Crises and Prices
Information Aggregation, Multiplicity and Volatility*

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Abstract

Crises, such as currency attacks, bank runs and riots, can be described as times of non-fundamental volatility. We argue that crises are also times when endogenous sources of information are closely monitored and thus an important part of the phenomena. We study the role of endogenous information in generating non-fundamental volatility by introducing a financial market in a coordination game where agents have heterogeneous information about the fundamental. The equilibrium price aggregates information without restoring common knowledge. In contrast to the case with exogenous information, we find that uniqueness may not be obtained as a perturbation from common knowledge: multiplicity is ensured when individuals observe fundamentals with small idiosyncratic noise. Moreover, multiplicity may also emerge in the financial price itself. When the equilibrium is unique, it becomes more sensitive to non-fundamental shocks as noise is reduced. Similar results are obtained when agents observe a signal of aggregate activity instead of a financial price, stressing that the main mechanism for our results is information aggregation.

JEL Codes: D8, E5, F3, G1.

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1 Introduction

It's a love-hate relationship, economists are at once fascinated and uncomfortable with multiple equilibria. On the one hand, economic and political crises involve large and abrupt changes in outcomes, but often lack obvious comparable changes in fundamentals. Many attribute an important role to more or less arbitrary shifts in 'market sentiments' or 'animal spirits', and models with multiple equilibria formalize these ideas. On the other hand, models with multiple equilibria can also be viewed as incomplete theories, which ultimately should be extended along some dimension to resolve the indeterminacy.

The first view is represented by a large empirical and theoretical literature. On the empirical side, Kaminsky (1999), for example, documents that the likelihood of a crisis is affected by observable fundamentals, but that a significant amount of volatility remains unexplained – crises are largely unpredictable. On the theoretical side, models featuring multiple equilibria attempt to address such non-fundamental volatility. Bank runs, currency attacks, debt crises, financial crashes, riots and political regime changes are modeled as a coordination game: attacking a regime – such as a currency peg or the banking system – is worthwhile if and only if enough agents are also expected to attack.[1]

Morris and Shin (1998, 2000, 2003), building on Carlsson and van Damme (1993), contribute to the second view by showing that a unique equilibrium survives in such coordination games when individuals observe fundamentals with small enough private noise. The result is most striking when seen as a perturbation around the original common-knowledge model, which is ridden with equilibria. As the noise in private information vanishes, agents become perfectly informed, yet the equilibrium outcome is uniquely determined. Most importantly, their contribution highlights the importance of the information structure in environments with complementarities.

The aim of this paper is to understand the role of information in crises. We focus on two distinct forms of non-fundamental volatility: the existence of multiple equilibria and the sensitivity of a unique equilibrium to non-fundamental disturbances. We argue that endogenizing public information is crucial for answering these questions.

Information is typically taken as exogenous in coordination models, but is largely endogenous in most situations of interest. Financial prices and macroeconomic indicators convey information regarding others' actions and their beliefs about the underlying fundamentals. Such indicators are monitored intensely during times of crises and appear to be an important part of the phenomena. As an example, consider the Argentine 2001-2002 crisis, which included devaluation of the peso, default on sovereign debt, and suspension of bank payments.

[1] See, for example, Diamond and Dybvig (1983), Obstfeld (1986, 1996), Velasco (1996), Calvo (1988), Cooper and John (1988), Cole and Kehoe (1996). Cooper (1998) provides an excellent review.

Leading up to the crisis throughout 2001, the peso-forward rate and bank deposits deteriorated steadily. These variables, and others, were widely reported by news media and investor reports and closely watched by people making crucial economic decisions.

These observations lead us to introduce endogenous sources of public information in a coordination game. In our baseline model, individuals observe their private signals and the price of a financial asset, whose dividend depends on the underlying fundamentals or the outcome of the coordination game. The rational-expectations equilibrium price aggregates disperse private information, but avoids perfect revelation due to noise in the aggregation process generated by unobservable supply shocks, as in Grossman and Stiglitz (1976).[2]

The main insight to emerge is that the precision of *endogenous* public information increases with the precision of *exogenous* private information. When private signals are more precise, individuals' asset demands are more sensitive to their information. As a result, the equilibrium price reacts relatively more to fundamental than to non-fundamental variables, thus conveying more precise public information.

This result has important implications for the determinacy of equilibria, as a horse-race between private and public information emerges. The direct effect of an increase in the precision of private information is that individuals find it harder to coordinate, as each relies more on her own distinct information. However, the resulting increase in the precision of endogenous public information facilitates coordination by helping individuals better forecast each others' actions. This indirect effect typically dominates and reverses the limit result: multiplicity is *ensured* when individuals observe fundamentals with small enough private noise.

Uniqueness therefore cannot be attained as a small perturbation around common knowledge. To illustrate this point, Figure 1 displays the regions of uniqueness and multiplicity in the space of exogenous levels of public and private noise, σ_ε and σ_x, respectively. Multiplicity is ensured when either noise is sufficiently small. In this sense, public and private noise have symmetric effects.

Interestingly, multiplicity may emerge not only in the regime outcome but also in the asset price. This occurs when the asset's dividend depends on the coordination game. Multiple equilibrium prices are then sustained by different self-fulfilling expectations about the dividend, which in turn are facilitated by the information conveyed by the price.

In regions where the equilibrium is unique, we perform comparative statics. We find that a reduction in exogenous noise, by helping agents better align their choices, may increase the sensitivity of the regime outcome to non-fundamental disturbances. When the dividend is endogenous, this may also increase volatility in the financial price. Thus, lower noise may

[2] Atkeson (2000) first pointed out that perfectly-revealing asset markets could restore common knowledge. By introducing noise in the aggregation process, we ensure that none of our results are driven by restoring common knowledge.

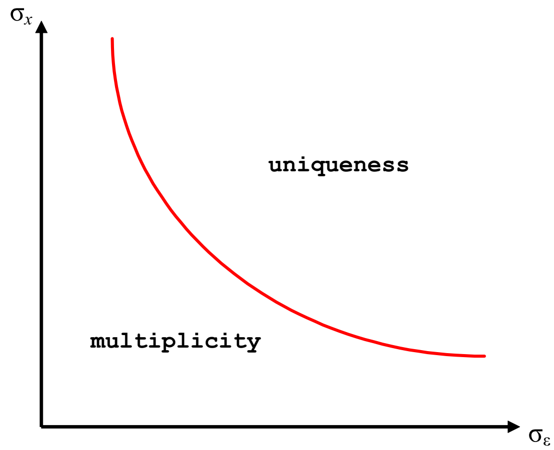

Figure 1: σ_x measures the exogenous noise in private information and σ_ε the exogenous public noise in the aggregation of information.

increase volatility by either introducing multiplicity or making the unique equilibrium more sensitive to non-fundamental shocks.

Motivated by bank run and riot applications, we also consider a model in which individuals do not trade a financial asset but instead directly observe a noisy public signal of the action of others. This model introduces endogenous public information in the Morris-Shin framework with minimal modifications. It also brings a main element of herding models, the observation of other players' actions, into coordination games. Our results on equilibrium multiplicity carry over here, illustrating that the key mechanism is information aggregation.

Related Literature. Our main interest is the role of information and coordination in non-fundamental volatility. Chari and Kehoe (2003) also focus on non-fundamental volatility as the distinct feature of crises, but within the context of a herding model. The model in Section 4, which allows individuals to observe signals of others actions, brings coordination games and herding models closer together.

Our analysis builds on Morris and Shin (1998, 2000, 2003), underscoring their general theme that the information structure is crucial in coordination games. Our focus is on endogenous sources of public information, such as financial prices and other macroeconomic indicators that aggregate private information dispersed in the economy. Atkeson (2000), in his discussion of Morris and Shin (2000), was the first to highlight the potential role of financial prices as endogenous sources of public information. Angeletos, Hellwig and Pavan (2003, 2004) also endogenize information, but in different ways: they examine the signaling and coordinating effects of policy, and the dynamics of information in a dynamic global game.

Closely related is Hellwig, Mukherji and Tsyvinski (2004), who consider a currency-crises model in which financial prices directly affect the coordination outcome. In particular, they focus on how the determinacy of equilibria depends on whether the central bank's decision to devalue is triggered by large reserve losses or high interest rates. In their model, multiple equilibria also survive for small levels of noise.

Tarashev (2003) also endogenizes interest rates in currency crises, but in a model where the equilibrium is always unique. Dasgupta (2003) introduces signals of others' actions in an investment game, but assumes that these signals are entirely private, instead of public as in our paper. Morris and Shin (2002) and Angeletos and Pavan (2004), on the other hand, consider the volatility and welfare effects of exogenous public information in a class of models where complementarities are weak enough that the equilibrium is always unique.

Finally, this paper contributes to the rational-expectations literature by introducing a coordinating role for financial prices. In rational expectations equilibria the payoff of an agent is typically independent of the actions of other agents given the price. Thus, the equilibrium price only provides information regarding the exogenous dividend. In contrast, in our framework the price also helps agents to predict each others' actions and align their choices beyond the financial market. This novel coordinating role is crucial for our results regarding price multiplicity and volatility.[3]

Section 2 introduces the basic model and reviews the exogenous information benchmark. Section 3 incorporates an asset market and examines the determinacy of equilibria. Section 4 considers comparative statics in regions with a unique equilibrium. Section 5 studies the model with direct signals on the actions of others. Section 6 concludes.

2 The Basic Model: Exogenous Information

Before introducing a financial price or other endogenous public signals, we briefly review the backbone of our model with exogenous information, as in Morris and Shin (1999, 2000).

Actions and Payoffs. There is a status quo and a measure-one continuum of agents, indexed by $i \in [0,1]$. Each agent i can choose between two actions, either attack the status quo $a_i = 1$, or not attack $a_i = 0$. The payoff from not attacking is normalized to zero. The payoff from attacking is $1 - c > 0$ if the status quo is abandoned and $-c$ otherwise, where $c \in (0,1)$ parametrizes the cost of attacking. The status quo, in turn, is abandoned if and only if $A > \theta$, where A denotes the mass of agents attacking and θ is the exogenous fundamental

[3] Barlevy and Veronessi (2004) consider a Grossman-Stiglitz environment that admits multiple equilibria, but the source of multiplicity there is entirely different from ours. In their model, the dividend is exogenous, so the price does not play a coordinating role. Instead, multiplicity emerges from the non-linearity of the inference problem faced by uninformed traders when interacting with informed and less risk-averse agents.

representing the strength of the status quo. It follows that the payoff of agent i is

$$U(a_i, A, \theta) = a_i(R(A, \theta) - c), \tag{1}$$

where $R(A, \theta)$ denotes the regime outcome, with $R(A, \theta) = 1$ if $A > \theta$ and $R(A, \theta) = 0$ otherwise.

The key property of the payoff structure is a coordination motive due to the strategic complementarity: $U(1, A, \theta) - U(0, A, \theta)$ increases with A, so the incentive to attack increases with the mass of agents attacking. If θ were commonly observed by all agents, both $A = 1$ and $A = 0$ are an equilibrium whenever $\theta \in (\underline{\theta}, \overline{\theta}] \equiv (0, 1]$. This interval represents the set of critical fundamentals over which the regime outcome depends on the size of the attack.

Interpretations. In models of self-fulfilling currency crises, as in Obstfeld (1986, 1996) and Morris and Shin (1998), the central bank is forced to abandon its peg when a sufficiently large group of speculators attacks the currency; θ then parametrizes the amount of foreign reserves or the ability and willingness of the central bank to maintain its peg. In models of bank runs, such as Goldstein and Pauzner (2000) and Rochet and Vives (2004), a regime change occurs when a large enough number of depositors decide to withdraw their deposits, forcing the banking system to suspend payments. Another possible interpretation of the model is an economy with investment complementarities, as in Cooper and John (1988) and Dasgupta (2003).[4]

Information. Following Morris-Shin, suppose θ is not common knowledge. In the beginning of the game, nature draws θ from a given distribution, which constitutes the agents' common prior about θ. For simplicity, the prior is taken to be the improper uniform over the entire real line. Agent i then receives a private signal $x_i = \theta + \sigma_x \xi_i$, with $\sigma_x > 0$ and $\xi_i \sim \mathcal{N}(0, 1)$ is independent of θ, and independently distributed across agents. Agents also observe an *exogenous* public signal $z = \theta + \sigma_z v$, where $\sigma_z > 0$ and $v \sim \mathcal{N}(0, 1)$ is common noise, independent of both θ and ξ.[5] The information structure is parametrized by the standard deviations σ_x and σ_z; or, equivalently, by $\alpha_x = \sigma_x^{-2}$ and $\alpha_z = \sigma_z^{-2}$, the *precisions* of private and public information.

Equilibrium. Throughout the paper, we focus on *monotone equilibria* defined as perfect Bayesian equilibria such that, for given a realization z of the public signal, an agent attacks if and only if the realization x of his private signal is less than some threshold $x^*(z)$.[6]

[4] Other applications include debt crises, financial crashes, and riots (Cole and Kehoe, 1996; Atkeson, 2000; Morris and Shin, 2004a, 2004b; Corsetti, Guimaraes, and Roubini, 2004).

[5] Normality makes the analysis of the effects of public information tractable (see Morris and Shin, 1999, 2000, 2003).

[6] Our main results concerning multiple equilibria are obtained even within this restricted class. Moreover, in the case of exogenous information, when monotone equilibria are unique it is known that there are no

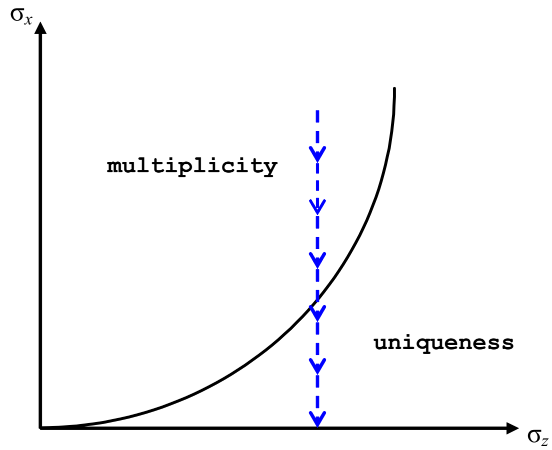

Figure 2: σ_x and σ_z parametrize the noise in private and public information; uniqueness is ensured for σ_x small enough.

Proposition 1 (Morris-Shin) *In the game with exogenous information, the equilibrium is unique if and only if* $0 < \sigma_x \leq \sigma_z^2\sqrt{2\pi}$.

Proof. See Appendix. ■

Figure 2 depicts the regions of (σ_x, σ_z) for which the equilibrium is unique. For any positive σ_z, uniqueness in ensured by a sufficiently small positive σ_x. The key intuition behind this result is that private information anchors individual behavior and limits the ability to forecast each others actions, which is required to coordinate on multiple equilibria.

The higher the precision of private information, for given precision of public information, the more heavily individuals condition their actions on it. Since private information is diverse this makes it more difficult for individuals to predict the actions of others. When σ_x is sufficiently small, relative to σ_z, this effect is strong enough and the ability to coordinate on multiple courses of action breaks down.

Moreover, as $\sigma_x \to 0$ individuals cease to condition their actions on the public signal so the equilibrium dependence on the common noise ε vanishes.

Corollary 1 *In the limit as* $\sigma_x \to 0$*, there is a unique equilibrium in which* $R(\theta, z) \to 1$ *if* $\theta < \hat{\theta}$ *and* $R(\theta, z) \to 0$ *if* $\theta > \hat{\theta}$*, where* $\hat{\theta} = 1 - c$.

This limit illustrates a sharp discontinuity of the equilibrium set around $\sigma_x = 0$: a small perturbation away from perfect information suffices to obtain a unique equilibrium. Moreover,

non-monotone equilibria.

it implies that crises, defined as situations displaying high non-fundamental volatility, cannot be addressed in the limit as $\sigma_x \to 0$.

3 Endogenous Information I: Financial Prices

The results above presume that the precision of public information remains invariant while varying the precision of private information. We argue that this is unlikely to be the case when public information is endogenous through financial prices.

To investigate the role of prices, we introduce a financial market where agents trade an asset prior to playing the coordination game. Because the dividend is a function of θ or A, the the equilibrium price will convey information that is valuable in the coordination game.

3.1 Asset market

As before, nature draws θ from an improper uniform distribution over the real line and each agent receives the exogenous private signal $x_i = \theta + \sigma_x \xi_i$. We avoid direct payoff linkages between the financial market and the coordination game to isolate and focus on information aggregation. Agents can be seen as interacting in two separate stages.

In the first stage agents trade over a risky asset with dividend $f = f(\theta, A)$ at price p. We assume a constant absolute risk aversion utility function over consumption generated from this portfolio choice. Thus, utility is $-e^{-\gamma c_i}/\gamma$ for $\gamma > 0$, where consumption is given by $c_i = w - pk_i + fk_i$, and k_i denotes the investment in the asset.

The net supply of the asset is uncertain and not observed, given by $K^s(\varepsilon) = \sigma_\varepsilon \varepsilon,$ where $\sigma_\varepsilon > 0$ and $\varepsilon \sim \mathcal{N}(0,1)$ and independent of θ and ξ_i. As in Grossman and Stiglitz (1976), the role of the unobserved shock ε is to introduce noise in the information revealed by the market-clearing price. In this way, σ_ε parameterizes the exogenous noise in the aggregation process.

The second stage is essentially the same as the benchmark model of the previous section: agents choose whether to attack or not; the status quo is abandoned if and only if the mass of agents attacking, A, exceeds θ; and the payoff of the agent from this stage is $U(a_i, A, \theta) = a_i(R(A, \theta) - c)$. The only difference is that agents now observe the price that cleared the financial market in stage 1. The regime outcome, the asset's dividend, and the payoffs from both stages are realized at the end of stage 2.

Individual asset demands and attack decisions are functions of x and p, the realizations of the private signal and the price. The corresponding aggregates are then functions of θ and p. We define an equilibrium as follows.

Definition 1 *An equilibrium is a price function, $P(\theta,\varepsilon)$, individual strategies for investment and attacking, $k(x,p)$ and $a(x,p)$, and their corresponding aggregates, $K(\theta,p)$ and $A(\theta,p)$, such that:*

$$k(x,p) \in \arg\max_{k\in\mathbb{R}} \mathbb{E}\left[\, V((f-p)\,k) \mid x,p \,\right] \tag{2}$$

$$K(\theta,p) = \int_x k(x,p)\phi\left(\tfrac{x-\theta}{\sigma_x}\right) dx \tag{3}$$

$$K\left(\theta, P(\theta,\varepsilon)\right) = K^s\left(\varepsilon\right) \tag{4}$$

$$a(x,p) \in \arg\max_{a\in[0,1]} \mathbb{E}\left[\, U(a, A(\theta,p),\theta) \mid x,p \,\right] \tag{5}$$

$$A(\theta,p) = \int_x a(x,p)\phi\left(\tfrac{x-\theta}{\sigma_x}\right) dx \tag{6}$$

Let $R\left(\theta,\varepsilon\right) = R\left(\theta, A\left(\theta, P\left(\theta,\varepsilon\right)\right)\right)$ denote the equilibrium regime outcome.

Equations (2)-(4) define a rational-expectations competitive equilibrium for stage 1. The price must clears the asset market (4), where demand is determined by individuals that condition on all available information as in (2), including anything inferable from the price realization $p = P(\theta,\varepsilon)$. Equations (5)-(6) define a perfect Bayesian equilibrium for stage 2.

3.2 Exogenous dividend

We consider first the case where the dividend depends only on the exogenous fundamental and $f = \theta$. Following Grossman and Stiglitz (1976), we focus on the unique equilibrium in the asset market with a linear price function that is not perfectly revealing.[7]

Observing the price realization is then equivalent to observing a Normal signal with some precision $\alpha_p \geq 0$. The posterior of θ conditional on x and p is then Normal with mean $\delta x + (1-\delta)p$ and precision α, where $\delta = \alpha_x/\alpha$ and $\alpha = \alpha_x + \alpha_p$. Individual's asset demands are

$$k(x,p) = \frac{\mathbb{E}[f|x,p] - p}{\gamma \mathrm{Var}[f|x,p]} = \frac{\delta\alpha}{\gamma}\left(x-p\right), \tag{7}$$

Aggregate demand is then $K(\theta,p) = \left(\delta\alpha/\gamma\right)\left(\theta - p\right)$ and market clearing, $K(\theta,p) = K^s\left(\varepsilon\right) = \sigma_\varepsilon \varepsilon$, implies

$$P\left(\theta,\varepsilon\right) = \theta - \sigma_p \varepsilon, \tag{8}$$

where $\sigma_p = (\delta\alpha/\gamma)^{-1}\sigma_\varepsilon$. This verifies the linear price function guess and using $\delta = \alpha_x/\alpha$ and $\alpha = \alpha_x + \sigma_p^{-2}$, we obtain

$$\sigma_p = \gamma\sigma_\varepsilon\sigma_x^2. \tag{9}$$

[7] In Grossman and Stiglitz's setup, the perfectly revelaing equilibrium seems implausible, and it is not known whether other non-linear equilibrium price functions exist.

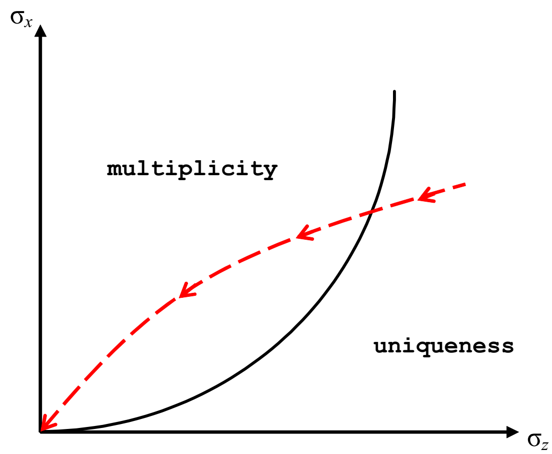

Figure 3: With endogenous public information, as σ_x decreases, σ_z also decreases; multiplicity is therefore ensured for sufficiently small σ_x.

Thus, the precision of public information increases with the precision of private information. This is the key observation of the paper and has important implications for the determination and characterization of equilibria in the coordination game.

Stage 2 is then equivalent to the benchmark model of Section 2, with the price p playing the role of the public signal z.

Proposition 2 *In the asset economy with exogenous dividend $f = \theta$, there are multiple equilibria if $\sigma_\varepsilon^2 \sigma_x^3 < \gamma^2 (2\pi)^{-1/2}$.*

Proof. Follows from equation (9) and Proposition 1. ■

In Proposition 1 the noise in public information, σ_z, was fixed, so a sufficiently low σ_x ensured uniqueness. In contrast, here better private information improves public information, and at a fast enough rate to ensure multiplicity. The result is illustrated in Figure 3. In contrast to Figure 2, as σ_x decreases, σ_z also decreases, eventually entering the multiplicity region.

An immediate implication is that uniqueness can no longer be seen as a small perturbation away from common knowledge: multiplicity is ensured when either σ_x or σ_ε are small, as illustrated in Figure 1. Indeed, both extreme common-knowledge outcomes can be recovered as *either* noise vanishes.

Corollary 2 *Consider the limit as $\sigma_x \to 0$ for given σ_ε, or the limit as $\sigma_\varepsilon \to 0$ for given σ_x. There exists an equilibrium in which $R(\theta, \varepsilon) \to 0$ whenever $\theta \in (\underline{\theta}, \overline{\theta})$, as well as an equilibrium in which $R(\theta, \varepsilon) \to 1$ whenever $\theta \in (\underline{\theta}, \overline{\theta})$. In every equilibrium, $P(\theta, \varepsilon) \to \theta$ for all (θ, ε).*

3.3 Endogenous dividend

We now consider the case where the asset's dividend is endogenously determined by the coordination game and $f = f(A)$. To preserve normality of the information structure we take $f(A) = -\Phi^{-1}(A)$.

As was the case with an exogenous dividend, the precision of the information conveyed endogenously by the price increases with the precision of exogenous private information. Again, this guarantees multiplicity for small levels of noise. A novel implication here is that multiplicity also emerges in the financial price.

The reason for this is the feedback between the asset market and the coordination game. As before, the price conveys information valuable for the coordination game. But now the outcome of the coordination game determines the dividend of the asset.

Proposition 3 *In the asset economy with endogenous dividend $f = f(A)$, there are multiple equilibria if $\sigma_\varepsilon^2 \sigma_x^5 < \gamma^2 (2\pi)^{-1/2}$. Multiplicity then emerges in both the regime outcome, $R(\theta, \varepsilon)$, and the price function, $P(\theta, \varepsilon)$.*

Proof. In monotone equilibria agents attack if and only if their private signal is below some threshold $x^*(p)$. Then $A(\theta, p) = \Phi\left(\sqrt{\alpha_x}(x^*(p) - \theta)\right)$ and the realized dividend is

$$f = \sqrt{\alpha_x}(\theta - x^*(p)). \tag{10}$$

Note that since p is observed agents can calculate $\tilde{p} = p/\sqrt{\alpha_x} + x^*(p)$, which represents the price of an asset that pays $\tilde{f} = f/\sqrt{\alpha_x} + x^*(p) = \theta$. We focus on equilibria with a one-to-one mapping between p and $\tilde{p}$, so that the observation of p is equivalent to the observation of $\tilde{p}$.

We guess and verify that the posterior for θ is Normal with mean $\delta x + (1 - \delta)\tilde{p}$ and precision α, where $\delta = \alpha_x/\alpha$ and $\alpha = \alpha_x + \alpha_p$, for some $\alpha_p = \sigma_p^{-2} \geq 0$. The asset demand is then given by

$$k(x, p) = \frac{\mathbb{E}[f|x, p] - p}{\gamma \mathrm{Var}[f|x, p]} = \frac{\delta\alpha}{\tilde{\gamma}}(x - \tilde{p}),$$

where $\tilde{\gamma} = \gamma\sqrt{\alpha_x}$, and market clearing gives

$$\tilde{p} = \theta - \sigma_p \varepsilon, \tag{11}$$

with $\sigma_p = (\delta\alpha/\tilde{\gamma})^{-1}\sigma_\varepsilon$. Our guess for the posterior for θ is then verified with

$$\sigma_p = \gamma \sigma_\varepsilon \sigma_x. \tag{12}$$

The precision of public information once again increases with the precision of private information. The thresholds $\theta^*(p)$ and $x^*(p)$ then solve the analogues of (22) and (23) from the benchmark model once we replace z with $\tilde{p} = p/\sqrt{\alpha_x} + x^*(p)$ and α_z with α_p. This gives a unique solution for $\theta^*(p)$ and $x^*(p)$:

$$\theta^*(p) = \Phi\left(\sqrt{\tfrac{\alpha_x}{\alpha_x+\alpha_p}}\Phi^{-1}(1-c) - \tfrac{\alpha_p}{\alpha_x+\alpha_p}p\right) \quad \text{and} \quad x^*(p) = \theta^*(p) + \tfrac{1}{\sqrt{\alpha_x}}\Phi^{-1}(\theta^*(p)) \tag{13}$$

To complete the analysis, we determine the mapping between p and $\tilde{p}$, which gives the equilibrium price function $p = P(\theta, \varepsilon)$. Since $x^*(p)$ is uniquely determined, the aggregate demand $K(\theta, p) = \left(\sqrt{\alpha_x}/\gamma\right)\left[\theta - p/\sqrt{\alpha_x} - x^*(p)\right]$ is also uniquely determined. Moreover, $K(\theta, p)$ is continuous in p, with $\lim_{p\to-\infty} K(\theta, p) = \infty$ and $\lim_{p\to+\infty} K(\theta, p) = -\infty$. Thus, the market clearing condition $K(\theta, p) = K^s(\varepsilon)$, or equivalently $\tilde{p} = p/\sqrt{\alpha_x} + x^*(p)$, necessarily admits a solution in p. However, since a component of the dividend $x^*(p)$ is decreasing in p, the demand $K(\theta, p)$ need not be monotonic in p. Thus, the equilibrium price need not be unique. Indeed,

$$sign\left\{\frac{\partial K(\theta, p)}{\partial p}\right\} = -sign\left\{\frac{\sqrt{\alpha_x}}{\alpha_p} - \phi\left(\Phi^{-1}(\theta^*)\right)\right\},$$

so that $K(\theta, p)$ is everywhere decreasing in p if and only if $\sqrt{\alpha_x}/\alpha_p \geq \sqrt{2\pi}$, or equivalently $\sigma_\varepsilon^2\sigma_x^3 \geq \gamma^2/\sqrt{2\pi}$. When instead, $\sigma_\varepsilon^2\sigma_x^3 < \gamma^2/\sqrt{2\pi}$ there is a non-empty interval $(\tilde{p}_1, \tilde{p}_2)$ where $\tilde{p} = p/\sqrt{\alpha_x} + x^*(p)$ admits three solutions for p, so there are multiple equilibrium price functions. ■

When the dividend was exogenous and given by $f = \theta$ the price played the role of a public signal of the fundamental θ. As a result, multiplicity emerged solely in the coordination stage, not in the asset market. In contrast, here the asset's dividend depends on the size of the attack, so its price acts as an anticipatory signal of A. Multiple equilibria in the coordination stage then feed back into multiple equilibrium prices.

In equilibrium, a higher price realization makes agents more inclined to attack, raising the dividend directly. This effect may make the asset demands non-monotonic in p over some region and give rise to multiple market-clearing prices. Since multiplicity occurs for low values of σ_x and σ_ε, a potential form of non-fundamental price volatility, occurs somewhat paradoxically, in situations of low exogenous noise.

Note that multiplicity emerges in the price and regime outcome, but not in individual strategies given the price realization. In this sense, price multiplicity is crucial for the equilibrium multiplicity. To gain some intuition for this result, consider the common-knowledge case with $\sigma_x = 0$. Then $x = \theta$ and $p = f = -\Phi^{-1}(A)$, so it is optimal to attack if and only if $A \geq \theta$, or equivalently $x \leq \Phi(-p)$. Thus, equilibrium strategies are uniquely determined even though the equilibrium price function $P(\theta, \varepsilon)$ and aggregate attack $A(\theta, \varepsilon)$ are not. Indeed, for every

$\theta \in (\underline{\theta}, \overline{\theta})$, both $(p, A) = (\infty, 0)$ and $(p, A) = (-\infty, 1)$ are consistent with an equilibrium.

Both extreme common-knowledge outcomes are approached as either noise vanishes.

Corollary 3 *Consider the limit as $\sigma_x \to 0$ for given σ_ε, or the limit as $\sigma_\varepsilon \to 0$ for given σ_x. There is an equilibrium in which $R(\theta, \varepsilon) \to 1$ and $P(\theta, \varepsilon) \to -\infty$ whenever $\theta \in (\underline{\theta}, \overline{\theta})$, as well as an equilibrium in which $R(\theta, \varepsilon) \to 1$ and $P(\theta, \varepsilon) \to -\infty$ whenever $\theta \in (\underline{\theta}, \overline{\theta})$.*

Our model with exogenous dividend purposely limits the interactions between the coordination and asset market to focus on information aggregation. The endogenous dividend case features a additional feedback from the coordination game to the asset market, which is the source of multiplicity in the asset price. Neither case however allows for the asset price to have a direct payoff effect in the coordination game. Hellwig, Mukherji, and Tsyvinski (2004), on the other hand, consider a currency-crises model in which the coordination game is embedded in the financial market and the price directly affects the agent's payoff from attacking and the central bank's devaluation decision. In their case, multiplicity is also ensured for small levels of noise.

3.4 Discussion

Our key result is that the precision of endogenous public information increases with the precision of exogenous private information. This feature is likely to be very robust and carries with it the important implication that lower values of private noise σ_x do not necessarily contribute towards uniqueness.

The simplest model featuring information aggregation, without financial prices, selects N individuals at random to be on a 'talk show'. Those on the show broadcast their signals to the rest of the population. This amounts to generating a public signal $z = \theta + \sigma_z v$ with $\sigma_z = \sigma_x / \sqrt{N}$. In this case, public communication links the precision of private and public information in such a way that equilibrium multiplicity is ensured for σ_x small enough.

In the model of the next section, there is neither a financial price nor direct communication, but a form of social learning. Once again, multiplicity is ensured for σ_x small enough.

In all these cases, a smaller σ_x facilitates multiplicity because the precision of public information increases at a faster rate than the square root of the precision of private information. We next show, however, that this property need not obtain in some variations of our asset-market model that introduce additional noise in the aggregation process.

Consider, in particular, an extension where the dividend is not perfectly correlated with the fundamental or the coordination outcome: $f = \theta + \eta$ or $f = f(A) + \eta$, where $\eta \sim \mathcal{N}\left(0, \sigma_\eta^2\right)$ is independent of $(\theta, \xi, \varepsilon)$. This extension is treated formally in Proposition A1 in the Appendix; here we discuss the main findings.

The equilibrium price continues to aggregate information, but the risk introduced by η limits the sensitivity of asset demands to changes in expected excess returns. With exogenous dividend, this effect implies a upper bound on the precision of the information revealed by the price. As a result, for any given $(\sigma_\eta, \sigma_\varepsilon) > 0$, multiplicity holds for an intermediate range of σ_x, but not in the limit as $\sigma_x \to 0$. With endogenous dividend, however, the sensitivity of the dividend itself to θ increases with the precision of private information, overturning the previous dampening effect. As a result, multiplicity now obtains even in the limit as $\sigma_x \to 0$.

Finally, with either endogenous or endogenous dividend, less noise in the form of smaller σ_ε or σ_η contributes to multiplicity. In particular, for any $(\sigma_x, \sigma_\varepsilon)$ for which multiplicity was obtained when $\sigma_\eta = 0$, multiplicity is again ensured as long as σ_η is positive but small enough.

We conclude that, while some extensions may qualify the limit result for σ_x, they are unlikely to modify our main conclusion that endogenous public information is important for understanding the determinacy of equilibria and the level of non-fundamental volatility.

4 Endogenous Information II: Signal on Aggregate Actions

In this Section, we remove the financial market and examine instead situations where information originates *within* the coordination game itself: agents observe a public signal of the aggregate attack. The resulting model bridges a gap between coordination and herding models that stress the observation of other player's actions.

We start with a version that requires a minimal modification of the exogenous-information benchmark: agents move simultaneously while observing a public signal about the *contemporaneous* aggregate attack. In this case, our equilibrium concept is a hybrid of game theoretic and rational-expectations concepts. We then show that similar results obtain in a sequential version of the model that allows for an equilibrium concept that is entirely game-theoretic.

4.1 Simultaneous moves

The model is similar to the benchmark model from Section 2. The prior and private signals are as before, but the public signal z is replaced with

$$y = s(A, \varepsilon)$$

where $s : [0,1] \times \mathbb{R} \to \mathbb{R}$ and ε is noise independent of θ and ξ. To preserve Normality of the information structure and obtain closed-form solution, we take $s(A, \varepsilon) = \Phi^{-1}(A) + \sigma_\varepsilon \varepsilon$ and

$\varepsilon \sim \mathcal{N}(0,1)$.[8] The exogenous information structure is parameterized by the pair of standard deviations $(\sigma_x, \sigma_\varepsilon)$.

Since the information agents posses include a signal of contemporaneous actions, our equilibrium concept, just as in the asset market model of Section 3, is a hybrid of a rational expectations and perfect Bayesian equilibrium.

Definition 2 *An equilibrium consists of an endogenous signal $y = Y(\theta, \varepsilon)$, an individual attack strategy $a(x, y)$, and an aggregate attack $A(\theta, y)$, that satisfy:*

$$a(x,y) \in \arg\max_{a \in [0,1]} \mathbb{E}\left[\, U(a, A(\theta,y), \theta) \mid x, y \,\right] \tag{14}$$

$$A(\theta,y) = \int_x a(x,y)\phi\left(\frac{x-\theta}{\sigma_x}\right) dx \tag{15}$$

$$y = s(A(\theta,y), \varepsilon) \tag{16}$$

for all $(\theta, \varepsilon, x, y) \in \mathbb{R}^4$.

Equation (14) requires the attack choice to be optimal given all available information, including the realized signal y of the aggregate attack. The fixed point in (16) is the rational-expectations element, that requires the signal y to be generated by individual actions that are, in turn, contingent on y.

In monotone equilibria, an agent attacks if and only if $x \leq x^*(y)$ and the status quo is abandoned if and only if $\theta \leq \theta^*(y)$, so an equilibrium is identified with a triplet of functions x^*, θ^*, and Y. As before, we focus on equilibria that preserve Normality of the information structure.[9]

As shown in the appendix, the equilibrium analysis is very similar to that of the endogenous dividend model from Section 3.3. Here agents receive a direct signal on A, while there the price was an indirect signal of A, but, in equilibrium, both y and p convey the same information.

Indeed, the noise in the endogenous public information generated by y turns out to be

$$\sigma_y = \sigma_x \sigma_\varepsilon,$$

implying that multiplicity once again survives for small levels of noise. Moreover, when multiplicity arises, it is with respect to aggregate outcomes and not with respect to individual behavior.

[8] This convenient specification was introduced by Dasgupta (2003) in a different environment.

[9] Formally, we consider equilibria such that $G(Y(\theta,\varepsilon)) = \lambda_1\theta + \lambda_2\varepsilon$ for some strictly monotone function G and non-zero coefficients λ_1, λ_2.

Proposition 4 *An equilibrium exists for all* $(\sigma_x, \sigma_\varepsilon)$ *and is unique if and only if* $\sigma_\varepsilon^2 \sigma_x \geq 1/\sqrt{2\pi}$. *If* $\sigma_\varepsilon^2 \sigma_x < 1/\sqrt{2\pi}$, *the equilibrium strategy* a *remains unique, but there are multiple signal functions* Y.

Proof. See Appendix. ■

Finally, common-knowledge outcomes once again obtain as either noise vanishes. On the other hand, the Morris-Shin limit is obtained as $\sigma_\varepsilon \to \infty$, since y then becomes completely uninformative.

4.2 Sequential moves

The model above assumed that agents can condition their decision to attack on a noisy indicator of *contemporaneous* aggregate behavior. Here we extend the model so that nobody has information about contemporaneous actions of others. This allows us to use standard game-theoretic equilibrium concepts.

The population is divided into two groups, 'early' and 'late' agents. Neither group observes contemporaneous activity. Early agents move first, on the basis of their private information alone. Late agents move second, on the basis of their private information as well as a noisy public signal about the aggregate actions of early agents.

Let $\mu \in (0,1)$ denote the fraction of early agents, and A_1 and A_2 be the fraction of early and late agents that attack, respectively. Late agents observe

$$y_1 = \Phi^{-1}(A_1) + \varepsilon, \tag{17}$$

where $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ is independent of θ and ξ. The regime changes if and only if the aggregate attack, $A = \mu A_1 + (1-\mu)A_2$, exceeds θ.

We look for monotone perfect Bayesian equilibria, which are summarized by is a scalar $x_1^* \in \mathbb{R}$ and a pair of functions $x_2^* : \mathbb{R} \to \mathbb{R}$ and $\theta^* : \mathbb{R} \to (0,1)$ such that: an early agent attacks if and only if $x \leq x_1^*$; a late agent attacks if and only if $x \leq x_2^*(y_1)$; and the regime is abandoned if and only if $\theta \leq \theta^*(y_1)$.

The aggregate attack of early agents is then $A_1(\theta) = \Phi\left(\sqrt{\alpha_x}\,[x_1^* - \theta]\right)$ implying $y_1 = \Phi^{-1}(A_1(\theta)) + \varepsilon = \sqrt{\alpha_x}\,[x_1^* - \theta] + \varepsilon$. In equilibrium then, the observation of y_1 is equivalent to the observation of

$$z = x_1^* - \frac{1}{\sqrt{\alpha_x}} y_1 = \theta - \sigma_x \varepsilon,$$

a public signal with precision $\alpha_z = \alpha_\varepsilon \alpha_x$, or equivalently $\sigma_z = \sigma_\varepsilon \sigma_x$. This feature is equivalent to the simultaneous-signal model.

Since y_1 and z are informationally equivalent, we write the strategy of late agents as a function of z instead of y_1. The attack from late agents is $A_2(\theta, z) = \Phi\left(\sqrt{\alpha_x}(x_2^*(z) - \theta)\right)$, and the overall attack from both groups is $A(\theta, z) = \mu A_1(\theta) + (1-\mu)A_2(\theta, z)$. The threshold $\theta^*(z)$ solves $A(\theta^*(z), z) = \theta^*(z)$, or equivalently

$$\mu\Phi\left(\sqrt{\alpha_x}\left[x_1^* - \theta^*(z)\right]\right) + (1-\mu)\Phi\left(\sqrt{\alpha_x}\left[x_2^*(z) - \theta^*(z)\right]\right) = \theta^*(z). \tag{18}$$

The threshold $x_2^*(z)$ for late agents then solves $\Pr\left[\theta \leq \theta^*(z) \mid x_2^*(z), z\right] = c$, or equivalently

$$\Phi\left(\sqrt{\alpha}\left(\delta x_2^*(z) + (1-\delta)z - \theta^*(z)\right)\right) = 1 - c, \tag{19}$$

where $\delta = \alpha_x/(\alpha_x + \alpha_z)$ and $\alpha = \alpha_x + \alpha_z$.

Early agents, on the other hand, do not observe z and face a double forecast problem in that they are uncertain about both the fundamental θ and the signal y_1 that late agents condition their attack on. The threshold x_1^* solves $\Pr\left[\theta \leq \theta^*(z) \mid x_1^*\right] = \mathbb{E}[\Pr\left[\theta \leq \theta^*(z) \mid x_1^*, z\right] \mid x_1^*] = c$, or equivalently

$$\int \Phi\left(\sqrt{\alpha}\left(\delta x_1^* + (1-\delta)z - \theta^*(z)\right)\right)\sqrt{\alpha_1}\phi\left(\sqrt{\alpha_1}\left[x_1^* - z\right]\right)dz = 1 - c, \tag{20}$$

where $\alpha_1 = \alpha_x\alpha_\varepsilon/(1+\alpha_\varepsilon)$.[10]

A monotone equilibrium is therefore a joint solution to (18)-(20). Let $\mathcal{D}$ denote the set of decreasing real functions with range in $[0,1]$. For any given function $\theta^* \in \mathcal{D}$, (20) defines a unique $x_1^* \in \mathbb{R}$. Given $x_1^* \in \mathbb{R}$, (18)-(19) has either unique or multiple solutions for θ^* and x_2^*, depending on $(\alpha_x, \alpha_\varepsilon, \mu)$. Different solutions to (18)-(19) for given x_1^* represent different continuation equilibria for the game between late agents defined by a fixed strategy for the early agents.

In the Appendix we show that, when (18)-(19) admits a unique solution for *every* $x_1^* \in \mathbb{R}$, the solution to the system (18)-(20) is also unique. On the other hand, when (18)-(19) admits multiple solutions for *every* $x_1^* \in \mathbb{R}$, we show that the system (18)-(20) admits multiple solutions. The analysis provides us with the following sufficient conditions for uniqueness and multiplicity.

Proposition 5 (**i**) *There exists a unique equilibrium if* $\sigma_\varepsilon^2\sigma_x \geq (1-\mu)/\sqrt{2\pi}$

(**ii**) *There exist multiple equilibria if* $\sigma_\varepsilon^2\sigma_x < (1-\mu-\mu\sigma_\varepsilon^2)/\sqrt{2\pi}$

Proof. See Appendix. ■

[10] To see this, note that $z = \theta - \sigma_x\varepsilon = x - \xi - \sigma_x\varepsilon$, so that $z|x \sim \mathcal{N}\left(0, \sigma_x^2 + \sigma_x^2\sigma_\varepsilon^2\right)$. That is, conditional on x, z is distributed normal with precision $\alpha_1 = \alpha_x\alpha_\varepsilon/(1+\alpha_\varepsilon)$.

Note that as $\mu \to 0$ the regions of multiplicity or uniqueness converge to those derived in Proposition 4. Indeed, the dependence of (18) on x_1^* vanishes as $\mu \to 0$, so the equilibria of the simultaneous-signal model are approximated by equilibria of the sequential model as $\mu \to 0$. More generally, for any $\mu < 1$, multiple equilibria exist if noise is sufficiently low, and uniqueness holds if noise is sufficiently high.

5 Non-fundamental Volatility

We now investigate the role of the information structure for *non-fundamental* volatility (i.e., volatility conditional on θ) in the regime outcome and the equilibrium price. We are interested in two sources of non-fundamental volatility: (1) multiple equilibria, since sunspots variables may be used to coordinate on different equilibria; and (2) if the equilibrium is unique the dependence on the non-fundamental shock ε.

With *exogenous* information, multiplicity disappears when agents observe the fundamentals with small idiosyncratic noise. By implication, there is no sunspot volatility when σ_x is small enough. Moreover, as $\sigma_x \to 0$, the size of the attack and the regime outcome become independent of ε. In this sense, all non-fundamental volatility vanishes as $\sigma_x \to 0$.

In contrast, with *endogenous* information, the impact of private noise on volatility is quite different. A sufficiently large reduction in σ_x may make the economy enter the region of multiplicity and non-fundamental volatility may emerge from the economy's reaction to sunspots. Indeed, Corollaries 2 and 3 imply that the potential sunspot volatility is greatest when either noise vanishes, $\sigma_x \to 0$ or $\sigma_\varepsilon \to 0$, since the regime can then either collapse or survive for any given $\theta \in (\underline{\theta}, \overline{\theta}]$ entirely depending on the sunspot realization. Moreover, when the dividend is endogenous, sunspot volatility also emerges in prices.

With endogenous information, less noise may increase volatility even without entering the region of multiple equilibria: when the equilibrium is unique, a reduction in σ_x or σ_ε may increase the sensitivity of equilibrium outcomes to the exogenous shock ε.

To see this, note that the regime is abandoned if and only if $\theta \leq \theta^*(p)$. As long as the equilibrium is unique, $\theta^*(p)$ is continuously decreasing in p and $P(\theta, \varepsilon)$ is continuously increasing in θ, so $R(\theta, \varepsilon) = 1$ if and only if $\theta \leq \hat{\theta}(\varepsilon)$, where $\hat{\theta}(\varepsilon)$ is the unique solution to $\hat{\theta} = \theta^*(P(\hat{\theta}, \varepsilon))$. We can thus examine non-fundamental volatility in the regime outcome by examining the sensitivity of $\hat{\theta}(\varepsilon)$ to ε.

Solving for $\hat{\theta}$ in this way we obtain $\hat{\theta}(\varepsilon) = \Phi(\psi + (\sigma_p/\sigma_x)\varepsilon)$, $\psi = (1+1/\sigma_p^2)^{1/2}\Phi^{-1}(1-c)$. It follows that

$$\left.\frac{\partial \hat{\theta}}{\partial \varepsilon}\right|_{\hat{\theta}(\varepsilon)=\hat{\theta}_0} = \frac{\sigma_p}{\sigma_x}\phi(\Phi^{-1}(\hat{\theta}_0)), \tag{21}$$

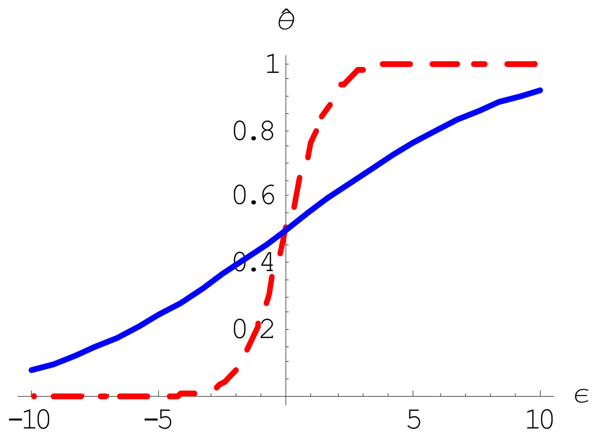

Figure 4: The regime-change threshold $\hat{\theta}(\varepsilon)$ as a function of the shock ε.

so that $\hat{\theta}(\varepsilon)$ satisfies a single-crossing property with respect to σ_p/σ_x. In this sense, the sensitivity of the regime outcome to the non-fundamental shock increases with σ_p/σ_x.

When the dividend is exogenous, $f = \theta$, then $\sigma_p/\sigma_x = 1/(\gamma\sigma_\varepsilon\sigma_x)$ and non-fundamental volatility increases with a reduction in either σ_x or σ_ε. This result is illustrated in Figure 4, which depicts the threshold $\hat{\theta}(\varepsilon)$ as a function of ε, with the dashed line corresponding to a lower σ_x or σ_ε than the solid line.

When instead $f = f(A)$, we have $\sigma_p/\sigma_x = 1/(\gamma\sigma_\varepsilon)$ and the sensitivity of $\hat{\theta}(\varepsilon)$ to ε increases with σ_ε but is invariant with σ_x. This result still contrasts with the case of exogenous information, where a reduction in σ_x reduces σ_p/σ_x, thus leading to a lower reaction to ε.

Consider next the implications for the volatility of prices, focusing again on regions where the equilibrium is unique. When $f = \theta$, we have $p = \theta - (\gamma\sigma_\varepsilon\sigma_x^2)\varepsilon$. The impact of noise on the sensitivity of the price to ε is then exactly as in Grossman-Stiglitz: a reduction in either σ_x or σ_ε implies lower non-fundamental volatility in prices. However, when the dividend is endogenous $f = f(A)$, we found that $p = f(A) - (\gamma\sigma_\varepsilon)\varepsilon$. Conditional on the attack – or equivalently, the dividend – the volatility of the price decreases with a reduction in σ_ε and is independent of σ_x. But since the attack A is a function of ε, a reduction in σ_ε may have an ambiguous overall effect on price volatility. We have verified numerically that this is indeed the case and that price volatility may increase with a reduction in σ_ε.

We conclude that less noise may increase volatility in both the regime outcome and the asset price even when the equilibrium is unique. Our earlier results on equilibrium multiplicity may thus be viewed as extreme versions this effect.

6 Conclusion

The main theme in Morris-Shin emphasizes the importance of the information structure for understanding the determinacy of equilibria and the volatility of outcomes. This paper contribution is to study the role of endogenous information aggregation. We model endogenous public information by either: (i) the price of a financial asset whose dividend depends on the underlying fundamental or the size of the attack in the coordination game; or (ii) a direct noisy signal of others' activity in the coordination game.

The most important feature in all cases is that the precision of endogenous public information rises with the precision of exogenous private information. This effect is typically strong enough to ensure multiplicity when the noise in either the individuals' private information about fundamentals or the aggregation process is small. Moreover, a reduction in noise may increase the sensitivity of equilibrium outcomes to exogenous non-fundamental shocks even when the equilibrium is unique.

These results may help understand crises phenomena such as currency attacks, bank runs, debt crises, and financial crashes. They suggest that, when coordination is important, the ability to trade in financial markets and otherwise exchange and aggregate information may have a destabilizing effect. Conversely, the presence of noise traders or policies that limit the information available to market participants may paradoxically have a stabilizing effect in financial markets.

Appendix

Proof of Proposition 1 (Morris-Shin) and Corollary 1. When agents attack if and only if $x < x^*(z)$, the aggregate size of the attack is $A(\theta, z) = \Phi\left(\sqrt{\alpha_x}\,(x^*(z) - \theta)\right)$. The status quo is then abandoned if and only if $\theta \leq \theta^*(z)$, where $\theta^*(z)$ solves $A(\theta, z) = \theta$, or equivalently

$$x^*(z) = \theta^*(z) + \tfrac{1}{\sqrt{\alpha_x}}\Phi^{-1}(\theta^*(z)). \tag{22}$$

It follows that the expected payoff from attacking is $\Pr[\theta \leq \theta^*(z)|x, z] - c$ and therefore $x^*(z)$ must solve the indifference condition $\Pr[\theta \leq \theta^*(z)|x, z] = c$. Since the posterior of the agent about θ is Normal with mean $\frac{\alpha_x}{\alpha_x+\alpha_z}x + \frac{\alpha_z}{\alpha_x+\alpha_z}z$ and precision $\alpha = \alpha_x + \alpha_z$, the indifference condition is

$$\Phi\left(\sqrt{\alpha_x + \alpha_z}\left(\theta^*(z) - \tfrac{\alpha_x}{\alpha_x+\alpha_z}x^*(z) - \tfrac{\alpha_z}{\alpha_x+\alpha_z}z\right)\right) = c. \tag{23}$$

Hence, an equilibrium is simply identified with a joint solution to (22) and (23).

Substituting (22) into (23) results in a single equation in $\theta^*(z)$:

$$-\frac{\alpha_z}{\sqrt{\alpha_x}}\theta^* + \Phi^{-1}(\theta^*) = g(z), \tag{24}$$

where $g(z) = \sqrt{1+\alpha_z/\alpha_x}\Phi^{-1}(1-c) - \left(\alpha_z/\sqrt{\alpha_x}\right)z$. It is easy to check that this equation always admits a solution and the solution is unique for every z if and only if $\alpha_z/\sqrt{\alpha_x} \leq 1/\sqrt{2\pi}$, or equivalently $\sigma_x \leq \sigma_z^2\sqrt{2\pi}$, which completes the proof.

Finally, consider the limits as $\sigma_x \to 0$ for given σ_z, or $\sigma_z \to \infty$ for given σ_x. In either case, $\alpha_z/\sqrt{\alpha_x} \to 0$ and $\sqrt{(\alpha_x+\alpha_z)/\alpha_x} \to 1$. Condition (24) then implies that $\theta^*(z) \to \hat{\theta} = 1-c$ for any z, so that the regime-change threshold is unique and independent of z. Similarly, $x^*(z) \to \hat{x}$, where $\hat{x} = \hat{\theta}$ if we consider the limit $\sigma_x \to 0$, and $\hat{x} = \hat{\theta} + \sigma_x\Phi^{-1}(\hat{\theta})$ if we instead consider the limit $\sigma_z \to \infty$. **QED.**

Proof of Propositions 2 and 3. In the main text.

Proposition A1 (noisy dividend). *Suppose the dividend has a random component η. For given $\sigma_\eta > 0$ and $\sigma_\varepsilon > 0$: (i) when $f = \theta + \eta$, a unique equilibrium survives for σ_x sufficiently small; (ii) when $f = f(A) + \eta$, multiple equilibria exist for σ_x sufficiently small. In either case, the region of $(\sigma_x, \sigma_\varepsilon)$ for which there are multiple equilibria decreases with σ_η.*

Proof. *Part (i).* Postulating, like in Section 3.2, that the posterior for θ conditional on (x,p) is Normal with mean $\delta x + (1-\delta)p$ and precision α, where $\delta = \alpha_x/\alpha$ and $\alpha = \alpha_x + \alpha_p$, we have the posterior for f is also Normal with the same mean and variance $\alpha^{-1} + \sigma_\eta$. Individual asset demands are thus given by

$$k(x,p) = \frac{\mathbb{E}[f|x,p] - p}{\gamma \text{Var}[f|x,p]} = \frac{\delta(x-p)}{\gamma\left(\alpha^{-1}+\sigma_\eta^2\right)}$$

and the equilibrium price by $p = \theta - \sigma_p\varepsilon$, where $\sigma_p = (\gamma/\delta)\left(\alpha^{-1}+\sigma_\eta^2\right)\sigma_\varepsilon = \gamma\left(\sigma_x^2 + \sigma_\eta^2/\delta\right)\sigma_\varepsilon$ Since $\delta \in [0,1]$ and $\sigma_x > 0$, we have immediately that that σ_p is bounded from below by $\gamma\sigma_\eta^2\sigma_\varepsilon > 0$. It follows that $\sigma_x < \left(\gamma\sigma_\eta^2\sigma_\varepsilon\right)^2\sqrt{2\pi}$ suffices for $\sigma_x < \sigma_p^2\sqrt{2\pi}$ and hence for the equilibrium to be unique. Moreover, σ_p is increasing in σ_x and σ_ε, as well as σ_η. A higher σ_η thus makes it more likely that the equilibrium is unique.

Part (ii). Like in Section 3.2, let the posterior for θ be Normal with mean $\delta x + (1-\delta)\tilde{p}$ and precision α, where $\tilde{p} = p/\sqrt{\alpha_x} + x^*(p)$, $\delta = \alpha_x/\alpha$, and $\alpha = \alpha_x + \alpha_p$. It follows that

$$k(x,p) = \frac{\mathbb{E}[f|x,p] - p}{\gamma \text{Var}[f|x,p]} = \frac{\sqrt{\alpha_x}\delta(x-\tilde{p})}{\gamma\left(\alpha_x\alpha^{-1}+\sigma_\eta^2\right)} = \frac{\delta}{\tilde{\gamma}\left(\alpha^{-1}+\sigma_\eta^2\alpha_x^{-1}\right)}(x-\tilde{p}),$$

where $\tilde{\gamma} = \gamma\sqrt{\alpha_x}$, and therefore $\tilde{p} = \theta - \sigma_p \varepsilon$, where $\sigma_p = \tilde{\gamma}\left(\alpha^{-1} + \sigma_\eta^2 \alpha_x^{-1}\right)\sigma_\varepsilon/\delta$. Using $\delta\alpha = \sigma_x^{-2}$, $\tilde{\gamma} = \gamma/\sigma_x$ and $1/\delta = 1 + \sigma_p/\sigma_x$, we get

$$\sigma_p = \tilde{\gamma}\left(1/(\delta\alpha) + \sigma_\eta^2\sigma_x^2/\delta\right)\sigma_\varepsilon = \tilde{\gamma}\sigma_x^2\left(1 + \sigma_\eta^2/\delta\right)\sigma_\varepsilon = \gamma\sigma_x\left(1 + \sigma_\eta^2\left(1 + \sigma_p/\sigma_x\right)\right)\sigma_\varepsilon$$

and therefore

$$\sigma_p = \frac{1 + \sigma_\eta^2}{1 - \gamma\sigma_\eta^2\sigma_\varepsilon}\gamma\sigma_\varepsilon\sigma_x.$$

Hence, a higher σ_η again makes it harder for multiple equilibria to exist, nevertheless multiplicity is ensured by a sufficiently small σ_x or σ_ε. **QED**

Proof of Proposition 4 (simultaneous signal). Given that an agent attacks if and only if $x \leq x^*(y)$, the aggregate attack is $A(\theta, y) = \Phi\left(\sqrt{\alpha_x}(x^*(y) - \theta)\right)$. Condition (16) then implies that the signal satisfies

$$x^*(y) - \sigma_x y = \theta - \sigma_x\sigma_\varepsilon\varepsilon. \tag{25}$$

Note that (25) is a mapping between y and $z = \theta - \sigma_x\sigma_\varepsilon\varepsilon$. Define the correspondence

$$\mathcal{Y}(z) = \{\ y \in \mathbb{R} \mid x^*(y) - \sigma_x y = z\ \}. \tag{26}$$

We will later show that $\mathcal{Y}(z)$ is non-empty and examine when it is single- or multi-valued.

Take any function $\tilde{Y}(z)$ that is a selection from this correspondence, $\tilde{Y}(z) \in \mathcal{Y}(z)$ for all z, and let $Y(\theta, \varepsilon) = \tilde{Y}(\theta - \sigma_x\sigma_\varepsilon\varepsilon)$. The observation of $y = Y(\theta, \varepsilon)$ is then equivalent to the observation of $z = \theta - \sigma_z\varepsilon = Z(y)$, where $Z(y) \equiv x^*(y) - \sigma_x y$ and

$$\sigma_z = \sigma_x\sigma_\varepsilon. \tag{27}$$

That is, it is as if agents observe a Normal public signal with precision proportional to precision of exogenous private information.

The individual attacks if and only if $x \leq x^*(y)$, where $x^*(y)$ solves the indifference condition

$$\Phi\left(\sqrt{\alpha_x + \alpha_z}\left(\theta^*(y) - \tfrac{\alpha_x}{\alpha_x + \alpha_z}x^*(y) - \tfrac{\alpha_z}{\alpha_x + \alpha_z}Z(y)\right)\right) = c. \tag{28}$$

The regime in turn is abandoned if and only if $\theta \leq \theta^*(y)$, where $\theta^*(y)$ solves $A(\theta, y) = \theta$, or equivalently

$$x^*(y) = \theta^*(y) + \tfrac{1}{\sqrt{\alpha_x}}\Phi^{-1}(\theta^*(y)). \tag{29}$$

Using $Z(y) = x^*(y) - \sigma_x y$ and substituting $x^*(y)$ from (29) into (28), we get

$$\theta^*(y) = \Phi\left(\sqrt{\tfrac{\alpha_x}{\alpha_x + \alpha_z}}\Phi^{-1}(1 - c) + \tfrac{\alpha_z}{\alpha_x + \alpha_z}y\right), \tag{30}$$

which together with (29) determines a unique pair $\theta^*(y)$ and $x^*(y)$. The strategy $a(x,y)$ and the corresponding aggregate $A(x,y)$ are thus uniquely determined.

We return to the equilibrium correspondence $\mathcal{Y}(z)$. Recall that this is given by the set of solutions to $x^*(y) - \sigma_x y = z$. Using (29) and (30) this reduces to $F(y) = z$, where

$$F(y) \equiv \Phi\left(\tfrac{\alpha_z}{\alpha_x+\alpha_z}y + q\right) + \tfrac{1}{\sqrt{\alpha_x}}\left(-\tfrac{\alpha_x}{\alpha_x+\alpha_z}y + q\right) \tag{31}$$

and $q \equiv \sqrt{\alpha_x/(\alpha_x+\alpha_z)}\Phi^{-1}(1-c)$. Note that $F(y)$ is continuous in y, and $F(y) \to -\infty$ as $y \to +\infty$, and $F(y) \to +\infty$ as $y \to -\infty$, which guarantees that $\mathcal{Y}(z)$ is non-empty and therefore an equilibrium always exists. Next, note that

$$sign\{F'(y)\} = -sign\left\{1 - \tfrac{\alpha_z}{\sqrt{\alpha_x}}\phi\left(\tfrac{\alpha_z}{\alpha_x+\alpha_z}y + q\right)\right\}$$

and therefore $F(y)$ is globally monotonic if and only if $\alpha_z/\sqrt{\alpha_x} \leq \sqrt{2\pi}$, in which case $\mathcal{Y}(z)$ is single valued. If instead $\alpha_z/\sqrt{\alpha_x} > \sqrt{2\pi}$, there is a non-empty interval $(\underline{z}, \overline{z})$ within which $\mathcal{Y}(z)$ takes three values. Different (monotone) selections then sustain different equilibria. Using $\alpha_z = \alpha_\varepsilon\alpha_x$ from (27) completes the proof. **QED**

Proof of Proposition 5 (non-simultaneous signal). We can reduce the dimensionality of the system by solving (18) for $x_2^*(z)$:

$$x_2^*(z) = \theta^*(z) + \tfrac{1}{\sqrt{\alpha_x}}\Phi^{-1}\left(\theta^*(z) + \tfrac{\mu}{1-\mu}\left\{\theta^*(z) - \Phi\left(\sqrt{\alpha_x}\left[x_1^* - \theta^*(z)\right]\right)\right\}\right).$$

Substituting the above into (19) and using $\delta = \alpha_x/(\alpha_x+\alpha_z)$ and $\alpha = \alpha_x + \alpha_z$, we obtain:

$$\Gamma(\theta^*(z), x_1^*) = g(z), \tag{32}$$

where

$$\Gamma(\theta, x_1) \equiv -\frac{\alpha_z}{\sqrt{\alpha_x}}\theta + \Phi^{-1}\left(\theta + \frac{\mu}{1-\mu}\left\{\theta - \Phi\left(\sqrt{\alpha_x}\left[x_1^* - \theta\right]\right)\right\}\right),$$

$g(z) = \sqrt{1+\alpha_z/\alpha_x}\Phi^{-1}(1-c) - \left(\alpha_z/\sqrt{\alpha_x}\right)z$, and $\alpha_z = \alpha_\varepsilon\alpha_x$. We conclude that an equilibrium is a joint solution of (20) and (32) for a threshold $x_1^* \in \mathbb{R}$ and a function $\theta^* : \mathbb{R} \to (0,1)$.

For any $\mu \in (0,1)$ and any $x_1 \in \mathbb{R}$, $\Gamma(\theta, x_1)$ is continuous in θ, with $\Gamma(\underline{\theta}, x_1, \mu) = -\infty$ and $\Gamma(\overline{\theta}, x_1, \mu) = \infty$, where $\underline{\theta} = \underline{\theta}(x_1, \alpha_x, \mu)$ and $\overline{\theta} = \overline{\theta}(x_1, \alpha_x, \mu)$ solves, respectively, $\theta + \frac{\mu}{1-\mu}\left\{\theta - \Phi\left(\sqrt{\alpha_x}\left[x_1 - \theta\right]\right)\right\} = 0$ and $= 1$, and therefore satisfy $0 < \underline{\theta} < \overline{\theta} < 1$. It follows that (32) always admits a solution; that is, for any given $x_1^* \in \mathbb{R}$, (32) defines at least one function $\theta^* : \mathbb{R} \to (\underline{\theta}, \overline{\theta})$.

We next examine under what conditions the function that solves (32) is unique. Note that

$$\begin{aligned}
\frac{\partial \Gamma}{\partial \theta} &= -\frac{\alpha_z}{\sqrt{\alpha_x}} + \Lambda\left(\theta; x_1, \alpha_x, \mu\right), \\
\Lambda\left(\theta; x_1, \alpha_x, \mu\right) &\equiv \frac{1}{\phi\left(\Phi^{-1}\left(\theta + \frac{\mu}{1-\mu}\left\{\theta - \Phi\left(\sqrt{\alpha_x}\left[x_1 - \theta\right]\right)\right\}\right)\right)} \left\{1 + \frac{\mu}{1-\mu}\left[1 + \sqrt{\alpha_x}\phi\left(\sqrt{\alpha_x}\left[x_1 - \theta\right]\right)\right]\right\}
\end{aligned}$$

As $\theta \to \underline{\theta}$ or $\overline{\theta}$ (equivalently, $z \to \pm\infty$), $\Lambda\left(\theta; x_1, \alpha_x, \mu\right) \to +\infty$. Let

$$K\left(x_1, \alpha_x, \mu\right) \equiv \inf_{\theta \in \left(\underline{\theta}, \overline{\theta}\right)} \Lambda\left(\theta; x_1, \alpha_x, \mu\right)$$

and note that, since $\phi\left(.\right)$ takes values in $(0, 1/\sqrt{2\pi}]$,

$$K\left(x_1, \alpha_x, \mu\right) \geq \frac{1}{1/\sqrt{2\pi}}\left\{1 + \frac{\mu}{1-\mu}\left[1 + \sqrt{\alpha_x}0\right]\right\} = \frac{\sqrt{2\pi}}{1-\mu} \equiv \underline{K}\left(\mu\right).$$

Moreover, letting $\hat{\theta} = \hat{\theta}\left(x_1, \alpha_x, \mu\right) \in \left(\underline{\theta}, \overline{\theta}\right)$ be the solution to $\theta + \frac{\mu}{1-\mu}\left\{\theta - \Phi\left(\sqrt{\alpha_x}\left[x_1 - \theta\right]\right)\right\} = 1/2 = \Phi\left(\phi^{-1}\left(1/\sqrt{2\pi}\right)\right)$, we have

$$\begin{aligned}
K\left(x_1, \alpha_x, \mu\right) &\leq \Lambda\left(\hat{\theta}; x_1, \alpha_x, \mu\right) = \frac{1}{1/\sqrt{2\pi}}\left\{1 + \frac{\mu}{1-\mu}\left[1 + \sqrt{\alpha_x}\phi\left(\sqrt{\alpha_x}\left[x_1 - \hat{\theta}\right]\right)\right]\right\} \\
&\leq \sqrt{2\pi}\left\{1 + \frac{\mu}{1-\mu}\left[1 + \frac{\sqrt{\alpha_x}}{\sqrt{2\pi}}\right]\right\} \equiv \overline{K}\left(\alpha_x, \mu\right).
\end{aligned}$$

Note, importantly, that the bounds $\overline{K}$ and $\underline{K}$ are the same across all $x_1^* \in \mathbb{R}$.

Part (i) : $\sigma_\varepsilon^2 \sigma_x \geq \frac{1}{\sqrt{2\pi}}\left(1-\mu\right)$. In this case, $\frac{\alpha_z}{\sqrt{\alpha_x}} \leq \underline{K}\left(\mu\right) \leq \inf_{x_1} K\left(\theta, x_1, \alpha_x, \mu\right) = \inf_{\theta, x_1} \Lambda\left(\theta, x_1, \alpha_x, \mu\right)$ and therefore $\Gamma\left(\theta, x_1\right)$ is globally increasing in θ for *every* x_1. It follows that (32) defines a unique function $\theta^* : \mathbb{R} \to \left(\underline{\theta}, \overline{\theta}\right)$ for any given x_1^*. Moreover, since Γ is decreasing in x_1 and g is decreasing in z, the function θ^* is decreasing in z and increasing in x_1^*. Finally, θ^* is continuous in both z and x_1^*.

Next, consider (20). For any given decreasing function $\theta^* : \mathbb{R} \to \left(\underline{\theta}, \overline{\theta}\right)$, (20) admits a unique solution $x_1^* \in \mathbb{R}$. Moreover, this solution is continuous and increasing in θ^*.

Let C be the set of decreasing (and bounded) functions $\theta^* : \mathbb{R} \to \left(\underline{\theta}, \overline{\theta}\right)$. Then, (32) is a mapping $\mathbb{R} \to C$ and (20) is a mapping $C \to \mathbb{R}$. Together, they define a continuous and increasing mapping $T : \mathbb{R} \to \mathbb{R}$.

It is easy to check that $T\left(-\infty\right) > -\infty$ and $T\left(+\infty\right) < \infty$. Hence, a fixed point always exists. Moreover, for arbitrary x_1^* and $a > 0$, let $x_1^{**} = x_1^* + a$ and let θ^* and θ^{**} be the solutions to (32) for x_1^* and x_1^{**}, respectively; that is,

$$\Gamma(\theta^*, x_1^*) = g(z) \quad \text{and} \quad \Gamma(\theta^{**}, x_1^{**}) = g(z).$$

The indifference condition (20) then gives

$$\Pi(\theta^*, Tx_1^*) = 1 - c \quad \text{and} \quad \Pi(\theta^{**}, Tx_1^{**}) = 1 - c,$$

where $\Pi : C \times \mathbb{R} \to [0,1]$ with

$$\Pi(\theta, x_1) \equiv \int \Phi\left(\sqrt{\alpha}\left[\delta x_1 + (1-\delta) z - \theta(z)\right]\right) \sqrt{\alpha_1}\phi\left(\sqrt{\alpha_1}\left[x_1 - z\right]\right) dz$$

Let $\tilde{\theta} \equiv \theta^* + a$ and $\tilde{x}_1$ be the solution to $\Pi\left(\tilde{\theta}, \tilde{x}_1\right) = 1 - c$. Since $\frac{\alpha_z}{\sqrt{\alpha_x}} < \frac{\sqrt{2\pi}}{1-\mu}$, we have that

$$\begin{aligned}\Gamma\left(\tilde{\theta}, x_1^{**}\right) &= \Gamma(\theta^* + a, x_1^* + a) = -\tfrac{\alpha_z}{\sqrt{\alpha_x}}(\theta^* + a) + \Phi^{-1}\left(\tfrac{1}{1-\mu}(\theta^* + a) - \tfrac{\mu}{1-\mu}\Phi\left(\sqrt{\alpha_x}\left[x_1^* - \theta^*\right]\right)\right) > \\ &> -\tfrac{\alpha_z}{\sqrt{\alpha_x}}\theta^* + \Phi^{-1}\left(\tfrac{1}{1-\mu}\theta^* - \tfrac{\mu}{1-\mu}\Phi\left(\sqrt{\alpha_x}\left[x_1^* - \theta^*\right]\right)\right) = \Gamma(\theta^*, x_1^*) = g(z) = \Gamma(\theta^{**}, x_1^{**});\end{aligned}$$

and since in this case $\Gamma(\theta, x_1)$ is increasing in θ, we get $\tilde{\theta} > \theta^{**}$. By the fact that $\Pi(\theta, x_1)$ is decreasing in θ and increasing in x_1, it then follows that $\tilde{x}_1 > Tx_1^{**}$. Next, note that

$$\begin{aligned}\Pi\left(\tilde{\theta}, Tx_1^* + a\right) &= \Pi(\theta^* + a, Tx_1^* + a) \\ &= \int \Phi\left(\sqrt{\alpha}\left[\delta Tx_1^* + (1-\delta)(z-a) - \theta^*(z)\right]\right) \sqrt{\alpha_1}\phi\left(\sqrt{\alpha_1}\left[Tx_1^* - (z-a)\right]\right) dz = \\ &= \int \Phi\left(\sqrt{\alpha}\left[\delta Tx_1^* + (1-\delta) z' - \theta^*(z'+a)\right]\right) \sqrt{\alpha_1}\phi\left(\sqrt{\alpha_1}\left[Tx_1^* - z'\right]\right) dz'\end{aligned}$$

where the last equality follows by changing the variable of integration from z to $z' = z - a$. But since θ^* is a decreasing function, Π in turn is decreasing in θ^*, and $a > 0$, we have

$$\begin{aligned}\Pi\left(\tilde{\theta}, Tx_1^* + a\right) &> \int \Phi\left(\sqrt{\alpha}\left[\delta Tx_1^* + (1-\delta) z' - \theta^*(z')\right]\right) \sqrt{\alpha_1}\phi\left(\sqrt{\alpha_1}\left[Tx_1^* - z'\right]\right) dz' = \\ &= \Pi(\theta^*, Tx_1^*) = 1 - c = \Pi\left(\tilde{\theta}, \tilde{x}_1\right)\end{aligned}$$

and therefore $Tx_1^* + a > \tilde{x}_1$. Combining with the earlier result that $\tilde{x}_1 > Tx_1^{**}$, we conclude that $Tx_1^{**} < Tx_1^* + a$, which proves that the slope of the mapping T is less than one for every x_1^*. It follows that T has a unique fixed point.

Part (ii): $\sigma_\varepsilon^2 \sigma_x < \frac{1}{\sqrt{2\pi}}\left(1 - \mu - \mu\sigma_\varepsilon^2\right)$. In this case, $\frac{\alpha_z}{\sqrt{\alpha_x}} > \overline{K}(\alpha_x, \mu) \geq \sup_{x_1} K(x_1, \alpha_x, \mu)$ and therefore $\Gamma(\theta, x_1)$ necessarily has a non-empty region of non-monotonicity in θ for *every* x_1. It follows that, for any x_1^*, there is a non-empty interval $Z = Z(x_1^*) = (\underline{z}(x_1^*), \overline{z}(x_1^*))$ such that (32) admits three distinct solutions whenever $z \in Z(x_1^*)$ and a unique solution otherwise. Let θ_L^* (resp., θ_H^*) be the function defined by selecting the lowest (resp., highest) solution whenever $z \in Z$ and the unique one whenever $z \notin Z$. Each of the functions θ_L^* and θ_H^* is decreasing in z. Next, let T_L (resp., T_H) be the associated mappings. Each of the mappings T_L and T_H are continuous and satisfy $T(-\infty) > -\infty$ and $T(+\infty) < \infty$. Hence, there exists a fixed point (at least one) for each mapping. Moreover, for any given x_1^*, $\theta_L^*(z) < \theta_H^*(z)$ for all $z \in Z$, which implies (because of the monotonicity in (20) and the fact that Z has positive measure) that $T_L(x_1^*) < T_H(x_1^*)$ for any x_1^*. It follows that the lowest fixed point of T_L is lower than the highest fixed point of T_H, which, together with the fact that $\theta_L^* < \theta_H^*$ for any given x_1^*, implies that the associated x_2^* and θ^* satisfy the same ordering.

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