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TRADE AND DOMESTIC DISTORTIONS: THE CASE OF INFORMALITY*

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Abstract

We examine the effects of international trade in the presence of a set of domestic distortions giving rise to informality, a prevalent phenomenon in developing countries. In our quantitative model, the informal sector arises from burdensome taxes and regulations that are imperfectly enforced by the government. In equilibrium, smaller, less productive firms face fewer distortions than larger, more productive ones, potentially leading to substantial misallocation. We show that in settings with a large informal sector, the gains from trade are significantly amplified, as reductions in trade barriers imply a reallocation of resources from initially less distorted to more distorted firms. We confirm findings from earlier reduced-form studies that the informal sector mitigates the impact of negative labor demand shocks on unemployment. Nonetheless, the informal sector can exacerbate the adverse real income effects of economic downturns, amplifying misallocation. Last, our research sheds light on the relationship between trade openness and cross-firm wage inequality.
JEL codes: F14, F16, J46, O17

1 Introduction

In the past 40 years, most developing countries have opened up to foreign competition, which has long been seen as a necessary step to modernize their economies and promote economic growth. However, these economies are also characterized by the presence of severe domestic frictions (Banerjee and Duflo, 2005; Hsieh and Klenow, 2009), which can shape the effects of international trade in important ways by either amplifying or mitigating the impacts of trade liberalization (Atkin and

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Khandelwal, 2020; Atkin and Donaldson, 2022; Bai et al., 2023). Thus, to understand whether trade hinders or promotes economic development, it is key to understand how it interacts with domestic frictions in developing countries.

This paper focuses on a set of domestic distortions that give rise to a phenomenon that is pervasive in developing economies: informality. The informal sector is broadly defined as the part of the economy (firms and/or workers) that evades burdensome taxes and labor market regulations. Remarkably, a substantial share of the labor force in many emerging and developing countries is employed informally; for example, in Latin America, the informally employed share of the labor force ranges from 35% in Chile to 80% in Peru (Perry et al., 2007). Given the informal sector's sheer magnitude and central role in the functioning of these economies, it is a priori likely that informality plays an important role in shaping the aggregate and distributional effects of trade. Indeed, recent empirical work has emphasized that shifts into or out of informality represent significant margins through which economies adjust to changes in the global environment.[1] However, the informal sector has received little attention in theoretical and empirical studies of international trade.

To model the impacts of trade in an economy with a large informal sector, we start with the premise that it arises from burdensome taxes and labor market regulations imposed on firms, which the government can only imperfectly enforce. Although the government imposes costs on informal firms through fines or the possibility of closure, these measures are not consistently applied to all firms. This lack of strict enforcement creates incentives for certain firms to operate informally, evading taxes, and bypassing compliance with labor market regulations. Importantly, this leads to a situation where firms face unequal exposure to distorting taxes and regulations.

Our framework considers a rich institutional setting, in which the government imposes onerous labor market regulations that discourage formal status. Past research has highlighted the importance of regulations such as firing costs and minimum wages in driving the size of the informal sector in Latin American countries (e.g., Heckman and Pagés, 2000). Moreover, formal firms that are legally registered with the authorities face value-added and payroll taxes, which further reduce their profitability. Finally, imported goods are subject to import tariffs.

The selection of the remaining ingredients in our model is guided by a number of facts on formal and informal firms and workers in Brazil.[2] In particular, the proportion of firms that are informal falls substantially with firm size, indicating that informal firms perceive increasing costs associated with informality as they grow. For example, as informal firms grow, they become more visible to the local authorities, leading to increased fines and threat of closure. Concurrently, in order to grow, informal firms face larger costs of capital (Catão et al., 2009), indicating larger opportunity costs of informality for larger firms. In addition, informal sector firms tend to be substantially smaller and exhibit lower worker productivity than their formal sector counterparts. These facts motivate a model based on firms that are heterogeneous in their total factor productivity and sort

[1] For example, see Goldberg and Pavcnik (2003), McCaig and Pavcnik (2018), Dix-Carneiro and Kovak (2019), Ponczek and Ulyssea (2022).

[2] See Meghir et al. (2015) and Ulyssea (2020) for a discussion of a large set of stylized facts involving the informal sector in Brazil.

into the formal and informal sectors accordingly.

If a firm is productive and large enough, it faces prohibitive costs of operating informally and sorts into the formal sector. However, less productive and smaller firms tend to sort into the informal sector, flying under the government's radar and avoiding the burdens of distortive taxes and regulations. In simple terms, this environment creates an economy with a size-dependent distortion: the larger (and more productive) firms in the formal sector are subject to taxes and regulations and experience larger distortions compared to the smaller scale (and lower productivity) informal sector. This implies that formal firms underproduce relative to the social optimum whereas informal firms overproduce.

Search and matching frictions in the labor market generate equilibrium unemployment and wage dispersion. These elements are important for three reasons. First, in the data, transitions from unemployment to informality are twice as likely as transitions from unemployment to formality. This implies that any policy or shock that leads to higher inflows into unemployment is also likely to lead to higher informality. Second, data on formal and informal firms reveal that the latter tend to pay lower wages—often below the official minimum wage—than their formal counterparts. This equilibrium wage dispersion is crucial for ensuring that the minimum wage meaningfully impacts firms' incentives to opt for either the formal or informal sector. Finally, we are motivated by the work by Dix-Carneiro and Kovak (2019) and Ponczek and Ulyssea (2022) who show the important role of the informal sector in mitigating the unemployment consequences of trade-induced negative labor demand shocks.

Our model encompasses two broad sectors: manufacturing and services. It is a priori important to carefully model both sectors for two reasons. The first is that informality is widespread in services. Second, although we do not allow for trade in service sector goods, service sector firms are also affected by trade through intersectoral linkages with manufacturing. This creates a potentially important channel linking trade to the aggregate level of informality. In turn, international trade plays a role in the model through two channels. First, imports impact all firms in the economy because of their direct impact on the price of imported manufacturing goods and the aforementioned intersectoral linkages. Second, manufacturing formal firms can export, subject to fixed export costs and variable trade costs (as in Melitz, 2003).

We estimate the model using multiple data sources, including matched employer-employee data from both formal and informal firms in Brazil, as well as firm- and worker-level data from household surveys, manufacturing and services censuses, and customs records. To ensure consistency, we restrict our sample to the six metropolitan regions covered by all datasets and exclude firms and workers in agriculture, mining, coal, and oil and gas, reflecting the urban focus of the data.[3]

Brazil provides an excellent setting for our work. Not only does it offer outstanding data sources for both the formal and informal sectors, but it also presents a particularly relevant environment

[3]Although agriculture is a significant sector of the Brazilian economy and exhibits high levels of informality, it is not covered in the data we use to measure informality. Specifically, the two datasets we rely on—*Economia Informal Urbana* and *Pesquisa Mensal de Emprego*—focus on urban businesses and workers, and therefore do not provide comprehensive coverage of agriculture or mining.

given its large informal sector. Nearly two-thirds of firms and 50% of workers are informal, and the informal sector generates 40% of GDP (Ulyssea, 2018). Moreover, there is a clear definition of what constitutes informality in the country: we define as informal workers those who do not hold a formal labor contract, clearly observable through the *carteira de trabalho*, a booklet logging each worker's (formal) employment history. Informal firms are those not registered with the tax authorities, indicated by the absence of a tax identification number (*Cadastro Nacional de Pessoa Jurídica*—CNPJ), which we can observe.

Armed with our model estimates, we begin by analyzing the effects of a reduction in bilateral trade barriers on the Brazilian economy. In the context of a small globalization shock, we decompose the impact on real income into a *mechanical effect* and a *reallocation effect* (as in Baqaee and Farhi, 2020; Atkin and Donaldson, 2022). The mechanical effect captures the real income gains from lower prices on imported goods, holding labor allocations, wages, aggregate income, and firm-level demand fixed. It isolates the immediate gains from reduced trade costs, without accounting for any reallocation. The reallocation effect, by contrast, captures the gains from reallocating resources across more or less distorted uses—here, with trade costs and technology held fixed—along with other general equilibrium responses.[4] In an open economy, terms-of-trade effects are embedded in the reallocation effect (Baqaee and Farhi, 2024). To further isolate the gains from reallocation in the presence of distortions, we also perform an alternative decomposition, separating gains from an undistorted economy and the remaining reallocation effect, net of terms-of-trade changes. In both decompositions, we find that the reallocation effect dominates: the distortions in our model imply gains more than twice as large as those in the undistorted case.

To better understand these results, we examine how informality, measured aggregate total factor productivity (TFP), and the dispersion of the log marginal revenue product of labor respond to changes in trade costs.[5] A reduction in trade costs leads to a contraction of informality in both the manufacturing and service sectors. This contraction of the less productive and less distorted informal sector is accompanied by a reallocation of resources to formal firms which are initially more productive and more distorted, leading to an increase in measured aggregate TFP. Simultaneously, we observe a decline in the dispersion of the log marginal revenue product of labor, indicating a more efficient allocation of resources following increases in globalization, further boosting the reallocation effect (Hsieh and Klenow, 2009). All these results suggest that reductions in trade costs trigger a reallocation of resources toward more efficient uses and that this reallocation has an important quantitative impact on real income.

We also simulate larger shocks, which are more relevant for policymakers, and we find that a 33% reduction in trade costs results in a real income gain of 24%. These results confirm that trade liberalization, by moving resources away from less distorted informal firms toward the initially more distorted formal sector, is able to substantially improve allocative efficiency. Although the

[4] This decomposition relies on a first-order approximation of changes in real income, following Baqaee and Farhi (2020) and Atkin and Donaldson (2022).

[5] We define "measured aggregate total factor productivity" as the average of firm-level TFP weighted by employment, similarly to Olley and Pakes (1996) and Pavcnik (2002).

impacts on the total number of informal workers are economically important, they are not enough to significantly curb informality. Specifically, as trade costs decline by 33%, the imports-to-GDP ratio increases from 7.7% to 32.5%, a substantial fourfold increase.[6] However, the share of informal workers declines much more modestly, from 48.7% to 43.6%.[7] This result is consistent with the casual observation that the informal sector has not substantially shrunk in middle-income economies despite the large-scale liberalization episodes these experienced in the 1980s and 1990s (see, for example, World Bank, 2019).

Given that our model includes several distortions beyond those causing informality, we examine to what extent the aggregate consequences of trade shocks can be attributed to the prevalence of informality in the economy. To this end, we compare the impact of a trade shock in the benchmark economy, representing the status quo in the data, to the outcomes in various scenarios in which we gradually increase government enforcement so that the size of the informal sector slowly declines, including the extreme case of perfect enforcement with zero informality. In all these counterfactual simulations, we hold the structural parameters fixed at the values estimated for the benchmark model and simply gradually increase the costs faced by informal firms. We find that the gains from trade, as measured by the increase in real income, are larger when enforcement is less strict (i.e., when there is more informality). This result is driven by changes in the magnitude of the reallocation effect, as the mechanical effect and the undistorted-economy effect are approximately constant across scenarios.[8] With a 33% reduction in trade costs, the total real income gains with no informality are less than half as a large as those under our benchmark scenario (11% vs. 24%).

Our main conclusion is that an increase in globalization leads to a reallocation of labor and output from less distorted, initially overproducing informal firms to more distorted, initially underproducing formal firms and that this boosts the reallocation effect. In an economy with stricter enforcement, trade openness has less room to accomplish this reallocation as the informal sector is initially smaller, resulting in a relatively smaller magnitude of the reallocation effect.

Our paper contributes to a nascent literature on the interactions between trade and informality, e.g., Goldberg and Pavcnik (2003), McCaig and Pavcnik (2018), Dix-Carneiro and Kovak (2019), Ponczek and Ulyssea (2022). This literature empirically shows that the informal sector constitutes an important margin of adjustment to trade shocks. To date, this literature has relied on reduced-form empirical methods, resorting to difference-in-differences identification strategies. In particular, the last two papers focus on the unilateral Brazilian trade liberalization of the early 1990s and adopt a local labor markets approach. Their specifications compare labor market outcomes of a region facing a smaller (trade-induced) negative labor demand shock to those of a region facing a larger negative shock. Given that common impacts of trade are absorbed into the intercept, this type

[6]For comparison, the current imports-to-GDP ratios in the United Kingdom, South Africa, and Colombia are all approximately 30%.

[7]While the overall share of informality employment decreases by 5 percentage points, employment in the informal manufacturing sector experiences a more substantial decline of 25%. This observation, coupled with our roundabout production structure and intersectoral linkages, helps explain the significant reallocation effect that we uncover.

[8]The reallocation effect is still important when informality is eradicated. As we elaborate in section 3.9, our model features other distortions, so that the no-informality equilibrium still exhibits inefficiencies.

of specification only isolates the impacts of negative labor demand shocks. From our model's perspective, this involves comparing the benchmark economy with an economy experiencing a negative labor demand shock.

We examine how the predictions of our model compare to the findings of Dix-Carneiro and Kovak (2019) and Ponczek and Ulyssea (2022). Specifically, we simulate the impacts of negative labor demand shocks on unemployment, informality, and real income. The outcomes of our counterfactual experiments align with the findings that the informal sector acts as an "unemployment buffer" during periods of adverse labor market conditions. Notably, while the impact on unemployment remains relatively small, if not slightly negative, there is a significant expansion in the informal sector, consistent with the findings of Dix-Carneiro and Kovak (2019). Furthermore, Ponczek and Ulyssea (2022) show that the impact of negative shocks on unemployment (informality) is larger (smaller) in locations where the government more strictly monitors compliance with labor market regulations. Once again, our model's predictions align with these findings, further validating the idea from both papers that the informal sector helps mitigate increases in unemployment during economic downturns.

However, we show that the aggregate real income losses from these negative labor demand shocks are larger in the benchmark economy with a larger informal sector than in the counterfactual economies with stricter enforcement and lower informality. Hence, the informal sector seems to serve as an unemployment buffer, but not as an aggregate real income buffer. The reason behind this result is that negative shocks push resources away from the more distorted formal sector toward the less productive and less distorted informal sector. This leads to a reallocation effect that is considerably more negative in the economy with a larger informal sector than in an economy in which the government more strictly monitors informal firms.

Given the scarcity of data on the informal sector, most empirical and quantitative studies on the impacts of trade have to focus on formal sector firms. In our final analysis, we investigate how incorporating the informal sector into these studies affects key insights and findings. We show that the standard measured aggregate productivity gains estimated from data on only the formal sector can substantially understate total aggregate productivity gains, which include the reallocation from low-productivity firms in the informal sector to high-productivity firms in the formal sector. Next, we revisit the effects of trade on cross-firm wage inequality. Prominent studies on this topic include Coşar et al. (2016) on Colombia and Helpman et al. (2017) on Brazil. Both papers center their analyses on data on formal manufacturing firms and find, from the lens of their quantitative structural models, that increases in openness were associated with increases in wage inequality across firms, driven by the behavior of the exporter wage premium and reallocation of labor from nonexporters to exporters. We replicate their finding that trade liberalization increases wage inequality in the formal manufacturing sector. However, we show that this result is reversed once one accounts for inequality in the whole economy, including the informal sector. Even though we account for only for between-firm wage inequality, our finding is particularly interesting in light of the recent literature that emphasizes the prominence of between-firm inequality in explaining overall trends in wage inequality in different countries (e.g., Card et al., 2013, 2016; Song et al., 2019), including Brazil (Alvarez et al., 2018).

The remainder of the paper is organized as follows. Section 2 describes the data we use in the analysis and presents five stylized facts on formal and informal workers and firms that we use to motivate our theoretical framework. Section 3 outlines our model. Section 4 details the estimation procedure, discusses identification and shows how the model fits key aspects of the data. Section 5 shows our counterfactual experiments, and section 6 presents our main takeaways.

2 Five Facts on Formal and Informal Firms and Workers in Brazil

We begin by highlighting five important facts about formal and informal workers and firms in Brazil that inform the selection of our model ingredients.[9] In anticipation of the notation introduced in the next section, we represent the manufacturing sector as C and services as S.

We make use of seven datasets containing information on formal and informal firms and their workers. An overview of these datasets and their main features is provided in Table 1. A key source of information on formal sector firms and workers is the *Relação Anual de Informações Sociais* (RAIS), which is a matched employer-employee dataset assembled by the Brazilian Ministry of Labor every year since 1976. RAIS is a high-quality panel that contains the universe of formal firms and workers.[10] With these data, we can provide a detailed cross-sectional picture of the formal labor market in the C and S sectors, and can generate important longitudinal statistics such as firm-level turnover and exit rates.

We also make use of three firm-level surveys conducted by the Brazilian National Statistics Agency (*Instituto Brasileiro de Geografia e Estatística*, IBGE) which cover the *formal* manufacturing, retail and service sectors: *Pesquisa Industrial Anual* (PIA), *Pesquisa Anual de Comércio* (PAC), and *Pesquisa Anual de Serviços* (PAS), respectively. These surveys collect detailed information on firms' revenues and inputs and combine a census of firms above a certain size threshold with a representative sample of smaller firms. Hence, longitudinal statistics can be computed for firms surveyed in the census. We identify exporters in RAIS and PIA by merging these datasets with administrative customs records from the *Secretaria de Comércio Exterior* (SECEX).

These five datasets provide a comprehensive view of the formal sector but omit information on the informal sector. Therefore, we use two additional sources providing data on informal firms and workers. The first is the *Pesquisa de Economia Informal Urbana* (ECINF), a cross-sectional survey collected by IBGE in 2003 and designed to be representative of the universe of *urban* firms with up to five employees (both formal and informal).[11] It is a matched employer-employee dataset that contains information on entrepreneurs, their businesses and their individual employees. Firms

[9] See Ulyssea (2020) for a more extensive discussion of stylized facts related to informality.

[10] The RAIS dataset has been increasingly used in different applications. For recent examples see Dix-Carneiro (2014), Dix-Carneiro and Kovak (2017), Helpman et al. (2017), and Ulyssea (2018), among others.

[11] Although a few firms in the dataset have more than five employees, we restrict our attention to those with five employees or fewer so that our sample is consistent with the population that the survey targets.

Table 1: Summary of Datasets

Dataset	Source	Description
Relação Anual de Informações Sociais *RAIS* Years: 2003–2005	Ministry of Labor	Administrative matched employer-employee dataset. Covers all formal firms and workers. Detailed information on firms and workers, but no information on firm-level revenues, capital and expenditures with intermediate inputs.
Pesquisa Industrial Anual *PIA* Years: 2003, 2004	IBGE	Survey data on Manufacturing firms. Firm-level information such as revenues, capital, investment, expenditures with intermediate inputs, employment. Covers all firms with 30 employees or more; random sample of smaller firms.
Pesquisa Anual dos Serviços *PAS* Years: 2003, 2004	IBGE	Survey data on Service-sector firms. Firm-level information such as revenues, capital, investment, expenditures with intermediate inputs, employment. Covers all firms with 20 employees or more; random sample of smaller firms.
Pesquisa Anual do Comércio *PAC* Years: 2003, 2004	IBGE	Survey data on Retail and Commerce firms. Firm-level information such as revenues, capital, investment, expenditures with intermediate inputs, employment. Covers all firms with 20 employees or more; random sample of smaller firms.
Secretaria de Comércio Exterior *SECEX* Year: 2003	Ministry of Industry, Foreign Trade and Services	Administrative customs data. Export and import values at the firm level.
Economia Informal Urbana *ECINF* Year: 2003	IBGE	Survey data. Matched employer-employee dataset with detailed information on formal and informal firms and their workers. Representative sample of small businesses (firms with 5 employees or less). Information on formal status of the firm and its workers.
Pesquisa Mensal de Emprego *PME* Year: 2003	IBGE	Survey data. Rotating panel of households that covers the 6 main metropolitan areas in Brazil.

are directly asked whether they are registered with the tax authorities and whether each of their workers has a formal labor contract. It is therefore possible to directly observe both firms' and workers' formal statuses. Given that the formality/informality statuses are self-reported, one might be concerned about measurement error and underreporting. However, IBGE has a long tradition of accurately measuring labor informality, and has very strict confidentiality clauses. The information that they collect cannot be used for auditing purposes by other government branches, in particular those responsible for enforcing the relevant laws and regulations. These characteristics, together with the high levels of informality observed in the data, make us confident that respondents are not systematically underreporting their informality status.[12]

Finally, we draw from the Monthly Employment Survey (*Pesquisa Mensal de Emprego*, PME) to obtain information on worker allocations and labor market flows. This is a rotating panel with a design similar to that of the Current Population Survey in the United States. The survey covers the six main metropolitan areas in Brazil and contains detailed information on individuals' sociodemographic characteristics and labor market outcomes, including informal employment status. We define a worker to be informal if she does not hold a formal labor contract—this variable is

[12] Additionally, Ulyssea (2018) shows that the ECINF survey reproduces very well statistics from RAIS on all the dimensions that are common to both datasets (e.g., size and sectoral distributions), which is reassuring with respect to ECINF's quality.

explicitly recorded in PME. If a worker is self-employed, she is also treated as an informal worker. Similarly, we treat self-employed workers in ECINF as informal firms employing one worker. We exploit the panel structure of PME to estimate one-year labor market transitions between formal employment, informal employment (in both C and S sectors) and unemployment status.[13]

We need to impose a few restrictions to make these seven datasets consistent with each other. First, because PME covers only six metropolitan regions, we restrict our samples to these regions whenever possible. Given the focus on metropolitan regions in PME and urban firms in ECINF, we remove firms and workers in the agriculture, mining, coal, and oil and gas industries.[14] Second, we restrict our attention to data from 2003, as ECINF is available only for that year. Whenever we need to compute dynamic statistics, we also employ data from 2004. Finally, we exclude firms in the public sector. Our data appendix provides additional details regarding data treatment—see Online Appendix L.

Having presented the various sources of data used in this paper, we are now ready to introduce the aforementioned five facts on formal and informal firms and workers in Brazil. These facts guide the construction of our model and are targeted in our estimation procedure.

Fact 1: Approximately 50% of the Brazilian labor force is informally employed. Transitions from unemployment to an informal job are almost twice as likely as transitions from unemployment to a formal job.

Table 2 uses data from PME to establish that 48.2% of all workers are informally employed. If we break down informality rates by sector, we find that 35.6% of C sector workers are informal, compared to 51.2% in the S sector. Moreover, transition rates from unemployment to informal jobs are almost twice as likely as those to formal jobs (45.7% compared to 26.4%).[15]

Fact 2: The probability that a firm is informal declines sharply with its employment size.

We estimate, for each sector, regressions relating a firm-level informal status indicator to the firm's number of employees. Table 3 uses data from ECINF to show that the fraction of informal firms rapidly declines with employment size.

Fact 3: Informal firms have, on average, lower revenue per worker than formal firms.

It has been widely documented that average worker productivity across firms is substantially lower in the informal sector than in the formal sector.[16] Our datasets confirm this insight. Note

[13] To minimize the effects of attrition in PME, we measure one-year transitions by annualizing four-month transitions.

[14] Incorporating the agriculture and mining sectors into the model would be desirable. These sectors play an important role in Brazil's international trade and they feature high prevalence of informality. Not being able to include them is an important limitation of our analysis.

[15] Using 2003 data from the *Pesquisa Mensal de Emprego*, we find that unskilled (skilled) workers are 2.2 (1.3) times more likely to transition from unemployment to informal rather than formal employment. Thus, transitions from unemployment to informal jobs exceed those to formal jobs for both groups, with the pattern being more pronounced among unskilled workers.

[16] For example, refer to La Porta and Shleifer (2008), Meghir et al. (2015), and Ulyssea (2018). In this literature,

Table 2: Employment Shares and Transition Rates

	Share of Workers	Transition Rates From Unemp.
Informal Manufacturing (Ci)	0.059	0.058
Formal Manufacturing (Cf)	0.106	0.059
Informal Services (Si)	0.351	0.399
Formal Services (Sf)	0.334	0.206
Unemployment	0.150	0.279
Share of Informal Employment	0.482	
Transition Rate from Unemp.		
to Informal Employment	0.457	
to Formal Employment	0.264	

Data source: 2003 PME.

Table 3: Firm-Level Informality Status vs. Firm-Level Employment

	Dep. Variable: $Informal\ Status\ Indicator_i$	
	C sector	S sector
Intercept	1.170	1.122
	(0.020)	(0.012)
ℓ_i	-0.212	-0.216
	(0.017)	(0.009)
Observations	1,811	10,532

Data source: 2003 ECINF. Standard errors in parentheses.

that the sample of informal firms from ECINF cannot be directly compared with the samples from PIA, PAS, and PAC as ECINF is designed to cover firms with at most five employees. However, we can compare firm-level labor productivity (measured as revenue per worker) across formal and informal firms by estimating linear regressions relating firm-level revenues to their employment size. The idea is that once we condition on employment size, the revenues of informal firms in ECINF can be compared to those of equal-sized formal firms in PIA, PAS, and PAC. The linear regression results reported in Panel A of Table 4 imply that, among C sector firms of size one, formal firms are on average more productive than informal ones by 1.76 log-points. For C sector firms of size five, this difference shrinks to 1.03 log-points. Similarly, for S sector firms of size one, formal firms are, on average, 1.23 log-points more productive than informal ones. Finally, for S sector firms of size five, this difference declines to 0.38 points.

Fact 4: The average informal worker is paid lower wages than the average formal worker.

It is a well-documented fact that average wages in the informal sector are lower than those in the formal sector. Although RAIS provides information on firm-level wages for the population of

worker productivity differences across firms are commonly measured by value added or revenue per worker. Whether this measure accurately reflects or is highly correlated with TFP at the firm level depends on the framework considered by the researcher. For example, in a standard Melitz model with heterogeneous firm-level TFP but no fixed costs of production, revenue per worker is constant across firms in equilibrium. However, the model that we discuss later in this paper, which incorporates labor market frictions and convex hiring costs, implies that firm-level productivity is predicted with high accuracy by revenue per worker. See Online Appendix P.

Table 4: Firm-Level log Revenue per Worker and log Wages *vs.* log Employment

	A. Dep. Variable: $\log(Revenue_i/\ell_i)$				B. Dep. Variable: $\log(wage_i)$			
Sector / Firm Type	Cf	Sf	Ci	Si	Cf	Sf	Ci	Si
Intercept	10.118	10.004	8.356	8.772	8.515	8.495	7.928	8.309
	(0.013)	(0.005)	(0.044)	(0.019)	(0.002)	(0.001)	(0.039)	(0.018)
$\log(\ell_i)$	0.000	-0.128	0.453	0.401	0.110	0.123	0.334	0.293
	(0.005)	(0.003)	(0.102)	(0.049)	(0.001)	(0.000)	(0.085)	(0.046)
$Exporter_i$ (Dummy)	1.462				0.473			
	(0.021)				(0.006)			
Observations	16,986	43,861	1,608	8,858	140,649	949,995	1,609	8,861
Dataset	PIA + SECEX	PAS + PAC	ECINF	ECINF	RAIS + SECEX	RAIS	ECINF	ECINF

Standard errors in parentheses.

formal firms, we do not have firm-level wages for the population of informal firms. ECINF contains wages for informal workers but covers only firms with up to five employees. We compare wages across datasets by regressing log wages on log size for each type of firm in the two sectors (C and S). Panel B of Table 4 shows that, after controlling for firm size, there is a significant informal wage penalty in both the C and S sectors. In the C sector, formal firms with only one employee pay wages that are, on average, 0.59 log points higher than those paid by informal firms of the same size. This wage gap narrows to 0.23 log points for firms with five employees. In the S sector, the corresponding wage gap starts at 0.19 log points for size-one firms and also narrows to 0.23 log points by size five. Interestingly, when conditioning on firms of size five, columns two and four of Panel B suggest that informal firms actually pay wages that are, on average, 0.09 log points higher than those of formal firms. Nonetheless, because informal firms are typically much smaller than their formal counterparts, the *average* informal worker still earns less than the *average* formal worker.[17]

Fact 5: Firm-level labor turnover tends to decline with firm-level employment size. However, conditional on size, exporters tend to have higher turnover.

In the context of Colombia, Coşar et al. (2016) show that labor turnover tends to decline with firm size but that, conditional on employment size, turnover tends to be larger for exporters. These relationships, replicated in Table 5 for Brazil, are potentially important for our quantitative analyses, as they establish a connection between trade openness and aggregate labor turnover. This link entails the reallocation of resources toward larger firms (which tend to exhibit lower turnover) and exporters (who, in contrast, tend to exhibit larger employment volatility). In a model incorporating search frictions and unemployment, turnover significantly influences the unemployment rate. Given Fact 1, these dynamics can, in turn, have substantial impacts on informality.

[17]Using 2003 data from the *Pesquisa Mensal de Emprego*, we find that, after controlling for age, gender, and education, informal workers earn 0.34 log points less than formal workers in Sector S, and 0.43 log points less in Sector C, confirming the robustness of Fact 4 to worker characteristics.

Table 5: Turnover, Firm Size and Export Status

	Dep. Variable: $Turnover_i$	
Intercept	0.713	0.633
	(0.003)	(0.001)
$\log(\ell_i)$	-0.128	-0.116
	(0.001)	(0.001)
$Exporter_i$ (Dummy)	0.084	
	(0.007)	
Observations	140,649	949,995

Data Sources: 2003 and 2004 RAIS and 2003 SECEX. Turnover of firm i between 2003 and 2004 measured as $Turnover_i = \frac{|\ell_{i,2004} - \ell_{i,2003}|}{0.5 \times (\ell_{i,2004} + \ell_{i,2003})}$. Standard errors in parentheses.

3 Model

3.1 Motivating the Model's Ingredients

Before we start the exposition of our quantitative framework, we motivate our selection of its main ingredients, based on the five facts documented in the previous section. We begin with the premise that the existence of the informal sector is attributed to burdensome taxes and labor market regulations, which are imperfectly enforced by the government. While the government, in practice, imposes costs on informal firms through fines or the threat of closure, these measures are not applied uniformly across all firms. This lack of strict enforcement creates incentives for certain firms to operate under the radar, evading taxes and avoiding compliance with labor market regulations. In this context, Fact 2 in section 2 suggests that firms perceive a significant increase in the costs of informality as they grow. This increase is presumably linked to the heightened visibility of larger firms, leading to a larger probability of their receiving fines from the government.

Turning to other crucial components of the model, we note that Facts 1 and 5 require a model that includes unemployment and accommodates firm-level dynamics, resulting in worker turnover. Modeling unemployment is important in light of the empirical findings of Dix-Carneiro and Kovak (2019) and Ponczek and Ulyssea (2022), who document how trade-induced local labor demand shocks impact informality and unemployment.[18] It is our objective to make the model's predictions align with these documented facts. In addition, firm-level dynamics and endogenous worker turnover are crucial for modeling the consequences of firing costs—a significant labor market regulation present in many Latin American countries, often blamed for the high levels of informality in the region (e.g., Heckman and Pagés, 2000).

To generate Fact 3, we need a model featuring firms that are heterogeneous in productivity. Furthermore, we allow firms to select their status (formal or informal), allowing for an endogenous

[18]In the context of the Brazilian trade liberalization of the early 1990s, Dix-Carneiro and Kovak (2019) show that (permanent) trade-induced negative labor demand shocks lead to long-run increases in informal employment but have no impact on unemployment. Ponczek and Ulyssea (2022) revisit these results and investigate the impact of the same trade-induced negative labor demand shocks on informality and unemployment across regions that enforce labor market regulations in varying degrees. They confirm that the informal sector helps mitigate the impact of adverse labor market conditions on unemployment.

response of the size of the informal sector to changes in the economic environment. Given Fact 2, smaller and less productive firms tend to choose to operate in the informal sector. This observation implies that, in equilibrium, less productive firms are subject to fewer distortions in their decisions from taxes and regulations than are more productive firms. This unequal exposure to domestic distortions can lead to considerable misallocation of resources. More precisely, higher-productivity formal firms face larger distortions, and, as a result, tend to underproduce relative to the social optimum. Meanwhile, lower-productivity informal firms face fewer distortions and tend to overproduce.

We next move on to Fact 4, to account for which the model must incorporate labor market frictions and equilibrium wage dispersion. The latter is particularly important for modeling the consequences of a minimum wage, another potentially significant labor market regulation influencing the incentives for firms to operate in the informal sector.

Finally, it is a priori important to model both the manufacturing and services sectors for two reasons. The first is the significant prevalence of informality in services (Table 2). The second is that service sector firms are also affected by trade through intersectoral linkages with manufacturing. This creates a potentially important channel linking trade to the aggregate level of informality.

Given the motivation for the model's ingredients, we start by considering a closed economy setup in sections 3.2 through 3.5 and then extend the model to an open economy in section 3.6. The equilibrium conditions are outlined in section 3.7. In section 3.8, we delve into the mechanisms through which trade can impact informality, and section 3.9 wraps up with a final discussion of the model.

3.2 Consumers

The economy is populated by homogeneous, infinitely lived workers-consumers. Individuals derive utility from a final good whose production will be described shortly. Preferences are given by:

$$U = \sum_{t=1}^{\infty} \frac{Y_t}{(1+r)^t}, \tag{1}$$

where Y_t is the total amount of the final good consumed at time t and $\frac{1}{1+r}$ is the discount factor. We denote the price of the final good at time t by P_{Yt}.

3.3 Technology and Firms

The economy is comprised of two broad sectors: manufacturing (C) and services (S). Four types of goods are available: (a) the final consumption good, (b) sector-specific intermediate differentiated varieties, (c) sector-specific composite goods, and (d) sector-specific intermediate inputs. Sector-specific intermediate differentiated varieties (b) are aggregated into the sector-specific composite goods (c). In turn, these sector-specific composite goods are used in two ways: to produce the final good (a) and to produce the sector-specific intermediate inputs (d).[19] The sector-specific intermediate inputs (d) are finally bundled with labor to produce the intermediate differentiated varieties (b).[20]

We now describe the technologies available to produce these different types of goods. The final good, which is used solely for final consumption, is assembled by a perfectly competitive producer. Its output at time t, denoted by Y_t, is given by a Cobb-Douglas aggregate of the two sector-specific composite goods:

$$Y_t = \widehat{C}_t^{\zeta} \widehat{S}_t^{1-\zeta}, \tag{2}$$

where ζ denotes the share of expenditures on the C sector composite good, $\widehat{C}_t$ is the total usage of the sector C composite good, and $\widehat{S}_t$ is the total usage of the sector S composite good.

In turn, sector-specific composite goods are produced by perfectly competitive producers operating in each sector $k \in \{C, S\}$. Total output of sector k is given by a constant elasticity of substitution (CES) aggregator over the output of a sector k specific continuum of differentiated varieties indexed by $n \in [0, N_{kt}]$, and where N_{kt} indicates the mass of available varieties in sector k at time t:

$$\mathcal{C}_t = \left(\int_0^{N_{Ct}} q_{Ct}(n)^{\frac{\sigma_C - 1}{\sigma_C}} \, dn \right)^{\frac{\sigma_C}{\sigma_C - 1}} \qquad \mathcal{S}_t = \left(\int_0^{N_{St}} q_{St}(n)^{\frac{\sigma_S - 1}{\sigma_S}} \, dn \right)^{\frac{\sigma_S}{\sigma_S - 1}}. \tag{3}$$

In (3), $\mathcal{C}_t$ denotes the total output of the sector C composite good, $\mathcal{S}_t$ represents the total output of the sector S composite good, $q_{kt}(n)$ is total usage of differentiated variety n from sector k, and $\sigma_k > 1$ is the elasticity of substitution across varieties within sector $k \in \{C, S\}$.[21] These sector-specific composite goods are solely used as inputs for the production of the final good in (2) or as inputs for the production of sector-specific intermediate inputs, which we describe next. As we focus on steady-state equilibria, we henceforth drop the time subscript t for notational convenience.

The sector k specific intermediate input is also produced competitively and bundles the sector-specific composite goods determined by equation (3) as follows:

$$\iota_k \equiv \widetilde{C}^{\lambda_k} \widetilde{S}^{1-\lambda_k}, \tag{4}$$

where λ_k is the share of total payments to the sector C composite good, $\widetilde{C}$ is the total usage of the sector C composite good, and $\widetilde{S}$ is the total usage of the sector S composite good.

Units of differentiated variety $n \in [0, N_k]$ for a particular sector $k \in \{C, S\}$ are produced by firms that combine homogeneous labor with sector-specific intermediate inputs. Importantly, these are the firms in our model that map to the firms we observe in the data. From now on, whenever we refer to "firms," we refer to these differentiated variety producers. Total output by a firm producing variety n in sector k with productivity z employing labor ℓ and sector-specific intermediate inputs

[19]Sector S composite goods are also used in the form of fixed costs of production, fixed costs of exporting and hiring costs.

[20]When we discuss the open economy model in section 3.6, we maintain that the final good (a), the sector-specific composite goods (c), and the sector-specific intermediate goods (d) cannot be traded across borders. However, the intermediate differentiated varieties (b) are allowed to be traded subject to trade costs.

[21]When we consider the open economy model, the set of available manufacturing varieties includes both domestically produced and foreign-produced varieties.

ι_k is given by:

$$q_k(z, \ell, \iota_k) = z\ell^{\delta_k}\iota_k^{1-\delta_k}, \tag{5}$$

where $\delta_k \in (0, 1)$.

Guided by Facts 1 and 5, we generate firm-level dynamics and endogenous worker turnover by allowing firms' idiosyncratic productivity z to evolve over time, following the AR(1) process below:

$$\ln z' = \rho_k \ln z + \sigma_k^z \varepsilon, \qquad \rho_k \in (0, 1), \qquad \varepsilon \sim N(0, 1), \tag{6}$$

where σ_k^z is the standard deviation of the stochastic shocks.[22]

Monopolistic competition among differentiated variety producers implies that gross revenues as a function of output q in sector $k \in \{C, S\}$ are given by:

$$\widetilde{R}_k(q) = \left(\frac{X_k}{P_k^{1-\sigma_k}}\right)^{\frac{1}{\sigma_k}} q^{\frac{\sigma_k - 1}{\sigma_k}}, \tag{7}$$

where X_k is total expenditure on the sector k specific composite good, $P_k = \left(\int_0^{N_k} p_k(n)^{1-\sigma_k} dn\right)^{\frac{1}{1-\sigma_k}}$ is the price of one unit of sector k's composite good, $k \in \{C, S\}$, and $p_k(n)$ is the price charged by firm n in sector k. To streamline the presentation, we henceforth refer to P_k as the price index of sector k.[23]

Aggregate expenditure on the sector C composite good is given by $X_C = \zeta I + X_C^{int}$, where I is aggregate income and X_C^{int} is the total expenditure from sector-specific intermediate input producers on the composite good from sector C. Similarly, aggregate expenditure on the sector S composite good is given by $X_S = (1-\zeta) I + X_S^{int} + E_S$, where X_S^{int} is total expenditure from sector-specific intermediate input producers on the composite good from sector S and E_S represents expenditures on the S sector composite good made by firms in order to cover entry, hiring, fixed and export costs (which we discuss below). Aggregate income is determined by total wages, government transfers and aggregate firm profits.

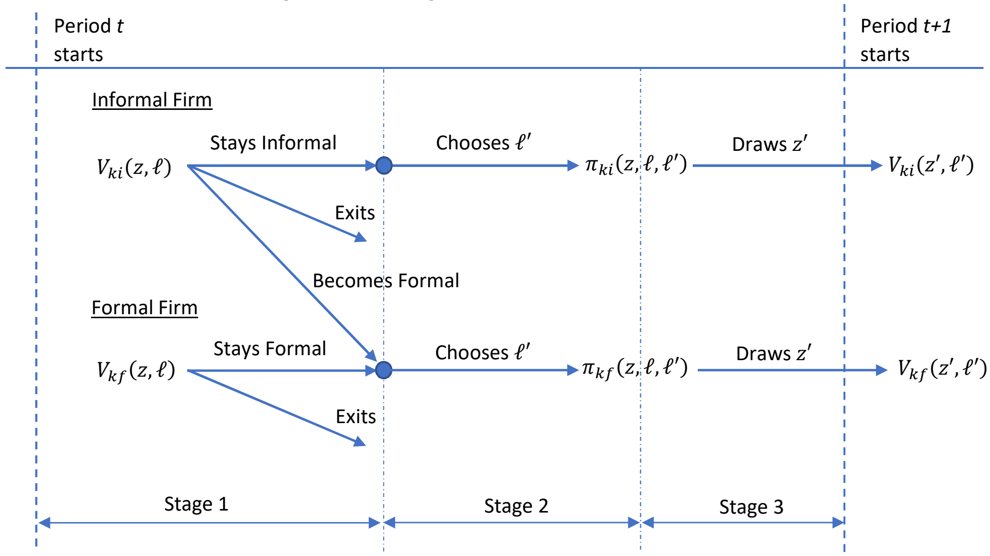
Firms can freely adjust their intermediate input usage. Denote by $\iota_k(z, \ell)$ the optimal intermediate input usage of a firm in sector k with productivity z and ℓ workers. This firm's gross revenue can then be written as $R_k(z, \ell) \equiv \widetilde{R}_k\left(z\ell^{\delta_k}\iota_k(z, \ell)^{1-\delta_k}\right)$. It is easy to show that expenditures on intermediates are proportional to gross revenues, resulting in the following expression for firm-level value added:

$$VA_k(z, \ell) = \frac{\sigma_k - (1-\delta_k)(\sigma_k - 1)}{\sigma_k} R_k(z, \ell). \tag{8}$$

[22]This process is imposed to be the same across formal and informal firms within sectors C and S. Unfortunately, we do not have longitudinal data on firms in the informal sector, so that this process cannot be separately identified for formal and informal firms.

[23]Given that P_k is the price of one unit of the sector k specific composite good, $k \in \{C, S\}$, the price of one unit of the final good is given by $P_Y = \frac{P_C^{\zeta} P_S^{1-\zeta}}{\zeta^{\zeta}(1-\zeta)^{1-\zeta}}$, and the price of one unit of the sector k specific intermediate input is given by $P_k^m = \frac{P_C^{\lambda_k} P_S^{1-\lambda_k}}{\lambda_k^{\lambda_k}(1-\lambda_k)^{1-\lambda_k}}$.

Figure 1: Diagram of Firms' Behavior



Timing

The timing of events is illustrated in Figure 1. Formal firms are indexed by f and informal firms by i. Consider an informal firm that starts period t with productivity z and employment level ℓ. In the first stage, this firm may: (i) stay informal; (ii) exit, either as a result of an endogenous decision or because of an exogenous shock that occurs with probability α_k; (iii) become formal. In the second stage, the firm decides whether to adjust its workforce (up or down) to ℓ' or not at all. Right after this decision the firm realizes profits and pays wages to its workers. In stage 3, the firm draws a new productivity value z' and starts period $t+1$ with state (z', ℓ'). The timing and sequence of events for formal firms is the same as that for informal ones, except that we do not allow them to become informal.

Hiring and Firing Costs

Both formal and informal firms in the C and S sectors face hiring costs, which are incurred in units of the S sector composite good. We parameterize hiring costs as a function of employment levels ℓ and number of posted vacancies υ:

$$C_k^h(\ell, \upsilon) = \left(\frac{h_k}{\gamma_{k1}}\right)\left(\frac{\upsilon}{\ell^{\gamma_{k2}}}\right)^{\gamma_{k1}}, \tag{9}$$

where h_k, $\gamma_{k1} > 1$ and $\gamma_{k2} \in (0, 1)$ are parameters to be estimated. If μ_{kj}^{υ} is the probability of filling a vacancy faced by a firm of type $j \in \{f, i\}$ in sector $k \in \{C, S\}$, then expanding from ℓ to ℓ' requires posting $\upsilon = \frac{\ell' - \ell}{\mu_{kj}^{\upsilon}}$ vacancies.[24] The cost of expanding from ℓ to ℓ' workers for a firm of type j in sector k is therefore given by:

$$H_{kj}(\ell, \ell') = (\mu_{kj}^{\upsilon})^{-\gamma_{k1}}\left(\frac{h_k}{\gamma_{k1}}\right)\left(\frac{\ell' - \ell}{\ell^{\gamma_{k2}}}\right)^{\gamma_{k1}}. \tag{10}$$

[24]Note that the probability of filling a vacancy μ_{kj}^{υ} is an endogenous object that depends on the aggregate number of vacancies in each sector $k \in \{C, S\}$ and firm type $j \in \{f, i\}$ and on the mass of unemployed workers.

The value of γ_{k2} controls the extent to which firm-level growth rates in employment decline with size, a fact that we discussed in section 2 (Fact 5). The parameter γ_{k1} governs the convexity of the hiring function. Allowing for convexity is important for the model to be able to generate wage dispersion across firms, as emphasized in Fact 4. To build intuition for this fact, we momentarily abstract from dynamic considerations. In this case, the wage determination process that we discuss in section 3.5 implies that wages are proportional to value added per worker, which is—by virtue of our assumptions—proportional to marginal value added. Firms set marginal value added equal to the marginal cost of an additional worker. With linear hiring costs, the marginal cost is constant and equal across firms, so that wages will also be equalized across firms. In contrast, with convex hiring costs, the marginal cost of an additional worker is increasing in the growth of employment, so that expanding firms tend to pay higher wages.

Firing costs are entirely driven by regulations and affect formal firms only. They take the linear form:

$$F\left(\ell, \ell'\right) = \kappa\left(\ell - \ell'\right), \tag{11}$$

where $\kappa > 0$ is the parameter governing the firing cost function. Consistent with the Brazilian labor market regulations, we assume that firing costs are equal across the C and S sectors. In our model, firing costs are collected by the government and are rebated back to consumers, while hiring costs are incurred in terms of the S sector composite good.

Profit and Value Functions

Formal firms are subject to payroll and value-added taxes, firing costs and the minimum wage regulation. The profit function of a formal firm in sector $k \in \{C, S\}$ is thus given by:

$$\pi_{kf}\left(z, \ell, \ell'\right) = \left(1 - \tau_y\right) VA_k\left(z, \ell'\right) - C_{kf}\left(z, \ell, \ell'\right) - \bar{c}_k, \tag{12}$$

where $\bar{c}_k$ denotes a per-period, fixed cost of operation, which we define in units of the S sector composite good; τ_y is a value added tax, collected by the government and rebated lump-sum to consumers. Due to hiring and firing costs, the total cost function for a formal firm adjusting from ℓ to ℓ' workers is given by the following expression:

$$C_{kf}\left(z, \ell, \ell'\right) = \begin{cases} \left(1+\tau_w\right)\max\left\{w_{kf}\left(z,\ell'\right), \underline{w}\right\}\ell' + H_{kf}\left(\ell,\ell'\right) & \text{if } \ell' > \ell \\ \left(1+\tau_w\right)\max\left\{w_{kf}\left(z,\ell'\right), \underline{w}\right\}\ell' + \kappa\left(\ell-\ell'\right) & \text{if } \ell' \leq \ell, \end{cases} \tag{13}$$

where the wage schedule $w_{kf}\left(z, \ell'\right)$ is the result of a bargaining problem between the firm and its workers that is detailed in section 3.5, $\underline{w}$ denotes the minimum wage and τ_w is the payroll tax, which is also assumed to be collected by the government and rebated lump-sum to consumers.

Since formal firms have to choose whether to stay or leave their industry, their value function is given by:

$$V_{kf}\left(z, \ell\right) = \left(1 - \alpha_k\right)\max\left\{0, \max_{\ell'}\left\{\pi_{kf}\left(z, \ell, \ell'\right) + \frac{1}{1+r}E_{z'|z}V_{kf}\left(z', \ell'\right)\right\}\right\}, \tag{14}$$

where α_k denotes the exogenous destruction probability that firms face every period for $k = C, S$. The solution of (14) leads to the employment policy function $\ell' = L_{kf}(z, \ell)$ and to the vacancy posting policy function $v_{kf}(z,\ell) = \frac{L_{kf}(z,\ell)-\ell}{\mu^v_{kf}} \times \mathcal{I}\left[L_{kf}(z,\ell) > \ell\right]$ (and to other policies such as the decisions to exit or stay active).

While informal firms do not incur any of the regulatory costs (taxes, minimum wages, firing costs), they do face a reduced-form expected cost of informality, which includes the probability of detection by the government and subsequent fines. It also includes a range of opportunity costs associated with informality such as scarce access to formal financial markets (e.g., credit lines), hampering the firms' ability to grow (Catão et al., 2009). As firms grow, they become more visible to the government and therefore face a higher probability of being inspected, which can entail costs in the form of fines or bribes or can lead to the firm's operations being shut down. Therefore, motivated by Fact 2, we allow the expected cost of informality as a fraction of revenues, p_{ki}, to depend on the firm's size ℓ'. Thus, the profit function of an informal firm is given by:

$$\pi_{ki}\left(z,\ell,\ell'\right) = VA_k\left(z,\ell'\right) - K^{inf}\left(z,\ell'\right) - C_{ki}\left(z,\ell,\ell'\right) - \bar{c}_k, \tag{15}$$

where $K^{inf}\left(z,\ell'\right) \equiv p_{ki}\left(\ell'\right) R_k\left(z,\ell'\right)$ are the expected costs associated with informality, which we assume proportional to gross revenues. We work with the following simple specification:

$$p_{ki}\left(\ell'\right) = \widetilde{a}_k \exp\left\{\widetilde{b}_k\left(\ell'-1\right)\right\}. \tag{16}$$

The costs of informality incurred by firms are assumed to be rebated lump-sum to consumers.

Since informal firms are not subject to firing costs or other regulations, their cost function is given by:

$$C_{ki}\left(z,\ell,\ell'\right) = \begin{cases} w_{ki}\left(z,\ell'\right)\ell' + H_{ki}\left(\ell,\ell'\right) & \text{if } \ell' > \ell \\ w_{ki}\left(z,\ell'\right)\ell' & \text{if } \ell' \leq \ell, \end{cases} \tag{17}$$

where $w_{ki}\left(z,\ell'\right)$ denotes the wage paid by an informal firm with productivity z and size ℓ'. The value functions of informal firms are similar to those of formal ones, except that they have the additional option of formalizing their businesses:

$$V_{ki}\left(z,\ell\right) = \left(1-\alpha_k\right)\max\left\{\begin{array}{c} 0, \max_{\ell'}\left\{\pi_{ki}\left(z,\ell,\ell'\right) + \frac{1}{1+r}E_{z'|z}V_{ki}\left(z',\ell'\right)\right\}, \\ \max_{\ell'}\left\{\pi_{kf}\left(z,\ell,\ell'\right) + \frac{1}{1+r}E_{z'|z}V_{kf}\left(z',\ell'\right)\right\} \end{array}\right\}. \tag{18}$$

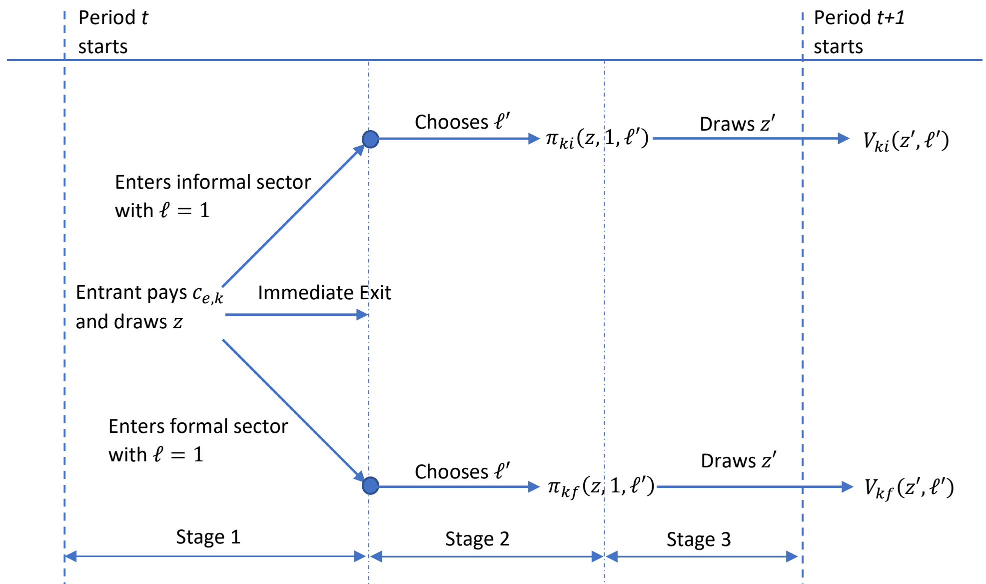
The solution of (18) leads to the employment policy function $\ell' = L_{ki}(z,\ell)$ and to the vacancy posting policy function $v_{ki}(z,\ell) = \frac{L_{ki}(z,\ell)-\ell}{\mu^v_{ki}} \times \mathcal{I}\left[L_{ki}(z,\ell) > \ell\right]$ (and to other policies such as the decisions to exit, formalize their status or remain informal).[25]

Entry

Firm entry is illustrated in Figure 2. In each period there is a mass M_k of entrants into the C and S sectors. In the first stage within the period, entrants observe their productivity z—drawn from the ergodic distribution g^e_k implied by (6)—after incurring a sunk cost $c_{e,k}$ of entry into sector

[25] This setup, where informal firms can transition to formal status but formal firms are not allowed to shift to informality, leads to overlapping distributions of productivity z within each sector $k \in \{C, S\}$ in equilibrium. This is an important feature of the data on formal and informal firms for many countries, as emphasized by Meghir et al. (2015). It is worth noting that once a firm becomes formal, it persists in operating formally as long as it remains profitable, even if its productivity drops below the threshold for entry into the formal sector.

Figure 2: Diagram of Entry Behavior



k. Based on this productivity draw, the entering firm chooses to be formal or informal or to exit immediately. Formal and informal entrants start their first period with workforce $\ell = 1$, whose recruitment cost is subsumed in $c_{e,k}$. Following entry, in stage 2, the firm decides to adjust its labor force to ℓ' just before the production stage. It then behaves as an incumbent, drawing productivity z' for the next period right after production (stage 3). The value functions for entrants in either sector are given by:

$$V_{kj}^{e}(z) = \max_{\ell'} \left\{ \pi_{kj}\left(z, 1, \ell'\right) + \frac{1}{1+r} E_{z'|z} V_{kj}\left(z', \ell'\right) \right\}, \tag{19}$$

where $j = i, f$. The value at entry together with the entry conditions is defined by

$$V_{k}^{e} = E_{z} \max \left\{ V_{ki}^{e}(z), V_{kf}^{e}(z), 0 \right\}, \tag{20}$$

whose solution leads to the entry policy functions. Assuming there are positive masses of entrants in each sector, free entry dictates that:

$$V_{k}^{e} = c_{e,k}. \tag{21}$$

3.4 Labor Market Frictions

Motivated by Facts 1 and 4 documented in section 2, both formal and informal labor markets are characterized by search and matching frictions. These frictions prevent unemployed workers from immediately finding open vacancies and contribute to hiring costs, as highlighted in equation (10). We assume random search, and therefore all unemployed workers form a pool of individuals who randomly meet with formal or informal firms in one of the sectors $k = C, S$. Thus, formal and informal firms operating in the C and S sectors compete for workers in the labor market. Given the total number of vacancies posted in each sector and type of firm $(\upsilon_{Cf}, \upsilon_{Ci}, \upsilon_{Sf}, \upsilon_{Si})$, and the mass of unemployed workers searching for jobs, L_u, the total number of matches formed is given by:

$$m\left(\upsilon_{Cf},\upsilon_{Ci},\upsilon_{Sf},\upsilon_{Si},L_u\right)=\phi\widetilde{\upsilon}^{\xi}L_u^{1-\xi}, \tag{22}$$

where $\widetilde{\upsilon}=\upsilon_{Cf}+\upsilon_{Ci}+\upsilon_{Sf}+\upsilon_{Si}$ aggregates vacancies across sectors and types of firms, $\phi>0$, and $0<\xi<1$. Matches are split across sectors and firm types in proportion to the number of vacancies posted, so that $m_{kj}=\frac{\upsilon_{kj}}{\widetilde{\upsilon}}\times m$ matches are formed with firms of type j in sector k. Thus, the probability of filling a vacancy ($\mu_{kj}^{\upsilon}=\frac{m_{kj}}{\upsilon_{kj}}$) is independent of sector and firm type. We denote it by μ^{υ}, and it is given by:

$$\mu^{\upsilon}=\phi\left(\frac{L_u}{\widetilde{\upsilon}}\right)^{1-\xi}. \tag{23}$$

This expression highlights that formal firms directly compete with informal ones in the labor market. Finally, unemployed workers face job-finding probabilities in each sector and firm type given by[26]:

$$\mu_{kj}^{e}\equiv\frac{m_{kj}}{L_u}=\frac{\upsilon_{kj}}{\widetilde{\upsilon}}\left(\frac{\phi}{\left(\mu^{\upsilon}\right)^{\xi}}\right)^{\frac{1}{1-\xi}}. \tag{24}$$

3.5 Wages

Wage setting takes place after hiring and fixed costs have been sunk and matching has taken place. We assume that a union engages in collective bargaining with the employer on behalf of the workers over the surplus of the match, determining a wage $w_{kj}\left(z,\ell'\right)$. The latter depends on the firm's size and productivity.

The surpluses of a formal firm in sector k (S_{kf}^{e}) and the union that it faces (S_{kf}^{u}) are each defined as:

$$S_{kf}^{e}\left(z,\ell'\right)=\left(1-\tau_y\right)VA_k\left(z,\ell'\right)-\left(1+\tau_w\right)w_{kf}\left(z,\ell'\right)\ell'+\frac{1}{1+r}E_{z'|z}V_{kf}\left(z',\ell'\right), \tag{25}$$

$$S_{kf}^{u}\left(z,\ell'\right)=\left[w_{kf}\left(z,\ell'\right)+\frac{1}{1+r}J_{kf}^{e}\left(z,\ell'\right)-\left(b_0\times P_Y+b^u+\frac{1}{1+r}J^u\right)\right]\ell', \tag{26}$$

where b_0 denotes the utility flow from being unemployed; P_Y is the price of one unit of the final good; b^u denotes the flow of unemployment insurance benefits, which are only received by formal workers; J^u is the expected present value of search; and $J_{kf}^{e}\left(z,\ell'\right)$ is the expected present value of a job in a formal firm in sector k with current productivity z and workforce ℓ'—see Online Appendix I for its derivation.[27]

Let β be the parameter that drives workers' bargaining power. If the joint surplus of the firm and workers is positive, the outcome of bargaining is given by:

$$S_{kf}^{u}\left(z,\ell'\right)=\beta\left(S_{kf}^{e}\left(z,\ell'\right)+S_{kf}^{u}\left(z,\ell'\right)\right). \tag{27}$$

Importantly, the overall surplus depends on the wage because of payroll taxes: in other words, the value of the surplus depends on how it is shared. This leads to a wage structure for formal workers

[26] μ_{kj}^{e} should be interpreted as the transition rate from unemployment to sector k and firm type j.

[27] Two observations about equations (25) and (26): (a) we assume that if all workers leave, the firm exits, and that hiring costs and fixed operating costs are already sunk at the bargaining process; and (b) note that b_0 is multiplied by P_Y in (26), converting it into a nominal value.

defined by:

$$(1+\beta\tau_w)\, w^u_{kf}\left(z,\ell'\right) = (1-\beta)\left(b_0 \times P_Y + b^u + \frac{1}{1+r}\left(J^u - J^e_{kf}\left(z,\ell'\right)\right)\right) + \beta\left((1-\tau_y)\frac{VA_k\left(z,\ell'\right)}{\ell'} + \frac{1}{1+r}E_{z'|z}\frac{V_{kf}\left(z',\ell'\right)}{\ell'}\right). \tag{28}$$

Thus, the bargained wage is proportional to a convex combination between the firm's value per worker and the worker's outside option net of the continuation value of the job to the worker, $J^e_{kf}(z,\ell')$.[28] If $w^u_{kf}(z,\ell')$ leads to a negative union surplus then we set the wage equal to its reservation value, that is, the wage $w^{res}_{kf}(z,\ell')$ that solves $S^u_{kf}(z,\ell') = 0$. Therefore the outcome of the Nash bargaining process leads to wage schedule:

$$w_{kf}\left(z,\ell'\right) = \max\left\{w^u_{kf}\left(z,\ell'\right), w^{res}_{kf}\left(z,\ell'\right)\right\}. \tag{29}$$

As highlighted in equation (13), formal firms cannot pay below the minimum wage, so that wages effectively paid are given by $\max\{w_{kf}(z,\ell'), \underline{w}\}$.

Wages in the informal sector are determined in a similar way. However, in the informal sector unemployment insurance benefits are not offered, taxes are not paid and firms face an expected cost of informality. Thus, the wage that informal firms pay their workers is given by:

$$w_{ki}\left(z,\ell'\right) = \max\left\{w^u_{ki}\left(z,\ell'\right), w^{res}_{ki}\left(z,\ell'\right)\right\}, \tag{30}$$

where

$$w^u_{ki}\left(z,\ell'\right) = (1-\beta)\left(b_0 \times P_Y + \frac{1}{1+r}\left(J^u - J^e_{ki}\left(z,\ell'\right)\right)\right) + \beta\left(\left(1 - \frac{\sigma_k}{\sigma_k - (1-\delta_k)(\sigma_k - 1)}\right)p_{ki}\left(\ell'\right)\frac{VA_k\left(z,\ell'\right)}{\ell'} + \frac{1}{1+r}E_{z'|z}\frac{V_{ki}\left(z',\ell'\right)}{\ell'}\right), \tag{31}$$

and $J^e_{ki}(z,\ell')$ is the analogous to $J^e_{kf}(z,\ell')$ in the informal sector, see Online Appendix I for details. Finally, the reservation wage, $w^{res}_{ki}(z,\ell')$, solves $S^u_{ki}(z,\ell') = 0$, where S^u_{ki} is the surplus of the union in the informal sector. This places a lower bound on wages in the informal sector.

3.6 Open Economy

We now extend the model to the open economy case. We assume that the home country is small relative to the rest of the world and therefore that foreign conditions do not react to the home country's policies. In the following analysis, we drop the formal/informal qualifier to simplify notation, as we assume throughout that informal firms cannot export.[29]

Price Indices and Aggregate Variables

Let $N_{F,C}$ denote the measure of foreign varieties available to domestic consumers. Given the small open economy assumption, this variable is assumed to be fixed. Without loss of generality, we normalize the price index of free-on-board imports to be one, since foreign prices are exogenous

[28] The factor of proportionality $1+\beta\tau_w$ highlights that workers also bear the cost of payroll taxes.

[29] This assumption comes from the fact that unregistered firms cannot undertake the necessary legal and bureaucratic procedures to export.

to our model. Thus, the price index of imports in domestic currency becomes $P_{F,C} = \epsilon \tau_a \tau_c$, where ϵ is the exchange rate, $\tau_a - 1 > 0$ is the ad valorem tariff and $\tau_c > 1$ the iceberg trade cost. The price index of domestically produced differentiated varieties $n \in (N_{F,C}, N_C]$ is given by $P_{H,C} = \left(\int_{N_{F,C}}^{N_C} p(n)^{1-\sigma_C} dn \right)^{\frac{1}{1-\sigma_C}}$, and the price index for the composite C sector good is given by $P_C = \left[P_{H,C}^{1-\sigma_C} + P_{F,C}^{1-\sigma_C} \right]^{\frac{1}{1-\sigma_C}}$.

The domestic demand for varieties produced domestically is given by $Q_{H,C}(n) = D_{H,C} p(n)^{-\sigma_C}$, for $n \in (N_{F,C}, N_C]$ with $D_{H,C} \equiv \left(\frac{X_C}{P_C^{1-\sigma_C}} \right)$; and the domestic demand for foreign-produced varieties is given by $Q_{H,C}(n) = D_{H,C} (\epsilon \tau_a \tau_c p^*(n))^{-\sigma_C}$, for $n \in [0, N_{F,C}]$—where $p^*(n)$ is the price of foreign variety n in foreign currency. Finally, foreign demand for domestically produced differentiated varieties is given by $Q_{F,C}(n) = D_F^* (p_x^*(n))^{-\sigma_C}$, for $n \in (N_{F,C}, N_C]$, where $p_x^*(n)$ is the price of domestic variety n in the foreign country, denominated in foreign currency, and D_F^* is an exogenous foreign demand shifter. If $\mathcal{I}_C^x(n)$ denotes an indicator function that equals one if variety n is exported, we have that the values of aggregate imports (before import tariffs) and exports are given by the following expressions:

$$Imports = \frac{D_{H,C} (\epsilon \tau_a \tau_c)^{1-\sigma_C}}{\tau_a} \quad \text{and} \quad Exports = D_F^* \epsilon \int_{N_{F,C}}^{N_C} \mathcal{I}_C^x(n) \, p_x^*(n)^{1-\sigma_C} dn. \tag{32}$$

Exporters

Given the expression of foreign demand for home differentiated variety n just described, $Q_{F,C}(n)$, revenues from exports are given by $\epsilon D_F^{*\frac{1}{\sigma_C}} (q_x/\tau_c)^{\frac{\sigma_C - 1}{\sigma_C}}$, where q_x is the total quantity exported. If a firm exports, it must decide which fraction η of its output to sell abroad. *Conditional on the firm's being an exporter*, total gross revenue for producing a total of q units and exporting a fraction η of this production is given by: $\widetilde{R}_C^x(q, \eta) = \exp(d_{H,C} + d_F(\eta)) \, q^{\frac{\sigma_C - 1}{\sigma_C}}$, where $d_{H,C} = \ln \left(D_{H,C}^{\frac{1}{\sigma_C}} \right)$ and $d_F(\eta) = \ln \left((1-\eta)^{\frac{\sigma_C - 1}{\sigma_C}} + \epsilon \left(\frac{D_F^*}{D_{H,C}} \right)^{\frac{1}{\sigma_C}} \left(\frac{\eta}{\tau_c} \right)^{\frac{\sigma_C - 1}{\sigma_C}} \right)$. It is easy to verify that all exporters optimally decide to export the same fraction, η^o, of their production. In what follows, we will refer to $d_F(\eta^o)$ as simply d_F.[30] Empirically, d_F is directly related to the fraction of gross revenues coming from exports among exporters, which is given by:

$$\frac{R_C^x(z, \ell') - R_C^{dom,x}(z, \ell')}{R_C^x(z, \ell')} = 1 - \exp(-\sigma_C \times d_F), \tag{33}$$

where $R_C^x(z, \ell') \equiv \widetilde{R}_C^x(q_C(z, \ell', \iota_C(z, \ell')), \eta^o)$ is the total gross revenue of an exporter with state (z, ℓ') and $R_C^{dom,x}(z, \ell')$ is the portion of an exporter's gross revenues coming from domestic sales.

The value added function for exporters takes the form:

$$VA_C^x(z, \ell') = \exp\left(\frac{\sigma_C}{\sigma_C - 1} \Lambda_C \times d_F \right) VA_C^d(z, \ell'), \tag{34}$$

[30] When we substitute η^o into $d_F(\eta)$, we obtain $d_F \equiv d_F(\eta^o) = \log \left(\left(1 + \frac{D_F^*}{D_{H,C}} \epsilon^{\sigma_C} \tau_c^{1-\sigma_C} \right)^{\frac{1}{\sigma_C}} \right)$.

where $\Lambda_C \equiv \frac{\sigma_C - 1}{\sigma_C - (1-\delta_C)(\sigma_C - 1)}$ and VA_C^d is the value-added function for nonexporters. It follows that the export decision is given by:

$$\mathcal{I}_C^x(z, \ell') = \begin{cases} 1 & \text{if } VA_C^x(z, \ell') - f_x > VA_C^d(z, \ell') \\ 0 & \text{otherwise,} \end{cases} \tag{35}$$

where $f_x > 0$ denotes the fixed cost of exporting, which is denominated in terms of the S sector composite good.

3.7 Equilibrium

We now summarize the equilibrium conditions below. Online Appendices A to I give further details.

1. Firms act optimally, make entry and exit decisions, and post vacancies according to equations (14), (18), (19) and (20). If entry is positive in sector k, the free-entry condition (21) holds with equality.

2. Wage schedules solve the bargaining problem between workers and the firm, as in equations (29) and (30).

3. Labor markets clear, that is, the sum of employment levels across sectors and firms types and the number of unemployed workers must be equal to the total labor force $\overline{L}$.

4. Product markets clear; the sum of expenditures on each sector-specific composite good, including consumption, intermediate goods, costs of entry, hiring, and export costs must add up to revenues in the sector (where relevant, this includes payment of tariffs).

5. Trade is balanced: $Imports = Exports$.

6. The government runs a balanced budget. All government revenues stemming from tax collection (including tariff revenues and fines on informal firms) and firing costs must exceed expenditures with unemployment benefits to all unemployed workers dismissed from formal employment. The budget surplus is directly rebated to consumers.

7. Aggregate income I is given by the sum of all wages and profits, plus the revenue from tariffs $(\tau_a - 1) \times Imports$, minus the total costs incurred by entering and hiring firms, fixed costs of operation and fixed costs of exporting.

8. We focus on steady-state equilibria, where the distributions of states (z, ℓ), by sector and firm type, and all aggregate variables remain constant. In particular, no sector can be expanding or contracting, which implies that: (i) the flow of workers out of unemployment and into the formal/informal and C/S sectors must be the same as the flow out of these sectors and into unemployment; (ii) the mass of firms entering the informal sector must be equal to the mass of informal firms that decide to exit or to formalize their businesses in either sector $k \in \{C, S\}$; and (iii) the sum of the number of firms entering the formal sector and those formalizing their businesses must equal the mass of formal firms that decide to exit either sector $k \in \{C, S\}$.

3.8 Trade and Informality: Discussion of Mechanisms

Our model includes several channels through which trade can impact informality, pushing the response to changes in trade openness in different directions. The first set of mechanisms linking trade to informality is what we call "Melitz-type" mechanisms, which operate through various channels. In the subsequent discussion, we abuse language to some extent, as firms' policy functions depend on both their productivity z and workforce size ℓ. However, for simplicity, we describe our mechanisms by focusing on selection based solely on productivity z.

Declines in import tariffs ($\tau_a - 1$) and iceberg trade costs (τ_c) lead to an increase in the size of the foreign market perceived by C sector firms (d_F). Consequently, initially higher-paying exporters experience greater profitability, prompting them to further raise wages and expand their workforce. Moreover, this shift encourages new firms to become high-paying exporters, further contributing to higher aggregate wages in sector C. These increased wages, in turn, elevate the value of search (J^u), exerting upward pressure on wages across all firms and sectors in the economy, as in Melitz (2003). These higher wages tend to push unproductive informal firms out of the market, raising the productivity threshold for operation in the informal sector. All else equal, this movement tends to lead to a decline in informality. Similarly, low-productivity formal firms find incentives to exit the market as their labor costs increase, and the productivity threshold for new firm entry into the formal sector is raised. Ceteris paribus, this tends to lead to an increase in informality. Taken together, these impacts lead to ambiguous effects of trade openness on aggregate informality.

Following a decline in the iceberg trade cost or tariff, exporters not only experience increased profitability but also see a higher likelihood that any firm in sector C can eventually become an exporter. Firms, being forward looking, observe these changes, leading to an increased present value of formality compared to informality in sector C. This, in turn, encourages high-productivity informal firms to transition to the formal sector. As a result, the productivity threshold for operating in the formal sector tends to decrease. When all other factors held constant, this force tends to reduce informality.

In addition, reductions in tariffs and trade costs affect the price of the intermediate composites ι_k, effectively altering optimal intermediate input usage by firms and, therefore, their workers' productivity. This effect on intermediate inputs amplifies the impact of tariff or trade cost reductions and is similar in spirit to the "magnification" effect highlighted in the work of Fieler et al. (2018) and Coşar et al. (2016) among others. A change in trade policy or other trade costs hence directly affect the decisions of firms, including their decisions to enter, exit, and produce formally or informally. Specifically, a reduction in trade barriers tends to make all firms more productive by improving access to cheaper intermediate goods. This encourages the most productive informal firms to grow and formalize. However, it can also lead to entry of lower-productivity informal firms, which have now become profitable. Furthermore, trade openness induces a reallocation of resources toward larger and more productive firms that export. This tends to further increase aggregate productivity and income, shifting aggregate demand up, generating incentives for productive informal firms to grow and formalize. The net effect of all these forces depends on the values of the parameters that we estimate.

The second mechanism linking openness to informality in our model is through its effect on unemployment. We show in section 2 that, in the Brazilian data, transitions from unemployment to informal employment are twice as large as transitions from unemployment to formal employment. Therefore, the channels in our model linking openness to unemployment have implications for the relative importance of the informal sector.

Accordingly, we now turn to the mechanisms linking trade to unemployment in our setup. All else equal, equation (34) shows that exporters' value added is magnified relative to nonexporters', since $\exp\left(\frac{\sigma_C}{\sigma_C-1}\Lambda_C \times d_F\right) > 1$. This implies that exporters' value added, and therefore their hiring and firing decisions, are more sensitive to productivity shocks z. Therefore, as in Coşar et al. (2016), reducing trade costs produces two opposing forces: (i) there is reallocation of workers toward larger and higher-productivity firms, which tend to be more stable and have lower worker turnover (as they face larger costs of growing the workforce); and (ii) due to the term $\exp\left(\frac{\sigma_C}{\sigma_C-1}\Lambda_C \times d_F\right)$, both new and old exporters become more sensitive to idiosyncratic shocks, which tends to increase turnover. We follow Coşar et al. (2016) and refer to these two forces as the distribution effect and sensitivity effect, respectively. The bottom line is that increasing openness can increase or decrease labor turnover depending on which of the two effects dominates. Labor turnover is tightly linked to unemployment, as workers who are fired must spend at least one period in unemployment. As a result, increasing openness can lead to increases or decreases in unemployment. Moreover, as explained in the previous paragraph, these unemployment changes generate corresponding changes in the share of informality.

3.9 Discussion

The key premise of our model is that the Brazilian government imposes burdensome and distortive taxes and regulations on the country's firms. These taxes and regulations distort the decisions of compliant formal firms. However, because of imperfect enforcement, firms are not equally exposed to these distortions: informal firms bypass taxes and labor market regulations, but face size-dependent costs of informality.

This environment implies that low-productivity and small-scale informal firms tend to overproduce relative to what is socially optimal while high-productivity formal firms tend to underproduce. In addition, according to Fact 4, informal firms enjoy a cost advantage, sustaining lower wages, thereby further amplifying misallocation.

The model features other distortions such as (a) monopolistic competition; (b) markups that differ across the C and S sectors; (c) roundabout production amplifying the aforementioned distortions; (d) search frictions, unemployment and Nash bargaining over wages. These distortions are not at the center of the analysis as they are not directly related to the phenomenon of informality, but we nevertheless include them in the analysis to have a realistic framework for the quantitative exercises.

In particular, labor market frictions are important to generate the unemployment and informality impacts documented in previous work (e.g., Dix-Carneiro and Kovak, 2019; Ponczek and Ulyssea, 2022). Intersectoral linkages are important to the model as they allow us to link trade shocks to labor market conditions in services, a large sector with a high prevalence of informality. Finally, the monopolistic competition environment that we adopt is a standard framework for analyzing the impacts of trade liberalization with heterogeneous firms (e.g., Melitz, 2003).

4 Estimation

We now quantify the model outlined in section 3 in two steps. First, we fix a subset of parameters based on a combination of aggregate data, estimates from previous papers, and the statutory value of institutional parameters, such as value-added and payroll taxes. Next, we estimate the remaining model parameters using an Indirect Inference procedure, which allows us to combine information from the different data sources discussed in section 2. Section 4.1 describes how we determine the parameters fixed throughout the estimation procedure. Section 4.2 discusses the estimation procedure, and section 4.3 addresses identification. Finally, section 4.4 presents the estimation results and discusses model fit.

4.1 Fixed Parameters

We describe the relevant labor market regulations in Brazil in Online Appendix N. As has been extensively discussed in previous work (Heckman and Pagés, 2000; Gonzaga et al., 2003; Botero et al., 2004), not only these labor market regulations impose significant costs on employers, but they also involve a certain level of complexity. Therefore, we need to make simplifying assumptions to map them to our model's payroll tax (τ_w), value-added tax (τ_y), firing cost (κ), and unemployment insurance benefit (b_u). We follow Ulyssea (2018) and set τ_w so that it reflects the main taxes that are proportional to firms' wage bill, namely, the employers' social security contribution (20%), payroll tax (9%), and contributions to the severance indemnity fund, the *Fundo de Garantia do Tempo de Serviço* (FGTS; 8.5%). We combine two VAT-like taxes to calculate τ_y: *Imposto sobre Produtos Industrializados* (IPI; 20%) and the Social Integration Program/Contribution for the Financing of Social Security PIS/COFINS (9.25%).[31]

Firing costs are set based on Heckman and Pagés (2000), who compute the cost of dismissing workers for several Latin American countries, including Brazil. Their calculation takes into account the specific features of dismissal costs that we review in Online Appendix N and shows that, on average, employers must pay approximately 1.9 months of wages to dismiss a worker. Considering that the *annual* average formal-sector wage in the 2003 RAIS data amounts to R$10,565, we obtain a firing cost κ of R$1,690 per worker. The minimum wage corresponds to the annualized value of the national monthly minimum wage in 2003: $\underline{w} = \text{R\$}2,880$. To compute unemployment insurance benefits, we assume that all workers receive the maximum duration of potential benefits (that is,

[31] We exclude state-level value-added taxes because these vary greatly across states and there is a complex system of tax substitution across the production chain, which would be impossible to properly capture.

5 monthly payments). This figure is very close to both the mean and median of the duration of actually received benefits (Gerard and Gonzaga, 2021). Finally, we use the average monthly value of benefits paid in 2003, as reported by the Ministry of Labor: 1.37 times the minimum wage. This amounts to unemployment insurance benefits $b_u = 1.37 \times 5$ times the *monthly* minimum wage = R$1,644.

The share of final consumption expenditure on the sector C composite good, ζ, and the share of sector k's intermediate input payments to the sector C composite good, λ_k, are extracted from the 2000 and 2005 Brazilian National Accounts. We obtain $\zeta = 0.283$, $\lambda_C = 0.65$, and $\lambda_S = 0.29$, suggesting that tariffs and iceberg trade costs can have a substantial effect on labor productivity in both sectors. The iceberg trade cost $\tau_c = 2.4$ is obtained based on Head and Ries index (Head and Mayer, 2014) applied to 2003 trade flows between Brazil and the rest of the world, and the average import tariff $\tau_a - 1 = 0.12$ comes from 2003 data in UNCTAD TRAINS.[32]

There are two sets of model parameters that are hard to identify given our data: the bargaining weight of workers β and the matching function parameters (ϕ and ξ)—see Flinn (2006) for a discussion. We follow Mortensen and Pissarides (1999) and Ljungqvist and Sargent (2017) and impose symmetric bargaining, i.e. $\beta = 0.5$. We assume a matching function elasticity of $\xi = 0.5$, which is in the middle of the range surveyed by Petrongolo and Pissarides (2001), and $\phi = 0.576$, a choice we discuss in section 4.3. Table 6 summarizes the parameter values fixed throughout the estimation and their sources.

Table 6: Fixed Parameters

Parameter	Description	Source	Value
τ_c	Iceberg Trade Cost	Head and Ries Index + 2003 WIOD	2.4
ζ	Share of final expend. on C	IBGE National Accounts (2000/2005)	0.283
λ_C	Prod. Function (Equation (4))	IBGE National Accounts (2000/2005)	0.645
λ_S	Prod. Function (Equation (4))	IBGE National Accounts (2000/2005)	0.291
r	Interest rate	Ulyssea (2010)	0.08
τ_y	Value Added Tax	Ulyssea (2018)	0.293
τ_w	Payroll Tax	Ulyssea (2018)	0.375
$\tau_a - 1$	Import Tariff	UNCTAD TRAINS	0.12
κ	Firing Costs (in R$)	Heckman and Pagés (2000)	1,956.7
$\underline{w}$	Min. Wage (in R$)	Annualized 2003 value	2,880
b_u	Unemployment Benefit	$1.37 \times 5 = 6.85$ monthly Min. Wage	1,644
ξ	Matching Function	Petrongolo and Pissarides (2001)	0.5
ϕ	Matching Function	Own Calculations, see section 4.3	0.576
β	Workers' Bargaining Weight	Mortensen and Pissarides (1999)	0.5

Parameters based on the IBGE National Accounts employ simple averages between 2000 and 2005. WIOD stands for "World Input-Output Database."

4.2 Estimation Procedure

We take the parameters described in Table 6 as given and estimate the remaining parameters using an Indirect Inference procedure. In this step, we estimate 27 parameters using 74 data moments

[32]See Online Appendix K for details.

and auxiliary model coefficients, ensuring that all equilibrium conditions listed in section 3.7 are met throughout the procedure. The estimation algorithm is described in detail in Supplementary Material I, but we highlight here a few important features of the procedure.

First, rewrite the value-added functions (8) and (34) as:

$$VA_k(z,\ell) = \Theta_k \Psi_k \left(\exp\left(\mathcal{I}_k^x(z,\ell)\, d_F\right)\right)^{\frac{\sigma_k}{\sigma_k-1}\Lambda_k} \left(z\ell^{\delta_k}\right)^{\Lambda_k}, \tag{36}$$

where $\Psi_k \equiv (P_k^m)^{-(1-\delta_k)\Lambda_k} (\exp(d_{H,k}))^{\frac{\sigma_k}{\sigma_k-1}\Lambda_k}$ for $k = C, S$.[33] In addition, define $\vartheta_{J_u} \equiv b_0 \times P_Y + \frac{1}{1+r} J^u$ as the expected discounted present value of unemployment. The estimation procedure treats the endogenous equilibrium objects Ψ_C, Ψ_S, and ϑ_{J_u} as "parameters" to be estimated.[34] Given a guess of these objects and of the remaining structural parameters, we are able to solve, for the mass of entrants, the mass of active firms, the firm-level policy functions, the steady-state distribution of states, and the unemployment rate. This solution is achieved by ensuring that the guesses and structural parameters are consistent with the equilibrium conditions.

To be precise, we minimize the loss function $\mathcal{L}$ given by: $\mathcal{L}(\Omega) = \sum_i W_i \left|m_i^{Model}(\Omega) - m_i^{Data}\right|$, where Ω denotes the set of parameters to be estimated, m_i^{Data} denotes the values of moments or auxiliay model coefficients measured in the data, $m_i^{Model}(\Omega)$ denotes the values of moments or auxiliary model coefficients generated by the model when the set of parameters is given by Ω, and W_i weights the importance of moment i in the loss function.[35] Ω includes the endogenous objects Ψ_C, Ψ_S, and ϑ_{J_u}, but excludes the structural parameters b_0 and D_F^*, which are obtained after the minimization of $\mathcal{L}$ is complete. It also excludes parameters δ_k $(k = C, S)$, which are directly determined as functions of the elasticities of substitutions σ_k and the share of gross revenues devoted to intermediate goods payments using: $\delta_k = 1 - \frac{\sigma_k}{\sigma_k-1}\left(\frac{Total\ Expenditures\ with\ Intermediates_k}{Total\ Gross\ Revenues_k}\right)_{Data}$, where the term in parentheses is computed from the Brazilian National Accounts compiled by IBGE. Finally, Ω also omits the foreign demand shifter d_F. Given a guess of σ_C, d_F can be directly recovered by means of equation (33) and data on the average share of exporters' gross revenues that is actually exported (which are obtained from PIA and SECEX).

Once the minimization of $\mathcal{L}$ over Ω is concluded, the flow utility of unemployment is recovered with:

$$b_0 \times P_Y = \vartheta_{J_u} - \frac{1}{1+r} J^u,$$

[33] $\Theta_k \equiv \left(\frac{1}{(1-\delta_k)\Lambda_k}\right)\left(\frac{(1-\delta_k)(\sigma_k-1)}{\sigma_k}\right)^{\frac{\sigma_k}{\sigma_k-1}\Lambda_k}$ and $\Lambda_k \equiv \frac{\sigma_k-1}{\sigma_k-(1-\delta_k)(\sigma_k-1)}$ are sector-specific constants; P_k^m is the price of one unit of sector k's intermediate bundle; and $d_{H,k} \equiv \log\left(\left(\frac{X_k}{P_k^{1-\sigma_k}}\right)^{\frac{1}{\sigma_k}}\right)$ are domestic demand shifters. Finally, we impose $\mathcal{I}_S^x(z,\ell) = 0$—no firm in the service sector exports—to simplify the notation.

[34] In counterfactual simulations, these are solved out under the new environment. Estimation relies on the assumption that the data come from an equilibrium allocation.

[35] We selected the weights W_i based on practical considerations. In theory, if the model captures the true data-generating process, the choice of weights is asymptotically irrelevant and the estimator remains consistent. In practice, however, matching a large number of moments inevitably requires prioritizing some over others. We assign greater weight to moments deemed more important *a priori*—such as the key facts discussed in Section 4.4.2—and prioritize matching the unemployment rate over firm-level turnover (see Section 4.4.2). Despite these trade-offs, the model fits the targeted moments reasonably well (see Online Appendix M).

where J^u can be computed by means of equation (I.3) in the Online Appendix, and P_Y is the price of the final good (which is also a by-product of the estimation procedure). In turn, D_F^* is recovered as:

$$D_F^* = \frac{\left(\exp\left(\sigma_C \times d_F\right) - 1\right)\left(P_C^m\right)^{(1-\delta_C)(\sigma_C - 1)} \Psi_C^{\frac{\sigma_C - 1}{\Lambda_C}}}{\overline{\epsilon}^{\sigma_C} \tau_c^{1-\sigma_C}},$$

where $\overline{\epsilon}$ is the exchange rate value that balances trade and P_C^m is the price of one unit of sector C's intermediate bundle.

We emphasize that the only differences between formal and informal firms are that the former are subject to regulations and taxes, and the latter face an expected cost of informality $K^{inf}(z, \ell')$. Otherwise, they have access to the same production and hiring technologies, and are subject to the same exogenous exit rates. However, these parameter restrictions do not significantly affect the final fit of the model.

4.3 Identification

To understand which moments in the data identify the parameters we estimate, first rewrite the hiring functions as: $H_k(\ell, \ell') = \widetilde{h}_k \left(\frac{\ell' - \ell}{\ell^{\gamma_{k2}}}\right)^{\gamma_{k1}}$, where $\widetilde{h}_k = (\mu^{\upsilon})^{-\gamma_{k1}} \left(\frac{h_k}{\gamma_{k1}}\right)$, for $k = C, S$. Our estimation procedure treats $\widetilde{h}_k$ as a "parameter" to be estimated. This term is identified based on the average level of turnover rates at the firm level as well as the unemployment rate, given that the two are closely related.[36] As we discussed in section 3.3, the auxiliary (linear) model relating firm-level turnover rates to log employment and an export indicator gives information on the scale economies γ_{k2}. The auxiliary (linear) model relating log wages to log employment and an export indicator gives information on the convexity of hiring costs γ_{k1}, as it relates to wage dispersion across firms with different characteristics.

Note that $\widetilde{h}_k$ is a combination of a structural parameter h_k with an endogenous term μ^{υ}. Following the estimation of $\widetilde{h}_k$ and γ_{k1}, we recover h_k by imposing that $\mu^{\upsilon} = 0.5$ in the equilibrium that we estimate.[37] In turn, we set ϕ so that we perfectly match the yearly transition rate from unemployment to employment in the data.[38]

The model needs the fixed costs of production $\overline{c}_k$ to match how the probability of firm-level exit rates declines with size. Exogenous destruction rates α_k are needed to match the average exit rates. Matching the relationship between firm-level revenues and firm size gives information on

[36] In our model, worker separations are followed by a period of unemployment. This mechanically ties turnover rates to unemployment rates.

[37] Simply put, we can identify the level of hiring costs $\widetilde{h}_k$ based on data on firm-level worker turnover. However, we cannot separate h_k from μ_v. We impose $\mu_v = 0.5$ to be able to recover h_k. Other assumptions could have been made. Similarly to how we treat Ψ_C, Ψ_S, and ϑ_{J_u}, μ^{υ} is solved out in our subsequent counterfactual exercises.

[38] Equation (24) implies that transition rates from unemployment to employment (in any sector), in the model, are given by: $\sum_{k,j} \mu_{kj}^e = \left(\frac{\phi}{(\mu^{\upsilon})^{\xi}}\right)^{\frac{1}{1-\xi}}$. We choose ϕ so that our model perfectly matches the transition from unemployment to employment in the data, $Transition_{Data}^{U \to E}$, conditional on $\mu^{\upsilon} = 0.5$ and $\xi = 0.5$. That is, $\phi = \frac{(\mu^{\upsilon})^{\xi}}{\left(Transition_{Data}^{U \to E}\right)^{\xi - 1}} = 0.576$, as shown in Table 6.

σ_k.[39] We pin down the AR(1) process for productivity by targeting two dynamic moments: the serial correlation in firm-level employment, and the serial correlation in firm-level revenues. The cross-sectional dispersions in firm-level employment size and revenues are also informative about the variance of shocks σ_k^z. The share of C sector firms that export pins down the fixed cost of exporting f_x.

Finally, we identify the cost-of-informality function $\widetilde{a}_k \exp\left\{\widetilde{b}_k(\ell-1)\right\}$ by matching the firm-size distribution in the informal sector, the share of employment in the informal sector, and the fraction of informal firms conditional on employment size. All the moments and auxiliary models used in the estimation procedure (as well as the datasets we have used to compute each of them) are listed in Appendix M.

We conclude by highlighting that, apart from these heuristic arguments for identification, Online Appendix Q includes Figures Q.1 to Q.4, illustrating the local behavior of the loss function around each parameter estimate. In addition, Tables Q.1 to Q.6 in the same Online Appendix compute the elasticity of each moment targeted in the estimation with respect to each element of Ω. These exercises corroborate that each parameter maps to each of the aforementioned moments.

4.4 Estimation Results

4.4.1 Estimates

Table 7 presents our parameter estimates. Informality costs as a fraction of revenues, $p_{ki}(\ell)$, differ across sectors. In the C sector, the informality costs start at a relatively low value but rapidly increase with size. On the other hand, the informality penalty starts at a larger value in the S sector, but increases with size at a slower pace. The different convexities of the informality penalty function in the C and S sectors are intuitive, as one would expect large manufacturing firms to face increasing hurdles to remaining invisible to the government.

Our hiring function estimates display convexity ($\gamma_C^1 = 1.7$, $\gamma_S^1 = 5.8$) and scale economies ($\gamma_C^2 = 0.06$, $\gamma_S^2 = 0.13$) in both sectors. These results are in the same ballpark as the recent estimates from Coşar et al. (2016), who use data from Colombia. Similarly, our estimates of the elasticities of substitution in the C and S sectors, given by $\sigma_C = 5.0$, and $\sigma_S = 3.0$, respectively, are within the range in earlier papers (see, for example, Coşar et al., 2016; De Loecker and Warzynski, 2012; De Loecker et al., 2016; Broda and Weinstein, 2006). In particular, our estimates are consistent with those of Gervais and Jensen (2019) who find that services have elasticities of substitution approximately one-quarter smaller than those of manufacturing.

Finally, we estimate a negative value for b_0, implying a significant disutility flow of unemployment. To aid its interpretation, Table 7 reports that b_0 amounts to approximately 80% of real income per capita. This is equivalent to 2.6 times the real value of the 2003 minimum wage

[39] In a standard Melitz model with zero fixed costs of production, revenue per worker pins down the elasticity of substitution. Our model is significantly more complex, but the intuition that the relationship between revenue and optimal size depends on the elasticity of substitution remains. Rows 68 to 70 of Table Q.4 and Rows 71 and 72 of Table Q.6 in Online Appendix Q show that the relationship between log revenue and log employment size in either sector is sensitive to the sector-specific elasticity of substitution, as we argue here.

Table 7: Parameter Estimates

Parameter	Description	$k = C$	$k = S$
$\widetilde{a}_k$	Cost of Informality, Intercept	0.205	0.379
$\widetilde{b}_k$	Cost of Informality, Convexity	0.071	0.010
h_k	Hiring Cost, Level	674.6	1,473.2
γ_k^1	Hiring Cost, Convexity	1.665	5.791
γ_k^2	Hiring Cost, Scale Economies	0.060	0.131
σ_k	Elasticity of Substitution	5.023	2.955
ρ_k	Productivity AR(1) Process, Persistence Coeff.	0.977	0.963
σ_k^z	Productivity AR(1) Process, Std. Dev. of Shock	0.210	0.409
α_k	Exogenous Exit Probability	0.069	0.081
$\overline{c}_k$	Fixed Cost of Operation	177.817	202.040
δ_k	Labor Share in Production	0.256	0.516
c_k^e	Entry Cost	4,616.5	4,893.8
f_x	Fixed Cost of Exporting	54,988.5	
b_0	Utility Flow of Unemployment	-0.801	
$(D_F^*)^{\frac{1}{\sigma_C}}$	Foreign Demand Shifter	1,847.3	

Notes: To aid interpretation, b_0 is expressed as a fraction of real income per capita according to the model.

$(\underline{w}/P_Y)$. As a benchmark, also in the context of a search model, Meghir et al. (2015) estimate a negative value for $b_0 \times P_Y$ for men in the São Paulo metropolitan region equal to 4.3 times the 2008 minimum wage.[40] As the search literature has shown, a negative value of unemployment is necessary to generate the magnitudes of wage dispersion typically found in the data (see, for example, Hornstein et al., 2011).

4.4.2 Model Fit

Tables M.1 through M.7 in Online Appendix M evaluate how our model-generated moments and auxiliary model coefficients compare to those obtained in the data. Overall, our model with 27 structural parameters fits our 74 target data moments or auxiliary model coefficients well. Importantly, Facts 1 through 5 highlighted in section 2 are all well matched. Specifically, Table M.1 shows that employment shares and transition rates from unemployment are on target, so that our model comfortably replicates Fact 1. The sharp relationship between informal status and size (Fact 2) is also well reproduced by our model, as shown in the lower panel of Table M.7. Tables M.5 and M.7 show that average wages in the formal sector indeed exceed those in the informal sector, mirroring Fact 4. Fact 5, relating firm-level turnover to size and export status, is also well replicated as Table M.2 shows. Also noteworthy is the ability of the model to match how turnover rates relate to employment and export status conditional on both expansions and contractions. We do not directly target Fact 3, but we verify that the model implies that less productive firms tend to sort into informal status. Finally, Table M.5 shows that the model recreates the strong wage size and exporter premia found in the data, in both sectors. These are important moments to replicate, as they give us confidence in the wage inequality counterfactuals we conduct in section 5.4.

[40] Table 5 in their paper estimates a (monthly) flow value of unemployment of -1,308 for men in São Paulo. The average monthly minimum wage during the period that they consider was of R$ 300.

Meghir et al. (2015) emphasize an important feature of the data on formal and informal firms, namely that the distributions of worker productivities across these two types of firms overlap. There are two reasons why our model generates this overlap. First, and more importantly, we do not allow formal firms to switch to informal status. This generates hysteresis: formal firms (which are more productive than informal firms at the entry stage) keep their status even if their productivity significantly declines over time, falling below the entry cutoff for formal firms. Second, selection into formality/informality depends on a two-dimensional state (z, ℓ). Therefore, for a fixed value of z, we can have selection into informality for some levels of ℓ and into formality for other values of ℓ, also contributing to the overlap in the two productivity distributions. Online Appendix O shows the overlap between these distributions within manufaturing and services.

Having discussed the successes of the model, we now turn to the moments that are not as well matched, and discuss reasons for some of these mismatches. Although the model is able to replicate very well the dependence between firm-level turnover rates, firm-level employment, and export status, the *average* turnover rate is not as well replicated. There is a tension between matching the *level* of turnover rates and matching the unemployment rate.

Our model also tends to underestimate the dispersion of firm size in the formal sector for both sectors C and S (see Table M.3). We hypothesize two reasons for this discrepancy. First, the data source that we employ has a thicker-than-usual left tail of the firm-size distribution.[41] Second, the assumed normal distribution of productivity shocks naturally makes it harder to match the right tail of the size distribution. Although modeling firm-level dynamics according to equation (6) is standard in the literature (see Hopenhayn and Rogerson, 1993; Alessandria and Choi, 2014; Coşar et al., 2016, among many others), we would need a more flexible specification for the productivity process to better match the firm-size distribution in Brazil.[42] Relatedly, the dispersion in revenues in Table M.6 is also underestimated.

Finally, we overestimate average productivity/revenues among informal C sector firms. In practice, there is a tension between matching this moment and the level of wages in the informal C sector, given that we currently underestimate the latter. Nonetheless, our model-generated moments are consistent with the fact that informal firms are, on average, substantially smaller and less productive than formal firms.

[41] The 20^{th} and 40^{th} percentiles of firm size in the formal C sector are given by, respectively, 2 and 4. In the formal S sector, these are 1 and 2.

[42] Although standard in the literature, imposing log-normality can have important quantitative implications for the gains from integration, as discussed in Adão et al. (2020).

5 Results

5.1 The Aggregate Consequences of Changes in Trade Costs

5.1.1 Small Changes in Trade Costs

We start by analyzing the behavior of the model in response to a small shock in iceberg trade costs τ_c. Specifically, we compare the outcomes of our model with $\tau_c = 2.3$ to those of the benchmark model under $\tau_c = 2.4$—holding all the remaining parameters constant at their calibrated values. This small shock allows us to decompose the total effect of a reduction in trade costs on real income or welfare into a *mechanical effect* and a *reallocation effect*, in the spirit of the decomposition of Baqaee and Farhi (2020).

The mechanical effect captures the direct and immediate impact of changes in trade costs or technology on firms' marginal costs, as well as their propagation through the model's input-output production network—all while holding allocations, markups, wages, and the exchange rate fixed at their initial equilibrium levels. In contrast, the reallocation effect reflects both general equilibrium adjustments and the reallocation of resources across more or less distorted uses, with trade costs and technology parameters held constant. If the underlying trade cost or technology shock is sufficiently small, we can employ the following first-order approximation, which serves as the point of departure in the analysis of Baqaee and Farhi (2020):

$$\Delta \log \mathcal{Y} \approx \Delta \log \mathcal{Y}^{ME} + \Delta \text{Reallocation Effect}, \tag{37}$$

where $\Delta \log \mathcal{Y}$ is the proportional change in real income (or welfare) induced by the shock, $\Delta \log \mathcal{Y}^{ME}$ is the mechanical effect, and ΔReallocation Effect is the reallocation effect. In our model, there is a distinction between aggregate real income and aggregate welfare because an additional utility term, b_0, is associated with unemployment status. More precisely, total welfare is given by $\mathcal{Y} = \text{Real Income} + b_0 L_u$, where L_u is the mass of unemployed workers.

In practice, we perform this decomposition by first computing the exact change in $\Delta \log \mathcal{Y}$ caused by the reduction in trade costs from $\tau_c = 2.4$ to $\tau_c = 2.3$ within the model. Next, we analytically characterize the $\Delta \log \mathcal{Y}^{ME}$ term, which depends on the share of manufacturing expenditure on imports and the input-output structure. The ΔReallocation Effect term is then backed out as the residual of equation (37).

In Online Appendix J.1.1, we show that for a small change in trade costs, the mechanical effect—when $\mathcal{Y}$ represents real income—is given by:

$$\Delta \log (\text{Real Income})^{ME} = -\left(\zeta \frac{\partial \log P_C}{\partial \log \tau_c} + (1-\zeta) \frac{\partial \log P_S}{\partial \log \tau_c}\right) d \log \tau_c,$$

where the price elasticities solve the system below, given the share of imports in manufacturing expenditure, $s \equiv \frac{\tau_a Imports}{X_C}$, in the benchmark equilibrium.[43]

$$\begin{bmatrix} 1-(1-\delta_C)\,\lambda_C\,(1-s) & -(1-\delta_C)\,(1-\lambda_C)\,(1-s) \\ -(1-\delta_S)\,\lambda_S & 1-(1-\delta_S)\,(1-\lambda_S) \end{bmatrix} \begin{bmatrix} \frac{\partial \log P_C}{\partial \log \tau_c} \\ \frac{\partial \log P_S}{\partial \log \tau_c} \end{bmatrix} = \begin{bmatrix} s \\ 0 \end{bmatrix} \tag{38}$$

In decomposition (37), terms-of-trade effects operating through exchange rate changes are embedded in the ΔReallocation Effect term (Baqaee and Farhi, 2024). To more precisely isolate the gains from reallocation in the presence of distortions, an alternative decomposition—also valid for small shocks—is given by:

$$\Delta \log \mathcal{Y} \approx \Delta \log \mathcal{Y}^{DF} + \Delta \text{Reallocation Effect}. \tag{39}$$

In this decomposition, $\Delta \log \mathcal{Y}^{DF}$ denotes the change in real income—or welfare—resulting from a small trade cost or technology shock in a perfectly competitive, distortion-free economy. This economy retains the same underlying production and input-output structure, as well as the same initial manufacturing import share s, as the benchmark economy. However, firms are required to price at marginal cost and earn zero profits, labor markets are frictionless and perfectly competitive, fixed costs are eliminated, and there are no regulations or taxes. In contrast to the mechanical effect, the term $\Delta \log \mathcal{Y}^{DF}$ also captures terms-of-trade effects arising from changes in the exchange rate that ensure trade balance. Following our previous strategy, we analytically characterize this term and recover ΔReallocation Effect as the residual in equation (39).

Online Appendix J.2 shows that the change in real income in response to a small trade shock in such distortion-free economy is given by:

$$\Delta \log (\text{Real Income})^{DF} = -\left(\zeta \frac{\partial \log P_C}{\partial \log \tau_c} + (1-\zeta)\frac{\partial \log P_S}{\partial \log \tau_c}\right) d \log \tau_c.$$

Here, the price elasticities solve the system below, given the share of imports in manufacturing expenditure, s, in the benchmark equilibrium.[44]

$$\begin{bmatrix} 1-(1-s)\,(1-\delta_C)\,\lambda_C & -(1-s)\,(1-\delta_C)\,(1-\lambda_C) & -s \\ -(1-\delta_S)\,\lambda_S & 1-(1-\delta_S)\,(1-\lambda_S) & 0 \\ -\left(\frac{(\sigma_C-1)}{\sigma_C} + \frac{(\sigma_C-1)(1-\delta_C)\lambda_C}{\sigma_C}\right) & -\frac{(\sigma_C-1)(1-\delta_C)(1-\lambda_C)}{\sigma_C} & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial \log P_C}{\partial \log \tau_c} \\ \frac{\partial \log P_S}{\partial \log \tau_c} \\ \frac{\partial \log \epsilon}{\partial \log \tau_c} \end{bmatrix} = \begin{bmatrix} s \\ 0 \\ 0 \end{bmatrix} \tag{40}$$

Note that, compared to system (38), system (40) includes an additional block that links exchange rate movements to the prices P_C and P_S. Specifically, imposing $\frac{\partial \log \epsilon}{\partial \log \tau_c} = 0$ in system (40) would bring us back to system (38).

Table 8 reports the results of decompositions (37) and (39) for the small shock under consideration. The table entries show percentage deviations relative to the benchmark equilibrium with $\tau_c = 2.4$. Reducing trade costs from $\tau_c = 2.4$ to $\tau_c = 2.3$ results in a real income (welfare) gain of

[43] The mechanical effect when $\mathcal{Y}$ represents aggregate welfare is then computed as $\Delta \log (\text{Welfare})^{ME} = \log\left(\frac{\text{Real Income}\times\left(1+\Delta \log (\text{Real Income})^{ME}\right)+b_0 L_u}{\text{Real Income}+b_0 L_u}\right)$ where the baseline values of Real Income and unemployment L_u are set to those in the benchmark economy.

[44] Similarly to the procedure computing the mechanical effect, when $\mathcal{Y}$ represents aggregate welfare, we obtain $\Delta \log (\text{Welfare})^{DF} = \log\left(\frac{\text{Real Income}\times\left(1+\Delta \log (\text{Real Income})^{DF}\right)+b_0 L_u}{\text{Real Income}+b_0 L_u}\right)$ where the baseline values of Real Income and unemployment L_u are set to those in the benchmark economy.

approximately 1.4% (1.5%).

Strikingly, across both decompositions and regardless of whether we focus on real income or total welfare, the reallocation effect accounts for the largest share of the overall gains. In all cases—regardless of the metric $\mathcal{Y}$ or the decomposition method—reallocation effects represent at least 55% of the gains from trade. This implies that the real income (welfare) gains in the full model are 2.4 (2.2) times larger than the corresponding distortion-free gains. Given the close similarity in results between real income and total welfare, we henceforth focus on real income. This choice reflects its widespread use as a welfare measure in the international trade literature and facilitates interpretation of our findings. Similarly, we will henceforth rely on decomposition (39) to quantify the role of distortions in shaping real income gains.

Table 8: Impacts of a Small Trade Cost Shock, τ_c declines from 2.4 to 2.3

	Real Income	Welfare
1. $\Delta \log \mathcal{Y}$	1.413	1.505
2. $\Delta \log \mathcal{Y}^{ME}$	0.422	0.493
3. Δ Reallocation Effect (37)	0.992	1.012
4. $\Delta \log \mathcal{Y}^{DF}$	0.580	0.678
5. Δ Reallocation Effect (39)	0.833	0.828
6. Δ Share Unemployment	0.162	
	C Sector	S Sector
7. Δ Share Informality$_k$	-1.127	-0.628
8. $\Delta \log TFP_k$	1.988	1.074
9. $\Delta \log Var\left(\log\left(\partial R_k(z,\ell)/\partial \ell\right)\right)$	-1.073	-1.299

All numbers are expressed as 100 × changes relative to the Benchmark with $\tau_c = 2.4$. Total welfare is given by $\mathcal{Y}$ =Real Income+$b_0 L_u$, where L_u is the mass of unemployed workers. Δ Reallocation Effect (37) is the reallocation effect obtained using equation (37), and Δ Reallocation Effect (39) is the reallocation effect obtained using equation (39). Share Informality$_k$ is the share of sector $k \in \{C, S\}$ employment in informal firms. TFP_k is is the aggregate total factor productivity in sector k, defined as the average of firm-level productivities z weighted by employment.

As discussed at length in Baqaee and Farhi (2020), in an economy without distortions, the reallocation effect would be negligible for small shocks. However, as we previously highlighted, our setting features many distortions. Crucially, because taxes and regulations are not perfectly enforced by the government, firms are not equally exposed to these domestic distortions. Consequently, smaller and less efficient informal firms experience lower distortions, leading them to produce above the socially optimal level. In contrast, larger and more productive formal firms face greater distortions, prompting them to produce below the socially optimal level.

A key consequence of a reduction in trade costs is a decline in the share of informal employment in both the C and S sectors, as illustrated in row 7 of Table 8. Concurrently, we observe a reallocation of labor from less distorted, low-productivity informal firms to more distorted, high-productivity formal firms. This is highlighted in row 8 of Table 8, which shows the behavior of measured aggregate TFP in both sectors.[45] Finally, row 9 shows a reduction in the dispersion of

[45] By "measured aggregate TFP," we refer to the weighted average of firm-level productivities z, where the weights are given by firm-level employment. This is a commonly used empirical measure of aggregate TFP in the literature

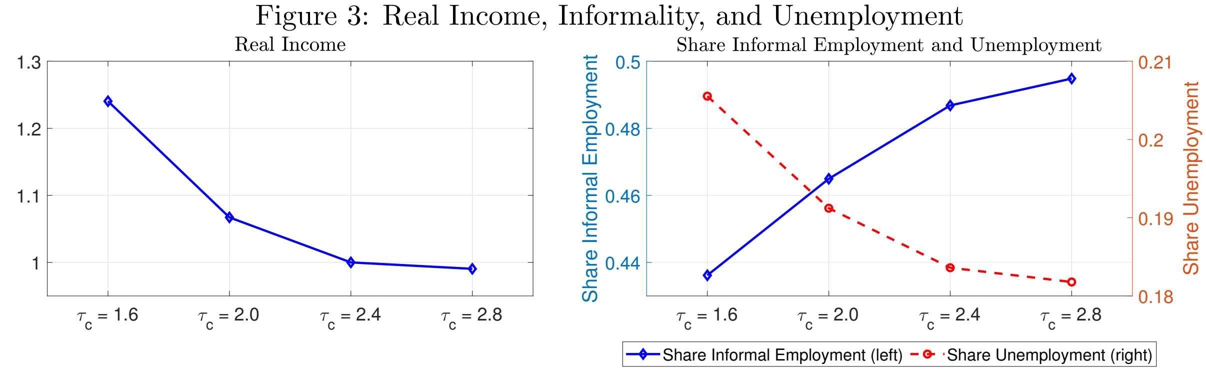
the log marginal revenue product of labor in both sectors—suggesting a more efficient allocation of resources following increased globalization, further boosting the reallocation effect (Hsieh and Klenow, 2009).

5.1.2 Larger Changes in Trade Costs

The impacts that we document in Table 8 focus on small changes in trade costs. Armed with this small shock, we are able to leverage the decompositions in equations (37) and (39) to back out the reallocation effect and compare it the total income or welfare effect of an increase in trade openness. However, with large shocks, the first-order approximations in equations (37) and (39) are no longer accurate and we are unable to back out the reallocation effect.

Nevertheless, we simulate the effects of larger trade cost shocks, as these better relate to extensive trade reforms. This section considers reductions in τ_c from 2.4 to 2.0 and 1.6, as well as an increase to 2.8. To gauge the magnitude of these changes, note that the benchmark equilibrium with $\tau_c = 2.4$ yields an imports-to-GDP ratio of 7.7%. Lowering τ_c to 2.0 raises this ratio to 16.7%, and a further reduction to 1.6 results in an openness ratio of 32.5%—comparable to levels observed in countries such as the United Kingdom, South Africa, Chile, and Colombia (World Bank, 2022).

Figure 3: Real Income, Informality, and Unemployment



Note: Real Income at the left panel is displayed relative to its benchmark value under $\tau_c = 2.4$.

Figure 3 (left panel) illustrates the real income gains resulting from reductions in trade costs. Lowering τ_c from 2.4 to 2.0 yields a total real income gain of 6.7%, while a larger reduction from $\tau_c = 2.4$ to $\tau_c = 1.6$ generates a gain of 24%. As trade costs decline, consistent with the patterns observed in Table 8, the informal sector shrinks (Figure 3, right panel). Labor reallocates from initially less distorted but less productive informal firms to more distorted but more productive formal firms. In addition, the dispersion in the log marginal revenue product of labor decreases, reflecting a more efficient allocation of resources.

We conclude this subsection by interpreting the magnitude of the reduction in informality resulting from lower trade costs. The right panel of Figure 3 shows that, while the effects on the total number of informal workers are economically meaningful, they are not sufficient to *substantially* reduce informality. Specifically, as τ_c falls from 2.4 to 1.6, the imports-to-GDP ratio rises from

(Olley and Pakes, 1996; Pavcnik, 2002). However, this measure is not necessarily informative about welfare, as discussed in Berthou et al. (2021).

7.7% to 32.5%—a substantial fourfold increase. Yet the share of informal workers declines only modestly, from 48.7% to 43.6%. This result is consistent with the fact that the informal sector has not substantially shrunk in middle-income economies despite the large-scale liberalization episodes these experienced in the 1980s and 1990s (see, for example, World Bank, 2019).

While the overall share of informality decreases by 5 percentage points, the fraction of workers employed in the manufacturing sector experiences a larger decline of 25%. This observation, coupled with our roundabout production structure and intersectoral linkages, helps to explain the significant reallocation effect that we uncover.

5.1.3 Trade, Informality and Unemployment: Mechanisms

We conclude this subsection by discussing how changes in trade costs affect informality and unemployment, as shown in Table 8 and the right panel of Figure 3. We begin by dissecting the effects of reducing trade costs on informality in sector C. When trade costs decrease, the foreign market size d_F increases, leading exporters to grow, become more profitable, and pay higher wages. In addition, the entry of new firms into the export market results in an increased share of firms paying higher wages. These impacts raise the value of search J^u, causing wages to rise across all firms in the economy. Consequently, these increased wages force the least productive informal firms out of the market. This mechanism tends to reduce informality in sector C, all else being equal.

Next, we have two opposing forces. The aforementioned higher wages also tend to push the least productive formal firms out of the market as their labor costs increase. Ceteris paribus, this would tend to increase informality. However, high-productivity informal firms anticipate that, at some point, they may grow and become exporters with higher probability. This increases the value of formality relative to informality, encouraging high-productivity informal firms to transition to the formal sector. This tends to decrease informality. Quantitatively, the combined effect of these forces is a higher productivity threshold for entry in the informal sector and a lower threshold for entry in the formal sector, unambiguously resulting in a reduction in the size of the informal sector in sector C.

The mechanisms at play in sector S are similar to those in sector C, except that high-productivity informal sector firms cannot export. As a result, we observe in our simulation an increase in the productivity threshold for entry in both the informal and formal sectors simultaneously. The impact of trade openness on informality in the S sector is therefore ambiguous. However, quantitatively, there is a net decline in the informal sector in the S sector as well.

Now, we turn to the unemployment impact of reducing trade costs. Table 8 and the right panel of Figure 3 both show that the unemployment rate increases following either small or large reductions in trade costs. Specifically, when the iceberg trade cost declines from its benchmark value of $\tau_c = 2.4$ to $\tau_c = 1.6$, the unemployment rate rises from 18.4% to 20.6%—a nontrivial 12% increase in the number of unemployed workers.

To understand this effect, recall that two opposing forces connect globalization to unemployment: (i) a reallocation of resources toward larger and more stable firms, and (ii) a reallocation toward exporters, whose revenues become more volatile due to the expanded foreign market size d_F and whose share in the economy increases via an extensive-margin adjustment. The net effect on aggregate turnover depends on the relative strength of these forces. Quantitatively, the second force dominates, leading to higher turnover in the economy. Given search frictions, this translates into higher unemployment rates. A second mechanism also contributes: rising wages reduce the number of vacancies posted, especially among less productive firms. Still, we emphasize that the resulting increase in unemployment only partially offsets the substantial real income gains from trade.

5.2 The Impact of Trade Costs under Various Enforcement Scenarios

The previous section showed that, in our high-informality context, trade openness triggers a large reallocation effect. This effect exceeds both the mechanical effect and the distortion-free gain in real income (or welfare) from a small reduction in trade costs, more than doubling the overall gains from trade. We now investigate the role of the informal sector in driving this substantial reallocation effect. To do so, we gradually increase the costs faced by informal firms, thereby reducing the share of firms and workers operating in the informal sector. We interpret these rising costs of informality as reflecting increasingly strict government enforcement, with informal activity facing progressively higher penalties.

We focus on *small* changes in enforcement by progressively increasing the convexity of the informality cost function. Specifically, recall that informality costs are given by

$$K^{inf}(z,\ell) = \widetilde{a}_k \exp\left\{\widetilde{b}_k\,(\ell-1)\right\} \times R_k(z,\ell), \quad k \in \{C,S\}.$$

We consider counterfactuals in which $\widetilde{b}_k$ is modestly and successively increased in both sectors. These incremental enforcement adjustments are intended to avoid comparing equilibria that differ too starkly. For completeness, we also examine a counterfactual scenario in which informality costs become prohibitive, effectively eliminating the informal sector. Throughout this exercise, all other model parameters are held constant at their benchmark calibrated values reported in Table 7.

Table 9 shows the impact of gradually increasing informality costs in both sectors on the share of informal employment. As the first row highlights, the *Benchmark* economy features 48.7% of employment in the informal sector. This rate declines to approximately 40.3% under the scenario *Stricter Enforcement 1*, then down to 33.5% under *Stricter Enforcement 2*, 26.7% under *Stricter Enforcement 3*, and finally 0 when the government perfectly enforces taxes and regulations under *No Informality*.[46] It is noteworthy that, as enforcement is gradually tightened, the unemployment rate rises from 18.4% under the *Benchmark* scenario to 21.4% under *Stricter Enforcement 3*. When informality is entirely eliminated, unemployment jumps to 27.8%. The primary driver of this sharp increase is that the minimum wage becomes increasingly binding as firms face higher costs of evading it by operating informally. Finally, the last row of Table 9 shows that all five scenarios feature similar

[46] Scenario *Stricter Enforcement 1* sets $\widetilde{b}_k$ such that $p_{ki}(\ell = 30) = 1$; scenario *Stricter Enforcement 2* sets $\widetilde{b}_k$ such that $p_{ki}(\ell = 14) = 1$; and scenario *Stricter Enforcement 3* sets $\widetilde{b}_k$ such that $p_{ki}(\ell = 6) = 1$. Finally, the *No Informality* scenario sets $\widetilde{b}_k$ such that $p_{ki}(\ell = 6) = 1$ and increases $\widetilde{a}_k$ tenfold.

manufacturing import shares s—especially the first four columns—suggesting that the mechanical and distortion-free effects of lower trade costs are largely comparable across scenarios.[47]

Table 9: Effects of Increasing the Cost of Informality

	Bench.	Strict Enf. 1	Strict Enf. 2	Strict Enf. 3	No Inf.
1. Share Informal Emp.	0.487	0.403	0.335	0.267	0.000
2. Unemployment Rate	0.184	0.193	0.202	0.214	0.278
3. $s = \tau_a Imports/X_C$	0.117	0.117	0.124	0.129	0.143

We now examine the effects of a small reduction in trade costs, from $\tau_c = 2.4$ to $\tau_c = 2.3$, across different enforcement scenarios. To isolate the role of informality in shaping the real income gains from trade, Table 10 reports the difference in gains *relative* to the Benchmark scenario:

$$100 \times (\Delta y_{\text{scenario}} - \Delta y_{\text{Bench}}), \quad y \in \left\{\log(\text{Real Income}), \log(\text{Real Income})^{DF}, \text{Reallocation Effect}\right\},$$
$$\text{scenario} \in \{\text{Bench, SE1, SE2, SE3, No Inf.}\},$$

where $\Delta y_{\text{scenario}}$ denotes the change in variable y under a given scenario in response to the reduction in trade costs. For example, Table 10 shows that the real income gain from reducing τ_c from 2.4 to 2.3 is 0.179 percentage points smaller in the *Stricter Enforcement 1* scenario than in the *Benchmark* scenario. This approach yields difference-in-differences comparisons that help quantify the contribution of distortions associated with informality.

We first note that the real income gains under the *Benchmark* economy, with a large informal sector, are larger than the gains in all the remaining scenarios when the informal sector is repressed. Furthermore, note that, as we anticipated, the distortion-free gains are similar across scenarios, meaning that the differences in real income gains from globalization are driven by differences in the magnitude of the reallocation effect.

For *modest* increases in enforcement, the reallocation effect declines monotonically, accounting for the reduction in real income gains observed between the *Benchmark* and *Stricter Enforcement 3* scenarios. For instance, the reallocation effect is 0.184 percentage points smaller in *Stricter Enforcement 1* relative to the *Benchmark*, and nearly 1 percentage point smaller in *Stricter Enforcement 2*. However, this pattern is not globally monotonic. In the *No Informality* scenario, shown in the last column, the reallocation effect increases relative to *Stricter Enforcement 3*. This reversal is partly due to a sharp drop in unemployment under *No Informality*. As noted earlier, the minimum wage is strongly binding in this case. When trade costs fall, equilibrium wages rise, making the minimum wage less restrictive and reducing unemployment. This decline in unemployment, in turn, contributes positively to the reallocation effect.

These lower reallocation effects arise from a weaker shift in labor and output across firm types. In the *Benchmark* economy, lower trade costs trigger a more substantial movement of resources

[47] Because all parameters except $\widetilde{b}_k$ are held constant, equations (38) and (40) indicate that any differences across scenarios in the mechanical and distortion-free effects of lower trade costs must arise from variation in the manufacturing import share s.

from low-productivity, less distorted informal firms to high-productivity, more distorted formal firms. This shift is more limited in scenarios with stricter enforcement, where informality is already repressed. Also consistent with the larger reallocation effect in the *Benchmark* scenario, the impact of trade openness on the dispersion of the log marginal revenue product of labor is more negative, reflecting greater efficiency gains.

Table 10: Impacts of a Small Trade Shock Across Enforcement Scenarios, τ_c Declines from 2.4 to 2.3—All Impacts Shown Are Relative to Those Under the Benchmark

	Bench	SE1	SE2	SE3	No Inf.
1. $\Delta \log$ (Real Income)	0	-0.179	-0.952	-1.040	-0.596
2. $\Delta \log$ (Real Income)^{DF}	0	0.005	0.044	0.074	0.169
3. Δ Reallocation Effect (39)	0	-0.184	-0.996	-1.114	-0.765

All effects are relative to the Benchmark and multiplied by 100. More precisely, each cell is given by $100 \times (\Delta y_{\text{scenario}} - \Delta y_{\text{Bench}})$, $y \in \{\log(\text{Real Income}), \log(\text{Real Income})^{DF}, \text{Reallocation Effect}\}$, scenario $\in$ {Bench, SE1, SE2, SE3, No Inf.}. Δ Reallocation Effect (39) is the reallocation effect obtained using equation (39).

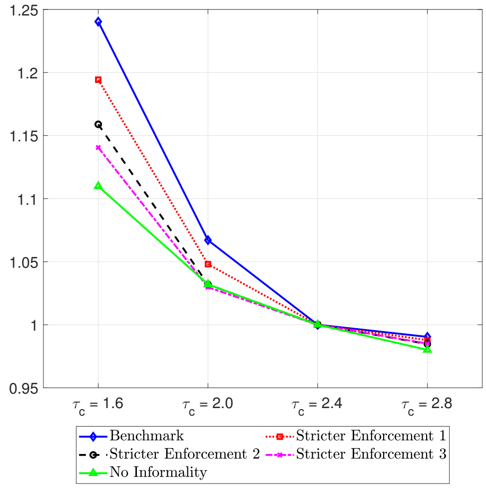
Figure 4 shifts the focus from small τ_c shocks to the larger reductions considered in Section 5.1.2. Consistent with Table 10, we confirm that the real income gains from trade are substantially larger in the *Benchmark* economy, which features a high degree of informality. As trade costs fall from $\tau_c = 2.4$ to $\tau_c = 1.6$, the *Benchmark* economy experiences gains of 24%. In comparison, the *Stricter Enforcement 1* scenario, which slightly represses the informal sector, yields gains of 19.4%. The *No Informality* scenario shows gains of just 11%, less than half of those in the *Benchmark* economy.

The key takeaway from this section is that gains from trade liberalization *tend* to be larger in economies with more pervasive informality. This is because trade reallocates resources away from overproducing, less distorted firms and toward underproducing firms that are initially more distorted. In settings with stricter enforcement of taxes and regulations, there is less scope for such beneficial reallocation. We emphasize, however, that—consistent with the last column of Table 10—gains from liberalization are not monotonic in the baseline level of informality.

Given the complexity of the model and the presence of multiple distortions, we view the first three columns of Table 10 as providing the most transparent analysis of the informal sector's role in amplifying the gains from trade. These exercises are local in scope: the economies differ only slightly in the costs of informality faced by firms and are otherwise identical in parameters—and nearly identical in equilibrium outcomes—except for modest differences in the share of informal employment, which declines with higher enforcement levels.

Online Appendix R presents complementary analyses comparing the aggregate consequences of trade in the *Benchmark* economy with those in alternative scenarios *without* informality. In addition to the *No Informality* scenario discussed above, we consider two additional scenarios: (a) an economy with prohibitive informality costs, re-estimated to match only formal-sector moments;

Figure 4: Effect of Trade on Real Income Relative to $\tau_c = 2.4$,
Various Enforcement Scenarios



and (b) an economy without taxes or labor market regulations, also re-estimated on formal-sector targets. Although these economies differ substantially—either in equilibrium outcomes or in underlying structural parameters—a robust result emerges: real income gains from trade are consistently larger in the *Benchmark* economy, driven by stronger reallocation effects.

5.3 Comparison with Earlier Reduced-Form Work: The Effects of Negative Labor Demand Shocks

Our paper also relates to a growing body of empirical work that highlights the responsiveness of the informal sector to trade shocks in developing countries (Goldberg and Pavcnik, 2003; McCaig and Pavcnik, 2018; Dix-Carneiro and Kovak, 2019). More broadly, there is an extensive literature exploring the local labor market impacts of foreign competition. However, a significant portion of these studies relies on differences-in-differences specifications, which are not conducive to assessing the aggregate effects of trade (see, e.g., Adão et al., 2023; Dix-Carneiro and Kovak, 2025). These analyses are focused on a distinct question, namely, how (trade-induced) sector- or region-specific *labor demand shocks* influence *relative* sector- or region-specific informality or other labor market outcomes.

It is therefore important to underscore that the trade-induced local impacts documented in, for example, Dix-Carneiro and Kovak (2019), do *not* measure the aggregate effects of trade on informality. Their specification compares labor market outcomes across regions exposed to different magnitudes of trade-induced negative labor demand shocks. Because common effects of trade are absorbed into the intercept, the estimated coefficients capture only the *differential* impacts of these shocks. From the perspective of our model, this is akin to comparing the *Benchmark* economy with one experiencing a negative labor demand shock.

This comparison is useful for validating some of our model's predictions with the quasi-experimental evidence in Dix-Carneiro and Kovak (2019) and Ponczek and Ulyssea (2022). We implement it by simulating an aggregate productivity shock that uniformly shifts the entire support of the productivity distribution to the left. Specifically, we focus on a small 1% shift, allowing us to explore the first-order approximation in equation (39). Online Appendix J.2.2 derives the expression for the distortion-free impact of this small aggregate productivity shock.

Table 11: Impact of a Negative Productivity Shock of 1 pct—Benchmark Scenario

	Benchmark
1. Δ Share Workforce Informal	5.377
2. Δ Share Unemployment	-1.058
3. $\Delta \log$ (Real Income)	-10.435
4. $\Delta \log$ (Real Income)^{DF}	-2.721
5. Δ Reallocation Effect (39)	-7.714

All numbers are expressed in 100 × changes relative to the initial equilibrium. Δ Reallocation Effect (39) is the reallocation effect obtained using equation (39).

Rows 1 and 2 of Table 11 show that an economy-wide negative labor demand shock has a relatively modest (and actually negative) effect on unemployment in the *Benchmark* economy.[48] However, it has a large positive impact on the share of informal employment, increasing it by 5 percentage points. These results parallel those of Dix-Carneiro and Kovak (2019) who show that, in the context of the Brazilian trade liberalization, regions facing more negative labor demand shocks experience no discernible responses of unemployment in the long run but do experience substantial relative increases in the share of labor informally employed. This result prompted the authors to propose a hypothesis: that the informal sector might have served as a fallback for displaced workers.

Table 12: Impacts of a Negative Productivity Shock of 1 pct Across Enforcement Scenarios—All Impacts Are Relative to Those Under the Benchmark

	Bench	SE1	SE2	SE3	No Inf.
1. Δ Share Workforce Informal	0	-1.883	-2.291	-3.617	-5.377
2. Δ Share Unemployment	0	0.461	0.642	1.190	2.517
3. $\Delta \log$ (Real Income)	0	3.852	4.882	6.000	5.198
4. $\Delta \log$ (Real Income)^{DF}	0	-0.003	-0.027	-0.044	-0.101
5. Δ Reallocation Effect (39)	0	3.855	4.909	6.045	5.299

All effects are relative to the Benchmark and multiplied by 100. More precisely, each cell is given by $100 \times (\Delta y_{\text{scenario}} - \Delta y_{\text{Bench}})$, $y \in \{\log(\text{Real Income}), \log(\text{Real Income})^{DF}, \text{Reallocation Effect}\}$, $\text{scenario} \in \{\text{Bench, SE1, SE2, SE3, No Inf.}\}$. Δ Reallocation Effect (39) is the reallocation effect obtained using equation (39).

This hypothesis is further explored by Ponczek and Ulyssea (2022), who revisit the findings of Dix-Carneiro and Kovak (2019). They find that the effects of trade-induced declines in labor demand on unemployment were more pronounced in regions where labor markets were closely

[48] Vacancies increase in response to the negative productivity shock, primarily due to informal firms. As resources shift toward the informal sector—which faces fewer distortions such as firing costs, minimum wages, and payroll taxes—these firms post more vacancies and pay lower wages, which in turn helps reduce unemployment despite the adverse shock.

monitored by the government (i.e., where enforcement of regulations was stricter), while the impacts on informality were less significant. These empirical results align with the findings presented in Table 12, indicating that in economies where the informal sector is more tightly monitored, the impact of the negative labor demand shock on informality tends to be smaller, while the effect on unemployment tends to be larger (see rows 1 and 2)—note that, as with Table 10 all impacts are expressed relative to those in the Benchmark. Taken as a whole, the results from the model corroborate the informal sector's role as an *unemployment buffer*, consistent with existing empirical evidence. This lends further credibility to our quantitative analyses in the previous sections.

Interestingly, the impact of the negative productivity shock on real income is more negative in the *Benchmark* economy than in the other economies where the informal sector is repressed—see row 3 of Table 12. The primary driver behind this result is the reallocation effect, which, in the *Benchmark* case, is almost 3 times larger than the distortion-free effect (rows 4 and 5 of Table 11). Row 5 of Table 12 highlights that the reallocation effect is substantially more negative in the Benchmark economy than in economies where there is stricter enforcement.

As aggregate productivity declines, we observe a reduction in aggregate TFP in both sectors C and S. This effect is significantly larger than the shock itself and is driven by a robust reallocation of labor to less distorted but less productive firms in the informal sector, thereby amplifying misallocation in the economy. Concurrently, we can also observe an increase in the dispersion of log marginal revenue product of labor in both sectors, further contributing to the misallocation of resources. The extent of this misallocation increase is mitigated in economies where the informal sector is repressed. The conclusion is that the informal sector indeed serves as a fallback for displaced workers, but this comes at the expense of an expansion of the informal sector, amplifying misallocation in the economy and further reducing welfare.

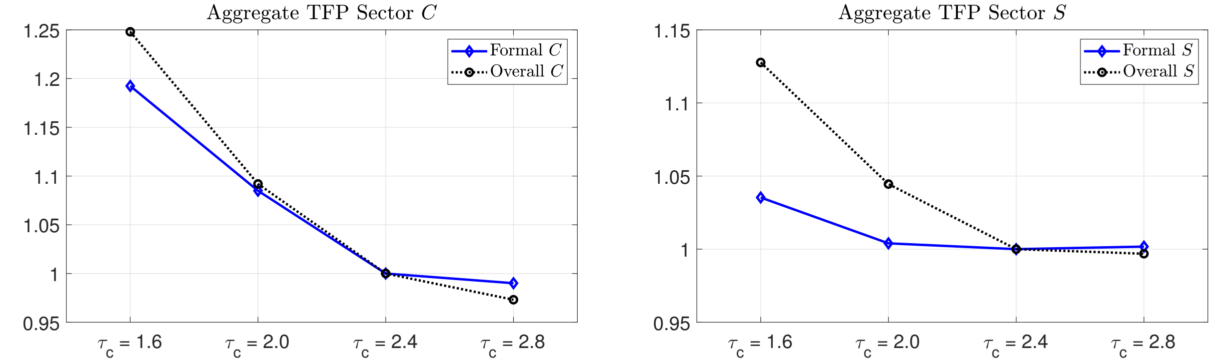
5.4 Reassessing the Impact of Trade on Aggregate TFP and Wage Inequality

We conclude this section by emphasizing that our work also informs the interpretation of a range of empirical results focused on the formal sector when the economy has a significant informal sector—typically the case in many developing countries (particularly in Latin America). We begin with Figure 5, which illustrates that the conventionally measured TFP gains, based on data solely from the formal sector, fall short of capturing the overall TFP gains including the informal sector.[49] Studies that focus solely on the formal sector miss that reductions in trade costs force unproductive informal firms out of the market, amplifying the reallocation of labor from low- to high-productivity firms.

Next, we shift gears and revisit a standard finding in the international trade literature: namely, that trade liberalization increases wage inequality across firms in the manufacturing sector when we focus on formal-sector outcomes. Examples of this finding appear in Helpman et al. (2017), who focus on Brazil, and Coşar et al. (2016), who study the Colombian experience. Both papers focus on

[49] As before, empirically measured aggregate TFP is computed as the employment-weighted average of firm-level productivities z.

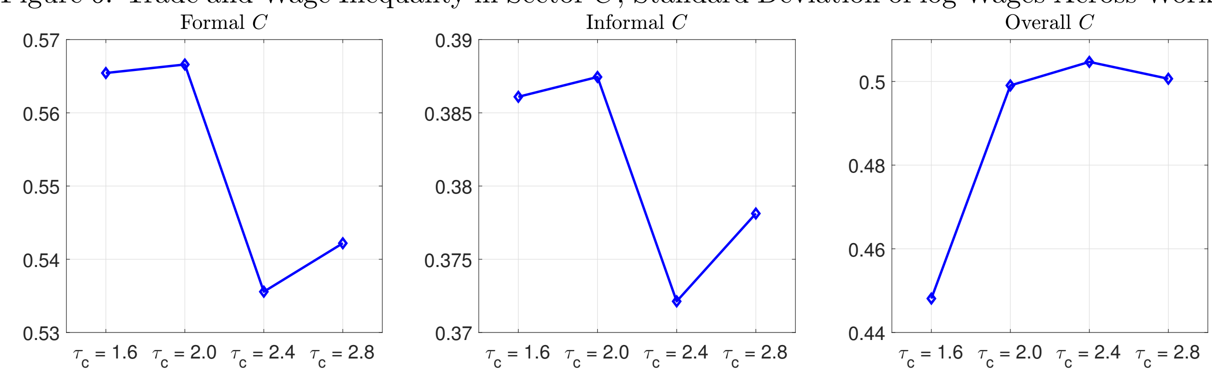
Figure 5: Trade and Aggregate Productivity, Relative to Benchmark ($\tau_c = 2.4$)



the effects of a reduction in trade costs on wage inequality within the formal manufacturing sector. They argue that in their respective contexts, the rise in trade openness has led to an increase in wage inequality across firms.

The two left panels of Figure 6 show that our model replicates these findings within both the formal and informal manufacturing sectors when focusing on the evolution of wage inequality across firms. In contrast, the rightmost panel shows that reductions in trade costs lead to a decline in overall wage inequality across firms, once the reduction in between-sector inequality is taken into account.[50]

Figure 6: Trade and Wage Inequality in Sector C, Standard Deviation of log Wages Across Workers



The results in this and previous sections underscore the importance of incorporating the informal sector into our models of trade. Not only does the magnitude of the gains from trade depend on the informal sector, but so do the interpretation and contextualization of a large range of empirical results for developing countries in which the researchers focused their attention on the formal manufacturing sector.

6 Conclusion

Understanding the implications of domestic distortions for the effects of international trade is an important area of research, particularly in developing countries where these distortions are expected

[50] Wage inequality across all firms, encompassing both the formal and informal sectors, is similarly reduced in the S sector. Additionally, we observe reductions in inequality within both sectors and across all workers in lifetime worker values $J^e(z, \ell)$, which incorporate the utility flow of unemployment b_0.

to be significant (Atkin and Khandelwal, 2020; Atkin et al., 2025). In this paper, we investigate a set of distortions associated with the phenomenon of informality, namely burdensome labor market regulations and taxes that are imperfectly enforced by the government. This imperfect enforcement implies that some firms are subject to these regulations, while others are not. The first set of firms constitutes the formal sector, which, due to this imperfect enforcement, is more distorted than the informal sector.

We show that trade liberalization has large, positive impacts in this environment. Trade openness helps reallocate resources from the initially less distorted informal firms to the initially more distorted formal firms, leading to a sizable "reallocation effect." Furthermore, we find that this effect is particularly pronounced in economies with a larger informal sector. Crucially, our analysis reveals that while the mechanical effect of trade remains relatively constant across scenarios, the reallocation effect diminishes as enforcement improves. In economies with stricter enforcement, the initial misallocation associated with the informal sector is smaller, thereby limiting the potential for trade openness to facilitate the reallocation from less to more distorted firms. These results suggest that globalization is particularly valuable when the misallocation resulting from imperfect enforcement is largest.

In comparisons with earlier quasi-experimental work, we replicate the finding that informality can act as an employment buffer in the presence of negative labor demand shocks. However, our analysis reveals that informality does not act as an aggregate welfare buffer, as real income declines less in the presence of stricter enforcement despite the larger increase in unemployment. This is explained by the fact that negative economic shocks reallocate resources from more to less distorted sectors, exacerbating misallocation.

Our results underscore the importance of incorporating the informal sector in analyses of trade policies in developing countries, both quantitatively and qualitatively. We hope that our paper provides a first step in this direction by analyzing the microfoundations of the informality phenomenon, specifically the unequal exposure to domestic distortions and their interactions with trade.

We end the paper highlighting that the quantitative framework we present here is very flexible and amenable to studying the impacts of tax policies and labor market regulations in an environment with widespread informality, tax evasion and noncompliance. However, an important limitation of this paper lies in the fact that workers are assumed to be homogeneous in our model. It has been documented that the informal sector tends to be more unskilled labor intensive than the formal sector (Perry et al., 2007; La Porta and Shleifer, 2014), so policies affecting this sector can have important distributional consequences. We leave this extension for future work.

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Online Appendix

Trade and Domestic Distortions: The Case of Informality

by Rafael Dix-Carneiro, Pinelopi Koujianou Goldberg, Costas Meghir, Gabriel Ulyssea

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A Steady-State Distribution of States

A.1 Informal Firms

Denote by $G_k(z'|z)$ the cumulative distribution function of z' conditional on z and $g_k(z'|z)$ its density. The period starts with N_{ki} informal firms and distribution of states $\psi_{ki}(z,\ell)$ at the very beginning of stage 1. After (endogenous and exogenous) exit, change in formal status, and entry, but before labor adjustment (end of stage 1 / beginning of stage 2) the distribution of states is:

$$\widetilde{\psi}_{ki}(z,\ell) \equiv \frac{\mathcal{I}[\ell=1]\, M_{ki}\psi^{e}_{ki}(z) + \mathcal{I}[\ell \geq 1](1-\alpha_k) N_{ki}\psi_{ki}(z,\ell)\, I^{stay}_{ki}(z,\ell)}{N_{ki}} \tag{A.1}$$

$$= \mathcal{I}[\ell=1] \frac{M_{ki}}{N_{ki}}\psi^{e}_{ki}(z) + \mathcal{I}[\ell \geq 1](1-\alpha_k)\, \psi_{ki}(z,\ell)\, I^{stay}_{ki}(z,\ell), \tag{A.2}$$

where M_{ki} is the mass of informal entrants into sector k, $\psi^{e}_{ki}(z)$ is the distribution of z productivities of entrants conditional on entry into the informal sector:

$$\psi^{e}_{ki}(z) \equiv \frac{g^{e}_{k}(z)\, I^{informal}_{k}(z)}{\int_{\widetilde{z}} g^{e}_{k}(\widetilde{z})\, I^{informal}_{k}(\widetilde{z})\, d\widetilde{z}}. \tag{A.3}$$

In equation (A.1) the numerator is the total mass of firms with state (z,ℓ). The denominator is the total mass of firms at the stage we consider. In steady state, entrants replace firms who exit, so that there are N_{ki} firms at that stage.

After firms make adjustment decisions, and at the production stage (end of stage 2 / beginning of stage 3), the distribution of states is:

$$\widehat{\psi}_{ki}(z,\ell') \equiv \int_{\ell} \widetilde{\psi}_{ki}(z,\ell)\, \mathcal{I}[L_{ki}(z,\ell)=\ell']\, d\ell. \tag{A.4}$$

At the end of the period, after production takes place, firms draw their productivity z' for the next period (stage 3). In steady state, the distribution of states at the very end of the period (end of stage 3) replicates the initial one (very beginning of stage 1):

$$\psi_{ki}(z',\ell') = \int_{z} \widehat{\psi}_{ki}(z,\ell')\, g_k(z'|z)\, dz. \tag{A.5}$$

To fix ideas, Table A.1 clarifies the notation for the distribution of states in different stages within a period.

A.2 Formal Firms

The period starts with N_{kf} formal firms and distribution of states $\psi_{kf}(z,\ell)$ at the very beginning of stage 1. After (endogenous and exogenous) exit, change in formal status, and entry, but before labor adjustment

Table A.1: Distributions of States at Different Stages

ψ_{kj}	Distribution of states at the very beginning, and at the very end of the period—very beginning of stage 1 and very end of stage 3
$\widetilde{\psi}_{kj}$	Distribution of states right after entry, exit, and change of formal status but **before** labor adjustment—very end of stage 1 / very beginning of stage 2
$\widehat{\psi}_{kj}$	Distribution of states after labor adjustment, at the production stage—end of stage 2

(end of stage 1 / beginning of stage 2) the distribution of states is:

$$\widetilde{\psi}_{kf}(z,\ell) \equiv= \frac{\begin{array}{c}\mathcal{I}[\ell=1]\,M_{kf}\psi^{e}_{kf}(z)\\ +\mathcal{I}[\ell\geq 1](1-\alpha_k)\,N_{ki}\psi_{ki}(z,\ell)\,I^{change}_{ki}(z,\ell)\\ +\mathcal{I}[\ell\geq 1](1-\alpha_k)\,N_{kf}\psi_{kf}(z,\ell)\,I^{stay}_{kf}(z,\ell)\end{array}}{N_{kf}} \tag{A.6}$$

$$= \begin{array}{c}\mathcal{I}[\ell=1]\,\frac{M_{kf}}{N_{kf}}\psi^{e}_{kf}(z)\\ +\mathcal{I}[\ell\geq 1](1-\alpha_k)\,\frac{N_{ki}}{N_{kf}}\psi_{ki}(z,\ell)\,I^{change}_{ki}(z,\ell)\\ +\mathcal{I}[\ell\geq 1](1-\alpha_k)\,\psi_{kf}(z,\ell)\,I^{stay}_{kf}(z,\ell)\end{array}, \tag{A.7}$$

where M_{kf} is the mass of formal entrants into sector k, $\psi^{e}_{kf}(z)$ is the distribution of z productivities of entrants conditional on entry into the formal sector:

$$\psi^{e}_{kf}(z) \equiv \frac{g^{e}_{k}(z)\,I^{formal}_{k}(z)}{\int_{\widetilde{z}} g^{e}_{k}(\widetilde{z})\,I^{formal}_{k}(\widetilde{z})\,d\widetilde{z}}. \tag{A.8}$$

The numerator in equation (A.6) is the total mass of firms with state (z,ℓ). The denominator is the total mass of firms at the stage we consider. In steady state, entrants replace firms who exit, so that there are N_{kf} firms at that stage. After firms make adjustment decisions, and at the production stage (end of stage 2 / beginning of stage 3), the distribution of states is:

$$\widehat{\psi}_{kf}(z,\ell') \equiv \int_{\ell} \widetilde{\psi}_{kf}(z,\ell)\,\mathcal{I}(L_{kf}(z,\ell)=\ell')\,d\ell. \tag{A.9}$$

At the end of the period, after production takes place, firms draw their productivity z' for the next period (stage 3). In steady state, the distribution of states at the very end of the period (very end of stage 3) replicates the initial one (very beginning of stage 1):

$$\psi_{kf}(z',\ell') = \int_{z} \widehat{\psi}_{kf}(z,\ell')\,g_k(z'|z)\,dz.$$

B Entry

Let M_k denote the mass of entrants in sector $k = C, S$. The fraction of entrants into the formal and informal sectors are given respectively by ω_{kf} and ω_{ki}:

$$\omega_{kf} \equiv \Pr\left(I_k^{formal}(z) = 1\right) = \int_z g_k^e(z)\, I_k^{formal}(z)\, dz, \tag{B.1}$$

$$\omega_{ki} \equiv \Pr\left(I_k^{informal}(z) = 1\right) = \int_z g_k^e(z)\, I_k^{informal}(z)\, dz. \tag{B.2}$$

Therefore, the masses of entrants in the formal and informal sectors are given by:

$$M_{ki} = \omega_{ki} M_k, \tag{B.3}$$

$$M_{kf} = \omega_{kf} M_k. \tag{B.4}$$

The masses of entrants into each sector, M_k, are pinned down by the free entry condition (assuming positive entry in both sectors):

$$c_{e,k} = V_k^e = \int_z \left[V_{ki}^e(z)\, I_k^{informal}(z) + V_{kf}^e(z)\, I_k^{formal}(z)\right] g_k^e(z)\, dz. \tag{B.5}$$

C Flow Conditions for Workers and Firms

In order to write the labor market clearing conditions, we first define the following quantities.

- Number of workers at the **beginning** of the period in sector k (before entry, exit, change of formal status and labor adjustment), working in formal or informal firms (T stands for "total"):

$$W_{kj}^T = N_{kj} \underbrace{\int_z \int_\ell \ell \psi_{kj}(z,\ell)\, d\ell dz}_{\text{avg. \# of workers per firm}} = L_{kj} \tag{C.1}$$

  for $j = f, i$ and $k = C, S$.

- Number of workers in sector (k, j) who are fired because their firms receive a destruction shock:

$$W_{kj}^{DS} = \alpha_{kj} N_{kj} \int_z \int_\ell \ell \psi_{kj}(z,\ell)\, d\ell dz = \alpha_{kj} L_{kj} \tag{C.2}$$

- Number of workers in sector (k, j) who are fired due to **endogenous** firm exit:

$$W_{kj}^{EE} = (1 - \alpha_{kj})\, N_{kj} \times \int_z \int_\ell \ell \psi_{kj}(z,\ell)\, I_{kj}^{exit}(z,\ell)\, d\ell dz \tag{C.3}$$

  where $(1 - \alpha_{kj})\, N_{kj}$ is the mass of firms that survive after the destruction shock hits.

- Number (mass) of **surviving incumbent** firms in sector (k, j) in the interim period:

$$N'_{kj} \equiv (1 - \alpha_{kj})\, N_{kj} \int_z \int_\ell \psi_{kj}(z,\ell)\, I_{kj}^{stay}(z,\ell)\, d\ell dz \tag{C.4}$$

- Number of workers initially in sector (k,j) who are fired due to downsizing at the interim stage:

$$W_{kj}^{D}=N'_{kj}\int_{z}\int_{\ell}\widetilde{\psi}_{kj}^{incumbent}\left(z,\ell\right)\left(1-I_{kj}^{hire}\left(z,\ell\right)\right)\left(\ell-L_{kj}\left(z,\ell\right)\right)d\ell dz \tag{C.5}$$

where $\widetilde{\psi}_{kj}^{incumbent}\left(z,\ell\right)$ is the distribution of states in the interim stage among **surviving incumbents**. Note that this is not the same distribution as $\widetilde{\psi}_{kj}\left(z,\ell\right)$ as it does not include entrants. It is obtained as follows:

$$\begin{aligned}\widetilde{\psi}_{kj}^{incumbent}\left(z,\ell\right) &\equiv \frac{\left(1-\alpha_{kj}\right)N_{kj}\psi_{kj}\left(z,\ell\right)I_{kj}^{stay}\left(z,\ell\right)}{N'_{kj}} \\ &= \frac{\psi_{kj}\left(z,\ell\right)I_{kj}^{stay}\left(z,\ell\right)}{\int_{\widetilde{z}}\int_{\widetilde{\ell}}\psi_{kj}\left(\widetilde{z},\widetilde{\ell}\right)I_{kj}^{stay}\left(\widetilde{z},\widetilde{\ell}\right)d\widetilde{\ell}d\widetilde{z}}\end{aligned} \tag{C.6}$$

- Total fraction of workers in the formal sector of sector k who are laid off, conditional on starting the period in a formal firm in sector k :

$$\begin{aligned}\chi_{kf}^{layoff} &= \frac{W_{kf}^{DS}+W_{kf}^{EE}+W_{kf}^{D}}{W_{kf}^{T}} \\ &= \alpha_k+\frac{\left(\begin{array}{c}\left(1-\alpha_k\right)\int_z\int_\ell \ell\psi_{kf}\left(z,\ell\right)I_{kf}^{exit}\left(z,\ell\right)d\ell dz \\ +\left(1-\alpha_k\right)\left(\int_z\int_\ell \psi_{kf}\left(z,\ell\right)I_{kf}^{stay}\left(z,\ell\right)d\ell dz\right)\times \\ \int_z\int_\ell \widetilde{\psi}_{kf}^{incumbent}\left(z,\ell\right)\left(1-I_{kf}^{hire}\left(z,\ell\right)\right)\left(\ell-L_{kf}\left(z,\ell\right)\right)d\ell dz\end{array}\right)}{\int_z\int_\ell \ell\psi_{kf}\left(z,\ell\right)d\ell dz}\end{aligned} \tag{C.7}$$

- Number of firms that start the period as informal firms, but end the period as formal firms (because they formalized).

$$N'_{ki\to f}\equiv\left(1-\alpha_k\right)N_{ki}\int_z\int_\ell \psi_{ki}\left(z,\ell\right)I_{ki}^{change}\left(z,\ell\right)d\ell dz, \tag{C.8}$$

where $\left(1-\alpha_k\right)N_{ki}$ is the mass of firms that survive after the destruction shock hits.

- Distribution of states among firms that switched from informal to formal, in the interim period—before adjusting the labor force.

$$\begin{aligned}\widetilde{\psi}_{ki\to f}\left(z,\ell\right) &\equiv \frac{\left(1-\alpha_k\right)N_{ki}\psi_{ki}\left(z,\ell\right)I_{ki}^{change}\left(z,\ell\right)}{N'_{ki\to f}} \\ &= \frac{\psi_{ki}\left(z,\ell\right)I_{ki}^{change}\left(z,\ell\right)}{\int_{\widetilde{z}}\int_{\widetilde{\ell}}\psi_{ki}\left(\widetilde{z},\widetilde{\ell}\right)I_{ki}^{change}\left(\widetilde{z},\widetilde{\ell}\right)d\widetilde{\ell}d\widetilde{z}}.\end{aligned} \tag{C.9}$$

- Number of workers who start the period in informal firms, but end the period in formal firms (their employers switched to formal, and they were not fired after the interim productivity was realized):

$$W_{k,i\to f}=N'_{ki\to f}\int_z\int_\ell \widetilde{\psi}_{ki\to f}\left(z,\ell\right)\left(\begin{array}{c}\ell\times I_{kf}^{hire}\left(z,\ell\right)+ \\ L_{kf}\left(z,\ell\right)\times\left(1-I_{kf}^{hire}\left(z,\ell\right)\right)\end{array}\right)d\ell dz \tag{C.10}$$

- Fraction of workers who start the period in informal firms, but end the period in formal firms:

$$\chi_{ki\to f}^{change} = \frac{W_{k,i\to f}}{W_{ki}^{T}}$$
$$= \frac{\left(\begin{array}{c} (1-\alpha_k)\left(\int_z \int_\ell \psi_{ki}(z,\ell)\, I_{ki}^{change}(z,\ell)\, d\ell dz\right) \times \\ \int_z \int_\ell \widetilde{\psi}_{ki\to f}(z,\ell) \left(\begin{array}{c} \ell \times I_{kf}^{hire}(z,\ell) + \\ L_{kf}(z,\ell) \times \left(1 - I_{kf}^{hire}(z,\ell)\right) \end{array}\right) d\ell dz \end{array}\right)}{\int_z \int_\ell \ell \psi_{ki}(z,\ell)\, d\ell dz} \tag{C.11}$$

- Number of workers who start the period in informal firms, but their employers switched to formal status:

$$W_{ki}^{SF} = (1-\alpha_k)\, N_{ki} \int_z \int_\ell \ell \psi_{ki}(z,\ell)\, I_{ki}^{change}(z,\ell)\, d\ell dz \tag{C.12}$$

- Fraction of workers who start employed in the informal sector and leave it in the interim period (became unemployed or employer switched to formal):

$$\chi_{ki}^{leave} = \frac{W_{ki}^{DS} + W_{ki}^{EE} + W_{ki}^{SF} + W_{ki}^{D}}{W_{ki}^{T}}$$
$$= \alpha_k + \frac{\left(\begin{array}{c} (1-\alpha_k)\int_z \int_\ell \ell \psi_{ki}(z,\ell)\, I_{ki}^{exit}(z,\ell)\, d\ell dz + \\ (1-\alpha_k)\int_z \int_\ell \ell \psi_{ki}(z,\ell)\, I_{ki}^{change}(z,\ell)\, d\ell dz + \\ (1-\alpha_k)\left(\int_z \int_\ell \psi_{ki}(z,\ell)\, I_{ki}^{stay}(z,\ell)\, d\ell dz\right) \times \\ \int_z \int_\ell \widetilde{\psi}_{ki}^{incumbent}(z,\ell)\left(1 - I_{ki}^{hire}(z,\ell)\right)(\ell - L_{ki}(z,\ell))\, d\ell dz \end{array}\right)}{\int_z \int_\ell \ell \psi_{ki}(z,\ell)\, d\ell dz} \tag{C.13}$$

With these objects, we can define the equilibrium conditions that refer to labor market flows:

$$\chi_{ki}^{leave} L_{ki} = L_u \mu_{ki}^{e} \tag{C.14}$$
$$\chi_{kf}^{layoff} L_{kf} = L_u \mu_{kf}^{e} + L_{ki} \chi_{ki\to f}^{change}. \tag{C.15}$$

These conditions state that the mass of workers in each sector (k, j) cannot be contracting or expanding in equilibrium (expressions (C.14) and (C.15)). Finally, the sum of unemployment and employment levels across sectors equals the total labor force $\overline{L}$:

$$L_{Cf} + L_{Ci} + L_{Sf} + L_{Si} + L_u = \overline{L}. \tag{C.16}$$

We can proceed in a similar way to define the equilibrium flow conditions for firms. The relevant objects follow.

- Fraction of formal firms exiting sector k:

$$\varrho_{kf}^{exit} = \alpha_k + (1-\alpha_k) \int_z \int_\ell I_{kf}^{exit}(z,\ell)\, \psi_{kf}(z,\ell)\, d\ell dz \tag{C.17}$$

- Fraction of informal firms exiting sector k:

$$\varrho_{ki}^{exit} = \alpha_k + (1-\alpha_k) \int_z \int_\ell \left(I_{ki}^{exit}(z,\ell) + I_{ki}^{change}(z,\ell)\right) \psi_{ki}(z,\ell)\, d\ell dz \tag{C.18}$$

- Fraction of informal firms changing status in sector k:

$$\varrho_{ki}^{change} = (1-\alpha_k) \int_z \int_\ell I_{ki}^{change}(z,\ell)\,\psi_{ki}(z,\ell)\,d\ell dz \tag{C.19}$$

Similarly to workers, the mass of firms in each sector (k,j) must be constant in steady state. This means that the inflow of firms must equal the outflow, which can be written as:

$$\varrho_{kf}^{exit} N_{kf} = M_{kf} + \varrho_{ki}^{change} N_{ki}, \tag{C.20}$$

$$\varrho_{ki}^{exit} N_{ki} = M_{ki}. \tag{C.21}$$

D Vacancies

Aggregate vacancies in sector kj are given by:

$$V_{kj} = N_{kj} \int_z \int_\ell v_{kj}(z,\ell)\,\widetilde{\psi}_{ki}(z,\ell)\,d\ell dz + \frac{M_{kj}}{\mu^v} \tag{D.1}$$

where $v_{kj}(z,\ell)$ is the number of vacancies a firm with productivity z and labor force ℓ posts and $\frac{M_{kj}}{\mu^v}$ is the number of vacancies posted at entry (and before adjustment in stage 2).

E Unemployment Benefits / Tax Collection / Transfers

Government Revenue is given by the sum of value-added taxes, payroll taxes, firing costs and import taxes:

$$\begin{aligned} G_{Rev} = & \sum_k N_{kf}\tau_y \int_z \int_{\ell'} VA_k(z,\ell')\,\widehat{\psi}_{kf}(z,\ell')\,d\ell' dz \\ & + \sum_k N_{kf}\tau_w \int_z \int_{\ell'} \max\{w_{kf}(z,\ell'), \underline{w}\}\,\ell\widehat{\psi}_{kf}(z,\ell')\,d\ell' dz \\ & + \sum_k N_{kf}\kappa \int_z \int_\ell \widetilde{\psi}_{kf}(z,\ell)\,(\ell - L_{kf}(z,\ell))\left(1 - I_{kf}^{hire}(z,\ell)\right) d\ell dz \\ & + (\tau_a - 1)\,\frac{D_{H,C}\,(\epsilon\tau_a\tau_c)^{1-\sigma_C}}{\tau_a}. \end{aligned} \tag{E.1}$$

Government spending with unemployment insurance is given by:

$$G_{UI} = b^u \times \underbrace{\sum_k \left(W_{kf}^{DS} + W_{kf}^{EE} + W_{kf}^{D}\right)}_{\text{mass of \textbf{formal} workers who transition to unemployment}} \tag{E.2}$$

We impose that

$$T = G_{Rev} - G_{UI} \geq 0 \tag{E.3}$$

and that T is rebated to consumers.

Important note: part of the aggregate informality costs

$$\sum_k N_{ki} \int_z \int_{\ell'} p_{ki}(\ell')\,R_k(z,\ell')\,\widehat{\psi}_{ki}(z,\ell')\,d\ell' dz \tag{E.4}$$

should be considered government revenue as these consist of fines. However, part of these costs should not, as they consist of opportunity costs associated with informality. Therefore, we do not add these costs to government revenue. However, the model redistributes these costs to consumers. One way to view this procedure is that these costs affect/distort the decisions of firms, but we do not consider these costs as wasted resources.

F Service Sector Market Clearing

Service sector goods are used for final consumption (consumers spend $(1-\zeta) I$ on it), intermediate inputs (firms spend X_S^{int} on it) and as inputs for hiring costs, fixed costs and entry costs (and fixed costs of exporting). The average (per firm) hiring costs in sector (k, j):

$$\overline{H}_{kj} = \int_z \int_\ell H_{kj}\left(\ell, L_{kj}\left(z,\ell\right)\right) I_{kj}^{hire}\left(z,\ell\right) \widetilde{\psi}_{kj}\left(z,\ell\right) d\ell dz, \tag{F.1}$$

and the fraction of manufacturing-sector goods firms that export is given by:

$$\mu_x = \int_z \int_{\ell'} \widehat{\psi}_{Cf}\left(z,\ell'\right) I_C^x\left(z,\ell'\right) d\ell' dz \tag{F.2}$$

Expenditure on entry and hiring costs, fixed costs of operations and export costs are given by:

$$E_S = \sum_{k=C,S\ ;\ j=i,f} N_{kj}\left(\overline{H}_{kj} + \overline{c}_{kj}\right) + N_{Cf}\mu_x f_x + \sum_{k=C,S} M_k c_{e,k} \tag{F.3}$$

G Aggregate Income

Aggregate income is given by total wages, government transfers and total profits:

$$\begin{aligned}
I = & \sum_k N_{ki} \int_z \int_{\ell'} w_{ki}\left(z,\ell'\right) \ell' \widehat{\psi}_{ki}\left(z,\ell'\right) d\ell' dz \\
& + \sum_k N_{kf} \int_z \int_{\ell'} \max\left\{w_{kf}\left(z,\ell'\right), \underline{w}\right\} \ell' \widehat{\psi}_{kf}\left(z,\ell'\right) d\ell' dz \\
& + \sum_k N_{ki} \int_z \int_{\ell'} \widetilde{\pi}_{ki}\left(z,\ell'\right) \widehat{\psi}_{ki}\left(z,\ell'\right) d\ell' dz \\
& + \sum_k N_{kf} \int_z \int_{\ell'} \widetilde{\pi}_{kf}\left(z,\ell'\right) \widehat{\psi}_{kf}\left(z,\ell'\right) d\ell' dz \\
& + G_{Rev} \\
& + \sum_k N_{ki} \int_z \int_{\ell'} p_{ki}\left(\ell'\right) R_k\left(z,\ell'\right) \widehat{\psi}_{ki}\left(z,\ell'\right) d\ell' dz \\
& - \sum_k N_{kf} \kappa \int_z \int_\ell \widetilde{\psi}_{kf}\left(z,\ell\right)\left(\ell - L_{kf}\left(z,\ell\right)\right)\left(1 - I_{kf}^{hire}\left(z,\ell\right)\right) d\ell dz \\
& - \sum_{k=C,S\ ;\ j=i,f} N_{kj} \overline{H}_{kj} \\
& - \sum_k M_k c_{e,k},
\end{aligned} \tag{G.1}$$

where profits $\widetilde{\pi}$ are computed **before** subtracting hiring costs.

H Trade Balance

Trade balance implies that total imports must equal total exports, which is given by:

$$\frac{D_{H,C}\left(\epsilon\tau_a\tau_c\right)^{1-\sigma_C}}{\tau_a} = Exports \tag{H.1}$$

I Worker Value Functions

Present value of a formal job at a firm with state (z,ℓ') at the production stage

$$\overline{J}^{e}_{kf}\left(z,\ell'\right) = w_{kf}\left(z,\ell'\right)$$
$$+\frac{1-\alpha_k}{1+r}E_{z'|z}\left(\begin{array}{c}\left(\frac{\alpha_k}{1-\alpha_k}+I^{exit}_{kf}\left(z',\ell'\right)\right)\times\left(b+b_u+\frac{1}{1+r}J^u\right)\\ +I^{stay}_{kf}\left(z',\ell'\right)\times I^{contract}_{kf}\left(z',\ell'\right)\times p^{fire}_{kf}\left(z',\ell'\right)\times\left(b+b_u+\frac{1}{1+r}J^u\right)\\ +I^{stay}_{kf}\left(z',\ell'\right)\times I^{contract}_{kf}\left(z',\ell'\right)\times\left(1-p^{fire}_{kf}\left(z',\ell'\right)\right)\times\overline{J}^{e}_{kf}\left(z',L_{kf}\left(z',\ell'\right)\right)\\ +I^{stay}_{kf}\left(z',\ell'\right)\times I^{expand}_{kf}\left(z',\ell'\right)\times\overline{J}^{e}_{kf}\left(z',L_{kf}\left(z',\ell'\right)\right)\end{array}\right)$$

$$p^{fire}_{kf}\left(z',\ell'\right)\equiv\frac{\ell'-L_{kf}\left(z',\ell'\right)}{\ell'}$$
$$I^{contract}_{kf}\left(z',\ell'\right)\equiv I\left(L_{kf}\left(z',\ell'\right)<\ell'\right)$$
$$I^{expand}_{kf}\left(z',\ell'\right)\equiv I\left(L_{kf}\left(z',\ell'\right)\geq\ell'\right)$$

Rewriting:

$$\begin{aligned}
&\overline{J}^{e}_{kf}\left(z,\ell'\right) = w_{kf}\left(z,\ell'\right)\\
&+\frac{1-\alpha_k}{1+r}\left(\frac{\alpha_k}{1-\alpha_k}+E_{z'|z}\left[I^{exit}_{kf}\left(z',\ell'\right)\right]\right)\times\left(b+b_u+\frac{1}{1+r}J^u\right)\\
&+\frac{1-\alpha_k}{1+r}E_{z'|z}\left[I^{stay}_{kf}\left(z',\ell'\right)\times I^{contract}_{kf}\left(z',\ell'\right)\times p^{fire}_{kf}\left(z',\ell'\right)\right]\times\left(b+b_u+\frac{1}{1+r}J^u\right)\\
&+\frac{1-\alpha_k}{1+r}E_{z'|z}\left[I^{stay}_{kf}\left(z',\ell'\right)\times I^{contract}_{kf}\left(z',\ell'\right)\times\left(1-p^{fire}_{kf}\left(z',\ell'\right)\right)\times\overline{J}^{e}_{kf}\left(z',L_{kf}\left(z',\ell'\right)\right)\right]\\
&+\frac{1-\alpha_k}{1+r}E_{z'|z}\left[I^{stay}_{kf}\left(z',\ell'\right)\times I^{expand}_{kf}\left(z',\ell'\right)\times\overline{J}^{e}_{kf}\left(z',L_{kf}\left(z',\ell'\right)\right)\right]
\end{aligned}$$

It will be convenient to work with:

$$J^{e}_{kf}\left(z,\ell'\right)\equiv\left(1+r\right)\left(\overline{J}^{e}_{kf}\left(z,\ell'\right)-w_{kf}\left(z,\ell'\right)\right) \tag{I.1}$$

Present value of an informal job at a firm with state (z,ℓ') at the production stage

$$\overline{J}^{e}_{ki}\left(z,\ell'\right) = w_{ki}\left(z,\ell'\right)$$

$$+\frac{(1-\alpha_k)}{1+r}E_{z'|z}\left(\begin{array}{c} \left(\frac{\alpha_k}{(1-\alpha_k)} + I^{exit}_{ki}\left(z',\ell'\right)\right)\times\left(b+\frac{1}{1+r}J^u\right) \\ +I^{stay}_{ki}\left(z',\ell'\right)\times I^{contract}_{ki}\left(z',\ell'\right)\times p^{fire}_{ki}\left(z',\ell'\right)\times\left(b+\frac{1}{1+r}J^u\right) \\ +I^{stay}_{ki}\left(z',\ell'\right)\times I^{contract}_{ki}\left(z',\ell'\right)\times\left(1-p^{fire}_{ki}\left(z',\ell'\right)\right)\times\overline{J}^{e}_{ki}\left(z',L_{ki}\left(z',\ell'\right)\right) \\ +I^{stay}_{ki}\left(z',\ell'\right)\times I^{expand}_{ki}\left(z',\ell'\right)\times\overline{J}^{e}_{ki}\left(z',L_{ki}\left(z',\ell'\right)\right) \\ +I^{change}_{ki}\left(z',\ell'\right)\times I^{contract}_{kf}\left(z',\ell'\right)\times p^{fire}_{kf}\left(z',\ell'\right)\times\left(b+b_u+\frac{1}{1+r}J^u\right) \\ +I^{change}_{ki}\left(z',\ell'\right)\times I^{contract}_{kf}\left(z',\ell'\right)\times\left(1-p^{fire}_{kf}\left(z',\ell'\right)\right)\times\overline{J}^{e}_{kf}\left(z',L_{kf}\left(z',\ell'\right)\right) \\ +I^{change}_{ki}\left(z',\ell'\right)\times I^{expand}_{kf}\left(z',\ell'\right)\times\overline{J}^{e}_{kf}\left(z',L_{kf}\left(z',\ell'\right)\right) \end{array}\right)$$

$$p^{fire}_{ki}\left(z',\ell'\right)\equiv\frac{\ell'-L_{ki}\left(z',\ell'\right)}{\ell'}$$

$$I^{contract}_{ki}\left(z',\ell'\right)\equiv I\left(L_{ki}\left(z',\ell'\right)<\ell'\right)$$

$$I^{expand}_{ki}\left(z',\ell'\right)\equiv I\left(L_{ki}\left(z',\ell'\right)\geq\ell'\right)$$

Rewriting

$$\begin{aligned}
&\overline{J}^{e}_{ki}\left(z,\ell'\right) = w_{ki}\left(z,\ell'\right) \\
&+\frac{(1-\alpha_k)}{1+r}\left(\frac{\alpha_k}{(1-\alpha_k)}+E_{z'|z}\left[I^{exit}_{ki}\left(z',\ell'\right)\right]\right)\times\left(b+\frac{1}{1+r}J^u\right) \\
&+\frac{(1-\alpha_k)}{1+r}E_{z'|z}\left[I^{stay}_{ki}\left(z',\ell'\right)\times I^{contract}_{ki}\left(z',\ell'\right)\times p^{fire}_{ki}\left(z',\ell'\right)\right]\times\left(b+\frac{1}{1+r}J^u\right) \\
&+\frac{(1-\alpha_k)}{1+r}E_{z'|z}\left[I^{stay}_{ki}\left(z',\ell'\right)\times I^{contract}_{ki}\left(z',\ell'\right)\times\left(1-p^{fire}_{ki}\left(z',\ell'\right)\right)\times\overline{J}^{e}_{ki}\left(z',L_{ki}\left(z',\ell'\right)\right)\right] \\
&+\frac{(1-\alpha_k)}{1+r}E_{z'|z}\left[I^{stay}_{ki}\left(z',\ell'\right)\times I^{expand}_{ki}\left(z',\ell'\right)\times\overline{J}^{e}_{ki}\left(z',L_{ki}\left(z',\ell'\right)\right)\right] \\
&+\frac{(1-\alpha_k)}{1+r}E_{z'|z}\left[I^{change}_{ki}\left(z',\ell'\right)\times I^{contract}_{kf}\left(z',\ell'\right)\times p^{fire}_{kf}\left(z',\ell'\right)\right]\times\left(b+b_u+\frac{1}{1+r}J^u\right) \\
&+\frac{(1-\alpha_k)}{1+r}E_{z'|z}\left[I^{change}_{ki}\left(z',\ell'\right)\times I^{contract}_{kf}\left(z',\ell'\right)\times\left(1-p^{fire}_{kf}\left(z',\ell'\right)\right)\times\overline{J}^{e}_{kf}\left(z',L_{kf}\left(z',\ell'\right)\right)\right] \\
&+\frac{(1-\alpha_k)}{1+r}E_{z'|z}\left[I^{change}_{ki}\left(z',\ell'\right)\times I^{expand}_{kf}\left(z',\ell'\right)\times\overline{J}^{e}_{kf}\left(z',L_{kf}\left(z',\ell'\right)\right)\right]
\end{aligned}$$

As before, it will be convenient to work with:

$$J^{e}_{ki}\left(z,\ell'\right)\equiv(1+r)\left(\overline{J}^{e}_{ki}\left(z,\ell'\right)-w_{ki}\left(z,\ell'\right)\right) \tag{I.2}$$

Value of Search

$$J^u=\sum_{k,j}\mu^e_{kj}\int_{\ell}\int_{z'}\overline{J}^{e}_{kj}\left(z,L_{kj}\left(z,\ell\right)\right)g_{kj}\left(z,\ell\right)dzd\ell+\left(1-\sum_{k,j}\mu^e_{kj}\right)\left(b+\frac{1}{1+r}J^u\right) \tag{I.3}$$

$$\widetilde{g}_{kj}(z,\ell) = N_{kj}\widetilde{\psi}_{kj}(z,\ell)\, v_{kj}(z,\ell) + \mathcal{I}[\ell=1]\frac{M_{kj}}{\mu^{\upsilon}}\psi^{e}_{kj}(z)$$
$$g_{kj}(z,\ell) = \frac{\widetilde{g}_{kj}(z,\ell)}{\int_z\int_\ell \widetilde{g}_{kj}(z,\ell)\, d\ell dz} = \frac{\widetilde{g}_{kj}(z,\ell)}{V_{kj}}$$

J Real Income Decompositions: Small Trade Cost or Technology Changes

J.1 The Mechanical Effect

This appendix derives the mechanical effect for real income in equation (37), $\Delta \log(\text{Real Income})^{ME}$. We derive the mechanical effect for both a small shock in iceberg trade cost and for a small shock in aggregate productivity.

Before we start the derivations, note that the price of sector C's composite good is given by:

$$P_C = \left(\int p_{Cf}(z,\ell)^{1-\sigma_C}\,\widehat{\psi}_{Cf}(z,\ell)\, N_{Cf} dz d\ell + \int p_{Ci}(z,\ell)^{1-\sigma_C}\,\widehat{\psi}_{Ci}(z,\ell)\, N_{Ci} dz d\ell + (\epsilon\tau_a\tau_c)^{1-\sigma_C}\right)^{\frac{1}{1-\sigma_C}},$$

and the price of sector S's composite good is given by:

$$P_S = \left(\int p_{Sf}(z,\ell)^{1-\sigma_S}\,\widehat{\psi}_{Sf}(z,\ell)\, N_{Sf} dz d\ell + \int p_{Si}(z,\ell)^{1-\sigma_S}\,\widehat{\psi}_{Si}(z,\ell)\, N_{Si} dz d\ell\right)^{\frac{1}{1-\sigma_S}},$$

where ϵ is the exchange rate, $p_{kj}(z,\ell)$ is the price charged by a firm with state (z,ℓ), $\widehat{\psi}_{kj}$ is the distribution of states at the production stage, and N_{kj} is the mass of active firms in sector k and status $j \in \{i, f\}$.

Recall that $P_{C,H}^{1-\sigma_C} \equiv \int p_{Cf}(z,\ell)^{1-\sigma_C}\,\widehat{\psi}_{Cf}(z,\ell)\, N_{Cf} dz d\ell + \int p_{Ci}(z,\ell)^{1-\sigma_C}\,\widehat{\psi}_{Ci}(z,\ell)\, N_{Ci} dz d\ell$ is the domestic component of $P_C^{1-\sigma_C}$, and $P_{C,F}^{1-\sigma_C} \equiv (\epsilon\tau_a\tau_c)^{1-\sigma_C}$ is the foreign component.

Conditional on wages $\widetilde{w}_{kf}(z,\ell) \equiv (1+\tau_w)\min\{w_{kf}(z,\ell), \underline{w}\}$ for formal firms, $\widetilde{w}_{ki}(z,\ell) \equiv w_{ki}(z,\ell)$ for informal firms, and the price of intermediates P_k^m, the minimum cost of producing 1 unit of output in sector kj is given by:

$$c_{kj}(z,\ell) = \frac{1}{\varsigma z}\left(\frac{\widetilde{w}_{kj}(z,\ell)}{\delta_k}\right)^{\delta_k}\left(\frac{P_k^m}{1-\delta_k}\right)^{1-\delta_k}, \tag{J.1}$$

where ς is a productivity shifter common to all firms in the economy (in the Benchmark economy, $\varsigma = 1$). This shifter will be used later when we derive the mechanical effect for an aggregate productivity shifter.

Write prices charged by a firm with state (z,ℓ) in sector $k \in \{C, S\}$ and status $j \in \{i, f\}$ as

$$p_{kj}(z,\ell) = \mu_{kj}(z,\ell)\, c_{kj}(z,\ell),$$

where $\mu_{kj}(z,\ell)$ is a wedge over the unit cost. To compute the mechanical effect of a trade cost shock $\tau_c \to \tau_c'$ or a technology shock $\varsigma \to \varsigma'$ we impose that all wedges $\mu_{kj}(z,\ell)$, allocations, wages and exchange rate are fixed at the initial equilibrium.

J.1.1 Small Change in Iceberg Trade Costs

Let us first derive the mechanical effect of a small change in trade cost τ_c. In this derivation, we fix the productivity shifter $\varsigma = 1$.

In the following derivations the following objects are held fixed: $\widehat{\psi}_{kj}$, N_{kj}, $\widetilde{w}_{kj}$ and ϵ. Taking the derivative of P_C with respect to τ_c leads to:

$$\frac{\partial P_C}{\partial \tau_c}\frac{\tau_c}{P_C} = P_C^{\sigma_C - 1}\left(\begin{array}{c} \int p_{Cf}\left(z,\ell\right)^{1-\sigma_C}\frac{\partial p_{Cf}(z,\ell)}{\partial \tau_c}\frac{\tau_c}{p_{Cf}(z,\ell)}\widehat{\psi}_{Cf}\left(z,\ell\right)N_{Cf}dzd\ell + \\ \int p_{Ci}\left(z,\ell\right)^{1-\sigma_C}\frac{\partial p_{Ci}(z,\ell)}{\partial \tau_c}\frac{\tau_c}{p_{Ci}(z,\ell)}\widehat{\psi}_{Ci}\left(z,\ell\right)N_{Ci}dzd\ell + \left(\epsilon\tau_a\tau_c\right)^{1-\sigma_C}\end{array}\right) \tag{J.2}$$

However, note that

$$\begin{aligned}\frac{\partial p_{kj}\left(z,\ell\right)}{\partial \tau_c} &= \mu_{kj}\left(z,\ell\right)\frac{\partial c_{kj}\left(z,\ell\right)}{\partial \tau_c} \\ &= \left(1-\delta_k\right)\frac{p_{kj}\left(z,\ell\right)}{P_k^m}\frac{\partial P_k^m}{\partial \tau_c},\end{aligned}$$

implying that

$$\frac{\partial p_{kj}\left(z,\ell\right)}{\partial \tau_c}\frac{\tau_c}{p_{kj}\left(z,\ell\right)} = \left(1-\delta_k\right)\frac{\partial P_k^m}{\partial \tau_c}\frac{\tau_c}{P_k^m}.$$

Substituting in (J.2) and simplifying:

$$\frac{\partial P_C}{\partial \tau_c}\frac{\tau_c}{P_C} = \left(1-\delta_C\right)\frac{\partial P_C^m}{\partial \tau_c}\frac{\tau_c}{P_C^m}\frac{P_{C,H}^{1-\sigma_C}}{P_C^{1-\sigma_C}} + \frac{\left(\epsilon\tau_a\tau_c\right)^{1-\sigma_C}}{P_C^{1-\sigma_C}}.$$

Using equation (32) we have that:

$$\frac{\left(\epsilon\tau_a\tau_c\right)^{1-\sigma_C}}{P_C^{1-\sigma_C}} = \frac{\tau_a Imports}{X_C} \equiv s.$$

And because

$$P_C^{1-\sigma_C} = P_{C,H}^{1-\sigma_C} + \left(\epsilon\tau_a\tau_c\right)^{1-\sigma_C},$$

We have:

$$\frac{P_{C,H}^{1-\sigma_C}}{P_C^{1-\sigma_C}} = 1 - s.$$

Therefore:

$$\frac{\partial P_C}{\partial \tau_c}\frac{\tau_c}{P_C} = \left(1-\delta_C\right)\frac{\partial P_C^m}{\partial \tau_c}\frac{\tau_c}{P_C^m}\left(1-s\right) + s.$$

Similar derivations for the price of the S sector composite good imply:

$$\frac{\partial P_S}{\partial \tau_c}\frac{\tau_c}{P_S} = \left(1-\delta_S\right)\frac{\partial P_S^m}{\partial \tau_c}\frac{\tau_c}{P_S^m}.$$

Now, remember that

$$P_k^m = \left(\frac{P_C}{\lambda_k}\right)^{\lambda_k}\left(\frac{P_S}{1-\lambda_k}\right)^{1-\lambda_k},$$

implying:

$$\frac{\partial P_k^m}{\partial \tau_c}\frac{\tau_c}{P_k^m} = \lambda_k \frac{\partial P_C}{\partial \tau_c}\frac{\tau_c}{P_C} + (1-\lambda_k)\frac{\partial P_S}{\partial \tau_c}\frac{\tau_c}{P_S}.$$

Let

$$\varrho_{C,\tau} \equiv \frac{\partial P_C}{\partial \tau_c}\frac{\tau_c}{P_C},$$

$$\varrho_{S,\tau} \equiv \frac{\partial P_S}{\partial \tau_c}\frac{\tau_c}{P_S}.$$

And so we have the following system in matrix format:

$$\begin{bmatrix} 1-(1-\delta_C)\lambda_C(1-s) & -(1-\delta_C)(1-\lambda_C)(1-s) \\ -(1-\delta_S)\lambda_S & 1-(1-\delta_S)(1-\lambda_S) \end{bmatrix}\begin{bmatrix} \varrho_{C,\tau} \\ \varrho_{S,\tau} \end{bmatrix} = \begin{bmatrix} s \\ 0 \end{bmatrix}$$

Once we solve for this system in elasticities $\varrho_{C,\tau}$ and $\varrho_{S,\tau}$, we obtain changes in price indices:

$$d\log P_C = \varrho_{C,\tau} d\log \tau_c,$$

$$d\log P_S = \varrho_{S,\tau} d\log \tau_c.$$

And the resulting first order change in real income given a small shock $d\log \tau_c$, and fixed wages and aggregate income, is given by:

$$-d\log P = -\left(\zeta \varrho_{C,\tau} + (1-\zeta)\varrho_{S,\tau}\right) d\log \tau_c.$$

J.1.2 Small Aggregate Productivity Shock

Now, let us obtain the mechanical effect for an infitesimal change in the common productivity shifter ς. Note that:

$$\frac{\partial P_C}{\partial \varsigma}\frac{\varsigma}{P_C} = P_C^{\sigma_C - 1}\left(\begin{array}{c} \int p_{Cf}(z,\ell)^{1-\sigma_C}\frac{\partial p_{Cf}(z,\ell)}{\partial \varsigma}\frac{\varsigma}{p_{Cf}(z,\ell)}\widehat{\psi}_{Cf}(z,\ell) N_{Cf} dz d\ell \\ +\int p_{Ci}(z,\ell)^{-\sigma_C}\frac{\partial p_{Ci}(z,\ell)}{\partial \varsigma}\frac{\varsigma}{p_{Ci}(z,\ell)}\widehat{\psi}_{Ci}(z,\ell) N_{Ci} dz d\ell \end{array}\right)$$

Using (J.1) we obtain:

$$\frac{\partial p_{kj}(z,\ell)}{\partial \varsigma}\frac{\varsigma}{p_{kj}(z,\ell)} = (1-\delta_k)\frac{\partial P_k^m}{\partial \varsigma}\frac{\varsigma}{P_k^m} - 1,$$

implying:

$$\frac{\partial P_C}{\partial \varsigma}\frac{\tau_c}{P_C} = (1-s)\left((1-\delta_C)\frac{\partial P_C^m}{\partial \varsigma}\frac{\varsigma}{P_C^m} - 1\right)$$

Note that:

$$\frac{\partial P_k^m}{\partial \varsigma}\frac{\varsigma}{P_k^m} = \lambda_k \frac{\partial P_C}{\partial \varsigma}\frac{\varsigma}{P_C} + (1-\lambda_k)\frac{\partial P_S}{\partial \varsigma}\frac{\varsigma}{P_S},$$

And let

$$\varrho_{C,\varsigma} \equiv \frac{\partial P_C}{\partial \varsigma}\frac{\varsigma}{P_C},$$
$$\varrho_{S,\varsigma} \equiv \frac{\partial P_S}{\partial \varsigma}\frac{\varsigma}{P_C}.$$

We then obtain the following system in matrix format:

$$\begin{bmatrix} 1-(1-s)(1-\delta_C)\lambda_C & -(1-s)(1-\delta_C)(1-\lambda_C) \\ -(1-\delta_S)\lambda_S & 1-(1-\delta_S)(1-\lambda_S) \end{bmatrix}\begin{bmatrix} \varrho_{C,\varsigma} \\ \varrho_{S,\varsigma} \end{bmatrix} = \begin{bmatrix} -(1-s) \\ -1 \end{bmatrix}$$

Once we solve for this system in elasticities $\varrho_{C,\varsigma}$ and $\varrho_{S,\varsigma}$, we obtain changes in price indices:

$$d\log P_C = \varrho_{C,\varsigma} d\log\varsigma,$$
$$d\log P_S = \varrho_{S,\varsigma} d\log\varsigma.$$

And the resulting first order change in real income, for fixed wages and aggregate income, is given by:

$$-d\log P = -\left(\zeta\varrho_{C,\varsigma} + (1-\zeta)\varrho_{S,\varsigma}\right) d\log\varsigma.$$

J.2 Distortion-Free Economy

We now turn our attention to the derivation of real income changes in a version of our economy without distortions—see equation (39). More precisely, consider a version of our model with the following modifications:

- Firms price at marginal cost and make zero profits.
- No fixed production or export costs, no search, no adjustment costs, no tariffs, no taxes, no labor market regulations.
- Perfect labor mobility across firms and sectors, implying a single wage w.
- Given no fixed costs, there are no extensive margin adjustments (no entry, exit or selection into export markets).
- No tariffs: $\tau_a = 1$.

The production function in sector $k \in \{C, S\}$ is still given by $q_k = z\ell^{\delta_k}\iota_k^{1-\delta_k}$. Firms are heterogeneous in their productivity z. Let $\psi_k(z)$ denote the mass of firms in sector k with productivity z. This mass is fixed given the absence of fixed costs.

We still keep our small open economy structure, with a fixed mass of imported varieties and fixed foreign demand D_F^*.

Under these assumptions, there are no distortions in the economy.

Firms price at marginal cost. Therefore, the price charged by a firm with productivity z in sector k is given by:

$$p_k(z) = \frac{1}{\varsigma z}\left(\frac{w}{\delta_k}\right)^{\delta_k}\left(\frac{P_k^m}{1-\delta_k}\right)^{1-\delta_k},$$

where w is the wage rate common across all firms and sectors and will be fixed at $w = 1$. ς is the aggregate productivity shock ($\varsigma = 1$ in the baseline). As before, P_k^m is the price of sector-k's intermediate input bundle and is given by:

$$P_k^m = \left(\frac{P_C}{\lambda_k}\right)^{\lambda_k}\left(\frac{P_S}{1-\lambda_k}\right)^{1-\lambda_k}.$$

As before, imports are given by:

$$Imports = \frac{X_C}{P_C^{1-\sigma_C}}(\epsilon\tau_c)^{1-\sigma_C},$$

but now $X_C = \zeta wL + X_C^{int}$ and $\tau_a = 1$. wL is total aggregate income and X_C^{int} is total expenditure on sector C's goods to form intermediate inputs. That is:

$$X_C^{int} = \lambda_C(1-\delta_C)Rev_C + \lambda_S(1-\delta_S)Rev_S, \tag{J.3}$$

where Rev_C is aggregat revenue in sector C and Rev_S is aggregate revenue in sector S. In sector S we have:

$$X_S^{int} = (1-\lambda_C)(1-\delta_C)Rev_C + (1-\lambda_S)(1-\delta_S)Rev_S. \tag{J.4}$$

In the next few lines, we show that because labor is the numeraire, X_C is also fixed.

We can write equations (J.3) and (J.4) in matrix format:

$$\begin{bmatrix} X_C^{int} \\ X_S^{int} \end{bmatrix} = \begin{bmatrix} \lambda_C(1-\delta_C) & \lambda_S(1-\delta_S) \\ (1-\lambda_C)(1-\delta_C) & (1-\lambda_S)(1-\delta_S) \end{bmatrix}\begin{bmatrix} Rev_C \\ Rev_S \end{bmatrix}$$

Market Clearing Dictates

$$\zeta wL + X_C^{int} = Rev_C$$

$$(1-\zeta)wL + X_S^{int} = Rev_S$$

Substituting (J.3) and (J.4) into the market clearing equations, we obtain:

$$\zeta wL = (1-\lambda_C(1-\delta_C))Rev_C - \lambda_S(1-\delta_S)Rev_S$$

$$(1-\zeta)wL = -(1-\lambda_C)(1-\delta_C)Rev_C + (1-(1-\lambda_S)(1-\delta_S))Rev_S$$

In matrix format:

$$\begin{bmatrix} 1-\lambda_C(1-\delta_C) & -\lambda_S(1-\delta_S) \\ -(1-\lambda_C)(1-\delta_C) & 1-(1-\lambda_S)(1-\delta_S) \end{bmatrix}\begin{bmatrix} Rev_C \\ Rev_S \end{bmatrix} = \begin{bmatrix} \zeta wL \\ (1-\zeta)wL \end{bmatrix}$$

This implies that Rev_C and Rev_S are linear functions of wL, which are fixed as labor is the numeraire.

Therefore, as we take derivatives with respect to τ_c or ς, $X_C = \zeta wL + X_C^{int}$ is fixed.

J.2.1 Small Change in Iceberg Trade Costs

Consider the welfare impact of a small shock in τ_c. Impose $\varsigma = 1$.

The price of sector C's composite good is given by:

$$P_C = \left(\int p_C(z)^{1-\sigma_C} \psi_C(z)\, dz + (\epsilon \tau_c)^{1-\sigma_C} \right)^{\frac{1}{1-\sigma_C}}$$

It will be useful to write

$$P_C^{1-\sigma_C} = P_{C,H}^{1-\sigma_C} + (\epsilon \tau_c)^{1-\sigma_C}$$

where

$$P_{C,H}^{1-\sigma_C} \equiv \int p_C(z)^{1-\sigma_C} \psi_C(z)\, dz$$

is the domestic component of the price of sector C's composite good.

The price of sector S's composite good is given by:

$$P_S = \left(\int p_S(z)^{1-\sigma_S} \psi_S(z)\, dz \right)^{\frac{1}{1-\sigma_S}}.$$

The following equations will be used in the derivation of welfare impacts:

$$\frac{\partial p_k(z)}{\partial \tau_c} \frac{\tau_c}{p_k(z)} = (1-\delta_k) \frac{\partial P_k^m}{\partial \tau_c} \frac{\tau_c}{P_k^m}$$

$$\frac{\partial P_k^m}{\partial \tau_c} \frac{\tau_c}{P_k^m} = \lambda_k \frac{\partial P_C}{\partial \tau_c} \frac{\tau_c}{P_C} + (1-\lambda_k) \frac{\partial P_S}{\partial \tau_c} \frac{\tau_c}{P_S}$$

$$s \equiv \frac{(\epsilon \tau_c)^{1-\sigma_C}}{P_C^{1-\sigma_C}} = \frac{Imports}{X_C}$$

$$\frac{P_{C,H}^{1-\sigma_C}}{P_C^{1-\sigma_C}} = 1 - \frac{(\epsilon \tau_c)^{1-\sigma_C}}{P_C^{1-\sigma_C}} = 1 - s$$

Differentiate P_C with respect to τ_c to obtain the following expression:

$$\frac{\partial P_C}{\partial \tau_c} \frac{\tau_c}{P_C} = (1-s)(1-\delta_C)\lambda_C \frac{\partial P_C}{\partial \tau_c} \frac{\tau_c}{P_C} + (1-s)(1-\delta_C)(1-\lambda_C) \frac{\partial P_S}{\partial \tau_c} \frac{\tau_c}{P_S} + s\left(1 + \frac{\partial \epsilon}{\partial \tau_c} \frac{\tau_c}{\epsilon}\right)$$

Similarly, differentiate P_S with respect to τ_c to obtain:

$$\frac{\partial P_S}{\partial \tau_c}\frac{\tau_c}{P_S} = (1-\delta_S)\,\lambda_S \frac{\partial P_C}{\partial \tau_c}\frac{\tau_c}{P_C} + (1-\delta_S)\,(1-\lambda_S)\,\frac{\partial P_S}{\partial \tau_c}\frac{\tau_c}{P_S}.$$

We now compute $\frac{\partial \epsilon}{\partial \tau_c}\frac{\tau_c}{\epsilon}$ using the balanced trade condition:

$$Imports = \frac{X_C}{P_C^{1-\sigma_C}}\,(\epsilon\tau_c)^{1-\sigma_C} = Exports = \epsilon D_F^* \tau_c^{1-\sigma_C}\int p_C\,(z)^{1-\sigma_C}\,\psi_C\,(z)\,dz.$$

We proceed by differentiating both sides with respect to τ_c, invoking that X_C is fixed given the choice of numeraire. Putting the derivatives of the LHS and RHS together, and imposing that $Imports = Exports$, we obtain an expression for $\frac{\partial \epsilon}{\partial \tau_c}\frac{\tau_c}{\epsilon}$:

$$\begin{aligned}\frac{\partial \epsilon}{\partial \tau_c}\frac{\tau_c}{\epsilon} = &\left(\frac{(\sigma_C-1)+(\sigma_C-1)\,(1-\delta_C)\,\lambda_C}{\sigma_C}\right)\frac{\partial P_C}{\partial \tau_c}\frac{\tau_c}{P_C} \\ &+\frac{(\sigma_C-1)\,(1-\delta_C)\,(1-\lambda_C)}{\sigma_C}\frac{\partial P_S}{\partial \tau_c}\frac{\tau_c}{P_S}.\end{aligned}$$

Therefore, we have the following system in elasticities $\frac{\partial P_C}{\partial \tau_c}\frac{\tau_c}{P_C}$, $\frac{\partial P_S}{\partial \tau_c}\frac{\tau_c}{P_S}$ and $\frac{\partial \epsilon}{\partial \tau_c}\frac{\tau_c}{\epsilon}$:

$$\begin{aligned}\frac{\partial P_C}{\partial \tau_c}\frac{\tau_c}{P_C} = &(1-s)\,(1-\delta_C)\,\lambda_C\frac{\partial P_C}{\partial \tau_c}\frac{\tau_c}{P_C} \\ &+(1-s)\,(1-\delta_C)\,(1-\lambda_C)\,\frac{\partial P_S}{\partial \tau_c}\frac{\tau_c}{P_S} + s\left(1+\frac{\partial \epsilon}{\partial \tau_c}\frac{\tau_c}{\epsilon}\right)\end{aligned}$$

$$\frac{\partial P_S}{\partial \tau_c}\frac{\tau_c}{P_S} = (1-\delta_S)\,\lambda_S\frac{\partial P_C}{\partial \tau_c}\frac{\tau_c}{P_C} + (1-\delta_S)\,(1-\lambda_S)\,\frac{\partial P_S}{\partial \tau_c}\frac{\tau_c}{P_S}$$

$$\begin{aligned}\frac{\partial \epsilon}{\partial \tau_c}\frac{\tau_c}{\epsilon} = &\frac{(\sigma_C-1)}{\sigma_C}\frac{\partial P_C}{\partial \tau_c}\frac{\tau_c}{P_C} \\ &+\frac{(\sigma_C-1)\,(1-\delta_C)}{\sigma_C}\left(\lambda_C\frac{\partial P_C}{\partial \tau_c}\frac{\tau_c}{P_C} + (1-\lambda_C)\,\frac{\partial P_S}{\partial \tau_c}\frac{\tau_c}{P_S}\right)\end{aligned}$$

To simplify, write $\varrho_{C,\tau} \equiv \frac{\partial P_C}{\partial \tau_c}\frac{\tau_c}{P_C}$, $\varrho_{S,\tau} \equiv \frac{\partial P_S}{\partial \tau_c}\frac{\tau_c}{P_C}$, $\varrho_{\epsilon,\tau} \equiv \frac{\partial \epsilon}{\partial \tau_c}\frac{\tau_c}{\epsilon}$, we obtain the following system in matrix format:

$$\begin{bmatrix} 1-(1-s)\,(1-\delta_C)\,\lambda_C & -(1-s)\,(1-\delta_C)\,(1-\lambda_C) & -s \\ -(1-\delta_S)\,\lambda_S & 1-(1-\delta_S)\,(1-\lambda_S) & 0 \\ -\left(\frac{(\sigma_C-1)}{\sigma_C}+\frac{(\sigma_C-1)(1-\delta_C)\lambda_C}{\sigma_C}\right) & -\frac{(\sigma_C-1)(1-\delta_C)(1-\lambda_C)}{\sigma_C} & 1\end{bmatrix}\begin{bmatrix}\varrho_{C,\tau}\\ \varrho_{S,\tau}\\ \varrho_{\epsilon,\tau}\end{bmatrix} = \begin{bmatrix} s\\ 0\\ 0\end{bmatrix}$$

Once we recover $\varrho_{C,\tau}$ and $\varrho_{S,\tau}$ we obtain the change in the price of the final good. Because $w = 1$, the welfare impact of a small shock in τ_c is given to a first order:

$$d\log \text{Real Income} = -d\log P = -\left(\zeta\varrho_{C,\tau} + (1-\zeta)\,\varrho_{S,\tau}\right)d\log \tau_c.$$

J.2.2 Small Aggregate Productivity Shock

We now consider shocks to aggregate productivity, changes in ς. As before, firms price at marginal cost:

$$p_k(z) = \frac{1}{\varsigma z}\left(\frac{w}{\delta_k}\right)^{\delta_k}\left(\frac{P_k^m}{1-\delta_k}\right)^{1-\delta_k}.$$

These equations will be useful in the derivations below:

$$\frac{\partial p_k(z)}{\partial \varsigma}\frac{\varsigma}{p_k(z)} = -1 + (1-\delta_k)\frac{\partial P_k^m}{\partial \varsigma}\frac{\varsigma}{P_k^m},$$

and

$$\frac{\partial P_k^m}{\partial \varsigma} = \lambda_k \frac{\partial P_C}{\partial \varsigma} + (1-\lambda_k)\frac{\partial P_S}{\partial \varsigma}.$$

The price of sector-C's composite good is given by:

$$P_C = \left(\int p_C(z)^{1-\sigma_C}\,\psi_C(z)\,dz + (\epsilon\tau_c)^{1-\sigma_C}\right)^{\frac{1}{1-\sigma_C}}$$

Therefore:

$$\frac{\partial P_C}{\partial \varsigma}\frac{\varsigma}{P_C} = \left(\left(-1 + (1-\delta_C)\frac{\partial P_C^m}{\partial \varsigma}\frac{\varsigma}{P_C^m}\right)\frac{P_{C,H}^{1-\sigma_C}}{P_C^{1-\sigma_C}} + \frac{(\epsilon\tau_c)^{1-\sigma_C}}{P_C^{1-\sigma_C}}\frac{\partial \epsilon}{\partial \varsigma}\frac{\varsigma}{\epsilon}\right)$$

Using $s = \frac{(\epsilon\tau_c)^{1-\sigma_C}}{P_C^{1-\sigma_C}}$ and $1-s = \frac{P_{C,H}^{1-\sigma_C}}{P_C^{1-\sigma_C}}$ we obtain:

$$\begin{aligned}\frac{\partial P_C}{\partial \varsigma}\frac{\varsigma}{P_C} =& -(1-s) + (1-s)(1-\delta_C)\lambda_C\frac{\partial P_C}{\partial \varsigma}\frac{\varsigma}{P_C}\\ &+ (1-s)(1-\delta_C)(1-\lambda_C)\frac{\partial P_S}{\partial \varsigma}\frac{\varsigma}{P_S} + s\frac{\partial \epsilon}{\partial \varsigma}\frac{\varsigma}{\epsilon}\end{aligned}$$

For the S sector, we have:

$$\frac{\partial P_S}{\partial \varsigma}\frac{\varsigma}{P_S} = -1 + (1-\delta_S)\lambda_S\frac{\partial P_C}{\partial \varsigma} + (1-\delta_S)(1-\lambda_S)\frac{\partial P_S}{\partial \varsigma}$$

As before, we differentiate both sides of the balanced trade condition with respect to ς. Writing $\varrho_{C,\varsigma} \equiv \frac{\partial P_C}{\partial \varsigma}\frac{\varsigma}{P_C}$, $\varrho_{S,\varsigma} \equiv \frac{\partial P_S}{\partial \varsigma}\frac{\varsigma}{P_S}$, and $\varrho_{\epsilon,\varsigma} \equiv \frac{\partial \epsilon}{\partial \varsigma}\frac{\varsigma}{\epsilon}$ we obtain the following system in matrix format:

$$\begin{bmatrix} 1-(1-s)(1-\delta_C)\lambda_C & -(1-s)(1-\delta_C)(1-\lambda_C) & -s \\ -(1-\delta_S)\lambda_S & 1-(1-\delta_S)(1-\lambda_S) & 0 \\ -\left(\frac{(\sigma_C-1)}{\sigma_C} + \frac{(\sigma_C-1)(1-\delta_C)\lambda_C}{\sigma_C}\right) & -\frac{(\sigma_C-1)(1-\delta_C)(1-\lambda_C)}{\sigma_C} & 1 \end{bmatrix}\begin{bmatrix}\varrho_{C,\varsigma}\\ \varrho_{S,\varsigma}\\ \varrho_{\epsilon,\varsigma}\end{bmatrix} = \begin{bmatrix}-(1-s)\\ -1\\ -\frac{(\sigma_C-1)}{\sigma_C}\end{bmatrix}$$

Once we recover $\varrho_{C,\varsigma}$ and $\varrho_{S,\varsigma}$ we obtain the change in the price of the final good. Because $w=1$, the welfare impact of a small shock in ς is given to a first order:

$$d \log \text{Real Income} = -d \log P = -\left(\zeta \varrho_{C,\varsigma} + (1-\zeta)\, \varrho_{S,\varsigma}\right) d \log \varsigma.$$

K Iceberg Trade Cost

We compute trade costs based on the Head and Ries index below (Head and Mayer, 2014):

$$\phi_{ni} \equiv \sqrt{\frac{X_{ni} X_{in}}{X_{nn} X_{ii}}}.$$

Using the 2003 World Input-Output Database (WIOD), $i = Rest\ of\ the\ World$, and $n = Brazil$, we obtain $\phi_{ni} = 0.02513$.

Parameterizing the index as:

$$\phi_{ni} = \left[(1 + t_{ni})\tau_c\right]^{\epsilon}$$

Where t_{ni} is the average tariff between Brazil and the Rest of the World ($\tau_a - 1 = 12\%$). For the trade elasticity, we use $\epsilon = -3.2$ (median) and $\epsilon = -4.5$ (mean)—see Table 3.5 of Head and Mayer (2014), who provide a survey of such estimates. This leads to $\tau_c = 2.82$ and $\tau_c = 2.02$, respectively. The average of these two numbers implies $\tau_c = 2.42$. We subsequently impose $\tau_c = 2.4$ in our new analyses.

L Data Appendix

We use six firm-level datasets containing information on formal and informal firms, as well on their workers. In addition to those, we use one worker-level dataset—*Pesquisa Mensal de Emprego* (PME)—which provides information on workers' allocations and labor market flows. We impose the following common filters across all datasets: we exclude firms and workers in the public sector, agriculture, mining, coal, oil and gas industries. 2003 is our reference year as the ECINF survey is only available for 1997 and 2003. All monetary values (e.g. revenues and wages) correspond to annual values. Finally, we rely on data from the 2000 and 2005 IBGE National Accounts to estimate utility and production function parameters. Sector C includes all manufacturing sectors (excluding mining, coal, oil and gas industries, as mentioned above). Sector S includes all services, commerce, construction, transportation, and utilities sectors. In the following sections, we describe the main variables we generate, as well as the moments and auxiliary models computed from each dataset.

L.1 RAIS and SECEX

RAIS (*Relação Anual de Informações Sociais*) is a matched employer-employee dataset assembled by the Brazilian Ministry of Labor every year since 1976. Establishments are identified by their *Cadastro Nacional de Pessoas Jurídicas* (CNPJ) number, which consists of 14 digits. To make RAIS data compatible with firm-level Census data (PIA, PAS, PAC), we aggregate establishments to the firm level using the first 8 digits of the CNPJ identifier. For multi-establishment firms featuring multiple 4-digit *CNAE* industry codes, we select the code accounting for the largest share of employment within the firm. A negligible share of firms (0.01 percent) have missing industry codes, so they are dropped from the analysis. Firm-level wages and employment are measured as of December of each year. December wages are subsequently annualized. We generate the following firm-level variables:

- *Exit indicator:* We pool RAIS data from 2003 through 2005 to create an exit indicator, which equals one if the firm operates in 2003 but is not found in the data in 2004 nor in 2005.
- *Firm-level employment:* the firm's number of employees, measured in December of each year. Let $\ell_{i,t}$ denote firm i's employment size in year t.
- *Average firm-level wage:* the firm's annual wage bill divided by number of employees, both measured in December of each year.
- *Firm-level Labor Turnover Rate:* for every firm i, we define

$$Turnover_i = \frac{|\ell_{i,2004} - \ell_{i,2003}|}{0.5 \times (\ell_{i,2004} + \ell_{i,2003})}.$$

SECEX (*Secretaria de Comércio Exterior*) is an administrative dataset from the federal government containing information on all export and import transactions. These transactions are identified at the firm-level (through the 8 first digits of the CNPJ identifier) and can be merged to the firm-level RAIS data. This procedure allows us to compute exporter indicators for all C-sector firms. This dummy variable equals one if the firm reports any export transaction in 2003 and zero otherwise (i.e. the firm is found in RAIS but not in SECEX). Using RAIS and SECEX, we compute the following moments and auxiliary models.

Exit Rate (Formal Firms) – see Table M.2
Separately for C- and S-sector firms, we compute the mean of the exit dummy variable across all firms.

Exit Regressions (Formal Firms) – see Table M.2
We estimate the following regressions separately for C- and S-sector firms:

$$Exit_i \quad = \quad \alpha_k + \beta_k \log(\ell_i) + u_i$$

where i denotes a firm, $k = C, S$ denotes sector, u_i is the error term, and $Exit_i$ indicates whether firm i, active in 2003, exits the market in 2004.

Average Turnover (Formal Firms) – see Table M.2
We compute, separately for C- and S-sector firms, mean turnover rates across all firms.

Turnover Regressions – see Table M.2
We separately estimate the following regressions, conditional on C- and S-sector firms, respectively:

$$\begin{aligned} Turnover_i &= \alpha_C + \beta_C \log(\ell_{i,2003}) + \gamma_C Exporter_{i,2003} + u_i \\ Turnover_i &= \alpha_S + \beta_S \log(\ell_{i,2003}) + u_i \end{aligned}$$

where i denotes a firm, $Exporter_{i,2003}$ indicates if firm i exports in 2003, u_i is the error term and the remaining variables are defined as above. These regressions are also separately estimated conditional on expansions and contractions.

Log-Employment Serial Correlations (Formal Firms) – see Table M.2

We compute, separately for C- and S-sector firms, the serial correlations:

$$Corr\left(\log \ell_{i,2003}, \log \ell_{i,2004}\right)_k \text{ for } k = C, S$$

Size Distribution of Formal Firms – see M.3

We compute, separately for C- and S-sector firms, the mean and standard deviation of log-employment across all firms, and the mean of log-employment across all C-sector exporters.

Fraction of Exporters – see Table M.4

We compute the share of all formal C-sector firms that export.

Log-Wages (Formal Firms) – see Table M.5

We compute, separately for C- and S-sector firms, the mean of log-wages across all formal firms.

Log-wage Regressions (Formal Firms) – see Table M.5

We estimate the following regressions, conditional on C- and S-sector firms, respectively (using data for 2003):

$$\begin{aligned}\log(w_i) &= \alpha_C + \beta_C \log(\ell_i) + \gamma_C Exporter_i + u_i \\ \log(w_i) &= \alpha_S + \beta_S \log(\ell_i) + u_i\end{aligned}$$

where i denote a firm, w_i is the (average) wage paid by firm i, u_i is the error term and the remaining variables are defined as above.

L.2 PIA, PAS and PAC (Firm-Level Surveys) and SECEX

Pesquisa Industrial Anual (PIA), *Pesquisa Anual de Comércio* (PAC), and *Pesquisa Anual de Serviços* (PAS) are firm-level surveys, covering the formal manufacturing, retail and service sectors, respectively. Conducted by the Brazilian Statistical Agency (IBGE), they contain detailed information on firms' inputs, output and revenues. They constitute a census for larger firms and a representative sample for smaller firms. In the manufacturing sector (PIA), all firms with at least 30 employees are part of the census and are surveyed every year, while firms with 5 to 29 employees are randomly sampled. The PAC (retail sector) and PAS (services) surveys have the same design, but have lower size thresholds for firms to be included in the census: firms with 20 employees or more are part of the census, while firms with up to 19 employees are randomly sampled. Finally, firms in PIA, PAS and PAC are also identified by their 8-digit CNPJ codes. Therefore, we are able to match SECEX with PIA to identify exporters. We use these datasets to obtain the following firm-level variables:

- *Annual gross revenues*
- *Export share:* for firm i, the share of revenues that comes from exports

$$Export\ Share_i = \frac{Value\ of\ Exports_i}{Revenues_i}$$

Using PIA, PAS, PAC and SECEX, we compute the following moments and auxiliary models.

Distribution of log-revenues – see Table M.6
We compute the mean and standard deviation of log-revenues across all firms in the C and S sectors.

Average Export Share
Average export share among all exporters, used to recover de value of d_F conditional on σ_C—see Step 4 in section I.1 for details. We obtain that the average export share among exporters equals 0.264.

Fraction of Aggregate Revenues in the Formal C-Sector that is Exported – see Table M.4
Ratio between total exports and total revenues in the (formal) C sector.[51]

Serial Correlation of log-Revenues – see Table M.6
$Corr\left(\log Revenues_{i,2004}, \log Revenues_{i,2003}\right)$ separately for the C and S sectors. These moments are computed conditional on firms with at least 30 employees for PIA, and conditional on firms with at least 20 employees for PAS and PAC, so that they are part of the census and therefore surveyed in both years.

Log-Revenues Regressions – see Table M.6
We estimate the following regressions, conditional on C- and S sector firms (data from 2003):

$$\begin{aligned} \log\left(Revenues_i\right) &= \alpha_C + \beta_C \log\left(\ell_i\right) + \gamma_C Exporter_i + u_i \\ \log\left(Revenues_i\right) &= \alpha_S + \beta_S \log\left(\ell_i\right) + u_i \end{aligned}$$

where i denotes a firm, u_i is the error term and the remaining variables are defined as above.

L.3 ECINF (*Pesquisa de Economia Informal Urbana*)

ECINF was collected by IBGE in 1997 and 2003, and was designed to be representative of the universe of *urban* firms with up to five employees (both formal and informal). It is a matched employer-employee dataset that contains information on entrepreneurs, their businesses and employees. We use the same filters for industries we described above. Although a few firms in the dataset have more than five employees, we restrict attention to those with five employees or less so that our sample is consistent with the population the survey targets. We define as informal firms those that do not have a tax registration number, which means that they are not formally registered as a firm.

ECINF is comprised of two main files. The first contains information on businesses (these are small businesses, so there are no multi-establishment firms and we can use firm and establishment interchangeably) and the second contains information on workers. Before merging these data sources, we drop workers who are younger than 18 and older than 64 years old from the individual level data (only 890 observations are dropped). We then aggregate these data up to the firm level, providing us with information on firms' size and wage bill.[52] We merge this information with the first (firm-level) file using a unique firm identifier. Finally, we trim observations below the first percentile of the revenue distribution, which amounts to dropping firms with revenues very close to zero. We generate the following firm-level variables with ECINF:

- *Informality Indicator:* Dummy variable that equals one if the firm is not registered with the tax authorities.

[51] The denominator comes from PIA's publication, Table 1.5 (pdf included in the replication folder). The two values used to compute the denominator correspond to the entries "Gross Revenues" and "Other Operational Revenues" of manufacturing firms (*Indústria da Transformação*).

[52] Thus, if a firm has employees older than 64 or younger than 18 years old they are not accounted for when we compute firm size.

- *Annual gross revenues*
- *Total number of employees*
- *Average wage*: firm's annual wage bill divided by number of people working at the firm. The wage bill includes the self-reported take-home earnings of the owner. For one-person firms, this is equal to the owner's take-home remuneration.

Using ECINF, we compute the following moments and auxiliary models.

Size Distribution (Informal Firms) – see Table M.7
We compute the following moments of firm-level log-employment separately for C- and S-sector informal firms: mean and standard deviation.

Distribution of Revenues (Informal Firms) – see Table M.7
We compute the mean of firms' log-revenues separately for C- and S-sector informal firms.

Log-Wages (Informal Firms) – see Table M.7
We compute the mean of firm-level log-wages separately for C- and S-sector firms.

Regression of Informal Status Indicator vs. Number of Employees – see Table M.7

$$Informal_i = \alpha_k + \beta_k \ell_i + u_i$$

where i denotes firms, $k = C, S$ denotes sector, and u_i is the error term.

L.4 PME

We use the *Pesquisa Mensal de Emprego* (PME) survey to obtain information on worker allocations and labor market flows. The PME is a rotating panel in which individuals in a given household are interviewed for four consecutive months, followed by an eight-month gap, after which they are interviewed again for another four consecutive months. This structure implies a maximum panel length of 16 months. As in the firm-level data, we exclude individuals employed in the public sector, as well as those working in agriculture, mining, coal, oil, and gas industries. As with ECINF, we retain only individuals aged between 18 and 64. Additionally, we exclude individuals who are out of the labor force, non-wage (unpaid) employees or employers.[53] Finally, we restrict our attention to the years of 2003 and 2004. Thus, there are three possible states in our sample:

(i) Formal workers: those who have a formal labor contract, which in Brazil is defined by having a booklet (*carteira de trabalho*) that has been signed by her employer and that registers workers' entire employment history in the formal sector

(ii) Informal workers: those who do not have a signed booklet (without a formal contract), which includes self-employed workers

(iii) Unemployed: those who are not employed, but are actively searching for a job

[53] As a result, transitions to and from these statuses are disregarded when computing the transition matrix. However, when computing cross-sectional moments, we include all observations that are not subject to these exclusions. That is, we do not use any longitudinal information to compute cross-sectional moments.

We employ PME to generate the following moments:

Transition Matrix – see Table M.1
To obtain the annual transition matrix between states, we first estimate the 3-month transition matrix using information from the first and fourth interviews. Denote this 3-month transition matrix by M. We then estimate the annual transition matrix by computing M^4. This is preferable to using information from the first and sixth interviews—which are 12 months apart—given the high attrition rates between the fourth and fifth interviews, which are 8 months apart. This high attrition is common in panel surveys that have similar designs, as the survey unit is a particular address (e.g. an apartment) and individuals may move in and out during the 8-month rest period.

Workers' Allocations – see Table M.1
We use PME's sample weights to obtain the total number and the shares of individuals in each of the possible labor market statuses: (i) formal worker in the C sector; (ii) informal worker in the C sector; (iii) formal worker in the S sector; (iv) informal worker in the S sector; and (v) unemployment.

Lowest Wage in the Informal Sector
In order to discipline wages in the informal sector, we use the PME to compute the lowest wage observed in the informal sector, as well as the 1st and 5th percentiles of the informal wage distribution.

L.5 IBGE National Accounts

We employ information available from IBGE's 2000 and 2005 National Accounts to compute the share of final expenditures on sector C goods, ζ, sector k's fraction of intermediate expenditures on sector C goods, λ_k, and statistics relevant for the estimation of δ_k, which drives the importance of labor in sector k's production.

We compute ζ using final demand information, excluding Agriculture and Mining to be consistent with the filters we implemented in the datasets above. We obtain $\zeta = 0.296$, as is reported in Table 6.

To obtain information on δ_k (conditional on σ_k), we compute:

$$\frac{Total\ Expenditures\ with\ Intermediates_C}{Total\ Gross\ Revenues_C} = 0.596,$$
$$\frac{Total\ Expenditures\ with\ Intermediates_S}{Total\ Gross\ Revenues_S} = 0.320.$$

See Step 3 of section I.1 for details on how to use these statistics to obtain δ_C and δ_S.

Finally, we compute λ_k, for $k = C, S$, as:

$$\lambda_k = \frac{Total\ Expenditure\ with\ Sector\ C\ Intermediates}{Total\ Expenditure\ with\ Intermediates\ (across\ C\ and\ S)},$$

leading to $\lambda_C = 0.645$ and $\lambda_S = 0.291$, as is reported in Table 6.

M Model Fit: Moments Generated by the Model vs. Data

This section compares the moments generated by the model, using our estimates, with those computed from the data. Tables M.1 through M.7 shows that our model is able to replicate several salient features of the

data.
It is worth noting that, although we generate a full annual transition matrix across states $\{\text{Unemp}, Ci, Cf, Si, Sf\}$, we only attempt to match transitions from unemployment. This is because the model does not allow for annual transitions from sector C to S, nor from formal to informal employment.

There are notable differences between the full transition matrix produced by the model and that observed in the data. Most strikingly, persistence in Ci is lower in the data than the model predicts. In the data, many individuals transition from Ci to Si, and to a lesser extent to Cf and Sf. The model captures the high persistence in Si and Sf relatively well, though it tends to overestimate them. It also overestimates transitions to unemployment.

Note, however, that the economy was not in steady state in 2003. Therefore, the steady-state allocation implied by the transition matrix in the data is quite different from the observed allocation. If we were to force the model to match the full transition matrix in the data, it would inevitably miss the observed allocation. Instead, by focusing only on matching transitions from unemployment, we retain enough flexibility to closely match the cross-sectional distribution of employment.

Table M.1: Employment Shares and Transition Rates from Unemployment

Moment	Dataset	Model	Data
Share of Employment Ci	PME	0.059	0.059
Share of Employment Cf	PME	0.100	0.106
Share of Employment Si	PME	0.339	0.351
Share of Employment Sf	PME	0.319	0.334
Share Unemployment	PME	0.184	0.151
Share Informal Workers (Conditional on Working)	PME	0.487	0.482
Trans. Rate from Unemp. to Ci	PME	0.059	0.058
Trans. Rate from Unemp. to Cf	PME	0.063	0.059
Trans. Rate from Unemp. to Si	PME	0.413	0.399
Trans. Rate from Unemp. to Sf	PME	0.186	0.206
Trans. Rate from Unemp. to Unemp.	PME	0.279	0.279
Ratio Trans. to Informal job / Trans. to Formal job	PME	1.895	1.728

N Background: The Cost of Labor Regulations in Brazil

The relevant laws and regulations that apply to formal labor relations in Brazil are contained in the Brazilian Labor Code (*Consolidação das Leis Trabalhistas*—CLT). According to the employment index in Botero et al. (2004), the cost of labor regulations in Brazil is around 20 percent above the mean and median of 85 countries and more than 2.5 times larger than in the United States.

The main aspects of labor regulations in Brazil regarding their magnitude and potential impacts on the labor market, are: (i) the presence of a national minimum wage; (ii) unemployment insurance that is only available to formal workers; (iii) substantial firing costs; and (iv) sizable payroll taxes. Since these play an important role in our model and counterfactuals, we provide a brief background discussion of each of them individually.

The nominal value of the national minimum wage is determined by the federal government once a year and is typically binding for many firms. For instance, in 2003 (the year we use in our empirical analysis),

Table M.2: Turnover-Related Moments and Auxiliary Models (Formal Sectors)

	Dataset	C sector Model	C sector Data	S sector Model	S sector Data
Exit Rate	RAIS	0.101	0.093	0.113	0.112
Average Firm-level Turnover	RAIS	0.221	0.485	0.220	0.495
$Corr\,(\log \ell_{i,t+1}, \log \ell_{i,t})$	RAIS	0.953	0.926	0.932	0.914
$Exit_i = \alpha + \beta \log(\ell_i)$					
Intercept	RAIS	0.149	0.179	0.164	0.177
$\log(\ell_i)$	RAIS	-0.022	-0.047	-0.040	-0.055
$Turnover_i = \alpha + \beta \log(\ell_i) + \gamma Exporter_i$					
Intercept	RAIS+SECEX	0.398	0.713	0.373	0.633
$\log(\ell_i)$	RAIS+SECEX	-0.085	-0.128	-0.121	-0.116
$Exporter_i$	RAIS+SECEX	0.084	0.084		
$Turnover_i = \alpha + \beta \log(\ell_i) + \gamma Exporter_i$, Conditional on Expansions					
Intercept	RAIS+SECEX	0.398	0.696	0.302	0.689
$\log(\ell_i)$	RAIS+SECEX	-0.108	-0.140	-0.109	-0.154
$Exporter_i$	RAIS+SECEX	0.150	0.126		
$Turnover_i = \alpha + \beta \log(\ell_i) + \gamma Exporter_i$, Conditional on Contractions					
Intercept	RAIS+SECEX	0.417	0.711	0.432	0.614
$\log(\ell_i)$	RAIS+SECEX	-0.070	-0.112	-0.094	-0.097
$Exporter_i$	RAIS+SECEX	0.062	0.062		

Table M.3: Firm-Size Distribution (Formal Sectors)

	Dataset	C sector Model	C sector Data	S sector Model	S sector Data
Avg. Firm-Level log-Employment	RAIS	2.203	1.817	1.265	1.192
Std Dev. Firm-Level log-Employment	RAIS	0.837	1.366	0.630	1.133
Avg. Exporter log-Employment	RAIS + SECEX	3.583	3.962		

Table M.4: Trade-Related Moments

	Dataset	Model	Data
Fraction of Exporters (among formal C-sector firms)	RAIS + SECEX	0.125	0.059
Total Exports / (Total Formal Manufacturing Revenue)	SECEX + IBGE	0.136	0.155

the minimum wage corresponds to 49 percent of the national average wage and 81.3 percent of the national median wage.[54]

Although the regulations governing unemployment insurance (UI) are intricate, typically, workers are eligible to receive UI benefits for a duration of 4 to 5 months. The amount of the benefit is determined by the worker's average wage during the three months preceding their layoff. The replacement rate is 100

[54]The mean and the median wages are computed using micro data from the National Household Survey (PNAD) and pooling together all formal and informal employees who are between 18 and 64 years old and work at least 20 hours per week.

Table M.5: Formal-Sector Wages

	Dataset	C sector Model	C sector Data	S sector Model	S sector Data
Avg. log-Wages	RAIS	8.702	8.743	8.599	8.641
$\log(w_i) = \alpha + \beta \log(\ell_i) + \gamma Exporter_i$					
Intercept	RAIS+SECEX	8.398	8.515	8.444	8.495
$\log(\ell_i)$	RAIS+SECEX	0.110	0.110	0.123	0.123
$Exporter_i$	RAIS+SECEX	0.487	0.473		

Table M.6: Formal-Sector Revenues

	Dataset	C sector Model	C sector Data	S sector Model	S sector Data
Avg. log-Revenues	IBGE	12.670	12.726	11.158	10.814
Std. Dev. log-Revenues	IBGE	1.189	1.874	0.948	1.440
$Corr(\log Rev_{i,t+1}, \log Rev_{i,t})$	IBGE	0.744	0.929	0.697	0.845
$\log(Rev_i) = \alpha + \beta \log(\ell_i) + \gamma Exporter_i$					
Intercept	IBGE+SECEX	10.101	10.118	9.682	10.004
$\log(\ell_i)$	IBGE+SECEX	1.138	1.000	1.167	0.872
$Exporter_i$	IBGE+SECEX	0.503	1.462		

Notes: The serial correlation of $\log(Rev)$ is conditional on the employment cutoffs the PIA (30 employees) and PAS (20 employees) panels.

Table M.7: Informal Sector Moments and Auxiliary Moments

	Dataset	C sector Model	C sector Data	S sector Model	S sector Data
Average log-Employment	ECINF	0.196	0.110	0.265	0.096
Std. Dev. log-Employment	ECINF	0.317	0.297	0.362	0.276
Avg. log-Revenue	ECINF	9.871	8.517	9.320	8.906
Avg. log-Wages	ECINF	7.828	7.965	7.684	8.337
$Informal_i = \alpha + \beta \ell_i$					
Intercept	ECINF	1.379	1.170	1.256	1.122
ℓ_i	ECINF	-0.212	-0.212	-0.222	-0.216

Notes: All statistics are computed conditional on firms with five employees or less, both in the data and in the model.

percent for individuals who earn one minimum wage, with an average replacement rate of 64 percent (all data come from Gerard and Gonzaga, 2021).[55]

As for the firing costs, the Brazilian labor regulation states that all formal workers with unjustified dismissal should receive a monetary compensation paid by the employer. In Brazil (and most Latin American countries), firms' outcomes (e.g. lack of businesses) are not considered a just cause for dismissal, thus any involuntary separation falls in this category (Heckman and Pagés, 2000). The magnitude of this compensation

[55] We focus on the rules in place before the 2015 reforms since our empirical analysis precedes them.

is determined as a fraction of the funds accumulated in the worker's *Fundo de Garantia por Tempo de Serviço* (FGTS), which is a job security fund proportional to job tenure and accumulates at a rate of roughly one monthly wage per year. Firms hand over additional severance payments to workers and a direct "penalty" to the government, which further increase the magnitude of the firing costs.[56]

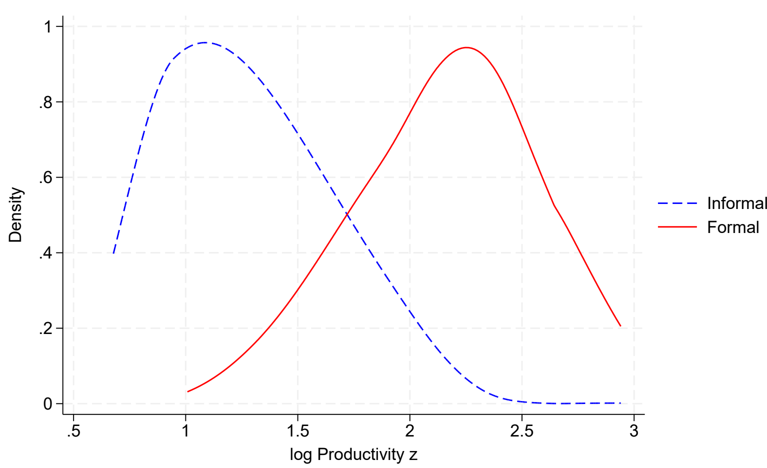
Finally, Brazil has a burdensome tax system, which is characterized not only by high tax rates, but also by a complex structure that implies large compliance costs. For instance, the time required to comply with labor taxes in Brazil is almost 5 times higher than in the U.S. (491 and 100 hours, respectively).[57]

O Additional Post-Estimation Results: Overlapping Distributions of Productivity

This section presents an important post-estimation result. Our model generates overlapping distributions of firm-level total factor productivity across formal and informal firms within sector $k \in \{C, S\}$, an empirical fact that has been emphasized in previous work (e.g., Meghir et al., 2015). Figure O.1 shows this overlap in sector C, and Figure O.2 shows this overlap in sector S.

There are two reasons why the model generates this overlap. First, and more importantly, we do not allow formal firms to switch to informal status. This generates hysteresis: formal firms (which are more productive than informal firms at the entry stage) keep their status even if their productivity significantly declines over time, below the entry cutoff for formal firms. Second, selection into formality/informality depends on a two-dimensional state (z, ℓ). Therefore, for a fixed value of z, we can have selection into informality for some levels of ℓ and into formality for other values of ℓ, also contributing to an overlap in the two productivity distributions.

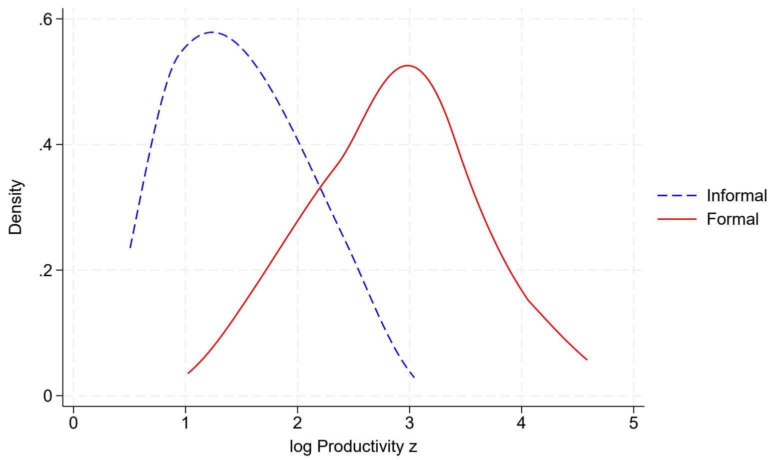
Figure O.1: Kernel Densities of Firm-Level log Productivity—$\log(z)$—in the Formal and Informal Sectors, sector C



[56] Gonzaga et al. (2003) provide an in depth discussion of the legislation on dismissal costs in Brazil.

[57] These data come from Doing Business (2007), which is the earliest report available on paying taxes in the Doing Business Initiative that provides comparability across a comprehensive set of countries.

Figure O.2: Kernel Densities of Firm-Level log Productivity—$\log(z)$—in the Formal and Informal Sectors, sector S



P Additional Post-Estimation Results: Revenue per Worker is Increasing with Firm-Level Total Factor Productivity

In another post-estimation exercise, we show that revenue per worker within sector $k \in \{C, S\}$ is informative about firm-level total factor productivity z. Therefore, from the perspective of our model, Fact 3 can be approximately stated as "informal firms are, on average, less productive than formal firms."

This post-estimation exercise is important because, in many models, equilibrium revenue per worker is not informative about firm-level productivity z. For example, in a standard Melitz model, revenue per worker is constant in equilibrium. In our model, revenue per worker varies across firms in equilibrium for many reasons. In particular, as discussed in section 3.3, convex hiring costs imply equilibrium dispersion in revenue per worker. Other reasons contributing to this dispersion include the unequal incidence of taxes and regulations.

Table P.1 shows that our model generates an equilibrium in which revenue per worker increases with productivity. It presents the results of regressions of $\log\left(\frac{R_k(z,\ell)}{\ell}\right)$ on $\log(z)$ for $k \in \{C, S\}$, using data generated by our model. Firm-level total factor productivity predicts revenue per worker with a high R^2.

Table P.1: Revenue per Worker Strongly Correlates with Productivity

	Dependent Variable: log Rev. per Worker							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	All C	All C	Informal C	Formal C	All S	All S	Informal S	Formal S
$\log(z)$	1.294	1.571	1.626	1.298	0.670	0.721	0.714	0.733
	(0.003)	(0.002)	(0.007)	(0.005)	(0.001)	(0.002)	(0.003)	(0.003)
Formal		-0.605				-0.151		
		(0.003)				(0.003)		
Observations	20,770	20,770	1,064	19,706	21,229	21,229	2,796	18,433
R^2	0.900	0.960	0.982	0.802	0.928	0.934	0.946	0.821

Table P.2 shows a sizable gap between revenue per worker across formal and informal firms. Average revenue per worker is substantially larger in the formal sector. There is also a large dispersion in revenue per worker within each sector/firm type combination.

Table P.2: Summary Statistics for Revenue per Worker

	Informal	Formal	Overall
Avg. log Rev per Worker, Sector C	8.311	9.103	8.430
Std. Dev. log Rev per Worker, Sector C	0.612	0.590	0.670
Avg. log Rev per Worker, Sector S	8.394	9.232	8.591
Std. Dev. log Rev per Worker, Sector S	0.431	0.607	0.596

Q Sensitivity Analysis

Q.1 Sensitivity of the Loss Function with Respect to Parameters

In Figures Q.1 to Q.4, we normalize the loss function relative to its optimal value, and we normalize each parameter relative to its estimated value. In each plot, we vary a specified parameter fixing the remaining ones at estimated values. Overall, the figures show that the loss function is sensitive to all the parameters we estimate.

Notice that some figures exhibit small discontinuities with respect to specific parameters. These small discontinuities are explained by the discrete grid we employ for both productivities and employment. As these grids become finer, these discontinuities become smaller and smaller. However, there is a trade-off between finer grids and computational time.

It is also worth mentioning that the estimation was conducted under the constraint $\widetilde{b}_S \geq 0.01$, which was binding at the optimum. We imposed this restriction to ensure the stability of the counterfactuals in Section 5.2. In particular, for some configurations of the cost of informality function, we were unable to find equilibria when increasing $\widetilde{b}_S$ from very small initial values.

Figure Q.1: Loss Function Sensitivity

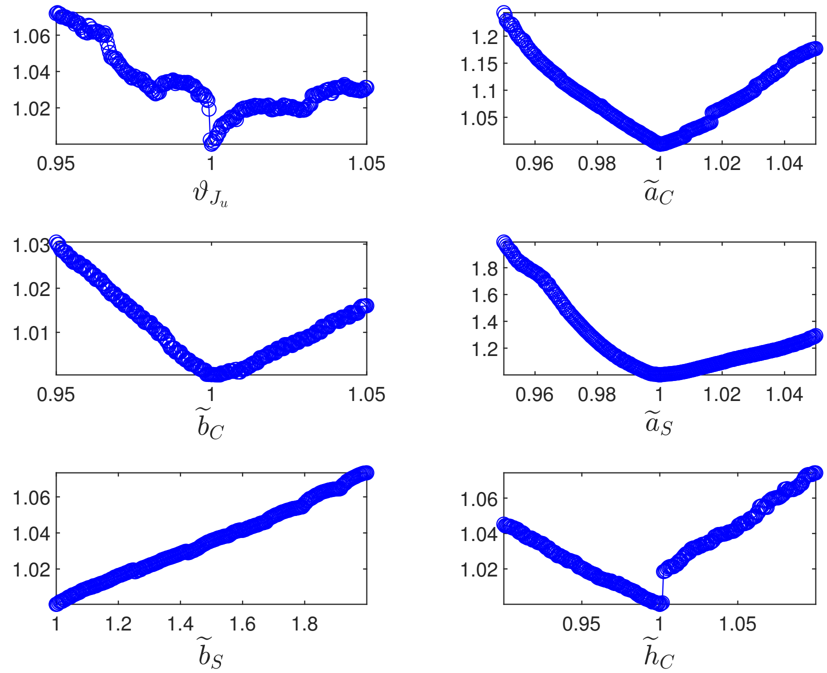


Figure Q.2: Loss Function Sensitivity

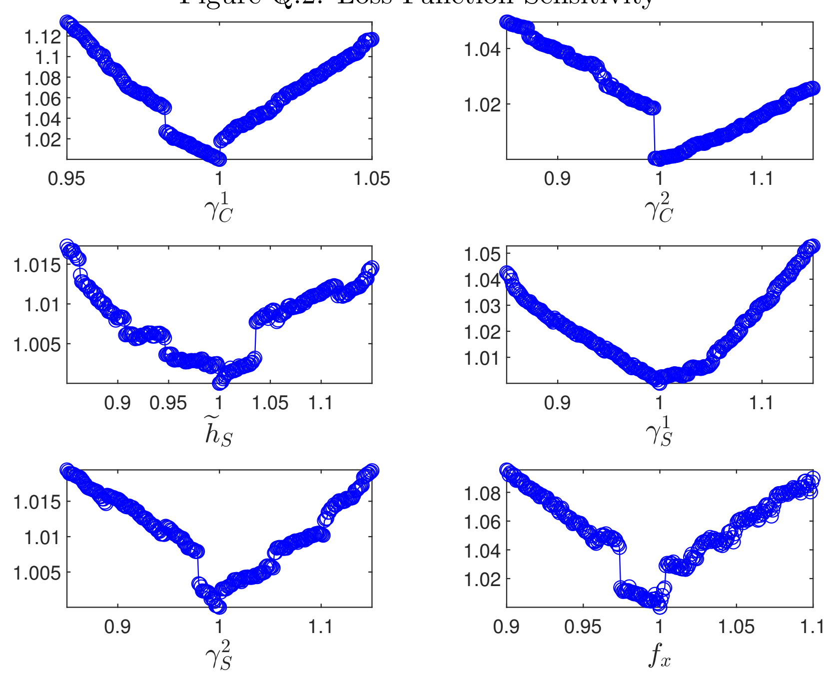


Figure Q.3: Loss Function Sensitivity

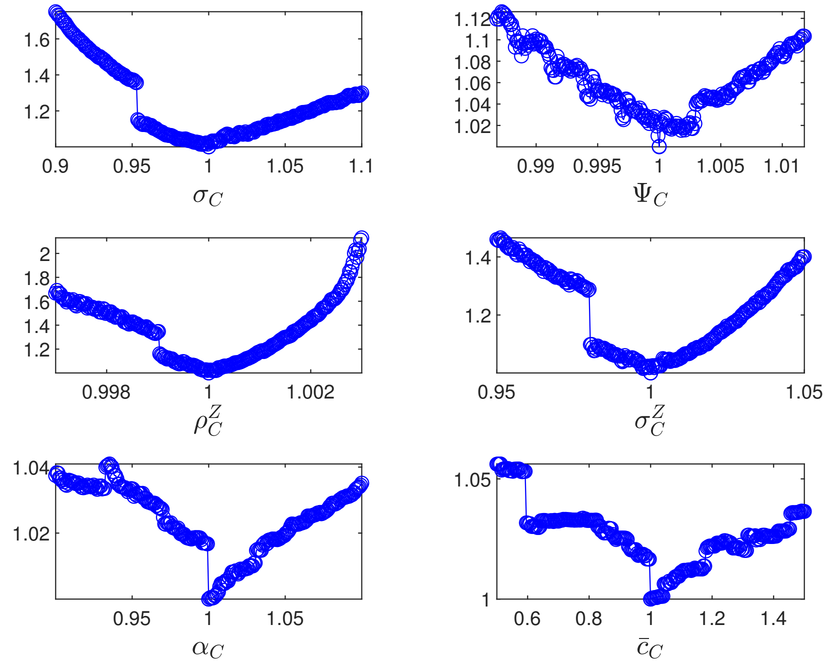
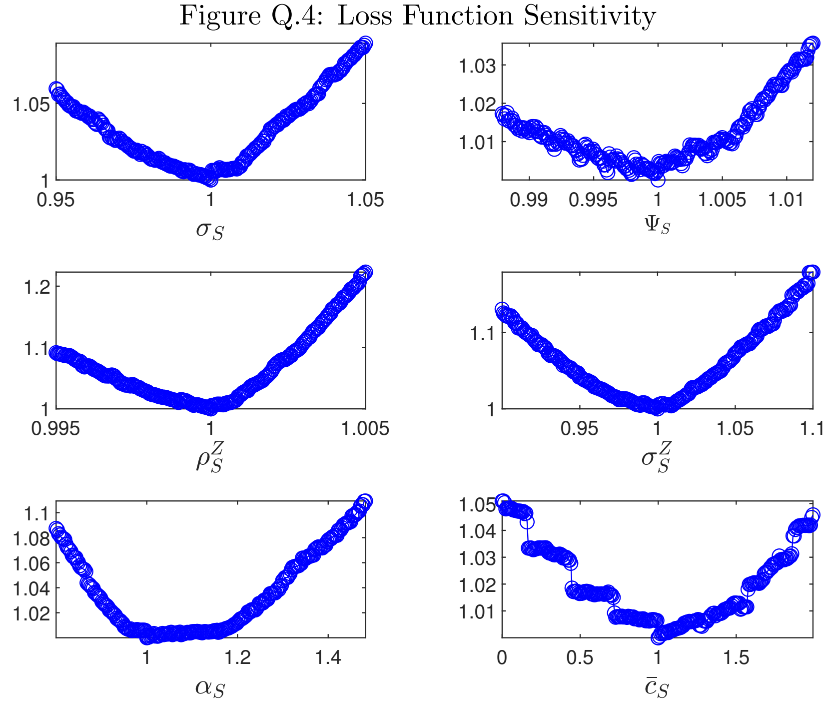


Figure Q.4: Loss Function Sensitivity



Q.2 Elasticities of Moments with Respect to Parameters

Tables Q.1 to Q.6 display the elasticity of each moment used in the loss function relative to each parameter. Here are a few lessons from these tables:

- ϑ_{J_u} measures the outside option of unemployed workers. In theory, this "parameter" directly affects the level of wages. Indeed, it is shown to affect the levels of Cf, Ci and Si wages. The impact is less strong on Sf wages, presumably because of minimum wages that are more binding in that sector. The behavior of the various elasticities with respect to this parameter illustrate the rich interactions in our model: moving this object also has large impacts on various employment shares as well as on turnover moments.
- The parameters determining the expected costs of informality, $\widetilde{a}_k$ and $\widetilde{b}_k$ for $k \in \{C, S\}$, strongly affect shares on employment in ki and they strongly impact the regression relating the probability of informal status to the size of the firm. They also affect wages in kf and ki.
- The hiring cost parameters $\widetilde{h}_k$, γ_k^1, γ_k^2, $k \in \{C, S\}$, affect turnover rates and wage regressions.
- The fixed cost of exporting f_x affects the fraction of exporters.
- The elasticity of substitution σ_k for $k \in \{C, S\}$, affects a large number of moments. They strongly affect the fraction of exporters (in the C sector) and the ratio of exports to revenue in Cf. Importantly, it affects the log-revenue regressions—both the constant and the slope.
- The persistence of the AR(1) process, ρ_k^Z for $k \in \{C, S\}$, also impacts many moments, but importantly, it strongly affects the serial correlation in revenues at the firm level.
- The standard deviation of the stochastic shocks in the AR(1) process, σ_k^Z for $k \in \{C, S\}$, is an important driver of firm size dispersion.
- Exogenous exit rates, α_k for $k \in \{C, S\}$, drive aggregate exit rates.
- Fixed costs of operation $\bar{c}_k$ for $k \in \{C, S\}$, impact the coefficient on size in the exit regressions.

Table Q.1: Elasticity Of Each Moment With Respect To Each Parameter

	ϑ_{J_u}	$\widetilde{a}_C$	$\widetilde{b}_C$	$\widetilde{a}_S$	$\widetilde{b}_S$	$\widetilde{h}_C$	γ_C^1	γ_C^2
1. Share Emp Ci	-0.036	-3.149	-0.448	0.622	0.010	0.057	0.136	-0.011
2. Share Emp Cf	0.222	1.762	0.253	0.622	0.010	-0.082	0.031	0.019
3. Share Emp Si	-1.049	0.006	0.002	-8.834	-0.112	0.010	-0.014	-0.002
4. Share Emp Sf	1.058	0.006	0.002	9.546	0.123	0.010	-0.014	-0.002
5. Share Emp U	-0.023	0.007	-0.001	-0.039	-0.004	-0.009	-0.008	0.002
6. Tr U Ci	0.182	-2.453	-0.439	0.661	0.014	0.068	0.727	-0.016
7. Tr U Cf	0.139	2.256	0.380	0.661	0.014	-0.244	-0.610	0.056
8. Tr U Si	-0.579	-0.001	0.003	-6.041	-0.056	0.019	-0.006	-0.004
9. Tr U Sf	1.180	-0.001	0.003	13.992	0.126	0.019	-0.006	-0.004
10. Tr U U	-0.000	-0.000	-0.000	0.000	0.000	-0.000	0.000	-0.000
11. Exit Rate Cf	0.141	0.194	0.020	0.000	0.000	-0.092	-0.217	0.028
12. Exit Rate Cf Const	0.064	0.088	-0.016	-0.000	-0.000	0.036	0.493	-0.027
13. Exit Rate Cf Size	-0.184	1.201	0.116	-0.000	-0.000	0.543	2.959	-0.206
14. Exit Rate Sf	0.235	-0.000	-0.000	1.336	0.016	-0.000	0.000	-0.000
15. Exit Rate Sf Const	-0.016	-0.000	-0.000	0.588	0.035	-0.000	-0.000	-0.000
16. Exit Rate Sf Size	-0.256	-0.000	-0.000	4.672	0.121	-0.000	-0.000	-0.000
17. exp(mean log(size) Cf)	0.190	-2.937	-0.459	0.000	-0.000	-0.516	-2.065	0.139
18. exp(SD log(size) Cf)	-0.455	0.277	0.041	0.000	-0.000	-0.015	-0.191	0.020
19. exp(mean log(size)—exp. Cf)	-0.418	-0.135	-0.002	0.000	-0.000	-0.506	-2.100	0.170
20. exp(mean log(size) Sf)	-0.408	0.000	0.000	-7.330	-0.059	0.000	0.000	0.000
21. exp(SD log(size) Sf)	-0.185	0.000	0.000	0.008	-0.015	0.000	0.000	0.000
22. Fraction Export	0.083	-3.950	-0.637	0.000	-0.000	-0.482	-2.695	0.169
23. mean Turnover Cf	-0.316	0.375	0.081	0.000	0.000	-0.120	-0.104	0.046
24. mean Turnover Sf	-0.179	-0.000	-0.000	2.257	0.017	-0.000	0.000	-0.000
25. Turn. Cf Const	0.006	-0.288	-0.020	-0.000	-0.000	-0.164	-0.141	0.052
26. Turn. Cf Size	0.343	0.184	0.049	-0.000	0.000	-0.030	0.461	0.020
27. Turn. Cf Exp	0.630	2.341	0.274	0.000	-0.000	-0.542	-2.507	0.323
28. Turn. Sf Const	-0.199	-0.000	-0.000	-1.103	0.002	-0.000	-0.000	-0.000
29. Turn. Sf Size	0.096	-0.000	-0.000	-0.064	0.028	-0.000	-0.000	-0.000
30. Turn. Cf — expand Const	0.779	-1.065	-0.051	0.000	0.000	-0.504	-1.603	0.139
31. Turn. Cf — expand Size	1.326	-1.276	-0.059	0.000	0.000	-0.535	-1.855	0.148
32. Turn. Cf — expand export	1.094	-0.224	-0.014	0.000	-0.000	-0.569	-2.957	0.247
33. Turn. Sf — expand Const	-0.041	-0.000	-0.000	-1.234	-0.026	-0.000	-0.000	0.000
34. Turn. Sf — expand Size	0.041	-0.000	-0.000	-1.160	-0.026	-0.000	-0.000	0.000
35. Turn. Cf — shrink Const	-0.241	-0.206	-0.037	-0.000	-0.000	-0.034	0.330	0.022
36. Turn. Cf — shrink Size	0.528	0.512	0.063	-0.000	0.000	0.078	0.893	-0.003
37. Turn. Cf — shrink Exp	1.409	3.376	0.484	0.000	0.000	-2.703	-7.914	0.999
38. Turn. Sf — shrink — Const	-0.277	-0.000	-0.000	-1.573	-0.001	-0.000	-0.000	0.000
39. Turn. Sf — shrink Size	1.650	-0.000	-0.000	1.506	0.066	-0.000	-0.000	-0.000

Table Q.2: Elasticity Of Each Moment With Respect To Each Parameter – Continued

	ϑ_{J_u}	$\widetilde{a}_C$	$\widetilde{b}_C$	$\widetilde{a}_S$	$\widetilde{b}_S$	$\widetilde{h}_C$	γ_C^1	γ_C^2
40. exp(mean log(Wages) Cf)	0.411	-0.754	-0.090	0.000	-0.000	-0.033	-0.433	0.016
41. exp(mean log(Wages) Sf)	0.103	0.000	0.000	-2.819	-0.041	0.000	-0.000	-0.000
42. exp(log(w) Cf Const)	0.902	-1.289	-0.120	0.000	0.000	-0.197	-1.225	0.112
43. log(w) fC Size	-2.385	5.066	0.539	-0.000	0.000	0.973	4.749	-0.484
44. log(w) fC Exp	0.933	-0.963	-0.193	-0.000	0.000	0.236	1.270	-0.064
45. exp(log(w) fS Const)	0.198	0.000	0.000	-4.657	-0.081	0.000	0.000	0.000
46. log(w) fS Size	-0.292	-0.000	-0.000	19.211	0.307	-0.000	-0.000	0.000
47. $Corr(emp_t, emp_{t+1})$ Cf	0.001	0.008	-0.003	0.000	0.000	0.002	-0.009	0.001
48. $Corr(emp_t, emp_{t+1})$ Sf	0.065	0.000	-0.000	-0.056	-0.004	0.000	0.000	-0.000
49. exp(mean log(size) Ci)	-0.108	-0.821	-0.222	0.000	-0.000	-0.026	-0.056	0.005
50. exp(SD log(size) Ci)	-0.092	-0.906	-0.252	0.000	-0.000	-0.037	-0.075	0.008
51. exp(mean log(size) Si)	-0.428	-0.000	0.000	-2.488	-0.044	0.000	0.000	-0.000
52. exp(SD log(size) Si)	-0.348	-0.000	0.000	-2.311	-0.049	0.000	0.000	-0.000
53. exp(mean log(rev) Ci)	0.660	-0.806	-0.249	0.000	-0.000	-0.059	-0.448	0.022
54. exp(mean log(rev) Si)	0.193	0.000	0.000	-4.093	-0.059	0.000	0.000	-0.000
55. exp(mean log(Wages) Ci)	0.861	-0.732	0.016	0.000	0.000	-0.040	-0.307	0.018
56. exp(mean log(Wages) Si)	0.779	0.000	0.000	-2.022	-0.024	0.000	-0.000	0.000
57. Inf Dummy C Const	0.064	1.125	0.182	0.000	0.000	0.062	0.032	-0.013
58. Inf Dummy C Slope	0.239	6.148	1.047	-0.000	-0.000	0.337	0.613	-0.087
59. Inf Dummy S Const	0.140	0.000	0.000	-0.429	0.064	0.000	-0.000	-0.000
60. Inf Dummy S Slope	1.462	0.000	-0.000	11.755	0.283	-0.000	-0.000	-0.000
61. exp(mean log(rev) Cf)	0.544	-3.761	-0.557	0.000	-0.000	-0.511	-2.403	0.148
62. exp(SD log(rev) Cf)	-0.462	0.207	0.018	-0.000	-0.000	0.063	0.091	-0.012
63. exp(mean log(rev) Sf)	-0.424	0.000	0.000	-10.542	-0.105	0.000	0.000	-0.000
64. exp(SD log(rev) Sf)	-0.233	0.000	0.000	-0.018	-0.008	0.000	0.000	0.000
65. ratio Exports Revenue Cf	-0.136	-1.018	-0.184	0.000	0.000	-0.236	-1.350	0.094
66. $Corr(rev_t, rev_{t+1})$ Cf	0.007	-0.003	-0.000	0.000	0.000	-0.021	-0.082	0.012
67. $Corr(rev_t, rev_{t+1})$ Sf	0.099	0.000	-0.000	0.000	0.000	0.000	0.000	-0.000
68. exp(log(rev) Const Cf)	0.890	-1.371	-0.124	0.000	-0.000	-0.192	-1.263	0.122
69. log(rev) log(size) Cf	-0.256	0.509	0.057	-0.000	-0.000	0.114	0.518	-0.056
70. log(rev) Exp Cf	1.178	-1.113	-0.216	-0.000	0.000	0.218	1.252	-0.053
71. exp(log(rev) Const Sf)	0.150	0.000	-0.000	-5.089	-0.093	0.000	0.000	-0.000
72. log(rev) log(size) Sf	-0.066	-0.000	0.000	2.071	0.037	-0.000	-0.000	0.000
73. Tr. Informal / Tr. Formal	-1.401	-0.871	-0.150	-15.357	-0.145	0.072	0.243	-0.017
74. Share Informal Workers	-0.904	-0.452	-0.065	-7.381	-0.096	0.015	0.006	-0.003

Table Q.3: Elasticity Of Each Moment With Respect To Each Parameter – Continued

	$\widetilde{h}_S$	γ_S^1	γ_S^2	f_x	σ_C	Ψ_C	ρ_C^Z	σ_C^Z
1. Share Emp Ci	0.017	-0.055	-0.010	0.066	-3.400	-5.782	-107.013	-4.780
2. Share Emp Cf	0.017	-0.055	-0.010	0.056	0.607	-0.848	-11.593	-0.825
3. Share Emp Si	-0.138	0.235	0.131	-0.014	0.201	0.664	11.632	0.544
4. Share Emp Sf	0.147	-0.263	-0.139	-0.014	0.201	0.664	11.632	0.544
5. Share Emp U	-0.014	0.073	0.008	-0.002	0.014	-0.083	-1.376	0.025
6. Tr U Ci	0.031	-0.129	-0.018	0.114	-3.035	-6.789	-115.804	-4.794
7. Tr U Cf	0.031	-0.129	-0.018	0.013	0.992	-0.857	-16.790	-0.498
8. Tr U Si	-0.103	0.227	0.087	-0.012	0.188	0.747	13.008	0.519
9. Tr U Sf	0.210	-0.422	-0.183	-0.012	0.188	0.747	13.008	0.519
10. Tr U U	0.000	-0.000	-0.000	-0.000	-0.000	0.000	0.000	0.000
11. Exit Rate Cf	-0.000	-0.000	-0.000	-0.017	0.468	-0.973	-16.485	-0.103
12. Exit Rate Cf Const	0.000	-0.000	-0.000	0.200	-0.851	-3.664	-59.156	-2.031
13. Exit Rate Cf Size	0.000	-0.000	-0.000	0.717	-5.358	-12.228	-207.982	-8.380
14. Exit Rate Sf	0.006	-0.023	-0.005	-0.000	-0.000	0.000	0.000	0.000
15. Exit Rate Sf Const	-0.028	-0.025	-0.012	-0.000	-0.000	0.000	0.000	0.000
16. Exit Rate Sf Size	0.101	-0.448	-0.212	-0.000	-0.000	0.000	0.000	0.000
17. exp(mean log(size) Cf)	-0.000	0.000	0.000	-0.119	3.539	6.222	110.803	4.637
18. exp(SD log(size) Cf)	-0.000	0.000	0.000	-0.126	1.329	2.912	48.087	2.200
19. exp(mean log(size)—exp. Cf)	-0.000	0.000	0.000	0.188	4.112	6.706	113.034	4.693
20. exp(mean log(size) Sf)	-0.260	0.532	0.234	0.000	0.000	0.000	-0.000	-0.000
21. exp(SD log(size) Sf)	-0.020	0.068	0.035	0.000	0.000	-0.000	-0.000	-0.000
22. Fraction Export	0.000	-0.000	0.000	-1.779	6.160	14.450	267.990	11.986
23. mean Turnover Cf	-0.000	-0.000	-0.000	-0.025	1.110	-0.054	-1.775	0.882
24. mean Turnover Sf	-0.009	0.053	0.019	-0.000	0.000	0.000	0.000	0.000
25. Turn. Cf Const	0.000	-0.000	-0.000	-0.008	0.710	-0.333	-3.586	0.202
26. Turn. Cf Size	0.000	-0.000	-0.000	-0.091	-1.190	-2.148	-34.916	-1.960
27. Turn. Cf Exp	0.000	-0.000	-0.000	-1.033	-1.510	9.047	100.640	1.934
28. Turn. Sf Const	-0.118	0.187	0.073	0.000	-0.000	0.000	0.000	0.000
29. Turn. Sf Size	-0.070	-0.042	-0.033	0.000	-0.000	0.000	0.000	0.000
30. Turn. Cf — expand Const	-0.000	-0.000	-0.000	-0.328	0.778	2.152	35.735	1.648
31. Turn. Cf — expand Size	0.000	-0.000	-0.000	-0.723	0.250	3.265	47.170	1.936
32. Turn. Cf — expand export	0.000	-0.000	-0.000	-1.121	1.936	9.902	135.777	5.535
33. Turn. Sf — expand Const	-0.161	0.473	0.086	-0.000	-0.000	-0.000	0.000	0.000
34. Turn. Sf — expand Size	-0.123	0.414	-0.028	0.000	-0.000	0.000	0.000	0.000
35. Turn. Cf — shrink Const	0.000	-0.000	-0.000	0.079	1.079	-0.882	-12.636	0.056
36. Turn. Cf — shrink Size	0.000	-0.000	-0.000	0.177	-3.274	-6.652	-105.809	-5.320
37. Turn. Cf — shrink Exp	-0.000	-0.000	-0.000	-2.946	-2.271	22.907	280.194	6.524
38. Turn. Sf — shrink — Const	-0.097	-0.005	0.104	0.000	-0.000	0.000	0.000	0.000
39. Turn. Sf — shrink Size	-0.141	-0.300	0.152	0.000	-0.000	-0.000	0.000	0.000

Table Q.4: Elasticity Of Each Moment With Respect To Each Parameter – Continued

	$\widetilde{h}_S$	γ_S^1	γ_S^2	f_x	σ_C	Ψ_C	ρ_C^Z	σ_C^Z
40. exp(mean log(Wages) Cf)	0.000	0.000	0.000	-0.101	0.473	2.034	34.361	1.713
41. exp(mean log(Wages) Sf)	-0.051	0.071	0.064	0.000	-0.000	-0.000	-0.000	-0.000
42. exp(log(w) Cf Const)	-0.000	0.000	-0.000	-0.509	1.238	3.819	66.367	3.022
43. log(w) fC Size	0.000	-0.000	-0.000	2.346	-6.426	-14.899	-316.859	-12.113
44. log(w) fC Exp	0.000	-0.000	-0.000	-0.176	0.252	2.341	35.601	1.863
45. exp(log(w) fS Const)	-0.109	0.228	0.178	0.000	0.000	-0.000	-0.000	-0.000
46. log(w) fS Size	0.579	-1.444	-0.917	0.000	-0.000	0.000	0.000	0.000
47. $Corr(emp_t, emp_{t+1})$ Cf	0.000	-0.000	-0.000	-0.012	0.102	0.248	4.088	0.138
48. $Corr(emp_t, emp_{t+1})$ Sf	-0.001	-0.000	0.006	-0.000	0.000	0.000	0.000	0.000
49. exp(mean log(size) Ci)	-0.000	0.000	0.000	-0.017	0.508	0.858	14.545	0.562
50. exp(SD log(size) Ci)	-0.000	0.000	0.000	-0.014	0.515	0.872	15.144	0.628
51. exp(mean log(size) Si)	-0.060	0.166	0.054	-0.000	0.000	-0.000	-0.000	-0.000
52. exp(SD log(size) Si)	-0.063	0.182	0.050	-0.000	0.000	-0.000	-0.000	-0.000
53. exp(mean log(rev) Ci)	0.000	0.000	0.000	-0.051	0.366	2.184	38.744	1.207
54. exp(mean log(rev) Si)	-0.106	0.272	0.110	0.000	0.000	-0.000	-0.000	-0.000
55. exp(mean log(Wages) Ci)	-0.000	0.000	0.000	-0.023	-0.075	0.932	17.381	0.558
56. exp(mean log(Wages) Si)	-0.033	0.073	0.042	0.000	0.000	-0.000	-0.000	-0.000
57. Inf Dummy C Const	-0.000	0.000	0.000	-0.015	-0.372	-0.548	-24.201	-0.886
58. Inf Dummy C Slope	0.000	-0.000	-0.000	0.013	-2.324	-3.709	-121.841	-4.448
59. Inf Dummy S Const	-0.013	-0.027	0.010	-0.000	-0.000	-0.000	-0.000	-0.000
60. Inf Dummy S Slope	0.229	-0.622	-0.212	-0.000	-0.000	0.000	0.000	0.000
61. exp(mean log(rev) Cf)	-0.000	0.000	0.000	-0.221	3.836	7.964	140.317	6.158
62. exp(SD log(rev) Cf)	-0.000	0.000	0.000	-0.092	1.183	3.184	50.536	2.498
63. exp(mean log(rev) Sf)	-0.308	0.591	0.296	-0.000	0.000	0.000	-0.000	-0.000
64. exp(SD log(rev) Sf)	0.009	0.009	-0.010	0.000	0.000	-0.000	-0.000	-0.000
65. ratio Exports Revenue Cf	0.000	-0.000	-0.000	-0.994	4.369	9.865	184.012	8.196
66. $Corr(rev_t, rev_{t+1})$ Cf	-0.000	-0.000	-0.000	0.138	1.087	2.065	32.534	1.058
67. $Corr(rev_t, rev_{t+1})$ Sf	0.009	-0.017	-0.020	-0.000	0.000	0.000	0.000	0.000
68. exp(log(rev) Const Cf)	-0.000	0.000	0.000	-0.555	1.241	3.950	68.625	3.090
69. log(rev) log(size) Cf	0.000	0.000	0.000	0.239	-0.740	-1.636	-29.075	-1.223
70. log(rev) Exp Cf	0.000	-0.000	-0.000	-0.191	0.025	1.631	26.520	1.407
71. exp(log(rev) Const Sf)	-0.119	0.230	0.198	-0.000	0.000	-0.000	-0.000	-0.000
72. log(rev) log(size) Sf	0.077	-0.176	-0.118	0.000	-0.000	0.000	0.000	-0.000
73. Tr. Informal / Tr. Formal	-0.252	0.530	0.215	0.009	-0.597	-0.521	-8.499	-0.402
74. Share Informal Workers	-0.118	0.209	0.112	-0.003	-0.318	-0.298	-6.087	-0.234

Table Q.5: Elasticity Of Each Moment With Respect To Each Parameter – Continued

	α_C	$\bar{c}_C$	σ_S	Ψ_S	ρ_S^Z	σ_S^Z	α_S	$\bar{c}_S$
1. Share Emp Ci	0.091	-0.093	0.645	2.009	17.382	1.221	-0.047	0.020
2. Share Emp Cf	-0.134	0.016	0.645	2.009	17.382	1.221	-0.047	0.020
3. Share Emp Si	-0.005	0.005	0.448	1.163	7.353	0.240	-0.350	-0.075
4. Share Emp Sf	-0.005	0.005	-0.784	-1.580	-12.113	-0.744	0.210	0.063
5. Share Emp U	0.062	0.002	0.002	-1.128	-7.563	-0.174	0.316	0.010
6. Tr U Ci	0.484	-0.050	0.643	3.137	24.945	1.395	-0.363	0.010
7. Tr U Cf	0.191	0.018	0.643	3.137	24.945	1.395	-0.363	0.010
8. Tr U Si	-0.067	0.003	0.296	0.365	3.204	0.109	-0.165	-0.039
9. Tr U Sf	-0.067	0.003	-1.059	-2.855	-23.681	-1.126	0.599	0.078
10. Tr U U	0.000	-0.000	-0.000	0.000	-0.000	0.000	-0.000	-0.000
11. Exit Rate Cf	0.688	0.023	0.000	0.000	-0.000	-0.000	-0.000	0.000
12. Exit Rate Cf Const	0.421	0.040	-0.000	0.000	-0.000	0.000	-0.000	0.000
13. Exit Rate Cf Size	-0.152	0.061	-0.000	0.000	-0.000	-0.000	-0.000	0.000
14. Exit Rate Sf	-0.000	0.000	-0.225	-2.176	-15.907	-0.513	0.767	0.030
15. Exit Rate Sf Const	0.000	0.000	-0.315	-2.054	-15.008	-0.387	0.376	0.027
16. Exit Rate Sf Size	-0.000	0.000	-2.508	-5.666	-46.107	-1.819	-0.110	0.023
17. exp(mean log(size) Cf)	0.014	0.032	0.000	0.000	0.000	0.000	0.000	-0.000
18. exp(SD log(size) Cf)	-0.253	-0.028	0.000	-0.000	0.000	0.000	0.000	-0.000
19. exp(mean log(size)—exp. Cf)	-0.165	0.003	0.000	-0.000	0.000	-0.000	0.000	-0.000
20. exp(mean log(size) Sf)	-0.000	-0.000	2.493	4.914	42.002	2.120	-0.491	-0.002
21. exp(SD log(size) Sf)	0.000	0.000	0.497	0.946	7.609	0.469	-0.167	-0.014
22. Fraction Export	-0.151	0.019	0.000	0.000	-0.000	-0.000	-0.000	0.000
23. mean Turnover Cf	0.043	0.014	0.000	0.000	-0.000	0.000	-0.000	0.000
24. mean Turnover Sf	0.000	0.000	0.573	-1.948	-15.281	0.011	0.154	0.030
25. Turn. Cf Const	0.208	0.059	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
26. Turn. Cf Size	0.403	0.101	-0.000	0.000	-0.000	0.000	-0.000	0.000
27. Turn. Cf Exp	0.447	0.095	-0.000	0.000	-0.000	-0.000	-0.000	0.000
28. Turn. Sf Const	-0.000	0.000	1.033	-0.379	-0.144	0.544	0.000	0.045
29. Turn. Sf Size	0.000	0.000	-0.288	-1.999	-11.577	-0.379	0.167	0.068
30. Turn. Cf — expand Const	0.299	0.021	0.000	0.000	-0.000	0.000	-0.000	0.000
31. Turn. Cf — expand Size	0.446	0.034	0.000	0.000	-0.000	-0.000	-0.000	0.000
32. Turn. Cf — expand export	0.408	0.037	0.000	-0.000	-0.000	-0.000	-0.000	0.000
33. Turn. Sf — expand Const	-0.000	0.000	0.634	0.861	7.901	0.537	0.017	-0.000
34. Turn. Sf — expand Size	0.000	0.000	0.218	0.123	1.662	0.166	0.109	0.002
35. Turn. Cf — shrink Const	0.225	0.086	-0.000	0.000	-0.000	0.000	-0.000	0.000
36. Turn. Cf — shrink Size	0.459	0.166	-0.000	0.000	-0.000	0.000	-0.000	-0.000
37. Turn. Cf — shrink Exp	-0.255	0.118	0.000	0.000	-0.000	0.000	-0.000	0.000
38. Turn. Sf — shrink — Const	0.000	0.000	1.729	-0.664	-0.015	0.815	0.098	0.093
39. Turn. Sf — shrink Size	0.000	-0.000	-4.689	-6.747	-41.791	-3.476	0.485	0.185

Table Q.6: Elasticity Of Each Moment With Respect To Each Parameter – Continued

	α_C	$\bar{c}_C$	σ_S	Ψ_S	ρ_S^Z	σ_S^Z	α_S	$\bar{c}_S$
40. exp(mean log(Wages) Cf)	0.128	0.009	0.000	-0.000	0.000	-0.000	0.000	-0.000
41. exp(mean log(Wages) Sf)	-0.000	-0.000	1.039	2.313	21.243	1.431	-0.009	0.001
42. exp(log(w) Cf Const)	0.343	0.049	0.000	-0.000	0.000	-0.000	0.000	-0.000
43. log(w) fC Size	-0.928	-0.202	-0.000	0.000	-0.000	0.000	-0.000	0.000
44. log(w) fC Exp	0.276	0.054	0.000	-0.000	-0.000	0.000	-0.000	0.000
45. exp(log(w) fS Const)	-0.000	-0.000	1.317	2.723	26.909	1.365	0.079	0.024
46. log(w) fS Size	-0.000	0.000	-3.752	-6.521	-70.147	-1.323	-0.152	-0.145
47. $Corr(emp_t, emp_{t+1})$ Cf	-0.019	-0.001	0.000	0.000	-0.000	-0.000	-0.000	0.000
48. $Corr(emp_t, emp_{t+1})$ Sf	0.000	-0.000	-0.033	0.087	1.178	-0.024	-0.014	0.001
49. exp(mean log(size) Ci)	-0.009	0.024	0.000	-0.000	0.000	0.000	0.000	-0.000
50. exp(SD log(size) Ci)	-0.025	0.012	-0.000	-0.000	-0.000	0.000	0.000	0.000
51. exp(mean log(size) Si)	-0.000	-0.000	1.307	2.134	15.701	0.908	-0.153	0.020
52. exp(SD log(size) Si)	-0.000	-0.000	0.974	1.684	12.675	0.730	-0.164	0.002
53. exp(mean log(rev) Ci)	0.081	0.111	-0.000	-0.000	0.000	-0.000	0.000	-0.000
54. exp(mean log(rev) Si)	-0.000	-0.000	1.187	3.144	24.627	1.083	-0.197	0.068
55. exp(mean log(Wages) Ci)	0.047	0.046	0.000	-0.000	0.000	-0.000	0.000	0.000
56. exp(mean log(Wages) Si)	0.000	-0.000	-0.165	0.421	4.332	0.072	-0.042	0.021
57. Inf Dummy C Const	0.042	0.002	0.000	-0.000	0.000	-0.000	0.000	-0.000
58. Inf Dummy C Slope	0.110	-0.000	-0.000	0.000	-0.000	0.000	-0.000	0.000
59. Inf Dummy S Const	-0.000	-0.000	-0.133	-0.164	-0.731	0.008	0.021	-0.002
60. Inf Dummy S Slope	-0.000	0.000	-3.006	-5.419	-43.506	-2.190	0.632	0.028
61. exp(mean log(rev) Cf)	0.178	0.043	0.000	0.000	0.000	-0.000	0.000	-0.000
62. exp(SD log(rev) Cf)	-0.302	-0.038	0.000	0.000	0.000	-0.000	0.000	-0.000
63. exp(mean log(rev) Sf)	-0.000	-0.000	3.611	7.317	64.083	3.632	-0.468	0.003
64. exp(SD log(rev) Sf)	0.000	-0.000	0.743	1.605	12.192	1.075	-0.212	-0.029
65. ratio Exports Revenue Cf	-0.146	0.003	0.000	0.000	-0.000	0.000	-0.000	0.000
66. $Corr(rev_t, rev_{t+1})$ Cf	-0.026	0.000	0.000	0.000	-0.000	0.000	-0.000	-0.000
67. $Corr(rev_t, rev_{t+1})$ Sf	0.000	0.000	1.091	2.785	21.702	1.044	-0.008	0.000
68. exp(log(rev) Const Cf)	0.420	0.063	0.000	0.000	0.000	-0.000	0.000	-0.000
69. log(rev) log(size) Cf	-0.107	-0.025	-0.000	-0.000	-0.000	-0.000	0.000	-0.000
70. log(rev) Exp Cf	0.334	0.068	0.000	0.000	-0.000	0.000	-0.000	0.000
71. exp(log(rev) Const Sf)	-0.000	-0.000	1.362	2.761	28.064	1.367	0.157	0.039
72. log(rev) log(size) Sf	-0.000	-0.000	-0.459	-0.801	-8.804	-0.168	-0.032	-0.023
73. Tr. Informal / Tr. Formal	0.003	-0.010	0.972	2.052	17.161	0.768	-0.547	-0.094
74. Share Informal Workers	0.023	-0.009	0.478	1.035	7.149	0.350	-0.234	-0.059

R Alternative Scenarios Without Informality

This appendix compares the effects of a small reduction in trade costs, τ_c, across four scenarios described below. The goal is to contrast the impact of this shock in the full baseline economy—featuring both informality-related and other distortions—with its impact in economies where informality-related distortions are removed.

1. **Benchmark** economy: the estimated baseline economy.
2. **No Informality** economy: the *Benchmark* economy with prohibitive informality costs (i.e., perfect enforcement).
3. **Formal** economy: a ***fully re-estimated*** economy with only a formal sector. All parameters are re-estimated, while firing costs, minimum wages, taxes, and unemployment benefits are set to the *Benchmark* (i.e., observed) values. This setup reflects current practice in which models are estimated using data from the formal sector alone, abstracting from informality but retaining other distortions. The counterfactuals in this scenario therefore illustrate how conclusions might differ when empirical work overlooks this particular aspect of the economy.
4. **Formal Dereg** economy: a ***fully re-estimated*** economy with only a formal sector, but fully deregulated. Taxes and firing costs are set to zero ($\tau_y = \tau_w = \kappa = 0$), with no unemployment benefits ($b_u = 0$) and no minimum wage ($\underline{w} = 0$).[58]

Table 7 in the main text reports the parameter estimates for the full model *Benchmark*. The parameters for the *No Informality* economy are the same as in Table 7, except that the cost of informality is set to be prohibitively high, effectively eliminating informality. Table R.1 presents estimates for the *Formal* model, which features only a formal sector and is estimated using formal-sector moments. Finally, Table R.2 reports estimates for the *Formal Dereg* model, which also includes only a formal sector but excludes taxes and regulations.[59]

We simulate a small reduction in trade costs from $\tau_c = 2.4$ to $\tau_c = 2.3$ in each of these economies. The objective here is to compare the impact of a reduction in trade costs in the full baseline economy with all distortions (those related to informality and those that are not) to the impact of a reduction in trade costs in economies without distortions related to informality.

Results are presented in Table R.3 below. Each cell reports changes induced by the small shock to τ_c, expressed ***relative*** to the *Benchmark* scenario. For instance, real income gains in the *No Informality* scenario are 0.596 percentage points smaller than in the *Benchmark* economy, while the reallocation effect in the *Formal* scenario is 0.193 percentage points smaller. *Benchmark* values are set to zero to facilitate comparisons across columns. Note that the second column of Table R.3 reproduces the last column of Table 10.

Table R.3 shows that real income gains in the *Benchmark* economy are larger than in any of the counterfactual economies without an informal sector, and that the smaller gains in these alternative economies are driven by weaker reallocation effects.

[58] To ensure comparability of distortion-free gains across the *Formal Dereg*, *Formal*, and *Benchmark* economies, we calibrate the *Formal Dereg* and *Formal* scenarios to match the share of manufacturing expenditure on imports, $\tau_a \cdot \text{Imports}/X_C$, implied by the *Benchmark* economy—see Table 9.

[59] The main differences in parameter estimates across Tables 7, R.1 and R.2 are the larger elasticities of substitution, as well as the higher fixed costs of operation and exporting required to match the data in *Formal* and *Formal Dereg*.

Table R.1: Parameter Estimates – Model with Formal Sector Only

Parameter	Description	$k = C$	$k = S$
h_k	Hiring Cost, Level	49.8	9.6
γ_k^1	Hiring Cost, Convexity	2.555	3.622
γ_k^2	Hiring Cost, Scale Economies	0.011	0.044
σ_k	Elasticity of Substitution	7.283	4.414
ρ_k	Productivity AR(1) Process, Persistence Coeff.	0.984	0.967
σ_k^z	Productivity AR(1) Process, Std. Dev. of Shock	0.185	0.369
α_k	Exogenous Exit Probability	0.092	0.064
$\bar{c}_k$	Fixed Cost of Operation	1,973.201	976.450
δ_k	Labor Share in Production	0.309	0.586
c_k^e	Entry Cost	2,954.1	1,319.0
f_x	Fixed Cost of Exporting	99,883.8	
b_0	Utility Flow of Unemployment	-0.784	
$(D_F^*)^{\frac{1}{\sigma_C}}$	Foreign Demand Shifter	142.0	

Notes: To aid interpretation, b_0 is expressed as a fraction of real income per capita according to the model.

Table R.2: Parameter Estimates – Model with Formal Sector Only, no Regulations or Taxes

Parameter	Description	$k = C$	$k = S$
h_k	Hiring Cost, Level	899.6	1490.6
γ_k^1	Hiring Cost, Convexity	1.745	4.993
γ_k^2	Hiring Cost, Scale Economies	0.048	0.432
σ_k	Elasticity of Substitution	8.000	6.050
ρ_k	Productivity AR(1) Process, Persistence Coeff.	0.985	0.982
σ_k^z	Productivity AR(1) Process, Std. Dev. of Shock	0.193	0.233
α_k	Exogenous Exit Probability	0.062	0.036
$\bar{c}_k$	Fixed Cost of Operation	3,849.424	926.739
δ_k	Labor Share in Production	0.319	0.617
c_k^e	Entry Cost	2,904.2	2,697.0
f_x	Fixed Cost of Exporting	95,306.5	
b_0	Utility Flow of Unemployment	-1.342	
$(D_F^*)^{\frac{1}{\sigma_C}}$	Foreign Demand Shifter	89.5	

Notes: To aid interpretation, b_0 is expressed as a fraction of real income per capita according to the model.

Table R.3: Impacts of a Small Trade Shock Across Scenarios, τ_c Declines from 2.4 to 2.3—All Impacts Shown Are Relative to Those Under the Benchmark

	Bench	No Inf.	Formal	Formal Dereg.
1. $\Delta \log$ (Real Income)	0	-0.596	-0.281	-0.649
2. $\Delta \log$ (Real Income)^{DF}	0	0.169	-0.088	-0.112
3. Δ Reallocation Effect (39)	0	-0.765	-0.193	-0.537

All effects are relative to the Benchmark and multiplied by 100. More precisely, each cell is given by $100 \times (\Delta y_{\text{scenario}} - \Delta y_{\text{Bench}})$, $y \in \{\log(\text{Real Income}), \log(\text{Real Income})^{DF}, \text{Reallocation Effect}\}$, scenario $\in \{\text{Bench, SE1, SE2, SE3, No Inf.}\}$. Δ Reallocation Effect (39) is the reallocation effect obtained using equation (39).

We highlight that the economies represented in Table R.3 differ substantially—either due to markedly different parameter values or because similar parameters lead to very different equilibrium outcomes. For example, the *Benchmark* and *No Informality* scenarios share the same parameters, except that the cost of informality is set to be prohibitive in the latter. Yet their equilibria diverge sharply, with large differences in unemployment rates (see Table 9 in the main text), the extent to which the minimum wage binds, average firm size, and other key variables.

By contrast, the *Benchmark*, *Formal*, and *Formal Dereg* economies display similar formal-sector equilibrium variables by construction, as the model is re-estimated to match the same formal-sector moments. However, they differ in structural parameters—such as the fixed costs of production and exporting, hiring costs, the labor share in production, and the elasticity of substitution. This makes it difficult to interpret differences in real income gains across models, as it is unclear whether these differences are driven by the absence of informality or by differences in underlying structural parameters.

Reassuringly, the gains from reducing trade costs are consistently larger in the *Benchmark* economy than in any of the counterfactuals without informality, driven by a stronger reallocation effect. Across all exercises, this is a robust finding: trade liberalization delivers larger gains when the informal sector is present than when it is repressed or entirely absent.

In conclusion, while each exercise has its strengths and limitations, the results point to a consistent message. Against this background, we believe the set of exercises presented in Section 5.2 offers the most transparent assessment of the role of informality in shaping the reallocation effect.

Supplementary Material – Not for Publication

I Estimation

I.1 Estimation Algorithm

In this section we describe the estimation algorithm in detail, which we break down into several steps for expositional clarity.

Before we proceed, remember that value added for domestic producers in sector k is given by:

$$VA_k(z,\ell) = \Theta_k (P_k^m)^{-(1-\delta_k)\Lambda_k} (\exp(d_{H,k}))^{\frac{\sigma_k}{\sigma_k-1}\Lambda_k} \left(z\ell^{\delta_k}\right)^{\Lambda_k},$$

where

$$P_k^m \equiv \frac{P_C^{\lambda_k} P_S^{1-\lambda_k}}{\lambda_k^{\lambda_k}(1-\lambda_k)^{1-\lambda_k}}, \tag{S.1}$$

$$\Theta_k \equiv \left(\frac{1}{(1-\delta_k)\Lambda_k}\right)\left(\frac{(1-\delta_k)(\sigma_k-1)}{\sigma_k}\right)^{\frac{\sigma_k}{\sigma_k-1}\Lambda_k},$$

and

$$\Lambda_k \equiv \frac{\sigma_k - 1}{\sigma_k - (1-\delta_k)(\sigma_k-1)}.$$

Rewrite value added for domestic producers as

$$VA_k(z,\ell) = \Theta_k \Psi_k \left(z\ell^{\delta_k}\right)^{\Lambda_k},$$

with

$$\Psi_k \equiv (P_k^m)^{-(1-\delta_k)\Lambda_k} (\exp(d_{H,k}))^{\frac{\sigma_k}{\sigma_k-1}\Lambda_k}. \tag{S.2}$$

Note that Θ_k is a solely a function of model's parameters. On the other hand, Ψ_k is a function of model's parameters but also of equilibrium objects such as P_C, P_S and $d_{H,k}$. In turn, value added for exporters is given by:

$$VA_C(z,\ell) = \Theta_C \Psi_C (\exp(d_F))^{\frac{\sigma_C}{\sigma_C-1}\Lambda_C} \left(z\ell^{\delta_C}\right)^{\Lambda_C}.$$

It will be convenient to define and work with

$$\vartheta_{J_u} \equiv b_0 \times P_Y + \frac{1}{1+r} J^u.$$

Ψ_C, Ψ_S, ϑ_{J_u} are treated as parameters to be estimated along with the remaining ones, but these are all endogenous variables. The procedure below makes sure that the values guessed for Ψ_C and Ψ_S are equilibrium outcomes (see Step 9 for details). The number of entrants M_C and M_S will be set to match Ψ_C and Ψ_S. Given knowledge of ϑ_{J_u} and the remaining parameters, we can recover the utility flow of unemployment b_0 and the value of unemployment J^u post-estimation (as P_Y is a byproduct of the estimation procedure depending on P_C, P_S and ζ).

Step 1a: λ_C and λ_S are obtained from input-output tables and fixed throughout.

Step 1b: Fix μ^{υ} and obtain ϕ using equation (24):

$$\phi = \left(\frac{\mu^{\upsilon}}{\left(Transition_{Data}^{U \to E}\right)^{\frac{\xi-1}{\xi}}} \right)^{\xi}$$

where $Transition_{Data}^{U \to E}$ is the transition rate from unemployment to employment in the data.

Step 2: Start with a parameter vector guess Ω, including values for Ψ_C, Ψ_S and ϑ_{J_u}.

Step 3: Obtain δ_k using $P_k^m \iota_k (z, \ell) = \frac{(1-\delta_k)(\sigma_k - 1)}{\sigma_k} R_k (z, \ell)$:

$$\left(\frac{Total\ Expenditures\ with\ Intermediates_k}{Total\ Gross\ Revenues_k} \right)_{Data} = \frac{(1-\delta_k)(\sigma_k - 1)}{\sigma_k}$$

$$\Rightarrow \delta_k = 1 - \frac{\sigma_k}{\sigma_k - 1} \left(\frac{Total\ Expenditures\ with\ Intermediates_k}{Total\ Gross\ Revenues_k} \right)_{Data}$$

$\left(\frac{Total\ Expenditures\ with\ Intermediates_k}{Total\ Gross\ Revenues_k} \right)_{Data}$ is obtained from input-output tables.

Step 4: Obtain d_F using equation (33):

$$E\left[Export\ Share | Exporter = 1\right]_{Data} = \left(1 - \exp\left(-\sigma_C \times d_F\right)\right)$$

$$\Rightarrow d_F = -\frac{1}{\sigma_C} \log\left(1 - E\left[Export\ Share | Exporter = 1\right]_{Data}\right)$$

$E\left[Export\ Share | Exporter = 1\right]_{Data}$ is the average share of exporters' gross revenues in sector C coming from exports, obtained from PIA and SECEX.

Step 5: This step solves for wage schedules $w_{kf}(z, \ell')$, $w_{ki}(z, \ell')$ as well as value functions $V_{kf}(z, \ell)$, $V_{ki}(z, \ell)$, $J_{kf}^e(z, \ell')$, $J_{ki}^e(z, \ell')$, and firms' policy functions.

Step 5a: Compute value added functions $VA_k(z, \ell)$.

Step 5b: Compute wage schedules $w_{kf}(z, \ell')$

- Guess a wage schedule $w_{kf}(z, \ell')$
- Compute the resulting $V_{kf}(z, \ell')$ using (14)
- Compute $J_{kf}^e(z, \ell')$ using (I.1)
- Compute $w_{kf}^u(z, \ell')$ using equation (28)
- Let $\widehat{w}_{kf}^u(z, \ell') = \omega_0 + \omega_1 \frac{VA_k(z, \ell')}{\ell'}$ be the linear projection of $w_{kf}^u(z, \ell')$ on $\left[1, \frac{VA_k(z, \ell')}{\ell'}\right]$
- Update $w_{kf}(z, \ell') = \max\left\{\widehat{w}_{kf}^u(z, \ell'), b_u + \vartheta_{J_u} - \frac{1}{1+r} J_{kf}^e(z, \ell'), \underline{w}\right\}$
- Restart until convergence

Step 5c: Compute wage schedules $w_{ki}(z, \ell')$

- Guess a wage schedule $w_{ki}(z,\ell')$
- Compute the resulting $V_{ki}(z,\ell')$ using (18)
- Compute $J^e_{ki}(z,\ell')$ using (I.2)
- Compute $w^u_{ki}(z,\ell')$ using equation (31)
- Let $\widehat{w}^u_{ki}(z,\ell') = \omega_0 + \omega_1\left(1 - \frac{\sigma_k}{\sigma_k-(1-\delta_k)(\sigma_k-1)}p_{ki}(\ell')\right)\frac{VA_k(z,\ell')}{\ell'}$ be the linear projection of $w^u_{ki}(z,\ell')$ on $\left[1, \left(1 - \frac{\sigma_k}{\sigma_k-(1-\delta_k)(\sigma_k-1)}p_{ki}(\ell')\right)\frac{VA_k(z,\ell')}{\ell'}\right]$
- Update $w_{ki}(z,\ell') = \max\left\{\widehat{w}^u_{ki}(z,\ell'), \vartheta_{J_u} - \frac{1}{1+r}J^e_{ki}(z,\ell')\right\}$
- Restart until convergence

Step 6: Solve for firms' entry decisions. Compute the fraction of entrants in the formal and informal sectors as follows:

$$\omega_{kf} \equiv \Pr\left(I^{formal}_k(z) = 1\right) = \int_z I^{formal}_k(z)\, g^e_k(z)\, dz$$

$$\omega_{ki} \equiv \Pr\left(I^{informal}_k(z) = 1\right) = \int_z I^{informal}_k(z)\, g^e_k(z)\, dz$$

Therefore, if M_k is the mass of entrants in sector k, the masses of formal and informal entrants in sector k are given by:

$$M_{ki} = \omega_{ki} M_k$$

$$M_{kf} = \omega_{kf} M_k$$

Finally, compute the distribution of z productivities among entrants, conditional on entry into sector kj.

$$\psi^e_{ki}(z) = \frac{g^e_k(z)\, I^{informal}_k(z)}{\int_{\widetilde{z}} g^e_k(\widetilde{z})\, I^{informal}_k(\widetilde{z})\, d\widetilde{z}},$$

$$\psi^e_{kf}(z) = \frac{g^e_k(z)\, I^{formal}_k(z)}{\int_{\widetilde{z}} g^e_k(\widetilde{z})\, I^{formal}_k(\widetilde{z})\, d\widetilde{z}}.$$

Step 7: Compute the steady-state distribution of states. For informal firms, start with a guess for ψ_{ki}. Then, compute

$$\varrho^{exit}_{ki} = \alpha_k + (1-\alpha_k)\int_z\int_\ell \left(I^{exit}_{ki}(z,\ell) + I^{change}_{ki}(z,\ell)\right)\psi_{ki}(z,\ell)\, d\ell dz.$$

In steady state $N_{ki} = \left(1-\varrho^{exit}_{ki}\right)N_{ki} + M_{ki}$. Therefore, set $\frac{M_{ki}}{N_{ki}}$, the fraction of sector k informal firms that are entrants, to:

$$\boxed{\frac{M_{ki}}{N_{ki}} = \varrho^{exit}_{ki} = \frac{\omega_{ki}M_k}{N_{ki}}.}$$

Now, compute $\widetilde{\psi}_{ki}$:

$$\begin{aligned}\widetilde{\psi}_{ki}(z,\ell) = {} & \mathcal{I}[\ell = 1] \times \varrho^{exit}_{ki} \times \psi^e_{ki}(z) \\ & + \mathcal{I}[\ell \geq 1] \times (1-\alpha_k) \times \psi_{ki}(z,\ell)\, I^{stay}_{ki}(z,\ell),\end{aligned}$$

and $\widehat{\psi}_{ki}$:

$$\widehat{\psi}_{ki}(z,\ell') = \int_{\ell} \widetilde{\psi}_{ki}(z,\ell)\, \mathcal{I}\left(L_{ki}(z,\ell) = \ell'\right) d\ell$$

Update ψ_{ki} with:

$$\psi_{ki}(z',\ell') = \int_{z} \widehat{\psi}_{ki}(z,\ell')\, g_k(z'|z)\, dz,$$

and repeat until convergence of ψ_{ki}. This converged value of ψ_{ki} will be used directly in the computation of ψ_{kf} below.

For formal firms, start with guess for ψ_{kf} and compute:

$$\varrho_{kf}^{exit} = \alpha_k + (1-\alpha_k)\int_z \int_\ell I_{kf}^{exit}(z,\ell)\,\psi_{kf}(z,\ell)\, d\ell dz,$$

$$\varrho_{ki}^{change} = (1-\alpha_k)\int_z \int_\ell I_{ki}^{change}(z,\ell)\,\psi_{ki}(z,\ell)\, d\ell dz.$$

In steady state:

$$\begin{aligned}
\varrho_{kf}^{exit} N_{kf} &= \varrho_{ki}^{change} \underbrace{N_{ki}}_{\frac{\omega_{ki} M_k}{\varrho_{ki}^{exit}}} + \omega_{kf} M_k \\
&= M_k \left(\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf} \right)
\end{aligned}$$

So that:

$$\boxed{\frac{M_{kf}}{N_{kf}} = \frac{M_k \omega_{kf}}{N_{kf}} = \frac{\varrho_{kf}^{exit}\omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}}\omega_{ki} + \omega_{kf}}}$$

Also, note that

$$\frac{M_{kf}}{N_{kf}} \times \frac{N_{ki}}{M_{ki}} = \frac{\varrho_{kf}^{exit}\omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}}\omega_{ki} + \omega_{kf}} \frac{1}{\varrho_{ki}^{exit}} = \frac{\varrho_{kf}^{exit}\omega_{kf}}{\varrho_{ki}^{change}\omega_{ki} + \varrho_{ki}^{exit}\omega_{kf}}$$

and

$$\frac{M_{kf}}{N_{kf}} \times \frac{N_{ki}}{M_{ki}} = \frac{\omega_{kf}}{\omega_{ki}} \frac{N_{ki}}{N_{kf}}$$

Therefore,

$$\boxed{\frac{N_{ki}}{N_{kf}} = \frac{\varrho_{kf}^{exit}\omega_{ki}}{\varrho_{ki}^{change}\omega_{ki} + \varrho_{ki}^{exit}\omega_{kf}}}$$

Compute $\widetilde{\psi}_{kf}$ as:

$$\widetilde{\psi}_{kf}(z,\ell) = \begin{array}{c} \mathcal{I}[\ell=1] \times \underbrace{\frac{\varrho_{kf}^{exit}\omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}}\omega_{ki} + \omega_{kf}}}_{\frac{M_{kf}}{N_{kf}}} \psi_{kf}^{e}(z) \\ +\mathcal{I}[\ell \geq 1] \times \left(\begin{array}{c} (1-\alpha_k)\,\psi_{kf}(z,\ell)\, I_{kf}^{stay}(z,\ell) \\ +(1-\alpha_k) \underbrace{\frac{\varrho_{kf}^{exit}\omega_{ki}}{\varrho_{ki}^{change}\omega_{ki} + \varrho_{ki}^{exit}\omega_{kf}}}_{\frac{N_{ki}}{N_{kf}}} \psi_{ki}(z,\ell)\, I_{ki}^{change}(z,\ell) \end{array} \right) \end{array}$$

and $\widehat{\psi}_{kf}$ as:

$$\widehat{\psi}_{kf}(z,\ell') = \int_{\ell} \widetilde{\psi}_{kf}(z,\ell)\,\mathcal{I}(L_{kf}(z,\ell) = \ell')\,d\ell.$$

Update ψ_{kf} with:

$$\psi_{kf}(z',\ell') = \int_{z} \widehat{\psi}_{kf}(z,\ell')\,g_k(z'|z)\,dz,$$

and repeat until convergence of ψ_{kf}.
At this point we have the following objects: ψ_{kj}, $\widetilde{\psi}_{kj}$, $\widehat{\psi}_{kj}$, ϱ_{ki}^{exit}, ϱ_{ki}^{change}, ϱ_{kf}^{exit}, $\chi_{ki\to f}^{change}$, χ_{kf}^{layoff}, and χ_{ki}^{leave} (see equations (C.7), (C.11) and (C.13)).

Step 8: Obtain the entry costs $c_{e,k}$ $(k = C, S)$:

$$c_{e,k} = V_k^e = \int_z \left[V_{ki}^e(z)\, I_k^{informal}(z) + V_{kf}^e(z)\, I_k^{formal}(z) \right] g_k^e(z)\,dz$$

These costs will be subtracted from aggregate income, and will be added to the expenditure on S-sector goods.

Step 9: This step solves for masses of entrants M_k's, masses of firms N_{kj}'s, aggregate vacancies V_{kj}'s and mass of unemployment L_u consistent with Ψ_C, Ψ_S, d_F and μ^{υ}.
Step 9a: Write aggregate income I as a function of masses of entrants M_C and M_S.
Step 9b: Write P_C and P_S as functions of M_C and M_S.
Step 9c: Write X_C^{int} as a function of M_C and M_S.
Step 9d: Solve for $\frac{M_S}{M_C}$ that matches Ψ_C.
Step 9e: Separately pin down M_C and M_S using the labor market clearing equation $\overline{L} - L_u = \sum_{k=C,S,j=i,f} L_{kj}$. Express M_C and M_S as functions of L_u.
Step 9fe: Express masses of firms N_{kj} as functions of L_u.
Step 9g: Express aggregate posted vacancies V_{kj} as functions of L_u.
Step 9h: Use equation for μ^{υ} (and the value initially guessed in Step 1 for μ^{υ}) to obtain L_u consistent with Ψ_C, Ψ_S, d_F and μ^{υ}.
Step 9i: Go back and obtain masses of entrants M_k's, masses of firms N_{kj}'s, and aggregate vacancies V_{kj}'s.
Step 9j: Recover price indices P_C and P_S.
Step 9k: Compute deviation between government revenues and spending with unemployment insurance Dev_T.

Step 10: Obtain job finding rates μ^e_{kj} using aggregate vacancies V_{kj}'s and mass of unemployment L_u obtained in Step 9.

$$\mu^e_{kj} = \frac{m_{kj}}{L_u} = \phi \frac{V_{kj}}{\widetilde{V}} \left(\frac{\widetilde{V}}{L_u} \right)^{\xi}$$

Step 11: Use equations (C.14)-(C.15) to obtain allocations L_{Cf}, L_{Ci}, L_{Sf}, L_{Si}.

$$\begin{aligned} L_{Ci} &= \frac{\mu^e_{Ci} L_u}{\chi^{leave}_{Ci}} \\ L_{Si} &= \frac{\mu^e_{Si} L_u}{\chi^{leave}_{Si}} \\ L_{Cf} &= \frac{\mu^e_{Cf} L_u + \chi^{change}_{Ci \to f} L_{Ci}}{\chi^{layoff}_{Cf}} \\ L_{Sf} &= \frac{\mu^e_{Sf} L_u + \chi^{change}_{Si \to f} L_{Si}}{\chi^{layoff}_{Sf}} \end{aligned}$$

Step 12: Compute deviation from the labor market clearing equation:

$$Dev_L = abs \left(\frac{\overline{L} - (L_{Cf} + L_{Ci} + L_{Sf} + L_{Si})}{\overline{L}} \right) ,$$

Step 13: Compute all moments to be matched with those in the data.

Step 14: Compute Loss Function. Add Model/Data deviations to equilibrium penalty $EQ\_Penalty$. The objective function is therefore given by

$$L = L_{mom} + EQ\_Penalty$$

Where L_{mom} penalizes deviations between moments in the data and $EQ\_Penalty$ penalizes deviations from the labor market clearing condition:

$$EQ\_Penalty = W_L Dev_L + W_T abs \left(\min \left\{ Dev_T, 0 \right\} \right)$$

With W_L and W_T denoting large weights and Dev_T is the relative deviation between government revenues and spending with unemployment insurance (see section I.2 for details). We highly penalize a negative Dev_T.

Step 15: Optimization routine picks new parameter vector Ω. Go back to Step 1 until convergence.

Step 16 (Post estimation): Obtain J^u using

$$\begin{aligned} J^u = &\sum_{k,j} \mu^e_{kj} \int_\ell \int_z \overline{J}^e_{kj} \left(z, L_{kj} \left(z, \ell \right) \right) g_{kj} \left(z, \ell \right) dz d\ell \\ &+ \left(1 - \sum_{k,j} \mu^e_{kj} \right) \vartheta_{J_u}. \end{aligned}$$

Step 17 (Post estimation): At this point, we know J^u and can compute

$$b_0 \times P_Y = \vartheta_{J_u} - \frac{1}{1+r} J^u,$$

Step 18 (Post-estimation): Obtain D_F^* (this is the parameter that we need for the counterfactuals as d_F is endogenous):

$$D_F^* = \frac{\left(\exp\left(\sigma_C \times d_F\right) - 1\right)\left(P_C^m\right)^{(1-\delta_C)(\sigma_C - 1)} \Psi_C^{\frac{\sigma_C - 1}{\Lambda_C}}}{\overline{\epsilon}^{\sigma_C} \tau_c^{1-\sigma_C}},$$

where $\overline{\epsilon}$ is the exchange rate value that balances trade:

$$\overline{\epsilon} = \frac{1}{\tau_a \tau_c} \left(P_C^m\right)^{(1-\delta_C)} \Psi_C^{\frac{1}{\Lambda_C}} \left(\tau_a Exports\right)^{\frac{1}{1-\sigma_C}}.$$

I.2 Estimation Algorithm – Further Details

This section details the steps within Step 9 of the estimation procedure.

Step 9: This step solves for masses of entrants M_k's, masses of firms N_{kj}'s, aggregate vacancies V_{kj}'s consistent with Ψ_C, Ψ_S and d_F.

We start with some definitions... Averages "per firm". All these quantities can be computed after Step 8, that is, after solving for the steady state distribution of states.

$$Avg\_wbill_{ki} = \int_z \int_{\ell'} \left[w_{ki}\left(z, \ell'\right) \ell'\right] \widehat{\psi}_{ki}\left(z, \ell'\right) d\ell' dz \text{ for } k = C, S$$

$$Avg\_wbill_{kf} = \int_z \int_{\ell'} \left[\max\left\{w_{kf}\left(z, \ell'\right), \underline{w}\right\} \ell'\right] \widehat{\psi}_{kf}\left(z, \ell'\right) d\ell' dz \text{ for } k = C, S$$

$$Avg\_Firing\_Costs_{kf} = \kappa \int_z \int_{\ell} \left[\left(\ell - L_{kf}\left(z, \ell\right)\right)\left(1 - I_{kf}^{hire}\left(z, \ell\right)\right)\right] \widetilde{\psi}_{kf}\left(z, \ell\right) d\ell dz \text{ for } k = C, S$$

$$Avg\_Hiring\_Costs_{kj} = \int_z \int_{\ell} \left[H_{kj}\left(\ell, L_{kj}\left(z, \ell\right)\right) I_{kj}^{hire}\left(z, \ell\right)\right] \widetilde{\psi}_{kj}\left(z, \ell\right) d\ell dz \text{ for } k = C, S;\ j = i, f$$

$$Avg\_Revenue_{kj} = \int_z \int_{\ell'} R_k\left(z, \ell'\right) \widehat{\psi}_{kj}\left(z, \ell'\right) d\ell' dz \text{ for } k = C, S;\ \ j = i, f$$

$$Avg\_InfPenalty_{ki} = \int_z \int_{\ell'} \left[p_{ki}\left(\ell'\right) R_k\left(z, \ell'\right)\right] \widehat{\psi}_{ki}\left(z, \ell'\right) d\ell' dz \text{ for } k = C, S$$

$$Avg\_Vacancies_{kj} = \int_z \int_{\ell} v_{kj}\left(z, \ell\right) \widetilde{\psi}_{kj}\left(z, \ell\right) d\ell dz \text{ for } k = C, S;\ j = i, f$$

$$Avg\_Exports_{Cf} = \left(1 - \exp\left(-\sigma_C \times d_F\right)\right) \int_z \int_{\ell'} \left[R_C\left(z, \ell'\right) \mathcal{I}_C^x\left(z, \ell'\right)\right] \widehat{\psi}_{kf}\left(z, \ell'\right) d\ell' dz$$

$$Fraction\_Export_{Cf} = \int_z \int_{\ell'} \mathcal{I}_C^x\left(z, \ell'\right) \widehat{\psi}_{Cf}\left(z, \ell'\right) d\ell' dz$$

$$Avg\_size_{kj} = \int_z \int_{\ell} \ell \psi_{kj}\left(z, \ell\right) d\ell dz \text{ for } k = C, S;\ j = i, f$$

Now, define

$$\begin{aligned} Avg\_Price_{kj} &= \int_z \int_{\ell'} p_{kj}\left(z,\ell'\right)^{1-\sigma_k} \widehat{\psi}_{kj}\left(z,\ell'\right) d\ell' dz \\ &= \int_z \int_{\ell'} \left(\frac{R_k\left(z,\ell'\right)}{q_k\left(z,\ell',\iota_k\left(z,\ell'\right)\right)} \right)^{1-\sigma_k} \widehat{\psi}_{kj}\left(z,\ell'\right) d\ell' dz \text{ for } k = C, S;\ j = i, f. \end{aligned}$$

We cannot compute Avg_Price_{kj}—given Ω, Ψ_C and Ψ_S. However, note that:

$$\begin{aligned} Avg\_Price_{kj} = &\left(\frac{\left(1-\delta_k\right)\left(\sigma_k-1\right)}{\sigma_k} \right)^{\left(1-\delta_k\right)\Lambda_k} \left(P_k^m\right)^{\left(1-\sigma_k\right)\left(1-\delta_k\right)} \Psi_k^{\left(1-\sigma_k\right)\delta_k} \times \\ &\int_z \int_{\ell'} \left(z \left(\ell'\right)^{\delta_k} \right)^{\Lambda_k} \widehat{\psi}_{kj}\left(z,\ell'\right) d\ell' dz, \end{aligned}$$

$$\begin{aligned} Avg\_Price_{Cf} = &\left(\frac{\left(1-\delta_C\right)\left(\sigma_C-1\right)}{\sigma_C} \right)^{\left(1-\delta_C\right)\Lambda_C} \left(P_C^m\right)^{\left(1-\sigma_C\right)\left(1-\delta_C\right)} \Psi_C^{\left(1-\sigma_C\right)\delta_C} \times \\ &\int_z \int_{\ell'} \left(z \left(\ell'\right)^{\delta_C} \right)^{\Lambda_C} \left(\exp\left(d_F \times \mathcal{I}_C^x\left(z,\ell'\right)\right)\right)^{-\delta_C \sigma_C \Lambda_C} \widehat{\psi}_{Cf}\left(z,\ell'\right) d\ell' dz. \end{aligned}$$

So, given Ω, Ψ_C and Ψ_S we can compute:

$$\begin{aligned} \widetilde{Avg\_Price}_{kj} &\equiv \left(\frac{\left(1-\delta_k\right)\left(\sigma_k-1\right)}{\sigma_k} \right)^{\left(1-\delta_k\right)\Lambda_k} \Psi_k^{\left(1-\sigma_k\right)\delta_k} \int_z \int_{\ell'} \left(z \left(\ell'\right)^{\delta_k} \right)^{\Lambda_k} \widehat{\psi}_{kj}\left(z,\ell'\right) d\ell' dz \\ &= \left(P_k^m\right)^{\left(\sigma_k-1\right)\left(1-\delta_k\right)} Avg\_Price_{kj}, \end{aligned}$$

$$\begin{aligned} \widetilde{Avg\_Price}_{Cf} \equiv &\left(\frac{\left(1-\delta_C\right)\left(\sigma_C-1\right)}{\sigma_C} \right)^{\left(1-\delta_C\right)\Lambda_C} \Psi_C^{\left(1-\sigma_C\right)\delta_C} \times \\ &\int_z \int_{\ell'} \left(z \left(\ell'\right)^{\delta_C} \right)^{\Lambda_C} \left(\exp\left(d_F \times \mathcal{I}_C^x\left(z,\ell'\right)\right)\right)^{-\delta_C \sigma_C \Lambda_C} \widehat{\psi}_{Cf}\left(z,\ell'\right) d\ell' dz \\ &= \left(P_C^m\right)^{\left(\sigma_C-1\right)\left(1-\delta_C\right)} Avg\_Price_{Cf}. \end{aligned}$$

At this point, we can compute the following variables, as functions of M_C and M_S

$$N_{Ci} = \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C \tag{S.3}$$

$$N_{Si} = \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S \tag{S.4}$$

$$N_{Cf} = \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \tag{S.5}$$

$$N_{Sf} = \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \tag{S.6}$$

$$M_{Ci} = \omega_{Ci} M_C$$

$$M_{Si} = \omega_{Si} M_S$$

$$M_{Cf} = \omega_{Cf} M_C$$

$$M_{Sf} = \omega_{Sf} M_S$$

Firm-level expenditures with sector S goods (fixed operating costs, hiring costs, entry costs, fixed export costs)

$$\begin{aligned}
E_S = & \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \left(Avg_Hiring_Costs_{Cf} + \overline{c}_{Cf}\right) \\
& + \frac{\omega_{Ci} M_C}{\varrho_{Ci}^{exit}} \left(Avg_Hiring_Costs_{Ci} + \overline{c}_{Ci}\right) \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \left(Avg_Hiring_Costs_{Sf} + \overline{c}_{Sf}\right) \\
& + \frac{\omega_{Si} M_S}{\varrho_{Si}^{exit}} \left(Avg_Hiring_Costs_{Si} + \overline{c}_{Si}\right) \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C Fraction_Export_{Cf} f_x \\
& + M_C c_{e,C} \\
& + M_S c_{e,S}
\end{aligned}$$

Define c_C:

$$\begin{aligned}
c_C \equiv \frac{E_{S,C}}{M_C} = & \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \left(Avg_Hiring_Costs_{Cf} + \overline{c}_{Cf}\right) \\
& + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \left(Avg_Hiring_Costs_{Ci} + \overline{c}_{Ci}\right) \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Fraction_Export_{Cf} f_x \\
& + c_{e,C},
\end{aligned} \tag{S.7}$$

Where $E_{S,C}$ is firm-level expenditures with sector S goods (fixed costs, etc) coming from C-sector activity.

Define c_S:

$$\begin{aligned}
c_S \equiv \frac{E_{S,S}}{M_S} = & \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \left(Avg_Hiring_Costs_{Sf} + \overline{c}_{Sf}\right) \\
& + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \left(Avg_Hiring_Costs_{Si} + \overline{c}_{Si}\right) \\
& + c_{e,S},
\end{aligned} \tag{S.8}$$

Where $E_{S,S}$ is firm-level expenditures with sector S goods (fixed costs, etc) coming from S-sector activity.

We can therefore write:

$$\begin{aligned} E_S &= E_{S,C} + E_{S,S} \\ &= c_C M_C + c_S M_S \end{aligned}$$

Market Clearing (C and S sectors)

Let I denote aggregate income. Then, market clearing in the C and S sectors must lead to:

$$\zeta I + X_C^{int} = Rev_C - Exports + \tau_a Imports$$

$$(1-\zeta) I + X_S^{int} + E_S = Rev_S$$

$$Imports = Exports$$

Note that expenditures on intermediates are proportional to gross revenues:

$$P_k^m \iota_k(z,\ell) = \frac{(1-\delta_k)(\sigma_k - 1)}{\sigma_k} R_k(z,\ell),$$

which leads to:

$$\begin{aligned} X_C^{int} &= \lambda_C \frac{(1-\delta_C)(\sigma_C-1)}{\sigma_C} Rev_C \\ &+ \lambda_S \frac{(1-\delta_S)(\sigma_S-1)}{\sigma_S} Rev_S \end{aligned}$$

$$\begin{aligned} X_S^{int} &= (1-\lambda_C) \frac{(1-\delta_C)(\sigma_C-1)}{\sigma_C} Rev_C \\ &+ (1-\lambda_S) \frac{(1-\delta_S)(\sigma_S-1)}{\sigma_S} Rev_S \end{aligned}$$

Where Rev_C and Rev_S are total gross revenues in sectors C and S respectively. Therefore:

$$\begin{aligned} I = &\left(1 - \frac{(1-\delta_C)(\sigma_C-1)}{\sigma_C}\right) Rev_C \\ &+ \left(1 - \frac{(1-\delta_S)(\sigma_S-1)}{\sigma_S}\right) Rev_S \\ &- E_S \\ &+ (\tau_a - 1) Exports \end{aligned}$$

Using

$$Rev_C = Avg\_Revenue_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C + Avg\_Revenue_{Ci} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C$$

$$Rev_S = Avg\_Revenue_{Sf} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S + Avg\_Revenue_{Si} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S$$

$$Exports = Avg\_Exports_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C$$

$$E_S = c_C M_C + c_S M_S$$

Step 9a: Write aggregate income I as a function of masses of entrants M_C and M_S.

$$\begin{aligned} I = & \left(1 - \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C}\right) \begin{pmatrix} Avg\_Revenue_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \\ +Avg\_Revenue_{Ci} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C \end{pmatrix} \\ & + \left(1 - \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S}\right) \begin{pmatrix} Avg\_Revenue_{Sf} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \\ +Avg\_Revenue_{Si} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S \end{pmatrix} \\ & - (c_C M_C + c_S M_S) \\ & + (\tau_a - 1) \left(Avg\_Exports_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \right) \end{aligned}$$

Therefore:

$$I = a_C M_C + a_S M_S \tag{S.9}$$

Where

$$\begin{aligned} a_C = & \left(1 - \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C}\right) \begin{pmatrix} Avg\_Revenue_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \\ +Avg\_Revenue_{Ci} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \end{pmatrix} \\ & + (\tau_a - 1) \left(Avg\_Exports_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \right) \\ & - c_C \end{aligned}$$

$$\begin{aligned} a_S = & \left(1 - \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S}\right) \begin{pmatrix} Avg\_Revenue_{Sf} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \\ +Avg\_Revenue_{Si} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \end{pmatrix} \\ & - c_S \end{aligned}$$

Step 9b: Write P_C and P_S as functions of M_C and M_S.

Price Index Sector C

$$P_C^{1-\sigma_C} = P_{H,C}^{1-\sigma_C} + P_{F,C}^{1-\sigma_C}$$

The domestic component is given by:

$$\begin{aligned}
P_{H,C}^{1-\sigma_C} &= N_{Cf} Avg_Price_{Cf} + N_{Ci} Avg_Price_{Ci} \\
&= \begin{pmatrix} \frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg_Price_{Cf} \\ +\frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg_Price_{Ci} \end{pmatrix} M_C \\
&= \begin{pmatrix} \frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \widetilde{Avg_Price}_{Cf} \left(P_C^m\right)^{-(\sigma_C-1)(1-\delta_C)} \\ +\frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \widetilde{Avg_Price}_{Ci} \left(P_C^m\right)^{-(\sigma_C-1)(1-\delta_C)} \end{pmatrix} M_C
\end{aligned}$$

We can therefore write $P_{C,H}$ as:

$$P_{H,C}^{1-\sigma_C} = b_C^1 \left(P_C^m\right)^{(1-\sigma_C)(1-\delta_C)} M_C,$$

Where

$$b_C^1 \equiv \frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \widetilde{Avg\_Price}_{Cf} + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \widetilde{Avg\_Price}_{Ci}$$

The foreign component is given by:

$$P_{F,C}^{1-\sigma_C} = \left(\epsilon\tau_a\tau_c\right)^{1-\sigma_C}.$$

Under Trade Balance:

$$Exports = \frac{D_{H,C}\left(\epsilon\tau_a\tau_c\right)^{1-\sigma_C}}{\tau_a},$$

$$\begin{aligned}
&\Rightarrow \left(\epsilon\tau_a\tau_c\right)^{1-\sigma_C} = \frac{\tau_a \times Exports}{D_{H,C}} \\
&= \frac{\tau_a \times N_{Cf} Avg_Exports_{Cf}}{D_{H,C}} \\
&= \frac{\tau_a \times Avg_Exports_{Cf}}{\exp\left(\sigma_C \times d_{H,C}\right)} \frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \\
&= \left(P_C^m\right)^{-(\sigma_C-1)(1-\delta_C)} \frac{\tau_a \times Avg_Exports_{Cf}}{\Psi_C^{\frac{\sigma-1}{\Lambda}}} \frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C.
\end{aligned}$$

Where we have used

$$\exp\left(\sigma_C \times d_{H,C}\right) = \left(\frac{\Psi_C}{\left(P_C^m\right)^{-(1-\delta_C)\Lambda_C}}\right)^{\frac{\sigma_C-1}{\Lambda_C}}.$$

Therefore:

$$P_{F,C}^{1-\sigma_C} = b_C^2 \left(P_C^m\right)^{(1-\sigma_C)(1-\delta_C)} M_C,$$

Where

$$b_C^2 \equiv \frac{\tau_a \times Avg\_Exports_{Cf}}{\Psi_C^{\frac{\sigma_C-1}{\Lambda_C}}} \frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}}.$$

Therefore:

$$\frac{P_C^{1-\sigma_C}}{\left(P_C^m\right)^{(1-\sigma_C)(1-\delta_C)}} = \underbrace{\left(b_C^1 + b_C^2\right)}_{b_C} M_C \tag{S.10}$$

Price Index Sector S

$$\begin{aligned} P_S^{1-\sigma_S} &= N_{Sf} Avg\_Price_{Sf} + N_{Si} Avg\_Price_{Si} \\ &= \left(\begin{array}{c} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg\_Price_{Sf} \\ + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg\_Price_{Si} \end{array} \right) M_S \\ &= \left(\begin{array}{c} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \widetilde{Avg\_Price}_{Sf} \left(P_S^m\right)^{-(\sigma_S-1)(1-\delta_S)} \\ + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \widetilde{Avg\_Price}_{Si} \left(P_S^m\right)^{-(\sigma_S-1)(1-\delta_S)} \end{array} \right) M_S \end{aligned}$$

$$\Rightarrow P_S^{1-\sigma_S} = b_S \left(P_S^m\right)^{(1-\sigma_S)(1-\delta_S)} M_S$$

Where

$$b_S \equiv \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \widetilde{Avg\_Price}_{Sf} + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \widetilde{Avg\_Price}_{Si}$$

Therefore:

$$\frac{P_S^{1-\sigma_S}}{\left(P_S^m\right)^{(1-\sigma_S)(1-\delta_S)}} = b_S M_S. \tag{S.11}$$

Step 9c: Write X_C^{int} as a function of M_C and M_S.

$$\begin{aligned} X_C^{int} &= \lambda_C \frac{(1-\delta_C)(\sigma_C-1)}{\sigma_C} \left(Avg\_Revenue_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} + Avg\_Revenue_{Ci} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \right) M_C \\ &+ \lambda_S \frac{(1-\delta_S)(\sigma_S-1)}{\sigma_S} \left(Avg\_Revenue_{Sf} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} + Avg\_Revenue_{Si} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \right) M_S \\ &= d_C M_C + d_S M_S \end{aligned}$$

Where

$$d_C = \lambda_C \frac{(1-\delta_C)(\sigma_C-1)}{\sigma_C} \left(Avg\_Revenue_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} + Avg\_Revenue_{Ci} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \right)$$

$$d_S = \lambda_S \frac{(1-\delta_S)(\sigma_S-1)}{\sigma_S} \left(Avg\_Revenue_{Sf} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} + Avg\_Revenue_{Si} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \right)$$

Step 9d: Solve for $\frac{M_S}{M_C}$ that matches Ψ_C.
Remember that:

$$\exp\left(d_{H,C}\right) = \left(\frac{\zeta I + X_C^{int}}{P_C^{1-\sigma_C}} \right)^{\frac{1}{\sigma_C}}$$

Using (S.9), (S.10), (S.2) and manipulating, we obtain:

$$\Psi_C^{\frac{\sigma_C-1}{\Lambda_C}} = \left(\frac{\zeta a_C}{b_C} + \frac{d_C}{b_C}\right) + \frac{\zeta a_S + d_S}{b_C}\frac{M_S}{M_C}$$
$$\frac{M_S}{M_C} = \frac{b_C}{\zeta a_S + d_S}\left(\Psi_C^{\frac{\sigma_C-1}{\Lambda_C}} - \left(\frac{\zeta a_C}{b_C} + \frac{d_C}{b_C}\right)\right)$$

Step 9e: Separately pin down M_C and M_S using the labor market clearing equation $\overline{L} - L_u = \sum_{k=C,S,j=i,f} L_{kj}$. Express M_C and M_S as functions of L_u.

To separately pin down M_C and M_S, use the labor market clearig equation.

$$\begin{aligned}
\overline{L} - L_u &= N_{Cf} Avg_Size_{Cf} + N_{Ci} Avg_Size_{Ci} + N_{Sf} Avg_Size_{Sf} + N_{Si} Avg_Size_{Si} \\
&= \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C Avg_Size_{Cf} + \frac{\omega_{Ci}M_C}{\varrho_{Ci}^{exit}} Avg_Size_{Ci} + \\
&\quad \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S Avg_Size_{Sf} + \frac{\omega_{Si}M_S}{\varrho_{Si}^{exit}} Avg_Size_{Si} \\
&= \left(\frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg_Size_{Cf} + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg_Size_{Ci}\right) M_C + \\
&\quad \left(\frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg_Size_{Sf} + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg_Size_{Si}\right) M_S
\end{aligned}$$

At this point, we can only express M_C and M_S as functions of L_u.
From now on write

$$\left(\frac{M_S}{M_C}\right)^* = \frac{b_C}{\zeta a_S + d_S}\left(\Psi_C^{\frac{\sigma_C-1}{\Lambda_C}} - \left(\frac{\zeta a_C}{b_C} + \frac{d_C}{b_C}\right)\right)$$
$$\Rightarrow M_S = \underbrace{\frac{b_C}{\zeta a_S + d_S}\left(\Psi_C^{\frac{\sigma_C-1}{\Lambda_C}} - \left(\frac{\zeta a_C}{b_C} + \frac{d_C}{b_C}\right)\right)}_{AA} M_C$$

Therefore:

$$\begin{aligned}
M_S &= AA \times M_C \\
AA &= \frac{b_C}{\zeta a_S + d_S}\left(\Psi_C^{\frac{\sigma_C-1}{\Lambda_C}} - \left(\frac{\zeta a_C}{b_C} + \frac{d_C}{b_C}\right)\right)
\end{aligned} \tag{S.12}$$

So that:

$$\begin{aligned}
\overline{L} - L_u &= \left(\frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg_Size_{Cf} + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg_Size_{Ci}\right) M_C + \\
&\quad \left(\frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg_Size_{Sf} + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg_Size_{Si}\right) AA \times M_C \\
&= BB \times M_C
\end{aligned}$$

$$BB = \left(\frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg\_Size_{Cf} + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg\_Size_{Ci} \right) +$$

$$\left(\frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg\_Size_{Sf} + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg\_Size_{Si} \right) AA \tag{S.13}$$

Finally:

$$M_C = \frac{\overline{L} - L_u}{BB} \tag{S.14}$$

$$M_S = \frac{AA}{BB} \left(\overline{L} - L_u\right) \tag{S.15}$$

Step 9f: Express masses of firms N_{kj} as functions of L_u.

Substituting (S.14) and (S.15) into (S.3)-(S.6) to obtain the masses of firms:

$$N_{Ci} = \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C = \underbrace{\frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \frac{1}{BB}}_{EE_C} \left(\overline{L} - L_u\right) = EE_C \left(\overline{L} - L_u\right)$$

$$N_{Si} = \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S = \underbrace{\frac{\omega_{Si}}{\varrho_{Si}^{exit}} \frac{AA}{BB}}_{EE_S} \left(\overline{L} - L_u\right) = EE_S \left(\overline{L} - L_u\right)$$

$$N_{Cf} = \underbrace{\frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \frac{1}{BB}}_{DD_C} \left(\overline{L} - L_u\right)$$

$$N_{Sf} = \underbrace{\frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \frac{AA}{BB}}_{DD_S} \left(\overline{L} - L_u\right)$$

Step 9g: Express aggregate posted vacancies V_{kj} as functions of L_u.

Now, substituting the expressions for the N_{kj}'s to obtain the number of vacancies in each sector as a function of L_u:

$$\begin{aligned} V_{Cf} &= N_{Cf} Avg\_Vacancies_{Cf} + \frac{\omega_{Cf} M_C}{\mu^{\upsilon}} \\ &= Avg\_Vacancies_{Cf} \times DD_C \left(\overline{L} - L_u\right) + \frac{\omega_{Cf}}{\mu^{\upsilon}} \frac{1}{BB} \left(\overline{L} - L_u\right) \\ &= \underbrace{\left(Avg\_Vacancies_{Cf} \times DD_C + \frac{\omega_{Cf}}{\mu^{\upsilon}} \frac{1}{BB} \right)}_{FF_C} \left(\overline{L} - L_u\right) \\ &= FF_C \times \left(\overline{L} - L_u\right) \end{aligned} \tag{S.16}$$

$$
\begin{aligned}
V_{Ci} &= N_{Ci} Avg\_Vacancies_{Ci} + \frac{\omega_{Ci} M_C}{\mu^{\upsilon}} \\
&= Avg\_Vacancies_{Ci} \times EE_C \left(\overline{L} - L_u\right) + \frac{\omega_{Ci}}{\mu^{\upsilon}} \frac{1}{BB} \left(\overline{L} - L_u\right) \\
&= \underbrace{\left(Avg\_Vacancies_{Ci} \times EE_C + \frac{\omega_{Ci}}{\mu^{\upsilon}} \frac{1}{BB}\right)}_{GG_C} \left(\overline{L} - L_u\right) \\
&= GG_C \times \left(\overline{L} - L_u\right)
\end{aligned} \tag{S.17}
$$

$$
\begin{aligned}
V_{Sf} &= N_{Sf} Avg\_Vacancies_{Sf} + \frac{\omega_{Sf} M_S}{\mu^{\upsilon}} \\
&= \underbrace{\left(Avg\_Vacancies_{Sf} \times DD_S + \frac{\omega_{Sf}}{\mu^{\upsilon}} \frac{AA}{BB}\right)}_{FF_S} \left(\overline{L} - L_u\right) \\
&= FF_S \times \left(\overline{L} - L_u\right)
\end{aligned} \tag{S.18}
$$

$$
\begin{aligned}
V_{Si} &= N_{Si} Avg\_Vacancies_{Si} + \frac{\omega_{Si} M_S}{\mu^{\upsilon}} \\
&= \underbrace{\left(Avg\_Vacancies_{Si} \times EE_S + \frac{\omega_{Si}}{\mu^{\upsilon}} \frac{AA}{BB}\right)}_{GG_S} \left(\overline{L} - L_u\right) \\
&= GG_S \times \left(\overline{L} - L_u\right)
\end{aligned} \tag{S.19}
$$

$$
\begin{aligned}
\widetilde{V} &= V_{Cf} + V_{Ci} + V_{Sf} + V_{Si} \\
&= \underbrace{\left(FF_C + GG_C + FF_S + GG_S\right)}_{JJ} \times \left(\overline{L} - L_u\right) \\
&= JJ \times \left(\overline{L} - L_u\right)
\end{aligned}
$$

Step 9h: Use equation for μ^{υ} to obtain L_u.

We have written each V_{kj} in terms of L_u. Now, note that

$$
\mu^{\upsilon} = \phi \left(\frac{L_u}{\widetilde{V}}\right)^{1-\xi}
$$

We can invert this equation to obtain L_u.

$$
\begin{aligned}
\mu^{\upsilon} &= \phi \left(\frac{L_u}{JJ \times \left(\overline{L} - L_u\right)}\right)^{1-\xi} \\
\Rightarrow L_u^* &= \frac{\left(\mu^{\upsilon}\right)^{\frac{1}{1-\xi}} \times JJ \times \overline{L}}{\phi^{\frac{1}{1-\xi}} + \left(\mu^{\upsilon}\right)^{\frac{1}{1-\xi}} \times JJ}
\end{aligned}
$$

Step 9i: Go back and obtain masses of entrants M_k's (equations (S.14) and (S.15)), masses of firms N_{kj}'s (equations (S.3)-(S.6)), and aggregate vacancies V_{kj}'s (equations (S.16)-(S.19)). We are now able to compute transitions out of unemployment μ_{kj}^e (Step 8).

Step 9j: Recover price indices P_C and P_S.

Equations (S.1) and (S.10) lead to:

$$P_C = \left((b_C M_C)^{\frac{1}{1-\sigma_C}} \left(\frac{1}{\lambda_C^{\lambda_C} (1-\lambda_C)^{1-\lambda_C}} \right)^{(1-\delta_C)} \right)^{\frac{1}{1-(1-\delta_C)\lambda_C}} P_S^{\frac{(1-\delta_C)(1-\lambda_C)}{1-(1-\delta_C)\lambda_C}}$$

Defining

$$\varpi_C = \left((b_C M_C)^{\frac{1}{1-\sigma_C}} \left(\frac{1}{\lambda_C^{\lambda_C} (1-\lambda_C)^{1-\lambda_C}} \right)^{(1-\delta_C)} \right)^{\frac{1}{1-(1-\delta_C)\lambda_C}}$$

and

$$\varkappa_C = \frac{(1-\delta_C)(1-\lambda_C)}{1-(1-\delta_C)\lambda_C}$$

Allows us to write

$$P_C = \varpi_C P_S^{\varkappa_C}$$

Equations (S.1) and (S.11) lead to:

$$P_S = \left((b_S M_S)^{\frac{1}{1-\sigma_S}} \left(\frac{1}{\lambda_S^{\lambda_S} (1-\lambda_S)^{1-\lambda_S}} \right)^{(1-\delta_S)} \right)^{\frac{1}{1-(1-\delta_S)(1-\lambda_S)}} P_C^{\frac{(1-\delta_S)\lambda_S}{1-(1-\delta_S)(1-\lambda_S)}}$$

Writing

$$\varpi_S = \left((b_S M_S)^{\frac{1}{1-\sigma_S}} \left(\frac{1}{\lambda_S^{\lambda_S} (1-\lambda_S)^{1-\lambda_S}} \right)^{(1-\delta_S)} \right)^{\frac{1}{1-(1-\delta_S)(1-\lambda_S)}}$$

and

$$\varkappa_S = \frac{(1-\delta_S)\lambda_S}{1-(1-\delta_S)(1-\lambda_S)}$$

Allows us to write

$$P_S = \varpi_S P_C^{\varkappa_S}$$

Solving the system leads to:

$$P_C = \left(\varpi_C (\varpi_S)^{\varkappa_C}\right)^{\frac{1}{1-\varkappa_S\varkappa_C}}$$

Step 9k: Compute deviation between government revenues and spending with unemployment insurance Dev_T.

Government Revenue

$$
\begin{aligned}
G_{Rev} = & \frac{\sigma_C - (1-\delta_C)(\sigma_C - 1)}{\sigma_C} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \tau_y Avg\_Revenue_{Cf} \\
& + \frac{\sigma_S - (1-\delta_S)(\sigma_S - 1)}{\sigma_S} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \tau_y Avg\_Revenue_{Sf} \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \tau_w Avg\_wbill_{Cf} \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \tau_w Avg\_wbill_{Sf} \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C Avg\_Firing\_Costs_{Cf} \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S Avg\_Firing\_Costs_{Sf} \\
& + (\tau_a - 1) \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C Avg\_Exports_{Cf}
\end{aligned}
$$

Government Spending with Unemployment Insurance

$$
G_{UI} = \underbrace{\left(b^u \times \underbrace{\sum_k \left(W_{kf}^{DS} + W_{kf}^{EE} + W_{kf}^{D} \right)}_{\text{mass of \textbf{formal} workers who transition to unemployment}} \right)}_{\text{Total Expenditure with Unemployment Benefits}}
$$

Government Transfers

$$
T = G_{Rev} - G_{UI}
$$

We impose in the objective function that $Dev_T \geq 0$—in other words, we highly penalize $Dev_T < 0$

$$
Dev_T = \frac{G_{Rev} - G_{UI}}{G_{Rev}}
$$

When we compute aggregate income, we implicitly assumed that $G_{Rev} - G_{UI} \geq 0$.

II Simulation

II.1 Simulation Algorithm

Fix P_S at $\overline{P}_S$. Write the value added function as:

$$
VA_k(z, \ell) = \Theta_k \left(\frac{\overline{P}_S^{1-\lambda_k}}{\lambda_k^{\lambda_k}(1-\lambda_k)^{1-\lambda_k}} \right)^{-(1-\delta_k)\Lambda_k} P_C^{-\lambda_k(1-\delta_k)\Lambda_k} \left(\exp\left(d_{H,k}\right)\right)^{\frac{\sigma_k}{\sigma_k - 1}\Lambda_k} \left(z \ell^{\delta_k} \right)^{\Lambda_k}
$$

Define

$$\Xi_k \equiv \Theta_k \left(\frac{\overline{P}_S^{1-\lambda_k}}{\lambda_k^{\lambda_k} (1-\lambda_k)^{1-\lambda_k}} \right)^{-(1-\delta_k)\Lambda_k},$$

and

$$\Phi_k \equiv P_C^{-\lambda_k(1-\delta_k)\Lambda_k} \left(\exp\left(d_{H,k}\right)\right)^{\frac{\sigma_k}{\sigma_k-1}\Lambda_k}.$$

Rewrite the value added function as:

$$VA_k(z,\ell) = \Xi_k \Phi_k \left(z\ell^{\delta_k}\right)^{\Lambda_k}.$$

μ^{υ}, ϑ_{J_u}, d_F, Φ_C, Φ_S are the endogenous variables to be determined in equilibrium. For a given value of these variables, Steps 1 through 11 below compute the deviations from equilibrium conditions given by $L_i\left(\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S\right)$ for $i = 1, ..., 5$. We then need to find values $\left(\mu^{\upsilon}\right)^*, \vartheta_{J_u}^*, d_F^*, \Phi_C^*, \Phi_S^*$ solving $L_i\left(\left(\mu^{\upsilon}\right)^*, \vartheta_{J_u}^*, d_F^*, \Phi_C^*, \Phi_S^*\right) = 0$ for all $i = 1, ..., 5$. We discuss potential solutions to this problem in Step 12.

We proceed by first imposing values for ϑ_{J_u}, μ^{υ}, d_F, Φ_C, Φ_S.

Step 1: This step solves for wage schedules $w_{kf}(z,\ell')$, $w_{ki}(z,\ell')$ as well as value functions $V_{kf}(z,\ell)$, $V_{ki}(z,\ell)$, $J_{kf}^e(z,\ell')$, $J_{ki}^e(z,\ell')$, and firms' policy functions.

Step 1a: Compute value added functions $VA_k(z,\ell)$.

Step 1b: Compute wage schedules $w_{kf}(z,\ell')$

- Guess a wage schedule $w_{kf}(z,\ell')$
- Compute the resulting $V_{kf}(z,\ell')$ using (14)
- Compute $J_{kf}^e(z,\ell')$ using (I.1)
- Compute $w_{kf}^u(z,\ell')$ using equation (28)
- Let $\widehat{w}_{kf}^u(z,\ell') = \omega_0 + \omega_1 \frac{VA_k(z,\ell')}{\ell'}$ be the linear projection of $w_{kf}^u(z,\ell')$ on $\left[1, \frac{VA_k(z,\ell')}{\ell'}\right]$
- Update $w_{kf}(z,\ell') = \max\left\{\widehat{w}_{kf}^u(z,\ell'), b_u + \vartheta_{J_u} - \frac{1}{1+r} J_{kf}^e(z,\ell'), \underline{w}\right\}$
- Restart until convergence

Step 1c: Compute wage schedules $w_{ki}(z,\ell')$

- Guess a wage schedule $w_{ki}(z,\ell')$
- Compute the resulting $V_{ki}(z,\ell')$ using (18)
- Compute $J_{ki}^e(z,\ell')$ using (I.2)
- Compute $w_{ki}^u(z,\ell')$ using equation (31)
- Let $\widehat{w}_{ki}^u(z,\ell') = \omega_0 + \omega_1 \left(1 - \frac{\sigma_k}{\sigma_k-(1-\delta_k)(\sigma_k-1)} p_{ki}(\ell')\right) \frac{VA_k(z,\ell')}{\ell'}$ be the linear projection of $w_{ki}^u(z,\ell')$ on $\left[1, \left(1 - \frac{\sigma_k}{\sigma_k-(1-\delta_k)(\sigma_k-1)} p_{ki}(\ell')\right) \frac{VA_k(z,\ell')}{\ell'}\right]$

- Update $w_{ki}(z,\ell') = \max\left\{\widehat{w}^u_{ki}(z,\ell'), \vartheta_{J_u} - \frac{1}{1+r} J^e_{ki}(z,\ell')\right\}$
- Restart until convergence

Step 2: Solve for firms' entry decisions. Compute the fraction of entrants in the formal and informal sectors as follows:

$$\omega_{kf} \equiv \Pr\left(I^{formal}_k(z) = 1\right) = \int_z I^{formal}_k(z)\, g^e_k(z)\, dz$$

$$\omega_{ki} \equiv \Pr\left(I^{informal}_k(z) = 1\right) = \int_z I^{informal}_k(z)\, g^e_k(z)\, dz$$

Therefore, if M_k is the mass of entrants in sector k, the masses of formal and informal entrants in sector k are given by:

$$M_{ki} = \omega_{ki} M_k$$

$$M_{kf} = \omega_{kf} M_k$$

Finally, compute the distribution of z productivities among entrants, conditional on entry into sector kj.

$$\psi^e_{ki}(z) = \frac{g^e_k(z)\, I^{informal}_k(z)}{\int_{\widetilde{z}} g^e_k(\widetilde{z})\, I^{informal}_k(\widetilde{z})\, d\widetilde{z}},$$

$$\psi^e_{kf}(z) = \frac{g^e_k(z)\, I^{formal}_k(z)}{\int_{\widetilde{z}} g^e_k(\widetilde{z})\, I^{formal}_k(\widetilde{z})\, d\widetilde{z}}.$$

Step 3: Compute the steady-state distribution of states. For informal firms, start with a guess for ψ_{ki}. Then, compute

$$\varrho^{exit}_{ki} = \alpha_k + (1-\alpha_k) \int_z \int_\ell \left(I^{exit}_{ki}(z,\ell) + I^{change}_{ki}(z,\ell)\right) \psi_{ki}(z,\ell)\, d\ell dz.$$

In steady state $N_{ki} = \left(1 - \varrho^{exit}_{ki}\right) N_{ki} + M_{ki}$. Therefore, set $\frac{M_{ki}}{N_{ki}}$, the fraction of sector k informal firms that are entrants, to:

$$\boxed{\frac{M_{ki}}{N_{ki}} = \varrho^{exit}_{ki} = \frac{\omega_{ki} M_k}{N_{ki}}.}$$

Now, compute $\widetilde{\psi}_{ki}$:

$$\begin{aligned}\widetilde{\psi}_{ki}(z,\ell) = {} & \mathcal{I}[\ell = 1] \times \varrho^{exit}_{ki} \times \psi^e_{ki}(z) \\ & + \mathcal{I}[\ell \geq 1] \times (1-\alpha_k) \times \psi_{ki}(z,\ell)\, I^{stay}_{ki}(z,\ell),\end{aligned}$$

and $\widehat{\psi}_{ki}$:

$$\widehat{\psi}_{ki}(z,\ell') = \int_\ell \widetilde{\psi}_{ki}(z,\ell)\, \mathcal{I}\left(L_{ki}(z,\ell) = \ell'\right) d\ell$$

Update ψ_{ki} with:

$$\psi_{ki}(z',\ell') = \int_z \widehat{\psi}_{ki}(z,\ell')\, g_k(z'|z)\, dz,$$

and repeat until convergence of ψ_{ki}. This converged value of ψ_{ki} will be used directly in the computation of ψ_{kf} below.

For formal firms, start with guess for ψ_{kf} and compute:

$$\varrho_{kf}^{exit} = \alpha_k + (1-\alpha_k)\int_z\int_\ell I_{kf}^{exit}(z,\ell)\,\psi_{kf}(z,\ell)\,d\ell dz,$$

$$\varrho_{ki}^{change} = (1-\alpha_k)\int_z\int_\ell I_{ki}^{change}(z,\ell)\,\psi_{ki}(z,\ell)\,d\ell dz.$$

In steady state:

$$\begin{aligned}\varrho_{kf}^{exit} N_{kf} &= \varrho_{ki}^{change}\underbrace{N_{ki}}_{\frac{\omega_{ki}M_k}{\varrho_{ki}^{exit}}} + \omega_{kf}M_k \\ &= M_k\left(\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}}\omega_{ki} + \omega_{kf}\right)\end{aligned}$$

So that:

$$\boxed{\frac{M_{kf}}{N_{kf}} = \frac{M_k\omega_{kf}}{N_{kf}} = \frac{\varrho_{kf}^{exit}\omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}}\omega_{ki} + \omega_{kf}}}$$

Also, note that

$$\frac{M_{kf}}{N_{kf}} \times \frac{N_{ki}}{M_{ki}} = \frac{\varrho_{kf}^{exit}\omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}}\omega_{ki} + \omega_{kf}}\frac{1}{\varrho_{ki}^{exit}} = \frac{\varrho_{kf}^{exit}\omega_{kf}}{\varrho_{ki}^{change}\omega_{ki} + \varrho_{ki}^{exit}\omega_{kf}}$$

and

$$\frac{M_{kf}}{N_{kf}} \times \frac{N_{ki}}{M_{ki}} = \frac{\omega_{kf}}{\omega_{ki}}\frac{N_{ki}}{N_{kf}}$$

Therefore,

$$\boxed{\frac{N_{ki}}{N_{kf}} = \frac{\varrho_{kf}^{exit}\omega_{ki}}{\varrho_{ki}^{change}\omega_{ki} + \varrho_{ki}^{exit}\omega_{kf}}}$$

Compute $\widetilde{\psi}_{kf}$ as:

$$\widetilde{\psi}_{kf}(z,\ell) = \begin{array}{c}\mathcal{I}[\ell=1] \times \underbrace{\frac{\varrho_{kf}^{exit}\omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}}\omega_{ki} + \omega_{kf}}}_{\frac{M_{kf}}{N_{kf}}}\psi_{kf}^{e}(z) \\ +\mathcal{I}[\ell \geq 1] \times \left(\begin{array}{c}(1-\alpha_k)\,\psi_{kf}(z,\ell)\,I_{kf}^{stay}(z,\ell) \\ +(1-\alpha_k)\underbrace{\frac{\varrho_{kf}^{exit}\omega_{ki}}{\varrho_{ki}^{change}\omega_{ki} + \varrho_{ki}^{exit}\omega_{kf}}}_{\frac{N_{ki}}{N_{kf}}}\psi_{ki}(z,\ell)\,I_{ki}^{change}(z,\ell)\end{array}\right)\end{array}$$

and $\widehat{\psi}_{kf}$ as:

$$\widehat{\psi}_{kf}(z,\ell') = \int_\ell \widetilde{\psi}_{kf}(z,\ell)\,\mathcal{I}(L_{kf}(z,\ell)=\ell')\,d\ell.$$

Update ψ_{kf} with:

$$\psi_{kf}(z',\ell') = \int_z \widehat{\psi}_{kf}(z,\ell')\,g_k(z'|z)\,dz,$$

and repeat until convergence of ψ_{kf}.

At this point we have the following objects: ψ_{kj}, $\widetilde{\psi}_{kj}$, $\widehat{\psi}_{kj}$, ϱ_{ki}^{exit}, ϱ_{ki}^{change}, ϱ_{kf}^{exit}, $\chi_{ki\to f}^{change}$, χ_{kf}^{layoff}, and χ_{ki}^{leave} (see equations (C.7), (C.11) and (C.13)).

Step 4: Compute the values of entry V_k^e $(k = C, S)$:

$$V_k^e = \int_z \left[V_{ki}^e (z)\, I_k^{informal} (z) + V_{kf}^e (z)\, I_k^{formal} (z)\right] g_k^e (z)\, dz$$

and compute the deviations

$$L_5 (\mu^\upsilon, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S) = L_5 (\vartheta_{J_u}, \mu^\upsilon, \Phi_S) = Dev_{entry,S} = \frac{V_S^e - c_{e,S}}{c_{e,S}}$$

$$L_4 (\mu^\upsilon, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S) = L_4 (\vartheta_{J_u}, \mu^\upsilon, d_F, \Phi_C) = Dev_{entry,C} = \frac{V_C^e - c_{e,C}}{c_{e,C}}$$

Step 5: This step solves for masses of entrants M_k's, masses of firms N_{kj}'s, aggregate vacancies V_{kj}'s consistent with Φ_C, Φ_S, ϑ_{J_u}, d_F and μ^υ.
Step 5a: Write aggregate income I as a function of masses of entrants M_C and M_S.
Step 5b: Write M_C as a functions of P_C and M_S as a function of M_C and $\overline{P}_S$.
Step 5c: Write X_C^{int} as a function of M_C and M_S
Step 5d: Pin down M_C using the equation defining Φ_C, then obtain M_S.
Step 5e: Obtain masses of firms N_{kj}.
Step 5f: Obtain aggregate posted vacancies V_{kj} and $\widetilde{V}$.
Step 5g: Save the values for P_C and $P_{F,C}$ to be used in Step 9.

Step 6: Compute L_u

$$L_u = \left(\frac{\mu^\upsilon}{\phi}\right)^{\frac{1}{1-\xi}} \widetilde{V}$$

Step 7: Obtain job finding rates μ_{kj}^e using aggregate vacancies V_{kj}'s and mass of unemployment L_u obtained in Steps 5 and 6.

$$\mu_{kj}^e = \frac{m_{kj}}{L_u} = \phi \frac{V_{kj}}{\widetilde{V}} \left(\frac{\widetilde{V}}{L_u}\right)^\xi$$

Step 8: Use equations (C.14)-(C.15) to obtain allocations L_{Cf}, L_{Ci}, L_{Sf}, L_{Si}.

$$L_{Ci} = \frac{\mu_{Ci}^e L_u}{\chi_{Ci}^{leave}}$$

$$L_{Si} = \frac{\mu_{Si}^e L_u}{\chi_{Si}^{leave}}$$

$$L_{Cf} = \frac{\mu_{Cf}^e L_u + \chi_{Ci\to f}^{change} L_{Ci}}{\chi_{Cf}^{layoff}}$$

$$L_{Sf} = \frac{\mu_{Sf}^e L_u + \chi_{Si\to f}^{change} L_{Si}}{\chi_{Sf}^{layoff}}$$

Step 9: Compute

$$\overline{\epsilon} = \frac{P_{F,C}}{\tau_a \tau_c},$$

where $P_{F,C}$ was determined in Step 5.
Compute:

$$d_F' = \log \left(\left(1 + \frac{D_F^*}{\exp\left(\sigma_C \times d_{H,C}\right)} \overline{\epsilon}^{\sigma_C} \tau_c^{1-\sigma_C} \right)^{\frac{1}{\sigma_C}} \right),$$

where

$$\exp\left(\sigma_C \times d_{H,C}\right) = \Phi_C^{\frac{\sigma_C - 1}{\lambda_C}} \left(P_C\right)^{\lambda_C (1-\delta_C)(\sigma_C - 1)},$$

and P_C was determined in Step 5. Compute the deviation

$$L_3\left(\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S\right) = Dev_{d_F} = \frac{d_F - d_F'}{d_F}$$

Step 10: Compute deviation from the labor market clearing equation:

$$L_1\left(\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S\right) = Dev_L = \frac{\overline{L} - \left(L_{Cf} + L_{Ci} + L_{Sf} + L_{Si} + L_u\right)}{\overline{L}}$$

Step 11: Compute the deviation

$$\begin{aligned} L_2\left(\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S\right) &= Dev_{J_u} \\ &= 1 - \frac{\left(\sum\limits_{k,j} \mu_{kj}^e \int_\ell \int_z \overline{J}_{kj}^e \left(z, L_{kj}\left(z, \ell\right)\right) g_{kj}\left(z, \ell\right) dz d\ell + \left(1 - \sum\limits_{k,j} \mu_{kj}^e \right) \vartheta_{J_u} \right)}{\left(1+r\right)\left(\vartheta_{J_u} - b\right)} \end{aligned}$$

Therefore, given $\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S$, we can compute deviations L_1, L_2, L_3, L_4, L_5.

Step 12: The equilibrium is given by $\left(\mu^{\upsilon}\right)^*, \vartheta_{J_u}^*, d_F^*, \Phi_C^*, \Phi_S^*$ solving

$$L_i\left(\left(\mu^{\upsilon}\right)^*, \vartheta_{J_u}^*, d_F^*, \Phi_C^*, \Phi_S^*\right) = 0 \text{ for all } i = 1, ..., 5$$

Step 13: Compute the price index for exports

$$P_X^* \equiv \left(\int_{N_{F,C}}^{N_C} \mathcal{I}_C^x\left(n\right) p_x^*\left(n\right)^{1-\sigma_C} dn \right)^{\frac{1}{1-\sigma_C}}$$

Note that

$$Exports = \epsilon D_F^* \left(P_X^*\right)^{1-\sigma_C}$$

So that:

$$P_X^* = \left(\frac{Exports}{\epsilon D_F^*}\right)^{\frac{1}{1-\sigma_C}}$$

A key difficulty is that, given the discrete approximations for the state space, the system above has discontinuities. We list a few solutions we implemented.

- Solve for the system using a sequential bisection method. This procedure has the drawback of being very slow.
- Solve for the system using an optimization routine minimizing the norm of the system. This procedure has the drawback of also being slow and to potentially be stuck in local minima.
- Our preferred solution is to approximate each function $L_i(\mu^v, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S)$ with a third degree polynomial on the arguments. To do so, we draw a large number of values for $(\mu^v, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S)$ and follow Steps 1 through 11 above to compute $L_i(\mu^v, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S)$ at each of these points. We then fit third degree polynomials for each L_i function $i = 1, ..., 5$. Finally, we can use an out-of-the shelf solver to find the root of this approximated system.

II.2 Simulation Algorithm – Details

This section details the steps within Step 5 of the estimation procedure.

Step 5: This step solves for masses of entrants M_k's, masses of firms N_{kj}'s, aggregate vacancies V_{kj}'s consistent with Φ_C, Φ_S, ϑ_{J_u}, d_F, and μ^v.

We start with some definitions... Averages "per firm". All these quantities can be computed after Step 4, that is, after solving for the steady state distribution of states.

$$Avg\_wbill_{ki} = \int_z \int_{\ell'} [w_{ki}(z,\ell')\,\ell']\,\widehat{\psi}_{ki}(z,\ell')\,d\ell' dz \text{ for } k = C, S$$

$$Avg\_wbill_{kf} = \int_z \int_{\ell'} [\max\{w_{kf}(z,\ell'), \underline{w}\}\,\ell']\,\widehat{\psi}_{kf}(z,\ell')\,d\ell' dz \text{ for } k = C, S$$

$$Avg\_Firing\_Costs_{kf} = \kappa \int_z \int_{\ell} \left[(\ell - L_{kf}(z,\ell))\left(1 - I_{kf}^{hire}(z,\ell)\right)\right] \widetilde{\psi}_{kf}(z,\ell)\,d\ell dz \text{ for } k = C, S$$

$$Avg\_Hiring\_Costs_{kj} = \int_z \int_{\ell} \left[H_{kj}(\ell, L_{kj}(z,\ell))\, I_{kj}^{hire}(z,\ell)\right] \widetilde{\psi}_{kj}(z,\ell)\,d\ell dz \text{ for } k = C, S;\ j = i, f$$

$$Avg\_Revenue_{kj} = \int_z \int_{\ell'} R_k(z,\ell')\,\widehat{\psi}_{kj}(z,\ell')\,d\ell' dz \text{ for } k = C, S;\ j = i, f$$

$$Avg\_InfPenalty_{ki} = \int_z \int_{\ell'} [p_{ki}(\ell')\,R_k(z,\ell')]\,\widehat{\psi}_{ki}(z,\ell')\,d\ell' dz \text{ for } k = C, S$$

$$Avg\_Vacancies_{kj} = \int_z \int_{\ell} v_{kj}(z,\ell)\,\widetilde{\psi}_{kj}(z,\ell)\,d\ell dz \text{ for } k = C, S;\ j = i, f$$

$$Avg\_Exports_{Cf} = (1 - \exp(-\sigma_C \times d_F)) \int_z \int_{\ell'} [R_C(z,\ell')\,\mathcal{I}_C^x(z,\ell')]\,\widehat{\psi}_{kf}(z,\ell')\,d\ell' dz$$

$$Fraction\_Export_{Cf} = \int_z \int_{\ell'} \mathcal{I}_C^x(z,\ell')\,\widehat{\psi}_{Cf}(z,\ell')\,d\ell' dz$$

$$Avg\_size_{kj} = \int_z \int_{\ell} \ell \psi_{kj}(z,\ell)\,d\ell dz \text{ for } k = C, S;\ j = i, f$$

Now, define

$$\begin{aligned} Avg\_Price_{kj} &= \int_z \int_{\ell'} p_{kj}(z,\ell')^{1-\sigma_k} \widehat{\psi}_{kj}(z,\ell')\, d\ell' dz \\ &= \int_z \int_{\ell'} \left(\frac{R_k(z,\ell')}{q_k(z,\ell',\iota_k(z,\ell'))} \right)^{1-\sigma_k} \widehat{\psi}_{kj}(z,\ell')\, d\ell' dz \text{ for } k = C,S;\ j = i,f. \end{aligned}$$

We cannot compute Avg_Price_{kj}—given Ω, Φ_C and Φ_S. However, note that:

$$Avg\_Price_{kj} = \widetilde{\Xi}_k P_C^{(1-\sigma_k)(1-\delta_k)\lambda_k} \Phi_k^{\delta_k(1-\sigma_k)} \int\int \left(z(\ell')^{\delta_k} \right)^{\Lambda_k} \widehat{\psi}_{kj}(z,\ell')\, dz d\ell'$$

$$Avg\_Price_{Cf} = \widetilde{\Xi}_C P_C^{(1-\sigma_C)(1-\delta_C)\lambda_C} \Phi_C^{\delta_C(1-\sigma_C)} \int\int \left(z(\ell')^{\delta_C} \right)^{\Lambda_C} \left(\exp\left(d_F \times \mathcal{I}_C^x(z,\ell')\right)\right)^{-\delta_C\sigma_C\Lambda_C} \widehat{\psi}_{Cf}(z,\ell')\, dz d\ell'$$

$$\widetilde{\Xi}_k = \left(\frac{\overline{P}_S^{1-\lambda_k}}{\lambda_k^{\lambda_k}(1-\lambda_k)^{1-\lambda_k}} \right)^{-(1-\delta_k)\Lambda_k} \left(\frac{(1-\delta_k)(\sigma_k - 1)}{\sigma_k} \right)^{(1-\delta_k)\Lambda_k}$$

So, given Ω, Φ_C and Φ_S we can compute:

$$\begin{aligned} \widetilde{Avg\_Price}_{kj} &\equiv \widetilde{\Xi}_k \Phi_k^{\delta_k(1-\sigma_k)} \int\int \left(z(\ell')^{\delta_k} \right)^{\Lambda_k} \widehat{\psi}_{kj}(z,\ell')\, dz d\ell' \\ &= \frac{Avg\_Price_{kj}}{P_C^{(1-\sigma_k)(1-\delta_k)\lambda_k}} \end{aligned}$$

$$\begin{aligned} \widetilde{Avg\_Price}_{Cf} &\equiv \widetilde{\Xi}_C \Phi_C^{\delta_C(1-\sigma_C)} \int\int \left(z(\ell')^{\delta_C} \right)^{\Lambda_C} \left(\exp\left(d_F \times \mathcal{I}_C^x(z,\ell')\right)\right)^{-\delta_C\sigma_C\Lambda_C} \widehat{\psi}_{Cf}(z,\ell')\, dz d\ell' \\ &= \frac{Avg\_Price_{Cf}}{P_C^{(1-\sigma_C)(1-\delta_C)\lambda_C}} \end{aligned}$$

At this point, we can compute the following variables, as functions of M_C and M_S:

$$N_{Ci} = \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C \tag{S.1}$$

$$N_{Si} = \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S \tag{S.2}$$

$$N_{Cf} = \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \tag{S.3}$$

$$N_{Sf} = \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \tag{S.4}$$

$$M_{Ci} = \omega_{Ci} M_C$$

$$M_{Si} = \omega_{Si} M_S$$

$$M_{Cf} = \omega_{Cf} M_C$$

$$M_{Sf} = \omega_{Sf} M_S$$

Firm-level expenditures with sector S goods (fixed operating costs, hiring costs, entry costs, fixed export costs)

$$\begin{aligned} E_S = & \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \left(Avg\_Hiring\_Costs_{Cf} + \bar{c}_{Cf} \right) \\ & + \frac{\omega_{Ci} M_C}{\varrho_{Ci}^{exit}} \left(Avg\_Hiring\_Costs_{Ci} + \bar{c}_{Ci} \right) \\ & + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \left(Avg\_Hiring\_Costs_{Sf} + \bar{c}_{Sf} \right) \\ & + \frac{\omega_{Si} M_S}{\varrho_{Si}^{exit}} \left(Avg\_Hiring\_Costs_{Si} + \bar{c}_{Si} \right) \\ & + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C Fraction\_Export_{Cf} f_x \\ & + M_C c_{e,C} \\ & + M_S c_{e,S} \end{aligned}$$

Define c_C:

$$\begin{aligned} c_C \equiv \frac{E_{S,C}}{M_C} = & \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \left(Avg\_Hiring\_Costs_{Cf} + \bar{c}_{Cf} \right) \\ & + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \left(Avg\_Hiring\_Costs_{Ci} + \bar{c}_{Ci} \right) \\ & + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Fraction\_Export_{Cf} f_x \\ & + c_{e,C}, \end{aligned}$$

Where $E_{S,C}$ is firm-level expenditures with sector S goods (fixed costs, etc) coming from C-sector activity.

Define c_S:

$$\begin{aligned} c_S \equiv \frac{E_{S,S}}{M_S} = & \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \left(Avg\_Hiring\_Costs_{Sf} + \bar{c}_{Sf} \right) \\ & + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \left(Avg\_Hiring\_Costs_{Si} + \bar{c}_{Si} \right) \\ & + c_{e,S}, \end{aligned}$$

Where $E_{S,S}$ is firm-level expenditures with sector S goods (fixed costs, etc) coming from S-sector activity.

We can therefore write:

$$\begin{aligned} E_S &= E_{S,C} + E_{S,S} \\ &= c_C M_C + c_S M_S \end{aligned}$$

Market Clearing (C and S sectors)

Let I denote aggregate income. Then, market clearing in the C and S sectors must lead to:

$$\zeta I + X_C^{int} = Rev_C - Exports + \tau_a Imports$$
$$(1-\zeta) I + X_S^{int} + E_S = Rev_S$$
$$Imports = Exports$$

Note that expenditures on intermediates are proportional to gross revenues:

$$P_k^m \iota_k(z,\ell) = \frac{(1-\delta_k)(\sigma_k - 1)}{\sigma_k} R_k(z,\ell),$$

which leads to:

$$\begin{aligned} X_C^{int} &= \lambda_C \frac{(1-\delta_C)(\sigma_C-1)}{\sigma_C} Rev_C \\ &+ \lambda_S \frac{(1-\delta_S)(\sigma_S-1)}{\sigma_S} Rev_S \end{aligned}$$

$$\begin{aligned} X_S^{int} &= (1-\lambda_C) \frac{(1-\delta_C)(\sigma_C-1)}{\sigma_C} Rev_C \\ &+ (1-\lambda_S) \frac{(1-\delta_S)(\sigma_S-1)}{\sigma_S} Rev_S \end{aligned}$$

Where Rev_C and Rev_S are total gross revenues in sectors C and S respectively. Therefore:

$$\begin{aligned} I &= \left(1 - \frac{(1-\delta_C)(\sigma_C-1)}{\sigma_C}\right) Rev_C \\ &+ \left(1 - \frac{(1-\delta_S)(\sigma_S-1)}{\sigma_S}\right) Rev_S \\ &- E_S \\ &+ (\tau_a - 1) Exports \end{aligned}$$

Using

$$\begin{aligned} Rev_C &= Avg\_Revenue_{Cf} \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C + Avg\_Revenue_{Ci} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C \\ Rev_S &= Avg\_Revenue_{Sf} \frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} M_S + Avg\_Revenue_{Si} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S \\ Exports &= Avg\_Exports_{Cf} \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C \\ E_S &= c_C M_C + c_S M_S \end{aligned}$$

Step 5a: Write aggregate income I as a function of masses of entrants M_C and M_S.

$$I = \left(1 - \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C}\right) \begin{pmatrix} Avg\_Revenue_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \\ + Avg\_Revenue_{Ci} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C \end{pmatrix}$$
$$+ \left(1 - \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S}\right) \begin{pmatrix} Avg\_Revenue_{Sf} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \\ + Avg\_Revenue_{Si} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S \end{pmatrix}$$
$$- (c_C M_C + c_S M_S)$$
$$+ (\tau_a - 1) \left(Avg\_Exports_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \right)$$

Therefore:

$$I = a_C M_C + a_S M_S \tag{S.5}$$

Where

$$a_C = \left(1 - \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C}\right) \begin{pmatrix} Avg\_Revenue_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \\ + Avg\_Revenue_{Ci} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \end{pmatrix}$$
$$+ (\tau_a - 1) \left(Avg\_Exports_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \right)$$
$$- c_C$$

$$a_S = \left(1 - \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S}\right) \begin{pmatrix} Avg\_Revenue_{Sf} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \\ + Avg\_Revenue_{Si} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \end{pmatrix}$$
$$- c_S$$

Step 5b: Write M_C as a functions of P_C and M_S as a function of M_C and $\overline{P}_S$.

Price Index Sector C

$$P_C^{1-\sigma_C} = P_{H,C}^{1-\sigma_C} + P_{F,C}^{1-\sigma_C}$$

The domestic component is given by:

$$\begin{aligned} P_{C,H}^{1-\sigma_C} &= N_{Cf} Avg\_Price_{Cf} + N_{Ci} Avg\_Price_{Ci} \\ &= \begin{pmatrix} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg\_Price_{Cf} \\ + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg\_Price_{Ci} \end{pmatrix} M_C \\ &= \begin{pmatrix} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \widetilde{Avg\_Price}_{Cf} P_C^{(1-\sigma_C)(1-\delta_C)\lambda_C} \\ + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \widetilde{Avg\_Price}_{Ci} P_C^{(1-\sigma_C)(1-\delta_C)\lambda_C} \end{pmatrix} M_C \end{aligned}$$

We can therefore write $P_{C,H}$ as:

$$P_{C,H}^{1-\sigma_C} = P_C^{(1-\sigma_C)(1-\delta_C)\lambda_C} b_C^1 M_C,$$

Where

$$b_C^1 \equiv \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}}\widetilde{Avg\_Price}_{Cf} + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}}\widetilde{Avg\_Price}_{Ci}$$

The foreign component is given by:

$$P_{F,C}^{1-\sigma_C} = (\epsilon\tau_a\tau_c)^{1-\sigma_C}.$$

Under Trade Balance:

$$Exports = \frac{D_{H,C}(\epsilon\tau_a\tau_c)^{1-\sigma_C}}{\tau_a},$$

$$\begin{aligned}
&\Rightarrow (\epsilon\tau_a\tau_c)^{1-\sigma_C} = \frac{\tau_a \times Exports}{D_{H,C}} \\
&= \frac{\tau_a \times N_{Cf}Avg_Exports_{Cf}}{D_{H,C}} \\
&= \frac{\tau_a \times Avg_Exports_{Cf}}{\exp(\sigma_C \times d_{H,C})}\frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}}M_C \\
&= (P_C)^{\lambda_C(1-\delta_C)(1-\sigma_C)}\frac{\tau_a \times Avg_Exports_{Cf}}{\Phi_C^{\frac{\sigma_C-1}{\lambda_C}}}\frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}}M_C.
\end{aligned}$$

Where we have used

$$\exp(\sigma_C \times d_{H,C}) = \Phi_C^{\frac{\sigma_C-1}{\lambda_C}}(P_C)^{\lambda_C(1-\delta_C)(\sigma_C-1)}.$$

Therefore:

$$P_{F,C}^{1-\sigma_C} = (P_C)^{\lambda_C(1-\delta_C)(1-\sigma_C)}b_C^2 M_C,$$

Where

$$b_C^2 \equiv \frac{\tau_a \times Avg\_Exports_{Cf}}{\Phi_C^{\frac{\sigma_C-1}{\lambda_C}}}\frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}}.$$

Rewriting:

$$\begin{aligned}
P_C^{1-\sigma_C} &= P_{C,H}^{1-\sigma_C} + P_{F,C}^{1-\sigma_C} \\
&= (P_C)^{\lambda_C(1-\delta_C)(1-\sigma_C)}b_C^1 M_C + (P_C)^{\lambda_C(1-\delta_C)(1-\sigma_C)}b_C^2 M_C
\end{aligned}$$

So that:

$$P_C^{1-\sigma_C} = (b_C M_C)^{\frac{1}{(1-\lambda_C(1-\delta_C))}} \tag{S.6}$$

where

$$b_C \equiv b_C^1 + b_C^2.$$

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$$\begin{aligned}
P_S^{1-\sigma_S} &= N_{Sf} Avg_Price_{Sf} + N_{Si} Avg_Price_{Si} \\
&= \begin{pmatrix} \frac{\varrho_{Si}^{change}\omega_{Si}+\varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg_Price_{Sf} \\ +\frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg_Price_{Si} \end{pmatrix} M_S \\
&= \begin{pmatrix} \frac{\varrho_{Si}^{change}\omega_{Si}+\varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \widetilde{Avg_Price}_{Sf} P_C^{(1-\sigma_S)(1-\delta_S)\lambda_S} \\ +\frac{\omega_{Si}}{\varrho_{Si}^{exit}} \widetilde{Avg_Price}_{Si} P_C^{(1-\sigma_S)(1-\delta_S)\lambda_S} \end{pmatrix} M_S
\end{aligned}$$

$$\Rightarrow P_S^{1-\sigma_S} = P_C^{(1-\sigma_S)(1-\delta_S)\lambda_S} b_S M_S$$

Where

$$b_S \equiv \frac{\varrho_{Si}^{change}\omega_{Si}+\varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \widetilde{Avg\_Price}_{Sf} + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \widetilde{Avg\_Price}_{Si}$$

Given that $P_S = \overline{P}_S$ is fixed, we can also write M_S as a function of P_C and model parameters.

$$M_S = \frac{\overline{P}_S^{1-\sigma_S}}{b_S P_C^{(1-\sigma_S)(1-\delta_S)\lambda_S}}, \tag{S.7}$$

and using (S.6):

$$M_S = \frac{\overline{P}_S^{1-\sigma_S}}{b_S \left(b_C M_C\right)^{\frac{(1-\sigma_S)(1-\delta_S)\lambda_S}{(1-\sigma_C)(1-(1-\delta_C)\lambda_C)}}}. \tag{S.8}$$

Step 5c:Write X_C^{int} as a function of M_C and M_S.

$$\begin{aligned}
X_C^{int} &= \lambda_C \frac{(1-\delta_C)(\sigma_C-1)}{\sigma_C}\left(Avg_Revenue_{Cf}\frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} + Avg_Revenue_{Ci}\frac{\omega_{Ci}}{\varrho_{Ci}^{exit}}\right) M_C \\
&+ \lambda_S \frac{(1-\delta_S)(\sigma_S-1)}{\sigma_S}\left(Avg_Revenue_{Sf}\frac{\varrho_{Si}^{change}\omega_{Si}+\varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} + Avg_Revenue_{Si}\frac{\omega_{Si}}{\varrho_{Si}^{exit}}\right) M_S \\
&= d_C M_C + d_S M_S
\end{aligned}$$

Where

$$d_C = \lambda_C \frac{(1-\delta_C)(\sigma_C-1)}{\sigma_C}\left(Avg\_Revenue_{Cf}\frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} + Avg\_Revenue_{Ci}\frac{\omega_{Ci}}{\varrho_{Ci}^{exit}}\right)$$

$$d_S = \lambda_S \frac{(1-\delta_S)(\sigma_S-1)}{\sigma_S}\left(Avg\_Revenue_{Sf}\frac{\varrho_{Si}^{change}\omega_{Si}+\varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} + Avg\_Revenue_{Si}\frac{\omega_{Si}}{\varrho_{Si}^{exit}}\right)$$

Step 5d: Pin down P_C using the equation defining Φ_C, and obtain M_C and M_S.

We can now express aggregate income I as a function of P_C using equations (S.5), (S.6) and (S.7) and solve for P_C. Remember that:

$$\exp\left(d_{H,C}\right) = \left(\frac{\zeta I + X_C^{int}}{P_C^{1-\sigma_C}}\right)^{\frac{1}{\sigma_C}}$$

Using the formula defining Φ_C and manipulating, we obtain:

$$P_C^{-\lambda_C(1-\sigma_C)(1-\delta_C)}\Phi_C^{\frac{\sigma_C-1}{\Lambda_C}} = \exp\left(\sigma_C \times d_{H,C}\right) = \frac{\zeta\left(a_C M_C + a_S M_S\right) + d_C M_C + d_S M_S}{P_C^{1-\sigma_C}},$$

which leads to

$$\Phi_C^{\frac{\sigma_C-1}{\Lambda_C}} = \frac{\left(\zeta a_C + d_C\right) M_C + \left(\zeta a_S + d_S\right) M_S}{b_C M_C},$$

which allows us to solve for M_C

$$M_C = \frac{1}{b_C}\left(\frac{\left(\zeta a_S + d_S\right)\overline{P}_S^{1-\sigma_S}}{b_S\left(\Phi_C^{\frac{\sigma_C-1}{\Lambda_C}} - \frac{\left(\zeta a_C + d_C\right)}{b_C}\right)}\right)^{\frac{1}{1+\frac{(1-\sigma_S)(1-\delta_S)\lambda_S}{(1-\sigma_C)(1-(1-\delta_C)\lambda_C)}}},$$

and then for M_S using (S.8).

Step 5e: Now that we have values of M_C and M_S, we obtain masses of firms N_{kj}.

$$N_{Ci} = \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C$$

$$N_{Si} = \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S$$

$$N_{Cf} = \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C$$

$$N_{Cf} = \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S$$

Step 5f: Obtain aggregate posted vacancies V_{kj}.
Now, substituting the expressions for the N_{kj}'s to obtain the number of vacancies in each sector as a function of L_u:

$$V_{Cf} = N_{Cf} Avg\_Vacancies_{Cf} + \frac{\omega_{Cf} M_C}{\mu^{\upsilon}}$$

$$V_{Ci} = N_{Ci} Avg\_Vacancies_{Ci} + \frac{\omega_{Ci} M_C}{\mu^{\upsilon}}$$

$$V_{Sf} = N_{Sf} Avg\_Vacancies_{Sf} + \frac{\omega_{Sf} M_S}{\mu^{\upsilon}}$$

$$V_{Si} = N_{Si} Avg\_Vacancies_{Si} + \frac{\omega_{Si} M_S}{\mu^{\upsilon}}$$

Step 5g: Save the values for P_C and $P_{F,C}$:

$$P_C^{1-\sigma_C} = \left(b_C M_C\right)^{\frac{1}{(1-(1-\delta_C)\lambda_C)}}$$

$$P_{F,C}^{1-\sigma_C} = (P_C)^{-\lambda_C(1-\delta_C)(\sigma_C-1)}\, b_C^2 M_C$$