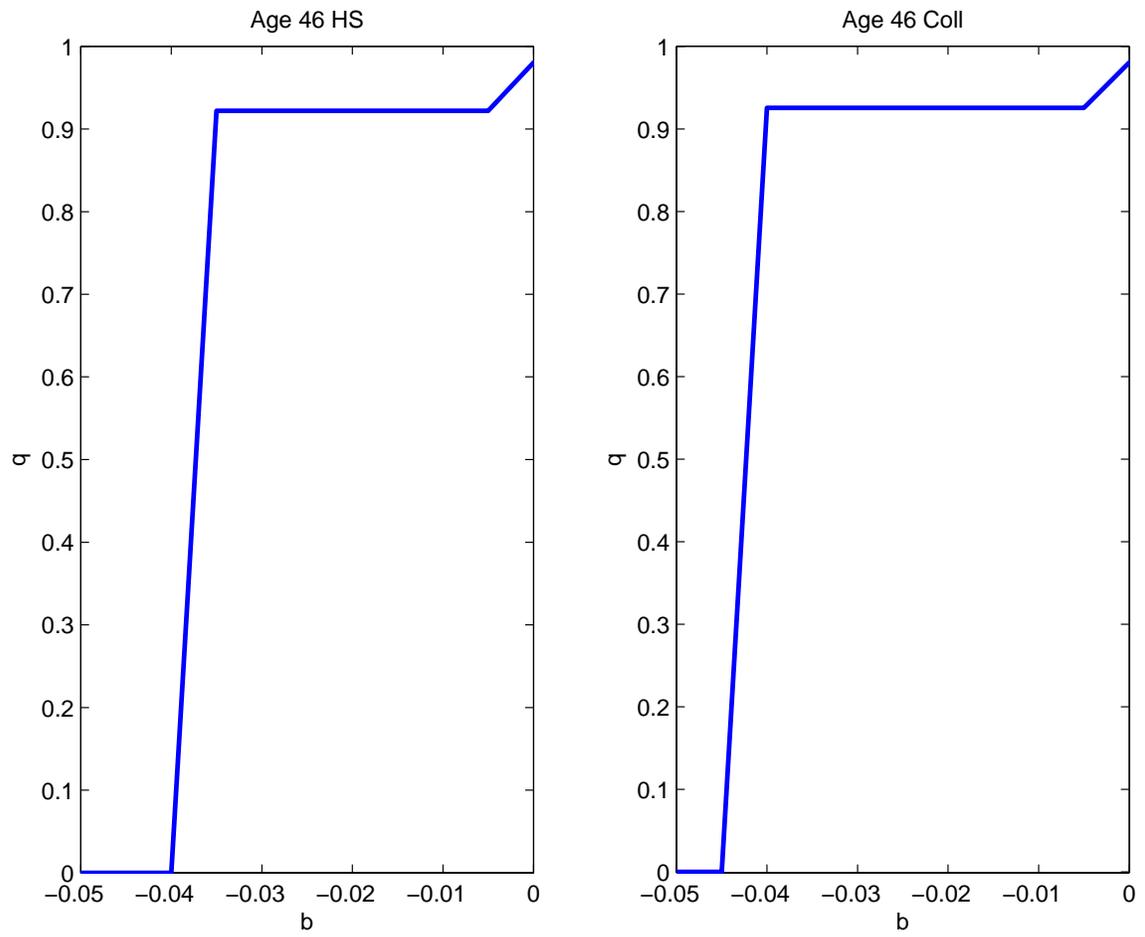
**Figure 10a**

Mean of consumption | Coll
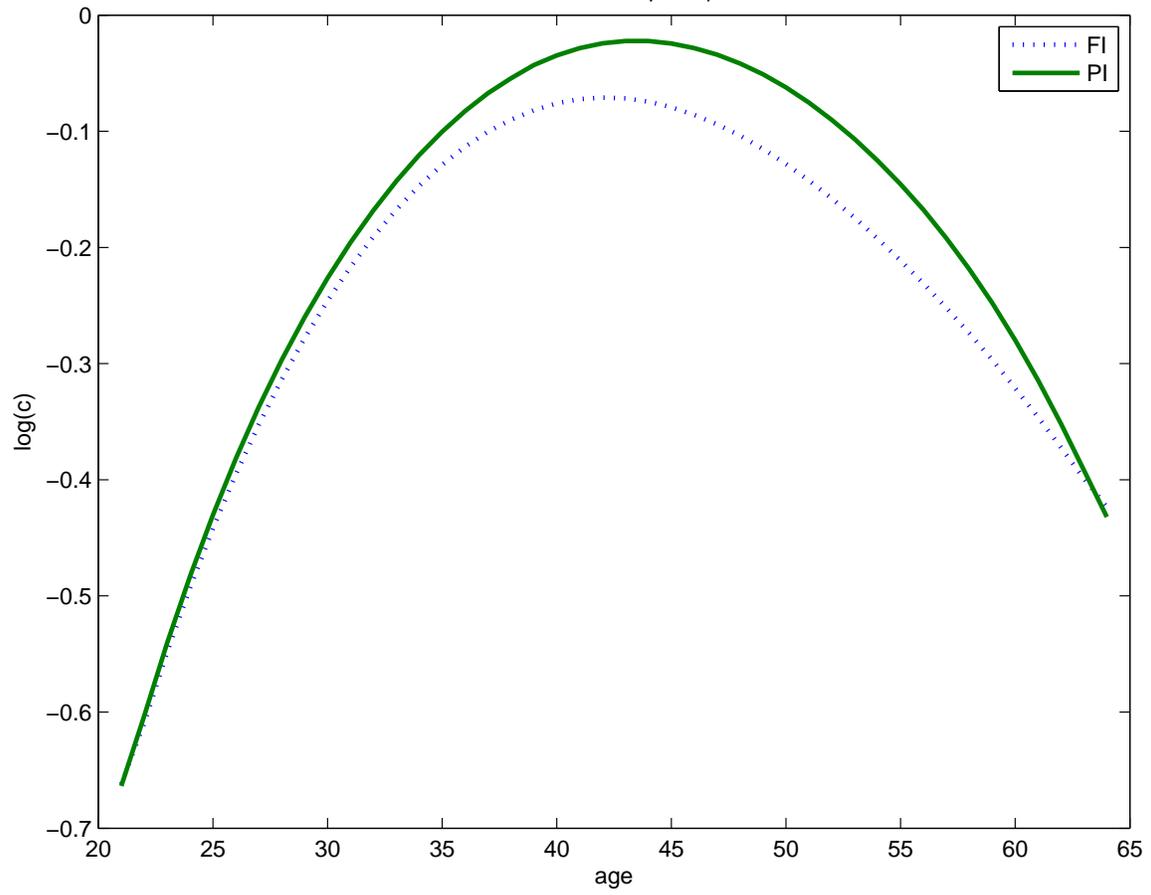
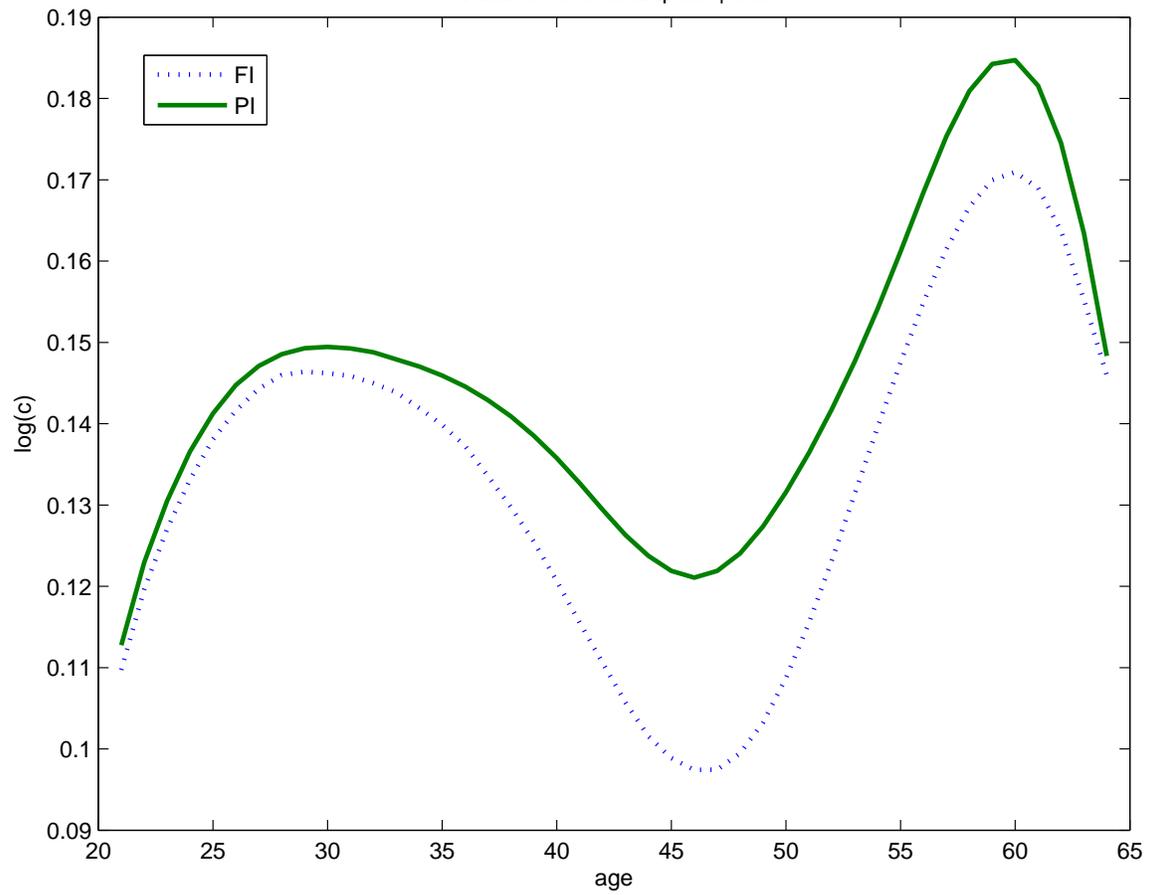**Figure 10b**

Mean of consumption | HS

# Figure 10c



Variance of consumption | Coll

# Figure 10d

Variance of consumption | HS