Aspects of Research Strategies for Time Series Analysis [1]

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1. Introduction.

Start out with some new construction such as a new model or specification or technique. A new nonlinear model of potential interest produces an example. This will typically be just a "construction of the mind," possibly suggested by an inter-twining of existing constructions. How do we consider its properties? The obvious alternatives are: using econometric theory, or using simulations, or by the use of empirical applications with actual data. The basic stages in an application are fit (estimate), evaluate, and interpret.

It is usual for a new model to be investigated by these alternative routes simultaneously by different workers. The natural question arises, how much attention should one group pay to the results being obtained by others?

There are often too many topics arising from a new construction so the question arises which topics to work on and which to neglect, for the time being. Do the results by one group (e.g. applications) affect research discussions of another in this direction (e.g. theorists)?

Quite often the "knife-edge" or "bifurcation" situations are particularly interesting when changing a parameter by a small amount results in a dramatic change in the properties of the process. A well known example is the AR(1) process

$$X_t = \alpha X_{t-1} + \varepsilon_t \ . \qquad (1)$$

ε_t is iid, zero mean, say, and α is greater than one. This particular bifurcation has been studied to a considerable extent. However, an equally interesting case arises from the fractionally integrated process

$$(1 - B)^d X_t = \varepsilon_t$$

where ε_t is now iid with mean m, and X_t starts at time zero which produces a process with the properties

	Mean	Variance
$d < ½$	$m\, t^{d}$	constant
$d = ½$	$m\, t^{½}$	$c_1 \log t$
$d > ½$	$m\, t^{d}$	$c_2\, t^{2d-1}$

where c_1, c_2 are constants. There is a bifurcation at $d = ½$ which has been little studied, an exception being K. Tanaka (1999).

Other topics relatively little studied include the stochastic unit root process

$$X_t = e^{\alpha_t} X_{t-1} + \varepsilon_t \tag{2}$$

$$E[e^{\alpha_t}] = 1 .$$

Here X_t does not cause α_t and α_t is stationary. They are discussed by Granger and Swanson (1997, *Journal of Econometrics*) and are found to be empirically useful. Another class of processes have been designated I(0^+) as they have the long-memory property of having a spectrum tending to zero in frequency becomes small, and are discussed in Ding and Granger (1996, *Journal of Econometrics*) and were re-discovered by M. Linden (1999, *Economic Letters*). An example is

$$[\log(1 - B)^{-1}]^{q} X_t = \varepsilon_t$$

which has a spectrum approximately $(\log w)^{2q}$ for small *w*, being dominated by the spectrum of an I(*d*) process for any positive *d*.

2. Links

2a. Theoretical Results to Which Applied Econometricians Should Pay Attention

There are many examples here, processes that are stationary for certain sets of parameters, or stable, or non-explosive, or not trending, etc. Methods of estimation have good properties and those that should be avoided. Applied econometricians have to pay attention to theoretical developments.

2b. Who Should Pay Attention to the Results From Simulations?

Everyone, to some extent as simulations may suggest problems and difficulties before theories or applications are publicized. Theorists can use simulation results to check the correctness of their own results.

2c. Should Theorists Pay Attention to the Results of Applied Studies?

I would claim that the answer is obviously "yes", but that in practice they often do not, or many times only do so selectively. The applied workers will find areas where there are particular types of difficulties and topics which are potentially useful to applied economists, decision markers, and others. They may also find no evidence that certain topics arise in the actual economy. However, some theorists may find certain topics of major interest even if they are of little practical value. I should emphasize that I strongly believe that everyone should be allowed to do whatever research they want. I also believe that it is socially desirable to do research that is practically useful. Certainly when any of us, applied or theoretical, apply for research grants we emphasize how widely useful the results will be. I believe that the very best theorists devise news tools to be able to investigate questions thrown up by applications or simulations, and could give examples. However most theorists chose topics that are claimed to be mathematically interesting, meaning that they can be tackled with the portfolio of techniques already available, combined with a set of simplifying assumptions, such as normality, which may not agree with reality.

The result can be that a great deal of work falls into the area of "empty boxes" - see the discussion by J.H. Clapham in the *Economic Journal* in 1922 or the entry of this title in the "New Palgrave Dictionary of Economics." This area is about theory -either economic or econometric - on topics that do not arise in the actual economy.

In my opinion such areas include:

i. infinite variance (stable distribution) processes which arose from a mis-interpretation of a correct piece of statistical theory by B. Mandelbrot. The mis-interpretation was pointed out by several writers son after the topic was introduced, including Osborne and myself (1969);

ii. deterministic chaotic processes which I understand is a controversial viewpoint and depends on how chaos is defined and therefore tested;

iii. and I will argue below that factional integrated processes, I(d) $0 < d < 1$, appear to fall into this same category.

3. Do fractionally Integrated Processes Occur in the Actual Economy?

The question asked is being considered empirically: is there any empirical evidence that they occur? As a reminder, a fractionally integrated process is considered to be generated by

$$(1-B)^{d} X_t = \varepsilon_t \tag{4}$$

where for simplicity, ε_t will be taken to be iid, but could be invertible or stationary. The process is assumed to start at time $t = 0$ and in practice is generated by the MA(t) process

$$\begin{aligned} X_t &= (1-B)^{-d}\varepsilon_t \qquad\qquad t \geq 0 \\ \text{i.}e.\ X_t &= \sum_{j=0}^{t} \theta_j(d)\,\varepsilon_{t-j} \end{aligned} \tag{5}$$

where $\theta_j(d) = \Gamma(j+d)[\Gamma(d)\Gamma(j+1)]$ so $\theta_j \simeq c_1 j^{\,d-1}$ for j not small.

The spectrum of X_t is approximately $c_2 w^{-2d}$ for small frequency w and t large, where here c_1, c_2 are constants. The fraction of the process that its spectrum is unbounded at zero frequency has been termed "long-memory". The autocorrelations also take a distinct shape, falling quicker with lag than an I(1) process but slower than a stationary AR(*p*) process with fixed, finite *p*. The distinguishing feature of the process is at very low spectral frequencies and information accumulates slowly there, so a long series is required to obtain a good estimate of *d*. It follows that most macroeconomic series are not long enough to provide estimates of *d* that are significantly different from the "attracting" and relatively easily interpreted values of 0 and 1. For an actual I(*d*) series to be correctly attributed, it would need to be quite long and to have a true (or apparent) *d* value rather different from 0 and 1. In my opinion only one class of data has presented such evidence, and that is the volatility series from speculative price series. Although the evidence is wide-spread I will deal just with daily absolute returns from stock indices, stock prices, commodity prices, and interest rates as examined in Ding, Granger, and Engle (1993) and Granger, Ding, and Spear (1999). Absolute returns show the strongest evidence of long memory and produce correlograms which remain positive and apparently significant up to lag 3000 and more. These absolute returns do include a number of substantial outliers, but removing values outside 4σ does not appreciably change the shape or magnitude of the correlogram. Some of the basic features of this data (which consist of 17,000 daily terms):

i. The correlation has the I(*d*) shape and using the full sample, the estimate of *d* is just below 0.5. Removing outliers slightly increases this estimate;

ii. if the sample is arbitrarily broken down into ten sub-samples of equal size (approximately 1700), the estimates of *d* vary considerably from under 0.3 to over 0.7;

iii. the distribution of the absolute returns, after removal of the outliers (which are 7 per 1000 terms), closely resembles the exponential, according to the first four moments (mean = standard deviation, skewness = 2, kurtosis = 9).

How do these "facts" correspond to the idea that absolute returns are I(d)? The first is that $|r_t|$ is measurably positive and so in the moving average representation (4). The input shocks must also be positive if they are iid and thus will have non-zero mean m. It follows, from the theory of I(d) processes that $|r_t|$ should have a trend mt^d which on average will be approximately $mt^{½}$. The plot of $|r_t|$ against time shows no apparent trend, and so this is one mark against the I(d) model.

A great deal of I(d) research is based on the assumption that the input shocks are iid and Gaussian, but it is seen that the best example of the process in economics cannot have this property. Actually, the situation is rather more extreme. Consider equation (5) and ask, what distribution must the ε_t have so that when placed into this long moving average with these given weights, for a particular d, will produce an x_t that is exponential (that is, even forgetting the outliers). Truncating at t = 150, Granger and Jeon (1999) solved this question numerically and found, for example

d	Skewness	Kurtosis
0.26	2.52	11.40
0.46	5.16	29.41

Thus, the shocks to the system would have to be drawn from an extraordinary distribution so that after being passed through the I(d) moving average filter, they produced an exponentially distributed output. An approximate distribution might be a chi-squared with fractional degrees of freedom, having ν = 1.27 degrees of freedom for d = .25 and ν = 0.30 for d = .45. It should be remembered that this is **after** removing the substantiall outliers from the absolute returns process.

If there are difficulties with the absolute returns having an I(d), fractional d, model, is there a plausible alternative model? The alternative clearly cannot be linear but there are a wealth of possible nonlinear models, one of which is discussed in Granger and Teräsvirta (1999), taking the very simple form

$$X_t = \text{ sign } X_{t-1} + \varepsilon_t$$

ε_t is iid $N(0, \sigma^2)$ for suitable choices of σ^2. This model can give a suitable appearing correlogram but the appearance of its plot against time is not appropriate, looking like a simple regime-switching model with just two regimes at 1 and -1.

A potentially more realistic model, with the desired correlogram and plot, is the multiple break model. If

$$X_t = M_t + \varepsilon_t$$
$$M_t = M_{t-1} + q_t \eta_t$$

where ε_t, η_t are both iid ($\varepsilon_t$could be stationary AR) and

$$\begin{aligned} q_t &= 0 \quad \text{with prob } p \\ &= 1 \quad \text{with prob } 1-p \end{aligned}$$

with $var\,\eta \gg var\,\varepsilon$ and p near 1, then M_t will be constant for lengthy periods. It will then, at random intervals, have a break of random size and this size will sometimes be large, so that persistent breaks are obtained. The frequency of these large breaks in a finite but long data sample will determine the shape of the correlogram. Often a shape similar to that of a fractionally integrated processes can be obtained. In a recent working paper a number of experiments were conducted:

i. A break process was generated and an I(d) process fitted, with $\hat{d}$ values ranging from 0.2 to 0.75 being obtained by varying p and σ_η^2 whilst keeping $\sigma_\varepsilon^2 = 1$.

ii. An I(d) process was generated, using an aggregation of AR(1) processes, for various values of d. A break process was estimated using the technique suggested by Bai (1997), which can produce several breaks. d was estimated for the original data X_t, for the estimated break process $\hat{M}_t$, and for $X_t - \hat{M}_t$. It was found that $\hat{d}$ for X_t was near the true value; on average $\hat{d}$ for $\hat{M}_t$ was much greater than the true value and $\hat{d}$ for the residual series was negative, as though removing breaks over differenced the data.

iii. The series of 17000 daily absolute returns for the S&P 500 index and a break process fitted to it using the Bai procedures to each of the ten equal sized samples. The absolute return series $|r_t|$ had estimates ranging from 0.154 to 0.715 and all appeared to be significantly different from both 0 and 1. Similarly the estimates of d from the break process B_t ranges 0.350 to 0.746 and all were significantly different from 0 and 1. However the different processes $|r_t| - B_t$ had estimates of d ranging from -0.335 to 0.008, with three significantly below 0, the remainder not significantly different from 0 and two positive and small. If the series $|r_t|/B_t$ is formed, which might be thought appropriate for a positive series, $\hat{d}$ is always positive, but for all ten sub-samples is never significantly bigger than zero.

The details of the analysis are not important in this discussion. What appears to be apparent is that a stochastic break process can produce many of the "long-memory" properties of the absolute return data without some of the other model difficulties. It thus becomes a plausible alternative model to I(d), and I am sure that there are many other possible non-linear candidates. The choice will be partly based on statistical criteria and economic interpretability. Further details can be found in the working paper by Granger and Hyung (1999).

4. Conclusion

I conclude that a good case can be made for fractional I(d) processes falling into the "empty box" category and that this should influence future research.

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