TITLE:
Test sets and the Scarf-Shallcross algorithm

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ABSTRACT:

The Frobenius number of a sequence of relative prime natural numbers is the largest natural number which is not a sum of these numbers. The Frobenius number of 4 and 5 is 11, whereas the Frobenius number of 5 and 7 is 23. Computing the Frobenius number in non-fixed dimension is an NP-hard problem. Kannan (1992) and later Barvinok-Woods (2003) proved the existence of very interesting polynomial algorithms for computing the Frobenius number when the dimension is fixed. For smaller numbers, the Frobenius number is readily computable using graph algorithms in group relaxations for integer programs.

Scarf and Shallcross (1993) related the Frobenius number to the geometry of maximal lattice free bodies and gave a surprising algorithm for the Frobenius number problem in dimension 3 using maximal lattice free triangles.

Recently (2006) Einstein, Lichtblau, Strzebonski and Wagon came up with a novel algorithm related to lattice point enumeration. This algorithm was further simplified by Roune using Gr\"obner bases for lattice ideals leading to record breaking computations of Frobenius numbers for random 11-digit numbers in dimension 13.

We describe an algorithm for computing maximal lattice free bodies inspired by revlex term orders in computational algebra. As an application we generalize the Scarf-Shallcross algorithm to arbitrary dimension. Using proper data structures (short rational functions) one may prove that this algorithm is also polynomial in fixed dimension.

This is joint work with A. Jensen and B. Roune.