CLASS PEERS AS COMPETITORS AND EDUCATORS: THE CONSEQUENCES OF RANK-BASED ACADEMIC REWARDS

By

Mark R. Rosenzweig and Bing Xu

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YALE UNIVERSITY
Box 208281
New Haven, Connecticut 06520-8281

http://cowles.yale.edu/

CLASS PEERS AS COMPETITORS AND EDUCATORS: THE CONSEQUENCES OF RANK-BASED ACADEMIC REWARDS

Mark R. Rosenzweig
Yale University and NBER

Bing Xu
Southwestern University of Finance and Economics

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This paper examines the theoretical and empirical consequences of rank-based reward systems in schools in which students' performance and effort are evaluated relative to their peers. In such environments, classmates act simultaneously as competitors—due to rank-determined rewards—and as educators through peer learning and assistance. Using nationally representative panel survey data from U.S. high schools, combined with administrative information on the location assignments of new refugee student cohorts, we exploit variation in school competition policies and class ability compositions to identify empirically their dual effects on student effort and peer learning. We develop a theoretical tournament model with heterogeneous students who adjust their effort in response to the effort of similar peers and in which students learn from peers. The model predicts that when rewards depend on relative standing, adding higher-ability students to a cohort will reduce both incumbent academic effort and peer assistance, particularly in schools emphasizing rank-based awards, while adding lower-ability students has the opposite effects. Empirical tests of the model confirm these predictions. In schools with strong rank-based reward policies, the addition of stronger peers reduces high-performing incumbent students' homework time and eliminates the positive spillovers from peer learning observed in less competitive settings. The adverse effects are concentrated among high-ability incumbents, while lower-ability students—who are less likely to win competitive awards—are largely unaffected. The results indicate that performance-based competition undermines cooperative peer learning and reduces student effort and overall academic performance, especially in institutions with high-ability students that explicitly emphasize relative ranking in determining academic recognition.

Bing Xu (corresponding author): xubing@swufe.edu.cn. The author's names are listed in alphabetical order.

I. Introduction

Academic administrators at Harvard University have recently proposed, as remedies for grade inflation, allocating academic honors based on student rankings rather than only on grade point averages as well as the establishment of a quota on A grades. These changes effectively transform academic honors and top grades into tournament prizes.[1] As grade inflation is a marked characteristic of many universities, particularly those with highly-selective admissions, universities are looking to Harvard for guidance on grade-inflation remedies.[2] It is thus important to obtain a better understanding of how the use of student ranking criteria for academic rewards affects student behavior and learning.

In this paper we examine theoretically and empirically the consequences of rewarding students based on their performance relative to their peers for both effort and academic performance in a context in which class peers can also directly enhance learning. We use data from a nationally-representative survey of students in US high schools to identify these peer competition effects. Students in US high schools already face rank-based award systems. Some high schools explicitly base academic awards on class rank and we leverage cross-school variation in the use of rank-based awards along with variation in class ability compositions to identify the consequences of rank-based rewards. However, financial aid and admission criteria for many colleges and universities explicitly include class rank, and these university policies thus affect the effort incentives for at least a subset of students in all US high schools.[3]

Student peers in a setting in which ranks determine rewards are thus both competitors, due to the reward rules based on relative performance, and educators augmenting the educational returns to effort by their fellow students via spillovers and direct assistance. Our principal aim is

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[1] Faculty Committee Proposes Cap on A Grades, New Internal Ranking System | News | The Harvard Crimson

[2] https://yaledailynews.com/articles/nodding-to-harvard-lewis-foresees-yale-effort-to-curb-grade-inflation

[3] The latest (2002) administrative data on admissions and test scores for 1,991 US colleges and universities from the Integrated Postsecondary Education Data System (IPEDS) of the National Center for Education Statistics [https://nces.ed.gov/ipeds/datacenter/DataFiles.aspx?year=2022&sid=90c08118-40e7-4c3a-8049-53d2324eb18b&rtid=7] indicate that 61% of students are enrolled in a post-secondary institution that considers class rank for admission (48% of all post-secondary schools). Of the schools that admit less than 50% of applicants, 65% use class rank as an admission criterion, weighted by enrollment (55% of these schools). All of the private colleges and universities ranked in 2024 by Forbes to be in the top 15 of all public and private colleges and universities in the United States used class rank as a criterion for admission (all eight Ivy League schools plus Stanford, MIT, Chicago, Johns Hopkins, Rice and Vanderbilt) [https://www.forbes.com/top-colleges/]. Based on our search of the websites of the state university systems in all 50 US states for the academic year 2023-24, nineteen systems explicitly indicated they used class rank for either admission or financial aid, covering more than 50% of all US high-school students. For example, 2009 legislation in Texas allocated automatic admission for 75% of available slots in the flagship University of Texas at Austin to all students based on class rank. In 2023-24, the criterion was top 6%.

to separately identify what we believe are the two main mechanisms by which peers affect student achievement – via competition effects resulting from rank-based rewards and peer learning – and how these vary by competition policy. We develop a game-theoretic tournament model with heterogeneous competitors and peer learning effects in which students match the efforts of their like-peers. The effort-matching competition model generates testable predictions for how, when rewards depend on relative performance, changes in ability composition within a grade affect both effort and exam outcomes across ability types and vary by the degree to which rankings criteria are explicitly used as reward criteria.[4]

We test the set of predictions from the model exploiting variation in both class composition and school policies that emphasize rank-based grade competition across a representative sample of US high schools. We combine the nationally-representative survey data, which describes both the behavior and academic outcomes of US high school students as well as the behavior of their parents and their teachers, with administrative data on the national distribution of students who are refugees. A key feature of the school survey data is the provision of information on the extent and characteristics of school competition policy. Our key finding, in accord with the competition cum peer learning model, is that when stronger students are added to the school cohort in a setting in which academic reward criteria are rank-based both the homework time of the strongest students and peer homework assistance are reduced. Moreover, these adverse academic effects are enhanced in schools that explicitly encourage student competition by giving more weight to the relative standing of students in awarding academic honors. Indeed, not only is student effort reduced among the strongest student incumbents but the positive peer learning effects on test scores, which are observed in less competitive schools, are completely nullified in high schools that emphasize ranking criteria. We also show that these adverse peer effect results are significantly weaker or absent among lower-ability incumbent students, who are substantially less likely to attain competitive awards, and that the effects on

[4] Fang *et al*. (2020) obtains their general result from a model with homogeneous competitors. In contrast to the model of Hopkins and Kornienko (2004) developed by Tincani (2024) to describe student responses to changes in the composition of the effort costs of students, in tournament models rank is not a status good but matters instrumentally because it affects the probability of rewards that are valued. There is evidence that rank may be intrinsically valued by students (Tran and Zeckhauser, 2012) and that, outside of schools, personal relative standing unrelated to extrinsic rewards can affect effort, risk-taking and outcomes (Ager *et al*. 2002). However, we show that in addition to our finding that being exposed to more close competitors decreases effort as predicted by tournament models with extrinsic rewards, variation in the degree of competition matters, and not all students behave as if they value rank intrinsically.

student effort among the higher-ability students are not driven by variation in the behavior of teachers or parents in response to the changes in cohort ability composition.

There is an established theoretical literature demonstrating that when rankings are used as reward criteria effort will be reduced. In particular, Fang *et al*. (2020) demonstrate that with convex effort costs, any system that allocates prizes based on one's relative standing will reduce effort.[5] In the school context, rank-based rewards thus may not only reduce student effort but may also turn class peers from educators into competitors (Drago and Garvey, 1998).[6] As pointed out in Chen and Hu (2024), however, the large number of empirical studies examining class peer effects on test scores assume that student peers are either educators who aid other students in learning or are disrupters, thereby ignoring the competitive aspects of peers and competition effects on effort or assistance.[7]

Competitor peer effects due to the employment of rank-based reward are also neglected in the recent studies of student effort. In Todd and Wolpin (2018), which is the first study to model and estimate student effort in the classroom, student effort is characterized by a Nash game between students and teachers, with the characteristics of peers only mattering for student effort and achievement to the extent that they influence teacher effort – there are neither direct peer learning effects nor a competition effect. In Cotton *et al*. (2026), academic rewards to effort are modeled as piece rates, only determined by individual achievement (grades), with the achievements of class peers irrelevant. And in the Del Boca *et al*. (2026) study of student academic effort the role of student peers is ignored, while in Agostinelli *et al*. (2026), a small number of mutually-chosen peers, who may or may not be classmates, are the only source of peer effects, and high-achieving peers are considered as strictly "favorable" to the child's human capital development.[8]

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[5] Xu and Pak (2021) analyzed the comparative statics of a contest model with symmetric input functions and resource constraints but in a context in which losing is so catastrophic that an interior equilibrium does not exist, and the equilibrium effort is always increasing.

[6] In courses that grade on a curve, fellow students in the classroom are also transformed into competitors such that better peers mechanically lower a student's grade even in the absence of effort effects (Calsamiglia and Laviglio, 2019).

[7] They cite Sacerdote (2001); Zimmerman (2003); Stinebrickner and Stinebrickner (2006); Lyle (2007); Carrell, Fullerton and West (2009); Imberman, Kugler and Sacerdote (2012); Abdulkadiroglu *et al*. (2014); Booij, Leuven and Oosterbeek (2017); Feld and Zölitz (2017); Zarate (2023) as examples of studies where peers are educators and Figlio (2007); Kling *et al*. (2007); Carrell *et al*. (2008); Gould *et al*. (2009); Carrell and Hoekstra (2010); Lavy and Schlosser (2011); Carrell *et al*. (2018) as examples of showing peers as disrupters.

[8] This is not unreasonable. Our competition mode implies that students and parents would avoid competitors who are peers, as is consistent with the school flight literature.

Studies of peer effects that focus on student effort when class rank is extrinsically rewarding are few and hampered by the absence of direct measures of student effort.[9] Important contributions include Grau (2019), who develops and structurally estimates a rank-order tournament model to explicitly assess how college admission policies affect student effort. However, in that model, student strategic behavior is ignored. Tincani (2017, 2024) uses the status-good model of Hopkins and Kornienko (2004), which, adapted to schools, assumes that students intrinsically value both their rank and absolute performance in optimally choosing effort. She finds that exogenous changes in relative effort costs affect both test scores and class rank, but her data do not have measures of student effort and the model is silent on how school competition policy would affect effort and test scores. Moreover, these studies of the role of ranks in schools on student effort are also limited because the models ignore the role of peers as educators and how competition affects that role. Effects of class composition on test scores alone cannot discriminate between models in which rank is a consumption good or is instrumental and affected by school rules.

Most prior empirical studies of peer effects on test scores stratify the treated incumbents and the peers into two groups by ability or parental background. Our model also contains two groups of students segmented by ability. We believe the dichotomous framework realistically depicts the stratification in high schools that results in part by rank-based rewards. One group consists of students with high socioeconomic backgrounds who have a high chance of success in competing for academic rewards inclusive of academic honors and college admission; the other consists of students who have significantly lower chances of achieving academic success in a competition and expend significantly less academic effort because the prospects of receiving awards from increased effort are minimal.

Figure 1 displays the fraction of senior US high school students who won an academic award in their senior year of high school by total parental years of schooling obtained from the nationally-representative survey of US high-school seniors in 1992 from the National Education Longitudinal Study of 1988. As can be seen, the probability of winning an award is significantly

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[9] Some empirical studies of sports events show that rankings-based rewards affect effort. For example, Genakos and Pagliero (2012) examine weight-lifting contests to show how revealing interim ranks affects both risk-taking and performance. Brown (2011) used the results of Stein's Nash tournament model to inform her empirical work on the effect of superstars on the effort levels of the competitors in golf tournaments. Guryan *et al*. (2009), however, find no peer effects in golf tournaments based on the random assignments of players differentiated by ability. The findings in Hickman and Metz (2018) from golf tournament data suggest that this may be because there are offsetting positive learning and negative competition effects.

higher among the students whose parents have a total schooling level greater or equal to 26. Notably, the award probability also rises sharply with parental schooling above that level, unlike for students below that cutoff for whom award probabilities only marginally vary by parental schooling levels.[10] As Figure 1a shows, student effort, measured by homework hours per week obtained from the same national survey, also exhibits the same dichotomous patterns as is seen for award probabilities.

Our focus is on the behavior and academic achievements of the top students, measured by family background, for whom academic rewards based on rankings are *ex ante* more plausible. Indeed, quotas on academic rewards lead to the stratification of students by ability in academic effort, with lower-ability students possibly not engaged in the competition. Given that relative standing matters and that students match the behavior of their peers, the behavior of the lower-ability students will also affect the behavior of the higher-background students, and we also examine the special, but empirically-relevant, case in which the weaker students do not compete at all and may drop out of school. Our model also permits (i) higher-ability peers to directly increase a student's achievement, given her effort, especially among those incumbents who have higher ability (as found by Chen and Hu) and (ii) schools to differ in competition policy. The resulting model delivers nine testable implications for how strong and weak incumbent students' effort and academic achievement vary with the addition of strong and weak students to the student cohort and how these vary by the degree of school-induced competition.

Among the key predictions of the model are that an increase in the number of strong students decreases the effort of the strong incumbent students, especially in schools that emphasize ranking criterion,[11] but an increase in the number of weaker students increases the effort levels of the strong incumbent students, again especially in more competitive schools.[12]

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[10] A similar pattern is observed by total family income. However, that variable is endogenous to student academic effort (homework time) because it includes the contribution to income by the students. 69.6% of the native-born high-school students in 1991-92 worked during their senior year; of those who worked, more than half worked more than 18 hours a week.

[11] This result is the same as in the Nash equilibrium but contrasts with the status good model of Hopkins and Kornienko in which, as shown by Tincani (2024), the returns to effort rise when there is a higher proportion of stronger students because a unit increase in effort improves class rank more strongly.

[12] The theoretical part of our work relates to the literature on contests with asymmetric competitors, most closely with Stein (2002) and Cornes and Hartley (2005). As noted, tournament models generally indicate that competition, which arises when rewards depend on one's performance relative to others, reduces individual performance. Stein derived the Nash equilibrium for linear input functions and showed that the equilibrium effort of all the contestants, except possibly the strongest one, weakly decreases when one of the competitors becomes stronger. In our model,

This is because in our model students take into account that their alike peers will match their own effort, and the effort of weaker students does not fully match (or match at all) those of the strong students. The addition of weaker students also directly decreases winning probabilities of the strong incumbent students less than the addition of strong students. However, because of the dual role of strong students as educators and competitors, the model is ambiguous on the effects of adding strong students on the academic achievement of strong incumbents. This means that empirical findings of peer effects based solely on the variation in the test scores of strong incumbents, however well-identified econometrically, cannot be interpreted as only measuring peer learning effects, as they may capture both negative competition and positive peer learning effects – the reduced-form test score effect is net of the two.

Due to the operation of the two mechanisms, test scores of the strong incumbent students may rise or fall when new entrants are high-ability, depending on whether the positive learning (educator) peer effect on test scores dominates the negative competition effect on effort. The balance of these may differ across grades and school policies, and may be responsible for the mixed findings in the literature of the net impact of peers on student outcomes. The opposite effects of strong peers on student effort and on learning, however, establishes a strong test for peers as educators and competitors. The dual roles are exposed when among the stronger incumbent students test scores rise while effort levels fall in response to an increase in the number of high-ability students. A challenge is to quantify both effects separately when they coexist, which is one of the aims of this paper.

Our main empirical strategy to identify the consequences of school competition in the presence of peer learning effects, for both incumbent student effort and test scores by ability type, is to exploit variation in the composition of students and in school competition policy. This variation is used to first test if we can empirically match the signs of the nine rejectable predictions of the model to those of the causal relationships we estimate from a large and nationally-representative panel data set that describes US high schools and contains information on student effort, test scores and school academic polices.[13] As indicated in the model, no single

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we provide for the first time comparative statics results for the general input functions considered in Cornes and Hartley (2005), and with a weaker assumption about cost curves than in Fang *et al*., but we also consider matching behavior, which gives rise to heterogeneous response of effort to the strength of peer students, in contrast to the Nash equilibrium in which adding competitors of any ability reduces effort.

[13] We take into account the bias in tests of individual coefficient statistical significance from multiple hypothesis testing.

empirical relationship between peer composition and test scores or effort uniquely identifies either competition or peer learning effects. Rather, the entire set of empirically matched sign predictions together identifies the presence of peer learning and peer competition effects. We also quantify the magnitude of the negative effects on the test scores of strong students at the school/grade level due purely to competitive student peer effects while controlling for the presence of positive peer learning effects. We do this by structurally estimating an achievement production function incorporating student effort and peer and parental homework assistance, net of school fixed effects. This allows us to isolate the effect of adding high-ability students on the test scores of high-ability incumbents through the competition channel alone, net of peer learning gains.

We use as our main data set the restricted-use U.S. Department of Education, National Center for Education Statistics, National Education Longitudinal Study of 1988 (NELS:88), which is the source for figures 1 and 1a.[14] The NELS:88 is a nationally-representative survey of students starting with a sample of over 25,000 8th-graders in 1988, a subset of whom were additionally surveyed in 1990 and 1992, the high school years, with additional sampling to attain representativeness by grade. As described in detail below, there are several advantages of this survey data set over the school administrative records that have been chiefly used to estimate student peer effects and the household survey data sets that have been used to examine parental-student interactions. In particular, the NELS:88 provides standardized test scores, student family characteristics, a measure of student effort, and information on teacher, student peer, and parent behavior.

An important additional feature of the data set is that it also provides information on school polices, including the level and characteristics of grade competition, admission selectivity, and class stratification, enabling us to test an important prediction of the competition model – that the negative and positive effects on student effort from changes in the ability composition of a class are stronger in schools that encourage competition net of other school policies. Importantly, we are able to verify empirically, based on the provided student information on class ranks, cumulative grade point averages, and academic awards, that in high schools where grade competition is said to be strongly encouraged by the school principal, academic rewards in the

[14] A condition of the use of these data is that reported sample sizes must be rounded so that the last digit is zero. All sample sizes in the tables in this paper conform to this restriction. The restricted-use version provides county-level location information and many extra student and parent behaviors compared with the public-use version.

school are evidently based more significantly on relative performance criteria, which conforms to the definition of school competitiveness we employ in the model.

To estimate the effects of changes in peer composition on student effort and scholarly achievement by school competitiveness, we append to the restricted-use NELS:88 data information on the number of Southeast Asian (SEA) refugees initially assigned to each county during 1990–1992, corresponding to the final two rounds of the NELS:88. These data come from the newly-available administrative records of the Office of Refugee Resettlement (ORR) compiled in Dreher *et al*. (2020). We use the number of high-school age Southeast Asian (SEA) refugees as the treatment variable because they arrived in large numbers during the NELS:88 panel period, were assigned to geographic locations by the US State Department rather than self-selecting, and are identifiable in the data by nativity, allowing direct comparison with the incumbent native-born students stratified by parental background characteristics.

In accord with the model, we divide up our treated group – native-born high school students – and our treatment group – SEA refugee students – into two segments based on the schooling of the parents. We use as our cutoff the criterion that both parents have more than a high-school education.[15] We find that the groups above the cutoff among both the native-born and the SEA refugees are similar to each other in academic achievement and effort, and have significantly higher test scores and spend significantly more time doing homework compared with the native-born students with lower-schooled parents.[16] We are thus able to show that the SEA refugee students are both credible competitive threats to and potential educators for the high-ability incumbent native-born.[17]

Our main econometric strategy uses a panel of native-born students who remained in the same school from 10th to 12th grade, employing first-differenced instrumental variables to identify the nine empirical peer effects and match them in sign to the comparative-static predictions from the model. Less than 3.3% of all US high-ability students changed high schools between those grades, so this is a minimally selective sample of all US high-school students that we show avoids significant biases due to the school-flight effects that are evident in studies of

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[15] By this criterion, no students in the higher-ability group have parent schooling below 26 years, the cutoff displayed in figures 1 and1a, but there are a few weak students by this definition whose parental total years exceeds 26 years.
[16] As indicated in Figure 1, students above the cutoff also have a significantly higher probability of receiving an academic honor.
[17] In Section 6, we test non-parametrically whether this cutoff is appropriate.

students in grade school.[18] With this sample, differencing eliminates not only all time-invariant characteristics of the students but also those of their schools. Tests of the model are derived from how the change in the number of assigned high-and low-ability refugee students across schools with different admission, class grouping, and competitiveness polices affects changes in the effort and test scores of the incumbent native-born students stratified by ability. To assess if the results are robust to the endogenous placement of refugees net of school and student fixed effects, we employ two instrumental variable approaches using historical refugee assignment patterns. The findings remain consistent across all specifications.

Our main findings, consistent with the predictions of the model, are that adding strong students to the grade reduces effort among strong incumbent students, while adding weaker students increases their effort.[19] We show that the responses of student homework time are not explained by changes in teacher homework assignments. Moreover, these effects are stronger, as predicted by the model, in strongly competitive high schools that encourage grade competition by according more weight to class rankings in awarding prizes. We also find, as in the previous literature indicating that competition discourages student assistance, that the increase in peer homework help when strong students are added to the grade occurs only in less competitive high schools and not in highly competitive schools. This result is also in accord with the finding in Kosse *et al*. (2025) that the introduction of class rankings for college admissions in Chile resulted in high-school students having lower prosociality later in life. Also consistent with the structure of the model and the descriptive evidence on award probabilities, we find evidence of the two-group stratification of the high-school student body by parental background, with the effort response to changes in class composition absent among weaker incumbents.

Consistent with the findings for strong students that affirm the presence of both negative competitive and positive peer learning effects, we find that adding strong students significantly

[18] Figlio *et al*. (2024) show that in their grade school data peer effects are biased negatively, especially for treated incumbent strong students as a result of school flight from immigrants, although this is not attributed to competition. Boustan *et al*. (2024) find that school enrollment declined in higher-income areas of California due to the entry of Asian immigrants, which the authors attribute to parents fearing increased competition. We test directly for selectivity bias in our high-school sample of strong native-born high-school students due to flight and find that it is not statistically significant for effort but is for test scores. The bias has the same sign as in Figlio *et al*.; however, the bias is economically insignificant because of the low rate of exit.

[19] These findings contradict the predictions of the classroom model of Todd and Wolpin (2018), which predict that adding strong students always increases incumbent achievement and effort while adding weak students always lowers effort and achievement, via the mechanisms of how these changes in the composition of students influence teacher effort. The findings also contradict the prediction of the model used by Tincani (2024), which predicts that adding a strong student increases incumbent effort.

increases the test scores of high-background incumbent students in less competitive schools. This finding on test scores due to an improvement in class quality is in accord with those in Todd and Wolpin (2018) and Tincani (2024), although we show that the mechanisms highlighted in those studies are absent in US high schools. We also find, however, there is no corresponding positive effect on incumbent test scores in strongly competitive schools. This latter result, along with our finding that adding strong students lowers the effort of incumbents more strongly in such schools, suggests that the negative competition effects on effort are sufficiently strong to eliminate peer learning effects when competition among students is encouraged by using ranking criteria.

To quantify the adverse effects of competition on student learning at the school/grade level net of any positive peer learning effects, we proceed in two steps. We first estimate peer effects on incumbent native-born student effort at the school/grade level, interacting the county-level refugee student variables with the number of high schools in each county. This allows us to project the peer effects to the school/grade level based on the counties with one high school per county.[20] We then obtain structural estimates of the achievement production, incorporating student homework time and parental and peer homework assistance in a specification allowing persistence of learning, effort productivity that differs by initial skill, and netting out all school characteristics. These estimates indicate the significant effects of student homework time on test scores and illuminate the dual roles of parents in directly helping with homework and encouraging homework. We then combine the imputed school/grade effort peer effect estimates with our structural estimates of the effects of homework time on test scores to estimate the counter-factual of how much test scores of strong students decline due to competition among strong peers in the absence of any peer learning effects.[21] The magnitudes of these reductions are stronger in absolute value than the typical estimates of effects of peers on the test scores of

[20] While peer learning effects may be stronger at the class rather than the grade level (e.g., Hanushek *et al*., 2009), prizes and college admission criteria are based on the ranking of all students in the same school/grade. Of course, teachers grading a class on a curve also induce competition between same-class students based on ranking, but class-level grading practices are not available in the NELS:88 data, or administrative data from schools.

[21] We find that homework time significantly positively affects test scores, net of the effects of peer and student help and school characteristics. We also find that parental homework assistance is at best ineffectual in increasing a student's test performance, net of her own homework time, even though parent homework help responds to changes in class composition. But parents' encouragement of homework is effective in increasing homework time as is increasing the amount of teacher homework assignment time.

strong incumbents in the literature, suggesting that such estimates of "peer" effects may strongly underestimate the potential for peer learning if competition effects can be mitigated.

Our study examining both peer learning and competitive behavior among students complements prior work on inputs to child human capital development, both the studies that focus on classroom peer effects and those that focus on parental roles in child-rearing and school-based incentives for effort. In using variation in the foreign-born to estimate student peer effects, our study also complements prior work on the effects of immigrants on the academic performance and attainment of the native-born (e.g., Hunt, 2016; Figlio and Ozek, 2019; Bossavie, 2020; Figlio *et al*., 2024). Our main purpose, however, is not to estimate the effects of immigrants or a particular refugee group on US native-born academic achievement, but to identify the two key mechanisms that we believe characterize peer effects in US high schools – competition effects on effort and achievement and peer learning effects, particularly among the stronger-background students. While the large number of empirical studies of classroom peer effects, including those examining the effects of foreign-born students on native-born test scores, are successful in establishing causation, for reasons discussed in Kremer and Levy (2008) most do not identify the specific mechanisms by which peers affect student achievement.

The most sophisticated and robust econometric strategy among the studies examining immigrant effects on native-born academic achievement (Figlio *et al*., 2024) exploits within-family, within-school, and over time variation in the exposure of native-born students to immigrants in grades 3 through 10 in Florida public schools. Convincing evidence is obtained of the negative bias in peer effects from school flight, especially among those native-born students from families with more resources, consistent with prior evidence on school shifting. However, none of the corrected estimates indicate any negative effects of immigrant exposure on this group's test scores, which are interpreted solely as peer learning effects. Given that interpretation, there is no explanation for why the stronger native-born students would avoid immigrant exposure by exiting. Our estimates of negative peer effects on the effort of incumbent strong students, and hence on their test scores, from exposure to strong refugee students suggest that the school flight effects identified in the study are likely to reflect the avoidance of the adverse effects of competition, which could not be identified using the available records on test scores alone.[22] We use simulations that change class composition similar to those in Figlio *et al*. and obtain comparable negative effects in absolute value on test scores due to competition when we net out any positive learning effects.

Our results also have implications for the impact on students who change schools and for the effects and remedies for grade inflation.[23] With respect to school transfers, for example, Abdulkadiroglu *et al*. (2014) find that shifting a student from a school with lower-ability students to one with high-ability students did not increase the test scores of the migrant student. Hoekstra *et al*. (2018) also find no evidence of peer effects from a person attending a school with more able peers. Our findings suggest that these null results, found also in Figlio *et al*. for the effects of immigrant exposure on higher-ability students, do not imply that there are no positive learning effects from stronger peers in such schools because, as our results show, when there is competition, increasing the strength of peers, in this case by moving to a school with better peers, also reduces own effort. We discuss the implications of class competition and peer learning for assessing the effects of grade inflation and their remedies in the conclusion.

II. Theory

a. *General setup*. There are N students in a class, $i = 1, \dots, N$, who seek to obtain educational success by exerting effort, $e_i \geq 0$. A positive effort produces academic achievement, $a_i = f_i(e_i)$, which is twice differentiable, increasing, and concave with $f_i(0) = 0$.[24] We initially assume that class peers do not directly contribute to a student's achievement (learning), an assumption we relax below. Obtaining educational success is competitive, so a student's probability of obtaining success is increasing in her own achievement and decreasing in others'. For tractability, we follow Tullock (1980)'s model of imperfectly-discriminating competition and assume that the success probability takes a ratio form:

$$P_i(e_1, \dots, e_N) = \frac{f_i(e_i)}{f_1(e_1) + \cdots + f_N(e_N)} \quad \text{if } e_i > 0. \tag{1}$$

[22] An alternative hypothesis is that well-off, better-educated parents dislike immigrants. However, polls and surveys have consistently shown that more-educated US citizens have more favorable views on immigration compared with the less educated (Pryce, 2018).

[23] Cullen *et al*. (2013) find that the introduction of the use of class rank as admission criterion at the University of Texas led to stronger students shifting to schools with under-performing students. They examined the impact of school shifting on the composition of the admitted college class but did not assess the impact on the learning of students who changed schools or on the incumbent students in those schools.

[24] Although achievement is assumed to be deterministic for simplicity, our model is compatible with achievements that are random functions of effort. For example, if achievement follows a gamma distribution with shape parameter $f_i(e_i)$ and scale parameter $\theta = 1$, then the resulting expected utility is identical to ours.

We assume that $P_i(e_1, \ldots, e_N) = 0$ if $e_i = 0$ so that a student cannot succeed without exerting some effort. Letting $1_{(\text{success})}$ be the indicator variable taking value 1 if the student succeeds, a student's (expected) utility is

$$\begin{aligned} u_i(e_1, \ldots, e_N) &= \mathrm{E}\left[v_i \times 1_{(\text{success})} - c_i(e_i)\right] \\ &= v_i P_i(e_1, \ldots, e_N) - c_i(e_i) = \frac{v_i f_i(e_i)}{f_1(e_1) + \cdots + f_N(e_N)} - c_i(e_i), \end{aligned} \quad (2)$$

where v_i is the student's valuation of success, and $c_i(e_i)$ is the cost of effort, which is twice-differentiable, increasing, and convex with $c_i(0) = 0$.[25] Although the ratio-form success probability we have adopted originally arose out of a winner-take-all contest, where contestants compete for a single prize, our application does not need such a strict interpretation, and we show in Appendix A.2 that our model only requires that the expected number of successful students is non-trivially limited.

The key feature of this model is that a student's expected utility depends on the effort levels of her peers because peer effort affects the success probability. The awarding of academic honors (inclusive of admission to prestigious universities or colleges) based on a student's class rank, for example, or grading on a curve would induce the behavior characterized by the model. However, unlike in models in which students care about rank *per se* (e.g., Tincani, 2017, 2024), rank does not enter into the utility function. Thus, in this model changes in a student's own rank only matter to the extent such a change affects the likelihood of success, and school competition policy thus matters.

Another key component of this framework is that students are heterogeneous – some students are academically stronger than other students, which we precisely define.

Definition. *Student i is said to be stronger than student j if $v_i \geq v_j$, $f_i' \geq f_j'$, $c_i' \leq c_j'$, and at least one of the inequalities hold strictly.*

That is, a student is stronger than another student if her value of success is greater, she has a higher marginal productivity of effort, or a lower marginal cost. It is convenient analytically to divide up the class into two groups, with $N_1 \geq 2$ students in group 1, $N_2 \geq 2$ students in group 2, with members within each group having identical achievement and cost functions. We assume without loss of generality that students in group 1 are stronger than those in group 2 and refer to group 1 and its members as strong and group 2 and its members as weak. Indeed, as we will

[25] This can be weakened. The existence of the equilibriums we employ only requires that the achievement function be more concave than the cost function.

show, and is consistent with Figure 1, students appear to empirically separate into two groups with distinct effort behaviors and family educational background. Because the members within a group are identical, we assume that they exert the same level of effort in an equilibrium. This means we can restrict attention to the representative members of the two groups, and henceforth we assume that students are indexed so that student 1 is the representative member of group 1 and student 2 is the representative of group 2. Thus, the equilibrium can be expressed more succinctly as (e_1^*, e_2^*). We also use i and j, with the understanding that unless otherwise stated $i \neq j$ whenever they appear together, to index the groups and their representative members.

There are different ways an equilibrium can be defined based on assumptions about student beliefs. A benchmark is the classic Nash equilibrium in which students will take as given the levels of effort of their peers. We will modify that model to assume that students believe their effort level will be matched by their close peers. Such a belief, for example, may be the result of learning dynamics in which individuals imitate the actions of their most successful group members (Bernhardt and Bergin, 2003). In both models strong students exert greater effort, have higher achievement and a greater chance of success, and enjoy higher utility than weak students in equilibrium, provided that the value of educational success is sufficiently greater than it is for weak students.[26] However, the comparative statics predictions of the two models are different, and the Nash model predictions are provided in Appendix A.1 for reference.

b. *The effort-matching competitive equilibrium*. The central assumption in our equilibrium model, which we call effort-matching competitive (EMC) equilibrium, is that students anticipate that their effort will be matched by the members of their own group, whom they regard as close peers, but not the members of the other group. This means, given two groups, that a representative student i believes that her success probability is

$$P_i(e_i, e_j, N_i, N_j) = \frac{f_i(e_i)}{N_i f_i(e_i) + N_j f_j(e_j)}. \tag{3}$$

The first-order necessary conditions characterizing the equilibrium are:

[26] The assumption of differing cross-group valuations or values of the prize ensures that strong students work harder than weak students even if they are much better at studying than weak students, and we retain this assumption throughout the paper. The nationally representative US data set we use to test the model is consistent with this assumption. It indicates that native-born senior high school students, who we define to be in the strong group based on parents' schooling, spend 24.6% more hours per week doing homework compared with the homework time of those in the lower group. The difference is statistically significant. Alternatively, assumptions can be placed on the achievement or cost functions instead, but such conditions are more complicated to state.

$$F(e_1^*, e_2^*, N_1, N_2) = \begin{bmatrix} \tilde{P}_1(e_1^*, e_2^*, N_1, N_2) - c_1'(e_1^*) \\ \tilde{P}_2(e_1^*, e_2^*, N_1, N_2) - c_2'(e_2^*) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad (4)$$

where $\tilde{P}_i$ is the marginal benefit of effort at the equilibrium.

$$\tilde{P}_i(e_i^*, e_j^*, N_i, N_j) = v_i \frac{\partial P_i(e_i, e_j, N_i, N_j)}{\partial e_i}\Big|_{(e_i^*, e_j^*)} = \frac{v_i f_i'(e_i^*) N_j f_j(e_j^*)}{\left(N_i f_i(e_i^*) + N_j f_j(e_j^*)\right)^2}. \quad (5)$$

The comparative statics of the equilibrium efforts with respect to group sizes are obtained by implicitly differentiating the above identity.[27] We obtain the following Propositions for the EMC equilibrium, (e_1^*, e_2^*):

Proposition 1. *The EMC equilibrium is unique.*

Proposition 2. *The entry of strong students induces the strong incumbents to decrease their effort; that is,* $\frac{\partial e_1^*}{\partial N_1} < 0$.

Proposition 3. *The entry of weak students induces the strong incumbents to increase their effort; that is,* $\frac{\partial e_1^*}{\partial N_2} > 0$.

Proposition 4. *The entry of strong students induces the weak incumbents to decrease their effort; that is,* $\frac{\partial e_2^*}{\partial N_1} < 0$.

Proposition 5. *The entry of weak students induces the weak incumbents to decrease their effort; that is,* $\frac{\partial e_2^*}{\partial N_2} < 0$.

Proofs. See Theorem A.7 in Appendix A.

In all cases but one, incumbent students reduce their effort when additional students of any type enter. The exception is that strong incumbent students increase their effort when additional weak students enter the competition. This is because every student in the EMC model anticipates that some of her effort will be matched by her competitors and be wasted. As a result, she withholds some of her effort in comparison to the effort level she would exert in absence of such anticipation. The withholding rate depends on the fraction of effort that will be wasted by matching, which turns out to be roughly proportional to the fraction of her group's aggregate effort relative to the total effort. When a non-matching student enters the competition, the proportion of students matching her effort declines. So, while the base amount of effort she

[27] The comparative statics are obtained in marginal terms, whereas group sizes change by a whole number. The comparative static for a whole number change can be derived by integrating the marginal effect, which would not change the sign of the effect because the marginal effect has a constant sign.

wants to exert declines due to the discouragement effect of increased competition, so does the withholding rate (and the amount of her effort wasted), and she may end up decreasing or increasing her effort depending on which of the two effects dominate.[28] The proof of the propositions show that for a strong incumbent, the second effect dominates because the competitive threat from additional weak students is not overwhelming. In contrast, for a weak incumbent, the threat from additional strong students dominates while for both incumbent types when a matching student enters, the proportion of students matching increases, thus unambiguously lowering the incumbent's effort.

c. *Weak student minimal effort and dropping out*. Although we have assumed that there is no lower bound (other than zero) on the effort students need to exert to attend school and participate in the academic competition, we can show that the effort-matching model has an equilibrium even when such a minimum-effort requirement is imposed. One alternative scenario is an equilibrium where weak students are only exerting minimal effort, for example to merely satisfy parental pressure to attend school. In that scenario, the comparative statics predictions for the strong students are same as in the model in which all students compete.[29] Weak students, however, do not react to any entries because they are constrained by the minimum-effort requirement (they would otherwise reduce their effort).[30] Thus, allowing for the possibility of minimal effort by weak students, Propositions 2 and 3, for strong student effort, are unchanged, but we get

Proposition 6. *The entry of strong students induces the weak incumbents to weakly decrease their effort; that is,* $\frac{\partial e_2^*}{\partial N_1} \leq 0$.

Proposition 7. *The entry of weak students induces the weak incumbents to weakly decrease their effort; that is,* $\frac{\partial e_2^*}{\partial N_2} \leq 0$.

Proofs. See Theorem A.8 in Appendix A.

We can also show that our comparative statics results for strong students are robust to the possibility of some students giving up and dropping out of school when their expected utilities from remaining in school fall below a reservation utility, for example, the utility from the

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[28] Theorem A.12 in the appendix shows that under suitable conditions the comparative statics results are identical when the added strong students replace weak students or vice versa (swapping).

[29] Weak students will exert minimum effort if their achievement function is too low or cost function is too high.

[30] The comparative statics results obviously remain the same for all students if the minimum effort is not binding.

average earning of high school dropouts. Provided that strong students do not have a lower reservation utility than weak students, strong students will not drop out in an equilibrium, except possibly in the extreme case where all the weak students drop out, because strong students receive a higher utility than weak students. Thus, as long as the rank order of the two groups' strengths remain the same, the comparative statics results for the strong students remain valid even when the possibility of dropping out is considered.[31] In contrast, the equilibrium number of weak students must be determined endogenously, making the comparative statics results for the weak students more complex. For this reason, we will focus empirically on the responses of the strong students, for whom dropping out is minimal and who, as shown in Figure 1, have significantly higher probabilities of winning academic prizes and thus are more likely affected by competition for academic awards.[32]

d. *Peer learning effects and competition.* In the models we have considered so far, a student's academic achievement depends only on her own effort level. Peers influence achievement only via their effects on effort. We now modify the EMC model to allow for the existence of direct peer learning effects. Peers can aid academic achievement in two ways, by direct assistance (e.g., peer-to-peer help with homework) or via setting examples or creating a better learning environment. We incorporate peer learning effects in the model by assuming that a student's achievement also depends directly and positively on the size of the strong group. Formally, the achievement function f_i is now $f_i(e_i, N_1) = a_i(N_1)b_i(e_i)$, with $a_i(N_1) > 0$ and $a_i'(N_1) \geq 0$. For given effort levels, adding strong students to the class directly increases individual achievement, raising the returns to effort.[33] We further assume that peer learning effects are characterized by complementarity with respect to ability. That is, strong students have

[31] When some of the weak students drop out, it may raise the achievement of the remaining students because, for example, they receive more individual attention or have less classroom disruptions. The comparative statics on the strong students will remain valid under such cases as long as the weak students do not benefit so much that they become stronger than the (original) strong students.

[32] The dropout rate of weak native-born students between 10th and 12th grades is 6.27%; in contrast only 1% of the strong native-born students dropped out in the same interval. In a separate project examining peer effects on weak students we find that adding strong peers reduces the dropout rates of the weak students. This increases the retention of the weakest students thereby biasing downward the peer effects on the test scores of the weak who stay in school.

[33] A useful extension of the model would be to add peer assistance as a choice along with own effort, as in Drago and Garvey (1998). In our data, however, while there is multi-round information on own student effort, there is no information in any round on student time spent helping others, only information on whether a student received homework assistance from classmates. We regard this model extension as valuable for future theoretical work along with more extensive data collection.

a higher peer effect elasticity, which makes peer effect stronger on strong students than weak students.[34]

We obtain the following:

Proposition 8. *Suppose* (e_1^*, e_2^*) *is the equilibrium in the EMC model with direct peer effects, in which* $\frac{a_1'(N_1)}{a_1(N_1)} > \frac{a_2'(N_1)}{a_2(N_1)}$. *Then* $\frac{\partial e_1^*}{\partial N_1} < 0$, $\frac{\partial e_1^*}{\partial N_2} > 0$, $\frac{\partial e_2^*}{\partial N_1} < 0$, *and* $\frac{\partial e_2^*}{\partial N_2} < 0$.

Proof. See Theorem A.10 in Appendix A.

Given peer learning complementarity, Propositions 2-5 remain intact. The predictions for how class compositional changes affect effort are robust to the presence of peer learning effects. In particular, when a strong competitor enters – even if providing direct positive peer learning effects - strong incumbents decrease their effort.

e. *Peer effects, competition, and student achievement*. The model now allows for two opposing forces to act on achievement for strong incumbent students when strong peers are added – negative effects from reductions in effort and positive peer learning effects. Thus, it is possible to observe an achievement increase even if student effort decreases when a strong student is added to the class if there are strong enough peer learning effects. Without direct information on effort, it can be falsely concluded that effort effects from an increase in the share of strong students in the class are positive if peer learning effects are ignored. Or the effects on learning from adding strong peers may be completely nullified by the negative effort effect, which can lead to the false conclusion that peer learning effects are absent if effort effects are ignored. Without information on student effort it may not be possible to separately identify peer learning or effort effects from co-movements between test scores and changes in class composition, even when causality is established.

The following results, however, provide an indirect test of the existence of peer learning effects for strong students when both effort and test scores are observed and strong student effort declines in response to the addition of strong peers:

Proposition 9. *In the setting of Proposition 8, the signs of* $\frac{df_1(e_1^*, N_1)}{dN_1}$ *and* $\frac{df_2(e_2^*, N_1)}{dN_1}$ *are ambiguous, but if* $\frac{\partial e_1^*}{\partial N_1} < 0$ *and* $\frac{df_1(e_1^*, N_1)}{dN_1} > 0$, *then* $a_i(N_1) > 0$ and $a_i'(N_1) > 0$.

[34] This assumption is consistent with the findings in Chen and Hu (forthcoming), who find that when students are not competing in the same major, high-ability students improve the grades of other high-ability students but have no effect on lower-ability students. We also find evidence that high-ability students are more likely to receive homework help from other students in less competitive environments.

Proof. See Theorem A.10 in Appendix A.

If effort decreases by strong incumbent students when there is an increase in the number of strong students, due to competition effects, but test scores improve, that implies there is peer learning. Of course, if it is found that test scores do not improve from the addition of strong peers, it is necessary to quantify the effects of effort on test scores directly to identify peer learning effects. We show below that by estimating the structural achievement function and the reduced-form effects of class composition on strong-student effort, the reduction in learning (test scores) among strong students due purely to effort effects from adding stronger peers to the class can be identified even when peer learning effects dominate effort reduction effects in the reduced-form.

f. *Competition intensity and student effor*t. Our data indicates that a large fraction of high schools implement a policy that specifically encourages grade competition among students by placing greater weight on class rankings. Our last result explores how the variation in competition intensity across schools affects student effort and achievement. We assume that a more competitive school gives a higher reward for student success, which increases students' valuation of success. Because success depends on a student's relative performance, this is equivalent to emphasizing relative standing as a reward criterion compared with a student's own mastery of educational material.[35] To keep the derivation simple, we focus on cases in which weak students' efforts do not respond to the entry of strong students, which appears to be the empirically-relevant case. In such a setting, it is enough to investigate how strong incumbent student behavior changes as v_1 changes because v_1 is an increasing function of the reward.

We derive the following additional propositions for the equilibrium in the EMC model with direct peer effects, (e_1^*, e_2^*), where $\frac{a_1'(N_1)}{a_1(N_1)} > \frac{a_2'(N_1)}{a_2(N_1)}$ and $\frac{\partial e_2^*}{\partial N_1} = 0$.[36]

Proposition 10. *The effect additional strong students have on the effort and achievement of incumbent strong students is less in competitive schools compared to non-competitive schools,* $\frac{\partial^2 e_1^*}{\partial N_1\, \partial v_1} \leq 0$ and $\frac{\partial^2 f_1(e_1^*, N_1)}{\partial N_1\, \partial v_1} \leq 0$.

[35]We show empirically below that high schools that have a stated policy of strongly encouraging student competition emphasize more a student's rank in class relative to grade-point average in rewarding academic honors compared with schools that do not or not strongly encourage competition.

[36]The inequalities in Propositions 10 and 11 are strict if $c_1(e_1)$ is strictly convex.

Proposition 11. *The effect additional weak students have on the effort and achievement of incumbent strong students is greater in competitive schools compared to non-competitive schools,* $\frac{\partial^2 e_1^*}{\partial N_2\, \partial v_1} \geq 0$ and $\frac{\partial^2 f_1(e_1^*, N_2)}{\partial N_2\, \partial v_1} \geq 0$.

Proofs. See Theorem A.11 in Appendix A

III. Data

The theoretical framework incorporating class competition and peer learning effects indicates how we can identify the distinct roles of class peers as competitors and educators. Our empirical strategy for the identification of peer competition and peer learning effects is to match the multiple signs of the predicted effects of class composition on student effort and achievement from the model to the signs of the relationships observed in the data. The model predictions are summarized in Table 1. There are seven predictions for the effects of adding strong and weak students, including interactions with school competition policy, on the effort and academic achievement of strong incumbent students. There are also two predictions on the difference in the effects of changing the classroom mix of students between strong and weak incumbents. In addition, we can identify the presence of peer learning effects if we verify that adding strong students lowers the effort of strong incumbents, as predicted by the model, but also increases their academic achievement.

The model highlights student peers and student effort and achievement as well as school competition policy but ignores the roles of parents and teachers. We need to take into account empirically the reaction of these agents to changes in class composition as well. For example, teacher homework assignments may strongly determine student homework time, and thus changes in observed student homework time, a key measure of effort, could merely reflect teacher responses to classroom composition for reasons outside of the model. To test the model thus requires rich data on the behavior of students, teachers, parents, and school policies as well as measures of student academic achievement. This comprehensive set of information is not available in administrative school records nor is it available in the major US panel data sets that have been recently used to examine student effort. To meet these requirements, we thus assemble a combination of survey and administrative data sets. We describe each data set we use in this section.

a. *Restricted-use National Education Longitudinal Study of 1988 (NELS:88) data*. The principal data set we use is the restricted-use version of the NELS:88.[37] This data set is a nationally representative panel survey of students containing over 25,000 8^{th}-graders in 1988, a random subset of whom were additionally surveyed in 1990 and 1992. It offers a unique combination of several key features that extend beyond what is available from school administrative data, which contain no information on student effort, and most household survey data, which focuses on behavior within households. First, the NELS:88 provides information on the behavior of students, parents and teachers in addition to nationally-standardized student test scores. Included among these is a key student effort variable, available in the data for all three survey rounds – the student's weekly homework hours. A second key feature of the data is that there is information elicited directly from parents and teachers regarding their inputs to student achievement, including parental homework assistance and encouragement, and teacher effort and assignments of homework, as well as information from students on peer student assistance with homework. Altogether, these variables enable us to verify the importance of a student's own effort, as measured by homework time, on individual academic achievement, isolating its causal effects from those of the inputs of other actors and of school characteristics. A third feature is the availability of parent and student characteristics, which permits us to stratify students based on their nativity and their pre-school human capital. In particular, we exploit the availability of parent information and use parental schooling attainment to stratify students by ability, as discussed below.

Another important feature of the data set that is absent in administrative school records and household survey data is information on school educational policies. The NELS:88 provides several school policy variables that were elicited from a questionnaire administered to the school principals, including the selectivity of admission, the grouping of students by ability, and the degree to which grade competition is encouraged. Variation in the latter enables testing Propositions 10 and 11 of the model, that encouraging competition exacerbates the effects of peer competition on student effort. Using the unique student information on academic honors received, senior-year grade-point averages, and class rank, we show that the principals' characterization of school competition policy is indeed reflected in the actual criteria by which

[37] A condition of the use of these data is that reported sample sizes must be rounded so that the last digit is zero. All sample sizes in the tables in this paper conform to this restriction. The restricted-use version provides county-level location information and many extra student and parent behaviors compared with the public-use version.

academic rewards are allocated, with grade competition schools emphasizing class rank as a criterion for obtaining academic rewards compared with other schools. That is, we provide direct evidence that academic rewards in high school do not depend solely on individual performance, as is assumed in Cotton *et al*. (2026), but depend also on the performance of student peers in the same school/grade.

Another key feature of the survey data we exploit is the panel – a random subset of students is followed from 8th grade through high school. We use the panel data to identify the effects of changes in class composition on homework time and test scores. We also use the panel to structurally estimate the achievement production function to identify the direct effect of effort on test scores net of the influence of parent and peer inputs and incorporating the persistence of skills over time. Our main sample for analysis is from the NELS:88 panel data providing information on the same student in 10th and 12th grades. One key advantage of this panel of high school students is that although many students in the NELS:88 changed schools between 8th grade and 10th grade, less than 3.7% of the native-born students changed schools after 10th grade. We thus focus on this sample of school stayers between 10th and 12th grade, using student fixed effects to eliminate the influence of both unmeasured time-invariant student and school characteristics. We also check for and estimate the remaining selectivity bias from any school flight.

As in the model, we divide the population of native-born students into strong and weak groups. We do not use test scores for student stratification, as is commonly done, because test scores are endogenous equilibrium outcomes reflecting student effort, school policies that are common to all students, and peer effects. As we show empirically and as our model demonstrates, this endogeneity is a central feature of the analysis.[38] Instead, we identify students as strong if both parents have schooling beyond high school, a common cutoff used to distinguish skilled from unskilled labor. Our principal treated group consists of native-born strong students. In the panel data set there are 3,990 strong native-born students with test scores in both the 10th and 12th grades, which is 37% of all native-born students in the panel.[39] We also assess non-

[38] Lagged test scores cannot be used to sort students by ability and examine the determinants of academic effort in any period to the extent that study habits – effort – are serially correlated. Conley *et al*. (2022) find using unique time-use data on college students that high school study time and study time in college are “strongly” correlated (p.4). We show that lagged test scores are endogenous to current effort in our panel data.

[39] School switching rates are lower for the strong native-born students (3.3%).We show that although there is evidence of selective school flight among the stronger native-born students in response to changes in the ability

parametrically, using the total schooling of parents as a running variable, that the results we obtain are not sensitive to our selection of the cutoff. We show that responses change distinctively and statistically significantly to changes in class composition across parental schooling levels around our cutoff.

The richness of the NELS:88 allows us not only to identify the mechanisms by which peers affect test scores, absent in the peer effects literature based on school administrative data, but also to assess recent studies examining the human capital development of high-school-aged students that focus on parent and student behavior. For example, the key implication of the classroom model of Todd and Wolpin (2018) that posits a game between students and teachers is that student effort and teacher effort will be positively correlated. The NELS:88 provides information from the teachers of the 10th-graders in the sample on the weekly hours they spend "planning and preparing for teaching," which we think is a good measure of teacher effort. However, we find no statistically significant correlation between this variable and the homework hours of students in the 10th grade. This is not evidence that teachers do not influence student effort. As we show below, the assigned amount of homework time by teachers is a significant determinant of student homework hours.

The NELS:88 data also provides information on parenting, including assisting students with their homework assignments, which we show below both responds to changes in classroom ability composition and affects student test scores. This variable as well as teacher homework assignment information is absent in the survey data used in the Del Boca *et al*. (2026) study of the determinants of student effort. The NELS:88 also has a question on parenting style that is similar to that used by Agostinelli *et al*. (2026) and is a key focus of their study of student peer selection. In particular, they focus on the consequences for children's human capital development from what they define as an authoritarian parent style. This is defined by a negative student response to the question "Do your parents let you make your own decisions about the people you hang around with?". The NELS:88 asks of 10th-graders "Who decides friends R [responding student] spends time with," to which only 4.6% of the high-background students whose behavior we focus on respond by saying it is parents only. We are thus unable to find a

composition of classmates, as in Boustan *et al*. (2023) and Figlio *et al*. (2021, 2024), the selectivity bias we quantify in our sample, consistent in sign to prior work, is small due to the low rate of school changing in the high school years.

significant effect of authoritarian parenting style on either the homework time or test scores of these students.

b. *Office of Refugee Resettlement data*. A limitation of the NELS:88 is that, like the household survey data sets used in the previous literature but unlike school administrative data, it does not provide detailed information on the classmates of the surveyed students. Specifically, we cannot characterize the composition of surveyed student classmates by ability or nativity at the school/grade level using only the NELS:88 data. However, in addition to information on students, teachers, parents, and schools, the restricted-use version (only) of the NELS:88 identifies the locations of the schools and residences of the students at the county level.

To test the competition model using the NELS:88 survey data, we estimate the impact of variation across counties and over time in the number of Southeast Asian (SEA) student-age refugees (Cambodian, Laotian, Vietnamese), divided into strong and weak groups by parental schooling, on strong and weak native-born student performance. Our main data source for the SEA refugees is the newly-released micro data set describing all refugees who were re-settled in the United States from 1975 through 2008 from the US Office of Refugee Resettlement (ORR), described in Dreher, *et al.* (2020). The data provide information on country of origin, month and year of resettlement, the county of initial location, age, gender and schooling attainment for the refugees.

We use the locations and flow of refugees as our treatment variables because the *initial* locations of refugees are not the choices of the refugees themselves. Unlike the locations of all non-refugee immigrants at a point in time, which reflect both their own initial location choices and subsequent internal migration, refugee initial locations are assigned by the US Department of State Bureau of Population, Refugees, and Migration in co-operation with a set of non-profit resettlement agencies working with approximately 200 affiliates located across the United States. Figure 2, from the ORR refugee administrative data, maps the initial locations of the SEA refugees in the period 1987-1990, showing that refugee location assignments are spread across the country.

Our key treatment effects on effort of the native-born students are econometrically identified from the variation in the *assignments* of additional student-age refugees, by ability type, across counties over time – between 1990 and 1992 – using the second and third rounds of the NELS:88 panel data. We discuss the power of the treatment variable and tests of the potential endogeneity of non-random refugee location assignments to changes in test scores and student effort below.

We focus on SEA refugees for three reasons. First, the magnitudes of SEA refugee flows into the United States since 1975 through the periods covered by our main data set describing students are very large: from 1975, the initial year refugee resettlement began, through 1990—the year of the NELS:88 second round used in our analysis—919,144 refugees were resettled in the United States. And another 102,090 SEA refugees were added between the second and third NELS:88 panel rounds, of whom 5,201 were of high-school age. The second reason for focusing on SEA refugees is that there were virtually no immigrants from the three SEA countries prior to 1975; thus, the SEA students in the NELS:88 and in Census data are the same students that are present in the ORR administrative records.[40]

The third and key reason we focus on SEA refugees is that the NELS:88 data provide information on a nationally-representative sample of SEA students, including their test scores and English language abilities. In particular, the NELS:88 identifies students who are born in Southeast Asian countries, excluding the Philippines. Refugees from Cambodia, Laos and Vietnam virtually make up the entire population of foreign-born Southeast Asians in the United States, excluding the Philippines, in the NELS:88 survey years, 1988-92. Thus, it is possible to obtain the characteristics of SEA refugee students from the representative NELS:88 survey and from the 1990 US Census that match those recorded in the ORR administrative records.

c. *Are SEA refugee students a threat to academically-strong native-born students?* The information on the characteristics of the foreign-born SEA students allows us to assess whether the refugee students with different family backgrounds are credible threats to (or suitable

[40] During the same period 151,424 refugees from the former Soviet Union, the second largest refugee group at that time, resettled in the United States. However, the birthplace information in the NELS:88 does not identify students from this geographic origin. Thus, it is not possible to compare the Soviet Union student refugees in the NELS:88 to the native-born in terms of family background, academic achievement, or language ability. Moreover, unlike for the foreign-born from the three Southeast Asian countries, many immigrants from the former Soviet Union resided in the United States before 1975, and a significant portion of the foreign-born entering the United States from that region entered as family-reunification immigrants after 1975. Data from the United States Department of Justice, Immigration and Naturalization Service data set "Immigrants Admitted to the United States" indicate that over 40% of immigrants from the former Soviet Union immigrated as non-refugees between 1975 and 1989. The location of these immigrants is not controlled by government policy. This compares to only 9% for immigrants from the three SEA countries over the same period. Thus, less than 60% of the former Soviet Union foreign born are represented in the ORR records. And because US Census data also do not distinguish foreign-born by their visa status at entry, it is not possible to use Census data to characterize Soviet Union refugee students.

"educators" of) the incumbent native-born students.[41] Columns 1-4 of Table 2 describe the mean 8th-grade math and reading scores for 10th-grade native-born and SEA refugee students from the 1988 and 1990 NELS:88 rounds. Students are stratified by the parental schooling cutoff that we use to distinguish strong and weak native-born students. As can be seen, the stronger-background students have higher test scores for both mathematics and reading compared with weak students in both nativity groups, with only the difference between weak and strong SEA refugee students not being statistically significant for the reading test. With respect to competitiveness between strong students by nativity, the test scores indicate that strong SEA refugee students perform at levels comparable to strong native-born students.

Weak SEA refugees and weak native-born are also comparable, with SEA refugee scores slightly higher for both tests in this group. However, the difference is only weakly statistically significant for mathematics. Higher effort by the Southeastern Asians is evidently partly responsible for these results. Columns 5 and 6 of Table 2 display the average weekly homework hours for the four student groups. These indicate that the homework hours per week are 70% higher for the weak SEA refugee students compared with those of the weak native-born students. Strong SEA refugee students also expend 23% more effort compared with their native-born counterparts, but this difference, unlike for the weak students, is not statistically significant. These descriptive statistics hint at the important role of student effort in academic performance, which we will test directly.

A concern is that later-arrival SEA refugees, due to lower English ability, may be less of a threat to the native-born. The NELS:88 provides four self-reported English abilities: speaking, reading, writing and understanding English in four categories: not at all, not well, well, and very well in the 1988 and 1990 rounds. The 1988 round also provides the dates of arrival in the United States for all foreign-born.

Figure 3 displays the English ability (performing 'well' or 'very well') of the high-background SEA refugees in 8th grade (1988) among those who arrived within the last five years and those who arrived earlier, by the four English ability types. As expected, English ability is higher for those who have been in the United States longer, with 92 to 100% performing well or very well among those in the United States for more than five years. Among the newer arrivals,

[41] The 1990 US Census, which provides birthplace by individual origin countries, indicates that among 15-16 year-olds (10th-graders) from Southeast Asian countries excluding those from the Philippines, 91.1% are from the three refugee countries. Most of the remaining students are from Thailand.

however, over 70% say they perform well or very well in understanding and speaking English. Writing in English performance is least good among the four skills both for new and earlier arrivals, with 62% of new arrivals and 92% of earlier arrivals reporting writing performance as well or very well.[42]

The comparisons of early and later arrivals in 8^{th} or 10^{th}-grade are confounded by cohort effects - those who arrived earlier, even within parental background groups, may have different abilities at entry compared with the earlier cohorts. The panel data in the NELS:88, however, enable the assessment of how quickly a single cohort of SEA refugees acquires English skill over time because English abilities are asked in both the 1988 and 1990 rounds. Figure 4, from the panel data, displays the change in the ability to write in English, evidently the most difficult skill, between 8^{th}- and 10^{th}-grade for 8^{th}-graders in 1988, by date of arrival.

The figures indicate rapid achievement in writing English for new arrivals – among those SEA refugees who arrived within two years of 1988 only 35% could write in English well or very well, but two years later 100% of them reported they could do so. Over the same period, the control group of earlier arrivals also slightly increased their ability to write in English, but they started the interval in 1988 with 89% able to write in English well or very well. Test score performance also increased substantially for the same recent-arrival cohort between 1988 and 1990, as seen in Figure 5.

The average composite math and reading test score for SEA refugee students rose 39%, from 46 in 1988 (after two years of U.S. residence) to 64 in 1990 - score higher than those who had arrived earlier. Interestingly, for the cohort of refugee students who arrived in the United States between 1986 and 1988, their test scores in 1990 are also higher than those of the strong native-born students although the difference is not statistically significant. The fast learning of the SEA refugees suggests that even new arrivals among this refugee group pose significant academic threats to the native-born engaged in academic competition.

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[42] One concern is that the self-reports may inflate English knowledge. The 1990 Census provides assessment of (only) English-speaking ability for the foreign-born using the same four categories for ability. For the 15–16-year-olds who correspond to the respondents in the NELS:88 in 1990, it is likely that the Census language assessments are not self-reports but are provided by a parent who fills in the long-form Census questionnaire. Appendix Table A reports from the 5% sample of the US Census in 1990 the proportions of foreign-born (refugee) SEA 10^{th}graders who speak English well or very well, stratified by strong and weak students and by arrival interval. The results appear comparable to those in the NELS:88 – 75% of the strong SEA students who arrived in the United States in the five years prior to 1990 speak English at least well, while 97% of those in the United States longer than five years speak English at least well. Performance of the more recent arrivals among weak students is weaker, with 64% speaking English well. But, among the earlier arrivals in this group, 94% are able to speak English well or very well.

d. *Key limitations of the data*. The principal shortcoming of our county-level refugee treatment variable is that the effects on the individual students in the NELS:88 will be diluted due to the fact that there are multiple high schools in each county. The United States Department of Education, National Center for Education Statistics (NCES) Common Core of Data Public Elementary and Secondary School Universe: 1989-90 and NCES Private School Universe Survey, 1989-90 provide the total number of public and private schools with students in the 10th-12th grades at the zip code level, from which we obtained school counts at the county level. Figure 6 displays the distribution of the number of such schools by county for the full United States and in the NELS:88 sample, which indicates that the NELS:88 oversampled counties with a larger number of schools.[43]

The median (mean) number of schools with high-school students per county in the NELS:88 is 6 (9.5), compared with 4 (7.5) for the whole country. Both figures are from the 1989-90 academic year, corresponding to the second round of NELS:88 data collection (10th grade). Changes in the number of high school-age SEA refugees assigned to a county may therefore have economically small average effects on the native students sampled in the NELS:88 high schools within a county. We thus first use the relationships between county-level SEA refugee students by ability group on native student, parent, teacher, and peer behavior, as well as student test scores, to test the set of nine qualitative predictions (coefficient signs) of the model, using appropriate one-tailed tests to assess statistical significance. To assess economic significance, we then estimate specifications that take into account the number of county-level schools. These, along with estimates of effort effects on test scores at the student level, enable us to quantify the effects of changes in student composition by ability at the grade/school level on student test scores due to changes in own effort net of peer learning effects.

A shortcoming of the ORR refugee records is that family members are not linked. Thus, it is not possible to stratify school-age SEA refugees by parental schooling directly. We take advantage of the coincidence of the 1990 Census with the second round of the NELS:88 in 1990 to remedy this shortcoming. In particular, we use the 1990 Census data, which enable direct

[43] To ensure confidentially the graph omits counties with less than three NELS:88 respondents, which are 25% of the total sample. These counties have half the number of high schools than the counties with more than two NELS:88 respondents and thus the graph underestimates the selectivity of the NELS:88 sample. These observations/counties are not omitted for any of samples from which the econometrics estimates are obtained. The NELS:88 includes sample weights to obtain representativeness, reflecting both sampling design and any panel attrition. Our panel-sample estimates use the sample weights.

stratification of students by parental schooling, to divide the student-age SEA refugees from the ORR records into strong and weak.

The Public-Use 1990 Census samples enable the identification of SEA refugees based on country-of-birthplace information, given that almost all SEA foreign-born in 1990 are refugees. Because most 10th-grade school-age refugees (ages 15-16) co-reside with parents, and co-residing family members are linked to parents in the Census samples, we can stratify the SEA student-age refugees in the Census using the same parental schooling criterion as that of the native-born students in the NELS:88 data based on the personal and household identifiers in the Census data. For each county represented in the Census, we apply the fraction of strong SEA students based on the Census data to the total number of SEA students assigned to that county in the ORR data.

There are two relevant limitations of the Census data for our analyses. First, the IPUMS public-use samples only provide county location information for counties with a population size of 250,000 or more. Although the Census-identified counties make up only 36% of the counties represented in the NELS:88 1990 round, they account for 61.4% of the student sample. All states are represented, so we multiply the fraction of 10th-grade SEA refugees whose parents had more than a high school education at the state level to stratify the ORR SEA school-age refugees into strong and weak when the Census county-level information is missing. This introduces error into the ORR refugee student variables, and we use an instrumental variables strategy, exploiting the long time-series of SEA refugee assignments by schooling and age, to both minimize bias from measurement error and the potential bias arising from endogenous changes in refugee locations assignments. The panel includes 2,140 strong native-born students with test scores in both 10th and 12th grades attending the same high school across both academic years, located in 190 counties that received new SEA refugee assignments between 1990 and the end of 1991-92. Table 3 provides the descriptive statistics for the panel sample, which includes all 10th-grade native-born strong students who remained in the same high-school through 12th grade in counties with at least one student-age SEA refugee added after 1990 and through the first 6 months of 1992.[44]

[44] There are also limitations of the NELS:88 data relative to the data sets used in prior studies. The survey data in Del Boca *et al*. (2026) divide up student homework time into time with and without a parent present, information absent in the NELS:88. However, as noted, we have direct information on parental homework assistance. Unlike in Agostinelli *et al*. (2026), there is no information in the NELS:88 on the friends chosen by respondent students (or

IV. Econometric Identification Strategy and the Basic Specification

To test the model predictions indicated in Table 1, we first estimate the effects of the assignment of SEA student-age refugees to counties, separated by student strength, on the log homework time of strong native-born students, on whether the students received help from student peers, and on their test scores. The specification we use is:

$$y_{i0jk} = \alpha_0 + \alpha_1 R_{0k}^{w} + \alpha_2 R_{0k}^{s} + \alpha_3 C_{0jk} + \sum_{l} \alpha_l s_{0ijk} + \varsigma_{i0jk}, \qquad (6)$$

where y_{i0jk} = the log of weekly homework hours, composite math and English test score, and an indicator variable for being helped by a peer student for a strong native-born student *i* at time 0 in school *j* located in county *k*; R_{0k}^{w}, R_{0k}^{s} = the cumulative number of weak (*w*) and strong (*s*) SEA refugees in *i*'s cohort up to time 0; C_{0jk} = an indicator variable for whether the school encourages grade competition; the s_{0jkl} = set of *L* school characteristics, and ς_{i0jk} is an iid error. The model (Propositions 2 and 3) indicates that α_1>0 and α_2<0 for strong native-born students. In addition, the model with marginal weak students (Propositions 6 and 7) indicates that from the same specification estimated for weak students, adding strong or weak students to the class should have little or no effect, α_1 and α_2<=0.

There are three problems in identifying α_1 and α_2. First, the NELS:88 data provide information only on students in 8th, 10th and 12th grades. It is not possible to know therefore where the students were located prior to 8th grade in order to match the assigned locations of the SEA refugees over the full life-cycle of the respondent students. The histories of student exposure to the refugees cannot be constructed. The current location of the assigned SEA refugees, at any time *t*, thus measures with error the cumulative lifetime exposure of a student at time *t* to SEA refugees. Second, students may have selectively chosen schools/locations in response to the influx of the SEA refugee students; for example, stronger native-born students with, say, a strong work ethic may have avoided or moved from areas with stronger SEA refugee students, creating a negative bias to α_2, as found in Figlio *et al*. (2024). Third, SEA refugees may

their parents) so we cannot assess the influence of friends apart from the influence of class peers. It is unclear whether the effects they find of friends on test scores merely proxies for class peers, since that information is not available in their data. Finally, as pointed out in Cotton *et al*. (2026), what matters for academic achievement is mastery of the material, not time spent learning it. Homework time does measure effort, our key behavioral variable, but the productivity of that time will depend on student ability. In their data they have information on assignments completed as well as time. In our estimation of the achievement production function, however, our specification, consistent with our model, permits the productivity of homework time to vary with the measured skill of the student as well as unobserved shocks to learning. Figure 1a also shows that students with more highly-educated parents, who have higher skill, spend more total time doing homework.

be assigned to areas according to the characteristics or behavior of the native-born students. The α coefficients in that case would reflect the placement rules of the refugee agencies rather than the behavioral effects of refugee variation on incumbent-student behavior.

To avoid these sources of bias we use the NELS:88 panel data on students who did not change schools between the 10th and 12th grades (between 1990 and 1992) and difference equation (6) across the two time periods:

$$\Delta y_{ijk} = \alpha_1 \Delta R_k^w + \alpha_2 \Delta R_k^s + \sum_l \alpha_l \Delta s_{jkl} + \Delta \varsigma_{ijk} , \tag{7}$$

where Δ is the time-difference operator and time zero is 1990 (10th grade) and time 1 is 1992 (12th grade). To estimate (7) we thus only need the new assignments of refugees between 10th and 12th grades. This sample choice and procedure also differences out all fixed school (and personal student and parent) characteristics, including the school's policies, that might influence refugee assignments and the student outcomes. We include as time-varying variables the change in the size of the class between 1990 and 1992 at the school and the county level.[45]

There are two remaining concerns. The first is that the sub-sample of native-born students who stay in the same high school is selective. However, only 3.8% of the 10th-grade students changed high-schools after 10th grade. Even if school exit is selective, the small fraction of school changers means that the biases cannot be large. To assess this directly, in Appendix B we report our analysis of the determinants of school changing and test for and quantify selectivity bias. We find that, as expected, school leaving is increased when more strong SEA refugees are added to the county, although the effect is only marginally significant. We also find that among the strong native-born, those with higher test scores in 10th grade are less likely to subsequently change schools, and, *ceteris paribus*, students from higher-income families are more likely to switch schools. Employing a control function approach, we find no evidence of statistically significant selectivity bias in the estimates of SEA refugee effects on homework time, but there is evidence of school-leaving selectivity bias in test scores – students who experience a positive change in the test score shock (e.g., a negative shock in the first period) are more likely to leave. However, as expected given the small number of school-leavers, the selectivity-corrected test

[45] The school level class size in 1990 and 1992 are from the NELS:88 second and third rounds; the total number of students in the county in the 10th and 12th grades in 1990 and 1992, respectively, are from NCES Common Core of Data Public Elementary and Secondary School Universe: 1989-90 and 1991-92, and NCES Private School Universe Survey, 1989-90 and 1991-92. The estimated refugee effects are not sensitive to the presence of these variables.

score estimates are almost identical to the estimates we report below that do not correct for selectivity bias.

The second concern is that the county assignments of refugees between 10^{th} and 12^{th} grades, after 1990, may have been influenced by initial-period shocks, unobserved by the econometrician, to student test scores or other student behaviors. While we think it is unlikely that the individual-student shocks might influence county refugee assignments, we additionally employ an instrumental variables approach, using instruments to predict the flows of student-age refugee assignments between 1991 and 1992. In particular, using ORR administrative records of county refugee assignments by age and year that begins in 1975, we construct three county-level measures for 1989: the projected number of 10th grade cohort SEA refugee students, the projected number of SEA refugee women aged 35-50, and the projected number of SEA refugee women with at least a high school education. The past assignments of SEA refugees are strong determinants of subsequent assignments, and thus we expect that the instruments are strong. In addition, labor market conditions may also influence the assignment of refuge locations. To capture these, we also include among the set of instruments commuting-zone dummy variables.

A potential shortcoming of this set of instruments is that if the error terms in (7) are serially correlated then the exclusion restriction may be violated. As an alternative set of instruments, we also employ shift-share versions of each instrumental variable, taking the initial county distribution of the three instruments back in 1980 for the county-shares, and the aggregate US time series up through 1989 for each variable as the shifts. Thus, for instrument I_{n89k}:

$$I_{n89k} = r_{nt}[r_{n80k} / \sum_k r_{n80k}],$$

where I_{n89k} = instrument n for county k in year 1989, r_{nt} = the projected national refugee counts for instrument n in year 1989 based on the cumulated numbers of refugees since 1975, and r_{n80k} = the county-specific projected counts for instrument n in 1980.

V. Basic Results for Student Effort

The first and second columns of Table 4 report the first-differenced (FD) and IV first-differenced (FD-IV) of (7) for our key measure of student effort – the log of weekly homework hours. We use the log of homework hours because, as seen in Appendix Figure A1, weekly homework hours are strongly skewed to the right, and only 2.6% of the strong students report that they spend no time doing homework. The first-two column estimates exclude those students.

In the remaining columns, we also report estimates using both the inverse hyperbolic sine transformation and a linear specification of homework time, including all students.

For all sets of estimates the α parameter signs conform to those indicated by the model, with homework time decreasing when strong SEA refugee students are added and increasing when there are additional numbers of weak SEA refugee students in the county. All the coefficients are statistically significant at at least the 5% level, one-tailed test. For all measures of the dependent variable, the estimates are similar across the two estimation strategies, and indeed, while the instruments are strong and pass the exclusion test, we cannot reject the hypotheses that the SEA refugee assignments are uncorrelated with the second-stage error terms at conventional levels.[46] The point estimates are small, as expected, given that the refugee variables are at the county level, not the school/grade level, and there are multiple high schools in each county. We will assess the quantitative importance of the refugee effects on effort at the school/grade level below.

VI. Do Schools that Encourage Competition Exacerbate Effort Responses to Changes in Class Composition and Blunt Direct Student Peer Help?

a. *Measuring competition encouragement*. A key assumption of the model is that the effort responses to changes in cohort composition arise from competition. An additional set of key predictions from the model (Propositions 10 and 11) are that effort responses to the additions of weak and strong students will be enhanced where competition is especially acute. We use the answers from high-school principals on whether school policy encourages grade competition as our measure of school competitiveness. The NELS:88 questionnaires elicited information from high-school principals on whether they agreed with the statement that "students were encouraged to compete for grades," to which the principals could answer in five categories ranging from "Not accurate at all" to "very accurate." 57.6% of US high-school seniors in 1992 were in schools in which the principal indicated that the statement was accurate or very accurate. We will categorize such schools as competitive schools.

We first verify whether the principals' characterizations of school competition policy correspond to meaningful differences in academic reward policies across competitive and less

[46] Appendix Table B displays the FE-IV results using the shift-share instruments for all of the behavioral variable we examine. These instruments are weaker, but all results are similar to those reported. For example, the first-stage F-statistic using the three projected refugee and the commuting-zone instruments (F(40, 188)) predicting the addition of strong refugees between 1991 and 1992 is 584.7 and the corresponding F-statistic for the first stage using the shift-share projections is 355.5.

competitive schools. To assess this, we use NELS:88 data on individual senior-year student honors, class rank, and cumulative GPA to compare reward policies across school types. Accordingly, we estimate the following model for strong (high background) native-born students in their senior year:[47]

$$\theta_{ijk} = \beta_0 + \beta_1 R_{ijk} + \beta_2 GPA_{ijk} + \beta_3 C_{jk} + \beta_4 R_{ijk} \times C_{jk} + \beta_5 GPA_{ijk} \times C_{jk} + I_k + \varepsilon_{ijk}\,, \qquad (8)$$

where θ_{ijk}=a student i in school j in county k received an academic honor, R_{ijk}=student i's class rank (as a fraction of the class total size), GPA_{ijk}=student i's cumulative grade-point average (1.0-4.0), and C_{jk}= and indicator variable for whether i's high school promotes grade competition.

The specification also includes county fixed effects I_k. We expect that $\beta_1<0$ and $\beta_2>0$, students with a higher GPA and higher relative class standing (so lower R_{ijk}) will be more likely to receive an academic honor in all schools. We also expect, that in competitive schools compared with other schools (i) there are more rewards for academic performance (students with a given rank and GPA are more likely to receive an academic honor) and (ii) receiving an academic honor is more sensitive to class rank, given a student's GPA, so that $\beta_3>0$ and $\beta_4<0$. That class rank matters for rewards is the fundamental basis for the competition model.

The first column of Table 5 reports the estimates of specification (8) but excluding the policy interaction terms. As expected, strong native-born seniors with a higher GPA and with a superior class rank (lower R_{ijk}) are more likely to receive an academic honor. The point estimates, all of which are statistically significant, indicate that students that increase their GPA by one point (out of 4) increase the probability of an academic honor by 27%, while improving class rank by 0.1 increases the probability by 2.7%. The second column of the table displays the estimates of the full specification, including the interaction terms capturing the effects of the competition school policy. All of the competition policy interaction coefficients are statistically significant. These indicate that strong students in the more competitive high schools are more likely to win an academic honor, for given GPA and class rank, and that in such schools class rank has a stronger effect on winning an honor compared with other schools – relative performance matters more in competitive schools. Indeed, the estimates indicate that in high schools that do not encourage competition, class rank does not have a statistically significant

[47] 30% of the strong native-born students received an academic honor; among the weak students, only 18.7% were awarded an honor. The results reported in Table 5 are similar if all seniors, including the weak students with small probabilities of winning an academic honor, are included in the sample, but the estimates are less precise.

effect on receiving an academic honor for the strong students - only the GPA matters. The point estimates indicate that in competition schools, for given GPA, a 0.1 improvement in rank increases the likelihood of winning an honor at the mean rank (0.16) by1.4%.

Schools have multiple policies that may affect the distribution of honors and student performance. To assess if the school competition policy is merely a proxy for other policies, we add to the specification two additional policies that many high schools employ – the assignment to separate mathematics and English classes by student performance, and selective school admissions.[48] Column three of Table 5 reports the estimates with the added policy variables. The coefficients associated with competitive school policy are robust to the inclusion of the additional school policy variables – competitive schools reward class rank more than other schools and are more likely to provide academic honors for the same performance metrics. The estimates also indicate that in high schools that either employ selective admissions or stratify classes, there are fewer academic rewards offered compared with high schools that encourage grade competition or employ none of the three policies. Moreover, in both the stratifying and selective admission schools, class rank has no statistically significant effect on receiving an academic honor among the strong students, while in the grade-competition high schools, an improvement in class rank by just 0.1 increases the probability of an academic honor by a statistically significant 3.6%.

b. *Consequences of competition encouragement*. The results in Table 5 indicate that schools encouraging competition do so in a way that is consistent with the key assumption of the model – academic prizes are more strongly dependent on a student's rank and thus on the effort of other students. It is widely believed by educators that student rivalry and competition encourage students to work harder.[49] Our nationally-representative data indicate, however, that across US high schools differentiated by grade competition policy there is no evidence that student effort is greater in the schools that encourage grade competition policy. Indeed, consistent with the competition model, as shown in Table 6, across the school policy types there is no difference in the weekly mean homework times of the weak background students, but

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[48] 64.7% of seniors were in high schools that stratified classes in mathematics and English; 8.3% were in schools with selective admissions.

[49] A recent post on the first page from a Bing search on "why do schools encourage grade competition?" summarizes the reasons for the encouragement of student rivalry; the first is that it encourages students "to put in additional effort to outperform their peers." Curran (2025) Why Schools Should Encourage Academic Rivalry - BrazenDenver.

among the strong-background students who are more likely to be competing with each other, as we show below, homework time is about ½ hour (6.5%) a week *less* in the competitive schools, a difference that is statistically significant at the .01 level. Of course, these contrasts that accord with the competition model could be caused by other differences across schools correlated with competition policy, or with high-background students who are less inclined to work hard flocking to schools that encourage competition. The stronger test exploits the panel data, which control for county, school, and student fixed characteristics, and compares the contrasts by competition policy in incumbent-student effort responses to changes in class composition with those predicted by the model.

To assess whether the enhanced competition strengthens both the negative and positive effects of peers by type on incumbent student effort in accord with Propositions 10 and 11 of the model, we estimate an extended version of (7) that incorporates interaction terms of the SEA variables with the competitive school indicator. Suppressing the included time-varying school characteristics, the extended specification we estimate to test Propositions 10 and 11 is:

$$\Delta y_{ijk} = \alpha_1 \Delta R_k^w + \alpha_2 \Delta R_k^s + \alpha_3 \Delta R_k^w \times C_{jk} + \alpha_4 \Delta R_k^s \times C_{jk} + \Delta \varsigma_{ijk}\,, \qquad (9)$$

where y_{ijk} = effort (log homework time). Propositions 10 and 11 of the model indicates that $\alpha_3>0$, $\alpha_4<0$ for effort.

The first column of Table 7 reports the first-differenced estimates of (9) for log homework time separately for competitive and non- or weakly-competitive schools. Consistent with the model, the log homework time responses to the change in the numbers of strong and weak SEA refugee students are more than double in absolute value for those students in competitive schools compared to non-competitive schools. The differences are statistically significant – thus, enhancing school competition evidently produces a stronger reduction in the academic effort of incumbent strong students when strong refugee students are added to the cohort. Also consistent with the model, adding weak students increases the effort of strong incumbent students more substantially in the more competitive schools.

Adding additional interactions between the SEA refugee variables and whether school policy groups student into math and English classes does not significantly change the results with respect to the competition policy effects, and the additional interactions are not statistically significant either jointly or individually. In Appendix Table D we report test statistics for the nulls that refugee effects by ability on the behavior of students, student peers, teachers, and parents and test scores do not differ by ability-grouping school policy. For no dependent variable is the null rejected.

A concern is that the homework time response by the students is simply driven by the change in the amount of homework assigned by teachers in response to the change in composition of students, and homework assignment responses may also differ by school competition policy. The NELS:88 records assigned homework time in minutes per day, as reported by the math or science teacher of each student in both 10th and 12th grades.[50] We can thus estimate the teacher response to the refugee additions to assess if that might be the cause of the student effort changes seen in columns one and two. The column-two first-differenced estimates from Table 7 show that student effort responses to peer composition changes are not driven by differential teacher behavior: homework assignments do not differ significantly across school types, nor are the assignment response estimates to the refugee variables statistically significant within either school type.[51]

If school policy that encourages student competition does indeed increase competition among students, we would also expect to see that peer assistance from strong students will be reduced in the more competitive schools. The estimates in column 3 of Table 7 indicate that in the less competitive schools, adding strong SEA students significantly increases the probability that a strong incumbent student receives homework help from a peer. This is consistent with the assumption of our model that there are peer learning effects, the payoffs to peer help come from strong peers, and the payoffs are highest for strong students. The model is silent, however, on how the addition of weak peers affects the likelihood that strong incumbents receive assistance. The estimates indicate that when weak peers are added strong incumbents are less likely to receive assistance. One reason for this is suggested by the NELS:88 data - there is a statistically significant relationship between the proportion of free-lunch-eligible students in a class, an indicator that proxies for students with lower backgrounds, and the probability that a school has a formal tutoring program. As it is likely that strong students are enlisted in this program as tutors,

[50] In the 1990 round, two random teachers of the respondent student were interviewed; in 1992 only one teacher was interviewed. The set of teachers included those in math, reading, science and history. To maximize comparability among the teachers we use the sample of students with reports from science and math teachers in both rounds. Thus, the sample size is reduced. Inclusion of all teachers regardless of specialty yields similar results to those in Table 7 - no statistically significant responses of teacher homework assignments to the refugee students.

[51] We show below that teacher homework assignments do influence student homework time. The results in Table 7 indicate that homework assignments cannot account for the differential effort behavior of students across schools stratified by competitiveness.

the presence of additional weak students will divert strong students away from assisting strong students.[52]

The key result in the third column of Table7 is that, in contrast to what is observed in less competitive schools, adding either type of student to the cohort in competitive schools has no statistically significant effect on the peer assistance received by strong incumbents. The difference in peer help associated with the addition of strong SEA students across school types is statistically significant. Direct peer effects are thus evidently also weakened when competition among students is encouraged, consistent with the findings in Chen and Hu (2024) based on the behavior of college students differing by the extent to which they were in direct competition with each other. That our findings match theirs, based on their method that exploits random dorm-room assignments, gives us additional confidence that the school competition variable we use reflects the degree of student competitiveness induced by school policy.

Parents are another source of homework assistance for students, and parents may also react directly to the changes in composition of students or to their observations of their child's response to such changes. In assessing the effects of changing class composition on test scores, as we do below, it is necessary to see whether and how parents as well as students respond. The NELS:88 elicited from parents whether they helped their child with their homework in four categories: not at all, not often, often, and very often. We created an indicator variable for parental homework assistance that takes on the value of one if the parent answered the question in the top two categories. The estimates of (9) for this dependent variable are reported in the fourth column of Table 7. The estimates indicate that in competition-encouraging high schools, in which the strong native-born students significantly lowered their homework time in response to added strong SEA refugees and in which class peers lowered their homework help, parents were more likely to increase their homework assistance. Parents appear to attempt to counteract the effects of competition, as their likelihood of homework assistance was unaffected by added SEA refugees of either type in the schools that did not encourage competition, although the differential response is not statistically significant across the two school policies.[53]

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[52] In 10th grade (1990), 18.2% of students were classified as free-lunch eligible and 32% of schools had a peer tutoring program. Across schools, a one standard-deviation increase in the percentage of free-lunch eligible students is associated with a 23.5% increase in the likelihood that a school had a peer tutoring program.

[53] We show below that parental homework assistance is not effective; indeed, it has adverse effects on test scores, given a student's own effort.

A remaining question is whether encouraging competition also leads to direct *adverse* inter-student effects. The NELS:88 questionnaires elicited information from 10^{th}- and 12^{th}-grade students on whether they had been robbed in school or "someone threatened to hurt" them. We created a dummy variable for whether either of these events occurred ("bullying"). Column five of Table 7 reports estimates of the effects of the two types of SEA refugees on the probability of a strong native-born student being bullied, by school competition policy. The estimates indicate that these incidents were not affected by the addition of SEA refugees of either type, and the effects are not different by school competition policy.

VII. Are There Two Groups of Incumbent Students - Strong and Weak Contenders?

In this section, we assess if the response we observe of homework time by strong native-born students to the additions of the SEA refugees are attenuated or absent among the weak native-born students – that there are indeed two groups of students distinguished by whether they consider themselves significant contenders for academic prizes, key assumption of the model. Table 8 reports FD estimates from the full native-born student panel, examining the effects of adding the two SEA refugee student types on the log weekly homework time of the weak and strong native-born students by school competition policy. Coefficients are also allowed to differ between strong and weak incumbent native-born students for both types of schools. While the two SEA refugee coefficients are both individually and jointly statistically significant for the strong native-born students in both competitive and less competitive schools, the estimates for the weak native-born students are not jointly or individually statistically significant and are statistically significantly weaker than those for the strong native-born in both types of schools, consistent with the model predictions. Also consistent with the model and the results in Table 7, the point estimates of the refugee effects on the homework times of the strong native-born students are 2.5 times larger in absolute value in the competitive schools.

A concern is that the 2 x 2 results by student type and school type in Table 8 may be sensitive to the cutoff defining strong and weak native-born students that we have chosen *a priori.* To assess this, we estimated equation (9) across the full panel sample of native-born students in the competitive schools, in which the coefficients are more precise, allowing all coefficients to vary nonparametrically by the total years of schooling of both parents of the native-born incumbents using the method of Cai *et al*. (2006). Figures 7 and 8 display the α point estimates for log homework time and their associated 95% confidence intervals by total parent schooling associated with adding weak and strong SEA refugee students to the cohort, respectively.

As can be seen, all the coefficient estimates for students with parental schooling totals below approximately 26 years are not statistically significantly different from zero for either SEA treatment variable but become positive (negative) and statistically significant for total years beyond that point from adding weak (strong) SEA students. Note that the shapes of the effort responses of native-born students to SEA refugees by total years of parental schooling correspond to the relationship displayed in Figure 1 between the probability of a strong student achieving an academic prize and total parental schooling, with responses appearing only for students whose parents' total years of schooling exceed 26, which approximates our cutoff.[54]

VIII. Class Composition, School Competition and Test Scores

As illuminated in the model, adding strong students to a class when there is competition among students has offsetting effects on student learning. Consistent with the evidence so far, the addition of strong students in competitive schools reduces the effort levels of incumbent strong students and decreases direct peer assistance. On the other hand, adding strong students may also enhance learning via exposure or role-model effects. The net effect is ambiguous on learning (Proposition 9). In this section we estimate equations (7) and (9) with student test scores as the dependent variable. Specifically, we use the composite standardized reading and mathematics score, available from the NELS:88 at the end of the 10^{th} and 12^{th} grades, as the measure of student learning.

The first and second columns of Table 9 report the first-differenced and first-differenced IV estimates of changes in the numbers of SEA refugee students by type on the test scores of strong native-born students. Both sets of estimates indicate that on net adding strong SEA refugee students statistically significantly increases the test scores of the strong native born, while adding weak SEA refugees has no effect on test scores. We again cannot reject the hypothesis that the SEA refugee flows are exogenous to shocks to test score changes. Therefore,

[54] Total parental years of schooling do not correspond exactly to the criterion we use for the cutoff. However, all students above our strong student cutoff have parents whose total schooling is at least 26 years, with 54% at 30 years or above. Only 5% of weak students by our criterion also have parents whose total schooling exceeds 25 years. Similar relationships are seen when we distinguish the native-born high-school students by household income in 1988 (when the students were unlikely to be contributing to household income), with the effects only statistically significant for families with annual incomes above $43,000, representing the top third of all households with students in high school. These results are reported in Appendix Figures A2 and A3.

as stated in Proposition 9, these results suggest, given the negative effort effects, the presence of positive strong-student peer learning effects.[55]

Finally, in the third column of Table 9 the reduced-form FD estimates of the SEA refugee student effects on test scores stratified by school competition policy are displayed. These show that the positive peer effect on the strong native-born from adding strong refugee students is only present in the schools that do not encourage grade competition. The addition of weak SEA refugee students reduces the test scores of native-born students in the low competition high schools, although the effect is smaller. The negative weak-student SEA refugee effect is consistent with the finding in Table 7 that adding weak students lowers direct peer assistance among the strong native born, despite the positive effect they have on strong-student effort (Table 4). The results in Table 9 thus appear to show that the peer effects of adding strong classmates dominate effort effects in schools that do not encourage competition, but effort effects combined with the withholding of peer assistance dominate in schools where competition is encouraged.

In summary, we have found supporting evidence for the competition model incorporating peer learning effects based on the match between the signs of the estimated coefficients from empirical specifications determining effort and academic achievement and the model's nine prediction signs. This is a strong test because the probability of matching nine signs is $(0.5)^9$. We also found that each of the theoretically-relevant coefficients was individually statistically significant at at least the 0.05 level applying the relevant one-tailed test. However, the coefficient standard errors that formed the basis of the tests were obtained ignoring Type 1 errors arising from testing multiple hypotheses.

To assess the statistical significance of the individual coefficients taking account the increase in the likelihood of false rejections arising from multiple hypotheses, we re-estimated jointly the key homework time and test score equations across the multiple samples using bootstrap algorithms (Romano and Wolf, 2016) to obtain Romano and Wolf's (2005a,b) step-down adjusted p-values, which are robust to multiple hypothesis testing. We also include in the set of equations the determinants of peer homework assistance, which we used to directly confirm the effects of school competition policy on peer learning. The estimation method allows

[55] These are net of direct peer homework assistance, which we found on average across all schools to be unrelated to class compositional changes (not reported).

for the joint dependence structure of the errors across equations, and is thus superior to simple Bonferroni corrections that assume error independence**.**

The results, reported in Appendix Table F and discussed in Appendix C, indicate that all of the conclusions about statistical significance are confirmed when the family-wise error rate, the probability of making any false rejections, are taken into account. We also report randomization tests of the joint significance of our main treatment variables – the two ability-type SEA refugee assignments – for each equation and across all equations that take into the possibility of influential outliers, as discussed in Young (2019). These tests confirm that the key treatment variables are jointly significant, except for test scores in schools that strongly encourage competition policy, as is consistent with competition blunting peer learning effects from the withholding of effort.

IX. Quantification: How Important Are the Competitive Student Effort Effects on Learning?

a. *Identifying the magnitude of the effect of peers on student effort at the school/grade level.* Our primary interest is not in estimating the historical effects of immigrants or refugees on the academic performance of the native-born, but rather in testing for the effects of classroom competition on student effort and academic achievement using county-level refugee assignments to identify these effects. The reduced-form estimates for student effort and test scores reported in the previous tables are consistent with the competition model. While there is evidently enough power to reject nulls of no effects on own student effort and test scores from adding students of different strengths at the county level, the statistically significant point estimates are small because the county-level peer effects on individual student behaviors are diluted by the existence of multiple schools in each county. Moreover, while we find negative effects on incumbent effort from the addition of strong students, we do not from the reduced-form estimates find any negative effects on test scores, because evidently these are offset by positive peer learning effects. The issue we address here is the identification and quantification of the adverse effort effects of competition on test scores net of the positive peer learning effects at the school/grade level.

We first quantify the peer effects on effort at the school/grade level using the county-level information on number of schools with 10^{th} and 12^{th}-graders. In the next section we estimate the effect of effort on test scores at the student level net of peer assistance and school effects by estimating structurally the achievement function. We then combine the estimates to compute the short and long-run effects from increasing the concentration of strong students in a class on strong student test scores net of peer learning effects.

To estimate the effects of class composition at the school/grade level using the county-level measures of refugees, we add to the basic specification for homework time interactions between the county-level refugee variables and the number of schools in the county, which was obtained from the NCES data. The first-difference specification is thus:

$$\Delta y_{ijk} = \pi_1 \Delta R_k^w + \pi_2 \Delta R_k^s + \pi_3 \Delta R_k^w \times N_k + \pi_4 \Delta R_k^s \times N_k + \Delta \varsigma_{ijk}, \quad (10)$$

where y_{ijk} = log homework time and N_k = the number of private and public high schools in the county. From the model and based on the previous results we expect that $\pi_1>0$, $\pi_2<0$; but due to the dilution effects from school numbers, $\pi_3<0$, $\pi_4>0$. That is, we expect that the higher the number of high schools in the county the smaller will be the refugee effects in absolute value we find at the school level.

The estimates of (10) are reported in Table 10. The estimates indicate that the dilution effects are statistically significant and large – a one standard deviation increase in the number of high schools per county (32) reduces the positive effect on native-born homework time from adding weak SEA refugees at the county level by over one-third (34.9%). Similarly, dilution reduces in absolute value the negative student effort effect of adding stronger SEA refugees by 37.5%. To compute the refugee effects at the school/grade level, we evaluate $\mathrm{d}\Delta y_{\mathrm{ijk}}/\mathrm{d}\Delta R_{jk}^s = \pi_2 + \pi_4 N_k$ at $N_k=1$.[56] Accordingly, the estimated effects of adding SEA refugees at the school/grade level on the homework time of strong native-born students are 2.2 (1.4) times the estimated average effects across all counties using county-level refugee variation (Table 4) for the weak (strong) refugees. Both sets of estimates are statistically significant.

Equation (10) imposes linearity on the dilution effect. To see if the results in Table 10 are an artifact of that parametrization, we estimated the relationship between the SEA refugee effects on log homework time allowing all the coefficients to vary non-parametrically by the number of county high schools, again using the method of Cai *et al*. (2006). Figures 9 and 10 display the effects of the weak and strong SEA refugee effects on log homework time with the associated 95% confidence intervals, respectively, by the number of county schools. These, as expected,

[56] The median number of schools per county in the NELS:88 sample of 12th-graders is 18, the mean is 30.6. Only 2.4% of the sampled students are in counties with one high school, which is why we use the projection method based on the estimates of (10) to quantify the effect at the school/grade level.

generally display a monotonically-weakening of the effects as high school county numbers increase.

The point estimates for the effects of adding weak students decrease monotonically across the full distribution of county high school numbers while the effects of adding strong students are similar across counties with high schools numbering less than eight but become strongly less negative for all counties with high school numbers above eight. We also checked whether school competitiveness policy is related to the number of high schools in the county. As shown in Appendix Figure A4, we found no relationship between the number of county high schools and grade competition policy. This result is important because school competitiveness also affects the strength of SEA refugee effects on effort, so we need to verify that the number of high schools in the county does not proxy for this key school characteristic that mediates class compositional effects.

b. *Structural estimates of the achievement function.* We next estimate the effect of homework time (effort) on test scores directly, net of peer, parent, and home and school characteristics, by estimating the structural achievement function. We need to be sure that effort as measured by homework time actually matters for learning so that effort responses matter for test scores. The achievement function of the model characterizes one period. As discussed in Todd and Wolpin (2007), academic achievement at time t in a child's life cycle reflects the accumulation of prior investments and effort in all prior years of the child's life. To identify the effects of behaviors and inputs in a single period therefore requires controlling for the skill level at the beginning of the period, which summarizes the influence of all prior inputs (e.g., Attanasio *et al*. (2020)). We therefore estimate the achievement function using the panel data characterizing behavior during the senior year of the high school students and their test score performance at the end of that year, which enables us from the panel data to control for initial-period (10^{th}-grade) test performance.

As in Attanasio *et al*. (2020) and Del Boca *et al*. (2026), we parameterize the production function as Cobb-Douglas. In addition to quantifying the effect of the student's own effort on her academic achievement, we allow for the direct influences of the student's parents, class peers, home environment, and school characteristics:

$$\ln a_{ijt} = \ln \tau_{ij} + \gamma_1 \ln E_{ijt} + \gamma_2 \ln a_{ijt-1} + \gamma_3 h^p_{ijt} + \gamma_4 h^s_{ijt} + u_{ijt}\,, \tag{11}$$

where a_{ijt} and $a_{ijt\text{-}1}$ = the standardized mathematics and reading composite test score for student i in school j at the conclusions of times t (12th grade) and t-1 (10th grade); τ_{ij} = factor-neutral productivity parameter (TFP), which we represent by a set of dummy indicator variables representing the school j in which the student is enrolled, capturing the influence of all school characteristics of the student, and aspects of the home environment i (whether there is a home computer)[57]; E_{ijt} = the weekly homework time of the student (effort) during time t; h^p_{ijt}, h^s_{ijt} = indicators for whether the student's parents and class peers, respectively, helped the student with homework; and u_{ijt} = unobserved shocks to test score performance that occur during period t.

The key parameters are γ_1, the effect of homework time on test scores, and γ_2, which captures the persistent effects of prior inputs. There are two challenges to the identification of the γ coefficients. First, the initial-period test score $a_{ijt\text{-}1}$ may measure the mathematics and reading skill of the student at the beginning of the period with error (the student was ill that day, was affected by the weather, etc.). This would bias all the coefficients in (11) if the student and parental inputs are correlated with $a_{ijt\text{-}1}$. Second, the student, parents, and student peers might directly respond to the shocks u_{ijt}. For example, parents or peers might increase their assistance if they, but not the econometrician, observe that the student is experiencing adverse shocks to academic performance, and the student herself might exert more effort, or may be less able to exert effort. Of course, each of the student and parental inputs may also be measured with error. It is not possible to confidently construct priors on the directions of the biases.

To correct for the biases from measurement errors in and the endogeneity of the behavioral variables, we employ instrumental variables.[58] The set of instruments captures (i) the resources of the family - an indicator variable for whether the household is in the top quarter of the distribution of a socioeconomic index, mother's years of schooling, and the number of the student's siblings, which may dilute resources and parental attention; (ii) the influence of parents and teachers in affecting the student's effort – the number of minutes of homework per day assigned by the student's math or science teacher, whether the father expressed hope to the

[57] The NELS:88 provides a large set of home characteristics (e.g., whether the student has her own room, whether there is an encyclopedia, books, etc.). Using lasso, we found that among the set of 12 home characteristics only the presence of a home computer was significantly correlated with test score performance. As discussed below, we treat the presence of a computer as a choice variable possibly correlated with the shocks to academic performance.

[58] In using IV to identify the parameters of the achievement function associated with the endogenous choice variables our strategy is similar to Attanasio *et al*. (2020). The parametric structure of the model is the source of identification in Del Boca et al. (2026) for student effort.

student that she attend college, and whether the parents imposed a homework rule, and (iii) student exogenous characteristics – race, gender, nativity. Note that the set of equations captures the two principal roles of parents in affecting student achievement – directly assisting the student with homework help, in (11), and providing incentives for the student to exert effort, as captured in the first-stage specifications. Finally, we include the student's standardized composite math and reading test score in the 8th grade (a_{ijt-2}) among the instruments. This variable serves as a suitable instrument for the 10th-grade test score a_{ijt-1} if the measurement errors in each are uncorrelated and the error term in (11) is iid.

Table 11 reports the descriptive statistics for the variables used in the estimation of the achievement production function. Table 12 reports the estimates of the first-stage instrumental-variable equations. The set of first-stage excluded instruments in the table (the included second-stage determinants are the set of school-specific dummy variables) are jointly statistically significant for each endogenous variable. The estimates for student effort indicate that teacher-assigned homework times do influence the homework time of the student (even though not responsive to the refugee composition) but the relationship is not 1:1 – a doubling of assigned daily minutes only increases the hours of student homework time per week by 29.3%; father's aspirations for the student have a statistically-significant positive effect on effort, but not parental rules for homework time; and student effort is greater in higher socioeconomic-status households but not in households in which the mother has more schooling. These effects are all net of the student's prior-period measured skill, which is strongly and positively associated with subsequent student homework time. Mother's schooling is, however, positively correlated with whether a parent helped the student with homework as is whether the household is in the top quarter of the socioeconomic index distribution. Household resources thus matter for inputs to a student's achievement, as the higher-status household are also 11.4% more likely to have a computer.

The first and second columns of Table 13 report the OLS and IV estimates of the achievement function, respectively, where achievement is measured by the 12th-grade composite mathematics and reading test score. The χ^2 test statistics reject that the set of inputs are uncorrelated with the error term, even excluding the initial-period test score, which is also correlated significantly with the error term. The IV estimates also pass the over-identification and under-identification tests. The IV γ estimates indicate that a student's homework time statistically significantly affects her test score – an increase in homework hours per week by one standard deviation increases the test score by 2%, which at the national mean test score of 50 and a standard deviation of 10 represents an increase of 0.1 test score standard deviations. The last four columns of the table also indicate that a student's cumulative grade point average and class rank, important determinants of academic honors, are also statistically significantly improved by the student's own effort.

The achievement function IV estimates also indicate that a student helping with homework also adds to the test score, by 0.06 of a standard deviation, which is however not statistically significantly different from zero. In contrast, a parent helping with homework reduces the student's test score, given the student's homework time, by a statistically significant -0.17 standard deviations. This latter result suggests that parental help may be in the form of substituting parent- for own-student assignment completion, which reduces the student's learning. We found that parents are more likely to help with homework when there are added strong students in schools that encourage competition. These results together indicate that in the competitive schools, the combined effects of own student reductions in homework time, parents aiding in student homework, and peer students reducing their homework assistance all contribute to a reduction in test scores when stronger peers are added to the class. All of these behavioral responses thus contribute to negating the positive peer learning effects, seen in less competitive schools, when strong students are added to the class in schools that encourage competition. Finally, given own student effort and parent and peer interventions, having a computer in the home increases test score performance by a statistically significant 5.6%, or a third of a standard deviation.[59]

A concern is that unobserved cognitive skill other than that specific to reading and mathematics may influence the reading/math test score and this general skill may also be correlated with the instruments, including the prior-period reading/mathematics test score and parental characteristics. In prior studies estimating achievement production functions, the issue of biases emanating from measurement errors in skill are dealt with using a measurement-error

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[59] We also estimated a specification allowing TFP to be affected by the log of the mother's schooling, the log of the percentage of time the student's math or science teacher devoted to focusing on the whole class (rather than to individuals or groups), the log of the amount of time the teacher spend in preparation, and whether the parents were authoritarian. The coefficients of all four variables were not statistically significant. Mother's schooling therefore evidently affects her actions – helping with homework (Table 12) – but does not directly affect the productivity of the inputs to her child's learning.

structure, based on the assumption that there exist multiple test performance measures of one general latent ability. We believe that for characterizing the behavioral determinants of achievement in math/reading the prior-period composite math/reading test score captures the relevant entry-period skill and that the existence of a single skill measured by different assessments is an untestable assumption. However, we can as a robustness check also implement the latent factor approach for initial-period skill. To do this we use four individual test scores that are available at the end of tenth grade for each student in the second survey round – for reading, mathematics, history and science. That is, following the latent factor approach, we assume a factor structure in which each of the four 10th-grade test scores measures latent skill with iid measurement errors; that is,

$$m_{jt} = \mu_{jt} + \lambda_{jt} \ln \tau_t + \varepsilon_{jt}, \qquad (12)$$

where m_{jt}=test score j at time t, μ_{jt}=the intercept, λ_{jt}=the factor loading for test measure j at time t, τ_t=latent skill at time t, and ε_{jt}=measurement error. Identification requires that one factor be normalized, and we choose the reading score to set λ_{jt}=1, and that cov(ln τ_t, ε_{jt})=0 and cov(ε_{jt}, ε_{it})=0, for all $j \neq i$.

Appendix Table G reports the estimates of (12), including the proportion s_j of the variance in each test score j that is signal, i.e.,

$$s_j = \lambda_j^2 \operatorname{var}(\ln \tau) / (\lambda_j^2 \operatorname{var}(\ln \tau) + \operatorname{var}(\varepsilon_j)) .$$

Each of the four test scores appears to be equally reliable as a measure of latent skill, with noise being approximately 26% of the total variance of each test score. Appendix Table H reports the achievement function estimates for the reading/mathematics test score, GPA and student rank, which use the latent skill measure in 10th grade as a substitute for the 10th-grade composite reading and mathematics test score and excludes the lag reading/mathematics test score from among the first-stage instruments. The results for the effort, peer, and parent effects are similar to those reported in Table 13, with own student effort and peer student help positively affecting the test score and parental homework help reducing achievement. And we again reject that student, parent and peer behavior is orthogonal to shocks to academic performance, net of initial skill.

c. *Quantifying the economic significance of the effects on test scores from changes in class composition due to competitive effort effects*. In this section we quantify the effects of changing the ability composition of a class on the test scores of high-ability incumbent native-born test scores that occur solely due to changes in own student effort; that is, net of any positive peer learning effects. To do this we combine (i) the structural within-school estimates of the achievement function, which identify the direct effect of homework time on test scores and the effect of initial test scores on end-of-period test scores, from Table 13, with (ii) the school/grade-level estimates of the effects of the SEA refugees by ability on the homework time of strong native students from Table 10, to obtain:

$\phi^w = \frac{d\,ln\,a_{ijt}}{dR^w_{jk}} = \frac{d\,ln\,a_{ijt}}{d\,ln\,E_{ijt}} \times \frac{d\,ln\,E_{ijt}}{dR^w_{jk}}|_{N_k=1} = \frac{d\,ln\,a_{ijt}}{dy_{ijk}} \times \frac{dy_{ijk}}{dR^w_{jk}}|_{N_k=1} = \gamma_1(\pi_1 + \pi_3) = 0.000210$ and

$\phi^s = \frac{d\,ln\,a_{ijt}}{dR^s_{jk}} = \frac{d\,ln\,a_{ijt}}{d\,ln\,E_{ijt}} \times \frac{d\,ln\,E_{ijt}}{dR^s_{jk}}|_{N_k=1} = \frac{d\,ln\,a_{ijt}}{dy_{ijk}} \times \frac{dy_{ijk}}{dR^s_{jk}}|_{N_k=1} = \gamma_1(\pi_2 + \pi_4) = -0.000544$.

To benchmark our estimates, we will use the simulations from Figlio *et al*. (2024), which provides credible estimates of class compositional change on student test scores for a US population of native-born students using changes in native-born exposure to immigrant classmates. In that study, the effect on the average test scores in mathematics and reading from their estimates of immigrant exposure effects is quantified by a simulation that increases average native-born student immigrant exposure by 12 percentage points. To carry out a simulation in which the proportion of strong students in the school/grade increases by 12 percentage points holding class size constant means that we are swapping weak for strong students.

In the NELS:88 data, the average class size of high-school seniors is 262, with 29% of the students strong by our definition.[60] The effect on the log homework time of the strong students, increasing that proportion by 12 percentage points, is therefore $(\phi^s - \phi^w)\times 31$ students. The result is that test scores of the strong students due to reductions in effort and net of any positive learning peer effects decline by 2.3%, which is 0.14 test score standard deviations for the strong native-born students. Figlio *et al*. (2024) find that their simulation based on their reduced-form estimates increases test scores on average by only 0.017 to 0.028 standard deviations. However, our findings suggest that the quantitative effects in Figlio *et al*. confound positive peer learning effects and negative competition effects on effort.

Moreover, the Figlio *et al*. simulation is based on estimates for the full population of students. When the student population is split by race and a proxy for income, they find that the net effect of immigrant exposure on the test scores of the strong group (White, not eligible for free lunches) is not significantly different from zero. Our estimates imply that positive effects

[60] This percentage was computed from the third-round sample (seniors) using sample weights, which are meant to render the sample statistics nationally representative.

from peer learning on the stronger group of students could have been cancelled out by the reductions in effort we find. If so, positive peer learning effects for the strong incumbent students from increasing the proportion of immigrants, in the absence of competition effects, could be quite large, if not offset due to competition.

ϕ^w and ϕ^s represent the one-period effects of adding one weak and one strong student, respectively, on incumbent strong-student effort. Because of the persistence of skills and dynamic complementarities, as indicated by the fact that initial log test scores affect current log test scores net of (log) inputs in the subsequent period (γ_2 = 0.93 in Table 13), the effects of class compositional changes on test scores from the effort response accumulate over time.[61] The cumulative effect on the test score in the G^{th} year for an incumbent student who has first experienced the added ability group l in year 1 with no further changes in class composition is:

$$\frac{d\, ln\, a_{ijkG}}{dR^l_{jk1}} = \sum_{g=0}^{G-1} \left(\gamma_2^g \gamma_1 \frac{d\, ln\, E_{ijg}}{dR^l_{jk1}} \right) = \left(\sum_{g=0}^{G-1} \gamma_2^g \right) \times \phi^l.$$

For example, if the 12 percentage point increase in the proportion of strong students occurs in 9th grade and is permanent through the rest of high school, our achievement function estimates indicate that the test scores of high-ability students by the end of their senior year would be lower due to yearly effort reductions by 8.3%, or by 0.52 test score standard deviations. Therefore, single-year reduced-form estimates of changes in class composition on test scores may significantly underestimate positive peer learning effects due to student competition and dynamic persistence of skill effects, especially when high-schools utilize rank-based academic reward policies and universities employ rank-based admission and financial aid policies that induce students to more strongly compete.

X. Conclusion and Implications for School Grade Compression

In this paper, we investigated how rank-based rewards systems used in high schools, and rank-based college admissions criteria adversely affect human capital formation through two peer effects channels: peer influence on student effort and the returns to effort. When academic

[61] The main immigrant treatment variable in Figlio *et al*. (2024), the average cumulative exposure (fraction of immigrants in the native-born incumbents' class) of a native-born student in grade *G* assumes that contemporaneous and past exposure effects are equal. In a robustness check they also allow contemporaneous effects to be greater than past exposure effects. These assumptions contradict the implications of the persistence of skills and dynamic complementarities found in estimates of the human capital production function in the literature (i.e., Cunha and Heckman, 2007), including those reported in Table 12. These estimates imply that earlier treatments have stronger effects than later treatments.

payoffs hinge on relative performance, peers face conflicting incentives: as collaborators they enhance mutual learning, but as rank competitors they are discouraged from exerting own effort and benefit from withholding their assistance to others. We employed a Tullock-type tournament model incorporating direct peer learning effects, heterogeneity in student abilities, within-ability group effort matching, and differential school academic award policies to assess empirically to what extent basing rewards on ranking alters student effort and affects peer learning in US high schools.

An important feature of the model, which appears to realistically mimic the stratification observed in US high schools, is that there are two tiers of students. One group is composed of students from families with highly-educated parents who strongly compete for academic rewards. The other group, composed of students with less-educated parents, win significantly less academic awards, exert significantly less academic effort, and appear not to engage or to engage substantially less in competitive behavior compared with the first group. We therefore principally focused on the high-background, strongly-competing students as our principal treated group who our estimates suggest make up about a third of a high-school class. Our strategy for identifying the effects of competition centers on their differential effort responses and test score changes when low- or high-background students are added to the class, differentiated across schools by grade competition policy.

To test the model we used the National Education Longitudinal Survey of 1988 (NELS:88), which uniquely combines information on student backgrounds, student homework time, student test scores, peer assistance, school competition policy, and parent and teacher behavior, much of which is absent in school administrative records used in many prior studies of peer effects. We tested whether the effects on test scores, homework time, and peer assistance among high-school students, stratified by ability and school competition policy, matched the model's nine predictions, leveraging exogenous variation in class composition from county-level Southeast Asian refugee assignments by the US State Department. We also used information on the number of high schools in each US county to show that the effects of competition at the school/grade level were economically significant.

The key takeaway from our findings is that student competition induced by rankings-based rewards induces student peers to become competitors, thereby reducing learning by both lowering student effort and by blunting peer learning effects. These competition-induced effects are significantly stronger in high schools that directly encourage grade competition and are confined to those students from families with highly-educated parents. Our results thus may help explain why in the prior literature there is mixed evidence on the reduced-form effects of peers on test scores, since such effects depend on the responses of student effort, direct peer assistance, the behavior of parents and teachers, and the characteristics of the treated and treatment students, all mediated by school competition policy, information that is absent in most data sets that provide student test scores.

Our findings that classmates are both competitors, induced by rank-based rewards, and educators with positive learning externalities thus have implication for educational policy. It is not likely that the search for the "best and brightest" by schools and employers will ever be curtailed or that students will not seek ways to differentiate themselves from their peers.[62] However, there is a trend in both high schools and colleges that would appear to reduce student competition and therefore its adverse effects on effort and peer learning – the increase in grade compression (grade inflation), which reduces distinctions among students. In elite universities, grade compression has increased as admission selectivity has also increased. For example, at Yale University, which recently issued a public report based on administrative data on grades, the fraction of students receiving A grades rose from 41% in 2011 to 58% in 2022 as seen in Figure 11.[63] At the same time, admission selectivity also rose substantially at Yale, as also seen in Figure 11, such that between 2011 and 2022 the fraction of students with mathematics SAT's between 700 and 800 rose from 79% to over 93%.

Our results imply that the rise in the quality of students increased the potential returns to own effort and peer learning effects at Yale. Most educators, however, have bemoaned the trend in grade compression, pointing to the fact that students get less feedback and have less incentive to work hard and, as noted, Harvard administrators have proposed to remedy grade inflation by moving to a system that uses ranking criteria.[64] Our data indicate, however, that student effort among high-background students, but not lower-background students, is lower in US high

[62] Indeed, the use of class rank to distribute rewards is increasing. The state of Ohio in 2023 introduced the Governor's Merit Scholarship (GMS), which provides up to a $20,000 scholarship to Ohio high school students in the top 5% of their class if they attend college in Ohio. https://highered.ohio.gov/educators/financial-aid/sgs/gms/.

[63] At Princeton, the percentage of students receiving an A grade rose from 47.9% in 2002-3 to 66.7% in December. In 2004, the Princeton faculty rejected pacing a cap on A grades because of it led to a "competitive campus environment" (As peer institutions consider changes, 'no plans' to cap A grades at Princeton - The Princetonian).

[64] For example, "Easy A's signal to students they don't need to work hard to succeed…" https://www.forbes.com/sites/frederickhess/2023/09/05/grade-inflation-is-not-a-victimless-crime/.

schools that encourage competition by giving more weight to the relative performance of students. And our econometric estimates show that decreased competition among the top students results in their exerting more effort, being more likely to receive help from classroom peers, and achieving higher test scores when strong students are added to the class. While we do not know all of the reasons for grade inflation, our findings imply that grade inflation and compression may not be all bad if they reduce the adverse effects of competition, especially in schools and colleges with higher-ability students, for whom reducing own effort and peer learning have the highest costs. Indeed, of the 1,040 US high schools represented in the second round of the NELS, 60% of the non-selective high schools have a policy encouraging grade competition, while only 47% of high schools with selective admission policies do so; the 27% difference in competition policy is statistically significant. And, the association between student body selectivity or quality and grade compression, as shown in Appendix Figure A5 from data compiled in Rojstaczer and Healy (2012), is also is evident across universities.

APPENDIX

A Model details and proofs

This appendix provides the details of the theoretical models that frame our empirical work, including the proofs of the results presented in the main text. Section A.1 provides the theoretical results for the Nash equilibrium model, which serves as a comparative statics benchmark, and Section A.2 provides the results for our effort-matching competitive (EMC) equilibrium model. Section A.3 incorporates direct peer effect into the EMC model.

A.1 Nash equilibrium

A (pure-strategy) Nash equilibrium is an effort profile $(e_1^*, ..., e_N^*)$, where e_i^* maximizes $u_i(e_i, e_{-i}^*)$ for all i. It is easy to see, by appealing to Cornes and Hartley (2005) for example, that a unique, interior Nash equilibrium exists in our setting, provided that the value of success is high enough to induce all students to actively vie for success, which we assume. The equilibrium can be characterized by a set of N first order conditions:

$$\text{for all } i,\ \frac{\partial u_i(e_1, ..., e_N)}{\partial e_i}\Big|_{e_1^*,...,e_N^*} = v_i \frac{\partial P_i(e_1, ..., e_N)}{\partial e_i}\Big|_{e_1^*,...,e_N^*} - c_i'(e_i)\Big|_{e_i^*} = \frac{v_i f_i'(e_i^*)\sum_{k\neq i} f_k(e_k^*)}{\left(\sum_k f_k(e_k^*)\right)^2} - c_i'(e_i^*) = 0.$$

Since the members of each group are identical and the equilibrium is unique, the equilibrium is symmetric, where all the members of the same group exert the same effort level. This means we can restrict attention to the representative members of the two groups, and henceforth we assume that students are indexed so that student 1 is the representative member of group 1 and student 2 is the representative of group 2. We also use i and j, with the understanding that unless otherwise stated $i \neq j$ whenever they appear together, to index the groups and their representative members. Thus, the equilibrium can be expressed more succinctly as (e_1^*, e_2^*). Letting

$$\widetilde{P}_i(e_i^*, e_j^*, N_i, N_j) = v_i \frac{\partial P_i(e_1, ..., e_N)}{\partial e_i}\Big|_{(e_i^*, e_j^*)} = \frac{v_i f_i'(e_i^*)\left((N_i - 1) f_i(e_i^*) + N_j f_j(e_j^*)\right)}{\left(N_i f_i(e_i^*) + N_j f_j(e_j^*)\right)^2} \tag{1}$$

denote the marginal benefit of effort at the equilibrium, the first-order-condition characterization of the equilibrium reduces to:

$$F(e_1^*, e_2^*, N_1, N_2) = \begin{bmatrix} \widetilde{P}_1(e_1^*, e_2^*, N_1, N_2) - c_1'(e_1^*) \\ \widetilde{P}_2(e_1^*, e_2^*, N_1, N_2) - c_2'(e_2^*) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

The comparative statics of the equilibrium efforts with respect to group sizes can be obtained by implicitly differentiating the above identity. Let $\boldsymbol{e^*} = (e_1^*, e_2^*)$ and $\boldsymbol{N} = (N_1, N_2)$. Then the implicit differentiation theorem yields,

$$D_{\boldsymbol{N}}\boldsymbol{e^*} = \begin{bmatrix} \frac{\partial e_1^*}{\partial N_1} & \frac{\partial e_1^*}{\partial N_2} \\ \frac{\partial e_2^*}{\partial N_1} & \frac{\partial e_2^*}{\partial N_2} \end{bmatrix} = -\left[D_{\boldsymbol{e^*}} F(e_1^*, e_2^*, N_1, N_2)\right]^{-1}\left[D_{\boldsymbol{N}} F(e_1^*, e_2^*, N_1, N_2)\right]$$

$$= -\begin{bmatrix} \frac{\partial}{\partial e_1^*}\left(\widetilde{P}_1 - c_1'\right) & \frac{\partial}{\partial e_2^*}\left(\widetilde{P}_1 - c_1'\right) \\ \frac{\partial}{\partial e_1^*}\left(\widetilde{P}_2 - c_2'\right) & \frac{\partial}{\partial e_2^*}\left(\widetilde{P}_2 - c_2'\right) \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial}{\partial N_1}\left(\widetilde{P}_1 - c_1'\right) & \frac{\partial}{\partial N_2}\left(\widetilde{P}_1 - c_1'\right) \\ \frac{\partial}{\partial N_1}\left(\widetilde{P}_2 - c_2'\right) & \frac{\partial}{\partial N_2}\left(\widetilde{P}_2 - c_2'\right) \end{bmatrix}$$

$$= -\begin{bmatrix} \frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1'' & \frac{\partial \widetilde{P}_1}{\partial e_2^*} \\ \frac{\partial \widetilde{P}_2}{\partial e_1^*} & \frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2'' \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial \widetilde{P}_1}{\partial N_1} & \frac{\partial \widetilde{P}_1}{\partial N_2} \\ \frac{\partial \widetilde{P}_2}{\partial N_1} & \frac{\partial \widetilde{P}_2}{\partial N_2} \end{bmatrix}.$$

Letting DET $= \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)$, we have

$$D_{\boldsymbol{N}}\boldsymbol{e}^* = \frac{-1}{\text{DET}} \begin{bmatrix} \frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2'' & -\frac{\partial \widetilde{P}_1}{\partial e_2^*} \\ -\frac{\partial \widetilde{P}_2}{\partial e_1^*} & \frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1'' \end{bmatrix} \begin{bmatrix} \frac{\partial \widetilde{P}_1}{\partial N_1} & \frac{\partial \widetilde{P}_1}{\partial N_2} \\ \frac{\partial \widetilde{P}_2}{\partial N_1} & \frac{\partial \widetilde{P}_2}{\partial N_2} \end{bmatrix}$$

$$= \frac{-1}{\text{DET}} \begin{bmatrix} \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) & \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right) \\ \left(-\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) + \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) & \left(-\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) + \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right) \end{bmatrix}$$

$$= \frac{-1}{\text{DET}} \begin{bmatrix} \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) & \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right) \\ \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) & \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) \end{bmatrix}.$$

The following result gives the signs of its partial derivatives in the above expression.

Lemma A.1. *Let (e_1^*, e_2^*) be a Nash equilibrium. For all i and $j \neq i$, we have*

$$\frac{\partial \widetilde{P}_i(e_1^*, e_2^*, N_1, N_2)}{\partial e_i^*} - c_i''(e_i^*) < 0, \quad \frac{\partial \widetilde{P}_i(e_1^*, e_2^*, N_1, N_2)}{\partial e_j^*} < 0,$$
$$\frac{\partial \widetilde{P}_i(e_1^*, e_2^*, N_1, N_2)}{\partial N_i} < 0, \quad \textit{and} \quad \frac{\partial \widetilde{P}_i(e_1^*, e_2^*, N_1, N_2)}{\partial N_j} < 0.$$

In the proof below and elsewhere, we drop the arguments in the function notation to conserve space when it does not interfere with clarity.

Proof. First, $N_i \geq 2$ means we have

$$\frac{(N_i - 1)f_i + N_j f_j}{N_i f_i + N_j f_j} > \frac{1}{2}. \tag{2}$$

To see this, note that

$$\frac{(N_i - 1)f_i + N_j f_j}{N_i f_i + N_j f_j} > \frac{1}{2} \iff 2N_i f_i - 2f_i + 2N_j f_j > N_i f_i + N_j f_j \iff N_i f_i + N_j f_j > 2f_i,$$

which is always true. Inequality (2) further implies the following.

$$\frac{N_i - 1}{N_i} < 1 < \left(\frac{2(N_i - 1)f_i + N_j f_j}{N_i f_i + N_j f_j}\right) \implies \frac{N_i - 1}{(N_i - 1)f_i + N_j f_j} < \frac{2N_i}{N_i f_i + N_j f_j}. \tag{3}$$

In addition, the first order condition for utility maximization yields

$$v_i \widetilde{P}_i - c_i' = 0 \iff \frac{v_i f_i' \left((N_i - 1)f_i + N_j f_j\right)}{\left(N_i f_i + N_j f_j\right)^2} - c_i' = 0 \iff \frac{f_i'}{c_i'} = \frac{\left(N_i f_i + N_j f_j\right)^2}{v_i \left((N_i - 1)f_i + N_j f_j\right)}.$$

Next, we have

$$\frac{\partial \widetilde{P}_i}{\partial e_i^*} = v_i \left(\frac{\left(f_i'' \left((N_i - 1)f_i + N_j f_j\right) + f_i'(N_i - 1)f_i'\right)\left(N_i f_i + N_j f_j\right)^2 - f_i' \left((N_i - 1)f_i + N_j f_j\right) 2 \left(N_i f_i + N_j f_j\right) N_i f_i'}{\left(N_i f_i + N_j f_j\right)^4} \right)$$

$$= v_i \left(\frac{f_i'' \left((N_i - 1) f_i + N_j f_j \right)}{\left(N_i f_i + N_j f_j \right)^2} + \frac{(N_i - 1) f_i' f_i'}{\left(N_i f_i + N_j f_j \right)^2} - \frac{2 N_i f_i' f_i' \left((N_i - 1) f_i + N_j f_j \right)}{\left(N_i f_i + N_j f_j \right)^3} \right).$$

Thus,

$$\begin{aligned}
\frac{\partial \widetilde{P}_i}{\partial e_i^*} - c_i'' < 0 \iff & v_i \left(\frac{f_i'' \left((N_i - 1) f_i + N_j f_j \right)}{\left(N_i f_i + N_j f_j \right)^2} + \frac{(N_i - 1) f_i' f_i'}{\left(N_i f_i + N_j f_j \right)^2} - \frac{2 N_i f_i' f_i' \left((N_i - 1) f_i + N_j f_j \right)}{\left(N_i f_i + N_j f_j \right)^3} \right) - c_i'' < 0 \\
\iff & f_i'' + \frac{(N_i - 1) f_i' f_i'}{(N_i - 1) f_i + N_j f_j} - \frac{2 N_i f_i' f_i'}{(N_i f_i + N_j f_j)} < \frac{c_i''}{v_i} \left(\frac{\left(N_i f_i + N_j f_j \right)^2}{(N_i - 1) f_i + N_j f_j} \right) \\
\iff & f_i'' + \frac{(N_i - 1) f_i' f_i'}{(N_i - 1) f_i + N_j f_j} - \frac{2 N_i f_i' f_i'}{(N_i f_i + N_j f_j)} < c_i'' \left(\frac{f_i'}{c_i'} \right) \text{ by above} \\
\iff & \underbrace{\frac{(N_i - 1) f_i'}{(N_i - 1) f_i + N_j f_j} - \frac{2 N_i f_i'}{(N_i f_i + N_j f_j)}}_{(-) \text{ by inequality (3)}} < \underbrace{\frac{c_i''}{c_i'} - \frac{f_i''}{f_i'}}_{(\geq 0)}, \text{ which is always true.}
\end{aligned}$$

Since $N_i \geq 2$, the remaining derivatives are also negative by inequality (2):

$$\begin{aligned}
\frac{\partial \widetilde{P}_i}{\partial e_j^*} &= v_i \left(\frac{f_i' N_j f_j' \left(N_i f_i + N_j f_j \right)^2 - f_i' \left((N_i - 1) f_i + N_j f_j \right) 2 \left(N_i f_i + N_j f_j \right) N_j f_j'}{\left(N_i f_i + N_j f_j \right)^4} \right) \\
&= v_i \left(\frac{N_j f_i' f_j'}{\left(N_i f_i + N_j f_j \right)^2} - \frac{2 N_j f_i' f_j' \left((N_i - 1) f_i + N_j f_j \right)}{\left(N_i f_i + N_j f_j \right)^2 \left(N_i f_i + N_j f_j \right)} \right) < 0 \\
\frac{\partial \widetilde{P}_i}{\partial N_i} &= v_i \left(\frac{f_i' f_i \left(N_i f_i + N_j f_j \right)^2 - f_i' \left((N_i - 1) f_i + N_j f_j \right) 2 \left(N_i f_i + N_j f_j \right) f_i}{\left(N_i f_i + N_j f_j \right)^4} \right) \\
&= v_i \left(\frac{f_i' f_i}{\left(N_i f_i + N_j f_j \right)^2} - \frac{2 f_i' f_i \left((N_i - 1) f_i + N_j f_j \right)}{\left(N_i f_i + N_j f_j \right)^2 \left(N_i f_i + N_j f_j \right)} \right) < 0 \\
\frac{\partial \widetilde{P}_i}{\partial N_j} &= v_i \left(\frac{f_i' f_j \left(N_i f_i + N_j f_j \right)^2 - f_i' \left((N_i - 1) f_i + N_j f_j \right) 2 \left(N_i f_i + N_j f_j \right) f_j}{\left(N_i f_i + N_j f_j \right)^4} \right) \\
&= v_i \left(\frac{f_i' f_j}{\left(N_i f_i + N_j f_j \right)^2} - \frac{2 f_i' f_j \left((N_i - 1) f_i + N_j f_j \right)}{\left(N_i f_i + N_j f_j \right)^2 \left(N_i f_i + N_j f_j \right)} \right) < 0.
\end{aligned}$$

□

The following theorem provides the comparative statics result for Nash equilibrium. Its proof uses the derivatives of $\widetilde{P}_i$ obtained in the proof of Lemma A.1. As the theorem shows, in a Nash framework no student responds by increasing her effort when new students enter the competition. Everyone's equilibrium effort decreases no matter which group's size increases. Moreover, the result is robust and does not rely on any assumption on the achievement function or the cost function beyond the ones used to guarantee that a non-trivial equilibrium exists.

Theorem A.2. *Let (e_1^*, e_2^*) be a Nash equilibrium. Then $\frac{\partial e_i^*}{\partial N_j} < 0$ for all $i = 1, 2$ and $j = 1, 2$.*

Proof. Let $g_i = \frac{v_i f_i'' ((N_i - 1) f_i + N_j f_j)}{(N_i f_i + N_j f_j)^2}$. Then $g_i - c_i'' = \frac{v_i f_i'' ((N_i - 1) f_i + N_j f_j)}{(N_i f_i + N_j f_j)^2} - c_i'' \leq 0$. We also have

$$\frac{\partial \widetilde{P}_i}{\partial e_i^*} - g_i < 0 \iff \frac{v_i (N_i - 1) f_i' f_i'}{\left(N_i f_i + N_j f_j \right)^2} - \frac{2 v_i N_i f_i' f_i' \left((N_i - 1) f_i + N_j f_j \right)}{\left(N_i f_i + N_j f_j \right)^3} < 0$$

$$\iff \frac{N_i - 1}{(N_i-1)f_i + N_j f_j} < \frac{2N_i}{N_i f_i + N_j f_j}, \text{ which holds by inequality (3).}$$

The following shows that $\left(\frac{\partial \widetilde{P}_i}{\partial e_i^*} - g_i\right)\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j\right) > \left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial e_i^*}\right)$:

$$\left(\frac{\partial \widetilde{P}_i}{\partial e_i^*} - g_i\right)\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j\right) > \left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial e_i^*}\right) \iff \frac{1}{v_i v_j}\left(\frac{\partial \widetilde{P}_i}{\partial e_i^*} - g_i\right)\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j\right) > \frac{1}{v_i v_j}\left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial e_i^*}\right)$$

$$\begin{aligned}
&\iff \left(\frac{(N_i-1)f_i' f_i'}{(N_i f_i + N_j f_j)^2} - \frac{2N_i f_i' f_i'\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)^3}\right)\left(\frac{(N_j-1)f_j' f_j'}{(N_i f_i + N_j f_j)^2} - \frac{2N_j f_j' f_j'\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)^3}\right) \\
&\quad > \left(\frac{N_j f_i' f_j'}{(N_i f_i + N_j f_j)^2} - \frac{2N_j f_i' f_j'\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)^3}\right)\left(\frac{N_i f_j' f_i'}{(N_i f_i + N_j f_j)^2} - \frac{2N_i f_j' f_i'\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)^3}\right) \\
&\iff \left((N_i-1) - \frac{2N_i\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)}\right)\left((N_j-1) - \frac{2N_j\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)}\right) \\
&\quad > \left(N_j - \frac{2N_j\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)}\right)\left(N_i - \frac{2N_i\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)}\right) \\
&\iff (N_i-1)(N_j-1) - (N_j-1)\frac{2N_i\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)} - (N_i-1)\frac{2N_j\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)} \\
&\qquad + \left(\frac{2N_i\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)}\right)\left(\frac{2N_j\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)}\right) \\
&\quad > N_j N_i - N_i\frac{2N_j\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)} - N_j\frac{2N_i\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)} \\
&\qquad + \left(\frac{2N_j\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)}\right)\left(\frac{2N_i\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)}\right) \\
&\iff (N_i N_j - N_i - N_j + 1) - (N_j-1)\frac{2N_i\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)} - (N_i-1)\frac{2N_j\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)} \\
&\quad > N_j N_i - N_i\frac{2N_j\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)} - N_j\frac{2N_i\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)} \\
&\iff 1 + \frac{2N_i\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)} - N_i + \frac{2N_j\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)} - N_j > 0, \text{ which holds by inequality (2).}
\end{aligned}$$

This implies that

$$\begin{aligned}
\text{DET} &= \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right) \\
&= \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - g_1 + g_1 - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - g_2 + g_2 - c_2''\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right) \\
&= \underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - g_1\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - g_2\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)}_{(+)} \\
&\qquad + \underbrace{(g_1 - c_1'')\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - g_2\right)}_{(\geq 0)} + \underbrace{(g_2 - c_2'')\left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - g_1\right)}_{(\geq 0)} + \underbrace{(g_1 - c_1'')(g_2 - c_2'')}_{(\geq)} > 0.
\end{aligned}$$

Next, we show that $\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_i}\right) > \left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial N_i}\right)$. We have

$$\begin{aligned}
&\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_i}\right) > \left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial N_i}\right) \iff \frac{1}{v_i v_j}\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_i}\right) > \frac{1}{v_i v_j}\left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial N_i}\right) \\
\iff & \left(\frac{(N_j-1)f_j' f_j'}{(N_i f_i + N_j f_j)^2} - \frac{2N_j f_j' f_j'\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)^3}\right)\left(\frac{f_i' f_i}{(N_i f_i + N_j f_j)^2} - \frac{2f_i' f_i\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)^3}\right) \\
& > \left(\frac{N_j f_i' f_j'}{(N_i f_i + N_j f_j)^2} - \frac{2N_j f_i' f_j'\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)^3}\right)\left(\frac{f_j' f_i}{(N_i f_i + N_j f_j)^2} - \frac{2f_j' f_i\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)^3}\right) \\
\iff & \left((N_j-1) - \frac{2N_j\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)}\right)\left(1 - \frac{2\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)}\right) \\
& > \left(N_j - \frac{2N_j\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)}\right)\left(1 - \frac{2\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)}\right) \\
\iff & (N_j-1) - \frac{2N_j\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)} - N_j\frac{2\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)} + \frac{2\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)} \\
& \qquad + \left(\frac{2N_j\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)}\right)\left(\frac{2\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)}\right) \\
& > N_j - \frac{2N_j\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)} - N_j\frac{2\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)} \\
& \qquad + \left(\frac{2N_j\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)}\right)\left(\frac{2\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)}\right) \\
\iff & -1 + \frac{2\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)} > 0, \text{ which holds by inequality (2).}
\end{aligned}$$

This in turn means that

$$\begin{aligned}
\frac{\partial e_i^*}{N_i} &= \frac{-1}{\mathrm{DET}}\left(\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - c_j''\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_i}\right) - \left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial N_i}\right)\right) \\
&= \frac{-1}{\mathrm{DET}}\left(\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j + g_j - c_j''\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_i}\right) - \left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial N_i}\right)\right) \\
&= \underbrace{\frac{-1}{\mathrm{DET}}}_{(-)}\left(\underbrace{\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_i}\right) - \left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial N_i}\right)}_{(+)} + \underbrace{(g_j - c_j'')\left(\frac{\partial \widetilde{P}_i}{\partial N_i}\right)}_{(\geq 0)}\right) < 0.
\end{aligned}$$

We next show that $\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_j}\right) > \left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial N_j}\right)$. We have

$$\begin{aligned}
&\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_j}\right) > \left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial N_j}\right) \\
\iff & \left(\frac{(N_j-1)f_j' f_j'}{(N_i f_i + N_j f_j)^2} - \frac{2N_j f_j' f_j'\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)^3}\right)\left(\frac{f_i' f_j}{(N_i f_i + N_j f_j)^2} - \frac{2f_i' f_j\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)^3}\right) \\
& > \left(\frac{N_j f_i' f_j'}{(N_i f_i + N_j f_j)^2} - \frac{2N_j f_i' f_j'\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)^3}\right)\left(\frac{f_j' f_j}{(N_i f_i + N_j f_j)^2} - \frac{2f_j' f_j\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)^3}\right) \\
\iff & \left((N_j-1) - \frac{2N_j\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)}\right)\left(1 - \frac{2\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)}\right)
\end{aligned}$$

$$
\begin{aligned}
&> \left(N_j - \frac{2N_j\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)}\right)\left(1 - \frac{2\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)}\right) \\
&\iff (N_j - 1) - \frac{2N_j\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)} - N_j \frac{2\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)} + \frac{2\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)} \\
&\qquad\qquad + \left(\frac{2N_j\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)}\right)\left(\frac{2\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)}\right) \\
&> N_j - N_j \frac{2\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)} - \frac{2N_j\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)} \\
&\qquad\qquad + \left(\frac{2N_j\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)}\right)\left(\frac{2\left((N_j-1)f_j + N_i f_i\right)}{(N_i f_i + N_j f_j)}\right) \\
&\iff -1 + \frac{2\left((N_i-1)f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)} > 0, \text{ which holds by inequality (2).}
\end{aligned}
$$

Thus, we have

$$
\begin{aligned}
\frac{\partial e_i^*}{N_j} &= \frac{-1}{\text{DET}}\left(\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - c_j''\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_j}\right) - \left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial N_j}\right)\right) \\
&= \frac{-1}{\text{DET}}\left(\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j + g_j - c_j''\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_j}\right) - \left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial N_j}\right)\right) \\
&= \underbrace{\frac{-1}{\text{DET}}}_{(-)}\left(\underbrace{\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_j}\right) - \left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial N_j}\right)}_{(+)} + \underbrace{\left(g_j - c_j''\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_j}\right)}_{(\geq 0)}\right) < 0.
\end{aligned}
$$

□

The following result shows that strong students exert greater effort, have higher achievement, and enjoy greater utility than weak students, provided that they value educational success sufficiently more than weak students.

Theorem A.3. *Let (e_1^*, e_2^*) be a Nash equilibrium. Then $e_1^* > e_2^*$, $f_1(e_1^*) > f_2(e_2^*)$, and $u_1(e_1^*, e_2^*) > u_2(e_1^*, e_2^*)$ if v_1 is sufficiently larger than v_2.*

Proof. We first obtain the comparative statics results with respect to students' valuation of success by expressing the equilibrium condition as an implicit function of efforts and values:

$$
F(e_1^*, e_2^*, v_1, v_2) = \begin{bmatrix} \widetilde{P}_1(e_1^*, e_2^*, v_1, v_2) - c_1'(e_1^*) \\ \\ \widetilde{P}_2(e_1^*, e_2^*, v_1, v_2) - c_2'(e_2^*) \end{bmatrix} = \begin{bmatrix} \frac{v_1 f_1'(e_1^*)((N_1-1)f_1(e_1^*) + N_2 f_2(e_2^*))}{\left(N_1 f_1(e_1^*) + N_2 f_2(e_2^*)\right)^2} - c_1'(e_1^*) \\ \frac{v_2 f_2'(e_2^*)((N_2-1)f_2(e_2^*) + N_1 f_1(e_1^*))}{\left(N_1 f_1(e_1^*) + N_2 f_2(e_2^*)\right)^2} - c_2'(e_2^*) \end{bmatrix} = \begin{bmatrix} 0 \\ \\ 0 \end{bmatrix}.
$$

Thus, letting $\boldsymbol{e^*} = (e_1^*, e_2^*)$ and $\boldsymbol{v} = (v_1, v_2)$, we have

$$
D_{\boldsymbol{v}}\boldsymbol{e^*} = \begin{bmatrix} \frac{\partial e_1^*}{\partial v_1} & \frac{\partial e_1^*}{\partial v_2} \\ \\ \frac{\partial e_2^*}{\partial v_1} & \frac{\partial e_2^*}{\partial v_2} \end{bmatrix} = -\left[D_{\boldsymbol{e^*}}F\right]^{-1}\left[D_{\boldsymbol{v}}F\right] = \frac{-1}{\text{DET}}\begin{bmatrix} \frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2'' & -\frac{\partial \widetilde{P}_1}{\partial e_2^*} \\ \\ -\frac{\partial \widetilde{P}_2}{\partial e_1^*} & \frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1'' \end{bmatrix}\begin{bmatrix} \frac{\partial \widetilde{P}_1}{\partial v_1} & \frac{\partial \widetilde{P}_1}{\partial v_2} \\ \\ \frac{\partial \widetilde{P}_2}{\partial v_1} & \frac{\partial \widetilde{P}_2}{\partial v_2} \end{bmatrix},
$$

where $\text{DET} = \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right) > 0$, as we have shown in the proof of Theorem A.2. Using Lemma A.1 and the fact that $\frac{\partial \widetilde{P}_1}{\partial v_2} = \frac{\partial \widetilde{P}_2}{\partial v_1} = 0$, we obtain

$$
\frac{\partial e_1^*}{\partial v_1} = \underbrace{\left(\frac{-1}{\text{DET}}\right)}_{(-)}\underbrace{\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)}_{(-)}\underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial v_1}\right)}_{(+)} > 0 \quad \text{and} \quad \frac{\partial e_2^*}{\partial v_1} = \underbrace{\left(\frac{-1}{\text{DET}}\right)}_{(-)}\underbrace{\left(-\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)}_{(+)}\underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial v_1}\right)}_{(+)} < 0.
$$

Therefore, $e_1^* > e_2^*$ if v_1 is sufficiently larger than v_2, which we will assume.

Since $e_1^* > e_2^*$, we have $f_1(e_1^*) > f_1(e_2^*) \geq f_2(e_2^*)$ since $f_1(0) = f_2(0)$ and $f_1' \geq f_2'$. Because e_1^* is the (unique) best response of a strong student to e_2^*, we have $u_1(e_1^*, e_2^*) > u_1(e_2^*, e_2^*)$. Thus,

$$
\begin{aligned}
u_1(e_1^*, e_2^*) > u_1(e_2^*, e_2^*) &= \frac{v_1 f_1(e_2^*)}{N_1 f_1(e_2^*) + N_2 f_2(e_2^*)} - c_1(e_2^*) \\
&\geq \frac{v_2 f_2(e_2^*)}{N_1 f_1(e_1^*) + N_2 f_2(e_2^*)} - c_2(e_2^*) = u_2(e_1^*, e_2^*).
\end{aligned}
$$

The second inequality follows because $c_1(0) = c_2(0)$ and $c_1' \leq c_2'$ means $c_1(e_2^*) \leq c_2(e_2^*)$. □

We now consider how equilibrium efforts change when members of one group are swapped for those of the other group while keeping the population size constant. To reduce confusion, we use $\hat{e}_i$ to denote the equilibrium effort when swapping is being considered: $\hat{e}_i(N_1) = e_i^*(N_1, N - N_1)$. Then $\frac{d\hat{e}_i}{dN_1} = \frac{\partial e_i^*}{\partial N_1} + \frac{\partial e_i^*}{\partial N_2}\left(\frac{dN_2}{dN_1}\right) = \frac{\partial e_i^*}{\partial N_1} - \frac{\partial e_i^*}{\partial N_2}$. Because both strong and weak students are discouraged by an increased competition from any group in a Nash equilibrium, the effect of a swap is ambiguous without further conditions:

$$
\text{sign}\left(\frac{d\hat{e}_i}{dN_1}\right) = \text{sign}\left(\underbrace{\frac{\partial e_i^*}{\partial N_1}}_{(-)} - \underbrace{\frac{\partial e_i^*}{\partial N_2}}_{(-)}\right) = \text{ ambiguous.}
$$

That is, an increased competition from having more strong students makes students lower their effort while reduced competition from having fewer weak students encourages them to raise their effort. The direction of the combined effect depends on which of the two effects dominate. The following result shows that if the cost function of the weak students is sufficiently convex, then the strong students' effort will decrease when weak students are replaced with the same number of strong student.

Theorem A.4. *There exists $m > 0$ such that $\frac{d\hat{e}_1(N_1)}{dN_1} < 0$ if $c_2''(e_2^*) > m$.*

Proof. We first show that $\frac{\partial \widetilde{P}_i}{\partial N_1} < \frac{\partial \widetilde{P}_i}{\partial N_2} < 0$. (That is, $\left|\frac{\partial \widetilde{P}_i}{\partial N_1}\right| > \left|\frac{\partial \widetilde{P}_i}{\partial N_2}\right|$). From the proof of Lemma A.1, we have

$$
\begin{aligned}
\frac{\partial \widetilde{P}_i}{\partial N_i} &= v_i\left(\frac{f_i' f_i}{(N_i f_i + N_j f_j)^2} - \frac{2 f_i' f_i\left((N_i - 1) f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)^2 (N_i f_i + N_j f_j)}\right) < 0 \\
\frac{\partial \widetilde{P}_i}{\partial N_j} &= v_i\left(\frac{f_i' f_j}{(N_i f_i + N_j f_j)^2} - \frac{2 f_i' f_j\left((N_i - 1) f_i + N_j f_j\right)}{(N_i f_i + N_j f_j)^2 (N_i f_i + N_j f_j)}\right) < 0.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\frac{\partial \widetilde{P}_i}{\partial N_1} < \frac{\partial \widetilde{P}_i}{\partial N_2} &\iff \frac{f_i' f_1}{(N_1 f_1 + N_2 f_2)^2} - \frac{2 f_i' f_1\left((N_i - 1) f_i + N_j f_j\right)}{(N_1 f_1 + N_2 f_2)^3} < \frac{f_i' f_2}{(N_1 f_1 + N_2 f_2)^2} - \frac{2 f_i' f_2\left((N_i - 1) f_2 + N_j f_j\right)}{(N_1 f_2 + N_2 f_2)^3} \\
&\iff f_i' f_1 - \frac{2 f_i' f_1\left((N_i - 1) f_i + N_j f_j\right)}{N_1 f_1 + N_2 f_2} < f_i' f_2 - \frac{2 f_i' f_2\left((N_i - 1) f_i + N_j f_j\right)}{N_1 f_2 + N_2 f_2} \\
&\iff (f_i' f_1 - f_i' f_2)(N_1 f_1 + N_2 f_2) < 2(f_i' f_1 - f_i' f_2)\left((N_i - 1) f_i + N_j f_j\right) \\
&\iff N_1 f_1 + N_2 f_2 < 2 N_i f_i - 2 f_i + 2 N_j f_j \iff 0 < (N_i - 2) f_i + N_j f_j, \text{ which holds since } N_i \geq 2.
\end{aligned}
$$

We also have

$$
D_{\boldsymbol{N}} \boldsymbol{e}^* = \frac{-1}{\text{DET}} \begin{bmatrix} \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) & \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right) \\ \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) & \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) \end{bmatrix},
$$

where DET $= \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right) > 0$ as shown in the proof of Theorem A.2. Therefore, using the derivatives obtained in the proof of Lemma A.1, we have

$$\begin{aligned}
\frac{\partial e_1^*}{\partial N_1} < \frac{\partial e_1^*}{\partial N_2} &\iff \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) > \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right) \\
&\iff \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1} - \frac{\partial \widetilde{P}_1}{\partial N_2}\right) > \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1} - \frac{\partial \widetilde{P}_2}{\partial N_2}\right) \\
&\iff \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1} - \frac{\partial \widetilde{P}_1}{\partial N_2}\right) + c_2'' \underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial N_2} - \frac{\partial \widetilde{P}_1}{\partial N_1}\right)}_{(+)} > \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1} - \frac{\partial \widetilde{P}_2}{\partial N_2}\right)
\end{aligned}$$

If c_2'' is sufficiently large, the last inequality will be satisfied and $\frac{d\hat{e}_1}{dN_1} < 0$, as desired. □

The intuition behind the proof is that strong students pose a greater competitive threat than weak students. Thus, the change in the number of strong students have a greater effect on marginal benefits than the corresponding change in the number of weak students. This means that the direct effect of the swap on the strong students is to reduce their equilibrium effort. To understand the condition on c_2'', recall that weak students' efforts are determined by where their marginal benefit, $\widetilde{P}_2$, intersects the marginal cost, c_2'. If c_2' is very steep (c_2'' is large and positive), then the weak students' reaction to changes in N_1 and N_2 will be small.[1] This in turn means that the indirect effect on the strong students that arises as a reaction to the change in the weak students' effort will be small and not undo the primary effect that comes from the change in the strong students' marginal benefit.

A.2 Effort-matching competitive equilibrium

We begin by showing that our effort-matching competitive (EMC) model is equivalent to a model in which many student can succeed, provided that the expected number of successful students, M, is smaller than the group sizes. To see this, let $1 \leq M \leq \min\{N_1, N_2\}$, $\hat{v}_i = \frac{v_i}{M}$, and $\hat{u}_i(e_i, e_j, N_i, N_j) = \hat{v}_i \hat{P}_i(e_i, e_j, N_i, N_j) - c_i(e_i)$, where the success probability $\hat{P}_i$ is given by

$$\hat{P}_i(e_i, e_j, N_i, N_j) = \frac{M f_i(e_i)}{N_i f_i(e_i) + N_j f_j(e_j)}.$$

Then the expected number of successful students is $\sum_{i=1}^{N} \mathrm{E}\left[\mathbb{1}_{(\text{student } i \text{ succeds})}\right] = \sum_{i=1}^{N} \hat{P}_i = M$.[2] Moreover, $\hat{u}_i(e_i, e_j, N_i, N_j) = u_i(e_i, e_j, N_i, N_j)$ and $\hat{P}_i$ is a constant multiple of P_i. Thus, the models generated by $\hat{P}_i$ and P_i yield the same equilibrium and the comparative statics. We have chosen to use specification P_i in this paper for its conciseness.

As in the Nash equilibrium model, the comparative statics results on effort for the EMC model are obtained by implicitly differentiating the first order condition:

$$F(e_1^*, e_2^*, N_1, N_2) \equiv \begin{bmatrix} \widetilde{P}_1(e_1^*, e_2^*, N_1, N_2) - c_1'(e_1^*) \\ \\ \widetilde{P}_2(e_1^*, e_2^*, N_1, N_2) - c_2'(e_2^*) \end{bmatrix} = \begin{bmatrix} \frac{v_1 f_1'(e_1^*) N_2 f_2(e_2^*)}{\left(N_1 f_1(e_1^*) + N_2 f_2(e_2^*)\right)^2} - c_1'(e_1^*) \\ \frac{v_2 f_2'(e_2^*) N_1 f_1(e_1^*)}{\left(N_1 f_1(e_1^*) + N_2 f_2(e_2^*)\right)^2} - c_2'(e_2^*) \end{bmatrix} = \begin{bmatrix} 0 \\ \\ 0 \end{bmatrix},$$

[1] As an analogy, consider the demand (marginal benefit) and supply curve (marginal cost) analysis. A steeper supply curve induces a smaller change in the quantity (effort) when the demand curve shifts as a result of some parameter (group size) change.

[2] Note that $\sum_{i=1}^{N} \text{Prob}\,(i \text{ succeeds})$ need not equal one because one student succeeding does not necessarily preclude another student from succeeding.

where $\widetilde{P}_i(e_1^*, e_2^*, N_1, N_2) = \frac{v_i f_i'(e_i^*) N_j f_j(e_j^*)}{\left(N_i f_i(e_i^*) + N_j f_j(e_j^*)\right)^2}$ is the marginal benefit of effort at an EMC equilibrium. Letting $\boldsymbol{e^*} = (e_1^*, e_2^*)$ and $\boldsymbol{N} = (N_1, N_2)$, we have

$$D_{\boldsymbol{N}} \boldsymbol{e^*} = \begin{bmatrix} \frac{\partial e_1^*}{\partial N_1} & \frac{\partial e_1^*}{\partial N_2} \\ & \\ \frac{\partial e_2^*}{\partial N_1} & \frac{\partial e_2^*}{\partial N_2} \end{bmatrix} = -\left[D_{\boldsymbol{e^*}} F(e_1^*, e_2^*, N_1, N_2)\right]^{-1} \left[D_{\boldsymbol{N}} F(e_1^*, e_2^*, N_1, N_2)\right]$$
$$= \frac{-1}{\text{DET}} \begin{bmatrix} \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) & \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right) \\ & \\ \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) & \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) \end{bmatrix},$$

where $\text{DET} = \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)$. The following result gives the signs of the partial derivatives of $\widetilde{P}_i$.

Lemma A.5. *Let (e_1^*, e_2^*) be an EMC equilibrium. For all i and $j \neq i$, we have*

$$\frac{\partial \widetilde{P}_i(e_1^*, e_2^*, N_1, N_2)}{\partial e_i^*} - c_i''(e_i^*) < 0, \quad \frac{\partial \widetilde{P}_i(e_1^*, e_2^*, N_1, N_2)}{\partial N_i} < 0,$$
$$\textit{and} \quad \frac{\partial \widetilde{P}_i(e_1^*, e_2^*, N_1, N_2)}{\partial e_j^*} > 0 \iff N_i f_i(e_i^*) > N_j f_j(e_j^*) \iff \frac{\partial \widetilde{P}_i(e_1^*, e_2^*, N_1, N_2)}{\partial N_j} > 0.$$

Proof. The first order condition for utility maximization in the EMC model yields

$$\frac{v_i f_i'(e_i) N_j f_j(e_j)}{\left(N_i f_i(e_i) + N_j f_j(e_j)\right)^2} - c_i'(e_i) = 0 \iff \frac{f_i'(e_i)}{c_i'(e_i)} = \frac{\left(N_i f_i(e_i) + N_j f_j(e_j)\right)^2}{v_i N_j f_j(e_j)}. \tag{4}$$

We have

$$\frac{\partial \widetilde{P}_i}{\partial e_i^*} - c_i'' = v_i \left(\frac{f_i'' N_j f_j \left(N_i f_i + N_j f_j\right)^2 - 2 f_i' N_j f_j \left(N_i f_i + N_j f_j\right) N_i f_i'}{\left(N_i f_i + N_j f_j\right)^4}\right) - c_i''$$
$$< 0 \iff \frac{f_i'' N_j f_j}{\left(N_i f_i + N_j f_j\right)^2} - \frac{2 f_i' N_j f_j N_i f_i'}{\left(N_i f_i + N_j f_j\right)^3} < \frac{c_i''}{v_i}$$
$$\iff f_i'' < \frac{c_i''}{v_i}\left(\frac{\left(N_i f_i + N_j f_j\right)^2}{N_j f_j}\right) + \frac{2 f_i' N_i f_i'}{\left(N_i f_i + N_j f_j\right)}$$
$$\iff f_i'' < \frac{c_i'' f'}{c'} + \frac{2 f_i' N_i f_i'}{\left(N_i f_i + N_j f_j\right)} \text{ by equation (4)}$$
$$\iff \frac{f_i''}{f_i'} < \frac{c_i''}{c'} + \frac{2 N_i f_i'}{\left(N_i f_i + N_j f_j\right)}, \text{ which holds since } \frac{f_i''}{f_i'} \leq \frac{c_i''}{c'}.$$
$$\frac{\partial \widetilde{P}_i}{\partial e_j^*} = v_i \left(\frac{f_i' N_j f_j' \left(N_i f_i + N_j f_j\right)^2 - 2 f_i' N_j f_j \left(N_i f_i + N_j f_j\right) N_j f_j'}{\left(N_i f_i + N_j f_j\right)^4}\right)$$
$$= v_i \left(\frac{f_i' N_j f_j' (N_i f_i + N_j f_j) - 2 f_i' N_j f_j N_j f_j'}{\left(N_i f_i + N_j f_j\right)^3}\right) = \frac{v_i \left(N_i f_i - N_j f_j\right) f_i' N_j f_j'}{\left(N_i f_i + N_j f_j\right)^3} > 0 \iff N_i f_i > N_j f_j.$$
$$\frac{\partial \widetilde{P}_i}{\partial N_i} = v_i \left(\frac{0 - 2 f_i' N_j f_j \left(N_i f_i + N_j f_j\right) f_i}{\left(N_i f_i + N_j f_j\right)^4}\right) = -\frac{2 v_i f_i' N_j f_j f_i}{\left(N_i f_i + N_j f_j\right)^3} < 0.$$
$$\frac{\partial \widetilde{P}_i}{\partial N_j} = v_i \left(\frac{f_i' f_j \left(N_i f_i + N_j f_j\right)^2 - 2 f_i' N_j f_j \left(N_i f_i + N_j f_j\right) f_j}{\left(N_i f_i + N_j f_j\right)^4}\right) = v_i \left(\frac{f_i' f_j \left(N_i f_i + N_j f_j\right) - 2 f_i' N_j f_j f_j}{\left(N_i f_i + N_j f_j\right)^3}\right)$$

$$= \frac{v_i f_i' f_j \left(N_i f_i - N_j f_j\right)}{\left(N_i f_i + N_j f_j\right)^3} > 0 \iff N_i f_i > N_j f_j.$$

□

Our next result shows that strong students exert greater effort than weak students, attain higher achievement both individually and in the aggregate, and enjoy a higher (expected) utility in the equilibrium, provided that they value educational success sufficiently more than weak students..

Lemma A.6. *Let (e_1^*, e_2^*) be an EMC equilibrium. Then, $e_1^* > e_2^*$, $f_1(e_1^*) > f_2(e_2^*)$, $N_1 f_1(e_1^*) > N_2 f_2(e_2^*)$, and $u_1(e_1^*, e_2^*) > u_2(e_1^*, e_2^*)$ if v_1 is sufficiently larger than v_2.*

Proof. As in the Nash equilibrium model, we regard the equilibrium condition as an implicit function of efforts and values and then differentiate to obtain the comparative statics with respect to students' valuation of success.

$$F(e_1^*, e_2^*, v_1, v_2) = \begin{bmatrix} \widetilde{P}_1(e_1^*, e_2^*, v_1, v_2) - c_1'(e_1^*) \\ \\ \widetilde{P}_2(e_1^*, e_2^*, v_1, v_2) - c_2'(e_2^*) \end{bmatrix} = \begin{bmatrix} \frac{v_1 f_1'(e_1^*) N_2 f_2(e_2^*)}{\left(N_1 f_1(e_1^*) + N_2 f_2(e_2^*)\right)^2} - c_1'(e_1^*) \\ \frac{v_2 f_2'(e_2^*) N_1 f_1(e_1^*)}{\left(N_1 f_1(e_1^*) + N_2 f_2(e_2^*)\right)^2} - c_2'(e_2^*) \end{bmatrix} = \begin{bmatrix} 0 \\ \\ 0 \end{bmatrix}.$$

Letting $\boldsymbol{e^*} = (e_1^*, e_2^*)$ and $\boldsymbol{v} = (v_1, v_2)$, we have

$$D_{\boldsymbol{v}} \boldsymbol{e^*} = \begin{bmatrix} \frac{\partial e_1^*}{\partial v_1} & \frac{\partial e_1^*}{\partial v_2} \\ \\ \frac{\partial e_2^*}{\partial v_1} & \frac{\partial e_2^*}{\partial v_2} \end{bmatrix} = -\left[D_{\boldsymbol{e^*}} F\right]^{-1} \left[D_{\boldsymbol{v}} F\right] = \frac{-1}{\text{DET}} \begin{bmatrix} \frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2'' & -\frac{\partial \widetilde{P}_1}{\partial e_2^*} \\ \\ -\frac{\partial \widetilde{P}_2}{\partial e_1^*} & \frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1'' \end{bmatrix} \begin{bmatrix} \frac{\partial \widetilde{P}_1}{\partial v_1} & \frac{\partial \widetilde{P}_1}{\partial v_2} \\ \\ \frac{\partial \widetilde{P}_2}{\partial v_1} & \frac{\partial \widetilde{P}_2}{\partial v_2} \end{bmatrix},$$

where

$$\text{DET} = \underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)}_{(-)} \underbrace{\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)}_{(-)} - \underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)}_{(\leq 0)} > 0 \text{ by Lemma A.5.}$$

Using Lemma A.5 and the fact that $\frac{\partial \widetilde{P}_1}{\partial v_2} = \frac{\partial \widetilde{P}_2}{\partial v_1} = 0$, we obtain

$$\frac{\partial e_1^*}{\partial v_1} = \underbrace{\left(\frac{-1}{\text{DET}}\right)}_{(-)} \underbrace{\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)}_{(-)} \underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial v_1}\right)}_{(+)} > 0 \quad \text{and} \quad \frac{\partial e_1^*}{\partial v_2} = \underbrace{\left(\frac{-1}{\text{DET}}\right)}_{(-)} \underbrace{\left(-\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)}_{(*)} \underbrace{\left(\frac{\partial \widetilde{P}_2}{\partial v_2}\right)}_{(+)}.$$

$$\frac{\partial e_2^*}{\partial v_1} = \underbrace{\left(\frac{-1}{\text{DET}}\right)}_{(-)} \underbrace{\left(-\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)}_{(**)} \underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial v_1}\right)}_{(+)} \quad \text{and} \quad \frac{\partial e_2^*}{\partial v_2} = \underbrace{\left(\frac{-1}{\text{DET}}\right)}_{(-)} \underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)}_{(-)} \underbrace{\left(\frac{\partial \widetilde{P}_2}{\partial v_2}\right)}_{(+)} > 0.$$

If $N_1 f_1(e_1^*(v_1, v_2)) \leq N_2 f_2(e_2^*(v_1, v_2))$, then $(*) \geq 0$ by Lemma A.5, which means $\frac{\partial e_1^*}{\partial v_2} \leq 0$. Therefore, as v_2 decreases, $N_1 f_1(e_1^*(v_1, v_2))$ is non-decreasing while $N_2 f_2(e_2^*(v_1, v_2))$ is decreasing toward zero. Therefore, for sufficiently small v_2, we must eventually have $N_1 f_1(e_1^*(v_1, v_2)) > N_2 f_2(e_2^*(v_1, v_2))$. So, suppose $N_1 f_1(e_1^*(v_1, v_2)) > N_2 f_2(e_2^*(v_1, v_2))$, then $(**) > 0$ by Lemma A.5, which means $\frac{\partial e_2^*}{\partial v_1} < 0$. Therefore, as v_1 increases, e_1^* increases and e_2^* decreases. Therefore, when v_1 is sufficiently larger than v_2, we have $e_1^* > e_2^*$, $f_1(e_1^*) > f_2(e_2^*)$, and $N_1 f_1(e_1^*) > N_2 f_2(e_2^*)$.

For the utilities of the students, note that we have $f_1(e_1^*) > f_1(e_2^*) \geq f_2(e_2^*)$ since $f_1' \geq f_2'$ and $f_1(0) = f_2(0)$. We also have $c_1(e_2^*) \leq c_2(e_2^*)$ since $c_1' \leq c_2'$ and $c_1(0) = c_2(0)$. Because e_1^* is the (unique) best response of a strong student to e_2^*, we have

$$u_1(e_1^*, e_2^*) > u_1(e_2^*, e_2^*) = \frac{v_1 f_1(e_2^*)}{N_1 f_1(e_2^*) + N_2 f_2(e_2^*)} - c_1(e_2^*)$$

$$\geq \frac{v_2 f_2(e_2^*)}{N_1 f_1(e_1^*) + N_2 f_2(e_2^*)} - c_2(e_2^*) = u_2(e_1^*, e_2^*),$$

as required. □

The following theorem shows that EMC equilibrium is unique and presents our main comparative statics results. Its proof uses the derivatives of $\widetilde{P}_i$ obtained in the proof of Lemma A.5.

Theorem A.7 (Propositions 1-5 in the main text)**.** *The EMC equilibrium* (e_1^*, e_2^*) *is unique, and* $\frac{\partial e_1^*}{\partial N_1} < 0$, $\frac{\partial e_2^*}{\partial N_1} < 0$, $\frac{\partial e_1^*}{\partial N_2} > 0$, *and* $\frac{\partial e_2^*}{\partial N_2} < 0$.

Proof. To see that the equilibrium is unique, suppose there are two equilibria, (e_1^*, e_2^*) and (e_1^{**}, e_2^{**}). Since students' utility functions are strictly concave in effort, it must be that $e_1^* \neq e_1^{**}$. Otherwise, e_2^* would also equal to e_2^{**}, yielding $(e_1^*, e_2^*) = (e_1^{**}, e_2^{**})$. So, without a loss of generality, assume that $e_1^{**} > e_1^*$, which also means $e_2^* \neq e_2^{**}$. Suppressing dependence on N_1 and N_2 in the notation, we have the following by Lemma A.5 and Lemma A.6.

$$\begin{aligned}
e_2^{**} < e_2^* \implies \widetilde{P}_1(e_1^{**}, e_2^{**}) - c_1'(e_1^{**}) &< \widetilde{P}_1(e_1^{**}, e_2^*) - c_1'(e_1^{**}) \quad \text{since } \frac{\partial \widetilde{P}_1(e_1^{**}, e_2^{**})}{\partial e_2^{**}} > 0 \\
&< \widetilde{P}_1(e_1^*, e_2^*) - c_1'(e_1^*) \quad \text{since } \frac{\partial \widetilde{P}_1(e_1^*, e_2^*)}{\partial e_1^*} - c_1''(e_1^*) < 0 \\
&= 0 \quad \text{since } (e_1^*, e_2^*) \text{ is an equilibrium.} \\
e_2^{**} > e_2^* \implies \widetilde{P}_2(e_1^{**}, e_2^{**}) - c_2'(e_2^{**}) &< P_2(e_1^{**}, e_2^*) - c_2'(e_2^*) \quad \text{since } \frac{\partial \widetilde{P}_2(e_1^{**}, e_2^{**})}{\partial e_2^{**}} - c_2''(e_2^{**}) < 0 \\
&< \widetilde{P}_2(e_1^*, e_2^*) - c_2'(e_2^*) \quad \text{since } \frac{\partial \widetilde{P}_2(e_2^*, e_2^*)}{\partial e_1^*} < 0 \\
&= 0 \quad \text{since } (e_1^*, e_2^*) \text{ is an equilibrium.}
\end{aligned}$$

Therefore, (e_1^{**}, e_2^{**}) cannot be an equilibrium.

For comparative statics, first note that as shown in the proof of Lemma A.6, $\text{DET} = \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right) > 0$. Therefore,

$$\frac{\partial e_i^*}{\partial N_i} = \underbrace{\frac{-1}{\text{DET}}}_{(-)} \left(\underbrace{\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - c_j''\right)}_{(-)} \underbrace{\left(\frac{\partial \widetilde{P}_i}{\partial N_i}\right)}_{(-)} - \underbrace{\left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial N_i}\right)}_{(\leq 0)} \right) < 0.$$

Next, let $g_j = \frac{v_j f_j'' N_i f_i}{(N_i f_i + N_j f_j)^2}$. Using equation (4), we obtain

$$g_j - c_j'' = \frac{v_j f_j'' N_i f_i}{(N_i f_i + N_j f_j)^2} - c_j'' = \frac{f_j'' c_j'}{f_j'} - c_j'' < 0.$$

As a preliminary to determining the sign of $\frac{\partial e_i^*}{\partial N_j}$, we first show that $\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_j}\right) = \left(\frac{\partial \widetilde{P}_j}{\partial N_j}\right)\left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)$. We have

$$\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_j}\right) = \left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial N_j}\right)$$

$$\iff v_j \left(-\frac{2f_j' N_i f_i N_j f_j'}{(N_i f_i + N_j f_j)^3}\right) v_i \left(\frac{f_i' f_j (N_i f_i - N_j f_j)}{(N_i f_i + N_j f_j)^3}\right) = v_i \left(\frac{(N_i f_i - N_j f_j) f_i' N_j f_j'}{(N_i f_i + N_j f_j)^3}\right) v_j \left(-\frac{2f_j' N_i f_i f_j}{(N_i f_i + N_j f_j)^3}\right)$$
$$\iff \left(-2f_j' N_i f_i N_j f_j'\right)\left(f_i' f_j (N_i f_i - N_j f_j)\right) = \left((N_i f_i - N_j f_j) f_i' N_j f_j'\right)\left(-2f_j' N_i f_i f_j\right),$$

the last line of which is clearly true. Thus,

$$\begin{aligned}\frac{\partial e_i^*}{\partial N_j} &= \frac{-1}{\text{DET}}\left(\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j + g_j - c_j''\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_j}\right) - \left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial N_j}\right)\right) \\ &= \frac{-1}{\text{DET}}\left(\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_j}\right) - \left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial N_j}\right) + \left(g_j - c_j''\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_j}\right)\right) = \underbrace{\frac{-1}{\text{DET}}}_{(-)}\underbrace{\left(g_j - c_j''\right)}_{(-)}\underbrace{\left(\frac{\partial \widetilde{P}_i}{\partial N_j}\right)}_{(*)}\end{aligned}$$

Since $(*) > 0 \iff N_i f_i > N_j f_j$ by Lemma A.5, we have $\frac{\partial e_i^*}{\partial N_j} > 0 \iff N_i f_i > N_j f_j$. □

The following theorem shows that equilibrium exists in the EMC model when minimum-effort requirement is introduced and provides a comparative statics result.

Theorem A.8 (Propositions 6 and 7 in the main text)**.** *An equilibrium exists in the EMC model even if a minimum-effort requirement,* $e_i \geq \underline{e} > 0$*, is imposed and satisfies* $\frac{\partial e_1^*}{\partial N_1} < 0$, $\frac{\partial e_1^*}{\partial N_2} > 0$, $\frac{\partial e_2^*}{\partial N_1} \leq 0$, *and* $\frac{\partial e_2^*}{\partial N_2} \leq 0$. *Moreover, in an equilibrium* (e_1^*, e_2^*)*, where* $e_1^* > e_2^* = \underline{e}$*, we have* $\frac{\partial e_2^*}{\partial N_1} = 0$ *and* $\frac{\partial e_2^*}{\partial N_2} = 0$.

Proof. Since the comparative statics obviously remain the same as Theorem A.7 if the mininmum effort constraint is not binding, we focus on the binding case. Let $\hat{e}_1$ and $\hat{e}_2$ be the equilibrium effort levels in the absence of the minimum-effort constraint. As we have shown in Lemma A.6, $\hat{e}_1 > \hat{e}_2$. If $\hat{e}_1 > \hat{e}_2 \geq \underline{e}$, this is also the equilibrium of the model when the minimum-effort requirement is imposed. Thus, it is enough to consider two remaining cases, $\hat{e}_1 > \underline{e} > \hat{e}_2$ and $\underline{e} \geq \hat{e}_1 > \hat{e}_2$. In the following, we suppress the dependence of $\tilde{P}_i$ on N_1 and N_2 to keep the notation concise when derivatives with respect to these parameters are not being considered.

Case 1: Suppose $\hat{e}_1 > \underline{e} > \hat{e}_2$. Since $\tilde{P}_1(\hat{e}_1, \hat{e}_2) - c_1'(\hat{e}_1) = 0$ and $\tilde{P}_1(\hat{e}_1, e_2)$ is increasing in e_2 by Lemma A.5, we have $\tilde{P}_1(\hat{e}_1, \underline{e}) - c_1'(\hat{e}_1) > 0$. That lemma also implies that $\tilde{P}_1(e_1, \underline{e}) - c_1'(e_1)$ is decreasing in e_1. Thus, if $\tilde{P}_1(\tilde{e}_1, \underline{e}) - c_1'(\tilde{e}_1) < 0$ for some $\tilde{e}_1 > \hat{e}_1$, continuity then implies that there must be some $e_1^* \in (\hat{e}_1, \tilde{e}_1)$ such that $\tilde{P}_1(e_1^*, \underline{e}) - c_1'(e_1^*) = 0$. To see the former, suppose $f_1(e_1) \to \infty$ as $e_1 \to \infty$. Then f_1' being bounded above by $f'(0) > 0$ and below by zero implies

$$\tilde{P}_1(e_1, \underline{e}) = \frac{v_1 f_1'(e_1) N_2 f_2(\underline{e})}{(N_1 f_1(e_1) + N_2 f_2(\underline{e}))^2} \to 0 \text{ as } e_1 \to \infty.$$

If $f_1(e_1) \not\to \infty$, then $f_1'(e_1) \to 0$ because f_1 is increasing. Thus, $\tilde{P}_1(e_1, \underline{e}) \to 0$ in this case also. Therefore, either way, we must have $\tilde{P}_1(\tilde{e}_1, \underline{e}) - c_1'(\tilde{e}_1) < 0$ for some $\tilde{e}_1$ since c_1' is bounded below by $c_1'(0) > 0$.

Next, since $\tilde{P}_2(\hat{e}_1, \hat{e}_2) - c_2'(\hat{e}_2) = 0$ and $\tilde{P}_2(\hat{e}_1, e_2) - c_2'(e_2)$ is decreasing in e_2 by Lemma A.5, we have $\tilde{P}_2(\hat{e}_1, \underline{e}) - c_2'(\underline{e}) < 0$. Lemma A.5 also implies that $\tilde{P}_2(e_1, \underline{e})$ is decreasing in e_1, which means $\tilde{P}_2(e_1^*, \underline{e}) - c_2'(\underline{e}) < 0$. This, together with $\tilde{P}_1(e_1^*, \underline{e}) - c_1'(e_1^*) = 0$ means that $(e_1^*, \underline{e})$ is the equilibrium of the model with the minimum-effort requirement.

Case 2: Suppose $\underline{e} \geq \hat{e}_1 > \hat{e}_2$. By the same reasoning as in case 1, there is $e_1^* > \hat{e}_1$ such that $\tilde{P}_1(e_1^*, \underline{e}) - c_1'(e_1^*) = 0$ and $\tilde{P}_2(e_1^*, \underline{e}) - c_2'(\underline{e}) < 0$. If $e_1^* \geq \underline{e}$, then $(e_1^*, \underline{e})$ is the equilibrium, as in case 1. If $\underline{e} > e_1^*$, then $\tilde{P}_1(\underline{e}, \underline{e}) - c_1'(\underline{e}) < 0$ because $\tilde{P}_1(e_1, \underline{e}) - c_1'(e_1)$ is decreasing in e_1 by Lemma A.5. In addition, $P_2(\underline{e}, \underline{e}) - c_2'(\underline{e}) < 0$ since $P_2(e_1, \underline{e})$ is decreasing in e_1. Thus, $(\underline{e}, \underline{e})$ is the equilibrium of the model with the minimum-effort requirement in this case.

For the comparative statics, suppose the equilibrium (e_1^*, e_2^*) is such that $e_1^* > e_2^* = \underline{e}$. If $\tilde{P}_2(e_1^*, e_2^*, N_1, N_2) - c_2'(e_2^*) < 0$, the minimum-effort constraint on weak students is binding, which means $\frac{\partial e_2^*}{\partial N_i} = 0$ for all i. Suppose $\tilde{P}_2(e_1^*, e_2^*, N_1, N_2) - c_2'(e_2^*) = 0$ instead. Then, in the absence of the minimum-effort constraint, we would have had $\frac{\partial e_2^*}{\partial N_i} < 0$ for all i by Theorem A.7. However, the minimum-effort requirement means e_2^* cannot decrease further since $e_2^* = \underline{e}$. Therefore, $\frac{\partial e_2^*}{\partial N_i} = 0$ for all i.

Next, since $e_1^* > \underline{e}$, we have $\tilde{P}_1(e_1^*, e_2^*, N_1, N_2) - c_1'(e_1^*) = 0$. Differentiating both sides of the equation with respect to N_1 yields,

$$\frac{\partial}{\partial N_1}\Big(\tilde{P}_1(e_1^*, e_2^*, N_1, N_2) - c_1'(e_1^*)\Big) = \frac{\partial \tilde{P}_1}{\partial e_1^*}\frac{\partial e_1^*}{\partial N_1} + \frac{\partial \tilde{P}_1}{\partial e_2^*}\frac{\partial e_2^*}{\partial N_1} + \frac{\partial \tilde{P}_1}{\partial N_1} - \frac{\partial c_1'}{\partial e_1^*}\frac{\partial e_1^*}{\partial N_1} = 0.$$

Using $\frac{\partial e_2^*}{\partial N_1} = 0$ and Lemma A.5, we obtain

$$\underbrace{\left(\frac{\partial \tilde{P}_1}{\partial e_1^*} - c''(e_1^*)\right)}_{(-)}\frac{\partial e_1^*}{\partial N_1} = -\underbrace{\frac{\partial \tilde{P}_1}{\partial N_1}}_{(+)} \implies \frac{\partial e_1^*}{\partial N_1} < 0.$$

Similarly, differentiating with respect to N_2 yields,

$$\frac{\partial}{\partial N_2}\Big(\tilde{P}_1(e_1^*, e_2^*, N_1, N_2) - c_1'(e_1^*)\Big) = \frac{\partial \tilde{P}_1}{\partial e_1^*}\frac{\partial e_1^*}{\partial N_2} + \frac{\partial \tilde{P}_1}{\partial e_2^*}\frac{\partial e_2^*}{\partial N_2} + \frac{\partial \tilde{P}_1}{\partial N_2} - \frac{\partial c_1'}{\partial e_1^*}\frac{\partial e_1^*}{\partial N_2} = 0.$$

Using $\frac{\partial e_2^*}{\partial N_2} = 0$ and Lemma A.5 again, we obtain

$$\underbrace{\left(\frac{\partial \tilde{P}_1}{\partial e_1^*} - c''(e_1^*)\right)}_{(-)}\frac{\partial e_1^*}{\partial N_2} = -\underbrace{\frac{\partial \tilde{P}_1}{\partial N_2}}_{(-)} \implies \frac{\partial e_1^*}{\partial N_2} > 0.$$

□

A.3 EMC equilibrium with direct peer effects

The sole difference between the EMC model with peer effects and the base model is that achievement functions are now $f_1(e_1, N_1) = a_1(N_1)b_1(e_1)$ and $f_2(e_2, N_1) = a_2(N_1)b_2(e_2)$, where $a_i(e_i) > 0$ and $a_i'(N_i) \geq 0$. This means that expressions involving differentiation with respect to e_1, e_2, or N_2 do not change from the base model. In particular, the equilibrium condition remains the same, aside from the achievement functions, and hence $\widetilde{P}_i$, having an additional dependence on N_1. That is, the equilibrium condition is:

$$F(e_1^*, e_2^*, N_1, N_2) = \begin{bmatrix} \widetilde{P}_1(e_1^*, e_2^*, N_1, N_2) - c_1'(e_1^*) \\ \\ \widetilde{P}_2(e_1^*, e_2^*, N_1, N_2) - c_2'(e_2^*) \end{bmatrix} = \begin{bmatrix} \frac{v_1 f_1'(e_1^*, N_1) N_2 f_2(e_2^*, N_1)}{\left(N_1 f_1(e_1^*, N_1) + N_2 f_2(e_2^*, N_1)\right)^2} - c_1'(e_1^*) \\ \frac{v_2 f_2'(e_2^*, N_1) N_1 f_1(e_1^*, N_1)}{\left(N_1 f_1(e_1^*, N_1) + N_2 f_2(e_2^*, N_1)\right)^2} - c_2'(e_2^*) \end{bmatrix} = \begin{bmatrix} 0 \\ \\ 0 \end{bmatrix}.$$

The following results shows that if strong students have higher peer effect elasticity with respect to N_1 than weak students, then all the partial derivatives of the marginal benefits remain the same as the base model.

Lemma A.9. *Suppose (e_1^*, e_2^*) is the equilibrium in the EMC model with direct peer effects, in which $\frac{a_1'(N_1)}{a_1(N_1)} > \frac{a_2'(N_1)}{a_2(N_1)}$. Then*

$$\frac{\partial \widetilde{P}_i(e_1^*, e_2^*, N_1, N_2)}{\partial e_i^*} - c_i'' < 0 \textit{ for all } i,$$

$$\frac{\partial \widetilde{P}_1(e_1^*, e_2^*, N_1, N_2)}{\partial e_2^*} > 0 \iff N_1 f_1(e_1^*, N_1) > N_2 f_2(e_2^*, N_1) \iff \frac{\partial \widetilde{P}_2(e_1^*, e_2^*, N_1, N_2)}{\partial e_1^*} < 0,$$
$$\frac{\partial \widetilde{P}_1(e_1^*, e_2^*, N_1, N_2)}{\partial N_1} < 0 \text{ if } N_1 f_1(e_1^*, N_1) > N_2 f_2(e_2^*, N_1),$$
$$\frac{\partial \widetilde{P}_1(e_1^*, e_2^*, N_1, N_2)}{\partial N_2} > 0 \iff N_1 f_1(e_1^*, N_1) > N_2 f_2(e_2^*, N_1),$$
$$\frac{\partial \widetilde{P}_2(e_1^*, e_2^*, N_1, N_2)}{\partial N_1} < 0 \text{ if } N_1 f_1(e_1^*, N_1) > N_2 f_2(e_2^*, N_1),$$
$$\text{and} \quad \frac{\partial \widetilde{P}_2(e_1^*, e_2^*, N_1, N_2)}{\partial N_2} < 0.$$

□

Proof. To clarify, in this proof and elsewhere a_i' denotes differentiation with respect to N_1 and b_i' denote differentiation with respect to e_i. The partial derivatives of $\widetilde{P}_i$ with respect to e_1^*, e_2^*, and N_2 are as in the proof of Lemma A.5. That is, we have

$$\frac{\partial \widetilde{P}_i}{\partial e_i^*} - c_i'' = v_i \left(\frac{f_i'' N_j f_j \left(N_1 f_1 + N_2 f_2\right) - 2 f_i' N_j f_j N_i f_i'}{\left(N_1 f_1 + N_2 f_2\right)^3} \right) - c_i'' < 0$$
$$\iff \frac{f_i''}{f_i'} < \frac{c_i''}{c'} + \frac{2 N_i f_i'}{\left(N_1 f_1 + N_2 f_2\right)}, \text{ which holds since } \frac{f_i''}{f_i'} \leq \frac{c_i''}{c'}.$$
$$\frac{\partial \widetilde{P}_i}{\partial e_j^*} = \frac{v_i \left(N_i f_i - N_j f_j\right) f_i' N_j f_j'}{\left(N_1 f_1 + N_2 f_2\right)^3} > 0 \iff N_i f_i > N_j f_j.$$
$$\frac{\partial \widetilde{P}_1}{\partial N_2} = \frac{v_1 f_1' f_2 \left(N_1 f_1 - N_2 f_2\right)}{\left(N_1 f_1 + N_2 f_2\right)^3} > 0 \iff N_1 f_1 > N_2 f_2.$$
$$\frac{\partial \widetilde{P}_2}{\partial N_2} = -\frac{2 v_2 f_2' N_1 f_1 f_2}{\left(N_1 f_1 + N_2 f_2\right)^3} < 0.$$

The partial derivatives with respect to N_1 are derived below.

$$\frac{\partial \widetilde{P}_1}{\partial N_1} = v_1 \left(\frac{\left(\frac{\partial f_1'}{\partial N_1} N_2 f_2 + f_1' N_2 \frac{\partial f_2}{\partial N_1}\right) \left(N_1 f_1 + N_2 f_2\right)^2 - 2 f_1' N_2 f_2 \left(N_1 f_1 + N_2 f_2\right) \left(f_1 + N_1 \frac{\partial f_1}{\partial N_1} + N_2 \frac{\partial f_2}{\partial N_1}\right)}{\left(N_1 f_1 + N_2 f_2\right)^4} \right)$$
$$= v_1 N_2 \left(\frac{\left(\frac{\partial f_1'}{\partial N_1} f_2 + f_1' \frac{\partial f_2}{\partial N_1}\right) \left(N_1 f_1 + N_2 f_2\right) - 2 f_1' f_2 \left(f_1 + N_1 \frac{\partial f_1}{\partial N_1} + N_2 \frac{\partial f_2}{\partial N_1}\right)}{\left(N_1 f_1 + N_2 f_2\right)^3} \right)$$
$$= v_1 N_2 \left(\frac{\left(a_1' b_1' a_2 b_2 + a_1 b_1' a_2' b_2\right) \left(N_1 f_1 + N_2 f_2\right) - 2 a_1 b_1' a_2 b_2 \left(N_1 a_1' b_1 + N_2 a_2' b_2\right) - 2 f_1' f_2 f_1}{\left(N_1 f_1 + N_2 f_2\right)^3} \right)$$
$$= v_1 N_2 \left(\frac{b_1' b_2 \Big(\left(a_1' a_2 + a_1 a_2'\right) \left(N_1 f_1 + N_2 f_2\right) - 2 \left(a_1 a_2 N_1 a_1' b_1 + a_1 a_2 N_2 a_2' b_2\right) \Big) - 2 f_1' f_2 f_1}{\left(N_1 f_1 + N_2 f_2\right)^3} \right)$$
$$= v_1 N_2 \left(\frac{b_1' b_2 \Big(\left(a_1' a_2 + a_1 a_2'\right) \left(N_1 f_1 + N_2 f_2\right) - 2 \left(a_1' a_2 N_1 f_1 + a_1 a_2' N_2 f_2\right) \Big) - 2 f_1' f_2 f_1}{\left(N_1 f_1 + N_2 f_2\right)^3} \right)$$
$$= v_1 N_2 \left(\frac{b_1' b_2 \Big(a_1' a_2 \left(N_2 f_2 - N_1 f_1\right) + a_1 a_2' \left(N_1 f_1 - N_2 f_2\right) \Big) - 2 f_1' f_2 f_1}{\left(N_1 f_1 + N_2 f_2\right)^3} \right)$$

$$= v_1 N_2 \left(\frac{b_1' b_2 \overbrace{(a_1' a_2 - a_1 a_2')}^{(+)} \overbrace{(N_2 f_2 - N_1 f_1)}^{(-)} - \overbrace{2 f_1' f_2 f_1}^{(+)}}{(N_1 f_1 + N_2 f_2)^3} \right) < 0 \quad \text{if } \frac{a_1'}{a_1} > \frac{a_2'}{a_2} \text{ and } N_1 f_1 > N_2 f_2.$$

Next, we have,

$$\frac{\partial \widetilde{P}_2}{\partial N_1} = v_2 \left(\frac{\left(\frac{\partial f_2'}{\partial N_1} N_1 f_1 + f_2' f_1 + f_2' N_1 \frac{\partial f_1}{\partial N_1} \right) (N_1 f_1 + N_2 f_2)^2 - 2 f_2' N_1 f_1 (N_1 f_1 + N_2 f_2) \left(f_1 + N_1 \frac{\partial f_1}{\partial N_1} + N_2 \frac{\partial f_2}{\partial N_1} \right)}{(N_1 f_1 + N_2 f_2)^4} \right)$$

$$= v_2 \left(\frac{\left(\frac{\partial f_2'}{\partial N_1} N_1 f_1 + f_2' f_1 + f_2' N_1 \frac{\partial f_1}{\partial N_1} \right) (N_1 f_1 + N_2 f_2) - 2 f_2' N_1 f_1 \left(f_1 + N_1 \frac{\partial f_1}{\partial N_1} + N_2 \frac{\partial f_2}{\partial N_1} \right)}{(N_1 f_1 + N_2 f_2)^3} \right)$$

$$= v_2 \left(\frac{\left(\frac{\partial f_2'}{\partial N_1} N_1 f_1 + f_2' N_1 \frac{\partial f_1}{\partial N_1} \right) (N_1 f_1 + N_2 f_2) - 2 f_2' N_1 f_1 \left(N_1 \frac{\partial f_1}{\partial N_1} + N_2 \frac{\partial f_2}{\partial N_1} \right) + f_2' f_1 (N_2 f_2 - N_1 f_1)}{(N_1 f_1 + N_2 f_2)^3} \right)$$

$$= v_2 \left(\frac{(a_2' b_2' N_1 a_1 b_1 + a_2 b_2' N_1 a_1' b_1) (N_1 f_1 + N_2 f_2) - 2 a_2 b_2' N_1 a_1 b_1 (N_1 a_1' b_1 + N_2 a_2' b_2) + f_2' f_1 (N_2 f_2 - N_1 f_1)}{(N_1 f_1 + N_2 f_2)^3} \right)$$

$$= v_2 \left(\frac{b_2' N_1 b_1 \Big((a_2' a_1 + a_2 a_1') (N_1 f_1 + N_2 f_2) - 2 (a_2 a_1 N_1 a_1' b_1 + a_2 a_1 N_2 a_2' b_2) \Big) + f_2' f_1 (N_2 f_2 - N_1 f_1)}{(N_1 f_1 + N_2 f_2)^3} \right)$$

$$= v_2 \left(\frac{b_2' N_1 b_1 \Big((a_1 a_2' + a_1' a_2) (N_1 f_1 + N_2 f_2) - 2 (a_1' a_2 N_1 f_1 + a_1 a_2' N_2 f_2) \Big) + f_2' f_1 (N_2 f_2 - N_1 f_1)}{(N_1 f_1 + N_2 f_2)^3} \right)$$

$$= v_2 \left(\frac{b_2' N_1 b_1 \Big(a_1 a_2' (N_1 f_1 - N_2 f_2) + a_1' a_2 (N_2 f_2 - N_1 f_1) \Big) + f_2' f_1 (N_2 f_2 - N_1 f_1)}{(N_1 f_1 + N_2 f_2)^3} \right)$$

$$= v_2 \left(\frac{b_2' N_1 b_1 \overbrace{(a_1 a_2' - a_1' a_2)}^{(-)} \overbrace{(N_1 f_1 - N_2 f_2)}^{(+)} + \overbrace{f_2' f_1 (N_2 f_2 - N_1 f_1)}^{(-)}}{(N_1 f_1 + N_2 f_2)^3} \right) < 0 \quad \text{if } \frac{a_1'}{a_1} > \frac{a_2'}{a_2} \text{ and } N_1 f_1 > N_2 f_2.$$

□

The main comparative statics results for the EMC model with peer effects is given below.

Theorem A.10 (Proposition 8 in the main text)**.** *Suppose* (e_1^*, e_2^*) *is the equilibrium in the EMC model with direct peer effects, in which* $\frac{a_1'(N_1)}{a_1(N_1)} > \frac{a_2'(N_1)}{a_2(N_1)}$*. Then* $\frac{\partial e_1^*}{\partial N_1} < 0$, $\frac{\partial e_1^*}{\partial N_2} > 0$, $\frac{\partial e_2^*}{\partial N_1} < 0$, *and* $\frac{\partial e_2^*}{\partial N_2} < 0$*. In addition, the signs of* $\frac{df_1(e_1^*, N_1)}{dN_1}$ *and* $\frac{df_2(e_2^*, N_1)}{dN_1}$ *are ambiguous. If, however,* $\frac{df_1(e_1^*, N_1)}{dN_1} > 0$*, then* $a_i'(N_1) > 0$*.*

Proof. Let $\boldsymbol{e}^* = (e_1^*, e_2^*)$ and $\boldsymbol{N} = (N_1, N_2)$. Appealing to the implicit differentiation theorem yields,

$$D_{\boldsymbol{N}} \boldsymbol{e}^* = \begin{bmatrix} \frac{\partial e_1^*}{\partial N_1} & \frac{\partial e_1^*}{\partial N_2} \\ \frac{\partial e_2^*}{\partial N_1} & \frac{\partial e_2^*}{\partial N_2} \end{bmatrix} = - \left[D_{\boldsymbol{e}^*} F(e_1^*, e_2^*, N_1, N_2) \right]^{-1} \left[D_{\boldsymbol{N}} F(e_1^*, e_2^*, N_1, N_2) \right]$$
$$= \frac{-1}{\text{DET}} \begin{bmatrix} \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2'' \right) \left(\frac{\partial \widetilde{P}_1}{\partial N_1} \right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*} \right) \left(\frac{\partial \widetilde{P}_2}{\partial N_1} \right) & \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2'' \right) \left(\frac{\partial \widetilde{P}_1}{\partial N_2} \right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*} \right) \left(\frac{\partial \widetilde{P}_2}{\partial N_2} \right) \\ \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1'' \right) \left(\frac{\partial \widetilde{P}_2}{\partial N_1} \right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*} \right) \left(\frac{\partial \widetilde{P}_1}{\partial N_1} \right) & \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1'' \right) \left(\frac{\partial \widetilde{P}_2}{\partial N_2} \right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*} \right) \left(\frac{\partial \widetilde{P}_1}{\partial N_2} \right) \end{bmatrix},$$

where

$$\text{DET} = \underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)}_{(-)} \underbrace{\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)}_{(-)} - \underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)}_{(\leq 0)} > 0 \quad \text{by Lemma A.9.}$$

Combining the above result with Lemma A.9, we obtain

$$\frac{\partial e_1^*}{\partial N_1} = \underbrace{\frac{-1}{DET}}_{(-)} \left(\underbrace{\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)}_{(-)} \underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right)}_{(-)} - \underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial e_2}\right)}_{(+)} \underbrace{\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right)}_{(-)} \right) < 0.$$

$$\frac{\partial e_2^*}{\partial N_2} = \underbrace{\frac{-1}{DET}}_{(-)} \left(\underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)}_{(-)} \underbrace{\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right)}_{(-)} - \underbrace{\left(\frac{\partial \widetilde{P}_2}{\partial e_1}\right)}_{(-)} \underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right)}_{(+)} \right) < 0.$$

Next, let $g_i = \frac{v_i f_i'' N_j f_j}{(N_1 f_1 + N_2 f_2)^2}$. By equation (4), we have

$$g_i - c_i'' = \frac{v_i f_i'' N_j f_j}{(N_1 f_1 + N_2 f_2)^2} - c_i'' = \frac{f_i'' c_i'}{f_i'} - c_i'' < 0 \quad \text{since } \frac{f_i''}{f_i'} < \frac{c_i''}{c_i'} \text{ by assumption.}$$

In addition, we have

$$\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - g_2\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) = \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right).$$

To see this, note that the derivatives found in the proof of Lemma A.9 implies that the above equation is equivalent to:

$$\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - g_2\right)\left(\frac{v_1 f_1' f_2 (N_1 f_1 - N_2 f_2)}{(N_1 f_1 + N_2 f_2)^3}\right) = \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(-\frac{2 v_2 f_2' N_1 f_1 f_2}{(N_1 f_1 + N_2 f_2)^3}\right)$$

$$\iff -\frac{2 v_2 f_2' N_1 f_1 N_2 f_2'}{(N_1 f_1 + N_2 f_2)^3}\left(\frac{v_1 f_1' f_2 (N_1 f_1 - N_2 f_2)}{(N_1 f_1 + N_2 f_2)^3}\right) = \frac{v_1 (N_1 f_1 - N_2 f_2) f_1' N_2 f_2'}{(N_1 f_1 + N_2 f_2)^3}\left(-\frac{2 v_2 f_2' N_1 f_1 f_2}{(N_1 f_1 + N_2 f_2)^3}\right)$$

$$\iff f_2' N_1 f_1 N_2 f_2' (f_1' f_2 (N_1 f_1 - N_2 f_2)) = (N_1 f_1 - N_2 f_2) f_1' N_2 f_2' (f_2' N_1 f_1 f_2),$$

which is clearly true. Thus,

$$\begin{aligned}
\frac{\partial e_1^*}{\partial N_2} &= \frac{-1}{DET}\left(\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right)\right) \\
&= \frac{-1}{DET}\left(\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - g_2 + g_2 - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right)\right) = \underbrace{\frac{-1}{DET}}_{(-)} \underbrace{(g_2 - c_2'')}_{(-)} \underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right)}_{(+)} > 0.
\end{aligned}$$

Next, we have

$$\left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - g_1\right)\left(\frac{v_2 f_2' f_1 (N_2 f_2 - N_1 f_1)}{(N_1 f_1 + N_2 f_2)^3}\right) = \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{-2 v_1 N_2 f_1' f_2 f_1}{(N_1 f_1 + N_2 f_2)^3}\right) \tag{5}$$

This holds because the equation is equivalent to:

$$-\frac{2v_1 f_1' N_2 f_2 N_1 f_1'}{(N_1 f_1 + N_2 f_2)^3}\left(\frac{v_2 f_2' f_1 (N_2 f_2 - N_1 f_1)}{(N_1 f_1 + N_2 f_2)^3}\right) = \frac{v_2 (N_2 f_2 - N_1 f_1) f_2' N_1 f_1'}{(N_1 f_1 + N_2 f_2)^3}\left(\frac{-2v_1 N_2 f_1' f_2 f_1}{(N_1 f_1 + N_2 f_2)^3}\right)$$
$$\iff (f_1' N_2 f_2 N_1 f_1')(f_2' f_1)(N_2 f_2 - N_1 f_1) = (N_2 f_2 - N_1 f_1) f_2' N_1 f_1' (N_2 f_1' f_2 f_1),$$

which clears holds. Thus,

$$\frac{\partial e_2^*}{\partial N_1} = \frac{-1}{DET}\left(\left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right)\right)$$
$$= \frac{-1}{DET}\left(\underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - g_1\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right)}_{(*)} + \underbrace{\left(g_1 - c_1''\right)}_{(-)}\underbrace{\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right)}_{(-)}\right),$$

Using the expressions for $\frac{\partial \widetilde{P}_2}{\partial N_1}$ and $\frac{\partial \widetilde{P}_1}{\partial N_1}$ derived in the proof of Lemma A.9 and equation (5), we obtain:

$$(*) = \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - g_1\right)\left(v_2\left(\frac{b_2' N_1 b_1 (a_1 a_2' - a_1' a_2)(N_1 f_1 - N_2 f_2) + f_2' f_1 (N_2 f_2 - N_1 f_1)}{(N_1 f_1 + N_2 f_2)^3}\right)\right)$$
$$- \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(v_1 N_2\left(\frac{b_1' b_2 (a_1' a_2 - a_1 a_2')(N_2 f_2 - N_1 f_1) - 2 f_1' f_2 f_1}{(N_1 f_1 + N_2 f_2)^3}\right)\right)$$
$$= \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - g_1\right)\left(\frac{v_2 b_2' N_1 b_1 (a_1 a_2' - a_1' a_2)(N_1 f_1 - N_2 f_2)}{(N_1 f_1 + N_2 f_2)^3}\right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{v_1 N_2 b_1' b_2 (a_1' a_2 - a_1 a_2')(N_2 f_2 - N_1 f_1)}{(N_1 f_1 + N_2 f_2)^3}\right).$$

In addition,

$$(*) > 0 \iff \left(-\frac{2v_1 f_1' N_2 f_2 N_1 f_1'}{(N_1 f_1 + N_2 f_2)^3}\right)\left(\frac{v_2 b_2' N_1 b_1 (a_1 a_2' - a_1' a_2)(N_1 f_1 - N_2 f_2)}{(N_1 f_1 + N_2 f_2)^3}\right)$$
$$> \left(\frac{v_2 (N_2 f_2 - N_1 f_1) f_2' N_1 f_1'}{(N_1 f_1 + N_2 f_2)^3}\right)\left(\frac{v_1 N_2 b_1' b_2 (a_1' a_2 - a_1 a_2')(N_2 f_2 - N_1 f_1)}{(N_1 f_1 + N_2 f_2)^3}\right)$$
$$\iff -2 f_1' f_2 b_2' N_1 b_1 (a_1 a_2' - a_1' a_2) > -(N_2 f_2 - N_1 f_1) f_2' b_1' b_2 (a_1' a_2 - a_1 a_2')$$
$$\iff 2 a_1 b_1' a_2 b_2 b_2' N_1 b_1 (a_1' a_2 - a_1 a_2') > (N_1 f_1 - N_2 f_2) a_2 b_2' b_1' b_2 (a_1' a_2 - a_1 a_2')$$
$$\iff 2 a_1 N_1 b_1 (a_1' a_2 - a_1 a_2') > (N_1 f_1 - N_2 f_2)(a_1' a_2 - a_1 a_2')$$
$$\iff (N_1 f_1 + N_2 f_2)(a_1' a_2 - a_1 a_2') > 0 \iff \frac{a_1'}{a} > \frac{a_2'}{a_2}.$$

Thus when $\frac{a_1'}{a_1} > \frac{a_2'}{a_2}$, we have $\frac{\partial e_2^*}{\partial N_1} < 0$, as required. Finally, since

$$\frac{df_1(e_1^*, N_1)}{dN_1} = \underbrace{\left(\frac{\partial f_1}{\partial e_1^*}\right)}_{(+)}\underbrace{\left(\frac{\partial e_1^*}{\partial N_1}\right)}_{(-)} + \underbrace{\frac{\partial f_1}{\partial N_1}}_{(+)} \quad \text{and} \quad \frac{df_2(e_2^*, N_1)}{dN_1} = \underbrace{\left(\frac{\partial f_2}{\partial e_2^*}\right)}_{(+)}\underbrace{\left(\frac{\partial e_2^*}{\partial N_1}\right)}_{(-)} + \underbrace{\frac{\partial f_2}{\partial N_1}}_{(+)},$$

the signs of the two derivatives are ambiguous without further assumption on f_1 and f_2. However, it is clear that if $\frac{df_1(e_1^*, N_1)}{dN_1} > 0$, then $a_i'(N_1) > 0$ since $\frac{\partial e_1^*}{\partial N_1} < 0$. □

The next result compares student behavior across schools with different levels of competitiveness. We assume that a more competitive school gives higher reward for success, and we restrict attention to a setting where weak students do not respond to the entry of additional students to keep the derivation simple. Assuming that students' valuation for success is increasing in the reward, the comparative statics results can be found by differentiating with respect to v_1.

Theorem A.11 (Propositions 10 and 11 in the main text)**.** *Suppose* (e_1^*, e_2^*) *is the equilibrium in the EMC model with direct peer effects, where* $\frac{a_1'(N_1)}{a_1(N_1)} > \frac{a_2'(N_1)}{a_2(N_1)}$ *and* $\frac{\partial e_2^*}{\partial N_1} = 0$*. Then* $\frac{\partial^2 e_1^*}{\partial N_1 \partial v_1} \le 0$*,* $\frac{\partial^2 f_1(e_1^*, N_1)}{\partial N_1 \partial v_1} \le 0$*,* $\frac{\partial^2 e_1^*}{\partial N_2 \partial v_1} \ge 0$*, and* $\frac{\partial^2 f_1(e_1^*, N_2)}{\partial N_2 \partial v_1} \ge 0$*. The inequalities are strict if* $c_1(e_1)$ *is strictly convex.*

Proof. When e_2^* does not change, the equilibrium condition as a function of N_1, N_2, and v_1 reduces to

$$F(e_1^*, e_2^*, N_1, N_2, v_1) = \widetilde{P}_1(e_1^*, e_2^*, N_1, N_2, v_1) - c_1'(e_1^*) = \frac{v_1 f_1'(e_1^*, N_1) N_2 f_2(e_2^*, N_1)}{\left(N_1 f_1(e_1^*, N_1) + N_2 f_2(e_2^*, N_1)\right)^2} - c_1'(e_1^*) = 0.$$

Since v_1 enters $\widetilde{P}_1$ as a multiplicative scale factor, we have

$$\left(\frac{\partial^2 \widetilde{P}_1}{\partial e_1^* \partial v_1}\right) v_1 = \frac{\partial \widetilde{P}_1}{\partial e_1^*}, \quad \left(\frac{\partial^2 \widetilde{P}_1}{\partial N_1 \partial v_1}\right) v_1 = \frac{\partial \widetilde{P}_1}{\partial N_1}, \quad \text{and} \quad \left(\frac{\partial^2 \widetilde{P}_1}{\partial N_2 \partial v_1}\right) v_1 = \frac{\partial \widetilde{P}_1}{\partial N_2}.$$

Next, implicit differentiation of $F(e_1^*, e_2^*, N_1, N_2, v_1) = 0$ yields

$$\begin{aligned} \frac{\partial e_1^*}{\partial N_1} &= -\frac{\frac{\partial F}{\partial N_1}}{\frac{\partial F}{\partial e_1^*}} = -\frac{\frac{\partial \widetilde{P}_1}{\partial N_1}}{\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''(e_1^*)} = -\left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''(e_1^*)\right)^{-1} \left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) \\ \implies \frac{\partial^2 e_1^*}{\partial N_1 \partial v_1} &= \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''(e_1^*)\right)^{-2} \left(\frac{\partial^2 \widetilde{P}_1}{\partial e_1^* \partial v_1}\right) \left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''(e_1^*)\right)^{-1} \left(\frac{\partial^2 \widetilde{P}_1}{\partial N_1 \partial v_1}\right). \end{aligned}$$

From the proof of Lemma A.9 and Theorem A.10, we have $\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''(e_1^*) < 0$, $\frac{\partial \widetilde{P}_1}{\partial e_1^*} < 0$, and $\frac{\partial \widetilde{P}_1}{\partial N_1} < 0$ if $\frac{a_1'}{a_1} > \frac{a_2'}{a_2}$ and $N_1 f_1 > N_2 f_2$. Thus, under these conditions, we have

$$\begin{aligned} \frac{\partial^2 e_1^*}{\partial N_1 \partial v_1} \le 0 &\iff \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''(e_1^*)\right)^{-2} \left(\frac{\partial^2 \widetilde{P}_1}{\partial e_1^* \partial v_1}\right) \left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) \le \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''(e_1^*)\right)^{-1} \left(\frac{\partial^2 \widetilde{P}_1}{\partial N_1 \partial v_1}\right) \\ &\iff \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''(e_1^*)\right)^{-1} \left(\frac{\partial^2 \widetilde{P}_1}{\partial e_1^* \partial v_1}\right) \left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) \ge \left(\frac{\partial^2 \widetilde{P}_1}{\partial N_1 \partial v_1}\right) \\ &\iff \frac{\left(\frac{\partial^2 \widetilde{P}_1}{\partial e_1^* \partial v_1}\right) v_1}{\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''(e_1^*)} \le \frac{\left(\frac{\partial^2 \widetilde{P}_1}{\partial N_1 \partial v_1}\right) v_1}{\frac{\partial \widetilde{P}_1}{\partial N_1}} \\ &\iff \frac{\frac{\partial \widetilde{P}_1}{\partial e_1^*}}{\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''(e_1^*)} \le 1, \text{ which holds since } c_1'' \ge 0. \end{aligned}$$

Similarly, implicit differentiation with respect to N_2 yields

$$\begin{aligned} \frac{\partial e_1^*}{\partial N_2} &= -\frac{\frac{\partial F}{\partial N_2}}{\frac{\partial F}{\partial e_1^*}} = -\left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''(e_1^*)\right)^{-1} \left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) \\ \implies \frac{\partial^2 e_1^*}{\partial N_2 \partial v_1} &= \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''(e_1^*)\right)^{-2} \left(\frac{\partial^2 \widetilde{P}_1}{\partial e_1^* \partial v_1}\right) \left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''(e_1^*)\right)^{-1} \left(\frac{\partial^2 \widetilde{P}_1}{\partial N_2 \partial v_1}\right). \end{aligned}$$

From the proof of Peer effect theorem (Lemma A.9), we have $\frac{\partial \widetilde{P}_1}{\partial N_2} > 0 \iff N_1 f_1 > N_2 f_2$. Thus, we have

$$\frac{\partial^2 e_1^*}{\partial N_2 \partial v_1} \ge 0 \iff \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''(e_1^*)\right)^{-2} \left(\frac{\partial^2 \widetilde{P}_1}{\partial e_1^* \partial v_1}\right) \left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) \ge \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''(e_1^*)\right)^{-1} \left(\frac{\partial^2 \widetilde{P}_1}{\partial N_2 \partial v_1}\right)$$

$$\iff \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''(e_1^*)\right)^{-1} \left(\frac{\partial^2 \widetilde{P}_1}{\partial e_1^* \partial v_1}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) \leq \left(\frac{\partial^2 \widetilde{P}_1}{\partial N_2 \partial v_1}\right)$$

$$\iff \frac{\left(\frac{\partial^2 \widetilde{P}_1}{\partial e_1^* \partial v_1}\right) v_1}{\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''(e_1^*)} \leq \frac{\left(\frac{\partial^2 \widetilde{P}_1}{\partial N_2 \partial v_1}\right) v_1}{\frac{\partial \widetilde{P}_1}{\partial N_2}}$$

$$\iff \frac{\frac{\partial \widetilde{P}_1}{\partial e_1^*}}{\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''(e_1^*)} \leq 1, \text{ which holds since } c_1'' \geq 0.$$

Finally, we have

$$\frac{df_1(e_1^*, N_1)}{dN_1} = \left(\frac{\partial f_1}{\partial e_1^*}\right)\left(\frac{\partial e_1^*}{\partial N_1}\right) + \frac{\partial f_1}{\partial N_1} \implies \frac{d^2 f_1(e_1^*, N_1)}{dN_1 dv} = \left(\frac{\partial f_1}{\partial e_1^*}\right)\left(\frac{\partial^2 e_1^*}{\partial N_1 \partial v_1}\right) \leq 0.$$

$$\frac{df_1(e_1^*, N_1)}{dN_2} = \left(\frac{\partial f_1}{\partial e_1^*}\right)\left(\frac{\partial e_1^*}{\partial N_2}\right) \implies \frac{d^2 f_1(e_1^*, N_1)}{dN_2 dv} = \left(\frac{\partial f_1}{\partial e_1^*}\right)\left(\frac{\partial^2 e_1^*}{\partial N_2 \partial v_1}\right) \geq 0.$$

Moreover, all the inequalities we have shown are strict if $c_1'' > 0$. □

Finally, we consider how the equilibrium effort in the EMC model changes when the members of the weaker group are swapped for those of the stronger group while maintaining the same group sizes. Letting $\hat{e}_i$ denote the equilibrium effort when swapping is being considered, $\hat{e}_i(N_1) = e_i^*(N_1, N - N_1)$, we have

$$\frac{d\hat{e}_i}{dN_1} = \frac{\partial e_i^*}{\partial N_1} + \frac{\partial e_i^*}{\partial N_2}\left(\frac{dN_2}{dN_1}\right) = \frac{\partial e_i^*}{\partial N_1} - \frac{\partial e_i^*}{\partial N_2}.$$

In both the base and the peer-effect versions of the EMC model (Theorems A.7 and A.10), strong students work harder when faced with an increased competition from weak students ($\frac{\partial e_1^*}{\partial N_2} > 0$) but are discouraged by the increased competition from fellow strong students ($\frac{\partial e_1^*}{\partial N_1} < 0$). Thus, when the swap occur, both effects work in the same direction and unambiguously reduce strong students' effort:

$$\frac{d\hat{e}_1}{dN_1} = \underbrace{\frac{\partial e_1^*}{\partial N_1}}_{(-)} - \underbrace{\frac{\partial e_1^*}{\partial N_2}}_{(+)} < 0.$$

In contrast, weak students are discouraged by an increased competition from anyone, making the effect of a swap ambiguous:

$$\text{sign}\left(\frac{d\hat{e}_2}{dN_1}\right) = \text{sign}\left(\underbrace{\frac{\partial e_2^*}{\partial N_1}}_{(-)} - \underbrace{\frac{\partial e_2^*}{\partial N_2}}_{(-)}\right) = \text{ ambiguous.}$$

That is, an increased competition from having more strong students makes weak students lower their effort while reduced competition from having fewer fellow weak students encourages them to raise their effort. The direction of the combined effect depends on which of the two effects dominate. The following theorem gives sufficient conditions for the effort to decline.

Theorem A.12. *In the setting of Theorem A.7 and Theorem A.10, we have* $\frac{d\hat{e}_1(N_1)}{dN_1} < 0$. *Suppose in addition* $N_1 f_1 > N_2 f_2 + 2N_1 f_2$ *in the base model and* $N_1 f_1 > N_2 f_2 + 2N_1 f_2$ *or* $\frac{\partial f_2}{\partial N_1} > \frac{f_2}{N_2}$ *in the peer-effect model. Then there exists* $m > 0$ *such that* $\frac{d\hat{e}_2(N_1)}{dN_1} < 0$ *if* $c_1''(e_1^*) > m$.

Proof. We have shown that in both the base and the peer-effect versions of the EMC model, we have

$$\begin{bmatrix} \frac{\partial e_1^*}{\partial N_1} & \frac{\partial e_1^*}{\partial N_2} \\ \frac{\partial e_2^*}{\partial N_1} & \frac{\partial e_2^*}{\partial N_2} \end{bmatrix} = \frac{-1}{\text{DET}} \begin{bmatrix} \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) & \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right) \\ \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) & \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) \end{bmatrix},$$

where DET $= \left(\frac{\partial\widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial\widetilde{P}_2}{\partial e_2^*} - c_2''\right) - \left(\frac{\partial\widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial\widetilde{P}_2}{\partial e_1^*}\right) > 0$. In the base model, we showed that

$$\frac{\partial\widetilde{P}_i}{\partial N_i} = -\frac{2v_1 f_i' N_j f_j f_i}{(N_i f_i + N_j f_j)^3} < 0 \quad \text{and} \quad \frac{\partial\widetilde{P}_i}{\partial N_j} = \frac{v_i f_i' f_j (N_i f_i - N_j f_j)}{(N_i f_i + N_j f_j)^3} > 0 \iff N_i f_i > N_j f_j.$$

Thus,

$$\frac{\partial\widetilde{P}_2}{\partial N_1} < \frac{\partial\widetilde{P}_2}{\partial N_2} \iff \frac{v_2 f_2' f_1 (N_1 f_1 - N_2 f_2)}{(N_2 f_2 + N_1 f_1)^3} > \frac{2v_2 f_2' N_1 f_1 f_2}{(N_2 f_2 + N_1 f_1)^3} \iff N_1 f_1 > N_2 f_2 + 2N_1 f_2.$$

In the peer-effect model, we showed

$$\frac{\partial\widetilde{P}_2}{\partial N_1} = v_2 \left(\frac{\frac{\partial f_2'}{\partial N_1} N_1 f_1 (N_1 f_1 + N_2 f_2) - f_2' f_1 (N_1 f_1 - N_2 f_2) - 2 f_2' N_1 f_1 N_2 \frac{\partial f_2}{\partial N_1}}{(N_1 f_1 + N_2 f_2)^3}\right) < 0, \text{ and}$$

$$\frac{\partial\widetilde{P}_2}{\partial N_2} = -\frac{2v_2 f_2' N_1 f_1 f_2}{(N_2 f_2 + N_1 f_1)^3} < 0.$$

Thus,

$$\frac{\partial\widetilde{P}_2}{\partial N_1} < \frac{\partial\widetilde{P}_2}{\partial N_2} \iff -\frac{\partial f_2'}{\partial N_1} N_1 f_1 (N_1 f_1 + N_2 f_2) + f_2' f_1 (N_1 f_1 - N_2 f_2) + 2 f_2' N_1 f_1 N_2 \frac{\partial f_2}{\partial N_1} > 2 f_2' N_1 f_1 f_2,$$

which holds if $N_1 f_1 > N_2 f_2 + 2N_1 f_2$ or $N_2 \frac{\partial f_2}{\partial N_1} > f_2$.

In the following, we show that $\frac{\partial\widetilde{P}_2}{\partial N_1} < \frac{\partial\widetilde{P}_2}{\partial N_2}$ is consistent with $\frac{d\hat{e}_1}{dN_1} < 0$ and that $\frac{d\hat{e}_2}{dN_1} < 0$ if in addition c_1'' is positive and large. First, we have

$$\frac{\partial e_1^*}{\partial N_1} < \frac{\partial e_1^*}{\partial N_2} \iff \left(\frac{\partial\widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial\widetilde{P}_1}{\partial N_1}\right) - \left(\frac{\partial\widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial\widetilde{P}_2}{\partial N_1}\right) > \left(\frac{\partial\widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial\widetilde{P}_1}{\partial N_2}\right) - \left(\frac{\partial\widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial\widetilde{P}_2}{\partial N_2}\right)$$

$$\iff \underbrace{\underbrace{\left(\frac{\partial\widetilde{P}_2}{\partial e_2^*} - c_2''\right)}_{(-)}\underbrace{\left(\frac{\partial\widetilde{P}_1}{\partial N_1} - \frac{\partial\widetilde{P}_1}{\partial N_2}\right)}_{(-)-(+)}}_{(+)} > \underbrace{\left(\frac{\partial\widetilde{P}_1}{\partial e_2^*}\right)}_{(+)}\underbrace{\left(\frac{\partial\widetilde{P}_2}{\partial N_1} - \frac{\partial\widetilde{P}_2}{\partial N_2}\right)}_{(*)}.$$

Since $(*) < 0$ by above, the last inequality is satisfied as required. Therefore, $\frac{d\hat{e}_1}{dN_1} < 0$. Next, we have

$$\frac{\partial e_2^*}{\partial N_1} < \frac{\partial e_2^*}{\partial N_2} \iff \left(\frac{\partial\widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial\widetilde{P}_2}{\partial N_1}\right) - \left(\frac{\partial\widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial\widetilde{P}_1}{\partial N_1}\right) > \left(\frac{\partial\widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial\widetilde{P}_2}{\partial N_2}\right) - \left(\frac{\partial\widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial\widetilde{P}_1}{\partial N_2}\right)$$

$$\iff \underbrace{\underbrace{\left(\frac{\partial\widetilde{P}_1}{\partial e_1^*}\right)}_{(-)}\underbrace{\left(\frac{\partial\widetilde{P}_2}{\partial N_1} - \frac{\partial\widetilde{P}_2}{\partial N_2}\right)}_{(-)}}_{(+)} + c_1'' \underbrace{\left(\frac{\partial\widetilde{P}_2}{\partial N_2} - \frac{\partial\widetilde{P}_2}{\partial N_1}\right)}_{(+)} > \underbrace{\underbrace{\left(\frac{\partial\widetilde{P}_2}{\partial e_1^*}\right)}_{(-)}\underbrace{\left(\frac{\partial\widetilde{P}_1}{\partial N_1} - \frac{\partial\widetilde{P}_1}{\partial N_2}\right)}_{(-)-(+)}}_{(+)}.$$

If c_1'' is sufficiently large, the last inequality will be satisfied and $\frac{d\hat{e}_2}{dN_1} < 0$, as desired. □

Appendix B: Testing for the Selectivity of Strong-Student High-School Stayers

To test for selectivity bias in the estimates from our key specifications using a sample of strong-student native-born high-school stayers we adopt a control function approach. We jointly estimate the determinants of staying in the same high school after 10th grade and the effects of the SEA refugee students on both the changes in homework time and test score performance for the stayers between the 10th and 12th grades (Heckman, 1979).

The first column of Appendix Table E reports probit estimates of the determinants of staying in the same school in the last three years of high school. The specification includes the post 10th-grade county SEA refugees flows as well as the student's test score at the end of the 10th-grade academic year and the total income of the student's family in 1988.[1] The estimates indicate that while strong native-born students are less likely to remain in the same school when there is an increase in the number of strong SEA refugee students, consistent with "native-born flight," and to not exit when there are more weak SEA refugees, these effects are not statistically significant. On the other hand, among the strong students, those with higher test scores are statistically significantly more likely to stay but those in higher-income households are statistically-significantly more likely to switch schools. The point estimates indicate that a one-standard deviation increase in family income would decrease the probability of remaining in the same school by 0.91 percentage points (an 18.6% increase in the probability of switching), while a one-standard deviation in increase in the 10th-grade test score would increase the probability of staying in the same high school by 1.2 percentage points (a 26.5% decrease in the probability of switching).

The second column of the table displays the estimate of the SEA refugee effects on log homework time, using the same differenced specification as reported in Table 4, but now taking into account selection on unobservables. The results indicate that we cannot reject the hypothesis that there is no selectivity with respect to student effort among the high-school stayers – those

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[1] We use 1988 family income rather than income when the student is in 10th grade because a significant fraction of 10th-grade students contributes to family income – 26% of the 10th-grade native-born strong students report that they have jobs during the academic year. Thus, family income in 10th-grade would reflect in part the endogenous time allocation of the students, especially an issue if work time and homework time compete. There is no NELS:88 information on work when the students were in 8th-grade, but that percentage is presumably low given that the Federal minimum work age is 14 for non-agricultural workers.

strong students who supply more effort are no more likely to switch schools than other strong students. There is, however, selectivity with respect to test score performance. The estimates, in the third column of the table, indicate that students who experienced a positive change in period-specific test score shocks are more likely to stay in the same school. The selectivity-corrected estimates, however, are similar to those reported in Table 8, indicating that on average there is no detectable effect of county-level SEA refugee students of either ability type on test scores.

In the last two columns we allow the refugee effects on homework time and test scores to differ by school competition policy and jointly estimate the school-staying and test score and homework time estimates along with the probit determining school switching after 10th grade. In these specifications, we also allow the determinants of the probability of staying in the same high school to differ by competition policy (not shown). Again, there is no evidence of unobservable selectivity with respect to student homework time, but there is with respect to test scores. However, the selectivity-corrected estimates for both homework time and test scores are similar to those with no correction for selectivity bias, and the conclusions are the same: the effects of the SEA refugees on incumbent strong student effort are significantly weaker in absolute value in high schools that do not encourage competition, but only in such schools do we detect a positive effect of higher-ability SEA refugees on the test scores of the strong native-born.

Appendix C: Accounting for Multiple Hypotheses

We report in this appendix statistical significance tests that correct for the family-wise error rate that arises from testing multiple hypotheses. We also carry out randomization tests of the joint statistical significance of our treatment variables in each equation and overall that take into account the potential influence of outliers.

While we obtain coefficient sign estimates consistent with our model predictions, and statistical significance for each, the reported standard errors in the tables reported in the text do not take into account the probabilities of committing any Type I error among all of the hypotheses tested. We therefore jointly re-estimated six key equations, five containing the hypotheses derived from the model for effort and test scores and one that we used to provide ancillary evidence on the credibility of the effort and achievement test findings by school competition policy (peer student assistance). The joint estimates are obtained using bootstrapping algorithms (Romano and Wolf, 2016) to obtain Romano and Wolf's (2005a,b) step-down adjusted *p*-values for each coefficient that are robust to multiple hypothesis testing and that take into account the non-independence of error rates across equations that are ignored using a Bonferroni adjustment.

Appendix Table F reports the estimated coefficients and their associated Romano-Wolf Stepdown Adjusted *p*-values for six equations, with all equation errors clustered at the county level. As noted, when we are testing sign predictions, not just hypotheses of the existence of treatment effects, we report one-tail adjusted *p*-values to assess statistical significance. The first column replicates the estimates from equation (7) for student effort among strong native-born students across all high schools but with corrected p-values. The second column replicates the estimates from equation (9), which adds interaction terms that permit tests of whether the effects of peers on strong-student effort are stronger in schools encouraging competition. The adjusted p-values confirm the statistical significance conclusions of the text, indicating that adding strong (weak) students decreases (increases) strong-student incumbent effort on average in all high schools and the effects are amplified in schools encouraging competition. The third column reports results using the same interactive specification but for whether the student received homework assistance from a peer student. These confirm that adding strong peers increases the likelihood of peer assistance, but that these effects are statistically significantly less in more competitive schools.

Columns four and five report the estimates with corrected p-values for test scores in high schools that are less and more competitive, respectively. These results confirm that only in schools that encourage competition less do we observe a statistically significant positive effect of adding strong peers to the cohort of strong incumbent students. Finally, in the last column, effort effects are estimated across all incumbent native-born students to test whether the effects predicted by the model from adding strong and weak students are only statistically significant for strong students. These results are confirmed using the adjusted p-values.

Finally, the penultimate row in the table reports randomization p-values for the hypotheses that the two treatment variables – SEA refugees by ability type – are jointly significant in each equation. This is confirmed for all equations except for that determining test scores in schools that strongly encourage grade competition, consistent with negative competition effects offsetting the positive and significant strong-student peer learning effects seen in the less competitive high schools. The last row reports the-p-value for testing the null that the treatment variables do not have any effects in the full set of equations. These p-values are estimated using bootstrapping, as described in Young (2019), and take into account the potential existence of strongly influential outlier observations (leverage). We soundly reject the null that the two treatment variables together have no statistically significant effect in the complete set of equations.

Appendix Table A
Percent of SEA Refugee 10th-Grade Students Speaking English "Well" or "Very Well" in 1990, by Student Background

Category	All	Entered 1985-89	Entered 1975-88
Strong students	93.7	75.3	96.6
Weak students	86.8	64.2	93.8
N	2,030	447	1,583
t-statistic: strong = weak [p]	3.28 [0.001]	1.36 [0.173]	1.71 [0.087]

Source: 1990 Census 5% Public Use Sample. Student strength is defined by parental schooling: strong students have parents with schooling beyond high school. Weak students are the complement. Census person weights are used.

Appendix Table B
Panel First-Difference IV Estimates of the Effects of the Assigned Numbers of Student-Age SEA Refugees 1991-92 in the County by Type on the Log Student Homework Time, Log Teacher Assigned Homework Time, and Whether a Fellow Student Helped with Homework for Strong Native-Born High-School Students 1990-92 Using Shift Share Instruments

Dependent variable	Log Weekly Homework Hours Spent by the Student	Log Teacher Assigned Daily Homework Minutes per Day	Whether Fellow Student Helped with Homework	Standardized Composite Test Score
Number of assigned weak school-age SEA refugees in the county 1991-1992	0.00490** (0.00153)	-0.00123 (0.000951)	-0.000131 (0.000781)	-0.00614 (0.0543)
Number of assigned strong school-age SEA refugees in the county 1991-1992	-0.0212** (0.00708)	0.00785*[a] (0.00403)	0.000845 (0.00342)	0.0461** (0.0234)
School cohort size	-0.000258 (0.000294)	-0.000322 (0.000274)	-0.000273 (0.000225)	0.00238 (0.00234)
Size of cohort in the county	0.00000224 (0.00000286)	0.0000074**[a] (0.0000034)	-0.0000060**[a] (0.0000029)	0.0000137 (0.0000157)
N	2,130	1,190	1,790	1,800
Kleinbergen-Paap under-identification test $\chi^2(117)$ [p]	140.6 [0.086]	117.7 [0.386]	141.5 [0.061]	140.1 [0.064]
Cragg-Donald Wald F test of weak instruments	73.5	51.7	56.8	73.9
Hansen *J*-statistic over-identification test $\chi^2(116)$ [p]	122.1 [0.379]	111.3 [0.472]	122.8 [0.315]	118.4 [0.394]
Endogeneity test $\chi^2(2)$ [p]	0.739 [0.691]	9.69 [0.008]	0.003 [0.993]	2.60 [0.273]

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Education Longitudinal Study of 1988 (NELS:88), "Panel, Student, School, and Parent Survey, 1990 and 1992" and the Office of Refugee Resettlement. Standard errors in parentheses clustered by county. The strong student sample includes native-born students whose mother and father have schooling above high school, who attended the same school in 10th and 12th grade, and who resided in a county that experienced new assignments of school-age SEA refugees between 1990 and 1992. The teacher sample consists of a science or math teacher of the strong students. The specification includes whether or not the sampled teacher was a science teacher. The excluded instruments in columns 3 and 6 are commuting-zone fixed effects, the projected total number of SEA refugees in 1989 who would be ages 15-16 in 1989, the projected number of SEA female refugees with schooling greater than high school in the county who would be aged 35-50 in 1989, and the total number of SEA female refugees in the county in who would be 35-50 in 1989 based on the initial county assignments of the SEA refugees from 1980 through 1989 by ORR. *[a]10% significance, two-tailed test; **[a]5% significance at least, two-tailed test. *10% significance, one-tailed test; **5% significance at least, one-tailed test.

Appendix Table C
Panel Estimates of the Effects of the Assigned Numbers of Student-Age SEA Refugees 1991-92 in the County by Type on Student Weekly Homework Hours for Strong Native-Born High-School Students, by Measurement of Weekly Hours, Estimation Procedure and School Grade Competition Policy

Dependent variable measure		Inverse Hyperbolic Sine			Linear	
Variable/estimation procedure	First-difference	FD-IV	First-difference	First-difference	FD-IV	First-difference
Number of assigned weak school-age SEA refugees in the county 1991-1992	0.00406** (0.00151)	0.00411** (0.00161)	-	0.0206** (0.0108)	0.0204** (0.0123)	-
Number of assigned strong school-age SEA refugees in the county 1991-1992	-0.0163** (0.00623)	-0.0167** (0.00676)	-	-0.0812** (0.0451)	-0.0827* (0.0517)	-
School grade competition is encouraged						
Number of assigned weak school-age SEA refugees in the county 1991-1992	-	-	0.00779** (0.00236)	-	-	0.0452** (0.0118)
Number of assigned strong school-age SEA refugees in the county 1991-1992	-	-	-0.0337** (0.0106)	-	-	-0.211** (0.0546)
School grade competition is weakly or not encouraged	-	-				
Number of assigned weak school-age SEA refugees in the county 1991-1992	-	-	0.00352** (0.00157)	-	-	0.0205** (0.00856)
Number of assigned strong school-age SEA refugees in the county 1991-1992	-	-	- 0.0144** (0.0222)	-	-	-0.0777** (0.0353)
N	2,230	2,230	1,950	2,230	2,230	1.950
Kleinbergen-Paap under-identification test $\chi^2(117)$ [p]	-	141.5 [0.088]	-	-	141.5 [0.088]	-
Cragg-Donald Wald F test of weak instruments	-	70.2	-	-	70.2	-
Hansen *J*-statistic over-identification test $\chi^2(116)$ [p]	-	124.4 [0.349]	-	-	121.6 [0.417]	-
Endogeneity test $\chi^2(2)$ [p]	-	0.455 [0.801]	-	-	1.69 [0.429]	-

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Education Longitudinal Study of 1988 (NELS:88), "Panel, Student, School, and Parent Survey, 1990 and 1992" and the Office of Refugee Resettlement. Standard errors in parentheses clustered by county. See notes to Table 4 for descriptions of the sample criteria, included control variables, and instruments.*10% significance, one-tailed test; **5% significance at least, one-tailed test.

Appendix Table D

First-Difference Panel Estimates Test Statistics: Are the Strong and Weak Refugee Effects Across Schools by Student Ability Tracking Different for Strong Native-Born Students, by Dependent Variable

Dependent variable	Log Student Homework Time	Log Teacher Assigned Homework Time	Student Helped with Homework	Parent helped with Homework Often	Standardized Score: Math/Reading Test
H_0: Strong SEA refugee effects are = by tracking policy $\chi(1)$ [p]	0.03 [0.863]	0.09 [0.765]	0.14 [0.713]	0.61 [0.435]	0.84 [0.360]
H_0: Weak SEA refugee effects are = by tracking policy $\chi(1)$ [p]	0.01 [0.931]	0.21 [0.643]	0.36 [0.560]	0.96 [0.327]	0.75 [0.385]
N	2,140	1,380	1,820	1,970	1,850

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Education Longitudinal Study of 1988 (NELS:88), "Panel, Student, School, and Parent Survey, 1990 and 1992" and the Office of Refugee Resettlement. Standard errors in parentheses were clustered by county. The sample includes native-born students whose mother and father have schooling above high school, who attended the same school in 10^{th} and 12^{th} grade, and who resided in a county that experienced new assignments of school-age SEA refugees between 1990 and 1992. All specifications include the school cohort's class size and the number of students in the cohort at the county level. The teacher specification includes whether the sampled teacher is a science teacher.

Appendix Table E
ML Control Function Estimates of SEA Refugee Student Effects on Native-Born Strong Students' Log Homework Time and Test Scores

	Probit	First Difference			
Dependent variable	Same School 1990-1992	Log Homework	Composite Test Score	Log Homework	Composite Test Score
Number of assigned weak school-age SEA refugees in the county 1991-1992	0.00547 (0.00351)	0.00401** (0.00171)	-0.00409 (0.00622)	-	-
Number of assigned strong school-age SEA refugees in the county 1991-1992	-0.0201 (0.0146)	-0.0144** (0.00717)	0.0419 (0.0264)	-	-
Family income ($x10^{-3}$), 1988	- 0.00253*[a] (0.00146)	-	-	-	-
Composite test score, 1990	0.0218**[a] (0.0105)	-	-	-	-
School encourages competition					
Number of assigned county weak school-age SEA refugees 1991-92	-	-	-	0.00733** (0.00324)	0.00226 (0.0166)
Number of assigned county strong school-age SEA refugees 1991-92	-	-	-	-0.0299** (0.0144)	0.00481 (0.0761)
School does not encourage competition					
Number of assigned county weak school-age SEA refugees 1991-92	-	-	-	0.00393** (0.00168)	-0.00864 (0.00615)
Number of assigned county strong school-age SEA refugees 1991-92	-	-	-	-0.0150** (0.00718)	0.0608**[a] (0.0253)
ρ	-	0.0312 (0.0709)	0.336**[a] (0.0694)	0.00730 (0.410)	0.342**[a] (0.0918)
N	2,140	2,060	2,060	2,060	2,060

Standard errors in parentheses clustered at the county (191). Test score and homework time specifications include school-level and county-level cohort sizes. *[a]significant at the 10% level, two-tailed test. **[a]significant at at least the 5% level, two-tailed test. **significant at at least the 5% level, one-tailed test. Source: NELS:88, 1988, 1990 and 1992..

Appendix Table F
Panel First-Difference Coefficient Estimates and Romano-Wolf Step-down Adjusted p-values of the Effects of the Assigned Numbers of Student-Age SEA Refugees 1991-92 in the County by Type on the Log Student Homework Time, on Whether a Fellow Student Helped with Homework, and on the Composite Test Scores for Strong Native-Born High-School Students, by School Competition Policy, and the Effects of SEA Refugee Types on Incumbents, by Type, for All Students, 1990-92

Student Sample	Strong Native-born					All Native-born
Dependent variable	Log Weekly Homework Hours Spent by the Student		Whether Fellow Student Helped with Homework	Composite Math/Reading Test Score		Log Weekly Homework Hours Spent by the Student
High-school sample		All		Less Competitive	Strongly Competitive	All
Added number of assigned weak school-age SEA refugees	0.00459	0.00405	-0.00216	-0.00943	0.0060	-0.00210
Romano-Wolf Step-down Adjusted p-value	[0.0072]	[0.0430]	[0.0430]	[0.0797]	[0.8673]	[0.1820]
Added number of assigned strong school-age SEA refugees	-0.0170	-0.0132	0.00858	0.0613	-0.00108	0.0103
Romano-Wolf Step-down Adjusted p-value	[0.0132]	[0.0797]	[0.0034]	[.0193]	[0.9867]	[0.1593]
Weak SEA refugess*competitive school	-	0.00634	0.00312	-	-	-
Romano-Wolf Step-down Adjusted p-value		[0.0432]	[0.0427]			
Strong SEA refugees*competitive school	-	-0.0370	-0.0132	-	-	-
Romano-Wolf Step-down Adjusted p-value		[0.0139]	[0.0139]			
Weak SEA refugees* strong incumbent native-born	-	-	-	-	-	0.00715
Romano-Wolf Step-down Adjusted p-value						[0.0007]
Strong SEA refugees* strong incumbent native-born	-	-	-	-	-	-0.0298
Romano-Wolf Step-down Adjusted p-value						[0.0014]
N	2,230	1,950	1,590	680	940	4,840
Westfall-Young joint test of irrelevance of refugee effects p-value	0.00733	0.0277	0.0187	0.0237	0.920	0.00018
Westfall-Young joint test of irrelevance of refugee effects overall p-value				0.00015		

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Education Longitudinal Study of 1988 (NELS:88), "Panel, Student, School, and Parent Survey, 1990 and 1992" and the Office of Refugee Resettlement. All specifications include the school cohort size and the number of students in the same grade in the county. Strong students and school competition policy defined in the text. Number of iterations for both tests=2,999. All error terms are clustered at the county level.

Appendix Table G
Measurement Model Estimates for 10th-Grade Skill

Exam score measure	Constant	Factor loading λ_j	Proportion signal
Standardized reading	3.90	1.0	
	(0.00156)	(normalized)	0.725
Standardized mathematics	3.91	1.016	
	(0.00156)	(0.00673)	0.732
Standardized history	3.90	1.029	
	(0.00157)	(0.00669)	0.737
Standardized science	3.90	1.007	
	(0.00151)	(0.00156)	0.728
N		17,490	

Source: U.S. Department of Education, National Center for Education Statistics, National Education Longitudinal Study of 1988 (NELS:88), "Panel, Student, School, and Parent Survey, 1988, 1990, and 1992," second round. Standard errors in parentheses.

Appendix Table H
Achievement Function Estimates Using the Latent Factor Method and Instrumental Variables:
Effects of Log Homework Time, Parental Help with Homework, Fellow Student Help with Homework, and Presence of a Computer in the Home on Log Math and Reading Test Score Performance, Cumulative GPA and Percentile Rank in 12^{th} Grade, Conditional on Log Latent Skill in 10^{th}-Grade

Outcome	Log Math/Reading Test Score	Log Cumulative GPA	Log Percentile Class Rank
Log weekly homework hours	0.0496**	0.199**	-1.045**
	(0.0119)	(0.0371)	(0.189)
Parent helped with homework or projects	-0.0391*	0.0207	0.344
	(0.0219)	(0.0677)	(0.359)
Fellow student helped with homework	0.102**	-0.0565	0.796
	(0.0495)	(0.150)	(0.803)
Computer in the home	0.0300	0.0463	- 0.436
	(0.0252)	(0.0826)	(0.392)
Log 10^{th}-grade latent test score	0.931**	0.690**	- 2.99**
	(0.0320)	(0.108)	(0.500)
School fixed effects included	Y	Y	Y
N	4,530	3,800	3,820
Kleinbergen-Paap under-identification test $\chi^2(5)$ [p]	22.8 [0.001]	19.3 [0.004]	17.8 [0.001]
Hansen *J*-statistic over-identification test $\chi^2(4)$ [p]	8.14 [0.149]	7.47 [0.188]	6.01 [0.305]
Endogeneity test: all endogenous variables, $\chi^2(4)$ [p]	97.5 [0.000]	105.3 [0.000]	109.9 [0.000]

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Education Longitudinal Study of 1988 (NELS:88), "Panel, Student, School, and Parent Survey, 1988, 1990, and 1992."The sample consists of students attending the same school with test scores in the 10^{th} and 12^{th} grades. Only 3.7% of native-born students in the panel sample did not attend the same school in both survey years. Standard errors in parentheses clustered by school. ** at least 5% significance, two-tailed test.

Appendix Figures

Figure A1: Distribution of Weekly Homework Hours for 12th-Grade Native-born Strong Students with Normal Density Plot

SOURCE: US Department of Education, National Center for Education Statistics, National Education Longitudinal Study of 1988, Round 3

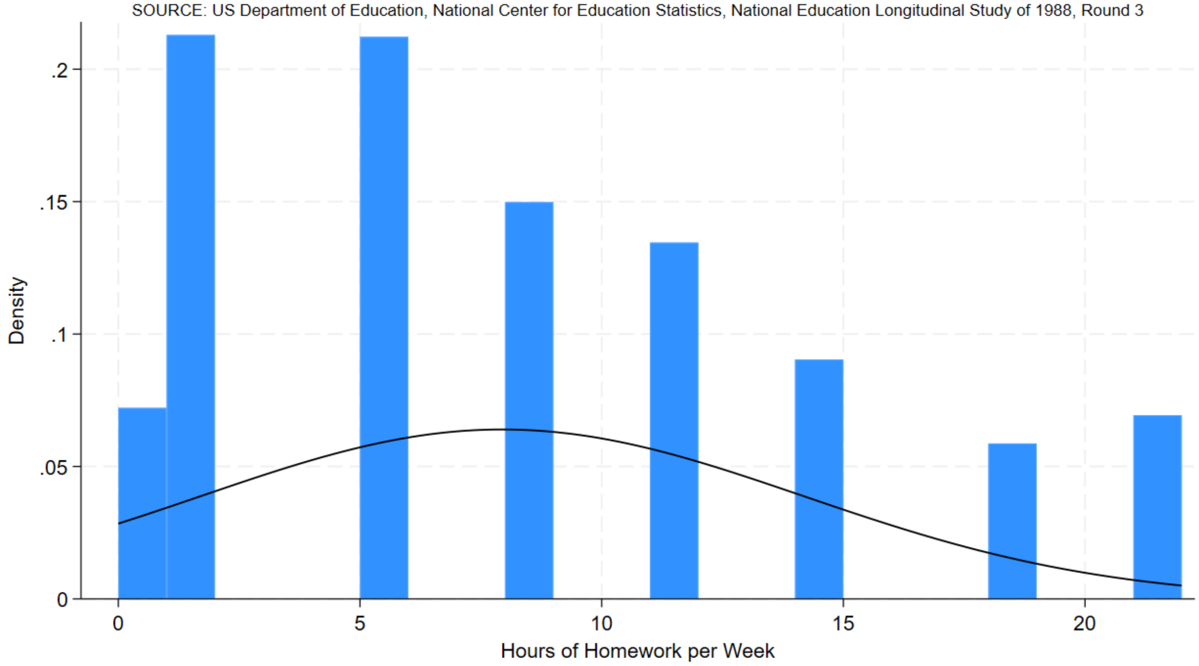


Figure A2: Estimated Effect of Increasing the Number of High-Ability SEA Refugee Students on Log Homework Hours of Native-Born Students in Competitive Schools, by Total Household Income

SOURCE: US Department of Education, NCES, National Education Longitudinal Study of 1988, Panel 1990-92

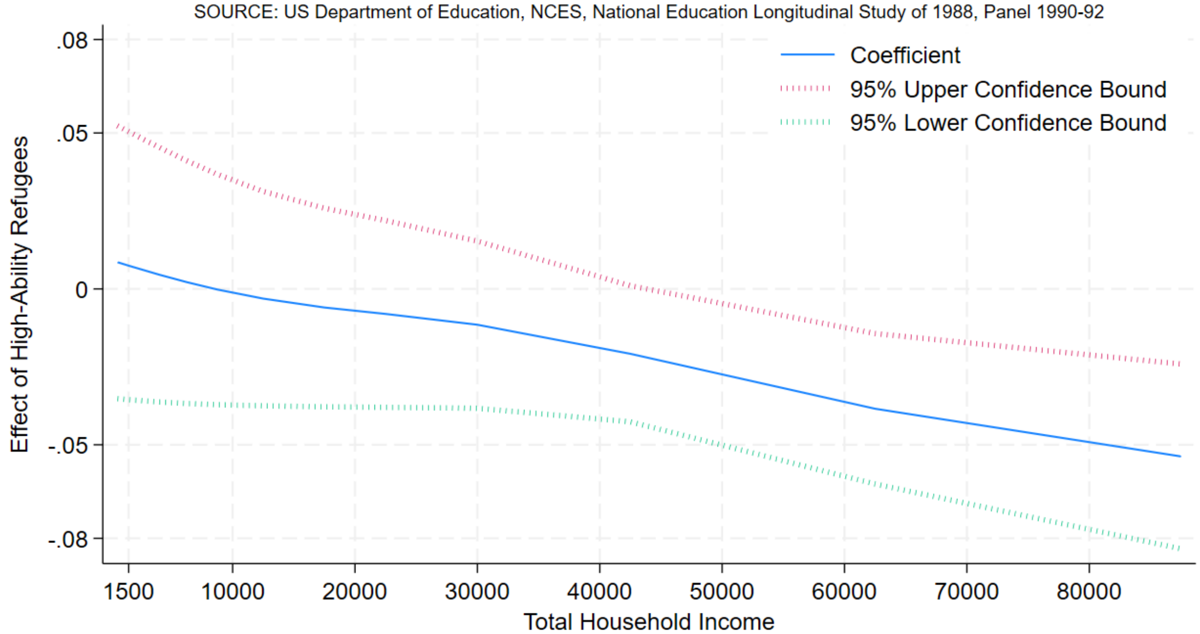


Figure A3: Estimated Effect of Increasing the Number of Low-Ability SEA Refugee Students
on Log Homework Hours of Native-Born Students in Competitive Schools, by Total Household Income

SOURCE: US Department of Education, NCES, National Education Longitudinal Study of 1988, Panel 1990-92

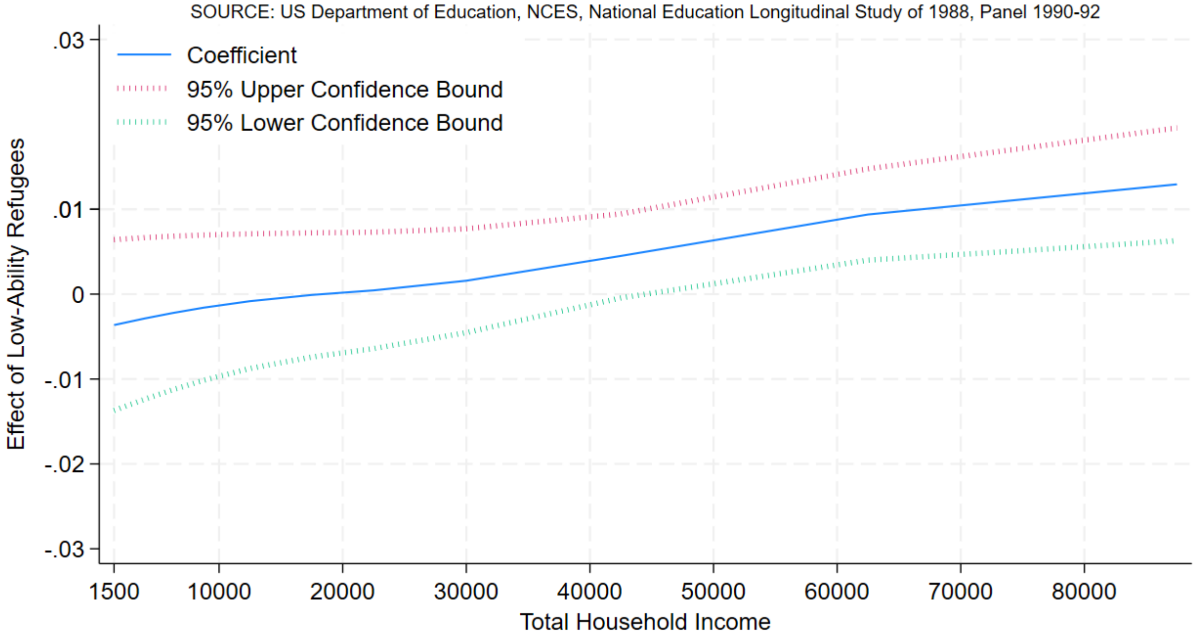


Figure A4. Kernal-weighted (Epanechnikov) Local Polynomial Smoothing Estimates Using Sample Weights (BW=6
Total Number of County High Schools and the Fraction Encouraging Grade Competition

SOURCE: US Department of Education, NCES, National Education Longitudinal Study of 1988, Round 3

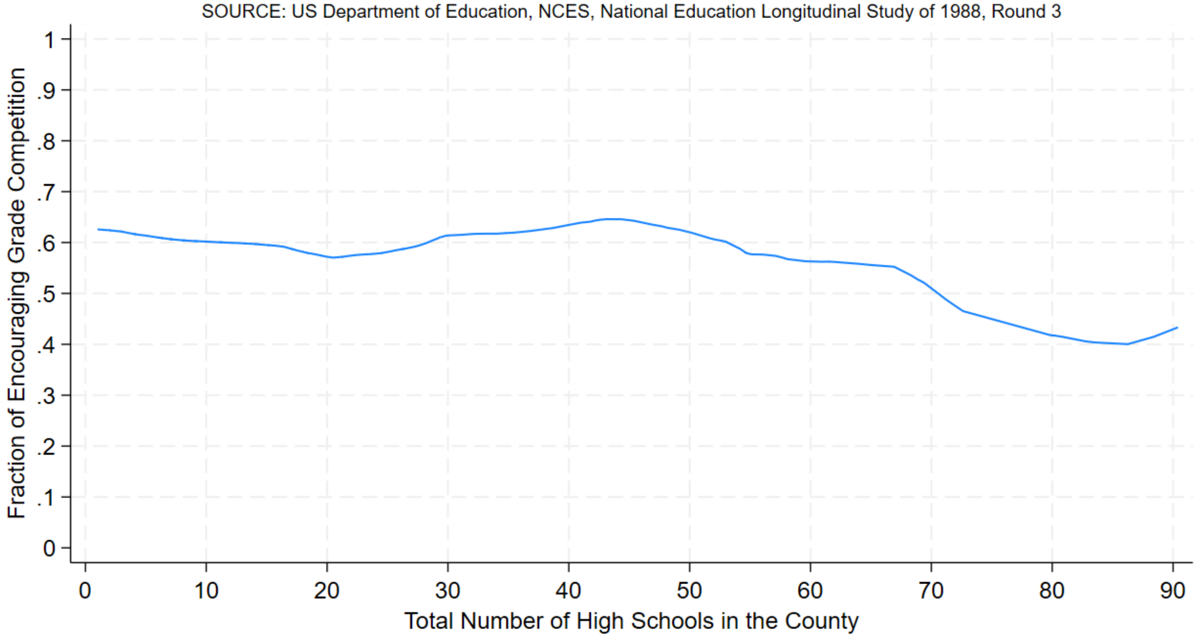


Figure A5. Fraction of Students Receiving A Grades by Tertiary Institution Type in 2009

Source: Rojstaczer and Healy, 2012

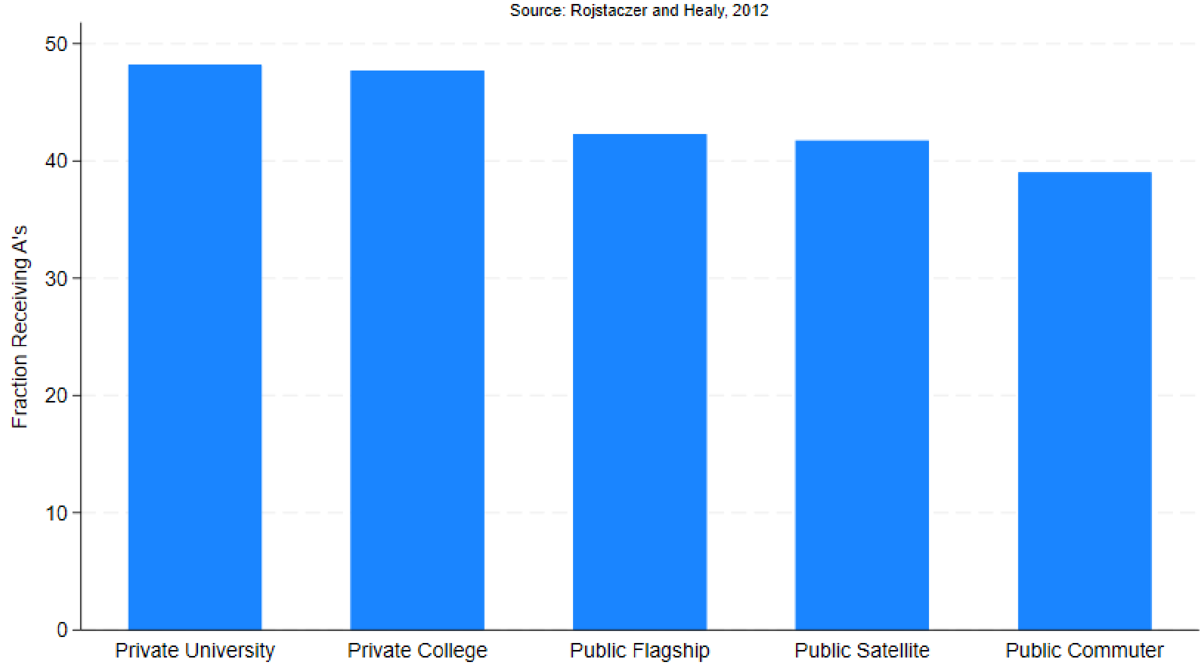


Table 1
Testable Predictions from the Matching Equilibrium Competition Model with Peer Learning:
Effects on the Effort and Academic Achievement of Strong Incumbent Students from Changes in Class Composition.

Added Strong Students		
Academic environment/outcome	Effort	Achievement
All schools	Negative	Unsigned
Difference between strongly competitive and less competitive school	Positive (in absolute value)[a]	Negative[b]
Difference between strong incumbent student and weak incumbent student	Negative	-
Added Weak Students		
Academic environment/outcome	Effort	Achievement
All schools	Positive	Positive
Difference between strongly competitive and less competitive school	Positive	Negative
Difference between strong incumbent student and weak incumbent student	Positive	-

[a] That is, there will be a more negative effect of adding strong students in more competitive schools compared with less competitive schools.

[b] Whatever the sign of the net (of peer learning and effort effects) influence of adding strong SEA students to the class on the test scores of the strong native born in each type of school, the difference in the effect across competitive and less competitive schools will always be negative because the effort effect is more negative and peer assistance is attenuated in the more competitive schools. There are therefore three possible cases consistent with the model: (i) the effect is negative in both type of schools: the effect should be more negative in the strongly competitive schools; (ii) the effect on incumbent test scores is positive in all schools: the effect should be less positive in the strongly competitive schools; and (iii) in less competitive schools the effect of adding strong students on incumbent test scores is positive and in more competitive schools the effect is negative, so the difference is negative.

Table 2
Comparison of Strong and Weak Students Using Parental Schooling Criterion for the Cutoff:
Native-Born and SEA Refugee Students in the 10th Grade

Outcome	Mean (SD) Standardized Math Test Score 1988		Mean (SD) Standardized Reading Test Score 1988		Mean (SD) Weekly Homework Hours		Percent Who Plan to Take the SAT or ACT	
Student type	Native-born	SEA Refugee	Native-born	SEA Refugee	Native-born	SEA Refugee	Native-born	SEA Refugee
Strong	56.4 (13.3)	59.1 (9.21)	55.8 (11.8)	52.3 (7.91)	5.60 (4.80)	6.90 (6.18)	80.1 (39.9)	75.0 (43.9)
Weak	50.6 (13.4)	54.0 (11.6)	50.8 (13.5)	51.6 (12.2)	3.93 (3.87)	6.69 (4.79)	58.6 (49.3)	73.3 (44.5)
t-statistic: within-group strong = weak[p]	25.8 [0.000]	2.43 [0.016]	21.5 [0.000]	0.34 [0.855]	22.3 [0.000]	0.220 [0.828]	25.2 [0.000]	0.190 [0.849]
t-statistic: strong SEA= strong native-born [p]	0.930 [0.355]		1.20 [0.230]		1.12 [0.263]		0.520 [0.605]	
t-statistic: weak SEA= weak native-born [p]	1.71 [0.087]		0.390 [0.699]		4.69 [0.000]		1.94 [0.053]	
Sample N	15,100	150	15,100	150	14,160	150	6,050	90

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Education Longitudinal Study of 1988 (NELS:88), "Panel, Student, School, and Parent Survey," 1988 and 1990 rounds. Strong students are defined by whether both the mother's and father's schooling exceed high school. Sample weights are used.

Table 3
Descriptive Statistics for Panel Sample of Strong Native-Born Students Staying in the Same High School and Experiencing a Change in Student-Age SEA Refugees in the County, 1990-92

Variable/year	1990 (10th grade)	1992 (12th grade)
Standardized composite math and reading test score	57.4 (8.76)	57.3 (8.57)
Weekly homework hours by student	6.68 (5.27)	8.48 (6.56)
Minutes per day of homework assigned by teachers	40.7 (25.2)	36.7 (17.3)
Student helped with homework	0.608	0.851
Parents helped with homework 'frequently'	0.093	0.200
Student was bullied in school (was threatened to hurt, was robbed)	0.236	0.165
School cohort size	329.2 (235.0)	283.6 (188.7)
Number of students in the county	14,752 (21,959)	10,206 (14,027)
School encourages grade competition (Principal agrees 'accurate' or 'very accurate')	0.563 (0.496)	
School groups English and math classes by student ability	0.642 (0.480)	
Number of new assigned weak school-age SEA refugees in the county 1991-92 (ORR)	35.4 (71.3)	
Number of new assigned strong school-age SEA refugees in the county 1991-92 (ORR)	7.18 (16.7)	
Number of counties	189	
Number of students	2,225	

SOURCES: U.S. Department of Education, National Center for Education Statistics, National Education Longitudinal Study of 1988 (NELS:88), "Panel, Student, School, and Parent Survey, 1988, 1990, and 1992" and the Office of Refugee Resettlement monthly refugee assignments by county in 1991 and 1992.

Table 4
Panel Estimates of the Effects of the Assigned Numbers of Student-Age SEA Refugees 1991-92 in the County by Type on Student Weekly Homework Hours for Strong Native-Born High-School Students 1990-92, by Estimation Procedure and Measure

Dependent variable transform	Log		Inverse Hyperbolic		Linear	
Variable/estimation procedure	First-difference	FD-IV	First-difference	FD-IV	First-difference	FD-IV
Number of assigned weak school-age SEA refugees in the county 1991-1992	0.00468** (0.00145)	0.00495** (0.00155)	0.00406** (0.00151)	0.00411** (0.00161)	0.0206** (0.0108)	0.0204** (0.0123)
Number of assigned strong school-age SEA refugees in the county 1991-1992	-0.0197** (0.00596)	-0.0212** (0.00654)	-0.0163** (0.00623)	-0.0167** (0.00676)	-0.0812** (0.0451)	-0.0827* (0.0517)
N	2,140	2,140	2,230	2,230	2,230	2,230
Kleinbergen-Paap under-identification test $\chi^2(117)$ [p]	-	144.2 [0.058]	-	141.5 [0.088]	-	141.5 [0.088]
Cragg-Donald Wald F test of weak instruments	-	68.6	-	70.2	-	70.2
Hansen *J*-statistic over-identification test $\chi^2(116)$ [p]	-	123.3 [0.350]	-	124.4 [0.349]	-	121.6 [0.417]
Endogeneity test $\chi^2(2)$ [p]	-	0.459 [0.795]	-	0.455 [0.801]	-	1.69 [0.429]

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Education Longitudinal Study of 1988 (NELS:88), "Panel, Student, School, and Parent Survey, 1990 and 1992" and the Office of Refugee Resettlement. Standard errors in parentheses clustered by county. The strong student sample includes native-born students whose mother and father have schooling above high school, who attended the same school in 10th and 12th grade, and who resided in a county that experienced new assignments of school-age SEA refugees between 1990 and 1992. All specifications include the change in class/cohort sizes at the school and county level. The excluded instruments in columns 3 and 6 are commuting-zone fixed effects, the projected total number of SEA refugees in 1989 who would be ages 15-16 in 1989, the projected number of SEA female refugees with schooling greater than high school in the county who would be aged 35-50 in 1989, and the total number of SEA female refugees in the county in who would be 35-50 in 1989 based on the initial county assignments of the SEA refugees from 1980 through 1989 by ORR. *a10% significance, two-tailed test; **a5% significance at least, two-tailed test. *10% significance, one-tailed test; **5% significance at least, one-tailed test.

Table 5
County Fixed-Effects Estimates: Determinants of Senior-Year Academic Honors, Native-born Strong Students, by School Policy

Variable	(1)	(2)	(3)
Class rank penalty	-0.177**	-0.00695	-0.201
	(0.0860)	(0.0837)	(0.133)
Cumulative grade point average (GPA)	0.270**	0.339**	0.212**
	(0.0354)	(0.0361)	(0.0546)
School encourages grade competition	-	0.512**	0.609**
		(0.244)	(0.212)
Class rank penalty x competition school	-	-0.359**	-0.420**
		(0.155)	(0.133)
GPA x competition school	-	-0.141**	-0.165**
		(0.0649)	(0.0552)
School stratifies classes by achievement in reading and math	-	-	-0.599**
			(0.215)
Class rank penalty x stratification	-	-	0.243*
			(0.135)
GPA x stratification	-	-	0.161**
			(0.0566)
School employs selective admission	-	-	-0.829**
			(0.425)
Class rank penalty x selective	-	-	0.408
			(0.252)
GPA x selective	-	-	0.237**
			(0.110)
Competition school class rank penalty effect at mean class rank	-	-0.143**	-0.361**
		(0.0693)	(0.120)
Stratification school class rank penalty effect at mean class rank	-	-	-0.109
			(0.0963)
Selective admissions school class rank penalty effect at mean class rank	-	-	-0.0461
			(0.163)
Number of students	2,380	2,380	2,380
Number of schools	540	540	540

Standard errors in parentheses clustered at the county and school level. All specifications include school senior-year class size, gender and race. **significant at the 10% level, two-tailed test. **significant at at least the 5% level, two-tailed test. Source: NELS:88, 1992 round.

Table 6
Mean (SD) Weekly Homework Hours of Native-Born High School Seniors, by Student Background and School Competition Policy

Student type	Weak Background Students		Strong Background Students	
School policy	Less Competitive	Competitive	Less Competitive	Competitive
Weekly hours	5.70 (5.53)	5.67 (5.50)	7.48 (6.25)	7.00 (5.97)
Difference	-0.034		-0.48	
H_0: t-test [p]	0.25 [0.802]		2.47 [0.014]	
N	2,830	3,680	1,740	2,410

Source: U.S. Department of Education, National Center for Education Statistics, National Education Longitudinal Study of 1988 (NELS:88), round three. Sample weights are used; the statistics are representative of the all native-born high-school seniors in the United States in the academic year 1991-2. Definitions of strong and weak backgrounds are in the text.

Table7

Panel First-Difference Estimates: The Effects of the Assigned Numbers of School-Age SEA Refugees1991-92 in the County by Type on Log Student Homework Time, Log Teacher Assigned Homework Time, Fellow Student Homework Help, Parent Homework Help, and Student Bullied for Strong High School Native-born Students, 1990-92, by School Grade Competition Policy

Variable	Log Weekly Homework Hours Spent by the Student	Log Teacher Assigned Daily Homework Minutes per Day	Whether Fellow Student Helped with Homework	Whether Parent Helped with Homework Often	Whether Student was Bullied
School grade competition is encouraged					
Number of assigned weak school-age SEA refugees in the county 1991-1992	0.00924** (0.00225)	-0.00191 (0.00206)	0.000822 (0.00127)	0.00174[a] (0.00104)	-0.00218 (0.00181)
Number of assigned strong school-age SEA refugees in the county 1991-1992	-0.0405** (0.0101)	0.0121 (0.00967)	-0.00618 (0.00573)	-0.00689 (0.00449)	0.0107 (0.00780)
School grade competition is weakly or not encouraged					
Number of assigned weak school-age SEA refugees in the county 1991-1992	0.00375** (0.00163)	-0.000160 (0.000842)	-0.0176**[a] (0.00060)	0.000552 (0.000555)	-0.000120 (0.00111)
Number of assigned strong school-age SEA refugees in the county 1991-1992	-0.0157* (0.00674)	0.00265 (0.00330)	0.00757**[a] (0.00244)	-0.00231 (0.00230)	0.000282 (0.000220)
N	1,880	1,080	1,570	1,690	1,920
H_0 test: Strong SEA refugee effects are = by grade competition regime $\chi(1)$ [p], one-tail test	4.71 [0.030]	0.74 [0.391]	4.72 [0.0298]	0.89 [0.347]	1.55 [0.318]
H_0 test: Weak SEA refugee effects are = by grade competition regime $\chi(1)$ [p], one-tail test	4.44 [0.035]	0.51 [0.473]	3.23 [0.0722]	1.11 [0.292]	1.00 [0.318]

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Education Longitudinal Study of 1988 (NELS:88), "Panel, Student, School, and Parent Survey, 1990 and 1992" and the Office of Refugee Resettlement. Standard errors in parentheses clustered by county. The sample includes native-born students whose mother and father have schooling above high school, who attended the same school in 10th and 12th grade, and who resided in a county that experienced new assignments of school-age SEA refugees between 1990 and 1992. *[a]10% significance, two-tailed test; **[a]5% significance at least, two-tailed test. *10% significance, one-tailed test; **5% significance at least, one-tailed test.

Table 8
Panel First-Difference Estimates, All Native-born Students: The Effects of the Numbers of Student-Age SEA Refugees in the County by Type on Log Student Homework Time, by Native-born Student Type and School Competition Policy

Student category and school type	Less Competitive	Strongly Competitive
Treated: Strong native-born students		
Number of assigned weak school-age SEA refugees in the county 1991-1992	0.00388**	0.00885**
	(0.00168)	(0.00227)
Number of assigned strong school-age SEA refugees in the county 1991-1992	-0.0160**	-0.0393**
	(0.00684)	(0.0102)
Treated: Weak native-born students		
Number of assigned weak school-age SEA refugees in the county 1991-1992	-0.00173	-0.00107
	(0.00184)	(0.00298)
Number of assigned strong school-age SEA refugees in the county 1991-1992	0.00790	0.00257
	(0.00745)	(0.0130)
N	1,730	2,110
H_0 test: Strong SEA refugee effect = by native-born ability type $\chi(1)$ [p]	5.72 [0.018]	8.51 [0.004]
H_0 test: Weak SEA refugee effect = by native-born ability type $\chi(1)$ [p]	5.07 [0.0244]	9.47 [0.002]
H_0 test: No native-born weak student response $\chi(2)$ [p]	1.13 [0.567]	2.40 [0.301]
H_0 test: No native-born strong student response $\chi(2)$ [p]	5.49 [0644]	16.0 [0.000]

SOURCES: U.S. Department of Education, National Center for Education Statistics, National Education Longitudinal Study of 1988 (NELS:88), "Panel, Student, School, and Parent Survey, 1990 and 1992" and the Office of Refugee Resettlement monthly refugee assignments by county. Standard errors in parentheses clustered by county. All coefficients are allowed to differ by the ability group of the native-born students. Strong native-born students are those whose mother and father has schooling above high school; weak native-born students are the remainder. All treated students in the panel sample attended the same school in 10^{th} and 12^{th} grade and resided in a county in which new school-age SEA refugees were assigned between 1990 and 1992. County-level strong and weak SEA refugee students are from the ORR administrative records for the panel.
*10% significance, one-tailed test; **5% significance, one-tailed test.

Table 9
Panel Estimates of the Effects of the Assigned Numbers of Student-Age SEA Refugees 1991-92 in the County by Type on the Standardized Composite Math and Reading Test Scores for Strong Native-Born High-School Students, by Estimation Procedure and School Grade Competition Policy

Variable/estimation procedure	First-difference	FD-IV	First-difference
Number of assigned weak school-age SEA refugees in the county 1991-1992	-0.00402 (0.00525)	-0.00612 (0.00558)	-
Number of assigned strong school-age SEA refugees in the county 1991-1992	0.0405*[a] (0.0224)	0.0478**[a] (0.00730)	-
School grade competition is encouraged			
Number of assigned weak school-age SEA refugees in the county 1991-1992	-	-	0.00522 (0.0153)
Number of assigned strong school-age SEA refugees in the county 1991-1992	-	-	-0.00766 (0.0715)
School grade competition is weakly or not encouraged	-	-	
Number of assigned weak school-age SEA refugees in the county 1991-1992	-	-	-0.00837 (0.00542)
Number of assigned strong school-age SEA refugees in the county 1991-1992	-	-	0.0568**[a] (0.0222)
N	1,810	1,810	1,610
Kleinbergen-Paap under-identification test $\chi^2(117)$ [p]	-	142.3 [0.049]	-
Cragg-Donald Wald F test of weak instruments	-	68.9	-
Hansen *J*-statistic over-identification test $\chi^2(116)$ [p]	-	118.5 [0.393]	-
Endogeneity test $\chi^2(2)$ [p]	-	0.299 [0.861]	-

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Education Longitudinal Study of 1988 (NELS:88), "Panel, Student, School, and Parent Survey, 1990 and 1992" and the Office of Refugee Resettlement. Standard errors in parentheses clustered by county. The sample includes native-born students whose mother and father have schooling above high school, who attended the same school in 10th and 12th grade, and who resided in a county that experienced new assignments of school-age SEA refugees between 1990 and 1992. All specifications include the school cohort's class size and the number of students in the cohort at the county level. The teacher specification includes whether the sampled teacher is a science teacher. The excluded instruments in columns 3 and 6 are commuting-zone fixed effects, the projected total number of SEA refugees in 1989 who would be ages 15-16 in 1989, the projected number of SEA female refugees with schooling greater than high school in the county who would be aged 35-50 in 1989, and the total number of SEA female refugees in the county in who would be 35-50 in 1989 based on the initial county assignments of the SEA refugees from 1980 through 1989 by ORR. *a10% significance, two-tailed test; **[a]5% significance at least, two-tailed test; *[a]10% significance, two-tailed test; *10% significance, one-tailed test; **5% significance at least0.00174
(0.0, one-tailed test.

Table 10
First-Difference Panel Estimates of County-Level SEA Refugee Effects on the Log Homework Time of Native-Born Strong Students, by the Number of High Schools in the County, 1990-92

Variable	
Number of assigned weak school-age SEA refugees in the county 1991-92 (π_1)	0.0103** (0.002788)
Number of assigned strong school-age SEA refugees in the county 1991-92 (π_2)	-0.0268** (0.0112)
School cohort size	-0.00119**[a] (0.000556)
County cohort size	0.0000273 (0.0000265)
Number of assigned weak school-age SEA refugees in the county 1991-92 x number of high schools in the county 1989-90 (π_3)	-0.00011** (0.0000456)
Number of assigned strong school-age SEA refugees in the county 1991-92 x number of high schools in the county 1989-90 (π_4)	0.000314** (0.000180)
School cohort size x number of high schools in the county 1989-90	0.00000226 (0.00000152)
County cohort size x number of high schools in the county 1989-90	-0.000000256**[a] (0.000000114)
Number of assigned weak school-age SEA refugees in the county 1991-92 in counties with one high school ($\pi_1 + \pi_3$ [1])	0.0102** (0.00274)
Number of assigned strong school-age SEA refugees in the county 1991-92 in counties with one high school ($\pi_2 + \pi_4$ [1])	-0.0264** (0.0110)
F(9, 188)	7.55
[p]	[0.000]
Number of students	2,230
Number of counties	190

SOURCES: U.S. Department of Education, National Center for Education Statistics, National Education Longitudinal Study of 1988 (NELS:88), Student, School, and Parent Survey, 1988, 1990, and 1992, NCES Common Core of Data Public Elementary and Secondary School Universe: 1989-90, and NCES Private School Universe Survey, 1989-90. Standard errors clustered at the county level. **[a]significant at at least the 5% level, two-tailed test. **significant at at least the 5% level, one-tailed test.

Table 11
Descriptive Statistics for Achievement Function Sample: All 12th-grade Students in 1992

Variable	Mean (SD)
Mean standardized composite math and reading test score in 1992	55.3 (8.87)
Mean cumulative senior year gpa	2.90 (0.682)
Mean percentile class rank	0.373 (0.268)
Mean weekly homework hours	7.45 (6.05)
Fraction receiving homework help from fellow student	0.860 (.347)
Fraction receiving help with homework or projects from parents	0.632 (0.482)
fraction with a computer in the home	0.503 (0.500)
Mean standardized composite math and reading test score in 1990	55.4 (9.09)
Mean standardized composite math and reading test score in 1988	55.5 (9.80)
Mean minutes per day of homework assigned by math and science teachers	34.9 (17.6)
Mean mother's years of schooling	13.6 (2.41)
Fraction whose father had hopes the student attends college	0.762 (0.426)
Fraction whose parents imposed a homework rule	0.757 (0.429)
Mean number of siblings	2.01 (1.40)
Fraction of students whose family is in the top quartile of the socioeconomic index	0.672 (0.469)
Fraction students native born	0.940 (0.238)
Fraction students Black	0.084 (0.277)
Fraction students male	0.497 (0.500)
Number of schools	860
Number of students	4,580

SOURCES: U.S. Department of Education, National Center for Education Statistics, National Education Longitudinal Study of 1988 (NELS:88), "Student, School, and Parent Survey, 1988, 1990, and 1992." All students attended the same school in 1990 and 1992.

Table 12
Achievement Function First-Stage Estimates

Variable/dependent variable	Log weekly Homework Hours	Parent Helped with Homework	Student Helped with Homework	Home Computer	Log Test Score 1990
Log test score in 1988	0.858**	-0.283**	-0.0878*	0.182**	0.784**
	(0.192)	(0.051)	(0.0451)	(0.0569)	(0.0113)
Log assigned daily minutes of homework by teachers	0.293**	0.00281	-0.0273	-0.00556	0.0212**
	(0.0738)	(0.0216)	(0.0169)	(0.0210)	(0.00404)
Mother's years of schooling	0.0114	0.0175**	-0.00633*	0.0164**	0.000932
	(0.142)	(0.00438)	(0.00331)	(0.00436)	(0.000784)
Whether father had hopes student attends college	0.147**	-0.000742	-0.0315**	0.0561**	0.0151**
	(0.0706)	(0.0213)	(0.0155)	(0.0222)	(0.00415)
Whether parents imposed a homework rule	0.0788	0.187**	-0.00110	-0.000444	-0.00620
	(0.0573)	(0.210)	(0.0150)	(0.0210)	(0.00340)
Number of siblings	0.0310	-0.0178**	-0.0154**	-0.00780	-0.00120
	(0.0210)	(0.00669)	(0.00444)	(0.00674)	(0.00117)
Family is in the top quartile of the socioeconomic index	0.131*	0.0530**	0.0104	0.114**	0.00285
	(0.0748)	(0.0231)	(0.0162)	(0.0230)	(0.00411)
Student native born	-0.215**	0.0817**	-0.0818**	0.0581	-0.0101
	(0.107)	(0.0397)	(0.0331)	(0.0379)	(0.00658)
Student black	-0.0592	0.0440	0.0535	0.0102	-0.0282**
	(0.141)	(0.0402)	(0.0363)	(0.0386)	(0.00816)
Student male	-0.415**	-0.0434**	0.0960**	0.103**	0.00144
	(0.0549)	(0.0109)	(0.0131)	(0.0176)	(0.00310)
Number of schools			860		
Number of students			4,580		
F (10,859) excluded instruments [p]	12.4 [0.000]	16.0 [0.000]	9.63 [0.000]	16.3 [0.000]	646.9 [0.000]
Sanderson-Windmeijer multivariate F (6, 859) [p]	3.79 [0.001]	10.2 [0.000]	3.46 [0.0022]	10.2 [0.000]	7.84 [0.000]

SOURCES: U.S. Department of Education, National Center for Education Statistics, National Education Longitudinal Study of 1988 (NELS:88), "Student, School, and Parent Survey, 1988, 1990, and 1992." All students attended the same school in 1990 and 1992. ** at least 5% significance, two-tailed test, ** 10% significance, two-tailed test.

Table 13
Achievement Function Estimates:
Effects of Log Homework Time, Parental Help with Homework, Fellow Student Help with Homework, and Presence of a Computer in the Home on Log Math and Reading Test Score Performance, Cumulative GPA and Percentile Rank in 12th Grade, Conditional on Log Test Performance in 10th-Grade, by Estimation Procedure

Outcome	Log Math/Reading Test Score		Log Cumulative GPA		Log Percentile Class Rank	
Estimation procedure	FE	FE-IV	FE	FE-IV	FE	FE-IV
Log weekly homework hours	0.00469** (0.000975)	0.0206** (0.00942)	0.0187** (0.00237)	0.176** (0.0349)	-0.0951** (0.0104)	-0.893** (0.179)
Parent helped with homework or projects	-0.000713 (0.00242)	-0.0336** (0.0164)	0.0123* (0.00742)	0.0414 (0.0635)	-0.110** (0.0318)	0.175 (0.325)
Fellow student helped with homework	0.00737* (0.00413)	0.0121 (0.0394)	-0.034** (0.0117)	-0.149 (0.141)	0.0335 (0.0524)	1.23* (0.747)
Computer in the home	0.00804** (0.00250)	0.0563** (0.0194)	0.00306 (0.00684)	0.0247 (0.0722)	-0.0533 (0.0333)	- 0.215 (0.340)
Log 10th-grade math/reading test score	0.861** (0.0927)	0.854** (0.0218)	0.844** (0.0301)	0.732** (0.0851)	-3.96** (0.111)	-3.51** (0.399)
School fixed effects included	Y	Y	Y	Y	Y	Y
N	4,580	4,580	3,840	3,840	3,860	3,860
Kleinbergen-Paap under-identification test $\chi^2(6)$ [p]	-	22.3 [0.001]	-	19.3 [0.004]	-	17.6 [0.007]
Hansen *J*-statistic over-identification test $\chi^2(5)$ [p]	-	5.65 [0.341]	-	7.67 [0.175]	-	6.98 [0.222]
Endogeneity test: all endogenous variables, $\chi^2(5)$ [p]	-	73.9 [0.000]	-	85.5 [0.000]	-	73.9 [0.000]
Endogeneity test: all endogenous variables except lagged test score $\chi^2(4)$ [p]	-	11.5 [0.021]	-	73.2 [0.000]	-	60.9 [0.000]

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Education Longitudinal Study of 1988 (NELS:88), "Panel, Student, School, and Parent Survey, 1988, 1990, and 1992."The sample consists of students attending the same school with test scores in the 10th and 12th grades. Only 3.7% of native-born students in the panel sample did not attend the same school in both survey years. Standard errors in parentheses clustered by school. Instruments for the estimates in columns 2, 4, and 6 are displayed in Table 11 containing the first-stage estimates. ** at least 5% significance, two-tailed test.

Figures

Figure 1. Lowess Estimate of the Probability of Winning an Academic Honor for Native-born High-School Seniors, by Total Parent Years of Schooling

SOURCE: US Department of Education, NCES, National Education Longitudinal Study of 1988 Round 3)

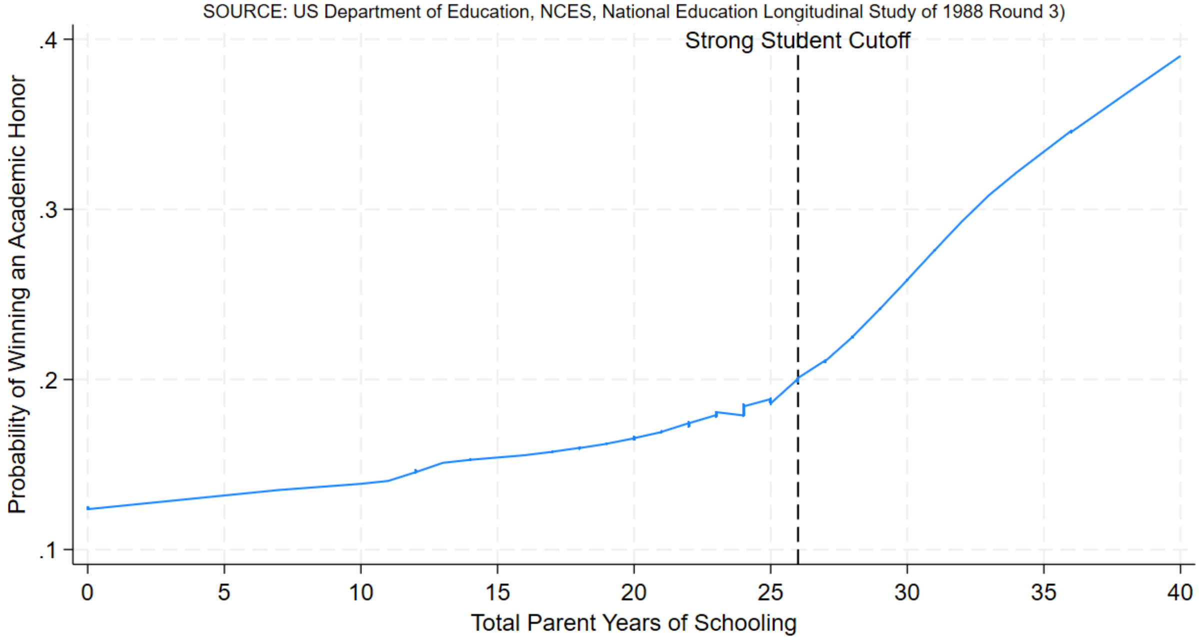


Figure 1a. Lowess Estimate of Weekly Homework Hours for Native-born High-School Seniors, by Total Parent Years of Schooling

SOURCE: US Department of Education, NCES, National Education Longitudinal Study of 1988, Round 3

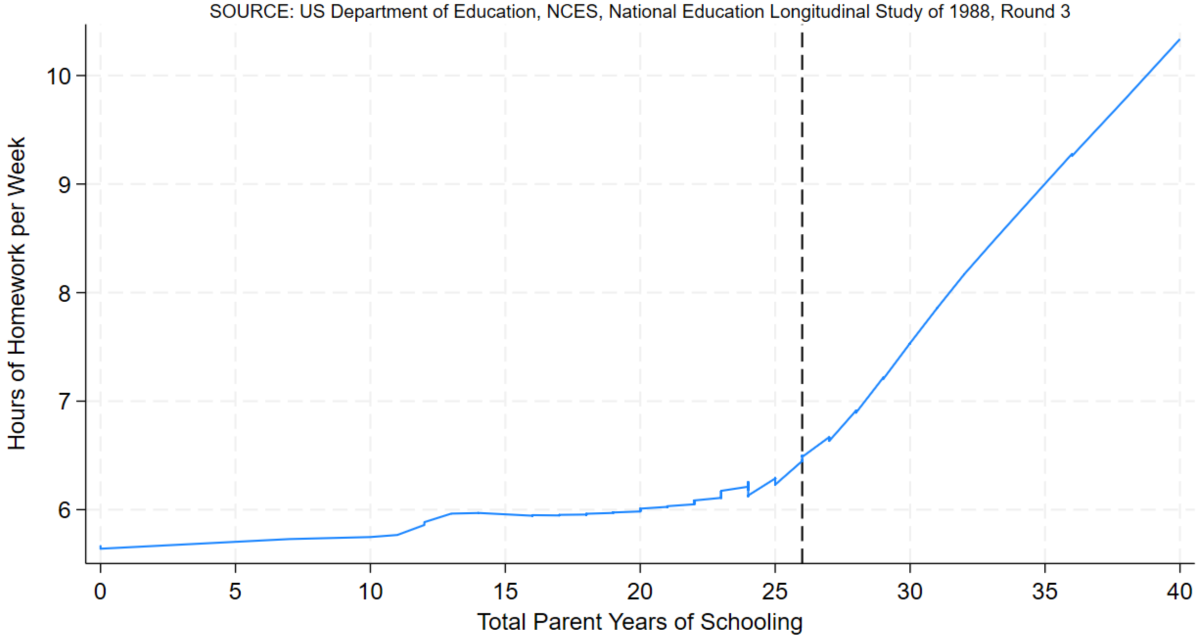


Figure 2. Initial Assigned Locations of SEA Refugees, 1987-90, by County

Source: Office of Refugee Resettlement

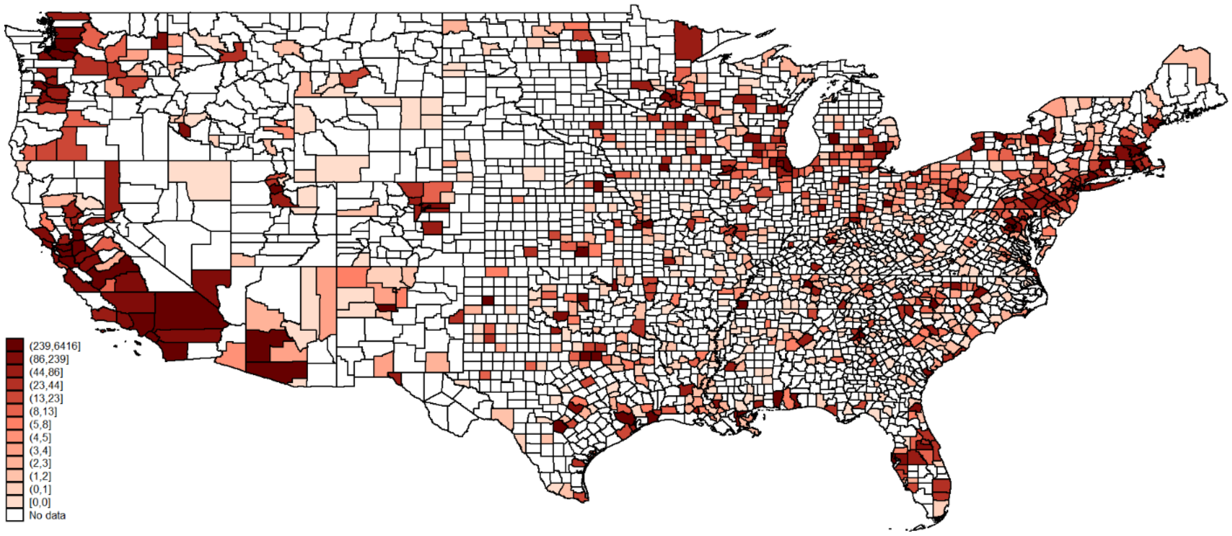


Figure 3. Percent of High-Background Foreign-Born SEA Eighth-Grade Students Who Understand, Speak, Read, and Write English 'Very Well' or 'Well,' by Years in the United States (Source: NELS:88 First Round)

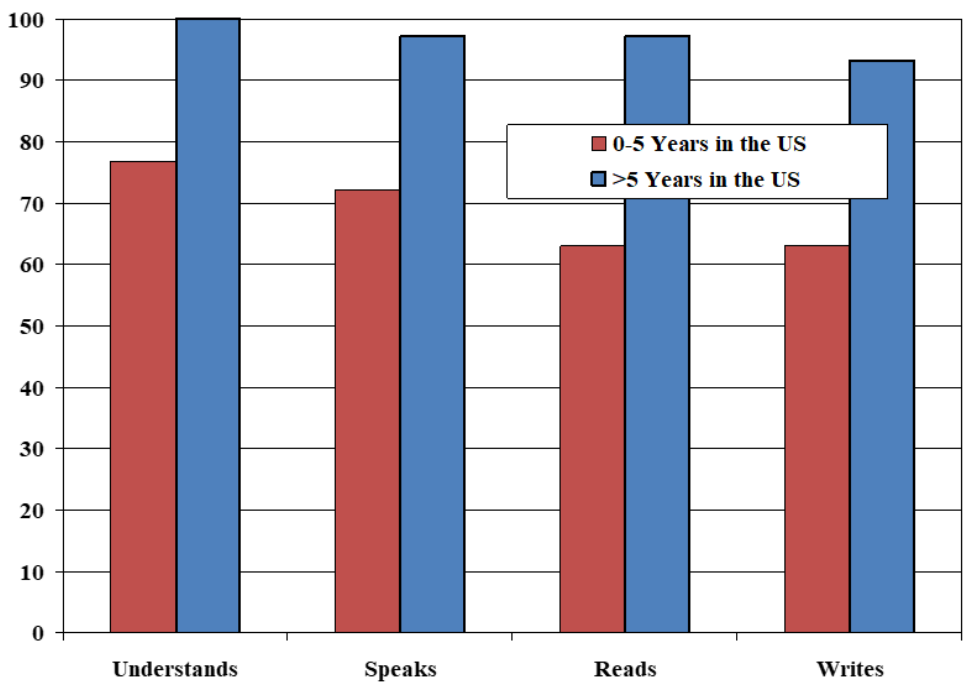


Figure 4. Change in the Ability to Write in English Between 8th and 10th Grade, for New and Earlier SEA Refugee Arrivals in 1988 (Source: NELS:88 First Round and Second Round)

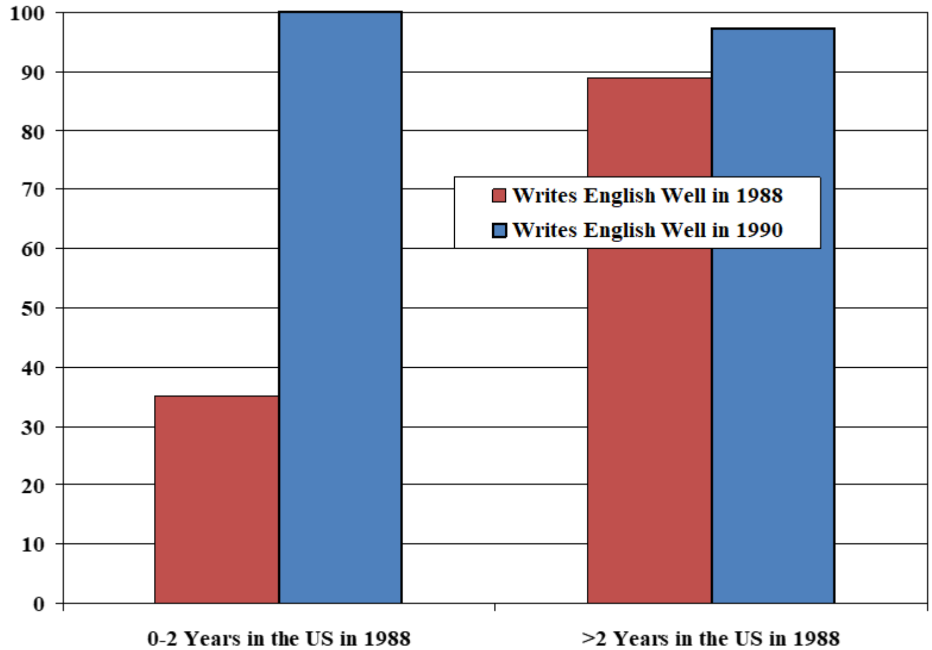


Figure 5. Change in Standardized Composite Math/Reading Test Scores Between 8th and 10th Grade, for New and Earlier SEA Refugee Arrivals in 1988 (Source: NELS:88 First Round and Second Round)

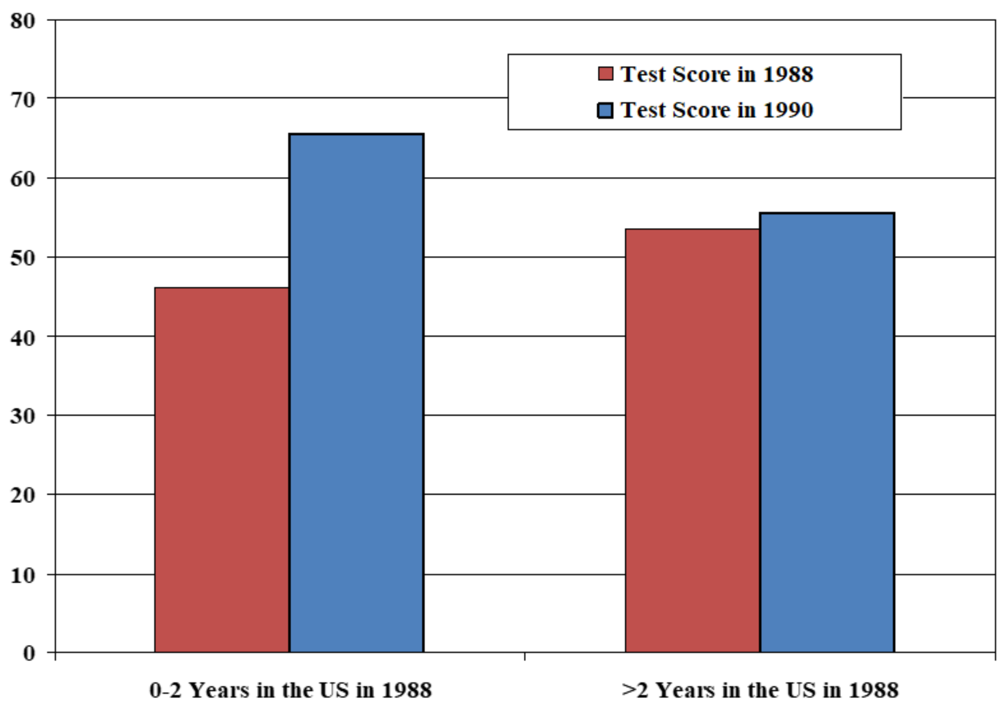


Figure 6. Distribution of the Total Number of High Schools in a County,
All US High Schools and NELS:88 Sampled High Schools

SOURCES: US Department of Education, National Center for Education Statistics, National Education Longitudinal Study of 1988, Rounds 2 and 3
NCES Common Core of Data Public Elementary and Secondary Schools, 1989-90

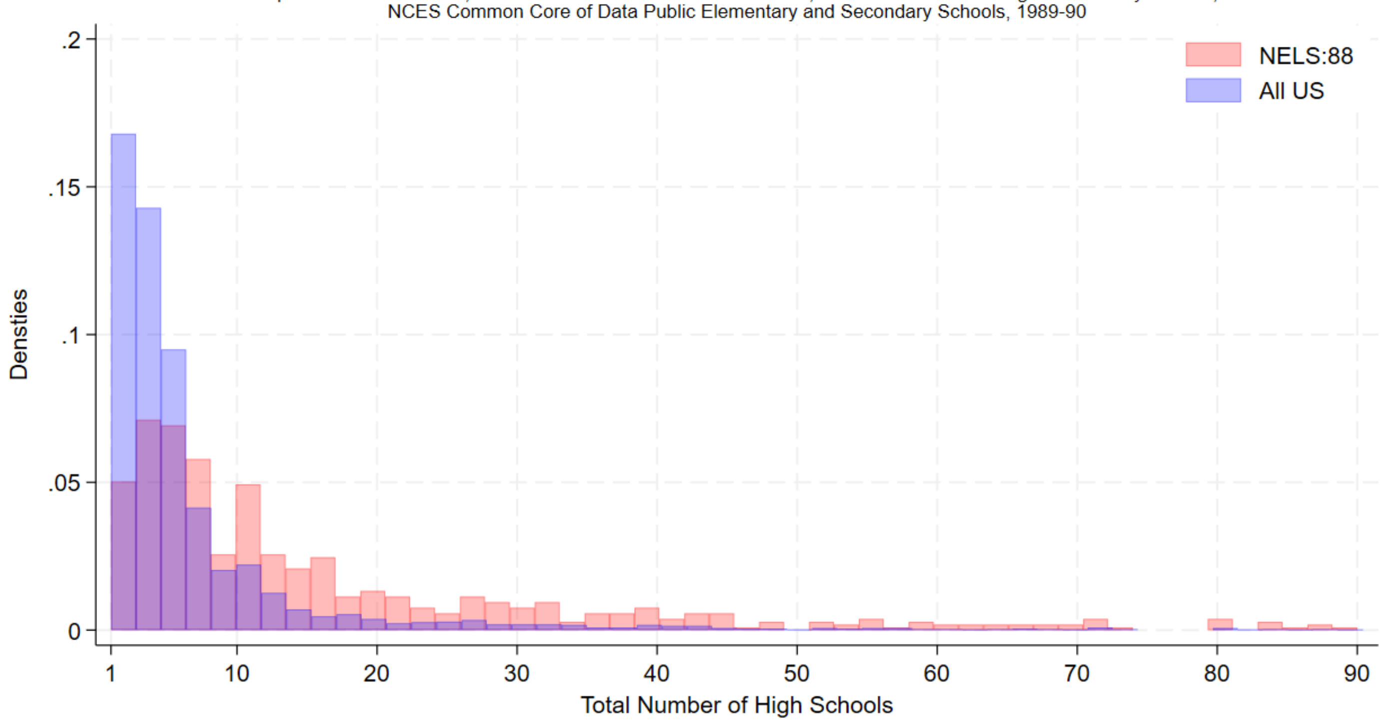


Figure 7: Estimated Effect of Increasing the Number of Low-Ability SEA Refugee Students
on Log Homework Hours of Native-Born Students in Competitive Schools, by Total Parent Years of Schooling

SOURCE: US Department of Education, NCES, National Education Longitudinal Study of 1988, Panel 1990-92

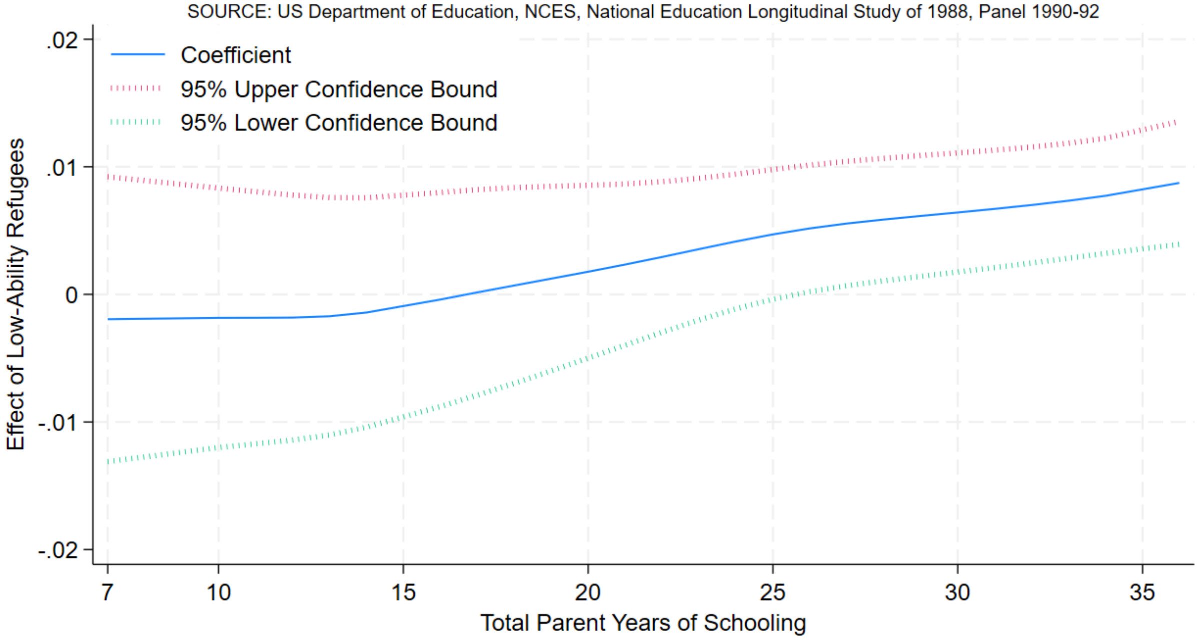


Figure 8: Estimated Effect of Increasing the Number of High-Ability SEA Refugee Students
on Log Homework Hours of Native-Born Students in Competitive Schools, by Total Parent Years of Schooling

SOURCE: US Department of Education, NCES, National Education Longitudinal Study of 1988, Panel 1990-92

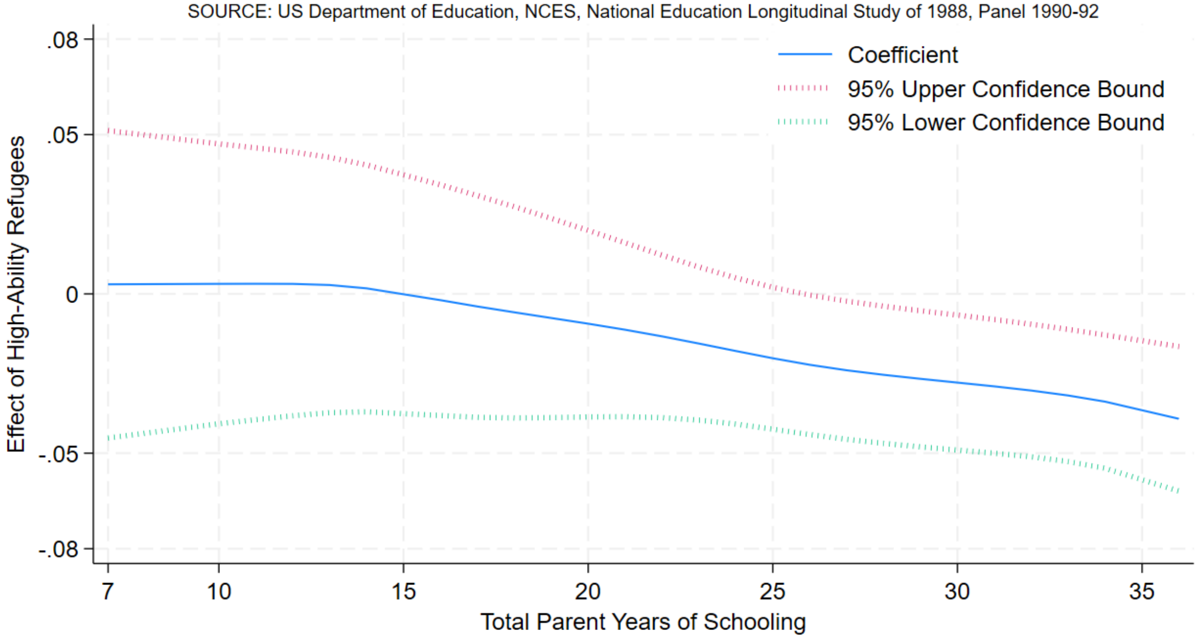


Figure 9: Estimated Effect of Increasing the Number of Low-Ability SEA Refugee Students in the County on Log Homework Hours of High-Ability Native-Born Students, by Total Number of High Schools in the County

SOURCES: US Department of Education, NCES, National Education Longitudinal Study of 1988, Rounds 2 and 3
NCES Common Core of Data Public Elementary and Secondary Schools, 1989-90

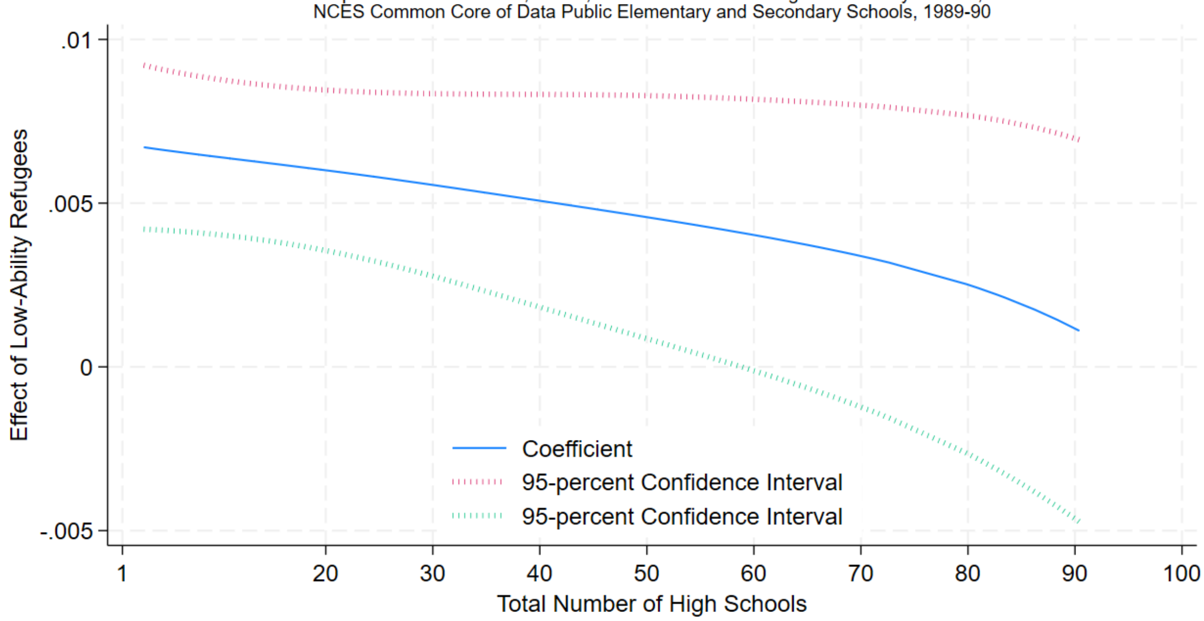


Figure 10: Estimated Effect of Increasing the Number of High-Ability SEA Refugee Students in the County on Log Homework Hours of High-Ability Native-Born Students, by Total Number of High Schools in the County

SOURCES: US Department of Education, NCES, National Education Longitudinal Study of 1988, Rounds 2 and 3
NCES Common Core of Data Public Elementary and Secondary Schools, 1989-90

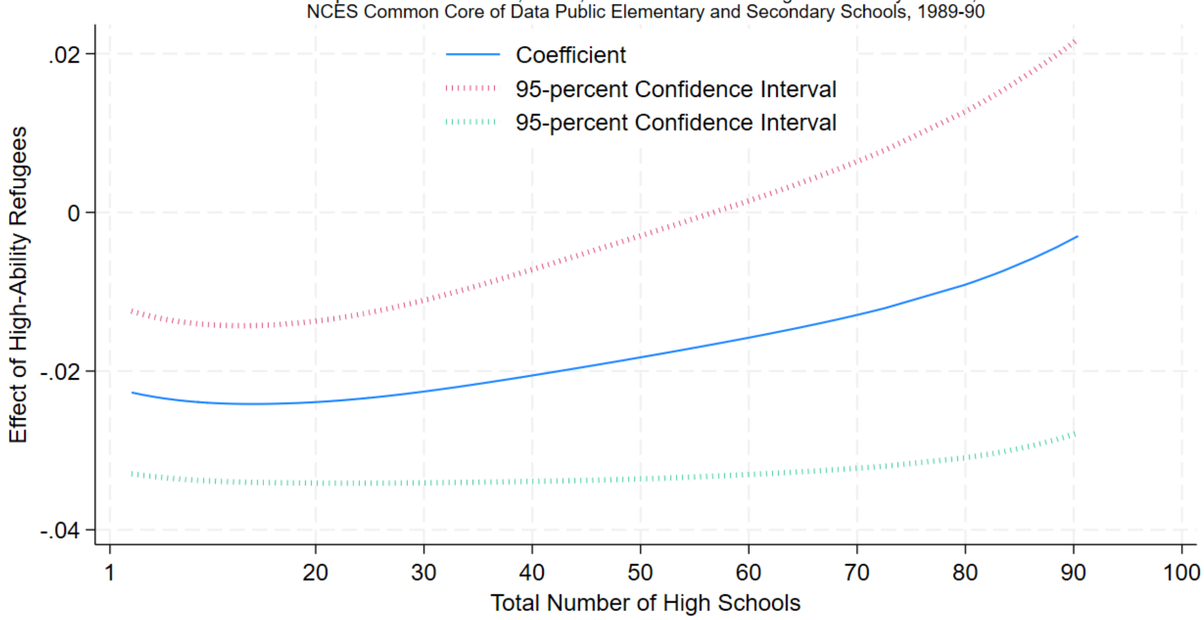
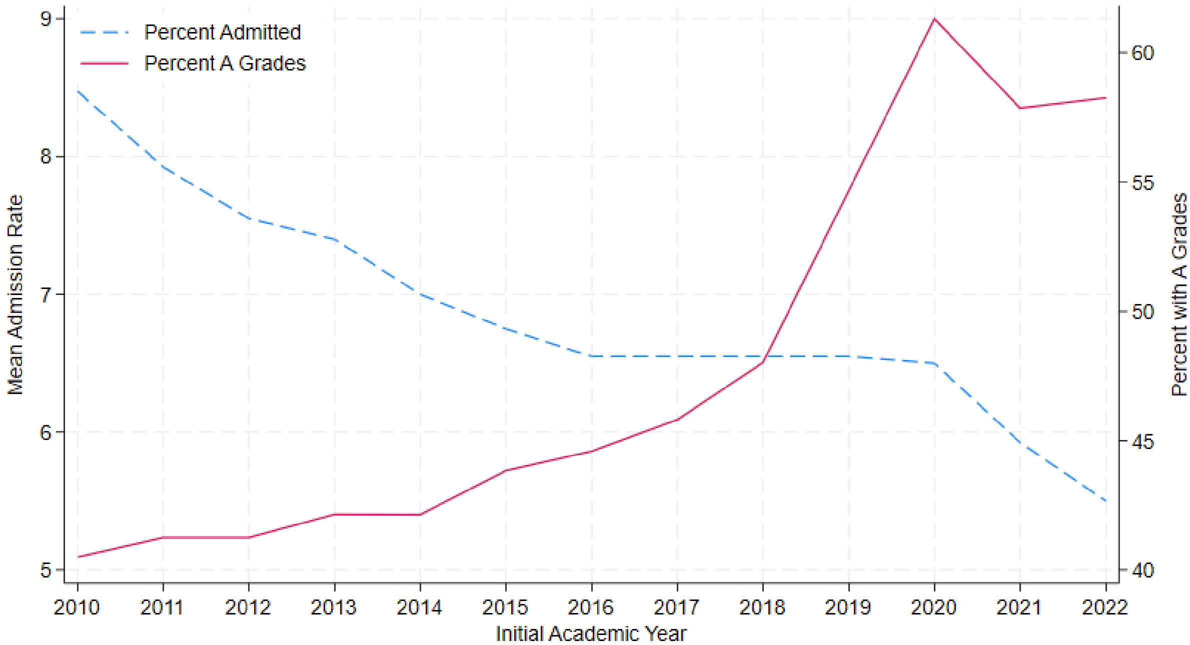


Figure 11. Grade Compression and Admission Selectivity of Yale College Students, 2010-2022

Source: Yale Office of Instructional Research



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