Abstract

This paper develops a dynamic model of mismatch. Workers and jobs are randomly assigned to labor markets. Each labor market clears at each instant but some labor markets have more workers than jobs, hence unemployment, and some have more jobs than workers, hence vacancies. As workers and jobs move between labor markets, some unemployed workers find vacant jobs and some employed workers lose or leave their job and become unemployed. The model is quantitatively consistent with the comovement of unemployment, job vacancies, and the rate at which unemployed workers find jobs over the business cycle. It can also address a variety of labor market phenomena, including duration dependence in the job finding probability and employer-to-employer transitions, and it helps explain the cyclical volatility of vacancies and unemployment.
1 Introduction

Why do unemployed workers and job vacancies coexist? What determines the rate at which unemployed workers find jobs? This paper advances the proposition that at any point in time, the skills and geographical location of unemployed workers are poorly matched with the skill requirements and location of job openings. The rate at which unemployed workers find jobs depends on the rate at which they retrain or move to locations with available jobs, the rate at which jobs open in locations with available workers, and the rate at which employed workers vacate jobs in locations with suitable unemployed workers.

My main finding is that such a model of mismatch is quantitatively consistent with two robust features of labor markets: the negative correlation between unemployment and vacancies at business cycle frequencies (the Beveridge curve) and the positive correlation between the rate at which unemployed workers find jobs and the vacancy-unemployment (v-u) ratio. The model-generated Beveridge curve has a slope of approximately $-1$, quantitatively consistent with evidence from the United States. The model predicts that a 1 percent increase in the v-u ratio should be associated with a 0.14 percent increase in the job finding rate. In particular, the elasticity of the job finding rate with respect to the v-u ratio is constant. Empirically the elasticity is constant but about twice as high. I also use the model to explore employer-to-employer transitions and duration dependence in the exit rate from unemployment.

The view of unemployment and vacancies that I advance in this paper is conceptually distinct from the one that search theory has advocated since the pioneering work of McCall (1970), Mortensen (1970), and Lucas and Prescott (1974). According to search theory, unemployed workers have left their old job and are actively searching for a new employer. In contrast, this paper emphasizes that unemployed workers are attached to an occupation and a geographic location in which jobs are currently scarce. Mismatch is a theory of former steel workers remaining near a closed plant in the hope that it reopens. Search, particularly as articulated in Lucas and Prescott (1974), is a theory of former steel workers moving to a new city to look for positions as nurses. These two theories are complementary and it is a priori reasonable to think that mismatch may be as important as search in understanding equilibrium unemployment.

1 A potential drawback to Lucas and Prescott (1974) is that they do not have a notion of job vacancies; however, Rocheteau and Wright (2005) have introduced vacancies into a monetary search model based on the Lucas-Prescott framework.
Indeed, the mismatch view of unemployment and vacancies is not new.\textsuperscript{2} Tobin (1972, p. 9) advances a theory of a “stochastic macro-equilibrium” in which “excess supplies in labor markets take the form of unemployment, and excess demands the form of unfilled vacancies. At any moment, markets vary widely in excess demand or supply, and the economy as a whole shows both vacancies and unemployment.”\textsuperscript{3} Drèze and Bean (1990) discuss important subsequent developments, including conditions on the joint distribution of workers and jobs across labor markets which ensure that the aggregation of many small markets yields a CES Beveridge curve. But both of these papers link mismatch with disequilibrium in local labor markets, where the wage does not clear the market. One point of this paper is to show that models of mismatch are quantitatively consistent with macro-labor facts even if each labor market is in equilibrium at every instant so the equilibrium is constrained Pareto optimal.

To that end, in Section 3 I develop an explicit model of heterogeneity. There are many local labor markets, each of which represents a particular geographic location and a particular occupation. The wage clears each labor market, but there may be unemployed workers in one labor market and job vacancies in another. As workers and jobs enter and exit labor markets, unemployed workers find jobs and employed workers lose jobs, sometimes moving directly to another job.

In Section 4, I show that fluctuations in aggregate productivity, in the cost of job creation, or in the duration of jobs affect the average number of jobs per labor market. An increase in this variable reduces the vacancy rate and raises the unemployment rate, moving the economy along a downward-sloping Beveridge curve. I compare the theoretical relationship with evidence from the Job Openings and Labor Turnover Survey (JOLTS) and the Conference Board Help Wanted Index, and show that the theoretical and empirical Beveridge curves are nearly indistinguishable.

I next show that vacancies and unemployment respond about 3.7 times as much to productivity shocks in the mismatch model as in Pissarides’s (1985) matching model. Shimer (2005a) argues that the matching model only explains about ten percent of the volatility in vacancies and unemployment, so this helps to reconcile the theory and the data. Moreover, fluctuations in the expected duration of jobs induce movements along a downward sloping Beveridge curve in the mismatch model. In the matching model, they induce a counterfactual positive co-movement of unemployment and vacancies (Abraham and Katz, 1986; Shimer, 2005a). Schioppa (1991) argues that there are four distinct meanings to the term mismatch. The notion of mismatch in this paper is closest to the second approach that he discusses. Tobin (1972) cites a number of previous authors in developing these ideas including Lipsey (1960) and Holt (1970). Hansen (1970) proposes a similar model of mismatch.

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\textsuperscript{3}Tobin (1972) cites a number of previous authors in developing these ideas including Lipsey (1960) and Holt (1970). Hansen (1970) proposes a similar model of mismatch.
Section 5 explores the relationship between the rate at which unemployed workers find jobs and the v-u ratio. Not only is this relationship increasing in the model, it is nearly indistinguishable from a Cobb-Douglas. An increase in productivity that raises the v-u ratio by 10 percent raises the instantaneous job finding rate by about 2 percent. This is roughly consistent with U.S. data, where it is impossible to reject the hypothesis of a constant elasticity, although the elasticity is closer to 0.3. This last fact is usually interpreted by search theorists as evidence in favor of a reduced-form Cobb-Douglas matching function (Petrongolo and Pissarides, 2001); this paper provides the first structural explanation for why the matching function appears to be Cobb-Douglas.

A careful examination of the job finding rate requires me to account for heterogeneity in the exit rate from unemployment, which I do in Section 6. The long-term unemployed are typically located in labor markets where jobs are particularly scarce, which makes their prospects for exiting unemployment unusually bleak. This dynamic sorting explains about half of the empirical duration dependence in the job finding probability. The remainder presumably reflects unmodeled worker heterogeneity. Accounting for duration dependence reduces the measured level of the job finding probability but has little effect on the elasticity of the job finding probability with respect to the v-u ratio.

In Section 7, I examine the model’s predictions for employer-to-employer transitions. Recent U.S. data indicates that employer-to-employer transitions are mildly procyclical and I find that the model is quantitatively consistent with these facts too. I conclude in Section 8.

2 Related Literature

2.1 Mismatch Models

A number of previous authors have developed formal models of mismatch as a source of unemployment. Many use an urn-ball structure, where workers (balls) are randomly assigned to jobs (urns); see Butters (1977) and Hall (1977) for early examples. The random assignment ensures that some jobs are unfilled, yielding vacancies, and some jobs are assigned multiple workers, only one of whom can be hired, yielding unemployment. Hall (2000) supposes that workers are randomly assigned to locations and then matched in pairs. One worker is necessarily unemployed in any location with an odd number of workers, linking the importance of matching (the number of workers per location) and the unemployment rate.
Den Haan, Ramey, and Watson (2000) offer an alternative model of matching frictions based on workers and firms searching in different “channels.” In their framework, the number of channels is assumed to be a CES function of unemployment and vacancies.

Stock-flow matching models offer another sensible theory of mismatch (Taylor, 1995; Coles and Muthoo, 1998; Coles and Smith, 1998; Coles and Petrongolo, 2003). According to these models, only a small proportion of worker-job matches are feasible. When a worker loses her job, she looks among the available stock of vacancies to see if her skills are suitable for any of them. If so, she is immediately paired with a suitable vacancy, while otherwise she remains unemployed. Symmetrically, entering job vacancies search for a match within the stock of unemployed workers.

Perhaps the most similar models of mismatch are Lagos’s (2000) model of the taxicab market and Sattinger’s (2005) model of queuing. According to Lagos (2000), there are a fixed set of locations and two types of economic agents, drivers and passengers. The short side of the market is served within each location and drivers optimally relocate to the best possible location. Nevertheless, Lagos finds that empty taxis and unserved riders can coexist in equilibrium if prices are fixed exogenously, yielding an aggregate Beveridge curve. Sattinger (2005) assumes workers are randomly assigned to job queues and wait to be “served.” A worker on a long queue experiences a longer unemployment spell. He shows that a combination of queuing and search is consistent with a downward sloping Beveridge curve. To generate mismatch, one must take one of the approaches adopted in these papers, either prices that do not clear markets or limited mobility of workers and jobs.

There are many small differences between these earlier approaches to mismatch and the model that I propose in this paper. For example, by making the notion of a labor market explicit, it is sensible to think about wages being determined by competition for labor within markets. The literature on urn-ball and stock-flow matching models has typically assumed that wages are either posted by firms as a recruiting device or bargained ex post by workers and firms. But the most important difference between this paper and the urn-ball and stock-flow literatures is one of emphasis. No previous paper has shown that a mismatch model is quantitatively consistent with the empirical comovement of unemployment, vacancies, and the job finding rate. Instead, the literature has focused on the theoretical shortcomings of the reduced-form matching function approach by arguing that mismatch models do not deliver a structural matching function.
2.2 Search and Matching Models

The issues this paper examines have traditionally been the realm of search models, especially Pissarides’s (1985) matching model and its variants. Under appropriate restrictions on the reduced-form matching function and on the nature of shocks, the matching model is quantitatively capable of describing the Beveridge curve (Abraham and Katz, 1986; Blanchard and Diamond, 1989) and the increasing relationship between the v-u ratio and the rate at which unemployed workers find jobs (Pissarides, 1986; Blanchard and Diamond, 1989).

Despite these successes, the matching model has two significant shortcomings. The first is the matching function itself. It is intended to represent “heterogeneities, frictions, and information imperfections” and to capture “the implications of the costly trading process without the need to make the heterogeneities and other features that give rise to it explicit” (Pissarides, 2000, pp. 3–4). But Lagos (2000) emphasizes that if the matching function is a reduced-form relationship, one should be concerned about whether it is invariant to policy changes. Addressing this issue requires an explicit model of heterogeneity that gives rise to an empirically successful reduced-form matching function.

The second is wage determination. In the matching model, workers and firms are typically in a bilateral monopoly situation, and so competitive theories of wage determination are inapplicable. Wages are instead set via bargaining. Some recent research has emphasized that the details of the bargaining protocol are quantitatively critical to the ability of the model to replicate business cycle fluctuations in unemployment and vacancies (Shimer, 2005a; Hall, 2005; Hall and Milgrom, 2005). The model I develop in this paper circumvents both of these issues.

3 A Model of Mismatch

3.1 Economic Agents

There are a $M$ workers and a large number of firms. Both are risk-neutral, infinitely-lived, and discount future income at rate $r$. Time is continuous.
3.2 Stocks

I start by looking at the state of the economy at each moment of time. I then examine the flow of workers and jobs and show that this is consistent with the stocks described here.

At any point in time, each worker is assigned to one of \( L \) labor markets. These assignments are independent across workers, so the distribution of workers across labor markets is a multinomial random variable.

Each firm may have zero, one, or more jobs. Let \( N \) denote the total number of jobs. Each job is assigned to one labor market. Again, these assignments are independent across jobs and independent of the number of workers assigned to the labor market. Thus the distribution of jobs across labor markets is an independent multinomial random variable.

Let \( M \equiv M/L \) and \( N \equiv N/L \). In the remainder of this paper, I fix \( M \) and \( N \) and take the limit as \( L \to \infty \). In this limit, the number of workers and jobs per labor market are independent Poisson random variables. In a standard abuse of the law of large numbers, I assume that the fraction of labor markets with \( i \) workers and \( j \) jobs is deterministic, hence equal to

\[
p(i, j) = \frac{e^{-(M+N)M^iN^j}}{i!j!} \tag{1}
\]

if \((i, j) \in \{0,1,2,\ldots\}^2 \) and \( p(i, j) = 0 \) otherwise. The cross-sectional distribution of workers and jobs is critical for what follows. It will prove useful to describe how changes in \( M \) and \( N \) affect this probability:

**Lemma 1** \( \frac{\partial p(i, j)}{\partial M} = p(i-1, j) - p(i, j) \) and \( \frac{\partial p(i, j)}{\partial N} = p(i, j-1) - p(i, j) \)

**Proof.** The results follow directly from differentiating \( p(i, j) \) in equation (1).

Workers and jobs must match in pairs in order to produce market output. One worker and one job in the same labor market can jointly produce \( x \) units of the numeraire homogeneous consumption good. A single worker (an unemployed worker) produces \( z < x \) units of the same good at home, while a single job (a vacancy) produces nothing. Workers and jobs are indivisible. These stark assumptions give a concrete notion of unemployment and vacancies.

There is competition within each labor market. Let \( i \) denote the number of workers in some labor market and \( j \) denote the number of jobs. If \( i > j \), \( i - j \) workers are unemployed but all workers are indifferent about being unemployed; the wage, \( w \), is driven down to the value of home production, \( w = z \). If \( i < j \), \( j - i \) jobs are vacant but all firms are indifferent about their jobs being vacant; the wage is driven up to the marginal product of labor, \( w = x \). If \( i = j \), there is neither unemployment nor vacancies in the market and the wage is not
determined. For notational simplicity I assume that if $i = j$, the wage is equal to workers’ reservation wage, $w = z$. The quantitative results are scarcely affected if I instead assume $w = x$ when $i = j$.

The competitive model of wage formation is stark, with wages only taking on two possible values. However, the movement of workers and jobs across markets, which I discuss next, ensures that the expected present value of wages differs continuously across markets depending on the current value of $i$ and $j$. If workers and firms can commit to long-term contracts, wage payments may be much smoother than is suggested by this spot-market model of wages.

### 3.3 Flows

Each worker suffers a shock to her human capital according to a Poisson process with arrival rate $q$. The arrival of this shock is exogenous, independent of the worker’s current employment status or wage. When the “quit” shock hits, the worker must quit her labor market and is randomly reassigned to a new one, independent of conditions in the new labor market. This means that the arrival rate of workers into a labor market is $qM$. This random inflow and outflow of workers is consistent with a Poisson distribution of workers across labor market at each instant.

Symmetrically, each job is destroyed according to a Poisson process with arrival rate $l$. When this “layoff” shock occurs, the job leaves the labor market and disappears. Conversely, a firm may create a new job by paying a fixed cost $k$. When it does so, the job is randomly assigned to a labor market. Again, both the entry and exit of jobs is independent of conditions in the local labor market, although the decision to create a job depends on aggregate labor market conditions. To maintain a steady state stock of $N$ jobs per labor market, the entry rate of jobs must be $n = lN$; however, out of steady state it may be higher or lower, with $\dot{N}(t) = n(t) - lN(t)$.

Whenever there are single workers and single jobs in a labor market, they are instantly paired off. For example, if a job enters a labor market with unemployed workers, one randomly selected worker is matched with the job and the pair starts producing output. I assume that a pair remains matched until either a quit or layoff hits the match, at rate $q + l$. This is consistent with a small unmodeled turnover cost.
3.4 Equilibrium

Firms create jobs whenever doing so is profitable, analogous to the free entry condition in Pissarides (1985). If all the parameters of the model are constant over time, the number of jobs per labor market must satisfy

$$k = \frac{x - z}{r + l} \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} p(i, j).$$

(2)

The cost of creating a job is $k$. It then yields profit $x - z$ whenever it is located in a labor market in which the number of other jobs, $j$, is strictly less than the number of workers $i$, a fraction $\sum_{i=1}^{\infty} \sum_{j=0}^{i-1} p(i, j)$ of the time in expectation. At other times it yields zero profits. These profits are discounted accounting both for the rate of time preference and for the rate at which jobs end, $r + l$.

This equation immediately yields intuitive comparative static results, consistent with the predictions of a search model (Pissarides, 2000):

**Proposition 1** The number of jobs per labor market $N$ is increasing in $\frac{x - z}{k(r + l)}$.

**Proof.** See Appendix A. ■

The proof simply establishes that the share of labor markets with excess workers is decreasing in $N$. The same logic implies that the equilibrium is stable. If the cost of creating a job is smaller than the expected profit from a job, firms create more jobs, raising $N$. This reduces the fraction of labor markets with excess workers, reducing profits and pushing the economy towards equilibrium. In fact, if $N$ is initially too low, equilibrium is attained instantaneously. Conversely, if the cost of creating a job is larger than the expected profit, there is no job creation. Instead, the stock of jobs falls gradually due to layoffs, $\dot{N}(t) = -lN(t)$. The fraction of labor markets in which jobs are scarce increases gradually, until in finite time the zero profit condition (2) is restored.

The choice of $N$ maximizes the expected present value of output in the economy net of the job creation costs, given the mobility restrictions. It is possible to prove this result directly by solving a social planner’s problem, but I omit the proof since it is also a simple application of the first welfare theorem.

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\(^4\)If the parameters follow a stochastic process, the free entry condition must be suitably modified. Most of my analysis in this paper focuses on comparative statics.
3.5 Discussion

This model is deliberately parsimonious. The only economic decision is one by firms, which must decide at each instant whether to create new jobs.\(^5\) In particular, the movement between labor markets is exogenous and random. While the reader may be accustomed to models in which mobility is endogenous, there are advantages to the approach I adopt here. On a theoretical level, it introduces relatively few free parameters and stresses that the main results are a consequence of limited mobility and aggregation.

There is also evidence that mobility at business cycle frequencies is primarily for idiosyncratic reasons. Kambourov and Manovskii (2004) show that gross occupational mobility is 10 to 15 percent per year at the one digit level while net mobility is only 1 to 3 percent. Blanchard and Katz (1992) argue that for 5 to 7 years following an adverse shock to regional employment, the impact is primarily on the local unemployment rate rather than on net migration.

Finally, there are substantial unmodeled costs to switching occupations or moving to a new location. In the numerical work that follows, no worker could increase her lifetime income by more than 5 percent if she moved to a random new location. If mobility is endogenous but mobility costs, including retraining costs, the loss of human capital, etc., exceed this amount, the analysis in this paper is applicable.

4 The Beveridge Curve

4.1 Theoretical Concepts

The number of unemployed workers per labor market is equal to the difference between the number of workers \(i\) and the number of jobs \(j\), summed across labor markets with more workers than jobs, and similarly for the number of vacancies per labor market:

\[
U = \sum_{i=1}^{\infty} \sum_{j=0}^{i} (i - j) p(i, j) \quad \text{and} \quad V = \sum_{j=1}^{\infty} \sum_{i=0}^{j} (j - i) p(i, j). \tag{3}
\]

The unemployment and vacancy rates are \(u \equiv U/M\) and \(v \equiv V/N\) and the \(v/u\) ratio is \(V/U\). These depend only on the contemporaneous number of workers and jobs per labor market, \(M\) and \(N\). This generates some simple comparative statics:

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\(^5\)This is also the only economic decision in Pissarides’s (1985) matching model.
Proposition 2 The unemployment rate \( u \) is increasing in the number of workers per labor market \( M \) and decreasing in the number of jobs per labor market \( N \):

\[
\frac{\partial u}{\partial \log M} = \frac{N}{M} \sum_{i=2}^{\infty} \sum_{j=0}^{i-2} p(i, j) \quad \text{and} \quad \frac{\partial u}{\partial \log N} = -\frac{N}{M} \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} p(i, j).
\]

(4)

The vacancy rate \( v \) is decreasing in \( M \) and increasing in \( N \):

\[
\frac{\partial v}{\partial \log M} = -\frac{M}{N} \sum_{j=1}^{\infty} \sum_{i=0}^{j-1} p(i, j) \quad \text{and} \quad \frac{\partial v}{\partial \log N} = \frac{M}{N} \sum_{j=2}^{\infty} \sum_{i=0}^{j-2} p(i, j).
\]

(5)

Proof. See Appendix A.

This proposition has a number of direct implications. First, an increase in the number of jobs per labor market \( N \), for example due to an increase in productivity \( x \), a decrease in the layoff rate \( l \), or a decrease in the cost of job creation \( k \), reduces the unemployment rate and raises the vacancy rate since additional jobs are increasingly less likely to locate in labor markets with unemployed workers. This induces movements along a downward-sloping v-u locus.

Second, a proportional increase in both \( M \) and \( N \) reduces both the unemployment and vacancy rates.\(^6\) Doubling \( M \) and \( N \) is equivalent to merging randomly selected pairs of labor markets. If both labor markets have unemployment, this merger does not affect the unemployment or vacancy rates, and similarly if both labor markets have vacancies. But merging a labor market with unemployment and a labor market with vacancies reduces the unemployment and vacancy rate in both markets. This comparative static suggests that the mismatch construction may be useful in other markets where the coexistence of unemployment and vacancies is more or less common. If matching is a more severe problem, as might be the case in marriage or housing markets, \( M \) and \( N \) should be modeled as relatively small numbers. If it is a less severe problem, as in commodity markets, \( M \) and \( N \) may be thought of as very large numbers.

Data on the unemployment and vacancy rates pin down the number of workers and jobs per labor market:

Proposition 3 For any \( u \in (0, 1) \) and \( v \in (0, 1) \), there is a unique \( M \in (0, \infty) \) and \( N \in (0, \infty) \) solving equation (3)

\(^6\)A proportional increase in \( M \) and \( N \) raises \( u \) by \( \frac{\partial u}{\partial \log M} + \frac{\partial u}{\partial \log N} = -\frac{N}{M} \sum_{i=1}^{\infty} p(i, i - 1) \) times the percentage change in \( M \) and \( N \), and similarly for \( v \).
Proof. See Appendix A. □

4.2 Measuring Vacancies: JOLTS

Since December 2000, the Bureau of Labor Statistics (BLS) has measured job vacancies using the JOLTS. This is the most reliable time series for vacancies in the U.S.. According to the BLS, “A job opening requires that 1) a specific position exists, 2) work could start within 30 days, and 3) the employer is actively recruiting from outside of the establishment to fill the position. Included are full-time, part-time, permanent, temporary, and short-term openings. Active recruiting means that the establishment is engaged in current efforts to fill the opening, such as advertising in newspapers or on the Internet, posting help-wanted signs, accepting applications, or using similar methods.”7 I measure the vacancy rate as the ratio of vacancies to vacancies plus employment.

The Bureau of Labor Statistics (BLS) uses the Current Population Survey (CPS) to measure the unemployment rate each month. The CPS measures employment and unemployment using a household questionnaire designed to determine whether an individual is working or, if she is not working, available for and actively seeking work. The ratio of unemployment to the sum of unemployment and employment is the unemployment rate. The brown dots in Figure 1 show the strong negative correlation between unemployment and vacancies over this time period, the empirical Beveridge curve.

In an average month from December 2000 to May 2005, the unemployment and vacancy rates were 5.4% and 2.3%, respectively. Using Proposition 3, these two numbers uniquely determine $M = 244.2$ and $N = 236.3$. To get a sense of whether these magnitudes are reasonable, observe that there are about 134 million workers in the U.S. according to the Current Employment Statistics (CES). Dividing by 244.2 gives about 550,000 labor markets. The Occupational Employment Statistics (OES) counts about 800 occupations, while there are 362 metropolitan statistical areas (regions with at least one urbanized area of 50,000 or more inhabitants) and 560 micropolitan statistical areas (regions with an urban area of 10,000 to 50,000 inhabitants). Together this gives a total of about 740,000 occupations and geographic areas. Although the sharp theoretical distinction between labor markets is less obvious in the data, this back-of-the-envelope calculation suggests that 244.2 workers per labor market is plausible and so I use it to pin down $M$.

I then consider how changes in productivity, the layoff rate, or the cost of job creation

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Figure 1: The brown dots show U.S. monthly data from December 2000 to May 2005. The unemployment rate is measured by the BLS from the CPS. The vacancy rate is measured by the BLS from the JOLTS. The solid blue line shows the model generated Beveridge curve with $M = 244.2$ and $N \in [233, 243]$.

affect the number of jobs per labor market $N$. Variation in these parameters, either deterministic or stochastic, forecastable or unexpected, induces variation in $N$, with $U$ and $V$ instantaneously adjusting so that equation (3) holds. This traces out the blue line in Figure 1, the model-generated Beveridge curve. The fit of the model to the data is remarkable.

The fact that the level of the model-generated Beveridge curve fits the data reflects a judicious choice of the number of workers per labor market. But the fact that the slope and curvature of the model-generated Beveridge curve also fits the data comes from the structure of the model. The model cannot generate a different Beveridge curve in response to fluctuations in productivity $x$, the layoff rate $l$, or the cost of job creation $k$.

4.3 Measuring Vacancies: Help-Wanted Index

A shortcoming of JOLTS is that it only covers one recession and subsequent expansion. Moreover, the recovery was unusual in that employment growth proceeded much slower than normal. While there is unfortunately no ideal measure of job vacancies over a longer time period, the Conference Board Help Wanted Index provides a crude one (Abraham, 1987). Since 1951, the Conference Board has constructed the index on a monthly basis by
Once again, the structure of the Section 5 Figure 2. Both are quarterly averages of monthly data, detrended using an HP Filter, with smoothing parameter $10^5$. The solid blue line shows the model generated Beveridge curve with $M = 244.2$ and $N \in [233, 243]$, expressed as a log deviation from $N = 238$.

measuring the number of help wanted advertisements in the largest newspaper in 51 major metropolitan areas. Consolidation of the newspaper industry, changes in newsprint costs, legally mandated changes in advertising like equal employment opportunity laws, and the rise of the internet likely all affected the help wanted index. To circumvent these issues, I compute the quarterly average of the help wanted advertising index, take logs, and detrend it using a low frequency HP Filter, with smoothing parameter $10^5$. The red dots in Figure 2 plots this against a similarly detrended measure of the unemployment rate, generating another empirical Beveridge curve.\footnote{Since January 1951, the average unemployment rate has been 5.4\%, the same as since December 2000.}

I compare the detrended empirical Beveridge curve with a demeaned theoretical Beveridge curve. For a given value of $N \in [233, 243]$, I compute the unemployment and vacancy rates using equation (3), take logs, and subtract the corresponding log unemployment and vacancy rates at $N = 238$.\footnote{In Section 5, I assume $N$ follows a well-specified stochastic process and look at the behavior of the detrended unemployment and vacancy rates. This gives a very similar result.} This gives the blue line in Figure 2. Once again, the structure of the model determines the slope and even the slightly concave shape of the theoretical Beveridge curve, both of which are consistent with the evidence.

Figure 2: The red dots show U.S. quarterly data from 1951 to 2004. The unemployment rate is measured by the BLS from the CPS. The vacancy rate is measured by the Conference Board Help Wanted Index. Both are quarterly averages of monthly data, detrended using an HP filter with smoothing parameter $10^5$. The solid blue line shows the model generated Beveridge curve with $M = 244.2$ and $N \in [233, 243]$, expressed as a log deviation from $N = 238$.\footnote{Since January 1951, the average unemployment rate has been 5.4\%, the same as since December 2000.}
In summary, this model is quantitatively consistent with the strong negative correlation between unemployment and vacancies and with the relative magnitude of changes in the two variables over time. This is true regardless of the source of shocks to the number of jobs per labor market. In contrast, while the matching model is able to produce a negative correlation between unemployment and vacancies, doing so is not trivial. For example, Mortensen and Pissarides (1994) report a theoretical correlation between unemployment and vacancies of $-0.26$. Merz (1995) reports the correlation is $-0.15$ if search intensity is exogenous and 0.32 if it moves endogenously over the business cycle. Shimer (2005a) finds that shocks to aggregate productivity induce a strong negative correlation between unemployment and vacancies and a judicious choice of the matching function yields the correct slope of the Beveridge curve as well. But even then, adding realistic fluctuations in the separation rate to the model induces a positive correlation between unemployment and vacancies.

4.4 Responsiveness to Shocks

Suppose there is an unanticipated permanent change in productivity $x$. The following proposition describes the resulting changes in unemployment and vacancies:

**Proposition 4** The responsiveness of the $v/u$ ratio to productivity is

$$\frac{\partial \log(V/U)}{\partial \log x} = \frac{(1-\alpha) + \frac{\alpha}{2}}{\sum_{i=1}^{\infty} p(i, i-1)} \cdot \frac{x}{x - z}, \quad (6)$$

where $\alpha \equiv \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} p(i, j)$ is the fraction of markets with unemployment. In particular, in the limit as $M = N \to \infty$,

$$\frac{\partial \log(V/U)}{\partial \log x} \to \frac{\pi x}{x - z}.$$

**Proof.** See Appendix A. □

It is difficult to interpret equation (6) directly. Instead, I illustrate the formula with a concrete example. Normalize $x = 1$ and set the value of home production at $z = 0.4$, the preferred number in Shimer (2005a); see Hagedorn and Manovskii (2005) for an analysis of the importance of this parameter. One way to pin it down is to look at the labor share of income, equal to 1 for employed workers in markets with $i < j$ and $z/x$ otherwise. A higher value of $z$ raises the labor share. In the parameterization I report here, about 60 percent of employed workers are paid $z = 0.4$ when $x = 1$, giving a labor share of $0.6 \cdot 0.4 + 0.4 \cdot 1 = 0.64$, a reasonable value.
Figure 3: The solid blue line shows the v-u ratio in the mismatch model as a function of productivity $x$. The dashed green line shows the same relationship in the Pissarides (1985) matching model using Shimer’s (2005a) calibration.

If $M = 244.2$, a 5 percent unemployment rate requires $N = 238.0$ and implies a v-u ratio of 0.49 (equation 3). Fluctuations in $x$ induce changes in $N$ and hence changes in the v-u ratio. As $x$ varies between 0.93 and 1.11, by about 18 log points, equation (2) implies that the number of jobs per labor market varies between 233 and 243, the range depicted in Figure 1 and Figure 2. This causes the v-u ratio to vary by about 114 log points, from 0.29 to 0.89, as shown by the solid blue line in Figure 3. Thus the elasticity of the v-u ratio with respect to labor productivity averages 6.3 in this example, somewhat larger than the theoretical value $\pi x/(x - z) = 5.2$ suggested by the limiting results in Proposition 6. Shimer (2005a) argues that the standard deviation of detrended labor productivity is about 0.02.\(^{10}\)

With an elasticity of 6.3, the model delivers a standard deviation of detrended v-u ratio of about 0.13, one third of the actual standard deviation of 0.38.

The failure to replicate the entire volatility in the v-u ratio is probably a success. Hall (2005), Mortensen and Nagypal (2005), and Rudanko (2005) argue that a one-shock model should explain the standard deviation of the projection of the detrended v-u ratio on detrended productivity, or equivalently the standard deviation of the detrended v-u ratio multiplied by the correlation between the two variables. Since the correlation is about 0.4, the proposed target for the volatility of the v-u ratio is 0.15, only slightly bigger than the model generated 0.13.

\(^{10}\)See Shimer (2005a) for the definition and measurement of the variables discussed here.
By contrast, Shimer (2005a, p. 36) argues that in a matching model calibrated with the same parameter values—in particular \( z = 0.4 \)—the elasticity of the v-u ratio with respect to labor productivity is approximately \( \frac{x}{x-z} = 1.7 \). This means that the same unanticipated permanent increase in productivity from 0.93 to 1.11 would raise the v-u ratio by about 30 log points. I graph this as the dashed green line in Figure 3. In short, the volatility of the v-u ratio in response to a given productivity shock is about 3.7 (6.3/1.7) times as high in the mismatch model as in the matching model.

What explains the rest of the volatility in the v-u ratio? As previously stated, shocks to the separation (layoff or quit) rate in the matching model induce a counterfactual upward sloping Beveridge curve, ruling out one plausible culprit. In contrast, shocks to the layoff rate induce movements along a downward sloping Beveridge curve in the mismatch model. Thus a countercyclical layoff rate in the mismatch model helps to explain some of the remaining comovement in unemployment and vacancies. This conclusion is unaffected by the procyclicality of the quit rate, which has no effect on entry (equation 2) or unemployment and vacancies (equation 3) in the mismatch model. Another possible candidate is shocks to the cost of investment goods \( k \) (Fisher, 2002).

# 5 The Job Finding Rate

## 5.1 Theoretical Concepts

This section explores the relationship between the rate at which unemployed workers find jobs and the v-u ratio. To do this, I need to describe how unemployed workers find jobs.

When a worker quits her labor market and moves to a new one or a job leaves its labor market and is replaced elsewhere by a new job, this may lead to one or more transitions between employment and unemployment. If an employed worker quits her labor market, an unemployed worker may take her old job (an unemployment-to-employment or UE transition) and she may fail to find a job in her new labor market (an employment-to-unemployment or EU transition). If an unemployed worker quits his labor market, he may find a job in his new labor market (UE transition). If a filled job leaves the labor market, its old employee may be left jobless (EU transition). But whenever a new job enters a labor market, it may hire a worker (UE transition). These events may also lead an employed worker to switch employers, a topic I defer until Section 7.

Let \( \pi_{q}^{UE} \) denote the probability that a quit leads to a UE transition. This occurs if either
the quitting worker is employed in a labor market with unemployed workers or if the worker
is unemployed and moves to a labor market with vacant jobs:

\[ \pi_{U}^{UE} \equiv \frac{1}{M} \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} j p(i, j) + u \sum_{i=0}^{\infty} \sum_{j=i+1}^{\infty} p(i, j). \] (7)

The first term is the fraction of workers who are employed in labor markets with unemployed
workers. This is equal to \( j \) workers in every labor market with \( i > j \). The second term is
the product of the fraction of workers who are unemployed and the fraction of labor markets
with vacancies, \( j > i \).

Add \( \sum_{i=0}^{\infty} \sum_{j=0}^{i} p(i, j) - \frac{1}{M} \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} ip(i, j) = 0 \) (see equation 19 in Appendix A) to
the right hand side of equation (7) and simplify using equation (3):

\[ \pi_{U}^{UE} = (1 - u) \sum_{i=0}^{\infty} \sum_{j=0}^{i} p(i, j). \] (8)

This is the product of the employment rate and the fraction of labor markets without vacan-
cies, i.e. the probability a quit shock leads an employed worker to move to a labor market
without vacancies, causing an EU transition: \( \pi_{U}^{UE} = \pi_{U}^{EU} \). Since the quit rate does not affect
the unemployment rate, the probability that a quit shock leads to a UE transition must be
the same as the probability that it leads to an EU transition.

I similarly let \( \pi_{n}^{UE} \) denote the probability that a job entering a labor market causes a UE
transition. This occurs whenever the job enters a market with unemployed workers, \( i > j \):

\[ \pi_{n}^{UE} = \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} p(i, j). \] (9)

Conversely, the probability that a job leaving a market causes an EU transition is equal to
fraction of jobs in markets without excess jobs \( i \geq j \):

\[ \pi_{l}^{EU} = \frac{1}{N} \sum_{j=1}^{\infty} \sum_{i=j}^{\infty} jp(i, j) = \sum_{j=0}^{\infty} \sum_{i=j+1}^{\infty} p(i, j). \] (10)

The second equation uses the same logic as going from equation (7) to equation (8). Re-
ordering the sum then proves that \( \pi_{l}^{EU} = \pi_{n}^{UE} \).

Putting these together gives the instantaneous transition rate from unemployment to
employment in steady state, i.e. the job finding rate for unemployed workers:

\[ f = \frac{qM \pi_{qUE} + n\pi_{nUE}}{U}. \]  

(11)

There are \( qM \) quit shocks per labor market, each leading to a UE transition with probability \( \pi_{qUE} \). Similarly, jobs enter at rate \( n \), leading to a UE transition with probability \( \pi_{nUE} \). This gives the total rate at which unemployed workers find jobs in an average labor market. Dividing by the total number of unemployed workers per labor market gives the instantaneous job finding rate for unemployed workers.

Fluctuations in the number of jobs per labor market \( N \) induced by changes in labor productivity \( x \) induce fluctuations in the v-u ratio (equation 3) and in the job finding rate \( f \). The next subsection discusses empirical measures of the job finding rate. I then use comparative statics and a simulation of aggregate shocks in a stochastic environment to discuss the comovement of unemployment, vacancies, and the job finding rate.

### 5.2 Measuring the Job Finding Probability

Suppose there are \( U_t \) unemployed workers at the start of month \( t \), indexed by \( i \in \{1, \ldots, U_t\} \). Let \( F_t^i \) denote the probability that worker \( i \) finds a job by the start of month \( t + 1 \). I assume that the randomness in the outcome of the job finding process cancels out, so \( \sum_{i=1}^{U_t} F_t^i \) is the number of workers who find a job during month \( t \). Then unemployment at the start of month \( t + 1 \) is

\[ U_{t+1} = U_t - \sum_{i=1}^{U_t} F_t^i + U_{t+1}^s, \]  

(12)

where \( U_{t+1}^s \) denotes the number of short-term unemployed, workers who are unemployed at the start of month \( t + 1 \) but worked at some point during month \( t \). Unemployment next month is equal to the number of unemployed workers this month, \( U_t \), minus the number of workers who find a job during the month, \( \sum_{i=1}^{U_t} F_t^i \), plus the number of workers who are unemployed but held a job at some time during the previous month, \( U_{t+1}^s \). Rearrange equation (12) to get a measure of the mean job finding probability among workers who are unemployed at date \( t \):

\[ F_t \equiv \frac{\sum_{i=1}^{U_t} F_t^i}{U_t} = 1 - \frac{U_{t+1}^s - U_{t+1}}{U_t}. \]  

(13)
The BLS measures unemployment and short-term unemployment from the CPS. I use these to construct $F_t$ from January 1951 to May 2005.

I compare this measure of the job finding probability with the $v-u$ ratio. Figure 4 uses the measure of vacancies from JOLTS, available since December 2000. The relationship is clearly positive. Although the exact shape is difficult to discern in this short sample, the point estimate suggests a Cobb-Douglas function with an elasticity of about 0.37, shown as a solid line. Figure 5 uses data from the Help Wanted Index, available since 1951. The detrended data show a linear relationship between the two variables, corresponding to a Cobb-Douglas function with an elasticity of about 0.28. Table 1 summarizes the joint behavior of detrended unemployment, vacancies, and the job finding probability since 1951.

5.3 Comparative Statics

To compare the model with the data, fix $M = 244.2$ and $q = l = 0.027$; I discuss this choice of $q$ and $l$ shortly. Let $N$ vary between 233 and 243, with the entry rate of jobs solving $n = Nl$ throughout. The solid blue lines in Figure 6 and Figure 7 show the resulting

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Figure 5: U.S. data, 1951Q1–2004Q4. Vacancies are measured from the help wanted advertising index, unemployment and the job finding probability from the CPS. All variables are quarterly averages of monthly data, detrended using an HP filter with smoothing parameter $10^5$.

Table 1: U.S. data, 1951–2004. Quarterly average of monthly data, detrended using an HP filter with smoothing parameter $10^5$. The measure of vacancies is the help wanted index.
relationship between the instantaneous job finding rate $f$ and the v-u ratio as $N$ varies.

A proportional increase in $q, l$ and $n$ does not affect the number of workers or jobs per labor market but simply results in a proportional increase in $f$ (equation 11). Hence it does not affect the curvature of the relationship between $f$ and $V/U$. In addition, Figure 6 shows that the relationship is insensitive to the composition of the total separation rate, $s ≡ q + l$, between quits and layoffs.

A striking feature of the relationship between $f$ and $V/U$ is that it is indistinguishable from a Cobb-Douglas. The dashed green line in Figure 7 shows the isoelastic relationship between the v-u ratio and the instantaneous job finding rate. When higher productivity raises the v-u ratio by ten percent, it also increases the instantaneous UE transition rate by just over two percent. Indeed one can prove in a manner analogous to Proposition 4 that if $M = N → ∞$, the elasticity converges to $\frac{1}{2} - \frac{1}{\pi} ≈ 0.18$ regardless of parameter values. Although the theoretical relationship is not exactly Cobb-Douglas, if the model were the data generating process, it would be virtually impossible to reject the hypothesis of a Cobb-Douglas empirically.
5.4 Aggregate Shocks

Although the comparative statics are suggestive about the comovement between the job finding rate and the v-u ratio, they are not definitive.\textsuperscript{12} For example, when a positive shock hits, the number of jobs per labor market increases, resulting in a surge of workers finding jobs. Conversely, a negative shock sharply reduces hiring for a period of time until the stock of job falls to its new equilibrium value. If these dynamic responses are important, they may swamp the static relationship depicted in Figure 6 and Figure 7, reducing or even eliminating the informativeness of the v-u ratio about the job finding probability.\textsuperscript{13}

I address this by exploring the behavior of an economy subject to aggregate shocks. More precisely, I fix $M = 244.2$ and $q = l = 0.027$ throughout. There are two state variables, the actual number of jobs per labor market $N$ and the target number of jobs per labor market $N^*$. I assume $N^*$ follows an exogenous discrete state Markov process in continuous time described precisely in Appendix B and calibrated to match the volatility and autocorrelation of the v-u ratio. This process reflects an unmodeled shock to productivity $x$ or the cost of job creation $k$ which, together with the free entry condition, determine the target number of jobs.

\textsuperscript{12}In contrast, the relationship between unemployment and vacancies is entirely static, since each depends only on the current number of jobs per labor market, $N$.

\textsuperscript{13}Coles and Petrongolo (2003) emphasize this theoretical possibility in a stock-flow matching model.
Table 2: Simulation of shocks to the target number of jobs $N^*$. The parameterization is described in the text. Bootstrapped standard errors are in parenthesis.

A shock hits the target $N^*$ according to a Poisson process, i.e. at exponentially distributed time intervals, whereupon the target takes on a new value $N^*$. If at any point in time $t$ the actual number of jobs per labor market, $N(t)$, is smaller than the target number of jobs, $N^*(t)$, there is a surge of entry until $N(t_+) = N^*(t)$. If $N(t)$ exceeds $N^*(t)$, no new jobs enter since entering firms would lose money. During this adjustment period, the actual number of jobs per labor market declines due to layoffs, $\dot{N}(t) = -lN(t)$, until $N(t)$ reaches $N^*(t)$. Finally, if $N(t) = N^*(t)$, the entry rate of jobs exactly offsets layoffs, $n(t) = lN(t)$.

I measure $N(t)$ and hence $U(t)$ and $V(t)$ at each date $t \in \{1, 2, \ldots, 648\}$. Between measurement dates I compute the total rate at which unemployed workers find jobs. This is $qM \pi_{q}^{UE} + n(t)\pi_{n}^{UE(t)}$ per unit of time at any $t$ with $N(t) \geq N^*(t)$, since unemployed workers find jobs following either a quit or the entry of a new job. In addition, whenever $N(t) < N^*(t)$, there is a surge of entry and hence a discrete drop in unemployment. I include the positive measure of unemployed workers who find jobs at these instants. I divide the total measure of unemployed workers finding jobs during the interval $[t, t+1)$ by unemployment at $t$ to get a discrete time measure of the job finding rate, consistent with the empirical measure of $F_t$.

The 648 time periods correspond to 54 years of monthly data. I take quarterly averages of the monthly data, detrend using an HP filter with smoothing parameter $10^5$, and compute summary statistics. I repeat this process 10,000 times to obtain reliable estimates of the mean and bootstrapped standard errors. Table 2 shows the results.
The stochastic model reproduces the strong negative correlation between unemployment and vacancies depicted in Figure 2 and quantified in the first three columns of Table 1. For example, the empirical correlation between unemployment and vacancies is −0.90, while the theoretical correlation is even stronger, −0.97. The model slightly overstates the volatility of vacancies and underestimates the volatility of unemployment, but it is close. The theoretical autocorrelations of the two variables are about equal, consistent with the empirical evidence. This last observation is notable since the equal persistence of unemployment and vacancies is a puzzle for matching models where unemployment is a state variable and vacancies are a jump variable (Shimer, 2005a; Fujita and Ramey, 2005).

The correlation between the instantaneous job finding rate and the v-u ratio is strongly positive, consistent with Figure 4 and Figure 5. Since the volatility of the detrended job finding rate is only a little more than 20 percent of the volatility of the detrended v-u ratio, a regression of the detrended variables would uncover an elasticity almost exactly equal to 0.2, consistent with the comparative statics. If one adds the squared v-u ratio as an additional regressor, one would reject the null hypothesis that the coefficient on the squared term is zero at a 5 percent confidence level only about 1.3 percent of the time, less frequently than one would expect if the job finding rate were proportional to the v-u ratio plus white noise. It is virtually impossible to distinguish the relationship between the job finding rate and the v-u ratio from a Cobb-Douglas using time series data.

The standard explanation for the comovement of the job finding rate and the v-u ratio comes from the matching model. If the total number of matches is a constant returns to scale Cobb-Douglas function of unemployment and vacancies, then the rate at which an unemployed worker finds a job is \( f = (V/U)^\alpha \), consistent with the empirical evidence. But left unexplained is why the reduced-form matching function appears to be Cobb-Douglas. The mismatch model provides an answer: aggregation.

6 Duration Dependence

The job finding rate for any particular unemployed worker may differ substantially from \( f \) in equation (11), depending on the number of workers \( i \) and the number of jobs \( j \) in her labor market. This gives rise to duration dependence in the job finding rate: if an econometrician observes a worker who has been unemployed for a long time but cannot observe local labor market conditions, he should infer that the worker is probably stuck in a labor market in which jobs are scarce and workers plentiful. The worker’s job finding rate is correspondingly
Conversely, the rate at which a newly unemployed worker finds a job is higher than the average job finding rate $f$.

One implication is that, if at the start of the month the average unemployed worker finds a job at rate $f$, by the end of the month the same worker’s job finding rate is less than $f$ assuming she is still unemployed. This means that the full month job finding probability is less than $1 - e^{-f}$, the probability of finding (at least) one job during a month if jobs arrive at a Poisson rate $f$. This section measures the monthly job finding probability, the theoretical counterpart of $F_t$ in equation (13). I look at the extent to which it varies with unemployment duration in the cross-section and the extent to which it varies with the $v-u$ ratio in response to changes in productivity.

### 6.1 Cross Section

I cannot find analytical expressions for the full month job finding probability either unconditionally or conditional on unemployment duration. Instead I simulate 1 million unemployment spells to recover these numbers numerically. In half the spells, I start with a “job leaver,” a worker who quit her labor market and moved to one in which there were more workers than jobs, $i > j$. In the other half of the spells, I start with a “job loser,” a worker whose job left a labor market that previously had $i$ workers and $j \leq i$ jobs. In both cases, I simulate the evolution of the worker’s local labor market, stochastic changes in the number of workers and jobs coming from entry and exit, until the worker finds a job either because a new job enters, an employed worker leaves, or our unemployed worker quits for a labor market with available jobs. I assume that whenever a job is available, each unemployed worker is equally likely to be hired, independent of unemployment duration. For example, if at some point our unemployed worker is in a labor market with $i$ workers and $j < i$ jobs and a new job enters, I assume that she gets the job with probability $1/(i - j)$.

I use the usual values for the number of workers per labor market, $M = 244.2$, and the quit and layoffs rate, $q = l = 0.027$. For now I fix $N = 236.3$ giving an unemployment rate of 5.4 percent. With these values, the instantaneous job finding rate, $f$ in equation (11), is 61.0 percent. If the job finding rate were constant during a month, the full month job

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The results in this section are sensitive to this assumption. If the most recently unemployed worker is always the first to get a job, the model generates significantly more duration dependence in the exit rate from unemployment. Conversely, if the unemployed queue for a job (Sattinger, 2005), duration dependence is inverted, with the long-term unemployed more likely to find a job than the short-term unemployed. Another approach would be to model heterogeneous workers, eliminating the current model’s ambiguity on who is hired.
Figure 8: Monthly job finding probability as a function of unemployment duration. The number of workers per labor market is $M = 244.2$, the number of jobs per labor market is $N = 236.3$. The quit and layoff rates are $q = l = 0.027$ per month.

The job finding probability would be $1 - e^{-f} = 45.6$ percent.

Figure 8 shows the theoretical monthly probability of finding a job—the fraction of workers who find a job during the next month—as a function of the current duration of an unemployment spell and the reason for unemployment. The wiggles are due to sampling variation. The job finding probability for job losers is slightly higher than for job leavers, at least during the initial few weeks of unemployment. This reflects slight differences in initial labor market conditions for the two groups.

To compute the theoretical counterpart of the job finding probability $F_t$, I take a weighted average of the job finding probability in Figure 8, with weights corresponding to the fraction of spells that do not end before a particular duration. 39.8 percent of job leavers and 40.2 percent of job losers find a job in a given month. The empirical counterpart of these numbers is 39.7 percent since December 2000 (Figure 4); the success of the model at matching this number is due to a judicious choice of $q = l = 0.027$. Once again, changes in $q$ and $l$ leaving $q + l$ constant scarcely affect this result.

The job finding probability of both job losers and job leavers declines sharply during an unemployment spell. They are about 55 percent when a worker first becomes unemployed but fall to less than 35 percent after three months of unemployment. I express this decline as a single number by looking at a weighted average of the job finding probability, where
weights correspond to unemployment duration. Shimer (2005b) shows that we can measure this empirically using time series on unemployment $U_t$ and mean unemployment duration $\bar{d}_t$:

$$\frac{\sum_{i=1}^{U_t} d_i^t F_i^t}{\sum_{i=1}^{U_t} d_i^t} = 1 - \frac{(\bar{d}_{t+1} - 1)U_{t+1}}{\bar{d}_t U_t},$$

where $d_i^t$ is worker $i$’s unemployment duration and $F_i^t$ is her probability of finding a job. This number averaged 24.3 percent in the U.S. since December 2000, while in the model it is 33.0 percent for job leavers and 33.2 percent for job losers. Since the model matches $F_t = 0.40$, by this metric it explains about half of the decline in the job finding probability as a function of unemployment duration; presumably the other half is a consequence of unmodeled heterogeneity among workers within labor markets.

6.2 Comparative Statics

I now explore how time aggregation and duration dependence affect the theoretical relationship between the job finding rate and the v-u ratio. I let $N$ vary from 233 to 243 with $M$, $q$, and $l$ fixed. At each value of $N$ I compute the v-u ratio and I simulate the fraction of unemployed workers who find a job within a month. The solid blue circles in Figure 9 show the results. There is again an increasing relationship between the v-u ratio and the measured job finding probability. The solid blue line depicts a Cobb-Douglas function through these points, analogous to Figure 7. Again the fit is remarkable, although the elasticity is somewhat lower, 0.13.

The hollow green circles in Figure 9 show the relationship between $1 - e^{-f}$, a full month measure of the job finding probability ignoring duration dependence, and the v-u ratio. This is systematically about 15 percent higher than $F$ but the quality of the Cobb-Douglas fit (dashed green line) and the elasticity (0.15) are similar. I conclude that accounting for time aggregation lowers the level of the theoretical job finding probability but does not affect the main conclusion of the earlier work on the matching function: the model is consistent with a reduced-form Cobb-Douglas matching function. The empirical elasticity of $F$ with respect to $V/U$ is about half the theoretical elasticity of 0.28 depicted in Figure 5.
Figure 9: Theoretical job finding probability as a function of the v-u ratio. The number of workers per labor market is fixed at $M = 244.2$ and the quit and layoff rates at $q = l = 0.027$. The entry rate of jobs $n$ varies so $N$ takes values between 233 to 243.

7 Employer-to-Employer Transitions

This is as much a model of employer-to-employer (EE') as it is one of UE and EU transitions. A quit shock leads to an EE' transition if it hits an employed worker and the worker moves to a labor market with vacancies, $i < j$:

$$
\pi^{EE'}_q = (1-u) \sum_{i=0}^{\infty} \sum_{j=i+1}^{\infty} p(i,j).
$$

(14)

Summing this and equation (8) implies a quit shock leads to either a EU transition or an EE' transition with probability $1 - u$, which is simply the probability that the shock hits an employed worker. Similarly, a layoff shock induces a worker to switch employers if it hits a filled job in a labor market with vacancies, $i < j$:

$$
\pi^{EE'}_l = \frac{1}{N} \sum_{j=1}^{\infty} \sum_{i=0}^{j-1} ip(i,j).
$$

Figure 10: The brown dots show U.S. monthly data from December 2000 to May 2005. The unemployment rate is measured by the BLS from the CPS. The vacancy rate is measured by the BLS from the JOLTS. The solid blue line shows the theoretical analog with $M = 244.2$, $q = l = 0.027$, and $N \in [233, 243]$.

Add to this $\sum_{j=0}^{\infty} \sum_{i=0}^{j} p(i, j) - \frac{1}{N} \sum_{j=1}^{\infty} \sum_{i=0}^{j-1} j p(i, j) = 0$, analogous to equation (19) in Appendix A. This gives

$$\pi_{i}^{EE'} = \sum_{j=0}^{\infty} \sum_{i=0}^{j} p(i, j) - v. \quad (15)$$

Summing this and equation (10) gives $\pi_{i}^{EU} + \pi_{i}^{EE'} = 1 - v$, so the probability that an $l$ shock leads to the end of a match is equal to the probability that the shock hits a filled job.

To construct an empirical measure of $EE'$ transitions, I use a relatively new question from the CPS. Since the switch to dependent interviewing in 1994, the CPS has asked respondents who are employed in consecutive months, “Last month, it was reported that you worked for $x$. Do you still work for $x$ (at your main job)?” Because the BLS does not tally the answer to this question, I follow Fallick and Fleischman (2004) and construct it from the CPS microdata, weighting individual answers, dividing by employment in the previous month, and seasonally adjusting using the Census X-12 algorithm.

The brown dots in Figure 10 show that in an average month from December 2000 to May 2005, 2.6 percent of employed workers switched employers. This fraction was higher at the start of the sample period when the v-u ratio was higher, generating an increasing relationship between the share of $EE'$ transitions in total separations and the v-u ratio.
The data in Figure 10 is time aggregated since a worker may lose a job and become unemployed but find a new job before the next measurement date. In order to compare the model with the data, I also have to time-aggregate the model. Indeed, the model suggests that there may be a significant number of unmeasured short unemployment spells since a large fraction of workers find a job during the first few weeks of unemployment (Figure 8). To assess the importance of this, I simulate 1 million separation episodes, half for job leavers and half for job losers. In some events, a worker may move directly to another job, as described above. Otherwise, the worker has a fraction of a month, a uniform random variable on [0, 1], to find a new job before experiencing a measured unemployment spell. In each case I compute the probability the worker is employed at the end of the month. Multiplying this by the quit or layoff rate gives the theoretical counterpart to the brown dots in Figure 10.

Variation in the number of jobs per labor market traces out an increasing relationship between the EE′ transition rate and the v-u ratio. The line in Figure 10 shows that the model predicts slightly too many EE′ transitions, although the relationship between the EE′ transition rate and the v-u ratio is about right. The model is broadly in line with the data on this measure as well.

8 Conclusions

This paper develops a mismatch model of unemployment, vacancies, and labor market transitions. It provides a coherent framework for exploring important macro-labor facts, including the comovement of unemployment, vacancies, the job finding rate, and the employer-to-employer transition rate.

The matching model (Pissarides, 1985) is an obvious alternative for addressing these facts. While the matching model can deliver a Beveridge curve with the right slope, the mismatch model must deliver such a Beveridge curve. Moreover, the mismatch model delivers a downward sloping Beveridge curve even if the layoff and quit rates fluctuate cyclically and it explains why the persistence of vacancies and unemployment is similar. Both of these are problematic in the matching model. Finally, the v-u ratio in the mismatch model is more responsive to productivity shocks than it is in the matching model, helping to address the puzzle explored by Shimer (2005a).

A partial success of the mismatch model is the relationship between the job finding rate and the v-u ratio. The mismatch model predicts that a one percent increase in the v-u ratio should raise the job finding probability by about 0.14 percent, about half of the empirically
relevant value. To my knowledge, this is the first explanation for the observation that the relationship between the job finding rate and the v-u ratio is well-described by a stable Cobb-Douglas matching function. In contrast, whether the matching function is Cobb-Douglas is exogenous in the matching model.

Finally, an integral part of the mismatch model is a theory of duration dependence in the job finding probability and of employer-to-employer transitions. In principle both of these can be tacked on to the matching model (Pissarides, 2000), although there is not much work on the business cycle properties of such models. By contrast, the simplest version of the mismatch model explains half the duration dependence in the job finding probability even if workers are homogeneous and it explains the weak procyclicality of employer-to-employer transitions.

There are other predictions of the mismatch model that I have not explored here. For example, the mismatch model predicts procyclical real wages since more workers are in labor markets with excess jobs during booms. It likewise predicts a stable link between local unemployment rates and wages, Blanchflower and Oswald’s (1995) “wage curve.” The mismatch model also provides a coherent theory of jobs and hence a model of job flows distinct from worker flows. In principle this means that the model could simultaneously be used to address facts about labor market flows and facts about job creation and job destruction (Davis, Haltiwanger, and Schuh, 1996). Preliminary work suggests that a simple feature of the labor market, the fact that the vacancy rate is less than the unemployment rate, may explain why job flows are systematically smaller than workers flows: it is easier to find a worker than to find a job.

I have kept this model deliberately simple and mechanical in order to highlight the main forces in a model of mismatch. My main findings are a consequence of aggregation and so it seems likely that aggregating other models of mismatch, e.g. the stock-flow matching model or the queuing model, would yield similar results. Exploring this possibility is a fruitful avenue for research.
A Omitted Proofs

Proof of Proposition 1. Lemma 1 implies

\[ \frac{\partial}{\partial N} \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} p(i, j) = \sum_{i=1}^{\infty} \sum_{j=1}^{i-1} p(i, j - 1) - \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} p(i, j) \]

\[ = \sum_{i=1}^{\infty} \sum_{j'=0}^{i-2} p(i, j') - \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} p(i, j) = -\sum_{i=1}^{\infty} p(i, i - 1) \] (16)

The second equality reindexes using \( j' = j - 1 \) while the third cancels common terms from the double sum. In other words, the double sum in equation (2) is decreasing in \( N \) and so the result follows immediately.

Proof of Proposition 2. I start with the response of \( u \) to \( M \):

\[ \frac{\partial u}{\partial \log M} = M \frac{\partial (U/M)}{\partial M} = \frac{1}{M} \left( M \frac{\partial U}{\partial M} - U \right) \] (17)

Equation (3) implies

\[ \frac{\partial U}{\partial M} = \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} (i - j)p(i - 1, j) - \sum_{i=1}^{\infty} \sum_{j=0}^{i} (i - j)p(i, j) \]

\[ = \sum_{i'=0}^{\infty} \sum_{j=0}^{i'} (i' + 1 - j)p(i', j) - \sum_{i=1}^{\infty} \sum_{j=0}^{i} (i - j)p(i, j) = \sum_{i=0}^{\infty} \sum_{j=0}^{i} p(i, j) \] (18)

The first equality uses Lemma 1. The second equality replaces \( i' = i - 1 \). The last equality eliminates common terms from the sums.

Next, observe that

\[ \sum_{i=0}^{\infty} \sum_{j=0}^{i-1} ip(i, j) = M \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} p(i - 1, j) = M \sum_{i'=0}^{\infty} \sum_{j=0}^{i'} p(i', j) = M \frac{\partial U}{\partial M} \] (19)

The first equality uses the definition of \( p(i, j) \) in equation (1), the second equality reindexes using \( i' = i - 1 \), and the third uses equation (18). Substituting this into equation (17) and
replacing $U$ using equation (3) gives

$$\frac{\partial u}{\partial \log M} = \frac{1}{M} \sum_{i=0}^{\infty} \sum_{j=0}^{i-1} j p(i, j).$$

Next, a logic similar to equation (19) establishes

$$\sum_{i=0}^{\infty} \sum_{j=0}^{i-1} j p(i, j) = N \sum_{i=0}^{\infty} \sum_{j=1}^{i} p(i, j - 1) = N \sum_{i=0}^{\infty} \sum_{j=0}^{i-2} p(i, j')$$

(20)

Substitute this into the previous equation to get $\partial u/\partial \log M$ in equation (4).

The response of $u$ to $N$ is simpler to compute. An argument analogous to equation (18) establishes that

$$\frac{\partial U}{\partial N} = - \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} p(i, j),$$

(21)

and so using $\frac{\partial u}{\partial \log N} = N \frac{\partial U/M}{\partial N} = \frac{N}{M} \frac{\partial U}{\partial N}$ gives the desired result. The partial derivatives of $v$ are computed symmetrically.

**Proof of Proposition 3.** Consider the locus of pairs $(M, N)$ that deliver a particular unemployment rate $u_0$. Equation (4) implies that this locus satisfies

$$\frac{\partial \log N}{\partial \log M} \bigg|_{u=u_0} = \frac{\sum_{i=2}^{\infty} \sum_{j=0}^{i-2} p(i, j) \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} p(i, j)}{1} < 1$$

(22)

That is, if $(M_1, N_1)$ and $(M_2, N_2)$ with $M_1 < M_2$ both yield the same unemployment rate $u_0$, $M_2/M_1 > N_2/N_1$. Similarly, equation (5) implies

$$\frac{\partial \log N}{\partial \log M} \bigg|_{u=u_0} = \frac{\sum_{j=1}^{\infty} \sum_{i=0}^{j-1} p(i, j)}{\sum_{j=2}^{\infty} \sum_{i=0}^{j-2} p(i, j)} > 1,$$

so if $(M_1, N_1)$ and $(M_2, N_2)$ with $M_1 < M_2$ both yield the same vacancy rate $v_0$, $M_2/M_1 < N_2/N_1$. This proves that there is at most one pair $(M, N)$ associated with each pair $(u, v)$. The proof of existence is standard and hence I omit it.

**Proof of Proposition 4.** I start by examining how $N$ responds to $x$. Consider the
following equations:
\[
\frac{\partial \alpha}{\partial (x - z)} = -\frac{\alpha}{x - z} \quad \text{and} \quad \frac{\partial N}{\partial \alpha} = -\frac{1}{\sum_{i=1}^{\infty} p(i, i - 1)}
\]

The first comes from differentiating equation (2) and the second inverts equation (16). Combine these equations with \(\frac{\partial (x - z)}{\partial \log x} = x\) to get

\[
\frac{\partial N}{\partial \log x} = \frac{\alpha}{\sum_{i=1}^{\infty} p(i, i - 1)} \cdot \frac{x}{x - z} \quad (23)
\]

Next I examine how the v-u ratio responds to \(N\):

\[
\frac{\partial \log(V/U)}{\partial N} = \frac{1}{V} \cdot \frac{\partial V}{\partial N} - \frac{1}{U} \cdot \frac{\partial U}{\partial N}.
\]

Since the number of employed workers is equal to the number of filled jobs, \(M - U = N - V\), \(\partial V/\partial N = 1 + \partial U/\partial N\). Equation (21) implies \(\partial U/\partial N = -\alpha\), giving

\[
\frac{\partial \log(V/U)}{\partial N} = \frac{1 - \alpha}{V} + \frac{\alpha}{U}.
\]

Combining with equation (23) gives equation (6).

To proceed further, rewrite equation (3) as follows:

\[
U = \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} i p(i, j) - \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} j p(i, j) = M \sum_{i=0}^{\infty} \sum_{j=0}^{i} p(i, j) - N \sum_{i=2}^{\infty} \sum_{j=0}^{i-2} p(i, j).
\]

The second equation uses equation (19) and equation (20). If \(M = N\), unemployment and vacancies are equal and this simplifies slightly:

\[
U = V = M \sum_{i=0}^{\infty} (p(i, i) + p(i + 1, i)).
\]

Now take limits as \(M = N \to \infty\):

\[
\lim_{M=N \to \infty} \sqrt{M} \sum_{i=0}^{\infty} p(i, i) = \lim_{M=N \to \infty} \sqrt{M} \sum_{i=1}^{\infty} p(i, i - 1) = \frac{1}{2\sqrt{\pi}} \quad \text{and} \quad \lim_{M=N \to \infty} \alpha = \frac{1}{2}
\]

Substituting these limits in the previous equation and then in equation (6) indicates that
the first fraction converges to $\pi$, yielding the desired result. ■

\section*{B Stochastic Process}

I model a second order autoregressive process for the target number of jobs per labor market $N^*$. The state includes the current and previous value of $N^*$, each of which may take on $2\nu + 1$ possible values. When an aggregate shock hits, $N^*$ may increase or decrease by one grid point. If the previous value of $N^*$ is higher than the current value, it is more likely that $N^*$ will continue to fall. If $N^*$ is lower, however, it is more likely to rise, giving a mean reverting second order autoregressive process.

More precisely, the state is a pair $\{N^*, \mu\}$ where $\mu \in \{-1, 1\}$,

$$\log \left( \frac{N^*}{N_0} \right) \in \{ -\nu \Delta, -(\nu - 1)\Delta, \ldots, -\Delta, 0, \Delta, \ldots, (\nu - 1)\Delta, \nu \Delta \},$$

and $N_0$ is the geometric mean of $N^*$. A shock arrives at Poisson rate $\lambda$. The new value $\{N^{*'}, \mu'\}$ is determined as follows: If $\mu = 1,$

$$\{N^{*'}, \mu'\} = \begin{cases} \{N^* e^\Delta, 1\} & \text{with prob. } \left( \frac{1}{2} - \frac{\log N^*/N_0}{2\nu \Delta} \right)^\eta \\ \{N^* e^{-\Delta}, -1\} & \text{otherwise} \end{cases}$$

while if $\mu = -1,$

$$\{N^{*'}, \mu'\} = \begin{cases} \{N^* e^{-\Delta}, -1\} & \text{with prob. } \left( \frac{1}{2} + \frac{\log N^*/N_0}{2\nu \Delta} \right)^\eta \\ \{N^* e^\Delta, 1\} & \text{otherwise} \end{cases}$$

If $\eta = 1$, this reduces to a first order autoregressive process. In that case, the parameter $\gamma = \lambda/\nu$ determines the mean reversion of the process and $\sigma = \Delta \sqrt{\lambda}$ determines the instantaneous volatility. As $\nu$ increases for given values of $\gamma$ and $\sigma$, the discrete state space process converges to an Ornstein-Uhlenbeck process (Shimer, 2005a).

If $\eta < 1$ a positive shock is more likely to be followed by another positive shock, which increases the persistence of shocks relative to the benchmark. The parameter values I use are $N_0 = 236.3$, matching the historical average unemployment and vacancy rates; $\nu = 100$; $\eta = 0.4$; $\gamma = 0.005$; and $\sigma = 0.0023$. The last three parameters are important for matching the standard deviation and first and second autocorrelation of the v-u ratio, while the results are insensitive to the choice of $\nu$. 

35
References


