Job and Wage Mobility in a Search Model with Non-Compliance (Exemptions) with the Minimum Wage

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Abstract

How well does a simple search on-the-job model fit the eighteen years of job and wage mobility of high school graduates? To answer this question we are confronted from the data with a prevalent non-compliance and exemptions from the minimum wage. We incorporate this observation in a job search model with three main ingredients: (i) search on-the-job; (ii) minimum wages, with potentially imperfect compliance or exemptions; and, (iii) exogenous wage growth on-the-job. We use panel data drawn from the NLSY79, US youth panel starting in 1979, to estimate the parameters of our simple job search model and, in particular, the extent of non-compliance/exemptions to the minimum wage. The model is solved numerically and we use simulated moments to estimate the parameters. The estimated parameters are consistent with the model and they provide a good fit for the observed levels and trends of the main job and wage mobility data. Furthermore, the estimated model indicates that the non-compliance and exemption rate with the federal minimum wage is of about 25 percent. Increase of the compliance/exemption rate or the minimum wage has a small effect on mean accepted wages but a significant effect on the non-employment rate.

Keywords: minimum wages, compliance, exemptions, job search, wage growth.
JEL: J42, J63, J64

1 Introduction

How well the standard Burdett (1978) and Lucas and Prescott (1974) search on-the-job models fit the first 18 years job and wage history data of high school graduates?[1] During the first eight years workers increase their employment rate and decrease their job-to-job and job-to-nonemployment mobility, which reach stability for the additional ten years. This higher mobility includes the mobility of: non-employment to employment, job-to-job, employment to non-employment and wage growth on the job and from job-to-job. Mean and variance of wages increases continuously and more at the early period. However, during the first six months after graduation from high school about 25 percent of white males earn an hourly wage below the US federal legal minimum wage. This percentage is reduced to 15 percent after a year following graduation, about 9 percent after three years and from the tenth year about three percent of the high school graduates earn an hourly wage below the minimum wage (see Figure 15).[2]

Despite the findings of Ashenfelter and Smith (1979) about the importance of compliance in the measurement of the impact of the law on wages and employment, recent studies on the minimum wage ignored the issue of compliance and exemptions with the law.[3] Ashenfelter and Smith (1979) computed compliance as the fraction of net additional workers earning exactly the minimum wage after the enactment of the law, out of workers that earn less than the minimum wage in the absence of the minimum wage law. Obviously, this fraction is not directly observed and it requires a model to be estimated. Ashenfelter and Smith (1979) concluded that "for the country as a whole the point estimate of the compliance rate is 69%, although a conventional confidence interval would include the range in the 63-75%". More recent work by Cortes (2004) studies whether immigrants are more likely to be paid less than the minimum wage than natives and, overall, she finds no systematic pattern of noncompliance between immigrants and natives. Finally, Weil et al. (2004) use data on apparel contractors in the Los Angeles area, and find that 54% of employers in 2000 did not comply with minimum wage laws, and that 27% of employees were paid below the minimum wage.

This paper uses the standard search on-the-job models to estimate the extent of non-

[1]The model we use is based on Mortensen (1986) survey. To answer this question one has to account for the main observed monthly facts on job and wage data for high school graduates who are observed monthly for about 18 years using the National Longitudinal Survey of the Youth of 1979 (NLSY79). These facts are summarized here and presented in detail in section 3.

[2]In Figure 15 we also use the extensive data on wages by age for high school graduates from the Census of Population Surveys (CPS, 1979-97). The CPS data is fully consistent with the facts from the NLSY79.

[3]We do not separate here between compliance with the law and exemptions included by the law. We view all actual wages paid below the federal minimum wage as either legal wage due to exemption or non-compliance by the worker and the employer. It should be noted that there are many exemptions based on particular occupation (e.g., farm workers in small farms, etc.) that are well documented by the U.S. department of Labor, Employment Standards Administration Wage and Hour Division at www.dol.gov/esa/.

compliance and exemptions with the US federal minimum wage.[4] Observed wages below the minimum wage can result from both non-compliance (exemptions) and/or measurement error (Ashenfelter and Smith, 1979). But while measurement error should apply throughout the wage distribution, non-compliance (exemptions), by definition, applies to actual accepted wages below the minimum wage. This distinction will be the basis of our identification strategy of the extent of compliance with the US federal minimum wage.

We construct a continuous time search model in a stationary labor market environment with the following ingredients: (i) search on the job; (ii) minimum wages with imperfect compliance by firms; (iii) endogenous search effort; (iv) exogenous wage growth on-the-job.[5] The model is solved numerically and we use simulated moments to estimate the parameters. The estimated parameters are consistent with the model and they provide a good fit for the level, the trend and the fluctuations of several moments that are used for estimation. We find that the arrival rate of job offers below the minimum wage is 75% lower than the arrival rate of job offers above the minimum wage, although there are reasons to believe that this is an over-estimate of the true non-compliance of firms with the minium wage federal law.

The literature on the minimum wage policy is large and we certainly do not attempt to cover it all. However, it is well recognized that the analysis of this policy requires an equilibrium model where firms and workers respond to the change in policy. This basic claim provides the reason that minimum wage policy was analyzed by Eckstein and Wolpin (1990), van den Berg and Ridder (1998), van den Berg (2003) and Flinn (2002, 2005), among others, in estimable search equilibrium models.[6] Here we use a simple search model where the equilibrium is interpreted as in Lucas and Prescott (1974). The main reason for our choice of the model is that by using data such as the NLSY79 it is not clear that one can empirically distinguish between the different search models (see e.g., Eckstein and van den Berg, 2005). The simple search model is a benchmark specification where the observed wage dispersion and labor market mobility of workers are due to productivity difference across firms, worker heterogeneity and worker search decisions. In this simple model wage dispersion and worker mobility are not a result of the response of firms and the endogenous wage formation within an equilibrium labor market with frictions (e.g.,

[4]Meyer and Wise (1983a,b) were first to estimate the employment and wage impacts of the minimum wage. They used a statistical model and individual data from CPS. They identified the minimum wage effects under the assumption that the observed wage distribution (under the minimum wage) is a distortion from the potential wage distribution. The model is static it imposes the restriction that employment decreases in response to an increase in the minimum wage.

[5]Bowlus and Neuman (2004) use a search equilibrium model to empirically analyze the wage growth using NLSY data. Wolpin (1992) uses the first eight years in quarterly format to fit a finite horizon discrete time search model with similar components.

[6]Eckstein and Wolpin (1990) use a posting wage equilibrium with only search from unemployment. van den Berg and Ridder (1998) and van den Berg (2003) use an extended version of Burdett and Mortensen (1998) posting wage equilibrium with search from unemployment and on the job. Flinn (2002, 2005) uses a search-matching-bargaining model with search from unemployment only.

Burdett and Mortensen, 1998). Here, the wage dispersion is due to the assumed productivity differences between potentially homogenous workers, but where additional dispersion is due to the worker dynamic search decision rules.

The model is estimated using the simulated method of moments. We use the monthly work history moments from NLSY79 for white males that graduated from the high school and did not attend college. We follow their employment status and wages for eighteen years after graduation and the model is set with two unobserved types of individuals to fit the data. The model is estimated using two separate set of moments, namely the 17 sequences of monthly moments of labor market states (non-employment and employment), transitions and wages, and moments computed on employment cycles, delimited by subsequent nonemployment spells. The model fits reasonably well both set of moments in terms of capturing the levels and trends in non-employment, transitions from non-employment to employment and from job-to-job. Furthermore, the model with the *same* wage offer distribution and an estimated constant annual wage growth of 2.4% on the same job (tenure), fits very well the eighteen years of mean hourly wage growth that grew from 8 to 16 dollars as well as the trend and level of the wage variance.

The parameter estimates have plausible magnitudes and are in line with previous estimates of the parameters of a search model with search on-the-job. That is, the arrival rates of job offers are higher for non-employed than for employed workers, and these rates differ between the two types of individuals, such that the mean hazard rate of leaving non-employment is decreasing. The estimated parameters from the two very different set of moments provide very similar set of estimated parameters. The model predicts that one type of workers is less employable, with relatively low arrival rates and mean wage offers, and a reservation wage of about three dollars, which is much below the federal minimum wage. The other type has much higher arrival rates of job offers and mean wage offers, and a reservation hourly wage of about eight dollars. This second type of invidual is thus not affected from the minimum wage regulation. The main result is that the estimated non-compliance and exemption rate is of about 25 percent. That is, the rate of job offer arrival at wages below the minimum wage is one fourth of the rate for wages above the minimum wage. This result is robust with respect to the moments that we use and it is not much affected from changing the value of the measurement error that we assume for the reported wages.

Counterfactual analysis of changes in the minimum wage and the non-compliance/exemption rate imply that both have a large change on non-employment and on the proportion of individuals working below the minimum wage. For example, a 20 percent increase in minimum wage increases non-employment by about 2 percent. These effects are in line with the reported time series estimates for the US reported in Kennan(1995, page 1954). However, we find that the impact of the minimum wage and compliance/exemption changes in the mean and variance of wages is very small.

The rest of the paper is organized as follows. Section 2 presents our job search model.

Section 3 describes our data set. Section 4 gives the estimation method, section 5 presents results and section 6 concludes.

2 The Search Model

We construct a continuous time search model in a stationary labor market environment with the following ingredients: (i) search on the job; (ii) minimum wages with imperfect compliance by firms; (iii) endogenous search effort; (iv) exogenous wage growth on-the-job.

Agents are infinitely lived, and at each moment in time they can be either non-employed (a state denoted by n) or employed (a state denoted by e). When they are non-employed, they enjoy some real return b (typically including the value of leisure and unemployment insurance benefits), and receive job offers at a Poisson rate λ_n. Generating job offers at rate λ_n requires some search effort, with related search costs $c_n(\lambda_n)$, with $c_n'(\lambda_n) > 0$ and $c_n''(\lambda_n) > 0$. When employed, they enjoy a real wage w, which is growing at an exogenous rate g, receive job offers at a Poisson rate λ_e, and bear search costs $c_e(\lambda_e)$, with $c_e'(\lambda_e) > 0$ and $c_n''(\lambda_e) > 0$. Existing jobs are hit by idiosyncratic shocks, which occur at a Poisson rate δ. The instantaneous discount rate is r. New wage offers for employed and unemployed are randomly drawn from some known, fixed distribution $F(w)$. Once an individual accepts a wage w, his wage on the same job grows with tenure, τ, such that $w_\tau = we^{g\tau}$.

Our modelling of the wage offer distribution closely resembles Lucas and Prescott (1974) islands' model as it is presented in Mortensen (1986). In the Lucas and Prescott formulation, the distribution of wage offers represents productivity differentials across different islands. As productivity in each island is subject to idiosyncratic shocks, workers need to spend some effort in order to locate better matching opportunities and eventually relocate across islands in pursuit of wage gains. In our model, the wage offer distribution represents productivity differentials across firms. Each firm's productivity is given, but better matching opportunities arise to workers through search on-the-job.

There is an exogenously set federal minimum wage in the economy, denoted by w_M. However, there may be less than perfect compliance of firms to the minimum wage due to existing exemptions included by the law or potential non-complinace such that some jobs are offered with wage below the federal minimum wage w_M.[7][8] Assuming that firms make

[7] We avoid the complexity of an explicit equilibrium model of non-compliance behavior of firms as this is not the focus of our work and in order to keep the wage offer distribution continuous and differentiable. A simple way to model non-compliance is to assume that firms choose whether to comply with the minimum wage where they face a probability of punishment for non-compliance (Ashenfelter and Smith, 1979 and Lott and Roberts 1995). In this type of model firms make profits. A model with these assumptions would generate a spike at the minimum wage and a non-continuity for wages close to the minimum wage. These elements complicate the search model.

[8] Exemptions are specific for workers under the age of 20, for students for their first 90 days of work and for tipped employees. These exemptions are spelled in a way that allow for a wide interpretation for

zero profit and cannot make negative profits, the firms with labor productivity below w_M would leave the market if they are forced to comply with the minimum wage. Let $(1-\alpha)$ be the proportion of firms that are forced to pay the minimum wage, and therefore, leave the market. Hence, workers face some positive probability to receive an offer which pays below the minimum wage. When $\alpha = 0$ there is full compliance of firms and workers with the economy minimum wage. When $\alpha = 1$ there is no effective minimum wage regulation in the economy. Hence, in this model the minimum wage policy has two parameters, the level of the minimum wage, w_M, and the level of non-compliance/emptions, α.

As said above, there exists a continuous wage offer distribution $F(w)$ before the introduction of a minimum wage, w_M. There are $F(w_M)$ firms that offer a wage below the minimum wage. After the minimum wage is imposed, a proportion $(1-\alpha)F(w_M)$ leave the economy. Assuming that initially the size of the market was normalized to one, the introduction of the minimum wage reduces the number of firm in the market to $1-(1-\alpha)F(w_M)$.To keep the distribution of the wage offer probability we have to adjust for the drop of offers below the minimum wage due to the reduction in number of firms. This adjustment to the effective wage density is given by $\gamma(\alpha) = \frac{1}{1-(1-\alpha)F(w_M)}$.

We suppose that each individual starts search for a job at the month the individual leaves school being unemployed. To solve for the optimal search, we compute the lifetime utilities for the employed and the non-employed. The value of employment τ periods on the same job is denoted by $V_e(w_\tau)$ and the value of non-employment clearly does not depend on specific job attributes and is denoted by V_n.

A worker who is currently non-employed enjoys a net flow of income $b - c_n(\lambda_n)$, receives job offers above or below the minimum wage at rates $\gamma(\alpha)\lambda_n$ and $\gamma(\alpha)\alpha\lambda_n$, respectively, which are accepted if the value attached to them exceeds the value of non-employment:

$$\begin{aligned} rV_n &= b - c_n(\lambda_n) + \gamma(\alpha)\lambda_n\{E_{w\geq w_M} \max\left[0, V_e(w) - V_n\right]\} \\ &\quad +\gamma(\alpha)\alpha\lambda_n \left\{E_{w<w_M} \max\left[0, V_e(w) - V_n\right]\right\}. \end{aligned} \tag{1}$$

A worker currently employed in a job with starting wage w and tenure τ receives net income $w_\tau - c_e(\lambda_e)$, enjoys wage growth at rate g, is forcibly separated from his employer at rate δ, and receives job offers above or below the minimum wage at rates $\gamma(\alpha)\lambda_e$ and $\gamma(\alpha)\alpha\lambda_e$, respectively, which are accepted if the value attached to them exceeds the lifetime utility in the current job:

$$\begin{aligned} rV_e(w_\tau) &= w_\tau - c_e(\lambda_e) + \delta\left[V_n - V_e(w_\tau)\right] \\ &\quad +\gamma(\alpha)\lambda_e E_{w\geq w_M} \max\left[0, V_e(w) - V_e(w_\tau)\right] + \gamma(\alpha)\alpha\lambda_e E_{w<w_M} \max\left[0, V_e(w) - V_e(w_\tau)\right] \\ &\quad +g w_\tau V_e'(w_\tau) \text{ for } w_\tau < w_M, \end{aligned}$$

non-compliance with the law. See the U.S. Department of Labor, Employment Standards Administration Wage and Hour Division at www.dol.gov/esa/.

and

$$
\begin{aligned}
rV_e(w_\tau) &= w_\tau - c_e(\lambda_e) + \delta\left[V_n - V_e(w_\tau)\right] \\
&\quad + \gamma(\alpha)\lambda_e E_w \max\left[0, V_e(w) - V_e(w_\tau)\right] + g w_\tau V_e'(w_\tau) \text{ for } w_\tau \geq w_M,
\end{aligned}
$$

where the last term in each case represents the change in value on the job, that is, $\frac{\partial V_e(w_\tau)}{\partial w_\tau}$. In either labor market state, agents set an acceptance rule for job offers and the optimal level of search effort. As job switching involves no cost, the optimal acceptance rule for the employed consists in accepting any job that pays more than their current wage, w. For the non-employed, the optimal acceptance rule consists in accepting all job offers which pay at least some reservation wage w^*, such that $V_n = V_e(w^*)$. Note that such reservation wage exists and is unique because, while the value of search is constant, the value of employment is monotonically increasing in w. If $w_M \leq w^*$, then minimum wages have no impact on agents' decisions or equilibrium outcomes. Therefore, we assume that the minimum wage is binding , i.e. $w_M > w^*$, such that the value functions in this model can be rewritten as:[9]

$$
rV_n = b - c_n(\lambda_n) + \gamma(\alpha)\lambda_n \int_{w_M} \left[V_e(w) - V_n\right] dF(w) + \gamma(\alpha)\alpha\lambda_n \int_{w^*}^{w_M} \left[V_e(w) - V_n\right] dF(w), \quad (2)
$$

and

$$
\begin{aligned}
rV_e(w_\tau) &= w_\tau - c_e(\lambda_e) + \delta\left[V_n - V_e(w_\tau)\right] \\
&\quad + \gamma(\alpha)\lambda_e \int_{w_\tau} \left[V_e(w) - V_e(w_\tau)\right] dF(w) + g w_\tau V_e'(w_\tau) \text{ for } w_\tau \geq w_M, \qquad (3) \\
rV_e(w_\tau) &= w_\tau - c_e(\lambda_e) + \delta\left[V_n - V_e(w_\tau)\right] + \gamma(\alpha)\lambda_e \int_{w_M} \left[V_e(w) - V_e(w_\tau)\right] dF(w) \\
&\quad + \gamma(\alpha)\alpha\lambda_e \int_{w_\tau}^{w_M} \left[V_e(w) - V_e(w_\tau)\right] dF(w) + g w_\tau V_e'(w_\tau) \text{ for } w_\tau < w_M, \qquad (4)
\end{aligned}
$$

Note that in (3) the probability of getting an offer below the minimum wage ($\gamma(\alpha)\alpha\lambda_e$) does not affect the value of employment (other than indirectly through V_n), as any such offer would be rejected by someone employed at $w \geq w_M$.

A non-employed worker will choose λ_n in order to maximize (2). The first-order condition for this optimization problem is given by

$$
c_n'(\lambda_n) = \gamma(\alpha) \int_{w_M} \left[V_e(w) - V_n\right] dF(w) + \gamma(\alpha)\alpha \int_{w^*}^{w_M} \left[V_e(w) - V_n\right] dF(w), \quad (5)
$$

[9] Note that we assume that the wage is drawn from the same distribution for the support below and above the minimum wage. This assumption simplifies the model by preventing discontinuity of the value function at the minimum wage rate.

thus equating the marginal cost of an extra job offer to its marginal benefit.

Similarly, the first order condition for the choice of search intensity for the employed is given by

$$c_e'(\lambda_e) = \gamma(\alpha)\int_w [V_e(w') - V_e(w)]\, dF(w'), \qquad \text{if } w \geq w_M, \tag{6}$$

$$c_e'(\lambda_e) = \gamma(\alpha)\int_{w_M} [V_e(w') - V_e(w)]\, dF(w') + \gamma(\alpha)\alpha\int_w^{w_M} [V_e(w') - V_e(w)]\, dF(w'), \tag{7}$$

$$\text{if } w < w_M. \tag{8}$$

By convexity of the search cost function, the unemployed will have a higher incentive to search for jobs, and, all else being equal, raise their arrival rate of job offers above that of the employed. Among the employed, search effort decreases with the current wage: in particular, those employed below the minimum wage will search more intensively than those employed above.

Given the acceptance rule $rV_n = rV_e(w^*)$, we can solve for the value of the reservation wage by setting equation (2) equal to equation (4) evaluated at $w = w^*$ and $\tau = 0$ (exploiting the continuity of $V_e(w_\tau)$ at w_M, a property that only holds if the wage offer distribution is the same above or below the minimum wage):

$$\begin{aligned} w^* &= b - c_n(\lambda_n) + c_e(\lambda_e) + \gamma(\alpha)(\lambda_n - \lambda_e)\int_{w^*} [V_e(w) - V_n]\, dF(w) \\ &\quad -\gamma(\alpha)(1-\alpha)(\lambda_n - \lambda_e)\int_{w^*}^{w_M} [V_e(w) - V_n]\, dF(w) - gw^* V_e'(w^*) \end{aligned} \tag{9}$$

$$\begin{aligned} &= b - c_n(\lambda_n) + c_e(\lambda_e) + \gamma(\alpha)(\lambda_n - \lambda_e)\int_{w^*} [1 - F(w)] V_e'(w)\, dw \\ &\quad -\gamma(\alpha)(1-\alpha)(\lambda_n - \lambda_e)\int_{w^*}^{w_M} [F(w_M) - F(w)] V_e'(w)\, dw - gw^* V_e'(w^*). \end{aligned} \tag{10}$$

To solve equation (10) one needs to know the function $V_e'()$. We prove in Appendix A that

$$V_e'(w_\tau) = e^{R(w_0,\tau)}\left[V_e'(w_0) - \int_0^\tau e^{-R(w_0,t)} dt\right] \tag{11}$$

and

$$V_e'(w_0) = \int_0^\infty e^{-R(w_0,\tau)} d\tau, \tag{12}$$

where

$$R(w_0,\tau) = (r + \delta - g)\tau + \gamma(\alpha)\lambda_e\int_0^\tau [1 - F(w_\tau)]\, d\tau, \text{ for } w_\tau \geq w_M, \tag{13}$$

$$R(w_0,\tau) = (r + \delta - g)\tau + \gamma(\alpha)\lambda_e\int_0^\tau [1 - \alpha F(w_\tau) - (1-\alpha)F(w_M)]\, d\tau, \text{ for } w_\tau < w_M. \tag{14}$$

The reservation wage can be numerically calculated substituting (11)-(14) into (10). If the job offer arrival rates are set exogenously, then the model is fully solved by calculating the reservation wage w^* using the solution to (10). Otherwise, the joint solution of w^*, λ_n and λ_e is found by solving jointly (10), (5), (6) and (**??**), and this solution enables us to simulate the dynamic decision sequence of the worker. This solution provides a joint dynamic distribution of labor market mobility from non-employment to work, from job-to-job and back to non-employment.

As standard in search models we get that the reservation wage is increasing in search efforts (i.e. in the arrival rate of job offers).[10] Furthermore, we show that the increase in the minimum wage implies that the reservation wage and the arrival rate decrease. On the other hand, the effect of compliance, $1-\alpha$, on reservation wage and arrival rate, is ambiguous. The reason is that changing compliance parameter α shifts the wage offer distribution. Intuitively, if the proportion of firms that offer a wage above the minimum wage is very large, then changing α has little effect on the wage distribution and offer rates. In this case, lower α (higher compliance) implies lower reservation wage almost the same as the increase in minimum wage. On the other hand, low proportion of firms paying above the minimum wage, a lower α increases the probability to get an offer above the minimum wage. Hence, it might be the case that both the offer arrival rates and the reservation wage are higher. When search effort is left exogenous (as in our empirical analysis below), the same results hold. Hence, the net impact of the minimum wage policy on non-employment is an empirical issue.[11]

Furthermore, this solution enables us the calculate by simulations the probability of all labor market states conditional on observed wages and, in particular, on whether the observed wage is below or above the minimum wage. Finally, the model provides a characterization of the probability to observe worker that are employed at a wage below the minimum wage and above the minimum wage. When $\alpha = 0$ the probability to observe a wage below the minimum wage is zero, hence, the only way to justify this observation if there is full compliance with the law, is to assume that wages are reported with error. This is the main alternative hypothesis to the claim that there is no full compliance ($0 < \alpha \leq 1$) with the minimum wage law.

3 Data

We use data drawn from the National Longitudinal Survey of Youths, which contains information on a sample of 12,686 respondents who were between 14 and 21 years of age in

[10]The proof of the comparative statics is in Appendix B.

[11]Card and Krueger (1995) emphasis this as an important feature of an empirical model for the analysis of the minimum wage. This is the case in most equilibrium search models (see Eckstein and van den Berg (2005) and Flinn (2005)). In our empirical work here we assume that efforts are exogenous. Hence, the policy impact is easier to analyze.

January 1979 (NLSY79). We attempt to obtain a sample from a fairly homogenous population, which is relatively likely to participate in the labor force and receive wage offers below the minimum wage. Hence, we restrict our sample to white males who are high school graduates, and never returned to school.[12] Specifically, we select non-black, non-hispanic, males who have completed at most 12 years of schooling and declare to hold high school degree. We exclude from our sample those who (i) ever went to the army; (ii) ever declared to be in college; (iii) ever declared to have a college or professional degree. We further restrict our sample to those who completed high school between age 17 and 19. These restrictions leave us with a sample of 577 individuals with almost 12(months)x18(years) of observations per-individual.

Information on selected respondents is available since calendar time January 1978. We construct individual monthly work histories using answers to retrospective questions. We assume that market entry coincides with the month an individual completed high school. Individuals in our sample completed high school between 1974 and 1984. More than 95% of them graduated in either May or June. We follow the individuals for 18 years after high school graduation and the data is organized to be consistent with the model's definitions and assumptions.

Labor Market States From the NLSY79 work history file, we obtain the monthly employment and non-employment status from January 1978 to December 1998. We define an individual as employed in a month if he works at least 10 hours per week and at least three weeks per month, or during the last two weeks in the month. Otherwise, an individual is classified as non-employed, and we do not further distinguish between unemployed and out of the labor force. Figure 1 shows the monthly proportion of employed and non-employed by time since high school graduation. The data shows clear pattern of seasonality.[13]

Among those who were found employed upon finishing high school, some started working before graduation. On average 55% of individuals in our sample worked during the year preceding graduation. This may happen because job search starts while in school or, more likely, because high school students may take up temporary and part time jobs while in school. The latter explanation seems also supported by the clear seasonal pattern of employment rates during the last year before graduation. We assume that individuals employed before graduation enter the "official" labor market upon graduation, but we will treat the proportion of individuals employed at labor market entry as an initial condition in our simulations.

The employment history information is employer-based. All references to a "job" should be understood as references to an employer. Multiple jobs held contemporaneously are treated as new jobs altogether: the associated wage is the average of the two hourly wages

[12] Flinn (2005) uses CPS sample of individuals at ages 16 to 24.

[13] We focus here on the growth of employment on the extensive margin. It should be noted that the average number of hours per-employed worker also shows a positive trend during the first eight years (Table 1). This intensive margin is not part of this paper but could be added to the search framework discussed here.

Table 1: Employment statistics by labor market experience

Year after graduation	Ave. months worked*	Ave. annual hours**	Ave. cumulative no. of jobs	Cumulative jobs
1	8.74 (3.96)	2031 (458)	1.52 (0.74)	1.52 (0.74)
2	9.54 (3.42)	2082 (440)	1.40 (0.64)	1.99 (1.14)
3	9.79 (3.51)	2130 (489)	1.36 (0.68)	2.41 (1.52)
4	10.10 (3.23)	2153 (443)	1.33 (0.65)	2.88 (1.84)
5	10.32 (3.07)	2200 (506)	1.31 (0.64)	3.28 (2.17)
6	10.48 (2.86)	2212 (469)	1.33 (0.65)	3.72 (2.53)
7	10.55 (2.90)	2195 (490)	1.29 (0.62)	4.07 (2.79)
8	10.86 (2.58)	2206 (492)	1.24 (0.55)	4.39 (3.03)
9	10.96 (2.49)	2226 (510)	1.29 (0.68)	4.72 (3.34)
10	10.86 (2.66)	2256 (569)	1.29 (0.64)	5.06 (3.61)
11	11.06 (2.34)	2292 (555)	1.21 (0.50)	5.29 (3.81)
12	11.16 (2.27)	2309 (548)	1.23 (0.56)	5.52 (4.00)
13	10.98 (2.44)	2337 (552)	1.21 (0.49)	5.74 (4.17)
14	10.93 (2.70)	2288 (549)	1.21 (0.56)	5.95 (4.32)
15	11.15 (2.32)	2352 (677)	1.21 (0.48)	6.15 (4.53)
16	11.10 (2.48)	2342 (616)	1.19 (0.47)	6.31 (4.68)
17	10.96 (2.84)	2360 (621)	1.19 (0.49)	6.45 (4.77)
18	11.00(2.79)	2356 (615)	1.14 (0.44)	6.55 (4.87)

Standard errors are in parentheses.

* The value is conditional on observations where all states are available in all months.

** Average hours are conditional on working in all months.

and the associated hours are the sum of the hours worked on the different jobs. Duration of a given job is considered as completed when a new job is recorded or the work is terminated and the individual is back to non-employment.

Table 1 gives employment statistics by years of labor market experience after graduation. Both the average number of months worked and average annual hours increase with experience. As expected, the average yearly cumulative number of jobs decreases with experience from 1.52 to 1.14 jobs, while cumulative jobs per worker reach about 6.5 after 18 years.[14]

Table 2 reports the duration of non-employment spells leading to the first 10 jobs in

[14] In the Bureau of Labor Statistics report on "Number of jobs held, labor market activity, and earnings growth among younger baby boomers: results from more than two decades of a longitudinal survey" (BLS 2002, Table 1), the average number of jobs held by white high school graduates is 9.2, which is higher than our figure. Such discrepancy stems from the different definitions of jobs. BLS define a job as an uninterrupted period of work with a particular employer, excluding recalls from temporary layoffs. In our definition we do not exclude recalls.

Table 2: Duration (months) of non-employment spells and job spells since high school graduation

Job No.	No. of obs.	Sample Mean duration (s.d.)	Kaplan-Meier restricted Mean duration (s.d.)	Kaplan-Meier extended Mean duration
NE*	148	8.86 (23.90)	9.93 (2.37)	10.74
1	574	32.74 (53.94)	38.69 (2.76)	44.85
2	508	33.77 (49.05)	41.03 (2.85)	47.66
3	457	27.82 (38.35)	39.37 (3.10)	46.98
4	387	24.71 (33.28)	34.06 (2.77)	37.73
5	337	22.22 (30.91)	31.23 (2.73)	35.05
6	276	20.38 (28.84)	32.33 (3.83)	37.95
7	236	23.23 (31.30)	34.59 (3.40)	40.31
8	182	17.92 (25.78)	24.82 (3.07)	29.51
9	150	17.58 (19.22)	21.51 (2.12)	22.12
10	131	16.96 (22.31)	24.19 (3.40)	28.10

* Mean duration of non-employment conditional on non-employment in $\tau = 1$.

individual careers, which seems to fall roughly monotonically with the job rank. 103 individuals had more than 10 jobs, 12 of whom had more than 20 jobs. The maximum number of jobs held is 27. As we have several censored spells in our sample, the sample mean duration is downward biased. In columns 4 and 5 we therefore also present the Kaplan-Meier nonparametric durations estimates.[15] The non-employment duration is on average about nine months and the corrections for the bias add one month. If the observation with the largest associated duration is censored, the Kaplan-Meier survivor function does not go to zero as duration goes to infinity. Consequently, the area under the curve still underestimates the mean duration. We thus extrapolate the survivor function using an exponential density function to compute the area under the entire curve, and from this one we obtain the Kaplan-Meier extended mean duration. This is reported in column 5 of table 2, and the mean duration for each job is estimated to be about one to six months longer using the extended (column 5) rather than the restricted (column 4) Kaplan-Meier estimates.[16] The duration increases from the first to the second job, but from the third job duration falls (after correction for biases in mean durations). Obviously, selection and sample attrition are important factors for these observations.

[15]Let n_t be the population alive at time t and d_t the number of failures. The nonparametric maximum likelihood estimate of the survivor function is: $\widehat{S}(t) = \Pi_{j|t_j \leqslant t}(\frac{n_j - d_j}{n_j})$. The Kaplan-Meier restricted mean duration is computed as the area under the Kaplan-Meier survivor function. And the associated standard error is given by the Greenwood formula: $\widehat{Var}\{\widehat{S}(t)\} = \widehat{S}^2(t) \sum_{j|t_j \leqslant t} \frac{d_j}{n_j(n_j - d_j)}$.

[16]Using job identifiers, individuals recalled by old employers after a nonemployment spell are considered as staying in the same job.

Figure 2 plots the job separation hazard by job tenure. We do not distinguish among job ranks, due to the insufficient number of observations for each rank. Duration is truncated at 10 years (and consequently 83 out of 2539 job spells are dropped). The monthly job hazard rate decreases significantly with duration, consistent with the above search model with wage growth on-the-job and/or endogenous search effort.

Figures 3 and 4 show labor market transition rates by experience. All the transitions have trend during the first 10 years and then they stay constant for the additional eight years. Furthermore, there are large monthly fluctuations with some evidence of seasonality. The probability of staying on the same job increases from 65 percent to 90 percent (Figure 3). The probability of staying non-employed decreases from 20 percent to about eight percent, and that of moving from employment to non-employment decreases from about five percent to about two percent. The probability of moving from non-employment to employment and job-to-job transitions (Figure 4) fall from about five percent to less than two percent and have relatively large fluctuations. One important goal of the search model above is to fit the trends and levels of these transition rates, although it is not meant to fit the monthly seasonal fluctuations.

Wages and Employment Cycles We next define employment cycles, in order to set the data in a way that is consistent with our search model (see Wolpin, 1992). Each cycle starts with non-employment and terminates with the last job before a subsequent non-employment spell. Since 55% of individuals in the sample started working before graduation, their first cycle started with their first job instead of non-employment. For an individual i, the sequence of cycles is denoted by

$$\{c_i^1(ne_i^1, J1_i^1, J2_i^1, \cdots), c_i^2(ne_i^2, J1_i^2, J2_i^2, \cdots), \cdots\},$$

where c_i^j denotes the cycle j for individual i, ne_i^j denotes non-employment spells, and $J1_i^j, J2_i^j, ...$ denote job spells within each cycle. We also record wages in each job spell.

The NLSY collects data on respondents' usual earnings (inclusive of tips, overtime, and bonuses, before deductions) during every survey year for each employer for whom the respondent worked since the last interview date. The amount of earnings, reported in dollars and cents, is combined with information on the applicable unit of time, e.g., per day, per hour, per week, per year, etc. Combining earnings and time unit data, the variable "hourly rate of pay job #1-5" in the work history file provides the hourly wage rate for each job. We use coded real hourly wage in 2000 dollars. Nominal wage data are deflated by monthly CPI from BLS CPI-U. We top code and bottom code the hourly wage at 150$ and 1.0$ before 1990, and at 200$ and 1.5$ afterwards. Note that, given the way in which the NLSY constructs wage information, we do not exactly have monthly wages. In particular, an individual's hourly wage is constant within a year unless he moves job. Clearly, when we convert nominal wage in real terms, real wages may decrease with inflation, but this may or may not be the correct actual pay for each month.

Table 3: Mean wages in the first three cycles of labor market careers

	Mean wage	Mean wage above w_M	Mean wage below w_M
First Cycle			
Job 1	8.22 (306)	9.16 (239)	4.89 (67)
Job 2	10.37 (192)	10.75 (179)	5.17 (13)
Job 3	11.85 (132)	11.96 (130)	4.99 (2)
Job 4	12.43 (77)	13.24 (70)	4.26 (7)
Job 5	12.64 (42)	12.83 (41)	4.87 (1)
Second Cycle			
Job 1	10.22 (311)	11.15 (264)	4.99 (47)
Job 2	11.02 (178)	11.50 (165)	4.96 (13)
Job 3	11.49 (84)	12.21 (77)	3.47 (7)
Job 4	13.13 (56)	13.27 (55)	5.46 (1)
Job 5	15.27 (29)	15.27 (29)	-
Third Cycle			
Job 1	9.95 (242)	10.61 (217)	4.29 (25)
Job 2	11.29 (125)	11.86 (115)	4.72 (10)
Job 3	11.81 (71)	12.22 (67)	5.02 (4)
Job 4	10.98 (39)	11.16 (38)	4.38 (1)
Job 5	14.20 (24)	14.20 (24)	-

Number of observations is in parentheses.

We focus on the data of accepted real hourly wage. Table 3 reports the mean accepted wage on the first five jobs in the first three cycles. As expected, mean accepted wages increase with job moves within cycles. When a new cycle starts, the mean accepted wage on the first job is lower than the accepted real wage on late jobs of previous cycles. Furthermore, the real accepted wage increases across jobs during the first two cycles but this does not hold clearly during the third cycle.

Minimum wage The federal minimum wage for covered nonexempt employees is currently at $5.15 an hour.[17] From Figure 5 we see that between 1978 and 2002, the nominal federal minimum wage increased from $2.65 to $5.15. However, the real minimum wage, deflated by monthly CPI-U and expressed in 2000 dollars, has been decreasing during this sample period. Several states also have state-level minimum wage laws. Where an employee is subject to both the state and federal minimum wage laws, he is entitled to the higher of the two. Seven states have no minimum wage law, namely Alabama, Arizona, Florida, Louisiana,

[17]The federal minimum wage provisions are contained in the Fair Labor Standards Act (FLSA).

Table 4: Number of months working below the minimum wage

No. of months	No. of obs.	%
0	309	53.55
1-6	125	21.67
7-12	62	10.74
13-24	43	7.45
25-36	21	3.64
36+	17	2.95
Total	577	100.00

Mississippi, South Carolina and Tennessee. Four states have minimum wage rates lower than the Federal level, namely Kansas, New Mexico, Ohio and Virgin Islands. All other states have minimum wage rates that are equal or higher than the Federal level. For the moment, we only take into account the time path of the federal minimum wage in our estimates.

Various minimum wage exceptions apply under specific circumstances to workers with disabilities, full-time students, youths under 20 in their first 90 consecutive calendar days of employment, tipped employees and student learners. A minimum wage of $4.25 per hour applies to young workers under the age of 20 during their first 90 consecutive calendar days of employment with an employer. After 90 days or when the employee reaches age 20, he or she must receive a minimum wage of $5.15. Full-time students can be paid not less than 85% of the minimum wage before they graduate or leave school for good. Student learners aged 16 or more can be paid not less than 75% of the minium wage for as long as they are enrolled in the vocational education program. Again in our baseline analysis, we do not take exemptions into consideration.

Tables 4 and 5 show statistics on pay below the minimum wage. 47 percent of the individuals are observed to work for a wage below the minimum wage for at least one month. For these workers that had a job below minimum wage, the average number of months worked below the minimum wage is 13.5, and the average number of jobs held below the minimum wage is 1.5. The mean job duration below the minimum wage is 8.9 months. These facts indicate that if wages are reported without error the violation of the minimum wage law is substantial among young high school graduate workers.

Table 6 presents the mean durations of non-employment and the first five jobs in the first three cycles, conditional on wages above or below the minimum wage. The mean duration from non-employment to the first job is lower for jobs paying at least the minimum wage. Also, mean duration on jobs paying at least the minimum wage is always longer than mean duration on jobs paying less than the minimum wage.

Table 7 gives the number of individuals making transitions from non-employment to

Table 5: Number of jobs paying below the minimum wage

No. of jobs	No. of obs.	%
0	309	53.55
1	172	29.81
2	66	11.44
3	22	3.81
4	6	1.04
6	2	0.35
Total	577	100.00

Table 6: Mean duration of nonemployment and jobs in months

	First Cycle	Second Cycle	Third Cycle
NE	7.3 (116)	5.8 (311)	5.8 (242)
To job 1 above w_M	7.2 (87)	5.3 (264)	5.0 (217)
To job 1 below w_M	7.7 (29)	8.7 (47)	12.6 (25)
Job 1	29.6 (306)	27.5 (311)	23.9 (242)
Above w_M	31.0 (239)	29.4 (264)	25.0 (217)
Below w_M	24.5 (67)	17.3 (47)	14.4 (25)
Job 2	40.3 (192)	27.5 (178)	27.8 (125)
Above w_M	42.3 (179)	27 (165)	29.8 (115)
Below w_M	12.9 (13)	34.4 (13)	5.2 (10)
Job 3	39.4 (132)	25.9 (84)	21.1 (71)
Above w_M	39.8 (130)	26.9 (77)	22.1 (67)
Below w_M	13.5 (2)	14.9 (7)	3.5 (4)
Job 4	31.2 (77)	21.5 (56)	19.6 (39)
Above w_M	31.0 (70)	21.7 (55)	18.9 (38)
Below w_M	32.7 (7)	12 (1)	45 (1)
Job 5	25.2 (42)	18.5 (29)	20.6 (24)
Above w_M	25.7 (41)	18.5 (29)	20.6 (24)
Below w_M	5 (1)	-	-

Number of observations is in parentheses. All statistics are conditional on wage being observed and this is why there is little discrepancy between moments in table 6 and 9.

jobs, and between jobs, again conditional on wages above or below the minimum wage. Most transitions to jobs paying less than the minimum wage originate in non-employment, and most workers earn wages above the minimum wage once they switch job. Very few workers move from a job paying more than the minimum wage to one paying less than the minimum wage, and in the model we assume that this could be only a result of measurement error in wages.

The data on wages and transitions below and above the minimum wage indicates that the evidence is consistent with the view that observations of individuals wages, below the minimum wage, are due to actual accepted jobs that violate the minimum wage law. We will test this hypothesis where the alternative will be that all observed wages below the minimum wage are due to measurement error.

On-the-Job Wage Growth The data show evidence of on-the-job wage growth, and following the model we assume a constant wage growth rate g in all jobs, which can be interpreted as the return to both general and job-specific experience. In order to estimate g, we consider wage observations in a given job :

$$\ln w_{i\tau} = \ln w_{i0} + \tau \ln(1+g)$$

where w_{i0} is the first wage observation for individual i and τ is job tenure. The OLS estimate of g is 0.2%. The corresponding annual growth rate is $(1+g)^{12}-1 = 2.43\%$ (see Figure 6a). This figure also shows the huge variance in wage growth, including negative wage growth. In fact, the inspection of the graph suggest that the hypothesis of no exogenous wage growth is not rejected. If we only use the first and the last wage observations on each job to compute the growth rate and take the average across jobs, the resulting annual growth rate is -0.75% (see Figure 6b). The two estimates are very different, as jobs with low or negative wage growth tend to last relatively shorter, and are thus assigned a lower weight when using the first method. In particular, jobs with tenure shorter than two years have negative annual growth on average (-1.62% for jobs with tenures shorter than one year, -0.19% for those with tenures between one and two years). Jobs with tenure longer than two years have an average annual growth rate 1.74%.[18]

4 Estimation

Specification. We estimate the model using simulated moments. We restrict the model to have exogenous arrival rates without search efforts and we allow for unobserved heterogeneity in arrival rates and the parameters of the wage offer distribution by assuming that there are two types of individuals in the population, with π denoting the proportion of type one. The

[18]It should be noted that the focus of this paper is not on wage growth, which requires a separate analysis. We just introduce this to the model and data analysis to let the model fit better the data.

Table 7: Transitions to employment and from job-to-job (no. of obs.)

	First Cycle	Second Cycle	Third Cycle
Unemployed	(116)	(311)	(242)
UE to J1 above w_M	87	264	217
UE to J1 below w_M	29	47	25
First Job above w_M	(239)	(264)	(217)
Move to J2 above w_M	93	123	88
Move to J2 below w_M	4	6	6
First Job below w_M	(67)	(47)	(25)
Move to J2 above w_M	22	14	11
Move to J2 below w_M	4	2	2
Second Job above w_M	(179)	(165)	(115)
Move to J3 above w_M	86	67	50
Move to J3 below w_M	2	5	2
Second Job below w_M	(13)	(13)	(10)
Move to J3 above w_M	4	5	5
Move to J3 below w_M	0	2	1
Third Job above w_M	(130)	(77)	(67)
Move to J4 above w_M	59	48	33
Move to J4 below w_M	6	0	1
Third Job below w_M	(2)	(7)	(4)
Move to J4 above w_M	2	2	1
Move to J4 below w_M	0	0	0
Fourth Job above w_M	(70)	(55)	(38)
Move to J5 above w_M	31	28	18
Move to J5 below w_M	1	0	0
Fourth Job below w_M	(7)	(1)	(1)
Move to J5 above w_M	6	0	1
Move to J5 below w_M	0	0	0

wage density function is assumed to be log normal, $\ln w \sim N(\mu, \sigma_w^2)$. The time preference parameter r is known to be 4% annually, which is 0.3% monthly. We allow for measurement error in observed wages, such that $w^o = w + u$, where w^o is the observed wage, w is the true wage and the error term is log-normally distributed: $u \sim N(0, \sigma_u^2)$.

As 55% of individuals in our sample worked before graduation, we assume that a separate labor market exists during pre-graduation periods, and we characterize this labor market by an initial (period 0) reservation wage w_0^*.[19] We estimate the reservation wage directly using equation (9). [20] The parameters of the model to be estimated are in the vector $\theta = [\lambda_{n1}, \lambda_{e1}, \lambda_{n2}, \lambda_{e2}, w_1^*, w_2^*, \mu_1, \mu_2, \sigma_{w1}, \sigma_{w2}, \delta_1, \delta_2, w_{01}^*, w_{02}^*, \pi, \sigma_u^2, \alpha]'$.

Data: As we have described above, we have a sample of white male high school graduates indexed by $i = 1, ..., 577$. We observe their employment status and wage if employed every month after high school graduation. The data do not allow to differentiate between unemployment and out of labor force, thus employment and non-employment are the only labor market states we consider. Let $d_{i\tau_i} = 1$ if the individual is working and $d_{i\tau_i} = 0$ if the individual is not working, where τ_i is the month after graduation or, equivalently, the month since entry in the labor market. We observe the following data: $[d_{i\tau_i}^D, w_{i\tau_i}^D]$ for $i = 1, \cdots, 577$ and $\tau_i = 1, \cdots, T_i$, where the superscript D denotes the data.

Simulations: We simulate both conditional moments, i.e. predicted values of wages and employment, conditional on the observed (data) values in the previous month, and unconditional moments, which only depend on the simulated values for the previous month. We take the 2.43% annual wage growth rate as given, as resulting from the estimates of the previous section.

In the first month ($\tau = 1$), individual i has 0.45 probability being nonemployed. If he is employed, we simulate a wage w_{i0} such that $w_{i0} \geq w_0^*$. In the month $\tau > 1$, if individual i is nonemployed, with probability λ_n, he receives a random wage draw from a truncted log normal distribution $\Phi(\cdot)$. The truncation is due to the minimum wage and the wage density function ϕ follows

$$\begin{aligned} \phi(w) &= \frac{f(w)}{1-(1-\alpha)F(w_M)} \quad \text{if } w \geq w_M \\ &= \frac{\alpha f(w)}{1-(1-\alpha)F(w_M)} \quad \text{if } w < w_M \end{aligned}$$

[19]Wolpin (1987) documents similar fact and argues that it is consistent with the notion that the search process begins prior to graduation. We make this assumption to match the initial wage. In the model, we assume 45% of the individuals non-employed and 55% of them employed at $\tau = 0$. To simulate their labor market status and wage in $\tau = 1$, we first draw one wage from the log normal distribution for each individual. If he is non-employed, we compare the wage to his reservation wage, which is determined by the underlying parameters of the model, and he accepts the offer as long as the wage draw exceeds the reservation wage. If he was employed, we draw his wage such that it is at least greater than w_0^*.

[20]Assuming measurement error in reported wages enable us to estimate the reservation wage by the moments rather than by the lowest observed wage.

where f is the log normal wage density function with mean μ and variance σ^2. If the wage offer is above w^*, he moves from nonemployment to employment. When he is employed at wage $w_{i\tau}$, he receives a random wage draw w' from $\Phi(\cdot)$. If $w' > w(1+g)$, he moves to the new job with wage w'. Otherwise he stays on current job and his wage increases to $w_{i\tau+1} = w_{i\tau}(1+g)$. He goes back to nonemployment with probability δ.

Let's first consider all individuals who have observations on $d^D_{i\tau_i}, w^D_{i\tau_i}$ from $\tau_i = 1, ...T_i$, i.e. that are not left-censored. In a conditional simulation s, the model predicts $d^s_{i\tau_i}$ and $w^s_{i\tau_i}$, conditional on $d^D_{i\tau_i-1}$ and $w^D_{i\tau_i-1}$. If an individual is working and a wage is observed, we simulate the measurement error to obtain the "true" wage according to $w^{TD}_{i\tau_i} = w^D_{i\tau_i} - u$.[21] Now TD indicates a predicted "true" wage that should be related to the observed wage. Conditional on the "true" wage in $\tau_i = 1$, we simulate the outcome for $\tau_i = 2$, i.e. $[d^s_{i\tau_i=2}, w^s_{i\tau_i=2}]$, again and again for 25 simulations. We thus generate a sequence of 25 simulated observations $[d^s_{i\tau_i}, w^s_{i\tau_i}]$ for $\tau_i = 1, ..., T_i$, that follow the true sequence $[d^D_{i\tau_i-1}, w^{TD}_{i\tau_i-1}]$ for $\tau_i = 1, ..., T_i$. When a wage is not observed, the simulated wage is dropped from the simulated sample for all simulations. In the unconditional simulation, the prediction of $[d^s_{i\tau_i}, w^s_{i\tau_i}]$ is conditional on the last period simulations $[d^s_{i\tau_i-1}, w^s_{i\tau_i-1}]$. For both conditional and unconditional cases, we simulate values for $[d^s_{i\tau_i}, w^s_{i\tau_i}]$ for $N^S = 25$ simulations. The simulations also take into account the unobserved heterogeneity of types and measurement error.

For the left censored observations, suppose that the first observation for individual i is available at $\tau_i = 2$. The simulation for period 2 is based on the sequence of two simulations: we first, simulate period 1 employment status and wages, and conditional on these we simulate period 2 employment status and wages, $[d^s_{i\tau_i=2}, w^s_{i\tau_i=2}]$.[22] Similarly if the first available observation is at time 3, and so on. Having said this, we have $N^S = 25$ simulations based on the parameter vector θ.

Monthly moments and identification. We use two sets of moments: the first set is computed by months and the second set is computed by employment cycle.[23]. Among the monthly moments, the conditional ones include the non-employment rate mne; the proportion of individuals that move from non-employment to employment mtr_1; the proportion of individuals that move from job to job mtr_2; the proportion of individuals that move from employment to non-employment mtr_3; the mean wage mw_1; its standard deviation mw_2; the mean wage below the minimum wage mw_3; and the standard deviation of the wage below the minimum wage mw_4. The unconditional moments include all previous 8 series of monthly moments plus the proportion of individuals that work below the minimum wage mp. All these moments are computed from the data and simulated 25 times, either conditionally or not. The simulated moments used in estimation are the averages across all simulations.[24]

[21]Note that the "true" data here, w^{TD}, has a simulated aspect the we do not specifically indicate.

[22]In this example the data starts at period 2, and, therefore, the initial period and period 1 data are simulated.

[23]We have 18 years' data. So each moment is a vector of 216 elements.

[24]See Appendix C for the exact defenitions of the moments.

Transition moments from non-employment to employment are used to identify the offer arrival rate when non-employed. Similarly, job-to-job transitions identify the offer arrival rate when employed and transitions from employment to non-employment identify the job destruction rate. The reservation wage is identified by the non-employment rate. Wage moments can identify the parameters of the wage distribution. In particular, the initial wage identifies the initial reservation wage. The mean and the variance of the wage are identified from the observed monthly mean and variance as well as from the transitions from job-to-job.

A key aspect of the paper is the identification of the noncompliance/exemption parameters $1 - \alpha$. The proportion of workers who earn below the minimum wage identify the compliance parameter α.But this moment also affect the measurement error. However, the measurement error in wages affect the variance of the monthly wage without affecting the transitions from job-to-job. Hence, conditional on the variance of the offered wage and the transitions from job-to-job the proportion of workers employed for a wage below the minimum wage and the variance of the monthly wage identify jointly the measurement error and α.

Cycle moments and identification. The second set of moments are based on employment cycles, as described in the previous section. In particular, we use moments from the first three employment cycles. We first use duration moments, namely mean non-employment and employment duration on the first three jobs in the first three employment cycles. Second, we use wage moments, namely mean and standard deviation of wages (either global or below the minimum wage) on the first three jobs in the first three cycles. Third, we use transition moments, including the proportion of individuals who start the first three cycles from non-employment, the proportion of individuals who move from the first to the second job, from the second to the third job in the first three cycles. Last, we also use the proportion of individuals who work below the minimum wage on the first three jobs in the first three cycles.

As with monthly moments, non-employment duration identifies the offer arrival rate when non-employed. Job-to-job transitions identify the offer arrival rate when employed. The reservation wage is identified by the wage on the first job. The mean and variance of the wage offer distribution are identified by the mean wage and its standard deviation. The job destruction rate is identified by the non-employment rate when new cycles start. The initial reservation wage is identified by the initial non-employment rate and the mean wage on the first job in the first cycle (significantly lower than wage on the first job in the second and third cycles). The monthly proportion below the minimum wage and the variance of the wage identifies the noncompliance parameter $1 - \alpha$ and the measurement error variance (see the discussion above).

Implementation: We implement the SGMM by using these two sets of moments and then compare results. Let mom_j^D be moment j in the data and $mom_j^S(\theta)$ be moment j from the model simulation, given the parameter vector θ. The moment vector is

$$g(\theta)' = [mom_1^D - mom_1^S(\theta), \cdots, mom_j^D - mom_j^S(\theta), \cdots, mom_J^D - mom_J^S(\theta)]$$

where J is the total number of moments. For first set of moments, $J = 3672$ and for the second set of moments, $J = 66$.[25] The objective function to be minimized with respect to θ is

$$J(\theta) = g(\theta)'Wg(\theta),$$

where the weighting matrix W is set to be diagonal.[26]

The Asymptotic Theory of the SGMM : Let $y_{i\tau} = [d_{i\tau_i}^D, w_{i\tau_i}^D]$ be a vector of observed data for individual i after τ months of market experience. Given the observed state vector at τ for individual i, $z_{i\tau}$, the simulated values of the random events at simulation s, $\varepsilon_s(z_{i\tau})$, and the value of the parameters θ^* the model implies that,

$$y_{i\tau}^s = G(z_{i\tau}, \varepsilon_s(z_{i\tau}); \theta^*).$$

The function $G(z_{i\tau}, \varepsilon_s(z_{i\tau}); \theta^*)$ is given by the solution to the model. We assume that the data $\{y_{i\tau}, z_{i\tau}\}_{i=1}^I$ for all τ are *i.i.d.* By the independence of the simulated random variables we have the orthogonality condition that $E[G(z_{i\tau}, \varepsilon_s(z_{i\tau}); \theta^*) - y_{i\tau} \mid z_{i\tau}] = 0$. Now for N^S simulations of $\varepsilon_s(z_{i\eta})$, we define $h(y_{i\tau})$ as the contribution of individual i for the vector of data moments at time τ, and $h(y_{i\tau}^s)$ as the contribution of simulation s of individual i for the vector of simulated moments at time τ.

$$\begin{aligned} g_{\tau I}(\theta) &= [\frac{1}{I_\tau}\sum_{i=1}^{I_\tau} h(y_{i\tau}) - \frac{1}{N^S}\sum_{s=1}^{N^S}(\frac{1}{I_\tau}\sum_{i=1}^{I_\tau} h(y_{i\tau}^s;\theta)] \\ &\equiv \frac{1}{I_\tau}\sum_{i=1}^{I_\tau} h_i(\theta) \end{aligned}$$

and we have the result that $g_{\tau I}(\theta) \longrightarrow 0$ as $I \longrightarrow \infty$. And under the standard regularity conditions $\theta \longrightarrow \theta^*$. Note that for any function of $z_{i\tau}$ that multiply $y_{i\tau} - \frac{1}{N^S}\sum_{s=1}^{N^S} G(z_{i\tau}, \varepsilon_s(z_{i\tau}); \theta)$ and the average of this product converges to zero as I converges to infinity.[27] The asymptotic variance is given by $(1 + \frac{1}{N^s})(A'WA)^{-1}A'W\Omega WA(A'WA)^{-1}/I$, where N^S is the number of simulations, $A \equiv E[\nabla_\theta h_i(\theta^*)]$ and $\Omega \equiv E[h_i(\theta^*) h_i(\theta^*)']$.

[25]The first set of moments include 8 series of conditional monthly moments and 9 series of unconditional monthly moments, so J=17*216=3672. The second set of moments consist of 12 duration moments, 36 wage moments, 9 transition moments and 9 moments on proportions below the minimum wage.

[26]In our estimates, the weight on each moment is set to be one over its sample mean for the monthly moments. We use the identity matrix as the weighting matrix for cycle moments.

[27]For a recent survey of the asymptotic distribution of the estimated parameters, tests and references see Carrasco and Florens (2002).

Table 8: Parameter estimates of search model with $g = 0.0243$

Parameters	Estimates (monthly moments)		Estimates (cycle moments)	
	coef.	(s.e.)	coef.	(s.e.)
λ_{n1}	0.428	(0.102)	0.451	(0.001)
λ_{n2}	0.958	(0.100)	0.958	(0.034)
λ_{e1}	0.097	(0.024)	0.129	(0.017)
λ_{e2}	0.237	(0.047)	0.264	(0.011)
π	0.432	(0.009)	0.473	(0.147)
w_1^*	2.696	(0.270)	2.695	(0.373)
w_2^*	8.520	(0.454)	10.309	(0.134)
μ_1	1.527	(0.057)	1.571	90.043)
μ_2	1.652	(0.047)	1.667	(0.017)
σ_{w1}	0.482	(0.067)	0.471	(0.027)
σ_{w2}	0.517	(0.022)	0.513	(0.010)
δ_1	0.061	(0.004)	0.040	(0.002)
δ_2	1.88e-3	(0.0003)	0.012	(0.0003)
w_{01}^*	2.843	(0.323)	3.128	(0.086)
w_{02}^*	6.791	(0.085)	6.610	(0.181)
σ_u	6.14e-3	(0.009)	0.012	(0.001)
α	0.249	(0.037)	0.254	(0.042)

Notes. The sample includes male high-school graduates from the NLSY. Number of observations: 577. Estimation methods: Simulated GMM.

5 Results

Parameters: The estimates of the parameters are presented in Table 8. In estimating the model we allow for two unobserved types of individuals (see, Heckman and Singer, 1984), who are different in all parameters but in the level of compliance (α) and the measurement error variance (σ_u).

Starting from monthly moments, the parameter estimates have plausible magnitudes and are in line with previous estimates of the parameters of a search model with search on-the-job. That is, the arrival rates of job offers is higher for non-employed than employed individuals and these rates are different across types of individuals, predicting a decreasing hazard rate. Type 1 individuals, including about 43% of workers, have lower arrival rates of offers while either non-employed or employed, a higher rate of job destruction, a lower mean wage offer and a lower reservation wage.[28] Hence, type 1 should be relatively more likely to work below

[28]It should be noted that the reservation wage depends on the level of the minmum wage, but using the data it has a negligible impact on the actual value of the reservation wage.

the minimum wage. As one would expect, reservation wages during the last few years of high school are lower than reservation wages after graduation (this is not true for type one), and they are lower for type 1 individuals. Type 2 individuals have reservation wages above the minimum wage and, therefore, for more than half of our sample the minimum wage is not binding for the entire period.

The novelty of our results consists in providing an estimate for the extent of compliance of firms' job offers to the minimum wage regulations, represented by the parameter α. We find that the arrival rate of job offers below the minimum wage is about a quarter of that above the minimum wage. However, there are reasons to believe that our estimate for α is an over-estimate of the true non-compliance of firms. First, there are (limited) categories of workers who are exempt from minimum wage regulations. And, second, we use as minimum wage the federal value at each date of the data, and assume that it is applied equally in all states. However, there are states that do not have minimum wage regulations, or that adopt a minimum wage below the federal level. On the other hand, there are also states that adopt a minimum wage above the federal level, and this should deliver an overestimate of α in our empirical model.

The estimates obtained on cycle moments are quite similar to those obtained on monthly moments, including the ranking of values for type 1 and type 2 individuals. The difference that is worthwhile mentioning is that under cycle moments we obtained a higher estimate of the measurement error variance. But the estimate for α remains virtually unchanged. Based on the model one might expect that the measurement error variance and the compliance parameter should be linked by the fact that, whenever one comes across a wage observation below the minimum wage, this should stem from either measurement error or non-compliance. However, the estimated value of α does not seem to be too sensitive in the practice to variations in σ_u.

Model Fit: Figures 7-14 show the fit of all the monthly moments that are used for estimation of the parameters. It is quite remarkable to see how well the conditional moments from the simulated model are able to fit the data moments both in terms of the eighteen years trends, levels and the seasonality variations.

The model fits well the life cycle decrease in non-employment (Figure 7), and the slight increase in the transition rate from non-employment to employment (Figure 8) during the first 10 years in the labor market, but fails to fit the increase in seasonal variations of these transition rates. The model fits well the decrease in job-to-job transitions (Figure 9): but while conditional moments also fit well its level, the unconditional ones seem to underpredicts mobility. Finally, the model fails to fit the decreasing trend in transition from employment to non-employment, but it does fit its level. Note that the only potential source of dynamics here is the unobserved heterogeneity in destruction rate: the implication is that the two-type heterogeneity is not sufficient to fit the job destruction decreasing trend.

The model with the *same* wage offer distribution for the entire eighteen years and a constant wage growth on the same job (tenure) of 2.43 annually, fits very well the eighteen years of mean hourly wage growth from about 8 to about 16 dollars (Figures 11). Note that the positive trend of the hourly wage variance is also well predicted by the model (Figure 12). Furthermore, the conditional moments of the model do well in predicting the mean and variance of wages below the minimum wage (Figures 13 and 14). The unconditional moments fit the trends in the data but do not fit the levels and fluctuations in wages.

Our estimated model measure the contribution of wage growth due to expereince on-the-job and the wage growth due to search on-the-job. During the first 18 years in the labor market, 55% of wage growth is due to on-the-job wage growth and the rest of 45% is due to job-to-job wage growth.[29] Topel and Ward (1992) find that wage gains attributable to job changing activity is about one third of total earnings growth during the first ten years of labor market experience. But their sample mostly work during 60's and consist of young men with all schooling levels. A more comparable recent study by Omer (2005) uses the same NLSY79 white male high school graduate sample and finds that wage growth between jobs accounts for about 45% of the worker's entire wage growth.

Figure 15 presents model and data moments for the proportion of individuals working below the minimum wage. In estimation we have used the unconditional prediction of the model as the conditional moments have no information on the compliance parameter, α. The fit of the model to the level and the trend of this proportion is remarkable. This result provides strong support for using the model as a good approximation for the explanation of the fact that large proportion of workers receive wages below the minimum wage. Hence, the analysis of the implications of changing the compliance rate on labor market outcome can be well trusted.

The fit of the cycle moments that are used in estimation (Table 9) are reproduced well by our estimates. Mainly the mean duration, mean wage and standard deviation of the wage (above and below the minimum wage) are captured quite well by the model for cycle one and somewhat less accurately for the other two cycles. The model fits well the non-employment duration and the job duration of each job in the cycles, as well as the mean and standard deviations of wages by jobs and cycles, both overall and below the minimum wage. The fit for the proportion of workers below the minimum wage is good in the first cycle but not as good for the second and third jobs in the later two cycles. This maybe mostly due to the low number of observations in later jobs/cycles.

The fit of transition moments is less accurate as it can be noted from the last three rows in Table 9. Mainly, the model predicts a much higher transition rate from non-employment to work in all three cycles. Job-to-job transitions also deviate from the data but to a lesser extent.

Counterfactual: We use the model to get quantitative implications for changing the level of the minimum wage (Table 10) and the rate of compliance with the minimum wage (Table

[29] Over the entire sample mean hourly wage grows by \$8.1, from \$8.32 in the first month to \$16.42 in the last month. Consider an individual staying on the same job, on average his wage will increase by \$4.46 from \$8.32 to $8.32 * (1 + 0.2\%)^{215} = 12.78$.

11).

An increase in the minimum wage by $1.35, from $5.15 to $6.50, increases non-employment rate by 2.1% to 2.4% and a decrease of the minimum wage by $2.15 decreases the non-employment rate by 2.9% to 3.5%. The same increase (decrease) in minimum wage increases (decreases) mean accepted hourly wages by 10-30 (30-50) cents. These results indicate a very small impact of the minimum wage level on unemployment and wages, consistently with the main findings reported by Card and Krueger (1995) and Kennan (1998). The impact of the minimum wage on inequality, measured as the ratio of the 90th to the 10th percentiles and standard deviation of wages is also small. However, the impact of the same change on the proportion of workers that are employed for a wage below the minimum wage is large. These findings from the model provide a very simple and convincing explanation to facts that a-priori seem to be inconsistent with an economic model. That is, potential large changes in the proportion of individuals working below the minimum wage due to changes in real minimum wage could be consistent with the fact that the same changes in minimum wage have small effect on the wage distribution and unemployment.

The effects of changing the non-compliance rate from the estimated level of 25 percent to full compliance ($\alpha = 0$), to less compliance ($\alpha = 0.5$) and no compliance ($\alpha = 1.0$) are qualitatively similar to the results on the change in the level of the minimum wage. That is, lower (higher) compliance decreases (increases) the non-employment rate by one to four percentage points depending on the change. Similarly, the hourly mean wage and the ratio of the 90th to 10th percentile in the wage distribution (opposite from mean wage!) decreases (increases) by few cents as compliance decreases (increases). However, the proportion of workers that are employed for a wage below the minimum wage changes dramatically (zero to 13 percent after ten years in the labor market) with the rate of the compliance policy.

Table 10: Counterfactual: Change the Level of the Minimum Wage

All individuals	Years in The Labor Market	Model $w_m = 5.15$	Counterfactuals $w_m = 3.0$	$w_m = 6.5$
Non-employment rate (percentage)	1	21.4	17.9	23.8
	2	10.7	7.8	12.8
	5-9	9.8	6.9	11.9
	10-18	9.9	6.9	12.2
Mean wage (2000 dollars)	1	9.2	8.8	9.3
	2	10.3	10.0	10.4
	5-9	12.8	12.4	13.0
	10-18	15.0	14.5	15.3
90p/10p wage	1	2.9	3.4	3.1
	2	2.8	3.4	3.0
	5-9	3.4	4.1	3.1
	10-18	4.1	5.1	3.8
Proportion below the minimum wage (percentage)	1	12.5	0.8	20.3
	2	6.8	0.3	13.7
	5-9	4.9	0.3	10.0
	10-18	4.9	0.2	10.2

Table 11: Counterfactual: Change the Level of Compliance

All individuals	Years in The Labor Market	Model $\alpha = 0.25$	Counterfactuals $\alpha = 0$	$\alpha = 0.5$	$\alpha = 1.0$
Non-employment rate (percentage)	1	21.4	23.3	19.9	17.5
	2	10.7	12.7	9.2	7.5
	5-9	9.8	11.7	8.4	6.6
	10-18	9.9	11.8	8.5	6.7
Mean wage (2000 dollars)	1	9.2	9.3	9.0	8.8
	2	10.3	10.6	10.1	9.9
	5-9	12.8	13.1	12.6	12.3
	10-18	15.0	15.3	14.7	14.5
90p/10p wage	1	2.9	2.7	3.1	3.4
	2	2.8	2.6	3.0	3.5
	5-9	3.4	3.1	3.6	4.2
	10-18	4.1	3.7	4.4	5.2
Proportion below the minimum wage (percentage)	1	12.5	9.0	15.1	20.8
	2	6.8	2.2	10.2	15.3
	5-9	4.9	0.0	8.4	12.4
	10-18	4.9	0.0	8.3	12.8

6 Conclusions

To be written

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Appendix A: Computation of $V_e'()$

Using integration by parts, (3) and (4) can be rewritten as:

$$
\begin{aligned}
rV_e(w_\tau) &= w_\tau - c_e(\lambda_e) + \delta\left[V_n - V_e(w_\tau)\right] \\
&+ \gamma(\alpha)\lambda_e \int_{w_\tau} [1 - F(w)]V_e'(w)\,dw + gw_\tau V_e'(w_\tau) \text{ for } w_\tau \geq w_M
\end{aligned} \quad (15)
$$

and

$$
\begin{aligned}
rV_e(w_\tau) &= w_\tau - c_e(\lambda_e) + \delta\left[V_n - V_e(w_\tau)\right] \\
&+ \gamma(\alpha)\lambda_e \int_{w_M} [1 - F(w)]V_e'(w)\,dw \\
&+ \gamma(\alpha)\lambda_e \int_{w_\tau}^{w_M} V_e'(w)\left[1 - \alpha F(w) - (1-\alpha)F(w_M)\right]dw \\
+gw_\tau V_e'(w_\tau) \text{ for } w_\tau &< w_M
\end{aligned} \quad (16)
$$

By differentiating (15) and (16):

$$
gw_\tau V''(w_\tau) = \left\{r + \delta + \gamma(\alpha)\lambda_e\left[1 - F(w_\tau)\right] - g\right\}V'(w_\tau) - 1, \text{ for } w_\tau \geq w_M \quad (17)
$$

$$
gw_\tau V''(w_\tau) = \left\{r + \delta + \gamma(\alpha)\lambda_e\left[1 - \alpha F(w_\tau) - (1-\alpha)F(w_M)\right] - g\right\}V'(w_\tau) - 1, \text{ for } w_\tau < w_M \quad (18)
$$

Consider the case $w_\tau \geq w_M$ first, and let $u(w_\tau) = -\frac{1}{gw_\tau}\left\{r + \delta - g + \gamma(\alpha)\lambda_e\left[1 - F(w_\tau)\right]\right\}$ and $q(w_\tau) = -\frac{1}{gw_\tau}$. Equation (18) implies

$$
\begin{aligned}
V_e'(w_\tau) &= e^{-\int u(w_\tau)dw_\tau}\left[A + \int q(w_\tau)e^{\int u(w_\tau)dw_\tau}dw_\tau\right] \\
&= e^{-\int_{w_0}^{w_\tau} u(s)ds}\left[A + \int_{w_0}^{w_\tau} q(s)e^{\int_{s_0}^{s_\tau} u(z)dz}ds\right]
\end{aligned} \quad (19)
$$

where A is an arbitrary constant. Now let

$$
\begin{aligned}
R(w_\tau; w_0) &= -\int_{w_0}^{w_\tau} u(s)ds \\
&= \int_{w_0}^{w_\tau} \frac{r + \delta - g + \gamma(\alpha)\lambda_e[1 - F(s)]}{gs}ds.
\end{aligned} \quad (20)
$$

This in turn implies

$$
\int_{w_0}^{w_\tau} q(s)e^{\int_{s_0}^{s_\tau} u(z)dz}ds = \int_{w_0}^{w_\tau} q(s)e^{-R(s)}ds = -\int_{w_0}^{w_\tau} \frac{1}{gs}e^{-R(s)}ds.
$$

Having set $\tau = 0$ in (19), one obtains $V_e'(w_0) = A = \int_{w_0}^{\infty} \frac{1}{gs} e^{-R(s)} ds$ and thus

$$\begin{aligned} V_e'(w_\tau) &= e^{R(w_\tau)} \left[\int_{w_0}^{\infty} \frac{1}{gs} e^{-R(s)} ds - \int_{w_0}^{w_\tau} \frac{1}{gs} e^{-R(s)} ds \right] && (21) \\ &= e^{R(w_\tau)} \int_{w_\tau}^{\infty} \frac{1}{gs} e^{-R(s)} ds && (22) \\ &= \int_{w_\tau}^{\infty} \frac{1}{gs} e^{R(w_\tau) - R(s)} ds && (23) \end{aligned}$$

where

$$R(w_\tau) - R(s) = \int_s^{w_\tau} \frac{r + \delta - g + \gamma(\alpha)\lambda_e[1 - F(z)]}{gz} dz.$$

Therefore:

$$V_e'(w_\tau) = \int_{w_\tau}^{\infty} \frac{1}{gs} \exp\left(\int_s^{w_\tau} \frac{r + \delta - g + \lambda_e[1 - F(z)]}{gz} dz \right) ds$$

Similarly, when $w_\tau < w_M$

$$V_e'(w_\tau) = \int_{w_\tau}^{\infty} \frac{1}{gs} \exp\left(\int_s^{w_\tau} \frac{r + \delta - g + \gamma(\alpha)\lambda_e[1 - \alpha F(z) - (1 - \alpha) F(w_M)]}{gz} dz \right) ds.$$

Appendix B: Comparative Statics

Optimal search efforts and reservation wage are jointly determined by equation (5) and equation (10). Combining these two equations together gives

$$w^* = b - c_n(\lambda_n) + c_e(\lambda_e) + (\lambda_n - \lambda_e) c_n'(\lambda_n) - gw^* V_e'(w^*).$$

Taking derivative with respect to λ_n, we have

$$\frac{\partial w^*}{\partial \lambda_n} = (\lambda_n - \lambda_e) c_n''(\lambda_n) - gV_e'(w^*) \frac{\partial w^*}{\partial \lambda_n} - gw^* V_e''(w^*) \frac{\partial w^*}{\partial \lambda_n}.$$

Thus

$$\begin{aligned} \frac{\partial w^*}{\partial \lambda_n} &= (\lambda_n - \lambda_e) c_n''(\lambda_n) [1 + gV_e'(w^*) + gw^* V_e''(w^*)]^{-1} \\ &= \frac{(\lambda_n - \lambda_e) c_n''(\lambda_n)}{\{r + \delta + \gamma(\alpha)\lambda_e[1 - \alpha F(w^*) - (1 - \alpha) F(w_M)]\} V'(w^*)} > 0. \end{aligned}$$

The reservation wage is increasing in search efforts, or the job arrival rate.

Differentiating (5) with respect to w_M gives

$$\frac{\partial \lambda_n}{\partial w_M} = \frac{-(1-\alpha)\int_{w^*}^{w_M} F'(w_M)\, V_e'(w)\, dw}{\frac{1}{\gamma(\alpha)} c_n''(\lambda_n) + [1-\alpha F(w^*) - (1-\alpha) F(w_M)] V_e'(w^*) \frac{\partial w^*}{\partial \lambda_n}} < 0.$$

Therefore when the minimum wage increases, both search effort and reservation wage decrease.

Similarly differentiating (5) with respect to α gives

$$\begin{aligned}
& \{c_n''(\lambda_n) + \gamma(\alpha)[1-\alpha F(w^*) - (1-\alpha)F(w_M)] V_e'(w^*) \frac{\partial w^*}{\partial \lambda_n}\} \frac{\partial \lambda_n}{\partial \alpha} \\
= \; & \gamma'(\alpha)[\int_{w^*} [1-F(w)] V_e'(w)\, dw - (1-\alpha) \int_{w^*}^{w_M} [F(w_M) - F(w)] V_e'(w)\, dw] \\
& + \gamma(\alpha) \int_{w^*}^{w_M} [F(w_M) - F(w)] V_e'(w)\, dw
\end{aligned}$$

The sign of the last term is ambiguous, so is the sign of $\frac{\partial \lambda_n}{\partial \alpha}$.

Appendix C: Moments

1. Monthly Moments

To compute the moments in the data, we use following formulas. Note that all moments are calculated by each month in the labor market $\tau = 1, 2, \cdots, 216$. For example, mne^D is a column vector of 216 dimensions and each element $mne^D(\tau)$ is determined by

$$mne^D(\tau) = \frac{\sum_i I(d_{i\tau}^D = 0)}{\sum_i I(d_{i\tau}^D = 0) + \sum_i I(d_{i\tau}^D = 1)};$$

$I(\cdot)$ is an indicator function, which equals one if the condition is satisfied and equals zero otherwise. Similarly

$$mtr_1^D(\tau) = \frac{\sum_i I(d_{i\tau}^D = 0, d_{i\tau+1}^D = 1)}{\sum_i I(d_{i\tau}^D = 0)}, \tau = 1, 2, \cdots, 215,$$

$$mtr_2^D(\tau) = \frac{\sum_i I(d_{i\tau}^D = 1, d_{i\tau+1}^D = 1)}{\sum_i I(d_{i\tau}^D = 1)}, \tau = 1, 2, \cdots, 215,$$

where $d_{i\tau}^D = 1$ and $d_{i\tau+1}^D = 1$ refer to two different jobs one after another.

$$mtr_3^D(\tau) = \frac{\sum_i I(d_{i\tau}^D = 1, d_{i\tau+1}^D = 0)}{\sum_i I(d_{i\tau}^D = 1)}, \tau = 1, 2, \cdots, 215, ;$$

$$mw_1^D(\tau) = \frac{\sum_i (w_{i\tau}^D | w_{i\tau}^D > 0)}{\sum_i I(d_{i\tau}^D = 1 | w_{i\tau}^D > 0)};$$

$$mw_2^D(\tau) = \sqrt{\frac{\sum_i ((w_{i\tau}^D - mw_1^D(\tau))^2 | w_{i\tau}^D > 0)}{\sum_i I(d_{i\tau}^D = 1 | w_{i\tau}^D > 0) - 1}};$$

$$mw_3^D(\tau) = \frac{\sum_i (w_{i\tau}^D | 0 < w_{i\tau}^D < w_{M\tau})}{\sum_i I(d_{i\tau}^D = 1 | 0 < w_{i\tau}^D < w_{M\tau})};$$

$$mw_4^D(\tau) = \sqrt{\frac{\sum_i ((w_{i\tau}^D - mw_3^D(\tau))^2 | 0 < w_{i\tau}^D < w_{M\tau})}{\sum_i I(d_{i\tau}^D = 1 | 0 < w_{i\tau}^D < w_{M\tau}) - 1}}$$

$$mp^D(\tau) = \frac{\sum_i I(w_{i\tau}^D < w_{M\tau} | w_{i\tau}^D > 0)}{\sum_i I(d_{i\tau}^D = 1 | w_{i\tau}^D > 0)},$$

where $w_{M\tau}$ is the minimum wage.

One period ahead conditional simulated moments are defined as following:

$$mne^S(\tau) = \frac{1}{N^S} \sum_{s=1}^{N^S} \frac{\sum_i I(d_{i\tau}^s = 0)}{\sum_i I(d_{i\tau}^s = 0) + \sum_i I(d_{i\tau}^s = 1)};$$

$$mtr_1^S(\tau) = \frac{1}{N^S} \sum_{s=1}^{N^S} \frac{\sum_i I(d_{i\tau}^D = 0, d_{i\tau+1}^s = 1)}{\sum_i I(d_{i\tau}^D = 0)}, \tau = 1, 2, \cdots, 215;$$

$$mtr_2^S(\tau) = \frac{1}{N^S} \sum_{s=1}^{N^S} \frac{\sum_i I(d_{i\tau}^D = 1, d_{i\tau+1}^s = 1)}{\sum_i I(d_{i\tau}^D = 1)}, \tau = 1, 2, \cdots, 215,$$

where $d_{i\tau}^D = 1$ and $d_{i\tau+1}^s = 1$ refer to two different jobs;

$$mtr_3^S(\tau) = \frac{1}{N^S} \sum_{s=1}^{N^S} \frac{\sum_i I(d_{i\tau}^D = 1, d_{i\tau+1}^s = 0)}{\sum_i I(d_{i\tau}^D = 1)}, \tau = 1, 2, \cdots, 215;$$

$$mw_1^S(\tau) = \frac{1}{N^S} \sum_{s=1}^{N^S} \frac{\sum_i (w_{i\tau}^s | w_{i\tau}^s > 0)}{\sum_i I(d_{i\tau}^s = 1 | w_{i\tau}^s > 0)};$$

$$mw_2^S(\tau) = \frac{1}{N^S} \sum_{s=1}^{N^S} \sqrt{\frac{\sum_i ((w_{i\tau}^s - mw_1^s(\tau))^2 | w_{i\tau}^s > 0)}{\sum_i I(d_{i\tau}^s = 1 | w_{i\tau}^s > 0) - 1}};$$

$$mw_3^S(\tau) = \frac{1}{N^S} \sum_{s=1}^{N^S} \frac{\sum_i (w_{i\tau}^s | 0 < w_{i\tau}^s < w_{M\tau})}{\sum_i I(d_{i\tau}^s = 1 | 0 < w_{i\tau}^s < w_{M\tau})};$$

$$mw_4^S(\tau) = \frac{1}{N^S}\sum_{s=1}^{N^S}\sqrt{\frac{\sum_i((w_{i\tau}^s - mw_3^s(\tau))^2|0 < w_{i\tau}^s < w_{M\tau})}{\sum_i I(d_{i\tau}^s = 1|0 < w_{i\tau}^s < w_{M\tau}) - 1}};$$

$$mp^S(\tau) = \frac{1}{N^S}\sum_{s=1}^{N^S}\frac{\sum_i I(w_{i\tau}^s < w_{M\tau}|w_{i\tau}^s > 0)}{\sum_i I(d_{i\tau}^s = 1|w_{i\tau}^s > 0)},$$

where $w_{M\tau}$ is the minimum wage and $N^S = 25$ is the total number of simulations.

All unconditional simulated moments are defined the same as the conditional simulated moments except for the transition moments:

$$mtr_1^S(\tau) = \frac{1}{N^S}\sum_{s=1}^{N^S}\frac{\sum_i I(d_{i\tau}^s = 0, d_{i\tau+1}^s = 1)}{\sum_i I(d_{i\tau}^s = 0)}, \tau = 1, 2, \cdots, 215;$$

$$mtr_2^S(\tau) = \frac{1}{N^S}\sum_{s=1}^{N^S}\frac{\sum_i I(d_{i\tau}^s = 1, d_{i\tau+1}^s = 1)}{\sum_i I(d_{i\tau}^s = 1)}, \tau = 1, 2, \cdots, 215,$$

where $d_{i\tau}^s = 1$ and $d_{i\tau+1}^s = 1$ refer to two different jobs;

$$mtr_3^S(\tau) = \frac{1}{N^S}\sum_{s=1}^{N^S}\frac{\sum_i I(d_{i\tau}^s = 1, d_{i\tau+1}^s = 0)}{\sum_i I(d_{i\tau}^s = 1)}, \tau = 1, 2, \cdots, 215.$$

2. Cycle Moments

Recall the way we construct employment cycles. To calculate the empirical moments, we follow each individual i for the first three cycle and the first three jobs in each cycle, i.e. $\{c_i^1(ne_i^1, J1_i^1, J2_i^1, J3_i^1), c_i^2(ne_i^2, J1_i^2, J2_i^2, J3_i^2), c_i^3(ne_i^3, J1_i^3, J2_i^3, J3_i^3)\}$. We convert our monthly data $[d_{i\tau}^D, w_{i\tau}^D]$ into $[\overline{d}_{it}^{Dcj}, \overline{w}_i^{Dcj}]$ where i denotes individual i, $c = 1, 2, 3$ denotes the number of cycle, $j = 0, 1, 2, 3$ corresponds to non-employment, first, second and third job, t is the tenure on each job (or non-employment). For example $\overline{d}_{i10}^{12} = 1$ means individual i works (otherwise equals 0) in the 10th month on the second job of his first employment cycle and $\overline{d}_{i5}^{20} = 1$ denotes fifth month non-employment in the second cycle. $\overline{w}_i^{Dcj}$ presents the accepted wage for job j in cycle c, which is the first wage observation on the job.

Data cycle moments are defined as following. Duration of cycle c job j for individual i is $\sum_t \overline{d}_{it}^{cj}$, thus mean duration

$$mdur^{Dcj} = \frac{\sum_i(\sum_t \overline{d}_{it}^{Dcj})}{\sum_i I(\sum_t \overline{d}_{it}^{Dcj} \geq 1)}, c = 1, 2, 3, j = 0, 1, 2, 3.$$

Mean accepted wage

$$mwage_1^{Dcj} = \frac{\sum_i(\overline{w}_i^{Dcj}|\overline{w}_i^{Dcj} > 0)}{\sum_i I(\overline{d}_{i1}^{Dcj} = 1|\overline{w}_i^{Dcj} > 0)}, c, j = 1, 2, 3.$$

Standard deviation of accepted wage

$$stdwage_1^{Dcj} = \sqrt{\frac{\sum_i ((\overline{w}_i^{Dcj} - mwage_1^{Dcj})^2 | \overline{w}_i^{Dcj} > 0)}{\sum_i I(\overline{d}_{i1}^{Dcj} = 1 | \overline{w}_i^{Dcj} > 0) - 1}}, c, j = 1, 2, 3.$$

Mean accepted wage below the minimum wage

$$mwage_2^{Dcj} = \frac{\sum_i (\overline{w}_i^{Dcj} | 0 < \overline{w}_i^{Dcj} < w_{M\tau})}{\sum_i I(\overline{d}_{i1}^{Dcj} = 1 | 0 < \overline{w}_i^{Dcj} < w_{M\tau})}, c, j = 1, 2, 3.$$

Standard deviation of accepted wage below the minimum wage

$$stdwage_2^{Dcj} = \sqrt{\frac{\sum_i ((\overline{w}_i^{Dcj} - mwage_2^{Dcj})^2 | 0 < \overline{w}_i^{Dcj} < w_{M\tau})}{\sum_i I(\overline{d}_{i1}^{Dcj} = 1 | 0 < \overline{w}_i^{Dcj} < w_{M\tau}) - 1}}, c, j = 1, 2, 3.$$

Proportion of workers paid below the minimum wage on job j in cycle c

$$prop^{Dcj} = \frac{\sum_i I(\overline{w}_i^{Dcj} < w_{M\tau} | \overline{w}_i^{Dcj} > 0)}{\sum_i I(\overline{d}_{i1}^{Dcj} = 1 | \overline{w}_i^{Dcj} > 0)}, c, j = 1, 2, 3.$$

Proportion of workers start cycle c as nonemployed

$$ne^{Dc} = \frac{\sum_i I(\overline{d}_{i1}^{Dc0} = 1)}{577}, c = 1, 2, 3.$$

Proportion of workers move from job 1 to job 2 in cycle c

$$tr_1^{Dc} = \frac{\sum_i I(\overline{d}_{i1}^{Dc1} = 1, \overline{d}_{i1}^{Dc2} = 1)}{\sum_i I(\overline{d}_{i1}^{Dc1} = 1)}, c = 1, 2, 3.$$

Proportion of workers move from job 2 to job3 in cycle c

$$tr_2^{Dc} = \frac{\sum_i I(\overline{d}_{i1}^{Dc2} = 1, \overline{d}_{i1}^{Dc3} = 1)}{\sum_i I(\overline{d}_{i1}^{Dc2} = 1)}, c = 1, 2, 3.$$

Simulated cycle moments are defined similarly for each simulation s and we take average over $N^s = 25$ simulations.

Figure 1: Monthly Employment and Nonemployment Rates

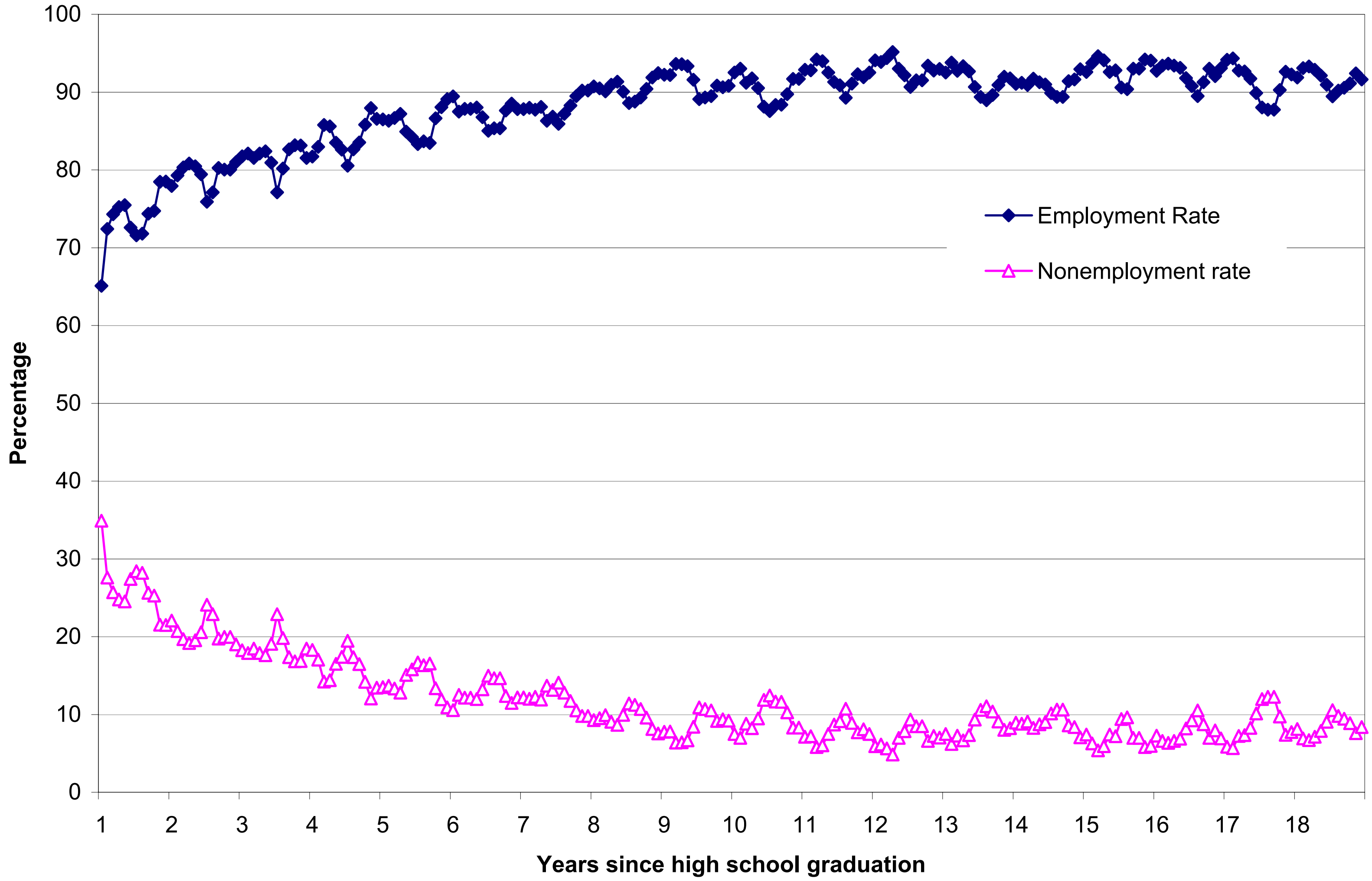


Figure 2: Job Separation Rate by Job Tenure (Months)

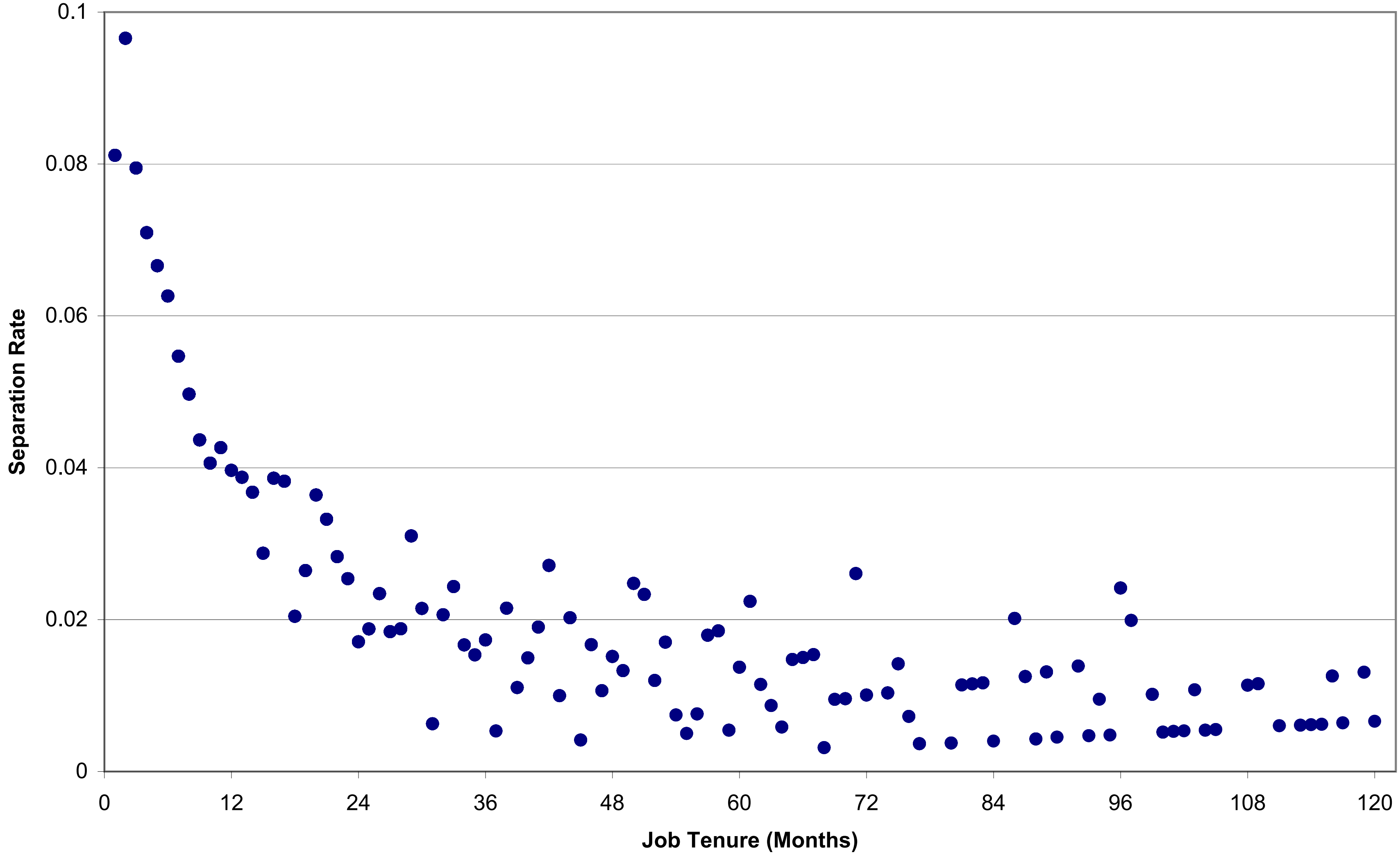


Figure 3: Transition Probabilities From and To Employment and Nonemployment

Figure 4: Transition Probabilities from Job to Job and from Nonemployment to Employment

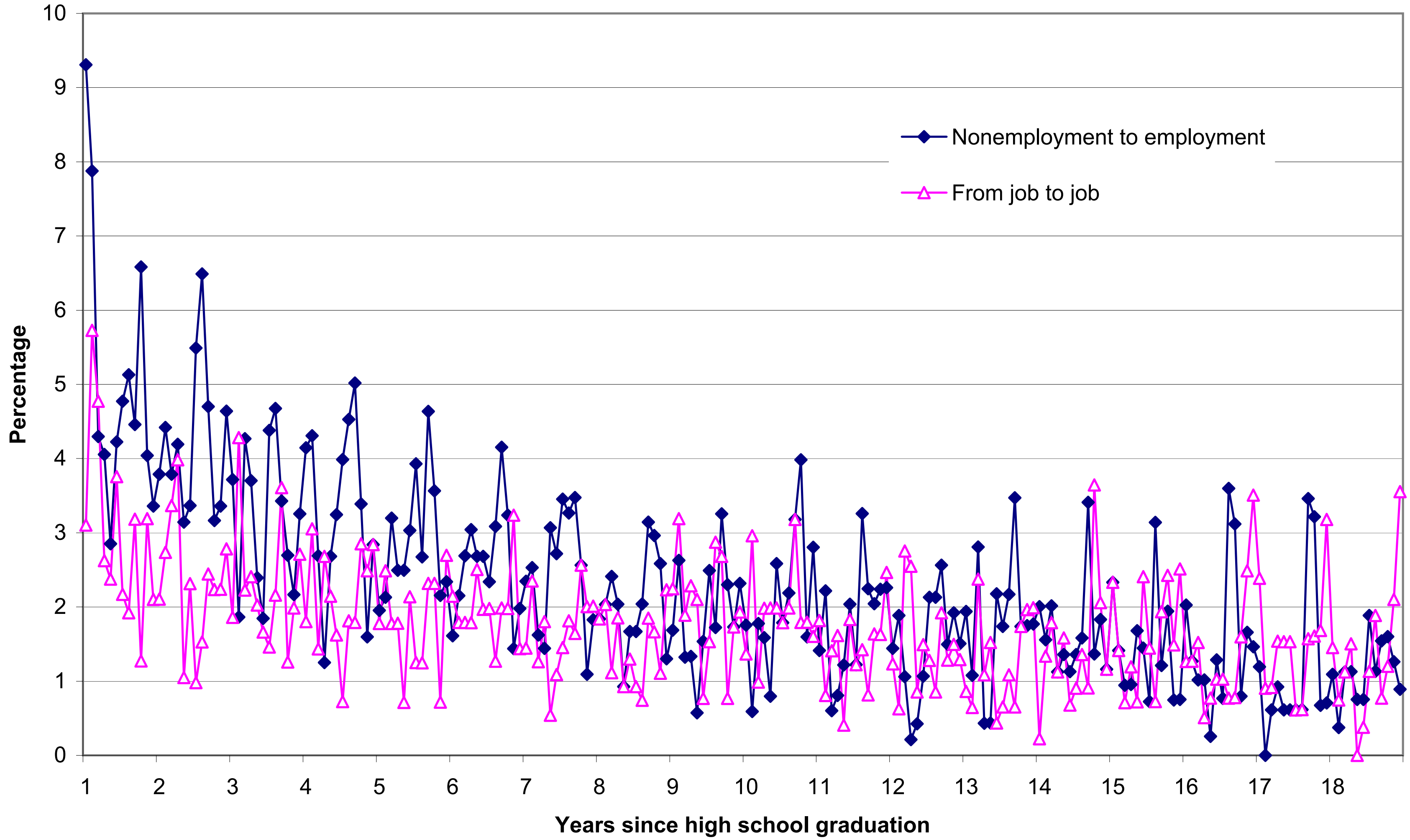


Figure 5: Federal Minimum Wage Under the Fair Labor Standards Act

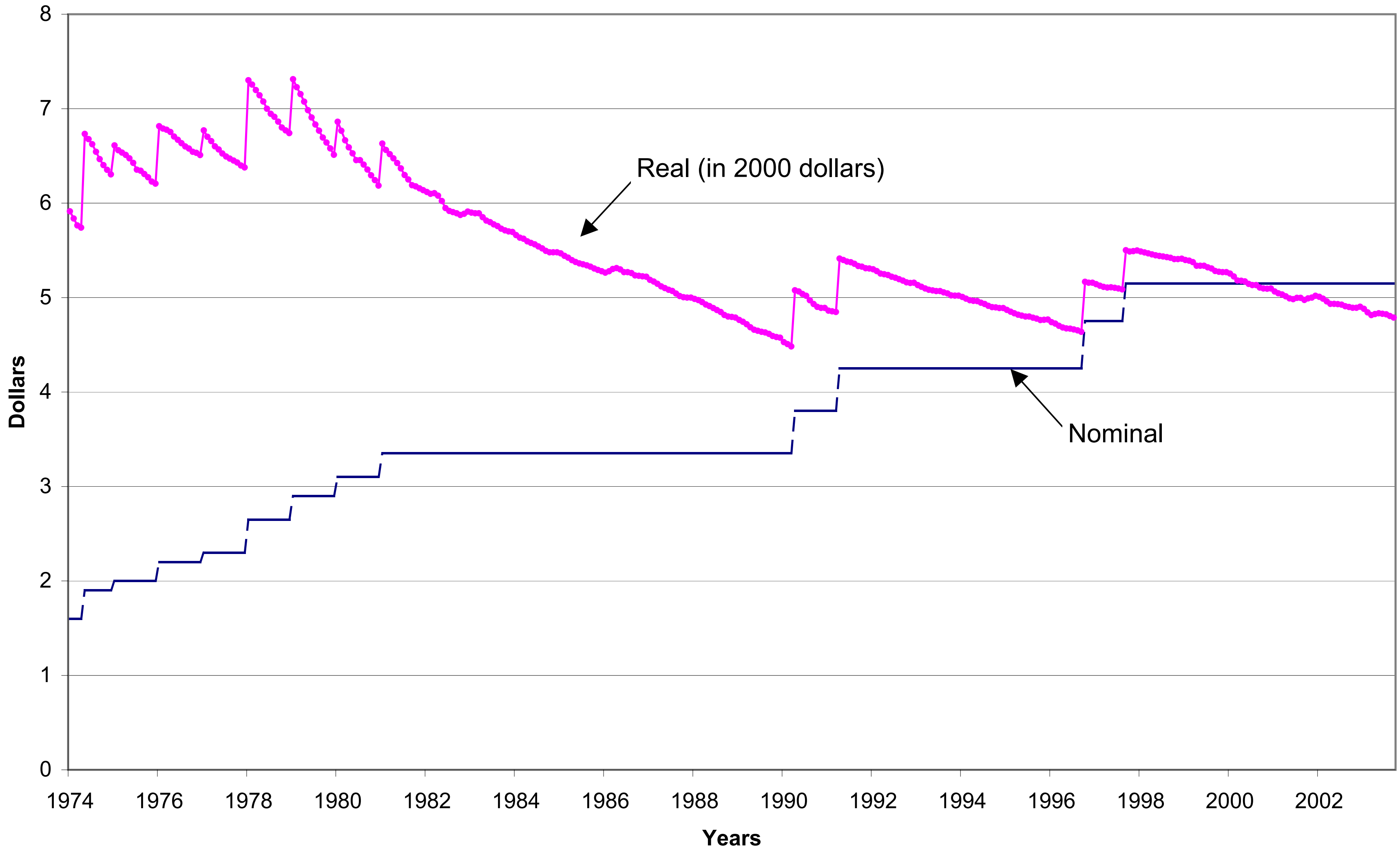


Figure 6a: Wage Growth From All Wage Observations

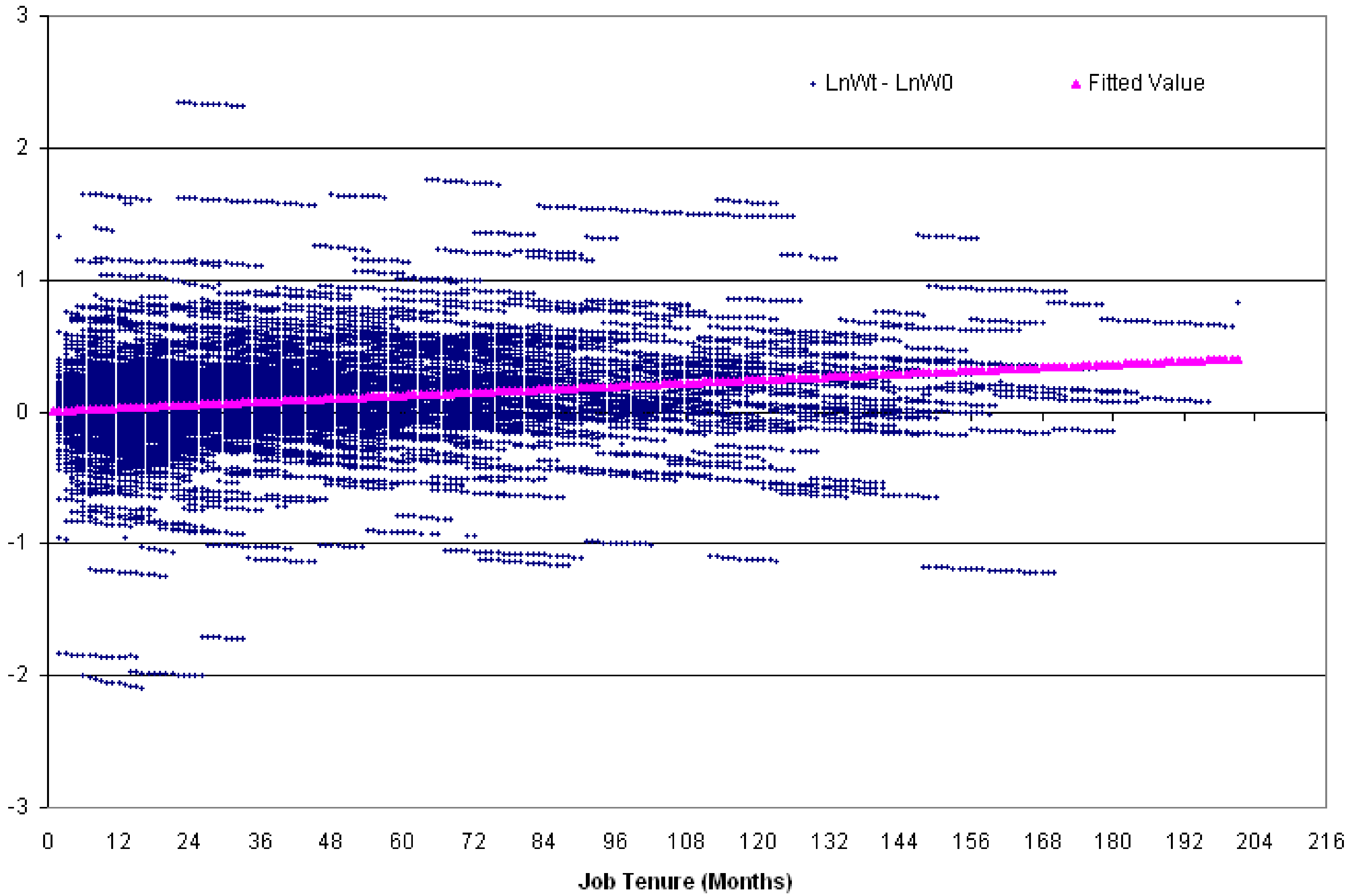


Figure 6b: Wage Growth on Each Job

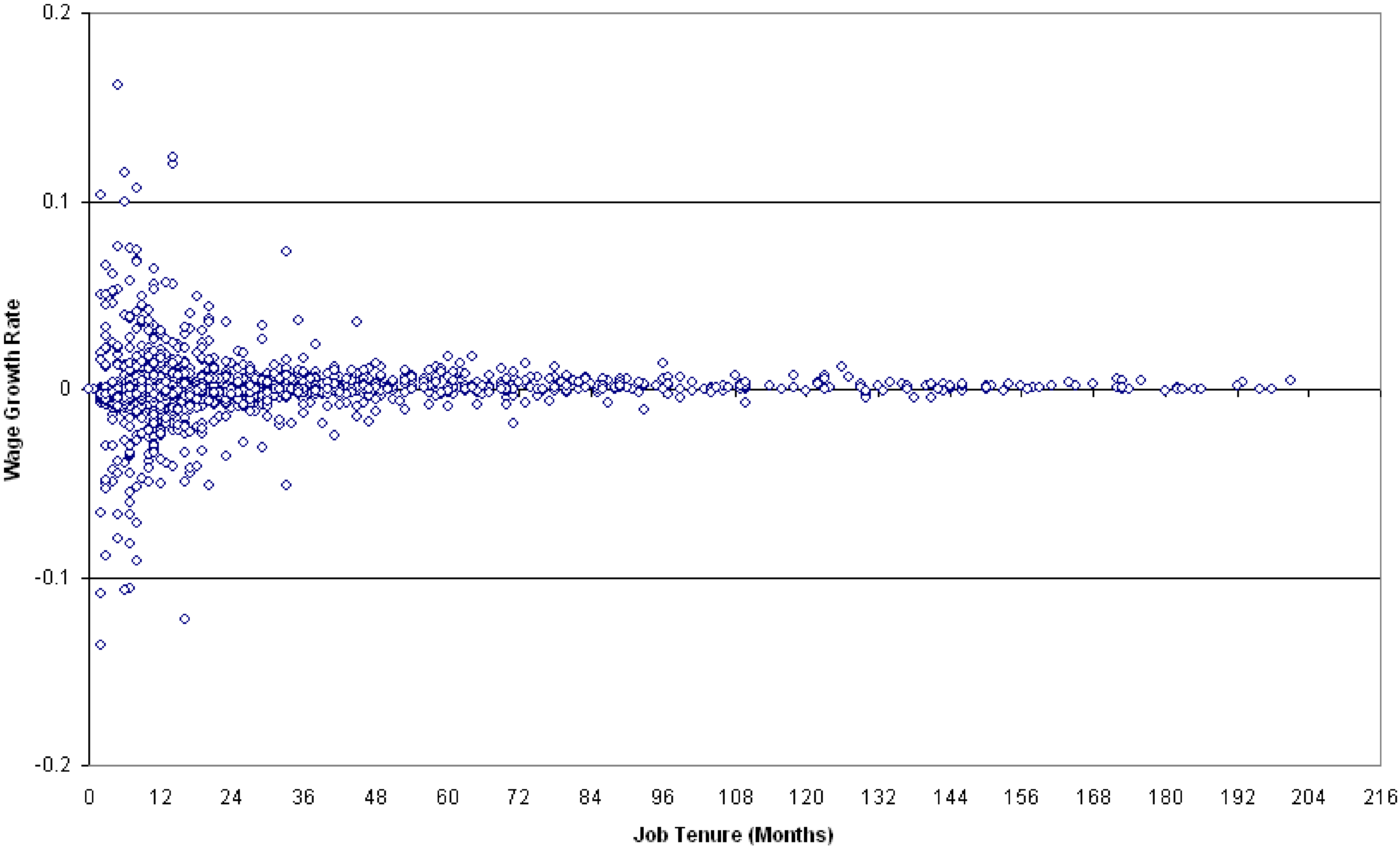


Figure 7: Actual and Predicted Monthly Nonemployment Rate

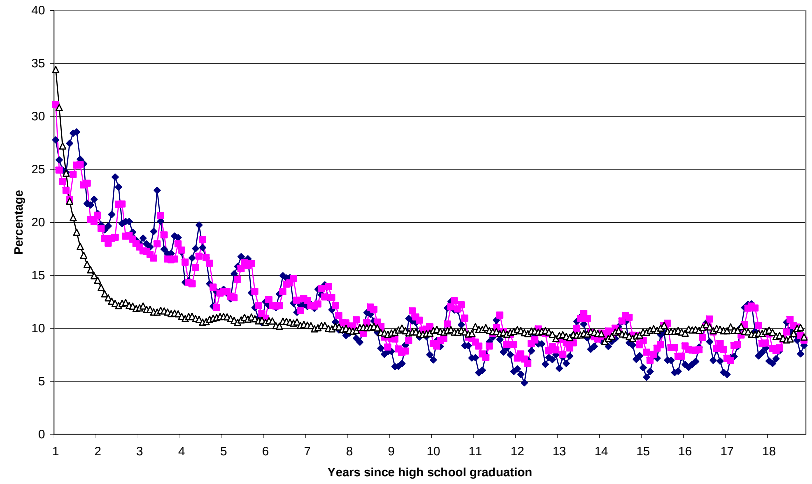


Figure 8: Actual and Predicted Monthly Transition Rate from Nonemployment to Work

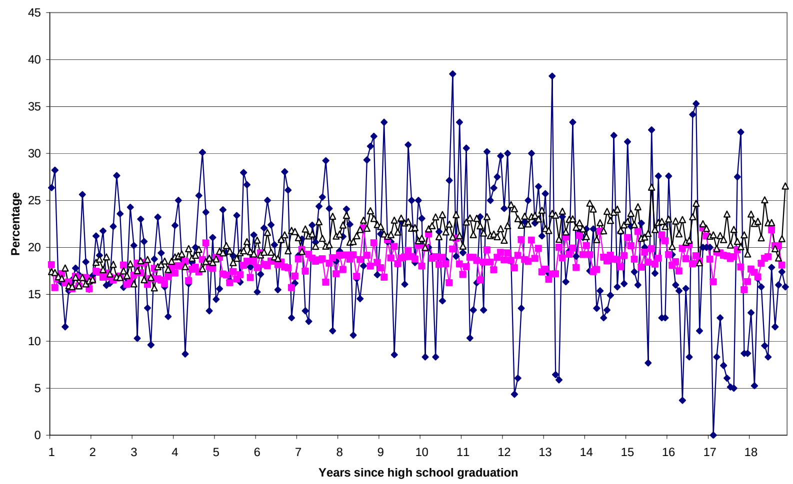


Figure 9: Actual and Predicted Monthly Transition Rate from Job-to-job

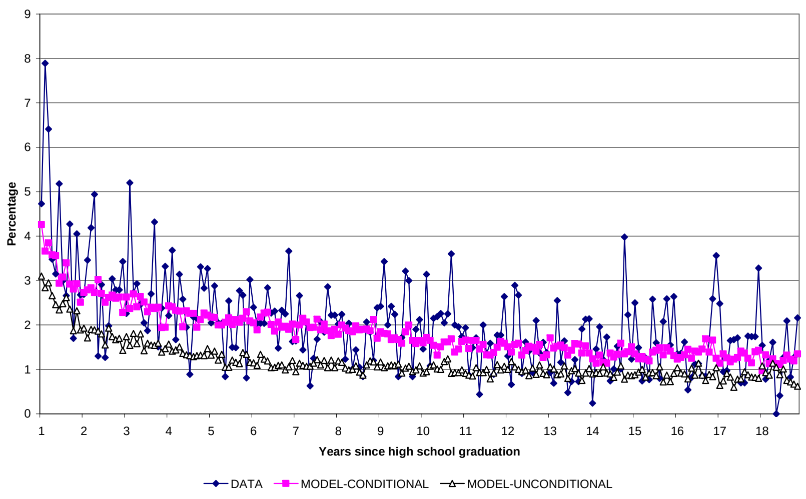


Figure 10: Actual and Predicted Monthly Transition Rate from Employment to Nonemployment

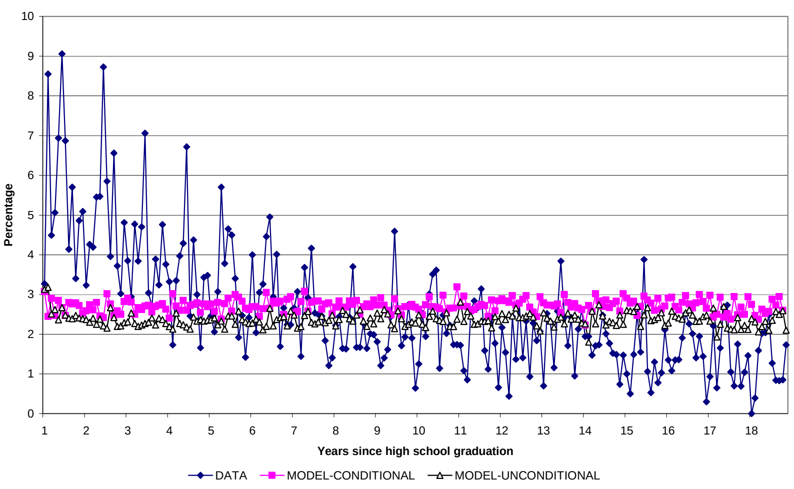


Figure 11: Actual and Predicted Hourly Mean Wage

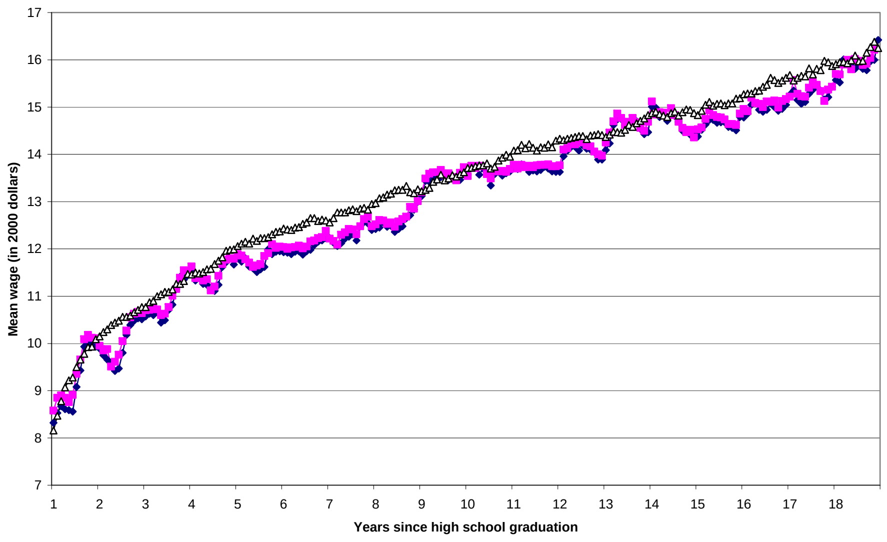


Figure 12: Actual and Predicted Standard Deviation of Hourly Wage

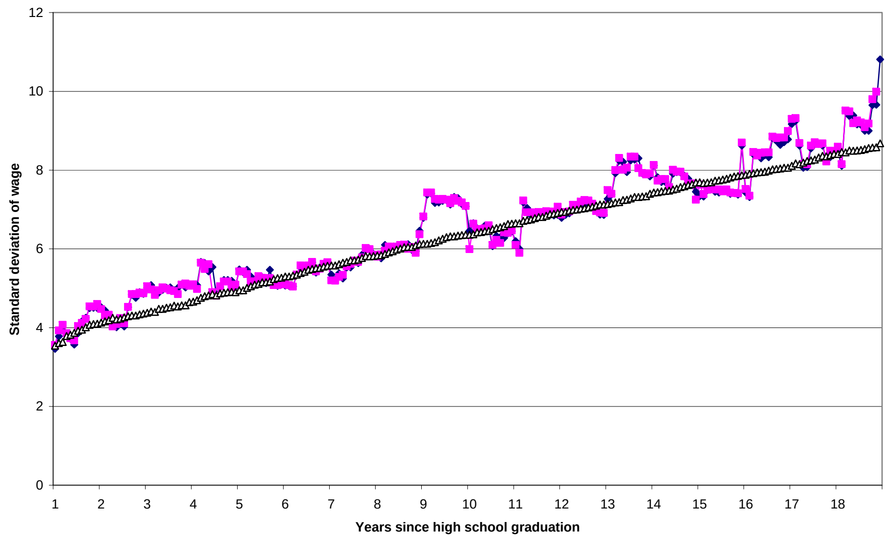


Figure 13: Actual and Predicted Hourly Mean Wage Below the Minimum Wage

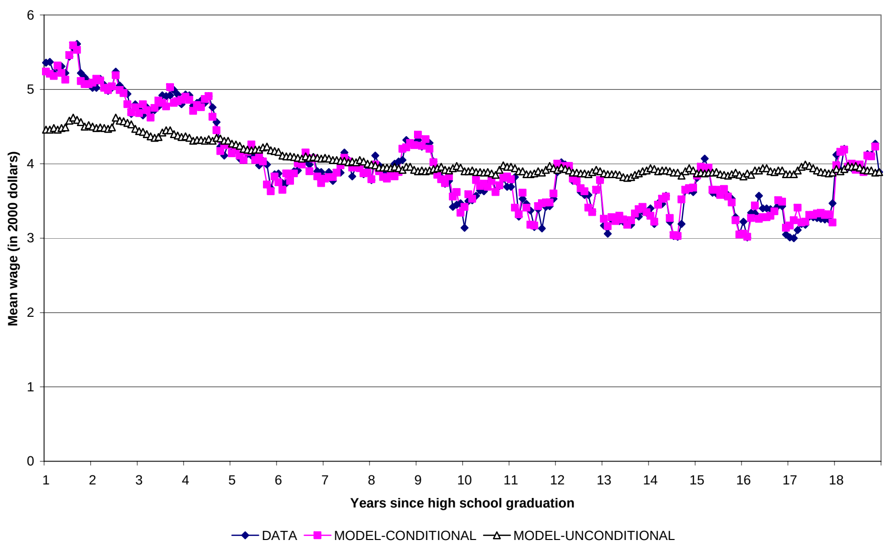


Figure 14: Actual and Predicted Standard Deviation of Hourly Wage Below the Minimum Wage

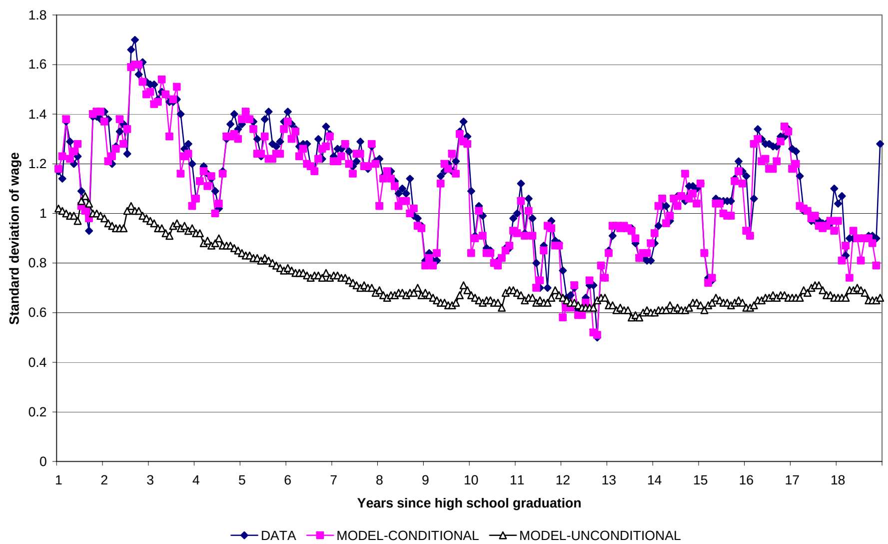
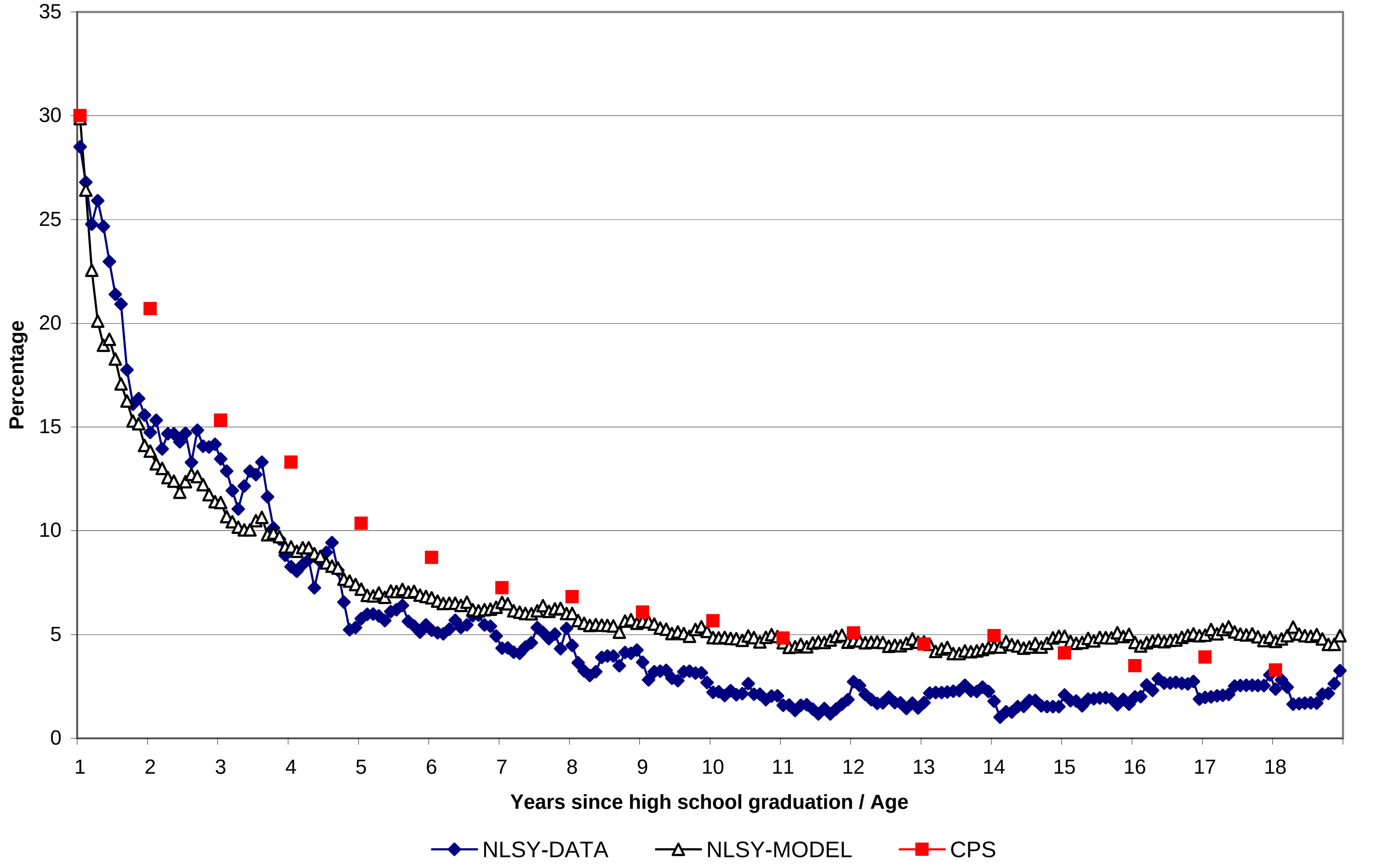


Figure 15: Percentage of Workers Paid Below the Minimum Wage



Note: CPS wage data has the same restriction as for NLSY wage data. We calculate the mean of proportions of workers paid below the minimum wage between 1979-1997 for youth aged between 19 to 36.

Table 9: Data and Predicted Moments

		Cycle One				Cycle Two				Cycle Three			
		NE	Job 1	Job 2	Job 3	NE	Job 1	Job 2	Job 3	NE	Job 1	Job 2	Job 3
Mean Duration	data	8.07	33.96	40.89	35.87	6.73	25.06	27.02	23.33	6.45	21.41	28.05	19.98
(months)	(model)	(8.14)	(26.09)	(32.06)	(37.47)	(7.65)	(27.21)	(30.53)	(26.31)	(6.78)	(22.00)	(22.97)	(21.37)
Mean Wage	data		8.22	10.37	11.85		10.22	11.02	11.49		9.95	11.29	11.81
(2000 dollars)	(model)		(9.32)	(11.96)	(13.25)		(10.41)	(12.42)	(13.06)		(9.54)	(11.03)	(11.72)
S.D. Wage	data		3.43	4.28	4.82		7.50	5.48	5.97		5.25	6.18	6.07
	(model)		(4.28)	(4.78)	(4.54)		(4.41)	(4.89)	(5.20)		(4.34)	(4.65)	(4.73)
Mean wage below w_M	data		4.89	5.17	4.99		4.99	4.96	3.47		4.29	4.72	5.02
(2000 dollars)	(model)		(4.57)	(4.94)	(5.12)		(4.36)	(4.80)	(4.88)		(4.13)	(4.48)	(4.53)
S.D. Wage Below w_M	data		1.38	1.21	1.35		1.20	1.17	1.10		1.56	0.85	1.00
	(model)		(1.00)	(0.91)	(0.94)		(0.97)	(0.81)	(1.10)		(0.87)	(0.73)	(1.00)
Proportion (%)	data		21.90	6.77	1.52		15.11	7.30	8.33		10.33	8.00	5.63
below w_M	(model)		(23.64)	(4.87)	(1.50)		(14.74)	(3.29)	(0.90)		(15.62)	(3.42)	(0.78)
	data	20.10*				53.90**				41.94**			
	(model)	(36.23)				(91.14)				(75.15)			
% moving from	data		40.20				46.62				44.21		
job 1 to job 2	(model)		(56.04)				(36.01)				(35.85)		
% moving from	data			47.92				44.38				46.40	
job 2 to job 3	(model)			(38.51)				(24.30)				(23.61)	

* The proportion of non-employed people in cycle one.

** Proportions of people moving to work from non-employment.