Secret Price Cutting and Strategic Buyers: An illustration of the Divide and Conquer Tactic*

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Abstract

This paper provides an analysis of when and how strategic buyers may undermine collusion between sellers. A key observation is that a buyer benefits from a price war between sellers. So a strategic buyer may be willing to shade his demand today in the hope that it will lead sellers to believe a secret price cut occurred, and that as a result, a price war will result. We perform our analysis in a stylized two stage model, and find that sustaining collusion in the first stage requires that buyers get a substantial fraction of the surplus. Comparative statics are performed on the number of sellers and buyers.

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1 Introduction

Our objective is to understand how and when strategic buyers may undermine collusion between sellers. We study an interaction where sellers publicly post prices, but are allowed to send secret price offers to buyers. And we also assume that the buyers' decisions are not public: in the spirit of Stigler (1964), sellers only observe the demand for their own product.

The main idea of the paper is that a secret price cut and a reduction in demand may have the same effect on the profile of private demands observed by the sellers. So, if deterring secret price cuts requires lower prices in the future, a strategic buyer may gain tomorrow from shading his demand today.

Sellers should thus worry that strategic buyers may affect the information received by sellers, in a way that would be detrimental to the sellers, and favorable to the buyers. To prevent this from happening, sellers have to make the decision to reduce demand a costly one for buyers, which amounts to leaving buyers enough surplus.

This paper attempts to capture this intuition in a simple model. In most of our analysis, the interaction between sellers and buyers is modelled as a two stage game: the first stage is a standard model of competition between two sellers and one buyer, which allows for secret price cuts; the second stage allows sellers to share information about what happened in the first stage, and results in payments contingent on the information transmitted by sellers.

The general question we address in this framework is the following: is it possible to find contingent payoffs that would allow sellers to collude on prices in the first round, and at what price level?[1]

[1] In more general settings, whether players may collude also depends on a similar trade-

More specifically, we consider first the case of myopic buyers, and derive the collusive equilibria for that case. Then we look for those equilibria which are robust to strategic behavior of the buyer. *Our main result is that in a robust collusive equilibrium, a strategic buyer must get a substantial fraction of surplus, and that this fraction increases with the number of sellers.*

Implicit collusion between a group of oligopolists has been the object of many theoretical and applied work. Our work differs from the literature in several respects. A first difference is that we do not take the demand faced by sellers as exogenous.[2] Rather, we assume that the buyer may be strategic, and that he may calculate how changing his demand may affect his future payoffs.

A second difference is the modelling strategy. We have opted for a very stylized two period model, rather the standard infinitely repeated games. Several reasons, beside simplicity, led to this choice. First, the logic of construction of equilibria in infinitely repeated games is now well understood, and we believe that avoiding the many technicalities of equilibrium construction permits to focus on why and when strategic buyers may undermine collusion between sellers. Second, the idea that repeated games may be viewed as two-stage games where second stage payoffs satisfy appropriate constraints is not new (see Fudenberg and Levine (1994)). In our framework, second-stage payments for sellers may a priori be set freely. However, endogenous constraints on second-stage payments will arise, as incentives to transmit information truthfully will have to be given to sellers.

off: players compare short term gains from deviating with the long term losses that the deviation might induce, and the question is whether one can find contingent continuation payoffs that deter players from deviating in the current period.

[2] One notable exception is Snyder (1996).

The question of collusion among a sub-group of players (here, the set of sellers) also opens difficult theoretical questions about the interpretation of equilibrium. Our approach consists in checking that the candidate collusive equilibria is robust to the degree of sophistication of the buyers. Our view is that a collusive agreement between sellers is a rule of behavior set by sellers, that performs well whether the buyers are strategic or not.

One alternative approach, which we would find less satisfactory, would be to derive the set of equilibria of the game played by sellers and buyers, and compute the total surplus that can be extracted by sellers in some equilibrium. However, we would be inclined to interpret one such equilibrium as a collusive agreement between the whole set of players (including both buyers and sellers).

One possible interpretation of our work is that we analyze the stability of a coalition that faces two pressures: pressure from the inside, because its members (the sellers) may deviate from high prices, and a pressure from the outside, because the outsiders (the buyers) have an interest in breaking up the coalition. The coalition solves the pressure from insiders by setting sanctions in case of deviations. The outsiders may try to behave in a way that trigger the sanctions, because they benefit from these sanctions. And they may do so by lowering their demand to one particular seller, who will then (erroneously) believe that a secret price cut occur. The coalition solves the pressure from outsiders by leaving enough surplus to outsiders, so that lower one's demand is a costly decision for a buyer.

The paper is organized as follows. In the next Section, we describe the basic version of our model, with two sellers and one buyer. Section 3 provides our main result. In Section 4, we extend our analysis to the case of n sellers and one buyer. The case of multiple buyers is analyzed in Section 5. We discuss our result and main assumptions in Section 6.

2 The Model

We start with the basic version of our two-stage model, with two sellers and one buyer.

The First stage: We consider two identical sellers $i \in \{1, 2\}$ selling homogenous products to one buyer. Each seller's marginal cost is normalized to 0. The buyer has a demand equal to two units. The value to the buyer of each unit is equal to v.

Each seller i publicly and simultaneously posts a price schedule $\pi_i = (\pi_i(1), \pi_i(2))$, where $\pi_i(k)$ is the public price for $k = 1, 2$ units. After these offers are made, each seller sends a secret price schedule offer $\widehat{\pi}_i = (\widehat{\pi}_i(1), \widehat{\pi}_i(2))$ to the buyer. We denote by $\pi = (\pi_1, \pi_2)$ the profile of public prices, and by $\widehat{\pi} = (\widehat{\pi}_1, \widehat{\pi}_2)$ the profile of secret prices.

The buyer then makes a purchase decision, based on the offers $(\pi, \widehat{\pi})$ received. We denote by $b_i \in \{0, 1, 2\}$ the number of units bought by the buyer to seller i, and we let $b = (b_1, b_2)$. We assume that the price paid by the buyer for the b_i units bought from seller i corresponds to the best conditions offered by seller i. That is, letting $q_i(b, \pi, \widehat{\pi})$ be the price paid by the buyer to seller i when the buyer makes the decision b and the offers are $(\pi, \widehat{\pi})$, we have:

$$q_i(b, \pi, \widehat{\pi}) = \min\{\pi_i(b_i), \widehat{\pi}_i(b_i)\}$$

and by $q(b, \pi, \widehat{\pi}) = q_1(b, \pi, \widehat{\pi}) + q_2(b, \pi, \widehat{\pi})$ the total price paid by the buyer.

We also let $u(b)$ denote the value to the buyer associated with the decision

b. Given our assumption that the buyer only values two units, we have

$$u(b) = v \max\{b_1 + b_2, 2\}.$$

Finally, we assume that each seller may only observe the demand b_i for his own product.

The second stage: In the second stage, sellers are allowed to share information about the demand for their respective products. We assume that each seller i makes a public announcement $m_i \in \{0, 1, 2\}$. Note however that messages are not costly to report. So for example, if we want the announcement m_i to coincide with the true demand b_i, we will need to provide sellers with the incentives to do so.

At the end of these two stages, the public information consists of (π, m), and we assume that payments contingent on this public information are made. We denote by $w = (w_1, w_2, w^b)$ the payments received by each agent. This payments are supposed to capture the continuation payoffs that each agent would be receiving in the continuing relationship.

A key assumption concerning these payments is that they must be balanced.

> **Assumption:** ***(balanced payments)*** *We assume that payments are balanced, that is, for any* (π, m),
>
> $$w_1(\pi, m) + w_2(\pi, m) + w(\pi, m) = 0$$

The fact that payments add up to 0 is not important. The key is that payments add up to a constant independent of (π, m), so that any decrease in the total payment received by sellers benefits to the buyer. We will discuss this assumption in Section 6. In particular, we will show that in the infinitely repeated version of the game, in equilibrium, all units must be sold in every period, so that in effect, the three players are sharing a surplus equal to $2v$. The intuition is very simple. If all units were not sold at some period, then there would be an incentive for a player to make a secret price cut allowing him to sell one additional unit. The observed demand for the other sellers would be unchanged, and the seller making the price cut could behave in the rest of the game as if he had not made the price cut. His payoffs in the rest of the game would thus be unaffected, while he would be able to make an extra gain by selling the additional unit.

In any event, this assumption is key to understanding the circumstances under which strategic buyers can make collusion between sellers more difficult to sustain. The assumption says that sellers cannot hurt themselves without the buyer benefitting from it.

In some cases, sellers might be engaged in some other relationships, and this could enable them to issue sanctions between themselves that would not *fully* benefit the buyer. In such cases, the effect we are about to describe will be reduced. We will discuss the case where payments satisfy a weaker constraint, that is, when *for any* (π, m),

$$-\Delta \leq w_1(\pi, m) + w_2(\pi, m) + w^b(\pi, m) \leq 0 \tag{1}$$

To conclude the presentation of the model, we define the strategies and the notion of equilibrium used.

A strategy $\mathbf{b}$ for the buyer specify a buying decision $\mathbf{b}(\pi, \widehat{\pi})$ function of the public and secret offers $(\pi, \widehat{\pi})$. In this Section, we will exclusively consider the case of *myopic buyers*, and we define now the strategies for buyers that are consistent with myopic behavior.

Definition 1 *We say that the strategy* $\mathbf{b}$ *is consistent with myopic behavior if for any* $(\pi, \widehat{\pi})$*, the choice* $b = \mathbf{b}(\pi, \widehat{\pi})$ *maximize first stage payoffs, that is, if*

$$\mathbf{b}(\pi, \widehat{\pi}) \in \arg\max_{b \in B} u(b) - q(b, \pi, \widehat{\pi})$$

We denote by B^* *the set of buyer's strategies that are consistent with myopic behavior.*

A strategy for seller i is denoted σ_i and consists of a public price π_i, a secret price $\widehat{\pi}$ that may be contingent on the public price profile offered π, and a message m_i that may be contingent on the public price profile π, the own secret price offer $\widehat{\pi}_i$ and the buyer's demand b_i for the product of seller i. For any payment schedule w, we let $v_i(\sigma, \mathbf{b}, w)$ be the total payoff received by player i when sellers conform to the strategy profile $\sigma = (\sigma_1, \sigma_2)$, when the buyer follows $\mathbf{b}$, and when the payment schedule is w. Throughout our analysis, we will restrict our attention to pure strategy equilibria.

Definition 2 *We say that the triplet* $(\sigma, \mathbf{b}, w)$ *constitute a perfect equilibrium if* $\mathbf{b}$ *is consistent with myopic behavior, if at each node where he moves, seller* i *finds that following* σ_i *is an optimal strategy, and if* w *is a balanced payment schedule*

We are now ready to present our definition of collusive equilibrium.

Definition 3 *We say that* σ *is a collusive equilibrium if there exists* w *such that for any buyer's strategy* $\mathbf{b}$ *consistent with myopic behavior,* $(\sigma, \mathbf{b}, w)$ *form a perfect equilibrium. We then say that* w *supports* σ*.*

Note that our definition of collusive equilibrium assumes that the buyer is myopic. We analyze the case where the buyer may be strategic in the next Section.

A benchmark: the myopic buyer case: Our first result says that when the buyer is not strategic, sellers may extract all the rent from the buyer.

Proposition 1 *For any* $v_0 \in [0, v)$*, there exists a collusive equilibrium* σ *under which the sum of the sellers first-stage payoffs equals* $2v_0$*.*

Note that when buyers are myopic, the constraint that payments are balanced is unimportant: Buyers do not take into account the effect of their buying decisions on their second-stage payment, so the buyer's payment becomes a free parameter. The balanced payment assumption will become important once we start analyzing the case of strategic buyers.

Proof. Choose $P_0 > 2v$ and $v_0 < v$. We consider the strategy for seller i defined by

$$\begin{aligned} \pi_i^* &= (v_0, P_0), \\ \widehat{\pi}_i^*(\pi) &= \begin{cases} \pi_i^* \text{ if } \pi = (\pi_1^*, \pi_2^*) \\ (0,0) \text{ otherwise} \end{cases} \\ m_i(b_i, \pi, \widehat{\pi}_i) &= \mathbf{b}_i \text{ for all } (b_i, \pi, \widehat{\pi}_i) \end{aligned}$$

The second condition means that seller i does not cut prices secretly, unless the public prices differ from π^*. The third condition says that seller i tells the truth about the demand for his own product.

We now choose $\bar{w} > v$ and define contingent payments as follows:

$$\begin{aligned} w_i(\pi, m) &= 0 \text{ if } \pi \neq \pi^*, \\ w_i(\pi^*, m_i, m_j) &= 0 \text{ if } m_j \in \{1, 2\} \\ w_i(\pi^*, m_i, m_j) &= -\bar{w} \text{ if } m_j = 0 \end{aligned}$$

The first condition says that payments are independent of the announcement made and equal to 0 when the public price π differ from π^*. Otherwise, seller i is punished when seller j announced that his demand was nul. Note that these payments preserve incentives to tell the truth as each seller i's payment is independent of his own announcement..

Finally, we consider any $\mathbf{b}^*$ consistent with myopic behavior.

Under the proposed strategies, each seller would sell one unit, at a price equal to v_0, and payments would be equal to 0. Let us now check incentives during the first stage. If $\pi \neq \pi^*$, then given that the opponent cuts prices, it is optimal to cut price as well. If $\pi = \pi^*$, then seller j is supposed not to cut prices. If seller i cuts prices and sell two units (cutting prices without selling an additional unit cannot be optimal), he obtains at most $2v_0$ in the first stage. In the second stage however, seller j would announce $m_j = 0$, and seller i's payment would then be equal to $-\bar{w}$. Since

$$2v_0 - \bar{w} < v_0,$$

it is optimal for seller i not to cut prices.

Finally, at the start of the first stage, any deviation from π_i^* would generate secret price cuts and no payments, hence zero profits. ■

An important feature of the equilibrium exhibited here is that in the event a player observe a drop in its demand, the total payments received by sellers decrease by $\bar{w}$. So the payment obtained by the buyer in the second stage is positive. Had the buyer anticipated this effect, he would have surely bought one unit less in the first stage.

3 Collusion with possibly strategic buyers

Our approach will consist in checking when collusive equilibria are robust to the buyer being strategic. Formally, we say that a collusive equilibrium σ is *robust to strategic behavior of the buyer* if there exists w that supports σ and $\mathbf{b} \in B^*$ such that the following condition holds:

$$v^b(\sigma, \mathbf{b}, w) \geq v^b(\sigma, \mathbf{b}', w) \text{ for any strategy } \mathbf{b}' \text{ of the buyer}$$

We have the following Proposition:

Proposition 2 *For any collusive equilibrium σ robust to strategic behavior of the buyer, the sum of sellers first-stage payoffs is at most equal to v.*

So the requirement that the equilibrium be robust to strategic behavior of the buyer decreases the rent that sellers can extract from the buyer. We will check that under the weaker constraint (1) on payments, the rent that sellers can extract is larger.

The rest of this section is devoted to the proof of this Proposition. In what follows, we consider a collusive equilibrium σ, and any strategy $\mathbf{b}$ consistent with myopic behavior. We let $\pi^*, \widehat{\pi}^*$, b^*, m^* be the public prices, secret prices, buying decision and announcement induced by $(\sigma, \mathbf{b})$. (Recall that we limit our attention to pure strategy equilibria.).

Assume that $b^* = (b_1^*, b_2^*)$ with $b_2^* \geq 1$. Let m_2^0 be the announcement made in equilibrium by seller 2 in case he observed $\pi^*, \widehat{\pi}_2^*$, and $b_2 = 0$. Also let q_i^* denote the price paid by the buyer to seller i for the b_i^* units. We are going to show that the two following inequalities hold:

$$\begin{aligned} w_2(m^*) &\geq w_2(m_1^*, m_2^0) \\ w_1(m^*) &\geq w_1(m_1^*, m_2^0) + q_2^* \end{aligned}$$

The first inequality holds trivially if $m_2^0 = m_2^*$. It also holds when $m_2^0 \neq m_2^*$ because otherwise, seller 2 would strictly prefer to announce m_2^0.

To check that the second inequality must hold, consider the following deviation by seller 1. Choose ε small, and assume that seller 1 chooses $\widehat{\pi}_1(2) = q_1^* + q_2^* - \varepsilon$ and m_1^*. The (myopic) buyer clearly prefers to buy two units from seller 1 rather than purchasing (b_1^*, b_2^*) because $b_1^* + b_2^* \leq 2$ and

$$2v - q_1^* - q_2^* + \varepsilon \geq v(b_1^* + b_2^*) - q_1^* - q_2^*$$

This deviation may affect second stage payoffs, because seller 2 will now report m_2^0. The total payoff obtained by seller 1 under the deviation should not exceed the payoff he obtains in equilibrium, that is, we should have:

$$q_1^* + q_2^* - \varepsilon + w_1(m_1^*, m_2^0) \leq q_1^* + w_1(m^*)$$

Since this inequality must hold for any $\varepsilon > 0$, we get the desired inequality.

We are now ready to check under what condition a collusive equilibrium is robust to strategic behavior of the buyer. Assume that the buyer only buys the b_1^* units from seller 1, and nothing from seller 2. He looses a surplus equal to $vb_2^* - q_2^*$ from doing so. However in the second stage, he gains

$$w^b(m_1^*, m_2^0) - w^b(m^*).$$

Given our balanced payment assumption and the inequalities just derived, the buyer's second stage gain satisfy:

$$\begin{aligned} w^b(m_1^*, m_2^0) - w^b(m^*) &= w_1(m^*) + w_2(m^*) - w_1(m_1^*, m_2^0) - w_2(m_1^*, m_2^0) \\ &\geq q_2^* \end{aligned} \tag{2}$$

Ensuring that the buyer does not have strict incentives to decrease his demand therefore requires

$$vb_2^* - q_2^* \geq q_2^*$$

or equivalently,

$$q_2^* \leq vb_2^*/2.$$

To conclude observe that either $b_1^* = 0$, and then $q_1^* = 0$, or $b_1^* \geq 1$, and the same argument as the one given above shows that $q_1^* \leq vb_1^*/2$. The total surplus that sellers can extract is therefore at most equal to v.

Comment: It is easy to check that under the weaker constraint (1) on payments, the bound on first-stage payoffs becomes $v+\Delta$. Indeed inequality (2) becomes

$$w^b(m_1^*, m_2^0) - w^b(m^*) \geq q_2^* - \Delta$$

which implies that deterring strategic behavior from the buyer only requires $q_2^* \leq vb_2^*/2 + \Delta/2$, which implies $q_1^* + q_2^* \leq v + \Delta$.

4 The n sellers case

The objective of this section is to generalize our main result to the case of n identical sellers. We assume that the demand of the buyer is K units, each valued at v. Marginal costs for sellers are still equal to 0. The vectors π_i (and $\widehat{\pi}_i$) specify as before a public price $\pi_i(k)$ (and a secret price $\widehat{\pi}_i(k)$) as a function of the number of units bought).

The purchase decision of the buyer is a vector $b = (b_1, ..., b_n)$, where b_i denotes the number of units bought to seller i.

Finally, our balanced payment assumption becomes:

$$\sum_{i=1,...,n} w_i(\pi, m) + w^b(\pi, m) = 0 \text{ for all } (\pi, m)$$

We prove the following Proposition:

Proposition 3 *In any collusive equilibrium σ robust to strategic behavior of the buyer, the sellers get a share of the surplus at most equal to $1/n$ in the first stage, that is, the sum of the payoffs received by sellers is at most equal to $\frac{Kv}{n}$.*

Consider a collusive equilibrium σ supported by w. We let $\pi^*, \widehat{\pi}^*$ be the public and secret equilibrium prices induced by σ. Consider any given buying decision $b^* = (b_1^*, ..., b_n^*)$ consistent with myopic behavior, when the public and secret prices are $\pi^*, \widehat{\pi}^*$, and we let m^* be the equilibrium announcement induced by $\pi^*, \widehat{\pi}^*$ and b^*, given σ. We also let q_i^* be the price paid by the buyer to seller i for the b_i^* units bought.

We first derive the constraints imposed by the fact that $(\sigma, \mathbf{b}, w)$ should be a perfect equilibrium for any $\mathbf{b} \in B^*$.

Assume $b_i^* > 0$, and let m_i^0 be the announcement that seller i would make in equilibrium if he were to observe no demand rather than a demand equal to b_i^*. We must have

$$w_i(m^*) \geq w_i(m_i^0, m_{-i}^*)$$

because otherwise, on the equilibrium path, seller i would be willing to make announcement m_i^0 instead of m_i^*.

Consider now a seller $j \neq i$ (supposed to sell b_j^* units in equilibrium). Assume that he makes the following deviation. He makes a secret offer $\widehat{\pi}_j$ for $b_i^* + b_j^*$ units such that

$$\widehat{\pi}_j(b_i^* + b_j^*) = q_j^* + q_i^* - \varepsilon,$$

and then makes the equilibrium announcement m_j^* as if no deviation occurred. Under the new secret and public price profile observed by the buyer, it is clearly strictly optimal for the buyer to buy $b_i^* + b_j^*$ from j, and his equilibrium purchase decisions from sellers other than i,j also remain optimal. So (for some $\mathbf{b} \in B^*$) the deviation by seller j only affect public information through the message sent by i. Deterring that deviation thus requires:

$$q_j^* + w_j(m^*) \geq q_j^* + q_i^* - \varepsilon + w_j(m_i^0, m_{-i}^*).$$

Since the latter inequality holds for all ε, and all $j \neq i$, we obtain

$$\sum_{j=1,\ldots,n} w_j(m^*) \geq \sum_{j=1,\ldots,n} w_j(m_i^0, m_{-i}^*) + (n-1)q_i^*.$$

By not buying from seller i, a strategic buyer loses a surplus equal to $kb_i^* - q_i^*$ in the first stage, but, thanks to the balanced payment assumption, he would gain $(n-1)q_i^*$ in the second stage. Deterring that deviation therefore requires that

$$(n-1)q_i^* \leq vb_i^* - q_i^*$$

that is,

$$q_i^* \leq \frac{v}{n} b_i^*.$$

Since the same argument holds for each seller i for which $b_i^* > 0$, we obtain that the total price $\sum_i q_i^*$ received by sellers is at most equal to $\frac{vK}{n}$, hence the sellers (as a whole) obtain a share of the surplus at most equal to $1/n$.

5 The Multiple buyers case

We have considered so far the case of a single buyer. We first wish to analyze the key role played by this assumption. If there are at least two buyers, and if the sellers can observe the identity of the buyers they sell to, a single buyer may not be able to secure an increase in second stage payoffs by shading his demand, even when a balanced payment assumption holds. The reason is as follows: by shading his demand, a buyer may induce a drop in second-stage payoffs for sellers. This drop will result in an increase in second-stage payoffs for buyers. However, this increase does not necessarily benefit to the deviating buyer: when sellers observe the identity of the buyer to which they (ought to) sell, a drop in demand due to buyer k not making the supposed equilibrium purchase may lead to all buyers but buyer k being rewarded in the second stage. So the buyers as a whole may benefit from the drop in demand, but buyer k may not necessarily benefit from it..

If the sellers cannot observe the identity of the buyers to wish they sell to, or if there are constraints on the extent to which other buyers may be rewarded (for example because other buyers are small), then our main insight will carry over: The effect will be limited because the deviating buyer will not fully benefit from the increase in second stage payoffs, but the effect will persist.

To illustrate this last point, let us consider the hypothetical case were the payments received by buyers would be shared equally (between buyers). Let the buyers be indexed by $l \in \{1, ..., m\}$, and assume that:

$$w_l^b(\pi, m) = -\frac{1}{m} \sum_{i=1,\ldots,n} w_i(\pi, m) \text{ for all } (\pi, m) \tag{3}$$

Let us also extend the definition of collusive equilibrium and collusive equilibrium robust to strategic behavior as follows: We say that σ is a collusive equilibrium if there exists w such that for any buyers strategy profile $\mathbf{b}$ consistent with myopic behavior, $(\sigma, \mathbf{b}, w)$ form a perfect equilibrium. A collusive equilibrium σ is said to be robust to strategic behavior if there exists a profile of strategies $\mathbf{b}$ consistent with myopic behavior and w supporting

σ such that, for any strategy b_l of buyer $l \in \{1, ..., m\}$,

$$v_l^b(\sigma, \mathbf{b}, w) \geq v_l^b(\sigma, \mathbf{b}_l', \mathbf{b}_{-l}, w).$$

We have the following Proposition.

Proposition 4 *Assume second stage payments satisfy (3), (i.e. balanced payment with equal sharing among buyers). Then in any collusive equilibrium σ robust to strategic behavior of the buyers, the sellers get a share of the surplus at most equal to $\frac{m}{n+m-1}$.*

The argument is similar to that developed in the proof of Proposition 3. Consider a particular buyer, say buyer l, and interpret b_i^*, q_i^* and m_i^0 defined in the proof of Proposition 3 as respectively the purchase decision of buyer l to seller i, the price paid by buyer l for the b_i^* units, and the report made by seller i when his demand is reduced by b_i^* (compared to his supposed equilibrium demand). With this interpretation in mind, the analysis of the proof of Proposition 3 dealing with the incentives of the sellers is unchanged, and we still obtain:

$$\sum_{j=1,\ldots,n} w_j(m^*) \geq \sum_{j=1,\ldots,n} w_j(m_i^0, m_{-i}^*) + (n-1)q_i^*.$$

The analysis of the incentives for buyer l are different, as he now only get a fraction $\frac{1}{m}$ of $(n-1)q_i^*$, so that deterring a deviation by buyer l now requires

$$\frac{(n-1)q_i^*}{m} \leq vb_i^* - q_i^*$$

or equivalently

$$q_i^* \leq \frac{m}{n+m-1} vb_i^*$$

which implies a bound $\frac{m}{n+m-1}$ on the share of the surplus that sellers may extract from buyers.

6 Discussion

There are two key ingredients in our model that explains why a coalition of players (the sellers) may suffer from the existence of a strategic agent outside the coalition (the buyer).

The first ingredient is that enforcing collusion requires that inefficiencies *across the members of the coalition* arise in some subgames. The second ingredient is that the overall interaction between all the agents is efficient, so that any loss of surplus for the members of the coalition benefits the agent exterior to the coalition.

Secret price cutting, as we shall now explain, is key in explaining the first ingredient and in motivating the second.

Secret price cutting and the balanced payment assumption. One interpretation of our balanced payment assumption is that there are no inefficiencies in the continuing interaction between sellers and buyers. In our framework, efficiency means that all units are sold. In the Appendix, we consider an infinitely repeated version of our two stage game, and show that indeed, in equilibrium, all units are sold in equilibrium.

To provide some preliminary intuition for this result, let us consider our two stage game with one myopic buyer, and show that the two units valued by the buyer must be bought in equilibrium. Assume for example only one unit is bought, say $b = (1, 0)$. Let q_1 be the price paid by the buyer for this. The surplus of the buyer is $v - q_1$. Assume that seller 1 makes a secret price offer for 2 units, at a price equal to $\widehat{\pi}_1(2) = q_1 + v/2$. For the buyer, purchasing two units from seller 1 yields a strictly higher surplus. If after this purchase, seller 1 behaves as if he had not deviated and the buyer had only bought one unit, then the public information generated by the first stage is unchanged, and therefore so are second stage payoffs. And the deviation is thus profitable for seller 1. We have therefore proved the following Proposition:

Proposition 5 *In any equilibrium of the two-stage game with one myopic buyer, the two units valued by the buyer must be bought in equilibrium.*

Secret price cutting and the inefficiencies induced among sellers. Another effect of secret price cutting is to make the enforcement of collusion between sellers difficult. In our model, enforcing collusion requires that sellers transmit information about their own demand to other sellers, so that deviators can be punished. Due to information revelation constraints, a seller cannot benefit from reporting a lower demand (otherwise he would always do so). This is why enforcing collusion requires inefficiencies among sellers in some subgames. This is true even when there are more than two sellers, because a seller cannot infer from a low demand who is responsible for that low demand, so that all sellers but the one reporting a low demand must be punished.

In such a context, a buyer is strong because i) enforcing collusion requires inefficient continuations among sellers in some subgames, and because ii) he may choose an action that lead precisely to such subgames.

Appendix

This appendix is devoted to the analysis of the infinite repetition of this two-stage in which the payments of the two-stage game are set equal to 0. Such infinite repeated games are difficult to analyze in general, because information about past demands and secret offers is private. The method and equilibrium concept used here to analyze this game are in the spirit of Compte (1998) and Kandori Matsushima (1998).

For simplicity, we return to the case of one buyer valuating only two units, and two sellers. The buyer is assumed to be myopic and we restrict attention to semi-public equilibria: a strategy for the seller is said to be semi-public if price offers only depend on past public information, and if reports depend only on past public information and current private observations.

Consider any myopic strategy for the buyer and a semi-public strategy profile σ for sellers. The strategies $(\sigma, \mathbf{b})$ form a semi-public equilibrium, if at any node where he moves, each seller i finds that following σ_i is optimal. The strategy profile σ is a collusive equilibrium if for any myopic strategy $\mathbf{b}$ for the buyer, the strategies $(\sigma, \mathbf{b})$ form a semi-public equilibrium.

Proposition 6 *For any $v_0 < v$, there exists a collusive equilibrium where the total surplus shared by the sellers is equal to $2v_0$. Besides, in any collusive equilibrium, the two units are sold in any period.*

Consider any collusive equilibrium, and choose a strategy for the buyer such that at any date his current decision depends only the current offers (and not on past information in case of ties). Then, at the start of the next period, continuation payoffs depend solely on the public history available at the start of next period. The analysis performed in the two stage game applies, and all units must be sold in equilibrium.

We now consider the following five public states $\{x_1, x_2, y_1, y_2, z\}$. The states x_1 and x_2 are interpreted as normal states, and the state y_i is interpreted as a punishment state for player i. The state z will be interpreted as a mutual punishment state. We consider the strategies defined as follows:

in state x_i, player i chooses $\pi_i(1) = \pi_i(2) = 2v_0$, and player $j \neq i$ chooses $\pi_j(1) = \pi_j(2) = P > 2v$; in state y_i, player j chooses $\pi_j(1) = 0$, $\pi_j(2) = 0$, and player i chooses $\pi_i(1) = -p$ and $\pi_i(2) = -2p$.[3] In state z, both players choose prices equal to 0. Also, players do not secretly cut prices.(We will check that they do not have incentives to secretly cut prices).

In state x_1, player 1 is supposed to sell the two units, at a total price $2v_0$. In state x_1, player 2 is supposed to sell the two units, at a total price $2v_0$. In state y_i player j is supposed to sell the two units, at a total price $-2p_0$ (below marginal costs). In state z, profits are 0 for both players.

To complete the description of the strategies, we explain how players move from one state to another, depending on the report made in the previous periods, and past public prices.

In case of a deviation in public prices, players move to state z for ever. So all deviations in public prices are deterred. If, in state x_i, player i reports a lower demand than expected, the next state is assumed to be y_j, followed by x_i. Otherwise, the next state is x_j.

We now check incentives to secretly cut prices. When player i reports a lower demand in state x_i, it does not affect the sequence of payoffs he receives: this sequence is $(0, 2v_0, 0, 2v_0....)$ whether he reports a lower demand or not. A bad report only affects the sequence of payoffs received by player j: this sequence is $(-2p, 0, 2v_0, 0....)$ instead of $(2v_0, 0, 2v_0, 0...)$. If p is sufficiently large, player j will thus be deterred from undercutting secretly in state x_i.

Finally, in state y_j, neither player wish to undercut the other. ■

[3] We allow for negative prices because we normalized costs to 0. Still one could argue that we should impose a lower bound on prices. Our result would still hold in that case: it would just require that we spread the punishment over more than one period.

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