Aggregate Labor Supply:
A Statement about Preferences, Technology, and their Interaction

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Abstract

The difference in labor supply across countries is often decomposed into hours worked per year per employed person and the fraction of persons employed. The difference between hours worked per employed person in Europe and the United States is due in large part to more weeks of vacation, more holidays, and more sick days in Europe than in the United States. However, sizable differences in labor supply remain after adjusting for hours per year per employed person. These differences are due to Europeans starting the working life later and, more importantly, retiring earlier. We develop a framework that incorporates this feature. In equilibrium what gets determined is how much people work over their lifetime, not when they work. As a result, policies intended to induce people to work a larger fraction of their lifetime, such as retirement systems that weight the last years of employment more heavily in determining benefits, only impact on when people work and not how much. Thus, most policies are offsetting.

Key words: Aggregate labor supply, labor supply elasticity, indivisible labor, extensive margin, labor taxation, retirement

Introduction

Perhaps the most important parameter for purposes of evaluating tax policies is the aggregate intertemporal substitution elasticity of leisure parameter. This parameter is a function of parameters that specify not only individuals' preferences but also a household's production technology as specified by its consumption sets. Aggregate general applied equilibrium studies using the stand-in household abstraction inevitably find this aggregate elasticity to be near one.

One class of studies is business cycles studies, which use variants of the neoclassical growth model to evaluate the contribution of various shocks to business cycle fluctuations. The finding of these studies is that only if this intertemporal elasticity parameter is near one do shocks, whatever their type, give rise to fluctuations of the business cycle variety. There are many such studies; see Cooley (1995) for a review of these studies completed prior to 1995.

More recently there have been a number of studies of large movements in output and market hours relative to the secular trend initiated by the study of Cole and Ohanian (1999).[1] Unlike business cycle studies, these studies of depressions and prosperities are concerned with the path of the economy and not with certain statistical properties of the aggregate time series. Another class of studies examines cross-country labor supply and large, relatively permanent changes in labor supply within some countries. All these studies require this intertemporal elasticity of leisure to be near one.

The intertemporal elasticity of substitution between consumption and leisure for the stand-in household implies a large elasticity of aggregate labor supply. A one percent change in the real wage, holding wealth constant, gives rise to a three percent change in

[1] See the Kehoe and Prescott volume (forthcoming) with 15 such studies.

hours worked for Australia, Chile, Japan and the United States and even higher for Western Europe where the fraction of the time endowment allocated to the market is smaller. Until recently, the macro finding was in apparent conflict with those of microeconomists who used observations on individual behavior and the prices this individual faces to estimate an individual's Frischian labor supply elasticity. This apparent conflict disappears once aggregation theory is used to derive the implication of the individual's preferences and technology for the preferences of the stand-in or aggregate household.

The fact that the sum of strictly convex technology sets asymptotically is a convex cone; that is, it displays constant returns to scale and has long been known, going back at least as far as Alfred Marshall and Knut Wicksell. But only recently has aggregation theory on the household side been developed.

In a seminal study, Rogerson (1984, 1988) discovered that essentially the same aggregation result holds for the stand-in household if people are constrained to work a standard workweek or not at all in a given week. In such worlds, the margin of adjustment is the fraction employed and not hours worked per employed person in a given period. Once finite lifetimes are introduced, what gets chosen is the fraction of the lifetime worked. This assumption that most people either work a standard workday or not at all matches well with observations. Most people work an eight-hour workday when they work.

The restriction to a standard workday was arbitrary and the question naturally arose as to why the principal margin of labor supply adjustment is in the fraction employed and not in the number of hours per person employed. Hornstein and Prescott

(1993) figure out why the principal margin of adjustment is the number working per day. Following Rosen (1978), there is a nonlinearity in the mapping from hours supplied to labor income. The nonconvexity arises because, for many, labor compensation increases overproportionally to the length of the workday, at least in the relevant range.

If the nature of the technology is such that an appropriate commodity space has market hours as the *labor* input, then aggregate labor supply is a statement about preferences only. But there is not a single labor input. Compensation typically is not proportional to the hours an individual works. The line between preferences and technology is not clean. When using general equilibrium theory in modeling, the way the data are reported along with the empirical facts dictates the way things are handled. It is desirable to have prices for the things exchanged, as prices are what we use to value things.

People's preferences are ordered by the expected value of time additively separable utility functions. We deal with a finite number of types with a measure of each type.

Rogerson uses the Prescott-Townsend (1984a, 1984b) lottery equilibrium, as did Hornstein and Prescott (1993). Equivalently, an Arrow-Debreu equilibrium could have been used to support that allocation (see Prescott and Shell (2002) and Kehoe, Levine, and Prescott (2002)), but then symmetry is lost and the notation becomes extremely complex. Here we show that it holds without the need to index to some sunspot or to introduce lotteries, provided there is borrowing and lending. If there is aggregate uncertainty, Arrow securities are needed as well. To summarize, the result that the aggregate supply elasticity for the stand-in household has large Frischian labor supply

elasticity is a robust one and does not require lottery allocations. The beauty of lottery competitive equilibria is that they are so easy to deal with, yet have the same equilibrium allocations as sequence-of-markets, recursive, and Arrow-Debreu completive equilibria.

The great labor economist Jacob Mincer pointed out that there was a good theory of how much secondary wage earners work over their lifetime, but not when they work. A significant fraction of these secondary wage earners are deciding what fraction of their potential working life to work in the market.

Becker and Ghez (1975) conclude that the intertemporal substitution of leisure is large based upon micro theoretic reasoning and individual time allocation studies. Lucas and Rapping (1969) come to the same conclusion based upon the time series behavior of real wages and aggregate hours. Their definition of the real wage is aggregate compensation divided by aggregate hours. In a recent study in which human capital investment being part of compensation in the tradition of Ben-Porath is taken into consideration, Imai and Keane (2004) estimate that the labor supply elasticity for males is 3.7. Ignoring this factor reduces the estimate to only about 0.3 (see McCurdy (1981) and Altonji (1986)). To summarize, the micro and macro evidence is that the aggregate elasticity of substitution between market time and consumption goods is large.

Section 1: Aggregation Theory and Aggregate Stand-in Household's Preferences

In static situations with labor indivisibilities, Prescott-Townsend lotteries or sunspot contracts are needed to achieve efficiency. In dynamic situations, this is not the case. Here we show that borrowing and lending contracts or Arrow securities, if there is aggregate uncertainty, suffice to achieve efficiency. Individuals have finite lives. What is

determined is the fraction of time the individual works, not when a person works. With the addition of some feature of reality that leads people to work the first part of their lifetime, the theory determines when individuals retire, that is, switch for the rest of their life from working some fixed number of hours per unit time to not working.

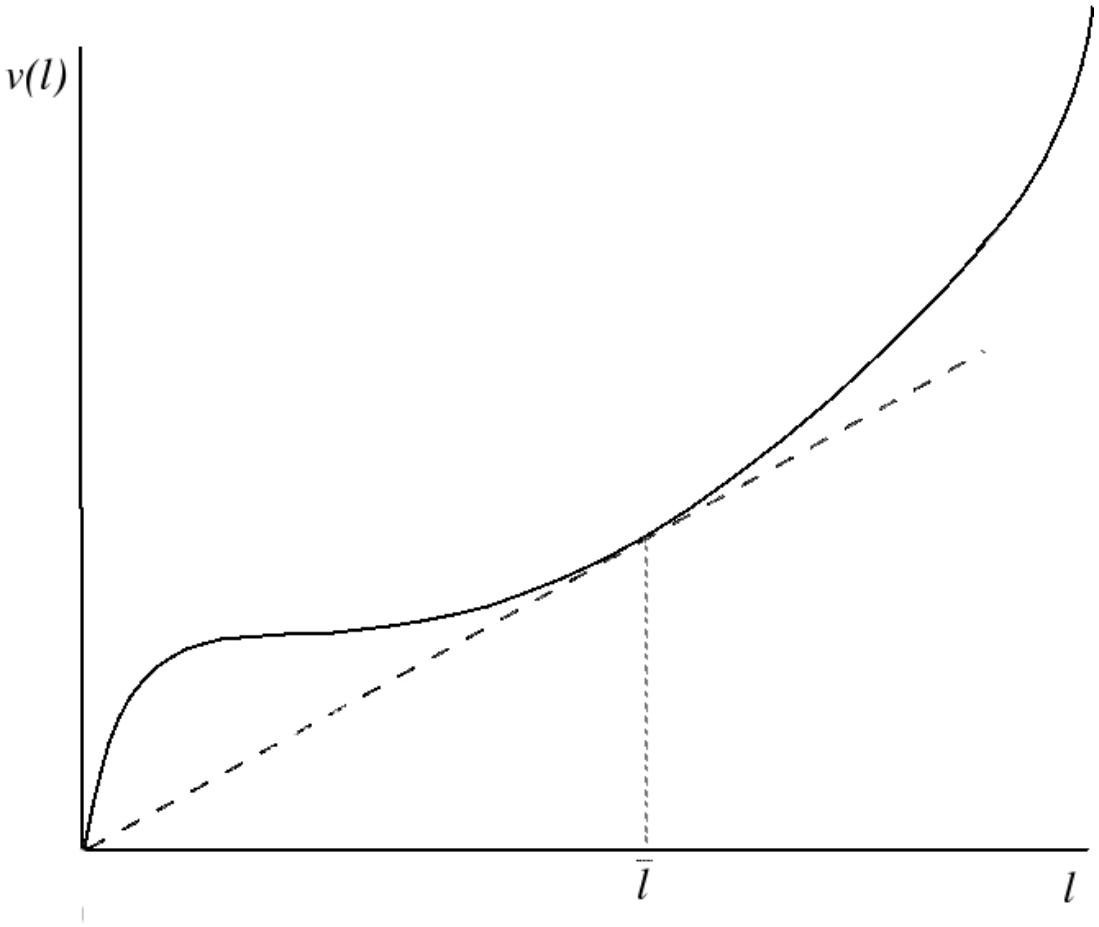
The nonconvex preferences arise because the disutility of labor supply flow, $v(h)$, is not convex. Labor supply and hours allocated to the market are not proportional. Thus, if the productive time endowment is normalized to 1, $1-h$ is not leisure, the reason being that the mapping from hours allocated to the market to labor services is not linear. The empirical evidence that this mapping is nonlinear in many cases is overwhelming. Someone who chooses to work 40 hours per week typically will earn more than twice as much as a similar person who chooses to work 20 hours per week. Often there are fixed costs associated with getting set up for a job. Needed supervisory time often falls when there only one full-time worker doing a job rather than two half-time workers.

Equivalently, following Hornstein and Prescott (1993), we could have treated workweeks of different lengths as different commodities with a technology that resulted in the demand price of a workweek of length *h* being increasing and initially convex. In particular, Hornstein and Prescott assume that a single worker produces hk^{θ}, where *k* is the number of units of capital that the individual uses. The demand reservation price is proportional to age and will be an increasing function of the hours worked per week (for workweek lengths not supplied in equilibrium, the supply reservation price will exceed the demand reservation price). This argument generalizes immediately to groups of workers working together to operate a production unit.

Time is denoted by $t \in [0,1]$, where the length of the lifetime has been normalized to 1. A person's preferences are ordered by

$$(1) \qquad \int_0^1 (\log(c) - v(l))\, dt ,$$

where $c(t)$ is his consumption path and $l(t)$ is his path of *labor* services supplied to the market. We do not assume that hours of labor supplied is proportional to hours *h* allocated to the market. In particular, the mapping from hours allocated to the market to labor supplied typically has a convex region and becomes concave only for large workweek lengths. The derivative of the function v is positive and strictly quasi convex. The function *v* satisfies $v(0) = 0$, is increasing and differentiable, is initially convex and then concave. Figure 1 depicts the shape of the function *v*.



The budget constraint is

(2) $$\int_0^1 (c - w\,l)\,dt \leq 0\,.$$

In the budget constraint the interest rate is 0 and the wage rate is constant over time. We deal with this simple case first in order to make as clear as possible what is going on.

The generalization is straightforward and is outlined. Solutions to this program are functions such as $c(t) = \bar{c}$ and $\int_0^1 l(t)\,dt = \bar{e}\,\bar{l}$ for some $\bar{e}$ and $\bar{c}$. The empirically relevant case is when $\bar{e} < 1$.

To find $\bar{e}$ and $\bar{c}$, two equations in these two unknowns can be solved. The first is the binding budget constraint. The second is obtained by equating the marginal rate of substitution between consumption and the fraction of time employed in some short interval of length dt; that is

(3) $$v(\bar{l})\,c = w\,g(\bar{l}).$$

If the solution to these two equations has an $\bar{e} > 1$, then $l(t) = \hat{l}$, where $\hat{l} \in \arg\max\{\log l - v(l)\}$ and $c(t) = w\hat{h}$ is the optimum.

If the interest rate is *r*, the discount rate is ρ, and the growth rate of consumption is γ, then consumption grows at rate γ rather than being constant. However, the labor supply correspondence does not change.

Proposition: The theory determines what fraction of the lifetime an individual works, not when.

With capital accumulation

With capital accumulation, provided that the equilibrium *e* is less than one at all times, an equilibrium condition is

(4) $$v(\bar{l}) = \lambda\,w(t),$$

where λ is the Lagrange multiplier on the budget constraint. An implication of this is that the wage rate will be constant, though there will be fluctuations in the employment rate $e(t)$. All those who work at instant t work $\bar{l}$.

Section 2: Decomposition of Hours Worked

The difference in labor supply across countries is often decomposed into hours per year per employed person and the fraction of persons employed. Table 1 reports such a decomposition for the United States and selected European countries. We see differences across countries in both hours per employed person per year and the fraction of employed people. The difference between French hours per employed person and U.S. hours per employed person is due in large part to more weeks of vacation, more holidays, and more sick days in France than in the United States. These differences are on the extensive margin, not on the intensive margin. An implication of this is that the marginal income tax rate, and not the average tax rate, is the relevant one insofar as people can vary the fraction of the accounting period they work.

Table 1

Decomposition of Hours per Person into
Hours per Employed Person and Employment Rate 2004*

Country	Hours per person	Hours per employed person	Employment rate
France	904	1441	.63
Germany	951	1443	.66
Italy	904	1585	.57
Spain	1102	1791	.61
United Kingdom	1195	1669	.72
United States	1305	1824	.72

Source: OECD Labor Database
*Data from 2003 used wherever observations for 2004 were missing.

Sizable differences in labor supply remain after adjusting for hours per year per employed person. These are due to the French starting the working life later and, more importantly, retiring earlier. The theory we develop does not say when people will work, just how much they will work over their lifetime.

Section 3: A Steady-State Life Cycle Model with Human Capital Accumulation

Ljungqvist and Sargent (2006) suggest that a framework with human capital accumulation with people borrowing and lending will not display the high elasticity of labor supply that the life cycle model we dealt with in Section 2 did. It is true that in the lottery equilibrium with human capital accumulation, some fraction of an age cohort make a large human capital investment and work the rest of their lives, while the more

fortunate have a life of leisure. With no lotteries, and just borrowing and lending, all make a smaller human capital investment and then work some fraction of their lives.

The Model

Density one of finite lived individuals are born at every instance. An individual born at date *s* has preferences over his consumption path *c*(*t*) and labor supply path *h*(*t*) ordered by

$$\int_s^{T+s} e^{-\rho t}(\log(c) - e(t)v(h))\,dt,$$

where $c:\Re \to \Re_+$ and $h:\Re \to \{0,\bar{h}\}$. We are implicitly restricting the parameters to the empirically interesting case that only the *e* margin adjusts, and as a consequence there is no need to distinguish between hours and labor services. We emphasize there is a Rogerson (1984, 1988) and Hansen (1985) labor indivisibility, the theoretical underpinnings of which can be found in Hornstein and Prescott (1993) and in Section 2 of this paper. Further, $v(0) = 0$ and $v(\bar{h}) > 0$.

It is convenient to introduce the employment function $e(t)$, which is 1 at instances in which an individual works $\bar{h}$ and 0 when an individual does not work. Individuals born at instance s spend the time interval $[s, s+T_1]$ in training. The time interval $[s+T_1, s+T_2]$ is the working life. The time interval $[s+T_2, s+T]$ is spent in retirement. T_1 and T_2 are policy parameters. Individuals decide what fraction of their working life to spend working. In equilibrium, what will be determined is $\int_{s+T_1}^{s+T_2} e(t)dt$.

An individual allocates his income toward consumption and saving. An individual receives labor income from the time devoted to work. Additionally, an individual

receives a retirement benefit conditional on being retired and dependent of the wage at the time of retirement. There is also a benefit that is common to all individuals. An individual born at instance *s* faces an intertemporal budget constraint

$$\int_{s}^{s+T} e^{-it}[c(t)+a(t+1)-(1+i)a(t)]dt \le \int_{s+T_1}^{s+T_2} e^{-it}(1-\tau_h)w(t)\bar{h}e(t)dt + \int_{s+T_2}^{s+T} e^{-it}b(w(s+T_2))dt + \int_{s}^{s+T} e^{-it}(Be^{0.02t})dt,$$

where $a(t)$ is assets at instance *t*, $w(t)$ is the wage at instance *t*, τ_h is the tax on labor income, $b(w(s+T_2))$ is the retirement benefit, and B is the benefit common to all individuals.

Note that $a(s)$ is the assets of a person born at date zero, at age *s*, and $a(s)1.02^{-s}$ is the assets of someone of age *s* at $t=0$. We sum $t=0$ assets over all ages to obtain $A(0)$, that is, aggregate assets at $t=0$. For an individual born at time zero, the asset position at date *t* is characterized by

$$a(t) = \frac{b-c}{0.02-i}\left(e^{0.02t}-e^{it}\right) \text{ for } t < T_1$$

$$a(t) = \frac{b-c+(1-\tau_h)w\bar{h}e}{0.02-i}\left(e^{0.02t}-e^{it}\right) \text{ for } T_1 \le t \le T_2$$

$$a(t) = \frac{b-c+b_R w(T_2)}{0.02-i}\left(e^{0.02t}-e^{it}\right) \text{ for } t > T_2,$$

which we get as solutions to three differential equations.

There is one firm in the economy that uses capital and labor to produce the consumption good with a Cobb-Douglas technology given by

$$Y(t) = K(t)^{\theta}(1.02^{t}N(t))^{1-\theta}.$$

Capital letters denote aggregate variables. We assume constant two percent growth in output. At instance *t* the aggregate labor supply is given by $N(t) = \bar{h}(T_2 - T_1) \int_{s+T_1}^{s+T_2} e(t)dt$.

We calibrate this model to U.S. data and then determine the consequences of changes in the marginal effective tax rate for labor supply. We find that the numbers are remarkably similar to those found in Prescott (2004), using a stand-in household construct. We calibrate to an interest rate of four percent, a capital to output ratio of three, a capital share parameter of 0.3, and an investment share of 0.18, or conversely a consumption share of 0.82. We assume constant growth of two percent per year in output. These facts are consistent with a return on capital of ten percent, annual depreciation of four percent, a capital income tax rate of 1/3, and a discount factor of two percent.

Section 4: Conclusion

The Frischian labor supply elasticity for individuals is not the same as that of the household standing in for these individuals. Care must be taken in drawing aggregate implications of substitution elasticities based upon micro observations whenever the extensive margin is the principal margin of adjustment.

An implication of our study is that various policies designed to induce individuals to work a greater fraction of their lifetime will fail. They will affect the timing of when people work, but not the total fraction of their lifetime that they work to a first approximation.

Our paper does not consider the implications of introducing home work into the framework.[2] This would be an interesting topic for future research. Recent studies

[2] For home production in the neoclassical growth model, see Benhabib, Rogerson and Wright (1991).

suggest that in order to correctly predict market labor supply in Scandinavia, it is important to consider how tax revenues are used (see Rogerson (2006)). More specifically, the public provision of substitutes for home produced goods appears to be a feature of Scandinavian welfare states (see Ragan (2005)) that increases labor supply by reducing allocations to home production.

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