# Efficiency and Welfare in Economies with Incomplete Information*

**George-Marios Angeletos**
MIT, NBER, and FRB of Minneapolis

**Alessandro Pavan**
Northwestern University

September 2005.

**Abstract**

This paper examines a class of economies with externalities, strategic complementarity or substitutability, and incomplete information. We first characterize efficient allocations and compare them to equilibrium. We show how the optimal degree of coordination and the efficient use of information depend on the primitives of the environment, and how this relates to the welfare losses due to volatility and heterogeneity. We next examine the social value of information in equilibrium. When the equilibrium is efficient, welfare increases with the transparency of information if and only if agents' actions are strategic complements. When the equilibrium is inefficient, additional effects are introduced by the interaction of externalities and information. We conclude with a few applications, including investment complementarities, inefficient fluctuations, and market competition.

*Keywords:* Social value of information, coordination, higher-order beliefs, externalities, transparency.

*An earlier version of this paper was entitled "Social Value of Information and Coordination." For useful comments we thank Daron Acemoglu, Gadi Barlevi, Olivier Blanchard, Marco Bassetto, Christian Hellwig, Kiminori Matsuyama, Stephen Morris, Thomas Sargent, and especially Ivàn Werning. *Email Addresses:* angelet@mit.edu, alepavan@northwestern.edu.

# 1 Introduction

Is coordination socially desirable? Do markets use available information efficiently? Is the private collection of information good from a social perspective? What are the welfare effects of the public information disseminated by prices, market experts, or the media? Should central banks and policy makers disclose the information they collect and the forecasts they make about the economy in a transparent and timely fashion, or is there room for "constructive ambiguity"?

These questions are non-trivial. Private incentives to align individual decisions need not be warranted from a social perspective. Externalities and strategic effects may introduce a wedge between private and social motives in the collection and use of information. Less noise in public information can increase aggregate volatility as agents use this information more heavily, and similarly less noise in private information can increase cross-sectional dispersion of individual activities.

This paper provides an analytical framework that may help answer these questions: we examine equilibrium allocations, efficiency and welfare in a tractable class of economies with externalities, strategic complementarity (or substitutability), and incomplete information.[1]

An economy is populated by a large number of small agents, each with incomplete information about the underlying economic fundamentals. Individual utility is given by $U = U(k, K, \tilde{\theta})$, where $k$ denotes the agent's own action, $K$ aggregate activity, and $\tilde{\theta}$ the exogenous fundamentals. Social welfare is given by expected utility behind the veil of ignorance (equivalently, by the utilitarian aggregator of individual payoffs). For tractability we assume that $U$ is quadratic and impose concavity at both the individual and the aggregate level, but otherwise allow for general externalities ($U_K \neq 0$) and for strategic complementarity or substitutability ($U_{kK} \neq 0$).[2]

Our first result is a characterization of efficient allocations. We find it useful to define the *optimal* degree of coordination as the sensitivity of individual actions to expected aggregate activity implicit in the efficient allocation. This is analogous to the slope of the best-response function in equilibrium and maps one-to-one to the relative sensitivity of efficient allocations to public information.

The welfare losses associated with incomplete information can be decomposed into those due to aggregate volatility and those due to cross-sectional dispersion. A higher relative sensitivity to public information trades off lower dispersion for higher volatility. It is this trade-off that underlies the efficient use of information and hence the optimal degree of coordination: the latter is higher the lower the social aversion to volatility.

[1] In this paper we only aim at identifing some of the general principles underlying the questions mentioned above. Although we discuss some applications in Section 6, a proper answer to any of these questions would require more careful attention to the details of the particular context of interest.

[2] For concave but non-quadratic environments, our results represent approximations that are better the smaller the noise in information. Convexities, on the other hand, may introduce novel effects about which our analysis is not appropriate. (For example, aggregate convexities can generate a social value for lotteries.)

Mapping this into the primitives of the environment, we have that the optimal degree of coordination is inversely related to the curvature of aggregate welfare. This in turn depends on both the complementarity ($U_{kK}$) and second-order external effects ($U_{KK}$).[3] The optimal degree of coordination is higher than the equilibrium one, and equivalently efficient allocations are more sensitive to public information than equilibrium allocations, if and only if the extra concavity introduced by the externality is small enough ($-U_{KK} < U_{kK}$).

Our second result is a characterization of the social value of information. For that purpose, we parametrize the information structure by its *accuracy* and its *transparency*. The former is defined as the overall precision of posterior beliefs; the latter as the extent to which beliefs are common across agents (equivalently, as the relative precision of public information). Since only the accuracy of information would be relevant in the absence of strategic effects and externalities, this parametrization seems most appropriate from a theoretical point of view.[4]

When the equilibrium is efficient, welfare necessarily increases with the accuracy of information. Moreover, welfare increases [decreases] with the transparency of information if and only if agents' actions are strategic complements [substitutes]. Efficiency thus implies a clear relationship between the form of strategic interaction and the social value of information.

When the equilibrium is inefficient, knowing that agents' actions exhibit complementarity or substitutability no longer suffices for signing the welfare effects of information. This is because information can now affect the gap between equilibrium and efficient allocations. We characterize how the overall effect depends on both a first-order externality, which determines the efficiency gap under complete information, and a second-order externality, which determines the gap between equilibrium and optimal coordination under incomplete information.

To gain further insight, we conclude the paper with some applications; different applications motivate different restrictions on the payoff structure.

In a typical model of production spillover where the individual return to investment increases with aggregate investment and productivity is uncertain, coordination in equilibrium is positive but inefficiently low. Agents thus put excess weight on public information, but the planner would like them to do even more so. Furthermore, welfare unambiguously increases with either the relative or the absolute precision of public information – a case for timely provision of relevant information by the government or the media.

[3] By "first-order external effects" we mean $U_K \neq 0$; by "second-order external effects" we mean $U_{KK} \neq 0$.

[4] This parametrization is also appropriate for some applied questions. Think for example of a central banker choosing whether to transmit information in a transparent or ambiguous way. This choice is rarely equivalent to the release of more or less information. Rather, it is a choice about the extent to which individuals adopt different idiosyncratic interpretations of the same publicly available information. Since, for given quality of available information, the dispersion of individual beliefs is decreasing in the *relative* precision of public information, we find the latter a good proxy for transparency.

In a competitive economy in which production decisions are made under incomplete information about aggregate demand, market competition implies that individual choices are strategic substitutes. Nevertheless, the equilibrium is efficient. Welfare thus increases with the accuracy of available information but decreases with its transparency – perhaps a case for "constructive ambiguity" in central bank communication.

Finally, in an economy where equilibrium fluctuations are inefficient, information, either private or public, that helps predict equilibrium behavior reduces welfare – from a social perspective, ignorance can be a bless!

**Related literature.** To the best of our knowledge, this paper is the first to characterize the optimal degree of coordination and the efficient use of information in the class of environments considered here. Perhaps the closest ascendant is Cooper and John (1988), who examine equilibrium and welfare in economies with complementarities but complete information.

This paper, however, is certainly not the first to examine the social value of information. Morris and Shin (2002) show that more precise public information can reduce welfare in a beauty contest where the private motive to coordinate is not warranted from a social perspective. Angeletos and Pavan (2004) and Hellwig (2005), on the other hand, provide counterexamples where public information is socially valuable despite the presence of strategic complementarities.[5] These three papers illustrate the non-triviality of the welfare effects of information, but fail to identify any general principles. We fill the gap in this paper by showing how the social value of information depends, not only on the form of strategic interaction, but also on the inefficiency of equilibrium.

Hayek (1945) advocated that a crucial function of the market mechanism is the aggregation of disperse private information. The welfare implications, however, are not always obvious. For example, Hirshleifer (1971) shows how more precise public information can reduce welfare by removing insurance possibilities; Messner and Vives (2001) show that the welfare properties of rational-expectation equilibria can be ambiguous; Angeletos and Werning (2004) argue that the information revealed by prices during financial crises can lead to more volatility and higher risk of coordination failures. Although the information structure here is exogenous, the paper provides an input into this line of research by characterizing the welfare effects of public information.

The paper also contributes to the debate about central bank transparency and communication.[6] Earlier work focused on incentive effects: among others, Canzoneri (1985), Cukierman and Meltzer (1986), and Atkeson and Kehoe (2001) have examined how transparency affects the ability of markets to detect policy deviations in Kydland-Prescott/Barro-Gordon environments. Pure informational effects have been highlighted more recently: Morris and Shin (2002, 2005) argue that

[5] The first paper considers a real economy with production externalities and complementarities in investment decisions; the second a monetary economy with demand externalities and complemnetarities in pricing decisions.

[6] See Geetra (2002) for a review of the literature on transparency and Woodford (2005) for a recent position on the issue.

central-bank disclosures can lead to welfare losses if markets engage in inefficient beauty contests. Svensson (2005) and Woodford (2005) question whether this result is relevant for US monetary policy, while Hellwig (2005) argues that public information can improve welfare by reducing price dispersion. Perhaps more surprisingly, the example in Section 6.5 shows that an argument for "constructive ambiguity" can be made even in efficient competitive environments.

The rest of the paper is organized as follows. We introduce the model in Section 2. We characterize equilibrium in Section 3 and efficiency in Section 4. We turn to the social value of information in Section 5 and to specific applications in Section 6. The Appendix includes the proofs omitted in the main text.

## 2 The Model

**Actions and Payoffs.** Consider an economy with a measure-one continuum of agents, distributed over $[0,1]$, each choosing an action $k \in \mathbb{R}$. For given state of nature, let $\Psi$ denote the c.d.f. of the distribution of $k$ in the population. Individual utility is given by

$$u = U(k, K, \tilde{\theta}),$$

where $K = \int k d\Psi(k)$ is the average action, $\tilde{\theta} = (\tilde{\theta}_1, ..., \tilde{\theta}_n) \subseteq \mathbb{R}^n$ are exogenous parameters, and $U : \mathbb{R}^{n+2} \to \mathbb{R}$. We can interpret $k$ as investment or effort (or some other economic activity) and $\tilde{\theta}$ as the underlying economic fundamentals for the relevant environment. We restrict $U$ to be a strictly concave quadratic function.[7]

**Social Welfare.** We define welfare as expected utility "behind the veil of ignorance," or equivalently as the utilitarian aggregator:

$$w = w(\tilde{\theta}; \Psi) = \int U(k, K, \tilde{\theta}) d\Psi(k).$$

Since $U$ is quadratic in $k$, the dependence of welfare on $\Psi$ can be summarized by the two first moments of $\Psi$, the cross-sectional mean $K = \int k d\Psi(k)$ and the cross-sectional dispersion $var = \int (k-K)^2 d\Psi(k)$. Indeed, a second-order Taylor expansion of $U$ around $k = K$, which is exact since $U$ is quadratic, gives

$$U(k, K, \tilde{\theta}) = U(K, K, \tilde{\theta}) + U_k(K, K, \tilde{\theta}) \cdot (k - K) + \frac{U_{kk}}{2} \cdot (k - K)^2.$$

Aggregating across agents then gives

$$w = W(K, \tilde{\theta}) + \frac{U_{kk}}{2} \cdot var$$

where $W(K, \tilde{\theta}) \equiv U(K, K, \tilde{\theta})$. This term represents welfare when all agents choose the same action; the other term captures the welfare loss associated with cross-sectional dispersion in $k$.

[7] That is, $U(k, K, \tilde{\theta}) = v\mathbf{U}v'$ where $\mathbf{U}$ is a $(n+3) \times (n+3)$ negative-definite matrix and $v = (1, k, K, \tilde{\theta})$.

**Externality, complementarity, and concavity.** Strategic complementarities (either positive or negative) emerge in this economy whenever $U_{kK} \neq 0$. An externality, on the other hand, is present whenever $U_K \neq 0$. We impose no restriction on $U_K$. We however restrict $U_{kk} < 0$ and $W_{KK} < 0$, which ensure concavity at the individual and the aggregate level; without concavity, either the individual or the social objective would be unbounded. We finally put bounds on the degree of complementarity by restricting $\alpha \equiv -U_{kK}/U_{kk}$ within $(-1, +1)$; these bounds are necessary and sufficient for the existence of a unique stable equilibrium.

Note that $W_{KK} = U_{kk} + 2U_{kK} + U_{KK}$. In the absence of complementarity and externality aggregate welfare inherits the same concavity as individual utility: $W_{KK} = U_{kk}$ when $U_{kK} = U_K = 0$.[8] Introducing a complementarity naturally affects the curvature of aggregate welfare by the addition of the term $2U_{kK}$ in $W_{KK}$. Allowing then for an external second-order effect ($U_{KK} \neq 0$) affects the curvature of aggregate welfare beyond the complementarity and without affecting individual incentives. The complementarity $U_{kK}$ and the external effect $U_{KK}$ will turn out to play a critical role in the social value of coordination and information. We find it useful to measure the intensity of second-order external effects with $\eta \equiv U_{KK}/U_{kk}$; the restriction $W_{KK} < 0$ can then be restated as $2\alpha - \eta < 1$.

**Information.** The fundamentals $\tilde{\theta} \in \mathbb{R}^n$ are not known at the time decisions are made; they are drawn from a joint Normal distribution with mean $\mu_{\tilde{\theta}}$ and covariance matrix $\Sigma_{\tilde{\theta}}$. Each agent then observes a private and a public signal about $\tilde{\theta}$. The private signal is given by $x = \theta + \sigma_x \xi$ and the public signal by $y = \theta + \sigma_y \varepsilon$, where $(\sigma_x, \sigma_y)$ are positive scalars and $(\theta, \xi, \varepsilon)$ are Normal random variables. $\theta$ is an affine transformation of $\tilde{\theta}$, defined by the unique solution to $U_k(\theta, \theta, \tilde{\theta}) = 0$ and thus representing the equilibrium activity under complete information.[9] $\xi$ and $\varepsilon$, on the other hand, are standard Normal noises, independent of each other and of any element of $\tilde{\theta}$ (and hence of $\theta$); $\xi$ is also independent across agents.

Assuming this signal structure simplifies the analysis and is motivated by the observation that what matters for best responses is merely the hierarchy of beliefs about $\theta$. Note also that the characterization of equilibrium and efficient allocations in part (i) of Propositions 1 and 3 does no depend on the specific information structure assumed here.

**Transparency and accuracy.** Let $\mu_\theta$ and $\sigma_\theta$ denote the prior mean and the standard deviation of $\theta$ (as implied by $\mu_{\tilde{\theta}}$ and $\Sigma_{\tilde{\theta}}$) and next let

$$\delta \equiv \frac{\sigma_y^{-2} + \sigma_\theta^{-2}}{\sigma_x^{-2} + \sigma_y^{-2} + \sigma_\theta^{-2}} \quad \text{and} \quad \sigma \equiv (\sigma_x^{-2} + \sigma_y^{-2} + \sigma_\theta^{-2})^{-1/2}.$$

The posterior belief about $\theta$ of an agent with private signal $x$ and public signal $y$ is Normal with

[8] Note that $U_K = 0$ for all $(k, K, \tilde{\theta})$ implies $U_{KK} = 0$, but not vice-versa.

[9] To see this, note that $\theta$ is the unique fixed point of the best-response condition $U_k(k, K, \tilde{\theta}) = 0$. Since $U$ is quadratic, $\theta = -(U_{kk} + U_{kK})^{-1}[U_k(0,0,0) + < U_{k\tilde{\theta}}, \tilde{\theta} >]$, where $U_{k\tilde{\theta}}$ is the gradient of $U_k(k, K, \tilde{\theta})$ with respect to $\tilde{\theta}$ and $< U_{k\tilde{\theta}}, \tilde{\theta} >$ the inner product of the two.

mean $\mathbb{E}[\theta|x,y] = (1-\delta)\,x + \delta z$ and variance $Var[\theta|x,y] = \sigma^2$, where $z = \lambda y + (1-\lambda)\mu_\theta$ and $\lambda = \sigma_y^{-2}/(\sigma_y^{-2} + \sigma_\theta^{-2})$.[10] For any given prior (and hence given $\sigma_\theta$), there is a one-to-one mapping between $(\sigma_x, \sigma_y)$ and $(\delta, \sigma)$. Thus, holding the prior constant, comparative statics with respect to $(\sigma_x, \sigma_y)$ are isomorphic to comparative static with respect to $(\delta, \sigma)$. We prefer to focus on comparative statics with respect to $(\delta, \sigma)$ for two reasons.

First, a change in $\sigma_y$ or $\sigma_x$ combines a change in the composition of information ($\delta$) with a change in overall posterior uncertainty ($\sigma$). If there were no externalities and strategic interactions, welfare would depend only on $\sigma$, not $\delta$. With strategic interactions, instead, the composition of information plays an important role since it affects the structure of higher order beliefs. From a theoretical point of view, it thus seems most interesting to separate these two effects.[11]

Second, and most importantly, the machinery of private and public signals should not be taken too literary; it is just a modeling device to introduce idiosyncratic and common components in the agents' beliefs about the fundamentals (and thereby in the system of higher-order beliefs). In this sense, $\sigma$ is merely an index of the total uncertainty agents face, and $\delta$ an index of the extent to which the available information is common across agents. We accordingly identify $\delta$ with the *transparency* of information and $\sigma^{-2}$ with the *accuracy* of information.

To recap, the class of economies that we examine are parametrized by $\left(U, \delta, \sigma, \mu_{\tilde{\theta}}, \Sigma_{\tilde{\theta}}\right)$, where $U$ is the utility function, $\delta$ the transparency of information, $\sigma$ the overall posterior uncertainty, and $\left(\mu_{\tilde{\theta}}, \Sigma_{\tilde{\theta}}\right)$ the prior about the fundamentals. We denote this class of economies by $\mathcal{E}$.

## 3 Equilibrium allocations

Each agent chooses $k$ so as to maximize his expected utility, $\mathbb{E}[U(k, K, \tilde{\theta})|x, y]$. The solution to this optimization problem gives the best response for the individual; the fixed point of the best response is the equilibrium.

**Proposition 1** *(i) An allocation $k : \mathbb{R}^2 \to \mathbb{R}$ is an equilibrium if and only if, for all $(x, y)$,*

$$k(x,y) \;=\; \mathbb{E}[\;(1-\alpha)\theta + \alpha K(\tilde{\theta}, y)\;|\;x, y\;], \tag{1}$$

*where $\theta$ is the unique solution to $U_k(\theta, \theta, \tilde{\theta}) = 0$, $K(\tilde{\theta}, y) = \mathbb{E}[k(x,y)|\tilde{\theta}, y]$, and $\alpha \equiv -U_{kK}/U_{kk}$.*

*(ii) The equilibrium exists, is unique, and is given $k(x,y) = \beta x + \gamma z$, where*

$$\beta = 1 - \delta - \rho, \qquad \gamma = \delta + \rho, \qquad \textit{and} \qquad \rho = \frac{\alpha\delta(1-\delta)}{1-\alpha(1-\delta)}. \tag{2}$$

[10] In the following, we will often identify public information with $z$ instead of $y$.

[11] In the context of specific applications, however, it is also interesting to translate the results in terms of comparative statics with respect to $(\sigma_x, \sigma_z)$. See Section 6 for some examples.

**Proof.** Part (i). Given any strategy $k : \mathbb{R}^2 \to \mathbb{R}$, aggregate investment is given by $K(\tilde{\theta}, y) = \mathbb{E}[k(x,y)|\tilde{\theta}, y]$. It follows that a best response is a function $k'(x,y)$ that solves

$$\mathbb{E}[U_k(k'(x,y), K(\tilde{\theta}, y), \tilde{\theta})|x,y] = 0$$

for any $(x,y)$. Using $U_k(\theta,\theta,\tilde{\theta}) = 0$ for all $\tilde{\theta}$, and the fact that $U$ is quadratic, the best-response function must satisfy

$$\mathbb{E}[U_{kk}(k'(x,y) - \theta) + U_{kK}(K(\tilde{\theta}, y) - \theta)|x,y] = 0,$$

for all $(x,y)$. In equilibrium, $k'(x,y) = k(x,y)$. Together with $\alpha \equiv -U_{kK}/U_{kk}$, this gives (1).

Part (ii). Given the linearity of (1) in $\mathbb{E}[\theta|x,y]$ and $\mathbb{E}[K|x,y]$ and the linearity of $\mathbb{E}[\theta|x,y]$ in $x$ and $z$, it is natural to look for a fixed point that is linear in $x$ and $z$. Thus suppose the equilibrium is $k(x,y) = \beta x + \gamma z$, for some $(\beta,\gamma) \in \mathbb{R}^2$. Then, $K(\tilde{\theta}, y) = \beta\theta + \gamma z$, which together with (1) gives

$$k(x,y) = (1 - \alpha + \alpha\beta)\left[(1-\delta)x + \delta z\right] + \alpha\gamma z.$$

It follows that $\beta$ and $\gamma$ must solve $\beta = (1-\alpha+\alpha\beta)(1-\delta)$ and $\gamma = (1-\alpha+\alpha\beta)\delta + \alpha\gamma$, or equivalently $\beta = (1-\alpha)(1-\delta)/[1-\alpha(1-\delta)] \in (0,1)$ and $\gamma = \delta/[1-\alpha(1-\delta)] \in (0,1)$. Clearly this is the unique linear equilibrium. Furthermore, since best responses are linear in $\mathbb{E}[\theta|x,y]$ and $\mathbb{E}[K|x,y]$, there do not exist equilibria other than this one.[12] ■

Condition (1) has a simple interpretation: an agent's best response is an affine combination of his expectation of the complete-information equilibrium activity ($\theta$) and his expectation of aggregate activity. We accordingly identify $\alpha$, the sensitivity of the best response to the expectation of others' activity, with the *equilibrium degree of coordination.*

The sensitivity of the equilibrium allocation to private and public information depends on both the degree of coordination and the transparency of information.When $\alpha = 0$, the weights on $x$ and $z$ are simply the Bayesian weights: $\beta = 1 - \delta$ and $\gamma = \delta$. The coefficient $\rho$ thus measures the excess sensitivity of equilibrium allocations to public information as compared to the case where there are no complementarities. Note that $\rho$ is increasing in $\alpha$. Stronger complementarities thus lead to a higher relative sensitivity to public information. This is a direct implication of the fact that, in equilibrium, the public signal is a relatively better predictor of aggregate behavior than the private signal. In other words, public information has also a coordinating role.

If information about $\theta$ were complete $(\sigma = 0)$, all agents would choose $k = K = \theta$. Incomplete information affects equilibrium behavior in two ways. First, aggregate noise generates "non-fundamental" volatility – variation in aggregate activity $K$ around the complete-information level

[12] Uniqueness follows from the same argument as in Morris and Shin (2002), for the structure of beliefs and best responses is identical.

$\theta$. Second, idiosyncratic noise generates dispersion – variation in the cross-section of the population. The first is measured by $Var(K-\theta)$ and the second by $Var(k-K)$; their dependence on the information structure is characterized below.

**Proposition 2** (**i**) *Non-fundamental volatility,* $Var(K-\theta)$*, increases with* $\delta$ *if and only if* $\alpha < 1/(1+\delta)$.

(**ii**) *Cross-sectional dispersion,* $Var(k-K)$*, necessarily decreases with* $\delta$.

(**iii**) *Both volatility and dispersion increase with* $\sigma$.

An increase in transparency thus always leads to a better alignment of individual choices, which is likely to increase welfare. But, as long as $\alpha$ is low enough, this comes at the expense of higher aggregate volatility, which may reduce welfare. We will examine in more detail the welfare effects of information later; we now turn to how the trade-off between volatility and dispersion is resolved in the efficient allocation.

## 4 Efficient allocations

In this section we characterize efficient allocations and show how the optimal degree of coordination depends on the primitives of the environment. We start by deriving a necessary and sufficient condition for efficiency.

**Lemma 1** *An allocation* $k:\mathbb{R}^2\to\mathbb{R}$ *is efficient if and only if, for almost all* $(x,y)$,

$$\mathbb{E}[\ U_k(k(x,y),K(\tilde{\theta},y),\tilde{\theta})+U_K(K(\tilde{\theta},y),K(\tilde{\theta},y),\tilde{\theta})\mid x,y\ ]\ =\ 0, \tag{3}$$

*where* $K(\tilde{\theta},y)=\mathbb{E}[k(x,y)|\tilde{\theta},y]$.

**Proof.** An efficient allocation can be characterized by looking for a pair of functions $k:\mathbb{R}^2\to\mathbb{R}$ and $K:\mathbb{R}^{n+1}\to\mathbb{R}$ that maximize

$$\mathbb{E}w=\int_{(\tilde{\theta},y)}\int_x U(k(x,y),K(\tilde{\theta},y),\tilde{\theta})dP(x|\tilde{\theta})dP(\tilde{\theta},y)$$

$$\text{subject to}\quad K(\tilde{\theta},y)=\int_x k\,(x,y)\,dP(x|\tilde{\theta})\quad\text{for all }(\tilde{\theta},y),$$

where $P(\tilde{\theta},x,y)$ stands for the joint distribution of $(\tilde{\theta},x,y)$, $P(\tilde{\theta},y)$ for the marginal distribution of $(\tilde{\theta},y)$ and $P(x|\tilde{\theta})=P(x|\tilde{\theta},y)$ for the conditional distribution of $x$ given $\tilde{\theta}$ (and hence also given $(\tilde{\theta},y)$). The Lagrangian of this problem is

$$\Lambda=\int_{(\tilde{\theta},y)}\int_x U(k(x,y),K(\tilde{\theta},y),\tilde{\theta})dP(x|\tilde{\theta})dP(\tilde{\theta},y)+$$

$$+\int_{(\tilde{\theta},y)}\lambda(\tilde{\theta},y)\left[K(\tilde{\theta},y)-\int_x k\,(x,y)\,dP(x|\tilde{\theta})\right]dP(\tilde{\theta},y).$$

and therefore the first order conditions for $K(\tilde{\theta}, y)$ and $k(x, y)$ are given by

$$\int_x U_K(k(x,y), K(\tilde{\theta}, y), \tilde{\theta}) dP(x|\tilde{\theta}) + \lambda(\tilde{\theta}, y) \;=\; 0 \tag{4}$$

$$\int_{\tilde{\theta}} \left[ U_k(k(x,y), K(\tilde{\theta}, y), \tilde{\theta}) - \lambda(\tilde{\theta}, y) \right] dP(\tilde{\theta}|x, y) \;=\; 0 \tag{5}$$

Noting that $U_K$ is linear in its arguments and that $K(\tilde{\theta}, y) = \int_x k(x, y)\, dP(x|\tilde{\theta})$, condition (4) can be rewritten as $-\lambda(\tilde{\theta}, y) = U_K(K(\tilde{\theta}, y), K(\tilde{\theta}, y), \tilde{\theta})$. Replacing this into (5) gives (3). Since $U$ is strictly concave and the constraint is linear, (3) is both necessary and sufficient, which completes the proof. ■

This result has a simple interpretation. Under complete information about $\tilde{\theta}$, the efficient (first-best) allocation solves $W_K(K, \tilde{\theta}) = 0$, or equivalently $U_k(K, K, \tilde{\theta}) + U_K(K, K, \tilde{\theta}) = 0$. Condition (3) is just the incomplete-information counterpart of this first order condition.

We next translate this result to a representation that facilitates a direct comparison of efficient and equilibrium allocations.

**Proposition 3** *(i) An allocation $k : \mathbb{R}^2 \to \mathbb{R}$ is efficient if and only if, for almost all $(x, y)$,*

$$k(x, y) \;=\; \mathbb{E}[\; (1 - \alpha^*)\theta_{FB} + \alpha^* K(\tilde{\theta}, y) \mid x, y \;] \tag{6}$$

*where $\theta_{FB}$ is the unique solution to $W_K(\theta_{FB}, \tilde{\theta}) = 0$, $K(\tilde{\theta}, y) \equiv \mathbb{E}[k(x, y)|\tilde{\theta}, y]$, and*

$$\alpha^* \equiv -\frac{2U_{kK} + U_{KK}}{U_{kk}} = 2\alpha - \eta. \tag{7}$$

*(ii) The efficient allocation exists, is essentially unique, and is given by $k^*(x, y) = T(\beta^* x + \gamma^* z)$, where*

$$\beta^* = 1 - \delta - \rho^*, \qquad \gamma^* = \delta + \rho^*, \qquad \rho^* = \frac{\alpha^* \delta (1 - \delta)}{1 - \alpha^*(1 - \delta)}. \tag{8}$$

*and $T(\theta) \equiv \mathbb{E}[\theta_{FB}|\theta]$.*

**Proof.** Part (i). Since $U$ is quadratic, (3) can be rewritten as

$$\mathbb{E}[\; U_k(\theta_{FB}, \theta_{FB}, \tilde{\theta}) + U_{kk} \cdot (k(x,y) - \theta_{FB}) + U_{kK} \cdot (K(\tilde{\theta}, y) - \theta_{FB}) + \\ + U_K(\theta_{FB}, \theta_{FB}, \tilde{\theta}) + (U_{Kk} + U_{KK}) \cdot (K(\tilde{\theta}, y) - \theta_{FB}) \mid x, y \;] \;=\; 0.$$

Using $W_K(\theta_{FB}, \tilde{\theta}) = U_k(\theta_{FB}, \theta_{FB}, \tilde{\theta}) + U_K(\theta_{FB}, \theta_{FB}, \tilde{\theta}) = 0$, the above can be rewritten as

$$\mathbb{E}[\; U_{kk}\,(k(x,y) - \theta_{FB}) + (2U_{Kk} + U_{KK})(K(\tilde{\theta}, y) - \theta_{FB}) \mid x, y \;] = 0,$$

which together with $W_{KK} = U_{kk} + 2U_{kK} + U_{KK} = U_{kk}(1 - 2\alpha + \eta)$ gives (6).

Part (ii). Since $U$ is quadratic, $\theta_{FB}$ is an affine transformation of $\tilde{\theta}$.[13] Furthermore, $\xi$ and $\varepsilon$ are independent of each element of $\tilde{\theta}$, implying that $\mathbb{E}[\theta_{FB}|x,y] = \mathbb{E}[\mathbb{E}[\theta_{FB}|\theta]|x,y] = \mathbb{E}[T(\theta)|x,y]$. Since in addition, $(\theta_{FB}, \theta)$ are jointly normal, $T(\theta)$ is simply the projection of $\theta_{FB}$ on $\theta$ and hence is also normal. Noting that $T(x)$ and $T(y)$ are additive signals of $T(\theta)$, and $\lambda T(y) + (1-\lambda)T(\mu_\theta) = T(\lambda y + (1-\lambda)\mu_\theta) = T(z)$, it is then immediate that the unique solution to (6) has the same structure as the equilibrium strategy of Proposition (1) with $\alpha^*$ replacing $\alpha$ and $T(\theta)$ replacing $\theta$, which completes the proof. ■

Hence, in the efficient allocation, each individual's action is an affine combination of his expectation of the first-best allocation and of the aggregate activity. In this sense, (6) is the analogue for efficiency of the best-response condition for equilibrium. A direct implication is the following.

**Proposition 4** *Given an economy* $\mathbf{e} = \left(U; \sigma, \delta, \mu_{\tilde{\theta}}, \Sigma_{\tilde{\theta}}\right) \in \mathcal{E}$, *let* $\mathcal{U}(\mathbf{e})$ *be the set of functions* $U'$ *such that, if agents perceived their payoffs to be* $U'$ *rather than* $U$, *the equilibrium would coincide with the efficient allocation for* $\mathbf{e}$.

*(i) For every* $\mathbf{e}$, $\mathcal{U}(\mathbf{e})$ *is non empty.*

*(ii) For every* $\mathbf{e}$ *such that the agents' signals are informative about* $\theta_{FB}$, $U' \in \mathcal{U}(\mathbf{e})$ *only if* $\alpha' \equiv -U'_{kK}/U'_{kk}$ *equals* $\alpha^*$.

Part (i) simply states that the efficient allocation can be implemented as the equilibrium of a game where individual incentives are appropriately manipulated – for example, the planner could use Pareto-like taxes and subsidies to fashion individual best responses. Part (ii), on the other hand, permits to identify $\alpha^*$ as the *optimal degree of coordination* – it describes the sensitivity of best responses to aggregate activity that the planner would have liked the agents to perceive.[14]

Proposition 3 thus tell us how the optimal degree of coordination depends on the primitives of the environment and how it compares to the equilibrium degree of coordination. In the absence of a second-order external effect ($U_{KK} = 0$), the optimal degree of coordination is twice as high (in absolute value) as the equilibrium one; otherwise, it is higher [lower] than the equilibrium one if and only if the contribution of the external effect to the concavity of aggregate welfare is small [big] enough, that is, if and only if $-U_{KK} < [>] U_{kK}$.

The counterpart of efficient coordination is the efficient use of information.

**Corollary 1** *The relative sensitivity of the efficient allocation to public information is higher than that of the equilibrium allocation if and only if the optimal degree of coordination is higher than the equilibrium one, which in turn is true if and only if the second-order external effect is weak enough:*

$$\gamma^*/\beta^* \geq \gamma/\beta \iff \alpha^* \geq \alpha \iff -U_{KK} \leq U_{kK}.$$

[13] Since $U$ and hence $W$ is quadratic, $\theta_{FB} = -(W_K(0,0) + < W_{K\tilde{\theta}}, \tilde{\theta} >)/W_{KK}$.

[14] If the signals $(x, y)$ are uninformative about $\theta_{FB}$, the efficient allocation does not depend on them and hence the relative sensitivity to public information (equivalently, the optimal degree of coordination) is not determined.

An alternative characterization of the efficient use of information is also useful. Welfare at the efficient allocation can be expressed as

$$\mathbb{E}w = \mathbb{E}W(\theta^*, \tilde{\theta}) + \frac{W_{KK}}{2}Var(K - \theta^*) + \frac{U_{kk}}{2}Var(k - K), \tag{9}$$

where $\theta^* \equiv \mathbb{E}\left[\theta_{FB}|\theta\right]$. (This follows from a Taylor expansion around $k = K = \theta^*$; see the Appendix.) Equivalently, $\mathbb{E}w = \mathbb{E}W(\theta^*, \tilde{\theta}) - \mathcal{L}^*$, where

$$\mathcal{L}^* = \frac{|W_{KK}|}{2}Var(K - \theta^*) + \frac{|U_{kk}|}{2}Var(k - K). \tag{10}$$

Note that $\mathbb{E}W(\theta^*, \tilde{\theta})$ is welfare under complete information (about $\theta$); $\mathcal{L}^*$ thus captures the welfare losses associated with incomplete information, namely those associated with aggregate volatility and cross-sectional dispersion.

That volatility and dispersion generate welfare losses is not surprising given the concavity of preferences; what is interesting is that their weights in welfare may differ. When there are no strategic and second-order external effects ($U_{kK} = U_{KK} = 0$), social welfare inherits the curvature of individual utility ($W_{KK} = U_{kk}$), so that volatility and dispersion contribute equally to welfare losses. Otherwise, complementarity ($U_{kK} > 0$) and convexity of external effects ($U_{KK} > 0$) reduce the relative weight on aggregate volatility, whereas substitutability ($U_{kK} < 0$) and concavity of external effects ($U_{KK} < 0$) increases it.

Volatility is generated by noise in public information, dispersion by noise in private information. Increasing the relative sensitivity of allocations to public information (equivalently, raising the degree of coordination) dampens dispersion at the expense of higher volatility; the efficient use of information simply reflects the efficient resolution of this trade-off.

**Corollary 2** *The optimal degree of coordination equals one minus the weight that welfare assigns to volatility relative to dispersion:*

$$\alpha^* = 1 - \frac{W_{KK}}{U_{kk}}.$$

We conclude this section with necessary and sufficient conditions for the equilibrium to be efficient (under incomplete information).

**Proposition 5** *The equilibrium is efficient if and only if* $\theta = \mathbb{E}\left[\theta_{FB}|\theta\right]$ *and* $\alpha^* = \alpha$.

The first condition means that the equilibrium under *complete* information coincides with the best predictor of the first best (which is of course naturally satisfied in the case that $\theta = \theta_{FB}$). But efficiency under complete information alone does not guarantee efficiency under incomplete information; what is also needed is efficiency in the use of information, or equivalently, efficiency in the degree of coordination.

**Remark.** In some applications the cross-sectional dispersion has a direct external effect on individual utility. For example, in New-Keynesian models with product differentiation and monopolistic competition, the property that different goods are imperfect substitutes implies that price dispersion has a negative effect on individual utility. We can easily accommodate such an effect as long as it enters linearly in the utility function: $u = U\left(k, K, var, \tilde{\theta}\right)$ with $U_{var}$ being a constant. Let $\omega \equiv U_{var}/(-2U_{kk})$ measure the intensity of this effect relative to the curvature of the utility function. Then all the results go through provided we replace (7) with

$$\alpha^* \equiv -\frac{2U_{kK} + U_{KK} - 2U_{var}}{U_{kk} + 2U_{var}} = \frac{2\alpha - \eta - \omega}{1 - \omega},$$

which is decreasing in $\omega$. Therefore, other things equal, a negative externality from dispersion ($U_{var} < 0$ or equivalently $\omega < 0$) contributes to a higher optimal degree of coordination. This is intuitive given the way such an externality tilts the trade off between volatility and dispersion.

# 5 Social value of information

In this section we characterize the impact of information on equilibrium welfare. As discussed in the introduction, we focus on the comparative statics of higher transparency (higher $\delta$ for given $\sigma$) and better accuracy of overall information (lower $\sigma$ for given $\delta$).

We start with economies in where the equilibrium is efficient, which provides a useful benchmark.

**Proposition 6** *Suppose the equilibrium is efficient. Then welfare necessarily decreases with $\sigma$, and increases [decreases] with $\delta$ if and only if agents' actions are strategic complements [substitutes].*

**Proof.** Since the equilibrium is efficient, equilibrium welfare coincides with welfare at the optimal allocation and hence the comparative statics of the former with respect to $(\delta, \sigma)$ coincide with the opposite of those of $\mathcal{L}^*$ in (10). Calculating volatility and dispersion (see the proof of Proposition 2) gives

$$Var(K - \theta^*) = Var(K - \theta) = \frac{\delta}{[1 - \alpha(1 - \delta)]^2}\sigma^2 \quad \text{and} \quad Var(k - K) = \frac{(1 - \alpha)^2(1 - \delta)}{[1 - \alpha(1 - \delta)]^2}\sigma^2.$$

Substituting the above into (10) and using $\alpha^* = \alpha$ (equivalently, $W_{KK}/U_{kk} = 1 - \alpha$), gives

$$\mathcal{L}^* = \frac{|U_{kk}|}{2}\frac{1 - \alpha}{1 - \alpha + \alpha\delta}\sigma^2.$$

It follows that $\partial\mathcal{L}^*/\partial\sigma > 0$ always, whereas $\partial\mathcal{L}^*/\partial\delta < (>)0$ if and only if $\alpha > (<)0$. ■

Thus, as long as the equilibrium is efficient, there is a perfect mapping between the social value of transparency and the degree of coordination: substituting private for public information increases welfare if and only if agents' actions are strategic complements. In other words, complementarity alone contributes to making public information *more* valuable.

Consider next economies where the equilibrium is inefficient. This affects the welfare effects of information for two reasons. First, the equilibrium degree of coordination may differ from the optimal, thus introducing an inefficiency in how the trade-off between volatility and dispersion is resolved in equilibrium. Second, the equilibrium level of activity may differ from the socially optimal one even under complete information, thus introducing first-order welfare losses in addition to those associated with volatility and dispersion.

In equilibrium, ex-ante welfare can be represented as $\mathbb{E}w = \mathbb{E}W(\theta, \tilde{\theta}) - \mathcal{L}$, where $\mathbb{E}W(\theta, \tilde{\theta})$ is expected welfare under complete information and

$$\mathcal{L} = -Cov[W_K(\theta, \tilde{\theta}), (K - \theta)] + \frac{|W_{KK}|}{2} \cdot Var(K - \theta) + \frac{|U_{kk}|}{2} \cdot Var(k - K) \tag{11}$$

measures the welfare losses, or gains, generated by incomplete information. (This follows from a Taylor expansion around $K = \theta$; see Appendix.)

The last two terms in $\mathcal{L}$ are the second-order effects associated with volatility and dispersion, familiar already from the case of efficient allocations. The covariance term, on the other hand, captures a novel first-order effect. When there is no externality, so that the complete-information equilibrium coincides with the first-best allocation, then $W_K(\theta, \tilde{\theta}) = 0$. In this case, the covariance term is zero; this is merely an implication of the fact that small deviations around a maximum have zero first-order effects. But when there is an externality, $W_K(\theta, \tilde{\theta}) \neq 0$. Welfare then increases [decreases] if $K - \theta$ and $W_K(\theta, \tilde{\theta})$ are positively [negatively] correlated, that is, if the "error" in aggregate investment due to incomplete information tends to move in the same [opposite] direction as the social return to investment, thus partly offsetting [exacerbating] the first-order loss associated with the externality.

As shown in the Appendix, this first-order effect can be decomposed as

$$Cov[W_K(\theta, \tilde{\theta}), (K - \theta)] = -\phi \cdot |W_{kk}| \cdot |Cov(\theta, K - \theta)|$$

where $\phi$ denotes the coefficient of the projection of $\theta_{FB} - \theta$ on $\theta$, namely

$$\phi \equiv \frac{Cov(\theta_{FB} - \theta, \theta)}{Var(\theta)}.$$

This coefficient measures how much the gap between the first-best and the complete-information equilibrium activity co-varies with the equilibrium activity; it depends on primitives of the environment but not the information structure. The term $Cov(\theta, K - \theta)$, on the other hand, captures the component of the first-order effect that is due to incomplete information. Since agents put a positive weight on the prior, $K$ moves less than one-to-one with $\theta$. It follows that $Cov(\theta, K - \theta)$ is negative and, as we show in the Appendix, decreases with $\sigma$ and increases [decreases] with $\delta$ if and only if $\alpha > 0$ $[\alpha < 0]$. How this affects welfare depends on $\phi$. If $\theta = \theta_{FB}$, or more generally if the variation in the efficiency gap is orthogonal to the variation in $\theta$, then $\phi = 0$ and first-order welfare losses are zero. If instead the efficiency gap tends to increase with $\theta$, then $\phi > 0$ and an increase in $\delta$ contributes to lower [higher] first-order losses if $\alpha > 0$ $[\alpha < 0]$; the converse is true for $\phi < 0$, in which case the first-order effect is actually a gain, not a loss.

To evaluate the overall impact of $\delta$ on welfare, these effects must be combined with those of volatility and dispersion. Recall that an increase in $\delta$ reduces dispersion, possibly at the expense of higher volatility. The rate at which dispersion can be substituted for volatility depends on $\alpha$; the rate at which the planner is willing to accept such as substitution depends on $W_{KK}/U_{kk}$, or equivalently on $\alpha^*$. The overall impact of $\delta$ on welfare thus depends on $\alpha$, $\alpha^*$, and $\phi$.

**Proposition 7** *There exist* $\underline{\phi}, \bar{\phi} : \mathbb{R}^2 \to \mathbb{R}$, *with* $\underline{\phi}(\alpha, \eta) = \bar{\phi}(\alpha, \eta)$ *if* $\alpha = \eta$ *(i.e.* $\alpha^* = \alpha$*) and* $\underline{\phi}(\alpha, \eta) < \bar{\phi}(\alpha, \eta)$ *otherwise, such that the following are true.*

1. *If* $\alpha = 0$ ***(strategic independence)****, then:*

   - *welfare increases [decreases] with* $\delta$ *if and only if* $\alpha^* > [<] 0$.

2. *If* $\alpha > 0$ ***(strategic complementarity)****, then:*

   - *welfare is increasing in* $\delta$ *for all* $\delta \in [0, 1]$ *if and only if* $\phi \geq \bar{\phi}$
   - *welfare is decreasing in* $\delta$ *for all* $\delta \in [0, 1]$ *if and only if* $\phi \leq \underline{\phi}$
   - $\bar{\phi} < 0$ *if and only if* $\alpha^* > \alpha^2$.

3. *If* $\alpha < 0$ ***(strategic substitutability)****, then:*

   - *welfare is increasing in* $\delta$ *for all* $\delta \in [0, 1]$ *if and only if* $\phi \leq \underline{\phi}$
   - *welfare is decreasing in* $\delta$ *for all* $\delta \in [0, 1]$ *if and only if* $\phi \geq \bar{\phi}$
   - $\bar{\phi} < 0$ *if and only if* $\alpha^* < -\alpha^2/(1 - 2\alpha)$.

*Finally, there exists* $\hat{\phi} : \mathbb{R}^2 \to \mathbb{R}$, *with* $\hat{\phi}(\alpha, \eta) < 0$ *for any* $(\alpha, \eta)$, *such that welfare is decreasing in* $\sigma$ *if and only if* $\phi < \hat{\phi}$.

The following sufficient conditions are then immediate for the case that the (complete-information) efficiency gap either is constant or co-varies positively with equilibrium.

**Corollary 3** *Suppose* $\phi \geq 0$. *The social value of transparency is positive if* $\alpha^* \geq \alpha > 0$ *and negative if* $\alpha^* \leq \alpha < 0$.

Finally, note that when $\phi$ is sufficiently negative, welfare *decreases* with the accuracy of information. That is, when the efficiency gap and the equilibrium level of investment are strongly negatively correlated, information per se can be undesirable from a social perspective (see also the example Section 6.4).

# 6 Applications

In this section, we show how our results may help understand the welfare effects of information in specific applications.

## 6.1 Investment complementarities

The canonical model of production externalities can be nested as follows:

$$U(k, K, \tilde{\theta}) = A(K, \tilde{\theta})k - c(k), \tag{12}$$

where $A(K, \tilde{\theta}) = (1-a)\tilde{\theta} + aK$ represents the private return to investment, with $a \in (0, 1/2)$ and $\tilde{\theta} \in \mathbb{R}$, and $c(k) = k^2/2$ the private cost of investment.[15] Variants of this specification appear in Bryant (1983), Romer (1986), Matsuyama (1992), Acemoglu (1993), and Benhabib and Farmer (1994), as well as models of network externalities and spillovers in technology adoption. The important ingredient is that the private return to investment increases with the aggregate level of investment.

In this example, the equilibrium level of investment under complete information is $\theta = \tilde{\theta}$, whereas the first best is $\theta_{FB} = \frac{1-a}{1-2a}\tilde{\theta}$, and hence the covariance between the efficiency gap and the equilibrium level under complete information is $\phi = \frac{a}{1-2a} > 0$. Furthermore, $U_{kk} = -1$, $U_{kK} = a$, and $U_{KK} = 0$, implying that $\alpha = a > 0$, $\eta = 0$, and $\alpha^* = 2\alpha$; that is, there is no second-order external effect and hence the socially optimal level of coordination is twice as much as the equilibrium one.

**Corollary 4** *In the investment example described above, coordination is inefficiently low and welfare unambiguously increases with both the accuracy and the transparency of information.*

Translating these results in terms of $\sigma_x$ and $\sigma_y$, it is easy to show that welfare unambiguously increases with a reduction in either $\sigma_x$ or $\sigma_y$.[16] We conclude that, in this example, any short of information is beneficial, and transparency is better than ambiguity.

[15] This was the example we examined in Angeletos and Pavan (2004); note however that there, by assuming an uninformative prior, we omitted the role of $Cov(\theta, K - \theta)$ on welfare.

[16] Expressing $\delta$ and $\sigma$ in terms of $(\sigma_x, \sigma_y, \sigma_\theta)$, and using $\eta = 0$ and $\phi = a/(1-2a)$, we have that $\mathcal{L} = \frac{1}{2}\left(\sigma_x^{-2} + \sigma_y^{-2} + \sigma_\theta^{-2} - a^2\sigma_x^{-2}\right)/(\sigma_x^{-2} + \sigma_y^{-2} + \sigma_\theta^{-2} - a\sigma_x^{-2})^2$, which is increasing in both $\sigma_x$ and $\sigma_y$.

### 6.2 No externalities

In the example considered above, the complementarity came together with externalities. Consider now economies where there is complementarity ($\alpha > 0$) but the complete-information equilibrium involves no externality of either the first or the second order in the sense that $U_K(\theta, \theta, \tilde{\theta}) = U_{KK} = 0$ (and hence $\phi = \eta = 0$).

**Corollary 5** *Consider an economy where the complete-information equilibrium is efficient, the complementarity is positive, and the second-order external effect is zero. Coordination is inefficiently low and welfare unambiguously increases with both the accuracy and the transparency of information.*

Therefore, it is not the presence of externality that makes transparency socially desirable in the previous application: the present example has no externalities and yet welfare unambiguously increases with $\delta$. What, however, the two examples above have in common is that the optimal level of coordination is higher than the equilibrium one. As we show next, this plays an important role.

### 6.3 Undesirable coordination

Consider an environment in which the complete-information equilibrium is efficient, but where the agents' desire to coordinate under incomplete information is not warranted from a social perspective.

The first condition is satisfied if and only if $U_K(\theta, \theta, \tilde{\theta}) = 0$, so that $\phi = 0$; the second if and only if $U_{KK} = -2U_{kK}$, so that $\alpha^* = 0$ and volatility and dispersion are equally weighted in welfare losses. It follows that[17]

$$\mathcal{L} = \frac{|U_{kk}|}{2} \{Var(K - \theta) + Var(k - K)\} = \frac{|U_{kk}|}{2} \left\{ \frac{\delta + (1-\alpha)^2(1-\delta)}{[1 - \alpha(1-\delta)]^2} \right\} \sigma^2,$$

which, for $\alpha > 0$, coincides with the welfare losses in Morris and Shin (2002).[18]

**Corollary 6** *Consider an economy where the complete-information equilibrium is efficient and the optimal degree of coordination is zero. In the case of strategic substitutes, welfare decreases monotonically with transparency; in the case of strategic complements, welfare increases with transparency if and only if $\delta \geq (1-\alpha)/(2-\alpha)$.*

As an example of this scenario, consider the investment game described in Section 6.1, but introduce an additional externality by adding the term $B(K, \tilde{\theta}) = (\tilde{\theta} - A(K, \tilde{\theta}))K$ in (12). Welfare

[17] Note that $\mathcal{L}$ is globally increasing in $\delta$ when $\alpha < 0$ and independent of $\delta$ when $\alpha = 0$. When $\alpha > 0$, $\mathcal{L}$ attains its minimum ($= \sigma^2$) at $\delta \in \{0, 1\}$ and its maximum at $\delta = \frac{1-\alpha}{2-\alpha}$; $\partial\mathcal{L}/\partial\delta > 0$ for $\delta < \frac{1-\alpha}{2-\alpha}$; and $\partial\mathcal{L}/\partial\delta < 0$ for $\delta > \frac{1-\alpha}{2-\alpha}$.

[18] In their model, individual payoffs are given by $U = -(1-r)(k-\tilde{\theta})^2 - r(k-K)^2 + r \cdot var$, so that $\tilde{\theta} = \theta = \theta_{FB}$, $U_{kk} = -2$, $U_{kK} = -2r$, $U_{KK} = -2r$ and hence $\alpha = r > 0$, $\eta = 2r$ and $\phi = 0$. It follows that $w = -(1-r)[(K-\tilde{\theta})^2 + var]$ and hence $\mathbb{E}w = -(1-r)\mathcal{L}$.

then reduces to $w = \mathbb{E}[\tilde{\theta}k - c(k)|\tilde{\theta}, y]$. From a social perspective, it is then as if utility were simply $u = \tilde{\theta}k - c(k)$, in which case of course there is no value to coordination. Welfare then decreases with transparency for low levels of $\delta$, which is essentially the same result as in the beauty contest model of Morris and Shin (2002).

### 6.4 Inefficient fluctuations

Consider next an economy in which the first-best level of activity is constant (or varies with $\tilde{\theta}$, but is orthogonal to the equilibrium level of activity $\theta$), so that the entire equilibrium fluctuations observed under either complete or incomplete information are inefficient. Formally, assume that $Cov\left(\theta_{FB} - \theta, \theta\right) = -Var\left(\theta\right)$ so that $\phi = -1$. To simplify, assume further that $\alpha^* = \alpha$. It follows that

$$\mathcal{L} = -\frac{|U_{kk}|}{2}\frac{1-\alpha}{1-\alpha+\alpha\delta}\sigma^2.$$

**Corollary 7** *In the example of inefficient fluctuations described above, welfare unambiguously decreases with the accuracy of information and decreases [increases] with transparency if and only if* $\alpha > 0$ $[\alpha < 0]$.

In this example, a reduction in $\sigma$, by bringing the incomplete-information equilibrium closer to the complete-information counterpart, induces more inefficient fluctuations. As a result, any short of information – either public or private – has the potential to reduce welfare. Moreover, the effect of $\delta$ is exactly the opposite than in the case that the equilibrium is efficient: transparency is harmful if $\alpha^* = \alpha > 0$ and beneficial if $\alpha^* = \alpha < 0$.

### 6.5 Competitive economies

The examples considered so far feature either positive complementarity or some form of inefficiency. We now turn to an environment where the agents' choices are strategic substitutes but where the equilibrium is efficient – a competitive market economy.

There is a continuum of households, each consisting of a consumer and a producer, and two commodities. Let $q_{1i}$ and $q_{2i}$ denote the respective quantities purchased by consumer $i$ (the consumer living in household $i$). His preferences are given by

$$u_i = v(q_{1i}, \tilde{\theta}) + q_{2i}, \tag{13}$$

where $v(q, \tilde{\theta}) = \tilde{\theta}q - bq^2/2$, $\tilde{\theta} \in \mathbb{R}$, and $b > 0$, while his budget is

$$pq_{1i} + q_{2i} = e + \pi_i, \tag{14}$$

where $p$ is the price of good 1 relative to good 2, $e$ is an exogenous endowment of good 2, and $\pi_i$ are the profits that producer $i$ makes denominated in terms of good 2. Profits in turn are given by

$$\pi_i = pk_i - c(k_i) \tag{15}$$

where $k_i$ denotes the quantity of good 1 produced by household $i$ and $c(k)$ the cost in terms of good 2, with $c(k) = k^2/2$.[19]

The random variable $\tilde{\theta}$ represents a shock in the relative demand for the two goods. Exchange and consumption take place once $\tilde{\theta}$ has become known; but production takes place at an earlier stage, when information is still incomplete.

Consumer $i$ chooses $(q_{1i}, q_{2i})$ so as to maximize (13) subject to (14), which gives $p = \tilde{\theta} - bq_{1i}$. Clearly, all households consume the same quantity of good 1, which together with market clearing gives $q_{1i} = K$ for all $i$ and $p = \tilde{\theta} - bK$. It follows that $i$'s utility can be restated as $u_i = v(K, \tilde{\theta}) - pK + e + \pi_i = bK^2/2 + e + \pi_i$, where $\pi_i = pk_i - c(k_i) = (\tilde{\theta} - bK)k_i - k_i^2/2$. This example is thus nested in our model with

$$U(k, K, \tilde{\theta}) = (\tilde{\theta} - bK)k_i - k_i^2/2 + bK^2/2 + e,$$

in which case $\theta_{FB} = \theta = \tilde{\theta}/(1+b)$, $\phi = 0$, $\alpha = -b$, $\eta = -b$, and therefore $\alpha^* = -b = \alpha < 0$.

That the complete-information equilibrium is efficient should not be a surprise: this is merely an example of a complete-markets competitive economy in which the first welfare theorem applies. What is interesting here is that the incomplete-information equilibrium is also efficient: the strategic substitutability ($\alpha < 0$) perceived by the agents coincides exactly with the one that the planner would have liked them to perceive $(\alpha^* = \alpha)$.

Since the equilibrium is efficient, Blackwell's theorem extends from the individual level to the aggregate, implying that welfare unambiguously increases with either a reduction in $\sigma_x$ for given $\sigma_y$ or a reduction in $\sigma_y$ for given $\sigma_x$. Nevertheless, because of the strategic substitutability introduced by market competition, substituting a lower $\sigma_y$ for a higher $\sigma_x$ reduces welfare.

**Corollary 8** *In the competitive economy described above, the equilibrium is efficient and welfare unambiguously decreases with both $\delta$ and $\sigma$.*[20]

[19] Implicit behind this cost function is a quadratic production frontier. The resource constraints are therefore given by $\int_i q_{1i}di = \int_i k_i di$ and $\int_i q_{2i}di = e - \int_i bk_i^2 di$ for good 1 and 2, respectively.

[20] The equilibrium is inefficient if we define welfare as producer surplus alone, which may be relevant for open economies that are net exporters of good 2. Nevertheless, welfare continues to decrease with both $\delta$ and $\sigma$. To see this, note that individual payoffs are then given by $u_i = \pi_i$, or equivalently $U(k, K, \tilde{\theta}) = (\tilde{\theta} - bK)k - k^2/2$. It follows that $\alpha = -b < 0$, $\eta = 0$, $\theta = \tilde{\theta}/(1+b)$ and $\theta_{FB} = \tilde{\theta}/(1+2b)$, so that $\phi = -b/(1+2b)(1+b) < 0$ and $\alpha^* = -2b = 2\alpha < 0$.

Consider the debate on transparency vs. constructive ambiguity in monetary policy. If we interpret "transparent" central bank disclosures as information that admits a single common interpretation and "ambiguous" disclosures as information that admits multiple idiosyncratic interpretations then the result above makes a case for constructive ambiguity.

This is reminiscent of Morris and Shin (2002). However, whereas the result there was driven by inefficiently high coordination ($\alpha^* = 0 < \alpha$), here it is due to efficient substitutability ($\alpha^* = \alpha < 0$). It is perhaps more surprising that a case for constructive ambiguity can be made even for efficient competitive economies.

Finally, this last result opens up the possibility that, in relatively efficient competitive economies, the informative role of prices may in some cases be welfare damaging. For example, suppose that an exogenous increase in the informativeness of prices (caused for example by a reduction in the impact of noisy traders) leads agents to reduce their (costly) collection of private information. Welfare may then decrease despite the fact that agents' overall information becomes more precise.

Clearly, our model does not examine information aggregation through prices. Extending the results in this direction is a promising line for future research.

# 7 Appendix

**Proof of Proposition 2.** From (2), $K-\theta = \gamma(z-\theta) = \gamma\left[\lambda(y-\theta)+(1-\lambda)(\mu_\theta-\theta)\right]$. Using $\lambda = \sigma_y^{-2}/[\sigma_y^{-2}+\sigma_\theta^{-2}]$, $\gamma = \delta/[1-\alpha(1-\delta)]$, and $\delta = (\sigma_y^{-2}+\sigma_\theta^{-2})/[\sigma_y^{-2}+\sigma_\theta^{-2}+\sigma_x^{-2}]$, we have that

$$Var(K-\theta) = \gamma^2(\lambda^2\sigma_y^2+(1-\lambda)^2\sigma_\theta^2) = \frac{\delta}{(1-\alpha+\alpha\delta)^2}\sigma^2,$$

which always increases with $\sigma$ and increases with $\delta$ if and only if $\alpha < 1/(1+\delta)$. Next, since $k-K = \beta(x-\theta)$, $\beta = (1-\alpha)(1-\delta)/[1-\alpha(1-\delta)]$, and $\sigma_x = \sigma/\sqrt{1-\delta}$, we have that

$$Var(k-K) = (\beta\sigma_x)^2 = \frac{(1-\alpha)^2(1-\delta)}{(1-\alpha+\alpha\delta)^2}\sigma^2,$$

which always increases with $\sigma$ and decreases with $\delta$. ■

**Proof of Condition (9).** Welfare at the efficient allocation is given by

$$\mathbb{E}w = \mathbb{E}[W(K,\tilde{\theta})] + \frac{U_{kk}}{2}\mathbb{E}[var],$$

where $k = k(x,y)$ and $K = K\left(\tilde{\theta},y\right)$ are shortcuts for the efficient allocation, and $var = \mathbb{E}[(k-K)^2|\tilde{\theta},y]$. A quadratic expansion of $W(K,\tilde{\theta})$ around $\theta^* \equiv \mathbb{E}[\theta_{FB}|\theta]$, which is exact since $U$ and thus

$W$ are quadratic, gives

$$W(K,\tilde{\theta}) = W(\theta^*,\tilde{\theta}) + W_K(\theta^*,\tilde{\theta}) \cdot (K-\theta^*) + \frac{W_{KK}}{2} \cdot (K-\theta^*)^2.$$

By the law of iterated expectations, the fact that $K-\theta^*$ is measurable with respect to $(\theta, y)$, the independence of $\theta^*$ and $\varepsilon$, and the linearity of $W_K$, we have that

$$\mathbb{E}[W_K(\theta^*,\tilde{\theta}) \cdot (K-\theta^*)] = \mathbb{E}[\mathbb{E}[W_K(\theta^*,\tilde{\theta}) \cdot (K-\theta^*)|\theta, y] = \mathbb{E}[\mathbb{E}[W_K(\theta^*,\tilde{\theta})|\theta, y] \cdot (K-\theta^*)]$$

where

$$\mathbb{E}[W_K(\theta^*,\tilde{\theta})|\theta, y] = \mathbb{E}[W_K(\theta^*,\tilde{\theta})|\theta] = \mathbb{E}[W_K(\theta_{FB},\tilde{\theta})|\theta] = 0.$$

It follows that

$$\mathbb{E}w = \mathbb{E}W(\theta^*,\tilde{\theta}) + \frac{W_{KK}}{2} \cdot \mathbb{E}[(K-\theta^*)^2] + \frac{U_{kk}}{2} \cdot \mathbb{E}[(k-K)^2].$$

At the efficient allocation, $\mathbb{E}k = \mathbb{E}K = T(\mu_\theta) = \mathbb{E}\theta^* = \mathbb{E}[\theta_{FB}]$ and therefore $\mathbb{E}[(K-\theta^*)^2] = Var(K-\theta^*)$ and $\mathbb{E}[(k-K)^2] = Var(k-K)$, which gives the result. ■

**Proof of Proposition 4.** Consider first part (ii). When agents perceive payoffs to be $U'$, the equilibrium is the unique function $k : \mathbb{R}^2 \to \mathbb{R}$ that solves

$$k(x,y) = \mathbb{E}[\ (1-\alpha')\theta' + \alpha' K(\tilde{\theta}, y) \mid x, y\ ], \tag{16}$$

for all $(x,y) \in \mathbb{R}^2$, where $\theta'$ is the unique solution to $U'_k(\theta',\theta',\tilde{\theta}) = 0$, $K(\tilde{\theta}, y) = \mathbb{E}[k(x,y)|\tilde{\theta}, y]$, and $\alpha' \equiv -U'_{kK}/U'_{kk}$. Since $\theta'$ is an affine function of $\tilde{\theta}$, $\mathbb{E}[\theta'|x,y] = \mathbb{E}[\mathbb{E}[\theta'|\theta] \mid x, y]$, where $\mathbb{E}[\theta'|\theta] = T'(\theta)$ is the projection of $\theta'$ on $\theta$. From the same arguments as in the proof of Proposition 5, the unique solution to (16) is given by $k(x,y) = T'(\beta' x + \gamma' z)$, where

$$\beta' = 1-\delta-\rho', \qquad \gamma = \delta + \rho', \qquad \text{and} \qquad \rho' = \frac{\alpha'\delta(1-\delta)}{1-\alpha'(1-\delta)}.$$

Similarly, the unique solution to (6) is given by $k(x,y) = T(\beta^* x + \gamma^* z)$, where $T(\theta) \equiv \mathbb{E}[\theta_{FB}|\theta]$,

$$\beta^* = 1-\delta-\rho^*, \qquad \gamma^* = \delta + \rho^*, \qquad \text{and} \qquad \rho^* = \frac{\alpha^*\delta(1-\delta)}{1-\alpha^*(1-\delta)}$$

For the two allocations to coincide it is necessary that $T(\theta) = T'(\theta)$; equivalently, that $\mathbb{E}[\theta'|x,y] = \mathbb{E}[\theta_{FB}|x,y]$ for all $(x,y)$. This is also sufficient if $T(\theta) \equiv \mathbb{E}[\theta_{FB}|\theta]$ is constant (equivalently, if $\mathbb{E}[\theta_{FB}|x,y]$ is constant), which is the case if and only if $Cov(\theta_{FB},\theta) = 0$. If instead $T(\theta)$ varies with $\theta$ (equivalently, if $\mathbb{E}[\theta_{FB}|x,y]$ varies with $x$ and $y$), then it is also necessary that $\alpha' = \alpha^*$, which proves part (ii).

For part (i) it suffices to let $U'(k,K,\tilde{\theta}) = U(k,K,\tilde{\theta}) + U_K(K,K,\tilde{\theta})k$, in which case $\theta' = \theta_{FB}$ and $\alpha' = \alpha^*$. ■

**Proof of Proposition 5.** Recall that the equilibrium allocation is $k(x,y) = (1-\gamma)x + \gamma z$, whereas the efficient allocation is $k(x,y) = (1-\gamma^*)T(x) + \gamma^* T(z) = T[(1-\gamma^*)x + \gamma^* z]$, where $T(\theta) = \mathbb{E}[\theta_{FB}|\theta]$ is an affine transformation of $\theta$. By the same argument as in the proof of Proposition 4, the two allocations coincide if and only if $\theta = \mathbb{E}[\theta_{FB}|\theta]$ and $\alpha = \alpha^*$. ■

**Proof of Condition (11).** Given $\theta$ and $y$, welfare is given by

$$w = W(K, \tilde{\theta}) + \frac{U_{kk}}{2} \cdot \mathbb{E}[(k-K)^2|\theta, y]$$

A quadratic expansion of $W(K, \tilde{\theta})$ around $K = \theta$ gives

$$W(K, \tilde{\theta}) = W(\theta, \tilde{\theta}) + W_K(\theta, \tilde{\theta}) \cdot (K - \theta) + \frac{W_{KK}}{2} \cdot (K-\theta)^2,$$

It follows that

$$\mathbb{E}w = \mathbb{E}W(\theta, \tilde{\theta}) + \mathbb{E}[W_K(\theta, \tilde{\theta}) \cdot (K-\theta)] + \frac{W_{KK}}{2} \cdot \mathbb{E}[(K-\theta)^2] + \frac{U_{kk}}{2} \cdot \mathbb{E}[(k-K)^2].$$

In equilibrium, $\mathbb{E}k = \mathbb{E}K = \mu_\theta = \mathbb{E}\theta$ and therefore $\mathbb{E}[(K-\theta)^2] = Var(K-\theta)$ and $\mathbb{E}[(k-K)^2] = Var(k-K)$, which gives the result. ■

**Proof of Proposition 7.** We prove the result in two steps. Step 1 calculates the welfare losses due to incomplete information. Step 2 derives the comparative statics.

*Step 1.* The fact that $W$ is quadratic together with $W_K(\theta_{FB}, \tilde{\theta}) = 0$ (by definition of the first best) gives

$$W_K(\theta, \tilde{\theta}) = W_K(\theta_{FB}, \tilde{\theta}) + W_{KK} \cdot (\theta - \theta_{FB}) = -W_{KK} \cdot (\theta_{FB} - \theta).$$

Next, projecting $\theta_{FB} - \theta$ on $\theta$ gives

$$\theta_{FB} - \theta = const + \phi\theta + error$$

where $\phi = Cov(\theta_{FB} - \theta, \theta)/Var(\theta)$ and $error$ is an affine function of $\tilde{\theta}$ that is orthogonal to $\theta$.[21] It follows that

$$\begin{aligned} Cov(W_K(\theta, \tilde{\theta}), K-\theta) &= -W_{KK} Cov(\theta_{FB} - \theta, K - \theta) \\ &= -W_{KK}[\phi Cov(\theta, K-\theta) + Cov(error, K-\theta)]. \end{aligned}$$

Since $K - \theta = \gamma[\lambda(\sigma_y \varepsilon) + (1-\lambda)(\mu_\theta - \theta)]$ is an affine function of $\theta$ and $\varepsilon$ and since the latter, by assumption, is orthogonal to each element of $\tilde{\theta}$, we have that $Cov(error, K-\theta) = 0$. Moreover, as shown in (19) below, $Cov(\theta, K-\theta) < 0$, whereas $W_{KK} < 0$ by concavity of $W$. It follows that

$$Cov(W_K(\theta, \tilde{\theta}), K-\theta) = -W_{KK}\phi Cov(\theta, K-\theta) = -\phi|W_{KK}||Cov(\theta, K-\theta)|. \tag{17}$$

[21] Note that both $\theta$ and $\theta^{FB}$ are affine functions of $\tilde{\theta}$, with $\theta = -(U_{kk} + U_{kK})^{-1}[U_k(0,0,0) + < U_{k\tilde{\theta}}, \tilde{\theta} >]$, and $\theta^{FB} = -[U_{kk}(1 - 2\alpha + \eta)]^{-1}[U_k(0,0,0) + U_K(0,0,0) + < U_{k\tilde{\theta}} + U_{K\tilde{\theta}}, \tilde{\theta} >]$.

Let $\mathcal{L} \equiv \mathbb{E}W(\theta, \tilde{\theta}) - \mathbb{E}w$ denote the welfare losses due to incomplete information. Using (11) with (17) and $W_{KK} = U_{kk} + 2U_{kK} + U_{KK} = U_{kk}(1 - 2\alpha + \eta)$, gives

$$\mathcal{L} = \frac{|U_{kk}|}{2} \{(1 - 2\alpha + \eta) [2\phi|Cov(\theta, K - \theta)| + Var(K - \theta)] + Var(k - K)\} \tag{18}$$

Finally, using $K - \theta = \gamma [\lambda(\sigma_y \varepsilon) + (1 - \lambda)(\mu_\theta - \theta)]$, we have that $Cov(\theta, K - \theta) = -\gamma(1 - \lambda)\sigma_\theta^2 = -\gamma\sigma_z^2 < 0$. Substituting $\gamma = \delta/[1 - \alpha(1 - \delta)]$ and using the expressions for volatility and dispersion derived in the proof of Proposition 2 gives

$$Var(k - K) = \frac{(1 - \alpha)^2(1 - \delta)\sigma^2}{(1 - \alpha\,(1 - \delta))^2},\; Var(K - \theta) = \frac{\delta\sigma^2}{(1 - \alpha\,(1 - \delta))^2},\; Cov(\theta, K - \theta) = -\frac{\sigma^2}{(1 - \alpha\,(1 - \delta))}. \tag{19}$$

Substituting into (18) gives

$$\mathcal{L} = \frac{|U_{kk}|}{2} \left\{ \frac{(1 - 2\alpha + \eta) [2\phi(1 - \alpha + \alpha\delta) + \delta] + (1 - \alpha)^2(1 - \delta)}{(1 - \alpha + \alpha\delta)^2} \right\} \sigma^2. \tag{20}$$

*Step 2.* Since $\mathbb{E}W(\theta, \tilde{\theta})$ is independent of $\delta$ and $\sigma$, the comparative statics of welfare with respect to the information structure coincide with (the reverse of) those of $\mathcal{L}$. Note that

$$\frac{\partial \mathcal{L}}{\partial \delta} = \frac{|U_{kk}|}{2} \left\{ \frac{-\alpha[2 - 3\alpha + \alpha^2(1 - \delta)] + \eta(1 - \alpha - \alpha\delta) - 2\alpha\phi(1 - 2\alpha + \eta)(1 - \alpha + \alpha\delta)}{(1 - \alpha + \alpha\delta)^3} \right\} \sigma^2$$

When $\alpha = 0$, in which case $\alpha^* = -\eta$, the above reduces to

$$\frac{\partial \mathcal{L}}{\partial \delta} = \frac{|U_{kk}|}{2} \eta\sigma^2$$

and hence $\partial\mathcal{L}/\partial\delta > [<]0$ if and only if $\alpha^* < [>]0$.

When instead $\alpha \neq 0$,

$$\frac{\partial \mathcal{L}}{\partial \delta} = \frac{|U_{kk}|}{2} \frac{2(1 - 2\alpha + \eta)\sigma^2}{[1 - \alpha(1 - \delta)]^2} \alpha[f(\alpha, \eta, \delta) - \phi],$$

where

$$f(\alpha, \eta, \delta) \equiv \frac{-\alpha[2 - 3\alpha + \alpha^2(1 - \delta)] + \eta(1 - \alpha - \alpha\delta)}{2\alpha(1 - \alpha + \alpha\delta)(1 - 2\alpha + \eta)}.$$

Now let

$$\underline{\phi}(\alpha, \eta) \equiv \min_{\delta \in [0,1]} f(\alpha, \eta, \delta) \quad \text{and} \quad \bar{\phi}(\alpha, \eta) \equiv \max_{\delta \in [0,1]} f(\alpha, \eta, \delta).$$

Next, note that

$$\frac{\partial f}{\partial \delta} = \frac{(1 - \alpha)(\alpha - \eta)}{(1 - \alpha + \alpha\delta)^2(1 - 2\alpha + \eta)} = \frac{(1 - \alpha)(\alpha^* - \alpha)}{(1 - \alpha + \alpha\delta)^2(1 - \alpha^*)}.$$

When $\eta < \alpha$ ($\alpha^* > \alpha$), $f$ is strictly increasing in $\delta$ and

$$\underline{\phi}(\alpha, \eta) = f(\alpha, \eta, 0) < \bar{\phi}(\alpha, \eta) = f(\alpha, \eta, 1)$$

When instead $\eta = \alpha$ ($\alpha^* = \alpha$), $f$ is independent of $\delta$ and

$$\underline{\phi}(\alpha,\eta) = \bar{\phi}(\alpha,\eta) = -\frac{1}{2} < 0.$$

Finally, when $\eta > \alpha$ ($\alpha^* < \alpha$), $f$ is strictly decreasing in $\delta$ and

$$\underline{\phi}(\alpha,\eta) = f(\alpha,\eta,1) < \bar{\phi}(\alpha,\eta) = f(\alpha,\eta,0).$$

Note that $sign(\frac{\partial \mathcal{L}}{\partial \delta}) = sign(f-\phi)$ if $\alpha > 0$ and $sign(\frac{\partial \mathcal{L}}{\partial \delta}) = -sign(f-\phi)$ if $\alpha < 0$. If $\phi \in (\underline{\phi}, \bar{\phi})$, then $\frac{\partial \mathcal{L}}{\partial \delta}$ alternates sign as $\delta$ varies between 0 and 1, no matter whether $\alpha > 0$ or $\alpha < 0$. Hence, $\phi < \underline{\phi}$ is necessary and sufficient for $\frac{\partial \mathcal{L}}{\partial \delta} > 0\ \forall \delta$ when $\alpha > 0$ and for $\frac{\partial \mathcal{L}}{\partial \delta} < 0\ \forall \delta$ when $\alpha < 0$, whereas $\phi > \bar{\phi}$ is necessary and sufficient for $\frac{\partial \mathcal{L}}{\partial \delta} < 0\ \forall \delta$ when $\alpha > 0$ and for $\frac{\partial \mathcal{L}}{\partial \delta} > 0\ \forall \delta$ when $\alpha < 0$.

Consider first the case $\alpha > 0$. If $\alpha^* > \alpha$, then

$$\bar{\phi}(\alpha,\eta) = f(\alpha,\eta,1) = \frac{-\alpha^2 - (1-2\alpha)\alpha^*}{2\alpha(1-2\alpha+\eta)} < 0.$$

To see this, let $g(\alpha,\eta) \equiv -\alpha[2-3\alpha]+\eta[1-2\alpha]$ denote the numerator of $f(\alpha,\eta,1)$. If $\alpha \leq 1/2$, $g(\alpha,\eta)$ is non-decreasing in $\eta$, in which case $\alpha^* > \alpha$ (equivalently, $\eta < \alpha$), implies $g(\alpha,\eta) < g(\alpha,\alpha) = -\alpha(1-\alpha)$. Since the denominator of $f(\alpha,\eta,1)$ is always positive, it follows that $\bar{\phi}(\alpha,\eta) < 0$. If instead $\alpha > 1/2$, $g(\alpha,\eta)$ is decreasing in $\eta$, in which case $\eta > -1+2\alpha$ (equivalently, $W_{KK} < 0$) implies $g(\alpha,\eta) < g(\alpha,-1+2\alpha) = -(1-\alpha)^2 < 0$ and hence again $\bar{\phi}(\alpha,\eta) < 0$. If instead $\alpha^* < \alpha$,

$$\bar{\phi}(\alpha,\eta) = f(\alpha,\eta,0) = \frac{\alpha^2 - \alpha^*}{2\alpha(1-2\alpha+\eta)}.$$

and hence $\bar{\phi} > 0$ if and only if $\alpha^* < \alpha^2$. Since $\alpha^2 < \alpha$, we conclude that, for the case $\alpha > 0$, $\bar{\phi} < 0$ if and only if $\alpha^* > \alpha^2$.

Next, consider the case $\alpha < 0$. If $\alpha^* > \alpha$, then

$$\bar{\phi}(\alpha,\eta) = f(\alpha,\eta,1) = \frac{-\alpha^2 - (1-2\alpha)\alpha^*}{2\alpha(1-2\alpha+\eta)}$$

and hence $\bar{\phi}(\alpha,\eta) < 0$ if and only if $\alpha^* \in (\alpha, -\alpha^2/(1-2\alpha))$. If instead $\alpha^* < \alpha$, then $\alpha^* < 0 < \alpha^2$ and hence

$$\bar{\phi}(\alpha,\eta) = f(\alpha,\eta,0) = \frac{\alpha^2 - \alpha^*}{2\alpha(1-2\alpha+\eta)} < 0.$$

We conclude that, for the case $\alpha < 0$, $\bar{\phi} < 0$ if and only if $\alpha^* < -\alpha^2/(1-2\alpha)$. ■

## References

[1] Acemoglu, Daron (1993), "Learning About Others' Actions and the Investment Accelerator," *Economic Journal* 103, 318-328.

[2] Angeletos, George-Marios, and Alessandro Pavan (2004), "Transparency of Information and Coordination in Economies with Investment Complementarities," *American Economic Review*, 94, 91-98.

[3] Angeletos, George-Marios, and Ivàn Werning (2004), "Crises and Prices: Information Aggregation, Multiplicity and Volatility," MIT mimeo.

[4] Atkeson, Andrew, and Patrick Kehoe (2001), "The Advantage of Transparent Instruments of Monetary Policy," NBER working paper.

[5] Benhabib, Jess, and Roger Farmer (1994), "Indeterminacy and Increasing Returns," *Journal of Economic Theory* 63, 19-41.

[6] Bryant, John (1983), "A Simple Rational Expectations Keynes-Type Model," *Quarterly Journal of Economics* 98, 3, 525-28.

[7] Canzoneri, Matthew (1985), "Monetary Policy Games and the Role of Private Information," *American Economic Review* 75, 1056-70.

[8] Cooper, Russell, and Andrew John (1988), "Coordinating Coordination Failures in Keynesian Models," *Quarterly Journal of Economics* 103, 441–463.

[9] Cukierman, Alex, and Allan Meltzer (1986), "A Theory of Ambiguity, Credibility, and Inflation under Discretion and Asymmetric Information," *Econometrica* 54, 1099-128.

[10] Gereatts, Petra (2002), "Central Bank Transparency," *Economic Journal* 112, F532-F565

[11] Hellwig, Christian (2005), "Heterogeneous Information and the Benefits of Transparency," UCLA mimeo.

[12] Hirshleifer, Jack (1971), "The Private and Social Value of Information and the Reward to Inventive Activity," *American Economic Review* 61, 561-574.

[13] Matsuyama, Kiminori (1992), "A Simple Model of Sectoral Adjustment," *The Review of Economic Studies* 59, 375-388.

[14] Messner, Simon, and Xavier Vives (2001), "Informational and Economic Efficiency in REE with Asymmetric Information," CEPR working paper 2678.

[15] Morris, Stephen, and Hyun Song Shin (2002), “The Social Value of Public Information”, *American Economic Review* 92, 1521-1534.

[16] Romer, Paul (1986), “Increasing Returns and Long-Run Growth,” *Journal of Political Economy* 94, 1002-1037.

[17] Svenson, Lars (2005), “The Social Value of Public Information: Morris and Shin (2002) is Actually Pro Transparency, Not Con,” forthcoming in *American Economic Review.*

[18] Woodford, Michael (2005), “Central Bank Communication and Policy Effectiveness,” Columbia University mimeo.