Adverse selection and market power in double auction markets: the case of milk quota

The goal of this project is to analyze bidding behavior in the monthly milk quota double auctions (termed "quota exchanges") run by the Dairy Farmers of Ontario, to allocate milk quota among its members. We have data on 11 auctions, from September 1997 to July 1998. The quota exchange operates on the premises that each producer who wishes to buy quota has a maximum price that he is willing to pay for it and, similarly, that each producer who has quota to sell has a minimum price that he is willing to accept. Units are traded at a market clearing price (MCP) at which the total quantity demanded (approximately) equals the total quantity supplied. Producers who offered quota for sale at a price greater than the MCP, and those bidding to purchase at a price lower than the MCP, are unsuccessful.

Background The existing (largely theoretical) double auction literature (eg. Wilson (1985), Williams (1991); surveyed in (Wilson, 1992, section 7) and Friedman (1993)) has focused on single-unit demand/supply private value auctions in which buyers/sellers differ in their valuation of the good being sold. Since winner's curse and, more generally, adverse selection concerns are not present in these models, they may be inappropriate for analyzing the market for quota, since a unit of quota — which grants its owner the right to produce and market a kilogram of butterfat per day, over a twelve-month period — is a risky asset whose common value (presumably a function of future dairy prices) is unknown by the bidders.

However, adverse selection issues play a key role in the finance literature on market microstructure. While the earlier literature (eg. Kyle (1985), Glosten and Milgrom (1985)) focused on adverse selection issues using a perfect competition assumption which is appropriate for large, anonymous financial markets, a more recent literature (Kyle (1989) (hereafter K89), Blais, Martimort, and Rochet (1998) (hereafter BMR), Röell (1998)) has analyzed the imperfectly competitive case which is more appropriate for the milk quota double auctions under study. Much of the empirical work in this literature to date (Hollifield, Miller, and Sandas (1999), Sandas (1999)) has analyzed bidding behavior in financial markets; as far as we are aware, we are the first to apply these models to non-financial settings.

Research goals We address two empirical questions. First we measure how much market power bidders have in this market, in terms of the markups (for sellers) and markdowns (for buyers) that participants earn in equilibrium. The question of strategic bidding and market power has recently generated much interests in the context of seller-bid auctions to supply electricity in deregulating power markets in the UK and California (cf. Wolak and Patrick (1997), Wolfram (1999)).

The second question we address concerns whether bidding behavior is more consistent with the common or private value paradigm. In other words, are the observed bids better explained by common uncertainty (but differentially informed bidders) about the value of a unit of quota (due perhaps to uncertainty about the demand for milk or future milk supply) or differences in (opportunity) costs of holding quota across producers? As in single-object auctions, common values provoke the winner's curse in a double auction context: for example, a given seller knows that it will sell a lot only when the (unknown) state of demand is high, and this consideration would raise the equilibrium ask prices for sellers wishing to sell a large amount of quota (cf. BMR, pp. 29ff).

Clearly, both market power and adverse selection potentially lead to market inefficiencies by reducing the quantity traded in equilibrium. The ultimate goal of this project is to address the efficiency of the uniform-price double auction relative to alternative trading mechanism (eg. a discriminatory "pay your bid" auction).

Empirical framework

Our market resembles the framework of the recent microstructure literature in that bidders submit "limit orders" (ie. demand (resp. supply) functions which are downward- (resp. upward-) sloping step functions). While the quota exchanges clear via a uniform-price mechanism, the continuous financial markets considered in the microstructure literature, in contrast, never "clear" — limit orders submitted by market makers remain active until they are either "picked off" by a speculator or withdrawn by the market maker. Given this consideration, the general model developed in K89 where (imperfectly) informed traders compete by submitting limit orders and the market clears via a uniform-price is most appropriate for our modeling purposes, and forms the basis of our empirical model.

Next we describe the simplest (and most stylized) version of the bidding model. We assume that there are M buyers and N sellers participating in the auction. Each seller is assumed is entry the auction with an endowment I of quota, while buyers' initial endowments are normalized to zero. We make the assumption that all bidders have exponential (CARA) utility functions with risk-aversion parameter γ. This assumption, standard in the literature (cf. K89, BMR), is required because otherwise rational risk-neutral bidders who receive a positive (resp. negative) signal would demand (resp. supply) an infinite quantity into the market.

Information structure The "fundamental value" $\tilde{v}$ of a unit of quota (a function of future milk prices) is unknown to the bidders, but each bidder i observes a noisy signal $\tilde{x}_i$ of $\tilde{v}$. Let $F_{\tilde{x}_i|\tilde{v}}(\cdot|\cdot)$ denote the distribution of bidder i's signal conditional on the fundamental value $\tilde{v}$. We assume that these functions are identical across all bidders i.

Equilibrium bidding In our dataset, the majority of the limit orders which bidders submit are single-step functions (i.e., for buyers, specifying a single price p below which they would be willing to purchase an amount q or any portion thereof; analogously for sellers). Therefore, we model each buyer i as choosing a price-quantity pair $(p_i = P_d(x_i), q_i = Q_d(x_i))$, each of which is a function of her signal x_i. Let $P_d(\cdot)$ and $Q_d(\cdot)$ denote the equilibrium price and quantity functions for buyers. (Analogously, let $P_s(\cdot)$ and $Q_s(\cdot)$ denote the equilibrium price and quantity functions for sellers.)

Given the equilibrium strategies $P_d(\cdot)$, $Q_d(\cdot)$, $P_s(\cdot)$, $Q_s(\cdot)$ and a realization $\mathsf{x} \equiv (x_1, \ldots, x_M, x_{M+1}, \ldots, x_{M+N})$, the market-clearing price $\mathcal{P}$ is completely determined as that which minimizes excess demand in the market:

$$\mathcal{P} = m\left(\mathsf{x}\right) \operatorname{argmin}_{\pi} \sum_{j=1}^{M} Q_d(x_j) \mathbf{1}\left(P_d(x_j) \leq \pi\right) - \sum_{j=1}^{N} Q_s(x_{M+j}) \mathbf{1}\left(P_d(x_{M+j}) \geq \pi\right) \tag{1}$$

The equilibrium strategies $P_d(\cdot)$ and $Q_d(\cdot)$ are determined from the profit maximization problem of buyer 1:

$$\begin{aligned} &(P_d(x_1), Q_d(x_1)) = \\ &\operatorname{argmax}_{p,q} E_{x_{-1}|x_1} \left\{-\exp\left(-\gamma\left[m\left(x_1, x_{-1}\right) - V\left(x_1, x_{-1}\right) * q_1\right]\right)\right\} * \mathbf{1}\left(p < m\left(x_1, x_{-1}\right)\right) \end{aligned} \tag{2}$$

where x_{-1} refers to the vector of signals corresponding to bidder 1's competitors and $V(\cdots)$ is the conditional expectation

$$V\left(x_1, x_{-1}\right) \equiv E_{x_{-1}|x_1}\left[\tilde{v}|x_1, x_{-1}\right].$$

A similar profit-maximization problem can be specified for the sellers. Given a particular parameterization of $F_{\tilde{x}|\tilde{v}}(|)$, then, equilibrium strategies $P_d(\cdot)$, $Q_d(\cdot)$, $P_s(\cdot)$, and $Q_s(\cdot)$ can be computed pointwise for each x_1 from equation (2). Clearly, there are computational challenges in maximizing this equation, due to the large-dimensional (M+N-1) integration required in computing the expectation. The pricing and quantity functions computed in this manner are the solution to the "relaxed" problem but, as in standard auction theory, incentive compatibility concerns may require that the equilibrium strategies are monotonic in the signals.

Common vs. Private Values Note that if both $P_d(\cdot)$ and $Q_d(\cdot)$ are monotonically increasing, this implies that the total transfer $T(x_i) \equiv P_d(x_i) * Q_d(x_i)$ would be strictly convex. Convexity implies that per-unit markups are increasing in q, and BMR interpret this as resulting from adverse selection ("winner's curse") considerations. On the other hand, BMR also show that in the private value case, the optimal transfer schedule is linear, implying constant $P_d(x_i)$ and zero correlation between (p, q) pairs. To the extent that these findings are applicable to our (slightly different) setting, a simple reduced-form test of the common value hypothesis would involve testing for positive correlation between buyers' prices and quantities in the data. A similar argument can be made for sellers. Preliminary results show that the correlation between p and q in-sample is indeed positive (but small) for both buyers and sellers, thus at a first glance confirming our modeling emphasis on common values.

Estimation Furthermore, we can exploit monotonicity of the equilibrium strategies in the signal s to aid our estimation procedure. We employ a *Monotone Quantile Estimator* which matches the sample quantiles of the observed bids to their population counterparts. Our estimator exploits the invariance property of conditional quantiles with respect to monotone transformations. Specifically, the equilibrium strategies are nonlinear but nevertheless monotone transformation of the underlying signals, so that $r_{\tau_k}(Z_i, \theta)$, the τ_kth quantile of the equilibrium ask quantity for the ith auction (including conditioning covariates Z_i) is just $Q_d(x_{\tau_k}, Z_i, \theta)$, the equilibrium quantity function evaluated at x_{τ_k}, the τ_kth quantile of the marginal distribution $F(x; Z_i, \theta)$ for a single signal. The objective function, then, for our estimator is:

$$\mathcal{Q}(\theta) = \sum_{i=1}^{T} \left[\sum_{j=1}^{M_i} \sum_{k=1}^{K} \rho_{\tau_k}\left(p_{ij} - P_d\left(x_{\tau_k}; Z_i, \theta\right)\right) + \rho_{\tau_k}\left(q_{ij} - Q_d\left(x_{\tau_k}; Z_i, \theta\right)\right) \right.$$
$$\left. + \sum_{j=1}^{N_i} \sum_{k=1}^{K} \rho_{\tau_k}\left(p_{ij} - P_s\left(x_{\tau_k}; Z_i, \theta\right)\right) + \rho_{\tau_k}\left(q_{ij} - Q_s\left(x_{\tau_k}; Z_i, \theta\right)\right) \right]$$

where $\rho_{\tau_k}(x) = (\tau_k - 1(x \leq 0))\, x$ is the "check" function familiar in quantile regression analysis, K is the number of quantile restrictions we employ to identify the parameters, T are the total number of auctions in the dataset, and M_i and N_i are, respectively, thte number of buyers and sellers in the ith auction. The invariance property has been previously exploited in the literature of semiparametric estimation of limited dependent variable models, but we believe this is the first attempt to use it in the estimation of structural equilibrium models.[1]

The main advantage of the monotone quantile estimator lies in its reduction in the computational burden. Consider the simulated nonlinear least squares estimator, in which the first moment of (say) the ask prices is matched with the conditional mean of the equilibrium $P_d(\cdot)$ function, which can be simulated by repeatedly solving equation 2 for different x_i. Our monotone quantile estimator requires only $T \times K$ calculations of (2) for each evaluation of the objective function, whereas the simulated nonlinear least squares estimator would require $T \times S$ calculations, where T is the number of auctions in the dataset, and S is the number of simulation draws used to evaluate the conditional mean of the equilibrium bids. Since the number of quantiles restrictions K would presumably be far less than the number of simulation draws S, this is a significant reduction in computational burden.

The question of market power would be addressed by calculating, for a given realization of s and a particular seller j, the markups $\frac{m(x_j, x_{-j}) - V(x_j, x_{-j})}{m(x_j, x_{-j})}$, and how they vary for different x_j.

The question of common vs. private values will be addressed first in reduced-form fashion (as described above). Subsequently, the model discussed above can be amended to allow

[1] We have previously used it in Hong and Shum (1999).

bidders' signals to be a combination of common and private value components. A formal parametric test of the common value hypothesis would proceed by testing the significance of the standard deviation of the common value component.

Agricultural controls — of which milk production quotas are one example — are a perennial bone of contention amongst the G7 countries. While this project does not directly address the desirability of these controls, it does shed light on the efficiency of a uniform-price double auction in allocating the subsequent rents among producers. For example, we can use the estimated structural parameters to simulate auction outcomes under alternative auction mechanisms, such as a non-uniform price "pay your bid" auction. Furthermore, double auctions are also used (and being considered for use) in emissions permit and deregulated electricity markets, and the research questions we address and methodology we develop in this project may also prove enlightening in those other contexts.

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