DSGE Model Econometrics

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Papers and software available at https://web.sas.upenn.edu/schorf/

1996-2000: Work by DeJong, Ingram, Whiteman, Otrok (Iowa); Geweke, Landon-Lane (Minnesota); myself (Yale) – Bayesian estimation and model evaluation; posterior simulation.


Early-mid 2000s: Incorporation of Bayesian estimation tools into DYNARE.

Mid 2000s: Central banks (Riksbank in particular) started to use / develop / take seriously DSGE models.


Subsequently: Widely-used in academia and policy-making institutions.
most people outside the discipline who take one look at these models immediately think they’re kind of a joke.

They contain so many unrealistic assumptions that they probably have little chance of capturing reality. Their forecasting performance is abysmal.

Some of their core elements are clearly broken. Any rigorous statistical tests tend to reject these models instantly, because they always include a hefty dose of fantasy.
The Trouble With Macroeconomics

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Wednesday 14th September, 2016

(...) To replicate the results from that model (the Smets and Wouters 2007 model), I read the User’s Guide for the software package, DYNARE, that the authors used. In listing the advantages of the Bayesian approach, the User’s Guide says: “Third, the inclusion of priors also helps identifying parameters.” This was a revelation. Being a Bayesian means that your software never barfs.

(...) It was news to me that priors are vectors for facts with unknown truth values (FWUTV), but once I understood this and started reading carefully, I realized it was an open secret among econometricians.
“Are you just pissing and moaning, or can you verify what you’re saying with data?”
Four Steps

1. Model Solution
2. Model Estimation
3. Model Assessment
4. Substantive Analysis with Estimated Models
Step 1 – Model Solution
Step 1: Model Solution

- **Log-linearization of equilibrium conditions**: (-) linear, (+) fast, (+) leads to linear state-space model, (+) likelihood is easy to compute.

- **Higher-order perturbation solution**: (0) a bit slower but numerically stable, (+) smooth nonlinear dynamics, good for welfare analysis, (-) likelihood evaluation requires nonlinear filter.

- **Global / projection methods**: (+) approximate decision rules by flexible fcn $\delta(S_t; \Theta)$, (+) can handle occasionally-binding constraints, (-) time-consuming, (-) delicate, (-) requires carefully chosen grid, (-) requires nonlinear filter to evaluate likelihood.

- Perturbation solutions capture some nonlinearities but not all → not well suited for occasionally-binding constraints.

- Example: ZLB/ELB for nominal interest rates

\[ R_t = \max \{1, R_t^* e^{εR,t} \}, \quad R_t^* = \left[ rπ_∗ \left( \frac{π_t}{π_∗} \right)^{ψ_1} \left( \frac{Y_t}{Y_∗} \right)^{ψ_2} \right]^{1−ρ_R} R_{t−1}^{ρ_R}. \]

- Three Challenges:
  1. capture “kinks” in decision rules;
  2. solution needs to be accurate in region of state-space that is relevant according to model AND according to data;
  3. multiple equilibria.
Challenge 1: Kinks... Sample Decision Rules - Small-Scale NK Model

- Interest Rate
- Inflation
- Output
- Consumption

F. Schorfheide

DSGE Model Econometrics
Choose $\Theta$ to minimize sum squared residuals from the (intertemporal) equilibrium conditions over particular grid of points in state space.
In a NK model with passive fiscal policy...
Step 2a – Model Estimation
Likelihood Evaluation
Bayesian Inference

- Implemented by sampling draws $\theta^i$ from posterior:

$$p(\theta|Y) = \frac{p(Y|\theta)p(\theta)}{p(Y)}.$$  

- Posterior samplers require evaluation of likelihood function:
  $\theta \rightarrow$ model solution $\rightarrow$ state-space representation $\rightarrow$ $p(Y|\theta)$.

- State-space representation $\rightarrow$ $p(Y, S|\theta)$:

$$y_t = \Psi(s_t, t; \theta) + u_t, \quad u_t \sim F_u(\cdot; \theta)$$
$$s_t = \Phi(s_{t-1}, \epsilon_t; \theta), \quad \epsilon_t \sim F_\epsilon(\cdot; \theta).$$

- In order to obtain $p(Y|\theta) = \prod_{t=1}^{T} p(y_t|Y_{1:t-1}, \theta)$ we need to integrate out latent states $S$ from $p(Y, S|\theta) \rightarrow$ use filter:

  - Initialization: $p(s_{t-1}|Y_{1:t-1}, \theta)$
  - Forecasting: $p(s_t|Y_{1:t-1}, \theta), p(y_t|Y_{t-1}, \theta)$
  - Updating: $p(s_t|y_t, Y_{1:t-1}, \theta) = p(s_t|Y_{1:t}, \theta)$. 
**Particle Filtering**: represent \( p(s_{t-1}|Y_{1:t-1}) \) by \( \{s^j_{t-1}, W^j_{t-1}\}_{j=1}^M \) such that

\[
\frac{1}{M} \sum_{j=1}^{M} h(s^j_{t-1}) W^j_{t-1} \approx \int h(s_{t-1}) p(s_{t-1}|Y_{1:t-1}) ds_{t-1}.
\]

**Example: Bootstrap particle filter**

- **Mutation/Forecasting**: turn \( s^j_{t-1} \) into \( \tilde{s}^j_{t} \): sample \( \tilde{s}^j_{t} \sim p(s_t|s^j_{t-1}) \).

- **Correction/Updating**: change particle weights to:
  \( \tilde{W}_t^j \propto p(y_t|\tilde{s}^j_{t}) W^j_{t-1} \).

- **Selection** (Optional): Resample to turn \( \{\tilde{s}^j_{t}, \tilde{W}_t^j\}_{j=1}^M \) into \( \{s^j_{t}, W^j_t = 1\}_{j=1}^M \).

**Problem**: naive forward simulation of Bootstrap PF leads to uneven particle weights

\( \rightarrow \) inaccurate likelihood approximation!
Smets-Wouters Model (Linearized)

Bootstrap PF \((M = 400,000)\) versus Cond. Optimal PF \((M = 4,000)\)

Density estimates of \(\hat{\Delta}_1 = \ln \hat{p}(Y|\theta) - \ln p(Y|\theta)\) based on \(N_{\text{run}} = 100\).

Bootstrap PF ($M = 40,000$) is dashed; Cond-opt. PF ($M = 400$) is dotted.

Step 2b – Model Estimation

Posterior Inference
We are trying to learn the parameters $\theta$ from the data.

Formal definitions... e.g., model is identified at $\theta_0$ if $p(Y|\theta) = p(Y|\theta_0)$ implies that $\theta = \theta_0$.

In the early DSGE days, lack of identification did not seem an issue.

Over time, it emerged as an important problem.

Without identification or with weak identification:

- use more/different data to achieve identification;
- use identification-robust inference procedures.

Lack of identification does not raise conceptual issues for Bayesian inference (as long as priors are proper), but possibly computational challenges.
Lack of Identification: Two Examples

1. Monetary policy rule coefficients

\[ \hat{R}_t = \psi \hat{\pi}_t + \epsilon_{R,t}. \]

2. Distinguishing internal propagation (e.g., partial indexation of prices to past inflation) from external propagation (e.g., persistent price mark-up shocks)
The Role of Priors

- **Ideally**: probabilistic representation of our knowledge/beliefs before observing sample $Y$.

- **More realistically**: choice of prior as well as model are influenced by some observations. Try to keep influence small or adjust measures of uncertainty.

- **DSGE model literature**: use priors to incorporate information from sources other than estimation sample. Useful to group parameters:
  1. steady state related;
  2. endogenous propagation;
  3. exogenous shock.

- **In other literatures**:
  1. keep them “uninformative” (???) so that posterior inherits shape of likelihood function;
  2. use them to regularize the likelihood function;
Lack of Identification as Computational Challenge

Remedy: Sequential Importance Sampling

\[
\pi_n(\theta) = \frac{[p(Y|\theta)]^{\phi_n} p(\theta)}{\int [p(Y|\theta)]^{\phi_n} p(\theta) d\theta}, \quad \phi_n = \left(\frac{n}{N_\phi}\right)^\lambda
\]

Smets-Wouters (Diffuse Prior) Posterior: Internal $\xi_w$ versus External $\rho_w$ Propagation

Once a reasonably accurate likelihood approximation has been obtained, it can be embedded in a posterior sampler.


Potential shortcuts:

- less accurate model solution;
- cruder state extraction / likelihood approximation;
- non-likelihood-based parameterization of model.

Schorfheide, Song, Yaron (2017): slight short-cut in model solution $\rightarrow$ conditionally-linear state-space representation $\rightarrow$ efficient particle filter approximation of likelihood $\rightarrow$ full Bayesian estimation.
Step III – Model Assessment
1980s: can DSGE model reproduce key sample correlations, e.g. between output and hours worked or output and inflation? Compare model-implied correlations and sample correlations computed from actual data.

1990s: do impulse responses to, say, unanticipated changes in monetary policy, from a DSGE model look like impulse responses from a vector autoregression (VAR)?

2000’s: can DSGE models track and forecast key macroeconomic time series?

The literature has developed numerous econometric tools to provide formalize the evaluation.

How to think about DSGE models...
Abysmal Forecasting Performance?

DSGE Model versus Blue Chip (1992-2011)

Output Growth

Inflation

Interest Rates

- $h = 1$ is current quarter nowcast.
- Growth rates, inflation rates, interest rates are QoQ %

Source: Del Negro and Schorfheide (2013): “DSGE Model-Based Forecasting,” In Handbook of Economic Forecasting.
Abysmal Forecasting Performance?

RMSE ratios: DSGE / AR(2)

Source: Del Negro and Schorfheide (2013): “DSGE Model-Based Forecasting,” In Handbook of Economic Forecasting.
We do not forecast because the DSGE is “good” at forecasting—we forecast with the DSGE to test the model.
Macroeconomists/econometricians have been criticized for relying on models that abstract from financial intermediation / frictions.

With hindsight it turned out that financial frictions were important to understand the Great Recession. But are they also important in normal times?

We need tools that tell us in real-time when to switch models...

Linear prediction pool:

\[ \text{Density Forecast}_t = \lambda_t \cdot \text{Forecast from “Normal” Model}_t + (1 - \lambda_t) \cdot \text{Forecast from “Fin Frictions” Model}_t \]

Determine weight \( \lambda_t \) in real time based on historical forecast performance.

“New” Models versus “Old” Models

Relative forecasting performance changes over time

“Old” Smets-Wouters Model vs. “New” DSGE with Financial Frictions

It’s easy to see with hindsight which model we should have used.
“New” Models versus “Old” Models

Time-Varying Weight $\lambda_t$ (Posterior Distribution) on “New” DSGE with Financial Frictions

It's more difficult to determine the best model in real time...
Techniques for determining the best model in real time are available.
Step IV – Substantive Analysis with Estimated Model
A Genuine Problem With Empirical Work in Economics

NK Phillips Curve

\[ \tilde{\pi}_t = \gamma b \tilde{\pi}_{t-1} + \gamma_f E_t[\tilde{\pi}_{t+1}] + \kappa \tilde{MC}_t \]
Literature on methods and applications for DSGE models is well and alive!

Significant progress in area of model solution and estimation techniques.

More work needed on the model assessment:

- Do DSGE models generate the right nonlinearities?
- Do DSGE models capture the interaction between cross-sectional distributions and macroeconomic aggregates?