Euler-Equation Estimation for Discrete Choice Models: A Capital Accumulation Application*

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Abstract

This paper studies capital adjustment costs at the establishment level. Our goal here is to characterize these adjustment costs, which are important for understanding both the dynamics of aggregate investment and the impact of various policies on capital accumulation. Our estimation strategy searches for parameters that minimize *ex post* errors in an Euler equation. This strategy is quite common in models for which adjustment occurs in consecutive periods. Here, we extend that logic to the estimation of parameters of dynamic optimization problems in which non-convexities lead to extended periods of investment inactivity. This methodology allows us to take the structural model directly to the data, avoiding simulation-based methods that rely on user-designated moments. To demonstrate the effectiveness of this methodology, we first undertake several Monte Carlo exercises using data generated by the structural model. We then estimate capital adjustment costs for U.S. manufacturing establishments in two sectors.

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1 Introduction

This paper estimates capital adjustment costs using an Euler-equation methodology. As in the recent literature, our model incorporates various forms of capital adjustment costs intended to capture the rich nature of capital adjustment at the plant-level. Our goal here is to characterize these adjustment costs, which are important for understanding both the dynamics of aggregate investment and the impact of various policies on capital accumulation.

Our estimation strategy searches for parameters which minimize *ex post* errors in an Euler equation. This strategy is quite common in models for which adjustment occurs in consecutive periods. Here, following Pakes (1994) and Aguirregabiria (1997), we extend that logic to the estimation of parameters of dynamic optimization problems in which non-convexities lead to extended periods of investment inactivity. We do so in the context of the capital adjustment problem.

This paper thus makes two contributions. First, we obtain parameter estimates for capital adjustment costs. Second, we obtain these estimates using a new methodology.

The paper begins by specifying the dynamic optimization problem at the plant-level. This problem is used to generate the Euler equation that underlies our empirical analysis. The empirical strategy is then laid-out in some detail. We provide some results using simulated data to guide us in terms of the choice of instruments and also in dealing with problems of censored observations.

Finally, estimates of adjustment costs using plant-level data for two sectors (transportation and steel) from the LRD are reported. Like other methodologies for estimating adjustment costs, the Euler equation based approach used here finds evidence of both quadratic and non-convex adjustment costs. As in the simulated method of moments estimated reported in Cooper and Haltiwanger (2006), the estimated profit function exhibits significant curvature, reflecting market power, and quadratic adjustment costs are relatively small. The Euler-equation approach finds less irreversibility and larger disruption costs than Cooper and Haltiwanger (2006) for the two sectors we study.

2 Model

The dynamic optimization model draws upon the results reported in Cooper and Haltiwanger (2006). The dynamic programming problem is specified as:

$$V(A,K) = \max\{V^i(A,K), V^a(A,K)\}, \qquad \forall(A,K) \tag{1}$$

where K represents the beginning of period capital stock and A is the profitability shock. The superscripts refer to active investment "a," where the plant undertakes investment to obtain capital stock K' in the next period, and inactivity "i," where no investment occurs. These options, in turn, are defined by:

$$V^i(A,K) = \Pi(A,K) + \beta E_{A'|A} V(A', K(1-\delta)) \tag{2}$$

and

$$\begin{aligned} V^a(A,K) &= \max_{K'} \Pi(A,K)\lambda - p_b(I>0)(K'-(1-\delta)K) \\ &+ p_s(I<0)((1-\delta)K - K') - \frac{\nu}{2}\left(\frac{K'-(1-\delta)K}{K}\right)^2 K + \beta E_{A'|A} V(A',K') \end{aligned} \tag{3}$$

The model includes three types of adjustment costs which, as reported in Cooper and Haltiwanger (2006), are the leading types of estimated adjustment costs. The first is a disruption cost parameterized by λ. If $\lambda < 1$, then any level of gross investment implies that a fraction of revenues is lost. The second is the quadratic adjustment cost parameterized by ν. The third is a form of irreversibility in which there is a gap between the buying, p_b, and selling, p_s, prices of capital. These are included in (3) by the use of the indicator function for the buying ($I > 0$) and selling of capital ($I < 0$).

Assume the profit function has the following form

$$\Pi(A, K) = AK^{\alpha}. \tag{4}$$

This is a reduced-form profit function which can be derived from an optimization problem over flexible factors of production (labor, materials). The parameter α will reflect factor shares as well as the elasticity of demand for the plant's output. Here A is a plant-specific profitability shock.[1]

The first-order condition for the investment decision is

$$p(I) + \nu \left(\frac{K' - (1 - \delta)K}{K} \right) = \beta E_{A'|A} V_2(A', K') \tag{5}$$

where $p(I) = p_b$ if $I > 0$ and capital is purchased and $p(I) = p_s$ if $I < 0$ and capital is sold. Here the expectation is with respect to A'. The uncertainty is thus over the future marginal profitability of capital as well as the likelihood of adjustment.

The left side of (5) is the marginal cost of adjustment. The right side is the expected marginal gain and includes the effects on both the intensive (the amount of capital) and extensive (to adjust or not) margins. Yet, the right side of (5) appears to ignore the effects of the choice of K' on the probability of adjustment. This is correct since the effect of capital adjustment on the probability of adjustment is evaluated just at point of indifference between adjusting and not adjusting. That is, for each K', there are critical values of A which characterize the boundaries between adjustment and non-adjustment. Though variations in K' influence these boundaries, since the boundaries are points of indifference between

[1] In the actual empirical implementation, A will have both a plant-specific and a common component.

adjustment and no-adjustment, there is no effect on the value of the objective function.[2]

In models without non-convex adjustment costs, investment activity occurs each period. Estimation of adjustment cost and profit function parameters then follows the procedure introduced by Hansen and Singleton (1982). *Ex post* errors are calculated using observations on capital and profit flows. Then parameters are estimated using orthogonality conditions.

The challenge is to use that approach when investment activity does not occur each period. It is not possible to use (5) directly since the marginal value of capital is not observable.

To evaluate (5) *ex post*, we expand the $E_{A'|A}V_2(A', K')$ term until the plant's next episode of capital adjustment is observed. With non-convex adjustment costs, $\lambda < 1$, adjustment will generally not occur each period. We then replace expectations with realizations to calculate the *ex post* errors from the Euler equation.

To see how this works, suppose the plant adjusts in two consecutive periods, t and $t+1$. Then the *ex post* error, denoted $\varepsilon_{t,t+1}$, from (5) is

$$\varepsilon_{t,t+1} = \nu \frac{I_t}{K_t} + p(I_t) - \beta \left[\lambda \Pi_2(A_{t+1}, K_{t+1}) + p(I_{t+1})(1-\delta) + \nu(1-\delta)\frac{I_{t+1}}{K_{t+1}} + \frac{\nu}{2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 \right] \tag{6}$$

where $I_t = K_{t+1} - K_t(1-\delta)$. The first two terms here are the period t marginal costs of capital and the remaining terms are the marginal gains for the next period, including the marginal profitability and the marginal effects on adjustment costs next period.

Of course, not all plants adjust every period. It is not appropriate due to selection bias to estimate parameters based solely on plants that choose to adjust in consecutive periods.[3]

[2] We thank Borghan Nezami Narajabad, Jean-Michel Grandmont and Guy Laroque for questions which lead to this explanation of (5). While the policy function, $K'(A, K)$ is not continuously differentiable at a point of indifference between activity and inactivity, the right side of (5) is a conditional expectation of the marginal value of capital and thus these points of non-differentiability are of measure zero. Put differently, $E_{A'|A}V(A', K') = \int_{adjust} V^a(A', K') + \int_{inactive} V^i(A', K')$. The effect of changes in K' on the boundaries of the sets of action and inaction disappear as the values of action and inaction are equal at these boundary points.

Thus we need a more general condition which allows estimation of the structural parameters.

In general, if the plant adjusts in period t and subsequently in period $t+\tau$, then the *ex post* error, denoted $\varepsilon_{t,t+\tau}$, from the first-order condition is

$$\begin{aligned}\varepsilon_{t,t+\tau} &= \nu\frac{I_t}{K_t} + p(I_t) - \sum_{i=1}^{\tau-1}\beta^i\Pi_2(A_{t+i},K_{t+i})(1-\delta)^{i-1} - \beta^\tau\lambda\Pi_2(A_{t+\tau},K_{t+\tau})(1-\delta)^{\tau-1} \\ &\quad -\beta^\tau\left[p(I_{t+\tau})(1-\delta)^\tau + \nu(1-\delta)^\tau\frac{I_{t+\tau}}{K_{t+\tau}} + \frac{\nu}{2}\left(\frac{I_{t+\tau}}{K_{t+\tau}}\right)^2(1-\delta)^{\tau-1}\right].\end{aligned} \tag{7}$$

From this general expression, the first term on the right is the marginal cost of adjustment and the second term is the gain in profitability in the period of adjustment. During the periods between adjustment, there is an effect of capital accumulation on marginal profitability. Finally, in the period of the next adjustment, i.e. when the spell of inactivity ends, there is a final term reflecting the effects of K_{t+1} on the marginal adjustment cost in period $t+\tau$. Note that the non-convex adjustment cost, λ, appears in (7), at the end of the spell of inaction. In addition, both the price of capital in the period of the initial adjustment and in the next adjustment are included as well.

As in the estimation of quadratic adjustment cost models, the *ex post* errors should not be predictable. In the next section we discuss estimation of all parameters, including the non-convex adjustment cost parameter using the orthogonality restrictions generated by optimization.

2.1 Extension

One extension worth considering is enriching the structure of non-convex adjustment costs so that they apply to only a subset of investment choices. So in this section we assume that

[3] We later characterize this bias.

the non-convex adjustment cost is incurred iff $K' \in \Sigma(A,K)$ where $\Sigma(A,K)$ is a subset of the capital state space. So, for example, if we assume that the non-convex adjustment costs apply iff the investment rate, $\frac{I}{K}$, exceeds a critical level, $\bar{\iota}$, then $\Sigma(A,K)$ is the set of K' such that $K' \geq \bar{\iota}K + (1-\delta)K$.

In the previous discussion, we set $\bar{\iota} = 0$. With $\bar{\iota} > 0$, the optimization problem becomes

$$V(A,K) = \max\{V^i(A,K), V^o(A,K)\} \tag{8}$$

The superscripts refer to active investment "i", for **in**, where the plant chooses $K' \in \Sigma(A,K)$, and "o", for **out**, where $K' \ni \Sigma(A,K)$. These options, in turn, are defined by:

$$V^i(A,K) \quad = \quad \max_{K' \in \Sigma(A,K)} \Pi(A,K)\lambda - C^i(K,K') + \beta E_{A'|A} V(A',K') \tag{9}$$

and

$$V^o(A,K) \quad = \quad \max_{K' \ni \Sigma(A,K)} \Pi(A,K) - C^o(K,K') + \beta E_{A'|A} V(A',K') \tag{10}$$

All of the differentiable adjustment costs are captured in $C^j(K,K')$ for $j = i, o$. They are indexed by i, o so that we can model them as we wish. In one case, we might assume that the costs are the same regardless of whether $K' \in \Sigma(A,K)$ or we may assume these costs apply only if $K' \in \Sigma(A,K)$ so that $C^o(K,K') \equiv 0$.

In deriving the first-order condition, which we need for the Euler equation, it is necessary to know if the constraint binds in the current and next periods.

Suppose the constraint $K' \in \Sigma(A,K)$ does not bind in the current period. The first order condition is then

$$C_2^j(K, K') = \beta E_{A'|A} V_2(A', K') \tag{11}$$

for $j = i, o$ indexing the two cases in the current period.

As in the development of (5), the key is to first expand the right side of (11) by substituting for $V(\cdot)$ to create an Euler equation. Then we calculate the *ex post* error.

As long as the constraint does not bind in the next period, $\beta E_{A'|A} V_2(A', K')$ can be separated into its two components and the derivative calculated using(8). Assuming we know *ex post* that the constraint did not bind and also know whether $K' \in \Sigma(A, K)$, we have two *ex post* errors to consider, $j' = i, o$.

In this first case, there is adjustment in the current period of type i or o followed by adjustment in the second period with $K' \in \Sigma(A, K)$. This leads to the following *ex post* error.

$$\varepsilon_{t,t+1} = C_2^j(K_t, K_{t+1}) - \beta[\lambda \Pi_2(A_{t+1}, K_{t+1}) + C_1^i(K_{t+1}, K_{t+2})]. \tag{12}$$

In the second case, there is adjustment in the current period of type i or o followed by adjustment in the second period with $K' \ni \Sigma(A, K)$. This leads to the following *ex post* error.

$$\varepsilon_{t,t+1} = C_2^j(K_t, K_{t+1}) - \beta[\Pi_2(A_{t+1}, K_{t+1}) + C_1^o(K_{t+1}, K_{t+2})]. \tag{13}$$

Again, for both of these cases, the constraint does not bind in either period. This allows us to construct these conditions without a multiplier.

If there is unconstrained investment in period t but period $t+1$ is constrained, then these conditions cannot be used. Instead, we search forward for period $t + \tau$ where investment is not constrained and then we compute $\varepsilon_{t,t+\tau}$ depending on whether the period $t+\tau$ investment is for $j = i, o$.

To implement this in a sample, we need to classify the investment rate along two dimensions:

- is $K_{t+1} \in \Sigma(A_t, K_t)$ or $K_{t+1} \ni \Sigma(A_t, K_t)$

- is the $K_{t+1} \in \Sigma(A_t, K_t)$ constraint binding or not

We only use observations where the constraint does not bind. This is the value of the second classification. The first classification is relevant for determining which {disruption,differentiable} adjustment costs to use: $\{\lambda, C^i(K_t, K_{t+1})\}$ or $\{1, C^o(K_t, K_{t+1})\}$.

There are again three key requirements for the estimation. First, the *ex post* errors must be composed of either data or parameters. Second, we must be able to determine $K_{t+1} \in \Sigma(A_t, K_t)$. Third, we need to know if the constraint $K_{t+1} \in \Sigma(A_t, K_t)$ binds.

The Monte Carlo and estimation discussion which follows assumes $\bar{\iota}$. We return to this extension in our concluding comments.

3 Euler Equation Estimation

Pakes (1994) argues that the logic of Hansen and Singleton (1982) can be applied to the estimation of the structural parameters in a dynamic discrete choice problems. The application in Pakes (1994) is investment coupled with an exit decision. Aguirregabiria (1997) considers a dynamic labor demand model. Here we discuss the estimation of the capital accumulation problem drawing on those contributions.

Using (7), we can compute the *ex post* errors between adjustment periods. The optimization condition of the firm from (5) imposes structure on these errors. Optimality implies that the period t expectation of the *ex post* errors should be zero:

$$E_{\tau|t}[\varepsilon_{t,t+\tau}] = 0 \qquad (14)$$

for all t.[4]

[4] To construct (5) from (14) to requires the use of (7) for all τ along with the associated probabilities that adjustment occurs in period $t + \tau$.

Here the expectation is conditional on all variables known in period t. The variable τ indicates the period of the first active capital adjustment after period t. Of course, in period t, τ is not known since the adjustment decision following period t is state dependent.

The estimation of the structural parameters comes from the condition that $\varepsilon_{t,t+\tau}$ ought to be uncorrelated with period t and prior variables. This orthogonality comes from expanding the right side of (5) which incorporates the uncertainty over the future realizations of the shocks and thus the future choices of whether to adjust and, if so, by how much.

Using a vector of N variables predetermined in period t, z_t, the following orthogonality condition can be used in a GMM estimation procedure.

$$E_\tau[z_t \varepsilon_{t,t+\tau}] = 0. \tag{15}$$

The sample analog of this condition is

$$m = \frac{1}{n} Z' \varepsilon(X, \theta) = m(\theta) \tag{16}$$

where n is the number of observations (investment spells), Z is the matrix of N variables over T periods, and $\varepsilon(X, \theta)$ are the *ex post* errors calculated using the sample data, X, and the parameter vector of interest, θ.

The minimum distance estimator is the $\hat{\theta}$ that minimizes

$$\begin{aligned} s &= m(\hat{\theta})' W^{-1} m(\hat{\theta}) \\ &= \frac{1}{n^2} [\varepsilon(X, \hat{\theta})' Z] W^{-1} [Z' \varepsilon(X, \hat{\theta})]. \end{aligned} \tag{17}$$

Hansen (1982) showed that for this estimator, the optimal choice for W is

$$
\begin{aligned}
W_{GMM} &= \operatorname{Var}\left(Z'\varepsilon(X,\theta)\right) \\
&= \frac{1}{n^2} Z'\Omega Z
\end{aligned} \tag{18}
$$

where $\Omega = E[\varepsilon\varepsilon']$. If the errors are uncorrelated, W can be estimated as shown by White (1980) using the following equation.[5]

$$
\left(\frac{1}{n}\right) S_0 = \frac{1}{n}\left[\frac{1}{n}\sum_{t=1}^{T} z_t z_t' \varepsilon(x_t,\hat{\theta})^2\right]. \tag{19}
$$

Finally, the estimated asymptotic covariance matrix of the GMM estimator is

$$
V(\hat{\theta}) = \left[G(\hat{\theta})\left(\frac{Z'\hat{\Omega}Z}{n^2}\right)^{-1} G(\hat{\theta})'\right]^{-1} \tag{20}
$$

where $G(\hat{\theta}) = \frac{\partial m(\hat{\theta})}{\partial \hat{\theta}}$ is numerically computed.

4 Monte Carlo

Before estimating this model, we construct a simulation-based exercise. There are a number of goals of this experiment. First, there is the issue of checking the methodology to be sure that we can consistently estimate the parameters of interest.[6] Second, there is the issue of instruments. One can solve (17) for any instruments and, at least in theory, obtain consistent parameter estimates. In practice, it is useful to find instruments that are effective across a broad range of parameterizations. This can be achieved by simulating different models of adjustment costs and evaluating alternative instrument sets.

[5] There is an unresolved issue concerning a correction in the case where the errors are correlated. Because the "observations" in this estimation are spells of different length it is not immediately apparent how to apply a correction similar to that of Newey and West (1987).

[6] This is partly a test of our programs and partly an evaluation of the logic associated with this extension of the method of Euler-equation estimation.

Third, the estimation strategy outlined above assumes that all investment spells are complete: for all plants adjusting in period t, there is a τ such that adjustment is observed in period $t+\tau$. In practice, spells may not all be complete. In that case, there are two issues. The first concerns the extent of the bias associated with estimation from completed spells only. The second is the development of a correction which is consistent with the dynamic optimization approach.

4.1 Creation of simulated dataset

A data set is simulated in the following steps. First, the structural parameters of the model are chosen and the investment policy functions of the dynamic programming problem are obtained through value function iteration. The parameters of interest in this exercise are those that can be estimated with GMM: $\Theta = \{\alpha, \nu, \lambda, p_s\}$.[7]

We consider three different parameterizations of Θ in order to assess the properties of the estimation procedure. The first case, $\Theta_a = \{0.6, 2, 1, 1\}$, includes only a quadratic cost of adjustment. The second case, $\Theta_b = \{0.6, 0.2, 0.95, 0.98\}$, adds asymmetry between the buying and selling prices of capital and disruption costs. This parameterization results in a much higher rate of inactivity due to the introduction of the non-convex costs associated with adjustment. The final case, $\Theta_c = \{0.6, 0.2, 0.8, 0.98\}$, has a much larger disruption cost and therefore leads to more inactivity. This parameterization most closely matches the estimates of Cooper and Haltiwanger.

The other structural parameters of the model are chosen to be similar to those used by Cooper and Haltiwanger.[8] The frequency of the model is annual, so the discount rate, β, is set at 0.95. The profitability shock, A, consists of an aggregate shock and an idiosyncratic shock. Each of these shocks follows a log-normal autoregressive process. The aggregate

[7] In this exercise, we normalize $p_b = 1$ and we have chosen not to focus on estimating the discount rate, β, at this point. In the estimation on manufacturing data, we may include an interest rate to allow for variation in the discount rate.

[8] These estimates are discussed in Cooper and Haltiwanger (2006).

shock process has a persistence of 0.75 and the innovation to this process has a standard deviation of 0.05. The idiosyncratic shock process has a persistence of 0.88 and the standard deviation of the innovation is 0.3. The depreciation rate, δ, is 0.07.

The simulated panel data set is created by using the investment policy functions in conjunction with the randomly drawn innovations to the two profitability shock processes. The data consists of observations on the shocks, investment, capital and profits. This allows us to construct the *ex post* errors in (7).

For these exercises, each data set contains 200 plants, the size of an average manufacturing sector. To explore the small sample properties of this estimation method, results are reported for 3 sample lengths: 19 periods, 50 periods, 100 periods.[9]

4.2 Parameter estimation

The parameter vector Θ is estimated by minimizing the weighted sum of squared moments statistic in (17). Two different sets of instruments are used to examine the impact of alternative instruments on the precision of the estimates. The first set of instruments is composed of current and once-lagged values of the state variables of the dynamic programming problem along with a constant; $Z_{1,t} = \{1, A_t, A_{t-1}, K_t, K_{t-1}\}$. The second set of instruments consists of current and once-lagged variables that are observed in the actual data. The variables include the investment rate ($\frac{I}{K}$), the profit rate ($\frac{\pi}{K}$), and the capital stock (K); $Z_{2,t} = \{1, \frac{I_t}{K_t}, \frac{I_{t-1}}{K_{t-1}}, \frac{\pi_t}{K_t}, \frac{\pi_{t-1}}{K_{t-1}}, K_t, K_{t-1}\}$.

The estimates are obtained using a two-stage procedure. In the first stage, an identity matrix is used to weigh the moments, and the simplex algorithm is used to obtain the parameter estimates. These first stage estimates are used to estimate a weighting matrix, W, based on the White specification. This weighting matrix is then used in the second stage estimation.

[9]The actual data set that is used for estimation in Section 5 contains 19 periods.

An important issue in the estimation is accounting for incomplete spells in the data set. Non-convex adjustment costs in the form of disruption costs and asymmetry between the prices of buying and selling capital result in periods of inaction for plants. The moment condition underlying this GMM estimation is based upon the expectation taken across *ex post* errors for all plants that adjusted in a given period. Therefore, if there is a plant that adjusted in a given period but did not adjust again before the end of the sample, then the *ex post* error expressed in (7) cannot be computed.

For the first set of estimates, we control for incomplete spells by only using observations, dated by the beginning of an investment spell, from periods prior to the first period in which there is an observed incomplete spell for any of the plants. This is an *ex ante* selection criteria. Since (14) holds for all the adjusting plants and all have complete spells, the data reproduces this *ex ante* condition. In the next section, we explore different ways of adding in observations with incomplete spells to create more data points.

The first set of results are shown in Tables 1, 2, and 3, corresponding to the three parameterizations being considered. For each estimation exercise, 1000 data sets were simulated. The parameter estimates reported in the tables represent the mean of the estimates from the 1000 data sets. The standard deviation of the estimates is reported in parentheses.

The results in Table 1 show that the GMM estimation procedure performs well in the case with only quadratic adjustment costs, Θ_a, even in the smallest sample exercise. In this case, the true value of the production function parameter, α, is 0.6, and the scalar on the quadratic adjustment cost, ν, is set at 0.2. Using the first instrument set, Z_1, the means of the parameter estimates across the 1000 samples of 200 plants and 19 periods are $\{\bar{\alpha}_{1,19}, \bar{\nu}_{1,19}\} = \{0.600, 1.971\}$. The respective standard deviations across the 1000 parameter estimates are $\{0.025, 0.136\}$. Due to the discrete nature of the value function iteration solution, there are some situations where firms choose to remain inactive. The average number of periods before the first incomplete spell is initiated is 13.6 (listed in the "uncensored periods" column). The average number of complete investment spells (observations) across datasets is 2615. All complete spells starting on or after the date on which the first incomplete spell in initiated are dropped.

Increasing the number of periods in the sample from 19 to 100 leads to a slight improvement in the estimates. The estimate of α is essentially unchanged while the mean estimate of ν is now 1.991. The standard deviations across both parameter estimates decreases by more than two-thirds in comparison to those from the smaller samples. The average number of observations in a sample is 18214.

The lower portion of Table 1 shows estimation results obtained using the second instrument set. The results here are almost identical to those in the upper panel. The mean estimate of ν in the largest data set, 1.978, is slightly further away from the the true value than that reported in the upper panel, but the difference is less than one standard deviation.

Table 2 shows results for the case where the true parameter vector is $\Theta_b = \{0.6, 0.2, 0.95, 0.98\}$, which includes both a non-convex adjustment cost and irreversibility through a capital price asymmetry. Here we see evidence that the size of the sample strongly affects the average and the precision of the estimates. In the smallest sample exercise, the mean parameter estimates using instrument set Z_1 are $\{\bar{\alpha}_{1,19}, \bar{\nu}_{1,19}, \bar{\lambda}_{1,19}, \bar{p_s}_{1,19}\} = \{0.63, 0.24, 0.89, 1.02\}$ with respective standard deviations of $\{0.11, 0.26, 0.32, 0.26\}$.

The imprecision of the estimates is due in large part to the length of investment inactivity induced by the non-convex adjustment costs. After omitting all periods beginning with the earliest observed incomplete spell, the average number of periods used in the estimation is only 4.4 and the average number of observations is 320. In the largest sample exercise, the mean parameter estimates are $\{\bar{\alpha}_{1,100}, \bar{\nu}_{1,100}, \bar{\lambda}_{1,100}, \bar{p_s}_{1,100}\} = \{0.602, 0.198, 0.946, 0.978\}$, which are very close to the true value of $\Theta_b = \{0.6, 0.2, 0.95, 0.98\}$. The precision of these estimates is also much improved as the average number of uncensored periods increases to 84 and the average number of observations is not 6207. The respective standard deviations

Table 1: GMM Estimates of Θ_a from Monte Carlo Exercise

	α	ν	T	uncensored periods	obs.
Θ_a	0.6	2.0			
GMM estimates using instrument set Z_1	0.600	1.971	19	13.6	2615.2
	(0.025)	(0.136)			
	0.601	1.988	50	44.6	8584.1
	(0.014)	(0.074)			
	0.601	1.991	100	94.6	18214.2
	(0.009)	(0.047)			
GMM estimates using instrument set Z_2	0.600	1.964	19	13.6	2615.2
	(0.025)	(0.129)			
	0.600	1.977	50	44.6	8584.1
	(0.014)	(0.072)			
	0.601	1.978	100	94.6	18214.2
	(0.009)	(0.044)			

Reported estimates are the mean value across estimates from 1000 simulated datasets. Standard deviations of the estimates are reported in parentheses. Each sample contains 200 plants. T denotes the length of the sample period. The last two columns report the mean number of periods in the sample before the first incomplete spell begins and the mean number of observations (completed spells) in the GMM estimation. Denoting $\bar{T}$ as the period in which the first incomplete spell is initiated, all completed spells initiated in periods $t \geq \bar{T}$ have been excluded from the estimation.

are $\{0.019, 0.050, 0.054, 0.046\}$.

The mean parameter estimates obtained using instrument set Z_2, shown in the lower portion of Table 2, are very similar to those based on Z_1. The precision of these estimates, however, is much improved. The respective standard deviations of the estimates using the smallest sample are one-third to two-thirds the size of those based on Z_1. In the largest sample, the standard deviations for the four parameter estimates are $\{0.014, 0.019, 0.044, 0.016\}$, representing a reduction of over 50 percent for the estimates of ν and p_s.

Table 3 shows results based on the parameterization that most closely matches the estimates of Cooper and Haltiwanger, $\Theta_c = \{0.6, 0.2, 0.80, 0.98\}$. This parameterization has a much larger disruption cost, $\lambda = 0.8$, than in the previous case. The larger disruption cost leads to more inactivity and longer observed incomplete spells, which translates into greater imprecision of the estimates due to the number of periods that must be excluded from the estimation. The mean parameter estimates in the smallest sample exercise when using instrument set Z_1 are $\{\bar{\alpha}_{1,19}, \bar{\nu}_{1,19}, \bar{\lambda}_{1,19}, \bar{p}_{s1,19}\} = \{0.61, 0.10, 0.1.17, 0.89\}$. The high degree of impression is reflected in the respective standard deviations of $\{0.28, 0.42, 1.33, 0.57\}$. After controlling for incomplete spells, the average number of periods in the sample is now only 1.8 periods and the average number of observations is 63. In the largest sample exercise, the mean parameter estimates are $\{\bar{\alpha}_{1,100}, \bar{\nu}_{1,100}, \bar{\lambda}_{1,100}, \bar{p}_{s1,100}\} = \{0.595, 0.189, 0.827, 0.967\}$. These mean estimates are not as close to the true values as the comparable estimates in Table 2, which is a reflection in part of the of greater impression of the estimates. The average number of uncensored periods is 76.6 and the average number of observations is 2841, approximately 30 percent fewer observations than in Table 2. The respective standard deviations are $\{0.034, 0.059, 0.127, 0.074\}$.

Table 2: GMM Estimates of Θ_b from Monte Carlo Exercise

	α	ν	λ	p_s	T	uncensored periods	obs.
Θ_b	0.6	0.2	0.95	0.98			
GMM estimates using instrument set Z_1	0.627	0.239	0.894	1.019	19	4.4	320.5
	(0.108)	(0.260)	(0.321)	(0.257)			
	0.603	0.197	0.943	0.979	50	35.2	2583.6
	(0.030)	(0.081)	(0.090)	(0.075)			
	0.602	0.198	0.946	0.978	100	84.5	6207.7
	(0.019)	(0.050)	(0.054)	(0.046)			
GMM estimates using instrument set Z_2	0.621	0.180	0.877	0.968	19	4.4	320.5
	(0.071)	(0.103)	(0.231)	(0.085)			
	0.604	0.194	0.938	0.978	50	35.2	2583.6
	(0.020)	(0.031)	(0.070)	(0.026)			
	0.602	0.197	0.945	0.978	100	84.5	6207.7
	(0.014)	(0.019)	(0.044)	(0.016)			

Reported estimates are the mean value across estimates from 1000 simulated datasets. Standard deviations of the estimates are reported in parentheses. Each sample contains 200 plants. T denotes the length of the sample period. The last two columns report the mean number of periods in the sample before the first incomplete spell begins and the mean number of observations (completed spells) in the GMM estimation. Denoting $\bar{T}$ as the period in which the first incomplete spell is initiated, all completed spells initiated in periods $t \geq \bar{T}$ have been excluded from the estimation.

Table 3: GMM Estimates of Θ_c from Monte Carlo Exercise

	α	ν	λ	p_s	T	uncensored periods	obs.
Θ_c	0.6	0.2	0.80	0.98			
GMM estimates using instrument set Z_1	0.610	0.099	1.170	0.894	19	1.8	63.2
	(0.275)	(0.422)	(1.332)	(0.572)			
	0.598	0.195	0.823	0.979	50	25.7	953.6
	(0.067)	(0.125)	(0.253)	(0.159)			
	0.595	0.189	0.827	0.967	100	76.6	2841.6
	(0.034)	(0.059)	(0.127)	(0.074)			
GMM estimates using instrument set Z_2	0.622	0.095	0.700	0.904	19	1.8	63.2
	(0.120)	(0.148)	(0.434)	(0.167)			
	0.595	0.185	0.808	0.966	50	25.7	953.6
	(0.029)	(0.044)	(0.125)	(0.047)			
	0.599	0.195	0.808	0.975	100	76.6	2841.6
	(0.017)	(0.027)	(0.072)	(0.030)			

Reported estimates are the mean value across estimates from 1000 simulated datasets. Standard deviations of the estimates are reported in parentheses. Each sample contains 200 plants. T denotes the length of the sample period. The last two columns report the mean number of periods in the sample before the first incomplete spell begins and the mean number of observations (completed spells) in the GMM estimation. Denoting $\bar{T}$ as the period in which the first incomplete spell is initiated, all completed spells initiated in periods $t \geq \bar{T}$ have been excluded from the estimation.

The estimates obtain using instrument set Z_2, shown in the lower portion of Table 3, are much more precisely estimated and the mean estimate is much closer to the true value in the largest sample. The standard deviations of the four parameter estimates are less that half the size obtained using Z_1 in the smallest sample. In the largest sample exercise, the mean parameter estimates are $\{\bar{\alpha}_{2,100}, \bar{\nu}_{2,100}, \bar{\lambda}_{2,100}, \bar{p}_{s2,100}\} = \{0.600, 0.196, 0.806, 0.976\}$, which are very close to the true values of $\Theta_c = \{0.6, 0.2, 0.8, 0.98\}$. The standard deviations are roughly half the size of those obtained with Z_1. One potential explanation for the improved results using Z_2 is that Z_2 contains two more variables than Z_1. With the additional variables,

Z_2 may provide more explanatory power than Z_1. However, instrument set Z_1 contains the current and once-lagged values of the state variables of the dynamic programming problem, which should be the only pieces of information needed to summarize the information set of the plant.

4.3 Summary of Findings

The simulation exercise leads to two conclusions. First, the methodology works: for large enough samples, the minimization of (17) reproduces the structural parameters underlying the simulated data. This is true for all three of the parameterizations. Second, instrument set Z_2 produces more precise estimates. This is good news in that most data sets will not have the realized values of shocks as in instrument set Z_1. The variables in instrument set Z_2 are more likely to be available.

However, there is one issue highlighted in this exercise: the dependence of the results on large samples. For the cases of non-convex adjustment costs and irreversibility, the point estimates were quite far from the true values when we used only 19 periods and the standard errors were quite large. This is an issue for empirical application since the plant level data we use for the estimation of the model has only 19 years.

In order to use this method for short data samples, we will need to incorporate the incomplete spells into the empirical analysis. We turn to this point next before describing empirical results.

5 Sample Selection and Incomplete Spells

The estimation procedure for the previous exercises employed a particular procedure for selecting the sample. The data used for the estimation was $\bar{T}$ periods long. This critical period, $\bar{T}$ had the property that all plants who invested up to and including period $\bar{T}$ invested again in the sample. All periods beginning with the earliest observed incomplete spell (period

$\bar{T}+1$) were omitted from the estimation. This is an *ex ante* selection device and, with a large enough sample, does not lead to any bias.

This criteria, however, often leads to several periods being omitted in which all investment spells initiated in those period are completed by the end of the sample. It also excludes observations in which plants invested in two consecutive periods. Further, this exercise excluded all incomplete spells.

In this section we discuss alternatives. The first are other sample selection procedures. As we shall see, they do not produce unbiased estimates for the small sample.

Then we propose a methodology for dealing with incomplete spells within our estimation procedure. The idea is to use the parameter estimates to characterize the marginal value of capital and thus calculate an *ex post* error for all plants.

5.1 Alternative Sample Selection Criteria

We could have implemented other ways to create a sample based on complete spells. We discuss two approaches.

One procedure selects plant-year observations in which investment occurs in two consecutive periods. This selection has an advantage of allowing the researcher to use conventional Euler-equation estimation techniques since, by construction, the data set does not include any inaction. The drawback is that this is an *ex post* selection procedure and leads to bias.

The row labeled "Consecutive" in Table 4 shows the results of this selection procedure when there are 19 periods. The second row of the table, labeled "Complete" are the results reported in Table 3 for the 19 period sample. Compared to the "Complete" case, the "Consecutive" selection has almost twice as many observations. In this sense, this procedure uses more of the data. Yet, clearly the selection of observations of consecutive adjustment generates parameter estimates quite far from truth, Θ_c.

The row labeled "All" in Table 4 goes a step further and includes all complete spells. So, relative to the sample selection used in the "Complete" case, all would include investment spells which started after $\bar{T}$ and were concluded within the 19 periods. The sample size increases to about 417 on average. Yet, once again, because of the *ex post* nature of the sample selection, the results still show considerable bias.

5.2 Controlling for incomplete spells

We propose a methodology for controlling for incomplete spells in the estimation procedure. The difficulty introduced by incomplete spells is that the *ex post* error expressed in (7) cannot be fully evaluated. However, the structure of the dynamic programming problem can potentially be used to approximate the unobserved portion of the incomplete spell by estimating the marginal value of capital.

We consider a multi-stage method for estimating the structural parameters, retaining the assumption that A is observed. We then turn to version of the procedure to deal with unobserved shocks.

In the first stage of this methodology, parameter estimates are obtained from (17) by including all complete spells as observations in the estimation. This first stage is the same as the "All" treatment in Table 4. We denote these first stage estimates as Θ_1. Assuming that we have obtained all of the other structural parameters of the model from other sources, we then solve the dynamic programming using Θ_1. From this solution, we can compute the expected derivative of the value function that appears in the first-order condition of the investment decision expressed in (5). This expected derivative is a function of the current profitability shock and the capital stock resulting from the investment decision in the current period, conditional on the parameterization Θ_1.

$$\psi(A, K'; \Theta_1) = E_{A'|A} V_2(A', K'; \Theta_1) \tag{21}$$

This function can then be evaluated using observations of A and $K' = (1-\delta)K + I$ from the final period of the sample, and *ex post* errors for all incomplete spells can be computed using the following specification.

$$\begin{aligned}\varepsilon_{t,incomplete} &= \nu \frac{I_t}{K_t} + p(I_t) - \sum_{i=1}^{T-t} \beta^i \Pi_2(A_{t+i}, K_{t+i})(1-\delta)^{i-1} \\ &\quad - \beta^{T-t+1}(1-\delta)^{T-t}\psi(A_T, K_{T+1}, \Theta).\end{aligned} \tag{22}$$

A second stage estimation including all complete and incomplete spells results in parameter estimates Θ_2. This process can be repeated by computing $\psi(A, K'; \Theta_2)$ and obtaining a third stage estimate, Θ_3. Additional repetitions can be computed until estimates of Θ converge.

Table 4: Dealing with a Small Sample

	α	ν	λ	p_s	T	obs.
Θ_c	0.6	0.2	0.8	0.98		
Complete	0.610	0.099	1.170	0.894	19	63.2
	(0.275)	(0.422)	(1.332)	(0.572)		
Consecutive	0.532	0.003	0.902	0.141	19	137.5
	(0.335)	(0.125)	(0.633)	(0.664)		
All	0.646	0.143	0.609	0.939	19	417.1
	(0.128)	(0.219)	(0.355)	(0.293)		
Incomplete	0.5809	0.1769	0.8228	0.9594	19	
	(0.0355)	(0.0474)	(0.1028)	(0.0531)		

The results are shown in row "Incomplete" of Table 4. Though the sample is only 19 periods long, the parameter estimates are quite close to truth and are estimated fairly precisely. This procedure clearly dominates the other for this sample.

6 Estimation

The estimation takes the procedures outlined above to plant-level manufacturing data. The LRD data set is described in some detail in Cooper and Haltiwanger (2006). Some pertinent aspects of the data are summarized Table 5, taken from that paper.

The approach in Cooper and Haltiwanger (2006) is to use these moments in a minimum distance estimation exercise. In doing so, for each vector of structural parameters, the dynamic programming problem was solved through value function iteration, a data set was simulated and moments were calculated. In addition, a fixed discount factor was assumed through the analysis.

Table 5: Moments from the LRD

Variable	LRD
Average Investment Rate	12.2% (0.10)
Inaction Rate: Investment	8.1% (0.08)
Fraction of Observations with Negative Investment	10.4% (0.09)
Spike Rate: Positive Investment	18.6% (0.12)
Spike Rate: Negative Investment	1.8% (0.04)
Serial correlation of Investment Rates	0.058 (0.003)
Correlation of Profit Shocks and Investment	0.143 (0.003)

The approach taken here is much faster as it does not require repeated solution of the dynamic programming problem. There is a considerable increase in the speed of the estimation exercise, though, in contrast to the approach of matching the moments in Table 5, the estimation requires access to the actual data rather than summary moments.

The estimation uses (17). As discussed in Cooper and Haltiwanger (2006), the LRD provides enough information to measure all the components of the second instrument set.

There are two difficulties posed by estimating the parameters of the model from the LRD. First, there is the relatively short sample length of 19 periods. This is again why we focused on the behavior of the estimates for short samples. Consequently, extending the approach to deal with incomplete spell is quite important to increase the number of data points.

Second, the analysis thus far assumes that A_{it} is observed. This is relevant for our ability to estimate λ from (7). Since measured profits would include λ, it is necessary to construct marginal profitability directly from observations on (A_{it}, K_{it}). Thus we need to measure

Table 6: 371 Results

parameter	GMM estimate	SMM estimate
λ	0.64 (0.11)	0.68
α	0.80 (0.02)	0.78
ν	0.0016 (0.02)	0.051
p_s	0.93 (0.06)	0.81
stat	135.3	

profitability at plant-level.

The estimation procedure has multiple stages to deal with the fact that A_{it} is not observed. The procedure is as follows:

- guess α, λ and infer A_{it} from observed revenues at the plant-level
- estimate $\rho_a, \sigma_a, \rho_\varepsilon, \sigma_\varepsilon$ from A_{it}
- fill in for incomplete spells using the procedure outlined in section 5.2
- use (K_{it}, A_{it}, I_{it}) as data to calculate ex post errors in (7).
- estimate parameters using (17) and the Z_2 instrument set
- use new parameter estimates in revenue function to extract new A_{it}
- iterate

6.1 Transportation (371)

Our initial results for this sector are reported in Table 6. These results were obtained using the procedure outlined above for **complete spells only**. Estimation of the model incorporating incomplete spells is in process.

The first column of the table shows the results the GMM estimation of the Euler equation. We find evidence of curvature in the profit function and a substantial disruption cost associated with changes in the capital stock. The estimated quadratic adjustment cost is quite small and there is some irreversibility.

The second column shows the results reported in Cooper and Haltiwanger (2006) using a SMM approach. For this sector, the results are fairly similar qualitatively across the two approaches. The parameter estimates of α and λ are quite close. One big difference in results is the larger estimate of irreversibility in the SMM approach.

6.2 Steel (331)

Following the procedures described in section 6.1, we estimated the parameters for Steel, sector 331. The results are reported in Table 7. As with the results for sector 371, we curvature in the profit function, relatively low quadratic adjustment costs, and substantial disruption costs. Once again, the Euler-equation approach does not find the substantial irreversibility detected in the SMM approach.

Table 7: 331 Results

parameter	GMM estimate	SMM estimate
λ	0.48 (0.05)	0.7
α	0.89 (0.01)	0.66
ν	0.000 (0.06)	0.015
p_s	0.9911 (0.07)	0.76
stat	36.0	

7 Conclusions

This paper had two purposes. The first was to analyze a methodology for using the logic of Euler-equation estimation, as in Hansen and Singleton (1982), to settings in which adjustment is infrequent. Our analysis indicates how these procedures can estimate underlying adjustment costs, including those that create the inaction. We have used a simulation environment to identify powerful instruments and to guide us in the analysis of incomplete spells.

The second part of the paper takes this approach to plant-level data for U.S. manufacturers. There we are successful in estimating the parameters of the model for complete spells.[10] The parameter estimates are qualitatively similar to those reported in Cooper and Haltiwanger (2006). One important difference is in the estimates of the irreversibility, which are much smaller in the Euler-equation-based approach.

As this research proceeds, we plant to supplement the estimation in two dimensions. First, as developed in Section 2.1, it is possible to analyze a model in which the non-convex adjustment costs are incurred for investment rates above a critical value, $\bar{\iota}$. Thus far, we have set that value at 0. One way to proceed is to estimate the model for different values of $\bar{\iota}$ and compare the specifications by how well they match the moments.

Second, the empirical analysis has focused on the *ex post* Euler-equation errors. But we have ignored additional information contained in the fact that in some states, the optimizing plant chooses inaction over action, $V^i(A, K) > V^a(A, K)$. There are two ways to use the information contained in this inequality. One is to see how well the estimated model matches the data along this dimension. The second is to formally incorporate these inequalities into the estimation.[11]

Finally, there are numerous other applications of this methodology. One in particular arises in dynamic choice problems with occasionally binding constraints, such as borrowing restrictions. The Euler equation does not hold in periods where the borrowing constraint binds. By the logic of the approach taken in this paper, the parameters of the optimization problem can be estimated by looking over periods in which the constraint does not bind.

[10]Extending the estimation to incomplete spells is in process.

[11]This is related to the procedures described in Pakes, Porter, Ho, and Ishii (2006).

References

AGUIRREGABIRIA, V. (1997): "Estimation of Dynamic Programming Models with Censored Dependent Variables," *Investigaciones Economicas*, 21(2), 167–208.

COOPER, R., AND J. HALTIWANGER (2006): "On the Nature of the Capital Adjustment Costs," *Review of Economic Studies*, 73, 611–34.

HANSEN, L. P. (1982): "Large Sample Properties of Generelized Method of Moments Estimators," *Econometrica*, 50, 1029–1054.

HANSEN, L. P., AND K. J. SINGLETON (1982): "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models," *Econometrica*, 50, 1269–1286.

NEWEY, W., AND K. WEST (1987): "Hypothesis Testing with Efficient Method of Moments Estimation," *International Economic Review*, 28, 777–87.

PAKES, A. (1994): "Dynamic Structural Models, Problems and Prospects: Mixed Continuous discrete controls and market interactions," in *Advances in Econometrics, Sixth World Congress*, ed. by C. Sims, vol. 2, pp. 171–259. Cambridge University Press.

PAKES, A., J. PORTER, K. HO, AND J. ISHII (2006): "Moment Inequalities and Their Application," `http://www.economics.harvard.edu/faculty/pakes/papers/Moments_November-06.pdf`, Harvard University, mimeo.

WHITE, H. (1980): "A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity," *Econometrica*, 48, 817–38.