

# CAPITAL FLOWS AND THE GLOBAL COLLATERAL CYCLE

By

Ana Fostel, John Geanakoplos and Gregory Phelan

April 2026

COWLES FOUNDATION DISCUSSION PAPER NO. 2521



COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

YALE UNIVERSITY  
Box 208281  
New Haven, Connecticut 06520-8281

<http://cowles.yale.edu/>

# Capital Flows and the Global Collateral Cycle

Ana Fostel and John Geanakoplos and Gregory Phelan\*

April 2026

## Abstract

Cross-country disparities in collateral technologies alone can account for large capital flows among mature economies, and allow the most advanced country to run a permanent trade deficit. When the collateral technology advantage is in creating negative beta (super safe) financial assets backed by positive beta assets, a Global Collateral Cycle emerges, with pro-cyclical gross and net flows and increased global asset price volatility. The supply of super safe assets is necessarily curtailed in downturns, providing a complementary (supply) channel to the flight to safety (demand) channel for explaining why US safe asset prices rise during crises.

**Keywords:** Global Collateral Cycle, gross capital flows, asset prices, trade balance, securitized markets, asset-backed securities, tranching, negative beta assets, uncertainty, pro-cyclicality.

**JEL Classification:** D52, D53, E32, E44, F34, F36, G01, G11, G12

---

\*Ana Fostel: University of Virginia, Department of Economics, NBER and CEPR. Monroe Hall 232, 248 McCormick Road, Charlottesville, VA 22904, e-mail: anafostel@virginia.com. John Geanakoplos: Yale University, Box 208281, New Haven, CT 06520-8281 and Santa Fe Institute and Ellington Capital Management, e-mail: john.geanakoplos@yale.edu. Gregory Phelan: Williams College, Department of Economics, Schapiro Hall, 24 Hopkins Hall Drive, Williamstown MA 01267, email: gp4@williams.edu. For valuable comments, we thank Sebnem Kalemli-Ozcan (Editor), anonymous referees, Valentina Bruno, Emmanuel Farhi, Matteo Maggiori, and Frank Warnock. Any errors are our own.

One standard textbook case for financial flows between regions is simple. Capital flows from capital-intensive advanced economies to high-return developing ones, eventually reversing as dividends flow back. This story, however, does not describe what we have observed in international financial markets in recent decades.

*Observation 1: There are substantial gross financial flows between rich, similarly-developed countries. These gross flows go in both directions, often dwarfing the net flows. The US has maintained net capital inflows and run a trade balance deficit every year, but one, since 1982.*

Lane and Milesi-Ferretti (2007); Obstfeld (2012, 2015); Bertaut et al. (2012); Forbes and Warnock (2012); Broner et al. (2013); Bruno and Shin (2014) show that there are substantial gross flows between the U.S. and Europe. In 2023, the biggest European countries, United Kingdom, France and Germany accounted for almost \$600 billion of US inflows, whereas the total trade balance deficit with these countries was below \$20 billion.<sup>1</sup> Andritzky (2012); Hale and Obstfeld (2016) show that there are also substantial gross flows within Europe.<sup>2</sup>

Another classical explanation of financial flows is that ex ante identical countries face idiosyncratic shocks, and they insure each other. One would expect capital gross flows driven by diversification motives to be associated with lower volatility and less amplification. But instead, the opposite is true.

*Observation 2: Financial integration tends to increase asset price volatility and co-movement, particularly depressing risky asset prices during financial crises while increasing super safe asset prices. Gross flows are pro-cyclical, particularly through banking flows and securitized markets. Trade imbalances in advanced economies are also pro-cyclical.*

The name Global Financial Cycle captures global fluctuations in financial activity, marked by co-movements in risky asset prices, leverage of financial intermediaries, credit growth, and gross flows.<sup>3</sup> Financial crises especially induce co-movements among financially integrated countries, and the reach of global banks and securitized markets—not net flows—seems to have marked the geography of financial crises.<sup>4</sup>

We propose that cross-country differences in the ability to use domestic assets as collateral can account, at least partly, for both observations. A collateral technology advantage is an exorbitant privilege that can drive capital flows and create a Global Collateral Cycle.

Collateral ensures the credibility of financial promises and depends on reliable property records and a court system that has the power and willingness to seize the collateral. Securitization, especially in which a pool of collateral backs multiple tranches, additionally requires an appropriate accounting system and tax code. As a result, collateral technologies differ across countries.<sup>5</sup> More-

---

<sup>1</sup>See Council of Economic Advisors 2025. The United States accounted for approximately 41 percent of global gross inflows in 2023. Capital inflows into the United States peaked at more than \$2 trillion on the eve of the global financial crisis in 2007, retreating in the wake of the crisis, and reaching again \$1.9 trillion in 2023. The growth of capital outflows has been less, reaching 979 billions outflows in 2023. The U.S. net IIP stood at negative \$21.3 trillion at the end of the first quarter of 2024, representing the difference between the stock of foreign assets (\$36.0 trillion) and foreign liabilities (\$57.1 trillion).

<sup>2</sup>From 2000–2007 core European banks increased their balance sheets, intermediating funds from the rest of the world, to finance the net current accounts of the periphery countries—Greece, Italy, Ireland, Portugal, and Spain (“GIIPS”). As a result, core banks borrowed from abroad to invest in GIIPS.

<sup>3</sup>Broner et al. (2013); Brunnermeier et al. (2012); Forbes and Warnock (2012); Rey (2015); Miranda-Agrippino and Rey (2020).

<sup>4</sup>Kalemli-Ozcan, Papaioannou and Perri (2013); Acharya and Schnabl (2010).

<sup>5</sup>Regulations in some countries also inhibit securitizations, while government agencies in the U.S. promote them

over, they are applied mostly to domestic collateral.<sup>6</sup> In the U.S., well over \$30 trillion in domestic physical assets—houses, cars, and corporate properties—back loans through leverage. Additionally, over \$15 trillion in domestic financial assets—primarily mortgages, other asset backed loans, corporate loans, and corporate bonds—are pooled and divided into distinct tranches through securitizations.<sup>7</sup> No other advanced economy approaches the U.S. in securitization volume or diversity, even relative to GDP. Many cash flow profiles created in U.S. securitizations either do not exist elsewhere or are in very short supply. In 2025 the value of outstanding American securitizations is over 50% of GDP, whereas in Europe, securitization value is less than 7% of GDP.

Securitization with tranching has two important consequences which will be key in our model: it creates securities that satisfy the needs of heterogeneous investors and it affects asset prices through deviations from the law of one price. A common kind of securitization splits the cash flows of a (mortgage or corporate) bond into a floater, whose coupon payments rise with interest rates, and an inverse floater, whose payments rise when interest rates fall. Agents who are vulnerable to higher interest rates, like the old savings and loans, might be willing to pay more than half the price of the bond to get the floater, while agents who are vulnerable to lower interest rates, like insurance companies, might be willing to pay more than half to get the inverse floater, creating a profit opportunity for the securitizer. Competition between securitizers will then push the price of the bond far enough above the price of otherwise identical bonds that cannot be tranced so that the profit opportunity disappears. Raising the price of mortgages in this way (in addition to serving heterogeneous clienteles) was precisely the goal of the securitization tax reform of 1986 that helped usher in the mortgage securitization boom of the late 1980s and 1990s.<sup>8</sup>

By far the most important and largest tranche in a typical U.S. securitization is a AAA tranche that is like a U.S. Treasury bond in its promised payments and its reliability. We think of these AAA tranches and U.S. Treasuries as negative beta assets.<sup>9</sup> In a global downturn their payments remain secure and their prices typically rise substantially relative to consumption.<sup>10</sup> One reason is the flight to safety motive (demand channel) that applies in downturns when the world gets more volatile. Another reason is that interest rates typically fall (policy channel) and these assets promise long term cash flows, so their present values rise. We will show another reason: during crises the supply of negative beta assets coming from securitization dramatically falls (supply channel).<sup>11</sup> Securitization had a big impact in the domestic US economy; we will show that securitization can also have important implications for international markets.

With all this in mind, we consider a two-country (or two-region) model with incomplete markets and collateralized financial markets. The two countries, Home and Foreign, are identical in every way except for the sophistication with which their financial systems can use their local col-

---

with their own securitizations.

<sup>6</sup>In the great American mortgage securitization boom of the 1980s and 1990s, American investment banks never tranced foreign mortgages.

<sup>7</sup>The total amount of collateral backed loans is not available. For American and European securitizations, see SIFMA, AFME, Bloomberg, Apollo Chief Economist. See also Pozsar et al. (2010); Gorton and Metrick (2012); Fostel and Geanakoplos (2012a).

<sup>8</sup>Higher mortgage prices means lower mortgage rates, which encourages people to own houses.

<sup>9</sup>In his pioneering paper showing how to drop the riskless asset from the CAPM model, Fisher Black first identified Treasuries as a negative beta asset.

<sup>10</sup>In the absence of default risk and prepayment risk, inverse floaters are more negative beta than bonds.

<sup>11</sup>The flow of new securitizations collapses, and many existing AAA assets are revealed to be vulnerable to further bad news and thus are no longer even safe assets.

lateral. The Home country has an advanced financial system that enables global investors to use a home risky asset as collateral to issue state-contingent financial promises. In contrast, in the Foreign country, global investors can use a foreign risky asset (with identical payoffs as the Home asset) as collateral to issue non-contingent promises only (collateralized debt). Given the collateral technology within each country, the collateral requirements are endogenously set by the market. Our model corresponds roughly to the United States as Home and Europe as Foreign. The collateral technology advantage of Home is that its assets alone can create negative beta tranches (backed by positive beta collateral). We shall say that Home can securitize (or tranche), and Foreign can merely leverage.<sup>12</sup>

Investors are heterogeneous within each country (with symmetry across countries). The heterogeneity is crucial, because it means that inside each country the agents would like to trade different financial promises with each other, if the securities existed.<sup>13</sup> We model the heterogeneity as differences in optimism, which we sometimes interpret as differences in risk aversion, since the pessimists put higher prior probability on the bad states where risk averse agents would put higher marginal utility. A second implication of heterogeneity emerges in the dynamic version of our model, because optimists lose disproportionately more wealth in the down state, which means that the effective pessimism of the economy increases after bad news.

We begin with a static analysis that yields the following theoretical results. First, we show that differences in collateral technologies across countries are enough to generate large gross international financial flows. There are no differences in preferences, endowments, or production technologies, and all shocks are common, not idiosyncratic. In the international equilibrium, all agents can buy the same set of financial instruments and goods. Cash flow profiles backed by Home collateral are desired by agents in Foreign, where they cannot be created. Foreign purchases of these tranches explains gross inflows into Home. Surprisingly, Home demands the more limited, non-contingent, cash flows created in Foreign. This Home gross outflow is present even when Foreign non-contingent cash flows can be replicated by a combination of contingent Home securities. Gross flows go in both directions, and dwarf net flows.

Second, differences in collateral technologies create violations of the law of one price. The superior technology raises the price of the scarce Home collateral, and lowers the value of identical Foreign collateral. Even more importantly, Foreign securities (bonds) sell for a cheaper price than the price of a replicating portfolio of finely tuned contingent tranches cut from collateral at Home.

Third, a more advanced collateral technology imparts a great advantage (or exorbitant privilege). The higher price of assets in Home gives it higher wealth and the ability to run a permanent trade deficit. In conventional general equilibrium models, a wealthier country cannot generally run a permanent trade deficit; it can buy more, but eventually it will also sell more. The difference here is that Foreign is willing in aggregate to sell consumption goods plus bonds in exchange for a portfolio of cash flows that in total pay strictly less than the bonds, because distinct securities in the

---

<sup>12</sup>We might think of a developing country like India as a region where, until recently, collateral was mostly non-existent. Homeowners generally couldn't get mortgages, because the courts were not willing to throw families out of their homes if they defaulted. We could have taken European collateral technology as Home and Indian collateral technology as Foreign. Our results would be similar. In this paper we will focus on the case where both Home and Foreign are advanced countries.

<sup>13</sup>The heterogeneity is crucial, but what causes it is not. For example, differences in individual risk aversion would give very similar conclusions. Therefore, for ease of computation, we assume linear (risk neutral) utilities with heterogeneous probabilities.

portfolio are bought by distinct Foreign consumers who particularly value their profiles and could not find the same domestically. Home then runs a balance of trade deficit today and in every state in the future. The collateral induced trade imbalance is of course much smaller than the collateral induced gross outflows and inflows, since it is defined as their difference.

Fourth, if the down risk of the collateral payoffs is worse, the opportunity for Home to use its superior collateral technology to create negative beta cash flows is less (because the collateral cannot guarantee much backing in the down state where the negative beta tranche is supposed to pay the most). We confirm that then there is less tranching (i.e. smaller securitizations), smaller gross flows, smaller violations of the law of one price, and smaller trade imbalances. The smaller supply of the negative beta assets raises the price of each one (while lowering their total value).

We next take our theoretical results and study their implications in a dynamic setting via numerical simulation. We show the presence of what we call a Global Collateral Cycle: increased volatility and pro-cyclical prices, gross flows and trade balance.<sup>14</sup> We assume that bad news is correlated with higher volatility. Previous work by Geanakoplos (2003; 2010) on the leverage cycle showed that this kind of “scary bad news” leads to large drops in asset prices because leverage endogenously falls so much; it becomes impossible to create as many safe assets. This is precisely the situation of Foreign autarky in our dynamic setting. Subsequent work by Fostel and Geanakoplos (2012*b*) on the securitization cycle showed that scary bad news leads to even bigger asset price crashes because securitizations endogenously plummet; it becomes impossible to create as many negative beta tranches when volatility rises. This is the situation of Home autarky. In the absence of international trade, asset prices would rise and fall together at Home and in Foreign as a result of their common shocks.

Our fifth finding is that with international trade between the two countries, with their disparate collateral technologies, the rise and fall of prices is bigger in both countries, so global volatility goes up, as suggested in Observation 2.

Sixth, we argue that pro-cyclical gross flows is a second aspect of the global collateral disparity cycle when Home’s collateral technology advantage consists in the ability to create negative beta securities, because scary bad news destroys that advantage, and thus destroys the *raison d’être* of the gross flows. The anticipation after bad news of an even steeper downside constrains the feasibility of creating counter-cyclical tranches. Thus Home’s differential capacity to create cash flows out of collateral diminishes in downturns. The collateral technology does not vary across the cycle, but the room for Home to use its superior technology does. The difference between Home and Foreign collateral usage is thus pro-cyclical, which in turn drives pro-cyclical gross flows, and pro-cyclical trade imbalances.

Seventh and finally, during the down phase of the Global Collateral Cycle, the lower supply of negative beta assets provides a supply channel that explains the increase in the value of negative beta (super safe) assets during crises. The creation of the AAA tranches that resemble U.S. Treasuries dramatically fell in 2008. This supply channel complements the traditional flight to safety – loss of risk appetite (demand) explanation for the spike in U.S. Treasuries during crises (see Gourinchas and Rey (2007); Jiang, Richmond and Zhang (2024)). Our model also includes this traditional demand channel via the endogenous rise in economy-wide risk aversion through the disproportionate effects of the crisis on the wealth of heterogeneous agents.

---

<sup>14</sup>The Global Collateral Cycle mirrors the terminology introduced by Rey (2015); Miranda-Agrippino and Rey (2020), while underscoring the Collateral channel of the international patterns in prices and flows.

**Related literature** First, our paper builds on the framework of collateral general equilibrium described by Geanakoplos (1997, 2003).<sup>15</sup> He showed that the splitting of collateral into different cash flows through leverage or securitization, as in the floater - inverse floater example, can raise the price of collateral. Fostel and Geanakoplos (2008) gave a theoretical formalization for this price increase and called it the collateral value. A crucial feature of collateral general equilibrium is that collateral requirements are endogenous.<sup>16</sup> In particular, Geanakoplos (2003; 2010) argued that scary bad news has driven many of the big crises in history because of its effect on endogenous leverage in what he called the leverage cycle. He also modeled heterogeneous investors and the evolution of economy-wide risk aversion through disproportionate wealth effects from bad news. Fostel and Geanakoplos (2012*a*; 2014) showed that when the tranching technology is unlimited, then scary bad news causes an even bigger drop in prices, which they called the securitization cycle. Fostel and Geanakoplos (2012*b*) shows how this type of uncertainty can arise endogenously. This paper extends these models to study the implications of disparate collateral technologies on excess price volatility and pro-cyclical gross flows.

Second, our model is based on three key ingredients. Two of them, investor heterogeneity within countries and differing collateral technology across countries, drive the whole paper. The third ingredient is the constraint scary bad news puts on the creation of negative beta assets out of positive beta collateral; this ingredient drives our global collateral cycle in the second half of the paper. The first ingredient relates to empirical evidence that shows that global investors, lenders, and banks behave heterogeneously. Avdjiev et al. (2022) highlight the importance of borrower-lender identity and document key patterns in gross flows. Bertaut et al. (2023) reveal a new allocative role for capital flows across sectors, reconciling previous puzzling findings on the link between capital flows and productivity. Faia, Lewis and Zhou (2024) study the role of investor heterogeneity on Emerging Market flows reversals. Di Giovanni et al. (2022*b*) show that capital flows in emerging markets are sensitive to risk perception of global heterogeneous investors. As for the second ingredient, there has been great progress in understanding how differences in institutional leverage (e.g., bank leverage) across countries affect capital flows (See for example Kalemli-Ozcan, Sorensen and Yesiltas, 2012 and Di Giovanni et al., 2022*b*). Evidence suggests differences in collateral use across advanced economies. As already noted, the U.S. uniquely securitizes trillions of dollars in assets each year. Bertaut et al. (2012) show that during the 2000s European investors purchased U.S. asset-backed securities and similar securities and provide consistent evidence of differential abilities to supply securitized assets. Geanakoplos (2003; 2010) coined the phrase scary bad news and showed how the scary news caused an endogenous reduction in leverage, also giving evidence of the decline in leverage following the jump in uncertainty in the 2008 crisis. Following him, Brunnermeier and Pedersen (2009) and Adrian and Boyarchenko (2012), and Adrian and Shin (2014) connected leverage to uncertainty. All these papers and Gorton and Metrick (2012) further documented the decline in leverage following the 2008 crisis. Leverage involves the creation of non-contingent debt, which is a weakly negative beta asset, and its curtailment in 2008 mirrors the simultaneous plummet in securitization. Fostel and Geanakoplos (2008) studied the implications of scary bad news across co-existing leverage cycles, explaining contagion and flight to collateral.

---

<sup>15</sup>Another early paper dealing with collateral equilibrium is Kiyotaki and Moore (1997). But that had no uncertainty, so neither the splitting of future cash flows nor the endogeneity of collateral requirements could arise.

<sup>16</sup>See also Fostel and Geanakoplos (2008, 2012*b,a*, 2015, 2016), Araujo, Kubler and Schommer (2012, 2009); Brumm et al. (2015); Gottardi and Kubler (2015), Simsek (2013), Geanakoplos and Zame (2014), Phelan (2015), and Gong and Phelan (2019, 2022)

More generally, Bloom (2009) suggested that uncertainty shocks are a key driver of economic fluctuations.

Third, our paper is related to a large literature on how differences in financial systems drive capital flows. This “global imbalances” literature has tended to focus on how *net* capital flows arise between developed and developing countries. The literature has broadly considered differences in (i) state-completeness, (ii) sharing idiosyncratic risk, (iii) the ability to supply financial assets, and (iv) funding costs. In this literature, financial flows are driven primarily by interest rate (or investment return) differentials that manifest in different savings across countries and therefore net flows and a trade imbalance.

Willen (2004) shows that differences in market incompleteness across countries cause trade imbalances because superior risk-sharing in Home leads to a lower precautionary demand for saving, raising the riskless interest rate; Home therefore attracts Foreign investment and runs a trade surplus which reverses in the final period. Mendoza, Quadrini and Rios-Rull (2009) and Angeletos and Panousi (2011) similarly emphasized how net capital flows arise when the developed country can better insure idiosyncratic risk, affecting autarkic interest rates. Within this literature, Phelan and Toda (2019) study how the risk-sharing qualities of securitized markets create capital flows from the high-margin to low-margin country.

Caballero, Farhi and Gourinchas (2008) is the paper closest in spirit to our own. In their paper, two countries U and R are identical in every way, including the productivity of their physical assets (trees). The degree of financial development is defined by the proportion  $\delta$  of the trees in each country that are tradable. Agents who want to save out of their dividends are forced to buy tradeable trees. When the proportion of tradable trees in R drops, the price of tradable trees rises (equivalently the interest rate falls), and since U has more of them is able to consume more and run a current account deficit in every period. Their paper focuses on net flows instead of gross flows. (Flows go in only one direction, from R to U, so gross flows are the same as net flows.) There is no uncertainty, no collateral, and no tranching. They emphasize trade between a developed country U and a developing country R; our subtler difference in collateral technologies allows us to explain gross flows between two developed countries.

Maggiore (2017) provides a model in which Home financiers can take on greater financial risk because, by assumption, they do not face borrowing constraints like Foreign. This leads Home to run persistent current account deficits financed by the risk-premium earned by its financial sector, which can better absorb aggregate shocks by borrowing out of trouble.

In our model every agent has access to the same insurance possibilities and all investment returns are the same in both countries. The key is that the cash flows that enable insurance are not in zero supply in our model, as they are in the prior literature, but are in positive supply and effectively owned by the original owners of the collateral that back the cash flows. Flows are driven by all risky assets, not just risk-free bonds. Moreover, analyzing gross flows are crucial to understanding the net flows. As Forbes and Warnock (2012) showed, something conceptually different is happening with gross flows that cannot be seen from net flows alone (gross inflows and outflows have grown more volatile while net flows remain stable).

Fourth, our focus on how financial integration leads to excess volatility and pro-cyclicality is related to several theoretical papers. Caballero and Simsek (2016) consider a model in which gross flows are driven by demands for liquidity, and the “fickle” reversal of capital flows creates instability. They focus on ex-ante policy implications taking fickle flows as given. Mendoza and Quadrini (2010) extend the model in Mendoza, Quadrini and Rios-Rull (2009) (flows driven by

precautionary savings) to include financial intermediaries and study how financial integration affects the consequences of a one-time non-anticipated shock to intermediary capital. They find that shocks propagate as a result of financial integration; importantly, however, asset price declines are smaller than they would be in autarky, and the crisis would have been worse for the U.S. if it had not been financially integrated. In contrast, in our model price crashes are larger with financial integration, not in autarky. Theoretical work by Devereux and Yetman (2010) and Ueda (2012) present models, with financial intermediaries or with leverage constraints, in which financial integration affects spillovers, propagation through interdependent portfolios, and business-cycle synchronization. Unlike our model, in the aforementioned papers considering financial frictions, collateral requirements are not endogenous.

**Organization** Section 1 presents the basic general equilibrium model with collateral. Section 2 studies the autarky equilibria in a static model. Section 3 studies the effects of financial integration on prices, gross and trade balance in a static model. Section 4 uses a numerical simulation of a three-period model to show the effect of financial integration on the volatility and cyclicity of prices and flows. Section 5 discusses the testable implications of our model and provides a “road map” for future empirical work. Supplemental material is presented in the Appendix.

## 1 General Equilibrium Model with Collateral

In this section we present a one-country general equilibrium model with collateral from Geanakoplos (2003), which we later extend to an international two-country setting in Sections 3 and 4. Following Fostel and Geanakoplos (2012a) we call this model the *C*-model.

**Time, Commodities and Assets** There are two time periods,  $t = 0, 1$ . Uncertainty is represented by a tree  $S = \{0, U, D\}$  with a root  $s = 0$  at time 0 and two terminal states of nature  $S_T = \{U, D\}$  at time 1. Let  $L_0 = \{c_0, Y\}$ ,  $L_U = \{c_U\}$ ,  $L_D = \{c_D\}$  be the set of commodities in states 0,  $U$  and  $D$ . Denote by  $L_T = \cup_{s \in S_T} L_s$  the set of commodities in terminal states.

Let  $F_s(c_0, Y) = c_0 + d_s Y$ ,  $s \in S_T$  be an inter-period durability function connecting any vector of commodities at state  $s = 0$  with the quantity of consumption goods  $c_s$  it becomes in each state  $s \in S_T$ . As shown in Figure 1,  $c_0$  is a (perfectly) durable consumption good and  $Y$  is a risky asset, which produces dividends (in units of the consumption good)  $d_U$  in state  $U$  and  $0 < d_D < d_U$  in state  $D$ . We normalize the price of consumption  $c_s$  in each state  $s \in S$  to 1. We denote the price of the asset  $Y$  at time 0 by  $p$ .

**Agents** Agents are uniformly distributed in the continuum  $I = [0, 1]$ . Each investor  $i \in I$  is risk-neutral, does not discount the future, and consumes only at time 1. We have taken the utilities to be linear to simplify the characterization of equilibrium, thereby making the forces creating gross flows as transparent as possible. The expected utility to agent  $i$  is

$$U^i(c_0, Y, c_U, c_D) = \gamma(i)c_U + (1 - \gamma(i))c_D \quad (1)$$

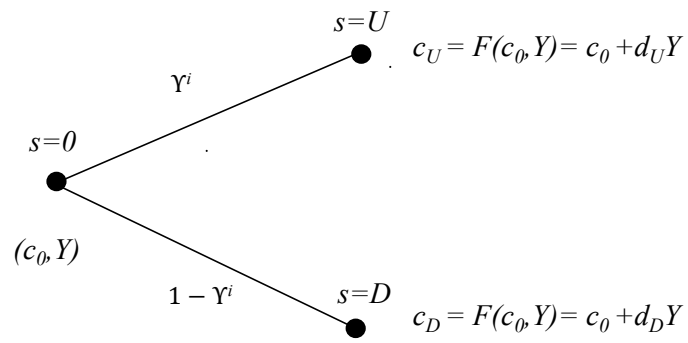


Figure 1: Time, commodities and assets.

$\gamma(i)$  is the subjective probability for the  $U$  state, which is strictly increasing and continuous in  $i$ .<sup>17</sup> Since only the output of  $Y$  depends on the state and  $0 < d_D < d_U$ , higher  $i$  denotes more optimism. Agent heterogeneity arises entirely from  $\gamma(i)$ 's dependence on  $i$ .<sup>18</sup> Each investor  $i \in I$  has initial endowments of commodities  $e = (e_{c_0}, e_Y, e_{c_U}, e_{c_D})$ .

**Financial Contracts and Collateral** We explicitly incorporate repayment enforceability problems, and we suppose that collateral acts as the only enforcement mechanism. Agents trade non-recourse collateralized financial contracts at time 0. A financial contract  $j = ((j_U, j_D), 1_Y)$  is defined by a pair consisting of a promise  $(j_U, j_D)$  of payment in units of the consumption good at each future state, and one unit of  $Y$ ,  $1_Y$ , serving as collateral.<sup>19</sup> Nothing else, such as consumption  $c_0$ , is allowed to serve as collateral.<sup>20</sup>

Borrowers, sellers of contracts, must own collateral in order to make promises. Lenders, buyers of contracts, have the right to seize the posted collateral, but no more. Hence, the delivery of the non-recourse contract  $j$  in each state  $s$ ,  $\delta_j(s)$ , is given by the minimum between the promise,  $j_s$ , and the value of the collateral,  $d_s$ , i.e.  $\delta_s(j) = \min(j_s, d_s)$ .

Each contract  $j \in J$  trades at price  $\pi_j$ . Purchasing contract  $j$  is tantamount to lending  $\pi_j$  in return for the promise  $(j_U, j_D)$  and more importantly, the delivery  $(\delta_U, \delta_D)$ . Selling one contract  $j$  is tantamount to borrowing  $\pi_j$  in return for promising  $(j_U, j_D)$  backed by one unit of collateral  $Y$ , i.e. delivering  $(\delta_U, \delta_D)$ . The borrower effectively pays  $p - \pi_j$  (to hold the collateral net of the contract sale price) in order to receive the residual payment  $(j_U - \delta_U, j_D - \delta_D)$ . We denote the total purchases of contract  $j$  by  $\varphi_j > 0$  and the total sales of the same contract by  $\varphi_j < 0$ .

The *collateral technology* is defined by the set  $J$  of available contracts.

**Budget Set** Given asset and contract prices at time 0,  $(p, (\pi_j)_{j \in J})$ , each agent  $i \in I$  chooses asset holdings,  $y$ , contract trades,  $\varphi_j$ , and consumption,  $c_0$  in state 0, and consumption in final states,  $c_U$  and  $c_D$ , in order to maximize utility (1) subject to the budget set defined by

<sup>17</sup>Relatively speaking, high  $i$  agents will prefer future cash flow streams that pay off predominantly in  $U$ , low  $i$  agents will prefer payoffs in  $D$ , and medium  $i$  agents will be happy with payoffs in both.

<sup>18</sup>We could easily extend our model to include consumption at time 0. However, because agents have linear utilities and do not discount the future, consumers would be indifferent between consumption today and stored consumption, and our analysis would remain essentially unchanged. For simplicity, in the rest of the paper we consider the case of future consumption only.

<sup>19</sup>Restricting contracts to be collateralized by one unit of  $Y$  is without loss of generality, e.g., a contract promising  $(0.4, 0.4)$  backed by two units of  $Y$  is identical to two units of a contract promising  $(0.2, 0.2)$  backed by one unit of  $Y$ .

<sup>20</sup>Notice that the consumption good  $c_0$  is not tranchable. Generally speaking it is difficult to tranche an asset that gives utility, because if the collateral needs to be seized, the consumption utility is disrupted. This explains why houses are not tranced but mortgages are.

$$B(p, \pi) = \left\{ (c_0, y, \varphi, c_U, c_D) \in R_+^{L_0} \times R^J \times R_+^{L_T} : \right. \\
c_0 + py + \sum_{j \in J} \varphi_j \pi_j \leq e_{c_0} + pe_Y, \\
\sum_{j \in J} \max(0, -\varphi_j) \leq y, \\
\left. c_s = e_{c_s} + F_s(c_0, Y) + \sum_{j \in J} \varphi_j \delta_s(j), s \in S_T \right\}.$$

At time 0, total expenditures on the consumption good, the asset and financial contracts have to be financed by the value of initial endowments. The second constraint is the collateral constraint, which states that the total short position on financial contracts cannot exceed the total asset holdings required as collateral; this collateral constraint is automatically satisfied for lenders since  $\varphi_j \geq 0$  and they cannot go short on the asset  $Y$ . Finally, consumption in the terminal states is derived from endowments, the receipts of storage, asset dividends and net deliveries from financial contracts.

The model encompasses the possibility of short selling the consumption good and asset  $Y$ . If  $(d_U, d_D)$  or  $(1, 1)$  is one of the promises available in  $J$ , then agents would be able to short  $Y$  or the consumption good. However, those promises would require posting collateral (see for example Fostel and Geanakoplos (2012a)).

**Collateral Equilibrium** A Collateral Equilibrium consists of prices and individual holdings  $((p, \pi), (c_0^i, y^i, \varphi^i, c_U^i, c_D^i)_{i \in I}) \in (R_+ \times R^J) \times (R_+^{L_0} \times R^J \times R_+^{L_T})^I$ , such that

- (1)  $\int_0^1 c_0^i di = e_c$
- (2)  $\int_0^1 y^i di = e_Y$
- (3)  $\int_0^1 \varphi_j^i di = 0 \forall j \in J$
- (4)  $(c_0^i, y^i, \varphi^i, c_U^i, c_D^i) \in B^i(p, \pi), \forall i$
- (5)  $(c_0, y, \varphi, c_U, c_D) \in B(p, \pi) \Rightarrow U^i(c_0, y, \varphi, c_U, c_D) \leq U^i(c_0^i, y^i, \varphi^i, c_U^i, c_D^i), \forall i$

In collateral equilibrium, agents trade the consumption good, whose market clear according to (1), the asset  $Y$ , whose market clears according to (2), and in all the contracts  $j \in J$ , whose markets clear according to (3). Agents optimize their utilities in their budget sets according to (4) and (5). By Walras' Law, markets for the consumption good in terminal nodes also clear. Geanakoplos and Zame (2014) show that equilibrium in this model always exists.

**The Collateral Technology and Intermediaries** We can think of the collateral technology as residing with the courts to enforce the delivery of certain promises  $j \in J$ . In that interpretation, investors borrow directly against assets, using them and the courts to issue a promise. This can occur when an agent endowed with  $Y$  decides to borrow, or when a buyer of  $Y$  wants to borrow some of the funds for the purchase by simultaneously selling a promise. The collateral technology enables each unit of collateral to back one promise, described in the formal equilibrium above.

Another interpretation of the collateral technology is that it captures the role of financial intermediaries in producing tranches backed by collateral. Selling a collateralized contract  $j = ((j_U, j_D), 1_Y)$  splits the future cash flows of  $Y$  into two pieces or tranches: one delivering  $(\delta_U(j), \delta_D(j))$

and the other delivering  $(d_U - \delta_U(j), d_D - \delta_D(j))$ . The lender ends up with the first (senior) tranche and the borrower/buyer of  $Y$  ends up with the residual (junior) second tranche. We could instead imagine an intermediary who buys  $Y$  and cuts its payments into those two tranches, and then sells one to the old lender and the other to the old borrower.

If tranching is costless, competition among intermediaries drives their profits to zero. The price  $p$  of  $Y$  becomes equal to the sum of the prices of its tranches, so the original owner of  $Y$  effectively owns all the tranches into which  $Y$  is split. In equilibrium the intermediaries disappear without a trace. The collateral technology (as described by  $J$ ) determines the tranches that can be traded. After that, equilibrium can be described more conventionally as a standard general equilibrium model (without collateral constraints) in which agents have endowments of tranches which they trade in lieu of trading  $Y$ . Although the collateral constraints ultimately disappear, the need for collateral is fundamental in determining exactly which tranches are traded.<sup>21</sup> In the following sections we shall use the intermediaries interpretation of equilibrium.

A much more advanced collateral technology would permit the same collateral to back many promises, splitting the asset  $Y$  into multiple tranches. The deliveries of all tranches would add up to the payoff of  $Y$  in every state. Tranching is generally performed by an intermediary who buys the collateral  $Y$  and then does the tranching, perhaps retaining some of the pieces himself.<sup>22</sup> In practice, multiple tranches involve an elaborate accounting system and a much more sophisticated court system to adjudicate defaults. In the United States this all came together in 1986 and gave rise to a securitization wave that is not close to being equaled anywhere else in the world even today.

Our simple notation, in which collateral can back only one promise, is nevertheless sufficient to represent not only (the equivalent) simple tranching into two pieces, but also the most advanced tranching technology, in which  $Y$  can be arbitrarily split into any number of pieces, by a clever choice of  $J$ . The reason, as Geanakoplos and Zame (2013) proved, is that unlimited tranching never needs to do more than splitting the payoffs of  $Y$  into distinct Arrow securities;  $S_T$  tranches suffice. When  $S_T$  has two states, the most advanced technology only needs to tranche  $Y$  into two pieces, just like when collateral can back only one promise. As we shall see, by positing one collateral technology  $J$ , we can interpret our equilibrium above as one with simple leverage, and by positing another collateral technology  $J$ , the same notation can describe intermediaries and perfect tranching.

## 2 Differing Collateral Technologies

We now consider a  $C$ -model with two countries (or regions), Foreign and Home, each defined as in Section 1. The key—and only—difference between the two countries is that Home has a more advanced collateral technology than Foreign, as defined by the set of financial contracts  $J$ . We can think of Foreign as Europe and Home as the US. In this Section we describe autarkic equilibrium for each country, and in the next Section we describe international trade between Foreign and

<sup>21</sup>In the equivalent intermediaries interpretation of equilibrium, agents trade every contract  $j$  and its residual and the consumption good, but do not trade  $Y$  at all. Their budget sets are standard Walrasian: expenditures on tranches and consumption equals revenue from sales of tranches and consumption.

<sup>22</sup>The (somewhat cumbersome) general notation for tranching and pyramiding was worked out in Geanakoplos and Zame (2013). See also Gong and Phelan (2019)

Home. We are able to completely characterize the features of international equilibrium and its differences from the autarkic equilibria. In what follows, we use (\*) to denote Foreign variables and ( $\hat{\cdot}$ ) to denote International Equilibrium (IE) variables in Section 4. Because of the linear utilities, the continuity of utility in  $i$  and the connectedness of the set of agents  $I$ , equilibria are easy to characterize. All the equations for Sections 2 and 3 are presented in Appendices A and B respectively.

## 2.1 Foreign Autarky: Leverage

Foreign can issue non-contingent promises using the asset as collateral. Formally,  $J^* = \{j : j = ((j, j), 1_{Y^*}) : j \geq 0\}$ . With slight abuse of notation, we let  $j$  denote both the contract and the promise for that contract. Each debt contract  $j$  promises  $j$  units of the consumption good at  $t = 1$ , and is collateralized by one unit of the asset  $Y^*$ . By buying  $Y^*$  and selling a contract  $j$ , agents borrow  $\pi_j$  and leverage their purchases of  $Y^*$ . The loan to value of contract  $j$  is  $LTV_j = \frac{\pi_j}{p}$ .

Figure 2a shows the payoff  $d = (d_U, d_D)$  of the asset  $Y^*$ , and the  $45^0$  line on which every contract promise must be chosen. All contracts are priced in equilibrium, but since collateral is scarce, a limited set is actively traded. As shown in Geanakoplos (2003) and Fostel and Geanakoplos (2012b), in equilibrium the only actively traded contract is the “maxmin” contract  $j^* = \min\{d_s\} = d_D$ , ruling out default in equilibrium. Its price is  $\pi_{j^*} = d_D$  (the risk-free rate is zero), implying a leverage ratio of  $LTV = \frac{d_D}{p}$ . Every Foreign agent leveraging  $Y^*$  buys the asset  $Y^*$  and uses it as collateral to sell the bond  $j^* = d_D$ . Leverage is thus endogenously determined. In the intermediaries interpretation, given its limited “non-contingent” collateral technology, Foreign endogenously tranches every asset  $Y^*$  into the same two pieces, a riskless tranche paying  $(d_D, d_D)$  and a  $U^*$  tranche paying  $(d_U - d_D, 0)$ , as indicated in the Figure 2b.<sup>23</sup> Foreign intermediaries thus create  $(d_U - d_D)e_Y$  Arrow  $U$  securities and  $e_Y d_D$  riskless bonds.

### 2.1.1 Foreign Autarky Equilibrium

In the intermediaries interpretation of equilibrium, there are just two distinct traded instruments: the Arrow  $U$  security and consumption/riskless bond, corresponding to the two tranches in 2a. Once  $Y$  is split into the two tranches (or Arrow securities), we can think of trading directly in tranches and not in  $Y$ .

In equilibrium, there is a marginal buyer  $i_1^*$  who is indifferent between holding the Arrow  $U$  securities and the consumption good  $c^*$ . As shown in Figure 2b, agents  $i > i_1^*$  buy all the Arrow  $U$  securities, and agents  $i < i_1^*$  hold the durable consumption good  $c^*$  and risk-free debt contract  $j^* = d_D$ .<sup>24</sup> Equilibrium is described by a system of two equations in two unknowns, the price of the asset,  $p^*$ , and the marginal buyer,  $i_1^*$ .

<sup>23</sup>We use the notation  $U^*$  tranche to underscore the fact that these are Arrow  $U$  securities created through the leverage of  $Y^*$ .

<sup>24</sup>In the collateral equilibrium interpretation, there is a marginal buyer  $i_1^*$  who is indifferent between leveraging  $Y^*$  and holding the consumption good  $c^*$ . As shown in Figure 2b, agents  $i > i_1^*$  buy all the  $Y^*$  in the economy with leverage: they borrow  $d_D$  by selling debt contract promising  $j = d_D$  using  $Y^*$  as collateral, thus effectively buying on net Arrow  $U$  securities. Agents  $i < i_1^*$  lend to the more optimistic agents, holding the durable consumption good  $c^*$  and risk-free debt contract  $j^* = d_D$ .

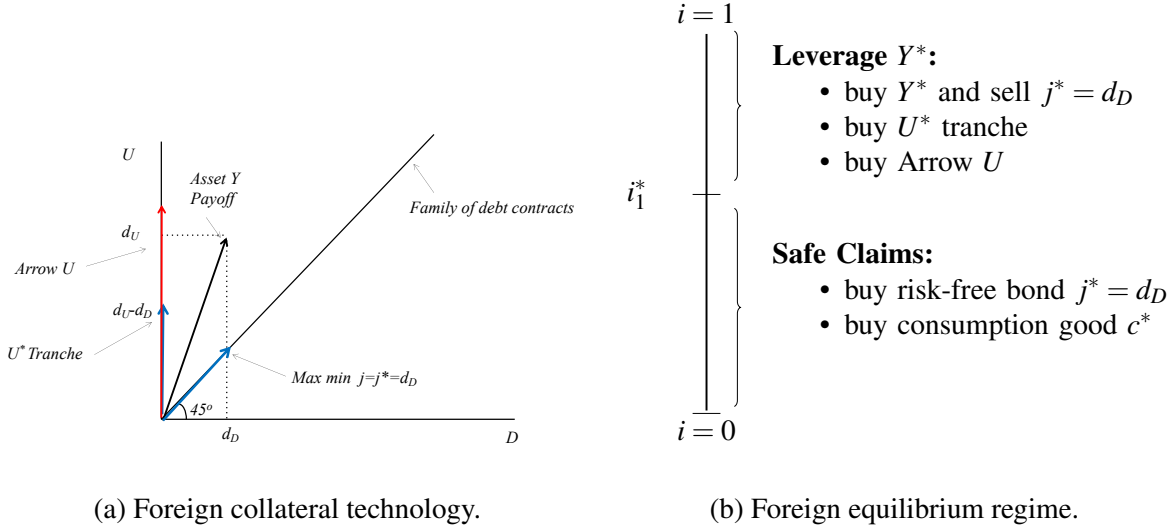


Figure 2: Panel 2a plots the collateral technology in Foreign. Panel 2b plots the equilibrium regime in Foreign.

For example, if  $\gamma(i) \equiv i$ ,  $(e_{c_0}, e_Y, e_{c_U}, e_{c_D}) = (1, 1, 0, 0)$  and  $d_U = 1 > d_D = 0.2$ , then Foreign autarky equilibrium is given by  $i_1^* = .69$ ,  $p^* = .75$ .

## 2.2 Home Autarky: Tranching

Home differs from Foreign in its more advanced collateral technology:  $Y$  can be used to back *any* state-contingent claim,  $J = \{((j_U, j_D), 1_Y) : (j_U, j_D) \geq (0, 0)\}$ . As shown in Geanakoplos and Zame (2014) and Fostel and Geanakoplos (2015), in equilibrium endogenous choices will lead every agent who holds  $Y$  to sell the same contract: the  $D$  tranche promising  $(0, d_D)$  i.e.  $d_D$  Arrow  $D$  securities.<sup>25</sup> This creates a residual tranche,  $(d_U, d_D) - (0, d_D) = (d_U, 0)$ , namely the  $U$  tranche promising  $d_U$  Arrow  $U$  securities.<sup>26</sup> In short, in the intermediaries interpretation of collateral equilibrium, every asset  $Y$  is endogenously tranching into Arrow securities, as shown in Figure 3a.

Every Foreign tranche is equal to a positive combination of tranches in Home, but not the reverse. There are effectively three traded instruments in Home: the Arrow  $U$  security, the Arrow  $D$  security, and consumption. As before, once  $Y$  is tranching we can think of trading directly in tranches and not in  $Y$ .

### 2.2.1 Negative Beta Assets

The  $D$  tranche is the archetypal negative beta asset, paying off exclusively in the bad state  $D$ . It is attractive to pessimists (low  $i$  agents). While the  $D$  tranche is risky, it resembles the archetypal “safe asset,” the U.S. Treasury bond. U.S. Treasury bonds behave similarly, often rising in dollar price during crises due to their low default risk and long duration. Additionally, the U.S. dollar

<sup>25</sup>More precisely, every equilibrium is equivalent to this one.

<sup>26</sup>We use the notation  $U$  tranche to underscore the fact that these are Arrow  $U$  securities created through the tranching of  $Y$ , instead of leveraging of  $Y$ .

typically appreciates during downturns, making these Treasuries valuable to foreign investors.<sup>27</sup> We argue that securitization creates negative beta assets, super safe assets, in addition to existing Treasuries.

The essential difference in collateral technologies between Home and Foreign is that only Home can create Arrow  $D$  securities. The quantity is limited by the payoff  $d_D$ , which is therefore a key measure of Home's collateral technology advantage. As  $d_D$  increases, Home's collateral advantage increases. The degree of sophistication of the collateral technology is seen not just in the *types* of promises that can be created, but also in their *quantity*. Foreign can create Arrow  $U$  securities, but Home has a second minor advantage because it can create more Arrow  $U$  securities since  $d_U > d_U - d_D$ .

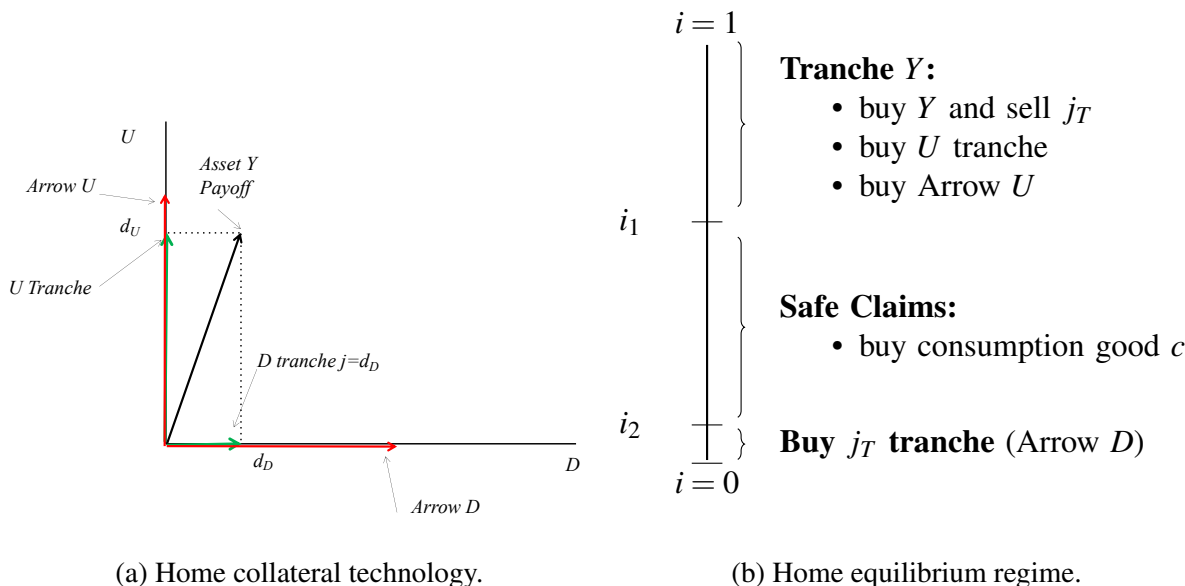


Figure 3: Panel 3a plots the collateral technology in Home. Panel 3b plots the equilibrium regime in Home.

### 2.2.2 Home Autarky Equilibrium

As shown in Figure 3b, in equilibrium there are two marginal buyers  $i_1$  and  $i_2$ . Optimistic agents  $i > i_1$  buy the Arrow  $U$  security. Moderate agents  $i \in [i_2, i_1]$  hold the consumption good  $c$ . The most pessimistic agents  $i < i_2$  buy the Arrow  $D$  security. Agent  $i_1$  is indifferent between holding an Arrow  $U$  security and the consumption good, and agent  $i_2$  is indifferent between holding the consumption good and the Arrow  $D$  security.

Equilibrium is described by a system of four equations in four unknowns: the price of the asset  $p$ , the price of the  $D$  tranche  $\pi_T$ , and two marginal buyers with  $i_2 < i_1$  (see Appendix A). Finally, note that although both Arrow securities can be created by tranching the asset, markets remain incomplete since they are generated only through  $Y$ , limiting their supply (the durable consump-

<sup>27</sup>This is a point made in Maggiori (2017), which, in a different financial context, describes the rest of the world buying “down state” Arrow securities from Home in order to achieve safer portfolios.

tion good cannot be tranced). Hence, collateral equilibrium fails to implement the Arrow-Debreu equilibrium.

In the same numerical example described earlier for Foreign, we now have  $i_1 = .65, i_2 = .10$  and  $p = .83, \pi_T = .18$ . The asset price rises in this example from .79 in the Foreign leverage economy to .83 in the more advanced Home tranching economy. In Section 3 we examine international equilibrium, for which we can prove general theorems.

### 3 A Static Model of Global Flows: International Equilibrium

We now consider the integrated economy where agents in both Foreign and Home trade all goods and financial securities, with collateral technology  $\hat{J} \equiv J^* \cup J$ .<sup>28</sup> The key asymmetry remains: Home assets can back state-contingent contracts (tranches), while Foreign assets can back only non-contingent debt. These collateral technologies correspond to the asset's origin, not the owner's nationality, because, for example, American courts have jurisdiction over American assets like houses even if they are owned by foreigners. Thus, Foreign investors can purchase and benefit from tranches backed by Home assets.

In the intermediaries interpretation of international equilibrium, IE, Home agents initially own the  $U$  and  $D$  tranches derived from  $Y$ , while Foreign agents own the debt and  $U^*$  tranches derived from  $Y^*$ . As shown in Figure 4a, effectively, only three distinct instruments are traded: Arrow  $U$  securities (created by Home and Foreign), Arrow  $D$  securities (created only by Home), and riskless claims (consumption in both countries, and Foreign bonds).

#### 3.1 International Equilibrium

In the IE the marginal agents across countries are the same because everybody has access to purchasing the same goods and because corresponding agents have the same homothetic preferences. As shown in Figure 4b, in each country there are two common marginal agents with  $\hat{i}_1 > \hat{i}_2$ . Optimistic agents  $i \geq \hat{i}_1$  in both countries hold Arrow  $U$  securities. Moderate agents  $\hat{i}_2 < i < \hat{i}_1$  in both countries hold consumption goods  $c$  and  $c^*$  and bonds  $j^* = d_D$ . Finally, pessimistic agents  $i \leq \hat{i}_2$  in both countries hold Arrow  $D$  securities. The marginal agent  $\hat{i}_1$  is indifferent between the Arrow  $U$  security and a safe position (consumption good or bond). The marginal agent  $\hat{i}_2$  is indifferent between the safe position and the Arrow  $D$  security.

The IE is described by a system of five equations in five unknowns: Home and Foreign asset prices  $\hat{p}$  and  $\hat{p}^*$ , the price of the  $D$  tranche  $\hat{\pi}_T$ , and two marginal buyers  $\hat{i}_1$  and  $\hat{i}_2$  (see Appendix B).

In the numerical example,  $\hat{p} = .86, \hat{p}^* = .73, \hat{\pi}_T = .19$ , and  $\hat{i}_1 = .67, \hat{i}_2 = .05$ . International trade has raised the price of the  $D$  tranche and the Home asset price, and lowered the Foreign asset price.

<sup>28</sup>Formally, international equilibrium is written exactly as in Section 1, except now there are twice as many agents, with Home agents initially owning Home goods and Foreign agents initially owning Foreign goods. Recall that all  $j \in J^*$  are backed by  $Y^*$  and all  $j \in J$  are backed by  $Y$ .

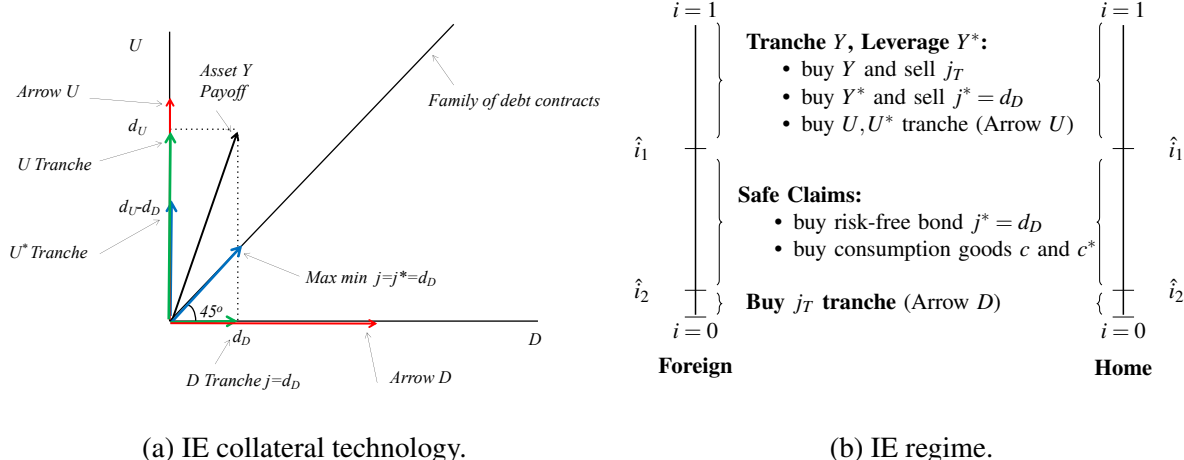


Figure 4: Panel 4a plots the collateral technology in Home. Panel 4b plots the equilibrium regime in Home.

### 3.2 International Equilibrium and Asset Prices

Standard international finance models predict convergence in asset prices with integration. Here, differences in collateral technologies move asset prices further apart—even though the payoffs are identical. As in Fostel and Geanakoplos (2008), Home and Foreign asset prices reflect a payoff value and also a non-negative collateral value. Trade occurs, despite identical asset payoffs across economies, because the assets have different collateral capacities. The Home asset, which can be tranced into state-contingent promises, is better collateral than the Foreign asset, which can only be used to issue non-contingent debt. And hence the Law of One Price fails.

Define the collateral gap  $\hat{\Delta} \equiv \hat{p} - \hat{p}^*$  as the difference between the Home and Foreign asset prices in IE. Since both assets have identical payoff values, this difference represents the equilibrium gap in collateral values. Proposition 1 shows that in any IE the collateral gap,  $\hat{\Delta}$ , is always positive.

**Proposition 1.** *Consider the C-model with two countries Home and Foreign. In any IE the Law of One Price fails. In particular,  $\hat{\Delta} = \hat{p} - \hat{p}^* = d_D(\gamma(\hat{i}_1) - \gamma(\hat{i}_2)) > 0$ .*

Proof: See Appendix C.

Intuitively, the same agent  $i^*$  prices both tranches in Foreign, whereas  $i_1$  prices the Arrow  $U$  and  $i_2$  prices the Arrow  $D$  security in Home. This raises the price  $Y$ , just like the split into floater and inverse floater tailored to two different buyers raised the price of mortgages. The collateral gap reflects Home’s superior collateral technology. The collateral gap is not only positive but it depends on the size of  $d_D$ . This characterization of the collateral gap will have important implications for the dynamic properties of prices and flows in Section 4. Proposition 1 has three corollaries that will prove important to understand behavior of gross and net flows described in Propositions 3, 4 and 5.

The first Corollary shows that price gaps also emerge between portfolios of securities that collectively pay off exactly the same as a single security that cannot be used as collateral. Define the basis as the difference between the sum of the Arrow security prices and the price of the riskless

position that pays 1:  $\beta = \hat{\pi}_U + \hat{\pi}_D - 1$ .<sup>29</sup> Then the following shows that the portfolio of two Arrow securities trades for a higher price than the riskless Foreign bond, exactly as with our example of the floater and inverse floater vs. a non-tranchable mortgage.

**Corollary 1.** *Consider the C-model with two countries Home and Foreign. In any IE the basis  $\beta$  is positive. Moreover,  $\hat{\Delta} = d_D\beta$ .*

Proof: See Appendix C.

Proposition 1 and Corollary 1 show how these two deviations from the Law of One Price depend on  $d_D$ , a key measure of the collateral technology advantage Home enjoys over Foreign.

Proposition 1 implies that Home is permanently wealthier. The initial wealth of each Home agent is  $(e_c + e_Y\hat{p})$ , which by Proposition 1 is greater than the initial wealth of each Foreign agent  $(e_c + e_Y\hat{p}^*)$ . Because Home and Foreign utilities are identical and homothetic, the following is true:

**Corollary 2.** *Consider the C-model with two countries Home and Foreign. In any IE, the value of Home's initial wealth in period 0 exceeds Foreign's by exactly  $e_Y(\hat{p} - \hat{p}^*)$ . Home collectively holds more (by the same multiple) of each traded instrument than Foreign collectively holds.*

This wealth gap persists into both future states  $U$  and  $D$ .

**Corollary 3.** *Consider the C-model with two countries Home and Foreign. In any IE, Home also holds strictly more wealth in states  $s = U, D$ .*

Proof: See Appendix C.

Define the autarky spread as  $\Delta = p - p^*$ . Whereas the collateral gap  $\hat{\Delta}$  measures the price difference *within* an equilibrium, the autarky spread measures the price difference *across* the two autarkic equilibria. Proposition 1 shows that the collateral gap is always positive even though the autarky spread may not be. Proposition 2 shows in addition that financial integration causes asset prices to diverge, with the Home price rising and the Foreign price falling as in the example, a result that will also have interesting consequences when we look at dynamics in Section 4.

**Proposition 2.** *Consider the C-model with two countries Home and Foreign. Then in any IE,  $\hat{p} > p$ ,  $\hat{p}^* < p^*$ , and  $\hat{\Delta} = \hat{p} - \hat{p}^* > \Delta = p - p^*$ . IE raises the asset price in Home and lowers the asset price in Foreign, and thus the collateral gap is bigger than the autarky spread.*

Proof: See Appendix C.

Intuitively, when international trade is possible, Foreign optimists shift from desiring the Foreign asset to desiring the Home asset in order to take advantage of its superior collateral capacity. Foreign pessimists buy the  $D$  tranche, raising its price. Together this boosts the price of collateral  $Y$ .

### 3.3 International Equilibrium, Gross Flows, Net Flows and Trade Balance

Home's collateral technology advantage enables the owners of Home collateral to create tranches that Foreign agents cannot get elsewhere, leading to the next result.

<sup>29</sup>The prices of Arrow  $U$  and Arrow  $D$  securities are given by  $\hat{\pi}_U = (\hat{p} - \hat{\pi}_T)/d_U$  and  $\hat{\pi}_D = \pi_T/d_D$  respectively.

**Proposition 3.** *Consider the C-model with two countries Home and Foreign. In any IE Home experiences gross inflows. In particular, Home is i) a seller of Arrow  $D$  securities, and ii) a net seller of Arrow  $U$  securities.*

Proof: See Appendix C.

The difference in collateral technologies across the two otherwise identical countries immediately explains why there must be cross country flows. First, Foreign agents can gain from holding  $D$  tranches, but only Home can create them, this explains (i). Second, despite the fact that Home holds more Arrow  $U$  securities because it is wealthier, its superior collateral technology enables it to create still more Arrow  $U$  securities than Foreign does, which explains (ii).

**Home-Biased Neutral IE** The equations for International Equilibrium, given in Appendix B, uniquely determine all prices, all individual final consumptions in states  $U$  and  $D$ , and all utilities. They also uniquely determine each individual's holdings of each of the effectively traded instruments, Arrow  $U$ ,  $D$  and riskless claims. Nevertheless, some individual holdings at 0 remain indeterminate because many commodities and financial instruments are perfect substitutes: Home and Foreign consumption goods, as well as the Foreign bond, are perfect substitutes. Also, both countries create Arrow  $U$  securities. This prevents us, for example, from uniquely identifying whether an agent is buying Home consumption or Foreign consumption or just holding the riskless bond, and thus prevents us from uniquely determining the gross flows in Foreign bonds and the trade balance.

We resolve this multiplicity by considering a refinement we call *home-biased neutral IE*, which we build in two steps. First, we say that an IE is neutral if agents seeking proportional payoffs across future states hold proportional portfolios. (If an agent wants to carry over twice as much wealth as another agent in every successor state, then she should hold twice as much of every asset, and not a radically different portfolio that also happens to have double the payoffs). Neutrality alone identifies a unique IE. This unique equilibrium in particular determines the gross flows in bond and the balance of trade. The neutral IE is not only simple to describe, but is also intuitive. Recognizing that the perfect substitutability is an artifact of building a model to maintain transparent simplicity, we could perturb (realistically) the commodities so that they are not perfect substitutes. This would also ensure that IE is unique. Gratifyingly, when we take the limit equilibrium as the perturbations go to zero, this equilibrium is precisely the neutral equilibrium.<sup>30</sup>

Unfortunately, Neutrality is not consistent with the empirically robust property of home bias, which means that countries prefer their own home goods relative to foreign ones. Ignoring home bias produces unrealistically large gross flows. To address this problem in the second step, we consider the unique *home-biased neutral IE*, which is obtained from the neutral IE by switching out foreign commodities or securities in favor of domestic holdings so that every agent in the net

---

<sup>30</sup>For example, the durable consumption good in Home could depreciate slightly differently from the durable Foreign consumption good, breaking the indifference between Home consumption, Foreign consumption, and Foreign bonds. Foreign could also face a binding leverage limit on its risky asset, so that the residual after paying the loan leaves a little bit of equity in the  $D$  state in addition to the big equity in the  $U$  state, rendering the Foreign  $U^*$  tranche not quite a perfect substitute for the Home  $U$  tranche. Assuming these types of frictions would imply that there would then be seven distinct instruments for holding wealth, and hence that each risk neutral individual (other than the marginal buyers) would make a unique choice of a single instrument to hold. The IE would be unique. It is easy to show that no matter what the perturbations that render the instruments distinct, letting the perturbation go to 0 picks out a unique (refined) IE in the original economy, which is indeed the neutral IE.

exporting country holds only domestic commodities or securities.<sup>31</sup> This re-allocation uniquely defines the *home-biased neutral IE*. The home bias shrinks the neutral trade of Arrow  $U$  and consumption, but preserves exactly the neutral allocation of bonds and Arrow  $D$  securities. The home-bias also maintains the neutral equilibrium balance of trade. The *home-biased neutral IE* minimizes the scale of trade, and yet countries still experience gross inflows (as guaranteed by Proposition 3) and outflows as the following proposition shows.

**Proposition 4.** *Consider the C-model with two countries Home and Foreign. In the unique home-biased neutral IE, Home experiences gross inflows and outflows. In particular, Home is i) a seller of Arrow  $D$  securities, ii) a net seller of Arrow  $U$  securities, and iii) a buyer of Foreign bonds.*

Proof: See Appendix C.

It is surprising that Home buys Foreign bonds when it can create those at Home using a replicating portfolio of Arrow securities. Corollary 1 helps us to understand this puzzle: Home buys the Foreign bond because the basis is positive, i.e. the foreign bond is cheaper than the replicating portfolio of Arrow securities. This explains how Home can start with the same real consumption possibilities as Foreign yet end up with more of everything. By selling high-valued Arrow securities for cheap Foreign bonds, Home can turn a profit on the transaction and also receive payments in both states in the future.

Finally, the next proposition brings us back to where we started, tying the trade balance to the collateral gap. By Corollary 2, the difference in wealth between Home and Foreign is equal to the collateral gap  $e_Y \hat{\Delta}$ . By homotheticity, this shows up as proportional to the initial balance of trade deficit for Home, and a permanent balance of trade deficit.

**Proposition 5.** *Consider the C-model with two countries Home and Foreign. In the unique home-biased neutral IE, Home runs a permanent balance of trade deficit, buying consumption goods from Foreign in every state  $0, U, D$ . Moreover, the trade balance of Home at time 0 is given by*

$$TB_0^H = \frac{e_Y \hat{\Delta}}{2e_{c_0} + e_Y(\hat{p} + \hat{p}^*)} = \frac{e_Y d_D \beta}{2e_{c_0} + e_Y(\hat{p} + \hat{p}^*)}.$$

Proof: See Appendix C.

Corollaries 2 and 3 are helpful to understand the basic intuition for this result: Home is wealthier than Foreign in every state, and hence it buys consumption goods from the rest of the world running a trade balance deficit. The trade balance is a function of the collateral gap and hence the basis. This will be key to explain the pro-cyclicality of gross and net flows in the dynamic model of Section 4.

### 3.4 Collateral Advantage and Comparative Statics

Our results on asset prices and international flows between Home and Foreign are all driven by the disparate ability of the Home financial system to create the distinct  $D$  tranche. To a first approximation, this disparity can be quantified by the down payoff of the assets,  $d_D$ . We solve the numerical

---

<sup>31</sup>Given agents' linear utility Home bias means that no country will hold any of the other country's goods or assets unless it is already holding all of its own. This extreme version will obviously reduce the scale of gross flows even further than in a model with some curvature.

example for all  $d_D$ , not just  $d_D = .2$ . Figure 5 plots asset prices, the collateral gap, gross flows, and Home trade balance deficit (net flows) as a function of  $d_D$ . The most important take-away is that all variables are increasing when  $d_D$  increases. The one exception is that the price of the Arrow  $D$  falls as  $d_D$  goes up and it becomes less scarce (even as the total value  $\pi_T$  of all the Arrow  $D$  rises). Note also that the total gross flows in each direction are much bigger than the net flows or trade imbalance.

It is noteworthy that for  $d_D$  approaching 1, the price of the collateral is higher than the price 1 of durable consumption (that always delivers more in the future than the collateral), because the consumption is not usable as collateral. This price gap is a manifestation of the collateral value we spoke about earlier.

In the dynamic model, the differential capacity of Home to create negative beta assets varies across time. Following bad news, the collateral capacity will decrease, and this will lead to a decrease in asset prices and international flows (but an increase in the Arrow  $D$  security). The variations in  $d_D$  in the static model provide a hint of how equilibrium behaves in the dynamic model over the cycle.

## 4 The Global Collateral Cycle

The static model in Section 3 illustrates how financial integration affects asset prices, gross flows and the trade balance. In this section we numerically solve a dynamic three-period variation of the C-model introduced in Section 1 that illustrates our Global Collateral Cycle. By hypothesis, when bad news  $D$  arrives in the intermediate period 1, it is accompanied by heightened uncertainty, earning the moniker scary bad news, while good news  $U$  arrives with reduced uncertainty. Consistent with Observation 2 from the introduction, we find that i) the prices of the positive beta assets  $Y$  and  $Y^*$  fall further from period 0 to  $D$  than they do in autarky, iia) the archetypal negative beta asset (namely the Arrow  $D$ ) is always worth more after financial integration than in Home autarky (exorbitant privilege), and iib) it is worth more after bad news at  $D$  than at 0, even though every one of the risk neutral agents believes the probability it pays is unchanged (flight to safety), and iii) gross flows as a percentage of contemporaneous wealth fall from period 0 to  $D$ , and iv) so do trade imbalances.

### 4.1 Dynamic C-Model

We consider a dynamic C-model introduced by Geanakoplos (2003). There are three periods  $t = 0, 1, 2$ . Uncertainty is represented by a tree  $S = \{0, U, D, UU, DU, DD\}$ , illustrated in Figure 6, with a root  $s = 0$  at time 0 and terminal states  $S_T = \{UU, DU, DD\}$  at time 2. Denote by  $s^*$  the unique predecessor of state  $s$ . Let  $L_s = \{c_s, Y_s\}$  be the set of commodities in each non-terminal state  $s \in \{0, U, D\}$ , and  $L_s = \{c_s\}$  the set of commodities in terminal states  $s \in S_T$ . Let the inter-period durability functions be  $F_s(c_0, Y) = (c_0, Y)$ , for  $s = U, D$  and  $F_s(c_{s^*}, Y_{s^*}) = c_{s^*} + d_s Y_{s^*}$  for  $s \in S_T$ . The consumption good is perfectly durable and perfectly durable assets pay dividends in units of the consumption good only in the terminal states.

Consider the same type of agents as in Section 1, with expected utility

$$U^i = ic_{UU} + i(1-i)c_{DU} + (1-i)^2c_{DD}, \quad (2)$$

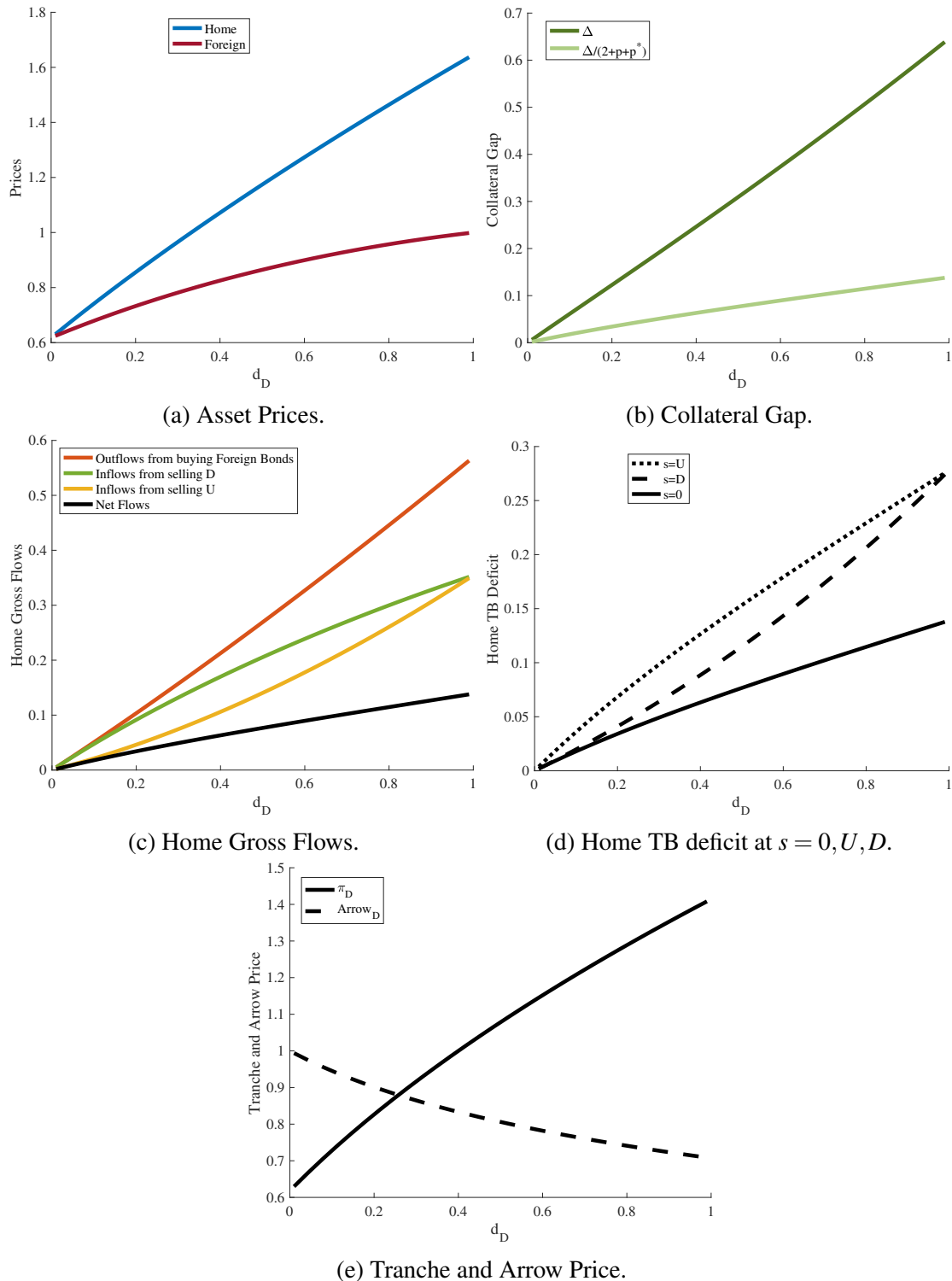


Figure 5: Comparative Statics in the static  $C$ -model. Panel 5a plots the price of  $Y^*$  and  $Y$ . Panel 5b plots the collateral gap (dark green) and the collateral gap divided by total wealth (light green). Panel 5c plots Home gross inflows, outflows and net flows: gross inflows from the sales of Arrow  $U$  securities (yellow), the gross inflows from the sales of Arrow  $D$  securities (green), gross outflows from the purchases of Foreign bonds (orange), and net flows (black). Panel 5d plots the Home TB deficit (net flows) at  $s = 0$  (solid), at  $s = U$  (dotted) and at  $s = D$  (dashed). 5e plots the prices of the tranche  $\pi_D$  (solid) and the Arrow price (dashed). All the variables are plotted as functions of  $d_D$ .

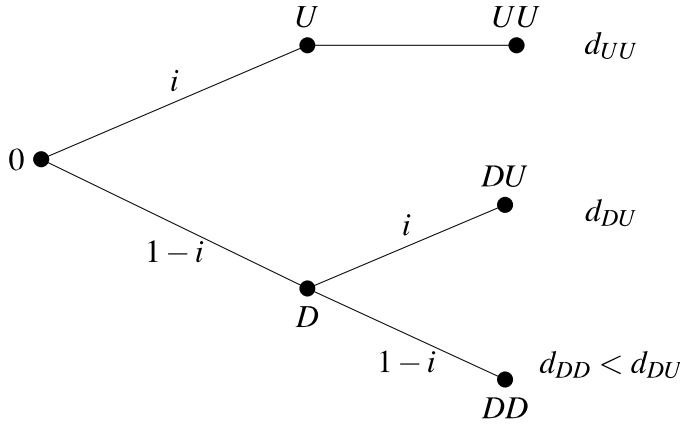


Figure 6: Uncertainty and risky asset payoffs in the dynamic  $C$ -model.

which corresponds to agent  $i$  having belief  $i$  of receiving good news at  $t = 1$ , and having belief  $i$  that, conditional on receiving bad news, assets will pay high dividends at  $t = 2$  (thus, with probability  $(1 - i)^2$  the economy receives bad news twice and the asset pays the low dividend  $d_{DD}$ ). Agents have endowments of the consumption good and the asset at all non-terminal nodes denoted by  $(e_{c_s}, e_{Y_s})$  for  $s = 0, U, D$ . All assets, whether from vintage  $t = 0$  or  $t = 1$ , pay identical dividends in  $t = 2$ .

At all non-terminal states  $s = 0, U, D$  agents can trade one-period financial contracts collateralized by  $Y$ . For each  $j \in J$ , the contract  $j = ((j_{sU}, j_{sD}), 1_Y)$  is defined by a promise of payments in units of the consumption good at each successor node collateralized by one unit of the risky asset.

We normalize the price of  $c_s$  in state  $s \in S$  to be one. The price of the asset  $Y$  at nodes  $0, U$  and  $D$  is denoted by  $p_0, p_U$  and  $p_D$  respectively. Since at  $U$  all uncertainty is resolved,  $p_U = d_{UU}$  and there is no trade; hence in what follows we only focus on  $s = 0, D$ . The budget set and collateral equilibrium are analogous to the static model and are presented in Appendix D.

The crucial assumption is that  $d_{DD} < d_{DU} = d_{UU}$ .  $D$  unambiguously signifies bad news compared to  $0$ .  $D$  also signifies a jump in uncertainty or volatility of prices across successor states. Geanakoplos (2003) and Geanakoplos (2010) called this combination *scary bad news*. The uncertainty about dividends from  $D$  is greater than the uncertainty about dividends from  $U$ , which is zero, hence it must be greater than the uncertainty from  $0$ .<sup>32</sup> Since every agent has the same belief of moving up or down conditional on being at  $D$  as she had at  $0$ , every agent regards  $D$  as scary bad news about next period's price of  $Y$ . Fostel and Geanakoplos (2012b) show that this type of uncertainty, where volatility increases after bad news, arises endogenously in models in which agents can choose which type of uncertain technologies to invest in with leverage. We consider an extreme case, where volatility after good news is zero, for clarity of exposition.

Geanakoplos (2003; 2010) argued that scary bad news has driven many of the big crises in history because of its effect on collateral. Scary bad news means that collateral may have an even bigger drop in the following period, hence the collateral can only guarantee a smaller non-contingent

<sup>32</sup>From  $s = 0$ , the price of  $Y$  might go up to  $p_U = d_{UU}$  in state  $s = DU$  or down to  $p_D > d_{DD}$  in state  $s = DD$ . From  $s = D$  the price of  $Y$  might go up to  $d_{UU}$  or down to  $d_{DD} < p_D$ , a bigger spread, with worse downside.

promise. After scary bad news, leverage endogenously plummets, which in turn causes a drop in  $p$  much bigger than warranted by the news alone. This is what he called the Leverage Cycle and we shall see it in Foreign Autarky. Fostel and Geanakoplos (2012a) showed that when the tranching technology is unlimited, then scary bad news reduces the amount of Arrow  $D$  securities that can be guaranteed, and hence causes an even bigger drop in price. That is what they call the Securitization Cycle and we shall see it in Home Autarky.

We shall see that when an autarkic leverage cycle in Foreign and an autarkic securitization cycle in Home interact after financial integration, the (pro-cyclical) price drop after scary bad news for positive beta assets gets bigger in both countries. At the same time, negative beta assets rise even more. Thus international trade exacerbates asset price volatility, at least over conditions that cause crises like scary bad news.

We shall also see that scary bad news not only creates pro-cyclical flows, but it also reduces gross flows and trade imbalances as a fraction of wealth. Gross flows emerge directly from the value of the tranches that Home exclusively creates. Trade imbalances in our model with identical agents stem from the collateral gap created by Home's superior collateral technology. Home's superior technology consists mostly in its ability to create negative beta tranches (Arrow  $D$  securities). Scary bad news curtails that advantage and therefore reduces gross flows, the collateral gap and therefore the trade imbalance.

## 4.2 Autarky Equilibrium

We next consider the 3-period  $C$ -model with two countries, Home and Foreign. Both countries are identical in every way except for the feasible financial contracts  $J$  available in each country. We use  $(^*)$  to denote Foreign variables and  $(^{\wedge})$  to denote variables in IE. As before, we first characterize the autarky equilibrium in Foreign and Home to demonstrate how leverage and tranching affect dynamics, and then we consider the IE in Section 4.3.

We assume that contract promises come due in one period. The dynamic equilibrium at  $s = 0$  and its two successors, as well as at  $s = D$  and its two successors  $s = DU$  and  $s = DD$ , each resemble the static 2-period equilibrium, with a high value of  $d_D$  and a low value of  $d_D$ , respectively.<sup>33</sup> Thus the comparative statics over  $d_D$  from the static model give us insight into what we should expect to happen in the dynamic model over time. And in fact, in line with the comparative static analysis in Section 3.4, we show that the behavior of prices and flows characterized in Propositions 1–5 hold for almost all values of the down payoff  $d_{DD}$ .

We solve the model numerically for  $(e_{c_s}, e_{Y_s}) = (1, 1)$ ,  $s = 0, U, D$ .  $d_{UU} = d_{DU} = 1$ , giving explicit solutions when  $d_{DD} = .2$  and presenting solutions graphically for all  $d_{DD} \in (0, 1)$ . For expositional ease, all equations characterizing the equilibria are provided in Appendix.D

### 4.2.1 Foreign Autarky: Leverage Cycle

At each non-terminal node, agents can trade one-period debt contracts  $j_s = ((j, j), 1_{Y^*})$  promising  $(j, j)$  units of consumption good in the following period, backed by one unit of  $Y^*$ . As before, in

---

<sup>33</sup>Of course there are some major differences from the static case. First, between  $s = 0$  and  $s = D$  there are large wealth transfers between individuals created by the bad news. Second, rational agents now have three period horizons, leading them to behave somewhat differently in period  $s = 0$  from how they would in a two period model.

equilibrium the only contract traded each period is the maxmin contract:  $j_0^* = p_D^* = 0.72$  at  $s = 0$  and  $j_D^* = d_{DD} = 0.2$  at  $s = D$ .<sup>34</sup> These contracts have prices  $\pi_0^* = p_D^*$  and  $\pi_D^* = d_{DD}$  respectively.

At  $s = 0$  there is a marginal buyer  $i_0^* = 0.88$  such that all agents  $i > i_0^*$  buy  $Y^*$  with leverage, holding the Arrow  $U$  security and agents  $i < i_0^*$  hold the consumption good and the safe bond  $j_0^* = p_D^*$ . In state  $D$  all agents that leveraged the asset lose their initial investments (the debt they owe is the value of their entire asset holdings) and their only wealth comes from the new endowments. By contrast, all less optimistic agents  $i < i_0^*$  carry over all their wealth from 0 to  $D$ , in addition to the new endowments they receive. As a result, the economy as a whole has gotten effectively less optimistic (or more risk averse) at  $D$  than it was at 0. More agents are required at  $D$  than at 0 to buy the entire asset supply and hence the new marginal buyer is a more pessimistic agent: in state  $D$  there is a marginal buyer  $i_D^* = 0.65 < 0.88 = i_0^*$ , such that all agents  $i > i_D^*$  leverage  $Y^*$  holding the Arrow  $U$ . Pessimistic agents  $i < i_D^*$  hold the consumption good and the safe bond  $j_D^* = d_{DD}$ .

The economy exhibits what Geanakoplos (2003) called the Leverage Cycle: leverage is procyclical and creates excess volatility above fundamentals. Figure 7a (blue lines) and figure 7c (blue solid line) show how the asset price falls after bad news (figure 7a shows the price drop and figure 7c shows the percentage change). In fact, the asset price falls 25% from  $p_0^* = 0.96$  at 0 to  $p_D^* = 0.72$  at  $D$  for three reasons. First, fundamentals are worse (a bad payoff is more likely). Second, after bad news there is a wealth redistribution away from optimistic buyers. Third, the increased down-risk at  $s = D$  (scary bad news) reduces the amount agents can leverage (i.e., required margins increase). Reasons two and three ensure that the marginal buyer at  $s = D$ ,  $i_D^* = 0.65$ , is much lower than the marginal buyer at  $s = 0$ ,  $i_0^* = 0.88$ . Since the price is equal to the marginal buyer's valuation, the price drop owing to the fall in the marginal buyer can be much bigger than the direct effect of bad news on any agent's valuation. For example, the marginal buyer  $i_0^* = 0.88$  thought at  $s = 0$  that the asset had expected payoff .99, and expected payoff .904 at  $s = D$ , for a drop of just over 8 points. Yet the price fell 24 points.<sup>35</sup>

## 4.2.2 Home Autarky: Securitization Cycle

As before, we assume that Home can create any arbitrary tranches. As before, that means without loss of generality, that in each non-terminal node agents will trade a one-period  $D$  tranche. At  $s = 0$  agents who hold a unit of  $Y$  as collateral can sell a  $D$  tranche  $j_0^T = ((0, p_D), 1_Y)$  promising  $(0, p_D)$  at  $t = 1$ , and at  $s = D$  agents holding the collateral can sell a  $D$  tranche  $j_D^T = (((0, d_{DD})), 1_Y)$  promising  $(0, d_{DD})$  at  $t = 2$ . These tranches have prices  $\pi_0^T$  and  $\pi_D^T$  respectively.

In Home autarky equilibrium, there are two marginal buyers in state  $s = 0$  and two in state  $s = D$ . In  $s = 0$ , agents  $i > i_0^1 = 0.70$  buy  $Y$  and issue a  $D$  tranche  $j_0^T$  promising  $p_D$  at  $t = 1$ , thus holding the residual Arrow  $U$  securities; agents  $i \in [i_0^2, i_0^1] = [0.25, 0.70]$  hold the consumption good, and agents  $i < i_0^2 = 0.25$  buy the  $D$  tranche at the price  $\pi_0^T = 0.55$ , thus holding Arrow  $D$  securities. After bad news, in  $s = D$ , agents  $i > i_0^1$  lose all their initial investment after repaying the  $D$  tranche. As in the Leverage Cycle, there is a redistribution of wealth at  $D$  from the more optimistic (or risk tolerant) to the less optimistic (risk averse) agents. Hence, there is a new marginal buyer  $i_D^1 = 0.55 < i_0^1$  such that all agents  $i > i_D^1 = 0.55$  buy the supply of asset  $Y$  and issue a new  $D$  tranche  $j_D^T$  promising  $d_{DD} = 0.2$ , thus holding the residual Arrow  $U$  securities. Moderate agents

<sup>34</sup>Since the contract promises made at time 0 come due in period 1, the relevant down payoff of the collateral from the point of view of time 0 is  $p_D^* = j_D^*$ .

<sup>35</sup>The price of the Arrow  $U$  security drops 27% from  $\frac{.96-.72}{1-.72} = .86$  at 0 to  $\frac{.72-.2}{1-.2} = .63$ .

$i \in [i_D^2, i_D^1] = [.08, .55]$  buy the consumption good and pessimistic agents  $i < i_D^2 = 0.08$  buy the  $D$  tranche at the price at price  $\pi_D^T = 0.18$ , holding Arrow  $D$  securities.

Figure 7b (blue lines) shows that with securitization in Home autarky, the  $Y$  asset price always starts much higher than it did with leverage in Foreign autarky, and falls much further after bad news, namely 39% from  $p_0 = 1.21$  at  $s = 0$  to  $p_D = 0.74$  at  $s = D$  when  $d_{DD} = 0.2$ .<sup>36</sup> Figure 7c shows that the price percentage crash in Home (blue dashed line) is always much larger than in the Foreign (blue solid line). The model with Home tranching exhibits what Fostel and Geanakoplos (2012a) called a Securitization Cycle. Because tranching greatly increases the collateral value of the asset, which decreases following bad news, tranching creates excess volatility even compared to a Leverage Cycle. It is noteworthy that the price of the Arrow  $D$  security, the archetypal short term negative beta asset, rises from  $.74 = \frac{.55}{.74}$  at 0 to  $.92 = \frac{.18}{.2} = 1 - .08$  at  $D$ . The simple reason is that the supply of Arrow  $D$  securities is smaller at  $D$ , relative to income, than at 0, because the collateral has a more severe downside loss potential over the next period starting from  $D$ .

### 4.3 International Equilibrium and the Global Collateral Cycle

In international equilibrium, Home and Foreign agents can buy the same set of financial instruments in every non-terminal state, though they are differentially created by Home and Foreign as before. As in Home autarky, in IE there are two pairs of marginal buyers, the same in both countries. At  $s = 0$  there are two marginal buyers,  $\hat{i}_0^1 = 0.78$  and  $\hat{i}_0^2 = 0.16$ , in both countries. The most optimistic investors  $i \geq \hat{i}_0^1 = 0.78$  buy assets and use them as collateral to finance their purchases: they buy the Home asset  $Y$ , selling the  $D$  tranche at price  $\hat{\pi}_0^T = .67$ ; and buy  $Y^*$  selling debt  $j_0^* = \hat{p}_D^* = 0.79$ . Optimistic agents end up holding residual  $U$  and  $U^*$  tranches (Arrow  $U$  securities). Moderate agents  $i \in [\hat{i}_0^2, \hat{i}_0^1] = [0.16, 0.78]$  hold the consumption goods and risk-free debt backed by  $Y^*$ . The most pessimistic agents  $i < \hat{i}_0^2 = 0.16$  buy  $D$  tranches (Arrow  $D$  securities).

As in autarky, important wealth redistributions occur after bad news: the optimists holding risky assets have reduced wealth after debt repayment/margin calls, and the pessimists holding down tranches have increased wealth. In  $s = D$  there are two marginal buyers,  $\hat{i}_D^1 = 0.60$  and  $\hat{i}_D^2 = 0.05$ , in both countries. As before, the most optimistic agents hold Arrow  $U$  securities, moderates the consumption good and bonds, and pessimists Arrow  $D$  securities. The top marginal buyer is much lower than at  $s = 0$ ,  $\hat{i}_D^1 = 0.60 < \hat{i}_0^1 = 0.78$ , partly because the original top group of investors,  $h \in [0.78, 1]$ , lost so much money from the bad news, and also because so many agents are needed to buy  $Y$  because the price of the  $D$  tranche has plummeted to  $\hat{\pi}_D^T = 0.19$  (because it pays only 0.2 after the bad news). The bottom marginal buyer at  $s = D$ ,  $\hat{i}_D^2 = 0.05 < \hat{i}_0^2 = 0.16$ , is so low because very few agents  $h \in [0, 0.05]$ , are required to buy the  $D$  tranche now that it pays only  $d_{DD} = .2$ . The original investors in assets  $Y$  and  $Y^*$  use their new endowments to purchase assets, but they have no other wealth after selling their initial asset holdings in order to repay their obligations due at  $t = 1$ . Moderate investors  $i \in [\hat{i}_D^2, \hat{i}_D^1] = [0.05, 0.60]$  hold the consumption good and bonds, and the most pessimistic investors  $i < \hat{i}_D^2 = 0.05$  buy  $D$  tranches.

<sup>36</sup>The Arrow  $U$  security drops only 15% from  $1.21 - .55 = .66$  to  $.74 - .18 = .56$ .

### 4.3.1 Global Collateral Cycle: Asset Prices

In the Global Collateral Cycle, the productive asset falls further from 0 to  $D$  than in autarky in both regions – global volatility is higher. The negative beta Arrow  $D$  security rises from 0 to  $D$ , and is always higher than in autarky.

First, as in the static case, Propositions 1 and 2 hold. International trade considerably raises the price of Home  $Y$  at 0 above its securitization price in Home autarky, from  $p_0 = 1.21$  to  $\hat{p}_0 = 1.40$ , and lowers the price of Foreign asset  $Y^*$  below its autarky price, from  $p_0^* = 0.96$  to  $\hat{p}_0^* = 0.91$ . Figures 7a and 7b show the same holds for all  $d_{DD}$ , and applies in state  $D$  as well. And hence, the collateral gap is always positive, and always bigger than the autarky spread.

Second, the collateral-driven international trades increase global volatility, since the drop in prices in IE after bad news both for Home  $Y$ , of 44% from  $\hat{p}_0 = 1.40$  to  $\hat{p}_D = 0.79$ , and for Foreign  $Y^*$ , of 25%, from  $\hat{p}_0^* = 0.91$  to  $\hat{p}_D^* = 0.68$ , is bigger than the corresponding autarkic drops.<sup>37</sup> As shown in Figure 7c, price crashes following bad news are generally higher for each country with IE than in autarky (both red lines, solid and dashed, are above their blue counterparts). The Securitization Cycle mechanisms amplifying volatility in the Home autarky equilibrium affect both Home and Foreign asset prices with financial integration. Financial integration amplifies volatility at Home because financial integration increases the value of tranching (given foreign demand for  $D$  tranches), which increases the collateral value of the Home asset at  $s = 0$ .<sup>38</sup> Moreover, since the Foreign asset is priced relative to the Home asset price, the excess volatility of the Home asset transfers to the Foreign asset. Thus, even though in the static model the Foreign asset price decreases with financial integration, which might suggest that the economy is more stable, the dynamic model underscores that the Foreign price following bad news is even lower with financial integration compared to autarky because of the additional amplifying mechanisms absorbed through financial integration.

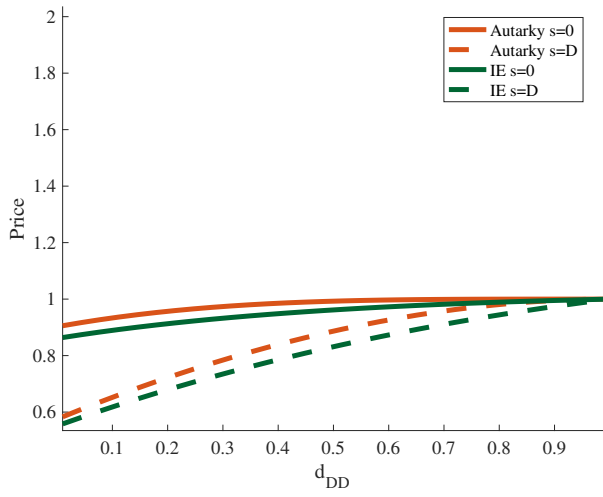
Third, the collateral gap is pro-cyclical. As shown in Figure 7d, the collateral gap decreases following bad news. This will have important implications for the behavior of gross and net flows.

Finally, one important feature of the Global Collateral Cycle is that whereas the prices of risky assets serving as collateral are pro-cyclical, the price of the negative beta (safe) asset is counter-cyclical. The price of Arrow  $D$  securities rises relative to consumption from  $\hat{\pi}_D^0 = 0.85 = \frac{.67}{.79}$  to  $\hat{\pi}_D^D = 0.96 = \frac{.19}{.2} = 1 - .04$ . Figure 8 shows the price of the Arrow  $D$  security at  $s = 0$  and  $s = D$  for different values of the final down state. The price of the Arrow  $D$  security goes up during the crisis. The reason for this is that the supply of negative beta assets falls. This underscores another, and complementary, channel to the traditional flight to safety, which relies on a demand channel.<sup>39</sup> The price of the Arrow  $D$  is always much higher in IE than it is in Home autarky because in IE Foreign and Home both demand them, but only Home produces them. This perhaps is the basic driver of exorbitant privilege that Home enjoys in our model.

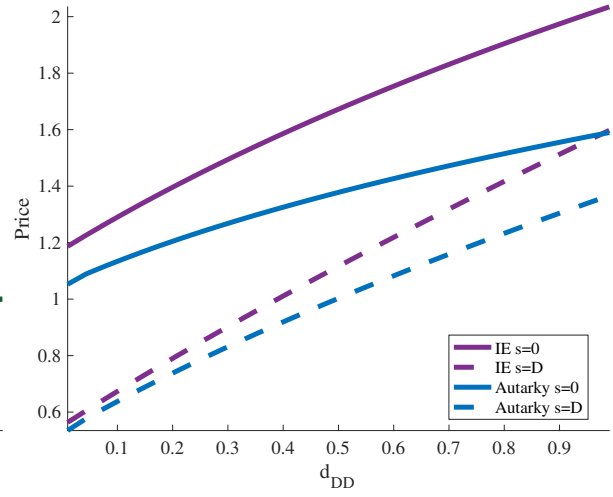
<sup>37</sup>The price of Arrow  $U$  drops 18% from  $1.40 - .67 = .73$  to  $.79 - .19 = .60$ , which is more than the drop in Home autarky, but less than the fall in Foreign autarky.

<sup>38</sup>Gong and Phelan (2019) derive a similar result in a closed-economy setting by studying how equilibrium changes when debt contracts can be used as collateral to make financial promises (“debt collateralization”). They show that debt collateralization increases the collateral value of the risky asset and increases the volatility of asset prices.

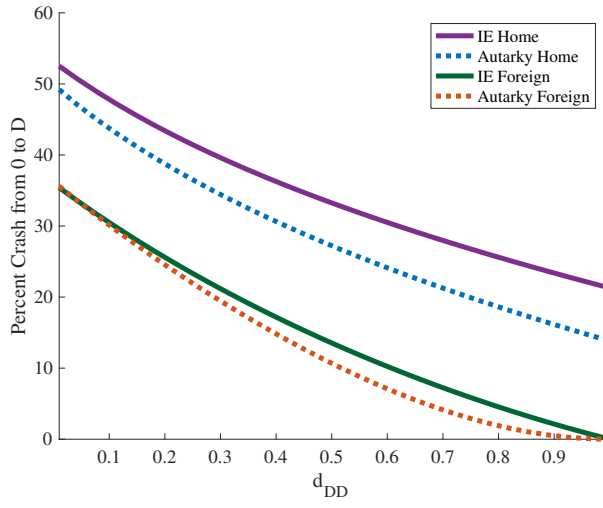
<sup>39</sup>Models like Gourinchas and Rey (2022) or Jiang, Richmond and Zhang (2024) explain flight to safety stemming from asymmetric levels of risk aversion across countries and across time.



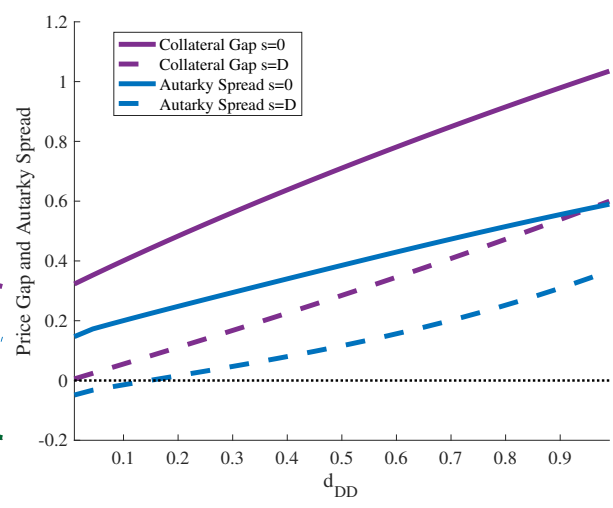
(a) Foreign Asset Price.



(b) Home Asset Price.



(c) Price Crash from 0 to  $D$ .



(d) Collateral Gap and Autarky Spread.

Figure 7: IE and Asset Prices. Panels 7a and 7b show asset prices in Foreign and Home in autarky (red and blue lines) and IE (pink and purple lines) across states. Panel 7c shows the price percentage crash from 0 to  $D$  in autarky and IE; solid lines correspond to IE and dashed lines correspond to autarky. Panel 7d plots the collateral gap in IE (purple lines) and the autarky spread (blue lines) in states 0 (solid lines) and  $D$  (dashed lines) . All the variables are plotted as functions of  $d_{DD}$ .

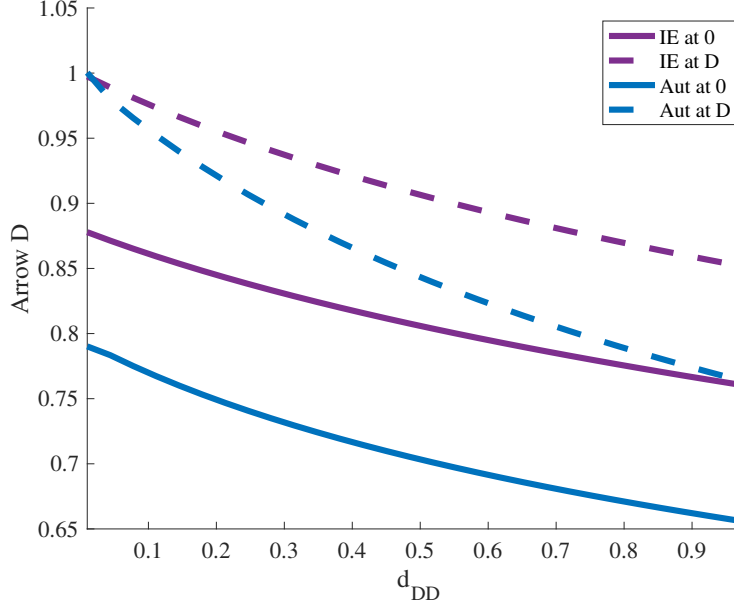


Figure 8: IE and negative beta (safe) asset price. The solid line corresponds to the price of the Arrow  $D$  security at  $s = 0$ . The dashed line corresponds to the price of the Arrow  $D$  security at  $s = D$ . All the variables are plotted as functions of  $d_{DD}$ .

#### 4.3.2 Global Collateral Cycle: Gross flows, Net Flows, and Trade Balance

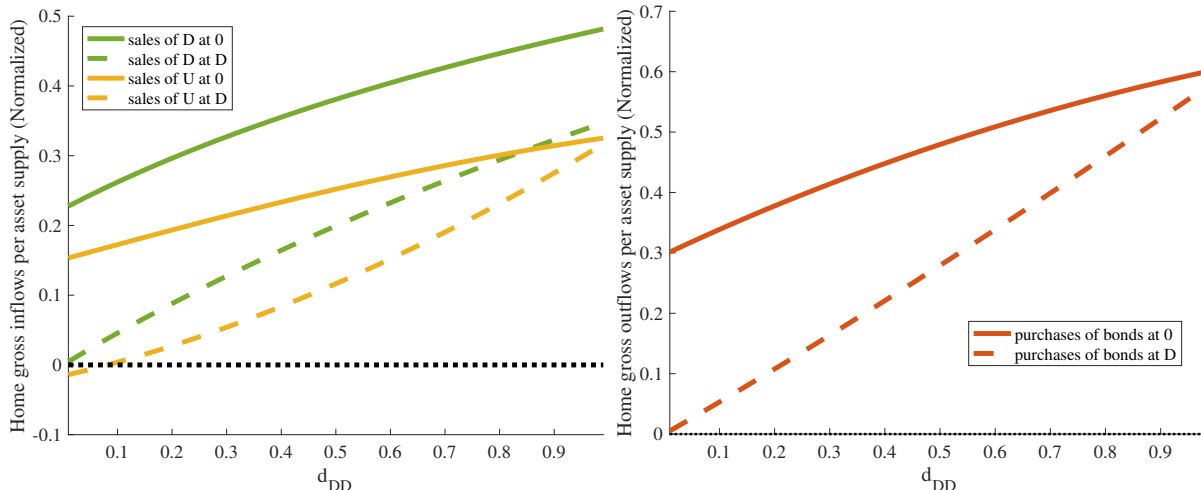
As we did in Section 3, we look at gross and net flows in the home-biased neutral IE.

Gross flows, net flows and the trade balance are pro-cyclical, declining from  $s = 0$  to  $s = D$ . The reason is that the scary bad news worsens the down risk at Home, from  $\hat{p}_D/\hat{p}_0 = 0.79/1.40 = 0.57$  at  $s = 0$ , to  $d_{DD}/\hat{p}_D = 0.2/0.79 = 0.35$  at  $s = D$ . Only Home can create negative beta tranches, and that collateral technology superiority exists at  $s = D$  as well as  $s = U$ . However, the collateral itself at  $s = D$  is now riskier, and so crucially supports the creation of fewer Arrow  $D$  securities (per unit of  $Y$ ) with the same technology. This explains the rise in the price of Arrow  $D$  securities relative to consumption. The decrease in quantity is bigger than the increase in price, so the total value of the Arrow  $D$  securities created at  $s = D$  is much lower than at  $s = 0$ . The gross inflows to Home for the sale of Arrow  $U$  and Arrow  $D$  securities drop from .266 to .173, and outflows from Home for purchases of Foreign bonds drops from 0.378 to 0.215.

The collateral advantage of Home plummets from  $s = 0$  to  $s = D$ , and as a result the collateral gap drops from  $\hat{\Delta}_0 = 0.49$  at  $s = 0$  to  $\hat{\Delta}_D = 0.11$  at  $s = D$ . With wealth between Home and Foreign closer to equal, the trade balances between the two countries shrinks. The trade balance deficit collapses from  $TB_0^H = 0.12$  at  $s = 0$  to  $TB_D^H = 0.04$  at  $s = D$ .

Figure 9 shows gross flows in  $s = 0$  and  $s = D$ , across different levels of  $d_{DD}$ . Since the supply of assets at  $s = D$  has doubled, we present gross flows as a fraction of outstanding supply in each state. These pictures confirm the qualitative features of IE at  $d_{DD} = 0.2$ , with one tiny twist for  $d_{DD} \approx 0$ . First, as stated by Propositions 3 and 4 for the static case, Home experiences gross inflows in both states since it is a seller of Arrow  $D$  securities and a net seller of Arrow  $U$  securities. Moreover, Home experiences gross outflows in both states since it buys the bond from Foreign. Second, as suggested by the comparative statics in Section 2, gross flows are pro-cyclical, i.e gross flows are

always bigger in  $s = 0$  than in  $s = D$ . The one twist is that Home becomes a slight net buyer of Arrow  $U$  securities for  $d_{DD} \approx 0$ . The reason is that in this case Home barely produces more of them, but carries extra wealth over from  $s = 0$ , and so still holds more of everything at  $s = D$ .



(a) Home Gross inflows from net sales of  $U$  and  $D$  tranches. (b) Home Gross outflows from purchase of Foreign bonds.

Figure 9: Home Gross Inflows and Outflows. Panel 9a plots Home gross inflows from net sales of Arrow  $U$  (yellow lines) and  $D$  securities (green lines) per unit of asset supply. Solid lines correspond to  $s = 0$  and dashed lines to  $s = D$ . Panel 9b plots Home gross outflows from purchases of Foreign bonds. Solid lines correspond to  $s = 0$  and dashed lines to  $s = D$ . All variables are plotted as functions of  $d_{DD}$  and scaled by asset supply.

Figure 9 shows Home trade balance deficit (net flows) in all states. First, as in the static case in Proposition 5, Home runs a permanent trade deficit in the dynamic setting. Second, as suggested by the comparative statics above, the Home trade deficit is pro-cyclical in our model, with the deficit decreasing following bad news.

## 5 Discussion and Empirical Implications

The three key ingredients in our model are investor heterogeneity within countries, the disparate collateral technologies across countries, and scary bad news for the global collateral cycle.

In our model, agent heterogeneity is captured, for tractability purposes, by different beliefs and degrees of “optimism.” Reasonable empirical counterparts would be within-country differences in demand for risk and insurance. Any real-world heterogeneity in hedging needs and risk-management would manifest in demands for different assets. Mutual funds, pensions funds, hedge funds, and financial intermediaries—not to mention shadow banks and commercial banks—have different hedging needs and different investment mandates, which are reflected in different portfolio holdings across and within asset classes. Sovereign wealth funds and central banks hold reserve assets due to macro hedging needs. Heterogeneity in our model captures these types of differences. There is ample empirical evidence that global investors, lenders, and banks behave

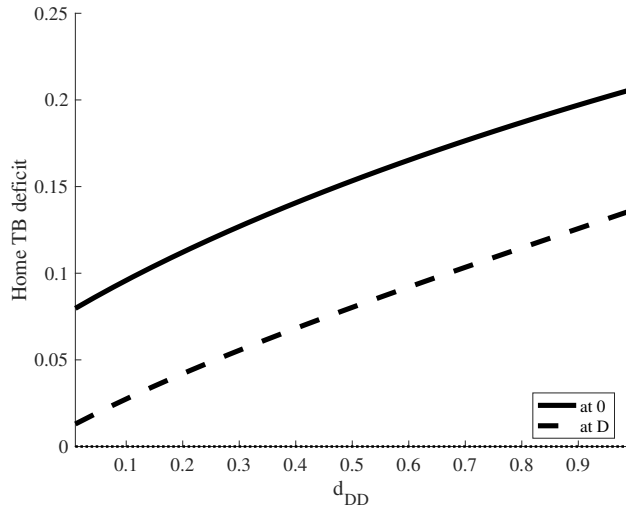


Figure 10: Home trade balance deficit ( $TB^H$ ). Solid line plots the initial TB deficit at  $s = 0$ , and the dashed line plots the Home trade deficit ( $TB^H$ ) at  $s = D$ . Both are in the dynamic model as a function of  $d_{DD}$ .

heterogeneously (see Avdjiev et al., 2022; Di Giovanni et al., 2022a; Bertaut et al., 2012, 2023; Faia, Lewis and Zhou, 2024). A serious empirical study would need to quantify the degree of heterogeneity.

Disparate collateral technologies are represented by the assumption that Foreign collateral can only back non-contingent promises, like the mortgages or corporate bonds that are featured in all advanced countries, while Home collateral can back any number of arbitrary promises, such as the tranches like CMOs and CLOs created in American securitizations out of pools of similar collateral, like mortgages or leveraged corporate loans. There is no doubt that the American securitization technology is in a different league from everybody else. A serious empirical study would have to quantify the disparity in the quantity and type of cash flows created via collateral in various countries. One measure might be the volume of securitizations. Another might be the number of loan types, and how different their cash flows are. Yet another might be the number of innovative structures, such as the seniority rules in subprime mortgage securitizations, or the pyramiding of loans where the collateral backs loans which back tranches which themselves are tranced, like the notorious American CDOs. A useful first step would be a taxonomy of all the world's collateral structures, together with their volumes by country of origin.

Scary bad news is represented in our model by a drop in expected output and a simultaneous increase in its anticipated variance, especially on the downside. The real world analogue of this is the jumps in indicators of uncertainty that accompany downturns, like the VIX.

The main international testable implications of our model are the following.

(1) Securitizing collateral in Home raises its price relative to the same unsecuritized collateral in Foreign.

Securitization raises the collateral price because the tranches can be sold to different buyers, as in our floater - inverse floater example, and because the tranches themselves become securities that can be resold so that the original buyer can unload them for cash if her needs change. One could test this price (collateral value) effect by seeing if the spread between the mortgage rate in America

and the 10-year Treasury yield is smaller than the spread between say the German mortgage rate and the German 10-year Bund rate. This is complicated because of different prepayment behavior in America and Germany, and fixed vs floating rate mortgages. Using the option adjusted spread might correct for much of this. It should be noted that a comparable study for the United States, across time instead of across countries, of the price effect of the mortgage securitization boom from 1970 - 2000 hasn't been done yet. One could refine this test by looking across many countries and time to see whether a degree of securitization creates a degree of price movement.

(2) As the securitization gap grows (measured as indicated above), gross financial flows grow in both directions.

This would require working with data on securitization trades between countries, which at present is difficult to find. Data on the loan terms for U.S. securities used as collateral in domestic funding markets are only beginning to be available and used in limited samples (see Copeland, Martin and Walker (2014) and Baklanova et al. (2017) for evidence).

(3) Larger securitization gaps should create larger trade imbalances, all else equal.

If collateral-based financial innovations are an important driver of gross flows, then a logical conjecture is that the wave of securitization beginning in the late 1990s could possibly explain the divergence between gross and net global flows. A preliminary indication, as we mentioned, is that America's permanent trade imbalance began at almost exactly the same time as America's securitization boom.

(4) Bigger cross-country differences in collateral technologies increases global asset price volatility (volatility increases in both countries).

(5) Changes in financial integration should also affect price volatility.

(6) Bigger cross-country differences in collateral technologies increase the pro-cyclicality of gross and net flows.

(7) Bigger cross-country differences in collateral technologies increases the counter-cyclicality of the price of super safe negative beta assets.

(8) Changes in financial integration should also affect gross and net flows cyclicality.

Finally, two observations about potential extensions.

First, our model does not allow for production and investment. For this reason we consider the trade balance as opposed to the current account balance, which is the difference between saving and investment. Fostel and Geanakoplos (2016) consider a closed economy model with collateral and production. They find that collateral value generates over investment. For this reason, we conjecture that a positive collateral gap in the international setting of this paper would provide a force leading to a Home current account deficit in an international model with production.

Second, our model does not consider exchange rates. We conjecture that if we were to add to our model differentiated Home and Foreign consumption goods, our Proposition 1 and Corollary 2 would imply that the Home currency would be stronger than the Foreign one, even though the countries are identical. In this sense our results would relate to the literature on exorbitant privilege (as in Gourinchas and Rey, 2022, and Jiang, Richmond and Zhang, 2024), and convenience yield (as in Jiang, Krishnamurthy and Lustig (2024)). However, the goal of our present paper is not to explain why the dollar is a negative beta asset. We do explain in our model how the US collateral technology uniquely enables the U.S. to create negative beta assets, and how this ability affects pricing of international assets and derivatives, and the behavior of gross and net flows. To the extent that other authors like Maggiori, 2017 are right that US Treasuries are negative beta assets, the huge production of near Treasury AAA tranches via securitization is evidence that the U.S.

does indeed create negative beta assets.

## References

- Acharya, Viral V, and Philipp Schnabl.** 2010. “Do global banks spread global imbalances? Asset-backed commercial paper during the financial crisis of 2007–09.” *IMF Economic Review*, 58(1): 37–73.
- Adrian, Tobias, and Hyun Song Shin.** 2014. “Procyclical leverage and value-at-risk.” *Review of Financial Studies*, 27(2): 373–403.
- Adrian, Tobias, and Nina Boyarchenko.** 2012. “Intermediary leverage cycles and financial stability.” Staff Report, Federal Reserve Bank of New York 567.
- Andritzky, Jochen R.** 2012. *Government bonds and their investors: What are the facts and do they matter?* International Monetary Fund.
- Angeletos, George-Marios, and Vasia Panousi.** 2011. “Financial Integration, Entrepreneurial Risk and Global Dynamics.” *Journal of Economic Theory*, 146(3): 863–896.
- Araujo, Aloisio, Felix Kubler, and Susan Schommer.** 2009. “Regulating Collateral-Requirements When Markets Are Incomplete.” mimeo.
- Araujo, Aloisio, Felix Kubler, and Susan Schommer.** 2012. “Regulating Collateral-Requirements When Markets Are Incomplete.” *Journal of Economic Theory*, 147(2): 450–476.
- Avdjiev, Stefan, Bryan Hardy, Şebnem Kalemli-Özcan, and Luis Servén.** 2022. “Gross capital flows by banks, corporates, and sovereigns.” *Journal of the European Economic Association*, 20(5): 2098–2135.
- Baklanova, Viktoria, Cecilia Caglio, Macro Cipriani, and Adam Copeland.** 2017. “The Use of Collateral in Bilateral Repurchase and Securities Lending Agreements.” Federal Reserve Bank of New York Staff Report 758.
- Bertaut, Carol, Laurie Pounder DeMarco, Steven Kamin, and Ralph Tryon.** 2012. “ABS inflows to the United States and the global financial crisis.” *Journal of International Economics*, 88(2): 219–234.
- Bertaut, Carol, Stephanie Curcuru, Ester Faia, and Pierre-Oliver Gourinchas.** 2023. “The global (mis) allocation of capital.” mimeo, NBER.
- Bloom, Nicholas.** 2009. “The Impact of Uncertainty Shocks.” *Econometrica*, 77(3): 623–685.
- Broner, Fernando, Tatiana Didier, Aitor Erce, and Sergio L Schmukler.** 2013. “Gross capital flows: Dynamics and crises.” *Journal of monetary economics*, 60(1): 113–133.
- Brumm, Johannes, Michael Grill, Felix Kubler, and Karl Schmedders.** 2015. “Collateral requirements and asset prices.” *International Economic Review*, 56(1): 1–25.

- Brunnermeier, Markus, José De Gregorio, Barry Eichengreen, Mohamed El-Erian, Arminio Fraga, Takatoshi Ito, PR Lane, J Pisani-Ferry, Eswar Prasad, Raghuram Rajan, et al.** 2012. “Banks and cross-border capital flows: Policy challenges and regulatory responses.” *Committee on International Economic Policy and Reform*.
- Brunnermeier, Markus K., and L. H. Pedersen.** 2009. “Market Liquidity and Funding Liquidity.” *The Review of Financial Studies*, 22(6): 2201–2238.
- Bruno, Valentina, and Hyun Song Shin.** 2014. “Cross-border banking and global liquidity.” *The Review of Economic Studies*, rdu042.
- Caballero, Ricardo J, and Alp Simsek.** 2016. “A Model of Fickle Capital Flows and Retrenchment: Global Liquidity Creation and Reach for Safety and Yield.” National Bureau of Economic Research.
- Caballero, Ricardo J., Emmanuel Farhi, and Pierre-Olivier Gourinchas.** 2008. “An Equilibrium Model of “Global Imbalances” and Low Interest Rates.” *American Economic Review*, 98(1): 358–93.
- Copeland, Adam, Antoine Martin, and Michael Walker.** 2014. “Repo Runs: Evidence from the Tri-Party Repo Market.” *The Journal of Finance*, 69(6): 2343–2380.
- Devereux, Michael B, and James Yetman.** 2010. “Leverage constraints and the international transmission of shocks.” *Journal of Money, Credit and Banking*, 42(s1): 71–105.
- Di Giovanni, Julian, Şebnem Kalemli-Özcan, Alvaro Silva, and Muhammed A Yildirim.** 2022a. “Global Supply Chain Pressures, International Trade, and Inflation.” National Bureau of Economic Research No. 30240.
- Di Giovanni, Julian, Şebnem Kalemli-Özcan, Mehmet Fatih Ulu, and Yusuf Soner Baskaya.** 2022b. “International spillovers and local credit cycles.” *The Review of Economic Studies*, 89(2): 733–773.
- Faia, Ester, Karen K Lewis, and Haonan Zhou.** 2024. “Do Investor Differences Impact Monetary Policy Spillovers to Emerging Markets?” National Bureau of Economic Research.
- Forbes, Kristin J, and Francis E Warnock.** 2012. “Capital flow waves: Surges, stops, flight, and retrenchment.” *Journal of International Economics*, 88(2): 235–251.
- Fostel, A., and J. Geanakoplos.** 2008. “Leverage Cycles and The Anxious Economy.” *American Economic Review*, 98(4): 1211–1244.
- Fostel, Ana, and John Geanakoplos.** 2012a. “Tranching, CDS, and Asset prices: How Financial Innovation Can Cause Bubbles and Crashes.” *American Economic Journal: Macroeconomics*, 4(1): 190–225.
- Fostel, Ana, and John Geanakoplos.** 2012b. “Why Does Bad News Increase Volatility And Decrease Leverage?” *Journal of Economic Theory*, 147(2): 501–525.

- Fostel, Ana, and John Geanakoplos.** 2014. “Endogenous collateral constraints and the leverage cycle.” *Annu. Rev. Econ.*, 6(1): 771–799.
- Fostel, Ana, and John Geanakoplos.** 2015. “Leverage and default in binomial economies: a complete characterization.” *Econometrica*, 83(6): 2191–2229.
- Fostel, Ana, and John Geanakoplos.** 2016. “Financial Innovation, Collateral, and Investment.” *American Economic Journal: Macroeconomics*, 8(1): 242–284.
- Geanakoplos, John.** 1997. “Promises Promises.” Vol. 27, 285–320, Addison-Wesley Publishing Co.
- Geanakoplos, John.** 2003. “Liquidity, Default and Crashes: Endogenous Contracts in General Equilibrium.” Vol. 2, 170–205, Econometric Society Monographs.
- Geanakoplos, John.** 2010. “The Leverage Cycle.” Vol. 24, 1–65. University of Chicago Press.
- Geanakoplos, John, and William R Zame.** 2013. “Security Pools and Efficiency.” mimeo.
- Geanakoplos, John, and William R Zame.** 2014. “Collateral equilibrium, I: a basic framework.” *Economic Theory*, 56(3): 443–492.
- Gong, Feixue, and Gregory Phelan.** 2019. “Debt collateralization, capital structure, and maximal leverage.” *Economic Theory*, 1–27.
- Gong, Feixue, and Gregory Phelan.** 2022. “Collateral Constraints, Tranching, and Price Bases.” *Economic Theory*.
- Gorton, Gary, and Andrew Metrick.** 2012. “Securitized banking and the run on repo.” *Journal of Financial Economics*, 104(3): 425 – 451.
- Gottardi, Piero, and Felix Kubler.** 2015. “Dynamic competitive economies with complete markets and collateral constraints.” *The Review of Economic Studies*, rdv002.
- Gourinchas, Pierre-Olivier, and Hélène Rey.** 2007. “International Financial Adjustment.” *Journal of Political Economy*, 115(4): 665–703.
- Gourinchas, Pierre-Olivier, and Helene Rey.** 2022. “Exorbitant privilege and exorbitant duty.”
- Hale, Galina, and Maurice Obstfeld.** 2016. “The Euro and the geography of international debt flows.” *Journal of the European Economic Association*, 14(1): 115–144.
- Jiang, Zhengyang, Arvind Krishnamurthy, and Hanno Lustig.** 2024. “Dollar safety and the global financial cycle.” *Review of Economic Studies*, 91(5): 2878–2915.
- Jiang, Zhengyang, Robert J Richmond, and Tony Zhang.** 2024. “A portfolio approach to global imbalances.” *The Journal of Finance*, 79(3): 2025–2076.
- Kalemli-Ozcan, Sebnem, Bent Sorensen, and Sevcan Yesiltas.** 2012. “Leverage across firms, banks, and countries.” *Journal of international Economics*, 88(2): 284–298.

- Kalemli-Ozcan, Sebnem, Elias Papaioannou, and Fabrizio Perri.** 2013. “Global banks and crisis transmission.” *Journal of international Economics*, 89(2): 495–510.
- Kiyotaki, Nobuhiro, and John Moore.** 1997. “Credit Cycles.” *Journal of Political Economy*, 105(2): pp. 211–48.
- Lane, Philip R, and Gian Maria Milesi-Ferretti.** 2007. “The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities, 1970–2004.” *Journal of international Economics*, 73(2): 223–250.
- Maggiore, Matteo.** 2017. “Financial Intermediation, International Risk Sharing, and Reserve Currencies.” *American Economic Review*, 107(10): 3038–71.
- Mendoza, Enrique G, and Vincenzo Quadrini.** 2010. “Financial globalization, financial crises and contagion.” *Journal of monetary economics*, 57(1): 24–39.
- Mendoza, Enrique G, Vincenzo Quadrini, and Jose-Victor Rios-Rull.** 2009. “Financial integration, financial development, and global imbalances.” *Journal of Political Economy*, 117: 3.
- Miranda-Agrippino, Silvia, and Hélène Rey.** 2020. “US monetary policy and the global financial cycle.” *The Review of Economic Studies*, 87(6): 2754–2776.
- Obstfeld, Maurice.** 2012. “Financial flows, financial crises, and global imbalances.” *Journal of International Money and Finance*, 31(3): 469 – 480. Financial Stress in the Eurozone.
- Obstfeld, Maurice.** 2015. “Trilemmas and trade-offs: living with financial globalisation.” *BIS working paper 480*.
- Phelan, Gregory.** 2015. “Collateralized Borrowing and Increasing Risk.” *Economic Theory*, 1–32.
- Phelan, Gregory, and Alexis Akira Toda.** 2019. “Securitized Markets, International Capital Flows, and Global Welfare.” *Journal Financial Economics*, 131(3): 571–592.
- Pozsar, Zoltan, Tobias Adrian, Adam Ashcraft, and Hayley Boesky.** 2010. “Shadow banking.” *New York*, 458(458): 3–9.
- Rey, Hélène.** 2015. “Dilemma not trilemma: the global financial cycle and monetary policy independence.” National Bureau of Economic Research.
- Simsek, Alp.** 2013. “Belief Disagreements and Collateral Constraints.” *Econometrica*, 81(1): 1–53.
- Ueda, Kozo.** 2012. “Banking globalization and international business cycles: Cross-border chained credit contracts and financial accelerators.” *Journal of international Economics*, 86(1): 1–16.
- Willen, Paul.** 2004. “Incomplete markets and trade.” FRB of Boston Working Paper, FRB of Boston Working Paper.

# Appendices

## A Autarky Equilibrium Equations (Section 2)

### A.1 Foreign (Subsection 2.1)

The price of the consumption good (or equivalently the riskless bond) is normalized to 1. Each Arrow  $U$  pays  $(1,0)$  and from the no-profit condition for the intermediaries, trades at a price of  $(p^* - d_D)/(d_U - d_D)$ .

$$(1 - i_1^*) \frac{(e_c + e_Y p^*)}{(p^* - d_D)/(d_U - d_D)} = (d_U - d_D) e_Y. \quad (3)$$

$$\frac{\gamma(i_1^*)}{(p^* - d_D)/(d_U - d_D)} = 1. \quad (4)$$

Equation (3) is the market clearing condition for Arrow  $U$  securities. The market for risk-free commodities (consumption goods and the bond) clears by Walras' Law. The top  $1 - i_1^*$  agents spend all of their initial endowment of  $e_c + e_Y p^*$  on Arrow  $U$  securities at a price of  $(p^* - d_D)/(d_U - d_D)$ . The aggregate demand has to equal the aggregate supply of Arrow  $U$  given by  $(d_U - d_D) e_Y$ . Equation (4) states that the marginal buyer is indifferent between holding an Arrow  $U$  (left hand side) and a riskless position (right hand side). Notice that these two equations also describe the collateral equilibrium formally presented in Section 2. Let us check that equation 3 implies that the marginal buyer  $i_1^*$  is indifferent between holding  $Y$ , holding a riskless position, and holding a leveraged position in  $Y$ . (Since there are only two states, indifference between any two of these three holdings guarantees indifference to the third.) Equation 3 can be written as  $p^* = \gamma(i_1^*) d_U + (1 - \gamma(i_1^*)) d_D$ , where the left hand side is the ratio  $(p^*/1)$  of the prices of  $Y^*$  and consumption, and the right hand side is the ratio of marginal their utilities, yielding the first indifference. Hence, the asset is priced according to the marginal buyer's beliefs. Equation  $p^* = \gamma(i_1^*) d_U + (1 - \gamma(i_1^*)) d_D$  can also be written as  $\frac{\gamma(i_1^*)(d_U - d_D)}{p^* - d_D} = 1$ , where the left-hand side is the return of the leveraged position and the right-hand side is the utility per dollar of the consumption good, yielding the second indifference. Note also that since the number of Arrow  $U$  securities is equal to a fixed multiple of the number of  $Y$  assets, market clearing of the former is equivalent to market clearing of the latter.

### A.2 Home (Subsection 2.2)

As before, we express all the equilibrium equations as if agents were directly trading in Arrow  $U$  and  $D$  securities that pay  $(1,0)$  and  $(0,1)$  with prices  $(p - \pi_T)/d_U$  and  $\pi_T/d_D$  respectively. Equation (5) is the market clearing condition for the Arrow  $U$  securities:

$$(1 - i_1) \frac{(e_c + e_Y p)}{(p - \pi_T)/d_U} = d_U e_Y, \quad (5)$$

The left hand side is the aggregate demand for Arrow  $U$  securities. The top  $1 - i_1$  agents each have wealth  $e_c + e_Y p$ , and the price of an Arrow  $U$  security is  $(p - \pi_T)/d_U$ . The right hand side is the aggregate supply of Arrow  $U$  securities

Equation (6) is the market clearing condition for the Arrow  $D$  securities:

$$i_2 \frac{(e_c + e_Y p)}{\pi_T / d_D} = d_D e_Y. \quad (6)$$

The left hand side is the aggregate demand for Arrow  $D$  securities. Agents  $i \leq i_2$  spend all their wealth  $e_c + e_Y p$  on Arrow  $D$  securities with a price of  $\pi_T / d_D$ . The right hand side is the aggregate supply of Arrow  $D$  which equals  $d_D e_Y$ .

Equation (7) states that the marginal buyer  $i_1$  is indifferent between the return from an Arrow  $U$  (left hand side) and the return from holding the consumption good (right hand side), and equation (8) states that the marginal buyer  $i_2$  is indifferent between the return from an Arrow  $D$  (left hand side) and the return from holding the consumption good (right hand side).

$$\frac{\gamma(i_1)}{(p - \pi_T) / d_U} = 1, \quad (7)$$

$$\frac{(1 - \gamma(i_2))}{\pi_T / d_D} = 1. \quad (8)$$

## B IE Equations (Section 3)

As before we express all the equations in terms of Arrow securities instead of  $U$  and  $D$  tranches. Equation (9) states that by no-arbitrage, the return of an Arrow  $U$  security obtained from tranching  $Y$  (left hand side) is equal to the return of an Arrow  $U$  security obtained from leveraging  $Y^*$  (right hand side).

$$\frac{1}{(\hat{p} - \hat{\pi}_T) / d_U} = \frac{1}{(\hat{p}^* - d_D) / (d_U - d_D)}. \quad (9)$$

Equations (10) and (11) are the market clearing conditions for Arrow  $U$  and  $D$  securities respectively:

$$(1 - \hat{i}_1) \frac{[(e_c + e_Y \hat{p}) + (e_c + e_Y \hat{p}^*)]}{(\hat{p} - \hat{\pi}_T) / d_U} = d_U e_Y + (d_U - d_D) e_Y. \quad (10)$$

$$\hat{i}_2 \frac{[(e_c + e_Y \hat{p}) + (e_c + e_Y \hat{p}^*)]}{\hat{\pi}_T / d_D} = d_D e_Y. \quad (11)$$

Compared to their autarky counterparts (equations (3), (5) and (6), the market clearing conditions in IE include wealth from both countries. The right hand side of both equations also underscores the fact that whereas both countries create Arrow  $U$  securities, only Home creates Arrow  $D$  securities.

Finally, equations (12) and (13) are the optimality conditions for marginal buyers  $\hat{i}_1$  and  $\hat{i}_2$ . Equation (12) states that the marginal buyer  $\hat{i}_1$  is indifferent between an Arrow  $U$  security (either via tranching the Home asset or leveraging the Foreign asset) and a safe position (holding consumption goods or riskless bonds)

$$\frac{\gamma(\hat{i}_1)}{(\hat{p} - \hat{\pi}_T) / d_U} = 1. \quad (12)$$

Equation (13) states that the marginal buyer  $\hat{i}_2$  is indifferent between an Arrow  $D$  security and a safe position (holding consumption goods and riskless bonds):

$$\frac{(1 - \gamma(\hat{i}_2))}{\hat{\pi}_T/d_D} = 1. \quad (13)$$

## C Proofs of Propositions

### C.1 Proof of Proposition 1

From equations (9), (12), and (13), we can write asset prices as

$$\begin{aligned} \hat{p} &= \gamma(\hat{i}_1)d_U + \hat{\pi}_T = \gamma(\hat{i}_1)d_U + (1 - \gamma(\hat{i}_2))d_D, \\ \hat{p}^* &= \gamma(\hat{i}_1)(d_U - d_D) + d_D. \end{aligned}$$

Hence we have  $\hat{\Delta} = d_D(\gamma(\hat{i}_1) - \gamma(\hat{i}_2))$ , which is positive since  $\hat{i}_1 > \hat{i}_2$  and beliefs are monotonic.

### C.2 Proof of Corollary 1

We define the basis as  $\beta = \hat{\pi}_U + \hat{\pi}_D - 1$ . Using equations (9), (12), and (13) we have that

$$\begin{aligned} \beta &= (\hat{p} - \hat{\pi}_T)/d_U + \hat{\pi}_T/d_D - 1 = \\ &= (\gamma(\hat{i}_1)d_U + \hat{\pi}_T - \hat{\pi}_T)/d_U + \hat{\pi}_T/d_D - 1 = \\ &= \gamma(\hat{i}_1)d_U/d_U + (1 - \gamma(\hat{i}_2)d_D)/d_D - 1 = \\ &= \gamma(\hat{i}_1) + (1 - \gamma(\hat{i}_2)) - 1 > 0 \text{ since } \hat{i}_1 > \hat{i}_2 \text{ and beliefs are monotonic.} \end{aligned}$$

Moreover, from Proposition 1 we have that,  $\hat{\Delta} = d_D\beta$  since  $\beta = \gamma(\hat{i}_1) + (1 - \gamma(\hat{i}_2)) - 1 = \gamma(\hat{i}_1) - \gamma(\hat{i}_2)$ .

### C.3 Proof of Corollary 2

The difference in wealth between Home and Foreign at  $s = 0$  is  $(e_c + e_Y\hat{p}) + (e_c + e_Y\hat{p}^*) = e_Y(\hat{p} - \hat{p}^*) > 0$ , which From Proposition 1 is positive.

### C.4 Proof of Corollary 3

Let  $f = \frac{e_c + e_Y\hat{p}}{[(e_c + e_Y\hat{p}) + (e_c + e_Y\hat{p}^*)]}$  be the fraction of wealth at  $s = 0$  collectively held by Home agents. From Proposition 1 and Corollary 2  $f > 1/2$ .

Since every Home agent owns the same fraction of wealth between him and his identical twin in Foreign, and since utilities are linear, Home will collectively own its endowment plus the fraction  $f > 1/2$  of all asset payoffs in each terminal state, while Foreign will own the same endowment plus the fraction  $1 - f < 1/2$  of all asset payoffs in each terminal state.

## C.5 Proof of Proposition 3

(i) Clearly Home must export Arrow  $D$  securities, and thus hold fewer than in autarky.

(ii) Note that the ratio of Home wealth to Foreign wealth is less than the ratio of Arrow  $U$  Securities created in Home to Arrow  $U$  securities created in Foreign. From equations (9), (12), and (13), we have that

$$w = \frac{\hat{p}e_Y + e_c}{\hat{p}^*e_Y + e_c} = \frac{[\gamma(\hat{i}_1)d_U + (1 - \gamma(\hat{i}_2))d_D]e_Y + e_c}{[\gamma(\hat{i}_1)d_U + (1 - \gamma(\hat{i}_1))d_D]e_Y + e_c} < \frac{d_U}{d_U - d_D}$$

because dividing both numerators and denominators by  $d_U$  and replacing  $\frac{d_U}{d_D}$  with  $R$ , that is equivalent to

$$\frac{[\gamma(\hat{i}_1) - \gamma(\hat{i}_1)R + (1 - R)(1 - \gamma(\hat{i}_2))R]e_Y + e_c(1 - R)}{[\gamma(\hat{i}_1) - \gamma(\hat{i}_1)R + R]e_Y + e_c} < 1$$

which is surely true. It follows from Corollary 2, that Home does not hold as many Arrow  $U$  securities as it creates, i.e. Home is a net exporter of Arrow  $U$  securities and holds fewer of them than it did in autarky.

## C.6 Proof of Proposition 2

To prove the proposition we will use the following Lemma.

**Lemma 1.** *International trade lowers Home's marginal buyer of  $D$  tranches and raises Home's marginal buyer of  $U$  tranches, while lowering Foreign's marginal buyer of  $U$  tranches: i)  $\hat{i}_2 < i_2$ , ii)  $\hat{i}_1 > i_1$ , and iii)  $\hat{i}_1 < i_1^*$ .*

*Proof:*

i)  $\hat{i}_2 < i_2$

Suppose the opposite. Then at least as many agents would be spending all their money on  $D$  tranches, yet buying fewer of them. This could only happen if  $D$  tranches are more expensive relative to wealth in global equilibrium than in autarky. Since by hypothesis  $1 - \gamma(\hat{i}_2) \leq 1 - \gamma(i_2)$ , the  $D$  tranche can only get relatively more expensive if  $\hat{i}_1 < i_1$ . But that means the number of agents buying safe instruments is less than in autarky,  $\hat{i}_1 - \hat{i}_2 < i_1 - i_2$ , and each agent is poorer (because the price of  $Y$  must have gone down), contradicting our starting observation that Home buys more riskless assets than it did in autarky.

ii)  $\hat{i}_1 > i_1$

Suppose the opposite. That means, according to Proposition 4, that at least as many Home agents are spending all their money yet buying fewer  $U$  tranches than in autarky. That means  $U$  tranches must have gotten more expensive relative to income, which (since  $1 - \gamma(\hat{i}_2) > 1 - \gamma(i_2)$ ), can only happen if  $\hat{i}_1 > i_1$ .

iii)  $\hat{i}_1 < i_1^*$

Suppose the opposite. That means fewer (or equal numbers) of agents must buy more  $U$  tranches than in autarky, when the price of  $U$  tranches relative to wealth has become more expensive, which is a contradiction.

The statement of proposition follow directly from the lemma and equations 9, 12, 13, 4 Since  $\hat{i}_1 > i_1$  and  $\hat{i}_2 < i_2$ ,  $\hat{p} > p$ . Similarly, since  $\hat{i}_1 < i_1^*$ ,  $\hat{p}^* < p^*$ .

## C.7 Proof of Proposition 4

The proposition follows directly from the Home-Bias Neutral equilibrium noting Home holds the fraction  $f > .5$  of the securities in *i*), *ii*) and *iii*) in the statement.

## C.8 Proof of Proposition 5

In Home-Bias Neutral equilibrium, Home holds the fraction fraction  $f > .5$  of world consumption, hence it runs a balance of trade deficit at 0. Furthermore, at 0 Home holds the fraction  $f$  of Arrow  $U$  securities and the same fraction  $f$  of bonds and Arrow  $D$  securities. Net cash flows from Foreign to Home at  $U$  are then

$$f(d_U + (d_U - d_D)) - d_U + fd_D = d_U(2f - 1) > 0.$$

Home spends this cash flow on consumption, running a trade deficit at  $U$ . Similarly, net cash flows from Foreign to Home at  $D$  are

$$-(1 - f)d_D + fd_D = (2f - 1)d_D > 0$$

Again, Home spends this cash flow on consumption, running a trade deficit at  $D$ .

Finally, the trade balance for Home at time 0 is

$$TB_0^H = \frac{(e_{c_0} + e_Y \hat{p})}{2e_{c_0} + e_Y(\hat{p} + \hat{p}^*)} - \frac{(e_{c_0^*} + e_{Y^*} \hat{p}^*)}{2e_{c_0} + e_Y(\hat{p} + \hat{p}^*)} = \frac{e_Y \hat{\Delta}}{2e_{c_0} + e_Y(\hat{p} + \hat{p}^*)}.$$

## D Dynamic Model

### D.1 Budget Set and Collateral Equilibrium in the Dynamic Model

#### Budget Set

Given asset and contract prices in each state  $s$ ,  $(p, (\pi_j)_{j \in J})$ , each agent  $i \in I$  chooses asset holdings  $y_s$  of  $Y$ , contract trades  $\varphi_{j,s}$  and consumption  $c_s$  in state  $s$ , subject to the budget set defined by

$$\begin{aligned}
B^i(p, \pi) = & \left\{ (c_s, y_s, \varphi_s) \in \mathbb{R}_+^L \times \mathbb{R}^J : \right. \\
& c_0 + p_0 y_0 + \sum_{j \in J} \varphi_{j,0} \pi_{j,0} \leq e_c + p_0 e_Y, \\
& \sum_{j \in J} \max(0, -\varphi_{j,0}) \leq y_0, \\
& c_U + p_U y_U + \sum_{j \in J} \varphi_{j,U} \pi_{j,U} \leq c_0 + p_U y_0 + \sum_{j \in J} \varphi_{j,U} \min(j_U, p_U), \\
& \sum_{j \in J} \max(0, -\varphi_{j,U}) \leq y_U, \\
& c_D + p_D y_D + \sum_{j \in J} \varphi_{j,D} \pi_{j,D} \leq c_0 + p_D y_0 + \sum_{j \in J} \varphi_{j,D} \min(j_D, p_D), \\
& \sum_{j \in J} \max(0, -\varphi_{j,D}) \leq y_D, \\
& \left. c_s = F_s(c_{s^*}, Y_{s^*}) + \sum_{j \in J} \varphi_j \min(j_s, d_s), s \in S_T \right\}.
\end{aligned}$$

### Collateral Equilibrium

A Collateral Equilibrium in this economy is a price of asset  $Y$ , contract prices, asset holdings, contract trades and consumption decisions by all the agents  $((p, \pi), (c_0^i, y^i, \varphi^i, c_U^i, c_D^i)_{i \in I}) \in (\mathbb{R}_+ \times \mathbb{R}_+^J) \times (\mathbb{R}_+^L \times \mathbb{R}^J)^I$ , such that

1.  $\int_0^1 c_s^i di = e_{c_s} \forall s \in \{0, U, D\}$
2.  $\int_0^1 c_s^i di = F(e_{c_{s^*}}, e_{Y_{s^*}}) \forall s \in S_T$
3.  $\int_0^1 y_s^i di = e_{Y_s} \forall s \in \{0, U, D\}$
4.  $\int_0^1 \varphi_{j,s}^i di = 0, \forall j \in J \text{ and } \forall s \in \{0, U, D\}$
5.  $(c_s^i, y_s^i, \varphi_s^i) \in B^i(p, \pi), \forall i \text{ and } (c_s, y_s, \varphi_s) \in B^i(p, \pi) \Rightarrow U^i(c_s, y_s) \leq U^i(c_s^i, y_s^i), \forall i$

## D.2 Equilibrium Equations (Section 4)

We solve the model numerically for  $(e_{c_s}, e_{Y_s}) = (1, 1), s = 0, U, D. d_{UU} = d_{DU} = 1$ . The following equations are the ones that characterize the equilibrium in each case. Unlike the static model in which we stated equations in the space of Arrow securities and the Foreign bond, here we state the equations in the space of the original commodities, assets used as collateral and consumptions. In the dynamic case it is easier to understand the changes of wealth trough time generated by interim scary bad news. Of course, as explained in the paper, the two approaches are equivalent.

### D.2.1 Autarky Foreign

The first two equations correspond to market clearing for the asset  $Y^*$  in states  $s = 0$  and  $s = D$ . The next two are the marginal agents' indifferent conditions at states  $s = 0$  and  $s = D$ .

$$\begin{aligned} (1 - i_0^1) \frac{1 + p_0}{p_0 - p_D} &= 1 \\ (i_0^1 - i_D^1) \frac{1 + p_0}{p_D - d_{DD}} + (1 - i_D^1) \frac{1 + p_D}{p_D - d_{DD}} &= 2 \\ \frac{i_0^1(1 - p_D)}{p_0 - p_D} &= i_0^1 + (1 - i_0^1) \frac{i_0^1(1 - d_{DD})}{p_D - d_{DD}} \\ i_D^1 + (1 - i_D^1)d_{DD} &= p_D \end{aligned}$$

### D.2.2 Autarky Home

The first four equations are the ones corresponding to state  $s = 0$ : the first two are market clearing for the asset  $Y$  and the  $D$  tranche, and the second two the marginal buyer indifference conditions. The last four equations are the analogous ones for state  $s = D$ .

$$\begin{aligned} (1 - i_0^1) \frac{(1 + p_0)}{p_0 - \pi_0^T} &= 1 \\ i_0^2(1 + p_0) &= \pi_0^T \\ \frac{i_0^1}{p_0 - \pi_0^T} &= i_0^1 + (1 - i_0^1) \frac{i_0^1}{p_D - \pi_D^T} \\ p_D(1 - i_0^2) &= \pi_0^T \\ \frac{(i_0^1 - i_D^1)(1 + p_0)}{p_D - \pi_D^T} + \frac{(1 - i_D^1)(1 + p_D)}{p_D - \pi_D^T} &= 2 \\ \frac{i_D^2}{i_0^2} p_D + i_D^2(1 + p_D) &= 2\pi_D^T \\ \frac{i_D^1}{p_D - \pi_D^T} &= 1 \\ d_{DD}(1 - i_D^2) &= \pi_D^T \end{aligned}$$

### D.2.3 International Equilibrium

The first five equations are the ones corresponding to state  $s = 0$ : the first two are market clearing for the asset  $Y$  and the  $D$  tranche, and the last three are the marginal buyer indifference conditions and the non arbitrage condition for the Arrow  $U$  security (it can be created via tranching  $Y$  or via leveraging  $Y^*$ ). The last five equations are the analogous ones for state  $s = D$ .

$$\begin{aligned}
(1 - \hat{i}_0^1) \frac{2 + \hat{p}_0 + \hat{p}_0^*}{\hat{p}_0 - \hat{\pi}_0^T + \hat{p}^* - \hat{p}_D^*} &= 1 \\
\hat{i}_0^2 (2 + \hat{p}_0 + \hat{p}_0^*) &= \hat{\pi}_0^T \\
\frac{\hat{i}_0^1}{\hat{p}_0 - \hat{\pi}_0^T} &= \hat{i}_0^1 + (1 - \hat{i}_0^1) \frac{\hat{i}_0^1}{\hat{p}_D - \hat{\pi}_D^T} \\
\frac{\hat{i}_0^1}{\hat{p}_0 - \hat{\pi}_0^T} &= \frac{\hat{i}_0^1 (1 - \hat{p}_D^*)}{\hat{p}_0^* - \hat{p}_D^*} \\
\hat{p}_D (1 - \hat{i}_0^2) &= \hat{\pi}_0^T \\
(\hat{i}_0^1 - \hat{i}_D^1) \frac{2 + \hat{p}_0 + \hat{p}_0^*}{\hat{p}_D - \hat{\pi}_D^T + \hat{p}_D^* - d_{DD}} + (1 - \hat{i}_D^1) \frac{2 + \hat{p}_D + \hat{p}_D^*}{\hat{p}_D - \hat{\pi}_D^T + \hat{p}_D^* - d_{DD}} &= 2 \\
\frac{\hat{i}_D^2}{\hat{i}_0^2} \hat{p}_D + \hat{i}_D^2 (2 + \hat{p}_D + \hat{p}_D^*) &= 2 \hat{\pi}_D^T \\
\frac{\hat{i}_D^1}{\hat{p}_D - \hat{\pi}_D^T} &= 1 \\
\frac{\hat{i}_D^1}{\hat{p}_D - \hat{\pi}_D^T} &= \frac{\hat{i}_D^1 (1 - d_{DD})}{\hat{p}_D^* - d_{DD}} \\
d_{DD} (1 - \hat{i}_D^2) &= \hat{\pi}_D^T;
\end{aligned}$$