OFFSETTING CARBON WITH LEMONS: ADVERSE SELECTION AND CERTIFICATION IN THE VOLUNTARY CARBON MARKET

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Offsetting Carbon with Lemons: Adverse Selection and Certification in the Voluntary Carbon Market

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To meet voluntary climate targets, firms often complement internal decarbonization efforts by purchasing carbon credits in the voluntary carbon market (VCM), which finance projects that reduce emissions elsewhere. However, these emissions reductions are difficult to verify, and growing evidence of overcrediting has cast doubt on the VCM's potential to genuinely offset emissions. We investigate how the VCM's defining features shape its climate effectiveness. Our model captures three central elements: adverse selection, as high-quality projects that truly reduce emissions are costlier yet difficult to distinguish from low-quality ones; imperfect third-party certification, as projects are screened based on a noisy signal of quality; and buyer preferences for non-carbon attributes, as some firms value credits that generate observable social or economic co-benefits beyond reducing emissions. We show that the market fails to sustain trade if certification is sufficiently noisy, as quality uncertainty erodes buyer confidence and triggers a market-for-lemons collapse. However, demand for co-benefits can sustain markets that would otherwise collapse. Yet in such cases, the market remains active but yields limited carbon abatement, as most traded credits are low-quality. We then examine policy and market design interventions reflecting recent developments in practice, such as penalizing buyers for greenwashing and offering credit portfolios. We show that these measures can be counterproductive for carbon mitigation if certification remains inaccurate. Accordingly, we demonstrate that the certifier's incentives for accuracy can be strengthened by modifying its fee structure so that its revenue is tied to the market value rather than the volume of credits.

1. Introduction

The continued rise of greenhouse gas emissions has pressured businesses to lower their environmental footprint, leading firms across industries to voluntarily pledge emissions reductions, in some cases setting ambitious net-zero targets within limited timeframes. While the preferred path to achieve these goals is to decarbonize their operations and transition to renewable energy sources, many industries face hard-to-abate emissions that are prohibitively costly or technically infeasible to eliminate. To offset these residual emissions, some firms turn to the voluntary carbon market (VCM), purchasing carbon credits that finance projects that reduce emissions elsewhere.

Carbon credits originate from a diverse array of projects that employ different methods to mitigate emissions. Examples include nature-based solutions (e.g., reforestation), community-focused initiatives (e.g., clean cookstove distribution), and technological projects ranging from industrial abatement (e.g., landfill gas capture) to engineered carbon removals (e.g., direct air capture). In

2024, firms globally offset approximately 180 million tons of CO_2-equivalent using credits valued at USD 1.4 billion. Analysts project the VCM to expand to USD 7–35 billion by 2030 and reach USD 45–250 billion by 2050 as pressure to meet climate targets grows (MSCI Carbon Markets 2025).

While carbon credits enable firms to claim that their residual emissions are offset, the credibility of such claims hinges on whether the underlying projects genuinely reduce emissions. In particular, projects are evaluated against several integrity criteria, such as *additionality* and *leakage*, which require that the credited emissions reductions would not have occurred without the project, and that the project does not simply shift emissions to other sources (Salzman and Weisbach 2024). For example, a forest conservation project must demonstrate not only that the protected area was truly at risk of deforestation, but also that preserving it does not lead to increased logging or land conversion elsewhere. Verifying these conditions is inherently challenging, as it requires assessing unobserved counterfactual scenarios (i.e., what would have occurred in the absence of the project), making it difficult to distinguish between high- and low-integrity credits.[1]

Given these challenges, carbon registries such as Verra and Gold Standard play a central role in the VCM by certifying projects against integrity standards and issuing credits intended to represent one ton of CO_2-equivalent emissions reductions. However, both academic research and investigative journalism have documented widespread instances of overcrediting, raising questions about the credibility of their certification processes (West et al. 2020, Greenfield 2023, Blake 2023, Probst et al. 2024, Gill-Wiehl et al. 2024). These concerns have sparked a crisis of confidence in the VCM, casting doubt on its potential to deliver genuine emissions reductions. In particular, imperfect certification creates scope for adverse selection, as buyers cannot reliably distinguish high-quality credits from low-quality ones, allowing spurious offsets to remain in the market.

Beyond these supply-side dynamics, the demand side of the VCM has its own distinct characteristics. Credits vary along several dimensions that buyers value differently, including price, project type, and geographic location. One particularly salient dimension is whether a project generates social or economic *co-benefits* in addition to reducing carbon emissions (Walsh and Toffel 2023). For instance, reforestation projects often provide employment opportunities for local communities, while clean cookstove initiatives improve public health. Such co-benefits are often valued by firms seeking to align offset purchases with their brand values and broader ESG objectives (Boston Consulting Group 2023). However, prioritizing these attributes introduces a tension between buyers' demand for non-carbon attributes and carbon integrity: projects with salient co-benefits (e.g., forest conservation, cookstoves) often face higher overcrediting risks than engineered removals, which are

[1] Notably, as the International Organization of Securities Commissions (IOSCO) has stated, "the projects underlying carbon credits appear to exhibit a wide variance in quality related to whether the project has actually avoided or removed the GHG emissions as claimed" (US Congressional Research Service 2024).

generally more reliable in reducing emissions but do not usually produce such co-benefits (Probst et al. 2024, Ferris 2025). As a result, an emphasis on co-benefits may shift demand towards credits that, despite generating social value through other channels, are not necessarily the most effective at delivering genuine emissions reductions.[2]

The distinctive features of the VCM, together with concerns about overcrediting and greenwashing, have sparked debate over whether the market can function effectively. While acknowledging these challenges, proponents view the VCM as a key mechanism for financing and scaling decarbonization efforts and argue that refinements in certification methodologies can address existing integrity concerns (Moore et al. 2023, Mak 2025). Recognizing this potential, policymakers have underscored that a high-integrity VCM can play an important role in achieving global climate targets.[3] Critics, however, argue that the VCM enables firms to make unfounded net-zero claims, diverts resources from more effective decarbonization efforts, and suffers from structural flaws that are difficult to overcome (Cullenward et al. 2023, Romm et al. 2025).

This paper contributes to this discussion by developing a theoretical framework to analyze how the VCM's climate impact and overall outcomes are shaped by its fundamental market characteristics. Specifically, we study how adverse selection, imperfect certification, and the nature of buyer preferences collectively influence the market's effectiveness, and how policy changes and market design interventions may affect the environmental integrity of the VCM. We next describe our main contributions.

Model of the VCM. We develop a stylized model that captures the key participants and information frictions in the VCM. Sellers represent offset projects that differ in their quality, which is privately known and reflects whether each credit genuinely offsets one ton of CO_2-equivalent, and in whether they generate (observable) co-benefits. Buyers are heterogeneous in their preferences over credits, with some placing greater value on co-benefits relative to genuine emissions reductions. The certification process is modeled through a third party that screens projects before they enter the market, admitting only those for which it observes a high-quality signal. The accuracy of this signal determines the extent to which low-quality projects are filtered out, shaping the composition of supply and the equilibrium prices that clear the market.

Consistent with canonical models of asymmetric information, the degree of adverse selection is determined by the effectiveness of the screening process. High-quality projects, which are more

[2] Berg et al. (2025) provide empirical evidence supporting this effect, as price patterns reflect a sizeable premium for co-benefits. They interpret this as evidence that buyers pay for "emotionally salient project features, possibly including positive externalities for local communities, rather than for emission reductions per se."

[3] For example, The White House et al. (2024) VCM Joint Policy Statement argues that the VCM "can and should play a meaningful role in [...] helping to reach global net-zero emissions by 2050," while noting that "integrity concerns must also be addressed in order for credits to truly drive decarbonization."

costly to implement, enter the market only if they anticipate prices to cover their costs, whereas low-quality projects attempt to enter even at low prices. If certification is perfectly accurate, adverse selection disappears as only high-quality projects are admitted. This gives buyers confidence in the integrity of credits, which supports high prices that attract broad participation by high-quality sellers, producing the maximum possible level of offset emissions. However, if certification is noisy, low-quality credits increasingly pass through, eroding buyers' confidence and leading to lower prices that drive high-quality supply out of the market. As a result, imperfect certification not only enables the circulation of credits that represent no genuine abatement but also reduces the volume of emissions that are truly offset. Finally, if the certification error exceeds a threshold, the unresolved quality uncertainty collapses the market—mirroring the classic *market for lemons* unraveling in Akerlof (1970)—with prices dropping to zero and all high-quality sellers exiting.

While market collapse is typically viewed as the worst-case outcome, our analysis distinguishes between market activity and effective carbon mitigation, which leads to distinct normative implications compared to traditional models of adverse selection. Specifically, we show that *market survival does not guarantee environmental efficacy.* Between the ideal scenario of a high-integrity market and the extreme of total collapse, we characterize an intermediate "climate-negative" region where certification is reliable enough to sustain trade, but too noisy to effectively filter out low-quality projects. Consequently, the market clears at low prices that largely crowd out high-quality supply, and trade is dominated by spurious credits. In this state, the market remains active but the social costs imposed by low-quality credits—interpreted as enabling greenwashing or the displacement of more effective abatement efforts—exceed the limited genuine emissions reductions achieved, resulting in a negative climate gain relative to the counterfactual of an inactive market.

Understanding the role of co-benefits. We then investigate how buyers' preferences for co-benefits shape market outcomes. On the one hand, stronger preferences for co-benefits can further decouple market activity from carbon integrity. By prioritizing attributes not directly tied to carbon abatement, buyers may sustain demand for credits with visible social appeal that, while desirable in other dimensions, may be less likely to deliver genuine emissions reductions. In particular, we show that as buyers place greater value on co-benefits, markets that would otherwise collapse due to inaccurate certification may instead remain active. However, in such marginal markets—which are sustained primarily by co-benefits—the majority of traded credits are low-quality, so the quantity of genuine emissions reductions is small relative to the volume of issued credits. In contrast, when certification is sufficiently accurate to support high-quality trade, stronger preferences for co-benefits can improve climate outcomes, as buyers' additional willingness to pay raises equilibrium prices and attracts more high-quality supply into the market. Altogether, our results indicate that *co-benefits can function as a double-edged sword within the VCM*: while preferences that diverge from the primary objective of carbon abatement can amplify the mitigation potential of well-functioning markets, they may also perpetuate greenwashing by maintaining the appearance of a functioning market that, in reality, delivers little or no emissions reductions.

Analysis of Policy and Market Design Interventions. Having characterized the conditions under which the market functions effectively, we apply our framework to evaluate the efficacy of select policy and market design interventions reflecting recent developments in the VCM. We first study a measure targeting the demand side. Motivated by growing scrutiny of firms' use of offsets and heightened reputational risks associated with greenwashing, we examine the effects of penalizing buyers for purchasing low-quality offsets.[4] Although such penalties aim to discourage greenwashing, we show that they can instead backfire by exacerbating adverse selection. Without improvements in certification, penalties reduce buyers' willingness to pay as they account for the risk of using low-quality credits. This lowers equilibrium prices, deterring participation by high-quality projects without improving the screening of low-quality supply.

Turning to the supply side, we examine the emerging practice of pooling credits into portfolios that give buyers exposure to a mix of projects rather than individual credits.[5] To understand the implications of this shift, we analyze an alternative market structure in which credits are bundled into a composite product, and contrast it with our baseline setting where individual credits trade separately. We find that the implications of pooling depend critically on certification accuracy. On one hand, pooling allows the premium that buyers pay for co-benefits to be shared across the portfolio. When certification is accurate, this acts as a subsidy that raises the price perceived by high-quality projects without co-benefits, incentivizing them to enter and increasing total abatement. However, under the pooled structure, all credits effectively trade at a single price, eliminating price differentiation across project types. This coarser price structure restricts the market's ability to reflect projects' quality differences through prices, generating an informational loss that becomes particularly consequential when certification is noisy. In such cases, pooling leads to reduced carbon abatement by amplifying the existing information frictions that remain unresolved by certification alone.

Taken together, our analysis of these interventions illustrates that measures targeting only the demand or supply side of the VCM can be counterproductive for carbon mitigation if not paired with more accurate certification. We therefore examine the certifier's incentives to maintain rigorous standards, modeling certification accuracy as a strategic choice rather than as exogenous.

[4] For example, California has enacted legislation mandating firms to disclose their offset portfolios (AB 1305), while Apple has faced litigation over carbon-neutrality claims that relied on allegedly low-quality offsets (Calma 2025).

[5] In particular, platforms such as Rubicon Carbon and Patch offer credit portfolios similar to ETFs in financial markets. Other organizations have experimented with blockchain-based carbon pools, which aggregate different credits into standardized tokens that trade as a single asset (e.g., Toucan BCT).

In practice, registries earn a fixed fee per issued credit, tying their revenue to the volume of credits they certify. Our analysis highlights that registries may have weak incentives to pursue fully accurate certification under this fee structure, as the volume of credits in the market is maximized at intermediate levels of accuracy, where some low-quality credits are admitted without eroding buyer confidence enough to trigger market collapse. This result is consistent with existing concerns about conflicts of interest in the certification process (Battocletti et al. 2024, Coglianese and Giles 2025). As a potential remedy, we consider an alternative scenario in which the certifier's objective is aligned with maximizing the total value of trade (i.e., the product of prices and quantities), as would occur if fees were set as a percentage of credit prices. This structure aligns the certifier's incentives toward perfect accuracy, since its revenue now depends on market prices, which, unlike quantities, closely reflect the overall quality of credits in circulation. Altogether, our results underscore that adopting mechanisms that compensate registries for rigorous screening is critical for both the success of other interventions and the effectiveness of the VCM itself.

1.1. Related Literature

Our work is related to the growing operations literature focused on environmental sustainability and decarbonization. This stream of research studies a range of sustainability practices within firms' internal operations and supply chains, including emissions accounting (e.g., Caro et al. 2013, Sunar and Plambeck 2016, Gopalakrishnan et al. 2021), environmental transparency and governance (e.g., Jira and Toffel 2013, Plambeck and Taylor 2016, Wang et al. 2016, Kraft et al. 2020, Kalkanci and Plambeck 2020), the adoption of energy-efficient technologies (e.g., Muthulingam et al. 2013, Wang et al. 2021), and the transition to renewable energy sources (e.g., Aflaki and Netessine 2017, Kök et al. 2018, Karaduman 2022, Singh and Scheller-Wolf 2022, Kaps et al. 2023); see Corbett and Klassen (2006), Agrawal et al. (2019) and Atasu et al. (2020) for comprehensive reviews of this literature. We contribute to this line of work by examining the market for voluntary carbon offsets, an external mechanism that firms use to complement internal abatement efforts as they pursue their emissions reduction goals.

More closely related is a stream of modeling work that studies the design of offset programs and markets for carbon credits. Earlier contributions in environmental economics focus on regulated offset initiatives (e.g., Van Benthem and Kerr 2013, Bento et al. 2015). More recently, as firms have increasingly set voluntary climate targets, a growing literature has begun to analyze operational aspects of carbon offsets, including firms' decisions to abate emissions internally or through offsets (Gao and Souza 2022), whether to purchase offsets at the firm level or offer customers the option to do so (Esenduran et al. 2026), and the choice of delivery terms and financing arrangements for project developers (Agrawal et al. 2024, Hou et al. 2025). Collectively, these papers highlight how the presence of eco-conscious consumers, project-yield uncertainty, and financing frictions shape the demand and supply sides of the VCM separately. However, they treat delivered offsets as homogeneous and fully effective, abstracting away from quality uncertainty and adverse selection, which are central in our analysis. A notable exception is Agrawal et al. (2026), who analyze how different registry rules shape developers' incentives to strategically report baselines and the resulting prevalence of non-additional offsets. Our work advances this stream by developing an equilibrium framework for the VCM that captures how supply-side quality uncertainty, imperfect certification, and buyers' preferences for co-benefits jointly shape the market's environmental effectiveness.

Our work also relates to the literature on adverse selection, quality uncertainty, and third-party certification. The classic paper by Akerlof (1970) shows how asymmetric information about quality can lead to market collapse, while subsequent works such as Spence (1973), Viscusi (1978), and Leland (1979) illustrate that costly signaling and quality certification can alleviate this friction. Building on these foundations, a large literature has studied intermediaries like certifiers, auditors, and rating agencies and their incentives to provide accurate information; Dranove and Jin (2010) provide a survey, while some recent contributions include Stahl and Strausz (2017), Hui et al. (2023), Haupt et al. (2024), and Feldman et al. (2025). Our work connects to this stream but differs in two respects. First, products in our setting are multi-dimensional: buyers care about both the carbon integrity of credits and their observable co-benefits, whereas adverse selection arises only along the (unobserved) carbon-quality dimension. Our model therefore captures how demand for attributes that are distinct from the dimension with quality uncertainty can sustain trade in markets with adverse selection. Second, we show that this interaction can shift the normative implications of adverse selection. While market collapse is typically treated as a worst-case outcome, our analysis shows that an active offset market can deliver limited emissions reductions relative to the volume of traded credits. In such cases, collapse may be preferable from a climate perspective, as it prevents low-quality offsets from supporting weak emissions reduction claims.

Finally, our analysis is motivated by the empirical literature that evaluates offsetting programs. While some payments-for-ecosystem-services programs have been shown to reduce deforestation and other environmentally harmful activities (e.g., Jayachandran et al. 2017, Jack et al. 2025), system-level evaluations of offsetting and conservation schemes have documented substantial non-additionality and adverse selection (e.g., Aspelund and Russo 2026, Calel et al. 2025, Chen et al. 2025). For the VCM specifically, recent studies find widespread overcrediting in various offset categories (e.g., West et al. 2020, Probst et al. 2024, Gill-Wiehl et al. 2024), highlight concerns about registry methodologies (e.g., Haya et al. 2023, West et al. 2024, Eash et al. 2025), identify discrepancies between credit prices and carbon impact (Berg et al. 2025), and discuss potential reforms to enhance the integrity of carbon credits (e.g., West et al. 2023, Anderegg et al. 2025, Romm et al. 2025). Our work complements this literature by developing a stylized equilibrium framework that captures the primary institutional features of the VCM, providing a formal structure for interpreting existing empirical findings and evaluating potential interventions in the VCM.

The rest of the paper is organized as follows. We introduce the model in §2 and analyze how certification accuracy and buyers' preferences for co-benefits shape equilibrium outcomes in §3. We then investigate policy and market design interventions in §4, study the certifier's incentives for accuracy in §5, and finally conclude in §6. All proofs are provided in the appendix.

2. Model Setup

We develop a stylized model of the voluntary carbon market (VCM) that captures the interactions between sellers, buyers, and a certifier. Sellers represent carbon-offsetting projects that vary in their *quality* (i.e., whether they genuinely offset emissions), their implementation cost, and whether they generate observable *co-benefits* beyond carbon mitigation. Buyers represent firms that consider purchasing carbon credits to offset their emissions. The certifier represents a carbon registry that evaluates projects and acts as a gatekeeper, allowing only those assessed to be high-quality to participate in the market by issuing credits.

Our framework is designed to capture three central features of the VCM. First, projects' quality and cost are private information to sellers, creating scope for adverse selection on the supply side. Second, certification is imperfect: the certifier observes only a noisy signal of quality, which determines the extent to which low-quality projects are screened out. Third, buyers value both offset emissions and co-benefits, reflecting that some firms place more weight on projects' non-carbon attributes, particularly when these align with their broader ESG goals (Boston Consulting Group 2023). We next describe our modeling framework in detail.

Sellers. There is a unit mass of potential sellers, each representing a carbon-offsetting project (we use "seller" and "project" interchangeably). Each seller is characterized by her type $\theta = (q, b, c) \in \{0,1\} \times \{0,1\} \times [0,1]$. The component q represents the project's quality: high-quality projects ($q = 1$) genuinely offset one ton of CO_2-equivalent, while low-quality projects ($q = 0$) do not result in actual emission reductions. The value of b indicates whether the project generates observable co-benefits ($b = 1$) or not ($b = 0$); we refer to class-b projects to distinguish between these two groups.[6] Finally, c denotes the project's opportunity cost, reflecting the economic value that is forgone by implementing it (e.g., alternative land use for a reforestation project).

[6] For example, cookstove programs and nature-based solutions (e.g., reforestation) fall under $b = 1$ as they often generate co-benefits such as local employment or health improvements. These attributes may produce social value but do not contribute to carbon mitigation. In contrast, engineered carbon removal methods such as direct air capture or biochar fall under $b = 0$, as they are primarily designed as technological solutions to remove CO_2 from the atmosphere.

We assume that the project class b is publicly observable, whereas quality q and cost c are privately known to the seller. Furthermore, we assume that only high-quality projects incur a positive cost, while low-quality projects have $c = 0$. This reflects that low-quality projects often correspond to non-additional activities, in the sense that the revenue from carbon credits does not generate additional emissions reductions relative to the counterfactual. Examples include renewable energy projects that are already profitable, as well as forest conservation efforts in areas that face low deforestation risk. Moreover, we assume that high-quality sellers are heterogeneous in their cost c, which is drawn from a known continuous distribution with CDF G. For simplicity, we assume that this cost distribution is the same across both project classes.

Let $\beta \in (0,1)$ denote the fraction of class-1 projects (i.e., those with co-benefits), and let $\phi_b \in (0,1)$ denote the share of high-quality projects within each class $b \in \{0,1\}$. That is,

$$\beta = \mathbb{P}[b=1], \quad \phi_1 = \mathbb{P}[q=1 \mid b=1], \quad \phi_0 = \mathbb{P}[q=1 \mid b=0].$$

The prior distribution over seller types is thus characterized by the parameters (β, ϕ_0, ϕ_1) and the cost distribution G, all of which are common knowledge. In our analysis, we assume that G is the uniform distribution on $[0,1]$.

Importantly, our framework allows for correlation between a project's quality and co-benefits, as the parameters ϕ_0 and ϕ_1 may differ. While our analysis does not require a particular ordering of these parameters, we occasionally illustrate our results in the empirically relevant regime where class-1 projects—those with co-benefits—have lower expected quality, i.e., $\phi_1 < \phi_0$. This reflects concerns that have emerged in practice: project categories that tend to emphasize their social co-benefits, such as reduced deforestation and clean cookstove initiatives, have been found to systematically overstate their carbon impact and exhibit greater additionality risk compared to engineered carbon removal methods such as biochar or direct air capture.[7]

After observing her type, each seller decides whether to enter the market. Those who enter incur their cost c and undergo the certification process (described below). Only sellers who pass the certification are allowed to issue a credit, which is then sold at the market price. A seller's total payoff equals the credit's price (if certified), net of her implementation cost. Sellers who opt not to enter receive a payoff of zero.

[7] West et al. (2020) and Gill-Wiehl et al. (2024) document substantial overcrediting in forest conservation and cookstove projects, while Ferris (2025) finds a negative correlation between project integrity ratings (a measure of carbon quality) and contributions toward sustainable development goals (which reflect co-benefits).

Certifier. The certifier observes a noisy signal of quality for each seller that enters the market. Signals are generated by a mechanism σ that classifies each project as either high- or low-quality ($s = H$ or $s = L$). We assume that high-quality projects are always classified correctly, while low-quality projects are misclassified with probability $\epsilon \in [0,1]$, which represents the certifier's false positive rate. Accordingly, quality signals are drawn from the following distribution:

$$\sigma(s = H \mid q = 1) = 1, \qquad \sigma(s = L \mid q = 1) = 0,$$
$$\sigma(s = H \mid q = 0) = \epsilon, \qquad \sigma(s = L \mid q = 0) = 1 - \epsilon.$$

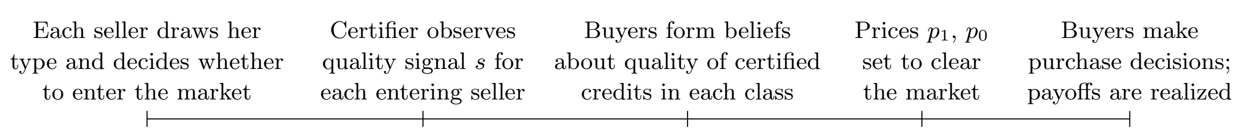
Note that the certification is perfectly accurate when $\epsilon = 0$ and completely uninformative when $\epsilon = 1$. For simplicity, we assume that the certifier's accuracy does not vary across project classes (i.e., the distribution σ is the same for projects with $b = 1$ and $b = 0$). We initially treat the certification error ϵ as fixed, and subsequently model it as a decision of the certifier in §5.

Buyers. There is a unit mass of buyers, each of whom decides whether to purchase a certified carbon credit. All buyers obtain a baseline utility of $u = 1$ when purchasing a high-quality credit, while a low-quality credit yields zero utility. In addition, buyers differ in the extent to which they value co-benefits. Each buyer is characterized by his type, $v \in [0,1]$, which captures his additional willingness-to-pay for a high-quality credit that also offers co-benefits. Specifically, a type-v buyer is willing to pay $1 + \gamma v$ for a high-quality class-1 credit, where the parameter $\gamma \geq 0$ scales the weight that buyers place on co-benefits. Buyer types are drawn from a known distribution $v \sim F$, which we take to be uniform on $[0,1]$. Given his type v, a buyer's net utility from purchasing a class-b credit with quality q at price p is thus given by

$$U(v,q,b,p) = \begin{cases} (1+\gamma v b)q - p, & \text{if he buys the credit,} \\ 0, & \text{otherwise.} \end{cases} \tag{2.1}$$

This specification captures three key features of the demand side in the VCM. First, assigning zero utility to low-quality credits reflects that firms generally recognize the risks associated with such credits (e.g., accusations of greenwashing) and seek to avoid them. Second, the multiplicative form of the utility captures lexicographic preferences over quality and co-benefits: buyers primarily purchase offsets to support a credible carbon-neutrality claim, and treat co-benefits as a complementary attribute that strengthens the narrative around such commitments. In particular, buyers derive value from co-benefits only if the underlying credit represents genuine emissions reductions. Finally, allowing buyers' valuation of co-benefits to vary reflects heterogeneity in firms' ESG priorities and their desire to align their offset purchases with broader sustainability goals.

Figure 1 **Timeline of events in the model**



Timeline of events. Figure 1 illustrates the sequence of events in the model. First, each seller observes her type $\theta = (q, b, c)$ and decides whether to enter the market. Sellers who enter incur their cost c and are evaluated by the certifier, which allows only those receiving a high signal to issue a credit. Buyers then form beliefs about the likelihood that a certified credit of each class is high-quality. Prices for credits of each class, denoted by p_1 and p_0, are then set to clear the market. Finally, buyers make purchase decisions and payoffs are realized.

2.1. Equilibrium Concept and Market Outcomes

Informally, an equilibrium in our model consists of (i) a pair of market-clearing prices $\boldsymbol{p} = (p_0, p_1)$ for certified credits of each class; (ii) a pair of buyer beliefs $\boldsymbol{\mu} = (\mu_0, \mu_1)$, representing the probability that a certified credit of each class is high-quality; and (iii) optimal entry and purchase decisions for sellers and buyers, given the prevailing prices and beliefs. We next introduce each of these components and formally define the conditions they must satisfy in equilibrium.

Buyers' decisions. A buyer strategy is a measurable function that maps each buyer type $v \in [0, 1]$ to an action $a(v) \in \{-1, 0, 1\}$. For $b \in \{0, 1\}$, the action $a(v) = b$ denotes purchasing a certified credit of class-b, while $a(v) = -1$ denotes refraining from purchase. Given beliefs $\boldsymbol{\mu} = (\mu_0, \mu_1)$ and prices $\boldsymbol{p} = (p_0, p_1)$, the expected utility of a type-v buyer when choosing action a is:

$$\tilde{U}(a, v; \boldsymbol{\mu}, \boldsymbol{p}) = \begin{cases} (1 + \gamma v)\mu_1 - p_1, & \text{if buying a class-1 credit } (a = 1; \text{ with co-benefits}), \\ \mu_0 - p_0, & \text{if buying a class-0 credit } (a = 0; \text{ without co-benefits}), \\ 0, & \text{otherwise } (a = -1). \end{cases} \tag{2.2}$$

The expected utility in (2.2) highlights two distinct factors that shape buyers' decisions: the difference in expected quality across credit classes, reflected in the beliefs μ_0 and μ_1, and the utility from co-benefits, captured by the term γv. As a result, buyers may face a tradeoff between higher expected quality and the utility derived from co-benefits when deciding which class of credits to purchase. In equilibrium, we require each buyer to maximize his expected utility:

$$a(v) \in \underset{x \in \{-1,0,1\}}{\arg\max}\ \tilde{U}(x, v; \boldsymbol{\mu}, \boldsymbol{p}), \text{ for all types } v \in [0, 1]. \tag{2.3}$$

Note that each strategy induces a pair of demand functions, which we denote by $D_b(\boldsymbol{p}, \boldsymbol{\mu})$, representing the fraction of buyers who choose to purchase a class-b credit given their beliefs and prices, for $b \in \{0, 1\}$. That is,

$$D_b(\boldsymbol{p}, \boldsymbol{\mu}) = \mathbb{P}[a(v) = b \mid \boldsymbol{p}, \boldsymbol{\mu}], \text{ for } b \in \{0, 1\}. \tag{2.4}$$

Sellers' decisions. A seller strategy is a measurable function that maps each type $\theta = (q, b, c)$ to an entry decision $e(\theta) \in \{0, 1\}$, where $e(\theta) = 1$ denotes entering the market and $e(\theta) = 0$ denotes staying out. Recall that sellers who enter incur their cost c, and are then allowed to issue a credit only if the certifier observes a high signal. Thus, given prices $\boldsymbol{p} = (p_0, p_1)$, the expected payoff for a seller of type $\theta = (q, b, c)$ choosing entry decision $e \in \{0, 1\}$ is:

$$\pi(e, \theta, p_b) = (\sigma(s = H \mid q) \cdot p_b - c) \cdot e = \begin{cases} p_b - c, & \text{if seller is high-quality } (q = 1) \text{ and enters } (e = 1), \\ \epsilon p_b, & \text{if seller is low-quality } (q = 0) \text{ and enters } (e = 1), \\ 0, & \text{if seller does not enter } (e = 0). \end{cases}$$

In equilibrium, sellers make entry decisions that maximize their expected payoff, taking prices as given. A high-quality seller strictly prefers to enter if her cost c is lower than the price p_b, so the fraction of high-quality sellers of class $b \in \{0, 1\}$ that enter is $G(p_b)$. In contrast, low-quality sellers face no cost and strictly prefer to enter when the price is positive and certification is not perfectly accurate, i.e., $\epsilon p_b > 0$. Thus, in equilibrium, sellers' entry decisions take the following form:[8]

$$e(\theta) = \begin{cases} 1 & \text{if } q = 1 \text{ and } p_b > c, \text{ or } q = 0 \text{ and } p_b > 0, \\ 0 & \text{otherwise.} \end{cases} \tag{2.5}$$

Given the entry rule in (2.5) and the certification error, the supply of certified credits of class $b \in \{0, 1\}$, denoted by $S_b(p_b)$, is given by:

$$\begin{aligned} S_1(p_1) &= \mathbb{P}[s = H, e(\theta) = 1, b = 1] = \beta \left(\phi_1 G(p_1) + \epsilon (1 - \phi_1) \mathbf{1}_{\{p_1 > 0\}} \right), \\ S_0(p_0) &= \mathbb{P}[s = H, e(\theta) = 1, b = 0] = (1 - \beta) \left(\phi_0 G(p_0) + \epsilon (1 - \phi_0) \mathbf{1}_{\{p_0 > 0\}} \right), \end{aligned} \tag{2.6}$$

which corresponds to the mass of high- and low-quality sellers who enter at price p_b and are then certified. Observe that this formulation captures adverse selection: low-quality sellers always enter when anticipating a positive price and some are certified if the screening process is noisy, whereas high-quality sellers only enter if the price exceeds their cost. As a result, the supply becomes predominantly low-quality when prices are low.

Buyers' beliefs. Buyers form beliefs using Bayes' rule, taking into account the composition of sellers of each class who enter the market and the certification error. Let μ_b denote the belief that a certified credit of class $b \in \{0, 1\}$ is high-quality:

$$\mu_b = \mathbb{P}[q = 1 \mid s = H, e(\theta) = 1, b] = \frac{\mathbb{P}[s = H, e(\theta) = 1 \mid q = 1, b] \cdot \mathbb{P}[q = 1 \mid b]}{\mathbb{P}[s = H, e(\theta) = 1 \mid q = 1, b] \cdot \mathbb{P}[q = 1 \mid b] + \mathbb{P}[s = H, e(\theta) = 1 \mid q = 0, b] \cdot \mathbb{P}[q = 0 \mid b]}.$$

From the entry rule in (2.5), high-quality class-b sellers enter the market if their cost is lower than the price p_b and are always certified, while low-quality sellers enter if $p_b > 0$ and are certified with probability ϵ. Therefore, in equilibrium, buyers' belief about credits of class $b \in \{0, 1\}$ is given by:

$$\mu_b = \frac{\phi_b G(p_b)}{\phi_b G(p_b) + \epsilon (1 - \phi_b)}. \tag{2.7}$$

This condition also applies when $p_b = 0$, in which case the belief can be defined as $\mu_b = 0$ since no sellers enter the market, and hence no high-quality credits are available.

[8] We refine the set of equilibria by assuming that low-quality sellers do not enter when indifferent, i.e., if their expected payoff is zero. In particular, this occurs when the market price of their class collapses to zero ($p_b = 0$). Since high-quality sellers also opt not to enter when anticipating a price of zero, this rules out equilibria where only low-quality credits are traded at a price of zero. Alternatively, this refinement can be justified by assuming that buyers refrain from purchasing credits they assess to be low-quality with certainty (i.e., when their belief is $\mu_b = 0$).

Equilibrium. Given the optimal decisions and belief formation rule described above, we now define an equilibrium in our setting.

DEFINITION 1. An *equilibrium* consists of (i) a pair of prices $\boldsymbol{p} = (p_0, p_1)$ for credits of each class, (ii) a pair of beliefs $\boldsymbol{\mu} = (\mu_0, \mu_1)$ about the quality of certified credits of each class, (iii) a buyer strategy $a : [0,1] \to \{-1, 0, 1\}$, and (iv) a seller strategy $e : \{0,1\} \times \{0,1\} \times [0,1] \to \{0,1\}$, such that:

1. Buyers' purchase decisions maximize their expected utility, given their beliefs and prices. That is, $a(v)$ satisfies (2.3).
2. Sellers' entry decisions maximize their expected payoff, taking prices as given. That is, $e(\theta)$ satisfies (2.5).
3. Buyers' beliefs are consistent with Bayes' rule. That is, μ_b satisfies (2.7) for $b \in \{0,1\}$.
4. The prices clear the market for each class of credits $b \in \{0,1\}$. That is,

$$D_1(\boldsymbol{p}, \boldsymbol{\mu}) = S_1(p_1) \quad \text{and} \quad D_0(\boldsymbol{p}, \boldsymbol{\mu}) = S_0(p_0),$$

   where the demand and supply functions are given by (2.4) and (2.6), respectively.

An equilibrium exists in our model, as the scenario in which both prices are zero and no trade occurs is always an equilibrium. However, when multiple equilibria exist, we focus on the one that maximizes trade volume, which in our setting coincides with the equilibrium with highest prices.

We will use this equilibrium framework to investigate how the underlying characteristics of the market—in particular the certification error (ϵ) and the weight buyers place on co-benefits (γ)—shape the functioning of the market and the climate impact it delivers. To that end, we now introduce two central concepts that will guide our analysis: the net climate gain generated by the market, and the conditions under which the market collapses.

Net Climate Gain and Market Collapse. To evaluate the effectiveness of the market in delivering genuine emissions reductions, we define the *net climate gain* generated by each market segment. This metric captures the aggregate climate impact of the market by accounting for the emissions that are truly offset (the volume of high-quality credits transacted), while penalizing the use of spurious offsets (the volume of low-quality credits that are traded). Specifically, for each class $b \in \{0,1\}$, the net climate gain is:

$$\mathcal{CG}_b = \mathcal{H}_b^* - \delta \cdot \mathcal{L}_b^*. \tag{2.8}$$

Here, $\mathcal{H}_b^* := \mathbb{P}[q = 1, s = H, e(\theta) = 1, b]$ denotes the mass of high-quality class-b credits traded in equilibrium, and $\mathcal{L}_b^* := \mathbb{P}[q = 0, s = H, e(\theta) = 1, b]$ is the corresponding mass of low-quality credits.

The parameter $\delta \geq 0$, which we refer to as the *greenwashing penalty*, determines how low-quality offsets are evaluated from a social planner's perspective. More precisely, δ represents the marginal rate of substitution between genuine and spurious offsets: the social planner is willing to tolerate $1/\delta$ spurious offsets in exchange for one ton of CO_2-equivalent that is genuinely abated. Alternatively, δ can be interpreted as the social opportunity cost of greenwashing, reflecting emissions abatement efforts that buyers might have pursued if they were not purchasing low-quality credits.

Throughout our analysis, we treat δ as fixed and exogenous, reflecting a given policy stance toward greenwashing rather than an object of comparative statics. Likewise, we hold the ex ante composition of potential projects (ϕ_0, ϕ_1, β) fixed, so that changes in net climate gain capture the effect of endogenous participation rather than shifts in the underlying supply base.

With this in mind, note that the net climate gain in (2.8) can be written in terms of the volume of class-b credits traded in equilibrium, S_b^*, the belief μ_b^* that such a credit is high-quality, and the greenwashing penalty δ as follows:

$$\mathcal{CG}_b = S_b^* \left(\mu_b^* - \delta(1-\mu_b^*)\right). \tag{2.9}$$

This expression highlights that the net climate gain generated by the class-b segment turns negative if such credits are traded but the fraction of high-quality supply falls below a threshold, i.e., if $\mu_b^* < \delta/(1+\delta)$ and $S_b^* > 0$. In such cases, the absence of trade in this segment may be socially preferable in terms of climate impact, as it prevents the circulation of low-quality offsets.

As a result, we are particularly interested in identifying conditions under which the market collapses due to adverse selection—i.e., when no trade occurs because quality uncertainty prevents a positive price from being sustained in equilibrium, so that no high-quality sellers enter and the VCM turns into a *market for lemons* (Akerlof 1970).

Definition 2. We denote a market by the underlying parameters $\Psi = (\epsilon, \gamma, \phi_0, \phi_1, \beta)$. We say *the market for class-b credits collapses* if, in any equilibrium for Ψ, the price p_b is zero.

Our subsequent analysis explores the conditions under which such scenarios arise.

2.2. Discussion of Modeling Assumptions

As a preface to our analysis, we note that our objective is to explore how market outcomes and the climate impact delivered by the VCM are shaped by its key characteristics—particularly the interplay between quality uncertainty, certification accuracy, and the value buyers attach to co-benefits. To this end, we developed a stylized framework that captures these essential features. We now discuss our main modeling assumptions and their implications.

One-sided certification error. We assume the certifier only makes false positive errors; i.e., it may mistakenly certify low-quality projects but never rejects high-quality ones. This reflects the primary concern that has undermined credibility in the VCM: the prevalence of low integrity credits, rather than the possible exclusion of some high-quality projects (Probst et al. 2024). For simplicity, we also assume the certification error is the same across project classes, although allowing it to vary would not change the mechanisms captured by the model or our qualitative findings.

Observability of co-benefits. We model co-benefits as publicly observable, whereas the emissions offset by a project are privately known by the seller and thus perceived as uncertain by the certifier and buyers. While project developers must also substantiate claims about co-benefits (e.g., job creation, local community development), these are generally easier to verify through measurable outcomes. In contrast, verifying whether a project truly reduces emissions is more difficult, as it requires evaluating an unobservable counterfactual scenario (Axelsson et al. 2024).

Cost structure. We normalize the cost of low-quality projects to zero, as they reflect non-additional activities that involve little or no foregone economic value (e.g., monetizing projects that are already profitable). We also assume that cost distributions are identical across classes. While this simplifies the analysis, it also reflects that co-benefits often arise naturally from a project's type or location (e.g., public health improvements from clean cookstoves, employment generated by community-led reforestation), rather than from separate investment. Nonetheless, we expect our findings to remain qualitatively valid even if cost distributions differ across classes, as this would alter the analytical expressions of the model but not its underlying economic forces. For instance, if projects with co-benefits were systematically more costly to implement, high-quality participation would be lower at any given price, making this market segment relatively more prone to collapse without altering the fundamental mechanics of adverse selection.

Buyer preferences. The premise that buyers value co-benefits is consistent with empirical evidence that credits with such attributes trade at a premium that is not explained by carbon impact alone (Berg et al. 2025). Moreover, the lexicographic utility specification implies that buyers derive value from co-benefits only if they purchase a high-quality credit. As a result, buyers do not knowingly purchase credits with no carbon impact solely for their associated co-benefits. We view this assumption as conservative: treating co-benefits as a standalone source of utility would accentuate the misalignment between preferences for carbon quality and co-benefits, allowing demand to persist even when carbon integrity is low. Finally, we abstract from intertemporal considerations, even though co-benefits are often realized in the short term whereas carbon quality is subject to long-term permanence risks. Incorporating this timing difference would make buyers prioritize immediate reputational gains over future quality risk, further sustaining demand in low-integrity environments. Abstracting from this channel is therefore conservative in the same sense.

Uniform distributions. For tractability, we assume buyer types and seller costs are drawn from uniform distributions. This allows us to obtain clean analytical characterizations while preserving the key tradeoffs in the model. However, the qualitative insights of the model do not hinge on these specific distributional assumptions. In particular, the equilibrium structure and main comparative statics can be formally extended to a broad class of cost distributions with bounded elasticity.

Offset purchases for residual emissions. We focus exclusively on buyers' decisions to purchase carbon offsets, abstracting away from potential tradeoffs with internal decarbonization efforts. This modeling choice is consistent with established industry guidelines, which emphasize that firms should prioritize cutting their own emissions and use offsets only to neutralize any residual emissions (Axelsson et al. 2024, Science Based Targets Initiative 2024). Under this view, offset purchases are treated as a separate decision made after all feasible internal abatement options have been exhausted, making it reasonable to model in isolation.

3. Equilibrium Analysis

We begin our analysis by characterizing the equilibrium in our model and identifying the conditions under which each market segment collapses. Building on this characterization, we then analyze how equilibrium outcomes and the resulting climate gain depend on the market's underlying features, focusing on certification accuracy and the value buyers place on co-benefits.

Proposition 1 **(Characterization of equilibrium)**. *For each credit class $b \in \{0,1\}$, there exists a threshold $\bar{\epsilon}_b$ and an equilibrium price p_b^* such that:*

(i) If the certification error is below the threshold (i.e., $\epsilon < \bar{\epsilon}_b$), then p_b^* is the unique positive equilibrium price for class-b credits, and the resulting trade volume for such credits is positive.

(ii) If the certification error exceeds the threshold (i.e., $\epsilon \geq \bar{\epsilon}_b$), the market for class-b credits collapses. That is, their price is $p_b^* = 0$ and no such credits are traded in any equilibrium.

Moreover, the thresholds and equilibrium prices are given as follows:

(a) For class-0 credits (without co-benefits),

$$\bar{\epsilon}_0 = \frac{\phi_0}{1-\phi_0} \quad \text{and} \quad p_0^* = \left(1 - \frac{1-\phi_0}{\phi_0}\epsilon\right)_+. \tag{3.1}$$

(b) For class-1 credits (with co-benefits),

$$\bar{\epsilon}_1 = \frac{\phi_1(1+\gamma)}{(1-\phi_1)(1+\phi_1\beta\gamma)} \quad \text{and} \quad p_1^* = \begin{cases} \dfrac{\phi_1(1+\gamma)}{\phi_1 + (1-\phi_1)\epsilon} - \phi_1\beta\gamma, & \text{if } \epsilon < \left(\frac{\gamma(1-\beta\phi_1)}{1+\gamma}\right)\bar{\epsilon}_1, \\ \left(\dfrac{1+\gamma}{1+\phi_1\beta\gamma} - \dfrac{1-\phi_1}{\phi_1}\epsilon\right)_+, & \text{otherwise.} \end{cases} \tag{3.2}$$

The equilibrium structure highlights the central role of certification in sustaining the market. If the error rate ϵ is lower than the threshold $\bar{\epsilon}_b$, the certification is sufficiently accurate to support a positive market price p_b^* for class-b credits. In this case, a fraction of high-quality sellers enter the market, while some low-quality credits are also traded due to certification errors. In contrast, if ϵ exceeds the threshold, the certification is too noisy to meaningfully mitigate quality uncertainty. Buyers' willingness to pay declines, driving high-quality sellers to refrain from joining the market in anticipation of a low price. Ultimately, this dynamic results in the equilibrium price collapsing to zero and the market segment shutting down.

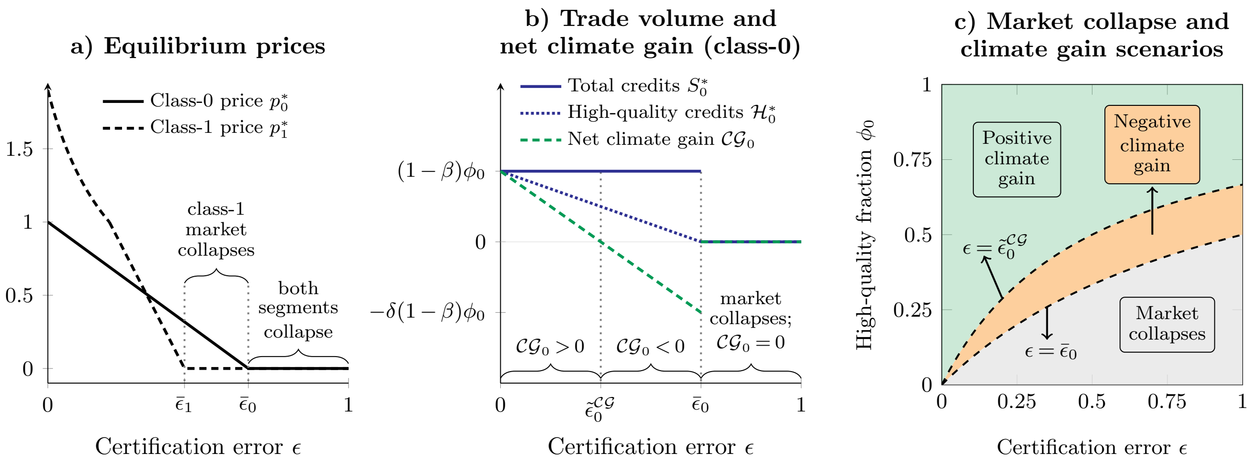
The underlying intuition mirrors the classic *market for lemons* argument (Akerlof 1970). When certification is perfectly accurate ($\epsilon = 0$), the adverse selection problem is fully resolved: low-quality entrants are perfectly screened out, giving buyers certainty that certified credits are high-quality. This results in equilibrium prices that are sufficiently high to support broad participation by high-quality sellers. In this ideal case, the market achieves full environmental integrity: only genuine offsets are traded, and the resulting net climate gain is maximized.

However, with less accurate certification, low-quality sellers are more likely to receive a favorable signal and participate in the market. Anticipating this dilution in quality, buyers lower their willingness to pay, driving down equilibrium prices. At lower prices, some high-quality sellers (those with higher costs) choose not to enter, further deteriorating the expected quality of credits in the market. This decline can trigger an amplifying feedback loop: lower prices discourage high-quality entry, which in turn further reduces buyers' willingness to pay. If the certification error ϵ is sufficiently large, this unraveling becomes complete: no positive price can simultaneously attract enough high-quality sellers and provide buyers with sufficient confidence to sustain their willingness to pay, driving the market to collapse. As we discuss below, however, buyer preferences for co-benefits can counteract this mechanism, potentially keeping the market active even when certification provides little assurance of carbon quality.

Figure 2a illustrates how this unraveling unfolds: the equilibrium prices for both credit classes decline if certification becomes noisier, and ultimately collapse to zero if the certification error ϵ exceeds the threshold $\bar{\epsilon}_b$. This plot also highlights the tension between quality uncertainty and the value of co-benefits. When certification is accurate (low ϵ), class-1 credits trade at a premium, as buyers are confident in their carbon quality and are willing to pay extra for co-benefits. However, this relationship may reverse if certification is noisier. If projects with co-benefits are less likely to reduce emissions (i.e., the empirically relevant regime where $\phi_1 < \phi_0$), the label "Class-1" effectively signals lower quality. When the certification is less informative (higher ϵ), this *informational discount* can outweigh the *preference premium* from co-benefits, in which case class-1 credits trade at a discount and are more prone to market collapse despite their added attributes.

The market collapse scenario constitutes a market failure, as the underlying information asymmetry and the certifier's inability to effectively screen quality preclude trade from taking place. However, from a climate standpoint, this outcome may be preferable to the alternative. If certification is noisy but the market remains active, trade may become dominated by low-quality credits, causing the net climate gain to turn negative, in which case a collapsed market with no transactions and zero net climate gain may be preferable.

Figure 2 Relationship between equilibrium outcomes and the certification error ϵ



Notes. **a)** Equilibrium prices as a function of the certification error. The price p_b^* for each class b collapses to zero if ϵ exceeds the threshold $\bar{\epsilon}_b$. Note that p_1^* drops below p_0^* when certification is noisy, reflecting that class-1 projects have lower prior quality ($\phi_1 < \phi_0$) in this example. **b)** Total and high-quality trade volumes in equilibrium and net climate gain for the class-0 segment. Net climate gain becomes negative in the intermediate region with $\epsilon \in (\tilde{\epsilon}_0^{\mathcal{CG}}, \bar{\epsilon}_0)$ as most traded credits are low-quality (Part (ii) of Proposition 2). **c)** Climate outcomes for the class-0 segment according to the certification error and the fraction of potential high-quality class-0 sellers (ϵ, ϕ_0). The three regions reflect whether net climate gain is positive, negative, or zero (market collapsed), as described in Proposition 2.
Parameter values: $\phi_0 = 0.4$, $\phi_1 = 0.2$, $\beta = 0.5$, $\gamma = 1$, $\delta = 1$

Figure 2b illustrates this point for the class-0 segment. As certification becomes less accurate, the volume of trade in this segment remains constant until the market collapses (solid line), but the share of high-quality credits declines steadily (dotted line). This pattern highlights how noisier certification exacerbates adverse selection both by allowing low-quality sellers to enter and by depressing prices, which deters participation from high-quality sellers. Consequently, as long as the market remains active, the net climate gain (dashed line) declines with ϵ and eventually turns negative as low-quality credits become more prevalent. Importantly, this climate-negative outcome may arise even though buyers derive utility from genuine carbon abatement and would avoid low-quality credits if they could identify them.

The following proposition formalizes the relationship between certification error and net climate gain, identifying the three possible scenarios that can arise for each market segment:

PROPOSITION 2 **(Climate gain and market collapse scenarios)**. *Suppose the greenwashing penalty is positive ($\delta > 0$), and let $\bar{\epsilon}_b$ denote the market collapse thresholds for each credit class $b \in \{0, 1\}$, as defined in Proposition 1. Then, for each $b \in \{0, 1\}$, there exists a threshold $\tilde{\epsilon}_b^{\mathcal{CG}} \in (0, \bar{\epsilon}_b)$ such that the class-b market falls into one of the following scenarios:*

(i) ***Climate-positive:*** *If $\epsilon < \tilde{\epsilon}_b^{\mathcal{CG}}$, the market is active and yields positive net climate gain, i.e., $\mathcal{CG}_b > 0$ and $S_b^* > 0$.*

(ii) ***Climate-negative:*** *If $\epsilon \in \left(\tilde{\epsilon}_b^{\mathcal{CG}}, \bar{\epsilon}_b\right)$, the market remains active but the net climate gain is negative, i.e., $\mathcal{CG}_b < 0$ and $S_b^* > 0$.*

(iii) ***Market collapse:*** *If $\epsilon \geq \bar{\epsilon}_b$, the class-b market collapses and yields zero net climate gain. In addition, the thresholds $\tilde{\epsilon}_b^{\mathcal{CG}}$ for each market segment $b \in \{0,1\}$ are given by:*

$$\tilde{\epsilon}_0^{\mathcal{CG}} = \frac{\bar{\epsilon}_0}{1+\delta}, \quad \textit{and} \quad \tilde{\epsilon}_1^{\mathcal{CG}} = \begin{cases} \dfrac{\bar{\epsilon}_1}{1+\delta}, & \textit{if } \delta \leq \dfrac{1+\phi_1\beta\gamma}{\gamma(1-\beta\phi_1)}, \\ \dfrac{\phi_1}{\delta(1-\phi_1)}, & \textit{otherwise.} \end{cases} \tag{3.3}$$

Figure 2c complements this result by illustrating how market and climate outcomes depend not only on the certification accuracy (ϵ), but also on the composition of the project pool, as captured by the share of potential high-quality projects (ϕ_b). The figure partitions the (ϵ, ϕ_0) parameter space for the class-0 segment into three distinct regions, corresponding to the scenarios outlined in Proposition 2: (i) a *functioning market*, where certification is sufficiently accurate or the share of high-quality projects ϕ_0 is high enough to sustain a market with positive net climate gain; (ii) a *low-integrity market* (intermediate region), where trade persists but the net climate gain turns negative, as noisy certification allows low-quality credits to become more prevalent and crowd out high-quality supply; and (iii) *market collapse*, where the combination of poor certification and a low fraction of high-quality projects ϕ_0 prevents trade from taking place. In particular, when ϕ_0 is low, even modest levels of certification error can trigger market collapse, as accurate screening becomes critical to filter out low-quality offsets and sustain buyer confidence.

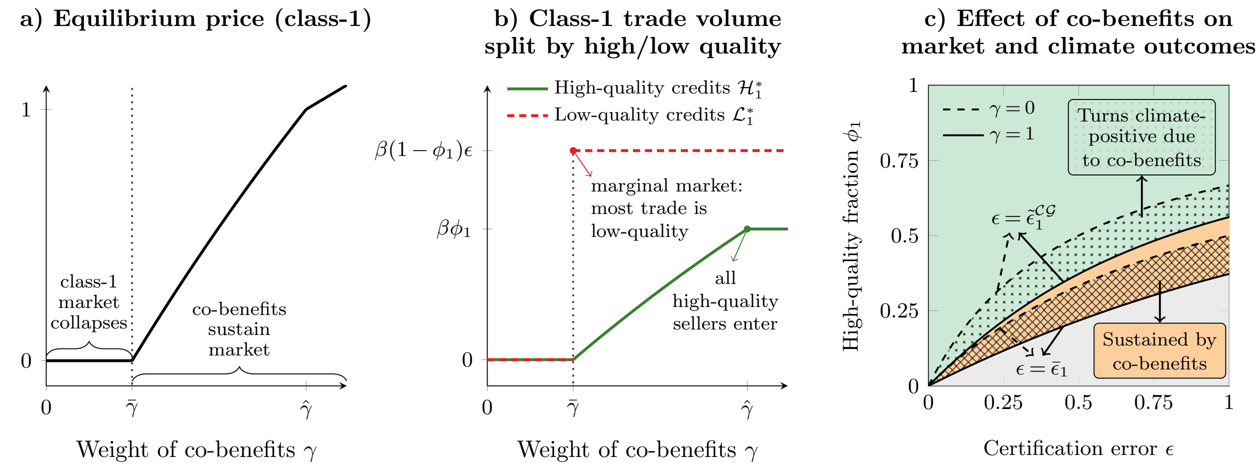
The role of co-benefits. We now examine how equilibrium outcomes and the resulting climate impact are shaped by the value buyers attach to co-benefits, as captured by the parameter γ.

While co-benefits are often viewed as desirable features of carbon offsets, as they enable buyers to advance broader sustainability goals alongside carbon mitigation, their influence on equilibrium behavior can be more nuanced. In particular, by shifting buyers' focus toward attributes that are not directly tied to carbon integrity, co-benefits may alter demand patterns and market viability. As we formalize in the next proposition, co-benefits can sustain a market that would otherwise collapse if credits were evaluated solely on the basis of their carbon integrity.

Proposition 3 **(Co-benefits can prevent market collapse)**. *Suppose that the certification error satisfies $\epsilon > \phi_1/(1-\phi_1)$. Then, there exists a threshold $\bar{\gamma} > 0$ such that the market for class-1 credits collapses if buyers place limited value on co-benefits (i.e., if $0 \leq \gamma \leq \bar{\gamma}$), but remains active if buyers place sufficiently high value on co-benefits (i.e., if $\gamma > \bar{\gamma}$).*

Figure 3a illustrates this result by plotting the equilibrium price of class-1 credits (p_1^*) as a function of γ. When certification is relatively noisy (i.e., $\epsilon \geq \phi_1/(1-\phi_1)$), the market collapses if buyers place little value on co-benefits (i.e., if $\gamma \leq \bar{\gamma}$). However, if γ exceeds the threshold $\bar{\gamma}$, the

Figure 3 Relationship between equilibrium outcomes in the class-1 market and the weight of co-benefits γ



Notes. **a)** Equilibrium price of class-1 credits (p_1^*) as a function of the weight buyers place on co-benefits (γ). The price is zero when $\gamma \leq \bar{\gamma}$, and increases with γ once the market is active. **b)** Equilibrium trade volume for high- and low-quality class-1 credits. When $\gamma \geq \hat{\gamma}$, the price p_1^* is sufficiently high to induce full participation by high-quality sellers. **c)** Climate outcomes for the class-1 segment according to the certification error and the fraction of potential high-quality class-1 sellers (ϵ, ϕ_1). Solid lines delineate the regions where net climate gain is positive (green), negative (brown), or zero due to market collapse (gray), when buyers place value on co-benefits ($\gamma = 1$). Dotted lines show the corresponding frontiers when buyers place no value on co-benefits ($\gamma = 0$), matching those in Figure 2c. The shaded areas highlight how co-benefits alter equilibrium outcomes. The crosshatched region identifies markets that would collapse when $\gamma = 0$ but are sustained when $\gamma = 1$, albeit with negative net climate gain. The dotted region captures markets that shift from negative to positive net climate gain when $\gamma = 1$ due to improved supply composition.
Parameter values: $\phi_1 = 0.2$, $\beta = 0.5$, $\epsilon = 0.4$, $\delta = 1$

market becomes active as the equilibrium price turns positive. The price then increases with γ, reflecting the additional premium buyers are willing to pay for co-benefits.[9]

This observation has a deeper implication: the value buyers place on co-benefits partially decouples the survival of the market from its carbon integrity, preventing the complete unraveling predicted by the classic *market for lemons* logic. As a result, the market for class-1 credits may be sustained entirely by non-carbon attributes (co-benefits) rather than by its potential for genuine carbon mitigation. In particular, when the market is only marginally active (i.e., when γ is just above $\bar{\gamma}$), trade is sustained but the equilibrium price remains low, attracting only a small share of high-quality sellers while low-quality credits dominate (Figure 3b). Thus, while co-benefits enable the market to function, the resulting net climate gain is negative, whereas the market would have otherwise collapsed if buyers valued only carbon mitigation.

Nonetheless, although co-benefits may sustain marginal markets with limited climate value, they can also improve the composition of supply when the market is already viable. In particular, higher

[9] We focus on the class-1 segment when analyzing the effect of co-benefits, as the equilibrium outcomes for class-0 are unaffected by γ in our baseline model. This results from two effects canceling out: higher γ shifts demand toward class-1 credits but also raises their equilibrium price. In a more general setting where these effects may not fully cancel out, we would expect an increase in γ to induce substitution away from class-0 and lower the price p_0^*. In that case, stronger preferences for co-benefits could have a sharper effect, not only sustaining the class-1 segment but also cannibalizing demand for, and potentially collapsing, the class-0 segment.

values of γ raise buyers' willingness to pay, which increases the equilibrium price p_1^* and attracts more high-quality sellers to enter, as shown in Figure 3b. In such cases, buyers' preference for co-benefits helps attenuate adverse selection and may even convert a climate-negative market into one that delivers positive climate gain.

Taken together, these contrasting scenarios highlight that *co-benefits can play a dual role* in shaping market outcomes, as illustrated in Figure 3c. On one hand, co-benefits may sustain markets that would otherwise collapse (represented by the brown crosshatched region) by attracting buyer demand even when certification is noisy and the share of high-quality supply is relatively low. These markets function only because buyers value co-benefits, yet deliver negative climate gain because most traded credits are low-quality. On the other hand, when the market is already active, stronger preferences for co-benefits raise equilibrium prices, attracting more high-quality supply. This effect is best exemplified by the parameter combinations within the dotted green region, where the market shifts from climate-negative when buyers do not value co-benefits ($\gamma = 0$) to climate-positive when $\gamma = 1$ as the composition of supply improves.

4. Policy and Market Design Levers in the VCM

Having examined how key features of the VCM shape its equilibrium outcomes, we now turn to analyze market design and policy interventions aimed at improving its climate effectiveness. We consider two such directions. First, we study the implications of penalizing buyers that purchase low-quality credits, inspired by recent regulatory efforts to increase transparency and deter greenwashing. Then, we investigate an alternative market structure where different credits are bundled into a portfolio, as some platforms now offer in practice, and contrast this setting with our baseline model where buyers select individual credits.

4.1. Penalizing Buyers for Low-Quality Offsets

Amid growing concerns about the integrity of carbon credits, buyers in the VCM are facing increased scrutiny over the offsets they purchase. The accumulated evidence of projects overstating their carbon impact has undermined firms' net-zero claims and exposed them to reputational and legal risks. For instance, Apple recently faced a class-action lawsuit for marketing one of its products as "carbon neutral" based on offsets whose quality was questioned (Calma 2025). More broadly, policymakers are pushing for greater transparency around offset use. For example, California recently passed legislation requiring firms to disclose details of the specific offsets used to support their environmental claims (California State Assembly 2023).

These measures aim to hold buyers accountable for the use of low-quality offsets, either through regulatory penalties or reputational damage, and ultimately seek to deter greenwashing by incentivizing the purchase of high-quality offsets that credibly support firms' environmental claims.

Motivated by these developments, we study the implications of penalizing buyers for low-quality purchases within our model. Specifically, we modify the buyer's utility function in (2.1) by introducing a penalty $\kappa \geq 0$ that is incurred when the purchased credit is low-quality:[10]

$$U(v,q,b,p) = \begin{cases} (1+\gamma v b)q - \kappa(1-q) - p, & \text{if he buys a credit } (q,b) \text{ at } p, \\ 0, & \text{otherwise.} \end{cases}$$

The penalty κ can be interpreted as the product of two factors: (i) the probability that the buyer's offset purchases will be scrutinized in the future, and (ii) the fine or reputational cost incurred if those credits are found to be of low quality. Note that although buyers do not observe the true quality of individual credits, they form beliefs about the average quality of certified credits and therefore internalize the expected penalty when making purchase decisions.

The next result characterizes how introducing a penalty for buyers affects equilibrium outcomes.

PROPOSITION 4 **(Equilibrium with buyer penalties)**. *Fix $\kappa > 0$. Then, for each credit class $b \in \{0,1\}$, there exists a threshold $\tilde{\epsilon}_b(\kappa)$ and an equilibrium price $p_b^*(\kappa)$ such that:*

(i) If the certification error is below the threshold (i.e., $\epsilon \leq \tilde{\epsilon}_b(\kappa)$), there exists an equilibrium in which class-b credits trade at a positive price $p_b^*(\kappa) > 0$. Moreover, $p_b^*(\kappa)$ is the unique price for class-b credits in any equilibrium with maximum trade volume.

(ii) If the certification error exceeds the threshold (i.e., $\epsilon > \tilde{\epsilon}_b(\kappa)$), the market for class-b credits collapses. That is, their price is $p_b^*(\kappa) = 0$ and no such credits are traded in any equilibrium.

Furthermore, both the equilibrium prices $p_b^*(\kappa)$ and the collapse thresholds $\tilde{\epsilon}_b(\kappa)$ are decreasing in the penalty κ. Closed-form expressions for these quantities are provided in Equations (A.10) *and* (A.11) *in Appendix A.2.1.*

Given a fixed penalty κ, the structure of equilibrium mirrors that of the baseline scenario with no penalty (Proposition 1, with $\kappa = 0$). For each credit class, trade occurs at a positive price if the certification error lies below a threshold, while the market collapses if the certification error exceeds it. More interestingly, Proposition 4 shows that *penalizing buyers for greenwashing can have unintended consequences for climate outcomes.* Specifically, penalties discourage high-quality participation and tighten the conditions required to sustain trade, as both the equilibrium prices $p_b^*(\kappa)$ and the market collapse thresholds $\tilde{\epsilon}_b(\kappa)$ decline as the penalty κ increases.

This pattern emerges because penalties amplify buyers' aversion to low-quality credits without improving the certification process. In other words, introducing a penalty $\kappa > 0$ reduces buyers' willingness to pay for offsets but does not enhance the certifier's screening ability, thus leading

[10] The private penalty imposed on buyers (κ) may differ from the parameter δ that penalizes greenwashing in the net climate gain calculation. While a regulator may set $\kappa \approx \delta$ to align private and social incentives, we allow κ to serve as a distinct policy lever.

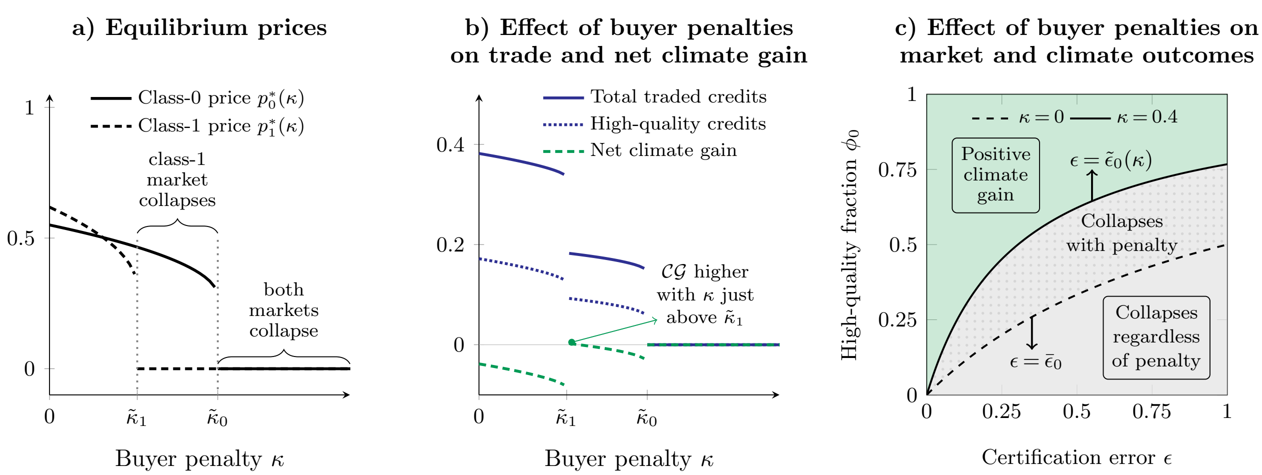
to lower equilibrium prices. In turn, lower prices erode the average quality of certified credits by discouraging entry from high-quality sellers. If the penalty κ is large enough, this feedback loop can trigger the self-reinforcing decline in prices and quality that drives the market to collapse, even if the underlying fundamentals would otherwise sustain active trade in the absence of penalties.

Figure 4a illustrates these points. For both credit classes, the equilibrium price $p_b^*(\kappa)$ declines as the penalty κ increases, and each segment collapses once κ exceeds a critical value $\tilde{\kappa}_b$ (the point at which the certification error equals the collapse threshold, i.e., $\epsilon = \tilde{\epsilon}_b(\kappa)$). Moreover, penalizing buyers also expands the range of scenarios in which the market collapses. As shown in Figure 4c, a subset of markets that support trade when $\kappa = 0$ instead collapse if a penalty is imposed.

These observations underscore a key insight: although penalizing buyers may be intended to discourage the use of low-quality offsets, doing so may instead exacerbate adverse selection and ultimately reduce the carbon abatement produced by the market or render it unviable.

In fact, within our model, buyer penalties may only improve climate outcomes if they collapse a market segment that would otherwise remain active but generate negative net climate gain. In such cases, eliminating low-quality trade outweighs the loss of some high-quality transactions. The next result formalizes this point.

Figure 4 **Relationship between equilibrium outcomes and the buyer penalty κ**



Notes. **a)** Equilibrium prices as a function of the penalty κ. The vertical lines at $\tilde{\kappa}_b$ indicate the maximum penalty that the class-b segment can sustain before collapsing (i.e., the value of κ satisfying $\epsilon = \tilde{\epsilon}_b(\kappa)$). **b)** Total trade volume, volume of high-quality credits, and net climate gain across both segments. This example illustrates Proposition 5: setting a penalty just above $\tilde{\kappa}_1$ increases net climate gain relative to $\kappa = 0$ by collapsing the climate-negative segment (class-1). In that case, both total trade and the volume of high-quality credits fall relative to $\kappa = 0$, but the share of traded credits that are high-quality increases. **c)** Equilibrium outcomes for the class-0 segment under no penalty ($\kappa = 0$) vs. a positive penalty ($\kappa = 0.4$). The penalty expands the market collapse region (dotted gray). In this example, the region where the market is active with negative climate gain disappears (brown region in Figure 2c), because these markets collapse under the penalty. At the same time, the markets that remain active after introducing the penalty generate lower (but still positive) net climate gain.
Parameter values: $\phi_0 = 0.4$, $\phi_1 = 0.2$, $\beta = 0.5$, $\gamma = 1$, $\epsilon = 0.3$, $\delta = 1$

PROPOSITION 5 **(Effect of penalties on climate outcomes)**. *Suppose the class-b market generates positive net climate gain in the absence of penalties. Then, any penalty $\kappa > 0$ reduces the net climate gain from this segment.*

Suppose instead that at least one segment generates negative net climate gain when $\kappa = 0$. Then, there exist parameter values for which a positive penalty $\kappa^{\mathcal{CG}} > 0$ yields higher total net climate gain. However, this improvement occurs only when the penalty induces the collapse of the segment(s) generating negative climate gain.

Proposition 5 implies that if both market segments are climate-positive in the absence of penalties, imposing a penalty $\kappa > 0$ on buyers only reduces net climate gain. Moreover, such penalties may improve climate outcomes only when they act as a selection mechanism—that is, by shutting down trade in credit classes that generate negative climate gain while allowing climate-positive segments to remain active. Figure 4b illustrates one such case, in which the class-1 segment is climate-negative while class-0 is climate-positive when $\kappa = 0$. In this example, net climate gain is highest when the penalty is set just above the value that collapses class-1 trade ($\tilde{\kappa}_1$), even though both the total volume of trade and the quantity of high-quality credits fall.

Even in such cases, imposing penalties on buyers turns out to be an imprecise policy lever for improving climate outcomes, as they lower prices across all credit classes rather than targeting only the low-integrity segments, thereby discouraging participation from high-quality sellers as well. More direct interventions are conceivable if the policy goal is to shut down specific low-quality segments; for example, having the certifier reject projects from those classes outright could achieve the same outcome without inflicting collateral damage on better-performing segments. Thus, while penalizing buyers for greenwashing may seem appealing from a regulatory standpoint, doing so may backfire by undermining the price-quality feedback mechanism that sustains market integrity.

4.2. Pooling Credits into Portfolios

Another recent trend in the VCM is the emergence of products that bundle credits into a single instrument, allowing buyers to acquire exposure to a pre-assembled mix of credits rather than purchasing specific credits from individual projects. For instance, several firms offer predefined credit portfolios, akin to exchange-traded funds (ETFs) in financial markets, framing them as a way to diversify exposure across credit classes while simplifying the buyer experience (e.g., Rubicon Carbon, Patch, CNaught, Carbon Direct). Another variant involves blockchain-based "carbon tokens," such as Toucan's Base Carbon Tonne (BCT), which convert a pool of credits into a standardized digital token that trades on a decentralized ledger, similar to a cryptocurrency.

Despite their differences, both of these business models share a common feature: they pool heterogeneous credits into a single product, offering buyers exposure to the aggregate mix rather than to individual projects. To study the implications of this shift, we analyze an alternative setting where all certified credits are pooled into a single portfolio. Under this market structure, buyers no longer select an individual credit from a specific class, but instead purchase a "share" of the portfolio, which reflects the mix of credits in the market and trades at a common price p. Accordingly, all certified sellers receive the same price p per credit, regardless of their class.

Formally, we represent the portfolio as a lottery over different credits, with the mix determined endogenously by the composition of sellers that enter the market and are certified. Let $\lambda \in [0,1]$ denote the fraction of class-1 credits (with co-benefits) in the portfolio. The expected utility for a buyer of type v is then

$$\tilde{U}(a, v; \lambda, \boldsymbol{\mu}, p) = \begin{cases} (1+\gamma v)\mu_1\lambda + \mu_0(1-\lambda) - p, & \text{if buying a share of the portfolio } (a=1), \\ 0, & \text{otherwise } (a=-1). \end{cases} \tag{4.1}$$

In contrast with the baseline setting where credits of each class trade separately (Equation (2.2)), the pooled market offers a single product (portfolio shares). Buyers can buy into the aggregate portfolio but cannot selectively choose credits from the class that best aligns with their preferences. As a result, they form beliefs about quality based on the overall composition of the portfolio. The equilibrium in this setting is characterized as follows.

Proposition 6 **(Equilibrium in the pooled market)**. *Consider the setting where credits are pooled into a portfolio. Let $\bar{\phi} = \beta\phi_1 + (1-\beta)\phi_0$ denote the overall fraction of high-quality projects, and define*

$$\bar{\epsilon}_{port} = \frac{\bar{\phi} + \gamma\beta\phi_1}{(1-\bar{\phi})(1+\gamma\beta\phi_1)}. \tag{4.2}$$

If the certification error is below this threshold (i.e., $\epsilon < \bar{\epsilon}_{port}$), there exists an equilibrium with positive trade in which the price of the portfolio is:

$$p^*_{port} = \begin{cases} \dfrac{\bar{\phi} + \gamma\beta\phi_1(1-\bar{\phi})(1-\epsilon)}{\bar{\phi} + (1-\bar{\phi})\epsilon}, & \text{if } \epsilon < \left(\frac{(1-\bar{\phi})\gamma\beta\phi_1}{\bar{\phi}+\gamma\beta\phi_1}\right)\bar{\epsilon}_{port}, \\ \left(\dfrac{\bar{\phi} + \gamma\beta\phi_1}{\bar{\phi}(1+\gamma\beta\phi_1)} - \dfrac{1-\bar{\phi}}{\bar{\phi}}\epsilon\right)_+, & \text{otherwise.} \end{cases} \tag{4.3}$$

Moreover, this is the unique equilibrium price that is positive.

Instead, if the certification noise exceeds the threshold (i.e., $\epsilon \geq \bar{\epsilon}_{port}$), the market collapses. That is, the price of the portfolio is $p^*_{port} = 0$ and no trade occurs in any equilibrium.

The equilibrium structure illustrates how seller outcomes across classes become intertwined in a pooled market. In particular, sellers' payoffs depend on the aggregate composition of the portfolio, which is jointly shaped by entry decisions in both classes. The equilibrium price p^*_{port} reflects this interdependence, as it is a function of the overall fraction of high-quality projects $\bar{\phi}$ (the weighted average across classes), while accounting for the extent to which co-benefits are bundled in the portfolio. Moreover, there is now a single condition for trade to be sustained: the market is active only if the certification error falls below the threshold $\bar{\epsilon}_{port}$ in (4.2), in which case sellers from both classes participate. Otherwise, if the error exceeds this threshold, the market collapses entirely and no trade occurs in either class.

A natural question is whether the pooled structure generates more favorable market conditions by offering buyers a single product that combines both carbon mitigation and co-benefits. Since pooling enforces a unified outcome across credit classes, one might expect equilibrium outcomes (e.g., prices and collapse thresholds) to lie "in between" those of the two classes under segmented trade. The next result shows that this is not always the case: pooling does not make market conditions simultaneously more favorable in both segments, and in some cases leads to outcomes that are less favorable across the board.

Proposition 7 **(Implications of pooling for market collapse and prices)**. *Fix the market parameters* $(\phi_0, \phi_1, \beta, \gamma)$ *with* $\phi_0, \phi_1, \beta \in (0,1)$, *and* $\gamma \geq 0$. *Then:*

(i) Pooling can never rescue both segments from collapse. In other words, if both segments collapse when credits trade separately, the pooled market also collapses. That is, $\bar{\epsilon}_{port} \leq \max\{\bar{\epsilon}_0, \bar{\epsilon}_1\}$.

(ii) However, the pooled market may collapse even if both credit classes would actively trade under the segmented structure. That is, there exist parameter values such that $\bar{\epsilon}_{port} < \min\{\bar{\epsilon}_0, \bar{\epsilon}_1\} \leq 1$.

An analogous result holds for equilibrium prices: the portfolio price never exceeds both class-specific prices, but is strictly below both in some parameter regions.

While Proposition 7 shows that pooling may lead to lower prices and a greater risk of market collapse, we are ultimately interested in whether net climate gain can increase by moving from a segmented to a pooled market. When the portfolio price falls below both class-specific prices under segmented trade, the supply of high-quality credits is strictly lower, and net climate gain is therefore unambiguously reduced (assuming all markets remain active). However, when the portfolio price lies between the two segmented prices, the effect of pooling on the overall supply composition (and hence on carbon abatement) is not immediately clear.

Intuitively, pooling affects market outcomes through two opposing forces. On one hand, pooling makes sellers' entry decisions interdependent across credit classes. In particular, the price perceived by class-0 sellers is influenced by the value buyers place on co-benefits. This can generate a cross-subsidy effect: when the pooled price exceeds the class-0 price under segmented trade, the presence of class-1 credits in the portfolio (and the co-benefits they offer) effectively subsidizes class-0 sellers, attracting high-quality entry within that segment. Even if pooling somewhat lowers the price perceived by class-1 sellers, buyers' willingness to pay for co-benefits can keep it high enough to sustain broad participation by high-quality class-1 sellers as well.

On the other hand, pooling reduces the market's flexibility to reflect quality uncertainty through prices. By allowing credits to trade at class-specific prices, the segmented structure better captures quality differences across classes. This granularity is lost in the pooled structure, as it effectively forces both classes to trade at a single price. The resulting informational loss can be particularly meaningful if certification is noisy, as buyers rely more heavily on prices as quality signals.

Therefore, whether pooling improves climate gain relative to segmented trade depends on the trade-off between these two opposing forces: the cross-subsidy that transmits buyers' willingness to pay for co-benefits across classes, and the loss of pricing flexibility that results from eliminating segmentation. As the next result shows, this trade-off hinges on the certification accuracy.

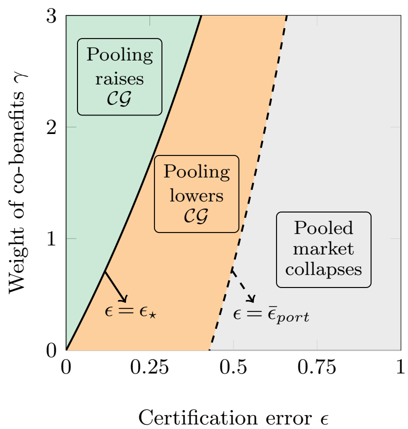
PROPOSITION 8 **(Effect of pooling on net climate gain)**. *Suppose the pooled market is active ($\epsilon < \bar{\epsilon}_{port}$). Then, the pooled market generates higher net climate gain than the segmented structure only if the certification error is sufficiently low and buyers place positive value on co-benefits. Formally, for any fixed parameters $(\phi_0, \phi_1, \beta, \delta)$ and $\gamma > 0$, there exists a unique threshold $\epsilon_\star \in (0, \bar{\epsilon}_{port})$ such that net climate gain is higher under the pooled structure if and only if $\epsilon < \epsilon_\star$.*

The intuition behind Proposition 8 is as follows. If certification is sufficiently accurate (low ϵ), the informational loss from reduced pricing flexibility is limited, as the certification already provides a strong signal of quality by filtering out most low-quality projects. In this case, the cross-subsidy effect sustains a high price that attracts broad participation from high-quality class-0 sellers while also maintaining high-quality class-1 sellers. This can expand the supply of high-quality credits, leading to higher net climate gain.

By contrast, if certification is noisier, much of the underlying quality uncertainty remains unresolved and segmentation plays a more important informational role. The single price in the pooled market provides a weaker signal of quality, and may fall too far below the class-specific price in one segment without sufficiently compensating in the other. As a result, pooling deters high-quality entry and delivers lower net climate gain than the segmented structure.

Figure 5 illustrates how this comparison depends on the value buyers place on co-benefits (γ). The region where pooling yields higher net climate gain (green) expands as γ increases, since greater willingness to pay for co-benefits strengthens the cross-subsidy effect within the portfolio. It is worth noting that pooling can only increase climate gain if buyers value co-benefits ($\gamma > 0$); otherwise, there is no cross-subsidy effect to counteract the loss of pricing flexibility. If certification is noisier (orange region), the informational loss from pooling becomes more pronounced and segmented trade yields higher net climate gain. Finally, the pooled market collapses if the certification is sufficiently

Figure 5 Net climate gain under the pooled and segmented market structures



Notes. The figure compares net climate gain under the pooled and segmented market structures across the (ϵ, γ) parameter space. As described in Proposition 8, pooling yields higher climate gain only when the certification error is below a threshold ($\epsilon < \epsilon_\star$). The region favorable to pooling expands with larger γ, reflecting a stronger cross-subsidy effect, as buyers' willingness to pay for co-benefits supports a high price that attracts high-quality class-0 projects to enter the market. The dashed line at $\epsilon = \bar{\epsilon}_{port}$ depicts the collapse threshold of the pooled market.
Parameter values: $\phi_0 = 0.4$, $\phi_1 = 0.2$, $\beta = 0.5$, $\delta = 1$

noisy (gray region). Similar to the discussion of §4.1, any apparent benefit of pooling in this region would arise only from shutting down trade entirely if the segmented market is climate-negative.

Our analysis underscores that accurate certification is crucial for pooling mechanisms to improve climate performance, and highlights the risk of eliminating the informational content embedded in the market structure when certification alone is insufficient. While our model is highly stylized, this insight helps interpret recent developments in practice. In particular, the main initiatives to tokenize carbon offsets (e.g., Toucan BCT and Moss Earth MCO2) have effectively collapsed, with their prices approaching zero amid concerns about the integrity of the credits represented in the pool.[11] This outcome is consistent with our model's intuition that pooling can exacerbate adverse selection when certification is noisy: by removing distinctions across credits, these tokens allowed low-quality offsets to remain in circulation by effectively "hiding" within the pool.

A similar concern applies to firms that offer pre-assembled credit portfolios, in that their products risk compounding adverse selection unless their internal curation and screening practices are sufficiently rigorous to compensate for inaccurate certification. Notably, some portfolio-based platforms emphasize these efforts as a way to provide additional quality assurance, effectively positioning their internal screening process as a form of enhanced certification.[12] Thus, although our model abstracts from many real-world considerations, it points to a key policy insight: pooling must be accompanied by reliable certification to improve carbon mitigation outcomes.

5. Incentivizing Accurate Certification

While the interventions we considered act on buyers' behavior and the market structure, we now turn to the certifier, who plays a central role in shaping market outcomes by deciding which projects

[11] See Badgley and Cullenward (2022) for an analysis of the quality concerns around Toucan's BCT, and related journalistic coverage by Rathi and White (2022), Mulder (2022), and Teixeira and Asher-Schapiro (2022).

[12] For instance, Rubicon uses independent ratings to substantiate the quality of its portfolios (Business Wire 2025).

are allowed to enter. Accurate certification is widely recognized as a key condition for the VCM to generate meaningful carbon abatement (West et al. 2023, Axelsson et al. 2024). As our model illustrates, accurate certification mitigates adverse selection by filtering out low-quality projects while supporting prices that attract high-quality supply. Reflecting this mechanism, a consistent feature across all the settings analyzed in §3 and §4 is that the market achieves its maximum net climate gain when certification is perfectly accurate (Appendix A.4 provides a formal proof).

So far we have treated the certification error ϵ as exogenous. In practice, however, carbon registries (who play the role of certifiers) design their own screening methodologies and exercise broad discretion over elements that ultimately determine their accuracy.[13] While the certification process is partly influenced by technical limitations in measurement, reporting, and verification (i.e., quantifying the actual emissions reduced by each project and assessing whether they are truly additional), registries have faced criticism over whether their incentives are aligned with achieving the highest possible accuracy (Giles and Coglianese 2025).

To investigate these concerns, we return to the baseline model and endogenize the certification error to analyze how the certifier's business model shapes its incentives for accuracy. Specifically, we introduce an initial stage preceding the timeline in Figure 1, in which the certifier chooses its error rate ϵ. For simplicity, we assume this choice is costless and unconstrained, so that ϵ can take any value in $[0, 1]$.

In practice, carbon registries typically charge a *flat fee per issued credit*, which ties their compensation to the *quantity* of credits in the market.[14] This fee structure has sparked debate over a potential conflict of interest: since their revenue grows with credit issuance, registries may benefit from approving marginal projects in order to expand the number of credits on the market (Battocletti et al. 2024, Coglianese and Giles 2025). Our model formalizes this concern through the following proposition, which shows that the certifier strictly prefers a positive error rate over perfect accuracy if its objective is to maximize trade volume.

Proposition 9 **(Noisy certification maximizes trade volume)**. *Suppose $\gamma > 0$. Then, there exists $\epsilon > 0$ for which the total trade volume is strictly higher than under perfect certification. Thus, the certifier-preferred error rate is positive if its objective is to maximize trade volume.*

[13] For example, registries define the assumptions that determine the baseline scenario used to benchmark emissions and assess whether projects would be economically viable without revenue from carbon offsets. Other dimensions include specifying what project-level data must be collected and how frequently it must be audited. Although some registries emphasize that their methodologies take a conservative approach, several studies have identified material scope for improvement (Gill-Wiehl et al. 2024, Probst et al. 2024, West et al. 2024).

[14] Verra and Gold Standard, the two largest registries representing over 75% of market share, charge between \$0.15 and \$0.25 per issued credit. While they also collect some other payments (e.g., yearly account and project listing fees), a material share of their revenue is directly tied to issuance volume (Verra 2024, Gold Standard 2025). Because these fees represent a small fraction of credit prices, they are unlikely to materially influence project developers' participation decisions. We therefore abstract from their effect on seller entry to focus on how the certifier's strategic objectives (maximizing volume vs. transaction value) shape its incentives for pursuing accuracy.

The mechanism behind this result is that, in general, the relationship between certification error and trade volume is ambiguous: as long as the market does not collapse, a higher error rate admits a larger mass of low-quality projects but also lowers prices, which can deter high-quality entry. Proposition 9 shows that the expansion effect dominates when moving from perfect accuracy to some positive error rate. For modest increases in ϵ, prices decline but remain high enough to attract most high-quality projects, so the entry of additional low-quality projects leads to a net increase in total credit volume. Figure 6a provides an illustration, showing that trade volume peaks at a positive error rate. Hence, if the certifier's objective aligns with maximizing trade volume (as occurs if it is paid a flat fee per credit), it has an incentive to tolerate a certain degree of noise. Note that this effect would be even stronger if improving accuracy were costly.

This misalignment arises because the quantity of credits transacted in the market is not an informative measure of their underlying quality. By contrast, prices directly reflect buyers' beliefs about quality and capture the extent to which certification resolves uncertainty, declining if the error rate ϵ is higher. Therefore, a fee structure that decouples the certifier's revenue from credit prices risks rewarding volume expansion at the expense of credit integrity.

A possible approach to mitigate this tension is to modify the certifier's fee structure. Specifically, instead of charging a flat fee per credit issued, the registry could collect a fixed percentage of each credit's market price as a commission. Under this structure, the certifier's revenue becomes proportional to the *total transaction value* (the product of price and quantity), as given by

$$\mathcal{TV}^* = p_0^* \cdot S_0^* + p_1^* \cdot S_1^*.$$

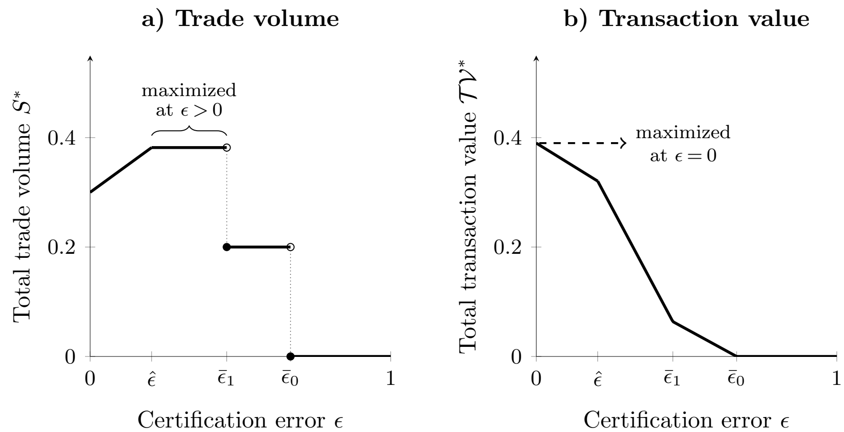
The next result shows that if the certifier seeks to maximize total transaction value, its incentives become fully aligned with pursuing perfect accuracy, and hence with maximizing the net climate gain produced by the market.

Proposition 10 **(Perfect certification maximizes transaction value)**. *Total transaction value is strictly maximized at $\epsilon = 0$. Consequently, the certifier pursues perfect accuracy if its objective is to maximize total transaction value.*

Figure 6b illustrates this result. Unlike trade volume, transaction value is maximized under perfect accuracy and declines monotonically with certification error since it captures the effect of prices. While increasing ϵ may raise trade volume by admitting additional low-quality supply, the accompanying decline in prices has a stronger effect, reducing the value of *all* transactions. Hence, the certifier is effectively rewarded for accuracy when its objective is tied to transaction value.

Finally, while our analysis suggests that linking registry fees to credit prices could mitigate the incentive misalignment associated with flat fees per credit, the potential downsides of implementing

Figure 6 **Trade volume and transaction value as functions of certification error**



Notes. Total trade volume and transaction value in equilibrium as functions of the certification error ϵ. As shown in Propositions 9 and 10, trade volume is maximized at some positive level of certification error (panel a), whereas transaction value is maximized under perfect accuracy (panel b). The discontinuities in trade volume occur at the thresholds where each market segment collapses ($\bar{\epsilon}_0$ and $\bar{\epsilon}_1$).
Parameter values: $\phi_0 = 0.4$, $\phi_1 = 0.2$, $\beta = 0.5$, $\gamma = 1$

this approach should also be considered. In particular, tying registries' revenue to prices may create new challenges, such as incentives to restrict or delay credit issuance to inflate prices. Although these dynamics lie outside the scope of our model, they highlight the importance of oversight mechanisms and competition among registries to promote efficiency and integrity in the certification process. These issues warrant further research as the market continues to evolve.

6. Concluding Remarks

The voluntary carbon market plays an important role in global decarbonization efforts, providing firms with a mechanism to offset their residual emissions by financing projects that reduce or remove carbon. Yet as evidence of credits overstating their climate impact has accumulated, doubts have emerged about whether the VCM can fulfill its intended role. To provide an analytical framework for this discussion, this paper develops a model of the VCM that links its defining institutional features—adverse selection on the supply side, imperfect certification, and buyers' preferences for non-carbon attributes such as co-benefits—to its effectiveness in genuinely reducing emissions.

Our results underscore that accurate certification is essential for a well-functioning market. Otherwise, unresolved quality uncertainty erodes buyer confidence, leading to lower prices that deter participation by high-quality supply. Ultimately, the market may collapse altogether if certification is too noisy, as persistent uncertainty prevents trade from taking place. However, unlike typical settings with adverse selection, the nature of demand in the VCM can alter these dynamics, as buyers' preferences for non-carbon attributes may sustain trade even when certification fails to adequately screen quality, giving rise to markets that appear to be functional but deliver little carbon mitigation. Conversely, in markets with accurate certification, stronger preferences for co-benefits can raise prices and attract additional high-quality supply, improving overall climate outcomes.

Our analysis of policy and market design interventions further highlights the importance of certification in shaping the market's climate effectiveness. To illustrate a demand-side intervention, we examine the effects of penalizing buyers for purchasing low-quality offsets and find that this policy tends to backfire, as it reduces buyers' willingness to pay without mitigating the underlying information asymmetry. On the supply side, we consider pooling credits into a single portfolio, and find that the implications of this shift depend on certification accuracy. Pooling can enhance carbon abatement when certification is accurate, as credits with co-benefits subsidize the entry of additional high-quality supply, but can instead reduce it when certification is noisy, as eliminating market segmentation amplifies information frictions.

Finally, by endogenizing accuracy as a decision of the certifier, we argue that registries face weak incentives to pursue accuracy under the prevailing industry practice of charging a flat fee per issued credit. This structure ties the certifier's revenue to the volume of issued credits rather than to their quality, creating a potential conflict of interest as trade volume is maximized at intermediate levels of error that allow some low-quality credits to enter the market. As a potential remedy, we show that linking the certifier's revenue to the value of transactions, which can be operationalized by setting registry fees as a percentage of credit prices, better aligns incentives by tying revenue to prices, which reflect overall credit quality and thus reward greater accuracy.

To an extent, this insight may inform ongoing discussions within the VCM on how to reward accuracy (Anderegg et al. 2025). While directly compensating certifiers for accuracy is difficult in practice, given that counterfactual outcomes are unobservable and projects have long time horizons, linking their compensation to credit prices could be an indirect but feasible mechanism to align financial incentives with the pursuit of accuracy. Nonetheless, future research should explore the potential downsides of this approach, including its effects on other dimensions of certifier behavior and its interaction with competitive dynamics among registries.

We have made a number of simplifying assumptions, adopting functional forms that allow for a clean characterization of equilibrium—which is often challenging in models with adverse selection (Wilson 1980, Gale 1992)—while preserving enough tractability to accommodate variations of the baseline setup. We have also abstracted from several features that are relevant in practice, such as how competition among certifiers influences market and climate outcomes, and how firms' adoption of net-zero pledges interacts with the perceived integrity of carbon offsets, both of which are interesting directions for future research.

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Appendix A: Proofs

This Appendix provides the proofs of the propositions stated in the paper. The proofs are organized in line with the structure of the main text: Appendix A.1 proves the results for the equilibrium analysis in §3. Appendices A.2 and A.3 provide the proofs for the analysis of buyer penalties and credit pooling from §4.1 and §4.2, respectively. Finally, Appendix A.4 covers the proofs for the analysis of certifier incentives in §5.

A.1. Proofs for §3 (Equilibrium Analysis)

This appendix provides the proofs for the equilibrium analysis in §3. Subsections A.1.1, A.1.2, and A.1.3 cover the equilibrium characterization (Proposition 1), the climate gain and market collapse scenarios (Proposition 2), and the role of co-benefits in preventing market collapse (Proposition 3), respectively.

A.1.1. Proof of Proposition 1 The proof follows three steps. First, in Lemma 1, we characterize the demand functions induced by buyers' utility-maximizing purchase decisions, taking prices and beliefs as given. Then, in Lemma 2, we establish necessary conditions that must hold in equilibrium and show that the unique prices that can support an equilibrium with positive trade are those defined in Equations (3.1) and (3.2). Finally, in Lemma 3, we construct an equilibrium with these prices, hence showing that they also provide sufficient conditions for equilibrium.

We proceed by stating and proving each of these lemmas, and subsequently conclude with a short arguments that proves Proposition 1 once these auxiliary results have been established. Throughout the argument, we focus on the case where $\gamma > 0$, and address the corner case $\gamma = 0$ separately at the end. We begin by characterizing the demand functions for each class of credits.

LEMMA 1. *Fix $\gamma > 0$ and suppose that given prices $\boldsymbol{p} = (p_0, p_1)$ and beliefs $\boldsymbol{\mu} = (\mu_0, \mu_1)$, buyers follow a utility-maximizing strategy that satisfies (2.3). Then, the demand functions $D_b(\boldsymbol{p}, \boldsymbol{\mu})$ for each credit class $b \in \{0, 1\}$ take the following form:*

(a) For class-0 credits:

$$D_0(\boldsymbol{p},\boldsymbol{\mu}) = \begin{cases} F\left(\frac{\mu_0 - p_0 + (p_1 - \mu_1)}{\gamma\mu_1}\right), & \text{if } \mu_0 > p_0 \text{ and } \mu_1 > 0, \\ \alpha_0(\boldsymbol{p},\boldsymbol{\mu}) \cdot F\left(\frac{\mu_0 - p_0 + (p_1 - \mu_1)}{\gamma\mu_1}\right), & \text{if } \mu_0 = p_0 \text{ and } \mu_1 > 0, \\ 1, & \text{if } \mu_0 > p_0 \text{ and } \mu_1 = 0, \\ \alpha_0(\boldsymbol{p},\boldsymbol{\mu}), & \text{if } \mu_0 = p_0 \text{ and } \mu_1 = 0, \\ 0, & \text{otherwise; namely if } \mu_0 < p_0. \end{cases} \tag{A.1}$$

(b) For class-1 credits:

$$D_1(\boldsymbol{p},\boldsymbol{\mu}) = \begin{cases} \bar{F}\left(\frac{(p_1 - \mu_1) + (\mu_0 - p_0)_+}{\gamma\mu_1}\right), & \text{if } \mu_1 > 0, \\ \alpha_1(\boldsymbol{p},\boldsymbol{\mu}), & \text{if } \mu_1 = 0,\ p_1 = 0, \text{ and } \mu_0 \le p_0, \\ 0, & \text{otherwise.} \end{cases} \tag{A.2}$$

Here, $\alpha_b(\boldsymbol{p},\boldsymbol{\mu})$ denote allocation probabilities that resolve indifference for each class of credits, while F and $\bar{F} = 1 - F$ denote the CDF and complementary CDF of buyer types, respectively.

Proof of Lemma 1. Recall that given prices $\boldsymbol{p} = (p_0, p_1)$ and beliefs $\boldsymbol{\mu} = (\mu_0, \mu_1)$, the expected utility of a buyer with type v is given by Equation (2.2):

$$\tilde{U}(a,v;\boldsymbol{\mu},\boldsymbol{p}) = \begin{cases} (1+\gamma v)\mu_1 - p_1, & \text{if } a=1 \text{ (buy a class-1 credit)},\\ \mu_0 - p_0, & \text{if } a=0 \text{ (buy a class-0 credit)},\\ 0, & \text{if } a=-1 \text{ (do not buy)}. \end{cases}$$

We derive the demand functions by characterizing the utility-maximizing action for each buyer type, considering separately the cases with $\mu_1 > 0$ and $\mu_1 = 0$, to obtain the expressions given in (A.1) and (A.2).

Case 1: $\boldsymbol{\mu_1 > 0}$. A buyer with type v strictly prefers to purchase a class-0 credit over both alternatives if:

$$\tilde{U}(0,v;\boldsymbol{\mu},\boldsymbol{p}) > \max\{\tilde{U}(1,v;\boldsymbol{\mu},\boldsymbol{p}), \tilde{U}(-1,v;\boldsymbol{\mu},\boldsymbol{p})\},$$

which is equivalent to

$$\mu_0 > p_0 \quad \text{and} \quad v < \frac{\mu_0 - p_0 + (p_1 - \mu_1)}{\gamma\mu_1}. \tag{A.3}$$

Conversely, a buyer with type v strictly prefers to buy a class-1 credit if and only if

$$v > \max\left\{\frac{p_1-\mu_1}{\gamma\mu_1}, \frac{\mu_0 - p_0 + (p_1-\mu_1)}{\gamma\mu_1}\right\},$$

which simplifies to the following condition:

$$v > \frac{p_1 - \mu_1 + (\mu_0 - p_0)_+}{\gamma\mu_1}. \tag{A.4}$$

When $\mu_0 > p_0$, the two conditions (A.3) and (A.4) partition the buyer type space (up to a measure-zero set of indifferent buyers). Thus, if $\mu_0 > p_0$, the demand split between the two credit classes depends only on the threshold in (A.4), and we have:

$$D_0(\boldsymbol{p},\boldsymbol{\mu}) = F\left(\frac{\mu_0 - p_0 + (p_1-\mu_1)}{\gamma\mu_1}\right), \quad \text{and} \quad D_1(\boldsymbol{p},\boldsymbol{\mu}) = \bar{F}\left(\frac{p_1-\mu_1+(\mu_0-p_0)_+}{\gamma\mu_1}\right).$$

Instead, if $\mu_0 < p_0$, no buyer purchases a class-0 credit, and the demand for class-1 is determined by the fraction of buyers who prefer buying those credits over refraining from purchase, which corresponds to types that satisfy (A.4). So in this case we have:

$$D_0(\boldsymbol{p},\boldsymbol{\mu}) = 0, \quad \text{and} \quad D_1(\boldsymbol{p},\boldsymbol{\mu}) = \bar{F}\left(\frac{p_1-\mu_1+(\mu_0-p_0)_+}{\gamma\mu_1}\right).$$

Finally, if $\mu_0 = p_0$, the demand for class-1 credits continues to be determined by condition (A.4), so the above expression still applies. On the other hand, buyers with type v below the threshold in (A.4) are indifferent between buying a class-0 credit and not purchasing. Let $\alpha_0(\boldsymbol{p},\boldsymbol{\mu})$ denote the probability that such buyers choose to purchase a class-0 credit in this case. Then, the demand for class-0 credits equals this probability times the fraction of buyers with type below the threshold. Thus, we have:

$$D_0(\boldsymbol{p},\boldsymbol{\mu}) = \alpha_0(\boldsymbol{p},\boldsymbol{\mu}) F\left(\frac{\mu_0 - p_0 + (p_1-\mu_1)}{\gamma\mu_1}\right), \quad \text{and} \quad D_1(\boldsymbol{p},\boldsymbol{\mu}) = \bar{F}\left(\frac{p_1-\mu_1+(\mu_0-p_0)_+}{\gamma\mu_1}\right).$$

In all cases, the demand for each class of credits aligns with Equations (A.1) and (A.2).

Case 2: $\boldsymbol{\mu_1 = 0}$. If $p_1 > 0$, then all buyers strictly prefer not to purchase class-1 credits, so $D_1(\boldsymbol{p},\boldsymbol{\mu}) = 0$. The decision between buying a class-0 credit or refraining from purchase the depends only on the sign of $\mu_0 - p_0$, as described in Equation (A.1).

If $p_1 = 0$, buyers may be indifferent between buying a class-1 credit and not purchasing, so we also have three subcases. If $\mu_0 > p_0$, all buyers strictly prefer to buy class-0 credits, so $D_0(\boldsymbol{p},\boldsymbol{\mu}) = 1$ and $D_1(\boldsymbol{p},\boldsymbol{\mu}) = 0$.

If $\mu_0 < p_0$, no buyers purchase class-0 credits, while buyers are indifferent between buying class-1 and not purchasing, so the demand for class-1 $D_1(\boldsymbol{p},\boldsymbol{\mu})$ is given by an allocation probability $\alpha_1(\boldsymbol{p},\boldsymbol{\mu}) \in [0,1]$ that represents the fraction of buyers that resolve this indifference in favor of class-1. Finally, if $\mu_0 = p_0$, buyers are indifferent between all three actions and the demand for each class-b is given by arbitrary allocation probabilities for each option, in line with Equations (A.1) and (A.2). □

We next characterize the equilibrium prices by solving the market-clearing conditions that equate supply and demand for each credit class. The following result shows that the prices in Equations (3.1) and (3.2) are the unique prices that can sustain an equilibrium with positive trade. In addition, we identify the threshold values for certification error beyond which each market segment collapses.

LEMMA 2. *Fix $\gamma > 0$ and let p_b^* and $\bar{\epsilon}_b$ be defined as in Equations* (3.1) *and* (3.2)*, for each credit class $b \in \{0,1\}$. Suppose there exists an equilibrium with prices $\boldsymbol{p} = (p_0, p_1)$. Then, for $b \in \{0,1\}$ we have that:*

(a) If $\epsilon < \bar{\epsilon}_b$ and $p_b > 0$, then $p_b = p_b^*$.

(b) Conversely, if $\epsilon \geq \bar{\epsilon}_b$, then $p_b = 0$.

The proof of Lemma 2 involves a series of claims that establish necessary conditions that must hold in any equilibrium. The first claim shows that in equilibrium, the expected utility from purchasing a class-0 credit must be zero, making buyers indifferent between purchasing class-0 credits and refraining from doing so.

CLAIM 1. *In any equilibrium, it holds that $\mu_0 = p_0$.*

Proof of Claim 1. First, suppose toward a contradiction that $\mu_0 < p_0$. Then, from Equation (A.1), the demand for class-0 credits is zero. Since the market clears in equilibrium, the supply for class-0 credits must also be zero. From Equation (2.6), the supply of class-0 credits is given by

$$S_0(p_0) = (1-\beta)\left(\phi_0 G(p_0) + \epsilon(1-\phi_0)\mathbf{1}_{\{p_0>0\}}\right).$$

This expression is strictly positive whenever $p_0 > 0$, so for supply to be zero, we must have $p_0 = 0$. But since $\mu_0 \in [0,1]$, we have that $\mu_0 \geq p_0 = 0$, a contradiction. This shows that $\mu_0 \geq p_0$.

Now suppose that $\mu_0 > p_0$. We complete the argument by considering two cases, based on whether μ_1 is positive or zero.

First, if $\mu_1 > 0$, we have from Lemma 1 that total demand satisfies $D_1(\boldsymbol{p},\boldsymbol{\mu}) + D_0(\boldsymbol{p},\boldsymbol{\mu}) = 1$. Since the markets for both credit classes clear in equilibrium, total supply equals total demand, and so we have:

$$\begin{aligned} D_1(\boldsymbol{p},\boldsymbol{\mu}) + D_0(\boldsymbol{p},\boldsymbol{\mu}) = 1 = S_1(p_1) + S_0(p_0) \\ \leq S_1(1) + S_0(1) = \beta(\phi_1 + (1-\phi_1)\epsilon) + (1-\beta)(\phi_0 + (1-\phi_0)\epsilon) \leq 1, \end{aligned}$$

where the inequalities hold because the supply functions $S_b(p_b)$ are increasing in the corresponding price when $p_b \in [0,1]$ and remain constant for $p_b \geq 1$. Therefore, for these inequalities to hold, both equilibrium prices must be greater or equal to one, i.e., $p_0 \geq 1$ and $p_1 \geq 1$. But this contradicts the assumption that $\mu_0 > p_0$, since $\mu_0 \leq 1$ by definition. Thus, if $\mu_1 > 0$, we must have $\mu_0 = p_0$, as desired.

To conclude, we consider the second case with $\mu_1 = 0$. If $\mu_0 > p_0$, we have $D_0(\boldsymbol{p},\boldsymbol{\mu}) = 1$ by Lemma 1. As before, market clearing requires $S_0(p_0) = 1$, which in turn implies $p_0 \geq 1$. Again, this contradicts the assumption that $\mu_0 > p_0$, completing the proof. □

The previous claim establishes that the condition $\mu_0 = p_0$ must hold in any equilibrium, making buyers must be indifferent between purchasing a class-0 credit or not. Building on this condition, the next claim characterizes the unique price at which class-0 credits can trade in equilibrium and identifies the threshold for certification error beyond which this segment collapses.

CLAIM 2. *Define, as in* (3.1),

$$\bar{\epsilon}_0 = \frac{\phi_0}{1-\phi_0}, \quad \textit{and} \quad p_0^* = \left(1 - \frac{1-\phi_0}{\phi_0}\epsilon\right)_+.$$

If $\epsilon < \bar{\epsilon}_0$, then p_0^* is the price of class-0 credits in any equilibrium where such credits trade at a positive price. Instead, if $\epsilon \geq \bar{\epsilon}_0$, the price for class-0 credits is $p_0 = 0$ (and coincides with p_0^*) in any equilibrium.

Proof of Claim 2. Suppose first that there exists an equilibrium with $p_0 > 0$. By Claim 1, we have $\mu_0 = p_0$. Since beliefs are pinned down by Bayes' rule, as given by (2.7), this condition implies:

$$p_0 = \frac{\phi_0 G(p_0)}{\phi_0 G(p_0) + (1-\phi_0)\epsilon}.$$

Moreover, since $\mu_0 \in [0,1]$ and $\mu_0 = p_0$, we must have $p_0 \leq 1$. Given that G is assumed to be the uniform distribution in $[0,1]$, we have $G(p_0) = p_0$, and so the above equation simplifies to

$$p_0 = \frac{\phi_0 p_0}{\phi_0 p_0 + (1-\phi_0)\epsilon}.$$

Since $p_0 > 0$, this condition uniquely pins down the equilibrium price as

$$p_0 = 1 - \frac{1-\phi_0}{\phi_0}\epsilon.$$

In particular, this price is positive if and only if $\epsilon < \bar{\epsilon}_0 = \phi_0/(1-\phi_0)$, which proves the first part of the claim.

Conversely, if $\epsilon \geq \bar{\epsilon}_0$, the expression above yields $p_0 \leq 0$, so there exists no positive price $p_0 > 0$ that can form an equilibrium, since it would otherwise solve the above equation. Hence, in any equilibrium, we must have $p_0 = 0$, as desired. □

The previous claim pins down the only possible equilibrium price for class-0 credits. We now turn to the class-1 segment. Before deriving its corresponding equilibrium price, we establish the following conditions, which later help simplify the analysis of market-clearing conditions in this segment.

CLAIM 3. *In any equilibrium, the price and belief for class-1 credits satisfy $0 \leq p_1 - \mu_1 \leq \gamma\mu_1$.*

Proof of Claim 3. First, suppose that $\mu_1 = 0$. In equilibrium, the belief μ_1 is pinned down by Bayes' rule, as given by Equation (2.7):

$$\mu_1 = \frac{\phi_1 G(p_1)}{\phi_1 G(p_1) + (1-\phi_1)\epsilon},$$

and so we must have $p_1 = 0$. Thus, if $\mu_1 = 0$, the inequalities we propose hold trivially.

Now suppose that $\mu_1 > 0$. By Claim 2, we must have $\mu_0 = p_0$. Substituting this condition into the demand function for class-1 credits in (A.2) yields:

$$D_1(\boldsymbol{p}, \boldsymbol{\mu}) = 1 - F\left(\frac{p_1 - \mu_1}{\gamma\mu_1}\right).$$

Suppose toward a contradiction that $p_1 - \mu_1 > \gamma\mu_1$. Then, $\frac{p_1-\mu_1}{\gamma\mu_1} > 1$, and since F is supported in $[0,1]$, we have that $D_1(\boldsymbol{p},\boldsymbol{\mu}) = 0$. Since each market segment clears in equilibrium, this implies that the supply of class-1 credits must also be zero, i.e. $S_1(p_1) = 0$. From Equation (2.6) we have

$$S_1(p_1) = \beta\left(\phi_1 G(p_1) + \epsilon(1-\phi_1)\mathbf{1}_{\{p_1>0\}}\right),$$

and so the condition $S_1(p_1) = 0$ implies that $p_1 = 0$. But this in turn implies that the buyers' belief also collapses to $\mu_1 = 0$, contradicting our assumption that $\mu_1 > 0$. This proves that $p_1 - \mu_1 \leq \gamma\mu_1$.

To show that $p_1 \geq \mu_1$, suppose for contradiction that $p_1 < \mu_1$. Plugging in the demand function gives $D_1(\boldsymbol{p},\boldsymbol{\mu}) = 1$. Market clearing implies that $S_1(p_1) = 1$, which may hold only if $p_1 \geq 1$. But then, we have $\mu_1 \leq 1 \leq p_1$, contradicting the assumed condition $p_1 < \mu_1$. Therefore, we must have $p_1 \geq \mu_1$. □

Following a similar line of reasoning as with the class-0 market, we now characterize the unique price at which positive trade for class-1 credits can be sustained in equilibrium, and the condition under which this market segment instead collapses.

Claim 4. *Define, as in* (3.2),

$$\bar{\epsilon}_1 = \frac{\phi_1(1+\gamma)}{(1-\phi_1)(1+\phi_1\beta\gamma)}, \quad \textit{and} \quad p_1^* = \begin{cases} \dfrac{\phi_1(1+\gamma)}{\phi_1+(1-\phi_1)\epsilon} - \phi_1\beta\gamma, & \textit{if } \epsilon < \left(\frac{\gamma(1-\beta\phi_1)}{1+\gamma}\right)\bar{\epsilon}_1, \\ \left(\dfrac{1+\gamma}{1+\phi_1\beta\gamma} - \dfrac{1-\phi_1}{\phi_1}\epsilon\right)_+, & \textit{otherwise.} \end{cases}$$

If $\epsilon < \bar{\epsilon}_1$, then p_1^* is the price of class-1 credits in any equilibrium where such credits trade at a positive price. Instead, if $\epsilon \geq \bar{\epsilon}_1$, the price for class-1 credits is $p_1 = 0$ (and coincides with p_1^*) in any equilibrium.

Proof of Claim 4. Suppose there exists an equilibrium with $\mu_1 > 0$. By Claim 1, this equilibrium satisfies $\mu_0 = p_0$. Plugging into the demand function D_1 in (A.2) gives that the demand for class-1 credits evaluated at the equilibrium prices and beliefs is:

$$D_1(\boldsymbol{p},\boldsymbol{\mu}) = 1 - F\left(\frac{p_1-\mu_1}{\gamma\mu_1}\right) = 1 - \frac{p_1-\mu_1}{\gamma\mu_1},$$

where the last equality follows since F is the uniform distribution over $[0,1]$ and, by Claim 3, we have $0 \leq p_1 - \mu_1 \leq \gamma\mu_1$. This condition also implies that $p_1 > 0$, since $p_1 \geq \mu_1 > 0$. Since p_1 is positive, all low-quality class-1 sellers enter the market (condition (2.5)), and the equilibrium supply of class-1 credits is

$$S_1(p_1) = \beta\left(\phi_1 G(p_1) + (1-\phi_1)\epsilon\right).$$

To derive the possible equilibrium prices p_1, we solve the market clearing condition $D_1(\boldsymbol{p},\boldsymbol{\mu}) = S_1(p_1)$. Plugging in the above expressions for supply and demand, we can write this equation as

$$\frac{(1+\gamma)\mu_1 - p_1}{\gamma\mu_1} = \beta\left(\phi_1 G(p_1) + (1-\phi_1)\epsilon\right).$$

In addition, the belief μ_1 must satisfy Bayes' rule as in Equation (2.7):

$$\mu_1 = \frac{\phi_1 G(p_1)}{\phi_1 G(p_1) + (1-\phi_1)\epsilon}.$$

By plugging in the expression for μ_1 into the above market clearing condition, the price p_1 is pinned down as the solution to:

$$\frac{(1+\gamma)\phi_1 G(p_1)}{\phi_1 G(p_1)+(1-\phi_1)\epsilon}-p_1=\beta\gamma\phi_1 G(p_1). \tag{A.5}$$

We next solve Equation (A.5) to obtain the possible equilibrium price by considering two cases.

Case 1: $\boldsymbol{p_1>1}$. Since G is the uniform distribution over $[0,1]$, we have that $G(p_1)=1$ in this case. Thus, Equation (A.5) simplifies to

$$\frac{(1+\gamma)\phi_1}{\phi_1+(1-\phi_1)\epsilon}-p_1=\beta\gamma\phi_1,$$

from where we immediately solve

$$p_1=\frac{(1+\gamma)\phi_1}{\phi_1+(1-\phi_1)\epsilon}-\beta\gamma\phi_1.$$

Straightforward algebra shows that this price satisfies the condition $p_1>1$ if and only if

$$\epsilon<\frac{\phi_1\gamma(1-\beta\phi_1)}{(1-\phi_1)(1+\phi_1\gamma\beta)}=\left(\frac{\gamma(1-\beta\phi_1)}{1+\gamma}\right)\bar{\epsilon}_1. \tag{A.6}$$

Finally, since the solution to Equation (A.5) is unique in this region, the above price is the unique possible equilibrium price that satisfies $p_1>1$.

Case 2: $\boldsymbol{0<p_1\le 1}$. Following a similar argument, within this region we have $G(p_1)=p_1$, and so Equation (A.5) simplifies to

$$\frac{(1+\gamma)\phi_1 p_1}{\phi_1 p_1+(1-\phi_1)\epsilon}-p_1=\beta\gamma\phi_1 p_1.$$

Since we are looking for a solution with $p_1>0$, this equation further simplifies to

$$\frac{(1+\gamma)\phi_1}{\phi_1 p_1+(1-\phi_1)\epsilon}=1+\beta\gamma\phi_1,$$

from where we obtain

$$p_1=\frac{1+\gamma}{1+\phi_1\beta\gamma}-\frac{1-\phi_1}{\phi_1}\epsilon.$$

Since the solution to the above equation is unique, p_1 above is the only possible equilibrium price that satisfies $0<p_1\le 1$. Simple algebra shows that this price satisfies both these conditions if and only if

$$\left(\frac{\gamma(1-\beta\phi_1)}{1+\gamma}\right)\bar{\epsilon}_1\le\epsilon<\bar{\epsilon}_1. \tag{A.7}$$

Note that the conditions in (A.6) and (A.7) define disjoint regions that correspond to Cases 1 and 2, and their union covers the cases with $\epsilon<\bar{\epsilon}_1$. Thus, Cases 1 and 2 prove the claim when $\epsilon<\bar{\epsilon}_1$.

Instead, if $\epsilon\ge\bar{\epsilon}_1$, it follows from the two above cases that no positive price p_1 solves the market-clearing condition for class-1 credits, $D_1(\boldsymbol{p},\boldsymbol{\mu})=S_1(p_1)$, provided that the belief satisfies $\mu_1>0$. Thus, if $\epsilon\ge\bar{\epsilon}_1$, the only possible equilibrium requires a belief of zero, $\mu_1=0$, which implies that $p_1=0$ as well. In particular, the expression for p_1 in (3.2) continues to hold since, when $\epsilon\ge\bar{\epsilon}_1$, we have

$$p_1=0=\left(\frac{1+\gamma}{1+\phi_1\beta\gamma}-\frac{1-\phi_1}{\phi_1}\epsilon\right)_+,$$

as desired. □

Having pinned down the unique prices that can sustain positive trade in equilibrium for each credit class, we now conclude the proof of Lemma 2.

Proof of Lemma 2. The result follows directly from Claims 2 and 4 for $b \in \{0,1\}$, respectively. □

While Lemma 2 establishes that the proposed prices define necessary conditions for any equilibrium with positive trade, we now show that these conditions are also sufficient by explicitly constructing an equilibrium supported by the prices p_0^* and p_1^*.

LEMMA 3. *Fix $\gamma > 0$. Then, there exists an equilibrium in which the price of class-b credits is equal to p_b^*, as defined in Equations* (3.1) *and* (3.2), *for $b \in \{0,1\}$.*

Proof of Lemma 3. To prove the result, we construct an equilibrium $(\boldsymbol{p}^*, \boldsymbol{\mu}^*, e^*, a^*)$, where $\boldsymbol{p}^* = (p_0^*, p_1^*)$ are the prices defined in Equations (3.1) and (3.2). Define the corresponding beliefs associated for each class of credit, $\boldsymbol{\mu}^* = (\mu_0^*, \mu_1^*)$, as in Equation (2.7):

$$\mu_b^* = \frac{\phi_b G(p_b^*)}{\phi_b G(p_b^*) + \epsilon(1-\phi_b)}.$$

For each seller type θ, define the strategy $e^*(\theta) = e(\theta)$ as in Equation (2.5). Finally, for each buyer type v, define the strategy $a^*(v)$ as follows:

$$a^*(v) = \begin{cases} 1, & \text{if } v > \tilde{v}, \\ 0, & \text{if } (1-\alpha_0^*)\tilde{v} \le v \le \tilde{v}, \\ -1, & \text{if } v < (1-\alpha_0^*)\tilde{v}, \end{cases} \tag{A.8}$$

where the quantities $\tilde{v}$ and α_0^* are given by:

$$\tilde{v} = \begin{cases} 1-\beta(\phi_1 + (1-\phi_1)\epsilon), & \text{if } \epsilon < \left(\frac{\gamma(1-\beta\phi_1)}{1+\gamma}\right)\bar{\epsilon}_1, \\ \dfrac{1-\beta\phi_1}{1+\beta\phi_1\gamma}, & \text{if } \left(\frac{\gamma(1-\beta\phi_1)}{1+\gamma}\right)\bar{\epsilon}_1 \le \epsilon < \bar{\epsilon}_1, \\ 1, & \text{if } \epsilon \ge \bar{\epsilon}_1, \end{cases}$$

and

$$\alpha_0^* = \frac{(1-\beta)\phi_0 \mathbf{1}_{\{\epsilon < \bar{\epsilon}_0\}}}{\tilde{v}}.$$

We next verify that the four conditions in Definition 1 are satisfied to show that $(\boldsymbol{p}^*, \boldsymbol{\mu}^*, e^*, a^*)$ forms an equilibrium. By construction, the sellers strategy e^* satisfies (2.5), and the buyers' beliefs $\boldsymbol{\mu}^*$ are consistent with Bayes' rule, in accordance with Equation (2.7). Therefore, it remains to show the buyers' strategy in (A.8) maximizes their expected utility, taking prices and beliefs as given, and that the prices clear the market for each class of credits.

To show that the buyers' strategy maximizes their expected utility, we first note that in the region where $0 \le \epsilon < \bar{\epsilon}_1$ (which corresponds to the condition $\mu_1^* > 0$), the threshold $\tilde{v}$ defined above satisfies:

$$\tilde{v} = \frac{p_1^* - \mu_1^* + (\mu_0^* - p_0^*)_+}{\gamma \mu_1^*},$$

which can be verified with straightforward algebra. So, given the prices $\boldsymbol{p}^*$ and beliefs $\boldsymbol{\mu}^*$, the demand for class-1 credits induced by the strategy a^* in (A.8) is equal to:

$$D_1^* = \begin{cases} \bar{F}\left(\frac{p_1^* - \mu_1^* + (\mu_0^* - p_0^*)_+}{\gamma \mu_1^*}\right), & \text{if } \mu_1^* > 0, \\ 0, & \text{if } \mu_1^* = 0, \end{cases}$$

which is consistent with the demand function D_1 in (A.2), evaluated at $\boldsymbol{p}^*$ and $\boldsymbol{\mu}^*$. Similarly, since the belief and price for class-0 credits satisfy $\mu_0^* = p_0^*$ by construction (see Claim 2), the demand for class-0 credits induced by the strategy a^* is equal to:

$$D_0^* = \begin{cases} \alpha_0^* F\left(\frac{\mu_0^* - p_0^* + (p_1^* - \mu_1^*)}{\gamma \mu_1^*}\right), & \text{if } \mu_1^* > 0, \\ \alpha_0^*, & \text{if } \mu_1^* = 0. \end{cases}$$

This quantity is also consistent with the demand function D_0 in (A.1) evaluated at $\boldsymbol{p}^*$ and $\boldsymbol{\mu}^*$, with the allocation probability that resolves indifference defined as $\alpha_0(\boldsymbol{p}^*, \boldsymbol{\mu}^*) = \alpha_0^*$. It remains to show that this allocation probability indeed satisfies $\alpha_0^* \in [0,1]$, which we verify at the end of the proof. Taking this as given, the same logic from the proof of Lemma 1 shows that the buyer's strategy a^* satisfies the utility maximization condition in (2.3).

We now verify that the prices $\boldsymbol{p}^* = (p_0^*, p_1^*)$ clear the market for each class of credits. For class-1 credits, this follows directly from the same argument of Claim 4. For class-0 credits, the supply given the price p_0^* is equal to (from Equation (2.6)):

$$\begin{aligned} S_0^* = S_0(p_0^*) &= (1-\beta)\left(\phi_0 G(p_0^*) + \epsilon(1-\phi_0)\mathbf{1}_{\{p_0^* > 0\}}\right) \\ &= (1-\beta)\left(\phi_0 \left(1 - \frac{1-\phi_0}{\phi_0}\epsilon\right)_+ + \epsilon(1-\phi_0)\mathbf{1}_{\{\epsilon < \bar{\epsilon}_0\}}\right) \\ &= (1-\beta)\phi_0 \mathbf{1}_{\{\epsilon < \bar{\epsilon}_0\}}. \end{aligned}$$

By construction, the corresponding demand equals:

$$D_0^* = \alpha_0^* \tilde{v} = \left(\frac{(1-\beta)\phi_0 \mathbf{1}_{\{\epsilon < \bar{\epsilon}_0\}}}{\tilde{v}}\right)\tilde{v} = (1-\beta)\phi_0 \mathbf{1}_{\{\epsilon < \bar{\epsilon}_0\}},$$

and thus the class-0 market clears as $D_0^* = S_0^*$.

Finally, we verify that $0 \le \alpha_0^* \le 1$. Note that

$$1 - \tilde{v} = \bar{F}(\tilde{v}) = D_1^* = S_1(p_1^*) = \beta(\phi_1 G(p_1^*) + (1-\phi_1)\epsilon \mathbf{1}_{\{\epsilon < \bar{\epsilon}_1\}}) \le \beta,$$

which implies that $\tilde{v} \ge (1-\beta)\phi_0$, and hence $\alpha_0^* \le 1$. Conversely, $\alpha_0^* \ge 0$ directly holds since $\beta \le 1$. ☐

Having established the supporting lemmas, we are now ready to prove Proposition 1.

Proof of Proposition 1. We consider two cases separately, depending on whether $\gamma > 0$ or $\gamma = 0$.

First, suppose that $\gamma > 0$. By Lemma 3, there exists an equilibrium where the prices for each credit class are given by Equations (3.1) and (3.2). Combining this with the uniqueness results from Lemma 2 establishes points (i) and (ii) of the proposition.

Suppose instead that $\gamma = 0$. In this case, the prices from (3.1) and (3.2) are:

$$p_0^* = \left(1 - \frac{1-\phi_0}{\phi_0}\epsilon\right)_+, \quad \text{and} \quad p_1^* = \left(1 - \frac{1-\phi_1}{\phi_1}\epsilon\right)_+. \tag{A.9}$$

Claims 1 and 2 continue to hold for class-0 since their proof is independent of the value of γ. Moreover, since $\gamma = 0$, the same arguments apply to class-1 credits. Specifically, in any equilibrium we must have $\mu_1 = p_1$ as in Claim 1, while the price derivation from Claim 2 holds analogously for class-1 in this particular case.

Thus, we only need to show that if $\gamma = 0$, there exists an equilibrium with the prices p_0^* and p_1^* as in (A.9). This can be done by following a similar argument to the proof of Lemma 3. Note that when $\gamma = 0$, the conditions $\mu_b = p_b$ must hold in equilibrium for $b \in \{0,1\}$ by Claim 1, and so buyers are indifferent between buying a credit from either class or refraining from purchase in equilibrium. Thus, we can construct a buyer strategy that assigns types to actions in a way that the mass of buyers is allocated to match supply with demand. That is, instead of a^* in (A.8), we define:

$$a^*(v) = \begin{cases} 1, & \text{if } v > 1-\alpha_1^*, \\ 0, & \text{if } 1-\alpha_1^*-\alpha_0^* \le v \le 1-\alpha_1^*, \\ -1, & \text{if } v < 1-\alpha_1^*-\alpha_0^*, \end{cases}$$

where α_0^* and α_1^* are given by:

$$\alpha_0^* = S_0(p_0^*) = (1-\beta)\phi_0 \mathbf{1}_{\{\epsilon<\bar{\epsilon}_0\}}, \quad \text{and} \quad \alpha_1^* = S_1(p_1^*) = \beta\phi_1 \mathbf{1}_{\{\epsilon<\bar{\epsilon}_1\}}.$$

Under this strategy, a mass of buyers α_b^* purchases class-b credits for $b \in \{0,1\}$. By construction, these quantities satisfy $\alpha_b^* = S_b(p_b^*)$, and thus markets clear. The beliefs, defined as $\mu_b^* = p_b^*$ for $b \in \{0,1\}$ satisfy Bayes' rule by the same argument of Claim 2. Moreover, the strategy a^* maximizes each buyer's expected utility since they are indifferent between all actions. The seller strategy e^* remains unchanged, as in (2.5).

Hence, by the above construction and the same reasoning as in Lemma 3, $(\boldsymbol{p}^*, \boldsymbol{\mu}^*, e^*, a^*)$ constitutes an equilibrium with prices as given in (A.9), concluding the proof. □

A.1.2. Proof of Proposition 2

Proof of Proposition 2. By Proposition 1, the market for class-b credits collapses if and only if $\epsilon \ge \bar{\epsilon}_b$, for $b \in \{0,1\}$. In this case, the volume of class-b credits traded in equilibrium is $S_b^* = 0$, resulting in a net climate gain of zero, which proves Part (iii).

To prove Parts (i) and (ii), we show that if the class-b market segment does not collapse (i.e., $\epsilon < \bar{\epsilon}_b$), its net climate gain ($\mathcal{CG}_b$) is positive when ϵ lies below the threshold $\tilde{\epsilon}_b^{\mathcal{CG}}$, and negative if we instead have $\epsilon > \tilde{\epsilon}_b^{\mathcal{CG}}$. Recall from Equation (2.9) that we can write the net climate gain generated by the class-b segment as

$$\mathcal{CG}_b = S_b^* \left(\mu_b^* - \delta(1-\mu_b^*)\right). \tag{2.9}$$

For the class-0 segment, we use Proposition 1 to obtain the equilibrium trade volume S_0^* and beliefs μ_0^*:

$$S_0^* = (1-\beta)\phi_0 \mathbf{1}_{\{\epsilon<\bar{\epsilon}_0\}}, \quad \text{and} \quad \mu_0^* = \left(1 - \frac{1-\phi_0}{\phi_0}\epsilon\right) \mathbf{1}_{\{\epsilon<\bar{\epsilon}_0\}}.$$

Plugging into (2.9) yields

$$\mathcal{CG}_0 = (1-\beta)\phi_0 \left(1 - \left(\frac{1+\delta}{\bar{\epsilon}_0}\right)\epsilon\right) \mathbf{1}_{\{\epsilon<\bar{\epsilon}_0\}}.$$

Note that for $\epsilon \in [0, \bar{\epsilon}_0)$, $\mathcal{CG}_0$ is strictly decreasing in ϵ and changes sign at $\epsilon = \tilde{\epsilon}_0^{\mathcal{CG}} = \frac{\bar{\epsilon}_0}{1+\delta}$. Since $\delta > 0$, we have $\tilde{\epsilon}_0^{\mathcal{CG}} < \bar{\epsilon}_0$. Thus, $\mathcal{CG}_0 > 0$ for $\epsilon < \tilde{\epsilon}_0^{\mathcal{CG}}$ and $\mathcal{CG}_0 < 0$ for $\epsilon \in (\tilde{\epsilon}_0^{\mathcal{CG}}, \bar{\epsilon}_0)$, proving Parts (i) and (ii) for $b = 0$.

We proceed similarly for the class-1 segment. The equilibrium trade volume S_1^* and belief μ_1^* are given by:

$$S_1^* = \begin{cases} \beta\left(\phi_1 + (1-\phi_1)\epsilon\right), & \text{if } \epsilon < \hat{\epsilon}_1, \\ \left(\frac{\beta\phi_1(1+\gamma)}{1+\phi_1\beta\gamma}\right) \mathbf{1}_{\{\epsilon<\bar{\epsilon}_1\}}, & \text{if } \epsilon \ge \hat{\epsilon}_1, \end{cases} \quad \text{and} \quad \mu_1^* = \begin{cases} \frac{\phi_1}{\phi_1+(1-\phi_1)\epsilon}, & \text{if } \epsilon < \hat{\epsilon}_1, \\ \left(1 - \frac{1}{\bar{\epsilon}_1}\epsilon\right) \mathbf{1}_{\{\epsilon<\bar{\epsilon}_1\}}, & \text{if } \epsilon \ge \hat{\epsilon}_1, \end{cases}$$

where the threshold $\hat{\epsilon}_1$ defining the two cases above is given by

$$\hat{\epsilon}_1 = \left(\frac{\gamma(1-\beta\phi_1)}{1+\gamma}\right)\bar{\epsilon}_1.$$

Plugging expressions into (2.9) yields the net climate gain for class-1:

$$\mathcal{CG}_1 = \begin{cases} \beta\left(\phi_1 - \delta(1-\phi_1)\epsilon\right), & \text{if } \epsilon < \hat{\epsilon}_1, \\ \beta(1-\phi_1)\left(\bar{\epsilon}_1 - (1+\delta)\epsilon\right), & \text{if } \hat{\epsilon}_1 \leq \epsilon < \bar{\epsilon}_1, \\ 0, & \text{if } \epsilon \geq \bar{\epsilon}_1. \end{cases}$$

Within the interval $\epsilon \in [0, \bar{\epsilon}_1)$, the net climate gain from class-1 $\mathcal{CG}_1$ is strictly decreasing in ϵ. We identify the threshold $\tilde{\epsilon}_1^{\mathcal{CG}}$ at which $\mathcal{CG}_1$ change sign by analyzing two cases based on the magnitude of δ.

Case 1: $\boldsymbol{\delta \leq \frac{1+\phi_1\beta\gamma}{\gamma(1-\beta\phi_1)}}$. Straightforward algebra shows that the following two conditions hold in this case:

$$\hat{\epsilon}_1 \leq \frac{\bar{\epsilon}_1}{1+\delta} < \bar{\epsilon}_1, \quad \text{and} \quad \frac{\phi_1}{\delta(1-\phi_1)} \geq \hat{\epsilon}_1.$$

These conditions imply that (a) $\mathcal{CG}_1 > 0$ if $0 \leq \epsilon < \hat{\epsilon}_1$ and that, (b) within the interval $\epsilon \in [\hat{\epsilon}_1, \bar{\epsilon}_1)$, $\mathcal{CG}_1$ is first positive and changes sign at $\epsilon = \tilde{\epsilon}_1^{\mathcal{CG}} = \bar{\epsilon}_1/(1+\delta)$.

Case 2: $\boldsymbol{\delta > \frac{1+\phi_1\beta\gamma}{\gamma(1-\beta\phi_1)}}$. Similar algebra shows that the following conditions hold in this case:

$$\frac{\bar{\epsilon}_1}{1+\delta} < \hat{\epsilon}_1, \quad \text{and} \quad \frac{\phi_1}{\delta(1-\phi_1)} < \hat{\epsilon}_1,$$

which imply that (a) within the interval $\epsilon \in [0, \hat{\epsilon}_1)$, the net climate gain $\mathcal{CG}_1$ is initially positive and changes sign at $\epsilon = \tilde{\epsilon}_1^{\mathcal{CG}} = \phi_1/(\delta(1-\phi_1))$, and (b) $\mathcal{CG}_1 < 0$ for $\epsilon \in [\hat{\epsilon}_1, \bar{\epsilon}_1)$.

The thresholds $\tilde{\epsilon}_1^{\mathcal{CG}}$ for two above cases correspond with the definition in Equation (3.3), and the above conditions directly show that $\tilde{\epsilon}_1^{\mathcal{CG}} < \bar{\epsilon}_1$. Finally, in either case, $\mathcal{CG}_1$ is positive for $\epsilon < \tilde{\epsilon}_1^{\mathcal{CG}}$, changing sign at $\epsilon = \tilde{\epsilon}_1^{\mathcal{CG}}$ and turning negative $\epsilon \in (\tilde{\epsilon}_1^{\mathcal{CG}}, \bar{\epsilon}_1)$, proving Parts (i) and (ii) for $b = 1$. □

A.1.3. Proof of Proposition 3

Proof of Proposition 3. By Proposition 1, the class-1 market collapses if and only if $\epsilon \geq \bar{\epsilon}_1$, where the collapse threshold (written explicitly as a function of γ) is given by

$$\bar{\epsilon}_1(\gamma) = \frac{\phi_1(1+\gamma)}{(1-\phi_1)(1+\phi_1\beta\gamma)},$$

Note that $\bar{\epsilon}_1(\gamma)$ is strictly increasing in γ. In addition, $\bar{\epsilon}_1(\gamma)$ converges to a limit strictly greater than 1 as $\gamma \to \infty$, ensuring that $\bar{\epsilon}_1(\gamma)$ can exceed any possible error level $\epsilon \in [0,1]$ for large enough γ.

Since we assumed that $\epsilon > \phi_1/(1-\phi_1) = \bar{\epsilon}_1(0)$, it follows that there exists a unique $\bar{\gamma} > 0$ that satisfies $\epsilon = \bar{\epsilon}_1(\gamma)$. Solving for $\bar{\gamma}$ in closed-form yields:

$$\bar{\gamma} = \frac{(1-\phi_1)\epsilon - \phi_1}{\phi_1\left(1 - \beta\epsilon(1-\phi_1)\right)}.$$

By monotonicity of $\bar{\epsilon}_1(\gamma)$, we have that the market collapses if and only if $0 \leq \gamma \leq \bar{\gamma}$, corresponding to $\epsilon \geq \bar{\epsilon}_1(\gamma)$. Instead, the market remains active (with $p_1^* > 0$) if $\gamma > \bar{\gamma}$, which is equivalent to $\epsilon < \bar{\epsilon}_1(\gamma)$. □

A.2. Proofs for §4.1 (Buyer Penalties)

This appendix contains the proofs for the analysis of buyer penalties presented in §4.1. Subsection A.2.1 establishes the equilibrium characterization (Proposition 4), while Subsection A.2.2 derives the implications of penalties for climate gain (Proposition 5).

A.2.1. Proof of Proposition 4 Before providing the proof, we restate the proposition including the closed-form expressions for the equilibrium prices and collapse thresholds.

PROPOSITION 4. Fix $\kappa > 0$. Then, for each credit class $b \in \{0,1\}$, there exists a threshold $\tilde{\epsilon}_b(\kappa)$ and an equilibrium price $p_b^*(\kappa)$ such that:

(i) If the certification error is below the threshold (i.e., $\epsilon \le \tilde{\epsilon}_b(\kappa)$), there exists an equilibrium in which class-b credits trade at a positive price $p_b^*(\kappa) > 0$. Moreover, $p_b^*(\kappa)$ is the unique price for class-b credits in any equilibrium with maximum trade volume.

(ii) If the certification error exceeds the threshold (i.e., $\epsilon > \tilde{\epsilon}_b(\kappa)$), the market for class-$b$ credits collapses. That is, their price is $p_b^*(\kappa) = 0$ and no such credits are traded in any equilibrium.

The collapse thresholds and equilibrium prices are given as follows:

(a) For class-0 credits (without co-benefits),

$$\tilde{\epsilon}_0(\kappa) = \left(1 + 2\kappa - \sqrt{(1+2\kappa)^2 - 1}\right)\bar{\epsilon}_0, \quad \text{and}$$
$$p_0^*(\kappa) = \frac{1}{2}\left(1 - \frac{1-\phi_0}{\phi_0}\epsilon + \sqrt{\left(1 - \frac{1-\phi_0}{\phi_0}\epsilon\right)^2 - 4\kappa\left(\frac{1-\phi_0}{\phi_0}\right)\epsilon}\right)\mathbf{1}_{\{\epsilon \le \tilde{\epsilon}_0(\kappa)\}}. \tag{A.10}$$

(b) For class-1 credits (with co-benefits),

$$\tilde{\epsilon}_1(\kappa) = \left(1 + \frac{2\kappa}{1+\gamma} - \sqrt{\left(1 + \frac{2\kappa}{1+\gamma}\right)^2 - 1}\right)\bar{\epsilon}_1, \quad \text{and} \tag{A.11}$$

$$p_1^*(\kappa) = \begin{cases} \dfrac{(1+\gamma)\phi_1 - \kappa(1-\phi_1)\epsilon}{\phi_1 + (1-\phi_1)\epsilon} - \beta\gamma\phi_1, & \text{if } \epsilon < \hat{\epsilon}_1(\kappa), \\ \dfrac{1}{2}\left(\dfrac{1+\gamma}{1+\beta\gamma\phi_1} - \dfrac{1-\phi_1}{\phi_1}\epsilon + \sqrt{\left(\dfrac{1+\gamma}{1+\beta\gamma\phi_1} - \dfrac{1-\phi_1}{\phi_1}\epsilon\right)^2 - 4\kappa\left(\dfrac{1-\phi_1}{\phi_1(1+\beta\gamma\phi_1)}\right)\epsilon}\right), & \text{if } \hat{\epsilon}_1(\kappa) \le \epsilon \le \tilde{\epsilon}_1(\kappa), \\ 0, & \text{if } \epsilon > \tilde{\epsilon}_1(\kappa), \end{cases}$$

where the intermediate threshold separating the above cases is

$$\hat{\epsilon}_1(\kappa) = \frac{\phi_1\gamma(1-\beta\phi_1)}{(1-\phi_1)(1+\phi_1\gamma\beta+\kappa)}.$$

Finally, both the equilibrium prices $p_b^*(\kappa)$ and the collapse thresholds $\tilde{\epsilon}_b(\kappa)$ are decreasing in the penalty κ.

To prove the result, we proceed similarly to the proof of Proposition 1. The next lemma derives the demand functions induced by buyers' utility-maximizing decisions, taking prices and beliefs as given, extending Lemma 1 to incorporate buyer penalties.

LEMMA 4. *Consider the setting of §4.1, where buyers pay a penalty $\kappa > 0$ if they purchase a low-quality offset. Fix $\gamma > 0$ and suppose that given prices $\boldsymbol{p} = (p_0, p_1)$ and beliefs $\boldsymbol{\mu} = (\mu_0, \mu_1)$, buyers follow a strategy that maximizes their expected utility. Then, the demand functions $D_b(\boldsymbol{p}, \boldsymbol{\mu})$ for each credit class $b \in \{0,1\}$ take the following form:*

(a) For class-0 credits:

$$D_0(\boldsymbol{p}, \boldsymbol{\mu}) = \begin{cases} F\left(\frac{\mu_0 - p_0 - \kappa(1-\mu_0) + (p_1 - \mu_1 + \kappa(1-\mu_1))}{\gamma\mu_1}\right), & \textit{if } \mu_0 > \frac{p_0+\kappa}{1+\kappa} \textit{ and } \mu_1 > 0, \\ \alpha_0(\boldsymbol{p}, \boldsymbol{\mu}) \cdot F\left(\frac{\mu_0 - p_0 - \kappa(1-\mu_0) + (p_1 - \mu_1 + \kappa(1-\mu_1))}{\gamma\mu_1}\right), & \textit{if } \mu_0 = \frac{p_0+\kappa}{1+\kappa} \textit{ and } \mu_1 > 0, \\ 1, & \textit{if } \mu_0 > \frac{p_0+\kappa}{1+\kappa} \textit{ and } \mu_1 = 0, \\ \alpha_0(\boldsymbol{p}, \boldsymbol{\mu}), & \textit{if } \mu_0 = \frac{p_0+\kappa}{1+\kappa} \textit{ and } \mu_1 = 0, \\ 0, & \textit{otherwise; namely if } \mu_0 < \frac{p_0+\kappa}{1+\kappa}. \end{cases} \tag{A.12}$$

(b) For class-1 credits:

$$D_1(\boldsymbol{p},\boldsymbol{\mu}) = \begin{cases} \bar{F}\left(\frac{p_1-\mu_1+\kappa(1-\mu_1)+(\mu_0-p_0-\kappa(1-\mu_0))_+}{\gamma\mu_1}\right), & \text{if } \mu_1>0, \\ 0, & \text{otherwise.} \end{cases} \tag{A.13}$$

Here, as in Lemma 1, $\alpha_0(\boldsymbol{p},\boldsymbol{\mu})$ denotes an allocation probability that resolves indifference for class-0 credits.

Proof of Lemma 4. The proof follows a similar structure to that of Lemma 1, so we only discuss how the argument is modified to take into account the penalty $\kappa \geq 0$.

First, note that the buyers' expected utility function from (2.2) is modified to incorporate the penalty as follows:

$$\tilde{U}(a,v;\boldsymbol{\mu},\boldsymbol{p}) = \begin{cases} (1+\gamma v)\mu_1-\kappa(1-\mu_1)-p_1, & \text{if } a=1 \text{ (buy a class-1 credit)}, \\ \mu_0-\kappa(1-\mu_0)-p_0, & \text{if } a=0 \text{ (buy a class-0 credit)}, \\ 0, & \text{if } a=-1 \text{ (do not buy)}. \end{cases}$$

With this utility function, the conditions that characterize the buyer types that strictly prefer to buy a class-0 or a class-1 credit over the alternatives, provided that $\mu_1>0$ (Case 1 in the proof of Lemma 1), are as follows. Instead of the conditions in (A.3), a buyer with type v strictly prefers to purchase a class-0 credit if and only if:

$$\mu_0 > \frac{p_0+\kappa}{1+\kappa} \quad \text{and} \quad v < \frac{\mu_0-p_0-\kappa(1-\mu_0)+(p_1-\mu_1+\kappa(1-\mu_1))}{\gamma\mu_1}.$$

Similarly, the condition that characterizes the types v who strictly prefer a class-1 credit, which plays the role of (A.4), is now given as:

$$v > \frac{p_1-\mu_1+\kappa(1-\mu_1)+(\mu_0-p_0-\kappa(1-\mu_0))_+}{\gamma\mu_1}.$$

The rest of the derivation follows the same reasoning as in Lemma 1, but considering the above two conditions to write the demand functions. In particular, instead of considering different cases based on the comparison between μ_0 and p_0, these same comparisons involve adjusting the price term as $(p_0+\kappa)/(1+\kappa)$ to incorporate the penalty. In addition, when $\mu_1=0$, the expected utility of purchasing a class-1 credit is negative since $\kappa>0$ (even if $p_1=0$), so the demand D_1 for class-1 credits in (A.13) involves only two cases. □

We now use the demand functions derived in Lemma 4 to characterize the equilibrium prices by solving the market-clearing conditions for each credit class. The argument is similar to that of Lemma 2, although the inclusion of the penalty κ makes the derivations somewhat more involved.

LEMMA 5. *Consider the setting of §4.1, and fix $\kappa>0$ and $\gamma>0$. Let $p_b^*(\kappa)$ and $\tilde{\epsilon}_b(\kappa)$ be defined as in Equations* (A.10) *and* (A.11)*, respectively, for each credit class $b\in\{0,1\}$. Suppose an equilibrium exists. Then, in any equilibrium that maximizes trade volume, the prices $\boldsymbol{p}=(p_0,p_1)$ satisfy:*

(a) If $\epsilon\leq\tilde{\epsilon}_b(\kappa)$ and $p_b>0$, then $p_b=p_b^*$.

(b) Conversely, if $\epsilon>\tilde{\epsilon}_b(\kappa)$, then $p_b=0$.

The proof of this result involves a series of claims, analogous to Lemma 2. As in that case, the first claim shows that, in any equilibrium where class-0 credits trade at a positive price, buyers must be indifferent between purchasing such a credit or not.

CLAIM 5. *In any equilibrium with $p_0>0$, it holds that $\mu_0=\frac{p_0+\kappa}{1+\kappa}$.*

Proof of Claim 5. Analogous argument to Claim 1. □

We next characterize the price for class-0 credits that can be sustained in a trade-maximizing equilibrium, and the corresponding threshold for this market segment to collapse as a function of the penalty κ.

CLAIM 6. *Let $\kappa > 0$ and define, as in (A.10),*

$$\tilde{\epsilon}_0(\kappa) = \left(1 + 2\kappa - \sqrt{(1+2\kappa)^2 - 1}\right)\bar{\epsilon}_0, \quad \text{where} \quad \bar{\epsilon}_0 = \frac{\phi_0}{1-\phi_0}, \quad \text{and}$$

$$p_0^* = \frac{1}{2}\left(1 - \frac{1-\phi_0}{\phi_0}\epsilon + \sqrt{\left(1 - \frac{1-\phi_0}{\phi_0}\epsilon\right)^2 - 4\kappa\left(\frac{1-\phi_0}{\phi_0}\right)\epsilon}\right).$$

If $\epsilon \le \tilde{\epsilon}_0(\kappa)$, p_0^* is the price of class-0 credits in any equilibrium where such credits trade at a positive price and trade volume is maximized. Instead, if $\epsilon > \tilde{\epsilon}_0(\kappa)$, the price for class-0 credits is $p_0 = 0$ in any equilibrium.

Proof of Claim 6. Let $\kappa > 0$ and suppose first that there exists an equilibrium with $p_0 > 0$. By Claim 5, it holds that

$$\mu_0 = \frac{p_0 + \kappa}{1 + \kappa}.$$

Moreover, the belief μ_0 is consistent with Bayes' rule, as in (2.7):

$$\mu_0 = \frac{\phi_0 G(p_0)}{\phi_0 G(p_0) + (1-\phi_0)\epsilon}.$$

The condition from Claim 5 implies that $p_0 = \mu_0 - \kappa(1-\mu_0) \le \mu_0 \le 1$. Hence, we have that $G(p_0) = p_0$. Plugging into the above expressions, we have that the price p_0 solves:

$$\frac{p_0 + \kappa}{1 + \kappa} = \frac{\phi_0 p_0}{\phi_0 p_0 + (1-\phi_0)\epsilon}.$$

This can be rewritten as the following quadratic equation:

$$\phi_0 p_0^2 - (\phi_0 - (1-\phi_0)\epsilon)\, p_0 + \kappa(1-\phi_0)\epsilon = 0. \tag{A.14}$$

Note that if $\epsilon \ge \bar{\epsilon}_0$, we have that $\phi_0 - (1-\phi_0)\epsilon \le 0$, and so the linear term in (A.14) is non-negative for $p_0 \ge 0$. As a result, no positive price $p_0 > 0$ solves Equation (A.14). Hence, a necessary condition for an equilibrium with $p_0 > 0$ to exist is that $\epsilon < \bar{\epsilon}_0$. If this condition holds, Equation (A.14) has two possible solutions:

$$p_0 = \frac{1}{2\phi_0}\left[\phi_0 - (1-\phi_0)\epsilon \pm \sqrt{(\phi_0 - (1-\phi_0)\epsilon)^2 - 4\kappa\phi_0(1-\phi_0)\epsilon}\right].$$

It is straightforward to verify with some calculus that both the positive and negative roots lie in $(0,1]$ if and only if the following conditions hold:

$$4\kappa\phi_0(1-\phi_0)\epsilon \le (\phi_0 - (1-\phi_0)\epsilon)^2, \quad \text{and} \quad \epsilon < \bar{\epsilon}_0 = \frac{\phi_0}{1-\phi_0}.$$

The first condition can be rewritten as a quadratic inequality in terms of ϵ. By doing so, and after some algebra, we can rewrite both the above conditions as:

$$\epsilon \le \tilde{\epsilon}_0(\kappa), \quad \text{where} \quad \tilde{\epsilon}_0(\kappa) = \left(1 + 2\kappa - \sqrt{(1+2\kappa)^2 - 1}\right)\bar{\epsilon}_0.$$

In particular, note that $0 < \left(1 + 2\kappa - \sqrt{(1+2\kappa)^2 - 1}\right) < 1$ for all $\kappa > 0$, and so we have $\tilde{\epsilon}_0(\kappa) < \bar{\epsilon}_0$.

Therefore, if $\epsilon \le \tilde{\epsilon}_0(\kappa)$ both the positive and negative roots of Equation (A.14) can be possible equilibrium prices. The positive root is the largest among these, and hence corresponds to the equilibrium with maximum trade volume, since the supply $S_0(p_0)$ is strictly increasing for $p_0 \in [0,1]$.

Conversely, if $\epsilon > \tilde{\epsilon}_0(\kappa)$, Equation (A.14) has no real solutions. In particular, there exists no positive price $p_0 > 0$ that can form an equilibrium. Hence, if $\epsilon > \tilde{\epsilon}_0(\kappa)$, we must have $p_0 = 0$ in any equilibrium. □

CLAIM 7. *In any equilibrium with $p_1 > 0$, the price and belief for class-1 credits satisfy:*

$$0 \leq p_1 - \mu_1 + \kappa(1-\mu_1) \leq \gamma\mu_1.$$

Proof of Claim 7. The proof is analogous to that of Claim 3, focusing only on the scenario where $p_1 > 0$ (which corresponds with the case where $\mu_1 > 0$). □

CLAIM 8. *Let $\kappa > 0$ and define, as in* (A.11),

$$\tilde{\epsilon}_1(\kappa) = \left(1 + \frac{2\kappa}{1+\gamma} - \sqrt{\left(1+\frac{2\kappa}{1+\gamma}\right)^2 - 1}\right)\bar{\epsilon}_1, \quad \text{where} \quad \bar{\epsilon}_1 = \frac{\phi_1(1+\gamma)}{(1-\phi_1)(1+\phi_1\beta\gamma)}, \quad \text{and}$$

$$\hat{\epsilon}_1(\kappa) = \frac{\phi_1\gamma(1-\beta\phi_1)}{(1-\phi_1)(1+\phi_1\gamma\beta+\kappa)}.$$

Then, if $\epsilon \leq \tilde{\epsilon}_1(\kappa)$, the price of class-1 credits in any equilibrium where such credits trade at a positive price and trade volume is maximized is:

$$p_1^* = \begin{cases} \dfrac{(1+\gamma)\phi_1 - \kappa(1-\phi_1)\epsilon}{\phi_1 + (1-\phi_1)\epsilon} - \beta\gamma\phi_1, & \text{if } \epsilon < \hat{\epsilon}_1(\kappa), \\ \dfrac{1}{2}\left(\dfrac{1+\gamma}{1+\beta\gamma\phi_1} - \dfrac{1-\phi_1}{\phi_1}\epsilon + \sqrt{\left(\dfrac{1+\gamma}{1+\beta\gamma\phi_1} - \dfrac{1-\phi_1}{\phi_1}\epsilon\right)^2 - 4\kappa\left(\dfrac{1-\phi_1}{\phi_1(1+\beta\gamma\phi_1)}\right)\epsilon}\right), & \text{if } \hat{\epsilon}_1(\kappa) \leq \epsilon \leq \tilde{\epsilon}_1(\kappa). \end{cases}$$

Instead, if $\epsilon > \tilde{\epsilon}_1(\kappa)$, the price for class-0 credits is $p_1 = 0$ in any equilibrium.

Proof of Claim 8. We derive the possible equilibrium prices by solving the market-clearing condition equating supply with demand for class-1 credits. First, suppose there exists an equilibrium with $p_1 > 0$. Then, since the belief μ_1 must be consistent with Bayes' rule, as given by (2.7), we must have $\mu_1 > 0$. Therefore, the demand function D_1 evaluated at the equilibrium prices and beliefs is (from (A.13)):

$$D_1(\boldsymbol{p}, \boldsymbol{\mu}) = 1 - F\left(\frac{p_1 - \mu_1 + \kappa(1-\mu_1) + (\mu_0 - p_0 - \kappa(1-\mu_0))_+}{\gamma\mu_1}\right).$$

Two observations allow us to simplify this expression. First, we claim that in equilibrium, the first term in the numerator vanishes, i.e., $(\mu_0 - p_0 - \kappa(1-\mu_0))_+ = 0$. If $p_0 > 0$, this follows directly from Claim 5. Instead, if $p_0 = 0$, then $\mu_0 = 0$ by (2.7), and so $(\mu_0 - p_0 - \kappa(1-\mu_0))_+ = (-\kappa(1-\mu_0))_+ = 0$. The second observation is that, by Claim 7, we have

$$0 \leq \frac{p_1 - \mu_1 + \kappa(1-\mu_1)}{\gamma\mu_1} \leq 1.$$

Due to these two observations, and since F is the CDF of the uniform distribution over $[0,1]$, we write the demand for class-1 credits in equilibrium as

$$D_1(\boldsymbol{p}, \boldsymbol{\mu}) = 1 - \left(\frac{p_1 - \mu_1 + \kappa(1-\mu_1)}{\gamma\mu_1}\right).$$

On the other hand, the supply for class-1 credits is given as in Equation (2.6). Since $p_1 > 0$, all low-quality sellers enter and we have:

$$S_1(p_1) = \beta\left(\phi_1 G(p_1) + (1-\phi_1)\epsilon\right).$$

To derive the possible equilibrium prices p_1, we solve the market clearing condition $D_1(\boldsymbol{p}, \boldsymbol{\mu}) = S_1(p_1)$. By plugging in the above expressions for supply and demand we obtain

$$\frac{(1+\gamma)\mu_1 - p_1 - \kappa(1-\mu_1)}{\gamma\mu_1} = \beta\left(\phi_1 G(p_1) + (1-\phi_1)\epsilon\right).$$

In addition, the belief μ_1 must satisfy Bayes' rule, as in Equation (2.7):

$$\mu_1 = \frac{\phi_1 G(p_1)}{\phi_1 G(p_1) + (1-\phi_1)\epsilon}.$$

After plugging in the expression for μ_1 into the above market clearing condition, the price p_1 is pinned down as the solution to:

$$\frac{(1+\gamma)\phi_1 G(p_1) - \kappa(1-\phi_1)\epsilon}{\phi_1 G(p_1) + (1-\phi_1)\epsilon} - p_1 = \beta\gamma\phi_1 G(p_1). \tag{A.15}$$

Like in the proof of Claim 4, we now solve Equation (A.15) to obtain the possible equilibrium price by considering two cases.

Case 1: $\boldsymbol{p_1 > 1}$. Since G is the uniform distribution over $[0,1]$, we have that $G(p_1) = 1$ in this case. Plugging into Equation (A.15) yields

$$\frac{(1+\gamma)\phi_1 - \kappa(1-\phi_1)\epsilon}{\phi_1 + (1-\phi_1)\epsilon} - p_1 = \beta\gamma\phi_1,$$

from where we directly solve

$$p_1 = \frac{(1+\gamma)\phi_1 - \kappa(1-\phi_1)\epsilon}{\phi_1 + (1-\phi_1)\epsilon} - \beta\gamma\phi_1.$$

Straightforward algebra shows that this price satisfies the condition $p_1 > 1$ if and only if

$$\epsilon < \hat{\epsilon}_1(\kappa) = \frac{\phi_1\gamma(1-\beta\phi_1)}{(1-\phi_1)(1+\phi_1\gamma\beta+\kappa)}. \tag{A.16}$$

Case 2: $\boldsymbol{0 < p_1 \le 1}$. In this case we have $G(p_1) = p_1$, and so Equation (A.15) becomes:

$$\frac{(1+\gamma)\phi_1 p_1 - \kappa(1-\phi_1)\epsilon}{\phi_1 p_1 + (1-\phi_1)\epsilon} - p_1 = \beta\gamma\phi_1 p_1,$$

which we rewrite as the following quadratic equation:

$$\phi_1(1+\beta\gamma\phi_1)p_1^2 - ((1+\gamma)\phi_1 - (1+\beta\gamma\phi_1)(1-\phi_1)\epsilon)p_1 + \kappa(1-\phi_1)\epsilon = 0. \tag{A.17}$$

Observe that if $\epsilon \ge \bar{\epsilon}_1$, then $(1+\gamma)\phi_1 - (1+\beta\gamma\phi_1)(1-\phi_1)\epsilon \le 0$, and so the linear term in (A.17) is non-negative for $p_1 \ge 0$. As a result, if $\epsilon \ge \bar{\epsilon}_1$, no positive price $p_1 > 0$ solves Equation (A.17). Further, it is easy to show that if $\epsilon \ge \bar{\epsilon}_1$, then condition (A.16) from Case 1 does not hold. Hence, a necessary condition for an equilibrium with $p_1 > 0$ to exist is that $\epsilon < \bar{\epsilon}_1$.

If $\epsilon < \bar{\epsilon}_1$, then Equation (A.17) has two possible solutions:

$$p_1 = \frac{(1+\gamma)\phi_1 - (1+\beta\gamma\phi_1)(1-\phi_1)\epsilon \pm \sqrt{((1+\gamma)\phi_1 - (1+\beta\gamma\phi_1)(1-\phi_1)\epsilon)^2 - 4\kappa\phi_1(1+\beta\gamma\phi_1)(1-\phi_1)\epsilon}}{2\phi_1(1+\beta\gamma\phi_1)}.$$

Moreover, for these solutions to be real-valued, the term within the square root must be non-negative. Thus, the following two necessary conditions must hold:

$$4\kappa\phi_1(1+\beta\gamma\phi_1)(1-\phi_1)\epsilon \le ((1+\gamma)\phi_1 - (1+\beta\gamma\phi_1)(1-\phi_1)\epsilon)^2, \quad \text{and} \quad \epsilon < \bar{\epsilon}_1 = \frac{\phi_1(1+\gamma)}{(1-\phi_1)(1+\phi_1\beta\gamma)}.$$

Similar to the argument of Claim 6, we can write the first condition as a quadratic inequality in terms of ϵ and, after some algebra, we rewrite both the above conditions as:

$$\epsilon \le \tilde{\epsilon}_1(\kappa), \quad \text{where} \quad \tilde{\epsilon}_1(\kappa) = \left(1 + \frac{2\kappa}{1+\gamma} - \sqrt{\left(1+\frac{2\kappa}{1+\gamma}\right)^2 - 1}\right)\bar{\epsilon}_1.$$

In particular, the term multiplying $\bar{\epsilon}_1$ is decreasing in κ and lies in $(0,1)$ for all $\kappa > 0$, which implies that $\tilde{\epsilon}_1(\kappa) < \bar{\epsilon}_1$.

Therefore, if $\epsilon \leq \tilde{\epsilon}_1(\kappa)$ both the positive and negative roots of Equation (A.17) can be possible equilibrium prices, provided they satisfy $0 < p_1 \leq 1$ (the condition assumed for Case 2). We next derive the conditions where this is the case.

To do so, let $h(p_1)$ denote the left-hand side of Equation (A.17), which is a convex parabola in p_1. Note that $h(0) \geq 0$. In addition, if $\epsilon < \bar{\epsilon}_1$, we have $h'(0) < 0$. Thus, provided that Equation (A.17) admits real-valued solutions (i.e. if $\epsilon \leq \tilde{\epsilon}_1(\kappa)$), the positive root satisfies $p_1 \leq 1$ if and only if both the roots of $h(p_1)$ lie in the interval $(0,1]$. This condition is equivalent to $h(1) \geq 0$, which can be directly written as the complement of (A.16). That is,

$$\epsilon \geq \hat{\epsilon}_1(\kappa) = \frac{\phi_1\gamma(1-\beta\phi_1)}{(1-\phi_1)(1+\phi_1\gamma\beta+\kappa)}.$$

Therefore, if (A.16) does not hold (i.e., if $\epsilon \geq \hat{\epsilon}_1(\kappa)$), both the positive and negative root of Equation (A.17) lie in $(0,1]$ provided that $\epsilon \leq \tilde{\epsilon}_1(\kappa)$. In addition, these are the only possible equilibrium prices and, among these two prices, the highest one (i.e., the positive root) results in larger trade volume.

Instead, if (A.16) holds (i.e., if $\epsilon < \hat{\epsilon}_1(\kappa)$), Case 1 admits a solution, which in particular satisfies $p_1 > 1$. Since any possible solution in Case 2 is constrained to $p_1 \leq 1$, the price in Case 1 is the highest and therefore corresponds to the trade-maximizing outcome.

Finally, we must also verify that $\hat{\epsilon}_1(\kappa) < \tilde{\epsilon}_1(\kappa)$. To do so, note from the earlier derivation that the interval $\{0 \leq \epsilon \leq \hat{\epsilon}_1(\kappa)\}$ is defined by the condition $h(1) \leq 0$, where h is the left-hand side of (A.17). Instead, the interval $\{0 \leq \epsilon \leq \tilde{\epsilon}_1(\kappa)\}$ corresponds to the cases where Equation (A.17) admits a real-valued solution, meaning that $h(p_1) \leq 0$ for some $p_1 \in (0,1]$, which is a weaker condition. Since these intervals must be nested, we have $\hat{\epsilon}_1(\kappa) \leq \tilde{\epsilon}_1(\kappa)$. To see that the inequality is strict, note that $\hat{\epsilon}_1(\kappa)$ is defined as the value of ϵ where $h(1) = 0$. Since the expression $h(1)$ is strictly increasing in ϵ, and $h(p_1)$ has two distinct roots in $(0,1]$ when $\epsilon \in [\hat{\epsilon}_1(\kappa), \tilde{\epsilon}_1(\kappa))$, it follows that this interval is non-empty, and thus $\hat{\epsilon}_1(\kappa) < \tilde{\epsilon}_1(\kappa)$.

To conclude, note that if $\epsilon > \tilde{\epsilon}_1(\kappa)$, there is no solution with $p_1 > 0$ to Equation (A.15) by the argument in Cases 1 and 2 above. Therefore, any equilibrium must have $p_1 = 0$. □

The derivations above characterize the prices that can arise in equilibrium. In some instances, there may be two candidate prices (corresponding to the solutions of the quadratic equations in Claims 6 and 8); in such cases, we select the higher price, consistent with our focus on the equilibrium that maximizes trade volume. Lemma 5 now follows directly from these arguments.

Proof of Lemma 5. Follows directly from Claims 6 and 8 for $b \in \{0,1\}$, respectively. □

The argument in the proof of Lemma 5 provides necessary conditions by deriving the prices that may arise in equilibrium. Next, we argue that an equilibrium with such prices can be constructed.

Lemma 6. *Consider the setting of §4.1, and fix $\kappa > 0$ and $\gamma > 0$. Then, there exists an equilibrium in which the price of class-b credits is equal to $p_b^*(\kappa)$, as defined in Equations* (A.10) *and* (A.11), *for each $b \in \{0,1\}$. Moreover, no other equilibrium yields higher trade volume for either credit class.*

Proof of Lemma 6. The result follows by adapting the construction in the proof of Lemma 3. Specifically, we construct an equilibrium $(\boldsymbol{p}^*, \boldsymbol{\mu}^*, e^*, a^*)$ where the prices $\boldsymbol{p}^* = (p_0^*(\kappa), p_1^*(\kappa))$ are given by Equations (A.10) and (A.11). The corresponding beliefs, seller strategies, and buyer strategies can be defined analogously, with the demand functions and buyer behavior derived from Lemmas 4 and 5.

We claim that this equilibrium maximizes trade volume. To see this, suppose there exists another equilibrium with prices $\tilde{\boldsymbol{p}} = (\tilde{p}_0, \tilde{p}_1)$. By Claims 6 and 8, $p_b^*(\kappa)$ is the highest price for class-b credits that can be sustained in equilibrium, and thus $p_b^*(\kappa) \geq \tilde{p}_b$ for $b \in \{0,1\}$. Since trade volume equals supply in equilibrium, and the supply function $S_b(p_b)$ is increasing in p_b (Equation (2.6)), it follows that $S_b(p_b^*(\kappa)) \geq S_b(\tilde{p}_b)$. □

Proposition 4 can now be proved as a consequence of the previous lemmas.

Proof of Proposition 4. Similar argument as Proposition 1, but now based on Lemmas 4, 5 and 6. □

A.2.2. Proof of Proposition 5 We begin with two key preliminary observations before proving the proposition. First, if the class-b market is collapsed in the absence of penalties (i.e., $\epsilon \geq \bar{\epsilon}_b$), it remains collapsed under any positive penalty $\kappa > 0$. This follows since the collapse threshold is lower in the presence of penalties, i.e., $\tilde{\epsilon}_b(\kappa) < \bar{\epsilon}_b$ for all $\kappa > 0$, where $\tilde{\epsilon}_b(\kappa)$ is given in Proposition 4. Thus, if $\epsilon \geq \bar{\epsilon}_b$, the net climate gain from class-b credits is zero for all $\kappa \geq 0$, including the baseline case without penalties.

Second, when $\epsilon < \bar{\epsilon}_b$, it is useful to express the market collapse condition as a threshold on the penalty κ rather than on the certification error ϵ. Specifically, the class-b segment is active at $\kappa = 0$ (by Proposition 1) and remains active for all $\kappa > 0$ such that $\epsilon \leq \tilde{\epsilon}_b(\kappa)$. Since $\tilde{\epsilon}_b(\kappa)$ is strictly decreasing in κ, we can invert this condition to define an upper bound $\tilde{\kappa}_b(\epsilon)$ such that the market is active if and only if $\kappa \leq \tilde{\kappa}_b(\epsilon)$. These thresholds are given by:

$$\tilde{\kappa}_0(\epsilon) = \frac{(\phi_0 - (1-\phi_0)\epsilon)^2}{4\phi_0(1-\phi_0)\epsilon}, \quad \text{and} \quad \tilde{\kappa}_1(\epsilon) = \frac{((1+\gamma)\phi_1 - (1+\beta\gamma\phi_1)(1-\phi_1)\epsilon)^2}{4\phi_1(1+\beta\gamma\phi_1)(1-\phi_1)\epsilon}. \tag{A.18}$$

Based on this observation, we can write the net climate gain generated by each market segment under a penalty $\kappa \geq 0$ (in the maximum-trade equilibrium) as follows:

$$\begin{aligned} \mathcal{CG}_0(\kappa) &= (1-\beta)\left(\phi_0 p_0^*(\kappa) - \delta(1-\phi_0)\epsilon\right)\mathbf{1}_{\{\kappa \leq \tilde{\kappa}_0(\epsilon)\}}, \text{ and} \\ \mathcal{CG}_1(\kappa) &= \beta\left(\phi_1 \min\{p_1^*(\kappa), 1\} - \delta(1-\phi_1)\epsilon\right)\mathbf{1}_{\{\kappa \leq \tilde{\kappa}_1(\epsilon)\}}, \end{aligned} \tag{A.19}$$

where these expressions result from plugging the prices of Propositon 4 into the net climate gain expression of (2.8). Note that $\mathcal{CG}_b(0)$ corresponds to the baseline net climate gain for class-b in the absence of penalties, since the prices $p_b^*(0)$ coincide with those in Proposition 1. We are now ready to prove Proposition 5.

Proof of Proposition 5 We begin with the first part of statement. Suppose the class-b market is climate-positive in the absence of penalties, i.e., $\epsilon < \tilde{\epsilon}_b^{\mathcal{CG}}$ where $\tilde{\epsilon}_b^{\mathcal{CG}}$ is defined in (3.3). We will show that any positive penalty $\kappa > 0$ reduces the net climate gain generated by this segment; that is, $\mathcal{CG}_b(\kappa) \leq \mathcal{CG}_b(0)$ for all $\kappa > 0$.

To do so, we show that the net climate gain $\mathcal{CG}_b(\kappa)$ is decreasing and continuous in κ over the interval $[0, \tilde{\kappa}_b(\epsilon)]$, and then constant and equal to zero for $\kappa > \tilde{\kappa}_b(\epsilon)$.

Consider first the interval $\kappa \in [0, \tilde{\kappa}_b(\epsilon)]$, where the market remains active. Observe from (A.19) that the net climate gain $\mathcal{CG}_b(\kappa)$ is increasing in the equilibrium price $p_b^*(\kappa)$. Moreover, the equilibrium prices $p_b^*(\kappa)$ are strictly decreasing and continuous in κ over the interval $[0, \tilde{\kappa}_b(\epsilon)]$, for both $b \in \{0,1\}$, as can be seen directly from the expressions in (A.10) and (A.11). It follows that $\mathcal{CG}_b(\kappa)$ is also decreasing and continuous in κ within the interval $[0, \tilde{\kappa}_b(\epsilon)]$.

For $\kappa > \tilde{\kappa}_b(\epsilon)$, the market collapses and results in zero net climate gain. Therefore, since $\epsilon < \tilde{\epsilon}_b^{\mathcal{CG}}$ implies $\mathcal{CG}_b(0) > 0$, we conclude that $\mathcal{CG}_b(0) \geq \mathcal{CG}_b(\kappa)$ for all $\kappa > 0$. Hence, any positive penalty $\kappa > 0$ reduces net climate gain in a segment that is climate-positive in the absence of penalties.

We now turn to the second part of the proposition. We will show that if at least one market segment is climate-negative in the absence of penalties, i.e., if $\tilde{\epsilon}_b^{\mathcal{CG}} < \epsilon < \bar{\epsilon}_b$ for some $b \in \{0,1\}$, there exist parameter values such that total net climate gain is strictly higher with a positive penalty $\kappa > 0$ than with no penalty. However, any such increase in climate gain necessarily occurs in a scenario where one or both market segments collapse as a result of the penalty.

Consider the total net climate gain across both segments:

$$\mathcal{CG}(\kappa) = \mathcal{CG}_0(\kappa) + \mathcal{CG}_1(\kappa),$$

and, for convenience, let us denote:

$$\tilde{\kappa}_{min} = \min\{\tilde{\kappa}_0(\epsilon), \tilde{\kappa}_1(\epsilon)\}, \quad \text{and} \quad \tilde{\kappa}_{max} = \max\{\tilde{\kappa}_0(\epsilon), \tilde{\kappa}_1(\epsilon)\}.$$

From the first part of the proof, it follows that $\mathcal{CG}(\kappa)$ is continuous and decreasing in κ within each of the intervals $[0, \tilde{\kappa}_{min}]$ and $(\tilde{\kappa}_{min}, \tilde{\kappa}_{max}]$, and is then constant and equal to zero for $\kappa > \tilde{\kappa}_{max}$.

In particular, no penalty in the range $\kappa \in (0, \tilde{\kappa}_{min}]$ can improve $\mathcal{CG}$ relative to the no-penalty case, since $\mathcal{CG}(\kappa)$ is decreasing within this region. Therefore, if a positive penalty $\kappa > 0$ yields higher net climate gain than $\kappa = 0$, it must lie strictly above $\tilde{\kappa}_{min}$ and induce the collapse of one or both market segments. It remains to show that there exist parameter values where such gains are possible, which we do by constructing a series of illustrative examples that reflect distinct possible scenarios in which a positive penalty may increase net climate gain. Figure 7 illustrates these examples.

Recall that we focus on the case where at least one segment is climate-negative in the absence of penalties; i.e., where $\tilde{\epsilon}_b^{\mathcal{CG}} < \epsilon < \bar{\epsilon}_b$ for some $b \in \{0,1\}$. We consider three possible cases where this condition is satisfied: Case 1 corresponds to the scenario where both market segments are climate-negative, in which case a sufficiently large penalty that collapses both markets unambiguously improves the net climate gain by eliminating negative contributions. Cases 2 and 3 examine mixed settings where one market segment is climate-positive and the other is climate-negative in the absence of penalties. In these cases, we identify conditions under which a positive penalty improves net climate gain by collapsing the climate-negative segment while preserving the one that is climate-positive.

Case 1: $\tilde{\epsilon}_0^{\mathcal{CG}} < \epsilon < \bar{\epsilon}_0$ and $\tilde{\epsilon}_1^{\mathcal{CG}} < \epsilon < \bar{\epsilon}_1$. Both market segments sustain trade but generate negative net climate gain in the absence of penalties; that is, $\mathcal{CG}_0(0) < 0$ and $\mathcal{CG}_1(0) < 0$, and hence $\mathcal{CG}(0) < 0$. Let $\kappa^{\mathcal{CG}}$ be any penalty higher than $\tilde{\kappa}_{max}$ so that both segments collapse under $\kappa = \kappa^{\mathcal{CG}}$, and so

$$\mathcal{CG}(\kappa^{\mathcal{CG}}) = 0 > \mathcal{CG}(0).$$

Moreover, $\kappa = \kappa^{\mathcal{CG}}$ attains the maximum achievable net climate gain, which is zero in this case, since any penalty $\kappa \in (0, \tilde{\kappa}_{max}]$ generates negative net climate gain. Thus, given any model parameters that fall in Case 1, there exists a positive penalty $\kappa^{\mathcal{CG}}$ that increases net climate gain.

Case 2: $\epsilon < \tilde{\epsilon}_0^{\mathcal{CG}}$ and $\tilde{\epsilon}_1^{\mathcal{CG}} < \epsilon < \bar{\epsilon}_1$. In this case, the class-0 market is climate-positive while the class-1 market is climate-negative in the absence of penalties, i.e., $\mathcal{CG}_0(0) > 0$ and $\mathcal{CG}_1(0) < 0$. A positive penalty may improve the total net climate gain relative to the no-penalty baseline only in two scenarios:

(a) *One segment collapses:* A penalty in the interval $(\tilde{\kappa}_{min}, \tilde{\kappa}_{max}]$ may improve net climate gain by collapsing the climate-negative class-1 segment while preserving the climate-positive class-0 market. Since $\mathcal{CG}(\kappa)$ is decreasing over this interval, it suffices to consider a penalty κ just above the collapse threshold $\tilde{\kappa}_{min}$. Such a penalty increases net climate gain if and only if

$$\lim_{\kappa \to \tilde{\kappa}_{min}^+} \mathcal{CG}(\kappa) > \mathcal{CG}(0),$$

where the limit is taken approaching $\tilde{\kappa}_{min}$ from the right as $\mathcal{CG}(\kappa)$ is potentially right-discontinuous at $\kappa = \tilde{\kappa}_{min}$, after which one segment collapses. The above condition can be rewritten equivalently as:

$$\tilde{\kappa}_1(\epsilon) < \tilde{\kappa}_0(\epsilon), \quad \text{and} \quad \mathcal{CG}_0(\tilde{\kappa}_1(\epsilon)) > \mathcal{CG}_0(0) + \mathcal{CG}_1(0).$$

The first inequality ensures that the class-1 segment collapses before the class-0 segment; i.e., that the class-0 segment remains active for values of κ just above $\tilde{\kappa}_{min}$. The second condition guarantees that the net climate gain from the surviving class-0 segment exceeds the aggregate climate gain under no penalties. If both conditions are satisfied (along with the assumptions of Case 2), a positive penalty $\kappa^{\mathcal{CG}}$ just above $\tilde{\kappa}_{min}$ increases net climate gain.

(b) *Both segments collapse:* Alternatively, a large enough penalty $\kappa > \tilde{\kappa}_{max}$ collapses both segments and yields zero net climate gain. This is an improvement over the no-penalty baseline if and only if total net climate gain is negative at $\kappa = 0$:

$$\mathcal{CG}_0(0) + \mathcal{CG}_1(0) < 0.$$

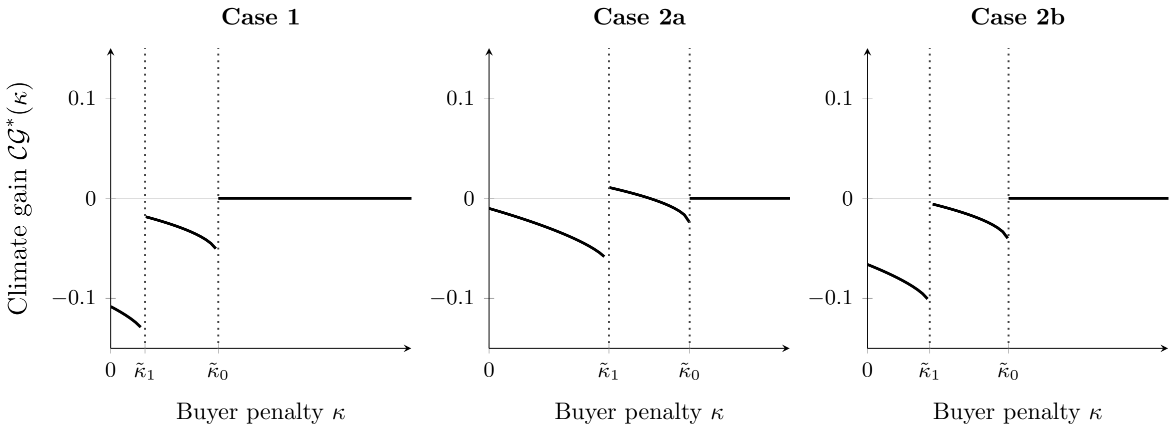
In other words, the same logic from Case 1 applies: if the aggregate net climate gain is negative despite one segment being climate-positive, then full market collapse increases net climate gain.

Case 3: $\tilde{\epsilon}_0^{\mathcal{CG}} < \epsilon < \bar{\epsilon}_0$ and $\epsilon < \tilde{\epsilon}_1^{\mathcal{CG}}$. This case mirrors the scenario in Case 2: the class-0 market generates negative climate gain, while the class-1 market is climate-positive, i.e., $\mathcal{CG}_0(0) < 0$ and $\mathcal{CG}_1(0) > 0$. The same logic from Case 2 applies, with the roles of the two segments reversed. A positive penalty can improve net climate gain either by collapsing the climate-negative class-0 segment while preserving the climate-positive class-1 segment, or by collapsing both segments when total net climate gain under no penalty is negative. As in the examples shown in Figure 7, one can identify parameter values that satisfy these conditions following the same reasoning as in Case 2. □

A.3. Proofs for §4.2 (Pooled Market)

This appendix presents the proofs for the results regarding the pooled market analyzed in §4.2. Subsection A.3.1 derives the equilibrium characterization (Proposition 6). Subsection A.3.2 compares the collapse thresholds and equilibrium prices between the pooled and segmented markets (Proposition 7). Finally, Subsection A.3.3 establishes the implications of pooling for net climate gain (Proposition 8).

Figure 7 Possible cases where a positive penalty increases net climate gain



Notes. Illustration of the scenarios where a positive penalty κ for buyers increases net climate gain relative to the no-penalty benchmark. Each plot corresponds to one of the cases described in the proof of Proposition 5. In these examples, the market collapse thresholds for each segment in the absence of penalties are $\bar{\epsilon}_0 = 2/3$ and $\bar{\epsilon}_1 = 0.454$. The thresholds that determine the sign of each segment's net climate gain are $\tilde{\epsilon}_0^{\mathcal{CG}} = 1/3$ and $\tilde{\epsilon}_1^{\mathcal{CG}} = 0.227$. Case 1 uses $\epsilon = 0.35$, so that both segments are climate-negative under $\kappa = 0$. The plots for Case 2(a) and 2(b) use $\epsilon = 0.28$ and $\epsilon = 0.32$, respectively. In both of these, the class-0 segment is climate-positive under no penalty, but the class-1 segment is climate-negative. Case 2(a) illustrates the scenario where net climate gain is highest with an intermediate penalty that sustains the class-0 segment while collapsing class-1. By contrast, this separation cannot be achieved in Case 2(b), where a high enough penalty that collapses both markets maximizes net climate gain.
Parameter values: $\phi_0 = 0.4$, $\phi_1 = 0.2$, $\beta = 0.5$, $\gamma = 1$, $\delta = 1$

A.3.1. Proof of Proposition 6 As a preface to the proof, we highlight the two main differences in the setting where credits are pooled into a portfolio, relative to the baseline model of §3 where credits of different classes trade separately. First, the market effectively functions as if there is a single product, which is the portfolio itself.[15] That is, buyers choose whether to purchase a share of the portfolio or abstain from purchasing altogether, and the key equilibrium object is the (single) market-clearing price for the portfolio, rather than separate prices for each credit class. Second, because credits of both classes are not offered separately, buyers form beliefs about the average quality of credits in the portfolio. These beliefs depend both on the mix of credit of each class in the portfolio (where λ denotes the share of class-1 credits), and the expected quality within each class, as captured by the conditional beliefs μ_0 and μ_1. Together, the terms (λ, μ_0, μ_1) summarize the buyer's beliefs, which must be consistent with Bayes' rule in equilibrium.

Taking these modifications into consideration, we establish Proposition 6 by following a similar three-step structure to the proof of Proposition 1 (see Appendix A.1.1). We first derive the buyers' demand function for "portfolio shares" in Lemma 7, and then characterize necessary and sufficient conditions that must hold in equilibrium in Lemmas 8 and 9, which yield the structure described in Proposition 6. We now present each of these components.

[15] In our model, a "portfolio share" refers to purchasing a claim on the pooled set of certified credits. We use this term informally to reflect that all credits trade as part of a common portfolio, rather than as individual assets.

Lemma 7. *Consider the setting of §4.2, where credits are pooled into a portfolio. Fix $\gamma > 0$ and suppose that given the portfolio price p and beliefs $(\lambda, \boldsymbol{\mu}) = (\lambda, \mu_0, \mu_1)$, buyers follow a strategy that maximizes their expected utility. Then, the demand function for portfolio shares, $D(p, \lambda, \boldsymbol{\mu})$, takes the following form:*

$$D(p,\lambda,\boldsymbol{\mu}) = \begin{cases} \bar{F}\left(\frac{p-(\lambda\mu_1+(1-\lambda)\mu_0)}{\gamma\lambda\mu_1}\right), & \text{if } \lambda\mu_1 > 0, \\ 1 & \text{if } (1-\lambda)\mu_0 > p \text{ and } \lambda\mu_1 = 0, \\ \alpha(p,\lambda,\boldsymbol{\mu}) \in [0,1] & \text{if } (1-\lambda)\mu_0 = p \text{ and } \lambda\mu_1 = 0, \\ 0, & \text{otherwise; namely if } (1-\lambda)\mu_0 < p \text{ and } \lambda\mu_1 = 0. \end{cases} \tag{A.20}$$

Here, as in Lemma 1, $\alpha(p, \lambda, \boldsymbol{\mu})$ denotes an allocation probability that resolves indifference.

Proof of Lemma 7. Let $a(v) \in \{-1, 1\}$ denote the action of a buyer with type v, where $a(v) = 1$ corresponds to purchasing a share of the portfolio, and $a(v) = -1$ represents no-purchase. From Equation (4.1), the buyers' expected utility function when considering these alternatives is given by:

$$\tilde{U}(a,v;\lambda,\boldsymbol{\mu},p) = \begin{cases} (1+\gamma v)\mu_1\lambda + \mu_0(1-\lambda) - p, & \text{if buying a share of the portfolio } (a=1), \\ 0, & \text{otherwise } (a=-1). \end{cases}$$

First suppose that the buyers' belief associated with the portfolio containing high-quality class-1 credits is positive, i.e., $\lambda\mu_1 > 0$. In this case, a buyer with type v strictly prefers to buy into the portfolio if

$$v > \frac{p-(\lambda\mu_1+(1-\lambda)\mu_0)}{\gamma\lambda\mu_1}.$$

Hence, the fraction of buyers who opt to purchase is given by the upper CDF $\bar{F}$ evaluated at this threshold, which coincides with the expression for $D(p, \lambda, \boldsymbol{\mu})$ when $\lambda\mu_1 > 0$.

The remaining case, where $\lambda\mu_1 = 0$, depends only on the comparison between the expected quality $(1-\lambda)\mu_0$ and the price p regardless of the buyer's type. The form of the demand function can be derived using the same argument as case 2 in Proposition 1. □

Next, we use the demand function derived above to solve for the market-clearing conditions that characterize the equilibrium price for the portfolio. As in Lemma 2, the argument proceeds through a series of claims that pin down the equilibrium beliefs and the price.

Lemma 8. *Consider the setting of §4.2, and fix $\gamma > 0$. Let $\bar{\epsilon}_{port}$ and p^*_{port} be as defined in Equations (4.2) and (4.3), respectively. Suppose there exists an equilibrium with portfolio price p. Then, we have that:*

(a) If $\epsilon < \bar{\epsilon}_{port}$ and $p > 0$, then $p = p^*_{port}$.

(b) Conversely, if $\epsilon \geq \bar{\epsilon}_{port}$, then $p = 0$.

The first step of the argument is to show that in any equilibrium with a positive price, all the belief terms are positive.

Claim 9. *In any equilibrium with $p > 0$, the beliefs λ, μ_0, and μ_1 are all strictly positive.*

Proof of Claim 9. Suppose $p > 0$ is an equilibrium price in the portfolio setting of §4.2. Then, by condition (2.5), all low-quality sellers enter the market and high-quality sellers with cost $c \leq p$ enter as well. In particular, since $p > 0$, a positive mass of high-quality sellers from each class enters the market. As a result, the beliefs μ_b for $b \in \{0,1\}$ are both strictly positive. To verify this, recall that the belief μ_b is determined by Bayes' rule (condition (2.7)):

$$\mu_b = \frac{\phi_b G(p)}{\phi_b G(p) + (1-\phi_b)\epsilon},$$

which is strictly positive since $G(p) > 0$ for any $p > 0$.

Moreover, as the belief λ must also be consistent with Bayes' rule in equilibrium, it must be equal to fraction of certified credits that are class-1. That is,

$$\lambda = \mathbb{P}\left[b = 1 \mid e(\theta) = 1,\, s = H\right] = \frac{\beta(\phi_1 G(p) + (1-\phi_1)\epsilon)}{\beta(\phi_1 G(p) + (1-\phi_1)\epsilon) + (1-\beta)(\phi_0 G(p) + (1-\phi_0)\epsilon)}, \tag{A.21}$$

from where we observe that $\lambda > 0$ since both the numerator and denominator are positive when $p > 0$. $\square$

The previous claim implies that, when characterizing the equilibria in which the market does not collapse and a positive price is sustained, it is sufficient to consider the case in which all belief terms are positive. As a result, the demand function in such an equilibrium involves only one expression (the first case in D in (A.20) above), rather than multiple cases. The following result further simplifies the equilibrium derivation by showing that, in any equilibrium with a positive price, the threshold that determines the buyers' demand is interior. This will help simplify the market-clearing conditions.

Claim 10. *In any equilibrium, the price and beliefs satisfy* $0 \le p - (\lambda\mu_1 + (1-\lambda)\mu_0) \le \gamma\lambda\mu_1$.

Proof of Claim 10. Consider first an equilibrium $p = 0$. Then, by condition (2.7), we have that $\mu_1 = 0$ and $\mu_0 = 0$, so the inequality holds trivially.

Suppose instead that we have an equilibrium with $p > 0$. By Claim 9, we have that $\lambda\mu_1 > 0$. Moreover, by Lemma 7, the demand function given the buyer beliefs takes the following form:

$$D(p, \lambda, \boldsymbol{\mu}) = \bar{F}\left(\frac{p - (\lambda\mu_1 + (1-\lambda)\mu_0)}{\gamma\lambda\mu_1}\right).$$

To prove the lower bound, suppose for contradiction that $p - (\lambda\mu_1 + (1-\lambda)\mu_0) < 0$. Then, we have $D(p, \lambda, \boldsymbol{\mu}) = 1$, and so market clearing requires that $S(p) = 1$, where

$$S(p) = S_1(p) + S_0(p) = \beta\left(\phi_1 G(p) + (1-\phi_1)\epsilon\right) + (1-\beta)\left(\phi_0 G(p) + (1-\phi_0)\epsilon\right).$$

For $S(p) = 1$ to hold, it is necessary that $p \ge 1$ and $\epsilon = 1$. Under these conditions, Equations (2.7) and (A.21) imply that the beliefs in equilibrium are $\lambda = \beta$ and $\mu_b = \phi_b$ for $b \in \{0,1\}$. Hence,

$$p - (\lambda\mu_1 + (1-\lambda)\mu_0) \ge 1 - (\beta\phi_1 + (1-\beta)\phi_0) = 1 - \bar{\phi} \ge 0,$$

contradicting the assumption that the expression in the LHS above is negative.

To prove the upper bound, suppose that $p - (\lambda\mu_1 + (1-\lambda)\mu_0) > \gamma\lambda\mu_1$ holds in equilibrium. This implies that $D(p, \lambda, \boldsymbol{\mu}) = 0$ and, for the market to clear, we must also have $S(p) = 0$. In turn, this implies that no high-quality sellers enter the market and $p = 0$, again a contradiction. $\square$

The previous two claims allow us to simplify the market-clearing conditions and explicitly derive the equilibrium price and the collapse threshold for the pooled market.

CLAIM 11. *Let $\bar{\epsilon}_{port}$ and p^*_{port} be as defined in Equations (4.2) and (4.3), respectively. If $\epsilon < \bar{\epsilon}_{port}$, then the price in any equilibrium where portfolio shares trade at a positive price is p^*_{port}. Instead, if $\epsilon \geq \bar{\epsilon}_{port}$, the price is $p = 0$ (and coincides with p^*_{port}) in any equilibrium.*

Proof of Claim 11. Suppose there exists an equilibrium with $p > 0$. By Claim 9, the beliefs λ, μ_0, and μ_1 are all strictly positive. Then, from Equation (A.20), the demand for portfolio shares evaluated at price p given these beliefs is

$$D(p, \lambda, \boldsymbol{\mu}) = 1 - F\left(\frac{p - (\lambda\mu_1 + (1-\lambda)\mu_0)}{\gamma\lambda\mu_1}\right) = 1 - \frac{p - (\lambda\mu_1 + (1-\lambda)\mu_0)}{\gamma\lambda\mu_1},$$

where the last equality follows from Claim 10 and since F is the uniform distribution over $[0,1]$.

Since $p > 0$, all low-quality sellers enter the market, and so the total supply of portfolio shares is

$$S(p) = \beta\left(\phi_1 G(p) + (1-\phi_1)\epsilon\right) + (1-\beta)\left(\phi_0 G(p) + (1-\phi_0)\epsilon\right) = \bar{\phi}G(p) + (1-\bar{\phi})\epsilon,$$

where $\bar{\phi} := \beta\phi_0 + (1-\beta)\phi_1$.

To derive the possible equilibrium prices p, we solve for the market-clearing condition $D(p, \lambda, \boldsymbol{\mu}) = S(p)$. After plugging in the above expressions for supply and demand, we can write this equation as

$$1 - \frac{p - (\lambda\mu_1 + (1-\lambda)\mu_0)}{\gamma\lambda\mu_1} = \bar{\phi}G(p) + (1-\bar{\phi})\epsilon.$$

The beliefs λ and μ_b must satisfy Bayes' rule, as in Equations (2.7) and (A.21). That is,

$$\lambda = \frac{\beta(\phi_1 G(p) + (1-\phi_1)\epsilon)}{\bar{\phi}G(p) + (1-\bar{\phi})\epsilon}, \quad \text{and} \quad \mu_b = \frac{\phi_b G(p)}{\phi_b G(p) + (1-\phi_b)\epsilon}, \quad \text{for } b \in \{0,1\}.$$

Substituting these expressions into the market-clearing condition and simplifying yields:

$$1 - \left(\frac{p(\bar{\phi}G(p) + (1-\bar{\phi})\epsilon) - \bar{\phi}G(p)}{\gamma\beta\phi_1 G(p)}\right) = \bar{\phi}G(p) + (1-\bar{\phi})\epsilon. \tag{A.22}$$

We now solve this equation to obtain the possible equilibrium prices by considering two cases.

Case 1: $p > 1$. Since G is the CDF of the uniform distribution over $[0,1]$, in this case we have $G(p) = 1$. Thus, Equation (A.22) simplifies to

$$1 - \left(\frac{p(\bar{\phi} + (1-\bar{\phi})\epsilon) - \bar{\phi}}{\gamma\beta\phi_1}\right) = \bar{\phi} + (1-\bar{\phi})\epsilon,$$

from where we immediately solve

$$p = \frac{\bar{\phi} + \gamma\beta\phi_1(1-\bar{\phi})(1-\epsilon)}{\bar{\phi} + (1-\bar{\phi})\epsilon}.$$

This solution satisfies the condition $p > 1$ if and only if

$$\epsilon < \frac{\gamma\beta\phi_1}{1 + \gamma\beta\phi_1} = \left(\frac{(1-\bar{\phi})\beta\gamma\phi_1}{\bar{\phi} + \beta\gamma\phi_1}\right)\bar{\epsilon}_{port}. \tag{A.23}$$

Case 2: $0 < p \leq 1$. Within this region, we have $G(p) = p$ and so Equation (A.22) becomes:

$$1 - \left(\frac{p(\bar{\phi}p + (1-\bar{\phi})\epsilon) - \bar{\phi}p}{\gamma\beta\phi_1 p}\right) = \bar{\phi}p + (1-\bar{\phi})\epsilon.$$

Solving this equation yields:

$$p = \frac{\bar{\phi} + \gamma\beta\phi_1}{\bar{\phi}\,(1 + \gamma\beta\phi_1)} - \left(\frac{1-\bar{\phi}}{\bar{\phi}}\right)\epsilon.$$

After some algebra, it is easy to show that this solution satisfies $0 < p \leq 1$ if and only if

$$\left(\frac{(1-\bar{\phi})\beta\gamma\phi_1}{\bar{\phi}+\beta\gamma\phi_1}\right)\bar{\epsilon}_{port} \leq \epsilon < \bar{\epsilon}_{port}. \tag{A.24}$$

Observe that the regions defined by conditions (A.23) and (A.24) are disjoint and cover the range with $\epsilon < \bar{\epsilon}_{port}$. Moreover, the previous derivations show that the solution to the market-clearing conditions in each of these cases is unique. Thus, Cases 1 and 2 show that there is a unique positive equilibrium price when the certification error satisfies $\epsilon < \bar{\epsilon}_{port}$.

Instead, if $\epsilon \geq \bar{\epsilon}_{port}$, neither Case 1 nor Case 2 allow for a positive price p that solve the market-clearing condition. Thus, if $\epsilon \geq \bar{\epsilon}_{port}$, the only possible equilibrium price is $p = 0$, which also pins down the beliefs as $\mu_b = 0$ for $b \in \{0,1\}$. In particular, the expression for the equilibrium price p^*_{port} in (4.3) remains valid, since with $\epsilon \geq \bar{\epsilon}_{port}$ we have

$$p = 0 = \left(\frac{\bar{\phi}+\gamma\beta\phi_1}{\bar{\phi}\,(1+\gamma\beta\phi_1)} - \frac{1-\bar{\phi}}{\bar{\phi}}\epsilon\right)_+ = p^*_{port},$$

as desired. This outcome corresponds to market collapse, where no high-quality sellers enter and buyers demand for credits is zero, consistent with the interpretation of $\epsilon \geq \bar{\epsilon}_{port}$ as a regime in which certification is too noisy to sustain trade. □

Now that the possible equilibrium prices are characterized, Lemma 8 follows immediately.

Proof of Lemma 8. The result follows directly from Claim 11. □

The last part of the proof consists of showing that the necessary conditions for equilibrium established in Lemma 8 are also sufficient. This involves explicitly constructing an equilibrium in which the price is given by Equation (4.3), following a similar approach to that used in Lemma 3 for the baseline model. We state this result formally and outline how the argument is adapted to the present setting below.

LEMMA 9. *Consider the setting of §4.2 where credits are pooled into a portfolio and fix $\gamma = 0$. Then, there exists an equilibrium in which the price of the portfolio is equal to p^*_{port} as defined in* (4.3).

Proof of Lemma 9. The result follows by adapting the argument of the proofs of Lemma 3 and Proposition 1 (see Appendix A.1.1) with a few modifications. Specifically, we construct a tuple $(p^*, \lambda^*, \boldsymbol{\mu}^*, e^*, a^*)$ that forms an equilibrium in the setting with portfolios. The definition of equilibrium is straightforward to adapt from Definition 1, by incorporating (i) the belief λ that buyers form about the likelihood that a credit in the portfolio is class-1, (ii) the buyer's expected utility, which is now a function of the decision to buy into the portfolio or not, and (iii) the market-clearing condition that ensures the total demand for portfolio shares equals the supply of certified credits.

To construct such an equilibrium, we set $p^* = p^*_{port}$. The buyer strategy $a^*(v)$ can be defined following the same logic as in (A.8), but involves only two possible actions: to buy into the portfolio if v exceeds a threshold, and to abstain from purchase otherwise. The beliefs μ^*_b retain the same structure as in the proof of Lemma 3 (but defined in terms of the price p^*_{port}). The belief λ, representing the probability that a project in the portfolio is class-1, is defined using Bayes' rule:

$$\lambda^* = \frac{\beta(\phi_1 G(p^*_{port}) + (1-\phi_1)\epsilon)}{\bar{\phi}G(p^*_{port}) + (1-\bar{\phi})\epsilon}.$$

The seller strategy $e^*(\theta)$ remains as in (2.5) but now defined in terms of the portfolio price p^*_{port}.

Verifying that this construction forms an equilibrium follows closely the reasoning of Lemma 3 and Proposition 1, taking into account the demand structure given in Lemma 7 and the necessary equilibrium conditions in Lemma 8. □

Finally, Proposition 6 follows directly from the preceding lemmas, while handling the case where $\gamma = 0$ separately.

Proof of Proposition 6. For $\gamma > 0$, the proof follows from Lemmas 7, 8, and 9, using the same logic as in the proof of Proposition 1.

It remains to address the case $\gamma = 0$. In this case, buyers place no value on co-benefits, making the setting analogous to a market with a single credit class where the fraction of high-quality sellers is $\bar{\phi} = \beta\phi_1 + (1-\beta)\phi_0$ (the weighted average across classes). The arguments from Claims 1 and 2 in Appendix A.1.1 apply directly to this case, resulting in a market collapse threshold and equilibrium price given by:

$$\bar{\epsilon} = \frac{\bar{\phi}}{1-\bar{\phi}}, \quad \text{and} \quad p^* = \left(1 - \frac{1-\bar{\phi}}{\bar{\phi}}\epsilon\right)_+.$$

Note that these expressions coincide with $\bar{\epsilon}_{port}$ and p^*_{port} in Equations (4.2) and (4.3) when $\gamma = 0$. □

A.3.2. Proof of Proposition 7 We prove two lemmas to establish the proposition. First, Lemma 10 makes the comparison for the market collapse thresholds, showing that if both segments collapse when credits trade separately, so does the pooled market. The converse, however, does not hold: the pooled market may collapse even though both segmented markets would remain active. Building on this result, Lemma 11 shows that equilibrium prices satisfy an analogous ordering: the pooled price never exceeds both class-specific prices, yet there exist parameter values for which it is strictly below both.

Lemma 10. *Fix the parameters $(\phi_0, \phi_1, \beta, \gamma)$ with $\phi_0, \phi_1 \in (0,1)$, $\beta \in (0,1)$, and $\gamma \geq 0$. Then:*

(i) If both segments collapse when credits trade separately, the pooled market also collapses. That is, $\bar{\epsilon}_{port} \leq \max\{\bar{\epsilon}_0, \bar{\epsilon}_1\}$.

(ii) However, there exist parameter values such that $\bar{\epsilon}_{port} < \min\{\bar{\epsilon}_0, \bar{\epsilon}_1\} \leq 1$. In other words, the pooled market may collapse even if both credit classes would actively trade under the segmented structure.

Proof of Lemma 10. We prove each part of the statement separately.

Part (i). Fix parameter values $(\phi_0, \phi_1, \beta, \gamma)$, and suppose towards a contradiction that $\bar{\epsilon}_{port} > \bar{\epsilon}_0$ and $\bar{\epsilon}_{port} > \bar{\epsilon}_1$ both hold. After substituting the expressions of these thresholds (from (3.1), (3.2) and (4.2)) and rearranging terms, we can rewrite these conditions as

$$(\bar{\phi} + \phi_1\beta\gamma)(1-\phi_0) > \phi_0(1-\bar{\phi})(1+\phi_1\beta\gamma), \tag{A.25}$$

$$(\bar{\phi} + \phi_1\beta\gamma)(1-\phi_1) > \phi_1(1-\bar{\phi})(1+\gamma). \tag{A.26}$$

Multiplying (A.25) by $(1-\beta)$ and (A.26) by β, and then adding, gives

$$(\bar{\phi} + \phi_1\beta\gamma)(1-\bar{\phi}) > (1-\bar{\phi})\left((1-\beta)\phi_0(1+\phi_1\beta\gamma) + \beta\phi_1(1+\gamma)\right).$$

Since $1-\bar{\phi} > 0$, we can divide both sides to obtain

$$\bar{\phi} + \phi_1\beta\gamma > (1-\beta)\phi_0(1+\phi_1\beta\gamma) + \beta\phi_1(1+\gamma) = \bar{\phi} + \phi_1\beta\gamma + (1-\beta)\phi_0\phi_1\beta\gamma.$$

This implies that $(1-\beta)\phi_0\phi_1\beta\gamma<0$, a contradiction since this term is the product of positive quantities. Thus, the inequalities in (A.25) and (A.26) cannot hold simultaneously, and therefore $\bar{\epsilon}_{port}\leq\max\{\bar{\epsilon}_0,\bar{\epsilon}_1\}$.

To conclude the argument, suppose that both market segments collapse if credits trade separately, i.e., that $\epsilon\geq\max\{\bar{\epsilon}_0,\bar{\epsilon}_1\}$. Since $\bar{\epsilon}_{port}\leq\max\{\bar{\epsilon}_0,\bar{\epsilon}_1\}$, it follows that $\epsilon\geq\bar{\epsilon}_{port}$, so the pooled market also collapses.

Part (ii). We now show that the region of parameters $(\phi_0,\phi_1,\beta,\gamma)$ where $\bar{\epsilon}_{port}<\min\{\bar{\epsilon}_0,\bar{\epsilon}_1\}<1$ is non-empty. To do so, we focus on the regime where carbon quality and co-benefits are negatively correlated, i.e., assume that $\phi_0>\phi_1$. In fact, this is a necessary condition: if $\phi_0\leq\phi_1$, one can show directly that $\bar{\epsilon}_{port}\geq\bar{\epsilon}_0$.

The condition $\bar{\epsilon}_{port}<\min\{\bar{\epsilon}_0,\bar{\epsilon}_1\}$ can be written by flipping the signs of the inequalities (A.25) and (A.26) above. That is, we want to show that there exist model parameters that satisfy:

$$(\bar{\phi}+\phi_1\beta\gamma)(1-\phi_0)<\phi_0(1-\bar{\phi})(1+\phi_1\beta\gamma), \tag{A.27}$$

$$(\bar{\phi}+\phi_1\beta\gamma)(1-\phi_1)<\phi_1(1-\bar{\phi})(1+\gamma). \tag{A.28}$$

We next solve conditions (A.27) and (A.28) for γ. After substituting $\bar{\phi}=\beta\phi_1+(1-\beta)\phi_0$ and some algebra, condition (A.27) is equivalent to

$$\phi_1\left((1-\phi_0)^2-\beta\phi_0(\phi_0-\phi_1)\right)\gamma\leq\phi_0-\phi_1.$$

Hence, provided that $(1-\phi_0)^2-\beta\phi_0(\phi_0-\phi_1)>0$, condition (A.27) is reduced to

$$\gamma<\bar{\gamma}_0:=\frac{\phi_0-\phi_1}{\phi_1\left((1-\phi_0)^2-\beta\phi_0(\phi_0-\phi_1)\right)}.$$

Analogously, condition (A.28) simplifies to the following lower bound on γ:

$$\gamma>\bar{\gamma}_1:=\frac{\phi_0-\phi_1}{\phi_1(1-\phi_0)}.$$

By the above derivations, a sufficient condition for $\bar{\epsilon}_{port}<\min\{\bar{\epsilon}_0,\bar{\epsilon}_1\}$ to hold is that

$$\bar{\gamma}_1<\gamma<\bar{\gamma}_0,\quad\text{and}\quad(1-\phi_0)^2-\beta\phi_0(\phi_0-\phi_1)>0.$$

We next show that these conditions hold for a broad range of parameters. In particular, the inequality $\bar{\gamma}_1<\bar{\gamma}_0$ always holds under $\phi_0>\phi_1$ and $(1-\phi_0)^2-\beta\phi_0(\phi_0-\phi_1)>0$. To verify this, note that $\bar{\gamma}_1<\bar{\gamma}_0$ holds iff.

$$(1-\phi_0)^2-\beta\phi_0(\phi_0-\phi_1)<1-\phi_0,$$

which, after rearranging, is equivalent to

$$-(1-\phi_0)<\beta(\phi_0-\phi_1),$$

which holds since $\phi_0>\phi_1$.

Finally, it remains to show that the condition $(1-\phi_0)^2-\beta\phi_0(\phi_0-\phi_1)>0$ also holds for a wide range of parameters. In particular, it is guaranteed when $\phi_0<1/2$, since

$$(1-\phi_0)^2-\beta\phi_0(\phi_0-\phi_1)>(1-\phi_0)^2-\beta\phi_0^2\geq(1-\phi_0)^2-\phi_0^2=1-2\phi_0>0.$$

Moreover, if $\phi_0<1/2$ then $\bar{\epsilon}_0=\phi_0/(1-\phi_0)<1$, and so $\min\{\bar{\epsilon}_0,\bar{\epsilon}_1\}<1$ holds as well. Thus, the region

$$\{(\phi_0,\phi_1,\beta,\gamma):\ 0<\phi_0<1/2,0<\phi_1<\phi_0,\ \bar{\gamma}_0<\gamma<\bar{\gamma}_1,\ 0<\beta<1\} \tag{A.29}$$

is non-empty and satisfies $\bar{\epsilon}_{port}<\min\{\bar{\epsilon}_0,\bar{\epsilon}_1\}<1$. □

We next prove a similar result for equilibrium prices.

Lemma 11. *Fix the parameters $(\phi_0, \phi_1, \beta, \gamma)$ with $\phi_0, \phi_1 \in (0,1)$, $\beta \in (0,1)$, and $\gamma \geq 0$, and fix the certification error $\epsilon \in [0,1]$. Then:*

(i) The price of the portfolio under the pooled structure never exceeds both the class-specific prices when credits trade separately. That is, $p^*_{port} \leq \max\{p^*_0, p^*_1\}$.

(ii) However, the portfolio price may be strictly below both segmented prices. That is, there exist parameters such that $p^*_{port} < \min\{p^*_0, p^*_1\}$.

Proof of Lemma 11. We begin by recalling the expressions for the equilibrium prices and introducing some notation. In the baseline model where credits of each class trade separately, the equilibrium prices are (from Proposition 1):

$$p^*_0 = \left(1 - \frac{1-\phi_0}{\phi_0}\epsilon\right)_+, \quad \text{and} \quad p^*_1 = \begin{cases} \dfrac{\phi_1(1+\gamma)}{\phi_1 + (1-\phi_1)\epsilon} - \phi_1\beta\gamma, & \text{if } \epsilon < \hat{\epsilon}_1, \\ \left(\dfrac{1+\gamma}{1+\phi_1\beta\gamma} - \dfrac{1-\phi_1}{\phi_1}\epsilon\right)_+, & \text{otherwise,} \end{cases}$$

where the threshold $\hat{\epsilon}_1$ above is

$$\hat{\epsilon}_1 = \frac{\phi_1\gamma(1-\beta\phi_1)}{(1-\phi_1)(1+\phi_1\gamma\beta)}. \tag{A.30}$$

On the other hand, under the pooled market structure of §4.2, the equilibrium price for the portfolio is (Proposition 6):

$$p^*_{port} = \begin{cases} \dfrac{\bar{\phi} + \gamma\beta\phi_1(1-\bar{\phi})(1-\epsilon)}{\bar{\phi} + (1-\bar{\phi})\epsilon}, & \text{if } \epsilon < \hat{\epsilon}_{port}, \\ \left(\dfrac{\bar{\phi} + \gamma\beta\phi_1}{\bar{\phi}\,(1+\gamma\beta\phi_1)} - \dfrac{1-\bar{\phi}}{\bar{\phi}}\epsilon\right)_+, & \text{otherwise,} \end{cases}$$

where the threshold $\hat{\epsilon}_{port}$ is

$$\hat{\epsilon}_{port} = \frac{\gamma\beta\phi_1}{1+\gamma\beta\phi_1}. \tag{A.31}$$

We now establish each part of the statement separately.

Part (i). We will show that $p^*_{port} \leq \max\{p^*_0, p^*_1\}$ for all parameters $(\phi_0, \phi_1, \beta, \gamma)$ and $\epsilon \in [0,1]$. The argument proceeds in five steps.

Step 1. First, consider the region with $\epsilon \geq \bar{\epsilon}_{port}$, where $\bar{\epsilon}_{port}$ is the collapse threshold for the portfolio market as given by (4.2). In this case, the result holds trivially since $p^*_{port} = 0$. In particular, $p^*_{port} = 0 \leq \max\{p^*_0, p^*_1\}$ since the equilibrium prices p^*_b are non-negative for $b \in \{0,1\}$.

Step 2. As an auxiliary claim, we show that $\hat{\epsilon}_{port} \leq \hat{\epsilon}_1$. Substituting from (A.30) and (A.31), this inequality is equivalent to

$$\gamma\beta\phi_1(1-\phi_1) \leq \phi_1\gamma(1-\beta\phi_1),$$

which holds trivially if $\gamma = 0$ and, if $\gamma > 0$, it reduces to $\beta(1-\phi_1) \leq 1 - \beta\phi_1$, which holds since $\beta < 1$.

Step 3. Next, we consider the region with $\epsilon \in [0, \hat{\epsilon}_{port}]$, and show that $p^*_{port} \leq p^*_1$ within this interval. By Step 2, we have $\hat{\epsilon}_{port} \leq \hat{\epsilon}_1$, and so the parameters in this region satisfy $\epsilon \leq \hat{\epsilon}_1$. Plugging in the relevant expressions for the equilibrium prices, we want to show that

$$p^*_{port} = \frac{\bar{\phi} + \gamma\beta\phi_1(1-\bar{\phi})(1-\epsilon)}{\bar{\phi} + (1-\bar{\phi})\epsilon} \leq \frac{\phi_1(1+\gamma)}{\phi_1 + (1-\phi_1)\epsilon} - \phi_1\beta\gamma = p^*_1.$$

After some algebra, this is equivalent to

$$(\bar{\phi}+\gamma\beta\phi_1)\left(\phi_1+(1-\phi_1)\epsilon\right) \le \phi_1(1+\gamma)\left(\bar{\phi}+(1-\bar{\phi})\epsilon\right).$$

Since both sides of the inequality are affine in ϵ, it suffices to show that this condition holds for $\epsilon=0$ and $\epsilon=\hat{\epsilon}_{port}$. Let $L(\epsilon)$ and $R(\epsilon)$ denote the left- and right-hand side terms above. At $\epsilon=0$ we have

$$R(0)-L(0)=\phi_1(1+\gamma)\bar{\phi}-\phi_1(\bar{\phi}+\gamma\beta\phi_1)=\phi_1\gamma(1-\beta)\phi_0\ge 0.$$

To show that the inequality also holds at $\epsilon=\hat{\epsilon}_{port}$, we note that this is the unique value of ϵ such that $p^*_{port}=1$ (see the derivation in Claim 11 in the proof of Proposition 6). In addition, we have that $p^*_1\ge 1$ for all $\epsilon\in[0,\hat{\epsilon}_1]$ (by the corresponding derivation in Claim 4 of Appendix A.1.1). Since $\hat{\epsilon}_{port}\le\hat{\epsilon}_1$ (Step 2), it follows that at $\epsilon=\hat{\epsilon}_{port}$, $p^*_1\ge 1=p^*_{port}$. So the inequality above also holds at this value, and hence $L(\epsilon)\ge R(\epsilon)$ for all $0\le\epsilon\le\hat{\epsilon}_{port}$. Therefore, within this region we have $p^*_1\ge p^*_{port}$.

Step 4. For $\epsilon\in[\hat{\epsilon}_{port},\hat{\epsilon}_1]$, the same argument of Step 3 shows that $p^*_1\ge 1$ and $p^*_{port}\le 1$ (by construction of $\hat{\epsilon}_1$ and $\hat{\epsilon}_{port}$). Hence, $p^*_1\ge p^*_{port}$.

Step 5. Finally, we cover the region with $\hat{\epsilon}_1<\epsilon<\bar{\epsilon}_{port}$. We show that it is impossible for p^*_{port} to exceed both p^*_0 and p^*_1 simultaneously.

First note that since $\epsilon<\bar{\epsilon}_{port}$, we have $p^*_{port}>0$. Thus, to show that both $p^*_{port}>p^*_0$ and $p^*_{port}>p^*_1$ cannot hold at the same time, we can ignore the positive part operator in the expressions for p^*_0 and p^*_1. In particular, it suffices to show that both the following inequalities cannot hold simultaneously:

$$\frac{\bar{\phi}+\gamma\beta\phi_1}{\bar{\phi}(1+\gamma\beta\phi_1)}-\frac{1-\bar{\phi}}{\bar{\phi}}\epsilon>\frac{1+\gamma}{1+\phi_1\beta\gamma}-\frac{1-\phi_1}{\phi_1}\epsilon, \quad\text{and}\quad \frac{\bar{\phi}+\gamma\beta\phi_1}{\bar{\phi}(1+\gamma\beta\phi_1)}-\frac{1-\bar{\phi}}{\bar{\phi}}\epsilon>1-\frac{1-\phi_0}{\phi_0}\epsilon.$$

After some algebra, these conditions are equivalent to

$$\frac{\gamma\phi_0\phi_1}{1+\phi_1\beta\gamma}<(\phi_0-\phi_1)\epsilon<\frac{\gamma\phi_0\phi_1(1-\bar{\phi})}{1+\phi_1\beta\gamma},$$

which is an impossibility since $1-\bar{\phi}>1$. Thus, we have $p^*_{port}\le\max\{p^*_0,p^*_1\}$.

Combining all five steps, we conclude that $p^*_{port}\le\max\{p^*_0,p^*_1\}$ for all $\epsilon\in[0,1]$ and all parameter combinations $(\phi_0,\phi_1,\beta,\gamma)$. This proves Part (i).

Part (ii). By Lemma 10(ii), there exists a nonempty set of parameters $(\phi_0,\phi_1,\beta,\gamma)$ such that $\bar{\epsilon}_{port}<\min\{\bar{\epsilon}_0,\bar{\epsilon}_1\}\le 1$. Fix any such parameters. At $\epsilon=\bar{\epsilon}_{port}$ we have $p^*_{port}=0$ while $p^*_0>0$ and $p^*_1>0$. Since all prices are continuous in ϵ, there exists $\eta>0$ such that for all $\epsilon\in(\bar{\epsilon}_{port}-\eta,\bar{\epsilon}_{port})$, the equilibrium prices satisfy $0<p^*_{port}<\min\{p^*_0,p^*_1\}$. Note that this construction can be made for all parameters in the set defined by (A.29). This proves part (ii). □

Finally, Proposition 7 follows directly from the two lemmas above.

Proof of Proposition 7. The result follows immediately from Lemmas 10 and 11. □

A.3.3. Proof of Proposition 8 Before proving the proposition, we obtain the net climate gain expressions under the two market structures. From Equation (2.8), the net climate gain generated by the pooled market in equilibrium is

$$\begin{aligned}\mathcal{CG}_{port} &= \mathcal{H}^*_{port} - \delta \cdot \mathcal{L}^*_{port} \\ &= \left(\bar{\phi} G\left(p^*_{port}\right) - \delta(1-\bar{\phi})\epsilon\right) \mathbf{1}_{\{\epsilon < \bar{\epsilon}_{port}\}}.\end{aligned}$$

Substituting the equilibrium price for the portfolio (Equation (4.3)) yields

$$\mathcal{CG}_{port} = \begin{cases} \bar{\phi} - \delta(1-\bar{\phi})\epsilon, & \text{if } \epsilon < \hat{\epsilon}_{port}, \\ (1-\bar{\phi})\left(\bar{\epsilon}_{port} - (1+\delta)\epsilon\right), & \text{if } \hat{\epsilon}_{port} \le \epsilon < \bar{\epsilon}_{port}, \\ 0, & \text{if } \epsilon \ge \bar{\epsilon}_{port}. \end{cases}$$

Under the baseline scenario where credits of each class trade separately, the net climate gain generated by each market segment is:

$$\mathcal{CG}_0 = (1-\beta)(1-\phi_0)\left(\bar{\epsilon}_0 - (1+\delta)\epsilon\right) \mathbf{1}_{\{\epsilon < \bar{\epsilon}_0\}},$$

and

$$\mathcal{CG}_1 = \begin{cases} \beta\left(\phi_1 - \delta(1-\phi_1)\epsilon\right), & \text{if } \epsilon < \hat{\epsilon}_1, \\ \beta(1-\phi_1)\left(\bar{\epsilon}_1 - (1+\delta)\epsilon\right), & \text{if } \hat{\epsilon}_1 \le \epsilon < \bar{\epsilon}_1, \\ 0, & \text{if } \epsilon \ge \bar{\epsilon}_1. \end{cases}$$

Here, the threshold $\hat{\epsilon}_1$ that separates the first two cases for class-1 is

$$\hat{\epsilon}_1 = \left(\frac{\gamma(1-\beta\phi_1)}{1+\gamma}\right)\bar{\epsilon}_1.$$

Proof of Proposition 8. Fix $\gamma > 0$, $\phi_0, \phi_1, \beta \in (0,1)$ and $\delta \ge 0$. Let $\Delta(\epsilon)$ denote the difference between the net climate gain generated under the pooled structure and that obtained when credits of each class trade separately. That is,

$$\Delta(\epsilon) = \mathcal{CG}_{port} - \left(\mathcal{CG}_0 + \mathcal{CG}_1\right).$$

We will show that on the domain where the pooled market sustains trade, $\epsilon \in [0, \bar{\epsilon}_{port})$, the function $\Delta(\epsilon)$ is initially positive and changes sign only once. The argument requires considering a series of cases and subcases. To simplify the exposition, we let

$$\bar{\epsilon}_{min} = \min\left\{\bar{\epsilon}_0,\, \bar{\epsilon}_1,\, \bar{\epsilon}_{port}\right\}$$

denote the smallest collapse threshold across the two segmented markets and under the pooled structure.

Case 1. Suppose that $\bar{\epsilon}_{min} = \bar{\epsilon}_1$. We evaluate the sign of $\Delta(\epsilon)$ across different intervals for ϵ:

(i) If $0 \le \epsilon < \hat{\epsilon}_{port}$, the relevant climate gain expressions are

$$\mathcal{CG}_{port} = \bar{\phi} - \delta(1-\bar{\phi})\epsilon, \quad \mathcal{CG}_0 = (1-\beta)(1-\phi_0)\left(\bar{\epsilon}_0 - (1+\delta)\,\epsilon\right), \quad \mathcal{CG}_1 = \beta\left(\phi_1 - \delta(1-\phi_1)\epsilon\right),$$

where the class-1 expression corresponds to the case where $\epsilon < \hat{\epsilon}_1$ since, by Step 2 in the proof of Lemma 11, we know that $\hat{\epsilon}_{port} \le \hat{\epsilon}_1$. The net climate gain gap is then

$$\Delta(\epsilon) = (1-\beta)(1-\phi_0)\epsilon \ge 0.$$

In particular, the pooled structure yields higher net climate gain for $\epsilon \in (0, \hat{\epsilon}_{port})$.

(ii) Next, consider the interval $\hat{\epsilon}_{port} \le \epsilon \le \hat{\epsilon}_1$. Then,

$$\mathcal{CG}_{port} = (1-\bar{\phi})\left(\bar{\epsilon}_{port} - (1+\delta)\epsilon\right),$$

while the expressions for $\mathcal{CG}_0$ and $\mathcal{CG}_1$ remain as above. The net climate gain gap is

$$\begin{aligned}\Delta(\epsilon) &= (1-\bar{\phi})\bar{\epsilon}_{port} - \bar{\phi} - \beta(1-\phi_1)\epsilon \\ &= \frac{\phi_1\beta\gamma(1-\bar{\phi})}{1+\phi_1\beta\gamma} - \beta(1-\phi_1)\epsilon.\end{aligned}$$

This quantity changes sign only once at

$$\epsilon_\star = \frac{\phi_1\gamma(1-\bar{\phi})}{(1-\phi_1)(1+\phi_1\beta\gamma)}, \tag{A.32}$$

provided this threshold lies in the interval under consideration, i.e., if $\hat{\epsilon}_{port} \le \epsilon_\star \le \hat{\epsilon}_1$. This is verified by direct algebra: after plugging in the expressions for $\hat{\epsilon}_{port}$ and $\hat{\epsilon}_1$, these conditions are equivalent to

$$\frac{\gamma\beta\phi_1}{1+\gamma\beta\phi_1} \le \frac{\phi_1\gamma(1-\bar{\phi})}{(1-\phi_1)(1+\phi_1\beta\gamma)} \le \frac{\phi_1\gamma(1-\beta\phi_1)}{(1-\phi_1)(1+\phi_1\gamma\beta)}.$$

Since $\gamma > 0$, after some cancellations, the left inequality becomes

$$\beta(1-\phi_1) \le 1-\bar{\phi} = \beta(1-\phi_1) + (1-\beta)(1-\phi_0),$$

which holds since $(1-\beta)(1-\phi_0) > 0$. Similarly, the right inequality is equivalent to

$$1-\bar{\phi} \le 1-\beta\phi_1,$$

which also holds.

Thus, $\Delta(\epsilon)$ is positive on $[\hat{\epsilon}_{port}, \epsilon_\star)$ and negative on $(\epsilon_\star, \hat{\epsilon}_1]$. To complete Case 1, it remains to show that $\Delta(\epsilon)$ remains negative for $\epsilon \in (\hat{\epsilon}_1, \bar{\epsilon}_{port})$.

(iii) Consider $\hat{\epsilon}_1 < \epsilon < \bar{\epsilon}_1$. Since $\bar{\epsilon}_{min} = \bar{\epsilon}_1$, both market segments and the pooled market are active within this interval. The relevant climate gain expressions are

$$\mathcal{CG}_{port} = (1-\bar{\phi})\left(\bar{\epsilon}_{port} - (1+\delta)\epsilon\right), \quad \mathcal{CG}_0 = (1-\beta)(1-\phi_0)\left(\bar{\epsilon}_0 - (1+\delta)\,\epsilon\right), \quad \mathcal{CG}_1 = \beta(1-\phi_1)\left(\bar{\epsilon}_1 - (1+\delta)\epsilon\right).$$

Hence, the net climate gain gap is

$$\begin{aligned}\Delta(\epsilon) &= (1-\bar{\phi})\bar{\epsilon}_{port} - (1-\beta)(1-\phi_0)\bar{\epsilon}_0 - \beta(1-\phi_1)\bar{\epsilon}_1 \\ &= \frac{\bar{\phi}+\gamma\beta\phi_1}{(1+\gamma\beta\phi_1)} - (1-\beta)\phi_0 - \beta\frac{\phi_1(1+\gamma)}{(1+\phi_1\beta\gamma)} \\ &= -\frac{\gamma\beta\phi_1(1-\beta)\phi_0}{1+\gamma\beta\phi_1} < 0,\end{aligned}$$

and so net climate gain is strictly lower under the pooled structure within this region.

(iv) To complete Case 1, we cover the interval $\bar{\epsilon}_1 \le \epsilon < \bar{\epsilon}_{port}$. This is the only remaining subcase to analyze since Proposition 7 implies $\bar{\epsilon}_{port} \le \max\{\bar{\epsilon}_0, \bar{\epsilon}_1\} = \bar{\epsilon}_0$. Since $\epsilon \ge \bar{\epsilon}_1$, the class-1 market is collapsed, so we compare an active pooled market with having only class-0 active. The net climate gain gap is

$$\Delta(\epsilon) = (1-\bar{\phi})\bar{\epsilon}_{port} - (1-\beta)(1-\phi_0)\bar{\epsilon}_0 - \beta(1-\phi_1)(1+\delta)\epsilon.$$

Since this expression decreases in ϵ, it suffices to verify that the gap is negative at $\epsilon = \bar{\epsilon}_1$:

$$\Delta(\bar{\epsilon}_1) = \underbrace{(1-\bar{\phi})\bar{\epsilon}_{port} - (1-\beta)(1-\phi_0)\bar{\epsilon}_0 - \beta(1-\phi_1)\bar{\epsilon}_1}_{<0 \text{ by Case 1(iii)}} - \beta(1-\phi_1)\delta\bar{\epsilon}_1 < 0,$$

which implies that $\Delta(\epsilon) < 0$, for all $\bar{\epsilon}_1 \le \epsilon < \bar{\epsilon}_{port}$.

This concludes Case 1. To summarize, if $\bar{\epsilon}_{min} = \bar{\epsilon}_1$, there exists a unique $\epsilon_\star \in (0, \bar{\epsilon}_{port})$ such that the pooled structure yields higher net climate gain if and only if $\epsilon < \epsilon_\star$.

Case 2. Now suppose that $\bar{\epsilon}_{\min} = \bar{\epsilon}_{port}$. We show that the same structure as in Case 1 holds: $\Delta(\epsilon)$ is positive for small ϵ, crosses zero at $\epsilon = \epsilon_\star$ (given by (A.32)), and is negative for $\epsilon \in (\epsilon_\star, \bar{\epsilon}_{port})$.

If $\hat{\epsilon}_1 < \bar{\epsilon}_{port}$, the argument from Case 1 applies directly. Specifically, the reasoning in Cases 1(i) and 1(ii) covers the interval $0 \le \epsilon < \hat{\epsilon}_1$, and the calculation in Case 1(iii) applies for $\hat{\epsilon}_1 < \epsilon < \bar{\epsilon}_{port}$.

Instead, if $\hat{\epsilon}_1 \ge \bar{\epsilon}_{port}$, Case 1(i) already shows that $\Delta(\epsilon) > 0$ for $\epsilon \in (0, \hat{\epsilon}_{port})$, so it suffices to show that $\Delta(\epsilon)$ changes sign only once within $[\hat{\epsilon}_{port}, \bar{\epsilon}_{port})$. The calculation is identical to Case 1(ii), implying that $\Delta(\epsilon)$ changes sign only at $\epsilon = \epsilon_\star$, provided that $\epsilon_\star$ lies in $[\hat{\epsilon}_{port}, \bar{\epsilon}_{port})$. We already established that $\hat{\epsilon}_{port} \le \epsilon_\star$, so it remains to check that $\epsilon_\star < \bar{\epsilon}_{port}$.

To do so, note that $\bar{\epsilon}_{\min} = \bar{\epsilon}_{port}$ implies $\bar{\epsilon}_{port} \le \bar{\epsilon}_0$. We show this condition in turn guarantees $\epsilon_\star < \bar{\epsilon}_{port}$. Suppose toward a contradiction that $\epsilon_\star \ge \bar{\epsilon}_{port}$. Since $\bar{\epsilon}_{port} \le \bar{\epsilon}_0$, we have

$$\frac{\phi_1\gamma(1-\bar{\phi})}{(1-\phi_1)(1+\phi_1\beta\gamma)} \ge \frac{\bar{\phi}+\gamma\beta\phi_1}{(1-\bar{\phi})(1+\gamma\beta\phi_1)}, \quad \text{and} \quad \frac{\phi_0}{1-\phi_0} \ge \frac{\bar{\phi}+\gamma\beta\phi_1}{(1-\bar{\phi})(1+\gamma\beta\phi_1)}.$$

Rearranging yields

$$\phi_1\gamma(1-\bar{\phi})^2 \ge (1-\phi_1)(\bar{\phi}+\gamma\beta\phi_1), \quad \text{and} \quad \phi_0(1-\bar{\phi})(1+\gamma\beta\phi_1) \ge (1-\phi_0)(\bar{\phi}+\gamma\beta\phi_1).$$

Multiplying the first inequality by β and the second by $1-\beta$, and then adding, gives

$$(1-\bar{\phi})\big(\beta\phi_1\gamma(1-\bar{\phi}) + (1-\beta)\phi_0(1+\gamma\beta\phi_1)\big) \ge (1-\bar{\phi})(\bar{\phi}+\gamma\beta\phi_1),$$

which simplifies to

$$(1-\beta)\phi_0 + \beta\phi_1\gamma(1-\beta\phi_1) \ge \bar{\phi} + \gamma\beta\phi_1.$$

Finally, we obtain

$$\gamma(1-\beta\phi_1) \ge 1+\gamma,$$

a contradiction since $0 < \beta\phi_1 < 1$. It follows that $\epsilon_\star < \bar{\epsilon}_{port}$, which concludes the argument for Case 2.

Case 3. We consider the remaining case where $\bar{\epsilon}_{\min} = \bar{\epsilon}_0$. If $\bar{\epsilon}_0 > \hat{\epsilon}_1$, the argument is analogous to Case 1: parts (i) and (ii) apply verbatim on $[0, \hat{\epsilon}_1)$, the calculation of part (iii) applies on the interval $(\hat{\epsilon}_1, \bar{\epsilon}_0)$, and the logic of part (iv) carries through for $[\bar{\epsilon}_0, \bar{\epsilon}_{port}]$ with class-0 now playing the role of the collapsed segment while class-1 remains active.

We therefore focus on the regime with $\bar{\epsilon}_0 \le \hat{\epsilon}_1$ and consider two subcases:

Case 3A. Suppose that $\hat{\epsilon}_{port} < \bar{\epsilon}_0$. We analyze different intervals for ϵ:

(i) If $0 \le \epsilon < \hat{\epsilon}_{port}$, the computation from Case 1(i) applies and we have $\Delta(\epsilon) = (1-\beta)(1-\phi_0)\epsilon \ge 0$, so the pooled structure yields higher net climate gain on this interval.

(ii) For $\hat{\epsilon}_{port} \le \epsilon < \bar{\epsilon}_0$, we have as in Case 1(ii):

$$\Delta(\epsilon) = \frac{\phi_1\beta\gamma(1-\bar{\phi})}{1+\phi_1\beta\gamma} - \beta(1-\phi_1)\epsilon,$$

which crosses zero at $\epsilon = \epsilon_\star$, as given by (A.32). If $\epsilon_\star$ lies in $[\hat{\epsilon}_{port}, \bar{\epsilon}_0)$, then $\Delta(\epsilon) > 0$ for $\epsilon < \epsilon_\star$ and $\Delta(\epsilon) < 0$ for $\epsilon > \epsilon_\star$ within this interval. If instead $\epsilon_\star \ge \bar{\epsilon}_0$, then $\Delta(\epsilon) > 0$ throughout $[\hat{\epsilon}_{port}, \bar{\epsilon}_0)$.

(iii) If $\bar{\epsilon}_0 \leq \epsilon < \min\{\hat{\epsilon}_1, \bar{\epsilon}_{port}\}$, the class-0 segment is collapsed while class-1 remains active with $p_1^* > 1$. Substituting the relevant expressions for climate gain yields

$$\begin{aligned}
\Delta(\epsilon) &= (1-\bar{\phi})\left(\bar{\epsilon}_{port} - (1+\delta)\epsilon\right) - \beta\left(\phi_1 - \delta(1-\phi_1)\epsilon\right) \\
&= (1-\bar{\phi})\bar{\epsilon}_{port} - \beta\phi_1 - \left(1-\bar{\phi} + \delta(1-\beta)(1-\phi_0)\right)\epsilon \\
&= \frac{\phi_1\beta\gamma(1-\bar{\phi})}{1+\phi_1\beta\gamma} + (1-\beta)\phi_0 - \left(1-\bar{\phi} + \delta(1-\beta)(1-\phi_0)\right)\epsilon.
\end{aligned}$$

We first show that if the change of sign occurs in part (ii), i.e., if $\epsilon_\star < \bar{\epsilon}_0$, then $\Delta(\epsilon) < 0$ throughout this interval. Since $\Delta(\epsilon)$ is decreasing in ϵ, it suffices to check that $\Delta(\bar{\epsilon}_0) < 0$. Indeed,

$$\begin{aligned}
\Delta(\bar{\epsilon}_0) &= \frac{\phi_1\beta\gamma(1-\bar{\phi})}{1+\phi_1\beta\gamma} + (1-\beta)\phi_0 - \left(1-\bar{\phi} + \delta(1-\beta)(1-\phi_0)\right)\bar{\epsilon}_0 \\
&= \frac{\phi_1\beta\gamma(1-\bar{\phi})}{1+\phi_1\beta\gamma} - \beta(1-\phi_1)\bar{\epsilon}_0 + (1-\beta)\phi_0 - (1-\beta)(1-\phi_0)\,(1+\delta)\,\bar{\epsilon}_0 \\
&= \underbrace{\left(\frac{\phi_1\beta\gamma(1-\bar{\phi})}{1+\phi_1\beta\gamma} - \beta(1-\phi_1)\bar{\epsilon}_0\right)}_{<0 \text{ since } \epsilon_\star < \bar{\epsilon}_0 \text{ in Part (ii)}} - (1-\beta)\delta\phi_0 < 0.
\end{aligned}$$

Thus, if $\epsilon_\star < \bar{\epsilon}_0$, then $\Delta(\epsilon) < 0$ for all $\epsilon \in [\bar{\epsilon}_0, \min\{\hat{\epsilon}_1, \bar{\epsilon}_{port}\})$.

If instead $\epsilon_\star \geq \bar{\epsilon}_0$, there are two possibilities. If $\Delta(\bar{\epsilon}_0) < 0$, the same monotonicity argument implies that $\Delta(\epsilon) < 0$ throughout this interval, so the sign change occurs discontinuously at $\epsilon = \bar{\epsilon}_0$. On the other hand, if $\Delta(\bar{\epsilon}_0) \geq 0$, then it suffices to verify that $\Delta\left(\min\{\hat{\epsilon}_1, \bar{\epsilon}_{port}\}\right) < 0$ to ensure we have a unique crossing in the interval, since $\Delta(\epsilon)$ is linear and strictly decreasing in ϵ. We next verify that this inequality holds, considering the two possible cases:

- If $\bar{\epsilon}_{port} \leq \hat{\epsilon}_1$, we directly have

$$\Delta\left(\bar{\epsilon}_{port}\right) = -\beta\phi_1 - \delta(1-\beta)(1-\phi_0)\bar{\epsilon}_{port} < 0.$$

- If instead $\hat{\epsilon}_1 < \bar{\epsilon}_{port}$, by continuity of $\mathcal{CG}_1$ at $\hat{\epsilon}_1$ we write

$$\begin{aligned}
\Delta(\hat{\epsilon}_1) &= (1-\bar{\phi})\left(\bar{\epsilon}_{port} - (1+\delta)\hat{\epsilon}_1\right) - \beta\left(\phi_1 - \delta(1-\phi_1)\hat{\epsilon}_1\right) \\
&= (1-\bar{\phi})\left(\bar{\epsilon}_{port} - (1+\delta)\hat{\epsilon}_1\right) - \beta(1-\phi_1)\left(\bar{\epsilon}_1 - (1+\delta)\hat{\epsilon}_1\right) \\
&= (1-\bar{\phi})\bar{\epsilon}_{port} - \beta(1-\phi_1)\bar{\epsilon}_1 - (1-\beta)(1-\phi_0)(1+\delta)\hat{\epsilon}_1
\end{aligned}$$

  Recall from Case 1(iii) that

$$(1-\bar{\phi})\bar{\epsilon}_{port} < (1-\beta)(1-\phi_0)\bar{\epsilon}_0 + \beta(1-\phi_1)\bar{\epsilon}_1.$$

  Moreover, since this case assumes $\bar{\epsilon}_0 \leq \hat{\epsilon}_1$, it follows that

$$\Delta(\hat{\epsilon}_1) < (1-\beta)(1-\phi_0)\bar{\epsilon}_0 - (1-\beta)(1-\phi_0)(1+\delta)\bar{\epsilon}_0 = -(1-\beta)\delta\phi_0 \leq 0.$$

Therefore, we have $\Delta\left(\min\{\hat{\epsilon}_1, \bar{\epsilon}_{port}\}\right) < 0$. As a result, if $\Delta(\bar{\epsilon}_0) \geq 0$, there is a unique zero-crossing in $[\bar{\epsilon}_0, \min\{\hat{\epsilon}_1, \bar{\epsilon}_{port}\})$.

To summarize, we have shown that if $\Delta(\epsilon)$ changes sign in the interval of part (ii), then it remains negative within $\bar{\epsilon}_0 \leq \epsilon < \min\{\hat{\epsilon}_1, \bar{\epsilon}_{port}\}$. Otherwise, if $\Delta(\epsilon)$ remains positive in part (ii), then it changes sign at a unique $\epsilon \in [\bar{\epsilon}_0, \min\{\hat{\epsilon}_1, \bar{\epsilon}_{port}\})$.

(iv) Finally, consider $\min\{\hat{\epsilon}_1, \bar{\epsilon}_{port}\} \le \epsilon < \bar{\epsilon}_{port}$. This is the only remaining interval to cover since, by Proposition 7, $\bar{\epsilon}_{port} \le \bar{\epsilon}_1$. If $\bar{\epsilon}_{port} \le \hat{\epsilon}_1$, this interval is empty and we conclude with part (iii). Instead, if $\bar{\epsilon}_{port} > \hat{\epsilon}_1$, following a similar argument to Case 1(iv) we have:

$$\begin{aligned}\Delta(\epsilon) &= (1-\bar{\phi})\left(\bar{\epsilon}_{port} - (1+\delta)\epsilon\right) - \beta(1-\phi_1)\left(\bar{\epsilon}_1 - (1+\delta)\epsilon\right)\\ &= (1-\bar{\phi})\bar{\epsilon}_{port} - \beta(1-\phi_1)\bar{\epsilon}_1 - (1-\beta)(1-\phi_0)(1+\delta)\epsilon\\ &< (1-\bar{\phi})\bar{\epsilon}_{port} - \beta(1-\phi_1)\bar{\epsilon}_1 - (1-\beta)(1-\phi_0)\bar{\epsilon}_0 < 0,\end{aligned}$$

where the last step evaluates at $\epsilon = \bar{\epsilon}_0$, which is sufficient since $\Delta(\epsilon)$ is decreasing in ϵ. The final expression is negative by Case 1(iii).

Case 3B. Now suppose that $\bar{\epsilon}_0 \le \hat{\epsilon}_{port}$. As before, we analyze different intervals for ϵ

(i) If $0 \le \epsilon < \bar{\epsilon}_0$, we have $\Delta(\epsilon) = (1-\beta)(1-\phi_0)\epsilon \ge 0$ as in Case 3A(i).

(ii) For $\bar{\epsilon}_0 \le \epsilon < \hat{\epsilon}_{port}$, we have

$$\Delta(\epsilon) = \bar{\phi} - \delta(1-\bar{\phi})\epsilon - \beta\left(\phi_1 - \delta(1-\phi_1)\epsilon\right) = (1-\beta)\left(\phi_0 - \delta(1-\phi_0)\epsilon\right).$$

Hence, $\Delta(\epsilon)$ changes sign at $\epsilon = \bar{\epsilon}_0/\delta$, if this quantity lies in $[\bar{\epsilon}_0, \hat{\epsilon}_{port})$. Three possibilities arise:

- If $\delta \le \bar{\epsilon}_0/\hat{\epsilon}_{port}$, then $\Delta(\epsilon) > 0$ throughout $[\bar{\epsilon}_0, \hat{\epsilon}_{port})$.
- If $\bar{\epsilon}_0/\hat{\epsilon}_{port} < \delta \le 1$, then $\Delta(\epsilon)$ crosses zero at $\epsilon = \bar{\epsilon}_0/\delta \in [\bar{\epsilon}_0, \hat{\epsilon}_{port})$.
- If $\delta > 1$, then $\Delta(\epsilon) < 0$ for all ϵ in this region, implying a discontinuous sign change at $\epsilon = \bar{\epsilon}_0$.

(iii) For $\min\{\hat{\epsilon}_1, \bar{\epsilon}_{port}\} \le \epsilon < \bar{\epsilon}_{port}$, the argument of Case 3A(iv) implies $\Delta(\epsilon) < 0$.

(iv) For $\hat{\epsilon}_{port} \le \epsilon < \min\{\hat{\epsilon}_1, \bar{\epsilon}_{port}\}$, the expression for $\Delta(\epsilon)$ remains as in Case 3A(iii), which is decreasing and linear in ϵ. Therefore, we have two possibilities:

- If $\Delta(\hat{\epsilon}_{port}) < 0$, then $\Delta(\epsilon)$ remains negative for all $\epsilon \in [\hat{\epsilon}_{port}, \min\{\hat{\epsilon}_1, \bar{\epsilon}_{port}\})$. Since $\mathcal{CG}_{port}$ is continuous at $\epsilon = \hat{\epsilon}_{port}$, this corresponds to the case where $\delta > \bar{\epsilon}_0/\hat{\epsilon}_{port}$, so a prior sign change occurred in part (ii) above.
- If $\Delta(\hat{\epsilon}_{port}) \ge 0$, then $\delta \le \bar{\epsilon}_0/\hat{\epsilon}_{port}$ by the same continuity argument, and so $\Delta(\epsilon) > 0$ for $\epsilon < \hat{\epsilon}_{port}$. Moreover, since $\Delta\left(\min\{\hat{\epsilon}_1, \bar{\epsilon}_{port}\}\right) < 0$ (by Case 3A(iii)) and $\Delta(\epsilon)$ is strictly decreasing and continuous within $[\hat{\epsilon}_{port}, \min\{\hat{\epsilon}_1, \bar{\epsilon}_{port}\})$, there must be a unique crossing in this interval.

Thus, if a change of sign occurs in part (ii), then $\Delta(\epsilon)$ remains negative in $[\hat{\epsilon}_{port}, \min\{\hat{\epsilon}_1, \bar{\epsilon}_{port}\})$. Otherwise, if $\Delta(\epsilon)$ remains positive in part (ii), then it changes sign once in the current interval.

(v) For $\min\{\hat{\epsilon}_1, \bar{\epsilon}_{port}\} \le \epsilon < \bar{\epsilon}_{port}$, the same argument from Case 3A(iv) implies $\Delta(\epsilon) < 0$.

This concludes Case 3. We established that if $\bar{\epsilon}_{min} = \bar{\epsilon}_0$, the pooled structure yields higher net climate gain only when ϵ lies below a unique threshold in $(0, \bar{\epsilon}_{port})$. Abusing notation, we denote this threshold by $\epsilon_\star$, extending (A.32) to encompass the subcases considered above.

Cases 1–3 exhaust all possible scenarios. In each case, $\Delta(\epsilon)$ is positive near zero, changes sign once at some $\epsilon_\star \in (0, \bar{\epsilon}_{port})$, and is negative thereafter on $[\epsilon_\star, \bar{\epsilon}_{port})$. This establishes the proposition. □

A.4. Proofs of §5 (Certifier Incentives)

Before proving Propositions 9 and 10, we show that perfect certification maximizes net climate gain.

Remark 1. Net climate gain is maximized under fully accurate certification.

Proof of Remark 1. Recall that the net climate gain generated by class-b credits is defined as the volume of high-quality traded class-b credits minus a penalty proportional to the number of low-quality offsets:

$$\mathcal{CG}_b = \mathcal{H}_b^* - \delta \cdot \mathcal{L}_b^*,$$

where $\delta \geq 0$ is the greenwashing penalty for low-quality offsets (as in Equation (2.8)).

We claim that $\mathcal{CG}_b$ is uniquely maximized when $\epsilon = 0$, for both $b \in \{0,1\}$. This follows because $\epsilon = 0$ simultaneously maximizes the participation of high-quality credits and minimizes the volume of low-quality sellers. On one hand, perfect certification ensures that no low-quality credits enter the market, so $\mathcal{L}_b^* = 0$. Moreover, when $\epsilon = 0$, the equilibrium price satisfies $p_b^* \geq 1$ (from Equations (3.1) and (3.2) in Proposition 1), which guarantees that all high-quality sellers participate, leading to the highest possible $\mathcal{H}_b^*$ (i.e., $\mathcal{H}_0^* = (1-\beta)\phi_0$ and $\mathcal{H}_1^* = \beta\phi_1$). Thus, $\mathcal{CG}_b$ is maximized at $\epsilon = 0$ for both $b \in \{0,1\}$, and the overall net climate gain is also maximized since it is the sum across credit classes.

Moreover, $\epsilon = 0$ is the unique maximizer: any $\epsilon > 0$ increases the volume of low-quality trade without expanding high-quality volume. We conclude by noting that the same argument applies for the settings with buyer penalties and credit portfolios studied in §4. □

We now turn to Proposition 9, which shows that the trade volume in equilibrium is not maximized under perfect certification.

Proof of Proposition 9. From the proof of Proposition 2 (Appendix A.1.2), the equilibrium trade volume for each credit class is given by

$$S_0^* = \begin{cases} (1-\beta)\phi_0, & \text{if } \epsilon < \bar{\epsilon}_0, \\ 0, & \text{if } \epsilon \geq \bar{\epsilon}_0, \end{cases} \quad \text{and} \quad S_1^* = \begin{cases} \beta\left(\phi_1 + (1-\phi_1)\epsilon\right), & \text{if } \epsilon < \hat{\epsilon}_1, \\ \frac{\beta\phi_1(1+\gamma)}{1+\phi_1\beta\gamma}, & \text{if } \hat{\epsilon}_1 \leq \epsilon < \bar{\epsilon}_1, \\ 0, & \text{if } \epsilon \geq \bar{\epsilon}_1, \end{cases} \tag{A.33}$$

where the above thresholds (from Proposition 1) are

$$\bar{\epsilon}_0 = \frac{\phi_0}{1-\phi_0}, \quad \bar{\epsilon}_1 = \frac{\phi_1(1+\gamma)}{(1-\phi_1)(1+\phi_1\beta\gamma)}, \quad \text{and} \quad \hat{\epsilon}_1 = \left(\frac{\gamma(1-\beta\phi_1)}{1+\gamma}\right)\bar{\epsilon}_1.$$

Without loss of generality, we assume both the market collapse thresholds $\bar{\epsilon}_0$ and $\bar{\epsilon}_1$ lie in $[0,1]$. If either threshold exceeds 1, the analysis proceeds essentially unchanged after truncating the domain of ϵ, with some conditions becoming vacuous.[16]

Let $S^* = S_0^* + S_1^*$ denote the total trade volume in equilibrium. We analyze the relationship between S^* and ϵ by considering two different cases, each of which leads to a different piecewise structure for S^*.

[16] For example, if both $\bar{\epsilon}_0$ and $\bar{\epsilon}_1$ exceed 1, neither market segment collapses even when certification is completely uninformative ($\epsilon = 1$). In such cases, the same arguments we follow show that trade volume is maximized at $\epsilon = 1$. We therefore focus on the more interesting cases where both segments collapse for sufficiently high certification error.

Case 1. Suppose that $\hat{\epsilon}_1 < \bar{\epsilon}_0$. Without loss, assume also that $\bar{\epsilon}_1 < \bar{\epsilon}_0$; the alternative subcase where $\bar{\epsilon}_1 \geq \bar{\epsilon}_0$ is analoguous. From Equation (A.33), the total trade volume equals

$$S^*(\epsilon) = \begin{cases} (1-\beta)\phi_0 + \beta(\phi_1 + (1-\phi_1)\epsilon), & \text{if } \epsilon < \hat{\epsilon}_1, \\ (1-\beta)\phi_0 + \beta\frac{\phi_1(1+\gamma)}{1+\phi_1\beta\gamma}, & \text{if } \hat{\epsilon}_1 \leq \epsilon < \bar{\epsilon}_1, \\ (1-\beta)\phi_0, & \text{if } \bar{\epsilon}_1 \leq \epsilon < \bar{\epsilon}_0, \\ 0, & \text{if } \epsilon \geq \bar{\epsilon}_0, \end{cases}$$

where we explicitly introduce the dependency on ϵ in the notation.

Observe that $S^*(\epsilon)$ is strictly increasing in ϵ within the interval $[0, \hat{\epsilon}_1)$, and that this interval is non-empty since $\gamma > 0$ implies $\hat{\epsilon}_1 > 0$. Hence, there exists a positive $\epsilon > 0$ that leads to strictly higher trade volume than perfect certification.

Moreover, note by inspection that $S^*(\epsilon)$ is piecewise constant and decreasing as a function of ϵ over the interval $[\hat{\epsilon}_1, 1]$. In addition, $S^*(\epsilon)$ is continuous at $\epsilon = \hat{\epsilon}_1$. Therefore, it follows that total trade volume is maximized at any $\epsilon \in [\hat{\epsilon}_1, \bar{\epsilon}_1)$, and any such ϵ delivers strictly higher trade volume than $\epsilon = 0$. This completes Case 1.

Case 2. Suppose instead that $\hat{\epsilon}_1 \geq \bar{\epsilon}_0$. Since $\hat{\epsilon}_1 < \bar{\epsilon}_1$, we also have $\bar{\epsilon}_0 < \bar{\epsilon}_1$. Adding the trade volume for each class from Equation (A.33) yields

$$S^*(\epsilon) = \begin{cases} (1-\beta)\phi_0 + \beta(\phi_1 + (1-\phi_1)\epsilon), & \text{if } \epsilon < \bar{\epsilon}_0, \\ \beta(\phi_1 + (1-\phi_1)\epsilon), & \text{if } \bar{\epsilon}_0 \leq \epsilon < \hat{\epsilon}_1, \\ \frac{\beta\phi_1(1+\gamma)}{1+\phi_1\beta\gamma}, & \text{if } \hat{\epsilon}_1 \leq \epsilon < \bar{\epsilon}_1, \\ 0, & \text{if } \epsilon \geq \bar{\epsilon}_1. \end{cases}$$

In this case, $S^*(\epsilon)$ is strictly increasing and continuous in ϵ over each of the two intervals $[0, \bar{\epsilon}_0)$ and $[\bar{\epsilon}_0, \hat{\epsilon}_1]$, but exhibits a downward jump discontinuity at $\epsilon = \bar{\epsilon}_0$, the point at which the class-0 market collapses. Moreover, $S^*(\epsilon)$ is then piecewise constant and non-increasing for $\epsilon \geq \hat{\epsilon}_1$. Hence $S^*(\epsilon)$ is highest either at $\epsilon = \hat{\epsilon}_1$, or when ϵ approaches $\bar{\epsilon}_0$ from below. In the latter case the maximum is not formally attained, but the supremum is given by $\lim_{\epsilon \to \bar{\epsilon}_0^-} S^*(\epsilon)$.

To pin down the value of ϵ that maximizes total trade, it follows from the argument above that $S^*(\epsilon)$ is maximized at $\epsilon = \hat{\epsilon}_1$ if and only if $S^*(\hat{\epsilon}_1) \geq \lim_{\epsilon \to \bar{\epsilon}_0^-} S^*(\epsilon)$. That is,

$$\frac{\beta\phi_1(1+\gamma)}{1+\phi_1\beta\gamma} \geq (1-\beta)\phi_0 + \beta(\phi_1 + (1-\phi_1)\bar{\epsilon}_0).$$

Straightforward algebra shows that this condition is equivalent to

$$\gamma \geq \frac{(1-\beta)\phi_0 + \beta(1-\phi_1)\bar{\epsilon}_0}{\beta\phi_1\left(1 - \bar{\phi} - \beta(1-\phi_1)\bar{\epsilon}_0\right)} \quad \text{and} \quad \bar{\phi} + \beta(1-\phi_1)\bar{\epsilon}_0 < 1, \tag{A.34}$$

where $\bar{\phi} = \beta\phi_1 + (1-\beta)\phi_0$ denotes the average quality across all sellers. Note that both conditions in (A.34) hold in scenarios with a predominantly low-quality seller base (low ϕ_0 and ϕ_1, which yield low $\bar{\phi}$ and $\bar{\epsilon}_0$) provided that buyers' value of co-benefits (γ) exceeds a threshold.

To summarize, in Case 2 the trade-maximizing ϵ equals $\hat{\epsilon}_1$ if condition (A.34) holds (in which case any $\epsilon \in [\hat{\epsilon}_1, \bar{\epsilon}_1)$ maximizes trade). Otherwise, $S^*(\epsilon)$ is highest as ϵ approaches $\bar{\epsilon}_0$ from the left (but the maximum is formally not attained due to the jump discontinuity at $\epsilon = \bar{\epsilon}_0$). Nonetheless, in either situation, there exists a positive $\epsilon > 0$ that yields strictly higher trade volume than perfect certification ($\epsilon = 0$).

In both Cases 1 and 2, we conclude that there exists $\epsilon > 0$ for which total trade volume exceeds the level under perfect certification. While there exist instances in which the maximum is not attained (as trade volume is discontinuous when one of the segments collapses), the certifier in all cases has an incentive to introduce certification error if its objective is aligned with maximizing trade volume. □

Finally, we turn to Proposition 10, demonstrating that unlike trade volume, the total value of transactions in the market is maximized under perfect certification.

Proof of Proposition 10. Let $\mathcal{TV}_b^*$ denote the total transaction value for class-b credits:

$$\mathcal{TV}_b^* = p_b^* \cdot S_b^*.$$

We will show that for both credit classes, $\mathcal{TV}_b^*$ is strictly maximized under perfect certification.

For class-0, multiplying the equilibrium price p_0^* and trade volume S_0^* from Equations (3.1) and (A.33) yields

$$\mathcal{TV}_0^*(\epsilon) = p_0^* \cdot S_0^* = \left(1 - \frac{1-\phi_0}{\phi_0}\epsilon\right)_+ \cdot (1-\beta)\phi_0 \mathbf{1}_{\{\epsilon < \bar{\epsilon}_0\}} = (1-\beta)(1-\phi_0)\left(\bar{\epsilon}_0 - \epsilon\right)_+,$$

where we explicitly denote the dependence of ϵ. Note that $\mathcal{TV}_0^*(\epsilon)$ is positive and strictly decreasing in ϵ over the interval $[0, \bar{\epsilon}_0)$, and constant and equal to zero for $\epsilon \geq \bar{\epsilon}_0$. Hence, $\mathcal{TV}_0^*(\epsilon)$ is strictly maximized at $\epsilon = 0$.

For class-1, multiplying the equilibrium price p_1^* in (3.2) and the trade volume S_1^* in (A.33) gives, after some algebra,

$$\mathcal{TV}_1^*(\epsilon) = \begin{cases} \beta\phi_1\left(1 + \gamma\left(1 - \beta\phi_1\right) - \beta(1-\phi_1)\gamma\epsilon\right), & \text{if } \epsilon < \hat{\epsilon}_1, \\ \frac{\beta(1-\phi_1)(1+\gamma)}{1+\phi_1\beta\gamma}\left(\bar{\epsilon}_1 - \epsilon\right), & \text{if } \hat{\epsilon}_1 \leq \epsilon < \bar{\epsilon}_1, \\ 0, & \text{if } \epsilon \geq \bar{\epsilon}_1. \end{cases}$$

As before, $\mathcal{TV}_1^*(\epsilon)$ is strictly decreasing in ϵ over the interval $[0, \bar{\epsilon}_1)$ and constant and equal to zero for $\epsilon \geq \bar{\epsilon}_1$. To verify this, note that $\mathcal{TV}_1^*(\epsilon)$ is positive and strictly decreasing in ϵ on both the intervals $(0, \hat{\epsilon}_1)$ and $[\hat{\epsilon}_1, \bar{\epsilon}_1)$, and it is continuous at $\epsilon = \hat{\epsilon}_1$ since both the price and the trade volume are continuous at that point. Therefore, $\mathcal{TV}_1^*(\epsilon)$ is also strictly maximized at $\epsilon = 0$.

Since transaction value in each class is maximized at $\epsilon = 0$, the total transaction value $\mathcal{TV}^* = \mathcal{TV}_0^* + \mathcal{TV}_1^*$ is likewise strictly maximized at $\epsilon = 0$. Therefore, it is optimal for the certifier to make certification perfectly accurate if its objective is aligned with maximizing total transaction value. □