Can stochastic discount factor models explain the FOREX risk premium?

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1 Introduction

A stylised fact of the foreign exchange (FOREX) market is that it is not efficient. But it is still not clear whether the rejection of efficiency is due to the FOREX risk premium (i.e. to omitting it entirely or failing to model it adequately), or to other causes such as a peso effect or non-rational expecations. Lewis (1995), in her comprehensive survey of the tests concluded that "no risk premium model with believable measures of risk aversion has yet been able to generate the variability in predictable excess returns that are observed in the data."(p1949), and that future research will need to integrate the various explanations for the rejection. Similar conclusions were reached by Engel (1996) in his survey. He identified four general directions in which the literature might go forward. One of these was to extend the analysis of the risk premium. This paper takes up the challenge of Engel and Lewis by examining whether the stochastic discount factor (SDF) model is able provide a measure of the foreign exchange risk premium that is consistent with FOREX market market efficiency. Instead of using standard SDF models in which the factors are unobservable latent variables, we pursue a different approach. We assume that the SDF can be proxied by observable macroeconomic variables that are jointly distributed with the excess return on foreign exchange. The tests surveyed by Engel and Lewis are based on a special case of the SDF model with observable factors, the inter-temporal consumption-based capital asset pricing model. Our more general framework enables us to examine a broader range of macroeconomic variables in a theoretically consistent way. Instead of using the familar Cox-Ingersoll-Ross (CIR) model to describe the factors, we employ a vector GARCH-in mean model.

The attractions of the SDF model are: it is consistent with most theories of asset pricing, including general equilibrium models of asset pricing as special cases; it does not depend on an explicit specification of risk aversion; and it has the flexibility to generate the required degree of variability in the discount factor. The best known of the SDF models is the Duffie-Kan (1996) class of affine models. This involves the use of unobservable factors which are extracted from the asset returns. SDF models of asset pricing are commonly used for the term structure. Papers that have used unobservable affine factor models for the term structure include Duffie and Kan (1996), Duffie and Singleton (1997), Backus, Foresi and Telmer (1998) and Remolona, Wickens and Gong (1999).

There are a number of new conceptual problems that arise in using the

SDF model to price currency compared with pricing bonds. The aim is to price currency risk and hence derive the foreign exchnage risk premium. Pure currency risk arises when the underlying assets are risk free in terms of their domestic currencies. When investors are risk-neutral, the arbitrage condition is given by uncovered interest parity. In this case their is no risk premium and the domestic and foreign investor is treated symmetrically. When investors are risk-averse, the currency risk premium for domestic investors may be different from that of the foreign investor. The relative size of domestic and foreign investors may also matter. In other words, there may be portfolio effects, and these could reflect differences in attitudes to risk between investors. In the case of complete markets these complications do not arise.

The use of the SDF model to price currency is new and there have been very few studies to date. Backus, Foresi and Telmer (1996) use the Duffie-Kan approach, not the observable SDF model. Hollifield and Yaron (1999) use a higher order expansion of the no–arbitrage condition with two observable factors (money and inflation) generated by a CIR model. They estimate the model using GMM on the moment condition. Hollifield and Yaron draw some interesting conclusions: the model must have significant real risk, and the monetary shocks should result in small inflation risk but lead to volatility in the real pricing kernel.

This paper can also be viewed from another perspective: it addresses the question of how to build econometric models of asset prices using the ARCH approach that are consistent with standard notions of finance. In general, unless investors are risk-neutral or asset returns have constant volatility, neither the standard linear models used in macroeconometrics nor ARCH models will not be appropriate because they are not arbitrage-free, and hence not consistent with finance theory.[1] This stricture applies notably to VAR models (co-integrated or not). To be arbitrage-free it is necessary for the econometric model to take account of any risk premium. For example, if the conditional covariance structure is assumed to be closely approximated by the ARCH (or GARCH) process, to be arbitrage-free in the absence of risk neutrality, it is necessary to include ARCH-in-mean effects too to represent the risk premium. Moreover, since the risk premium will be a conditional covariance, it will be necessary to use vector ARCH (or GARCH)-in-mean (VGARCHM) models. Thus the objective is to use finance theory to provide

[1] It is assumed that returns must be arbitrage-free otherwise there would be an opportunity for unlimited profits.

appropriate identifying restrictions on the joint distribution of the macro variables and the asset returns. An example of this approach to optimal asset allocation is provided by Flavin and Wickens (1999). This paper extends the methodology to the FOREX market.

Prior to the use of factor models most studies of the FOREX market were based on inter-temporal CAPM, a general equilibrium model. For example, Mark (1985) based a test of efficiency on the Euler condition and used GMM estimation. Implausibly large estimates of the coefficient of relative risk aversion (CRRA) were obtained, and the restrictions of the theory were rejected. But Kaminsky and Peruga (1990) adopted an approach to testing the general equilibrium model that is similar to the SDF model, and they employed a vector GARCH specification of the error structure. Their findings were similar to those of Mark in that they obtained an implausibly large estimate the CRRA, but they could not reject the theoretical restrictions. These two studies were based on monthly data. In commenting on Kaminsky and Peruga's findings, Baillie and Bollerslev (1991) argued that one reason for the weak results may be the lack of sufficient conditional heteroskedasticity in exchange rate data. Baillie and Bollerslev therefore used weekly data, and they allowed for moving average dynamics of the conditional mean of the excess return. They also used a univariate GARCH model for each variable from which they derived an estimate of the risk premium. Their findings were, however, similar to those of Kaminsky and Peruga in that all of the ARCH-in-mean effects were insignificant.

In this paper we generalize existing results in several respects. First, as noted, the use the SDF approach enables us to examine a broader range of macroeconomic variables than the general equilibrium model in a theoretically consistent way. Second, we use observable, not latent, factors which allows us to test the general equilibium model as a special case. Third, because we wish to price macroeconomic sources of risk, we use monthly data. Fourth, we employ the more general vector GARCH-in-mean model. This has both VAR and ARCH effects in the conditional mean, and has a multivariate GARCH structure.

The paper is set out as follows. In Section 2 we set out the theory. We explain the forward premium puzzle that we seek to resolve, and we provide the theoretical framework for the SDF approach. The econometric model is described in Section 3, and our findings are reported in Section 4. Our conclusions are given in Section 5.

1.1 The forward premium puzzle

Since one of the main aims of this paper is to address the forward premium puzzle, we begin by explaining what this is. Consider two countries (domestic and foreign) each of which issues a one-period bond that is risk-free in terms of its own currency. Let $R(t+1)$ denote the excess return to domestic investors from investing at time t in the foreign bond. Thus

$$R(t+1) = i^*(t) + \Delta s(t+1) - i(t) \tag{1}$$

where $i(t)$ and $i^*(t)$ are the domestic and foreign one-period nominal interest rates, respectively and $s(t)$ is the logarithm of the domestic price of foreign exchange. If investors are risk neutral and rational then the expectation of $R(t+1)$ conditional on information at time t is

$$E_t[R(t+1)] = 0 \tag{2}$$

the uncovered interest parity condition. But if investors are risk averse then

$$E_t[R(t+1)] = \phi(t) \tag{3}$$

where, for the moment, $\phi(t)$ will be given the interpretation of a risk premium. If only domestic investors are exposed to exchange risk (i.e. foreign investors only hold the foreign bond) then $\phi(t) \geq 0$. The sign is reversed if only foreign investors are exposed to exchange risk. More generally, $\phi(t)$ can be positive or negative depending on the relative magnitudes of these portfolio composition effects.

If the logarithm of the forward rate is denoted by

$$f(t) = s(t) + i(t) - i^*(t) \tag{4}$$

then the excess return can be written

$$\begin{aligned} R(t+1) &= s(t+1) - f(t) \\ &= \Delta s(t+1) - [f(t) - s(t)] \end{aligned} \tag{5}$$

where $f(t) - s(t)$ is the forward premium. If the rational expectations innovation is defined as

$$\varepsilon(t+1) = R(t+1) - E_t[R(t+1)] \tag{6}$$

then equation (3) can be expressed as

$$R(t+1) = \alpha + \beta[f(t) - s(t)] + e(t+1) \tag{7}$$

where

$$e(t+1) = \phi(t) + \varepsilon(t+1) \tag{8}$$

Equation (7) - or more commonly a variant in which $R(t+1)$ is replaced by $\Delta s(t+1)$ - is the basis of most tests of the efficiency of the foreign exchange market. Efficiency implies that $\alpha = \beta = 0$ and rationality implies that $E_t[\varepsilon(t+1)] = 0$. If, in addition, investors are risk neutral then $\phi(t) = 0$, and so $E_t[e(t+1)] = 0$. When all these assumptions hold the OLS estimators of α and β will be consistent.

Using the convention that the currency is measured as US dollars per unit of domestic currency (i.e. US bonds are the domestic asset), the forward premium puzzle is that in practice the OLS estimate of β is negative - even values as low as -4 have been obtained, see the surveys of Lewis(1995) and Engel (1996). In other words, although theory predicts that the dollar will depreciate if the forward premium is positive, the evidence shows that typically it depreciates. One explanation of this is that the estimate of β is biased downwards due to the presence of the risk premium in the error term of the regression. Assuming that the FOREX market is efficient and investors are rational, Fama (1984) has shown that the bias in β can be expressed as

$$\begin{aligned} bias &= cov[f(t) - s(t), \phi(t)]/var[f(t) - s(t)] \\ &= \rho \left[\frac{var[\phi(t)]}{var[f(t) - s(t)]} \right]^{\frac{1}{2}} \end{aligned} \tag{9}$$

where ρ is the correlation between $f(t) - s(t)$ and $\phi(t)$, (i.e. between the forward and risk premia). Negative bias implies, therefore, that $\rho < 0$. This can be interpreted as meaning that for US investors, the greater the expected depreciation of domestic currency, the lower is the required risk premium for holding foreign assets. In effect, therefore, the 45^0-line predicted by theory is shifting up or down due to changes in the risk premium: and the greater the expected depreciation, the smaller the shift. The result is a negative (or vertical) scatter diagram instead of a positive one as in Figure 1. This is a scatter diagram of the excess return $R(t+1)$ against $f(t)-s(t)$ for the US dollar-sterling exchange rate for the data used in this study. The 45^0-line and the regression line of $R(t+1)$ on $f(t)-s(t)$ are also depicted. This gives a rough idea of the order of magnitude of the shifts in the 45^0-line that a time-varying risk premium would need to induce to account for the forward premium puzzle.[2]

The puzzle is deepened by the fact that general equilibrium models of the risk premium typically do not produce a risk premium that is capable of generating the sorts of bias observed in practice. Since the estimate of β is typically negative and $\rho < 1$ the variance of the risk premium would need to be considerably greater than the variance of the forward premium. Since the maximum value of $\rho^2 = 1$, equation (9) implies that

$$\begin{aligned} var[\phi(t)] &= bias^2.var[f(t)-s(t)]/\rho^2 \\ &\geq bias^2.var[f(t)-s(t)] \end{aligned} \tag{10}$$

This gives a lower bound to the variance of $\phi(t)$, the equivalent of the Hansen-Jagannathan (1991) bound. For a typical bias of the order of -2, the variance of the risk premium would need to be a factor four greater than the variance of the forward premium.[3] The failure to generate from general equilibrium theories a risk premium with enough variation has led to other explanations that have focussed more on the formation of expectations, such as peso effects, regime switching and non-rationality. We turn next to generating risk premia from the SDF model.

1.2 Stochastic Discount Factor Theory

We consider how to obtain an expression for the foreign exchange risk premium using a version of the stochastic discount model based on observable, but stochastic, macroeconomic factors. These factors are jointly distributed with the excess return on foreign exchange, the only asset we consider. First

[2] The estimates of equation (7) are given in Table 1. The estimate of β is -2.706 and is highly significant. The estimate of α is -5.76 and is significant. The residual diagnostic statistics show both strong serial correlation and ARCH effects.

[3] For our estimate of equation (7) a lower bound to the proportion of the variation of $er(t+1)$ due to $\phi(t)$ is 6.5%($=V[\phi]/V[er]$)

we outline SDF theory as relevant for FOREX, we then take account of the measurement of the factors.

The SDF model can be expressed as

$$1 = E_t[M(t+1)(1 + R(t+1))] \tag{11}$$

where $M(t)$ is the discount factor, or pricing kernel (see Singleton(1990)). In other words, $M(t+1)$ is the discount factor required to make the present value of the total income $(1+i^*(t))$ from holding a foreign bond and converted to domestic currency equal to the certain income $(1 + i(t))$ from investing in the domestic bond. The only source of uncertainty here is the one-period ahead spot exchange rate as both $i(t)$ and $i^*(t)$ are known at time t. Taking logarithms of equation (11) and assuming log-normality gives

$$E_t[m(t+1) + R(t+1)] + \frac{1}{2}V_t[m(t+1) + R(t+1)] = 0 \tag{12}$$

where $m = \ln(M)$ and it is assumed that $\ln(1+x) \approx x$ for small x. Replacing $R(t+1)$ in equations (11) and (12) by the risk free rate $i(t)$ gives

$$E_t[m(t+1) + i(t)] + \frac{1}{2}V_t[m(t+1)] = 0 \tag{13}$$

Subtracting equation (13) from equation (12) gives

$$E_t[R(t+1)] + \frac{1}{2}V_t[R(t+1)] = -Cov_t[m(t+1), R(t+1)] \tag{14}$$

The last term on the left hand-side of equation (14) is the Jensen effect due to taking the expectations of a non-linear function of Normally distributed variables - i.e. the logarithm. The term of the right hand-side is the FOREX risk premium. Comparing equation (14) with equation (3) implies that $\phi(t)$ is not in fact just the risk premium but is

$$\phi(t) = -\frac{1}{2}V_t[R(t+1)] - Cov_t[m(t+1), R(t+1)] \tag{15}$$

This implies that $\phi(t)$ will have a higher variance than the FOREX risk premium which will be of some assistance in helping to generate the additional variability required in $\phi(t)$.[4]

[4]If logarithms are not taken, and the excess return is defined as $1 + R(t+1) = \frac{(1+i^*(t))S(t+1)}{(1+i(t))S(t)}$ then the arbitrage relation would be $E_t[R(t+1)] = -Cov_t[M(t+1), R(t+1)]$ which does not involve the Jensen effect.

Because equation (14) involves conditional expectations, and given equation (5), it can be expressed in other ways, for example, as[5]

$$E_t[R(t+1)] + \frac{1}{2}V_t[\Delta s(t+1)] = -Cov_t[m(t+1), \Delta s(t+1)] \tag{16}$$

This shows explicitly that the risk premium arises from uncertainty about the future spot exchange rate.

It has been implicitly assumed that the risk is being borne by domestic investors through their holding of foreign bonds. This would imply that the discount factor is that appropriate for domestic investors. In practice, of course, foreign investors are exposed to the same FOREX risk in reverse. Measuring returns and the discount factor in foreign currency would give

$$E_t[R^*(t+1)] + \frac{1}{2}V_t[R^*(t+1)] = -Cov_t[m^*(t+1), R^*(t+1)] \tag{17}$$

where m^* is the foreign investor's discount factor and is measured in foreign currency, and $R^* = -R$. Hence

$$E_t[R(t+1)] - \frac{1}{2}V_t[R(t+1)] = -Cov_t[m^*(t+1), R(t+1)] \tag{18}$$

Adding equations (14) and (18) gives

$$E_t[R(t+1)] = -Cov_t[\frac{1}{2}(m(t+1) + m^*(t+1)), R(t+1)] \tag{19}$$

Thus the Jensen effect disappears and the risk premium is due to covariation between the average of the discount factors of the domestic and foreign investors and the excess return defined for the domestic investor (or, equivalently, $\Delta s(t+1)$). Subtracting (18) from (14) gives

$$V_t[R(t+1)] = Cov_t[(m^*(t+1), R(t+1)] - Cov_t[m(t+1)), R(t+1)] \tag{20}$$

This implies that

$$R(t+1) = m^*(t+1) - m(t+1) + \eta(t+1)] \tag{21}$$

with $Cov_t[R(t+1), \eta(t+1)] = 0$. Equation (20) reveals that there is a linear relation between $V_t[R(t+1)]$, $Cov_t[(m^*(t+1), R(t+1)]$ and $Cov_t[m(t+$

[5] $\Delta s(t+1)$ could be replaced in equation (16) by $s(t+1)$.

$1)), R(t+1)]$ and only two terms are required, as in equation (19). There is an important proviso to this result. If, as is likely, there is measurement error in the proxy for the discount factor, then the data will not hold in practice.

In the case of complete markets the two discount factors are identical when measured in the same currency, see Backus, Foresi and Telmer (1996). Hence

$$m^*(t+1) = m(t+1) + \Delta s(t+1) \tag{22}$$

This would imply that equations (16) and (18) are then identical.

1.3 SDF model with observable factors

In general the stochastic discount factor is not observable. An exception is the general equilibrium pricing kernel in which the SDF equals the intertemporal marginal rate of substitution. To implement this it is necessary to make several additional assumptions, including specifying preferences or the marginal utility function. For example, for the power utility function $U[C(t)] = [C(t)^{1-\sigma} - 1]/(1-\sigma)$ with coefficient of relative risk aversion σ

$$\begin{aligned} M(t+1) &= \delta \left[\frac{U'[C(t+1)]}{U'[C(t)]} \right] \frac{P(t)}{P(t+1)} \\ &= \delta \left[\frac{C(t+1)}{C(t)} \right]^{-\sigma} \frac{P(t)}{P(t+1)} \end{aligned} \tag{23}$$

where $C(t)$ is nominal consumption, $P(t)$ is the price level and δ is the rate of discount of utility. Taking logarithms gives

$$m(t+1) = \ln \delta - \sigma \Delta c(t+1) - \Delta p(t+1) \tag{24}$$

Here we simply assume that $m(t)$ is a linear function of observable macroeconomic variables $z(t)$, namely

$$m(t+1) = \beta' z(t+1) + \xi(t+1) \tag{25}$$

$z(t+1)$ may include a constant. Clearly, equation (25) is a special case of (24) in which there is one macro factor, $\Delta c(t+1)$. The term $\xi(t)$ is included to represent the possibility that the macro factors do not capture $m(t)$ perfectly. It is assumed that $\xi(t)$ is orthogonal to $z(t)$. An alternative interpretation is that $z(t)$, a single variable, measures $m(t)$ with error. In this case $\xi(t)$ is correlated with $z(t)$ but not $m(t)$.

Equation (14) can now be written

$$\begin{aligned} E_t[R(t+1)] + \frac{1}{2}V_t[R(t+1)] &= -\beta' Cov_t[z(t+1), R(t+1)] \\ &\quad - Cov_t[\xi(t+1), R(t+1)] \end{aligned} \tag{26}$$

The aim is to proxy the risk premium by the first term on the right hand-side of equation (26) and compute the conditional covariance from the joint conditional distribution of $\{R(t+1),\ z(t+1)\}$ together with the conditional variance of $R(t+1)$. β will need to be estimated, and the last term in equation (26) is ignored.

In general, the presence of $\xi(t)$ would mean that the last term in equation (26) is not zero; nor will the two conditional covariances be uncorrelated. Thus omitting this term will introduce a bias and reduce the power of tests of the model. If, however, $\xi(t)$ is due to pure measurement error, then the last term may be expected to be zero. There will then be no bias, only estimation inefficiency.

2 The econometric model

The excess return is a function of three variables, the exchange rate and the two interest rates. After transforming these variables to stationarity we obtain the the excess return, the forward premium and the change in one of the interest rates. The joint conditional distribution of three variables is the starting point for the econometric model. To these variables we can add others to help proxy the SDF.

We assume that the conditional covariance structure of the FOREX market can be closely approximated by ARCH. Thus we model the joint conditional distribution of asset returns by a vector autoregressive GARCH-in-mean (VGARCHM) process. This allows the conditional mean of the distribution to be affected by lagged levels and by the conditional covariance matrix, and it models the conditional covariance matrix by a multivariate GARCH process.[6]

[6]For an extensive review of multi-variate ARCH models and alternative parameteriza-

This approach permits us to model the excess return and the macroeconomic factors jointly, and it can capture all of the features in the theory above. For the model to be consistent with the absence of arbitrage it is necessary to impose restrictions. These restrictions also provide a test of market efficiency. This will be broadly the correct way to specify models of asset prices when using ARCH. More generally, it will be necessary to use multivariate non-linear models with stochastic volatility, of which the CIR and Vasicek models are well-known univariate examples. The reason for including lagged variables (the VAR part of the model) is that we are able to obtain a better representation of the error terms. Also, by including the lagged forward premium in the equation for the excess return, we can compare the SDF model directly with equation (**??**) which is just a special case. In particular, we can directly observe whether, by including a measure of the risk premium, the forward premium becomes insignificant as SDF theory predicts.

We define the following vector of stationary variables $x(t+1) = \{R(t+1), z(t+1)'\}'$ and assume that x_t is generated by the VGARCHM

$$\mathbf{x}(t+1) = \boldsymbol{\alpha} + \boldsymbol{\Gamma}\mathbf{x}(t) + \boldsymbol{\Phi}\mathbf{g}(t) + \varepsilon(\mathbf{t+1}) \tag{27}$$

where the distribution of $\varepsilon(t+1)$ conditional on information available at time t, $\Psi(t)$, is

$$\boldsymbol{\varepsilon}(\mathbf{t+1}) \mid \Psi(t) \sim N[0, \mathbf{H}(t+1)] \tag{28}$$

$$\mathbf{g}(t) = vech\{\mathbf{H}(\mathbf{t+1})\} \tag{29}$$

$$\mathbf{H}(t+1) = \mathbf{V'V} + \mathbf{A}'[\mathbf{H}(t)\mathbf{H}(t)' - \mathbf{V'V}]\mathbf{A} + \mathbf{B}'[\varepsilon(t)\varepsilon(t)' - \mathbf{V'V}]\mathbf{B} \tag{30}$$

where $\mathbf{V}$ is lower triangular and $\mathbf{A}$ and $\mathbf{B}$ are diagonal matrices.

Equation (30) is a special case of the multivariate GARCH model of the BEKK model described and generalized in Engle and Kroner (1995). There are several advantages to this specification of the conditional variance. First, compared with more general formulations which do not restrict $\mathbf{V}$, $\mathbf{A}$ and $\mathbf{B}$, it offers a considerable reduction in the number of parameters to be estimated. Second, by formulating the conditional variance-covariance structure

tions see Bollerslev, Engle and Nelson (1994) and Pagan (1996). And for a discussion of the specification of multi-variate ARCH models in financial models see Flavin and Wickens (1997).

in this way, we are able to obtain an estimate of both the unconditional (long-run) covariance matrix and the conditional covariance matrix (the short-run dynamics). Third, this formulation guarantees a positive semi-definite unconditional and conditional covariance matrix. Fourth, but by no means least, the restriction that **A** and **B** are diagonal improves numerical convergence considerably but without affecting the general conclusions.

The final step in the specification of the data generating process is to impose on the joint conditional distribution the appropriate restrictions to avoid arbitrage possibilities. Assuming that $z(t)$ accurately approximates the stochastic discount factor, so that the last term of equation (26) can be ignored, the restrictions are given by equation (26). This determines the row of $\mathbf{\Phi}$ associated with $R(t+1)$. As, in general, β is unrestricted, the coefficients corresponding to the conditional covariances of the other variables with $R(t+1)$ will be unrestricted. The coefficient corresponding to the own conditional variance is restricted (to -0.5) and the other coefficients in this row are zero. There are also restrictions on Γ. All of the coefficients in the row corresponding to $R(t+1)$ will be zero. The intercept should not be restricted as it is part of the long-run risk premium.

The specification of the remaining equations is more problematic as the theory gives little guidance. The model will depend upon the choice variables for the SDF. We estimate a number of variants. Another potential problem is that the macroeconomic variables used to proxy the risk premium will need to display ARCH effects. Even for financial data ARCH effects are often only observed at high frequencies (a month or less), but the highest frequency for which macroeconomic data is available is usually one month, and more commonly at lower frequencies.

(a) Benchmark SDF model

If equation (7), the regression of $R(t+1)$ on $f(t)-s(t)$ is, our non-SDF benchmark model, our benchmark SDF model is closest to traditional finance in that it does not contain any non-essential macroeconomic variables. It consists of the three variables: the US dollar-sterling exchange rate and the US and UK euro-interest rates. These are transformed into three stationary variables: the excess return $R(t+1)$, the forward premium $f(t+1)-s(t+1) = i^{us}(t+1) - i^{uk}(t+1)$, and the change in the US interest rate $\Delta i^{us}(t+1)$. The change in the UK interest rate is not used because it is assumed that causality runs largely from the US to the UK rate. Both interest rates are permitted to have ARCH-in-mean effects and if they do, then the forward premium and the change in the US interest rate will display ARCH-in-mean effects. The excess return is allowed to have both conditional variance-in-mean and covariance-in-mean effects with the forward rate and US interest rate. In effect, therefore, this model assumes that the discount factor can be proxied by the two interest rates. If each country is assumed to base its discount factor on its own interest rate then $V_t[R(t+1)]$ could be omitted from the model as in equation (19).

(b) General equilibrium model

This model is an attempt to implement the general equilibrium pricing kernel, equation (24). This can be accomplished in four ways ways.

(i) It can be assumed that the domestic investor is the US investor

(ii) It can be assumed that the domestic investor is the UK investor

(iii) It can be assumed that there are complete markets

(iv) A general version can be used which encompasses all three.

In the absence of monthly data on consumption, real retail sales are used instead, and the CPI (for the US) and the RPI (for the UK) are used as the price series. Thus for (i) there are two additional variables in the benchmark model: the growth rate in real US retail sales Δc^{us} and US CPI inflation Δp^{us}. This implies that $Cov_t[\Delta c^{us}(t+1), R(t+1)]$ and $Cov_t[\Delta p^{us}(t+1), R(t+1)]$ should be included in the excess return equation. For (ii), the equivalent UK variables Δc^{uk} and $\Delta p^{uk}(t+1)$ are used instead. For (iii), cases (i) and (ii) should be the same. For (iv), all four of the extra conditional covariances should be included in the excess return equation. According SDF theory, and in the absence of measurement error in the proxies for the discount factors, one out of $V_t[R(t+1)]$ and the conditional covariances can be omitted because they are linearly related. $V_t[R(t+1)]$ could therefore be omitted from the excess return equation. But if the proxy for the SDF is poor (or if the general equilibrium model is wrong), then it would not in general be correct to omit $V_t[R(t+1)]$ as the conditional variance and the conditional covariances need not be linearly related.

(c) Generalized SDF model

We consider just one case: the use of base money. MB for the US and M0 for the UK. This case is like the general equilbrium model but with the rates of growth of the two money stocks, namely Δm^{us} and Δm^{uk}, forming the extra variables. It is also possible to combine the two cases by using all six additional variables. The main limitation on the number of variables that can be included in this way is the increased difficulty of achieving numerical maximization of the likelihood as the number of coefficients to be estimated increases at the rate n^3, where n is the number of variables. For these reasons we are only able to report selected cases.

3 Empirical evidence

3.1 The Data

The data set is monthly from 1975.1 to 1997.12 and is based on two countries, the US and the UK. It consists of the US dollar-sterling exchange rate , one month Eurocurrency interest rates, real retail sales (the nearest we can get to monthly real consumption) estimates, the CPI for the US and the RPI for the UK, the monetary base for the US and M0 for the UK. All data are expressed in annualized rates. The forward premium is calculated as the US minus the UK interest rate. Since a unit root cannot be rejected for each series, we use stationary transformations of the data either in the form of first differences or spreads.

The data used in the analysis are transformed to stationarity. This a unit root is rejected for changes in the interest rates, the rates of growth of the exchange rate, retail sales and money, the inflation rates, the forward premium and the excess return. The variables were also tested for ARCH effects.[7] All except the US monetary base showed strong ARCH. This provides some justification for the use of monthly data in GARCH-based models of risk premia.

3.2 Estimates

As already noted, Table 1 provides OLS estimates of the non-SDF model. These are also included in our summary in Table 3. We note that the estimate of the forward premium is negative and highly significant, and that the diagnostic statistics reveal both residual serial correlation and residual ARCH. As is usual in these equations, the fit of the model is very poor reflecting the importance of news in determining next period's exchange rate, and hence the excess return. We also recall that the lower bound estimate of the proportion of the variance in the excess return due to fluctuations in

[7]For each variable an LM test for ARCH was carried out on the residuals of a third order autoregression.

the risk premium is 6.5%. Essentially this is what we seek to explain in our other models with ARCH-in-mean effects.

In Table 2 we report our estimates of the benchmark SDF model. The equation is not restricted by the omission of $V_t[R(t+1)]$. The first rows of $\boldsymbol{\alpha}$, $\boldsymbol{\Gamma}$, $\boldsymbol{\Phi}$ contain the estimates for the excess return equation. The estimate of the coefficient of $V_t[R(t+1)]$ is not significant, but the estimates of the two conditional variances are more significant. This is broadly consistent with equation (19) and suggests that exchange risk may be associated with interest rate uncertainty. An expected rise in the future US interest rate is negatively correlated with the excess return to holding foreign assets and hence raises the risk premium. Nonetheless, the coefficient of the forward premium is still significantly negative and so the model clearly does not come close to fully capturing the FOREX risk premium.

Table 3 contains the estimates of the general equilibrium model and a more general SDF model. For ease of comparison, the estimates in Tables 1 and 2 are included in the columns labelled 1 and 2. Columns 3 and 4 contain the general equilibrium model estimates for the US and the UK, respectively. It is clear that neither equation is well determined, suggesting that general equilibrium model is unable to explain the FOREX risk premium. This is consistent with the findings of other studies based on the general equilibrium model that were discussed earlier.

The last column contains estimates of a generalisation of the benchmark SDF model to which the US and UK money growth rates have been added. This model produces some interesting results. The two conditional covariances of money with the excess return are both significant as is the coefficient of the conditional variance of the excess return, but the covariances between the interest rates and the excess return are now less important. This suggests that nominal uncertainty is a more fundamental source of FOREX risk. The model is not fully consistent with theory, however, as the lagged excess return and the forward premium are still significant.

4 Conclusions

In this paper we have examined whether SDF models based on observable macroeconomic factors and using VGARCHM to model the joint distribution of the variables can provide an explanation of the FOREX risk premium. The advantage of this approach is that it is consistent with no-arbitrage, it directly identifies the sources of FOREX risk and it provides an alternative to general equilibrium models of the FOREX risk premium. Our results offer some support for this approach. Money growth (domestic and foreign) is found to be the most important source of exchange risk, while the general equilbrium model based on consumption risk is clearly rejected.

The main problems with this approach are the availability of appropriate data and the estimation of the model. Macroeconomic data is not widely available and then at most at monthly intervals. This curtails the amount of heteroskedasticity in the explanatory variables. In order to provide an adequate representation of the theory, the VGARCHM model must be highly parameterized. This, together with the lack of heteroskedasticity in the data makes the numerical convergence difficult and the optimization a lengthy procedure. Further advances in the use of this general approach will depend in large part in finding a satisfactory solution to these problems.

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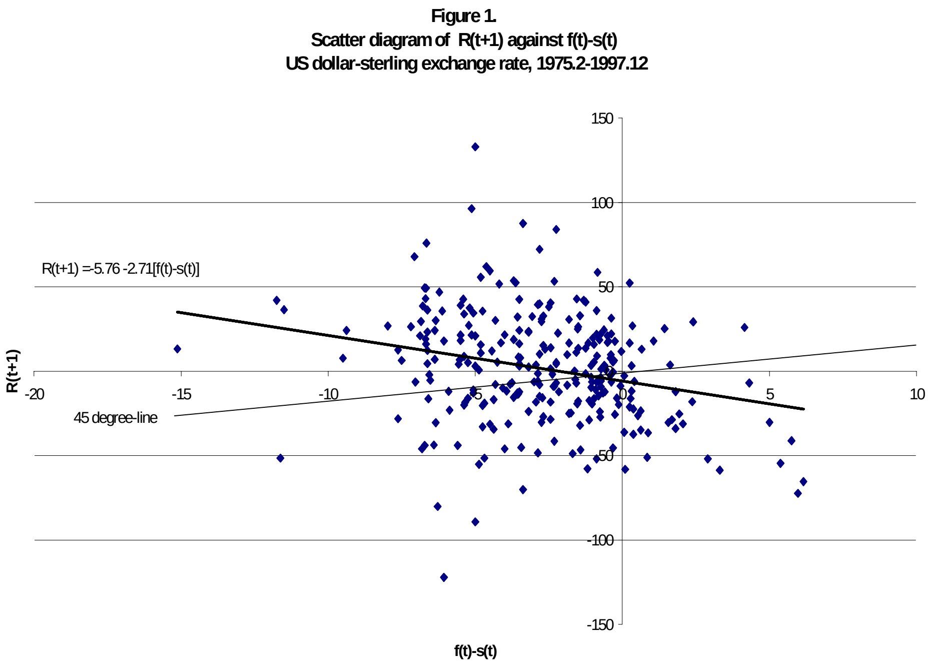

Figure 1:

Table 1
Estimates of the non-SDF model

$$R(t+1) = \alpha + \beta[f(t) - s(t)] + e(t+1)$$

	Estimate	t-statistic
α	-5.756	2.24
β	-2.706	4.29
R^2	0.063	
se	31.733	
LM^{AR}	44.1	
LM^{ARCH}	18.6	

Table 2
Estimates of the Benchmark SDF Model

$$\mathbf{x}(t+1) = \boldsymbol{\alpha} + \boldsymbol{\Gamma}\mathbf{x}(t) + \boldsymbol{\Phi}\mathbf{g}(t) + \varepsilon(\mathbf{t+1})$$
$$\boldsymbol{\varepsilon}(\mathbf{t+1}) \mid \Psi(t) \sim N[0, \mathbf{H}(t+1)]$$
$$\mathbf{g}(t) = vech\{H(t+1)\} = \{H11, H21, H31, H22, H32, H33\}'$$
$$\mathbf{H}(t+1) = \mathbf{V}'\mathbf{V} + \mathbf{A}'[\mathbf{H}(t)\mathbf{H}(t)' - \mathbf{V}'\mathbf{V}]\mathbf{A} + \mathbf{B}'[\varepsilon(t)\varepsilon(t)' - \mathbf{V}'\mathbf{V}]\mathbf{B}$$
$$x(t) = \{R(t), f(t) - s(t), \Delta i^{us}(t)\}'$$

Conditional Mean

$$\boldsymbol{\alpha} = \begin{bmatrix} 2.811 \\ (0.21) \\ -0.133 \\ (2.03) \\ 0.034 \\ (0.71) \end{bmatrix}, \quad \boldsymbol{\Gamma} = \begin{bmatrix} 0.293 & -3.037 & -2.233 \\ (4.09) & (4.14) & (0.66) \\ -0.001 & 0.9455 & -0.002 \\ (0.93) & (57.32) & (0.02) \\ 0.001 & 0.014 & 0.018 \\ (1.11) & (1.21) & (0.26) \end{bmatrix},$$

$$\boldsymbol{\Phi} = \begin{bmatrix} -0.010 & -1.780 & 0.918 & 0 & 0 & 0 \\ (0.62) & (2.31) & (1.47) & & & \\ 0 & -0.017 & 0 & 0.022 & 0.009 & 0 \\ & (1.10) & & (0.25) & (0.08) & \\ -0.002 & 0 & 0 & 0 & -0.078 & -0.040 \\ (0.23) & & & & (0.45) & (0.22) \end{bmatrix}$$

Conditional Variance

$$\mathbf{V} = \begin{bmatrix} 29.274 & 0 & 0 \\ (23.52) & & \\ 0.014 & 3.456 & 0 \\ (0.31) & (0.05) & \\ -0.010 & 0.068 & 0.748 \\ (0.26) & (0.05) & (2.13) \end{bmatrix},$$

$$diag\{\mathbf{A}\} = \begin{bmatrix} 0.095 & 0.937 & 0.776 \\ (0.42) & (208.21) & (19.22) \end{bmatrix}$$

$$diag\{B\} = \begin{bmatrix} 0.356 & 0.348 & 0.592 \\ (3.62) & (14.30) & (10.14) \end{bmatrix}$$

Table 3
Estimates of the the Excess Return Equation

$$E_t[R(t+1)] = \alpha + \beta V_t[R(t+1)] + \gamma' Cov_t[z(t+1), R(t+1)]$$

Full Model

$$\mathbf{x}(t+1) = \boldsymbol{\alpha} + \boldsymbol{\Gamma}\mathbf{x}(t) + \boldsymbol{\Phi}\mathbf{g}(t) + \varepsilon(\mathbf{t+1})$$
$$\boldsymbol{\varepsilon}(\mathbf{t+1}) \mid \Psi(t) \sim N[0, \mathbf{H}(t+1)]$$
$$\mathbf{g}(t) = vech\{H(t+1)\}$$
$$\mathbf{H}(t+1) = \mathbf{V}'\mathbf{V} + \mathbf{A}'[\mathbf{H}(t)\mathbf{H}(t)' - \mathbf{V}'\mathbf{V}]\mathbf{A} + \mathbf{B}'[\varepsilon(t)\varepsilon(t)' - \mathbf{V}'\mathbf{V}]\mathbf{B}$$
$$x(t) = \{R(t), z(t)'\}'$$

Variables	1	2	3	4	5
const	-5.756	2.811	-5.344	-8.311	6.627
	(12.24)	(0.21)	(0.43)	(0.53)	(1.00)
$R(t)$		0.293	0.441	0.352	0.346
		(4.10)	(7.78)	(5.75)	(7.45)
$f(t) - s(t)$	-2.706	-3.037	-2.474	-2.170	1.863
	(4.29)	(4.14)	(3.69)	(2.57)	(3.99)
$\Delta i^{us}(t)$			-1.275	-1.225	-1.719
			(0.45)	(0.33)	(0.74)
$\Delta c^{us}(t)_t$			-1.275		
			(0.31)		
$\Delta p^{us}(t)$			0.230		
			(0.47)		
$\Delta m^{us}(t)$					0.372
					(1.62)
$\Delta c^{uk}(t)$				0.306	
				(2.26)	
$\Delta p^{uk}(t)$				0.196	
				(0.54)	
$\Delta m^{uk}(t)$					-0.897
					(2.01)
$V_t[R]$		-0.010	-0.008	-0.000	-0.030
		(0.62)	(0.50)	(0.00)	(3.73)
$Cov_t[f - s, R]$		-1.780	-0.168	-0.040	0.239
		(2.31)	(0.44)	(0.10)	(1.44)
$Cov_t[\Delta i^{us}, R]$		0.918	0.140	-0.010	0.190
		(1.47)	(0.28)	(0.02)	(1.05)
$Cov_t[\Delta c^{us}, R]$			-0.097		
			(0.50)		
$Cov_t[\Delta p^{us}, R]$			-0.286		
			(0.50)		
$Cov_t[\Delta m^{us}, R]$					0.157
					(3.10)
$Cov_t[\Delta c^{uk}, R]$				-0.014	
				(0.27)	
$Cov_t[\Delta p^{uk}, R]$				0.021	
				(0.07)	
$Cov_t[\Delta m^{uk}, R]$					0.142
					(1.99)

The numbers in parentheses are t-statistics