INFORMATION DESIGN AND MECHANISM DESIGN: AN INTEGRATED FRAMEWORK

By

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Information Design and Mechanism Design: An Integrated Framework*

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Abstract

We develop an integrated framework for information design and mechanism design in screening environments with quasilinear utility. Using the tools of majorization theory and quantile functions, we show that both information design and mechanism design problems reduce to maximizing linear functionals subject to majorization constraints. For mechanism design, the designer chooses allocations weakly majorized by the exogenous inventory. For information design, the designer chooses information structures that are majorized by the prior distribution. When the designer can choose both the mechanism and the information structure simultaneously, then the joint optimization problem becomes bilinear with two majorization constraints. We show that pooling of values and associated allocations is always optimal in this case. Our approach unifies classic results in auction theory and screening, extends them to information design settings, and provides new insights into the welfare effects of jointly optimizing allocation and information.

JEL CLASSIFICATION: D44, D47, D82, D83.

KEYWORDS: Mechanism Design, Screening, Nonlinear Pricing, Information Design, Bayesian Persuasion, Majorization.

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1 Introduction

We offer an integrated perspective on mechanism design and information design, emphasizing the deep symmetry between these two classes of problems. We use the geometric language of majorization to provide a unified treatment that reveals structural parallels of information design and mechanism design.

We start with the classic screening problem of Mussa and Rosen (1978), where a monopolist sells goods of varying quality to buyers with heterogeneous values. Following Loertscher and Muir (2022), we assume the seller has an exogenous inventory of goods rather than producing them at convex cost. Instead of working directly with distributions of the values of the buyer and the qualities of the good, we work throughout with their inverses—that is, quantile functions. Thus, we describe the problem in terms of a quantile value function $V(t)$, giving the value V of the buyer at the t-th quantile, and a quantile allocation function $Q(t)$, giving the quality Q of the good at the t-th quantile.

The core contribution of this paper is to show how three fundamental problems in pricing and information provision can be understood as variations of a single mathematical structure. We provide an envelope formula expressing the seller's revenue as a function of quantile value and allocation functions. This bilinear representation allows us to analyze three distinct optimization problems:

First, we characterize the optimal screening mechanism for an exogenous information structure. The seller chooses a quantile allocation function that is weakly majorized by the exogenous inventory to maximize revenue. This turns the objective function of the seller into a linear functional of the allocation. This approach follows Kleiner, Moldovanu, and Strack (2021) who emphasized the role of majorization constraints in economic optimization problem (see also Kleiner, Moldovanu, and Strack (2026), their survey article in this volume). The solution involves concavifying an appropriately defined pointwise revenue function, naturally leading to pooling (or "ironing") in regions where monotonicity constraints bind.

Second, we consider the information design problem: choosing the optimal information structure given an exogenous mechanism that allocates goods efficiently. Here, the designer maximizes a linear functional subject to a (strict) majorization constraint—the set of feasible information structures consists of all mean-preserving contractions of the prior distribution of values. Remarkably, this problem mirrors the mechanism design exercise. The solution again involves a concavification procedure, but now applied to a different function and with a different majorization constraint.

Third, we examine the problem where the designer can choose both the information structure and the mechanism jointly. The optimization is now bilinear in the quantile value and quantile allocation functions, subject to two majorization constraints. We show that pooling information is always optimal when both dimensions are jointly chosen. By pooling information, we refer to any information structure that provides less than complete information to the buyer about their value; however, coarse information translates directly into the optimality of a mechanism that offers few distinct items. This encompasses any deterministic or stochastic information structure that sends the same signal for at least two distinct values. This result contrasts sharply with the separate optimization problems, where full disclosure or full separation can be optimal depending on the primitives.

Throughout the paper, we relate the screening problem to the classic single good auction setting with a finite number of bidders. Kleiner, Moldovanu, and Strack (2021) noted that when one restricts attention to symmetric bidder distributions and symmetric mechanisms, the optimal auction problem is equivalent to the screening problem via a generalization of Border's theorem (Border (1991)). Specifically, in an auction with N bidders, the relevant quantile allocation function is $Q(t) = t^{N-1}$. All three of our optimization exercises extend naturally to this auction interpretation.

In Bergemann, Heumann, Morris, Sorokin, and Winter (2022), we studied the information design problem in the auction case, showing that the optimal information structure depends only on the number of bidders and involves upper censoring. In Bergemann, Heumann, and Morris (2026), we analyzed the screening problem where the seller can choose both the mechanism and information structure. The auction version of this problem was earlier analyzed by Bergemann and Pesendorfer (2007); the symmetric case of the auction problem can thus be understood as a special case of our screening framework.

Our approach differs from the standard Bayesian persuasion framework of Kamenica and Gentzkow (2011), where the sender's payoff is a function of the receiver's posterior expected value (state). In the Bayesian persuasion setting, there exists a well-known characterization of the distribution of posterior beliefs that can be generated by any information structure: namely, the ones that are a mean-preserving contractions of the distribution of values. The characterization of the set of feasible distributions of expected values remains valid in our information design problem; however, it is not useful because our problem is linear in expected values but nonlinear in the corresponding probabilities. To get tractability, we work in the quantile space. Thus, while the objective function is the same as in the classic Bayesian persuasion framework, the distributions of expected values are re-

placed with the quantile distributions. Importantly, the set of feasible quantile distributions is given by the mean-preserving spreads of the prior quantile value function. Hence, we invert the direction of the constraint by moving from value distributions to quantile distributions. Methodologically, we follow Kleiner, Moldovanu, and Strack (2021) in their work on majorization constraints and therefore also use the corresponding terminology. Interestingly, this places our information design closer to the Myerson (1981) ironing technique, which also employed quantile functions. Recent work by Jewitt and Quigley (2023) shows that our problem is equivalent to Bayesian persuasion with Yaari (1987) preferences—linear in outcomes but non-linear in probabilities.

While our primary focus is revenue maximization, we also examine alternative objectives. We show that the consumer surplus maximization problem exhibits a natural duality with revenue maximization, allowing us to apply the same geometric techniques. We characterize the complete welfare frontier—the set of all feasible revenue and consumer surplus pairs under all possible information structures. For the auction case with an efficient allocation rule, we show that the frontier can be traced out by upper and lower censoring information structures, with the optimal structure switching discontinuously at the point where total surplus is maximized. These results illustrate the robust predictions approach described in Bergemann and Morris (2013), (2016) : we don't think that there is a literal information designer who is maximizing a weighted sum of revenue and consumer surplus. Rather, the information designer is metaphorical and by examining information design for all possible weights, we identify which combinations of revenue and consumer surplus are possible in equilibrium under some information structure.

We also examine comparative statics in the number of bidders in the auction problem. We show that the optimal information structure in revenue-maximizing auctions involves pooling a decreasing fraction of the value distribution as N grows. However, a striking regularity emerges: the expected number of bidders whose values are pooled at the top remains approximately constant at about two bidders, regardless of N. This suggests that optimal information design sustains competition among roughly two bidders at the margin, even in large auctions.

Our framework connects to several strands of literature beyond those already mentioned. The problem of jointly optimizing pricing and information relates to recent work on data markets and digital advertising, see Bergemann, Heumann, Morris, Sorokin, and Winter (2022), Bergemann and Bonatti (2024), Bergemann, Bonatti, and Wu (2025). The revenue guarantee problem—finding mechanisms that perform well across all information structures—connects to the informationally robust mechanism design literature surveyed by Brooks and Du (2025b). Finally, our use of quantile

methods links to the virtual value approach in auction theory by Myerson (1981) and Bulow and Klemperer (1996).

Our analysis focuses exclusively on mechanism design with transfers. Kleiner, Moldovanu, and Strack (2021) provide a connection between information design and delegation (mechanism design without transfers) in a linear-quadratic environment using majorization techniques; Kolotilin and Zapechelnyuk (2025) provide a more general connection between these two classes of problems. Lou (2023) analyzes the joint design of delegation and information design. Similarly, Ivanov (2010) considers the joint choice of information design and information transmission in the context of a game of strategic communication (cheap talk). He shows that information control increases the benefit for the sender relative to a delegation solution in the setting of Crawford and Sobel (1982).

The remainder of the paper proceeds as follows. Section 2 introduces the model and establishes notation for quantile functions, information structures, and mechanisms. Section 3 derives the envelope formula for revenue and consumer surplus in quantile space. Sections 4, 5 and 6 analyze mechanism design, information design, and joint design optimization of allocation and information, respectively. Section 7 examines consumer surplus maximization and weighted welfare objectives. Section 8 discusses robust predictions, characterizes the welfare frontier, and provides comparative statics in the number of bidders.

2 Setting

2.1 Quantile Representation of the Screening Problem

A seller supplies goods of varying quality $q \in \mathbb{R}_+$ to a continuum of buyers with mass 1 indexed by $t \in [0,1]$. Each buyer has unit demand and a willingness-to-pay, or value, $v \in \mathbb{R}_+$ for quality q. The utility net of the payment $p \in \mathbb{R}_+$ is:

$$v \cdot q - p. \tag{1}$$

The functional form - which follows Mussa and Rosen (1978) - is important because it ensures that buyers only care about the expected quality of the good they are supplied. The buyers' values $v \in [\underline{v}, \overline{v}]$ are distributed according to $F_v \in \Delta([\underline{v}, \overline{v}])$ where $0 \leq \underline{v} < \overline{v} \leq \infty$. Moreover, we can interpret q as the quality, but also a quantity or probability that a good is provided. In this second interpretation of q, we can link the problem to the auction problem where a single unit is provided

among many symmetric bidders.

We assume that there is an exogenous inventory of the good available to the seller. Specifically, the seller has a mass 1 of goods with qualities $q \in [\underline{q}, \overline{q}]$ distributed according to

$$F_q \in \Delta\left([\underline{q}, \overline{q}]\right), \tag{2}$$

where $0 \leq \underline{q} < \overline{q} \leq \infty$. Thus, the seller can offer a fixed and exogenously given distribution F_q of qualities.

It is without loss of generality to assume that the inventory has mass 1 as a smaller inventory can be augmented with goods of quality $q = 0$. For example, if the inventory consists of a mass $1/2$ of homogeneous goods of quality $q = 1$ (so only $1/2$ of the buyers can be served), this corresponds to the distribution:

$$F_q(q) = \begin{cases} \frac{1}{2}, & \text{if } q \in [0, 1); \\ 1, & \text{if } q \geq 1. \end{cases}$$

Hence, the mass of $1/2$ at $q = 0$ captures the fact that $1/2$ of the buyers cannot be served.

We denote the inverse of the each distribution, the corresponding quantile function, as follows:

$$V(t) \triangleq F_v^{-1}(t) \quad \text{and} \quad Q(t) \triangleq F_q^{-1}(t). \tag{3}$$

We will refer to V as a quantile value function, which captures the exogenous distribution of values among buyers. We will refer to Q as a quantile allocation function and it captures the exogenous inventory of the seller. Whenever one of the distributions has gaps in its support, we adopt the convention that V, Q are right continuous.

This is the classic screening environment of Mussa and Rosen (1978), with the proviso that, like Loertscher and Muir (2022), we take the inventory of goods sold to be exogenous. This is convenient for our survey. The complete analysis of Mussa and Rosen (1978) can then be restored if we endogenize the inventory, i.e., allow the seller to produce goods of different distributions of qualities at some convex cost.

We present our analysis in the quantile space $t \in [0, 1]$ rather than the value and quality space $v, q \in \mathbb{R}_+$. Mapping to quantile space is convenient and is done implicitly or explicitly in many contexts, see Myerson (1981), Hartline (2017), Bergemann, Heumann, Morris, Sorokin, and Winter (2022) and Bulow and Klemperer (1996).

2.2 Information Structures

While V captures the distribution of ex post values in the buyer population, buyers may be imperfectly informed about their values.

An information structure is given by a mapping $I : [\underline{v}, \overline{v}] \to \Delta(S)$, with the interpretation that $I(v)$ is a distribution of signals in some signal space S observed by buyers with a latent value v. A buyer's expected value conditional on the signal realization s is denoted by:

$$w(s) \triangleq \mathbb{E}[v \mid s]. \tag{4}$$

Importantly, a buyer's expectation of their value will be a sufficient statistic for their preferences. This is an assumption that is widely adopted in the Bayesian persuasion literature (see Kolotilin (2018)).

Frequently, we can omit the dependence of the expected value $w(s)$ on the signal s and simply write w for a generic expected value. We denote by F_w the distribution of expected valuations generated by an information structure, and denote by $W(t)$ the associated quantile expected value function:

$$W(t) \triangleq F_w^{-1}(t),$$

which, as before, we adopt the convention that it is right continuous. We will refer to W as a quantile value function as $W(t)$ is the expected value of the t-th quantile buyer in the population.

2.3 Selling Mechanisms

A selling mechanism is described by a menu of qualities and payments. The seller has the ability to shape the distribution of expected qualities sold by offering lotteries on goods of different qualities. If buyers are offered a distribution of qualities, or a stochastic bundle B in form of a lottery, with

$$B \in \Delta\left([\underline{q}, \overline{q}]\right),$$

then the expected quality x offered is:

$$x \triangleq \int_{\underline{q}}^{\overline{q}} q dB(q).$$

If $\int dB(q) < 1$, then the probability of receiving an object is less than 1. This is of course equivalent to receiving a zero quality good with probability $1 - \int dB(q)$. The expected utility of a bundle B net of the payment $p \in \mathbb{R}_+$ is:

$$w \int_{\underline{q}}^{\bar{q}} q dB(q) - p. \tag{5}$$

We now describe the set of possible mechanisms.

A menu indexed by $t \in [0,1]$ consists of bundles $B(\cdot|\, t)$ at prices $p(t)$:

$$M \triangleq \{(B(\cdot|\, t), p(t))\}_{t\in[0,1]}. \tag{6}$$

Without loss of generality, we assume that the lotteries with a higher index t correspond to higher expected qualities, and denote the quantile distribution of expected qualities by:

$$X(t) \triangleq \int_{\underline{q}}^{\bar{q}} q dB(q|\, t).$$

Without loss of generality, we assume that– in addition to being nondecreasing– it is right continuous. We say that a menu induces a quantile distribution of qualities $X(t)$; we formalize later the fact that this is indeed a quantile distribution. The menu must satisfy the feasibility constraint. That is, the seller can only bundle and discard goods, but cannot produce more goods. For now, we omit this constraint and make it explicit later.

2.4 The Auction Model

We will also discuss the translation of our results into a symmetric auction setting. There is a single object to be allocated. There are N bidders who have values drawn independently according to a common distribution H. Now, the probability that a bidder n with quantile t in the value distribution H has the highest value among all bidders is simply $Q(t) = t^{N-1}$, the probability that $N-1$ bidders have a quantile lower than t. Note that this expression does not depend on the common distribution H of values. It will turn out that the optimal symmetric auction solution is essentially identical to that of the screening problem when Q represents the exogenous inventory of different qualities. Thus, solutions to the auction model will be identical to solutions for a corresponding screening model.

Throughout the paper, we will illustrate our results for the case of symmetric power distributions

of values and qualities:

$$F_v(v) = v^{\frac{1}{4}}, \text{ and } F_q(q) = q^{\frac{1}{4}}. \tag{7}$$

The associated quantile functions are given by

$$V(t) = Q(t) = t^4.$$

In the case of the auction model, this corresponds to the case of five bidders i.e., $N = 5$ competing for a single unit of an object.

3 Revenue and Consumer Surplus: The Envelope Theorem in Quantile Space

We begin our analysis by finding the consumer surplus and profits generated by an arbitrary information structure and mechanism. For now, we abstract away from the signals or the lotteries in a menu, and instead write the consumer surplus and profits in terms of the induced quantile (expected) value function $W(t)$ and the induced quantile (expected) allocation function $X(t)$.

A buyer with expected value $W(t)$ will choose a quantile t' and associated item $X(t')$ in the mechanism to solve the following:

$$u(t) \triangleq \max_{t'} \{W(t)X(t') - p(t')\}.$$

By the revelation principle, it is without loss of generality to assume that the mechanism is incentive compatible, so that $t' = t$ is always the optimal choice for the consumer with the value $W(t)$. Following the Envelope theorem we find that $u(t)$ is determined by the allocation $X(t)$ as follows:

$$du(t) = X(t)\, dW(t),$$

and in integral form

$$u(t) = \int_0^t X(s)dW(s) + u(0).$$

As is standard, the lowest type is always left with zero consumer surplus, so we have $u(0) = 0$. We

thus have that the expected consumer surplus is given by:

$$\mathbb{E}[u(t)] = \int_0^1 \left[\int_0^t X(s)dW(s)\right] dt = \int_0^1 (1-t)X(t)dW(t). \tag{8}$$

The first equality is obtained by noting that the quantiles are always uniformly distributed in $[0,1]$; the second equality is obtained by integrating by parts.

The payment by a t-quantile buyer is given by the total surplus generated by this buyer less their indirect utility, so its given by:

$$p(t) = W(t)X(t) - \int_0^t X(s)dW(s). \tag{9}$$

The expected revenue of the seller is thus given by:

$$\mathbb{E}[p(t)] = \int_0^1 \left(W(t)X(t) - \int_0^t X(s)dW(s)\right)dt = \int_0^1 (1-t)W(t)dX(t) + X(0)W(0). \tag{10}$$

as before, the first equality is obtained by noting that the quantiles are always uniformly distributed in $[0,1]$; the second equality is obtained integrating by parts twice.

We thus denote the expected consumer surplus and expected revenue by:

$$U(W,X) \triangleq \mathbb{E}[u(t)], \tag{11}$$

$$R(W,X) \triangleq \mathbb{E}[p(t)]. \tag{12}$$

where the notation now emphasizes that revenue is solely dependent on the quantile value function (the information structure) and the quantile allocation function (the mechanism).

There are two important aspects of the description that we have provided in terms of quantile functions. First, revenue and consumer surplus are bilinear in the quantile functions. Second, the expression for revenue and consumer surplus show a remarkable symmetry in (8) and (11). The integrator $W(t)$ in the revenue is the integrand in the consumer surplus. Conversely, the integrator $X(t)$ in the consumer surplus is the integrand in the revenue. Moreover, if

$$X(0) = 0 \text{ or } W(0) = 0, \tag{13}$$

and this boundary condition typically holds as either the lowest type has zero value or receives zero

allocation, then we have a complete payoff symmetry in that

$$R(W, X) = U(X, W). \tag{14}$$

Thus, the revenue with quantile value function $W(t)$ and allocation quantile function $X(t)$ is the same as the consumer surplus if $X(t)$ is the quantile value function and $W(t)$ is the quantile allocation function. We shall return to the symmetry in particular in Section 7 when we consider solutions that focus on maximizing consumer surplus. Third, by expressing the revenue (or consumer surplus) directly in terms of the quantile allocation function and the quantile value function, the feasibility of the allocation rule becomes uncoupled from the feasibility of the information structure. In other words, the feasibility of either allocation or information policy become uncoupled. We will explore the usefulness of this uncoupling immediately in the analysis of the optimal mechanism.

4 Mechanism Design

We first identify the revenue-maximization mechanism, fixing an exogenous information structure $W(t)$. We characterized revenue for arbitrary information structures in the previous section, but our leading interpretation in this section is that buyers know their values and so $W(t) = V(t)$.

We begin the analysis by characterizing the set of feasible distributions of qualities that the seller can generate across all mechanisms. We show that an allocation is feasible if and only if the induced quantile allocation function X is weakly majorized by the exogenous inventory Q. The seller's revenue can then be expressed as a linear functional of X, and the optimal mechanism is characterized by the concavification of the pointwise revenue function, which we will define in due course. We then interpret the result in the context of auctions, provide an example and examine some of the classic results that establish when the optimal mechanism does not exhibit pooling. We conclude this section by providing a general discussion about the results we present and the corresponding literature.

4.1 Feasibility of Allocation

Following Kleiner, Moldovanu, and Strack (2021) (see Proposition 4), an allocation rule–which is mapping from quantiles to qualities– is feasible if and only if the quantile allocation rule X satisfies

$$\int_x^1 X(t)dt \leq \int_x^1 Q(t)dt, \, \forall x \in [0,1], \tag{15}$$

where we do not require that the inequality becomes an equality at $x = 0$. In this case, we say Q *weakly majorizes* X and we write $X \prec_w Q$ (the weak majorization order is equivalent to the increasing convex order). As discussed above, the seller has the ability to pool goods of different quality, which corresponds to choosing distributions X that are a mean preserving contraction of the distribution Q. However, the seller has the ability not to sell some products, which means that the qualities of these products are set to 0. This is equivalent to allowing downward shifts in the distribution of qualities in the sense of first-order stochastic dominance. And, in fact, for any $X \prec_w Q$, there exists a distribution of qualities $\widehat{X}$ such that $\widehat{X}$ is a mean-preserving contraction of Q and $\widehat{X}$ first-order stochastically dominates X (see Shaked and Shanthikumar (2007), Theorem 4.A.6.). Thus this part of the seller's problem corresponds to first pooling and then reducing qualities.

Here we emphasize a distinct advantage of our formulation in terms of quantile functions. The feasibility constraint stated in (15) – when expressed as a quantile function– does not depend on the information structure. If instead we were to consider an incentive compatible menu where the allocation rule is expressed as a function of the value, then the distribution of goods allocated by the mechanism will obviously depend on the distribution of (expected) values. Hence, the feasibility constraint becomes *coupled* with the information structure (and with the underlying distribution of values).

Feasibility in Auctions When the seller chooses the optimal selling mechanism of an indivisible object with N bidders, the allocation rule is a function $q_n(v_n, v_{-n})$ that determines the probability that agent n wins the object given the report of all bidders. The winning probability is $q_n(v_n) = \int q_n(v_n, v_{-n}) f(v_{-n}) dv_{-n}$. Hence, the quantile allocation is:

$$X(t) = q_n(V(t)).$$

In a symmetric auction, a non-decreasing allocation rule can be implemented if and only if

$$X(t) \prec_w t^{N-1},$$

which is sometimes referred to as Border's conditions (see Border (1991)). This argument was developed in detail and also extended to general rank auctions in Kleiner, Moldovanu, and Strack (2021). Thus with $Q(t) = t^{N-1}$, we can interpret the mechanism design problem as finding the allocation that maximizes the revenue in a symmetric auction.

4.2 Revenue-Maximizing Mechanism

We now fix the information structure to be complete disclosure– that is, we impose $W(t) = V(t)$– and find the optimal mechanism. We can express the problem of finding the optimal mechanism as solving

$$\max_{X \prec_w Q} R(V, X).$$

This is the problem studied by Loertscher and Muir (2022) who characterize the optimal selling policy for a distribution of (discrete) qualities to a continuum of buyers.

We define a function $r_W(t)$ as follows:

$$r_W(t) \triangleq W(t)(1-t),$$

and write the revenue using the earlier expression (10) as follows:

$$R(W, X) = X(0)r_W(0) + \int_0^1 r_W(t)dX(t). \tag{16}$$

We refer to r_W as the *pointwise revenue function*. If the seller makes a take-it-or-leave-it offer that is accepted by $(1-t)$ fraction of the consumers then the price must be $p = W(t)$, and thus the expected revenue is $r_W(t)$. If we interpret the distribution as a continuum of potential buyers, then $(1-t)$ is the quantity sold by the seller (the demand) and $V(t)$ is the inverse demand function. We thus have a linear optimization problem in $dX(t)$ where the weights are given by the pointwise revenue function.

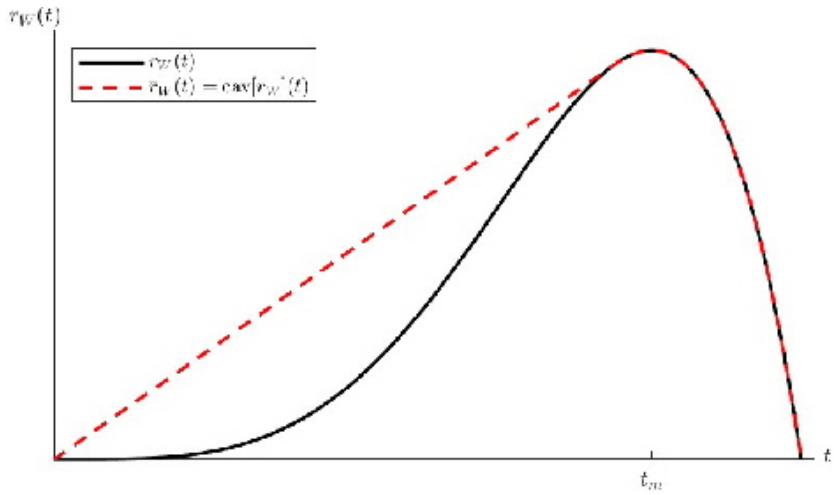

Figure 1: Revenue function $r_W(t)$ and its concavification $\overline{r}_W(t)$ when $W(t) = t^4$.

We define the quantile that maximizes the revenue function:

$$t_m \triangleq \arg\max_{t\in[0,1]} r_W(t),$$

which we assume is uniquely defined. Hence, a monopolist facing a demand $V(t)$ would serve a fraction $(1 - t_m)$ of the buyers.

We denote by $\overline{r}_W$ the concavification of r_W; the concavification of function r_W is the smallest concave function that is pointwise larger than r_W, which we illustrate in Figure 1.

To describe the optimal mechanism, it is useful to introduce the notation necessary to describe the concavification. There exists a collection of quantiles (and associated open intervals) $\left\{(\underline{t}_i, \overline{t}_i)\right\}_{i\in I}$ indexed by $i \in I$, with $\underline{t}_i, \overline{t}_i \in [0, 1]$ and

$$\underline{t}_i < \overline{t}_i \leq \underline{t}_{i+1}, \tag{17}$$

such that:

$$\overline{r}_W(t) \begin{cases} = r_W(t), & \text{if } t \notin \bigcup_{i\in I}(\underline{t}_i, \overline{t}_i); \\ > r_W(t), & \text{if } t \in (\underline{t}_i, \overline{t}_i). \end{cases}$$

Hence, the quantiles $(\underline{t}_i, \overline{t}_i)_{i\in I}$ define where r_W lies below its concavification. Thus, we can describe the optimal mechanism.

Theorem 1 (Optimal Mechanism via Concavification)

An optimal mechanism is given by:

$$X(t) = \begin{cases} 0, & \text{if } t < t_m; \\ Q(t), & \text{if } t \geq t_m \text{ and } t \notin \bigcup_{i \in I} (\underline{t}_i, \overline{t}_i); \\ \frac{\int_{\underline{t}_i}^{\overline{t}_i} Q(t)dt}{\overline{t}_i - \underline{t}_i}, & \text{if } t \geq t_m \text{ and } t \in (\underline{t}_i, \overline{t}_i). \end{cases}$$

The revenue generated by the optimal mechanism is:

$$\max_{X \prec_w Q} R(W, X) = \overline{r}_W(t_m)Q(t_m) + \int_{t_m}^{1} \overline{r}_W(t)dQ(t). \tag{18}$$

That is, the mechanism excludes buyers with quantiles $t < t_m$ (using a reserve price $p = V(t_m)$) and pools all quantiles (and values) $t > t_m$ whenever $r_W(t) < \overline{r}_W(t)$. By contrast all quantiles $t > t_m$ and outside the pooling interval are allocated the efficient allocation: $X(t) = Q(t)$. We provide a proof of the result in the Appendix that uses majorization (as explained later, similar results have appeared in the literature and have employed a variety of different approaches). If $\overline{r}_W(t)$ is strictly concave at any

$$t \notin \bigcup_{i \in I} (\underline{t}_i, \overline{t}_i),$$

then the mechanism we provide is the unique optimal mechanism; furthermore, this condition is satisfied "generically" (where we add the quotation marks to emphasize we have not formalized this).

This theorem characterizes the revenue obtained by the seller as well as the optimal mechanism that attains this revenue. We first note that the optimal revenue is obtained using expression (16) with three replacements. We can replace r_W with its concavification (which by construction is always weakly higher), we replace $X(t)$ with the exogenous inventory $Q(t)$, and the integral with the initial condition are evaluated at t_m instead of zero. The optimal mechanism excludes buyers below the revenue-maximizing quantile t_m, pools the allocation of all quantiles where $r_W(t) < \overline{r}_W(t)$ and buyers with quantiles t such that $r_W(t) = \overline{r}_W(t)$ receive their corresponding efficient allocation.

4.3 Single Unit Auction

When we interpret our model as an auction, the specific functional form $Q(t) = t^{N-1}$ does not play a role in determining the quantiles of goods that need to be pooled. As expressed in Theorem 1,

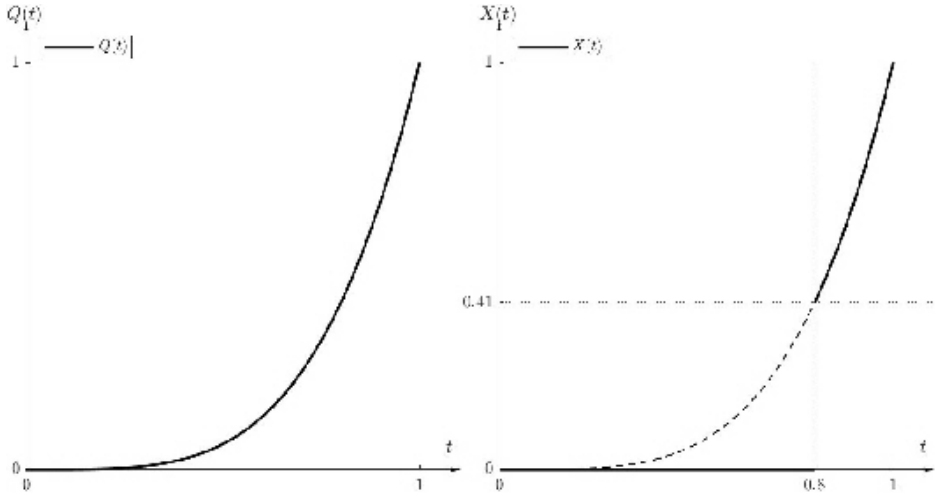

Figure 2: Allocation in the Optimal Auction: Feasible Allocation $Q(t)$ and Optimal Allocation $X(t)$, with $Q \succ_w X$.

this only depends on the distribution of values. Once the correct distribution of quantiles that need to be pooled is determined using the distribution of values, the optimal auction can be implemented as a direct mechanism where a buyer whose value is in quantile t receives the good with probability $X(t)$. In auctions, there are usually indirect implementations of interest (such as a first– or second–price auction), but we will not delve into this as it is outside the scope of our survey.

4.4 Example: Nonlinear Pricing and Single Unit Auction

We illustrate the result with our leading example. If $W(t) = V(t) = t^4$, then r_W is plotted in Figure 1. We can see that $r_W(t) < \overline{r}_W(t)$ in a range of quantiles $t \in [0, 3/4]$. However, the revenue-maximizing quantile is $t_m = 4/5$, so all values that would be pooled are, anyway, excluded as they are below the revenue maximizing quantile t_m. The unique seller-optimal allocation is:

$$X(t) = \begin{cases} 0, & \text{if } t < \frac{4}{5}; \\ Q(t), & \text{if } t \geq \frac{4}{5}. \end{cases}$$

Hence, the seller discards a fraction $4/5$ of the goods and assigns the remaining ones assortatively. In an auction, this allocation can be implemented using a second-price auction with a reserve price equal to $V(t_m)$. We present the resulting allocation in Figure 2.

4.5 Regular Distributions

Myerson (1981) identified a regularity assumption that is sufficient for no pooling, namely monotonicity of the virtual value in the value v:

$$v - \frac{1 - F(v)}{f(v)}.$$

We thus focus on when the virtual value $\phi(t)$, stated in terms of the quantile value function with

$$\phi(t) \triangleq W(t) - W'(t)(1-t) \tag{19}$$

is increasing in t. This is equivalent to requiring that the revenue function $W(t)(1-t)$ is concave in t.

Corollary 1 (Monotone Virtual Values)
If the virtual value $\phi(t)$ is increasing in t, then the optimal allocation is efficient for all quantiles $t \geq t_m$–that is $X(t) = Q(t)$– and excludes quantiles $t < t_m$–that is, $X(t) = 0$.

This proposition provides sufficient conditions so that the optimal selling mechanism consists of discarding a fraction of the goods and assigning the remaining ones assortatively. This is the same classic condition that is traditionally given so that the optimal auction can be implemented as a second-price auction with a reserve price. Our main example does not satisfy the condition in this proposition as the virtual utility is first decreasing and then increasing. But the optimal auction is still a second-price auction with a reserve price. As explained above, this is because the section where the allocation would be pooled corresponds to buyers that are excluded anyway as the virtual utility is indeed negative.

4.6 Ironing and Feasibility in Mechanism Design

Our analysis most directly combines two recent papers. The model with a continuum of buyers and an exogenous inventory of goods is essentially equivalent to that of Loertscher and Muir (2022), except for the fact that we consider a continuum of different qualities. However, to analyze the problem, we follow Kleiner, Moldovanu, and Strack (2021) in expressing the feasibility constraint as a majorization constraint and then using their characterization of maximization of linear functionals subject to majorization constraints. We also follow Kleiner, Moldovanu, and Strack (2021) in

mapping the model with a continuum of buyers of Loertscher and Muir (2022) to optimal symmetric auctions (Myerson (1981)).

The general problem of maximizing a function subject to a monotonicity constraint has appeared in various environments. Early on, Mussa and Rosen (1978) analyze the problem of a monopolist that can engage in second-degree price discrimination and provides a solution when monotonicity constraints binds. In their model, the quality provision is costly, and so this corresponds to an endogenous inventory, which we do not address in this survey. The seminal work of Myerson (1981) also provides a solution to the maximization problem when the monotonicity constraint binds, but in the context of auctions. As we explained earlier, this can be mapped to our model with an exogenous inventory when we restrict attention to symmetric auctions, for a more general analysis of ironing see Toikka (2011) and Noeldeke and Samuelson (2007).

Now, a version of the geometric analysis we have provided here already appears in Myerson (1981), however the interpretation of the concavification (convexification) method appears later on. The connection between the virtual values and the marginal revenue of a monopolist appears in Bulow and Klemperer (1996); the connection between the ironing procedure and the concavification of the revenue function appears in Loertscher and Muir (2022). And, earlier versions of Loertscher and Muir (2022) showed that the same concavification allows finding the optimal mechanism when the cost is endogenous. Hence, when the distribution of qualities is endogenous, similar geometric techniques can be applied (even though there is no relevant feasibility constraint for the inventory of goods).

The description we have provided is not meant to be exhaustive, but simply to sample some of the literature to illustrate the different variations and interpretations of the problem we study that have arisen in the context of optimal mechanism design.[1] Our focus on exogenous inventory allows us to bring together a couple of different interpretations and use the language of majorization to describe both the feasible and optimal distributions of qualities. Most importantly, we show that this framework allows us to closely connect the analysis to the seemingly unrelated literature on information design, which we analyze next.

[1] Similar ironing techniques have also appeared in Dworczak, Kominers, and Akbarpour (2021), Akbarpour, Dworczak, and Kominers (2024), Kolotilin, Corrao, and Wolitzky (2025) and Kolotilin and Wolitzky (2025) among others.

5 Information Design

We now turn to the information design problem, fixing the mechanism and optimizing over the information structure. Analogously to our previous section, our results apply to any distribution of qualities X, but the results can be most naturally interpreted when the mechanism is efficient. That is, when $X(t) = Q(t)$.

We begin by establishing the distribution of expected values that can be induced by some information structure, which follows from Blackwell's theorem. The seller's problem again becomes a linear optimization over the quantile value function W. We show that the optimal information structure is obtained by concavifying an appropriately constructed function. We then examine a broad class of distributions of qualities where the inventory of qualities exhibits a monotone hazard rate. In this case, the analysis yields a simple dichotomy: a decreasing hazard rate of the quality distribution implies complete disclosure of information, while increasing hazard rate implies complete pooling in a single interval, thus no disclosure. The following subsection connects these results to efficient auctions, showing that the optimal information structure depends only on the number of bidders, and then illustrates the results with an example. In the context of auctions, the corresponding distribution of qualities does not exhibit a monotone hazard rate, so we get that partial disclosure is optimal. The final subsection interprets the problem as a form of nonlinear Bayesian persuasion, closely related to Myerson's ironing argument and to persuasion problems with Yaari (1987)-type preferences.

5.1 Feasibility of Information Structure

The buyer information structure is summarized by the distribution of values F_v. By Blackwell (1951), Theorem 5, there exists an information structure that induces a distribution F_w of expected values if and only if F_w is a mean-preserving contraction of F_v, that is,

$$\int_v^{\overline{v}} F_v(x)dx \leq \int_v^{\overline{v}} F_w(x)dx, \, \forall v \in [\underline{v}, \overline{v}], \tag{20}$$

with equality for $v = \underline{v}$. If F_w is a mean-preserving contraction of F_v (or F_w majorizes F_v), we write $F_v \prec F_w$. Following Shaked and Shanthikumar (2007), Section 3.A, we have that F_w majorizes F_v if and only if the ex ante quantile value function $V(t)$ majorizes the quantile (expected) value

function $W(t)$:

$$W \prec V.$$

We thus obtain a direct constraint on the quantile functions.

5.2 Revenue-Maximizing Information Structure

We now consider the problem of finding the information structure that maximizes revenue assuming that the mechanism is implementing the efficient outcome. We can express this problem as follows:

$$\max_{W \prec V} R(W, Q). \tag{21}$$

Here we evaluate the second argument of the revenue X at $X = Q$ because we are considering an efficient mechanism.

To describe the revenue-maximizing information structure, we define:

$$e_X(t) \triangleq \begin{cases} \int_t^1 (1-s)dX(s), & \text{if } t > 0; \\ \int_t^1 (1-s)dX(s) + X(0), & \text{if } t = 0. \end{cases} \tag{22}$$

Here, the definition is discontinuous at $t = 0$ to account for the rents extracted from the minimum quality that is offered to all buyers. For now we omit the discontinuity at $t = 0$ to provide an interpretation. Integrating by parts, we get that for all $t > 0$:

$$e_X(t) = \int_t^1 (X(s) - X(t))\, ds.$$

This is a measure of the excess quality in the inventory at t. In particular, it is the total additional quality that can be offered to quantiles above t.

This is related to a well-known stochastic order: the excess wealth order (see Section 3.C in Shaked and Shanthikumar (2007)). Say we have two inventories $X, \widehat{X}$, then $\widehat{X}$ is greater than X in the excess wealth order if and only if $e_X(t) \leq e_{\widehat{X}}(t)$ for all t. The excess wealth order is a measure of variability. It is precisely the stochastic order that determines when one inventory leads to more informational rents for every distribution of expected values. With the notion of excess quality, we

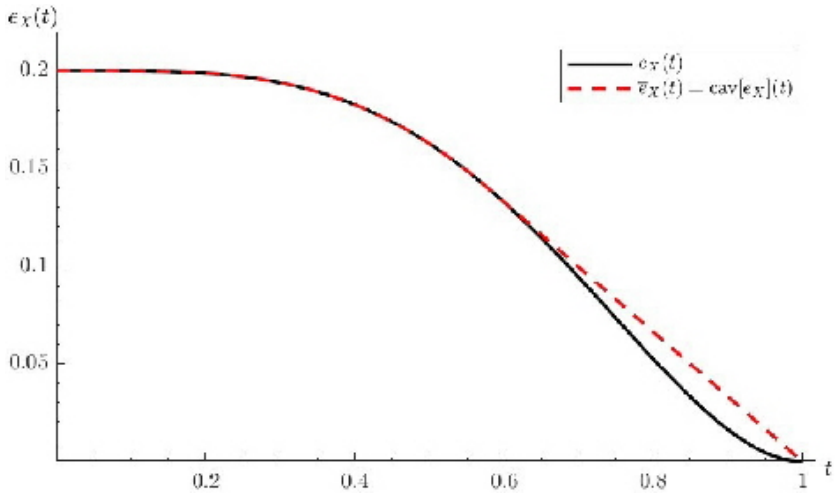

Figure 3: The excess function e_X and its concavification $\overline{e}_X$ when $X(t) = t^4$

can rewrite the revenue (12) as follows:

$$R(W, X) = e_X(0)W(0) + \int_0^1 e_X(t)dW(t). \tag{23}$$

Here we obtained the same expression as in (16), but we replaced the revenue function $r_W(t)$ with the excess quality function, $e_X(t)$, and we maximize with respect to the value quantile function $W(t)$ rather than the allocation quantile $X(t)$. Thus, as we solve for the optimal information design, we permute the integrator from $dX(t)$ to $dW(t)$, and conversely permute the integrand from $r_W(t)$ to $e_X(t)$ (as displayed in (23) relative to the earlier problem (16) in mechanism design.

We can interpret the new expression (23) as follows. The seller sells all goods at a baseline price $W(0)$, which generates a baseline revenue $W(0)\, e_X(0)$, and then adds a surcharge. The surcharge consists of an additional price increment $dW(t)$ per unit of quality to all goods of quality higher than $Q(t)$. Hence, the price increment $dW(t)$ generates a revenue $e_X(t)$, which is the total excess quality relative to $Q(t)$. Thus, the linear weight of the maximization problem are now the excess qualities $e_X(t)$.

Additionally, we also have that the maximization problem (21) is subject to a strong majorization constraint. In Figure 3 we plot $e_X(t)$ for our canonical example where $X(t) = t^4$. Although the quantile distributions of $W(t)$ and $X(t)$ are symmetric, the linear coefficients in the integrals, $r_W(t)$ and $e_X(t)$, respectively are not the same.

To obtain an intuition for why we have a discontinuity in the definition of (22) suppose that the

seller offers a homogeneous quality, that is, $X(t) = 1$. We then have that $dX(t) = 0$ for all quantiles $t \in (0, 1)$, and so also $e_X(0) = 0$ in this interval. However, in this case, the revenue obtained is exactly the quality $X(0)$ times the minimum value $W(0)$. Hence, the discontinuity allows us to correctly account for the fact that the optimal mechanism extracts $X(0)W(0)$ from all buyers, and then the seller extracts extra revenue from each quality increment $dX(t)$.

We can thus describe the optimal information structure as follows. We denote by $\overline{e}_X(t)$ the concavification of e_X (smallest concave function that is pointwise larger than the function itself). To describe the optimal mechanism it is useful to introduce notation necessary to describe the concavification. As before in the description of the mechanism design problem (see (17)), there exists a sequence of quantiles $\left\{\underline{t}_i, \overline{t}_i\right\}_{i\in I}$, with $\underline{t}_i, \overline{t}_i \in [0, 1]$ and

$$\underline{t}_i < \overline{t}_i \leq \underline{t}_{i+1},$$

such that:

$$\overline{e}_X(t) \begin{cases} = e_X(t), & \text{if } t \notin \bigcup\limits_{i\in I}(\underline{t}_i, \overline{t}_i); \\ > e_X(t), & \text{if } t \in (\underline{t}_i, \overline{t}_i). \end{cases} \tag{24}$$

Hence, the quantiles $(\underline{t}_i, \overline{t}_i)_{i\in I}$ define where e_X lies below its concavification. We can thus describe the optimal information structure.

Theorem 2 (Optimal Information Structure via Concavification)
The optimal information structure generates revenue:

$$\max_{W\prec V} R(W, X) = \overline{e}_X(0)V(0) + \int_0^1 \overline{e}_X(t)dV(t). \tag{25}$$

The optimal information structure is given by:

$$W(t) = \begin{cases} V(t), & \text{if } t \notin \bigcup\limits_{i\in I}(\underline{t}_i, \overline{t}_i); \\ \frac{\int_{\underline{t}_i}^{\overline{t}_i} V(s)ds}{\overline{t}_i - \underline{t}_i}, & \text{if } t \in (\underline{t}_i, \overline{t}_i). \end{cases}$$

That is, the optimal information structure pool values whenever $e_X(t) < \overline{e}_X(t)$. This theorem characterizes the revenue obtained by the seller as well as the optimal information structure that attains this revenue. We first note that the optimal revenue is obtained using expression (23) but we replaced e_X with its concavification (which by construction is always weakly higher), and

we replaced the distribution of expected values $W(t)$ with the exogenous distribution of values $V(t)$. The optimal information structure pools the values of all quantiles where $e_X(t) < \overline{e}_X(t)$ and all quantiles such that $e_X(t) = \overline{e}_X(t)$ learn their value. We provide a proof of the result in the Appendix, where, as in Theorem 1, we use results and ideas from majorization theory.

Also, as before, if $\overline{e}_X(t)$ is strictly concave at any

$$t \notin \bigcup_{i \in I} (\underline{t}_i, \overline{t}_i),$$

then the mechanism we provide is the unique optimal information structure; furthermore, this condition is satisfied "generically" (where we add the quotation marks to emphasize we have not formalized this).

5.3 Inventories with Monotone Hazard Rate

We now consider the problem of finding the revenue-maximizing information structure when the mechanism is efficient. Hence, we replace $X(t) = Q(t)$ and proceed to find the solution as a function of $Q(t)$. For simplicity, we assume that $Q(0) = 0$.

We can infer the solution to (25) by looking at the curvature of e_Q. If e_Q is concave, then the solution is full disclosure. If e_Q is convex, then the solution is no disclosure. If e_Q is linear, then any information structure is a solution. We can then provide conditions for e_Q to be convex or concave.

If $Q'(t)(1-t)$ is decreasing, then the optimal information structure displays no disclosure. If $Q'(t)(1-t)$ is increasing, then the optimal information structure is to provide full disclosure of values. We can express the monotonicity condition regarding $Q'(t)(1-t)$ in terms of the hazard rate:

$$Q'(s)(1-s) = \frac{1 - F_q(q)}{F_q'(q)}.$$

Hence, this corresponds to the inverse hazard rate of the distribution of qualities.

Proposition 1 (Optimal Disclosure)
If the distribution of qualities has an increasing hazard rate, then the optimal information structure is no disclosure. If the distribution of qualities has a decreasing hazard rate, then the optimal information structure is full disclosure.

We thus have that the optimal information structure critically depends on the hazard rate of the distribution of qualities. The reason is that the allocation is efficient $X(t) = Q(t)$ so revenue is

determined by competition from the nearby value. For example, if the qualities are all the same, say $Q(t) = 1$, then the revenue is simply the lowest value, so full pooling is optimal. On the other extreme, if the distribution of qualities is Pareto, say $Q(t) = (1-t)^{-\alpha}$ with $\alpha > 1$, then the surplus gains from allocating the good efficiently outweigh the informational rents that are generated.

5.4 Single Unit Auction

As we noted earlier, when $Q(t) = t^{N-1}$ the model can be interpreted as an auction with N bidders. Furthermore, since the allocation is the efficient one, in the context of an auction, this allocation can be implemented with a second-price auction without a reserve price. We thus recover the analysis of Bergemann, Heumann, Morris, Sorokin, and Winter (2022). We will also inherit from Theorem 2 the feature that the optimal information structure depends only on quantiles and not on the distribution of values. In contrast to the optimal mechanism setting, we now have that the quantiles Q determine the quantiles that need to be pooled. Hence, under the interpretation of an auction environment, the revenue-maximizing information structure will depend only on the number of bidders. The optimal information structure will not be either of the cases discussed in Proposition 1, as the hazard rate associated with $Q(t) = t^{N-1}$ is not monotone. We illustrate this in the following section with our main example.

5.5 Example: Information Design in Second Price Auction

We now illustrate the result of the optimal information structure for the case of an exogenous inventory given by the quantile allocation function $Q(t) = t^4$. We present our results with the auction interpretation as in Bergemann, Heumann, Morris, Sorokin, and Winter (2022), but the reader should keep in mind that the results also apply to the screening problem with exogenous inventory.

We characterize the structure of the optimal information policy. For this, we first plot e_Q in Figure 3. We can see that it is concave at low quantiles and convex at high quantiles. We then have that the optimal information structure consists of pooling high values and providing full disclosure to low values, as suggested by the concavification in Figure 3. We thus have that in our main

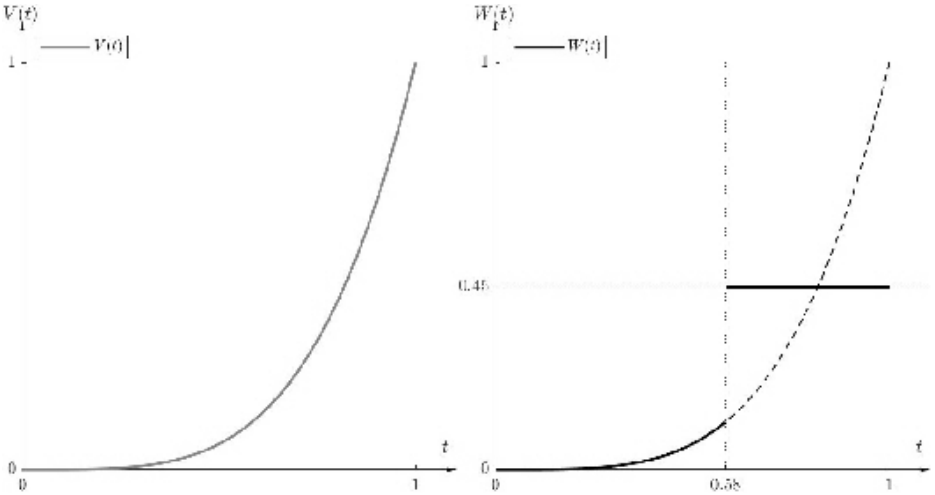

Figure 4: Optimal Information Design with $V(t)$ and $W(t)$, where $V(t) \succ W(t)$

example, the optimal information structure is:

$$W(t) = \begin{cases} V(t), & \text{if } t < t^*; \\ \frac{\int_{t^*}^1 V(s)ds}{1-t^*}, & \text{if } t \geq t^*; \end{cases} \tag{26}$$

where the optimal threshold $t^* \approx 0.6$ is the solution to:

$$e'_X(t^*)(1-t^*) = e_X(1) - e_X(t^*).$$

Hence, this threshold is just the threshold that determines the concavification of e_X.

Intuitively, the optimal structure reveals the exact value of all bidders whose values fall below a fixed threshold, determined by the quantile t^*. For values above this threshold, the mechanism reveals only the information that the value lies above the cutoff, effectively pooling these types at a common posterior mean. This amounts to an "upper censorship" of the value distribution.

The role of this information structure is to sustain competition at the upper tail of the distribution, at the cost of efficiency. In particular, all values above the threshold $F^{-1}(q^*)$ are bundled together into a single mass point. In doing so, the seller depresses the information rent of the winning bidder, thereby raising expected revenue.

5.6 Non-Linear Bayesian Persuasion

In the classic Bayesian persuasion problem of Kamenica and Gentzkow (2011), a sender chooses an information structure - a distribution of posteriors - to maximize her expectation of her indirect utility from the action of a receiver informed of the posterior. Because the sender is an expected utility maximizer, the objective is linear in the indirect utility of the sender. Several papers have examined the case where the receiver only cares about their expectation of the state, e.g., Kolotilin (2018) and Dworczak and Martini (2019). In this case, all the sender cares about is the distribution over expectations of the state that can be induced by an information structure. The set of such distributions is the set of mean-preserving contractions of the ex ante distribution of expected values. In the language of majorization, it is the set of distributions that majorize the ex ante distribution of values. Kleiner, Moldovanu, and Strack (2021) unify and generalize these results in their Theorem 2, which characterizes the extreme points of the set of mean preserving contractions of a distribution.

The information design problem that we study in this section does not belong to this class. We analyze the problem in quantile space, working with the inverse of the distribution over values. We observed that the seller's revenue is linear in the quantile value function and the seller is choosing among quantile value functions that are majorized by the ex ante quantile value function. Thus, we appeal to Theorem 1 of Kleiner, Moldovanu, and Strack (2021) which characterizes extreme points of the mean preserving spreads of a distribution. Note that Myerson (1981) already characterized optimal auctions using quantile value functions, which is why the information design in this section essentially follows Myerson (1981).

The information design in this section is equivalent to a non-linear Bayesian persuasion problem, i.e., one where the sender's preferences are not expected utility. Yaari (1987) considered a dual theory of choice under risk where the decision maker's preferences were linear in lottery payments but non-linear in probabilities. Our problem is equivalent to a Bayesian persuasion problem with Yaari preferences.[2] Jewitt and Quigley (2023) analyzes this problem and shows its relevant for a number of applications (with expected utility maximizers). Gershkov, Moldovanu, Strack, and Zhang (2021) consider an auction model with endogeneous value that leads to a reduced-from model where agents have nonexpected utility with Yaari preferences. Gershkov, Moldovanu, Strack, and Zhang (2022) investigate the optimal auction for non-expected utility models which include the dual preferences of Yaari.

[2] We are grateful to Roberto Corrao for pointing this out to us.

Kolotilin and Wolitzky (2025) show that the problem of gerrymandering congressional districts corresponds to a linear Bayesian persuasion problem when the gerrymanderer cares about maximizing the expected number of seats. An early version of their paper (Kolotilin and Wolitzky (2020)) showed that one could solve the problem if the gerrymanderer cares about a non-linear function of the number of seats in a special case where their aggregate shock had a uniform distribution. They solved the problem adapting Myerson's ironing result and this result too fits our setting.

Also related are a number of papers where one assumes that the state is a real number and information is restricted to take the form of a partition, so that signals are a monotonic deterministic function of the state: Rayo (2013) and Onuchic and Ray (2023).

6 Combining Mechanism Design and Information Design

We now combine mechanism design and information design by allowing the seller to choose both the allocation and the information structure.

We first define the problem and discuss its interpretation. We then show that optimal mechanisms can be represented by monotone partitional structures, intervals in which types are either fully separated or pooled, with common support between the distributions of expected values and qualities. The main result establishes that pooling is always optimal: whenever both information and allocation are jointly optimized, every interval in the quantile space involves pooling. We then revisit the interpretation of the auction and provide the solution to our leading parametrization. We conclude by arguing that the main takeaway– that is, that pooling is optimal– is robust and provide alternative ways to argue for this.

6.1 Maximizing Over Information and Allocation

Suppose that the seller can choose both the allocation and the information structure that are chosen to maximize revenue. The joint optimization problem can be succinctly expressed as solving:

$$\max_{\substack{W \prec V \\ X \prec_w Q}} R(W, X). \tag{27}$$

We thus obtain a maximization problem over two quantile functions subject to two majorization constraints. We now refer to a *joint mechanism* as an information structure W and an allocation

X:

$$M \triangleq (W, X).$$

Here there is some abuse of notation because in previous sections a mechanism only referred to and allocations and associated transfers. But, since the seller can now also control the information disclosure, we augment the concept of a mechanism to incorporate both dimensions of the problem.

Before we proceed it is useful to briefly discuss the implementation of a mechanism. One way to think of a mechanism is that the seller first designs the information structure and then, given this fixed information structure, design an optimal mechanism. Under this interpretation it is without loss of generality to assume that the mechanism is a direct mechanism. An alternative interpretation is that the seller first chooses a price menu $T : q \to \mathbb{R}_+$ specifying the price at which each good is sold and then, given this menu, provides some information to buyers about their own value. Under this alternative interpretation it is without loss of generality to think of the information as being a recommendation of which quality should the buyer purchase. Furthermore, this opens the door to different interpretations of why the seller might be able to design the information without being able to engage in explicit discrimination. For the purpose of this survey we will simply examine the reduced-form description (27); a more detailed discussion of the interpretation and implementation is given in Bergemann, Heumann, and Morris (2026).

Our main analysis proceeds in two steps. We first leverage the analysis we have carried out so far to provide general properties of the optimal mechanism. We then provide the main qualitative property of the solution which is that there is always pooling. We then interpret the result in the context of auctions and provide an illustrative example. Finally, we conclude this section by explaining in what sense the pooling that arises in this problem is more pervasive and sustained than the pooling that arises when we maximize over distribution of values or allocations, but not both. The results we provide here can be found in the published version of Bergemann, Heumann, and Morris (2026). Here we follow more closely the exposition found in an earlier working paper version Bergemann, Heumann, and Morris (2024b).[3]

[3]The published version allowed for a general class of production functions, so the distribution of qualities is allowed to be endogeneous. In the earlier working paper we focused on the case in which the seller had an exogeneous inventory.

6.2 Structure of the Optimal Mechanism

We now provide a characterization of the optimal mechanism. The mechanism specifies a distribution of expected values W as the choice of information and a distribution of (expected) qualities X as the choice of available qualities. The bilinear structure allows us to use Propositions 1-2 to obtain that the quantile functions of expected values and qualities have monotone partitional structure. Furthermore, a simple argument allows us to show that both quantile functions must have the same support. The common support can be obtained from using the revelation principle and the taxation principle simultaneously. In other words, it is without loss of generality to focus on mechanisms in which there is a bijection between the number of possible expected values and expected qualities offered to buyers.

We now introduce some language for critical properties of the value and quality distributions. A quantile function $W$is said to be *monotone partitional* if $[0,1]$ is partitioned into countable intervals $[\underline{t}_i, \bar{t}_i)_{i\in I}$ and, writing I for the labels, we have

$$W(t) \triangleq \begin{cases} V(t), & \text{if } t \notin \bigcup_{i\in I} [\underline{t}_i, \bar{t}_i) \ ; \\ \frac{\int_{\underline{t}_i}^{\bar{t}_i} V(s)ds}{\bar{t}_i - \underline{t}_i}, & \text{if } t \in [\underline{t}_i, \bar{t}_i) \text{ for some } i \in I. \end{cases} \tag{28}$$

and each interval either has pooling, i.e., buyers know only that their value corresponds to a quantile in the interval $\left(\underline{t}_i, \bar{t}_i\right)$ or full disclosure, that is, all buyers with values corresponding to quantiles not in any interval $[\underline{t}_i, \bar{t}_i]$ know their value. The expectation can be written explicitly in terms of the quantile function as follows:

$$w_i = \frac{\int_{\underline{t}_i}^{\bar{t}_i} V(t)\, dt}{\bar{t}_i - \underline{t}_i}.$$

The distribution of expected values W is said to be *monotone pooling* if all quantiles are pooled, i.e.:

$$\bigcup_{i\in I} [\underline{t}_i, \bar{t}_i) = [0,1].$$

We can similarly define monotone partitional and monotone pooling distributions X of qualities. However, the quality distribution X needs to be only *weakly* majorized by Q. We say X is a *weak* monotone partitional distribution if there exists a monotone partitional distribution $\widehat{X}$ and an

indicator function $\mathbb{I}_{t\geq t_m}$ such that:

$$X(t) = \widehat{X}(t) \cdot \mathbb{I}_{\ t\geq t_m}(t),$$

where $\mathbb{I}_{t\geq t_m}(t)$ is the indicator function:

$$\mathbb{I}_{t\geq t_m}(t) \triangleq \begin{cases} 1 & \text{if } t \geq t_m; \\ 0 & \text{if } t < t_m. \end{cases} \tag{29}$$

In other words, a weak monotone partitional distribution is generated by first taking all qualities corresponding to quantiles below t_m and reducing these qualities to zero, and then generating a monotone partitional distribution (where some low qualities have been reduced to 0). A *weak* monotone pooling distribution is defined in the analogous sense, where all intervals are pooling.

To simplify the exposition, henceforth we omit the qualifier "weak". Thus, when we say a distribution of qualities X is a monotone partitional (or monotone pooling) distribution X, it is always in the weak sense.

Proposition 1 in Kleiner, Moldovanu, and Strack (2021) shows that the set $\{W : W \prec V\}$ is a compact and convex set, and their Theorem 1 shows that the extreme points of this set are given by (28). The same analysis applies for X, the only difference is that the set of extreme points of the set $\{X : X \prec_w Q\}$ is the set of weak partitional structures (see Corollary 2 in Kleiner, Moldovanu, and Strack (2021)). The objective function (27) is bilinear in (W, X). Hence, following Bauer's maximum principle (Bauer (1958)), the maximization problem attains its maximum at an extreme point of $\{W : W \prec V\}$.

We say that W and X have *common support* if the induced *partitions of quantiles* that generate the monotone partition distributions are the same. We obtain the following result.

Proposition 2 (Monotone Partitional Distribution)
There exists a solution (W^*, X^*) *to the joint optimization problem* (27) *that has monotone partitional structure with common support.*

Hence, the bilinear structure of the problem guarantees that there exists a solution in which the quantile distribution of expected values and qualities are extreme points of their respective feasible sets. The intuition for the common-support result is that it is (weakly) suboptimal to provide information if that does not lead to different choices– which implies that W^* has the same support

as X^*– and it is (weakly) suboptimal to provide multiple options to buyers with the same expected value – which implies that X^* has the same support as W^*. In other words, the result is obtained by simultaneously applying the revelation principle and the taxation principle to the solution of the problem.

6.3 Optimality of Pooling

We next show that, indeed, every interval is a pooling interval in an optimal mechanism.

Theorem 3 (Optimality of Pooling)
Every solution (W^*, X^*) *to the joint optimization* (27) *consists of pooling intervals only.*

We provide the main arguments of the proof. Consider a mechanism in which there is full disclosure and full separation in some interval $[t_1, t_2]$ (i.e., $W(t) = V(t)$ and $X(t) = Q(t)$ for all $t \in [t_1, t_2]$). Suppose that the seller pools the allocation of all values in an interval $[t_1, t_2]$, so that all values get the average quality on this interval. How much lower would the profit be? We begin the argument with the virtual values given by:

$$\phi(t) \triangleq V(t) - V'(t)(1-t). \tag{30}$$

The profit generated is the expectation of the product of the virtual values and the qualities:

$$\Pi \triangleq \int_0^1 Q(t)\phi(t)dt.$$

We denote the (conditional) mean and variance of virtual values and qualities in the interval $[t_1, t_2]$ by:

$$\mu_\phi \triangleq \frac{\int_{t_1}^{t_2} \phi(t)dt}{t_2 - t_1}; \qquad \mu_q \triangleq \frac{\int_{t_1}^{t_2} Q(t)dt}{t_2 - t_1}; \tag{31}$$

$$\sigma_\phi^2 \triangleq \frac{\int_{t_1}^{t_2} (\phi(t) - \mu_\phi)^2 dt}{t_2 - t_1}; \qquad \sigma_q^2 \triangleq \frac{\int_{t_1}^{t_2} (Q(t) - \mu_q)^2 dt}{t_2 - t_1}. \tag{32}$$

As we only compute conditional mean and variance in the interval $[t_1, t_2]$, we can safely omit an index referring to the interval $[t_1, t_2]$ in the expression of μ and σ. The first step of the proof shows that the revenue losses due to the pooling of qualities in the interval $[t_1, t_2]$ are bounded by the

following:

$$\sigma_\phi \sigma_q (t_2 - t_1).$$

Hence, pooling the qualities generates third-order profit losses when the interval is small (since each of the terms multiplied is small when the interval is small).

If in addition to pooling the qualities, we pool the values in this interval, we can reduce the buyers' information rent. When only qualities are pooled– but not the values– then the quality increase that the values assigned to the pooled quality get relative to the values just below the pool is the quality difference $\mu_q - Q(t_1)$ priced at t_1. After pooling the values, the price of the quality increase is computed using the expected value conditional on being in this interval:

$$\mu_v \triangleq \mathbb{E}[V(t) \mid t \in [t_1, t_2]].$$

Hence, pooling the values increases the payments for every quantile value higher than t_1 by an amount:

$$(\mu_v - V(t_1))(\mu_q - Q(t_1))(1 - t_1).$$

Here the first two terms being multiplied are small when the interval is small. However, payments are marginally increased for all values higher than t_1, which is a non-negligible mass of values (i.e., $(1 - t_1)$ is not small). In other words, pooling values increases the price of the quality improvement $(\mu_q - X(t_1))$ for all values higher than t_1. Hence, pooling values generates a second-order benefit which always dominates the third-order distortions.

6.4 Example: Nonlinear Pricing and Product Design

In Figures 5 we illustrate the optimal mechanism in an example. In sharp contrast to the earlier analysis of either the optimal allocation or the optimal information structure, the optimal joint mechanism now leads to pooled intervals everywhere as established by Theorem 3. In the current example, this leads to there being just two qualities offered and all values being assigned to one of two pools of values. We can see that the jumps of W^* and X^* occur on the same quantiles, as established in Proposition 2.

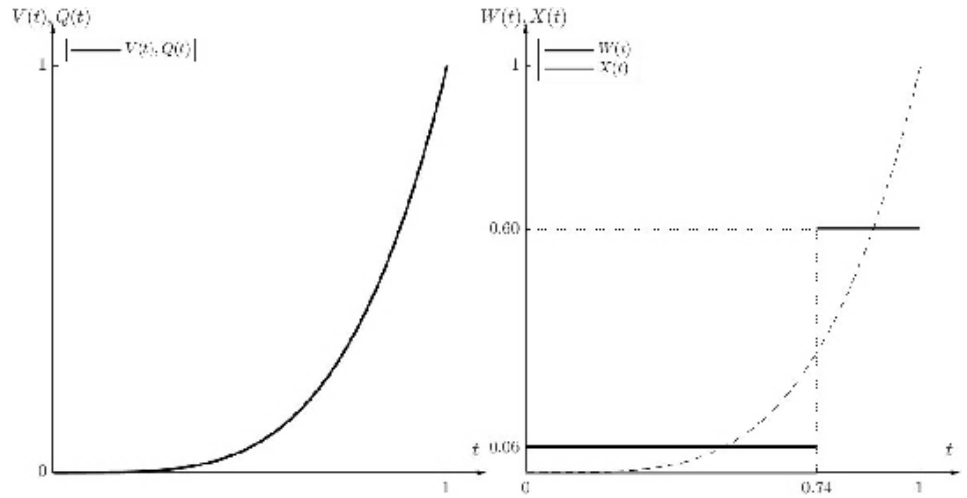

Figure 5: The given distributions of values $V(t) = t^4$ and qualities $Q(t) = t^4$ are depicted on the left. The associated optimal distributions $W(t)$ and $X(t)$, which are monotone partitional distributions are depicted on the right.

6.5 Auctions

We can interpret the model as a seller that designs both the information structure and an optimal symmetric auction. This problem is studied in greater generality by Bergemann and Pesendorfer (2007), where they do not restrict attention to symmetric auctions. And, in fact, they show that asymmetric information structures and auctions are optimal. With the restriction to symmetric outcomes, we then find that with five bidders, there will be two categories of bidders, those with low and high (expected) value.

6.6 The Robustness of Pooling

We conclude this section by arguing the pooling we obtain from maximizing bilinear environments is a more robust feature than the results presented here. For expositional purposes we analyzed the situation where the seller has an exogenous inventory, but in the published version of our work we allow for the inventory to be endogenous (for example, as in Mussa and Rosen (1978)). While the result of a monotone partition information structure is based on the linear utility in v, some versions of our main results continue to hold in a nonlinear utility environment (see Theorem 5 in the working paper version Bergemann, Heumann, and Morris (2024b)). In the published version, Bergemann, Heumann, and Morris (2026), we show that even with discrete values some pooling is optimal as long as the entropy of the distribution is not lower than log(8) (this is the entropy of a uniform

distribution with 8 values). Hence, we get that pooling arises even in already informationally coarse environments. Overall, we find that the results pointing towards pooling are of qualitatively different nature than those presented in Sections 4-5 when only one dimension of the design problem was controlled by the seller.

7 Consumer Surplus

So far, our analysis has focused on examining how controlling the allocation and/or information can improve the revenue of the seller. We now examine how these choices can also be deployed to improve consumer welfare. In fact, we show that the objective function which represents consumer surplus, as suggested earlier in the equality (14) towards the end of Section 3 turns out to be the same as those studied in Sections 4-5 after a simple exchange of the quantile functions. Moreover, the majorization constraints will only be mildly adjusted relative to those studied in the sections of revenue maximization. We use these adjustment from weak to strict majorization (and conversely) to emphasize their role in the optimization problems. We conclude this section by analyzing the case in which both the information and the mechanism are chosen but the objective is not necessarily revenue maximization. We argue that the problem becomes more nuanced with a variety of different problems that can be interpreted differently, while keeping much of the structure of the original problem.

7.1 Consumer Surplus

We begin by highlighting the symmetry between buyers and sellers. Whenever the boundary condition $W(0) = 0$ or $X(0) = 0$, given earlier in condition (13) and the distributions are absolutely continuous, we have that:

$$R(W, X) = U(X, W). \tag{33}$$

That is, for a given pair of distributions (W, X) the seller obtains the same revenue as the consumer surplus that the buyers would have obtained if we had exchanged the roles of the distributions. Thus, the objective are identical up to an exchange symmetry that requires the swapping of the quantile function W and X. Throughout this section we assume that the boundary conditions, $W(0) = 0$ or $X(0) = 0$, hold to simplify the exposition.

We now briefly provide an interpretation of the symmetry in the objective functions. The

problem of consumer surplus maximization can be found as a linear optimization, where we only need to modify the weights. Table 1 summarize the different objectives with the different possible instruments' being controlled.

		Objective Function	
		R	CS
Instrument	Allocation	$(r_W, \prec_w)$	$(e_W, \prec_w)$
	Information	$(e_X, \prec)$	$(r_X, \prec)$

Table 1: Exchange Symmetry of Instruments for Revenue and Consumer Surplus

The above table summarizes the relationship between the design optimization problems and their corresponding representation in terms of linear weights and type of majorization constraints.

While the expressions for revenue and consumer surplus are symmetric with respect to the roles of allocation and information, two aspects of the problem modify the symmetry in a subtle manner when controlling either allocation or information. First, so far, we have focused on revenue maximization, which breaks the symmetry between allocation and information. In the interpretation of a two-sided matching, controlling the allocation is the same as pooling sellers, which is the side of the market being maximized, while controlling the information is pooling buyers, which is the opposite side of the market being maximized. A second aspect that modify the symmetry is that, when controlling the allocation, we consider a weak majorization constraint rather than a strong one. This difference in the constraint is minor and only results in discarding some goods or, equivalently, excluding some consumers.

7.2 Symmetry and Two-Sided Matching

We have interpreted the model as a seller (monopolist) that offers a variety of goods to buyers. The transfer payments are then determined by the allocation and the requirement that the mechanism must be incentive compatible. However, we could have also interpret the model as consisting of a continuum of different sellers indexed by $t \in [0, 1]$, each offering a good of specific quality $Q(t)$. We would then have a two-sided matching market and look for the prices that arise in a competitive equilibrium. In this case, we would have reached the same formulas (see Chiappori, McCann, and Nesheim (2010) or Galichon (2016) for a detailed connection in the context of matching markets under complete information with quasilinear utility).

We obtain complete symmetry only when $W(0) = 0$ or $X(0) = 0$. The reason is that we have assumed that the lowest-value buyer obtains zero surplus. This condition is implied by the participation constraint when $W(0) = 0$ or $X(0) = 0$. However, if $W(0),\ X(0) > 0$, then the lowest-value buyer obtains zero surplus only because we assumed that the seller has the all the bargaining power. Hence, when $W(0),\ X(0) > 0$, the transfers in a two-sided matching market are determined by the allocation and the bargaining power, where the latter determines the surplus of the lowest seller and buyer. The bargaining power also becomes relevant when we have discontinuities in the allocation or the information structure, which will become clear as we provide our analysis. We do not delve into this interpretation any further, and instead simply leverage the symmetry to study the consumer-optimal allocation and mechanism.

7.3 Consumer-Optimal Design

We now find the consumer-optimal mechanism and the consumer-optimal information structure.

Consumer-Optimal Allocation We begin by considering the consumer-optimal mechanism where the choice variable is the quantile allocation function X:

$$\max_{X \prec_w Q} U(W, X), \tag{34}$$

where as mentioned we assume that $W(0) = 0$. Appealing to the symmetry argument in (33), we obtain that analytically this is the same problem as

$$\max_{X \prec Q} R\left(X, W\right),$$

studied in Section 5, except for the fact that the majorization constraint in the consumer surplus maximization problem is weak while earlier in the revenue maximization problem we had a strict majorization inequality associated with the Bayesian consistency . However, it is easy to verify that in the optimum the majorization constraint will be binding. Formally, this can be proved by noting that $e_X(t)$ is non-increasing. Intuitively, discarding goods is never optimal for the buyers. Hence, the solution will indeed be the same as the one in Section 5.

While we analytically obtain the same problem as finding the revenue-maximizing information, we now have a different interpretation. In this case, we recover the problem of finding the pooling of qualities that maximizes buyers' surplus in a two-sided matching market. The problem can be

interpreted as a planner that wants to provide goods to consumers to maximize their surplus but can only screen buyers using a costly screening device, like wasteful screening (Kleiner, Moldovanu, and Strack (2021) or Condorelli (2012)). A general solution is provided by Kleiner, Moldovanu, and Strack (2021) (see Proposition 4 for the result, and the discussion therein for a broader discussion on contest design). In fact, in Proposition 1 in Section 5.3 we recover the same solution as Kleiner, Moldovanu, and Strack (2021) (see Proposition 4.3). They show that the consumer-optimal allocation depends on the hazard rate of the distribution of values.

Consumer-Optimal Information Structures We now examine the consumer-optimal information structure where the choice variable is the quantile value function W :

$$\max_{W \prec V} U(W, X), \tag{35}$$

where as mentioned we assume that $X(0) = 0$. Appealing to the symmetry argument in (33) we obtain that analytically this is the same problem as the mechanism design problem of maximizing the revenue

$$\max_{W \prec_w V} R(X, W)$$

studied in Section 4, except for the fact that that majorization here is strict while earlier we had a weak inequality. In this case, this difference can lead to different predictions. But of course, if were to insist on an efficient allocation given the information structure then we turn the weak majorization constraint into a strict majorization constraint anyhow.

For concreteness, we consider our leading example of a second price auction for a single good. Then the unique bidder-optimal symmetric information structure is given by:

$$W(t) = \begin{cases} \frac{\int_0^{t^*} V(s)ds}{t^*}, & \text{if } t < t^*; \\ V(t), & \text{if } t \geq t^*; \end{cases}$$

where the cutoff $t^* = \frac{3}{4} \in [0, 1)$ is independent of the distribution F_v. Here we obtained the same threshold to determine the concavification as in Section 4.4. This is not a coincidence as we are in fact in both cases concavifying exactly the same function (here it was useful that $Q = V$ so $r_Q = r_V$). However, we do not obtain the same solution as in Section 4.4 because the weak and strong majorization constraint make a difference regarding the nature of the solution in each instance. When optimizing over information structures, it is not possible to discard buyers, and the solution

pools low values. When optimizing over allocations, it is possible to discard goods, so those quantiles that are pooled in the consumer-optimal information structure are discarded when analyzing the seller-optimal allocation. Hence, in this case we get some pooling of values while in Section 4.4 we obtained that some qualities are discarded, which would have otherwise being pooled.

8 Robust Predictions and Other Comparative Statics

This section discusses robust predictions and comparative statics. It interprets the preceding results as characterizing the set of feasible welfare outcomes under all possible information structures. The section derives the welfare frontier in efficient auctions, showing that the optimal information structure alternates between upper and lower censoring, depending on the weight placed on revenue versus consumer surplus. Finally, it examines comparative statics in the number of bidders, showing that the optimal pooling threshold rises monotonically with N and that the number of effectively competing bidders remains approximately constant at about two, even as N grows large.

8.1 Robust Predictions

We have so far considered the problem of mechanism design and information design with a various different objectives. The mechanism design problem typically has a very natural literal interpretation but the information design problem frequently does not have a straightforward interpretation. We can think of stories where the information design can have a literal interpretation: perhaps there is a platform that offers information to sellers in order to maximize the surplus of consumers on their platform (see Bergemann, Brooks, and Morris (2015) and others).

However, we became interested in information design via a different, fully metaphorical, interpretation of the information design problem. The first generation of information economics made highly parametric assumptions about information. These assumptions translated into extreme structural assumptions in empirical work that are hard to test. But we asked: what are the set of possible outcomes that can arise in a fixed game: see Bergemann and Morris (2013), Bergemann and Morris (2016), Bergemann and Morris (2019). For applications, see Bergemann, Brooks, and Morris (2015), Bergemann, Heumann, and Morris (2015), Bergemann, Brooks, and Morris (2017), Bergemann, Brooks, and Morris (2025). For applications with twists, see Bergemann, Brooks, and Morris (2021) and Bergemann, Heumann, and Morris (2020).

8.2 Robust Predictions in Efficient Auctions

We begin by considering the environment studied in Section 5.4, but we now consider all possible linear combinations of revenue and consumer surplus (possibly with negative weight). Hence we solve:

$$\max_{W \prec V} \{ \ m\left((1-|\lambda|)R(W,Q)+\lambda U(W,Q)\right)\} \tag{36}$$

for all $\lambda \in [-1,1],\ m \in \{-1,1\}$ and $Q(t) = t^{N-1}$. We note that $Q(0) = 0$ so we do not need to worry about the discontinuity at (22). If $\lambda \in [0,1],\ m = 1$ then the objective function places positive weight on both consumer surplus and revenue; if $\lambda \in [0,1],\ m = -1$ then the objective function places negative weight on both consumer surplus and revenue; if $\lambda \in [-1,0],\ m = 1$ then the objective function places positive weight on revenue but negative weight on consumer surplus; if $\lambda \in [-1,0],\ m = -1$ then the objective function places positive weight on consumer surplus but negative weight on revenue.

Using the symmetry observed earlier in (33) we have that:

$$U(W,Q) = R(Q,W).$$

We can thus write the social welfare function as follows:

$$m\left((1-|\lambda|)R(W,Q)+\lambda U(W,Q)\right) = m((1-|\lambda|)e_Q(0)+\lambda r_Q(0))W(0)+\int_0^1 m((1-|\lambda|)e_Q(t)+\lambda r_Q(t))dW(t).$$

To make the notation more compact, we define the weighted social welfare, or simply surplus function as:

$$S_{\lambda,m,Q}(t) = m((1-|\lambda|)e_Q(t)+\lambda r_Q(t))$$

The analysis can be done the same way as we did in Section 5, but replacing the excess quality function $e_Q(t)$ in (23) with the surplus function $S_{\lambda,m,Q}(t)$. It turns out that $S_{\lambda,m,Q}(t)$ always has at most one inflection point in $[0,1]$.

There are two types of optimal information structures that can track the whole frontier of the welfare set: *upper and lower censorship*. As we construct the concavification of $S_{\lambda,m,Q}(t)$, we always find an interval of full disclosure and an interval of pooling, thus a binary and monotone partition. The interval of pooling can be at the top of the distribution (as when we maximized revenue in Section 5.5) or it can be at the bottom (as when we maximized consumer surplus in Section 7.3). We say the optimal information structure is upper censorship if there exists a threshold quantile t_c

such that:

$$W(t) = \begin{cases} V(t), & \text{if } t \leq t_c; \\ \frac{\int_{t_c}^{1} V(t)}{1-t_c}, & \text{if } t \geq t_c. \end{cases}$$

That is, quantiles $t < t_c$ learn their values while quantiles above t_c only learn that their value is above $V(t_c)$. The solution we obtain in Section 5.4 was upper censorship. We say the optimal information structure is lower censorship if there exists t_c such that:

$$W(t) = \begin{cases} \frac{\int_{0}^{t_c} V(t)}{t_c}, & \text{if } t \leq t_c; \\ V(t), & \text{if } t \geq t_c. \end{cases}$$

That is, quantiles $t > t_c$ learn their values while quantiles below t_c only learn that their value is below $V(t_c)$. Complete disclosure and no disclosure are particular instances of upper and lower censorship

Theorem 4 (Optimal Information Structure for Weighted Social Welfare)
The optimal information structure generates welfare:

$$\max_{W \prec V} S_{\lambda,m,Q} = \overline{S}_{\lambda,m,Q}(0)V(0) + \int_0^1 \overline{S}_{\lambda,m,Q}(t)dV(t). \tag{37}$$

If $m = 1$ and $\lambda \in [-1, 1/2]$, then the optimal information structure is upper censorship; otherwise, the optimal information structure is lower censorship.

We now show the boundary of the feasible welfare pairs in Figure 6, where the different colors represent the different solutions. The figure also shows when full disclosure and when no disclosure is optimal. When $m = 1$ and $\lambda = 1/2$, then the objective function is to maximize total surplus so complete disclosure is optimal; $m = -1$ and $\lambda = 1/2$, then the objective function is to minimize total surplus so no disclosure is optimal. We can see the boundary is non-differentiable at these points, which means the full disclosure and no disclosure is optimal also for other values close to $\lambda = 1/2$.

8.3 Allocation and Information Design with Misaligned Objectives

So far we have either maximized over the allocation or the information while keeping the other fixed. The only exception was Section 6 in which we analyzed the problem when both allocation and

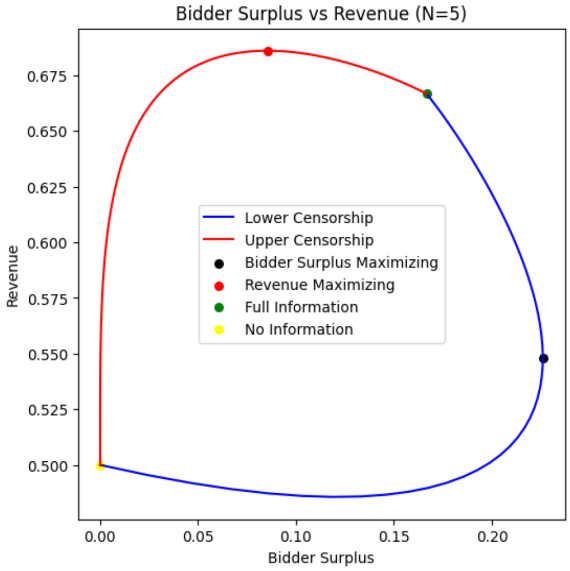

Figure 6: Revenue and Consumer Surplus Frontier across all social welfare weights λ, m.

information were chosen with a common objective. Due to the exchange symmetry argument in (33), it is easy verify that the same solution would go through if both were chosen to maximize consumer surplus. However, we may be interested in situations in which both allocation and information are endogenous to the game, but they are chosen by different agents with different objectives. An important class are adversarial or robust setting where the seller chooses the mechanism and adversarial nature chooses the information structure. We address this problem in our working paper Bergemann, Heumann, and Morris (2022), (2024). The solution differs significantly from the analysis carried out so far. In particular, we will not be able to use the results about extreme points developed by Kleiner, Moldovanu, and Strack (2021), which we presented in Section 6.2. We now briefly discuss our work in this direction to illustrate some of novel interpretations and associated results.

To begin, consider the situation in which the information structure is chosen to maximize consumer surplus and then, after knowing what the information structure is, the seller chooses the allocation to maximize revenue. This has a natural literal interpretation where a third party platform can modify the information provided about consumers, but then sellers can respond optimally to the demand they face. The problem can be written as follows:

$$\max_{\substack{W \prec V \\ X \prec_w Q}} U(W, X), \tag{38}$$

subject to

$$X \in \arg\max_{X \prec_w Q} R(W, X).$$

If $Q(t) = t^{N-1}$ then the problem can be interpreted as finding the consumer-optimal information structure when the seller uses the revenue-maximizing mechanism (knowing the information structure). If $Q(t) = 1$ for all $t \in [0, 1]$ then we recover the problem studied by Roesler and Szentes (2017).

Now, we could also consider the revenue-minimizing information structure. The problem can be written as follows:

$$\min_{\substack{W \prec V \\ X \prec_w Q}} R(W, X) \text{ subject to } X \in \arg\max_{X \prec_w Q} R(W, X). \tag{39}$$

Here is is unclear whether there is a literal interpretation, but one can interpret this problem as giving bounds on the revenue that a seller can generate regardless of the information structure.

Furthermore, one can expect to find a saddle point of (39), and then the mechanism would have the property of providing the optimal revenue guarantee across all information structure. Early examples of this approach are Bergemann, Brooks, and Morris (2016); Bergemann, Brooks, and Morris (2019) while a more general approach has been recently developed by Brooks and Du (2021), Brooks and Du (2024) and Brooks and Du (2025a) and further papers is surveyed in a chapter of this volume, i.e., Brooks and Du (2025b). Consistent with this literature, we show a saddle point exists and provide the corresponding optimal revenue-guarantee mechanism. In contrast to this early literature we are able to use more basic intuitions of majorization to construct the saddle point, hence, bringing the tools of majorization to finding the optimal robust mechanism.

8.4 Impact of the Number of Bidders in Auctions

We now analyze how the optimal information structure in an auction changes with the number of bidders. That is, we revisit the analysis in Section 5.5 but we see how the solution changes with N. A striking feature of the analysis is that the optimal information structure can be characterized solely in terms of the number of bidders. In contrast to the general case of an exogenous inventory, in this case, the exogenous inventory distribution has a particularly simple form, summarized by a single parameter, which allows us to derive results with clear economic interpretations. By contrast, in general, either in the information design or mechanism design problem, the optimal ironing procedure depends intricately on the distribution of values or qualities. This makes it harder to

describe the relevant distribution qualitatively.

The optimal information structure is upper censorship, as in (26). The quantile threshold $t^*(N)$ that determines the optimal information structure is equal to zero when there are two bidders, increases monotonically with N, and converges to one as the number of bidders tends to infinity. For $N \geq 3$, it is implicitly defined as the solution to the equation

$$e'_Q(t)(1-t) \;=\; e_Q(1) - e_Q(t),$$

which is a polynomial of degree N in q. Numerical solutions for selected values of N are reported below:

N	2	3	4	5	10	100
$t^*(N)$	0	0.25	0.46	0.58	0.81	0.98

It is possible to verify that the expected number of bidders whose values are pooled at the top is approximately constant across N. The expected number of bidders whose value is at the top of the distribution is:

$$N(1 - t^*(N)).$$

One can then show that:

$$N(1 - t^*(N)) \in (1.79, 2.25) \approx 2 \quad \text{for all } N.$$

Hence, the optimal information policy consistently maintains competition among about two bidders at the top of the distribution.

9 Conclusion

This paper has presented a unified framework for analyzing mechanism design and information design using the language of majorization and quantile functions. By working in quantile space, we have shown that both problems reduce to maximizing linear functionals subject to majorization constraints—with mechanism design involving weak majorization (allowing discarding of goods) and information design involving strict majorization (requiring that all types be informed).

Several key insights emerge from our analysis. First, there is a deep symmetry between the mechanism design and information design problems. Both involve concavifying an appropriately defined function, and both lead to pooling or ironing in optimal solutions. However, the nature of

the majorization constraints differs, leading to distinct economic interpretations: mechanism design excludes low types through reserve prices, while information design pools their information.

Second, when the designer can optimize over both information and allocation simultaneously, pooling becomes ubiquitous. Unlike the separate optimization problems—where full separation or full disclosure can be optimal under appropriate conditions—joint optimization always involves some pooling. This reflects the bilinear structure of the objective and the interaction between the two majorization constraints. Intuitively, even small amounts of information pooling generate second-order gains in extracting surplus from inframarginal types, while creating only third-order allocative distortions.

Third, our framework provides a natural approach to robust predictions and welfare analysis. By characterizing the set of feasible outcomes under all possible information structures, we can identify predictions that hold regardless of informational assumptions. In the auction context, we have shown that the welfare frontier can be completely characterized by upper and lower censoring policies, with complete disclosure being optimal when total surplus is maximized.

Fourth, the comparative statics in auctions reveal a striking regularity: as the number of bidders grows, the optimal information structure pools a decreasing fraction of the value distribution, yet maintains competition among approximately two bidders at the top. This suggests that marginal competition, rather than total competition, is the relevant force in determining optimal disclosure.

Our analysis can be extended in several directions. First, while we have focused on exogenous inventories, the framework naturally accommodates endogenous production decisions with convex costs, as in the original Mussa and Rosen (1978) model. Second, although we have emphasized symmetric settings for tractability, the techniques extend to asymmetric environments, as shown by Bergemann and Pesendorfer (2007) for auctions. Third, our approach to consumer surplus maximization and weighted welfare objectives opens connections to platform design and regulation, where the designer may not fully control both information and allocation rules.

More broadly, our framework contributes to the emerging second generation of information economics. The first generation, beginning with Akerlof (1970), Spence (1973), and Rothschild and Stiglitz (1976), employed highly parametric models of information. Empirical work incorporating private information therefore relied on strong structural assumptions. The development of data-intensive tools and the digital economy has both increased the practical importance of information design and made fully parametric approaches less tenable. Recent theoretical work has increasingly shifted from simple parametric models to allowing for general information structures, both in models

of costly information acquisition (Sims (2003)) and in Bayesian persuasion and information design (Kamenica and Gentzkow (2011); Bergemann and Morris (2019)).

Our contribution has been to show that this generality need not come at the cost of tractability. By working in quantile space and exploiting the geometry of majorization, we can characterize optimal mechanisms and information structures, derive robust predictions, and compute welfare frontiers—all without making restrictive parametric assumptions about the underlying distributions. The key is recognizing that majorization provides the right mathematical structure for capturing feasibility constraints in both mechanism design and information design problems.

The techniques we have developed—concavification, ironing, and the characterization of extreme points—have classical antecedents in auction theory and optimal taxation. What is new is the recognition that these same tools apply, with appropriate modifications, to information design problems. Moreover, when mechanism and information design are combined, the bilinear structure leads to qualitatively different conclusions: pooling becomes optimal even when the primitives would not suggest it in either problem separately.

As markets become increasingly data-driven and platforms gain greater ability to control both pricing and information provision, understanding the joint optimization of these instruments becomes ever more important. Our framework provides a foundation for analyzing these problems and for deriving welfare implications that are robust to specific informational assumptions. The finding that joint optimization leads systematically to pooling—and the robustness of this result across different specifications—suggests that concerns about excessive information manipulation may be warranted when designers control multiple instruments.

10 Appendix

Proof of Theorem 1. We want to maximize:

$$\max_{X \prec_w Q} \left\{ X(0) r_W(0) + \int_0^1 r_W(t) dX(t) \right\}. \tag{40}$$

We cannot apply Proposition 2 in Kleiner, Moldovanu, and Strack (2021) because we have a weak majorization instead of a strong majorization. However, with minor adjustments, we can use their

results to identify the optimal solution. We first define:

$$Q_\theta(t) = Q(t) \cdot 1\{t \geq \theta\}.$$

That is, $\{Q_\theta\}_{\theta\in[0,1]}$ is the set of functions that equal $Q(t)$ at quantiles larger than θ and is zero otherwise. Since (40) is a linear maximization over a convex domain, it attains its optimum at one of the extreme points of the domain. Following Corollary 2 in Kleiner, Moldovanu, and Strack (2021), the set of extreme points of the set:

$$\{X : X \prec_w Q\},$$

is the union of all the extreme points of the sets of the form:

$$\{X : X \prec Q_\theta\},$$

for some $\theta \in [0,1]$. That is, we change the weak majorization for a strong majorization and we change Q for Q_θ and then take the union over all θ. Hence, we have that:

$$\max_{X\prec_w Q} X(0)r_W(0) + \int_0^1 r_W(t)dX(t) = \max_{\theta\in[0,1]} \left\{ \max_{X\prec Q_\theta} X(0)r_W(0) + \int_0^1 r_W(t)dX(t) \right\}.$$

We can now apply Proposition 2 in Kleiner, Moldovanu, and Strack (2021) to the innermost maximization. We obtain:

$$\begin{aligned}\max_{X\prec Q_\theta} X(0)r_W(0) + \int_0^1 r_W(t)dX(t) =& Q_\theta(0)\overline{r}_W(0) + \int_0^1 \overline{r}_W(t)dQ_\theta(t) \\ =& Q(\theta)\overline{r}_W(\theta) + \int_\theta^1 \overline{r}_W(t)dQ(t). \end{aligned} \tag{41}$$

The first equality corresponds to inequality (16) in Kleiner, Moldovanu, and Strack (2021) (which in the same proof is shown to be satisfied with equality); the second equality is just writing the integral explicitly. By construction, $\overline{r}_W$ is concave, so (41) is maximized at $\theta = t_m$:

$$t_m \in \arg\max_{\theta\in[0,1]} Q(\theta)r_W(\theta) + \int_\theta^1 \overline{r}_W(t)dQ(t).$$

where we also used that $r_W(t_m) = \overline{r}_W(t_m)$. ■

Proof of Theorem 2. We seek to maximize:

$$\max_{W \prec V} \left\{ e_X(0)W(0) + \int_0^1 e_X(t)dW(t) \right\}.$$

Now, we cannot directly apply Proposition 2 in Kleiner, Moldovanu, and Strack (2021) because $e_X(t)$ is discontinuous at $t = 0$. However, we have that:

$$\lim_{t \downarrow 0} e_X(t) < e_X(0).$$

Hence, $e_X(t)$ is upper-hemicontinuous; thus, the proof in Kleiner, Moldovanu, and Strack (2021) proceeds in the same way. More precisely, we have that:

$$\lim_{t \downarrow 0} e_X(t) < e_X(0).$$

Hence, there exists $t_1 > 0$ such that $\overline{e}_X(t) > e_X(t)$ for all $t \in [0, t_1]$. We can thus apply Proposition 2 in Kleiner, Moldovanu, and Strack (2021) by considering a limit of continuous functions. In particular, there exists a family of functions $\{e_\ell(t)\}_{\ell \in [0,1]}$, that are continuously differentiable such that

$$e_\ell(t) \begin{cases} \in [e_X(t), \overline{e}_X(t)], & \text{for all } t \in [0, t_1]; \\ = e_X(t), & \text{for all } t \in [t_1, 1]; \end{cases}$$

such that in the limit as $\ell \to 1$, the functions converge to e_X:

$$\lim_{l \to 1} e_\ell(t) = e_X(t).$$

By construction, we have that for all $\ell \in [0, 1]$:

$$\overline{e}_\ell(t) = \overline{e}_X(t).$$

Since the solution is defined solely in terms of the support of the concavification, we have that the optimal solution is the same for all $\ell \in [0, 1]$ and is the same as in Theorem 2. Hence, this is also the solution for e_X. ■

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