Testing for a Shift in Trend when Serial Correlation is of Unknown Form

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Abstract

In this paper test statistics are proposed that can be used to test for shifts in the trend function of a univariate time series. The tests are valid in the presence of general forms of serial correlation in the errors and can be used without having to estimate the serial correlation parameters either parametrically or nonparametrically. The tests are valid for both $I(0)$ and $I(1)$ errors. The tests are designed to detect a single break at a known or unknown date. Asymptotic distributions are tabulated. A local asymptotic analysis is used to evaluate the size and power of the tests. Local asymptotic power indicates that the new tests have nontrivial asymptotic power. If the supremum statistic is used when the break date is unknown, one of the new tests has greater power than currently available statistics. Simulations are used to assess the finite sample size and power of the tests. A discussion is given on computing confidence intervals for trend function parameters when there is a trend shift at an unknown date. Such confidence intervals are computed for GNP growth rates of 16 countries using historical data.

JEL Classification: C12, C22

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1. INTRODUCTION

In this paper tests are proposed that can be used to test for a shift in the trend function of a univariate time series. The results in this paper make use of the general theorems on trend function hypothesis testing given by Vogelsang (1998a). The break date of the shift in trend can be modeled as known or unknown. The innovations of the time series may be serially correlated and contain up to one unit root. A priori knowledge as to whether the innovations are $I(0)$ or $I(1)$ is not required. In fact, the tests can be applied without any knowledge of the exact form of the serial correlation provided a functional central limit theorem can be applied to partial sums of the innovations. In addition the tests do not require estimation of the serial correlation parameters

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either parametrically or nonparametrically. Therefore, the sometimes subjective choices like lag length, information criteria, kernel or truncation lag can be completely avoided.

The results in the paper also build upon the literature on testing for structural change in time series models. The approaches of Andrews (1993) and Andrews and Ploberger (1994) are applied here when the break date is unknown. Related papers on the subject of testing for structural change in trend functions in the presence of serial correlation include Bai, Lumsdaine and Stock (1998), Chu (1989), Chu and White (1992), Kramer, Ploberger and Alt (1988), Perron (1991) and Vogelsang (1997) among others.

The importance of uncovering instability of parameters in time series is well known. Failure to account for structural change in parameters can lead to inconsistent parameter estimates and biased forecasts. From the work of Perron (1989) it is well known that failure to account for a shift in trend can bias unit root tests towards nonrejection. In some cases the existence of a shift in trend can be of interest itself. For example, the debate on whether GNP growth rates have declined since 1973 can be modeled as a shift in the slope of the trend function of the natural logarithm of GNP.

The rest of the paper is organized as follows. In Section 2 the models and statistics are presented. Asymptotic critical values are tabulated. In Section 3 the size and power of the tests are analyzed using a local asymptotic framework. It is found that the tests have nontrivial power. Interestingly, when the break date is unknown, one of the new supremum statistics for detecting a shift in slope has greater power than some popular supremum statistics. Section 4 contains results on the finite sample properties of the tests. Section 5 discusses construction of confidence intervals for estimates of the trend parameters. In Section 6 an empirical application is given where confidence intervals are constructed for growth rates of historic real GNP series. Finally, Section 7 gives some concluding comments.

2. THE MODELS, THE STATISTICS, AND ASYMPTOTIC CRITICAL VALUES

2.1 The Model and Assumptions

Consider the following models for the time series processes $\{y_t\}$ and $z_t = \sum_{j=1}^{t} y_j$, $t = 1, 2, ..., T$:

$$
\begin{aligned}
y_t &= \mu + \beta t + \delta DU_t + u_t, && \text{(1a)}\\
z_t &= \mu t + \beta[\tfrac{1}{2}(t^2 + t)] + \delta DT_t + S_t, && \text{(1b)}\\
y_t &= \mu + \beta t + \gamma DT_t + u_t, && \text{(2a)}\\
z_t &= \mu t + \beta[\tfrac{1}{2}(t^2 + t)] + \gamma[\tfrac{1}{2}(DT_t^2 + DT_t)] + S_t, && \text{(2b)}\\
y_t &= \mu + \beta t + \delta DU_t + \gamma DT_t + u_t, && \text{(3a)}\\
z_t &= \mu t + \beta[\tfrac{1}{2}(t^2 + t)] + \delta DT_t + \gamma[\tfrac{1}{2}(DT_t^2 + DT_t)] + S_t, && \text{(3b)}
\end{aligned}
$$

where $DU_t = 1$ if $t > T_b'$ and 0 otherwise, $DT_t = t - T_b'$ if $t > T_b'$ and 0 otherwise. T_b' denotes the date of a break should it occur and $S_t = \sum_{j=1}^{t} u_j$. The $\{z_t\}$ models are obtained by partial summing the $\{y_t\}$ models. Model (1) allows for a shift in the intercept of a trending time series. Model (2) allows for a shift in slope in a trending time series but requires the segments of the trend to be joined before and after the break. Model (2) is often used to test the hypothesis that GNP growth has slowed since 1973. Model (3) allows for both a shift in intercept and slope. For the asymptotics it is assumed that $\lambda' = T_b'/T$ remains fixed as the sample size increases.

The following assumptions on the error term $\{u_t\}$ are sufficient to obtain the main results of the paper,

$$\begin{aligned}(1 - L\alpha_T)u_t &= v_t, t = 2, 3, ..., T, && \text{(A1)}\\ u_1 &= \Sigma_{i=0}^{[\kappa T]}\alpha_T^i v_{1-i}, && \text{(A2)}\\ v_t &= d(L)e_t, \quad d(L) = \Sigma_{i=0}^{\infty} d_i L^i, \quad \Sigma_{i=0}^{\infty} i|d_i| < \infty \quad \text{and} \quad d(1)^2 > 0, && \text{(A3)}\end{aligned}$$

where $\{e_t\}$ is a martingale difference sequence with $E(e_t^2|\ e_{t-1}, e_{t-2}, ...) = 1$ and $\sup_t Ee_t^4 < \infty$, L is the lag operator, *i.e.* $Le_t = e_{t-1}$, and $[x]$ denotes the integer part of x. These assumptions are the same as those used by Canjels and Watson (1997). In particular, A3 implies that a functional central limit theorem applies to the partial sums of $\{e_t\}$ so that $T^{-1/2}\sum_{t=1}^{[rT]} e_t \Rightarrow w(r)$ where $w(r)$ is a standard Wiener process, and $\Rightarrow$ denotes weak convergence. It follows from A3 that $T^{-1/2}\sum_{t=1}^{[rT]} v_t \Rightarrow d(1)w(r)$. The restriction $d(1)^2 > 0$ is equivalent to bounding the spectral density of $\{v_t\}$ at frequency zero above zero. This is required to rule out nondegenerate cases. Other conditions on $\{u_t\}$, such as mixing conditions popularized by Phillips (1987a), could be considered without changing any of the results that follow provided that $\{S_t\}$ satisfies a functional central limit theorem. The assumptions include both $I(0)$ and $I(1)$ errors, and explicitly model the effect of the initial condition of $\{u_t\}$. When $\alpha_T = \alpha$ and $|\alpha| < 1$, then $\{u_t\}$ is $I(0)$. When $\alpha_T = (1 - \bar{\alpha}/T)$, $\{u_t\}$ is a nearly $I(1)$[1] process and a pure $I(1)$ process when $\bar{\alpha} = 0$. Throughout the paper $I(1)$ denotes errors with $\alpha_T = (1 - \bar{\alpha}/T)$. Under these assumptions a functional central limit theorem applies to $\{S_t\}$. When $\{u_t\}$ is $I(0)$, $T^{-1/2}S_{[rT]} \Rightarrow \sigma w(r)$ where $\sigma^2 = d(1)^2/(1-\alpha)^2$, and when $\{u_t\}$ is $I(1)$, $T^{-1/2}u_{[rT]} \Rightarrow d(1)V_{\bar{\alpha}}(r)$ where $V_{\bar{\alpha}}(r) = w_{\bar{\alpha}}(r) + \exp(-r\bar{\alpha})\widetilde{w}_{\bar{\alpha}}(\kappa)$, $w_{\bar{\alpha}}(r) = \int_0^r \exp(-\bar{\alpha}(r-s))dw(s)$, $\widetilde{w}_{\bar{\alpha}}(\kappa) = \int_0^\kappa \exp(-\bar{\alpha}(\kappa - s))d\widetilde{w}(s)$ and $\widetilde{w}(r)$ is a standard Wiener process independent with $w(r)$. Assumption A2 incorporates the effects of the initial condition into the asymptotic distribution theory. The κ parameter governs the variance of the initial condition. When $\kappa = 0$, then u_1 is an $O_p(1)$ random variable. When $\kappa > 0$, then u_1 is $O_p(1)$ when $\{u_t\}$ is $I(0)$, but is $O_p(T^{1/2})$ when $\{u_t\}$ is $I(1)$.

2.2 The Statistics

The hypotheses of interest are $\delta = 0$ in Model 1, $\gamma = 0$ in Model 2, and $\delta = 0$, $\gamma = 0$ or $\delta = \gamma = 0$ in Model 3. The statistics of interest are related to Wald statistics applied to the regressions given by (1) — (3). In order to define the statistics some notation is useful. Let the $(T \times k)$ matrices X_y and X_z generically denote matrices of regressors in the $\{y_t\}$ and $\{z_t\}$ regressions respectively (note that $k = 3$ in models 1 and 2 and $k = 4$ in model 3). Let ν generically denote the $(k \times 1)$ vectors of parameters from a regression (note that for a particular model ν is the same in the $\{y_t\}$ and $\{z_t\}$ regressions). Models (1) — (3) can be generically written as

[1] Local to unity asymptotics is becoming standard in time series analysis involving integrated data. Theoretical background can be found in Bobkoski (1983), Chan and Wei (1987) and Phillips (1987b). In this paper nearly I(2) asymptotics as developed by Nabeya and Perron (1994) is also used.

$$\begin{aligned} y_t &= X'_{yt}\nu + u_t, \\ z_t &= X'_{zt}\nu + S_t, \end{aligned}$$

where X_{yt} and X'_{zt} are the t^{th} rows of X_y and X_z respectively. Let $\widehat{\nu}$ and $\widetilde{\nu}$ denote the OLS estimates from the $\{y_t\}$ and $\{z_t\}$ regressions respectively. Let RSS_y and RSS_z denote the OLS residual sum of squares from the $\{y_t\}$ and $\{z_t\}$ regressions. Define $s_y^2 = T^{-1}RSS_y$ and $s_z^2 = T^{-1}RSS_z$. Let R be a $(q \times k)$ matrix of constants and r a $(q \times 1)$ vector of constants that correspond to a hypothesis of interest written as $H_0 : R\nu = r$.

Using this notation, the statistics are defined as,

$$\begin{aligned} T^{-1}W_T &= T^{-1}(R\widehat{\nu} - r)'[R(X'_yX_y)^{-1}R']^{-1}(R\widehat{\nu} - r)/s_y^2, \\ PS_T &= T^{-1}(R\widetilde{\nu} - r)'[R(X'_zX_z)^{-1}R']^{-1}(R\widetilde{\nu} - r)/(s_z^2 \exp(bJ_T(m))), \\ PSW_T &= (R\widehat{\nu} - r)'[R(X'_yX_y)^{-1}R']^{-1}(R\widehat{\nu} - r)/[100T^{-1}s_z^2 \exp(bJ_T(m))], \end{aligned}$$

where b is a constant. $J_T(m)$ is define as follows. Let RSS_J denote the residual sum of squares from the regression $y_t = X'_{yt}\nu + \sum_{i=2}^{m} c_i t^i + u_t$. Then $J_T(m) = (RSS_y - RSS_J)/RSS_J$. Note that T times $J_T(m)$ is the Wald statistic for testing the joint hypothesis that $c_2 = c_3 = ... = c_m = 0$. The $J_T(m)$ statistic is in the class of the unit root tests proposed by Park and Choi (1988) and Park (1990). The 100 is included in the denominator of PSW_T to normalize the critical values and is done for computation reasons. Obviously, this will not affect the size or power of PSW_T. If the 100 is not used, the critical values of PSW_T can take on large values. When the break date is unknown, one of the statistics that is used is an average over all possible break dates of the exponential of PSW_T. When PSW_T takes on large values, $\exp(PSW_T)$ takes on very large values that can cause overflows on some computers. The 100 in the denominator is a solution to this problem that has no effect on the performance of the tests in practice.

The $T^{-1}W_T$ statistic is designed to have power when the errors are $I(1)$ but remain a conservative test when the errors are $I(0)$. The PS_T and PSW_T statistics are designed to have power when the errors are $I(0)$. But, when the errors are $I(1)$, the critical values of the PS_T and PSW_T are much larger than when the errors are $I(0)$. Therefore, if the $I(0)$ critical values are used in practice, and the errors have persistent correlation, the tests will have inflated size because the $I(0)$ asymptotic approximation is inadequate. To control size when the errors are $I(1)$ (or highly persistent), for a desired percentage point b is chosen so that the asymptotic critical values are the same in the $I(0)$ and $I(1)$ cases. This makes the statistics robust to $I(1)$ errors. Therefore, the $J_T(m)$ statistic is used to smooth the distributions of PS_T and PSW_T as the errors go from $I(0)$ to $I(1)$.

When $q = 1$, so that only one restriction is being tested, t-statistic versions of the tests can be defined as

$$\begin{aligned} T^{-1/2}t - W_T &= T^{-1/2}(R\widehat{\nu} - r)[R(X'_yX_y)^{-1}R's_y^2]^{-1/2}, \\ t - PS_T &= T^{-1/2}(R\widetilde{\nu} - r)[R(X'_zX_z)^{-1}R's_z^2]^{-1/2}\exp(-bJ_T(m)), \\ t - PSW_T &= (R\widehat{\nu} - r)[R(X'_yX_y)^{-1}R'100T^{-1}s_z^2]^{-1/2}\exp(-bJ_T(m)). \end{aligned}$$

These statistics are useful for constructing confidence intervals for point estimates. See Section 5.

As a practical consideration, it may be useful for some readers to express the statistics in terms of more familiar quantities. Let W_y and W_z denote standard Wald statistics for testing a hypothesis of interest from the $\{y_t\}$ and $\{z_t\}$ regressions respectively. Likewise let t_y and t_z denote standard OLS t-statistics from the $\{y_t\}$ and $\{z_t\}$ regressions respectively for testing a hypothesis of interest. The statistics can be computed as:

$$\begin{aligned}
T^{-1}W_T &= T^{-1}W_y, \\
T^{-1/2}t-W_T &= T^{-1/2}t_y, \\
PS_T &= T^{-1}W_z \exp(-bJ_T(m)), \\
t-PS_T &= T^{-1/2}t_z \exp(-bJ_T(m)), \\
PSW_T &= W_y[s_y^2/(100T^{-1}s_z^2)] \exp(-bJ_T(m)), \\
t-PSW_T &= t_y[s_y^2/(100T^{-1}s_z^2)]^{1/2} \exp(-bJ_T(m)).
\end{aligned}$$

When the break date is known, the above statistics can be used directly. However, in some cases it is more appropriate to model the break date as unknown. Two recent papers have provided frameworks that can be used to test for structural change at an unknown break date. Andrews (1993) proposed supremum type statistics, and Andrews and Ploberger (1994) proposed optimal exponential average-type statistics. These approaches involve computing statistics for a range of possible break dates and then taking the supremum or some form of average.

Let T_b denote the break date used in estimating models (1) — (3). Note that T_b may be different from T_b'. Let $h_T(T_b)$ generically denote $T^{-1}W_T$, PS_T or PSW_T where PS_T and PSW_T are computed using $b = 0$. Suppose that possible break dates are restricted to the set $\Lambda = (T_b^*, T_b^* + 1, ..., T - T_b^*)$ where $\lambda^* = T_b^*/T$ remains fixed as $T \to \infty$. The parameter λ^* is the amount of trimming. Following Andrews (1993) and Andrews and Ploberger (1994) define,

$$\begin{aligned}
Meanh_T &= [T^{-1} \sum_{T_b \in \Lambda} h_T(T_b)] \exp(-bJ_T(m)), \\
Exph_T &= \log[T^{-1} \sum_{T_b \in \Lambda} \exp(\tfrac{1}{2} h_T(T_b))] \exp(-bJ_T(m)), \\
Suph_T &= [Sup_{T_b \in \Lambda} h_T(T_b)] \exp(-bJ_T(m)),
\end{aligned}$$

where $log(\cdot)$ is the natural logarithm and $J_T(m) = \inf_{T_b \in \Lambda} J_T(T_b, m)$ where $J_T(T_b, m)$ denotes the $J_T(m)$ statistic defined in a regression with the break date T_b. Note that when $h_T(T_b)$ represents the $T^{-1}W_T$ statistic, it is always the case that $b = 0$. The $Meanh_T$ and $Exph_T$ statistics were proposed by Andrews and Ploberger (1994) and are in a class of optimal statistics under certain regularity conditions including stationary errors and no trending regressors. Therefore, none of the statistics considered here are in that optimal class. The $Suph_T$ statistic is taken from Andrews (1993). The $Suph_T$ statistic is not in the optimal class but can be viewed as an extreme version of a statistic in the optimal class. In practice the $Suph_T$ statistic is useful since it provides an estimate of the break date.

2.3 Asymptotic Distributions and Critical Values

The following notation is used in the representations of the asymptotic distributions of the statistics. In model (1) let $F(r,\lambda) = [1, r, du(\lambda)]'$, $G(r,\lambda) = [r, \frac{1}{2}r^2, dt(\lambda)]$ and $R^* = [0,0,1]$ where $du(\lambda) = 1$ if $r > \lambda$ and 0 otherwise, $dt(\lambda) = r - \lambda$ if $r > \lambda$ and 0 otherwise and $\lambda = T_b/T$. In model (2) let $F(r) = [1, r, dt(\lambda)]'$, $G(r) = [r, \frac{1}{2}r^2, \frac{1}{2}dt(\lambda)^2]$ and $R^* = [0,0,1]$. In model (3) let $F(r,\lambda) = [1,\ r, du(\lambda), dt(\lambda)]'$, $G(r,\lambda) = [r,\ \frac{1}{2}r^2, dt(\lambda)\frac{1}{2}dt(\lambda)^2]$ and $R^* = [0,0,1,0]$ for testing $\delta = 0$, $R^* = [0,0,0,1]$ for testing $\gamma = 0$ and $R^* = \left[\begin{smallmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{smallmatrix}\right]$ for testing $\delta = \gamma = 0$. Let $\widetilde{w}(r)$ denote the residuals from the projection of $w(r)$ onto the space spanned by $G(r,\lambda)$ on [0,1]. Let $\widehat{V}_{\bar{\alpha}}(r)$ denote the residuals from the projection of $V_{\bar{\alpha}}(r)$ onto the space spanned by $F(r,\lambda)$ on [0,1]. Let $V_{\bar{\alpha}}^*(r)$ denote the residuals from the projection of $V_{\bar{\alpha}}(r)$ onto the space spanned by $(F(r,\lambda)', r^2, r^3, ..., r^m)$ on [0,1]. Define $Q(r) = \int_0^r V_{\bar{\alpha}}(s)ds$, and let $\widetilde{Q}(r)$ denote the residuals from the projection $Q(r)$ onto the space spanned by $G(r,\lambda)$ on [0,1]. Finally, define $J(\lambda, m) = \int_0^1 \widehat{V}_{\bar{\alpha}}(r)^2 dr / \int_0^1 V_{\bar{\alpha}}^*(r)^2 dr - 1$ where the dependence on λ is through $G(r,\lambda)$. The asymptotic null distributions of the statistics in the case where the break date is known are given by the following theorems. Results for t-statistic versions of the tests follow analogously. Proofs of the theorems follow directly from Theorems 1 and 2 of Vogelsang (1998a).

Theorem 1 *Suppose that $\alpha_T = \alpha$, $|\alpha| < 1$, and T_b' is known. Then, under the respective null hypotheses, as $T \to \infty$,*

$$
\begin{aligned}
T^{-1}W_T &\Rightarrow 0, \\
PS_T &\Rightarrow [R^*(\int_0^1 G(r,\lambda')G(r,\lambda')'dr)^{-1}\int_0^1 G(r,\lambda')w(r)dr]'[R^*(\int_0^1 G(r,\lambda')G(r,\lambda')'dr)^{-1}R^{*\prime}]^{-1} \\
&\quad \times[R^*(\int_0^1 G(r,\lambda')G(r,\lambda')'dr)^{-1}\int_0^1 G(r,\lambda')w(r)dr]/[\int_0^1 \widetilde{w}(r)^2 dr], \\
PSW_T &\Rightarrow [R^*(\int_0^1 F(r,\lambda')F(r,\lambda')'dr)^{-1}\int_0^1 F(r,\lambda')dw(r)]'[R^*(\int_0^1 F(r,\lambda')F(r,\lambda')'dr)^{-1}R^{*\prime}]^{-1} \\
&\quad \times[R^*(\int_0^1 F(r,\lambda')F(r,\lambda')'dr)^{-1}\int_0^1 F(r,\lambda')dw(r)]/[100\int_0^1 \widetilde{w}(r)^2 dr].
\end{aligned}
$$

Theorem 2 *Suppose that $\alpha_T = (1 - \bar{\alpha}/T)$ and T_b' is known. Then, under the respective null hypotheses, as $T \to \infty$*

$$
\begin{aligned}
T^{-1}W_T &\Rightarrow [R^*(\int_0^1 F(r,\lambda')F(r,\lambda')'dr)^{-1}\int_0^1 F(r,\lambda')V_{\bar{\alpha}}(r)dr]'[R^*(\int_0^1 F(r,\lambda')F(r,\lambda')'dr)^{-1}R^{*\prime}]^{-1} \\
&\quad \times[R^*(\int_0^1 F(r,\lambda')F(r,\lambda')'dr)^{-1}\int_0^1 F(r,\lambda')V_{\bar{\alpha}}(r)dr]/[\int_0^1 \widehat{V}_{\bar{\alpha}}(r)^2 dr], \\
PS_T &\Rightarrow [R^*(\int_0^1 G(r,\lambda')G(r,\lambda')'dr)^{-1}\int_0^1 G(r,\lambda')Q(r)dr]'[R^*(\int_0^1 G(r,\lambda')G(r,\lambda')'dr)^{-1}R^{*\prime}]^{-1} \\
&\quad \times[R^*(\int_0^1 G(r,\lambda')G(r,\lambda')'dr)^{-1}\int_0^1 G(r,\lambda')Q(r)dr]\exp(-bJ(\lambda',m))/\int_0^1 \widetilde{Q}(r)^2 dr], \\
PSW_T &\Rightarrow [R^*(\int_0^1 F(r,\lambda')F(r,\lambda')'dr)^{-1}\int_0^1 F(r,\lambda')V_{\bar{\alpha}}(r)dr]'[R^*(\int_0^1 F(r,\lambda')F(r,\lambda')'dr)^{-1}R^{*\prime}]^{-1} \\
&\quad \times[R^*(\int_0^1 F(r,\lambda')F(r,\lambda')'dr)^{-1}\int_0^1 F(r,\lambda')dw(r)]\exp(-bJ(\lambda',m))/[100\int_0^1 \widetilde{Q}(r)^2 dr].
\end{aligned}
$$

Asymptotic null distributions in the case where the break date is unknown are as follows. Let $h(\lambda)$ generically denote the limiting distributions of $h_T(T_b)$ given by Theorems 1 and 2 where $T_b = [\lambda T]$. Let $J^*(m) = \inf_{\lambda\in[\lambda^*,1-\lambda]} J_T(\lambda, m)$ denote the limiting distribution of $J_T(m)$. Then, using the continuous mapping theorem,

$$\begin{aligned}
Meanh_T &\Rightarrow [\int_{\lambda^*}^{1-\lambda^*} h(\lambda)d\lambda]\exp(-bJ^*(m)),\\
Exph_T &\Rightarrow \log[\int_{\lambda^*}^{1-\lambda^*} \exp(\tfrac{1}{2}h(\lambda))d\lambda]\exp(-bJ^*(m)),\\
Suph_T &\Rightarrow [Sup_{\lambda\in[\lambda^*,1-\lambda^*]}h(\lambda)]\exp(-bJ^*(m)),
\end{aligned}$$

where for the $T^{-1}W_T$ statistics $b = 0$ always.

The asymptotic distributions are nonstandard and critical values have to be simulated (see below). When the errors are $I(0)$, $T^{-1}W_T$ converges to zero while when the errors are $I(1)$, $T^{-1}W_T$ has a nondegenerate limiting distribution. Thus, $T^{-1}W_T$ is designed to have power when errors are $I(1)$, but the statistic remains robust (conservative) when errors are $I(0)$. In contrast the PS_T and PSW_T statistics have nondegenerate limiting distributions for both $I(0)$ and $I(1)$ errors. But notice that the distributions only depend on b and the $exp(\text{-}bJ)$ scaling factor when errors are $I(1)$. This is true because $J_T(m) \to 0$ and $\exp(-bJ_T(m)) \to 1$ when errors are $I(0)$. Therefore, b can be chosen so that the critical values are the same in the $I(0)$ and $I(1)$ cases for a given percentage point.

When the errors are $I(1)$, the limiting distributions depend on $\bar{\alpha}$ and κ. Unreported simulations indicate that the critical values are not sensitive to κ in any practical way so $\kappa = 0$ critical values can be used. The critical values also depend on $\bar{\alpha}$. In the next section asymptotic size with respect to $\bar{\alpha}$ is examined. It is shown that using $\bar{\alpha} = 0$ critical values provides tests that are approximately conservative for $\bar{\alpha} \geq 0$. Finally, note that the $I(1)$ limiting distributions of PS_T and PSW_T depend on m through the $J_T(m)$ statistic. Using simulations, Vogelsang (1998a) found that $m = 9$ results in good power and stable size in a model with a simple linear trend. Using similar simulations it was found $m = 9$ is a reasonable choice in models (1) — (3). Details are available upon request.

Using simulation methods critical values were computed. For the case of a known break date, critical values are tabulated in Tables 1, 2 and 3 (Models 1, 2, and 3 respectively) for $\lambda' = 0.1$, 0.2, ..., 0.9. For hypotheses that involve one parameter only critical values for the t-statistic version of the tests are also reported. Critical values for the $T^{-1/2}t - W_T$ and $T^{-1}W_T$ statistics are reported for the case when the errors are $I(1)$ with $\bar{\alpha} = 0$ and $\kappa = 0$. The critical values for the $t - PS_T$, PS_T, $t - PSW_T$ and PSW_T statistics are valid for $I(0)$ and $I(1)$ errors ($\bar{\alpha} = 0$, $\kappa = 0$) provided $m = 9$ is used when computing $J_T(m)$ and the $b's$ given in parentheses below the critical values are used. The $t - PS_T$, PS_T, $t - PSW_T$ and PSW_T statistics are used in practice in the following way. A significance level for the test is chosen which determines the percentage point and critical value. The statistics should then be computed using $\exp(-bJ_T(9))$ with the b given in parentheses below the critical value.

Critical values for the mean, exponential-mean and supremum statistics were computed using simulations and are tabulated in Table 4. As before, the values of b for each percentage point needed to compute the PS_T and PSW_T statistics are given in parentheses under the relevant critical value.

3. ASYMPTOTIC SIZE AND POWER

To assess the stability of the size of the statistics as the errors go from $I(1)$ to $I(0)$, local asymptotic size was simulated using $I(1)$ asymptotic distributions for $\bar{\alpha} = 0, 2, 4, ..., 20$. In all cases the $\bar{\alpha} = 0$, $\kappa = 0$ and $m = 9$ values of b, and the $I(0)$ critical values were used. The nominal size was 5%. Results are reported for a known break date with $\lambda' = 0.5$ in Table 5. Results for other break dates and unknown break date are qualitatively similar. With the exception of Model (1), the asymptotic size of the PS_T and PSW_T statistics is quite stable and close to or below 0.05. In Model (1) asymptotic size of the PS_T statistic rises to 0.079 with $\bar{\alpha} = 16$. This size distortion could be reduced by choosing b so that the $I(0)$ and ($I(1)$, $\bar{\alpha} = 16$) critical values are the same. The $T^{-1}W_T$ statistic has size 0.05 by construction for $\bar{\alpha} = 0$ and is greatly undersized for $\bar{\alpha} > 0$ resulting in a conservative test.

In order to compare the power of the statistics, the following local alternatives were used. In the case of $I(0)$ errors the local alternatives are given by $H_1 : \delta = T^{-1/2}c$ in Model (1) and $H_1 : \gamma = T^{-3/2}c$ in Model (2). In Model (3) the local alternatives are given by $H_1 : \delta = T^{-1/2}c_1$ and $H_1 : \gamma = T^{-3/2}c_2$. In Model (3) c_1 is the local intercept shift parameter and c_2 is the local slope shift parameter. In the case of $I(1)$ errors nondegenerate local asymptotic distributions can be obtained for the statistics that detect slope shifts. In these cases the local alternatives are $\gamma = T^{-1/2}c$ in Model (2) and $\gamma = T^{-1/2}c_2$ in Model (3).

When the break date is known, limiting distributions under these local alternatives follow directly from Theorems 3 and 4 of in the working paper Vogelsang (1996). Using these results, asymptotic power was simulated using a nominal level of 5%. The true break date was $\lambda' = 0.5$ in all cases. As a benchmark for when the errors are $I(0)$, asymptotic power of the W_T statistic (the unnormalized Wald statistic based on the $\{y_t\}$ regressions) was also computed. When the errors are $I(0)$, W_T can have optimality properties. When the errors are $I(1)$, W_T diverges to ∞. Thus, W_T cannot be used in practice if it is not known whether the errors are $I(0)$ or $I(1)$. But, the power of W_T does provide a useful benchmark.

The results for $I(0)$ errors are given in Figures 1 — 6. Generally speaking, the W_T statistic has the greatest power for all models. For statistics that detect intercept shifts (Figures 1, 3, and 5), the PSW_T statistic is more powerful than the PS_T statistic with the differences sometimes nontrivial, although the PSW_T statistic has power comparable to that of W_T. For the statistics that detect slope changes (Figures 2, 4, and 6) the difference in power among the three statistics is much smaller with the power of the PS_T and PSW_T statistics very similar and only slightly below the power of W_T.

In the case of $I(1)$ errors asymptotic power was simulated for the statistics that detect slope shifts using $\bar{\alpha} = 0, 5, 10$. Results are only reported for Model (2) in Figures 7a, 7b and 7c. Results for Model 3 are very similar and are not reported. When $\bar{\alpha} = 0$, $T^{-1}W_T$ is the most powerful statistic with PS_T and PSW_T having low power. But, recall that $T^{-1}W_T$ is designed to have power when errors are $I(1)$, whereas PS_T and PSW_T are designed to have power when errors are $I(0)$. As $\bar{\alpha}$ increases, PS_T and PSW_T are more powerful than $T^{-1}W_T$ in detecting small slope shifts. Therefore, the statistics have complementary power.

When the break date is unknown, local asymptotic distributions no longer follow directly from the general theorems in Vogelsang (1996). The reason is that the models are now estimated using break dates different from the true break date. Using an incorrect break date in a regression results in a misspecified model, and different distribution results are needed. Local asymptotic distributions were obtained for Models (2) and (3) but are only reported here for Model (3) to save space as the limiting representations are complex. Define the (2×1) vector $C = [c_1, c_2]'$.

Theorem 3 *Suppose that $\alpha_T = \alpha, |\alpha| < 1$ so $\{u_t\}$ is I(0) and $\delta = T^{-1/2}c_1$, $\gamma = T^{-3/2}c_2$ in Model (3). If T_b is the break date used in estimating the respective regressions then the following holds as $T \to \infty$. If $T_b \leq T_b'$,*

$$\begin{aligned} PS_T &\Rightarrow \Omega_1 \left(\Omega_1 - [C'\Phi(\lambda',\lambda) + \eta(\lambda)']\Phi(\lambda,\lambda)^{-1}[\Phi(\lambda',\lambda)'C + \eta(\lambda)]\right)^{-1} - 1, \\ PSW_T &\Rightarrow [C'\Pi(\lambda',\lambda) + \zeta(\lambda)']\Pi(\lambda,\lambda)^{-1}[\Pi(\lambda',\lambda)'C + \zeta(\lambda)] \\ &\quad \times\{\Omega_1 - [C'\Phi(\lambda',\lambda) + \eta(\lambda)']\Phi(\lambda,\lambda)^{-1}[\Phi(\lambda',\lambda)'C + \eta(\lambda)]\}^{-1}. \end{aligned}$$

if $T_b \geq T_b'$,

$$\begin{aligned} PS_T &\Rightarrow \Omega_1 \left(\Omega_1 - [C'\Phi(\lambda,\lambda')' + \eta(\lambda)']\Phi(\lambda,\lambda)^{-1}[\Phi(\lambda,\lambda')C + \eta(\lambda)]\right)^{-1} - 1, \\ PSW_T &\Rightarrow [C'\Pi(\lambda,\lambda')' + \zeta(\lambda)']\Pi(\lambda,\lambda)^{-1}[\Pi(\lambda,\lambda')C + \zeta(\lambda)] \\ &\quad \times\{\Omega_1 - [C'\Phi(\lambda,\lambda')' + \eta(\lambda)']\Phi(\lambda,\lambda)^{-1}[\Phi(\lambda,\lambda')C + \eta(\lambda)]\}^{-1}, \end{aligned}$$

where

$$\begin{aligned} \zeta(\lambda) &= \begin{bmatrix} \int_\lambda^1 dw(r) \\ \int_\lambda^1 (r-\lambda)dw(r) \end{bmatrix} - \begin{bmatrix} 1-\lambda & \int_\lambda^1 rdr \\ \int_\lambda^1 (r-\lambda)dr & \int_\lambda^1 r(r-\lambda)dr \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix}^{-1} \begin{bmatrix} w(1) \\ \int_0^1 rdw(r) \end{bmatrix}, \\ \eta(\lambda) &= \begin{bmatrix} \int_\lambda^1 (r-\lambda)w(r)dr \\ \int_\lambda^1 (r-\lambda)^2 w(r)dr \end{bmatrix} - \begin{bmatrix} \int_\lambda^1 r(r-\lambda)dr & \int_\lambda^1 r(r-\lambda)^2 dr \\ \int_\lambda^1 r^2(r-\lambda)dr & \int_\lambda^1 r^2(r-\lambda)^2 dr \end{bmatrix} \begin{bmatrix} \frac{1}{3} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{5} \end{bmatrix}^{-1} \begin{bmatrix} \int_0^1 rw(r)dr \\ \int_0^1 r^2 w(r)dr \end{bmatrix}, \\ \Pi(\gamma,\rho)' &= \begin{bmatrix} (1-\rho) & \int_\rho^1 (r-\rho)dr \\ \int_\lambda^1 (r-\lambda)dr & \int_\lambda^1 (r-\rho)(r-\gamma)dr \end{bmatrix} \\ &\quad - \begin{bmatrix} (1-\gamma) & \int_\gamma^1 rdr \\ \int_\gamma^1 (r-\gamma)dr & \int_\gamma^1 r(r-\gamma)dr \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix}^{-1} \begin{bmatrix} (1-\rho) & \int_\lambda^1 (r-\rho)dr \\ \int_\rho^1 rdr & \int_\rho^1 r(r-\rho)dr \end{bmatrix}, \\ \Phi(\gamma,\rho)' &= \begin{bmatrix} \int_\rho^1 (r-\rho)(r-\lambda)dr & \int_\rho^1 (r-\rho)^2(r-\lambda)dr \\ \int_\rho^1 (r-\rho)(r-\lambda)^2 dr & \int_\rho^1 (r-\rho)^2(r-\lambda)^2 dr \end{bmatrix} \\ &\quad - \begin{bmatrix} \int_\gamma^1 r(r-\lambda)dr & \int_\gamma^1 r^2(r-\lambda)dr \\ \int_\gamma^1 r(r-\lambda)^2 dr & \int_\gamma^1 r^2(r-\lambda)^2 dr \end{bmatrix} \begin{bmatrix} \frac{1}{3} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{5} \end{bmatrix}^{-1} \begin{bmatrix} \int_\rho^1 r(r-\rho)dr & \int_\rho^1 r^2(r-\rho)dr \\ \int_\rho^1 r(r-\rho)^2 dr & \int_\rho^1 r^2(r-\rho)^2 dr \end{bmatrix}, \\ \Omega_1 &= \int_0^1 w(r)^2 dr - 48[\int_0^1 rw(r)dr]^2 + 120\int_0^1 rw(r)dr \int_0^1 r^2 w(r)dr - 80[\int_0^1 r^2 w(r)dr]^2. \end{aligned}$$

Theorem 4 *Suppose that $\alpha_T = (1 - \bar{\alpha}/T)$ so $\{u_t\}$ is I(1) and $\delta = T^{1/2}c_1$, $\gamma = T^{-1/2}c_2$ in Model (3). Set $b = 0$. If T_b is the break date used in estimating the respective regressions then the following holds as $T \to \infty$, if $T_b \leq T_b'$,*

$$\begin{aligned} T^{-1}W_T &\Rightarrow \Omega_2 \left(\Omega_2 - [C'\Pi(\lambda',\lambda) + \psi(\lambda)']\Pi(\lambda,\lambda)^{-1}[\Pi(\lambda',\lambda)'C + \psi(\lambda)]\right)^{-1} - 1, \\ PS_T &\Rightarrow \Omega_3 \left(\Omega_3 - [C'\Phi(\lambda',\lambda) + \varphi(\lambda)']\Phi(\lambda,\lambda)^{-1}[\Phi(\lambda',\lambda)'C + \varphi(\lambda)]\right)^{-1} - 1, \\ PSW_T &\Rightarrow [C'\Pi(\lambda',\lambda) + \psi(\lambda)']\Pi(\lambda,\lambda)^{-1}[\Pi(\lambda',\lambda)'C + \psi(\lambda)] \\ &\quad \times\{\Omega_3 - [C'\Phi(\lambda',\lambda) + \varphi(\lambda)']\Phi(\lambda,\lambda)^{-1}[\Phi(\lambda',\lambda)'C + \varphi(\lambda)]\}^{-1}. \end{aligned}$$

if $T_b \geq T_b'$,

$$\begin{aligned}
T^{-1}W_T &\Rightarrow \Omega_2\left(\Omega_2 - [C'\Pi(\lambda,\lambda')' + \psi(\lambda)']\Pi(\lambda,\lambda)^{-1}[\Pi(\lambda,\lambda')C + \psi(\lambda)]\right)^{-1} - 1,\\
PS_T &\Rightarrow \Omega_3\left(\Omega_3 - [C'\Phi(\lambda,\lambda')' + \varphi(\lambda)']\Phi(\lambda,\lambda)^{-1}[\Phi(\lambda,\lambda')C + \varphi(\lambda)]\right)^{-1} - 1,\\
PSW_T &\Rightarrow [C'\Pi(\lambda,\lambda')' + \psi(\lambda)']\Pi(\lambda,\lambda)^{-1}[\Pi(\lambda,\lambda')C + \psi(\lambda)]\\
&\quad \times\{\Omega_3 - [C'\Phi(\lambda,\lambda')' + \varphi(\lambda)']\Phi(\lambda,\lambda)^{-1}[\Phi(\lambda,\lambda')C + \varphi(\lambda)]\}^{-1},
\end{aligned}$$

where

$$\begin{aligned}
\psi(\lambda) &= \begin{bmatrix} \int_\lambda^1 V_{\bar{\alpha}}(r)dr \\ \int_\lambda^1 (r-\lambda)V_{\bar{\alpha}}(r)dr \end{bmatrix} - \begin{bmatrix} 1-\lambda & \int_\lambda^1 rdr \\ \int_\lambda^1 (r-\lambda)dr & \int_\lambda^1 r(r-\lambda)dr \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix}^{-1} \begin{bmatrix} \int_0^1 V_{\bar{\alpha}}(r)dr \\ \int_0^1 rdw(r) \end{bmatrix},\\
\varphi(\lambda) &= \begin{bmatrix} \int_\lambda^1 (r-\lambda)Q(r)dr \\ \int_\lambda^1 (r-\lambda)^2 Q(r)dr \end{bmatrix} - \begin{bmatrix} \int_\lambda^1 r(r-\lambda)dr & \int_\lambda^1 r(r-\lambda)^2 dr \\ \int_\lambda^1 r^2(r-\lambda)dr & \int_\lambda^1 r^2(r-\lambda)^2 dr \end{bmatrix} \begin{bmatrix} \frac{1}{3} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{5} \end{bmatrix}^{-1} \begin{bmatrix} \int_0^1 rQ(r)dr \\ \int_0^1 r^2 Q(r)dr \end{bmatrix},,\\
\Omega_2 &= \int_0^1 V_{\bar{\alpha}}(r)^2 dr - 4[\int_0^1 V_{\bar{\alpha}}(r)dr]^2 + 6\int_0^1 V_{\bar{\alpha}}(r)dr \int_0^1 rV_{\bar{\alpha}}(r)dr - 12[\int_0^1 rV_{\bar{\alpha}}(r)dr]^2,\\
\Omega_3 &= \int_0^1 Q(r)^2 dr - 48[\int_0^1 rQ(r)dr]^2 + 120\int_0^1 rQ(r)dr \int_0^1 r^2 Q(r)dr - 80[\int_0^1 r^2 Q(r)dr]^2.
\end{aligned}$$

The proofs are tedious but straightforward extensions of the proofs of Theorems 3 and 4 of Vogelsang (1996) and are omitted. The limiting distributions of the *Mean*, *Exp* and *Sup* statistics follow directly from the continuous mapping theorem and $J_T^*(m) \Rightarrow J^*(m)$. Using Theorem 3 asymptotic power curves were simulated for Model (3) for $I(0)$ errors with $\lambda' = 0.5$. Again, power of statistics based on W_T is also reported as a benchmark. Figures 8a-8c depict local asymptotic power where there is a shift in intercept only ($c_1 \neq 0$, $c_2 = 0$). From these figures it is apparent that the W_T statistics result in more powerful tests than the PS_T and PSW_T statistics. When the *Mean* functional is used, power using PS_T and PSW_T is quite low but is much higher when using the *Exp* and *Sup* functionals. Similar results were obtained for Model (1) and are available upon request.

When there is a shift in slope, however, the power results are strikingly different. Figures 9a-9c show local asymptotic power in the presence of a shift in slope only ($c_1 = 0$, $c_2 \neq 0$). First consider the *Mean* statistics given in Figure 9a. As can be seen in the figure, the power of all three statistics is very close with the W_T statistic having the highest power. For the *Exp* statistics in figure 9b, power differences are even smaller among the three statistics with power essentially the same. Finally, as figure 9c depicts, when the *Sup* statistics are used, the PS_T statistic has the highest power followed by W_T and PSW_T. A similar result was obtained by Vogelsang (1998b) in models without trending data. So, if the *Sup* statistic is used, the PS_T statistic provides a more powerful test than the W_T statistic. It should be noted, though, that overall the *Mean* and *Exp* tests are slightly more powerful than the *Sup* tests. Similar results were obtained for Model (2).

Local asymptotic power was also simulated in the case of I(1) errors for the tests that detect a change in slope, and the results were qualitatively similar to the results obtained when the break is known. The figures are not included to save space and are available upon request.

4. FINITE SAMPLE PROPERTIES

To assess the finite sample properties of the statistics size and power of the statistics in Model (2) was simulated using the following DGP:

$$\begin{aligned} y_t &= \gamma DT_t + u_t, \\ u_t &= \alpha u_{t-1} + v_t + \theta v_{t-1}, \end{aligned}$$

where v_t is iid $N(0,1)$ with $v_0 = 0$. The parameters μ and β were set to zero as the statistics are exactly invariant to their values. The errors are modeled as ARMA(1,1) processes, and when $\alpha = 1$, the errors are $I(1)$ while for $\alpha < 1$ the errors are $I(0)$. Sample sizes of $T = 100$ and 250 were used for the size simulations and $T = 100$ for the power simulations. 2,000 replications were used in all cases. Results are reported for the case of an unknown break date with $\lambda' = 0.5$ and 1% trimming ($\lambda^* = 0.01$). In all cases 5% asymptotic critical values are used in computing rejection probabilities and power is not size adjusted. Rejection probabilities under the null ($\gamma = 0$) are reported for $\alpha = 0.8, 0.9, 0.95, 1.0$ and $\theta = -1.0, -0.8, -0.4, 0.0, 0.4, 0.8$. Rejection probabilities under the alternative are reported for the same values of $\alpha, \theta = -0.4, 0.0, 0.4$ and $\gamma = 0.0, 0.1, 0.3, 0.5$. Results are reported for the $Mean/Sup\ PS_T$ and $T^{-1}W_T$ statistics which are denoted by SPS, MPS, STW and MTW. Results for the PSW_T statistics are similar to the PS_T statistics and are not reported.

For comparison, rejection probabilities are also reported for the QD_T statistic proposed by Perron (1991) and the supremum and mean Wald statistics (SW_T, MW_T) analyzed by Vogelsang (1997). These statistics are useful benchmarks since they are also valid for both $I(0)$ and $I(1)$ errors as long as their $I(1)$ asymptotic critical values are used. In the results that follow, 5% asymptotic $I(1)$ critical values were always used for these statistics. These statistics are based on dynamic regression models where lags of $\{y_t\}$ are added to the $\{y_t\}$ regression. The lag lengths were chosen using a data dependent method suggested by Ng and Perron (1995) where significance of the coefficient on the last included lag is used. The maximal lag length for the data dependent procedure was 5 in all cases. The statistics from Vogelsang (1997) are based on the approach of Andrews (1993) and Andrews and Ploberger (1994) and require trimming which was 1% for the simulations. The precise definitions of these statistics are not included here to keep the discussion brief, and the interested reader should consult the references for details.

Rejection probabilities under the null that $\gamma = 0$ are given in Table 6. For the STW and MTW statistics rejection probabilities are close to or below the nominal level of 0.05. Therefore, these statistics have approximately the correct size. When $\alpha < 1$, rejection probabilities are close to zero which illustrates the conservative nature of the tests. This low size results in comparatively lower power when $\alpha < 1$ as seen below. For the other statistics, when $\theta \geq 0$, rejection probabilities are close to or below the nominal level 0.05. When $\theta < 0$ and in particular, when $\theta = -0.8$ and $\alpha = 1.0$, rejection probabilities are often inflated above 0.05. These size distortions do not disappear as T increases from 100 to 250. In unreported simulations it was found that the size distortions do disappear slowly as T increases further. But, as α becomes smaller (moves away from 1), these size distortions disappear. So, if it is suspected that the errors are nearly integrated with a root near one in the moving average component, these tests should be used with caution.

Power results are reported in Table 7. Some patterns emerge from the table. First, when $\alpha < 1.0$ and the slope shift is small to moderate ($\gamma = 0.1, 0.3$), the *SPS* and *MPS* statistics have the highest power while for large slope shifts ($\gamma = 0.5$) the STW and MTW statistics have the highest power. When $\alpha = 1$ the STW and MTW statistics generally have the highest power. Therefore, the *SPS*, *MPS* and *STW*, *MTW* statistics have complementary power and generally dominate *QD*, *SW* and *MW* for the parameterizations considered here.

5. CONFIDENCE INTERVALS

When the break date is known, a nice feature of the $T^{-1/2}t{-}W_T$, $t{-}PS_T$ and $t{-}PSW_T$ statistics (henceforth, the normalized t-statistics) is that they are easily inverted to compute confidence intervals for the trend parameters. When the break date is unknown, computing confidence intervals is more complicated. One approach is to estimate the break date (assuming there is a shift in trend), treat that estimated break date as known, and then invert the normalized t-statistics. As long as the estimated break date is consistent and converges to the true break date at a fast enough rate (this may difficult to prove), this approach can yield asymptotically valid confidence intervals. Of course, the approximation in finite samples may not be satisfactory if the estimated break has large sampling variability.

In this section an alternate approach is taken which provides conservative confidence intervals for the trend parameters when the break date is unknown. First, the break date is estimated by choosing the date which maximizes $T^{-1}W_T$ for $T_b \in \Lambda$. That is, $\widehat{T}_b = argmax_{T_b \in \Lambda} T^{-1}W_T(T_b)$. This is equivalent to estimating T_b by maximizing $W_T(T_b)$ for $T_b \in \Lambda$. Bai (1997) has shown that this procedure consistently estimates λ' in stationary regression models. Second, using $\widehat{T}_b$ the models are estimated and confidence intervals formed by inverting the normalized t-statistics. What makes this a conservative approach is that asymptotic critical values for the normalized t-statistics are derived under the assumption that there is no break in the trend function. This approach results in wider confidence intervals than if the estimated break date were treated as known, but these confidence intervals remain conservative when there is no break in the trend function.

In order to provide confidence intervals that have a direct interpretation, it is useful to rewrite Models (1) — (3) in the following way:

$$y_t = \delta_1 DU_{1t} + \delta_2 DU_{2t} + \beta t + u_t, \tag{4a}$$

$$z_t = \delta_1 SDU_{1t} + \delta_2 SDU_{2t} + \beta[\tfrac{1}{2}(t^2 + t)] + S_t, \tag{4b}$$

$$y_t = \mu + \gamma_1 DT_{1t} + \gamma_2 DT_{2t} + u_t, \tag{5a}$$

$$z_t = \mu t + \gamma_1 SDT_{1t} + \gamma_2 SDT_{2t} + S_t, \tag{5b}$$

$$y_t = \delta_1 DU_{1t} + \delta_2 DU_{2t} + \gamma_1 DT_{2t} + \gamma_2 DT_{2t} + u_t, \tag{6a}$$

$$z_t = \delta_1 SDU_{1t} + \delta_2 SDU_{2t} + \gamma_1 SDT_{2t} + \gamma_2 SDT_{2t} + S_t, \tag{6b}$$

where $DU_{1t} = 1$ if $t \le T_b'$ and 0 otherwise, $DU_{2t} = 1$ if $t > T_b'$ and 0 otherwise, $DT_{1t} = t$ if $t \le T_b'$ and 0 otherwise, $DT_{2t} = t - T_b'$ if $t > T_b'$ and 0 otherwise, $SDU_{it} = \Sigma_{i=1}^{t} DU_{it}$, and $SDT_{it} = \Sigma_{i=1}^{t} DT_{it}$, $i = 1, 2$. The parameters in Models (4)-(6) are before and after break intercepts and slopes. For example, γ_1 gives the growth rate of y_t before the break and γ_2 gives the growth rate after the break.

Asymptotic critical values for the $t - PS_T$, $t - PSW_T$ and $T^{-1/2}t - W_T$ statistics appropriate for testing the significance of each of the parameters in Models (4) — (6) were simulated using techniques similar to those used for Tables 1-5. As before, $m = 9$ was used for $t - PS_T$ and $t - PSW_T$. Using 10% trimming, the break date estimated by maximizing the $T^{-1}W_T$ statistic for the following hypotheses: $\delta_1 = \delta_2$ in Model (4), $\gamma_1 = \gamma_2$ in Model (5) and $\delta_1 = \delta_2$, $\gamma_1 = \gamma_2$ in Model (6). The critical values are tabulated in Table 8. Only right tail critical values are given since the distributions are symmetric around zero. For the $t - PS_T$ and $t - PSW_T$ statistics the values of b that result in the $I(0)$ and $I(1)$ critical values being the same are given in parentheses below each critical value. The $T^{-1/2}t - W_T$ critical values were simulated using $I(1)$ errors.

Confidence intervals are constructed as follows. Let cv_π generically denote the right tail critical value for a one tailed test with significance level π for a test regarding the parameter η (which generically denotes μ, β, δ_i, γ_i). Let $\widehat{\eta}$ and $\widetilde{\eta}$ denote the OLS estimates of η from the $\{y_t\}$ and $\{z_t\}$ regressions respectively. Let se_y and se_z denote the standard errors for the OLS estimates of η from the $\{y_t\}$ and $\{z_t\}$ regressions respectively. Then, the end points of $100(1-2\pi)\%$ confidence intervals for η can be constructed as follows:

$$\begin{aligned} t-PS_T &: \widetilde{\eta} \pm T^{1/2} se_z \cdot \exp(bJ_T(9)) \cdot cv_\pi, \\ t-PSW_T &: \widehat{\eta} \pm 10 se_y (T^{-1} s_z^2 / s_y^2)^{1/2} \cdot \exp(bJ_T(9)) \cdot cv_\pi, \\ T^{-1/2} t - W_T &: \widehat{\eta} \pm T^{1/2} se_y \cdot cv_\pi. \end{aligned}$$

6. EMPIRICAL APPLICATION

Using data from Maddison (1991), Ben-David and Papell (1995) found that many industrialized countries have experienced shifts in average growth rates of GNP. In particular they found that over the years 1860 — 1989 (some countries had later starting dates) many countries experienced a sharp drop in output during either WWI, WWII or the great depression but had GNP growth rates higher after the break compared to before. They illustrate their findings by reporting estimates of growth rates before and after the break. They did not, however, report confidence intervals for these estimates. It would be useful to have an idea of how much sampling variability there is in estimates of GNP growth rates.

Model (6) was fitted to the natural logarithms of the same series examined by Ben-David and Papell (1995). The break date was chosen to maximize $T^{-1}W_T$ (using 1% trimming) for testing the joint hypothesis $\delta_1 = \delta_2$, $\gamma_1 = \gamma_2$. 90% confidence intervals were constructed as described in Section 5. The results are reported in Tables 9 and 10. Table 9 reports results for real GNP series and Table 10 reports results for real per capita GNP series. For most countries, the point estimates of γ_2 are larger than γ_1 and the estimated break dates closely correspond to those of Ben-David and Papell (1995). The confidence intervals indicate that for most countries estimates of growth rates have substantial sampling variability as confidence intervals are relatively wide. This is to be expected as most GNP series have substantial serial correlation and persistence which makes inference on trend function parameters less precise. An exception is the U.K. where confidence intervals are relatively narrow using $t-PS_T$ and $t-PSW_T$. In general, confidence intervals are tighter when using $T^{-1/2}t - W_T$ than when using $t-PS_T$ or $t-PSW_T$. This is not surprising since $T^{-1/2}t - W_T$ is designed to be powerful when errors are highly persistent (having a near unit root). There are exceptions, though. Tighter confidence intervals are obtained with $t-PS_T$ and $t-PSW_T$ for Canada, Germany, U.K. and U.S.A. It is important to keep in mind that these confidence intervals are conservative regarding the correlation in the errors and the location of the break date. Given the conservative nature of the confidence intervals, it is perhaps surprising how narrow the confidence intervals are in many cases.

7. CONCLUSION

By applying general theorems in Vogelsang (1998a), new tests for detecting shifts in the trend function of a univariate time series were proposed. The tests are asymptotically valid in the presence of general serial correlation and do not require estimation, either parametrically or nonparametrically., of the serial correlation parameters. The tests are also asymptotically valid whether the errors are $I(0)$ or $I(1)$, and are, for practical purposes, invariant to the variance of the initial condition.

Three models of trend shifts were analyzed. Asymptotic critical values were tabulated. Using nearly integrated asymptotics, the size of the tests was shown to be quite stable in terms of the local to unity parameter with the exception of the PS_T and PSW_T tests when used to test for a shift in the intercept. Local asymptotic power of the statistics was examined, and it was shown that the tests have nontrivial local asymptotic power and compare favorably with standard Wald tests which can be optimal tests when the errors are $I(0)$. Surprisingly, the PS_T statistic is more powerful than the standard Wald statistic when used to test for a shift in slope when the break date is unknown using the supremum statistic. On the other hand, the PS_T and PSW_T statistics have low power in detecting a shift in the intercept of a trending time series. Overall, the power of the PS_T and PSW_T statistics is quite good and size is well behaved except in the case where the errors have a an autoregressive unit root and near unit root in the moving average term. In this situation size is inflated and the PS_T and PSW_T tests should be used with caution.

The statistics should prove very useful in practice as they remove the need to specify the form of serial correlation, are valid whether the series is stationary or has a unit root, do not depend on the variance of initial conditions, and are relatively easy to compute.

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Table 1: Asymptotic Distributions Model (1), Known Break Date, $H_0 : \delta = 0$
$t - PS_T$ and $t - PSW_T$ I(0) Errors; $T^{-1/2}t - W_T$ I(1) Errors with $\bar{\alpha} = 0, \kappa = 0$.

		λ'								
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$t - PS_T$	90.0%	0.842	0.926	0.904	0.682	0.699	0.912	0.911	0.766	0.529
	b	0.192	0.323	0.288	0.168	0.132	0.209	0.295	0.273	0.172
	95.0%	1.067	1.238	1.189	0.895	0.902	1.177	1.187	0.972	0.667
	b	0.243	0.379	0.309	0.221	0.210	0.263	0.349	0.365	0.201
	97.5%	1.308	1.477	1.425	1.099	1.090	1.410	1.442	1.172	0.784
	b	0.283	0.461	0.389	0.274	0.275	0.331	0.410	0.432	0.235
	99.0%	1.526	1.829	1.748	1.315	1.331	1.673	1.719	1.446	0.963
	b	0.414	0.509	0.476	0.345	0.351	0.447	0.477	0.539	0.279
$t - PSW_T$	90.0%	0.722	0.728	0.730	0.691	0.680	0.723	0.731	0.694	0.668
	b	0.102	0.273	0.244	0.105	0.071	0.134	0.221	0.243	0.087
	95.0%	0.939	0.977	0.958	0.917	0.908	0.946	0.954	0.907	0.880
	b	0.144	0.323	0.255	0.154	0.121	0.183	0.286	0.335	0.108
	97.5%	1.137	1.213	1.161	1.124	1.102	1.150	1.174	1.111	1.056
	b	0.196	0.378	0.334	0.206	0.194	0.246	0.332	0.405	0.174
	99.0%	1.400	1.474	1.411	1.398	1.400	1.382	1.422	1.388	1.316
	b	0.316	0.455	0.462	0.282	0.252	0.326	0.396	0.516	0.237
$T^{-1/2}t - W_T$	90.0%	0.598	0.759	0.738	0.617	0.533	0.614	0.728	0.758	0.595
	95.0%	0.747	0.942	0.918	0.770	0.693	0.763	0.894	0.938	0.741
	97.5%	0.877	1.099	1.048	0.896	0.832	0.890	1.025	1.091	0.862
	99.0%	1.021	1.259	1.196	1.040	0.992	1.048	1.179	1.279	1.000

Note: The critical values were calculated via simulation methods using $N(0,1)$ iid random deviates to approximate the Wiener processes defined in the respective distributions. The integrals were approximated by the normalized sums of 1,000 steps using 10,000 replications. For a given percentage point, the $t - PS_T$ and $t - PSW_T$ critical values are the same for I(0) and I(1) errors provided they are calculated using the values of b given below each critical value. Left tail critical values follow by symmetry around zero.

Table 2: Asymptotic Distributions Model (2), Known Break Date, $H_0 : \gamma = 0$
$t - PS_T$ and $t - PSW_T$ I(0) Errors; $T^{-1/2}t - W_T$ I(1) Errors with $\bar{\alpha} = 0, \kappa = 0$.

		λ'								
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$t - PS_T$	90.0%	0.794	0.899	0.967	0.987	0.966	0.890	0.793	0.635	0.452
	b	0.144	0.273	0.417	0.500	0.526	0.469	0.365	0.248	0.138
	95.0%	1.021	1.180	1.234	1.289	1.242	1.159	1.002	0.813	0.565
	b	0.163	0.376	0.627	0.730	0.755	0.700	0.542	0.314	0.155
	97.5%	1.209	1.430	1.554	1.546	1.528	1.400	1.206	0.966	0.653
	b	0.193	0.469	0.833	1.022	1.027	0.978	0.744	0.395	0.179
	99.0%	1.440	1.745	1.925	1.878	1.818	1.676	1.493	1.169	0.794
	b	0.223	0.682	1.095	1.570	1.471	1.322	1.007	0.500	0.193
$t - PSW_T$	90.0%	0.701	0.746	0.745	0.747	0.738	0.719	0.698	0.679	0.649
	b	0.049	0.233	0.425	0.519	0.565	0.512	0.406	0.229	0.039
	95.0%	0.941	0.972	0.973	0.983	0.972	0.941	0.917	0.890	0.850
	b	0.068	0.344	0.616	0.757	0.786	0.746	0.603	0.331	0.052
	97.5%	1.160	1.178	1.216	1.198	1.175	1.163	1.107	1.105	1.056
	b	0.095	0.467	0.830	1.075	1.105	1.067	0.844	0.420	0.087
	99.0%	1.415	1.451	1.513	1.461	1.423	1.412	1.398	1.346	1.279
	b	0.158	0.675	1.106	1.598	1.548	1.462	1.144	0.632	0.015
$T^{-1/2}t - W_T$	90.0%	0.534	.0763	0.933	1.020	1.052	1.016	0.923	0.778	0.537
	95.0%	0.669	1.003	1.209	1.337	1.391	1.352	1.186	0.983	0.668
	97.5%	0.800	1.197	1.480	1.650	1.716	1.646	1.461	1.159	0.784
	99.0%	0.953	1.417	1.801	2.063	2.094	1.972	1.794	1.375	0.934

Note: The critical values were calculated via simulation methods using $N(0,1)$ iid random deviates to approximate the Wiener processes defined in the respective distributions. The integrals were approximated by the normalized sums of 1,000 steps using 10,000 replications. For a given percentage point, the $t - PS_T$ and $t - PSW_T$ critical values are the same for I(0) and I(1) errors provided they are calculated using the values of b given below each critical value. Left tail critical values follow by symmetry around zero.

Table 3A: Asymptotic Distributions Model (3), Known Break Date, $H_0 : \delta = 0$
$t - PS_T$ and $t - PSW_T$ I(0) Errors; $T^{-1/2}t - W_T$ I(1) Errors with $\bar{\alpha} = 0, \kappa = 0$.

		λ'								
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$t - PS_T$	90.0%	0.397	0.572	0.702	0.754	0.772	0.731	0.634	0.525	0.350
	b	0.108	0.175	0.296	0.384	0.406	0.358	0.270	0.153	0.107
	95.0%	0.503	0.729	0.892	0.983	0.990	0.924	0.816	0.661	0.441
	b	0.100	0.198	0.360	0.454	0.486	0.454	0.320	0.166	0.100
	97.5%	0.594	0.887	1.088	1.199	1.198	1.100	0.974	0.785	0.521
	b	0.093	0.236	0.447	0.557	0.584	0.547	0.370	0.180	0.088
	99.0%	0.701	1.034	1.289	1.435	1.429	1.297	1.184	0.918	0.621
	b	0.096	0.284	0.597	0.738	0.719	0.736	0.470	0.213	0.081
$t - PSW_T$	90.0%	0.735	0.767	0.822	0.820	0.830	0.791	0.773	0.724	0.669
	b	0.087	0.041	0.195	0.313	0.325	0.295	0.144	-0.003	-0.099
	95.0%	0.961	1.023	1.068	1.095	1.094	1.045	1.012	0.965	0.891
	b	-0.080	0.052	0.250	0.369	0.385	0.360	0.204	0.011	-0.098
	97.5%	1.192	1.245	1.288	1.322	1.321	1.262	1.246	1.171	1.069
	b	-0.080	0.082	0.361	0.487	0.497	0.459	0.254	0.030	-0.092
	99.0%	1.494	1.547	1.551	1.606	1.594	1.538	1.527	1.462	1.364
	b	-0.080	0.097	0.489	0.628	0.581	0.620	0.365	0.032	-0.094
$T^{-1/2}t - W_T$	90.0%	0.331	0.458	0.571	0.649	0.649	0.634	0.552	0.443	0.306
	95.0%	0.391	0.578	0.708	0.801	0.818	0.787	0.700	0.561	0.384
	97.5%	0.453	0.663	0.833	0.937	0.962	0.921	0.820	0.656	0.449
	99.0%	0.520	0.779	0.970	1.066	1.107	1.082	0.946	0.763	0.509

Note: The critical values were calculated via simulation methods using $N(0,1)$ iid random deviates to approximate the Wiener processes defined in the respective distributions. The integrals were approximated by the normalized sums of 1,000 steps using 10,000 replications. For a given percentage point, the $t - PS_T$ and $t - PSW_T$ critical values are the same for I(0) and I(1) errors provided they are calculated using the values of b given below each critical value. Left tail critical values follow by symmetry around zero.

Table 3B: Asymptotic Distributions Model (2), Known Break Date, $H_0 : \gamma = 0$
$t - PS_T$ and $t - PSW_T$ I(0) Errors: $T^{-1/2}t - W_T$ I(1) Errors with $\bar{\alpha} = 0, \kappa = 0$.

		λ'								
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$t - PS_T$	90.0%	0.309	0.509	0.754	1.051	1.032	0.708	0.503	0.364	0.027
	b	0.010	0.071	0.412	0.688	0.770	0.644	0.317	0.036	0.000
	95.0%	0.393	0.659	0.983	1.630	1.344	0.926	0.656	0.466	0.298
	b	-0.008	0.122	0.595	0.999	1.058	0.915	0.469	0.046	0.014
	97.5%	0.471	0.790	1.196	1.672	1.612	1.122	0.799	0.569	0.345
	b	-0.020	0.209	0.809	1.348	1.414	1.161	0.624	0.079	0.005
	99.0%	0.552	0.936	1.458	2.049	1.933	1.377	0.953	0.671	0.411
	b	-0.027	0.356	1.158	1.877	1.973	1.486	0.888	0.156	-0.007
$t - PSW_T$	90.0%	0.738	0.780	0.826	0.841	0.843	0.814	0.761	0.726	0.670
	b	-0.203	0.069	0.443	0.725	0.815	0.697	0.405	0.024	-0.216
	95.0%	0.989	1.036	1.088	1.102	1.095	1.075	1.011	0.949	0.882
	b	-0.198	0.133	0.631	1.014	1.122	0.972	0.583	0.075	-0.205
	97.5%	1.196	1.226	1.303	1.335	1.319	1.283	1.222	1.169	1.087
	b	-0.180	0.228	0.876	1.399	1.431	1.270	0.813	0.173	-0.196
	99.0%	1.455	1.537	1.614	1.660	1.584	1.542	1.544	1.422	1.349
	b	-0.160	0.371	1.285	1.978	2.030	1.173	0.983	0.247	-0.175
$T^{-1/2}t - W_T$	90.0%	0.206	0.468	0.773	1.048	1.156	1.034	0.769	0.468	0.206
	95.0%	0.270	0.628	1.035	1.376	1.517	1.384	1.000	0.608	0.272
	97.5%	0.331	0.777	1.297	1.675	1.835	1.665	1.207	0.735	0.335
	99.0%	0.422	0.961	1.553	2.110	2.265	2.043	1.508	0.923	0.409

Note: The critical values were calculated via simulation methods using $N(0,1)$ iid random deviates to approximate the Wiener processes defined in the respective distributions. The integrals were approximated by the normalized sums of 1,000 steps using 10,000 replications. For a given percentage point, the $t - PS_T$ and $t - PSW_T$ critical values are the same for I(0) and I(1) errors provided they are calculated using the values of b given below each critical value. Left tail critical values follow by symmetry around zero.

Table 3C: Asymptotic Distributions Model (2), Known Break Date, $H_0 : \delta = \gamma = 0$
PS_T and PSW_T I(0) Errors: $T^{-1/2}t - W_T$ I(1) Errors with $\bar{\alpha} = 0, \kappa = 0$.

		λ'								
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
PS_T	90.0%	1.349	1.937	2.451	2.676	2.641	2.352	1.826	1.158	0.526
	b	0.440	1.035	1.586	1.982	1.984	1.824	1.389	0.827	0.368
	95.0%	1.904	2.771	3.374	3.769	3.633	3.225	2.555	1.651	0.738
	b	0.536	1.346	2.164	2.678	2.554	2.340	1.815	1.056	0.435
	97.5%	2.488	3.660	4.469	4.885	4.712	4.157	3.336	2.185	0.977
	b	0.699	1.715	2.825	3.463	3.420	3.073	2.358	1.361	0.519
	99.0%	3.398	5.131	6.162	6.517	6.330	5.642	4.579	2.901	1.260
	b	0.901	2.239	3.768	4.615	4.641	4.174	3.110	1.763	0.612
PSW_T	90.0%	1.644	1.858	1.989	2.124	2.093	1.976	1.801	1.579	1.404
	b	0.086	0.757	1.426	1.846	1.897	1.769	1.322	0.668	0.025
	95.0%	2.292	2.584	2.766	2.932	2.843	2.708	2.505	2.237	1.994
	b	0.161	1.040	1.917	2.545	2.489	2.292	1.837	0.948	0.086
	97.5%	2.969	3.238	3.529	3.732	3.647	3.510	3.210	3.006	2.652
	b	0.311	1.436	2.594	3.345	3.233	2.992	2.398	1.248	0.165
	99.0%	4.014	4.536	4.612	4.759	4.829	4.620	4.486	4.049	3.473
	b	0.493	1.943	3.500	4.421	4.411	4.031	3.099	1.651	0.322
$T^{-1}W_T$	90.0%	0.630	1.371	2.017	2.490	2.689	2.486	1.975	1.317	0.601
	95.0%	0.876	1.922	2.847	3.596	3.769	3.539	2.818	1.826	0.828
	97.5%	1.116	2.476	3.811	4.770	4.948	4.693	3.651	2.437	1.061
	99.0%	1.439	3.329	5.278	6.473	6.817	6.422	5.098	3.383	1.392

Note: The critical values were calculated via simulation methods using $N(0,1)$ iid random deviates to approximate the Wiener processes defined in the respective distributions. The integrals were approximated by the normalized sums of 1,000 steps using 10,000 replications. For a given percentage point, the PS_T and PSW_T critical values are the same for I(0) and I(1) errors provided they are calculated using the values of b given below each critical value.

Table 4: Asymptotic Distributions Model (2), Unknown Break Date
1% Trimming ($\lambda^* = 0.01$)
PS_T and PSW_T I(0) Errors: $T^{-1}W_T$ I(1) Errors with $\bar{\alpha} = 0, \kappa = 0$.

		PS_T			PSW_T			$T^{-1}W_T$		
		Mean	Exp	Sup	Mean	Exp	Sup	Mean	Exp	Sup
Model (1)	90.0%	0.888	0.536	3.151	0.499	0.326	4.237	0.441	0.235	1.858
H_0 :	b	1.025	2.044	1.646	0.949	1.862	0.868			
$\delta = 0$	95.0%	1.134	0.742	4.205	0.586	0.418	5.115	0.492	0.267	2.181
	b	1.040	2.377	1.836	1.067	2.413	1.129			
	97.5%	1.373	0.979	5.360	0.681	0.536	6.093	0.531	0.924	2.476
	b	1.052	2.835	2.103	1.168	3.083	1.384			
	99.0%	1.671	1.352	7.002	0.794	0.753	7.437	0.568	0.324	2.839
	b	1.138	3.496	2.495	1.342	3.794	1.729			
Model (2)	90.0%	1.031	0.582	2.416	0.720	0.407	2.432	1.087	0.678	3.244
H_0 :	b	2.484	3.972	3.406	2.523	4.012	2.985			
$\gamma = 0$	95.0%	1.389	0.838	3.387	0.953	0.566	3.138	1.421	0.979	4.518
	b	3.211	5.631	4.722	3.341	5.760	4.297			
	97.5%	1.764	1.167	4.553	1.185	0.756	3.894	1.754	1.404	6.024
	b	3.950	7.377	6.059	4.281	7.887	5.880			
	99.0%	2.289	1.675	6.217	1.546	1.069	5.218	2.180	2.088	8.109
	b	5.187	10.15	8.411	5.592	10.75	7.939			
Model (3)	90.0%	1.597	0.969	3.970	1.243	0.838	6.273	1.471	0.923	3.838
H_0 :	b	3.103	4.807	3.939	2.931	4.644	2.603			
$\delta = \gamma = 0$	95.0%	2.125	1.388	5.321	1.534	1.093	7.450	1.885	1.312	5.168
	b	3.946	6.216	5.153	3.930	6.220	3.709			
	97.5%	2.702	1.851	6.732	1.821	1.429	8.615	2.359	1.832	6.685
	b	4.850	7.814	6.359	4.978	7.844	4.928			
	99.0%	3.386	2.539	8.571	2.252	1.959	10.522	2.937	2.600	8.778
	b	6.426	10.52	8.597	6.588	10.77	7.065			

Note: The critical values were calculated via simulation methods using $N(0,1)$ iid random deviates to approximate the Wiener processes defined in the respective distributions. The integrals were approximated by the normalized sums of 1,000 steps using 10,000 replications. For a given percentage point, the PS_T and PSW_T critical values are the same for I(0) and I(1) errors provided they are calculated using the values of b given below each critical value.

Table 5: Asymptotic Size I(1) Errors, Known Break Date with $\lambda' = 0.5$. Nominal Size = 0.05 using $\bar{\alpha} = 0, \kappa = 0$ critical values.

	Model (1), $H_0 : \delta = 0$			Model (2), $H_0 : \gamma = 0$			Model (3),$H_0 : \delta = \gamma = 0$		
$\bar{\alpha}$	PS_T	PSW_T	$T^{-1}W_T$	PS_T	PSW_T	$T^{-1}W_T$	PS_T	PSW_T	$T^{-1}W_T$
0	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050
2	0.053	0.053	0.049	0.046	0.044	0.033	0.049	0.048	0.035
4	0.061	0.061	0.044	0.041	0.039	0.015	0.045	0.042	0.015
6	0.065	0.064	0.038	0.038	0.036	0.006	0.041	0.040	0.005
8	0.071	0.067	0.031	0.034	0.033	0.002	0.040	0.037	0.002
10	0.075	0.070	0.024	0.033	0.030	0.001	0.037	0.036	0.001
12	0.076	0.071	0.018	0.031	0.029	0.000	0.037	0.034	0.000
14	0.077	0.072	0.014	0.030	0.029	0.000	0.037	0.033	0.000
16	0.079	0.073	0.011	0.032	0.029	0.000	0.035	0.033	0.000
18	0.077	0.073	0.008	0.033	0.029	0.000	0.034	0.033	0.000
20	0.077	0.071	0.006	0.034	0.030	0.000	0.035	0.032	0.000

Notes: $\kappa = 0$ for all simulations.
Model (1): $y_t = \mu + \beta t + \delta DU_t + u_t,$ $z_t = \mu t + \beta[\frac{1}{2}(t^2 + t)] + \delta DT_t + S_t$.
Model (2): $y_t = \mu + \beta t + \gamma DT_t + u_t$, $z_t = \mu t + \beta[\frac{1}{2}(t^2 + t)] + \gamma[\frac{1}{2}(DT_t^2 + DT_t)] + S_t$.
Model (3): $y_t = \mu + \beta t + \delta DU_t + \gamma DT_t + u_t$, $z_t = \mu t + \beta[\frac{1}{2}(t^2 + t)] + \delta DT_t + \gamma[\frac{1}{2}(DT_t^2 + DT_t)] + S_t$.

Table 6: Finite Sample Null Hypothesis Rejection Probabilities ARMA(1,1) Errors Using 5% Asymptotic Critical Values.
Model (2): $y_t = \gamma DT_t + u_t, \quad z_t = \gamma[\frac{1}{2}(DT_t^2 + DT_t)] + S_t, \quad u_t = \alpha u_{t-1} + v_t + \theta v_{t-1}.$
$H_0 : \gamma = 0$, **2,000 Replications, Trimming 10% where applicable.**

				$T = 100$				
α	θ	QD	SW	MW	SPS	MPS	STW	MTW
0.8	-1.0	.000	.002	.000	.002	.000	.000	.000
	-0.8	.010	.004	.004	.036	.038	.000	.000
	-0.4	.029	.030	.018	.049	.046	.000	.000
	0.0	.010	.010	.009	.028	.029	.000	.000
	0.4	.011	.014	.013	.023	.023	.000	.000
	0.8	.009	.015	.011	.020	.021	.000	.000
0.9	-1.0	.006	.004	.003	.041	.037	.000	.000
	-0.8	.058	.042	.037	.107	.100	.000	.000
	-0.4	.019	.072	.038	.055	.054	.000	.000
	0.0	.007	.023	.020	.030	.028	.001	.000
	0.4	.010	.026	.024	.023	.021	.002	.001
	0.8	.010	.028	.021	.020	.020	.002	.001
0.95	-1.0	.025	.008	.015	.080	.078	.000	.000
	-0.8	.092	.108	.111	.171	.161	.000	.000
	-0.4	.018	.091	.057	.065	.065	.000	.000
	0.0	.015	.033	.030	.030	.030	.007	.005
	0.4	.012	.041	.030	.025	.027	.010	.006
	0.8	.012	.041	.031	.023	.026	.011	.007
1.0	-1.0	.009	.005	.003	.029	.029	.000	.000
	-0.8	.111	.200	.188	.236	.228	.000	.000
	-0.4	.026	.134	.098	.103	.101	.018	.018
	0.0	.023	.056	.038	.052	.049	.050	.048
	0.4	.015	.064	.045	.042	.041	.059	.060
	0.8	.012	.061	.042	.040	.062	.062	.061

Table 6 Continued

$T = 250$

α	θ	QD	SW	MW	SPS	MPS	STW	MTW
0.8	-1.0	.000	.000	.000	.000	.000	.000	.000
	-0.8	.020	.001	.004	.044	.040	.000	.000
	-0.4	.024	.010	.009	.045	.041	.000	.000
	0.0	.013	.002	.006	.033	.032	.000	.000
	0.4	.015	.004	.006	.030	.030	.000	.000
	0.8	.008	.005	.002	.028	.028	.000	.000
0.9	-1.0	.000	.004	.000	.046	.015	.000	.000
	-0.8	.098	.055	.038	.085	.072	.000	.000
	-0.4	.022	.013	.012	.038	.035	.000	.000
	0.0	.008	.005	.006	.023	.024	.000	.000
	0.4	.008	.006	.006	.021	.022	.000	.000
	0.8	.003	.006	.006	.019	.019	.000	.000
0.95	-1.0	.053	.022	.027	.198	.145	.000	.000
	-0.8	.145	.184	.105	.140	.124	.000	.000
	-0.4	.015	.018	.017	.033	.034	.000	.000
	0.0	.007	.010	.008	.021	.021	.000	.000
	0.4	.008	.013	.011	.019	.020	.000	.000
	0.8	.006	.012	.009	.018	.019	.000	.000
1.0	-1.0	.020	.001	.004	.045	.039	.000	.000
	-0.8	.131	.311	.234	.282	.272	.000	.000
	-0.4	.030	.054	.041	.077	.073	.038	.036
	0.0	.041	.041	.032	.050	.049	.053	.048
	0.4	.037	.045	.034	.046	.046	.056	.051
	0.8	.045	.044	.031	.046	.044	.056	.052

Table 7: Finite Sample Power
ARMA(1,1) Errors Using 5% Asymptotic Critical Values.
Model (2): $y_t = \gamma DT_t + u_t, \quad z_t = \gamma[\frac{1}{2}(DT_t^2 + DT_t)] + S_t, \;\; u_t = \alpha u_{t-1} + v_t + \theta v_{t-1}$.
$H_0 : \gamma = 0$, $T = 100$, $T_b^c = 50$, **2,000 Replications, Trimming 10% where applicable.**
Power is not size adjusted.

α	θ	γ	QD	SW	MW	SPS	MPS	STW	MTW
0.9	-0.4	0.0	.019	.072	.038	.055	.054	.000	.000
		0.1	.017	.175	.161	.191	.194	.002	.001
		0.3	.000	.436	.318	.539	.540	.353	.386
		0.5	.000	.527	.247	.691	.670	.977	.977
0.9	0.0	0.0	.007	.023	.020	.030	.028	.001	.000
		0.1	.006	.034	.033	.053	.055	.002	.001
		0.3	.001	.104	.069	.215	.212	.140	.143
		0.5	.000	.167	.052	.328	.317	.655	.653
0.9	0.4	0.0	.010	.026	.024	.023	.021	.002	.001
		0.1	.006	.036	.033	.033	.034	.002	.001
		0.3	.003	.091	.087	.110	.111	.062	.062
		0.5	.000	.149	.091	.207	.204	.317	.324
0.95	-0.4	0.0	.018	.091	.057	.065	.065	.000	.000
		0.1	.018	.162	.132	.154	.149	.014	.012
		0.3	.002	.345	.229	.417	.418	.358	.381
		0.5	.001	.427	.186	.547	.527	.926	.921
0.95	0.0	0.0	.015	.033	.030	.030	.030	.007	.005
		0.1	.010	.041	.036	.045	.041	.018	.018
		0.3	.007	.088	.063	.141	.139	.179	.180
		0.5	.003	.142	.055	.227	.220	.566	.571
0.95	0.4	0.0	.012	.041	.030	.025	.027	.010	.006
		0.1	.010	.046	.039	.029	.031	.016	.016
		0.3	.007	.086	.065	.072	.070	.104	.104
		0.5	.002	.124	.079	.130	.122	.310	.302
1.0	0.4	0.0	.026	.134	.098	.103	.101	.018	.018
		0.1	.020	.171	.123	.154	.149	.044	.046
		0.3	.007	.294	.192	.338	.329	.378	.390
		0.5	.004	.376	.179	.465	.447	.861	.489
1.0	0.0	0.0	.023	.056	.038	.052	.049	.050	.048
		0.1	.023	.063	.048	.061	.057	.064	.065
		0.3	.016	.088	.060	.109	.104	.240	.238
		0.5	.008	.137	.071	.174	.165	.528	.531
1.0	-0.4	0.0	.015	.064	.045	.042	.041	.059	.060
		0.1	.016	.066	.052	.047	.044	.067	.068
		0.3	.014	.089	.062	.064	.061	.165	.164
		0.5	.009	.112	.077	.105	.099	.338	.338

Table 8: Asymptotic (Null of no break) Critical Values for t-statistics Break Date Chosen by Maximizing $T^{-1}W_T$ using 1% Trimming.

		Model(1)			Model (2)			Model(3)			
		δ_1	δ_2	β	μ	γ_1	γ_2	δ_1	δ_2	γ_1	γ_2
$t-PS_T$	90.0%	1.87	1.33	1.43	1.25	0.90	0.80	1.57	1.14	1.33	0.94
	b	0.44	0.73	1.04	0.14	0.11	0.41	0.53	1.84	1.34	1.31
	95.0%	2.51	1.67	1.84	1.76	1.25	1.07	2.19	1.50	1.76	1.27
	b	0.54	0.94	0.42	0.19	0.15	0.66	0.62	2.49	1.90	1.82
	97.5%	3.07	1.98	2.24	2.25	1.64	1.30	2.79	1.82	2.18	1.55
	b	0.68	1.16	1.87	0.21	0.18	0.97	0.71	3.35	2.47	2.43
	99.0%	3.80	2.32	2.75	2.92	2.14	1.59	3.52	2.24	2.69	1.94
	b	0.91	1.48	2.55	0.31	0.22	1.49	0.89	4.38	3.33	3.39
$t-PSW_T$	90.0%	1.16	1.63	1.53	0.55	0.42	0.68	1.21	1.65	1.58	1.54
	b	0.27	0.37	0.72	0.32	0.05	0.46	0.26	1.47	0.89	0.92
	95.0%	1.45	1.86	1.79	0.73	0.60	0.93	1.52	1.92	1.88	1.81
	b	0.35	0.56	1.10	0.44	0.09	0.70	0.35	2.15	1.37	1.48
	97.5%	1.70	2.07	2.03	0.92	0.79	1.13	1.81	2.18	2.12	2.05
	b	0.45	0.72	1.56	0.57	0.14	1.04	0.47	3.06	1.99	2.12
	99.0%	2.00	2.39	2.32	1.11	1.00	1.39	2.15	2.47	2.42	2.38
	b	0.60	0.96	2.33	0.79	0.18	1.54	0.58	4.36	2.91	3.02
$T^{-1/2}t-W_T$	90.0%	0.84	1.32	1.82	1.01	0.45	1.37	0.67	2.37	1.47	1.48
	95.0%	1.06	1.70	2.38	1.25	0.66	1.87	0.88	3.00	2.00	2.01
	97.5%	1.26	1.97	2.88	1.43	0.87	2.34	1.05	3.61	2.49	2.50
	99.0%	1.48	2.27	3.50	1.66	1.09	2.95	1.26	4.37	3.11	3.17

For a given percentage point, the $t-PS_T$ and $t-PSW_T$ critical values are the same for I(0) and I(1) errors provided they are calculated using the values of b given below each critical value. Left tail critical values follow by symmetry. $T^{-1/2}t-W_T$ is valid for I(1) errors. If $b=0$ is used, the critical values for $t-PS_T$ and $t-PSW_T$ are valid only for I(0) errors.

Table 9: 90% Confidence Intervals for Annual Growth Rates of Real GNP

$T^{-1/2}t - W_T$

Country	$\widehat{T}_b$	$\gamma_{1_{\min}}$	$\widehat{\gamma}_1$	$\gamma_{1_{\max}}$	$\gamma_{2_{\min}}$	$\widehat{\gamma}_2$	$\gamma_{2_{\max}}$
Australia	1928	1.66	3.06	4.46	2.20	3.90	5.59
Austria	1944	-0.29	1.41	3.11	1.05	4.73	8.41
Belgium	1939	0.89	1.70	2.51	1.88	3.53	5.17
Canada	1930	2.34	3.80	5.27	3.12	4.67	6.22
Denmark	1939	1.97	2.63	3.28	2.12	3.46	4.79
Finland	1916	1.37	2.56	3.74	3.09	3.91	4.73
France	1940	-0.05	1.18	2.40	2.09	4.70	7.31
Germany	1954	1.04	2.17	3.29	-1.74	3.31	8.36
Italy	1943	0.72	1.87	3.02	2.05	4.85	7.66
Japan	1944	1.47	3.15	4.83	4.71	7.30	9.90
Netherlands	1939	-0.72	2.70	6.12	1.83	4.29	6.75
Norway	1946	1.60	2.26	2.93	2.28	4.04	5.80
Sweden	1916	1.08	2.09	3.10	2.81	3.51	4.22
Switzerland	1945	0.25	2.04	3.83	1.15	3.14	5.12
U.K.	1919	1.16	1.91	2.66	1.68	2.28	2.88
U.S.A.	1929	2.14	3.61	5.08	1.89	3.41	4.93

$t - PS_T$

Country	$\widehat{T}_b$	$\gamma_{1_{\min}}$	$\widehat{\gamma}_1$	$\gamma_{1_{\max}}$	$\gamma_{2_{\min}}$	$\widehat{\gamma}_2$	$\gamma_{2_{\max}}$
Australia	1928	<-10	3.14	>10	<-10	4.10	>10
Austria	1944	<-10	1.41	>10	<-10	5.16	>10
Belgium	1939	0.73	1.74	2.76	0.87	3.85	6.83
Canada	1930	2.28	3.79	5.30	3.07	4.72	6.36
Denmark	1939	-2.82	2.60	8.01	<-10	3.54	>10
Finland	1916	-3.79	2.58	8.94	0.47	3.90	7.33
France	1940	-0.28	1.11	2.50	0.51	4.83	9.15
Germany	1954	1.88	2.20	2.51	0.55	3.81	7.07
Italy	1943	<-10	1.84	>10	<-10	4.96	>10
Japan	1944	<-10	3.03	>10	<-10	7.54	>10
Netherlands	1939	-3.32	2.62	8.56	1.02	4.55	8.08
Norway	1946	<-10	2.22	>10	<-10	3.81	>10
Sweden	1916	-9.42	2.09	>10	-2.36	3.62	9.59
Switzerland	1945	<-10	1.99	>10	<-10	3.34	>10
U.K.	1919	1.65	1.94	2.23	2.08	2.29	2.51
U.S.A.	1929	2.43	3.58	4.73	2.26	3.48	4.69

Table 9: Continued

$t - PSW_T$

Country	$\widehat{T}_b$	$\gamma_{1_{\min}}$	$\widehat{\gamma}_1$	$\gamma_{1_{\max}}$	$\gamma_{2_{\min}}$	$\widehat{\gamma}_2$	$\gamma_{2_{\max}}$
Australia	1928	<-10	3.06	>10	<-10	3.90	>10
Austria	1944	<-10	1.41	>10	<-10	4.73	>10
Belgium	1939	0.72	1.70	2.68	1.45	3.53	5.61
Canada	1930	2.47	3.80	5.14	3.22	4.67	6.12
Denmark	1939	-0.40	2.63	5.65	-3.73	3.46	>10
Finland	1916	-0.76	2.56	5.87	1.34	3.91	6.49
France	1940	-0.05	1.18	2.40	1.91	4.70	7.49
Germany	1954	1.70	2.17	2.63	1.26	3.31	5.36
Italy	1943	<-10	1.87	>10	<-10	4.85	>10
Japan	1944	<-10	3.15	>10	<-10	7.30	>10
Netherlands	1939	-1.58	2.70	6.98	1.05	4.29	7.52
Norway	1946	-7.33	2.26	>10	<-10	4.04	>10
Sweden	1916	-2.56	2.09	6.74	-0.30	3.51	7.33
Switzerland	1945	<-10	2.04	>10	<-10	3.14	>10
U.K.	1919	1.62	1.91	2.20	2.05	2.28	2.51
U.S.A.	1929	2.52	3.61	4.71	2.26	3.41	4.55

Table 10: 90% Confidence Intervals for Annual Growth Rates of Real Per Capita GNP

$T^{-1/2}t - W_T$

Country	$\widehat{T}_b$	$\gamma_{1_{\min}}$	$\widehat{\gamma}_1$	$\gamma_{1_{\max}}$	$\gamma_{2_{\min}}$	$\widehat{\gamma}_2$	$\gamma_{2_{\max}}$
Australia	1929	-0.70	0.48	1.66	0.94	2.13	3.32
Austria	1944	-0.69	0.84	2.38	1.14	4.45	7.77
Belgium	1940	0.05	0.84	1.62	1.76	3.14	4.52
Canada	1930	0.65	2.06	3.47	1.37	2.86	4.36
Denmark	1939	0.90	1.54	2.18	1.78	2.84	3.91
Finland	1916	-0.20	1.39	2.98	2.39	3.22	4.04
France	1940	-0.30	1.04	2.38	1.53	3.87	6.21
Germany	1949	0.25	1.52	2.78	-0.27	3.33	6.93
Italy	1942	0.01	1.29	2.57	1.78	4.26	6.75
Japan	1944	0.35	1.93	3.52	3.75	6.20	8.64
Netherlands	1940	-1.75	1.24	4.24	0.91	3.21	5.52
Norway	1945	0.78	1.51	2.25	1.68	3.37	5.05
Sweden	1916	0.26	1.39	2.51	2.81	3.51	4.22
Switzerland	1945	0.09	1.50	2.92	0.59	2.16	3.74
U.K.	1920	0.05	0.99	1.93	1.28	1.88	2.47
U.S.A.	1940	0.31	1.42	2.52	-0.31	1.62	3.56

$t - PS_T$

Country	$\widehat{T}_b$	$\gamma_{1_{\min}}$	$\widehat{\gamma}_1$	$\gamma_{1_{\max}}$	$\gamma_{2_{\min}}$	$\widehat{\gamma}_2$	$\gamma_{2_{\max}}$
Australia	1929	-1.18	0.45	2.07	0.49	2.10	3.71
Austria	1944	<-10	0.78	>10	<-10	4.62	>10
Belgium	1940	0.44	0.83	1.23	2.34	3.28	4.22
Canada	1930	0.88	2.08	3.28	1.55	2.86	4.18
Denmark	1939	0.49	1.53	2.57	0.74	2.95	5.15
Finland	1916	<-10	1.45	>10	-7.37	3.18	>10
France	1940	-1.09	0.98	3.05	-0.73	3.99	8.71
Germany	1949	0.53	1.44	2.35	-1.29	3.18	7.65
Italy	1942	<-10	1.30	>10	<-10	4.49	>10
Japan	1944	<-10	1.84	>10	<-10	6.45	>10
Netherlands	1940	-3.44	1.17	5.78	0.36	3.40	6.44
Norway	1945	<-10	1.47	>10	<-10	3.12	>10
Sweden	1916	<-10	1.40	>10	-3.56	2.99	9.53
Switzerland	1945	-4.08	1.49	7.05	-3.88	2.28	8.45
U.K.	1920	0.48	1.03	1.57	1.58	1.86	2.14
U.S.A.	1940	0.54	1.53	2.51	-0.45	1.87	4.18

Table 10: Continued

$\underline{t - PSW_T}$

Country	$\widehat{T}_b$	$\gamma_{1_{\min}}$	$\widehat{\gamma}_1$	$\gamma_{1_{\max}}$	$\gamma_{2_{\min}}$	$\widehat{\gamma}_2$	$\gamma_{2_{\max}}$
Australia	1929	-0.81	0.48	1.76	0.77	2.13	3.49
Austria	1944	-7.55	0.84	9.24	<-10	4.45	>10
Belgium	1940	0.41	0.84	1.26	2.39	3.14	3.90
Canada	1930	0.95	2.06	3.17	1.67	2.86	4.06
Denmark	1939	0.70	1.54	2.38	1.34	2.84	4.34
Finland	1916	-9.99	1.39	>10	-3.81	3.22	>10
France	1940	-0.62	1.04	2.71	0.73	3.87	7.01
Germany	1949	0.55	1.52	2.49	0.48	3.33	6.18
Italy	1942	-7.77	1.29	>10	<-10	4.26	>10
Japan	1944	<-10	1.93	>10	<-10	6.20	>10
Netherlands	1940	-2.22	1.24	4.71	0.43	3.21	6.00
Norway	1945	<-10	1.51	>10	<-10	3.37	>10
Sweden	1916	-5.36	1.39	8.13	-1.29	3.51	7.13
Switzerland	1945	-2.06	1.50	5.07	-2.21	2.16	6.53
U.K.	1920	0.51	0.99	1.48	1.57	1.88	2.18
U.S.A.	1940	0.45	1.42	2.39	-0.13	1.62	3.38

Figure 1:

Figure 2:

Figure 3:

Figure 3:

Figure 4:

Figure 5:

Figure 6:

Figure 7:

Figure 8: