CORRELATED INFORMATION, MECHANISM DESIGN AND INFORMATIONAL RENTS

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Abstract

The optimal mechanism literature shows that the principal can extract all the equilibrium payoffs at a given Bayesian Game when the players' information is correlated. For this result to hold, the principal needs to know how informative are the agents' signals. If the principal does not know how noisy the signal of a player is, or equivalently, when the signals available to a player can be ranked in order of informativeness by Blackwell's criterion, any informational rents obtained at the game are left to the players. Conversely, whenever the optimal mechanism leaves a positive surplus to some player, it is always possible to interpret the model as if, the principal were uncertain regarding how noisy is the signal of that player. Finally, although signals may be highly correlated, Blackwell's ranking implies that the degree of informativeness of a player's signal ought to be independent on the degree of informativeness of others' signals.

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1. INTRODUCTION

Can a principal – without observing the agents' investment in information gathering – construct a mechanism that extracts all of the surplus agents obtain at a given Bayesian game? A key condition for full surplus extraction requires the principal to be certain regarding how noisy the signal of a player is. When the principal is uncertain regarding the quality of an agent's signal, or put it formally, when the signals' distributions conceivably available to an agent can be ranked in order of informativeness by Blackwell's criterion, the principal must leave to the agent any informational rent obtained at the Bayesian game.

To illustrate, consider an investment bank preparing an initial public offering. The typical potential subscribers have correlated information regarding the future profitability of the venture. Therefore, in theory (Crémer and McLean 1988, McAfee and Reny 1992), the bank should be able to design a selling mechanism that yields the expected present-value of the venture. Yet, extensive empirical evidence indicates that initial public offerings are often underpriced[1].

Also, consider the regulation of an industry. Although the regulator can induce firms to produce the efficient quantity by resorting

[1] See Smith (1986).

to subsidies, social loss occur whenever the subsidies are financed by distortionary taxes. Although costs are privy to firms, costs in a particular industry are likely to be correlated. That allows the regulator to recoup the subsidies. Through benchmarking, that is – by requiring firms to buy licenses with payments coupled to the performance of competitors, the regulator is able to avoid the shadow cost of public funds and so, in theory, the social efficient outcome is always implementable[2].

Accounts of why the principal fails to extract all the surplus rely on risk-aversion and limited liability (Robert 1991), or competition among sellers (Peters 2001). Another explanation pursued here argues that the optimal mechanism requires too much knowledge from the principal. It is rather unrealistic that the amount of effort each potential buyer devotes to gather information would be perfectly known. As Crémer and McLean (1988, p. 1254) originally pointed,

> In 'nearly all' auctions the seller should be able to extract the full surplus, which implies that asymmetry of information between sellers and buyers should be of no practical importance. Economic intuition and informal evidence suggest that this result is counterfactual, and several explanations can be suggested. First,

[2]See Faure-Grimaud and Reiche (2001)

the assumption that a common knowledge probability distribution exists is very strong.

Consider a simple common-value example in which the object may be of a low or a high value. Initially, buyers are uninformed but, previously to the sale, at a monetary cost, buyers can covertly discover the true value of the object.

If all buyers are uninformed, the second-price auction extracts all the surplus. In this instance, however, provided the cost of acquiring information is not too high, a buyer should acquire information. So, when the value is low, the buyer does not participate; when the value is high, the buyer enjoys a positive surplus. But if all buyers become informed, the second-price auction again extracts all the surplus. Moreover, since information acquisition costs are sunk, from the ex-ante point of view, informed buyers do not recoup information acquisition costs. As a result, buyers should remain uninformed to avoid the hold-up problem. In sum, in any equilibrium, buyers acquire information with positive probability but not with certainty.

At the second-price auction, informed buyers enjoy informational rents. Nevertheless, since buyers are employing mixed strategies in equilibrium when acquiring information, from the ex-ante point of view, they get zero expected surplus. The seller, however, can not fully extract the surplus since buyers must be compensated for the information acquisition costs.

In this paper, the underlying acquisition of information stage is left unmodelled. As a starting point, it is just posited that the the informativeness of the players' signal is not know with certainty. But, for the reader, it may be useful to keep in mind the above acquisition of information story. Section 3 presents an example where the moral hazard stage is modelled and it discusses the results of Obara (2003).

As in McAfee and Reny (1992), this paper is concerned with the rent extraction problem in a given game Γ. That is, the principal wants to extract the equilibrium payoffs agents get at Γ. The reason why the Principal may fail to extract the surplus when agents' quality of information is unknown is the following: For simplicity, let's say that agents may be either poorly or well informed. In order to extract the surplus, the principal offers menus of contracts, possibly distinct menus, to each agent. By choosing a contract from the menu, an agent gains the right to participate into the game under consideration. Thus, a contract is simply an entry fee to the game. The monetary transfers that the contract stipulates are contingent on the contractual choices of other agents. Insofar as the choices of the other agents depend on their private information, from an individual agent's perspective, a contract is just a lottery whose payoff is determined by the realization of the other agents' types.

A well informed agent can always mimic the contractual choice of a poorly informed agent without incurring a higher expected entry fee. An well informed agent can simply add noise to his/her private signal and then randomize over the equilibrium choices that the poorly informed would make. As a result, the principal is unable to screen agents and charge a higher expected entry fee to the well informed. In the ensuing game, however, the well informed agent is able to leverage on the information advantage. The entry fee does not depend the strategy played during the game, and so, at the game, a well informed agent can always guarantee to herself a higher payoff than a poorly informed gets. Consequently, the optimal mechanism must leave informational rents to the well informed agent.

Two conditions are crucial for the above accounting of the existence of informational rents: One is that information must be valuable at the game the agents play. Two is that the signals' distributions available to an agent can be weekly ranked in order of informativeness by

Blackwell's criterion.

The necessity of the first condition should be clear. If information were to be worthless, the principal would still be able extract all of the surplus because pooling would not be a problem.

Again, in this short paper, the underlying acquisition of information stage is left unmodelled. But, it appears hard to explain why agents would decide to become informed if information were to be worthless at games they play.

The second condition is seemly, a non-generic property. If Nature were to randomly pick the distribution of an agent's signal, the event that two feasible distributions would be Blackwell comparable has zero probability. Although, it might be convenient to model the primitives, utilities and signals, as exogenously given, it appears somewhat odd to conceive primitives as being randomly given. Primitives are shaped by history and, clearly, no endogenously determined 'primitive' needs to be generic. For example, Bergemann and Valimaki (2002) and Bergemann and Pesendorfer (2001) present 'non-generic', yet endogenously generated, information structures.

2. RELATED LITERATURE

The full surplus extraction result defies the natural economic intuition that owners of private information receive informational rents[3]. A pivotal assumption for this result requires the distribution of the signals to be common-knowledge (Crémer and McLean 1988, p. 1254). Recent research has focusing on easing such knowledge burden.

Bergemann and Morris (2001) study 'large' type spaces where a type is a description of the player's payoff-relevant characteristics and his belief regarding the other players' types. 'Large' in that context means that there are types with identical payoff characteristics but holding distinct beliefs. Also, Amarante (2001) examines 'large' type spaces where the seller is restricted to 'simple' mechanisms. In contrast, Neeman (1999) looks at type spaces where a player may have many types with identical beliefs but with different payoff-relevant characteristics. Neeman's work is closely related to the this paper, in section 4, the contrasts and similarities of his model with our model are discussed.

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[3]Additional criticism is addressed not to the full surplus extraction result *per se* but rather to the mechanism that implements it. The optimal mechanism is sensitive to changes in fine details of the model. As a result, many papers have analyzed robust mechanisms. Few examples are: McAfee, McMillan, and Reny (1989), **?**, Lopomo (1998, 2001), Amarante (2001) and Bergemann & Morris (2001).

3. RENT EXTRACTION IN A NUTSHELL

This section introduces most of the notation used in the rest of the paper, and revisits the results of Crémer and McLean (1988) and McAfee and Reny (1992), henceforth C&M–M&R.

Consider a finite Bayesian game $\Gamma = (N, (C_i)_{i\in N}, (T_i)_{i\in N}, (p_i)_{i\in N}, (u_i)_{i\in N})$ where, N is the set of players, and for each player $i \in N$: C_i is the set of possible actions, T_i is the set of types, p_i is the belief over the types of other players (conditional on i's own type), and u_i is the payoff function.

Let σ be a Bayes-Nash equilibrium of Γ, and $u_i^\sigma = (u_i(\sigma|t_i))_{t_i\in T_i}$ be the corresponding vector of *interim* equilibrium payoffs of player i.

The principal problem is to design a mechanism to extract the rents of i. Formally, the rent extraction problem is to find a finite collection of stochastic entry fees, $f_i^k : T_{-i} \to \mathrm{R}$ with $k \in T_i$, in order to maximize

$$\sum_{t_i\in T_i} \left(\min_k \sum_{t_{-i}\in T_{-i}} f_i^k(t_{-i}) p_i(t_{-i}|t_i) \right) p(t_i)$$

subject to the participation constraints,

$$\forall t_i \in T_i, \quad \min_k \sum_{t_{-i}\in T_{-i}} f_i^k(t_{-i}) q_i(t_{-i}|t_i) \leq u_i^\sigma(t_i).$$

The basic idea is exactly as in C&M–M&R: The principal offers the menu of stochastic entry fees to player i. By picking an entry fee from the menu, player i is allowed to play the game Γ. The expected payments stipulated by an entry fee do not depend upon the action she chooses to play later on in the game. Hence, conditional on her type, she must select the entry fee with the lowest expected payment, provided that the payment does not exceed her equilibrium payoff.

C&M–M&R identify a property of information structures that is necessary and sufficient for the full extraction of the surplus for any possible payoff structure. The condition requires that, for any player, there are no types whose beliefs, about the types of other players, are a convex combination of beliefs of other types of him/herself.

Theorem 1 ***Crémer & McLean (1988)*** *The optimal mechanism yields full surplus extraction for any* u_i^σ *if and only if* $\forall\, t_i \in T_i$, $\nexists$ $(\lambda_\tau)_{\tau \in T_i \setminus \{t_i\}} \geq 0$ *such that*

$$p_i\left(t_{-i}|t_i\right) = \sum_{\tau \in T_i \setminus \{t_i\}} \lambda_\tau \; p_i\left(t_{-i}|\tau\right), \forall\; t_{-i} \in T_{-i}. \tag{C1}$$

When the beliefs of a type are not a convex combination of the beliefs of other types, there is a hyperplane that separates the beliefs of this type from the beliefs of other types. In other words, there is a contingent entry fee $g_i^{t_i}(\cdot)$ such that $\left\langle g_i^{t_i},\, p_i(\cdot|t_i)\right\rangle = 0$ and $\left\langle g_i^{t_i},\, p_i(\cdot|\tau_i)\right\rangle > 0$ for all $\tau_i \neq t_i$. The inner product of the entry fee with the beliefs of a give type is just the expected payment that this type incurs under this entry fee. Consequently, by offering the menu of entry fees $\left\{g_i^{t_i}\right\}_{t_i \in T_i}$, the principal is able to induce the players to reveal their beliefs. A type who chooses an entry fee intended to another type incurs positive payments but, no payments are incurred by choosing the entry fee tailored to one's own type. In the language of Neeman (1999), it is possible to extract the players' beliefs.

In order to extract the players' surpluses, the principal simply constructs another menu of any fees: $\left\{f_i^{t_i} = u_i^{\sigma}(t_i) + \omega\, g_i^{t_i}\right\}_{t_i \in T_i}$. Any type that chooses the entry fee $f_i^{t_i}$ pays the interim equilibrium payoffs of type t_i with certainty and, in addition, pays the expected penalties of misreporting, which are zero if no misreporting occurs but otherwise can be made arbitrarily large by increasing ω_i.

5. AN EXAMPLE

The example presented here is a static version of a multi-period Baron-Myerson model of procurement studied by Faure-Grimaud and Reiche (2001). Two firms, have to produce a quantity q of consumption good. Firm 2 produces q units of an intermediate good which are then are used by firm 1 to produce q units of a final good.Firm $i \in \{1,2\}$ has a constant average total cost c^i and so the total cost of producing q units of the final good is $(c^1 + c^2)\, q$. The consumers' surplus generated by the production of q units is $S(q)$ where S satisfies the following properties: $S' > 0\,, S'' < 0, S'(0) = +\infty$ and $S'(+\infty) = 0$.

The marginal cost c^i of firm i takes values in $\{c_l, c_h\}$ where $c_h > c_l$. The actual value of the marginal cost is proprietary information of the firm. The joint probability distribution of the marginal costs – $p = (p_{rs})$ where $r, s \in \{l, h\}$, is common-knowledge among the regulator and the firms.

The regulatory agency offers contracts to the firms that specify a monetary transfer to each firm as well as a quantity to be produced, (m, q): m^i_{rs} is the monetary transfer that firm i gets when it reports marginal cost c_r and the other firm reports marginal cost c_s. The contracted quantity contingent on the reports is defined analogously.

The goal of the regulatory agency is find a menu of contracts in order to maximize the sum of consumer and producer surplus minus the shadow costs of public funds:

$$\sum_{r,\,s} p_{rs} \left(S(q_{rs}) - (1+\lambda)(m^1_{rs} + m^2_{sr}) + m^1_{rs} - c^1_r q^1_{rs} + m^2_{sr} - c^2_s q^2_{sr} \right)$$

The efficient level of production equates the marginal benefit with the marginal social cost of production, $S'(q_{rs}) = (1+\lambda)(c_r+c_s)$, where λ is the shadow cost of public funds. In this setting, the optimal mechanism implements the social efficient outcome.

Consider a modification of the above scenario. Prior to accepting the contract offered by the regulator and observing its current marginal cost, firm 1 may have an opportunity to covertly invest in a cost-saving technology. More specifically, investing k shifts down the firm 1's marginal cost by d. The principal and the firm 2 are uncertain weather that firm 1 invested or not, let's say that σ is the belief they attach to the probability investment occurred.

Capital letters are used to denote the types of the firm 1 that invested.

In this scenario is trivial to Blackwell-rank signals of firm 1. Regardless weather it invested or not, its beliefs regarding the costs of firm 2 remain unchanged. As a result, any firm 1 type that invested can always pool with the respective firm 1 type who did not invest. Pooling with other types, however, is not feasible since the principal can extract beliefs at zero cost. In sum, the only incentive compatible constrains relevant are regarding reporting investment or not investment. Furthermore, by investing, firm 1 obtains a higher return since it has lower costs.

As in the usual mechanism design setting, the individual rationality constrain binds at the 'bottom', the downward incentive compatible constraints are bidding, and there is no distortion at the 'top'. More precisely, if investment occurs, the principal sets an efficient quantity: $q_{Rs} = S'^{-1}((1+\lambda)(c_r+c_s-d))$. Otherwise, the quantities are distorted: $q_{rs} = S'^{-1}((1+\lambda)\,(c_r + c_s + \frac{\sigma}{1-\sigma}d))$.

Finally, in order for an equilibrium in mixed strategy to exist, the gains from investment must recoup exactly the outlays in the cost saving technology:

$$d\,\sigma \sum_{r,\,s} p_{rs}\, q_{rs} = k\,,$$

and the assumptions regarding S guarantee that there is $\sigma \in (0,1)$ that satisfies this condition for k sufficient low.

The above story fits precisely the theoretical model of Neeman (1999) where, the key condition, for the failure of the full surplus extraction in an auction, was the existence of an agent with two types who share identical beliefs but differ in their valuations. This paper shows that identical beliefs are sufficient but not necessary for pooling – different degrees of informativeness will do as well as in allowing agents to pool. In the story above, as again in Neeman, the difference in cost/valuations was crucial for the result. In this paper, that role is played by condition 4 that guarantees that a player gets a strictly higher expected payoff when being well informed than when being poorly informed. That is, information is valuable at the game Γ.

Notice that in the above game the principal is not allowed to commit to a mechanism before agents decide to invest or not. The results would change if one allowed for that. Obara (2003) considers a general model where: the principal commits to a mechanism; agents take hidden actions that affect their utilities and/or signals; agents privately observe the realization of their mixed strategies and the realization of their signals; agents report their private information to the principal; and finally, the principal implements the allocation and monetary transfers dictated by the mechanism. A main result is that the ability to commit to a mechanism prior to the moral hazard stage allows the principal to eliminate the uncertainty and henceforth, the full surplus extraction outcome is implementable.

Under the light of Obara (2003)'s results, one may tempted to attribute the existence of informational rents to an *ad-hoc* restriction imposed on the mechanism, namely the principal's lack of ability to commit. That misses that the main point of this paper that is unlikely that the principal would know all information necessary to fully extract the agents' surplus, still holds even if the mechanism is augmented. More exactly, in order to design a *grand* mechanism, as in Obara (2003), the principal must know the cost of acquiring information; the principal must know which the set of signals is available to the agents. If there is uncertainty regarding costs of acquiring information or, regarding which sets of signals' are available to the agents, the full surplus extraction result breaks down again. It is equally true, however, that if the uncertainty is created by moral hazard, by allowing the principal to design a larger mechanism, its possible to restore the result. But, clearly, once more, it is always possible, to consider an additional layer of uncertainty.

4. SIGNAL INFORMATIVENESS

In most applications, a type is simply the realization of a single dimensional signal. To contemplate the fact that signals vary in their informativeness/quality, this identification is no longer useful. Henceforth, the type of a player shall be a description of both: the probability distribution from where the player's signal is drawn from – the *informativeness* of the player's signal, and also the particular extraction, or realization, of that signal.

From the point of view of the principal, the informativeness of an agent's information is not known with certainty, possibly, due to covert acquisition of information. Here, not as in Bergemann and Pesendorfer (2001), the principal is not able to control the buyers' acquisition of information, nor choose their information structures, otherwise, it would be trivial to fully extract the surplus.

In this paper, however, the underlying model of acquisition of information is left unmodelled. For simplicity, player i has at her disposal only two signals; put simply, with positive probability, her signal is either X^{θ} or X^{η}.

Signals are ranked in order of informativeness by Blackwell's criterion [4]: signal X^{θ} is said to be more informative, about the others players' types t_{-i}, than signal X^{η} is, if and only if, there are scalars $\beta_{(x,y)} \geq 0$ such that, for all y realizations of X^{η} and all $t_{-i} \in T_{-i}$,

$$\frac{p\left(t_i = (y,\eta)|t_{-i}\right)}{p\left(t_{i2} = \eta\right)} = \sum_{x \in \ \mathtt{supp} \ X^{\theta}} \beta_{(x,y)} \quad \frac{p\left(t_i = (x,\theta)|t_{-i}\right)}{p\left(t_{i2} = \theta\right)}, \qquad \text{(C2)}$$

where p is the common-prior probability distribution and

$$\sum_{y \in \ \mathtt{supp} \ X^{\eta}} \beta_{(x,y)} = 1.$$

This definition says that signal X^{η} is a garbling of X^{θ}. Intuitively, X^{η} is a further randomization over the possible outcomes of X^{θ}, with

[4]See Crémer (1982) for a clear exposition of Blackwell's Theorem.

the $\beta_{(x,y)}$ being the weights employed by this 'randomization' as depicted in the figure below.

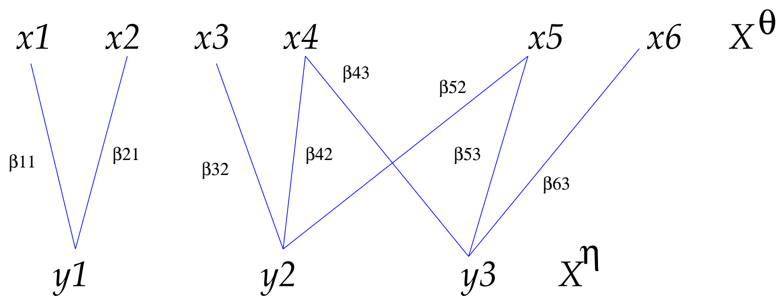

Figure 1: X^{η} is obtained by adding 'noise' to X^{θ}

The next proposition establishes that equivalence of the Blackwell's ranking of signals and the Crémer & McLean's condition (C1).

Proposition 2 *The equality in (C1) holds if and only if, it is possible to write the set of types of player i as $T_i = \{x_1, \ldots, x_{n_i}\} \times \{\theta, \eta\}$ and find signals' X_i^{θ} and X_i^{η} such that: 1) The distribution of types can be expressed as $p(t_i, t_{-i}) = \Pr\left(X_i^{t_{i2}} = t_{i1}, t_{-i}\right) \Pr(t_{i2})$ and 2) The signal X_i^{θ} is sufficient for the signal X_i^{η} regarding t_{-i}.*

Proof. First, we prove that if X_i^{θ} is sufficient for the signal X_i^{η} and the type space of i is $T_i = \{x_1, \ldots, x_{n_i}\} \times \{\theta, \eta\}$ then, the equality in $(C1)$ must hold.

Let $\lambda_{(x,y)} = \beta_{(x,y)} \dfrac{p\left(t_i = (x, \theta)\right)}{p\left(t_i = (y, \eta)\right)}$ where $\beta_{(x,y)}$ are the coefficients implied by Blackwell's ranking (C2). Bayes' rule implies that,

$$\begin{aligned}
\frac{p\left(t_{-i}|t_i=(y,\eta)\right)}{p(t_{i2}=\eta)} &= \frac{p(t_{-i})}{p(t_i=(y,\eta))}\frac{p\left(t_i=(y,\eta)|t_{-i}\right)}{p(t_{i2}=\eta)} = \\
&= \frac{p(t_{-i})}{p(t_i=(y,\eta))}\sum_{x\in\ \mathtt{supp}\ X^\theta}\beta_{(x,y)}\frac{p\left(t_i=(x,\theta)|t_{-i}\right)}{p\left(t_{i2}=\theta\right)} = \\
&= \frac{p(t_{-i})}{p(t_i=(y,\eta))}\sum_{x\in\ \mathtt{supp}\ X^\theta}\beta_{(x,y)}\frac{p\left(t_{-i}|t_i=(x,\theta)\right)}{p\left(t_{i2}=\theta\right)}\frac{p\left(t_i=(x,\theta)\right)}{p\left(t_{-i}\right)} = \\
&= \sum_{x\in\ \mathtt{supp}\ X^\theta}\beta_{(x,y)}\frac{p\left(t_i=(x,\theta)\right)}{p(t_i=(y,\eta))}\frac{p\left(t_{-i}|t_i=(x,\theta)\right)}{p\left(t_{i2}=\theta\right)} = \\
&= \sum_{x\in\ \mathtt{supp}\ X^\theta}\lambda_{(x,y)}\frac{p\left(t_{-i}|t_i=(x,\theta)\right)}{p\left(t_{i2}=\theta\right)},
\end{aligned}$$

which is precisely the equality in (C1).

To prove the converse, consider $\mathcal{T}_i$, a subset of T_i that is maximal with respect the property that the beliefs about t_{-i} of any type $t_i \in \mathcal{T}_i$ cannot be expressed as a convex combination of beliefs of types in the complementary set $T_i \backslash \mathcal{T}_i$. For finite T_i, the set $\mathcal{T}_i$ can be obtained by a constructive proof. In the case that the type space of i is infinite, Zorn's Lemma is needed to prove the existent of $\mathcal{T}_i$.

By construction, the beliefs of any type in the complementary set can be expressed as a convex combination of beliefs of types in $\mathcal{T}_i$. To construct the more informative signal, X_i^θ, set $\Pr(t_{i2}=\theta) = \Pr\left(\mathcal{T}_i\right)$ and $\Pr\left(X_i^\theta = t_i, t_{-i}\right) = \begin{cases} 0 & \text{if } t_i \notin \mathcal{T}_i \\ \dfrac{p(t_i,t_{-i})}{\Pr\left(\mathcal{T}_i\right)} & t_i \in \mathcal{T}_i \end{cases}$. The construction of the less informative signal, X_i^η is analogous. Finally, to verify that X_i^θ is sufficient for X_i^{η}, we just use Baye's rule in the equation given by (C1) to obtain Blackwell's condition.

Observe that the proof of proposition above can be easily adapted for the case of continuous distributions, as the common-value example given below shows.

Example 1 *As Kagel and Levin (1999), the signals belong to a family of uniform distributions parameterized by* ε*, more exactly*

$$X^{\varepsilon}|V \sim U[V-\varepsilon, V+\varepsilon].$$

Since, $f^{2\varepsilon}(y|\nu) = \frac{1}{2}f^{\varepsilon}(x_1|\nu) + \frac{1}{2}f^{\varepsilon}(x_2|\nu)$ *where* $x_1 = y - \varepsilon$ *and* $x_2 = y + \varepsilon$*, it can be easily checked that* X^{ε} *is more informative than* $X^{2\varepsilon}$*.*

Also notice that, since the probability of the type factors into the product of the probability of the degree of informativeness and the realization of the signal, Blackwell's ordering implies the informativeness of a player's signal must be independent of the others' informativeness.

Corollary 3 *If player* i *signal may be either* X^{η} *or* X^{θ} *with positive probability and signal* X^{θ} *is more informative than* X^{η} *then full surplus extraction for any* u_i^{σ} *is not possible.*

5. INFORMATIONAL RENTS

It is important to emphasize that the definitions of well or poorly informed buyers refers to the state of buyers *before* they learn the realization of their signals. As a result, the informed buyers' payoff refers to the expected payoff of buyers who have a more informative signal. Information rents exist when a well informed buyer expected payoff is higher than the payoff she would obtain in the case she were poorly informed. It does not make sense to measure information rents by ex-post payoffs. For instance, a poorly informed high signal type may get a higher payoff than a well informed low signal type.

When condition C2 of Proposition 3 holds, full extraction is not achievable for *arbitrary* equilibrium payoffs. But, equilibrium payoffs are not arbitrary, they depend on the information structure. One should ask, what restrictions C2 imposes on the equilibrium payoffs? Condition C2 implies that the expected the payoff of player i must satisfy:

$$u_i(c|t_i = (y, \eta)) = \sum_{x \in \ \mathtt{supp} \ X^\theta} \lambda_{(x,y)} \, u_i(c|t_i = (x, \theta)), \quad \forall c \in C.$$

Summing over t_{-i} in C1 gives $\sum_x \lambda_{(x,y)} = 1$, which says that the payoff of any poorly informed type is an average of the payoff of some well informed types. Thus, any equilibrium payoff profile must satisfy: for all $y \in$ `supp` X^{η},

$$u_i^{\sigma}(y,\eta) \leq \sum_{x \in \texttt{ supp } X^{\theta}} \lambda_{(x,y)}\, u_i^{\sigma}(x,\theta) = \lambda_y^{\top}\, u_i^{\sigma}(\theta) \tag{C3}$$

The condition below is quite natural, it just says information is valuable at the game Γ.

Condition 4 *There exists at least one poorly informed type y, such that C3 holds as a strictly inequality.*

The corollary below establishes the existence of informational rents.

Corollary 5 *Assume 4 and also that no type is excluded by the optimal mechanism, then player i obtains a higher expected surplus at the optimal mechanism when her signal is X^{θ} than when it is X^{η}.*

Proof. Let $\{f_i^{t_i}\}_{t_i \in T_i}$ be an optimal entry fee schedule. Consider the following alternative strategy: each well-informed type (x,θ), instead of choosing the fee $f_i^{(x,\theta)}$ designated to her own type, chooses, with probability $\beta_{(x,y)}$, a fee intended for a poorly informed type (y,η) where $\beta_{(x,y)}$ is the weight given by C2. This strategy guarantees that, the expected entry fee of the well-informed buyer θ is equal to the one that the poor-informed buyer η incurs. In the following game, however, C3 and condition 4 assure that the well-informed buyer obtains a strictly higher surplus. ■

6. CONCLUSION

This paper tells a natural story for the existence of informational rents under the optimal mechanism. It also suggest an interpretation on the role of independence. Remember that the informativeness of a player's signal must be independent of the others' informativeness. Otherwise, it would be impossible to order signals of a given player by informativeness. In other words, independence, of informativeness, is built into Blackwell's ranking. This remark corroborates the view that informational rentals are a product of independent signals but, Blackwell's ranking, or independence of informativeness, is far from being an arbitrary presumption. If players decide how much information to acquire, the signals' informativeness ought to be independent as a consequence of Nash play. Instead of viewing independence a 'knife-edge' property, it is possible to interpret it just as a convenient way of writing a continuation game, without need to specify the previous history.

The two-dimensional type space considered in this model is obviously very parsimonious. It was the simplest short-cut to point that uncertainty over the quality of the agents' information matters. But, it is still unrealistic to presume that the principal, or the other agents, would know which set of possible signals' distributions is available to the agent. Or even which is the set of available sets of possible signals' distributions. To describe a more realistic environment, a richer language as employed by Epstein and Peters (1999) or Bergemann and Morris (2001) is required.

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