Slides for introductory talk for

Cowles Foundation Conference

on

Robust Mechanism Design

Stephen Morris

May 2004

Example

Neeman: "The Relevance of Private Information in Mechanism Design"

Two individuals

	L	H
L	$\frac{6}{14}$	$\frac{3}{14}$
H	$\frac{3}{14}$	$\frac{2}{14}$

Myerson, Cremer-McLean:

a simple mechanism allocates the object efficiently and extracts full surplus: direct mechanism where agents bet on others' types

Mechanism

allocate object to agent with highest value (flip a coin if equal) and make following payments to agent 1

	L	H
L	M	$-2M$
H	$-2M$	$3M$

agent 1's posterior beliefs are

	L	H
L	$\frac{2}{3}$	$\frac{1}{3}$
H	$\frac{3}{5}$	$\frac{2}{5}$

for sufficiently large M, truth-telling is an equilibrium....

Richer Type Space

	t_2	t_2'	t_2''	
t_1	$\frac{2}{14}$	$\frac{1}{14}$	$\frac{1}{14}$	L
t_1'	$\frac{1}{14}$	$\frac{2}{14}$	$\frac{2}{14}$	L
t_1''	$\frac{1}{14}$	$\frac{2}{14}$	$\frac{2}{14}$	H
	L	L	H	

Conditional beliefs of 1:

	t_2	t_2'	t_2''	
t_1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	L
t_1'	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$	L
t_1''	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$	H
	L	L	H	

Full surplus extraction impossible. Type t_1'' must earn informational rent.

Type Space

- Agents $1, ..., I$
- Payoff types $\Theta_1, ..., \Theta_I$
- Types $T_1, ..., T_I$

Types are implicitly defined by

$\widehat{\theta}_i : T_i \rightarrow \Theta_i$

$\widehat{\pi}_i : T_i \rightarrow \Delta(T_{-i})$

Let Π_i be the support of $\widehat{\pi}_i$

Properties of Beliefs

1. Belief Extraction Property (BEP): for each $\pi_i \in \Pi_i$, there exists $\theta_i \in \Theta_i$ such that

$$\widehat{\pi}_i (t_i) = \pi_i \Rightarrow \widehat{\theta}_i (t_i) = \theta_i.$$

2. Linear Independence of Beliefs (LIB): beliefs in Π_i are linearly independent...

- BEP is necessary condition for full surplus extraction

- BEP and LIB are sufficient for full surplus extraction

Natural Belief Properties

- BEP + LIP hold if we fix a finite type space and "generically" choose a (common knowledge) common prior on that type space; c.f. Cremer-McLean and McAfee-Reny

One response: find natural assumptions that give rise to failure of BEP and/or LIB

- BEP fails under independence (LIB holds)

- Parreiras' conference paper gives natural reason why LIB fails: uncertainty about accuracy of opponent's signal (with Blackwell ordering). REAL informational rents.

- McLean/Postlewaite (REStud forthcoming, conference paper)

Common Value payoff states Φ. Each agent observes signal in S_i about Φ and idiosyncratic payoff component in X_i. Value depends on ϕ and x_i.

Common prior $h \in \Delta(\Phi \times S_1 \times X_1 \times ... S_I \times X_I)$

$$h(\phi, s_1, x_1, ..., s_I, x_I) = \pi^*(\phi) \prod_{i=1}^{I} g_i(s_i|\phi) f_i(x_i)$$

Not independent, fails BEP (satisfies LIP for generic distributions...)

Genericity

- BEP fails on the universal type space

What about with the common prior assumption? BEP holds for some types. Need a notion of genericity.

- Topological notions. Both BEP and non-BEP common prior types are dense in product topology. Dekel/Fudenberg/Morris define (stronger) strategic topology. Doesn't help?

- Prevalence. Heifetz and Neeman conference paper.

Mechanism Design on Rich Type Spaces: Efficiency I

- Allocation utility $v_i : A \times \Theta \rightarrow \mathbb{R}$

- Allocation rule $f : \Theta \rightarrow A$

For each $\pi_i \in \Pi_i$, $\widehat{\psi}_i(\pi_i) \in \Delta(\Theta_{-i})$ is defined by

$$\psi_i(\theta_{-i}) = \sum_{\left\{t_{-i} : \widehat{\theta}_{-i}(t_{-i}) = \theta_{-i}\right\}} \pi_i[t_{-i}]$$

Mechanism Design on Rich Type Spaces: Efficiency II

PROPOSITION: Allocation rule can be implemented with transfers if and only if for each $\pi_i \in \Pi_i$ and $\psi_i = \widehat{\psi}_i(\pi_i)$, there exists $y_i : \Theta_i \to \mathbb{R}$ such that

$$\begin{aligned} & \sum_{\theta_{-i}} \psi_i(\theta_{-i}) u_i(f(\theta), \theta) + y_i(\theta_i) \\ \geq & \sum_{\theta_{-i}} \psi_i(\theta_{-i}) u_i\left(f\left(\theta_i', \theta_{-i}\right), \theta\right) + y_i\left(\theta_i'\right) \end{aligned}$$

for all i, θ_i and θ_i'.

- McLean/Postlewaite: unextractable component is conditionally independent, Vickrey auction (and with info. smallness, extract beliefs with small transfers).

Implementing a Social Choice Correspondence I

- Look at all type spaces within class......

- Social Choice Correspondence $F : \Theta \to 2^A \big/ \varnothing$

- ex post implementation: there exists $f : \Theta \to A$ such that $f(\theta) \in F(\theta)$

$$u_i(f(\theta), \theta) \geq u_i\left(f\left(\theta_i', \theta_{-i}\right), \theta\right)$$

for all i, θ_i and θ_i'.

Implementing a Social Choice Correspondence II

- Bergemann/Morris "Robust Mechanism Design": when is ex post (dominant strategies) implementation equivalent to interim implementation on all type spaces in some class?

- For "separable environments" (functions, quasi-linear without budget constraints), strong equivalence

- Otherwise (e.g., quasi-linear w/o budget balance), equivalences may break down. Thus ex post (dominant strategies) implementation is impossible, interim (Bayes Nash) implementation is possible.

- Making transfers depend on payoff irrelevant higher order beliefs relaxes incentive constraints.

An Old Debate

PRACTICAL OBJECTION TO BAYESIAN IMPLEMENTATION: Bayesian Implementation requires unrealistic assumptions about planner's knowledge of type space, prior. Assuming mechanism independent of these things implies dominant strategies (ex post equilibrium with interdependent values).

THEORETICAL OBJECTION TO PRACTICAL OBJECTION: As type space / prior common knowledge among agents, easy to extract that information. If infeasible (say, for computational or bounded rationality reasons) model the reason....

1. Even if prior / type space can be extracted, sometimes cannot use that information

2. Sometimes Bayesian implementation is possible when dominant strategies is impossible

3. Sometimes Bayesian implementation is possible for all priors on a fixed type space, but not on all possible type spaces....

Full Implementation

- Bergemann/Morris conference paper: full implementation, all equilibria deliver right outcome

- If ex post equilibrium is attractive because of rich type robustness properties, ex post full implementation is NOT enough

Mechanism Design on Rich Type Space: Revenue Maximization I

In previous problems, planner's beliefs are irrelevant.

Conceptual formulation is a little messier.

Seller has beliefs $h \in \Delta(\Theta)$. Mechanism M will be used. Agents have type space T consistent with h.

Mechanism Design on Rich Type Space: Revenue Maximization II

Ex post revenue R_{EP}: maximal revenue can be achieved with ex post equilibrium

Maxmin revenue:

$$R_{MAXMIN} = \max_{M} \min_{T} R(T, M)$$

Minmax revenue:

$$R_{MINMAX} = \min_{T} \max_{M} R(T, M)$$

Now

$$R_{MINMAX} \geq R_{MAXMIN} \geq R_{EP}.$$

Chung/Ely conference paper: classic conditions under which $R_{EP} \geq R_{MAXMIN}$, counterexample in other cases.....

Themes on Day 2

- Informational smallness creates robustness to private information, McLean-Postlewaite conference paper

- Worst case analysis in auctions, Goldberg conference paper

- Multiple priors, Bose-Ozdenoren-Pape and Lopomo-Rigotti-Shannon conference papers

- Collusion-resistance, Goldberg-Hartline conference paper

Rich Types and Worst Case Analysis

Maxmin always has Bayesian interpretation with adversary....

Mukerji-Shin: relating multiple priors in game theory to richer types....

Rich Types and Collusion

- Scope for collusion in VCG: major theme in literature and Goldberg-Hartline.

  Common knowledge of independent private values of true agents; lack of common knowledge about bidders: bidders may be shills or coalitions.....

More Robustness Issues

- bounds
- continuity (Postlewaite/Wettstein, Duggan)
- irrational types (Eliaz)
- simplicity
- computational issues....