SOFT-FLOOR AUCTIONS: HARNESSING REGRET TO IMPROVE EFFICIENCY AND REVENUE

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YALE UNIVERSITY
Box 208281
New Haven, Connecticut 06520-8281

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Abstract. A soft-floor auction asks bidders to accept an opening price to participate in an ascending auction. If no bidder accepts, lower bids are considered using first-price rules. Soft floors are common despite being irrelevant with standard assumptions. When bidders regret losing, soft-floor auctions are more efficient and profitable than standard optimal auctions. Revenue increases as bidders are inclined to accept the opening price to compete in a regret-free ascending auction. Efficiency is improved since having a soft floor allows for a lower hard reserve price, reducing the frequency of no sale. Theory and experiment confirm these motivations from practice.

Keywords: auctions, market design, reserve price, experiments
JEL Classification: C91, D44, D47

*Bergemann: Yale University, dirk.bergemann@yale.edu; Breuer: kbreuer0@gmail.com, Cramton: Max Planck Institute for Research on Collective Goods and University of Maryland, pcramton@gmail.com, Hirsch: Harvard University, jhirsch1@g.harvard.edu, Ndiaye: University of Cologne and Max Planck Institute for Research on Collective Goods,ndiaye@wiso.uni-koeln.de, Ockenfels: University of Cologne and Max Planck Institute for Research on Collective Goods, ockenfels@uni-koeln.de. Support from the Center for Social and Economic Behavior (C-SEB), the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany´s Excellence Strategy – EXC 2126/1– 390838866, and from the European Research Council (ERC, European Union's Horizon 2020 research and innovation program, GA No. 741409) is gratefully acknowledged.

Conventional auction theory advises sellers to set a binding *hard* reserve price that exceeds their value of the auctioned item (Myerson 1981 and Riley and Samuelson 1981). If the seller can credibly commit to not reselling the item or accepting bids beneath the reserve, she earns more expected revenue from an auction. In practice, however, sellers often fail to commit to a binding reserve price. Instead, they lower the opening price of unsold items or later re-auction them (Burguet and Sakovics 1996). On the other hand, theory predicts that a non-binding reserve price—also known as a *soft* floor—does not affect revenue. Nonetheless, many real-world settings frequently feature a soft floor, as in eBay's best-offer program or informal auctions such as house sales (Huang et al. 2013). In such soft-floor auctions, sellers ask bidders to accept an opening price to participate in an ascending auction and allow lower bids. If no bidder accepts the opening price, the alternative bids are considered using first-price auction rules.

This paper shows that soft-floor auctions are more profitable and efficient than standard auctions if bidders regret losing an auction at a favorable price. We introduce a standard private-value auction framework in which bidders maximize a weighted average of their gain less loser's regret. While bidders could also experience winner's regret, the winner would need feedback about losing bids to compute forgone gain. This is not typically provided in practice. On the other hand, when bidders lose, they learn the winning price, allowing the calculation of forgone gain. We therefore focus on the more salient loser's regret.[1]

When bidders anticipate experiencing regret upon seeing the auction results, they may change their bidding strategy and, thus, the auction's revenue. However, bidders never experience regret in the truth-telling equilibrium of a second-price auction. When they bid their value, they would not wish to change their bid regardless of the auction's outcome. On the other hand, a bidder may regret her bid in a first-price auction if she could have bid higher and won at a price beneath her value for the object. The desire to avoid regret induces more aggressive bidding. Regret can, therefore, explain the empirical observation of overbidding in first-price auctions. Proposition 2 formally proves that when bidders anticipate regret, a first-price auction revenue dominates a second-price or ascending auction.[2]

While a revenue-maximizing seller prefers a first-price auction, bidders prefer a second-price auction because they do not experience regret. soft-floor auctions exploit this fact by offering bidders the opportunity to pay a premium to participate in a second-price auction instead of a first-price auction. Proposition 6 shows that soft-floor auctions earn strictly more expected

[1] Such feedback would only reduce revenues (Isaac and Walker 1985, Dufwenberg and Gneezy 2002, Ockenfels and Selten 2005). That said, our analysis in Appendix A.1 generalizes the model and shows how including winner's regret, if made salient, would change our predictions.

[2] Engelbrecht-Wiggans (1989) provided the first proof using a comparative statics result. Proposition 2 explicitly quantifies how bidders change their behavior in first-price auctions.

revenue than first-price auctions. Proposition 7 then shows that a soft-floor auction with a hard reserve strictly outperforms a first-price auction with a hard reserve in terms of revenue and efficiency.[3] Intuitively, the optimal hard reserve in a first-price auction increases with the support of values. However, adding a soft floor to the auction reduces the range of values at which bidders submit first-price bids. Adding a soft floor accordingly lowers the optimal hard reserve and, thus, the inefficiency of the hard reserve.

Our model also suggests that regret, not risk aversion, drives overbidding in first-price auctions. Models with risk aversion and regret aversion predict that first-price auctions yield higher revenues than second-price auctions. However, Proposition 8 demonstrates that when agents are risk averse, soft-floor auctions yield lower revenues than first-price auctions.

We test our hypotheses in a controlled laboratory experiment. Soft-floor auctions can significantly increase revenues compared to first-price auctions without the efficiency loss associated with a hard reserve. This suggests that regret aversion, not risk aversion, drives overbidding. Moreover, our experiments show that the soft floor is even more attractive than our model of regret suggests. We confirmed our prediction that a larger weight on regret increases the likelihood of accepting the opening bid. However, we also found that many bidders accept any opening price so long as it does not exceed their value. When we added a soft floor in settings with a hard reserve, the revenue and efficiency of the best soft-floor auction exceeded that of the best first-price auction.

Our findings explain the widespread use of soft-floor auctions by emphasizing the role of bidder regret in market design. Soft-floor auctions exploit bidders' desire to avoid regret by allowing them to pay a premium to participate in a second-price auction. However, our experimental framework intentionally abstracts from several advantages of a soft floor over standard hard reserve auctions. First, in practice, a seller who wants to use a hard reserve may find it difficult to commit to not selling the item if the reserve is not met (Coase 1972, Caillaud and Mezzetti 2004, Skreta 2015, Liu et al. 2019). A soft floor, on the other hand, does not require such a commitment.[4] Our laboratory experiments ensure that all hard reserves are credible commitments, which would otherwise make soft floors comparatively more appealing. Second, a hard reserve discourages participation (Bajari and Hortaçsu 2003). Indeed, some auction houses, such as Ritchie Brothers, require no reserve to attract more bidders and provide certainty of sale. We have exogenous participation across treatments, which removes the comparative advantage

[3]More precisely, if both auctions have optimally-chosen reserves, then a soft-floor auction outperforms a first-price auction in terms of revenue, and if the first-price auction has a hard reserve exceeding 0, the soft-floor auction is also more efficient.

[4]This holds both when soft floors are used *instead* of hard reserves or if soft floors are *added* to a hard reserve since it allows for a lower hard reserve, thus making the hard reserve binding less often.

of soft floors. Third, in real-world auctions, bidders often act on behalf of others and are driven by a desire to avoid blame. Accepting a favorable opening price avoids the possibility of blame since the final price will be higher than the bidder's value for the item. Our experiments did not include such agency problems. Our experiments likely underestimate the relative appeal of soft floors.

Related Literature. Our study contributes to the literature on auctions. Engelbrecht-Wiggans (1989) introduced bidder regret into an auction framework, showing that in a first-price auction, loser's regret increases bids while winner's regret decreases bids. Many studies have since confirmed the role of loser's regret in explaining overbidding and high revenues in first-price auctions when compared to English and second-price auctions (Ockenfels and Selten 2005, Filiz-Ozbay and Ozbay 2007, Engelbrecht-Wiggans and Katok 2007; Engelbrecht-Wiggans and Katok 2008, 2009).[5] Our research builds upon this empirical and theoretical work. We incorporate regret as a linear and additively separable penalty to the utility function and use it to explain the attractiveness of soft-floor auctions.

Moreover, we add to the literature on the role of risk aversion in auctions.[6] Some studies have suggested that risk aversion can explain overbidding in first-price auctions (Cox et al. 1988). However, controlled laboratory evidence often contradicts this claim. Studies find no correlation between risk preferences and overbidding (Isaac and James 2000, Berg et al. 2005). Additionally, studies find no difference in overbidding when the risk involved is changed (Engelbrecht-Wiggans and Katok 2009, Füllbrunn et al. 2019). This suggests that risk aversion does not explain overbidding in practice. Our study contributes to this literature, as risk aversion would predict *lower* auction revenues in a soft-floor auction, which is the opposite effect of what our model of regret predicts and what we experimentally find.

Zeithammer (2019) was the first to analyze soft-floor auctions. He showed that, with symmetric bidders, soft-floor auctions preserve revenue equivalence. Although bidders adjust their behavior in response to the soft floor, the resulting equilibrium still features monotonic symmetric bidding strategies. The Revenue Equivalence Theorem applies because the highest bidder wins while bidders with minimal values receive zero expected utility. Zeithammer perturbs revenue equivalence by extracting bidder surplus through the occasional participation of high-value bidders. In contrast, we maintain symmetric bidders and introduce loser's regret to perturb revenue equivalence.

[5]See Kagel and Levin (2016) for a survey.
[6]See Vasserman and Watt (2021) for a survey.

Lastly, our paper relates to the literature on optimal reserve prices. Early seminal work suggested that hard reserve prices can be used to increase revenues both when values are independent and interrelated if sellers commit not to resell (Myerson 1981, Riley and Samuelson 1981, Engelbrecht-Wiggans 1987, Levin and Smith 1996). When such commitments are impractical, Coase conjectured a price-setting monopolist would lose her monopoly power and set prices close to marginal cost (Coase 1972). In sequential auctions with resetting of hard reserve prices, many studies find that revenues above the efficient revenues are difficult to obtain (McAfee and Vincent 1997, Liu et al. 2019, Liu et al. 2025). We contribute by showing that if bidders anticipate experiencing regret, a soft-floor auction maximizes revenue while mitigating efficiency loss and circumventing commitment problems.

The paper is structured as follows. Section 1 develops a model of salient regret. Section 2 analyzes the model and derives our main predictions. Section 3 considers risk aversion and derives differences from the predictions of the regret model. Section 4 describes the experimental design and our laboratory results. Section 5 discusses the results and concludes.

1. Model

Section 1 presents the model and shows how we incorporate regret into an auction setting. We also explain why we focus our analysis on loser's regret.

1.1. **Set-Up.**

Consider a single seller with one unit of an indivisible good she values at $v_0 = 0$. There are n risk-neutral bidders indexed $i \in \{1, 2, \cdots, n\}$. Bidders have private values v_i drawn independently and identically from the distribution F with full support on the interval $[0, 1]$. We assume F is non-decreasing and differentiable on its support and has density function f. The seller implements a soft-floor auction with hard reserve r and soft floor s with:

$$s \geq r \geq 0.$$

Bidders submit sealed bids exceeding r or decline to participate. The highest bid wins, and if the winning bid exceeds s, the winner pays the maximum of the second highest bid and s, as in a second-price auction. Otherwise, the winner pays her bid.[7] Ties are broken by randomly assigning the object to one of the highest bidders. We incorporate regret as a linear and additively separable component of the utility function so that the utility of a bidder is the weighted average of her gain less her regret. In particular, let $\beta \in (0, 1)$ be the weight on the gain, and $1 - \beta$ be

[7]Observe that if $s = 1$, we recover a first-price auction, if $s = r$, we recover a second-price auction, and if $r = 0$, we have a soft-floor auction without reserve.

the weight on regret. Before we describe a soft-floor auction, it is useful to establish the effects of regret on first and second-price auctions.

1.2. **Regret in a First-Price Auction.** Consider a first-price auction in which each agent i bids b_i. Denote the winning bid

$$b^1 := \max_i \{b_i\}. \tag{1}$$

We similarly use b^2 to denote the second highest bid and v^1 and v^2 to denote the highest and second highest values, respectively. A bidder only experiences regret if she loses the auction at a favorable price. Thus a bidder with value v_i has regret r_i:

$$r_i\left(v_i, b_i, b^1\right) = \begin{cases} v_i - b^1 & \text{if} \quad b_i < b^1 < v_i \\ 0 & \text{else.} \end{cases}$$

In other words, if agent i loses, she experiences regret if her value exceeds the price of the winning bid. Otherwise, she does not experience regret.[8] She has gain π_i:

$$\pi_i(v_i, b_i, b^1) = \begin{cases} v_i - b_i & \text{if} \quad b_i = b^1 \\ 0 & \text{else.} \end{cases}$$

We can now write utility of agent i as a convex combination of gain and regret.

$$u_i\left(v_i, b_i, b^1\right) = \begin{cases} \beta(v_i - b_i) & \text{if} \quad \text{agent } i \text{ wins} \\ -(1-\beta)\max\{v_i - b^1, 0\} & \text{if} \quad \text{agent } i \text{ loses.} \end{cases}$$

1.3. **Regret in a Second-Price Auction.** If bidders in the second-price auction are bidding truthfully, then agent i has regret

$$r_i\left(v_i, v^1, v^2\right) = \begin{cases} \max\{v_i - v^2, 0\} & \text{if} \quad v_i < v^1 \\ 0 & \text{else.} \end{cases}$$

If $v_i = v^1$ and bidder i wins, she does not experience loser's regret. If $v_i \leq v^1$ and she does not win, then $v_i \leq v^2$, which means that regret is $\max\{v_i - v^2, 0\} = 0$.[9] Intuitively, no losing bidder prefers to have won the object and, therefore, does not experience regret. Thus, bidders do not experience regret in the truth-telling equilibrium of a second-price auction. She has gain

$$\pi_i(v_i, v^1, v^2) = \begin{cases} v_i - v^2 & \text{if} \quad v_i > v^2 \\ 0 & \text{else.} \end{cases}$$

[8] We use mnemonic notation as much as possible, and therefore choose r_i for regret. We also follow standard notation and use r for the hard reserve price. They are clear from context.

[9] This includes the zero probability case in which two bidders have the same value, and bidder i loses the tie-break.

Once again we write the utility of agent i as a convex combination of gain and regret.

$$u_i\left(v_i, v^1, v^2\right) = \begin{cases} \beta(v_i - v^2) & \text{if} \quad v_i = v^1 \\ 0 & \text{if} \quad v_i \neq v^1. \end{cases}$$

The fact that there is no regret in the truth-telling equilibrium of a second-price auction is critical for our later analysis; high-value bidders are willing to accept a premium on the winning price to participate in a second-price auction and avoid the exposure to regret in the first-price auction.

1.4. **Remarks on Regret.** Regret theory models choice under uncertainty when an agent's utility function depends negatively on the best possible outcome. When agents make decisions based on incomplete information, they base their decisions in expectation of what they think will happen. After all information is revealed, an agent evaluates the results of her choices based on the realized outcome and the optimal outcome that could have transpired had she acted differently. In particular, a losing bidder in an auction experiences loser's regret when her value exceeds the sale price. Similarly, a winning bidder experiences winner's regret when the sale price exceeds the next highest bid. In both cases, regret is the difference between gain under complete and incomplete information.[10] In particular, we incorporate regret as a linear and additively separable component of the utility function.

We restrict our analysis to loser's regret for two reasons: salience and explanation of overbidding. Loser's regret is salient because the final sales price of an auction is almost always public knowledge. As a result, losing bidders can calculate the magnitude of their lost gains and thus regret. Bidders therefore *anticipate* loser's regret, which may increase their bids. On the other hand, losing bids are not usually disclosed, meaning the winner never learns by how much they overpaid. Indeed, auction houses and other institutions have little incentive to publish losing bids. Suppose bidders know that they will learn the losing bids upon winning. In that case, they have increased anticipation of winner's regret, and winner's regret induces bidders to shave bids, both theoretically and empirically. Thus, from a market design perspective, auction houses prefer not to publish information about losing bids.

Loser's regret can also explain why agents tend to overbid in first-price auctions, while winner's regret cannot. In Appendix A, we derive analogous results for a utility function that incorporates both loser's and winner's regret. We show that if bidders place more weight on winner's regret than loser's regret, they will underbid in first-price auctions. Since we empirically

[10] Our definition of regret follows that of Engelbrecht-Wiggans (1989).

and experimentally observe overbidding, it is reasonable to assume bidders more heavily consider loser's regret.

2. Analysis

In Section 2, we derive monotonic bidding functions for first-price and soft-floor auctions with regret and use them to calculate the seller's expected revenue. We prove that so long as the weight on regret $1 - \beta > 0$, optimal soft-floor auctions provide strictly more expected revenue than first-price auctions. Further, if the first-price auction has optimal hard reserve $r > 0$, soft-floor auctions are also more efficient.

2.1. **First-Price Auction.**

In the presence of loser's regret, the bidding behavior of agents will reflect their concern for failing to win at a favorable price. We assume that $b(\cdot)$ is a symmetric, monotonically increasing, and continuously differentiable equilibrium bidding strategy. Say bidder i bids $b_i = b(v_i)$. Let $Q(v) = F(v)^{n-1}$ denote the distribution of the maximal value of the other $n-1$ bidders, and let $Q'(v) = q(v)$. Then bidder i has expected utility

$$U(v_i, b_i) = \beta(v_i - b_i)Q(v_i) - (1-\beta)\int_{b_i}^{v_i}(v_i - b_j)dQ(v_j). \tag{2}$$

Using (2), we can characterize the unique symmetric Nash equilibrium bidding function.

Proposition 1 (First-Price Auction Bidding Function).
The unique symmetric Nash equilibrium bidding function in a first-price auction with hard reserve r is given by

$$b(v) = v - \int_r^v \left(\frac{F(z)}{F(v)}\right)^{\frac{n-1}{\beta}} dz. \tag{3}$$

All proofs are in the appendix. When $\beta = 1$, (3) recovers the standard first-price auction equilibrium bidding function. As β falls and bidders care more about regret, the integrand decreases, and agents bid higher. As β approaches 0, agents bid close to their true value.

2.2. **Profit in First and Second-Price Auctions.**

To find the seller's expected revenue in first and second-price auctions, define the density functions of the largest and second largest values. By independence, the largest value v_1 has a distribution

$$F_1(x) = Pr[v_1 \le x] = F(x)^n,$$

and density function

$$f_1(x) = nF(x)^{n-1}f(x). \tag{4}$$

To derive the distribution of the second largest value v_2, break it into two mutually exclusive cases. The probability that all values are less than x is $Pr[v_2 \leq x] = F(x)^n$, and the probability that exactly one value exceeds x is $(1 - F(x))F(x)^{n-1}$. There are n choices for which the value exceeds x. Together,

$$F_2(x) = Pr[v_2 \leq x] = F(x)^n + nF(x)^{n-1}(1 - F(x)).$$

This yields a density function

$$f_2(x) = n(n-1)F(x)^{n-2}(1 - F(x))f(x). \tag{5}$$

It is now easy to compute the revenues generated from first and second-price auctions. The revenue generated from a first-price auction with reserve price r is given by

$$\int_r^1 b(v)f_1(v)dv = \int_r^1 \left(v - \int_r^v \left(\frac{Q(z)}{Q(v)} \right)^{\frac{1}{\beta}} dz \right) nF(v)^{n-1}f(v)dv. \tag{6}$$

The revenue of a second-price auction with reserve r is the probability that only one person participates and thus pays r plus the probability that at least two people participate; the winner pays the second highest value.

$$n(1 - F(r))F(r)^{n-1}r + \int_r^1 vf_2(v)dv. \tag{7}$$

Proposition 2 (First-Price vs. Second-Price Auction)**.**
In the presence of regret, a first-price auction revenue dominates a second-price auction.

So long as bidders place positive weight on regret, first-price auctions outperform second-price auctions. Intuitively, since there is no regret in a second-price auction, we can compare the revenue of a first-price auction with and without regret. Since loser's regret induces more aggressive bidding, a first-price auction with regret has greater revenue.

2.3. **soft-floor auction.** We return to our general model set-up with a soft floor $s \geq r$. A soft-floor auction has two components: if the winning bid exceeds s, payment is determined by second-price auction rules, and if the winning bid is less than s, payment is determined by first-price auction rules. The expected utility of a bidder can therefore be described as a composite of the expected utility they receive in the two regions.

Say bidder i has value v_i and bids b_i. As in (1), we let b^1 denote the maximum bid, and b^2 the second highest bid. Considering the possibility of winning in both the first and second-price auctions, bidder i has gain

$$\pi_i\left(v_i, b_i, b^1, b^2\right) = \begin{cases} v_i - b^2 & \text{if} \quad s < b^2 < b^1 = b_i = v_i \\ v_i - s & \text{if} \quad b^2 \leq s < b_i = b^1 \\ v_i - b_i & \text{if} \quad b^1 = b_i < s \\ 0 & \text{else.} \end{cases}$$

Bidder i can gain if she has the highest bid and it exceeds the soft floor (the first two lines), or if she has the highest bid and it is less than the soft floor (the third line). She can only experience regret, however, if she bids beneath the soft floor. In particular, she has regret

$$r_i\left(v_i, b_i, b^1, b^2\right) = \begin{cases} v_i - b^1 & \text{if} \quad b_i < b^1 < v_i \\ v_i - s & \text{if} \quad b^2, b_i < s < v_i, b^1 \\ 0 & \text{else.} \end{cases}$$

The first condition says that bidder i loses in the first-price auction, but the winning bid does not exceed her value. The second condition says that the winning bidder is alone in the second-price auction and therefore pays the soft floor s, but that the value of bidder i exceeds s and she thus wishes she had participated in the second-price auction.

In the symmetric equilibrium of a soft-floor auction, a threshold value $w > s$ determines an agent's bidding behavior. Proposition 4 characterizes the relationship between s and w, but first, we must determine how the threshold shapes the bidding behavior.[11] If the bidder has value $v_i > w$, she participates in a second-price auction and bids truthfully. If she has value $v_i \leq w$, she bids as she would in a first-price auction and, therefore, bids according to (3).[12] Let $b(\cdot)$ denote (3), the first-price auction bidding function. Define its inverse function

$$\phi(\cdot) := b^{-1}(\cdot), \tag{8}$$

so that $\phi(x)$ is the value a bidder must have to bid x. Since $b(\cdot)$ is strictly increasing, its inverse $\phi(\cdot)$ is well-defined. We use $\phi(\cdot)$ to help characterize the soft-floor auction bidding function. So long as at least some bidders wish to participate in the second-price auction ($w < b(1)$), the soft-floor auction bidding function is a composite of truthful bidding in the second-price auction component, and bidding according to $b(\cdot)$ in the first-price auction component (see Figure 1 for an illustration). If no bidder wishes to participate in the second-price auction ($w \geq b(1)$), then the auction reduces to a first-price auction and all agents bid according to $b(\cdot)$.

[11] One way to think about this is to pretend that the seller picks threshold w instead of soft floor s, and then to back out the necessary s to achieve that w.

[12] Formally, if $v_i = w$, she is indifferent between the two auctions.

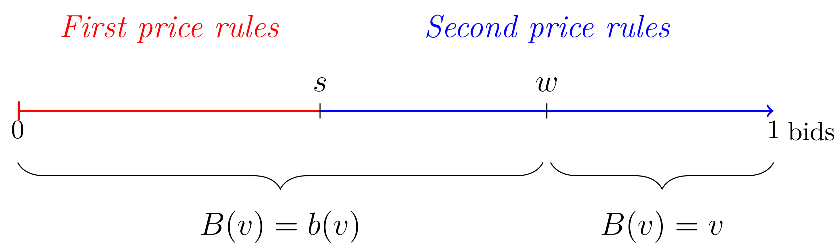

FIGURE 1. The structure of the bidding strategies when the threshold $w < b(1)$

Proposition 3 (Soft-Floor Auction Bidding Function)**.**
Let $w < b(1)$. Then the soft-floor auction has a unique symmetric Nash equilibrium

$$B(v) = \begin{cases} b(v) & \text{if } v \leq \phi(w) \\ v & \text{if } v > \phi(w), \end{cases} \tag{9}$$

If $w \geq b(1)$, the soft-floor auction reverts to a first-price auction and $B(v) = b(v)$.

Proposition 3 says that bidders with values above the threshold bid as they would in a second-price auction, and bidders with values below the threshold bid how they would in a first-price auction. If the threshold is too high, all bidders enter the first-price auction.

The chosen soft floor s and the exogenous level of regret determine the threshold w. When bidders have no regret, $b(w) = s$, but introducing regret causes w to rise. To determine the exact relationship between s and w, consider when bidder i has value $v_i = w$ and is therefore indifferent between participating in the first or second-price auction. Equating her expected utility in each portion of the soft-floor auction derives a relationship between s and w.

Proposition 4 (Soft Floor Participation Threshold)**.**
The relationship between s and w is given by

$$s(w) = w - \frac{\beta(w - b(w))F(w)}{\beta F(w) + (1 - \beta)(n - 1)(1 - F(w))}. \tag{10}$$

Further, $w \geq s$, with equality if and only if $w = s = r$ or $\beta = 0$.

The structure of Proposition 4 is counter-intuitive in that s is exogenous and w is endogenous. However, it is generally impossible to solve for w, and isolating $s(w)$ allows for the direct comparison of revenues between auctions.

2.4. **Soft-Floor Auction Revenue.** To calculate the expected revenue in a soft-floor auction, consider three cases:

(1) $v_j \leq w$ for all j. Bidders participate in the first-price auction, giving the item to the highest bidder.

$$\int_r^w b(v) f_1(v) dv = \int_r^w b(v) n F(v)^{n-1} f(v) dv.$$

(2) Exactly one bidder j has a value higher than w, so j wins at price s. There are n choices for who wins, the probability the winner has a value greater than w is $1 - F(w)$, and the probability all other bidders have a value less than w is $F(w)^{n-1}$. Expected revenue is

$$n(1 - F(w))F(w)^{n-1} s(w).$$

(3) At least two bidders have values greater than w, and the winner pays the second-highest bid.

$$\int_w^1 v f_2(v) dv = \int_w^1 v n(n-1) F(v)^{n-2} (1 - F(v)) f(v) dv.$$

Combining these three possible scenarios gives expected revenue as a function of w.

$$R(w) = \int_r^w b(v) f_1(v) dv + n(1 - F(w))F(w)^{n-1} s(w) + \int_w^1 v f_2(v) dv. \tag{11}$$

We use (11) to prove that a soft-floor auction is revenue-equivalent to a first and second-price auction when bidders maximize their gain ($\beta = 1$).

Proposition 5 (Soft-Floor Auction No Regret)**.**
If bidders only maximize their gain, then a soft-floor auction with hard reserve r and soft floor $s \geq r$ is revenue-equivalent to a first-price auction with hard reserve r.

The Revenue Equivalence Theorem proves Proposition 5, but it is also possible to calculate their revenues using (11) explicitly. The latter approach allows us to compare the revenues of various auction formats when bidders care about both gains and regret.

2.5. **Comparing Auction Revenues.** Using (11), we can directly compare the revenues of a soft-floor auction and first and second-price auctions in the presence of regret.

Proposition 6 (Soft-Floor Auction Beats First and second-price auctions)**.**
Consider a first-price, second-price, and soft-floor auction with hard reserve $r \in [0, 1)$. Then, in the presence of regret, a soft floor $s > r$ exists such that the soft-floor auction earns more revenue than the first or second-price auction.

Proposition 6 says that so long as bidders care about monetary gain and regret, if all three auctions share the same hard reserve, the seller can choose a soft floor such that the soft-floor auction yields strictly more revenue in expectation. The proof proceeds by showing that when $w = 1$—which reduces the soft-floor auction to a first-price auction—lowering w slightly increases revenue compared to a first-price auction. Proposition 2 shows that a first-price auction outperforms a second-price auction, which completes the proof. It is not true, however, that *any* soft floor suffices. Indeed, setting $s = r$ reduces the soft-floor auction to a second-price auction, which is worse than a first-price auction. We use Proposition 6 in our proof of Proposition 7.

Proposition 7 (Soft-Floor Auction More Efficient and Profitable)**.** *Consider a first-price auction with optimal hard reserve r and a soft-floor auction with optimal hard and soft reserves. Then, in the presence of regret, a soft-floor auction earns strictly more revenue and is weakly more efficient than the first-price auction. Further, if $r > 0$, the soft-floor auction is strictly more efficient. The result follows identically for a second-price auction.*

Proof. Proposition 6 proves introducing a soft floor increases revenue. To establish efficiency, it suffices to show that the optimal hard reserve in a soft-floor auction is lower than the optimal reserve in both first and second-price auctions. Indeed, the optimal reserve price in a first-price auction increases with the support of the values. Conversely, if the range over which bidders enter the first-price auction decreases, so does the reserve price. ■

Proposition 7 shows that the optimal soft-floor auction outperforms the optimal first and second-price auctions concerning revenues and efficiency. Further, so long as the optimal first or second-price auction is inefficient—if the reserve exceeds zero, the good might not be sold—the soft-floor auction is more efficient.

Proposition 7 says nothing about the optimal choice of hard reserve r. Figure 2 illustrates Proposition 7 in the case of two bidders with uniform distributions, which is a standard experimental setting (which we also use). Even in this simple case, calculating the optimal hard reserve and soft floor is difficult. In Appendix A.2, we derive parametrization for all three auction formats, assuming two bidders with independent values drawn from a uniform distribution. (Figure 2 illustrates that in our experimental setting, the optimal hard reserves in the first-price and soft-floor auctions are very similar, so we do not expect significant efficiency differences in our laboratory study and therefore focus on revenue comparisons.)

We showed that for any level of regret $\beta \in (0, 1)$, an optimal soft-floor auction increases revenue and improves efficiency compared to a first-price auction. Nonetheless, there may be a different auction format that earns more revenue. Incorporating loser's regret makes analyzing the optimal auction format difficult and would most likely require a novel approach beyond the tools used in Riley and Samuelson (1981).

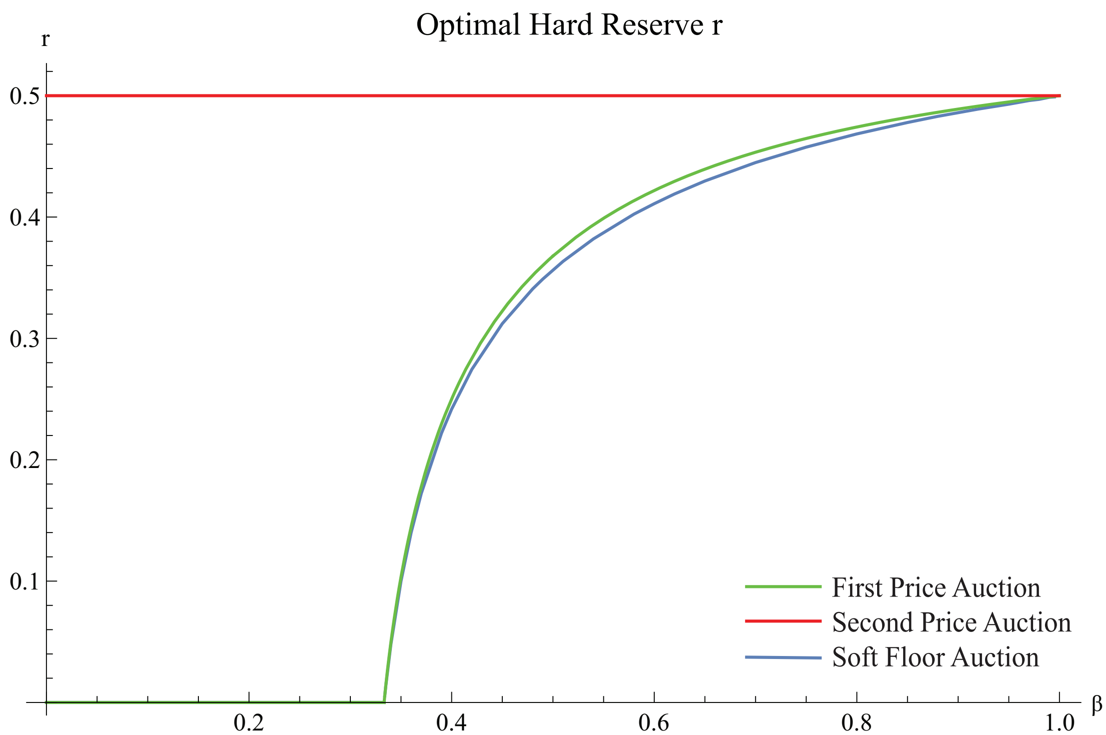

Figure 2. Optimal r for Two Bidders, Uniform Distribution

3. Risk Aversion vs. Regret

Risk aversion is frequently presented as a competing explanation for overbidding in first-price auctions. However, laboratory evidence often finds that regret is more important. Our work contributes to this evidence by showing that soft-floor auctions with risk-averse bidders should yield *lower* auction revenues.

Consider the same utility-maximizing model set-up with $\beta = 1$, but assume bidders have thrice differentiable utility function $u(x)$ for strictly concave $u(\cdot)$, and normalize $u(0) = 0$. Let $b(\cdot)$ denote the unique symmetric Nash equilibrium bidding strategy of a first-price auction, and note that it must be increasing.[13] In equilibrium, a type-v bidder has expected utility

$$Q(v)u(v - b(v)).$$

We now derive the relationship between the soft floor s and the threshold w for participating in the second-price auction. Let bidder i have type $v_i = w$. Participating in the first-price auction yields expected utility

$$Q(w)u(w - b(w)).$$

If bidder i instead participates in the second-price auction, she faces two possible scenarios.

[13]The requirements on $u(\cdot)$ are innocuous, and there is in fact a unique (symmetric) equilibrium in a first-price auction: see Maskin and Riley (1984).

(1) Bidder i is alone in the second-price auction. This occurs with probability $Q(w)$; her gain is $w - s$.

(2) Bidder i is not alone in the second-price auction. Then, the winning bid is no smaller than w, so bidder i cannot gain.

Together, this means she has expected utility

$$Q(w)u(w - s).$$

Since a type w bidder is indifferent between participating in the first or second-price auction,

$$Q(w)u(w - s) = Q(w)u(w - b(w)).$$

Since $u(\cdot)$ is strictly increasing, we conclude

$$\begin{aligned} s &= b(w) \\ w &= b^{-1}(s). \end{aligned} \tag{12}$$

In other words, the threshold w is determined by the bidder type who would bid exactly s in a first-price auction. For (12) to hold, it must be that $s \leq b(1)$, and indeed this is the case, as then bidders will only choose to participate in the first-price auction (since bidding $b(1)$ guarantees the object).

Proposition 8 (Risk Aversion with Soft Floor)**.**
If bidders are risk averse, then a first-price auction revenue dominates a soft-floor auction.

Proposition 8 demonstrates that when facing risk-averse bidders, a soft-floor auction is *less* attractive to the seller than a first-price auction.

4. Experimental Design and Results

In Section 4, we test our theoretical predictions. First, we illustrate—as suggested by Proposition 6—that a soft-floor auction with a sufficiently high soft floor consistently yields statistically and economically significantly higher revenues than a first-price auction while being equally efficient (Section 4.2). We also find that the best soft-floor auction with a non-zero hard reserve in our experiment revenue dominates all first-price auctions with a non-zero hard reserve price. It also efficiency dominates the first-price auction with the highest revenue (Section 4.3). Lastly, we show that soft floors are even more attractive than predicted by our model (Section 4.4). While our model implies that increased regret makes bidders more likely to accept the opening bid, in our experiment many participants will accept any opening price that does not exceed their value. All reported p-values for our main treatments come from non-parametric directed permutation tests based on independent matching group averages with Holm corrections for multiple hypothesis testing.

4.1. **Experimental Design.** Table 1 describes the three stages of our laboratory soft-floor auction.

Table 1. Laboratory Soft-Floor Auction

Stage I	Auctioneer announces an opening price of s.
Stage II	Bidders decide whether to participate in a second-price auction with a minimum bid of s.
Stage III	Participating bidders compete in a second-price auction by submitting a bid equal to or larger than s.[14] Non-participating bidders submit a bid larger than hard reserve r and smaller than s.
Payment Rule	The highest bid in the second-price auction wins and pays the second highest bid (or, in the case of a single bidder, the opening price). If no bidder participates, the highest bid smaller than s and larger than r wins, and the winner pays her winning bid.

Otherwise, our laboratory auction environments follow standard procedures. Two bidders compete for a single item. We focus on the two-bidder case because hard reserves are especially important when there are few bidders. Indeed, our model predicts that the reserve price policy becomes irrelevant as the number of bidders increases.[15] Bidders have private values independently and uniformly distributed between 0 and 100 Experimental Currency Units (ECU). Values are randomly drawn before the experiment and remain identical across all sessions and treatments. Each subject participates only in a single treatment (between-subject design) and competes for 48 rounds in a strangers' matching protocol. Subjects participating in treatments with a soft floor, a hard reserve, or both face four levels of s, r, and (s, r) for 12 rounds each (within-subject design) in random order. Sellers are computerized. Subjects accumulate earnings during all 48 rounds, and the final payoff is converted to Euros, which are paid out immediately

[14]In practice, an ascending auction instead of a sealed-bid second-price auction would follow if multiple bidders accepted the opening price. We use a second-price auction in Stage III for convenience in the laboratory. Second-price auctions can be conducted much more quickly in the laboratory than in the same number of ascending auctions. In our private value setting, a second-price auction yields nearly the same outcome as an ascending auction, both in theory and in the laboratory, when properly explained (see Ariely et al. 2005, Shachat and Wei 2012). Thus, we anticipate that our results will apply when Stage III is ascending rather than when a second-price auction is conducted. If at all, the common preference among bidders for ascending auctions (Cramton 1998) suggests that using an ascending auction may strengthen our results.

[15]As $n \to \infty$ in (3), the winning bid converges to 1, as bidders compete with more bids and thus lose the opportunity to shave their bids.

after the experiment. After each round, bidders learn whether they won the auction, the final price of the item, and their earnings.

TABLE 2. Auction Formats and Parametrization

Auction format	**Level of hard reserve r and soft floor s**
FPA	$r = 0$
SFA	$s \in \{40, 46, 52, 58\}$
FPA_R	$r \in \{32, 38, 44, 50\}$
SFA_R	$(s, r) \in \{(50, 50), (56, 44), (62, 38), (68, 32)\}$

This framework considers four auction formats: FPA, SFA, FPA_R, and SFA_R. Table 2 summarizes the parametrization. FPA is a standard, efficient first-price auction with a hard reserve price of zero. Bidders submit bids between 0 and 100 ECU. The bidder with the highest bid wins the auction and pays her bid. SFA is a soft-floor auction with a hard reserve of zero. Based on the estimated weight on regret $1 - \beta$ in the FPA sessions, we varied soft floors around the implied optimal s of 46 and considered $s \in \{40, 46, 52, 58\}$. We note that ex-post calibration based on data from actual SFA sessions suggests a higher optimal soft floor s of 51.

Comparing the performance of FPA and SFA allows us to distinguish between three competing predictions. Under standard assumptions, FPA and SFA should lead to the same revenue (and full efficiency) by the Revenue Equivalence Theorem. However, according to our model, the optimal SFA revenue dominates the optimal FPA (Proposition 6). Lastly, with risk aversion rather than regret aversion, FPA revenue dominates SFA (Proposition 8).

FPA_R is a first-price auction with a non-zero hard reserve price r. Bidders submit bids between r and 100 ECU. SFA_R is a soft-floor auction with a non-zero hard reserve price. Comparing FPA_R and SFA_R allows us to test the dominance of soft-floor auctions over first-price auctions with non-zero hard reserve prices (Proposition 7). Again, we chose parameters to reflect optimal levels of r in FPA_R and for (s, r) in SFA_R based on the estimated weight on regret $1 - \beta$ from FPA treatment and our model. We varied the level of hard reserve around the estimated optimal level of the hard reserve $r = 44$ and considered $r \in \{32, 38, 44, 50\}$. Based on the ex-post calibration, the optimal r is 39. The estimated optimal levels of the soft floor and the hard reserve (s, r) in SFA_R are $(56, 43)$ according to the FPA sessions and $(59, 38)$ based on the ex-post calibration.

Adding the soft floor to FPA_R only slightly reduces the optimal hard reserve (from 44 to 43 given the estimated weight on regret), and the corresponding predicted efficiency effect is less than one percentage point (from 80.64 percent in FPA_R to 81.51 percent in SFA_R). The impact of such effects is likely too small to be detectable in laboratory data. Thus, we slightly deviate from the estimated optimum and instead choose $(s, r) = (56, 44)$ for SFA_R. Our analysis is therefore focused on the effect of adding a soft floor to auctions with a given hard reserve. As a result, we consider $(s, r) \in \{(50, 50), (56, 44), (62, 38), (68, 32)\}$. Note that SFA_R with (50,50) equals a second-price auction with a hard reserve of 50. Thus, we disregard FPA_R with $r = 50$ and SFA_R with $(50, 50)$ in the analysis.

All sessions were conducted in April 2023 in the Cologne Laboratory for Economic Research (CLER).[16] Participants were students from the University of Cologne invited via ORSEE (Greiner 2015). The experiment was programmed with a z-tree (Fischbacher 2007). Participants were randomly matched within matching groups utilizing a stranger's matching protocol. One matching group consisted of four bidders. In total, we collected 11,712 bids from 244 subjects. We collected 14 independent observations for SFA_R, 15 for FPA and SRA, and 17 for FPA_R. Some invited participants failed to attend each treatment with less than 17 observations. Sessions lasted around 90 minutes, and the average payoff was approximately 16.26 € with a standard deviation of 3.78 €.

4.2. **A soft-floor auction beats a first-price auction in terms of revenue.** Figure 3 shows the revenues for the FPA and SFA treatments. For a soft floor of $s = 52$, closest to the optimal level according to our model, revenue increases by 14 percent relative to FPA ($p = 0.002$). Thus, a soft floor increases revenue, as suggested by our model (Proposition 6) and contradicting the predictions of standard theory and risk aversion (Proposition 8). Notably, we find that *for all high enough soft floors* s, the SFA revenues exceed the FPA revenue. This includes some soft floors below and all above the estimated optimal level. The differences are economically large, ranging from six percent for $s = 46$ ($p = 0.162$) to 17 percent for $s = 58$ ($p < 0.001$). However, they are only statistically significant at or above the estimated optimal level. As the revenue is increasing in the level of the soft floors considered, an even larger soft floor ($s > 58$) may have resulted in still greater revenues.

As predicted by our model, the increase in revenue does not come at the expense of efficiency. Measuring realized efficiency as a fraction of maximum efficiency, the efficiency of all auctions is high at about 97 percent. There are no significant differences in efficiency between FPA and SFA for all levels of the soft floor. Figure 4 shows the frequency of allocation types. While one might have expected that efficiency would increase when there are more bids under the second-price rule than under the first-price rule—because second-price bidding is in dominant strategies

[16]Pilot sessions were conducted between December 2016 and October 2017 in the Cologne Laboratory for Economic Research (CLER). See Appendix B.3 for a detailed protocol and results.

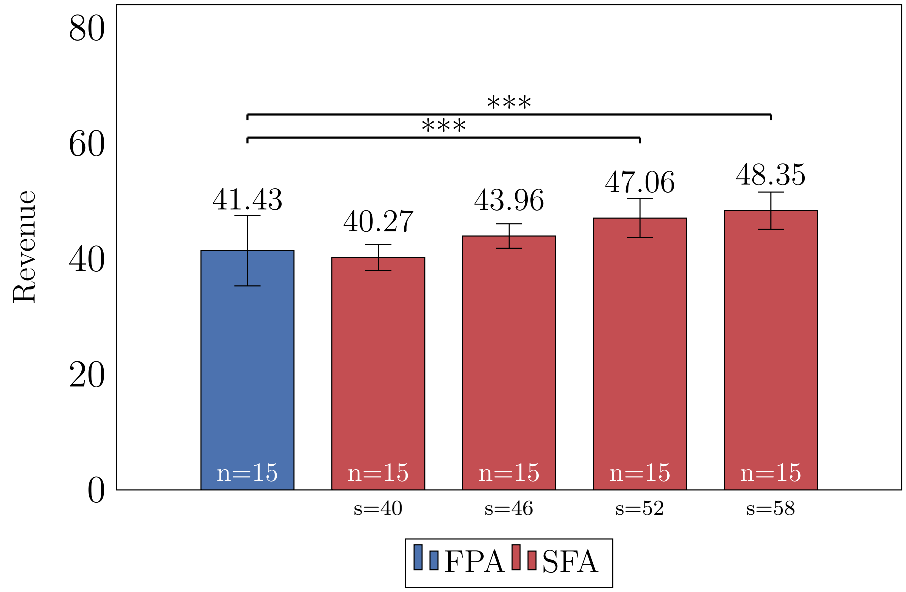

FIGURE 3. Revenue in FPA and SPA

Notes. The figure reports average revenue on the observation group level. Significance levels are based on permutation tests with Holm-correction and *, **, and *** denote significance at the 10 percent, 5 percent, and 1 percent levels, respectively.

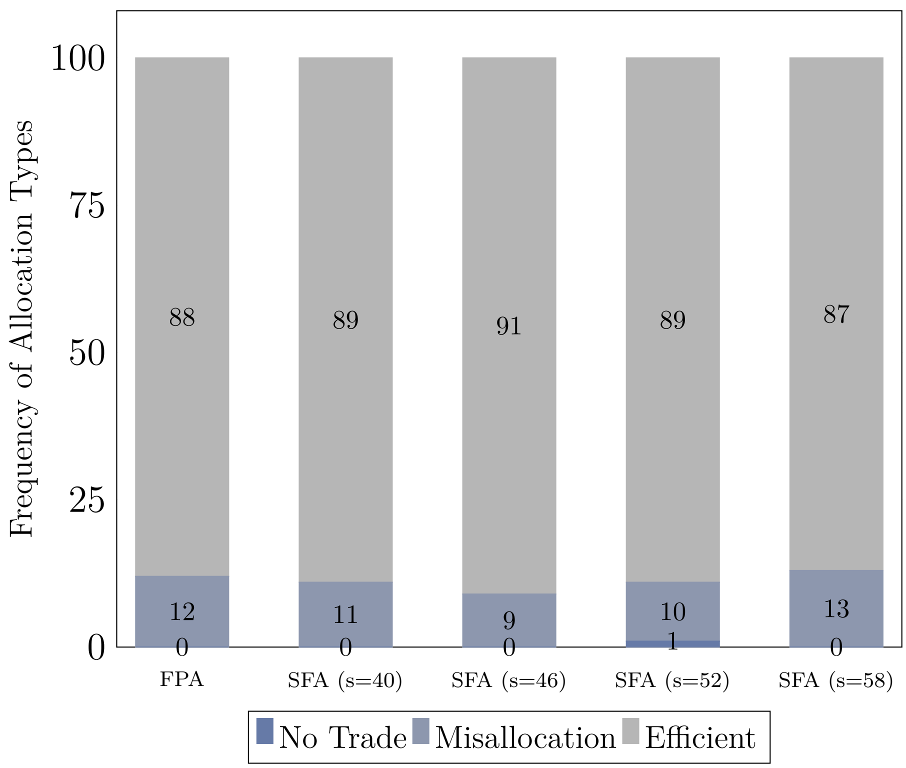

FIGURE 4. Frequency of Allocations in FPA and SFA

Notes. The figure reports for each treatment achieved efficiency as a share of maximum achievable efficiency (gray), efficiency loss due to no trade (dark blue), and efficiency loss due to allocation to the low-value bidder (light blue).

(Pezanis-Christou 2002)—we do not find that the share of items allocated or the highest-valued bidder winning decreases in s.

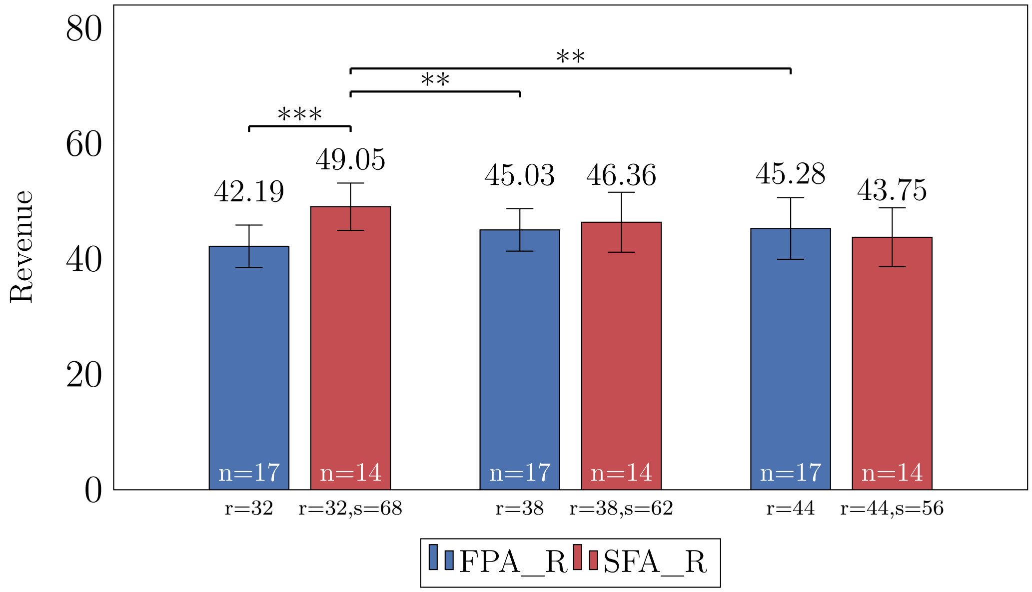

FIGURE 5. Revenue in FPA_R and SFA_R

Notes. The figure reports average revenue on the observation group level. Significance levels are based on permutation tests with Holm-correction and *, **, and *** denote significance at the 10 percent, 5 percent, and 1 percent levels, respectively.

The results are fully consistent with an initial laboratory experiment conducted as a part of a pilot study in a similar laboratory environment before the theory was fully developed. In that study, with a total of 348 subjects, adding a soft floor $s = 50$ (66) to the first-price auction significantly increases revenues by more than ten percent, from 40.11 to 44.41 (46.01), without compromising efficiency (see Appendix B.3 for details).[17] We conclude—consistent with Proposition 6—that adding an optimal (or sufficiently high) soft floor to the first-price auction robustly increases revenues without sacrificing efficiency. Further, within the range tested, higher soft floors increase revenues.

4.3. **The best soft-floor auction beats the best first-price auction in terms of revenue and efficiency.** Proposition 7 suggests that, with salient regret, an optimal soft-floor auction with a non-zero hard reserve revenue dominates and efficiency dominates an optimal first-price auction with a non-zero hard reserve. Recall, however, that the efficiency improvements are predicted to be negligible in our simulations and thus controlled away in our laboratory environment.

[17]The initial study also included sellers as subjects, in a further treatment. The sellers could choose to implement a second-price auction, a first-price auction, and a soft-floor auction with different levels of soft floors and hard reserves. See Appendix B.3 for a detailed protocol and results. Across all auctions, twice as many sellers prefer the soft-floor auction (66.81 percent) over a first-price auction with a hard reserve (33.19 percent) ($p < 0.001$). Moreover, they also choose higher levels of soft floors than hard reserves. This suggest that our results are robust to including (human) sellers, anticipating the positive effects of (larger) soft floors on revenue.

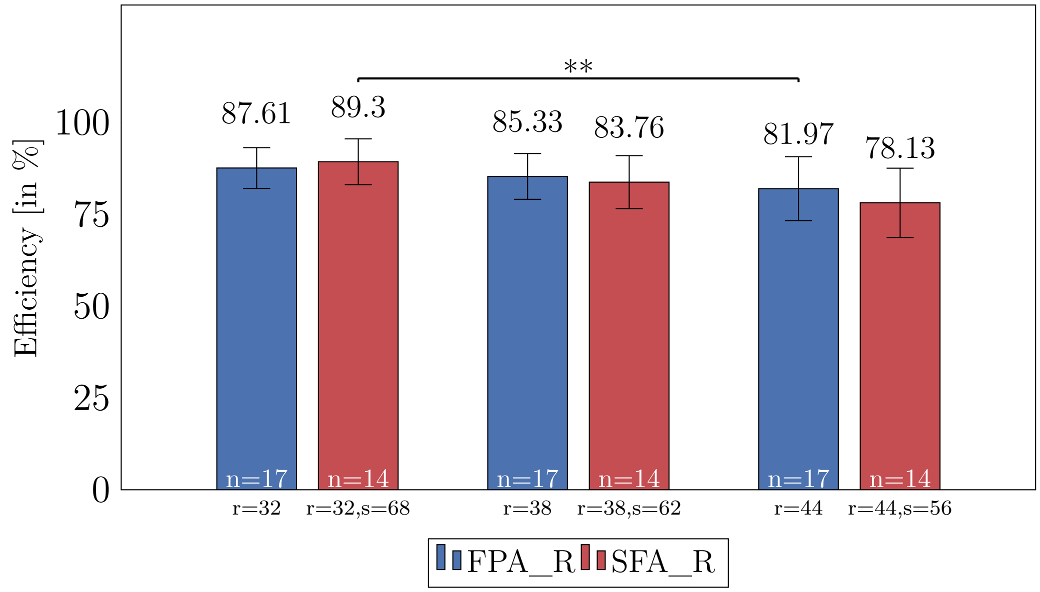

FIGURE 6. Efficiency in FPA_R and SFA_R

Notes. The figure reports average efficiency on the observation group level. Significance levels are based on permutation tests with Holm-correction and *, **, and *** denote significance at the 10 percent, 5 percent, and 1 percent levels, respectively.

Figure 5 illustrates the revenue for the different levels of hard reserve r and soft floor s. According to our model, the soft-floor auction with $(s, r) = (62, 38)$, closest to the optimal level, has larger revenue than all first-price auctions. This increase, however, is insignificant, providing only directional evidence for Proposition 7.

In the experiment, the revenue of SFA_R strongly increases as the soft floor increases and the hard reserve decreases. It is maximized for the largest soft floor considered, $s = 68$, and not $s = 62$. The best SFA_R significantly revenue dominates *any* FPA with substantial increases in revenue between eight percent ($p = 0.049$) and 16 percent ($p < 0.001$). Interestingly, this suggests that trading off a larger soft floor for a smaller hard reserve will likely cost less in revenue than suggested by our model.

Figure 6 shows the efficiency in SFA_R and FPA_R for each level of hard reserve and soft floor. Efficiency decreases as the hard reserve increases but does not depend on the presence or size of the soft floor. The decrease in efficiency is driven by non-trades, with their proportion rising from roughly ten percent for $r = 32$ to roughly 25 percent for $r = 50$ in both SFA_R and FPA_R. The revenue of SFA_R increases as the soft floor increases and the hard reserve decreases. Therefore, the soft-floor auction provides a tool to reduce the efficiency loss from using hard reserves while still increasing revenue. The highest-revenue SFA_R not only revenue dominates all FPA_R, but also leads to significantly higher efficiency than the revenue-maximizing FPA_R. Notably,

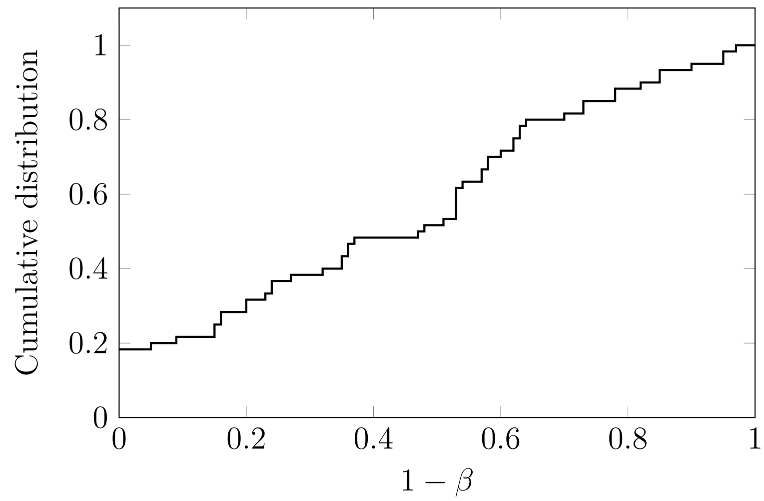

FIGURE 7. Distribution of the Weight on Regret $1-\beta$

Notes. The figure reports the cumulative distribution of the estimated regret parameter $1-\beta$. For each subject participating in SFA ($n=60$), we run a regression of values on bids below s, resulting in 60 regression parameters. Based on the bidding function (see 29), we calculate $(1-\beta)$. We censored 11 estimations smaller than 0.

the experiment suggests potential efficiency gains are much larger as the best soft-floor auction has a lower hard reserve (and higher soft floor) than our model suggests.

4.4. **The surprising attractiveness of the soft floor to the bidders.** Given the predictions of our model, we find that high soft floors are surprisingly effective at increasing revenues. One reason is that the soft floor is remarkably attractive to bidders. The estimated average weight on regret $1-\beta$ in our sample is 0.41. Our model predicts that 22 percent of bids would accept the opening price. In the experiment, more than twice this number, 46 percent of bids, do. To further investigate, we analyze individual bidding behavior and find that the bidder's regret aversion varies substantially. We use the bidding function from our model (see 29) to estimate each bidder's weight $(1-\beta)$ by running a regression of their values on bids below s in the soft-floor auctions. This yields the cumulative distribution of weights across our subjects shown in Figure 7.[18] More than 80 percent of bidders experience regret, and the regret weight $1-\beta>0$ is roughly uniformly distributed on $(0,1]$ according to our measure.[19]

Recall that our model parameter w is defined as the cutoff value at which a bidder is willing to accept the soft floor. While the cutoff without regret in our auction environment is 100, the model predicts that the cutoff w decreases with the weight on regret $1-\beta$. The model thus suggests that the smaller a bidder's cutoff value, the more willing she is (in terms of a larger range of values assigned to her) to accept the soft floor. We compute the individual cutoffs most consistently, i.e., leading to the fewest violations, with each bidder's decision to accept the

[18] As the function simplifies if and only if $r=0$, we only estimate it for the subjects in SFA treatment.
[19] The estimated weight on regret significantly differs from zero for 80 percent of these subjects.

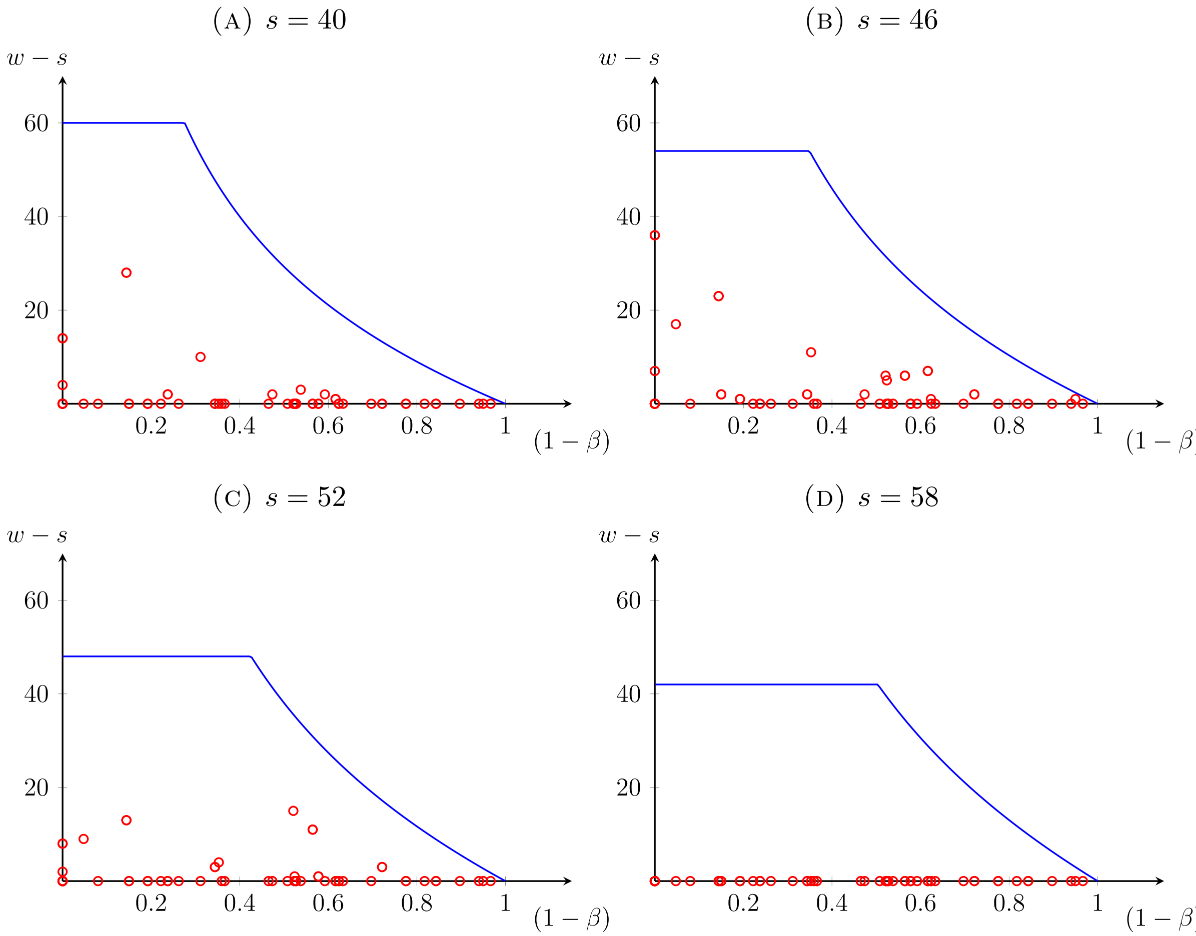

FIGURE 8. Normalized Cutoff Value and Weight on Regret

Notes. The figures report cutoff values that are most consistent with our data for each subject in SFA ($n = 60$) and the cutoff value function conditional on the weight on regret predicted by the model (see the bidding function (see 29)). We derive the cutoff values by finding values minimizing violations of observed acceptances of opening prices.

opening bid.[20] The data confirm the predicted negative relationship between w and $(1-\beta)$ with Spearman correlations of $-0,29$ if $s = 40$ ($p = 0.027$), $-0,33$ if $s = 46$ ($p = 0.011$), and $-0,16$ if $s = 52$ ($p = 0.219$).[21]

Figure 8 compares the cutoff values predicted by our model (line) and estimated cutoffs based on our laboratory data (points). We use normalized cutoffs ($w - s$) to ensure comparability across soft floors. As all estimated cutoffs (points) are below the predicted cutoffs (line), the model strongly underestimates the willingness to accept the opening price for all bidders.

The attractiveness of the soft floor beyond the predictions of our model contributes to explaining why the soft floor outperforms the hard reserve. We found that combining the soft

[20]Note that our model assumes a uniform weight on regret that is common knowledge. Our empirical analysis supposes that the weight on regret is neither uniform nor common knowledge.

[21]The correlation is not defined for $s = 58$ as all bidders accept any opening price above their value for this soft floor.

floor and hard reserve can raise revenue more than a hard reserve alone, in particular for rather high soft floors.

The soft floor mirrors the hard reserve's positive effects on revenue (by pushing bidders with sufficiently high values above the opening price). At the same time, it mitigates against the hard reserve's negative effects (by avoiding efficiency losses when values are below the opening price). In our experiment, roughly 98 percent of bids in SFA_R and FPA_R accept non-zero hard reserves if the hard reserve is below the bidder's value. Notably, 83 percent of bids in SFA_R and 88 percent in SFA also accept soft floors if the floor is below the bidder's value. In this sense, the soft floor captures more than four-fifths of the revenue-increasing mechanism of the hard reserve. In contrast to a hard reserve, however, bidders with values below the soft floor still bid a positive amount in 80 percent of cases in SFA_R and 87 percent in SFA. These effects explain why the soft floor does better than the hard reserve in terms of revenue and efficiency. Lastly, this suggests that while it may be hard to determine the optimal combination of soft floor and hard reserve when designing the auction, it's better to err on the side of setting a soft floor too high and a hard reserve too low, than soft floor too low and a hard reserve too high.

We can think of several other reasons why the soft floor is more attractive to bidders than our model predicts. First, not accepting the opening price implies exclusion from the auction competition, which may induce additional regret and a "fear of missing out" beyond what our model of salient regret (based only on forgone monetary gains) captures. Second, the soft floor may serve as a salient reference point that anchors bidders' reasoning and subsequent bidding strategies (Kahneman and Tversky 2013). This is similar to how negotiators anchor their counteroffers to the first offer received (Galinsky and Mussweiler 2001) or bidders anchor their bids to previous prices (Beggs and Graddy 2009). There are also higher cognitive costs associated with determining optimal bids in first-price auctions compared to second-price auctions (Kagel and Levin 1993, Kirchkamp and Reiß 2011), with or without regret. Bidders may prefer to avoid these costs by accepting the soft floor, which provides a cognitively simple bidding strategy. This also aligns with the concept of "satisficing" introduced by Simon (1955), where decision-makers opt for satisfactory solutions rather than optimal ones to reduce cognitive burden.

In conclusion, the attractiveness of soft floors in auctions stems not only from salient regret but likely from a complex interplay of psychological, cognitive, and strategic factors—which we cannot disentangle in the context of our laboratory experiment. However, such additional factors only strengthen our conclusion that the soft floor is behaviorally robust enough to increase auction revenues without sacrificing efficiency.

5. Conclusion

Conventional auction theory advises that reserve prices should be used to increase auction revenues—especially in instances with few bidders. The use of hard reserves, however, has downsides. It reduces efficiency if the highest bidder's value is above the seller's value but less than the reserve price. In practice, many sellers find it difficult to commit to a hard reserve. Unsold items are often re-auctioned at a lower opening price as suggested by the Coase Conjecture. Soft floors are widely used, typically without explicit reference, to encourage the sale of the object. This paper provides a rationale beyond the intuition of Coase.

We show that when bidders regret losing, soft-floor auctions can circumvent undesirable efficiency loss and commitment problems associated with hard reserves while improving revenues. Intuitively, whereas bidders may experience regret in a first-price auction—in which they shade bids—when the good is sold at a price below their value, they never experience regret in a second-price auction if they bid their value. Thus a second-price or ascending auction motivates bidders to accept the opening price and avoid exposure to regret.

We suggest that two factors may contribute to the effectiveness of the soft-floor auction in practice. The first is that the high bidder wins and has positive gains, and all losing bidders prefer to lose. Accepting the opening price rewards the bidder with entry into a regret-free competition. The second is the regret of losing at a favorable price. In a setting in which the bidder is representing a private company or a public entity, entering the second-price auction allows the bidder to avoid having to explain why they failed to compete.

Our model makes three key predictions. First, regret aversion explains the empirical observations that the revenue in first-price auctions is larger than in second-price auctions. We note that risk aversion could also explain overbidding in first-price auctions. Second, regret aversion suggests that the revenue of the soft-floor auction dominates the revenue of a first-price auction without a hard reserve price. This prediction is contrary to the prediction of risk aversion, which suggests the first-price auction will lead to a larger revenue. Third, the soft-floor auction with a hard reserve revenue and efficiency dominates the first-price auction.

We test our main hypotheses in a controlled laboratory experiment. In a setting with no hard reserve price, we find that sufficiently high soft floors lead to higher revenue than first-price auctions. We can therefore rule out risk aversion as an alternative explanation for overbidding in first-price auctions. Further, our experiment shows that the soft floor is even more attractive than our model suggests. While our model implies increased regret makes bidders more likely to accept the opening bid, many accept any opening price below their value. In a setting with a hard reserve price, we find that the best soft-floor auction leads to higher revenue and higher efficiency than the best first-price auction with a hard reserve. Indeed, the soft floor captures most of the revenue-increasing effects of the hard reserve while mitigating the hard reserve's revenue-decreasing effects. Thus, increasing revenue is combined with almost full efficiency.

We test our theory in a situation most favorable to the standard reserve price theory because we abstract from four practical advantages of soft-floor auctions: 1) the soft floor can protect the seller from weak competition without commitment to a hard reserve price, 2) the soft floor encourages participation by not excluding low-value bidders, 3) bidders acting as agents for others can avoid blame by accepting the opening price, and 4) the soft-floor auction may have other behavioral reasons for its performance, such as anchoring and satisficing. Taken together, our model and laboratory analyses help explain why regret contributes to the widespread use of soft-floor auctions.

A task for future research is extending our analysis to multi-item auctions where the central insights from the one-item setting should extend. In an upcoming multi-band spectrum auction in Thailand, the regulatory agency considered a specific version of a soft-floor auction tailored to multiple units (Cramton et al. 2025): First, bidders state demands at or below the opening price. Second, for products with demand less than supply at the opening price, the auctioneer, after receiving the bids, may select a lower starting price for the subsequent ascending auction.

Although our analysis is limited to private value auctions, we anticipate that our results would extend to the case of interdependent values as in Ausubel et al. (2014). Like with demand reduction, our results deliver strict improvements with soft-floor auctions and thus should remain when marginal interdependent value elements are introduced.

References

Ariely, D., A. Ockenfels, and A. E. Roth (2005). "An Experimental Analysis of Ending Rules in Internet Auctions". In: *RAND Journal of Economics*, pp. 890–907.

Ausubel, L. M., P. Cramton, M. Pycia, M. Rostek, and M. Weretka (2014). "Demand Reduction and Inefficiency in Multi-Unit Auctions". In: *The Review of Economic Studies* 81.4, pp. 1366–1400.

Bajari, P. and A. Hortaçsu (2003). "The Winner's Curse, Reserve Prices, and Endogenous Entry: Empirical Insights from eBay Auctions". In: *RAND Journal of Economics*, pp. 329–355.

Beggs, A. and K. Graddy (2009). "Anchoring Effects: Evidence from Art Auctions". In: *American Economic Review* 99.3, pp. 1027–1039.

Berg, J., J. Dickhaut, and K. McCabe (2005). "Risk Preference Instability across Institutions: A Dilemma". In: *Proceedings of the National Academy of Sciences* 102.11, pp. 4209–4214.

Burguet, R. and J. Sakovics (1996). “Reserve Prices without Commitment”. In: *Games and Economic Behavior* 15.2, pp. 149–164.

Caillaud, B. and C. Mezzetti (2004). “Equilibrium reserve prices in sequential ascending auctions”. In: *Journal of Economic Theory* 117.1, pp. 78–95.

Coase, R. H. (1972). “Durability and monopoly”. In: *The Journal of Law and Economics* 15.1, pp. 143–149.

Cox, J. C., V. L. Smith, and J. M. Walker (1988). “Theory and Individual Behavior of First-Price Auctions”. In: *Journal of Risk and Uncertainty* 1, pp. 61–99.

Cramton, P. (1998). “Ascending auctions”. In: *European Economic Review* 42.3-5, pp. 745–756.

Cramton, P., E. Bohlin, D. Hoy, D. Malec, J. Dark, P. Sujarittanonta, and C. Wilkens (Mar. 2025). *A Study on Designing IMT Multiband Auction Formats for Thailand*. Tech. rep. Office of the National Broadcasting and Telecommunications Commission.

Dufwenberg, M. and U. Gneezy (2002). “Information disclosure in auctions: an experiment”. In: *Journal of Economic Behavior & Organization* 48.4, pp. 431–444.

Engelbrecht-Wiggans, R. and E. Katok (2007). “Regret in Auctions: Theory and Evidence”. In: *Economic Theory* 33, pp. 81–101.

Engelbrecht-Wiggans, R. (1987). “On Optimal Reservation Prices in Auctions”. In: *Management Science* 33.6, pp. 763–770.

– (1989). “The Effect of Regret on Optimal Bidding in Auctions”. In: *Management Science* 35, pp. 685–692.

Engelbrecht-Wiggans, R. and E. Katok (2008). “Regret and Feedback Information in First-Price Sealed-Bid Auctions”. In: *Management Science* 54.4, pp. 808–819.

– (2009). “A Direct Test of Risk Aversion and Regret in First Price Sealed-Bid Auctions”. In: *Decision Analysis* 6.2, pp. 75–86.

Filiz-Ozbay, E. and E. Ozbay (Sept. 2007). “Auctions with Anticipated Regret: Theory and Experiment”. In: *American Economic Review* 97, pp. 1407–1418.

Fischbacher, U. (2007). “z-Tree: Zurich toolbox for ready-made economic experiments”. In: *Experimental economics* 10.2, pp. 171–178.

Füllbrunn, S., D.-J. Janssen, and U. Weitzel (2019). *Does Risk Aversion cause Overbidding? New Experimental Evidence from First Price Sealed Bid Auctions*. SSRN.

Galinsky, A. D. and T. Mussweiler (2001). “First Offers as Anchors: The Role of Perspective-Taking and Negotiator Focus”. In: *Journal of personality and social psychology* 81.4, p. 657.

Greiner, B. (2015). “Subject pool recruitment procedures: organizing experiments with ORSEE”. In: *Journal of the Economic Science Association* 1.1, pp. 114–125.

Huang, C.-I., J.-R. Chen, and C.-Y. Lee (2013). "Buyer behavior under the Best Offer mechanism: A theoretical model and empirical evidence from eBay Motors". In: *Journal of Economic Behavior & Organization* 94, pp. 11–33.

Isaac, R. M. and D. James (2000). "Just Who Are You Calling Risk Averse?" In: *Journal of Risk and Uncertainty* 20, pp. 177–187.

Isaac, R. M. and J. M. Walker (1985). "Information and Conspiracy in Sealed Bid Auctions". In: *Journal of Economic Behavior & Organization* 6.2, pp. 139–159.

Kagel, J. H. and D. Levin (1993). "Independent Private Value Auctions: Bidder Behaviour in First-, Second-and Third-Price Auctions with Varying Numbers of Bidders". In: *The Economic Journal* 103.419, pp. 868–879.

– (2016). "Auctions: A Survey of Experimental Research". In: *The Handbook of Experimental Economics* 2, pp. 563–637.

Kahneman, D. and A. Tversky (2013). "Prospect Theory: An Analysis of Decision under Risk". In: *Handbook of the fundamentals of financial decision making: Part I*. World Scientific, pp. 99–127.

Kirchkamp, O. and J. P. Reiß (2011). "Out-Of-Equilibrium Bids in First-Price Auctions: Wrong Expectations or Wrong Bids". In: *The Economic Journal* 121.557, pp. 1361–1397.

Levin, D. and J. L. Smith (1996). "Optimal Reservation Prices in Auctions". In: *The Economic Journal* 106.438, pp. 1271–1283.

Liu, Q., K. Mierendorff, and X. Shi (2025). "Coasian equilibria in sequential auctions". In: *European Economic Review*, p. 104960.

Liu, Q., K. Mierendorff, X. Shi, and W. Zhong (2019). "Auctions with limited commitment". In: *American Economic Review* 109.3, pp. 876–910.

Maskin, E. and J. Riley (Nov. 1984). "Optimal Auctions with Risk Averse Buyers". In: *Econometrica* 52.6, p. 1473.

McAfee, R. P. and D. Vincent (1997). "Sequentially optimal auctions". In: *Games and Economic Behavior* 18.2, pp. 246–276.

Myerson, R. (Feb. 1981). "Optimal Auction Design". In: *Mathematics of Operations Research* 6.1, pp. 58–73.

Ockenfels, A. and Selten (Apr. 2005). "Impulse balance equilibrium and feedback in first price auctions". In: *Games and Economic Behavior* 51.1, pp. 155–170.

Pezanis-Christou, P. (2002). "On the impact of low-balling: Experimental results in asymmetric auctions". In: *International Journal of Game Theory* 31, pp. 69–89.

Riley, J. and W. Samuelson (June 1981). “Optimal Auctions”. In: *The American Economic Review* 71.3, pp. 381–392.

Shachat, J. and L. Wei (2012). “Procuring Commodities: First-Price Sealed-Bid or English Auctions?” In: *Marketing Science* 31.2, pp. 317–333.

Simon, H. A. (1955). “A Behavioral Model of Rational Choice”. In: *The Quarterly Journal of Economics*, pp. 99–118.

Skreta, V. (2015). “Optimal auction design under non-commitment”. In: *Journal of Economic Theory* 159.PB, pp. 854–890.

Vasserman, S. and M. Watt (2021). “Risk aversion and auction design: Theoretical and empirical evidence”. In: *International Journal of Industrial Organization* 79, p. 102758.

Zeithammer, R. (2019). “Soft Floors in Auctions”. In: *Management Science* 65.9, pp. 4204–4221.

Appendix A. Theory

A.1. **Auxiliary Results and Proofs.** The appendix contains several auxiliary results and proofs.

Proof of Proposition 1. We prove a more general result that incorporates both winner's regret and loser's regret. Bidder i has value v_i and bids $b_i = b(v_i)$. As in the main text, she has expect gain

$$\Pi(v_i, b_i) = (v_i - b_i)Q(v_i)$$

and expected loser's regret

$$R^L(v_i, b_i) = \int_{b_i}^{v_i} (v_i - b_j)dQ(v_j).$$

She also has expected winner's regret

$$R^W(v_i, b_i) = \int_0^{v_i} (b_i - b_j)dQ(v_j).$$

Let $\alpha, \beta \in [0, 1]$ be the weights on winner's and loser's regret such that $(1 - \alpha - \beta) \in [0, 1]$. Define the expected utility of bidder i as

$$\begin{aligned} U(v_i, b_i) &= (1 - \alpha - \beta)\Pi(v_i, b_i) - \alpha R^W(v_i, b_i) - \beta R^L(v_i, b_i) \\ &= (1 - \alpha - \beta)(v_i - b_i)Q(v_i) - \alpha \int_0^{v_i} (b_i - b_j)dQ(v_j) - \beta \int_{b_i}^{v_i} (v_i - b_j)dQ(v_j). \end{aligned}$$

Bidder i chooses b_i to maximize $U(v_i, b_i)$. Since the bidding function is monotonically increasing and continuously differentiable, it has a well-defined inverse $\phi(b_i) = v_i$. Use it to find first-order conditions.

$$\max_{b_i}(1 - \alpha - \beta)(v_i - b_i)Q(\phi(b_i)) - \alpha \int_0^{v_i} (b_i - b_j)dQ(\phi(b_j)) - \beta \int_{b_i}^{v_i} (v_i - b_j)dQ(\phi(b_j))$$

$$0 = -(1 - \beta)Q(\phi(b_i)) + (1 - \alpha - \beta)(v_i - b_i)q(\phi(b_i))\phi'(b_i) + \beta(v_i - b_i)q(\phi(b_i))\phi'(b_i)$$

$$\phi'(b_i) = \frac{(1 - \beta)Q(\phi(b_i))}{(1 - \alpha)(v_i - b_i)q(\phi(b_i))}.$$

Since $v_i = \phi(b_i(v_i))$ and $\phi'(b_i(v_i)) = 1/b_i'(v_i)$,

$$\frac{db}{dv} = \frac{1}{\phi'(b)} = \frac{1 - \alpha}{1 - \beta}(v - b)\frac{q(v)}{Q(v)}. \tag{13}$$

We drop the subscript i to emphasize that bidding formula is symmetric. Note (13) is a linear first-order ordinary differential equation. Let $b_i = y$ and $v_i = x$. Further, define

$$P(x) = \frac{(1-\alpha)q(x)}{(1-\beta)Q(x)},\ R(x) = x\frac{(1-\alpha)q(x)}{(1-\beta)Q(x)},\ y = uv \text{ and } y' = u\frac{dv}{dx} + v\frac{du}{dx}.$$

Then by (13),

$$R(x) = y' + P(x)y$$

$$R(x) = u\frac{dv}{dx} + v\left(\frac{du}{dx} + P(x)u\right). \tag{14}$$

Since $y = uv$, (14) has a degree of freedom. Set $\frac{du}{dx} + P(x)u = 0$ to obtain

$$0 = \frac{du}{dx} + P(x)u$$

$$\frac{du}{u} = -P(x)dx$$

$$\ln(u) = -\int P(x)dx$$

$$u = e^{-\int P(x)dx}. \tag{15}$$

Insert (15) into (14).

$$R(x) = \frac{dv}{dx} \times e^{-\int P(x)dx}$$

$$dv = \frac{R(x)}{e^{-\int P(x)dx}}dx$$

$$v = \int \left(\frac{R(x)}{e^{-\int P(x)dx}}\right) dx. \tag{16}$$

Substituting (15) and (16) for u and v gives

$$y = uv = e^{-\int P(x)dx} \int \left(\frac{R(x)}{e^{-\int P(x)dx}}\right) dx. \tag{17}$$

Insert $P(x)$ and $R(x)$ into (17) to get

$$y = e^{-\int P(x)dx} \times \int \left(\frac{R(x)}{e^{-\int P(x)dx}}\right) dx$$

$$y = e^{-\int \frac{(1-\alpha)q(x)}{(1-\beta)Q(x)}dx} \times \int \left(\frac{\frac{(1-\alpha)xq(x)}{(1-\beta)Q(x)}}{e^{-\int \frac{(1-\alpha)q(x)}{(1-\beta)Q(x)}dx}}\right) dx. \tag{18}$$

Observe that

$$-\int \frac{(1-\alpha)q(x)}{(1-\beta)Q(x)}dx = -\frac{(1-\alpha)\log(Q(x))}{(1-\beta)} + c = \log\left(Q(x)^{-\frac{1-\alpha}{1-\beta}}\right) + c.$$

Thus (18) becomes

$$\begin{aligned} y &= e^{\log\left(Q(x)^{-\frac{1-\alpha}{1-\beta}}\right)+c} \times \int \left(\frac{\frac{(1-\alpha)xq(x)}{(1-\beta)Q(x)}}{e^{\log\left(Q(x)^{-\frac{1-\alpha}{1-\beta}}\right)+c}} \right) dx \\ &= e^c \times Q(x)^{-\frac{1-\alpha}{1-\beta}} \times \int \left(\frac{\frac{(1-\alpha)xq(x)}{(1-\beta)Q(x)}}{e^c \times Q(x)^{-\frac{1-\alpha}{1-\beta}}} \right) dx \\ &= Q(x)^{-\frac{1-\alpha}{1-\beta}} \times \int \left(\frac{(1-\alpha)xq(x)}{(1-\beta)Q(x)^{1-\frac{1-\alpha}{1-\beta}}} \right) dx. \end{aligned}$$

Integrate by parts to obtain

$$\begin{aligned} y &= Q(x)^{-\frac{1-\alpha}{1-\beta}} \times \int \left(\frac{(1-\alpha)xq(x)}{(1-\beta)Q(x)^{1-\frac{1-\alpha}{1-\beta}}} \right) dx \\ &= Q(x)^{-\frac{1-\alpha}{1-\beta}} \left(xQ(x)^{\frac{1-\alpha}{1-\beta}} - \int Q(x)^{\frac{1-\alpha}{1-\beta}} dx \right) \\ &= x - Q(x)^{-\frac{1-\alpha}{1-\beta}} \times \int Q(x)^{\frac{1-\alpha}{1-\beta}} dx. \end{aligned}$$

Finally let $y = b$ and $x = v$, and replace the integral variable with a dummy variable z integrated from our lower bound r to the value v to obtain

$$b(v) = v - \int_r^v \left(\frac{F(z)}{F(v)} \right)^{(n-1)\frac{1-\alpha}{1-\beta}} dz. \tag{19}$$

To recover (3) in Proposition 1 in the text, set $\alpha = 0$ and switch the labels on the coefficients for the gain and loser's regret. ■

Remark. When $\alpha = \beta$, (19) confirms that winner's and loser's regret offset—as argued in Engelbrecht-Wiggans (1989)—and agents bid according to the standard monetary gain-maximizing bidding formula for first-price auctions. Interestingly, even if $\alpha = \beta = 1/2$ and bidders ignore gain and only aim to minimize expected regret, they still bid *as if* maximizing expected gain.

We prove a more general version of Proposition 2. In particular, we prove that when bidders face regret, a first-price auction revenue dominates a second-price auction if $\beta > \alpha$ and is revenue dominated by a second-price auction if $\alpha > \beta$.

Proof of Proposition 2. With both winner's and loser's regret, the revenue generated from a first-price auction with reserve price r is given by

$$\int_r^1 b(v)f_1(v)dv = \int_r^1 \left(v - \int_r^v \left(\frac{Q(z)}{Q(v)} \right)^{\frac{1-\alpha}{1-\beta}} dz \right) nF(v)^{n-1}f(v)dv. \tag{20}$$

A second-price auction has the same expected revenue with or without regret. By the Revenue Equivalence Theorem, without regret, a first-price auction and a second-price auction generate the same expected revenue. Hence, we compare the expected revenue of a first-price auction with regret to that of a first-price auction without regret. The expected revenue of a first-price auction without regret is given by

$$\int_r^1 \left(v - \int_r^v \left(\frac{Q(z)}{Q(v)} \right) dz \right) nF(v)^{n-1} f(v) dv. \tag{21}$$

Subtracting (21) from (20) gives

$$\int_r^1 \left(\int_r^v \left(\frac{Q(z)}{Q(v)} \right) - \left(\frac{Q(z)}{Q(v)} \right)^{\frac{1-\alpha}{1-\beta}} dz \right) nF(v)^{n-1} f(v) dv.$$

Since $Q(\cdot)$ is non-decreasing, $Q(z) \leq Q(v)$. If $\beta > \alpha$, then $1-\alpha > 1-\beta$ and

$$\frac{Q(z)}{Q(v)} > \left(\frac{Q(z)}{Q(v)} \right)^{\frac{1-\alpha}{1-\beta}},$$

so the integrand is positive. Conversely, $\beta < \alpha$ means $1-\alpha < 1-\beta$ and

$$\frac{Q(z)}{Q(v)} < \left(\frac{Q(z)}{Q(v)} \right)^{\frac{1-\alpha}{1-\beta}},$$

so the integrand is negative. ■

Remark. Conceptually, the proof is easy, and Engelbrecht-Wiggans (1989) and Filiz-Ozbay and Ozbay (2007) both provide non-explicit versions. Loser's regret induces aggressive bidding, whereas winner's regret causes conservative bidding. Since these effects exactly offset when $\alpha = \beta$, if one outweighs the other, the seller's profit either rises or falls. However, we appear to provide the first explicit calculation to prove it, which has the added practical benefit that it is then easy to compare revenues for given α and β.

Proof of Proposition 4. Let bidder i have type $v_i = w$. Participating in the first-price auction yields expected gain

$$(w - b_i)F(w)^{n-1}.$$

Bidder i has three potential sources of loser's regret when she participates in the first-price auction.

(1) If $v_j \leq w$ for all $j \neq i$, bidder i experiences regret if she loses but the winning bid is beneath her value. Since $v_i = w$, any winning bid exceeds her value, and this term vanishes.

(2) If $v_j \geq w, v_k < w$ for all $k \neq j$, by default j wins the second-price auction at price s, and bidder i has regret of $w - s$.

(3) At least two other bidders participate in the second-price auction. However, they both have values of at least w, so the winning price is at least w. Hence, this term also vanishes.

Thus, bidder i only experiences loser's regret if exactly one bidder has a value greater than w, and the other $n-2$ have values less than w. There are $n-1$ choices for who has a value greater than w, so the probability that this occurs is

$$(n-1)(1-F(w))F(w)^{n-2}.$$

Expected loser's regret is

$$(n-1)(1-F(w))F(w)^{n-2}(w-s).$$

The expected utility bidder i receives from participating in the first-price auction is

$$U(w, b(w)) = \beta(w - b(w))F(w)^{n-1} - (1-\beta)(n-1)(1-F(w))F(w)^{n-2}(w-s). \tag{22}$$

If bidder i instead participates in the second-price auction, she faces two possible scenarios.

(1) Bidder i is alone in the second-price auction. This occurs with probability $Q(w) = F(w)^{n-1}$ and her gain is $w - s$.

(2) Bidder i is not alone in the second-price auction. Then the winning bid is no smaller than w, so bidder i cannot gain.

There is no regret in a second-price auction, so her expected utility is just her gain.

$$U(w, b(w)) = \beta F(w)^{n-1}(w-s). \tag{23}$$

Since a type w bidder is indifferent between participating in the first or second-price auction, set (22) equal to (23) to obtain

$$\beta F(w)^{n-1}(w-s) = \beta(w-b(w))F(w)^{n-1} - (1-\beta)(n-1)(1-F(w))F(w)^{n-2}(w-s).$$

Isolating $s(w)$ gives

$$s(w) = w - \frac{\beta(w-b(w))F(w)}{\beta F(w) + (1-\beta)(n-1)(1-F(w))}.$$

To prove $w \geq s$, rearrange terms to get

$$w - s = \frac{\beta(w-b(w))F(w)}{\beta F(w) + (1-\beta)(n-1)(1-F(w))}.$$

Thus $w = s$ if $\beta = 0$. For $\beta \neq 0$,

$$w = s \iff \frac{\beta(w - b(w))F(w)}{\beta F(w) + (1-\beta)(n-1)(1-F(w))} = 0.$$

This requires $F(w) = 0$ or $w = b(w)$. In the former case, $s = w = 0 \leq r$ and in the latter, by (3), $w = r$ so that $b(r) = r$. Since $s \geq r$, $s = r$. ■

Proof of Proposition 5. The revenue of a first-price auction without regret is given by (21). Set $\beta = 1$ and subtract (21) from (11) to obtain

$$\int_r^1 b(v)f_1(v)dv - \left(\int_r^w b(v)f_1(v)dv + n(1-F(w))F(w)^{n-1}s(w) + \int_w^1 vf_2(v)dv\right) =$$

$$\int_w^1 b(v)f_1(v)dv - \left(n(1-F(w))F(w)^{n-1}s(w) + \int_w^1 vf_2(v)dv\right) \tag{24}$$

By (10), since $\beta = 1$, $s = b(w)$. The term in parentheses in expression (24) simplifies to

$$n(1-F(w))F(w)^{n-1}b(w) + \int_w^1 vf_2(v)dv. \tag{25}$$

If a first-price auction has hard reserve w, then $b(w) = w$. By (7), if $b(w) = w$, then (25) is precisely the revenue of a second-price auction with hard reserve w. Similarly,

$$\int_w^1 b(v)f_1(v)dv$$

is the revenue of a first-price auction with hard reserve w. By the Revenue Equivalence Theorem, they are equal. Therefore, expression (24) equals zero, proving that a soft-floor auction is revenue-equivalent to a first-price auction. ■

Proof of Proposition 6. Proposition 2 proves that a first-price auction revenue dominates a second-price auction. It is, therefore, sufficient to show that a soft-floor auction revenue dominates a first-price auction. First observe that by evaluating (10) when $w = 1$, we get

$$s(1) = b(1). \tag{26}$$

We can now use (11) to take the derivative of $R(w)$ with respect to w and evaluate it at $w = 1$.

$$\begin{aligned}
\frac{\partial R}{\partial w} &= b(w)nF(w)^{n-1}f(w) + s'(w)n(1-F(w))F(w)^{n-1} \\
&\quad + ns(w)F(w)^{n-1}f(w)\left(n-1-nF(w)\right) - wn(n-1)F(w)^{n-2}(1-F(w))f(w) \\
\left.\frac{\partial R}{\partial w}\right|_{w=1} &= nb(1)f(1) + ns(1)f(1)(-1) \\
&= 0.
\end{aligned}$$

The penultimate line uses (26) to substitute $s(1) = b(1)$. Hence, $w = 1$ is a critical point. To verify it is a minimum, take the derivative again and evaluate at $w = 1$.

$$\left.\frac{\partial^2 R}{\partial w^2}\right|_{w=1} = \frac{nf(1)}{\beta} \times \left(-\beta b'(1) + 2(n-1)(1-b(1))f(1) - \beta(n-1)(1-b(1))f(1)\right).$$

Use (13) to obtain an expression for $b'(1)$.

$$\begin{aligned} b'(v) &= \frac{v-b}{\beta} \times \frac{f(v)}{F(v)}(n-1) \\ b'(1) &= \frac{1-b(1)}{\beta} f(1)(n-1). \end{aligned}$$

Hence

$$\begin{aligned} \left.\frac{\partial^2 R}{\partial w^2}\right|_{w=1} &= \frac{nf(1)^2}{\beta}\left(-\beta\left(\frac{1-b(1)}{\beta}\right)(n-1) + 2(n-1)(1-b(1)) - \beta(n-1)(1-b(1))\right) \\ &= \frac{n(n-1)(1-\beta)(1-b(1))f(1)^2}{\beta} > 0. \end{aligned}$$

Since $w = 1$ is a minimum, a slightly smaller w increases revenue. Since $w = 1$ is the revenue with only a first-price auction, a soft-floor auction revenue dominates a first-price auction. ■

Proof of Proposition 8. Let the revenue from a soft-floor auction derived in (11) be denoted $RSFA(r, w)$. Similarly, let $RFPA(r)$ and $RSPA(r)$ denote equations (6) and (7), the revenue in a first and second-price auction, respectively. For risk-averse bidders, a first-price auction weakly revenue dominates a second-price auction.[22] Hence we have the following chain of inequalities.

$$\begin{aligned} RFPA(r) &= \int_r^1 b(v)f_1(v)dv \\ &= \int_r^w b(v)f_1(v)dv + RFPA(w) \\ &\geq \int_r^w b(v)f_1(v)dv + RSPA(w) \\ &= \int_r^w b(v)f_1(v)dv + n(1-F(w))F(w)^{n-1}w + \int_w^1 vf_2(v)dv \\ &\geq \int_r^w b(v)f_1(v)dv + n(1-F(w))F(w)^{n-1}b(w) + \int_w^1 vf_2(v)dv \\ &= \int_r^w b(v)f_1(v)dv + n(1-F(w))F(w)^{n-1}s(w) + \int_w^1 vf_2(v)dv \\ &= RSFA(r, w). \end{aligned}$$

[22] See Maskin and Riley (1984).

The first inequality follows from the fact that for risk-averse bidders, a first-price auction generates more revenue than a second-price auction. The second inequality follows from the fact that $b(x) \leq x$ for all x, and the penultimate equality uses (12). ■

Proof of Proposition 11. We tackle the slightly more general problem of implicitly solving for the optimal r in terms of w and β for the uniform distribution $F(v) = v$ with n bidders. By (3), the bidding function is:

$$b(v) = v - \int_r^v \left(\frac{z^{\frac{n-1}{\beta}}}{v^{\frac{n-1}{\beta}}} \right) dz = v - \frac{\beta \left(v - r^{\frac{\beta+n-1}{\beta}} v^{\frac{1-n}{\beta}} \right)}{\beta + n - 1} = \frac{\left(v(n-1) - \beta r^{\frac{\beta+n-1}{\beta}} v^{\frac{1-n}{\beta}} \right)}{\beta + n - 1}. \tag{27}$$

Further, (10) gives the relationship between s and w.

$$s(w) = \frac{w \left(\beta - \beta n - \frac{\beta^2 \left(w - r^{\frac{\beta+n-1}{\beta}} w^{\frac{1-n}{\beta}} \right)}{\beta+n-1} + \beta n w - n w + n + w - 1 \right)}{\beta + (\beta-1)n(w-1) + w - 1}$$

$$s(w) = w - \frac{w \left(\frac{\beta^2 \left(w - r^{\frac{\beta+n-1}{\beta}} w^{\frac{1-n}{\beta}} \right)}{\beta+n-1} \right)}{(n-1)(1-\beta)(1-w) + \beta w} \tag{28}$$

Insert (27) and (28) into (11) to obtain the revenue as a function of r and w.

$$R(r,w) = \frac{n w \left(\frac{\beta^2 r^{\frac{\beta+n-1}{\beta}} w^{\frac{(\beta-1)(n-1)}{\beta}}}{(\beta-1)n+1} + \frac{(n-1)w^n}{n+1} \right)}{\beta + n - 1} + \frac{(n-1)\left((n(w-1)-1)w^n + 1\right)}{n+1}$$
$$- \frac{n(\beta + (\beta-1)n + 1) r^{n+1}}{(n+1)((\beta-1)n+1)} + n(1-w)w^n - \frac{n(1-w)w^n \left(\frac{\beta^2 \left(w - r^{\frac{\beta+n-1}{\beta}} w^{\frac{1-n}{\beta}} \right)}{\beta+n-1} \right)}{(n-1)(1-\beta)(1-w) + \beta w}$$

Although complicated, the first-order conditions on r give a manageable expression.

$$\frac{\partial R}{\partial r} = \frac{n \left(\frac{\beta^2 r^{\frac{n-1}{\beta}} w^{\frac{(\beta-1)n+1}{\beta}}}{\beta + (\beta-1)n(w-1) + w - 1} - (\beta + (\beta-1)n + 1) r^n \right)}{(\beta-1)n + 1} = 0$$

$$r^{\frac{n\beta - n + 1}{\beta}} = w^{\frac{n\beta - n + 1}{\beta}} \frac{\beta^2}{(1 + \beta + n\beta - n)((n-1)(1-\beta)(1-w) + w\beta}$$

$$r^* = w \left(\frac{\beta^2}{(1 + \beta + n\beta - n)((n-1)(1-\beta)(1-w) + w\beta)} \right)^{\frac{\beta}{1+n\beta-n}}.$$

When $n = 2$,

$$r^* = w\left(\frac{\beta^2}{(3\beta-1)((1-\beta)(1-w)+\beta w)}\right)^{\frac{\beta}{2\beta-1}}.$$

Hence if $\beta \in [0, 1/3]$, $3\beta - 1 < 0$ and optimally $r = 0$. If $\beta \in (1/3, 1]$, r^* is as above. ■

A.2. **Two Bidders with Uniform Distribution.** We consider the case of two bidders with values drawn from the uniform distribution. Since β is determined endogenously, the seller chooses the reserve price(s) as a function of β. In the first-price auction, we solve explicitly for the optimal hard reserve $r^*(\beta)$ and show that it strictly increases in β for $\beta \in (1/3, 1]$. Then, we determine the corresponding maximal revenue. The soft-floor auction's optimal hard reserve and soft floor are interdependent. We solve for $r^*(\beta)$ in terms of w, and then we solve several implicit functions to graph the optimal reserves $r^*(\beta)$ and $s^*(\beta)$ and the corresponding threshold w^* and maximal expected revenue.

A.3. **First-Price Auction.** Inserting $F(v) = v$ and $n = 2$ into (3) gives the bidding function in a first-price auction with hard reserve r.

$$b(v) = \frac{v}{1+\beta} + r\left(\frac{r}{v}\right)^{1/\beta}\left(\frac{\beta}{1+\beta}\right). \tag{29}$$

By (4), $f_1(v) = 2v$. Insert (29) into (11) to find the seller's expected revenue.

$$\begin{aligned} RF(r) &= \int_r^1 b(v)f_1(v)dv \\ &= \int_r^1 \left(\frac{v}{1+\beta} + r\left(\frac{r}{v}\right)^{1/\beta}\frac{\beta}{1+\beta}\right)2vdv \end{aligned}$$

$$RF(r) = \frac{2}{3(1+\beta)} + \frac{2r^{\frac{\beta+1}{\beta}}\beta^2}{(1+\beta)(2\beta-1)} + \frac{2r^3(1-3\beta)}{3(2\beta-1)}. \tag{30}$$

For given β, the seller picks r to maximize (30).

Proposition 9 (Optimal r in a first-price auction)**.**
The optimal hard reserve $r^*(\beta)$ is given by

$$r^*(\beta) = \begin{cases} 0 & \text{if } \beta \in [0, 1/3] \\ r(\beta) = \left(3 - \frac{1}{\beta}\right)^{\frac{\beta}{1-2\beta}} & \text{if } \beta \in (1/3, 1]. \end{cases} \tag{31}$$

Further, $r(\beta)$ is strictly increasing for $\beta \in (1/3, 1]$.

Proof. The first term of (30) is independent of r. For $\beta \in [0, 1/3]$, the second and third terms of (30) are weakly negative, and their sum is strictly negative.

$$\frac{2r^{\frac{\beta+1}{\beta}}\beta^2}{(1+\beta)(2\beta-1)} + \frac{2r^3(1-3\beta)}{3(2\beta-1)} = -\frac{2r^{\frac{\beta+1}{\beta}}\beta^2}{(1+\beta)(1-2\beta)} - \frac{2r^3(1-3\beta)}{3(1-2\beta)} < 0.$$

Setting $r = 0$ is thus maximal and gives the total expected revenue

$$\frac{2}{3(1+\beta)}. \tag{32}$$

Now let $\beta \in (1/3, 1]$. Taking the derivative of (30) with respect to r gives

$$\frac{\partial RF}{\partial r} = \frac{2\left(\beta r^{1/\beta} + (1-3\beta)r^2\right)}{2\beta - 1} = 0. \tag{33}$$

There are two solutions to (33):

$$r \in \left\{0, \left(\frac{\beta}{3\beta-1}\right)^{\frac{\beta}{2\beta-1}}\right\}.$$

If $r = 0$, the revenue is given by (32). Rewrite the non-trivial solution as

$$r(\beta) = \left(3 - \frac{1}{\beta}\right)^{\frac{\beta}{1-2\beta}}. \tag{34}$$

For $\beta \in (1/3, 1]$, (34) strictly increases (see Figure A.1). Insert (34) into (30) to obtain the expected revenue.

$$RF(r(\beta)) = \frac{2}{3(\beta+1)} + \frac{2\left(\frac{3\beta^2\left(\left(\left(\frac{\beta}{3\beta-1}\right)^{\frac{1}{2\beta-1}}\right) - \left(\frac{\beta}{3\beta-1}\right)^{\frac{2\beta}{2\beta-1}}\right)\left(\frac{\beta}{3\beta-1}\right)^{\frac{\beta}{2\beta-1}}}{2\beta-1} - \left(\frac{\beta}{3\beta-1}\right)^{\frac{3\beta}{2\beta-1}}\right)}{3(\beta+1)}$$

$$RF(r(\beta)) = \frac{2}{3(\beta+1)} + \frac{2\beta\left(\frac{\beta}{3\beta-1}\right)^{\frac{\beta+1}{2\beta-1}}}{3(\beta+1)}. \tag{35}$$

The second term in (35) is strictly positive for $\beta > 1/3$, and the first is the revenue when $r = 0$. Hence, (34) gives the optimal r for $\beta \in (1/3, 1]$.[23] ■

Proposition 10 (First-Price vs. Second-Price Auction)**.**
For any $\beta \in [0, 1)$, the maximal revenue of a first-price auction is strictly greater than that of a second-price auction.

[23] Second order conditions are difficult to prove explicitly but are easily verified graphically.

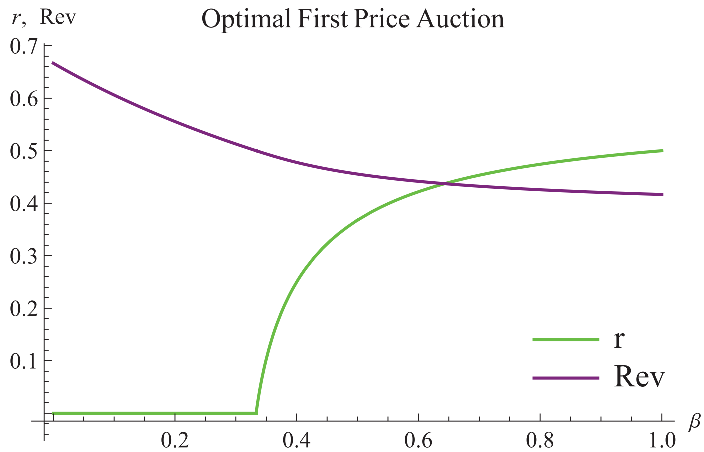

FIGURE A.1. First-Price Auction for Two Bidders, Uniform Distribution

Proof of Proposition 10. By (5), $f_2(v) = 2(v - v^2)$. The revenue of a second-price auction with reserve r is independent of β and given by

$$\int_r^1 v f_2(v) dv = \frac{1}{3}(4r^2 + r + 1)(1 - r),$$

which achieves its maximal value at $r = 1/2$ and gives a revenue $5/12$. In the first-price auction, the revenue function is strictly decreasing in β and is exactly equal to $5/12$ when $\beta = 1$. ■

Proposition 10 also follows from Proposition 2.

A.4. **Soft-Floor Auction.** In a soft-floor auction, the seller optimizes r and s simultaneously. However, the revenue also depends on w, which, in turn, depends on s. By (11), the revenue as a function of $r, s,$ and w is given by

$$R(r,s,w) = 2sw(1-w) + \frac{2w^3 - 3w^2 + 1}{3} + \frac{2\left((w^3 - r^3) + \frac{3\beta^2 r\left(r^2 - \left(\frac{r}{w}\right)^{\frac{1}{\beta}} w^2\right)}{1-2\beta}\right)}{3(1+\beta)} \tag{36}$$

By (10),

$$s(w) = \frac{w\left(1 - w + w\beta - \beta^2 + r\left(\frac{r}{w}\right)^{\frac{1}{\beta}}\beta^2 + w\beta^2\right)}{(1+\beta)(1 - w - \beta + 2w\beta)}.$$

Eliminating s from (36) gives

$$R(r,s(w),w) = \frac{w\left(1 - w + w\beta - \beta^2 + r\left(\frac{r}{w}\right)^{\frac{1}{\beta}}\beta^2 + w\beta^2\right)}{(1+\beta)(1 - w - \beta + 2w\beta)}(2w - 2w^2)$$
$$+ \frac{2w^3 - 3w^2 + 1}{3} + \frac{2\left((w^3 - r^3) + \frac{3\beta^2 r\left(r^2 - \left(\frac{r}{w}\right)^{\frac{1}{\beta}} w^2\right)}{1-2\beta}\right)}{3(1+\beta)}. \tag{37}$$

First-order conditions on r give the optimal hard reserve for w.

Proposition 11 (Soft-Floor Auction r^*)**.**
In a soft-floor auction with two bidders with uniform values, the optimal hard reserve is given by

$$r^* = \begin{cases} 0 & \textit{if } \beta \in [0, 1/3] \\ w\left(\frac{\beta^2}{(3\beta-1)(-\beta+2\beta w-w+1)}\right)^{\frac{\beta}{2\beta-1}} & \textit{if } \beta \in (1/3, 1]. \end{cases}$$

The proof presented above solves for r^* in the slightly more general case of n bidders with values drawn from the uniform distribution. As anticipated, the optimal hard reserve depends on the threshold value w, which depends on the chosen soft floor s. The seller's choice of s uniquely determines w, and each w corresponds to a unique s. It is, therefore, sufficient for the seller to choose w, solve the first order conditions of (37) for r and w, and then use (10) to determine the corresponding s. Unfortunately, the first order conditions of (37) for w prove complicated, but we use Mathematica to construct Figure A.2, which plots the optimal r, s, and w, and the corresponding maximal expected revenue, as a function of β.

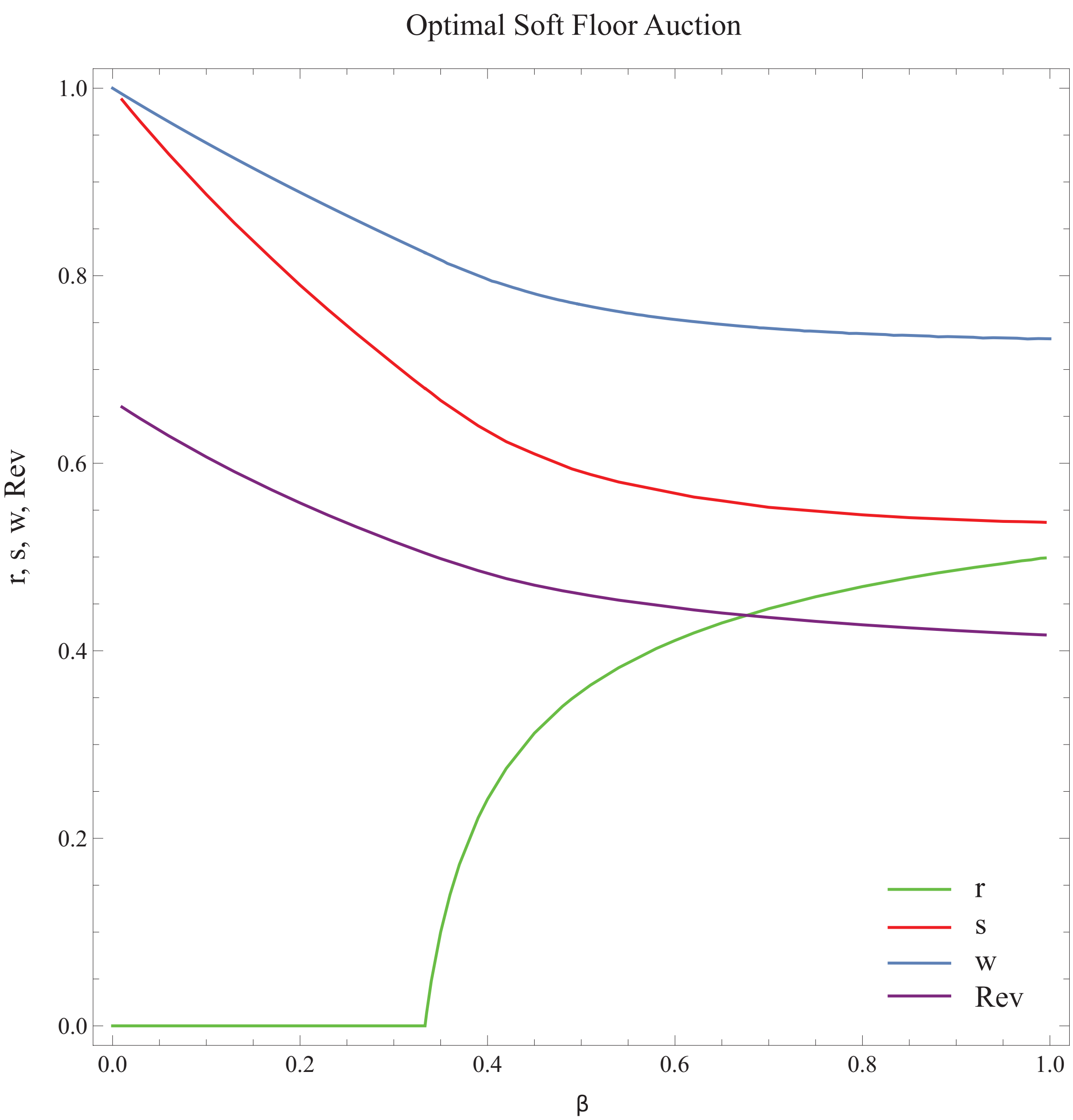

FIGURE A.2. Soft-Floor Auction for Two Bidders with Uniform Distribution

We also consider a soft-floor auction with no hard reserve $r = 0$. For $\beta \in [0, 1/3]$, the optimal s and w, and expected revenue, are unchanged since $r = 0$. For $\beta \in (1/3, 1]$, set $r = 0$ and take first-order conditions of (36) to get

$$w^* = \frac{2\beta^2 + \sqrt{5\beta^2 - 4\beta^4} + \beta - 2}{2\left(2\beta^2 + \beta - 1\right)}.$$

Substituting into (28) gives

$$s^* = \frac{-4\beta^6 - 2\beta^5 + 10\beta^4 - 6\beta^2 - 2\sqrt{5\beta^2 - 4\beta^4}\beta + \sqrt{5\beta^2 - 4\beta^4} + \left(2\sqrt{5\beta^2 - 4\beta^4} + 2\right)\beta^3 + \beta}{\left(2\beta^2 + \beta - 1\right)^2\left(\sqrt{5\beta^2 - 4\beta^4} + \beta\right)}.$$

The optimal revenue in a soft-floor auction with no hard reserve is given by substituting $r = 0, s^*$, and w^* into (36). The result yields Figure A.3.

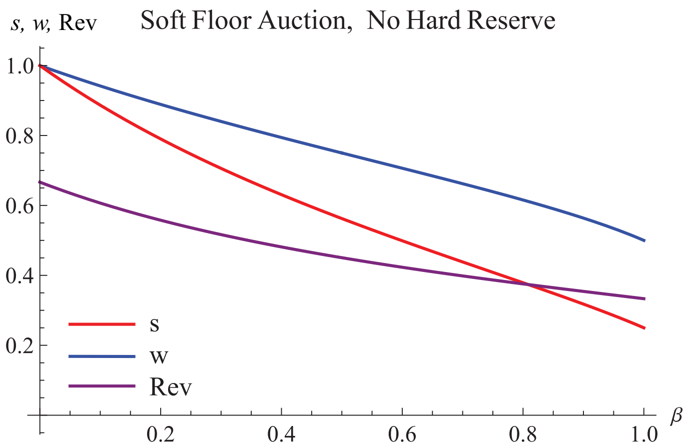

FIGURE A.3. Optimal s, w, and Revenue for Two Bidders, Uniform Distribution

Combining the revenues from the optimal first-price auction, soft-floor auction with no hard reserve, and soft-floor auction allows for comparing revenues across auctions.

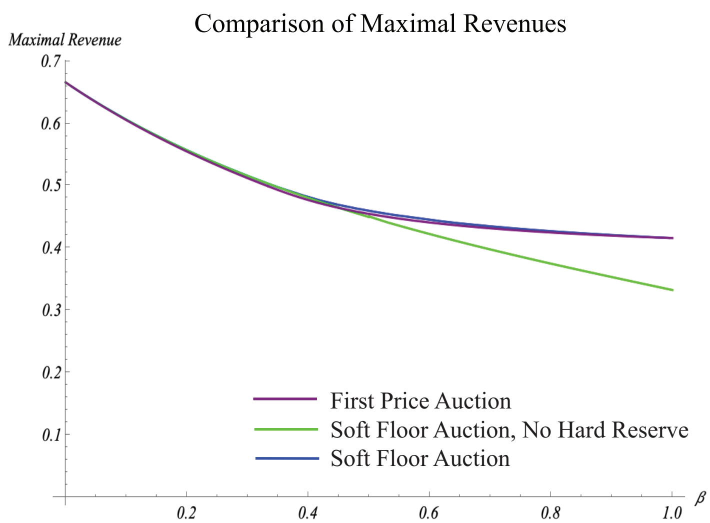

FIGURE A.4. Comparison of Revenues for Two Bidders with Uniform Distribution

The revenues of the soft-floor auction with and without a hard reserve coincide for $\beta \in [0, 1/3]$ because the optimal hard reserve is $r = 0$ in that range. For $\beta > 1/3$, the performance of the soft-floor auction with no hard reserve sharply decreases. By contrast, the first-price auction always generates strictly lower revenue than the soft-floor auction, confirming Proposition 6, but by relatively small margins. The introduction of a soft floor indeed increases expected revenue, but the effect is apparent only for β near 1/2.

APPENDIX B. EXPERIMENT

B.1. **Dynamics.** Tables B.1 and B.2 show that revenues in the different treatments and parametrizations are stable across time.

Table B.1. Revenue in SFA and FPA by round

	FPA	**SFA**			
		s = 40	**s = 46**	**s = 52**	**s = 58**
Period 1 to 12	41.04 (6.88)	39.66 (7.73)	40.31 (8.10)	47.56 (6.03)	50.81 (4.83)
Period 13 to 24	42.99 (6.79)	37.83 (5.98)	45.27 (2.94)		47.56 (7.79)
Period 25 to 36	41.10 (6.42)	39.35 (7.39)	43.37 (7.68)	48.41 (3.68)	47.27 (6.57)
Period 37 to 48	40.61 (8.26)	42.36 (3.48)	48.18 (9.50)	44.14 (5.92)	47.74 (5.58)

Table B.2. Revenue in SFA_R vs. FPA_R by round

	FPA			**SFA**		
	$r = 32$	$r = 38$	$r = 44$	$s = 68$	$s = 62$	$s = 56$
Period 1 to 12	46.28 (7.62)	46.14 (8.11)	45.86 (11.19)	49.85 (9.87)	44.27 (7.72)	41.37 (11.19)
Period 13 to 24	40.56 (6.12)		44.42 (6.88)	48.81 (7.30)		44.46 (6.58)
Period 25 to 36	40.11 (9.08)	44.65 (5.93)	47.32 (11.16)	49.42 (6.53)	45.70 (9.28)	46.16 (10.83)
Period 37 to 48	41.80 (6.17)	44.19 (6.72)	44.66 (12.99)	48.13 (10.49)	50.23 (9.25)	41.86 (24.19)

B.2. **Instructions.**

General part

Treatment dimensions are: FPA / FPA_R / SFA / SFA_R

Welcome to our experiment!

Please read the following instructions carefully. If you have a question, please raise your hand. We will then come to you and answer your questions. Communication with other participants is not allowed during the whole experiment. If you violate this rule, we might exclude you from the experiment and all payouts.

All participants receive a 4.00 Euro show-up fee. In addition, you can earn further payoffs depending on your decisions and those of the other participants.

The currency used in this experiment is experimental currency units (ECU). At the end of the experiment, all ECUs will be converted into euros and paid out in cash. The conversion rate is

40 ECU = 1 Euro. All decisions and payoffs in this experiment will be treated anonymously. All participants receive identical instructions.

Experiment

The experiment consists of a total of 48 rounds. In each round, the participants are randomly matched into groups of two. We ensure that nobody is matched with the same participant in two consecutive rounds. All rounds are identical and independent of each other. All round earnings are added and paid out at the end of the experiment. Possible losses will be offset by the 4.00 Euro show-up fee.

In each round, the participants can bid in an auction. A round consists of two stages. The first stage, the bidders decide participation in the auction [SFA/SFA_R/FPA_R: with a minimum bid] On the second level, bidders can bid on a fictitious good if they have decided to participate in the auction [SFA/SFA_R/FPA_R: with a minimum bid]

The personal value of the fictitious good (the value) of each bidder varies between the bidders and the rounds. At the beginning of each round, each bidder is told how much the good is worth to him in that round. The values are determined randomly and independently for each bidder, and each ECU-Amount between 0.00 ECU and 100.00 ECU (with two decimal places) is equally probable.

Participation decision. [FPA / FPA_R: In the first stage, both bidders decide whether they wish to participate in the auction [FPA_R: with a minimum bid.]

[FPA_R: Bids in the auction must be at least as high as the opening bid. The opening bid varies between rounds and can take the values of 32 ECU, 38 ECU, 44 ECU, or 50 ECU.]

If both bidders decide not to participate in the auction, none will receive the goods.]

[SFA / SFA_R: In the first stage, both bidders decide whether to participate in the auction with an opening bid. *Auction bids* must be at least as high as the opening bid.] [SFA: The opening bid is determined before each round of the experiment. It can take the following values: 40 ECU, 46 ECU, 52 ECU, or 58 ECU.]

[SFA / SFA_R: If a bidder does not want to participate in the auction, they can make a *purchase price suggestion*, which must be between [SFA: 0 ECU][SFA_R: the minimum price] and the opening bid. Anyone who does not want to make a purchase price suggestion can decline this option.]

[SFA_R: The opening bid and the minimum price were set for each round ahead of the experiment. They can take the following values: (50 ECU, 50 ECU), (44 ECU, 56 ECU), (38 ECU, 62 ECU), and (32 ECU, 68 ECU). The first value in the parentheses corresponds to the minimum price, and the second value corresponds to the opening bid. (If the minimum price equals the opening bid, no purchase price suggestions can be made.)]

[SFA / SFA_R: If both bidders decide against participating in the auction with an opening bid, the bidder with the higher purchase price suggestion receives the good. In this case, they pay their purchase price suggestion. The buyer is determined randomly if both bidders suggest the same purchase price. If both bidders decide against participating in the auction with an opening bid and neither of them makes a purchase price suggestion, neither bidder receives the good, and both bidders end up empty-handed.]

Auction. [FPA: Bidders who participate in the auction place their bids for the goods at the second stage. Bidders who have decided not to participate do not submit a bid. The bid must be a minimum of 0 ECU and a maximum of 100.00 ECU.]

[FPA_R: Bidders who participate in the auction with minimum bid place their maximum bid for the good at the second stage. Bidders who have decided not to participate do not submit a bid. The maximum bid must be at least as high as the opening bid and may not exceed 100.00 ECU.]

[FPA / FPA_R: The bidder who places the higher maximum bid wins the auction. The price he has to pay corresponds to his bid. If both bidders place the same bid, a random decision is made about who wins the auction.]

[SFA / SFA_R: In the second stage, bidders who participate in the auction with an opening bid submit their maximum bid for the good. Bidders who have decided not to participate cannot submit a bid. The maximum bid is the maximum price the bidder will pay for the good. This maximum bid must be at least as high as the opening bid and can be a maximum of 100.00 ECU. The bidder who submits the higher maximum bid wins the auction. The price he has to pay corresponds to the second highest bid plus 0.01 ECU.

Exceptions:

- ⇒ *If only one bidder participates in the auction with an opening bid, the price corresponds to the opening bid, regardless of his bid.*
- ⇒ *If both bidders submit the same bid, the auction winner is decided randomly. In this case, the price corresponds exactly to the maximum bid of the auction winner.*

The maximum bid thus works like a proxy that bids for you at the auction. The proxy always bids as much as is necessary to be the highest bidder. It does this until your maximum bid is reached; after that, it drops out. Therefore, the price the winner has to pay never exceeds the second highest bid plus 0.01 ECU.]

Round payoff. If a bidder receives the good, the round payoff corresponds to his value minus the price. For a bidder who does not receive the good, the round payoff is 0.00 ECU.

$$\text{Round payoff} = \begin{cases} \text{value} - \text{Price}, & \text{if the bidder receives the good} \\ 0, & \text{if the bidder does not receive the good} \end{cases}$$

Feedback. At the end of each round, all bidders receive information about the price, the bidder who received the good, and their round payoff.

B.3. **Pilot.** Our pilot experiments were conducted prior to the development of the theory. Thus, parametrization differs. However, experimental protocols were the same. We had two kinds of pilot sessions. One focuses on buyers in the auctions - as in the main part of the paper - to determine whether SFA increases revenue (and efficiency), and another focuses on sellers to investigate whether sellers indeed opt for a soft-floor auction if given the chance.

Pilot sessions were conducted between December 2016 and October 2017 in the Cologne Laboratory for Economic Research (CLER). Participants were students from the University of Cologne invited via ORSEE (Greiner 2015). The experiment was programmed with z-tree (Fischbacher 2007). We conducted two sessions for each of the main treatments, with exogenous sellers, consisting of 32 (with one exception of 28) participants in each session. Participants were randomly matched within matching groups utilizing a stranger's matching protocol. One matching group consisted of four bidders; thus, we collected 16 independent observations for each treatment. In FPA, we only had 15 independent observations because some invited participants failed to attend. For ENDO, we collected data from 96 subjects in four sessions. One matching group consisted of four bidders and two sellers. Thus, we have 16 independent observations for the endogenous seller treatment. We collected 15,800 bids and 1,600 soft floor or hard reserve price decisions from 348 subjects.

B.3.1. *Buyer results.* In the pilot, we considered treatments FPA, SFA, and FPA_R. All treatments were conducted as described in the main part of the study in Section 4.1. Table B.3 summarizes the parametrization. Note that the treatments were not parametrized according to our model, as the experiments were run before the models were derived. In FPA, we naturally consider the same setting as in the main experiment. In SFA, we consider two soft floor levels only. A soft floor of $s = 50$ roughly at the midpoint of values considered in the main experiments and a soft floor of $s = 66$ larger than any value considered in the main experiment. Lastly, for FPA_R, we only considered the optimal hard reserve according to the standard theory of $r = 50$, which we also consider in the main experiment.

Table B.3. Auction Formats and Parametrization

Auction format	**Level of hard reserve r and soft floor s**
FPA	$r = 0$
SFA	$s \in \{50, 66\}$
FPA_R	$r = 50$

A soft-floor auction, with a soft floor of $s = 50$ and $s = 66$, significantly increases revenue compared to a first-price auction without a hard reserve. Similar to our results in Section 4, the increase is more than 10 percent for $s = 50$ and more than 15 percent for $s = 66$. Notably, behavior in the pilot leads to an estimated weight on regret $1 - \beta$ of 0.41 and implies an optimal soft floor of $s = 51$. As in the main experiment, we find that the revenue-increasing effect of s maintains even above the theoretically optimal level, also suggesting that the the attractiveness of s goes beyond what even our model captures.

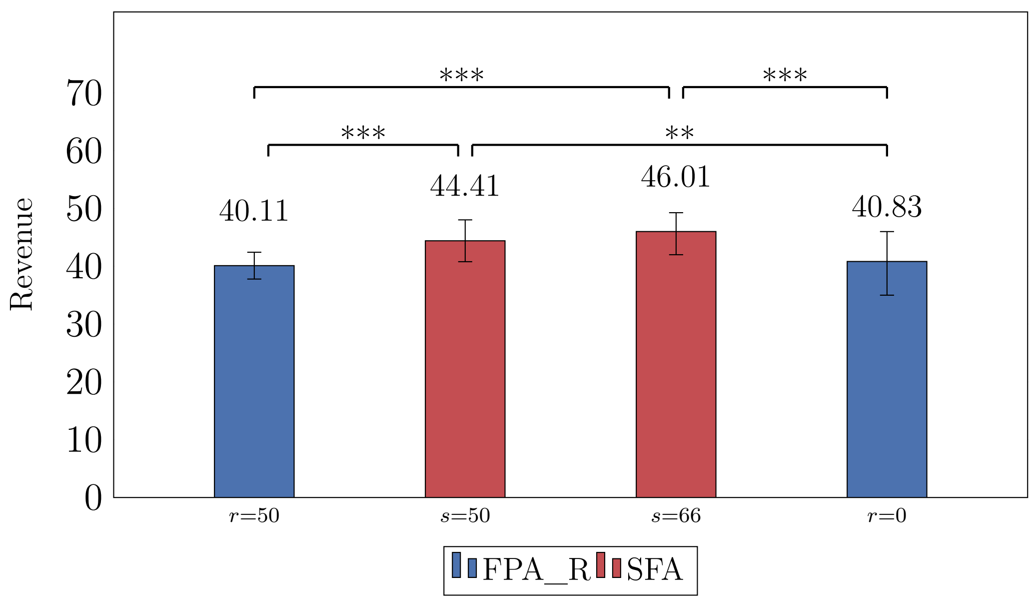

Figure B.5. Revenue in FPA_R and SFA in pilot

Notes. The figure reports average revenue on the observation group level. Significance levels are based on permutation tests with Holm-correction, and *, *, and * denote significance at the 10 percent, 5 percent, and 1 percent level, respectively.

In the pilot, we also compared the first-price auction with a non-zero hard reserve price to a soft-floor auction with a hard reserve of zero. Here, we find that the soft-floor auction for both

$s = 50$ and $s = 66$ also leads to a significant increase in revenue of around 10 percent. However, note that the revenue for a first-price auction with $r = 50$ is almost 10 percent smaller than in the main experiment.

B.3.2. *Sellers: Endogenous Auction Format Choice.* The second part of our pilot study focused on seller choices. In each period, we let sellers choose among the auctions from the main experiment as explained in Section 4.1. Table B.4 summarizes the menu of choices, which includes a first-price auction, a second-price auction, and a soft-floor auction with different levels of soft floors and hard reserve prices. In each period„ the seller must choose the format and the soft floor or hard reserve level.

TABLE B.4. Seller Choice Menu

Auction format	**Level of hard reserve r or soft floor s**
SPA	$r = 0$
FPA	$r = 0$
SFA	$s \in \{33.33, 50, 66.67, 100\}$
FPA_R	$r \in \{33.33, 50, 66.67, 100\}$

Across all auctions, a sizable majority of sellers prefer the soft floor, 66.81 percent, over the hard reserve, 33.19 percent ($p < 0.001$). The same holds if we restrict ourselves to intermediate reserve price levels ($s, r = \{33.33; 50; 66.67\}$), where 60.83 percent of our sellers prefer the soft floor while 39.17 percent prefer the hard reserve ($p = 0.049$).

Not only do sellers prefer the soft floor, but they also choose higher soft floors than hard reserves. The average hard reserve is 53.70 ECU, slightly above the optimal reserve for risk-neutral bidders of 50 ECU, while the average soft floor is 71.80 ECU ($p < 0.001$). If we restrict to intermediate reserve levels, the average hard reserve, 55.36 ECU, is smaller than the average soft floor, 60.87 ECU ($p = 0.08$).

Figure B.6 shows the relative frequencies with which sellers choose soft floors and hard reserves across levels. Here, we exclude the rarely chosen second-price auction. Note that they could be implemented indirectly by choosing a soft floor and a hard reserve of $s = r = 0$. The figure shows that a low hard reserve of 33.33 ECU is more popular than a correspondingly low soft floor, yet with only 5 percent of all cases, both are hardly chosen. The absolute and relative attractiveness of the soft floors increases in s. The soft floor is more popular for a level of 50 ECU, although the effect is not statistically significant ($p = 0.736$). For $r = s = 66.67$, the difference becomes highly significant both economically (more than twice as many sellers choose the soft floor) and statistically ($p = 0.035$). Not surprisingly, the popularity difference between

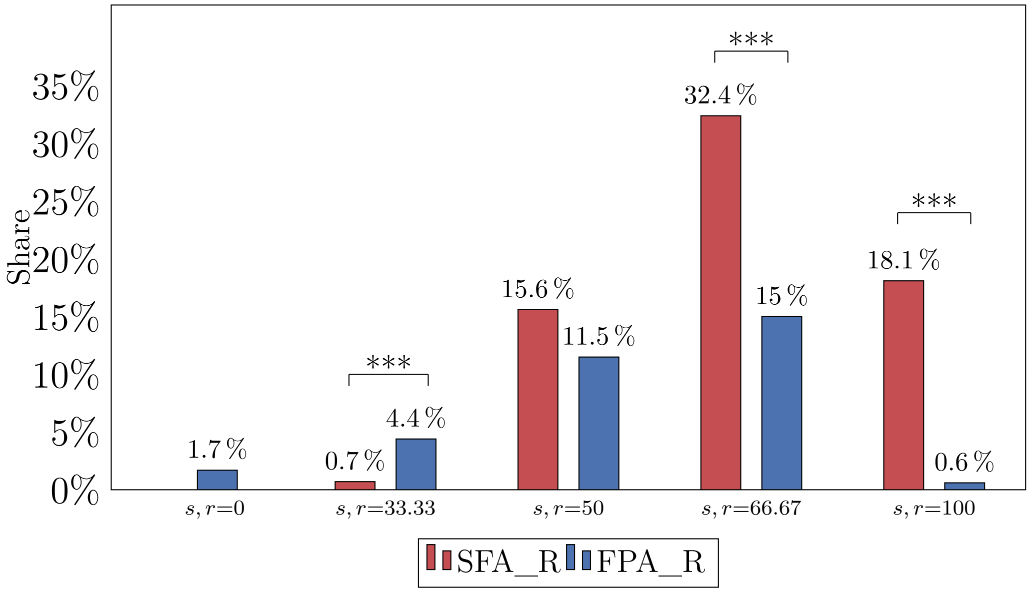

FIGURE B.6. Seller choices of FPA_R and SFA_R in ENDO treatment

Notes. The figure reports average revenue on the observation group level. Significance levels are based on permutation tests with Holm-correction, and *, *, and * denote significance at the 10 percent, 5 percent, and 1 percent level, respectively.

corresponding soft floors and hard reserves becomes even larger for $s = r = 100$, because all other auction formats obviously dominate the $r = 100$ auction.

Sellers' choices reflect that soft-floor auctions generally lead to higher revenues. Figure B.7 displays the average revenue for the different soft floors and hard reserves levels in our ENDO treatment. Soft-floor auctions never perform significantly worse. However, they perform better than first-price auctions for a high enough level of soft floors. Comparing the soft-floor and first-price auction among the different levels, we find weakly significantly higher revenue for a soft floor of 50 ECU ($p = 0.063$) and highly significantly higher revenue for 66.67 ECU ($p = 0.001$), but no significant difference for 33.33 ECU ($p = 0.399$). Computing the average revenue across all soft floor and hard reserve price levels, we find the soft-floor auction revenue (48.29 ECU) beats the first-price auctions' revenue with a hard reserve price (36.11 ECU) by a significant margin ($p < 0.001$). Restricting the data to intermediate levels of soft floors and hard reserves only, confirms our findings with an average soft-floor auction revenue of 48.26 ECU and an average first-price auction revenue of 37.31 ECU ($p < 0.001$).

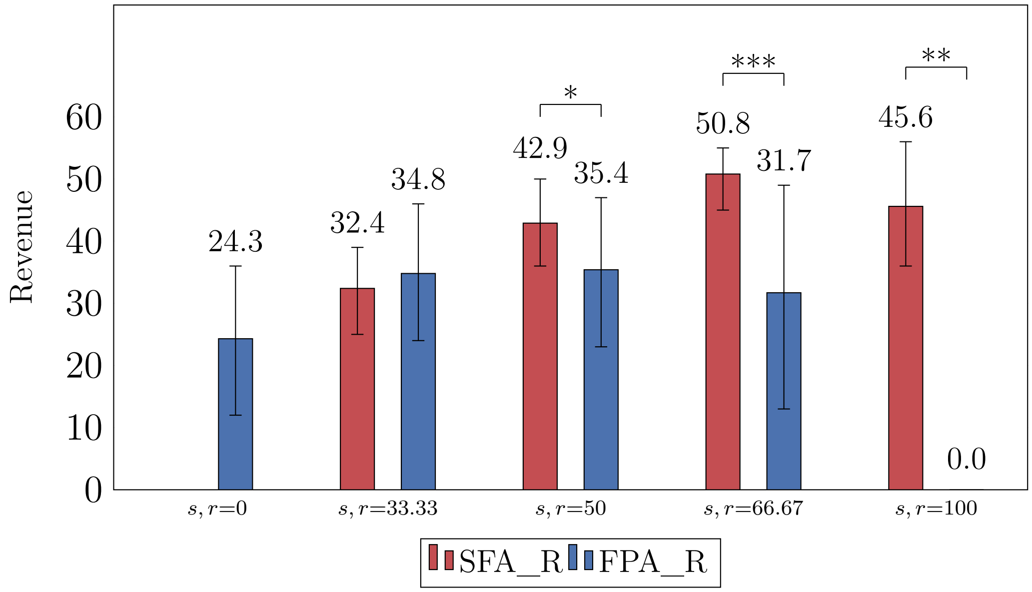

Figure B.7. Revenue of FPA_R and SFA_R in ENDO treatment

Notes. The figure reports average revenue on the observation group level. Significance levels are based on permutation tests with Holm-correction, and *, *, and * denote significance at the 10 percent, 5 percent, and 1 percent level, respectively.