Diagreement about the Term Structure of Inflation Expectations*

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Abstract

We develop a panel Nelson-Siegel model of individual inflation expectations across different forecasting horizons: an individual term-structure of inflation expectations. Utilizing panel data from the Survey of Professional Forecasters, we uncover significant heterogeneity in the dynamic patterns of the term structure across forecasters and over time. We decompose the dispersion of inflation expectations into three components: 1) prior beliefs, 2) heterogeneous responses to common information, and 3) idiosyncratic information. Our findings suggest that in normal times, long-horizon inflation disagreement is predominantly driven by prior beliefs, while short-horizon disagreement stems mainly from idiosyncratic information. However, during economic recessions, heterogeneous responses to common information become more important in driving disagreement at all horizons. This result suggests that clear communication about aggregate policies can more effectively anchor long-term inflation expectations of economic agents during periods of significant uncertainty.

JEL classification: E17, E31, E37, E52, E58, E65.
Keywords: Inflation Expectations, Term structure of interest rates, Monetary policy

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"Of course, an extended period of high goods and services inflation resulting from a series of demand and supply shocks associated with the pandemic and the war could lead to a rise in inflation expectations, which would make it much more difficult to bring inflation down. That is why it has been important for monetary policy to take a risk-management posture to defend the expectations anchor. And the evidence from market- and survey-based measures suggests that longer-term inflation expectations are well anchored, while year-ahead measures have recently declined but remain elevated." — **Lael Brainard (January 19, 2023)**

1 Introduction

Maintaining stable inflation expectations among economic agents is crucial for achieving the FOMC's mandate of price stability and for managing macroeconomic risk. To gauge how well anchored economic agents' inflation expectations are, economists and policy makers have paid attention to the mean or median of inflation expectations— so-called the consensus. A less utilized metric is the disagreement about inflation expectations: Economic agents would not disagree much about future inflation if their expectations are well-anchored. In this context, a less known fact is that during the 2000s and over the course of the Great Recession, when the consensus inflation expectations stayed low and stable, the disagreement showed substantial volatility comparable to that seen in the early 1990s — the period of high inflation and likely unanchored inflation expectations.[1] This observation suggests the disagreement may serve as an independent measure of anchored inflation expectations, supplementing the consensus.

Not all the disagreements are alike. For monetary policy, it is important to consider two aspects of heterogeneity in disagreement about inflation expectations: forecast horizons and the source of disagreement. A practical question a policy maker may ask is *"Which type of disagreement or what portion of the disagreement is relevant to monetary?"* Disagreements about long-run inflation expectations are more relevant to monetary policy than those about short-run inflation expectations, since the monetary authority is primarily interested in anchoring long-run inflation expectations. Long-run inflation expectations matter more for the price and wage setting, while short-run inflation expectations largely reflect the realized inflation or effects of transitory factors like oil price changes.

In addition, different sources of disagreement also matter for the management of macroe-

[1] In 2010 when the consensus long-term inflation expectations stayed at around two percent in the survey of professional forecasters (henceforth, SPF), the variance across forecasters was at the level seen in the early 1990s when the consensus expectation was five percent— the period of unstable inflation.

conomic expectations. Forecasters may have different priors on average inflation Patton and Timmermann (2010), or different private information on future inflation that is not shared with others Herbst and Winkler (2021). Or simply, forecasters may interpret publicly available information differently Lahiri and Sheng (2008); Andrade et al. (2016). These different sources have direct implications for the effectiveness of monetary policy. Effective forward guidance, for example, may reduce the portion of disagreement driven by heterogeneous reactions to public information. If changes in the disagreement are driven by private information or a forecaster's long-run prior, there is not much room for monetary policy to reduce disagreement of these margins. In this context, not only the disagreement but also its source is important for monetary policy.

This paper develops a new individual-level model of inflation expectations over forecasting horizons–what we call *"the individual term-structure of inflation expectation."* This model is designed to comprehensively characterize disagreement about inflation expectations relevant to macroeconomic policy. Inspired by Nelson and Siegel's term structure of interest rates (Nelson and Siegel, 1987), we model an individual's trajectory of inflation forecasts over forecasting horizons with three factors—*the level, slope, and curvature*. The level captures the long-run inflation projection, which we call *the long end*. The slope captures the overall difference between the long end and the inflation nowcast of the current quarter. The curvature depicts residual nonlinearity in the forecast horizon and inflation projection between the current-quarter nowcast and the long end, not captured the other two factors. We do not consider the curvature for parsimonious modeling because its role in the observed individual-level term structure of inflation expectations is limited.

Each factor of an individual forecaster, the level, for instance, is composed of three components: (1) the constant term capturing individual prior; (2) the term capturing forecast reactions to public or common information; (3) the term reflecting forecast reactions to private or idiosyncratic information. The first term is estimated with the individual fixed effect and hence time invariant. The second and third terms are time-varying and are described as the common and idiosyncratic components of a dynamic factor structure. Likewise, the individual's slope is also composed of the three analogous components. Our analytical framework decomposes the disagreement of each forecasting horizon at each point in time into the contributions from (1) individuals' prior beliefs, (2) heterogeneous reactions to common information, and (3) heterogeneous reactions to idiosyncratic information. It is noteworthy that this decomposition of disagreement is entirely new in the literature.

We take the model to the forecaster-level data from the Survey of Professional Forecasters (the SPF, henceforth) and estimate the model with maximum likelihood. The sample period is 1990:Q1-2023:Q2. The SPF provides a rough snapshot of a forecaster's term structure of inflation projections, not a complete trajectory. Specifically, the forecasting horizons are available in grouped forms such as "the average over the next 5 years" or "the average over the next 10 years." From the incomplete observations, our model recovers a forecaster's complete trajectory of quarter-over-quarter inflation forecasts from the current quarter to 10 years out at each point in time. This new model is so flexible that it can also be estimated with other survey data on inflation expectations with different forecasting horizons.

The estimated level and slope effectively capture the long-run and transitory dynamics in inflation expectations, respectively. The mean level trends down in the 1990s and then stays at a near-constant level from 2000. This mean level edges down during the Great Recession and the pandemic recession, and recovers after the end of both recessions. However, the mean slope factor is quite volatile, reflecting transitory developments such as oil price fluctuations and changes in economic activity. During the Covid-19 pandemic, the mean slope fluctuates drastically, capturing the run-up in inflation in 2021 and 2022. The mean level and slope also have unique forecast implications for the pandemic era. In particular, the mean level does not change much during the pandemic. The mean slope factor normalizes in 2022 but has not yet reached the prepandemic level. These two distinct movements in mean level and slope suggest that the run-up in inflation during the pandemic is less likely to reflect permanent changes in long-run trend inflation.

Consistently, the recovered mean inflation expectations 10 years out from the current quarter, *the 10-year consensus forecast*, trend down in the 1990s and then stabilize at around 2-2.5% from 2000 including the Great Recession period. This estimated consensus is quite similar to the trajectory of popularly cited trend inflation (e.g., Stock and Watson, 2016). Even during the Covid-19 pandemic, the 10-year consensus exhibits muted variations—edging lower in 2020, rising fast by a half percentage point in 2021, and then stepping down to 2.5 percent, the level a little elevated relative to the pre-pandemic level. Quite differently, the consensus forecast with a horizon shorter than 1 year largely reflects the realized CPI inflation. Specifically, during the Covid-19 pandemic, the consensus short-term inflation projections spiked above 4 percent, much higher than that seen in the early 1990s.

Although the estimated consensus forecasts seem to suggest well-anchored long-term inflation expectations, the recovered disagreement over the forecast horizon tells us a completely different story. Forecasters disagree more about the level during the Great Recession and its recovery than they did in the early 1990s. Notably, during the pandemic, there are quite a few forecasters who were estimating dramatically high long-run inflation expectations higher than 4 percent, signalling a substantial upside risk in the long-term projection. In addition, the disagreement about slope is also larger in the Great Recession and the pandemic than that in the early 1990s. In 2021 and 2022, there are quite a few forecasters who expect the elevated inflation to slow down sluggishly than what is suggested by the mean slope, again indicating the upside risk in inflation projection. This observation suggests that the term structure of disagreement provides information about the degree of anchored inflation expectations independent of that of the consensus.

We further decompose changes in the disagreement into the distinct sources. To begin with, prior beliefs and idiosyncratic information account for the bulk of disagreement among forecasters and across the forecasting horizons, while the role of common information is relatively small. This decomposition result is in line with previous findings, including those of Patton and Timmermann (2010) and Lahiri and Sheng (2008). During an economic downturn, however, common information is responsible for the increased disagreement. For example, at the peak of the Great Recession, common information accounts for about 60 percent of the long-term disagreement. During the pandemic, the disagreement about short-run and long-run inflation forecasts increased substantially. Common information is responsible for the bulk of increased disagreement.

Our novel findings have unique implications for policy. The importance of idiosyncratic information and permanent heterogeneity in the disagreement suggests that monetary policy may be limited in reducing the disagreement substantially. However, this is not necessarily bad news. In particular, the portion of long-run disagreement driven by common information is larger before 2000 than in the post-2000 period. This may suggest better anchored expectations in the 2000s than in the pre-2000 period, and not much room is left for monetary policy to further reduce the disagreement. More importantly, the increased importance of public information during an economic downturn suggests clear communication about monetary policy may be more effective in expectation management in times of economic uncertainty than in normal times.

Furthermore, we empirically examine the link between monetary policy and disagreement in inflation forecasts. Specifically, we estimate responses of the disagreement to a forward-guidance shock and Fed's reactions to economic news Bauer and Swanson (2023). Contractionary forward guidance or Fed's reaction, which aims inflation stabilization, reduces the disagreement driven by common information, in line with the previous studies including Ehrmann et al. (2019) and Glas and Hartmann (2016). However, their effects are limited in reducing the idiosyncrasy-driven disagreement. This empirical result confirms the economic interpretation of the disagreement created by heterogeneous reactions to common information as the portion of disagreement that is reducible with clear communication on monetary policy.

Finally, we explore the implications of our empirical findings for the theory of expectation formation and learning. First, our novel disagreement decomposition provides the empirical validation of existing theories on information acquisition. Specifically, our result highlights the importance of long-term priors (Patton and Timmermann, 2010; Farmer et al., 2021) and the equilibrium disagreement driven by heterogeneous interpretations of public information (Lahiri and Sheng, 2008) in the learning models of the finance literature. In addition, our results suggest a next step for the information acquisition model in the macro-literature. The individual factor loadings– parameters capturing the heterogeneous responses to public information–can be interpreted as the information precision heterogeneous across forecasters. Note that we have the loadings for both the level and the slope for each individual. The two-dimensional precision measures may be necessary to capture changes in the commonality-driven disagreement across different forecasting horizons, which has not yet been considered in the literature.

Related Literature This paper makes novel contributions to a few different strands of economic research. The first is the literature on *aggregate* the term structure of interest rate or inflation expectations. Aruoba (2020) models the term structure of inflation expectations using a Nelson-Siegel (Nelson and Siegel, 1987) yield curve. Aruoba treats the level, slope, and curvature factors as latent states and estimates them with a linear state-space model using median SPF forecasts and Blue Chip consensus forecasts. Clark et al. (2022) construct the term structure of inflation expectations and uncertainty using a state space model in which stochastic volatility and persistent biases in the forecasts are allowed. Bundick and Smith (2020) analyze the effect of monetary policy on inflation expectations and find that inflation became more anchored after the announcement of the inflation target in 2012. However, the empirical analysis is conducted at the aggregate level and is based on the Michigan Survey of Consumers and TIPS break-even rates.

Recent studies focus on modeling individuals' inflation expectations (Herbst and Winkler, 2021; Crump et al., 2021; Fisher et al., 2022).[2] This paper is closest to Herbst and Winkler (2021) in that inflation forecasts over the forecasting horizon are modeled and estimated at the individual level. However, the two studies are different for the following reasons. First, our focus is the term structure of inflation projection at the individual level, and hence we model the complete term structure of inflation forecasts from the current quarter through 10 years out with flexible Laguerre polynomials employed in Nelson-Siegel's model. Meanwhile, Herbst and Winkler characterizes the individual-level inflation projection over relatively short-term forecast horizons (up to one year), since their goal is to characterize the comprehensive joint dynamics of various macroeconomic variables.[3] Second, we compute the extent to which public and private information accounts for the changing disagreement about future inflation at each forecasting horizon with a particular focus on forecasters' heterogeneous responses to public information. These heterogeneous responses are not allowed in Herbst and Winkler (2021). Last, we provide the unique decomposition that summarizes the role of prior beliefs, and public and private information in the disagreement at each point in time, a methodological advancement from Herbst and Winkler's.

Our research directly speaks to the literature on disagreement and learning models. Lahiri and Sheng (2008) emphasizes the importance of prior information in disagreement and proposes a Bayesian learning model.[4] Similarly, Patton and Timmermann (2010) find that the key source of persistent disagreement stems from heterogeneity in priors and show that the differences in opinion move countercyclically. A recent study by Farmer et al. (2021) claims the importance of prior beliefs in the formation of macroeconomic expectations by professional forecaters. Our paper is distinguished from the previous studies in that the model has the component capturing heterogeneous responses to public information.

This paper also speaks to the literature on heterogeneous effects of monetary policy on

[2]Crump et al. (2021) estimate a multivariate trend-cycle model using the universe of professional forecasts for the U.S. and show that the multivariate model better fits the data than estimating individual univariate models on each time series. Fisher et al. (2022) model people's expectations of inflation using a trend-cycle decomposition and estimate the term structure of expectations using the full panel structure of the SPF assuming agents receive private and public signals.

[3]They adopt three common factors and the idiosyncratic component, and estimate the model with a Bayesian method.

[4]Lahiri and Sheng show that in the presence of strong prior and uncertain public information, the heterogeneous response to public information is an equilibrium outcome.

the disagreement of economic agents. Andrade et al. (2016) provide empirical evidence for heterogeneous beliefs about forward guidance and analyze the effect of monetary policy in the context of a new Keynesian model. Glas and Hartmann (2016) distinguish individual inflation uncertainty and disagreement between forecasters, and show that the overall disagreement increases during the period of contractionary policy. Our finding is distinguished from that of Glas and Hartmann in that we further show that the common-information driven disagreement reduces to a contractionary monetary policy shock. Ehrmann et al. (2019) shows long-horizon time-contingent and state-contingent forward guidance effectively reduces disagreement, while short-horizon time-contingent forward guidance does not.

The paper is organized as follows. Section 2 discusses the data on inflation expectations in the Survey of Professional Forecasters. Section 3 introduces the individual-level semiparametric model that we use for the term structure of inflation expectations. Section 4 decomposes the dispersion of inflation expectations into components driven by public and private information at each forecasting horizon. Section 5 discusses the implications of empirical findings for policy and theory. Section 6 concludes.

2 Data: The Survey of Professional Forecasters

This section discusses inflation expectations data from the Survey of Professional Forecasters (SPF).

2.1 Notation

First, we define some notation that we will use throughout the rest of the paper. Denote the price level at time t by P_t (in our case this will refer to the consumer price index). Let $\pi_{s\to t}$ be the continuously compounded inflation rate between time s and time t:

$$\pi_{s\to t} \equiv log(P_t) - log(P_s) \tag{1}$$

Throughout we will work with continuously compounded inflation rates because of their time-additive properties. Define the forecast, made at time t, of the inflation rate between times r and s, as $\pi_{r\to s|t}$. Finally, let $q_{a-1}(t)$ denote the final quarter of the year prior to the year time t is in.

2.2 Definition of Forecasted Quantities

We collect data on CPI inflation forecasts from the SPF, conducted by the Federal Reserve Bank of Philadelphia. The survey is sent out in the first month of each quarter and responses are collected around the middle of the quarter, e.g. mid-February in Q1. Survey participants are asked to forecast the average quarterly level of the CPI (or transformations of this quantity) at various horizons. The SPF CPI Inflation Forecasts can be broken into 4 categories:

- 1-period backcasts to 4-quarter ahead forecasts of annualized quarter-over-quarter CPI inflation :

  $$100 \times \left[\left(\frac{P_{t+h}}{P_{t+h-1}} \right)^4 - 1 \right]$$

  for $h = -1, \ldots, 4$.

- 1 to 3-year ahead forecasts of Q4 over Q4 CPI inflation:

  $$100 \times \left[\frac{P_{q_{a-1}(t)+4j}}{P_{q_{a-1}(t)+4(j-1)}} - 1 \right]$$

  for $j = 1, \ldots, 3$.

- Forecasts of average Q4 over Q4 CPI inflation over the next 5 years:

  $$100 \times \left[\left(\prod_{j=1}^{5} \frac{P_{q_{a-1}(t)+4j}}{P_{q_{a-1}(t)+4(j-1)}} \right)^{\frac{1}{5}} - 1 \right]$$

- Forecasts of average Q4 over Q4 CPI inflation over the next 10 years:

  $$100 \times \left[\left(\prod_{j=1}^{10} \frac{P_{q_{a-1}(t)+4j}}{P_{q_{a-1}(t)+4(j-1)}} \right)^{\frac{1}{10}} - 1 \right]$$

The first type of forecasts is what's known as *fixed horizon* forecasts and the other three types are known as *fixed event* forecasts. We assume that these forecasts correspond to forecasts of continuously compounded inflation so that they line up with our model specification directly.[5]

[5] This turns out to be relatively innocuous assumption, see Aruoba (2020) for a discussion.

That is, we assume

$$
\begin{aligned}
100 \times \mathbb{E}_t \left[\left(\frac{P_{t+h}}{P_{t+h-1}} \right)^4 - 1 \right] &\approx 400 \times \pi_{t+h-1 \to t+h|t} \\
100 \times \mathbb{E}_t \left[\frac{P_{q_{a-1}(t)+4j}}{P_{q_{a-1}(t)+4(j-1)}} - 1 \right] &\approx 100 \times \pi_{q_{a-1}(t)+4(j-1) \to q_{a-1}(t)+4j|t} \\
100 \times \mathbb{E}_t \left[\left(\prod_{j=1}^{5} \frac{P_{q_{a-1}(t)+4j}}{P_{q_{a-1}(t)+4(j-1)}} \right)^{\frac{1}{5}} - 1 \right] &\approx 20 \times \pi_{q_{a-1}(t) \to q_{a-1}(t)+19|t} \\
100 \times \mathbb{E}_t \left[\left(\prod_{j=1}^{10} \frac{P_{q_{a-1}(t)+4j}}{P_{q_{a-1}(t)+4(j-1)}} \right)^{\frac{1}{10}} - 1 \right] &\approx 10 \times \pi_{q_{a-1}(t) \to q_{a-1}(t)+39|t}
\end{aligned}
$$

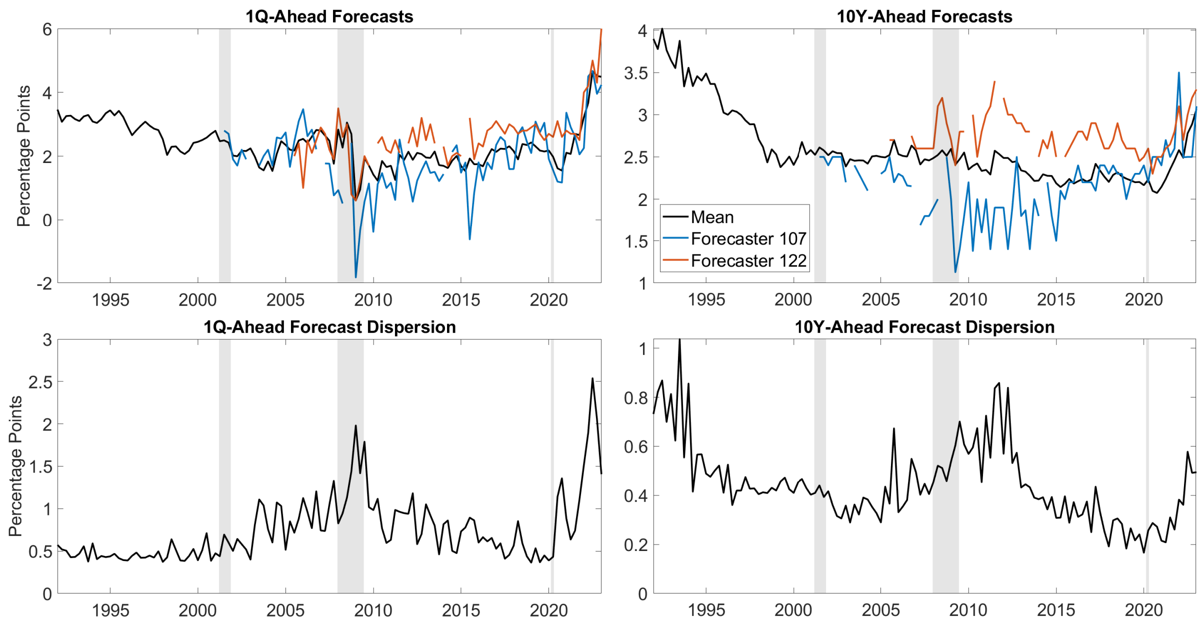
Our sample begins in 1991Q4, which is the first time that forecasts of average Q4 over Q4 inflation over the subsequent ten years become available, and runs through 2022Q4. This is necessary to be able to estimate the long-end of the term structure. Three-year ahead forecasts of Q4 of Q4 inflation and forecasts of average Q4 over Q4 inflation over the subsequent five years first become available in 2005Q3. We restrict our sample to forecasters who report nowcasts to four-quarter ahead forecasts and either a 5-year or 10-year average forecast in at least one quarter to ensure that we can identify long-run forecasts.

2.3 Properties of SPF CPI Forecasts

There are 172 unique forecasters in the data set during our sample period. In any given quarter, there are between 28 and 53 forecasters who report a forecast and a median of 37. Forecasters remain in the data set for between 1 and 112 quarters with a median tenure of 14 quarters (3 and a half years).

Figure 1 displays the distribution of 1-quarter ahead forecasts and 10-year ahead forecasts. The upper panels display the mean (which we will refer to as consensus) of the forecasts and the projections of two individual forecasters. The upper-left panel shows 1-quarter ahead forecasts and the upper-right panel reports 10-year ahead forecasts. The gray bars represent the NBER recessions. In the first ten to fifteen years of the sample there is a downward trend in both short and long-run inflation forecasts from around 4% to 2%. Short-run forecasts tend to exhibit more time series volatility than long-run forecasts as they react more strongly to transitory shocks. The bottom panels also show the cross-sectional standard deviation (which we will refer to as dispersion) of projections a quarter ahead and a 10-year ahead over time. Short-run forecasts

Figure 1: SPF FORECAST SUMMARY STATISTICS AND EXAMPLES



Notes: Black lines in the upper two panels show the average of inflation forecasts 1 quarter (left panel) and 10 years (right panel) ahead. The colored lines capture the projection of two forecasters whose IDs are 107 and 122 in the survey. The bottom two panels report the standard deviation of the projections.

Sources: SPF and authors' calculation.

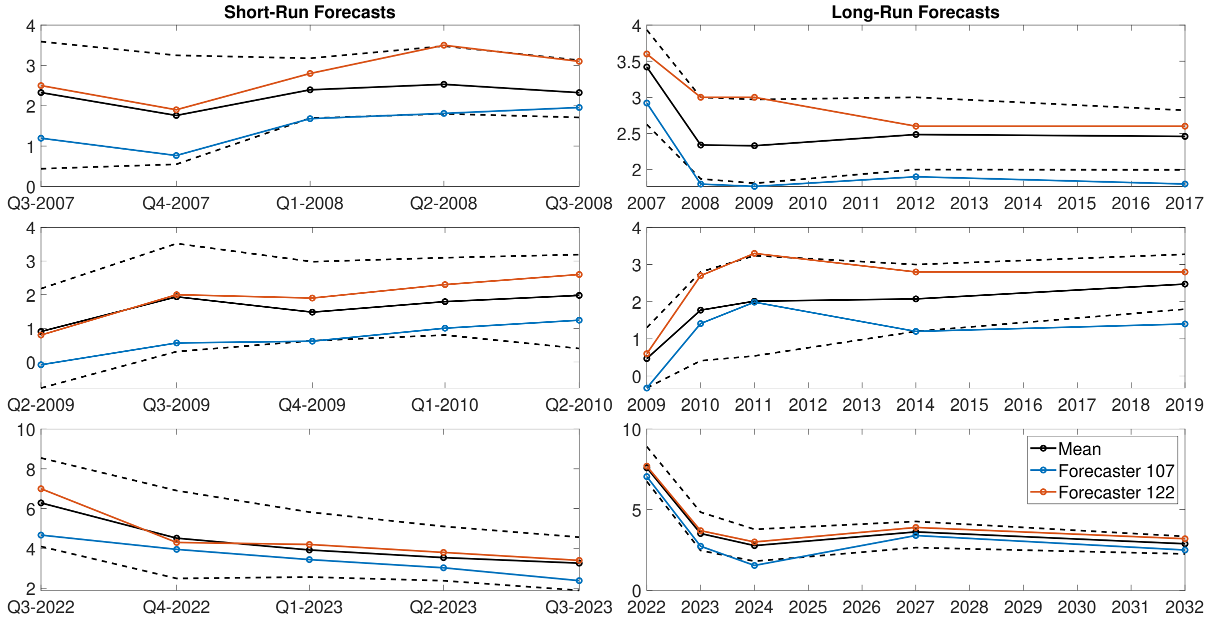
typically exhibit higher dispersion than long-run forecasts, with a notable exception being the early 1990s.

In addition to Figure 1 which captures the aggregate properties of the SPF forecasts, we also wish to explore how the expectations of the individual forecasters evolve over time. To this end, Figure 2 plots the reported forecasts over all horizons of two different forecasters for three different time periods: 1) 2007:Q3, 2) 2009:Q2 and 3) 2022:Q3. The solid black line is the consensus, and the dashed lines capture the forecasts at 5% and 95% of the distribution. The three dates reported in the figure help illustrate the wide variety of shapes the term structure of inflation expectations can take and how different an individual forecaster can be relative to the consensus. The levels of forecasts, as well as the trajectories over the forecasting horizons, are all different.

3 An Individual Term Structure of Inflation Forecasts

In this section, we build and estimate a model to recover the complete path of inflation forecasts over a 10-year horizon at each point in time.

Figure 2: THREE TERM STRUCTURES OF SPF FORECASTS



Notes: Black lines capture the consensus projections. The colored lines capture the projections of two forecasters whose IDs are 107 and 122 in the survey. The dashed lines capture the forecasts at 5 percent and 95 percent of the distribution.

Sources: SPF and authors' calculation.

3.1 Model

Following Aruoba (2020), we set up a Nelson-Siegel model for the term structure of inflation expectations:

$$\pi_{i,t\to t+h|t} = L_{i,t} - \left(\frac{1-e^{-\lambda_i h}}{\lambda_i h}\right) S_{i,t} + \left(\frac{1-e^{-\lambda_i h}}{\lambda_i h} - e^{-\lambda_i h}\right) C_{i,t}. \tag{2}$$

Based on this representation, the forecast of inflation between any two horizons h_1 and h_2 is given by

$$\pi_{i,t+h_1\to t+h_2|t} = L_{i,t} - \left(\frac{e^{-\lambda_i h_1} - e^{-\lambda_i h_2}}{\lambda_i (h_2 - h_1)}\right) S_{i,t} + \left(\frac{(1+\lambda_i h_1)e^{-\lambda_i h_1} - (1+\lambda_i h_2)e^{-\lambda_i h_2}}{\lambda_i (h_2 - h_1)}\right) C_{i,t}.$$

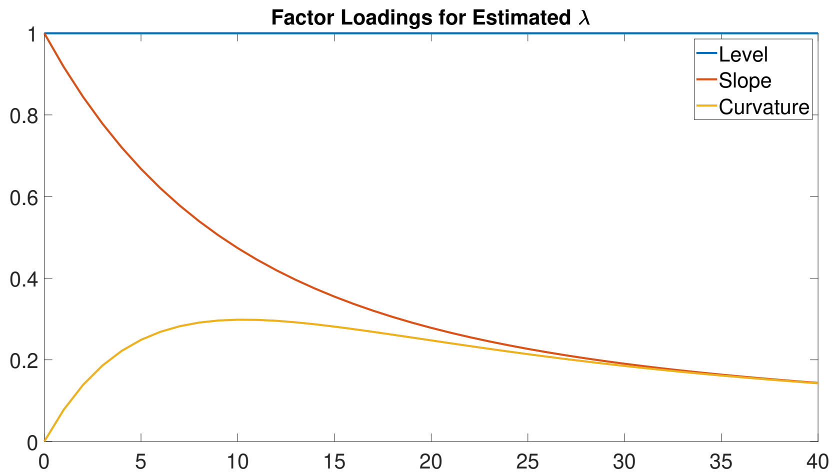
Following Diebold et al. (2008), we specify the following decomposition for the factors

$$L_{i,t} = \alpha_{i,L} + \beta_{i,L} L_t + \varepsilon_{i,L,t} \tag{3}$$

$$S_{i,t} = \alpha_{i,S} + \beta_{i,S} S_t + \varepsilon_{i,S,t} \tag{4}$$

$$C_{i,t} = \alpha_{i,C} + \beta_{i,C} C_t + \varepsilon_{i,C,t} \tag{5}$$

Figure 3: THREE FACTOR LOADINGS OF NELSON-SIEGEL MODEL



Notes: The figure captures the level, slope, and curvature factors in Equation (2).
Sources: Authors' calculation

where L_t, S_t, and C_t are level, slope, and curvature factors which are common to all forecasters. $\alpha_{i,L}, \alpha_{i,S}$, and $\alpha_{i,C}$ are forecaster-specific constant terms which govern the overall level of their individual factors. $\beta_{i,L}, \beta_{i,S}$, and $\beta_{i,C}$ are forecaster-specific loadings on the common factors. λ_i are forecaster-specific shape parameters.

We make several simplifying assumptions in our baseline model. Namely, we assume that there is no curvature factor for parsimony (we explore this further in robustness). We assume that the constant terms are the same for all forecasters so that $\alpha_{i,L} = \alpha_L$ and $\alpha_{i,S} = \alpha_S$ for all i. We assume that the shape parameter $\lambda_i = \lambda$ for all i. As λ mostly governs where the maturity at which the curvature factor is maximized, this does not greatly impact our results.

We assume that the common factors follow a VAR(1) process as follows:

$$\begin{bmatrix} L_t \\ S_t \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} L_{t-1} \\ S_{t-1} \end{bmatrix} + \begin{bmatrix} u_{L,t} \\ u_{S,t} \end{bmatrix}. \tag{6}$$

Since the levels of the common factors and the factor loadings are not separately identified, we normalize the shocks to the common factors $u_{L,t}$ and $u_{S,t}$ to have unit variance. We assume the shocks are uncorrelated:

$$\begin{bmatrix} u_{L,t} \\ u_{S,t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right). \tag{7}$$

In addition, we assume that the idiosyncratic factors follow VAR(1) processes such that

$$\begin{bmatrix} \varepsilon_{i,L,t} \\ \varepsilon_{i,S,t} \end{bmatrix} = \begin{bmatrix} b_{i,11} & b_{i,12} \\ b_{i,21} & b_{i,22} \end{bmatrix} \begin{bmatrix} \varepsilon_{i,L,t-1} \\ \varepsilon_{i,S,t-1} \end{bmatrix} + \begin{bmatrix} u_{i,L,t} \\ u_{i,S,t} \end{bmatrix}. \tag{8}$$

In our baseline specification, we assume that all of the VAR coefficients and covariance matrices are the same across forecasters, and that the covariance matrix is diagonal:

$$\begin{bmatrix} u_{i,L,t} \\ u_{i,S,t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_L^2 & 0 \\ 0 & \sigma_S^2 \end{bmatrix} \right). \tag{9}$$

3.2 State space representation

Since the model is specified for continuously compounded inflation at the quarterly frequency, this makes mapping the model predictions to observed SPF forecasts straightforward. In this section, we reformulate our model into a linear state-space model that is estimable with the Kalman filter and the maximum likelihood. It is noteworthy that our nonlinear term-structure model is translated into a linear representation because the nonlinear portion is parsimoniously expressed into a linear combination of Laguerre polynomials. This section describe the full state-space representation of the model.

3.2.1 State equation

We first start with the state equation. Let the state vector x_t be defined as

$$x_t = \begin{bmatrix} L_t, S_t, \varepsilon_{1,L,t}, \varepsilon_{1,S,t}, \cdots, \varepsilon_{n,L,t}, \varepsilon_{n,S,t} \end{bmatrix}' \tag{10}$$

Define the transition matrix F as

$$F = \begin{bmatrix} a_{11} & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & a_{22} & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & b_{11} & 0 & 0\dots & 0 & 0 \\ 0 & 0 & 0 & b_{22} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & b_{11} & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & b_{22} \end{bmatrix}. \tag{11}$$

Let u_t be the vector of shocks to the state vector defined as follows:

$$u_t = \begin{bmatrix} u_{L,t}, u_{S,t}, u_{1,L,t}, u_{1,S,t}, \cdots, u_{n,L,t}, u_{n,S,t} \end{bmatrix}'. \tag{12}$$

The covariance matrix of the shocks is given by:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \sigma_L^2 & 0 & 0\dots & 0 & 0 \\ 0 & 0 & 0 & \sigma_S^2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \sigma_L^2 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & \sigma_S^2 \end{bmatrix}. \tag{13}$$

We finally arrive at the following state equation of standard form:

$$x_t = F x_{t-1} + u_t, \quad u_t \sim N(0, Q). \tag{14}$$

3.2.2 Measurement equation

This section introduces the measurement equation that describes the observations with the state variables. Our measurement equation will have the following form:

$$y_t = \mu_y + \mathbf{H} x_t + v_t, \quad v_t \sim N(0, \mathbf{R}), \tag{15}$$

where μ_y is the vector composed of forecaster fixed effects, $\mathbf{H}$ is the matrix composed of factor loadings on the aggregate level and slope factors, and υ_t is the vector of measurement errors. The rest of the section details each component in Equation (15).

A. The vector of observations: y_t

For estimation, we use one-quarter to four-quarter ahead fixed-horizon forecasts and two-year forward, three-year forward, five-year average, and ten-year average fixed event forecasts. For the 5-year and 10-year average forecasts, we use observed nowcasts and one-quarter back-casts when available, and realized inflation two-quarter and three-quarter prior of the most recent CPI vintage.

The observation vector for any period y_t is given by

$$y_t = \begin{bmatrix} \pi_{1,t\to t+1|t}, \quad \pi_{1,t+1\to t+2|t}, \quad \pi_{1,t+2\to t+3|t}, \quad \pi_{1,t+3\to t+4|t}, \quad \cdots \\ \pi_{1,t+3\to t+7|t}, \quad \pi_{1,t+2\to t+6|t}, \quad \pi_{1,t+1\to t+5|t}, \quad \pi_{1,t\to t+4|t}, \quad \cdots \\ \pi_{1,t\to t+7|t+11}, \quad \pi_{1,t+6\to t+10|t}, \quad \pi_{1,t+5\to t+9|t}, \quad \pi_{1,t+4\to t+8|t}, \quad \cdots \\ \pi_{1,t\to t+19|t}, \quad \pi_{1,t\to t+18|t}, \quad \pi_{1,t\to t+17|t}, \quad \pi_{1,t\to t+16|t}, \quad \cdots \\ \pi_{1,t\to t+39|t}, \quad \pi_{1,t\to t+38|t}, \quad \pi_{1,t\to t+37|t}, \quad \pi_{1,t\to t+36|t}, \quad \cdots \\ \cdots\cdots \\ \cdots \quad \pi_{n,t\to t+37|t}, \quad \pi_{n,t\to t+36|t} \end{bmatrix}'. \tag{16}$$

The first four elements of y_t correspond to fixed horizon forecasts of one to four quarters ahead and are typically observed every period. Only four of the final sixteen elements of y_t are observed in any given quarter. These final sixteen elements correspond to fixed event forecasts, where each group of four correspond to the fixed event correctly mapped to the right forecast for the given quarter in which the survey is being conducted.

For the final eight elements, which correspond to forecasts of average inflation over five and ten year periods including the current calendar year, we must adjust them to directly with our model. Specifically,

- In Q1, we define

$$\pi_{i,t\to t+19|t} = \frac{4}{19}\left(5\pi_{i,t-1\to t+19|t} - \frac{1}{4}\pi_{i,t-1\to t|t}\right)$$
$$\pi_{i,t\to t+39|t} = \frac{4}{39}\left(10\pi_{i,t-1\to t+19|t} - \frac{1}{4}\pi_{i,t-1\to t|t}\right)$$

- In Q2, we define

$$\pi_{i,t\to t+18|t} = \frac{4}{18}\left(5\pi_{i,t-1\to t+19|t} - \frac{1}{4}\pi_{i,t-1\to t|t} - \frac{1}{4}\pi_{i,t-2\to t-1|t}\right)$$
$$\pi_{i,t\to t+38|t} = \frac{4}{38}\left(10\pi_{i,t-1\to t+19|t} - \frac{1}{4}\pi_{i,t-1\to t|t} - \frac{1}{4}\pi_{i,t-2\to t-1|t}\right)$$

- In Q3, we define

$$\pi_{i,t\to t+17|t} = \frac{4}{17}\left(5\pi_{i,t-1\to t+19|t} - \frac{1}{4}\pi_{i,t-1\to t|t} - \frac{1}{4}\pi_{i,t-2\to t-1|t} - \frac{1}{4}\pi_{i,t-3\to t-2|t}\right)$$
$$\pi_{i,t\to t+37|t} = \frac{4}{37}\left(10\pi_{i,t-1\to t+19|t} - \frac{1}{4}\pi_{i,t-1\to t|t} - \frac{1}{4}\pi_{i,t-2\to t-1|t} - \frac{1}{4}\pi_{i,t-3\to t-2|t}\right)$$

- In Q4, we define

$$\pi_{i,t\to t+16|t} = \frac{4}{16}\left(5\pi_{i,t-1\to t+19|t} - \frac{1}{4}\pi_{i,t-1\to t|t} - \frac{1}{4}\pi_{i,t-2\to t-1|t} - \frac{1}{4}\pi_{i,t-3\to t-2|t} - \frac{1}{4}\pi_{i,t-4\to t-3|t}\right)$$
$$\pi_{i,t\to t+36|t} = \frac{4}{36}\left(10\pi_{i,t-1\to t+19|t} - \frac{1}{4}\pi_{i,t-1\to t|t} - \frac{1}{4}\pi_{i,t-2\to t-1|t} - \frac{1}{4}\pi_{i,t-3\to t-2|t} - \frac{1}{4}\pi_{i,t-4\to t-3|t}\right)$$

For the nowcasts $\pi_{i,t-1\to t|t}$ and backcasts $\pi_{i,t-2\to t-1|t}$, we use the reported values from the SPF. For the two and three period backcasts $\pi_{i,t-3\to t-2|t}$ and $\pi_{i,t-4\to t-3|t}$, we use the most recently available vintage of the CPI at the time that the forecast was made.

B. The vector of forecaster fixed effects: μ_y

We define the loading function on the slope factor for forecasts of inflation between horizons at two dates $t+h_1$ and $t+h_2$ as follows.

$$f_S(h_1,h_2) = \frac{e^{-\lambda h_1} - e^{-\lambda h_2}}{\lambda(h_2-h_1)} \tag{17}$$

This expression is used in μ_y and $\mathbf{H}$.

We define the constant vector in the measurement equation, μ_y, as the following:

$$\mu_y = \begin{bmatrix} \alpha_L - \alpha_S f_S(0,1), \quad \alpha_L - \alpha_S f_S(1,2), \quad \alpha_L - \alpha_S f_S(2,3), \quad \alpha_L - \alpha_S f_S(3,4), \\ \alpha_L - \alpha_S f_S(3,7), \quad \alpha_L - \alpha_S f_S(2,6), \quad \alpha_L - \alpha_S f_S(1,5), \quad \alpha_L - \alpha_S f_S(0,4), \\ \alpha_L - \alpha_S f_S(7,11), \quad \alpha_L - \alpha_S f_S(6,10), \quad \alpha_L - \alpha_S f_S(5,9), \quad \alpha_L - \alpha_S f_S(4,8) \\ \alpha_L - \alpha_S f_S(0,19), \quad \alpha_L - \alpha_S f_S(0,18), \quad \alpha_L - \alpha_S f_S(0,17), \quad \alpha_L - \alpha_S f_S(0,16), \\ \alpha_L - \alpha_S f_S(0,39), \quad \alpha_L - \alpha_S f_S(0,38), \quad \alpha_L - \alpha_S f_S(0,37), \quad \alpha_L - \alpha_S f_S(0,36), \\ \cdots\cdots \\ \cdots\cdots \alpha_L - \alpha_S f_S(0,37), \quad \alpha_L - \alpha_S f_S(0,36) \end{bmatrix}'. \tag{18}$$

We define the constant vector in the measurement equation, v_t, as the following:

$$v_t = \begin{bmatrix} v_{1,1,t}, \quad v_{1,2,t}, \quad v_{1,3,t}, \quad v_{1,4,t}, \\ v_{1,5,t}, \quad v_{1,6,t}, \quad v_{1,7,t}, \quad v_{1,8,t}, \\ v_{1,9,t}, \quad v_{1,10,t}, \quad v_{1,11,t}, \quad v_{1,12,t}, \\ v_{1,13,t}, \quad v_{1,14,t}, \quad v_{1,15,t}, \quad v_{1,16,t}, \\ v_{1,17,t}, \quad v_{1,18,t}, \quad v_{1,19,t}, \quad v_{1,20,t}, \\ \cdots\cdots \\ \cdots\cdots v_{n,19,t}, \quad v_{n,20,t} \end{bmatrix}. \tag{19}$$

The covariance matrix of the the measurement error vector (v_t), R, is given by the following diagonal matrix.

$$R = \text{diag}\left(\begin{bmatrix} \sigma^2_{v,1}, \quad \sigma^2_{v,2}, \quad \sigma^2_{v,3}, \quad \sigma^2_{v,4}, \\ \sigma^2_{v,5}, \quad \sigma^2_{v,6}, \quad \sigma^2_{v,7}, \quad \sigma^2_{v,8}, \\ \sigma^2_{v,9}, \quad \sigma^2_{v,10}, \quad \sigma^2_{v,11}, \quad \sigma^2_{v,12}, \\ \sigma^2_{v,13}, \quad \sigma^2_{v,14}, \quad \sigma^2_{v,15}, \quad \sigma^2_{v,16} \\ \sigma^2_{v,17}, \quad \sigma^2_{v,18}, \quad \sigma^2_{v,19}, \quad \sigma^2_{v,20} \\ \cdots\cdots \\ \cdots\cdots, \sigma^2_{v,19}, \quad \sigma^2_{v,20} \end{bmatrix}' \right) \tag{20}$$

Note that the argument in the square bracket is a vector.

Last, we define the measurement equation mapping matrix $\mathbf{H}$ as

$$\mathbf{H} = \begin{bmatrix} \beta_{1,L} & -\beta_{1,S} f_S(0,1) & 1 & f_S(0,1) & \dots & 0 & 0 \\ \beta_{1,L} & -\beta_{1,S} f_S(1,2) & 1 & f_S(1,2) & \dots & 0 & 0 \\ \beta_{1,L} & -\beta_{1,S} f_S(2,3) & 1 & f_S(2,3) & \dots & 0 & 0 \\ \beta_{1,L} & -\beta_{1,S} f_S(3,4) & 1 & f_S(3,4) & \dots & 0 & 0 \\ \beta_{1,L} & -\beta_{1,S} f_S(3,7) & 1 & f_S(3,7) & \dots & 0 & 0 \\ \beta_{1,L} & -\beta_{1,S} f_S(2,6) & 1 & f_S(2,6) & \dots & 0 & 0 \\ \beta_{1,L} & -\beta_{1,S} f_S(1,5) & 1 & f_S(1,5) & \dots & 0 & 0 \\ \beta_{1,L} & -\beta_{1,S} f_S(0,4) & 1 & f_S(0,4) & \dots & 0 & 0 \\ \beta_{1,L} & -\beta_{1,S} f_S(7,11) & 1 & f_S(7,11) & \dots & 0 & 0 \\ \beta_{1,L} & -\beta_{1,S} f_S(6,10) & 1 & f_S(6,10) & \dots & 0 & 0 \\ \beta_{1,L} & -\beta_{1,S} f_S(5,9) & 1 & f_S(5,9) & \dots & 0 & 0 \\ \beta_{1,L} & -\beta_{1,S} f_S(4,8) & 1 & f_S(4,8) & \dots & 0 & 0 \\ \beta_{1,L} & -\beta_{1,S} f_S(0,19) & 1 & f_S(0,19) & \dots & 0 & 0 \\ \beta_{1,L} & -\beta_{1,S} f_S(0,18) & 1 & f_S(0,18) & \dots & 0 & 0 \\ \beta_{1,L} & -\beta_{1,S} f_S(0,17) & 1 & f_S(0,17) & \dots & 0 & 0 \\ \beta_{1,L} & -\beta_{1,S} f_S(0,16) & 1 & f_S(0,16) & \dots & 0 & 0 \\ \beta_{1,L} & -\beta_{1,S} f_S(0,39) & 1 & f_S(0,39) & \dots & 0 & 0 \\ \beta_{1,L} & -\beta_{1,S} f_S(0,38) & 1 & f_S(0,38) & \dots & 0 & 0 \\ \beta_{1,L} & -\beta_{1,S} f_S(0,37) & 1 & f_S(0,37) & \dots & 0 & 0 \\ \beta_{1,L} & -\beta_{1,S} f_S(0,36) & 1 & f_S(0,36) & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \beta_{n,L} & -\beta_{n,S} f_S(0,37) & 0 & 0 & \dots & 1 & f_S(0,37) \\ \beta_{n,L} & -\beta_{n,S} f_S(0,36) & 0 & 0 & \dots & 1 & f_S(0,36) \end{bmatrix}. \tag{21}$$

C. Remark

In the measurement equation, each series in y_t is assumed to be observed with measurement error. The fixed event forecasts are treated separately in each quarter throughout the calendar year. This leaves us with a total of 20 observables for each forecaster in each quarter, 12 of which are missing by construction.

3.3 Estimation

Since our model is a Gaussian state-space model, we employ the Kalman filter to make inference on the latent variables and form the likelihood function. The model is estimated with maximum likelihood. Our baseline model has a total of 431 parameters consisting of the following.

- Forecaster-specific means $\{\alpha_{i,L}, \alpha_{i,S}\}_{i=1}^{n}$
- Forecaster-specific factor loadings $\{\beta_{i,L}, \beta_{i,S}\}_{i=1}^{n}$
- Factor autocorrelation parameters a_{11}, a_{22}, b_{11}, and b_{22}
- Idiosyncratic factor conditional variances σ_L^2 and σ_S^2
- Shape parameter λ
- Measurement error variances $\sigma_{v,1}, \ldots, \sigma_{v,20}^2$.

The parameter vector is denoted as

$$\theta = [\alpha_{1,L}, \ldots, \alpha_{n,S}, \beta_{1,L}, \ldots, \beta_{n,S}, a_{11}, a_{22}, b_{11}, b_{22}, \sigma_L^2, \sigma_S^2, \lambda, \sigma_{v,1}^2, \ldots, \sigma_{v,20}^2]'.$$

4 Estimation Results

This section reports and discusses our results from the estimation of the term-structure model. We estimate our model for every forecaster and every time period available in the SPF for whom we have sufficient data.[6]

4.1 Consensus

Figure 4 plots the mean of each forecaster's smoothed factor and slope estimates. The top panel plots the smoothed common level factor. Note that the unit does not have a particular economic meaning because of the normalization. Overall, the estimate tracks variations in the long-run inflation expectations. The estimate exhibits a sharp downward trend in the 1990s. In the later part of the sample, the estimate drifts down even further after the Great Recession. During the

[6] So long as we observe non-missing values for $\pi_{t-1\to t|t}$, $\pi_{t+3\to t+4|t}$, and so long as two elements of both $\hat{\varepsilon}_{s,t}$ and $\hat{\varepsilon}_{l,t}$ are well-defined for forecaster i at time t, we estimate an individual-level term structure of inflation expectations for forecaster i at time t.

Table 1: MAXIMUM LIKELIHOOD PARAMETER ESTIMATES

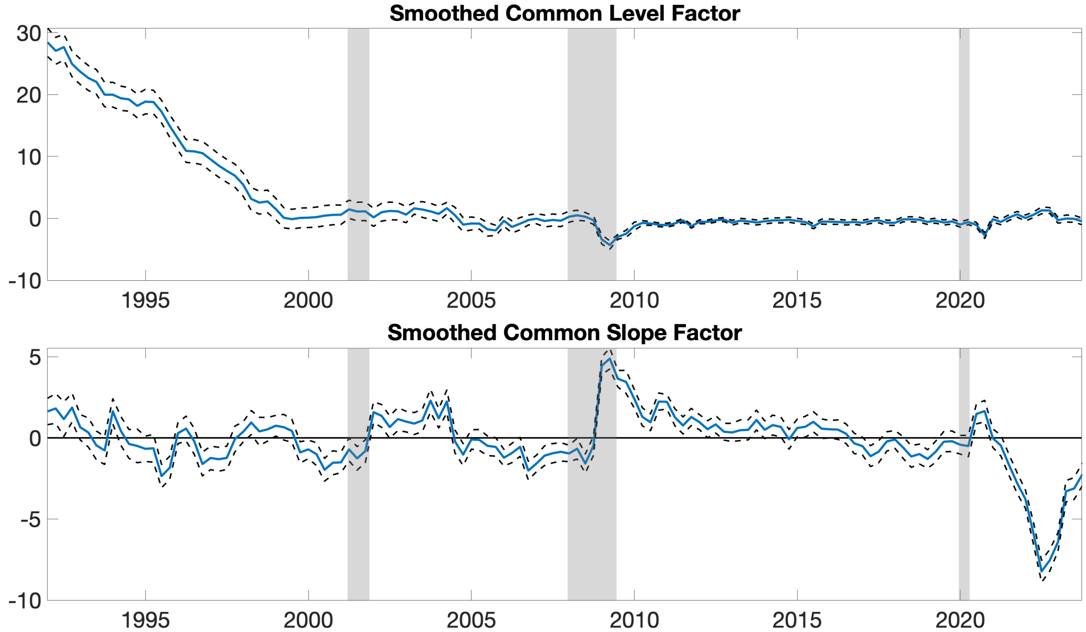
For both $\alpha_{i,L}$ and $\alpha_{i,S}$, the "Estimate" column refers to the average estimate across forecasters and the "95% CI" column refers to the 2.5th and 97.5th percentiles of the estimates.

Parameter	Estimate	95% CI
$\alpha_{i,L}$	2.429	[1.867,3.567]
$\alpha_{i,S}$	0.382	[-0.434,1.559]
a_{11}	0.999	
a_{22}	0.852	
b_{11}	0.868	
b_{22}	0.605	
σ_L	0.137	
σ_S	0.512	
λ	0.285	
$\sigma_{v,1}$	0.639	
$\sigma_{v,2}$	0.457	
$\sigma_{v,3}$	0.474	
$\sigma_{v,4}$	0.455	
$\sigma_{v,5}$	0.294	
$\sigma_{v,6}$	0.297	
$\sigma_{v,7}$	0.248	
$\sigma_{v,8}$	0.011	
$\sigma_{v,9}$	0.300	
$\sigma_{v,10}$	0.328	
$\sigma_{v,11}$	0.308	
$\sigma_{v,12}$	0.314	
$\sigma_{v,13}$	0.183	
$\sigma_{v,14}$	0.193	
$\sigma_{v,15}$	0.229	
$\sigma_{v,16}$	0.255	
$\sigma_{v,17}$	0.210	
$\sigma_{v,18}$	0.210	
$\sigma_{v,19}$	0.194	
$\sigma_{v,20}$	0.231	

COVID pandemic, the estimate dips in the early phase of the pandemic, quickly recovers from it, and does not exhibit a run-up or a notable swing afterward.

The bottom panel plots the smoothed common slope factor. Again, the unit does not have economic interpretations because of the normalization. The estimate tracks expected changes in inflation between the current quarter and the long run at each point in time. The positive slope estimates indicate that the term structure of inflation expectations is typically upward

Figure 4: Mean Smoothed Factors



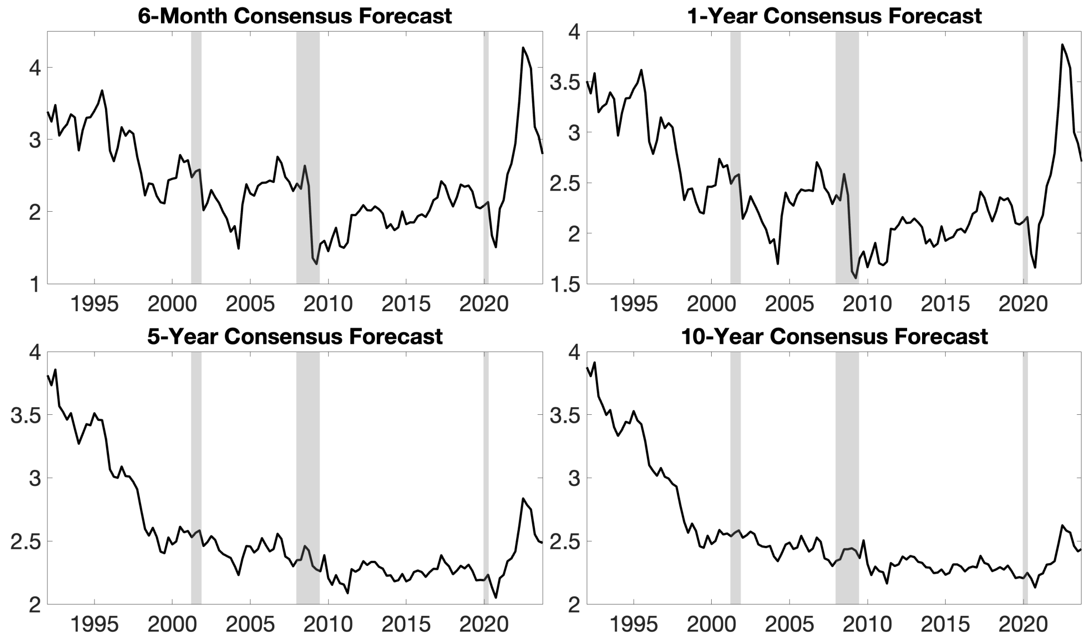
Notes: The upper panel shows the smoothed common level factor (blue line) along with the 95 percent confidence intervals (black dashed line). The bottom panel shows the smoothed common slope factor (blue line) along with the 95 percent confidence intervals (black dashed line). The shaded areas denote the NBER recessions.

Sources: Authors' calculation

sloping. The steepest positive slopes occur in the years following the dotcom bubble and in the Great Recession. The slope has systematically declined over the 2010s, reflecting the decade of low and stable inflation after the Great Recession. The slope increases with the onset of the Covid-19 pandemic. This reflects that forecasters expect inflation to increase relative to that in the current quarter, given that inflation plunged in the early phase of the pandemic and the long-run expectation did not change much. From 2021 when inflation picked up rapidly, the slope estimate turned negative and plunged further, reflecting forecasters' expectation that inflation would eventually decline.

Figure 5 plots the mean of each forecaster's implied forecast on four different forecast horizons. The top panels plot the mean 6-month and 1-year ahead inflation expectations, which track the consensus inflation nowcast and thus realized inflation to a large extent. The bottom panels display the man 5-year and 10-year ahead inflation projections, which are quite different from the short-term projections. The 10-year-ahead inflation projection reflects less of the variations in realized inflation than the 5-year-ahead inflation expectation.

Figure 5: MEAN FORECAST ESTIMATES



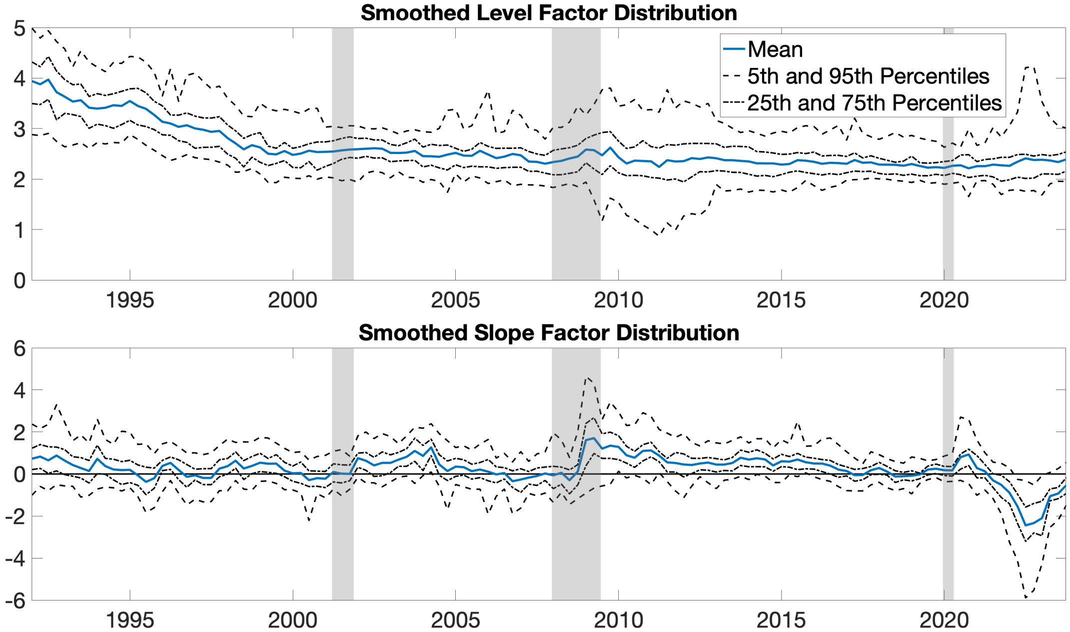
Notes: The black lines display the average inflation forecasts recovered from the individual-level term structure model. The shaded areas denote the NBER recessions.

Sources: Authors' calculation

4.2 Dispersion: A proxy for disagreement

Figure 6 plots the distributions of each forecaster's smoothed factor and slope estimates. Note, for instance, that the individual level factor includes both the common and idiosyncratic components with the loadings on common factor considered for the level, and hence the unit is comparable to that of consensus inflation expectations. The same applies to the slope factor. The top panel displays the distribution of individual smoothed level estimates. It is noteworthy that even when the long-run inflation expectation stays at a low level after 2005, the dispersion of estimates changes substantially over time. In particular, the dispersion seen after the Great Recession is larger than that seen in the early 1990s, when the mean long-run inflation expectations are around 4 percent. During the Covid pandemic, the mean level edged up slightly, but the distribution dramatically skewed to the right, reflecting the increased upside risk to long-run inflation expectations. The bottom panel displays the distribution of individual smoothed slope estimates. Similar to the level estimate, the dispersion of slope increases substantially over the course of the Great Recession and the Covid-19 pandemic. In particular, the distribution becomes skewed to the right at the onset of the pandemic, but it becomes skewed to the left dramatically following a rapid rise in inflation in 2021.

Figure 6: SMOOTHED FACTOR DISTRIBUTIONS



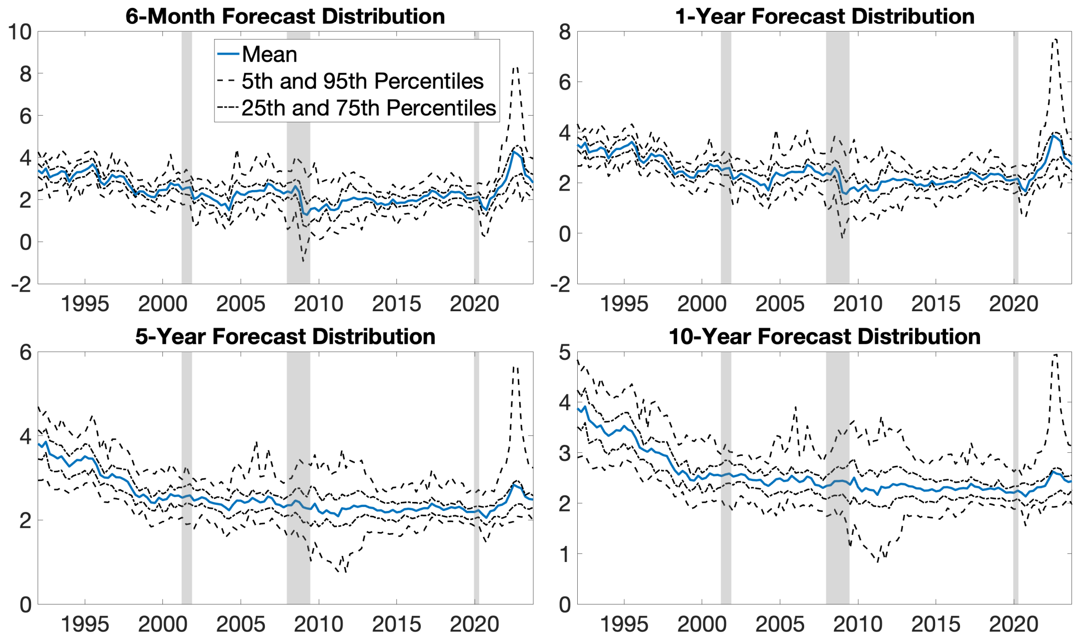
Notes: The figure shows the distribution of the individual level factor (upper panel) and that of individual slope factor (bottom panel). The dotted lines depict the confidence intervals at 25th and 75th percentiles. The dashed lines depict the confidence intervals at the 5th and 95th percentiles.

Sources: Authors' calculation

Figure 9 displays the distribution of inflation forecasts at four different horizons. Across all four forecasting horizons, the dispersions vary independently of the corresponding mean forecasts. In particular, when the mean inflation projections are low and stable, say 2005 onward, the dispersions become larger than those observed in the 1990s, when the level of inflation is generally high. This observation suggests that the disagreement measured with the dispersion carries information about the degree of anchored inflation expectation quite different from what the consensus suggests.

Long-run inflation factors were more dispersed than short-run expectations early in our sample, but since around the Great Recession, that pattern has flipped and short-run factors have tended to be more dispersed. The dispersion of all factors spiked during the Great Recession but interestingly long-run factors have remained relatively stable during the COVID-19 pandemic, suggesting strong credibility of central bank policy and communication. In the most recent couple of quarters there has been an increase in long-run dispersion, which may be an early warning sign of this pattern reversing.

Figure 7: DISAGREEMENT ABOUT FORECASTS



Notes: The figure shows the distribution of recovered individual inflation forecasts at four different forecast horizons. The dotted lines depict the confidence intervals at the 25th and 75th percentiles. The dashed lines depict the confidence intervals at the 5th and 95th percentiles.

Sources: Authors' calculation

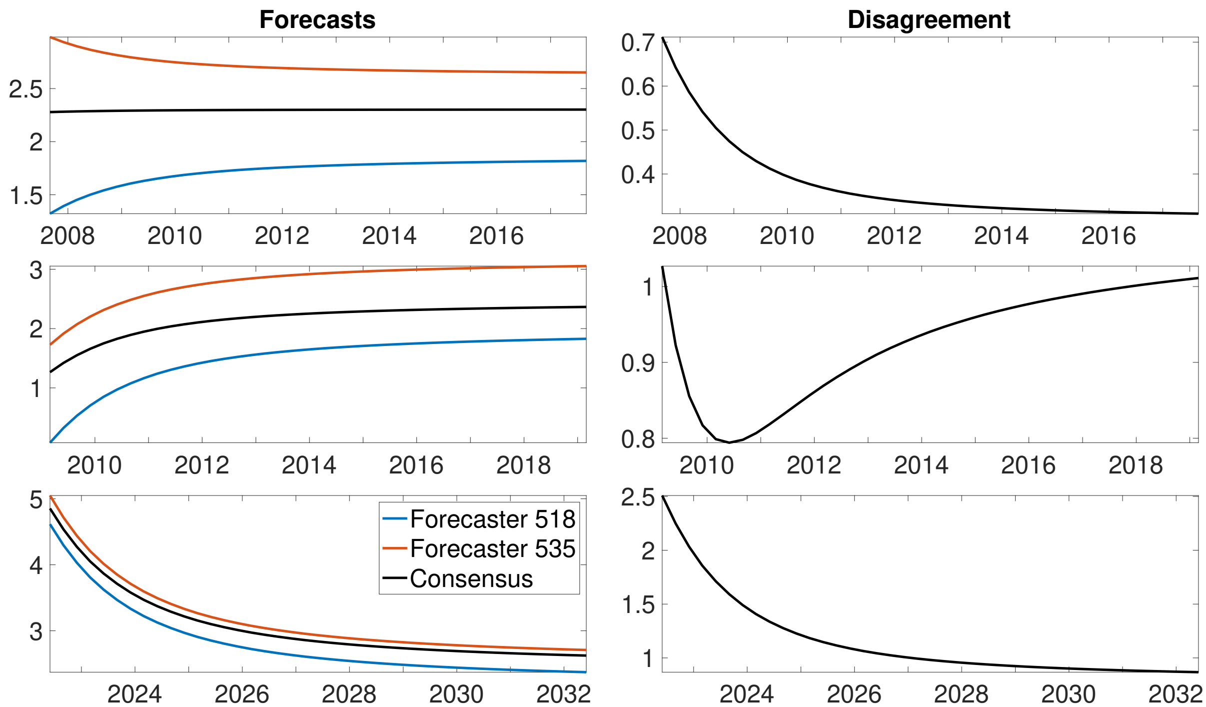
4.3 Individual dynamics

We look closely at the individual-level term structure of inflation expectations and its implications for the aggregate term structure of disagreement at each point in time. We select three dates for illustration.

Figure 8 shows the term structure of the inflation forecasts of two forecasters along with the consensus (left panels) and the implied aggregate disagreement over the forecasting horizons (right panels). First, we consider 2007Q3 (upper panels). This was the final survey conducted before the start of the Great Recession. The consensus term structure is relatively flat, with the nowcast and 10-year ahead inflation expectations being around 2.25%. Meanwhile, the two forecasters disagree about the nowcasts and the direction of slopes, as shown in the downward sloping term structure of Forecaster 535 and the upward sloping term structure of Forecaster 518. Overall, the disagreement is largest in the short run but reduces over the forecasting horizon, yielding the downward-sloping term structure of disagreement (upper right panel).

A quite different pattern is observed in 2009Q2 — the peak of Great Recession. The term structures are upward-sloping for both forecasters and consensus. Although the two forecasters

Figure 8: TERM STRUCTURE OF INFLATION EXPECTATIONS AT THREE DATES



Notes: The left panel shows the estimated term structure of the inflation expectations of the consensus (black line) and two forecasters (colored lines). The right panel shows the overall term structure of disagreement over the forecast horizon.

Sources: Authors' calculation

significantly disagree about near-term inflation, they are still expecting a rise in inflation over the forecast horizon, in line with the consensus. The disagreement is largest in the short run, reduces in the near term, and then widens in the long run (middle right panel). This non-linearity signals substantially high uncertainty in the immediate future and in the long run.

Finally, we consider 2022Q1 (bottom panels). Inflation expectations are monotonically decreasing for both forecasters and in the consensus. The disagreement is largest in the current quarter but reduces over the forecasting horizon, producing the downward-sloping term structure of disagreement (lower right panel). Although the term structure of disagreement is similar to that prior to the Great Recession, the magnitude of disagreement is three times as large as that seen in 2007.

5 Three sources of disagreement

Our model decomposes the cross-sectional dispersion at each point in time into three components of distinct sources: (1) individual prior belief (individual fixed effect); (2) heterogeneous responses to common (or public) information; (3) idiosyncratic (or private) information. The goal of this decomposition is to identify the extent to which macroeconomic policies affect disagreement. For example, monetary policy, which is public information, may affect forecasters' disagreement through the common-information channel.

5.1 Model for the disagreement decomposition

The decomposition is made up of two steps. We first decompose the forecaster i' inflation forecast of each forecasting horizon into the portions of three sources. With the estimated portion, we decompose the cross-sectional variance into the contributions from the three sources at each point in time.

Equations (3) and (4) essentially decompose the individual level and slope factors into the three information sources. Here, we give economic interpretations to each component as follows.

$$L_{i,t} = \underbrace{\alpha_{i,L}}_{\text{prior belief}} + \underbrace{\beta_{i,L} L_t}_{\text{common}} + \underbrace{\varepsilon_{i,L,t}}_{\text{idiosyncratic}} . \tag{22}$$

The level factor of forecaster i, $L_{i,t}$, is decomposed into the portions representing *prior belief*

($\alpha_{i,L}$), *common or public information* ($\beta_{i,L}L_t$), and *idiosyncratic or private information* ($\varepsilon_{i,L,t}$).

Define the prior belief component of $L_{i,t}$ to be $L_i^{pb} = \alpha_{i,L}$, the common component of $L_{i,t}$ to be $L_{i,t}^c = \beta_{i,L}L_t$, and the idiosyncratic component of $L_{i,t}$ to be $L_{i,t}^{id} = \varepsilon_{i,L,t}$. We then rewrite $L_{i,t}$ into

$$L_{i,t} = L_i^{pb} + L_{i,t}^c + L_{i,t}^{id}. \tag{23}$$

Likewise, we give similar economic interpretations to each component of $S_{i,t}$ in Equation (4), and $S_{i,t}$ is rewritten into the following:

$$S_{i,t} = S_i^{pb} + S_{i,t}^c + S_{i,t}^{id}. \tag{24}$$

With the decompositions in Equations (23) and (24), forecaster i's h-quarter-ahead inflation forecast at time t, $\pi_{i,t\to t+h|t}$, is expressed into the portions representing prior beliefs ($\pi_{i,t\to t+h|t}^{pb}$), common information ($\pi_{i,t\to t+h|t}^{c}$), and idiosyncratic information ($\pi_{i,t\to t+h|t}^{id}$) as follows.

$$\begin{aligned}
\pi_{i,t\to t+h|t} &= L_{i,t} - \left(\frac{1-e^{-\lambda h}}{\lambda h}\right) S_{i,t} \\
&= \alpha_{i,L} + \beta_{i,L}L_t + \varepsilon_{i,L,t} - \left(\frac{1-e^{-\lambda h}}{\lambda h}\right)\left(\alpha_{i,S} + \beta_{i,S}S_t + \varepsilon_{i,S,t}\right) \\
&= \alpha_{i,L} - \left(\frac{1-e^{-\lambda h}}{\lambda h}\right)\alpha_{i,S} + \beta_{i,L}L_t - \left(\frac{1-e^{-\lambda h}}{\lambda h}\right)\beta_{i,S}S_t + \varepsilon_{i,L,t} - \left(\frac{1-e^{-\lambda h}}{\lambda h}\right)\varepsilon_{i,S,t} \\
&= \underbrace{L_i^{pb} - \left(\frac{1-e^{-\lambda h}}{\lambda h}\right) S_i^{pb}}_{\text{prior belief component}} + \underbrace{L_{i,t}^{c} - \left(\frac{1-e^{-\lambda h}}{\lambda h}\right) S_{i,t}^{c}}_{\text{common component}} + \underbrace{L_{i,t}^{id} - \left(\frac{1-e^{-\lambda h}}{\lambda h}\right) S_{i,t}^{id}}_{\text{idiosyncratic component}} \\
&= \pi_{i,t\to t+h|t}^{pb} + \pi_{i,t\to t+h|t}^{c} + \pi_{i,t\to t+h|t}^{id}.
\end{aligned}$$

The factor structure leads us to decompose the cross-sectional dispersion of $\pi_{i,t\to t+h|t}$ into the dispersion components driven by the three information sources.[7]

$$var_i(\pi_{i,t\to t+h|t}) \approx var_i(\pi_{i,t\to t+h|t}^{pb}) + var_i(\pi_{i,t\to t+h|t}^{c}) + var_i(\pi_{i,t\to t+h|t}^{id}) \tag{25}$$

If the pool of forecasters does not change over time, the variance of individual prior beliefs

[7] This variance decomposition is possible, because innovations to the common component and innovations to the idiosyncratic component are independent with each other. The realized shocks to common and idiosyncratic component may show some comovements at times, but this ex-post covariation still does not invalidate the assumption that the two are independent processes.

(the first term in (25)) should not create changes in the dispersion. Since each forecaster has different average forecasts, the churns of the forecasters or the compositional changes of the forecasters likely create variations in the dispersion.[8]

5.2 Decomposition result

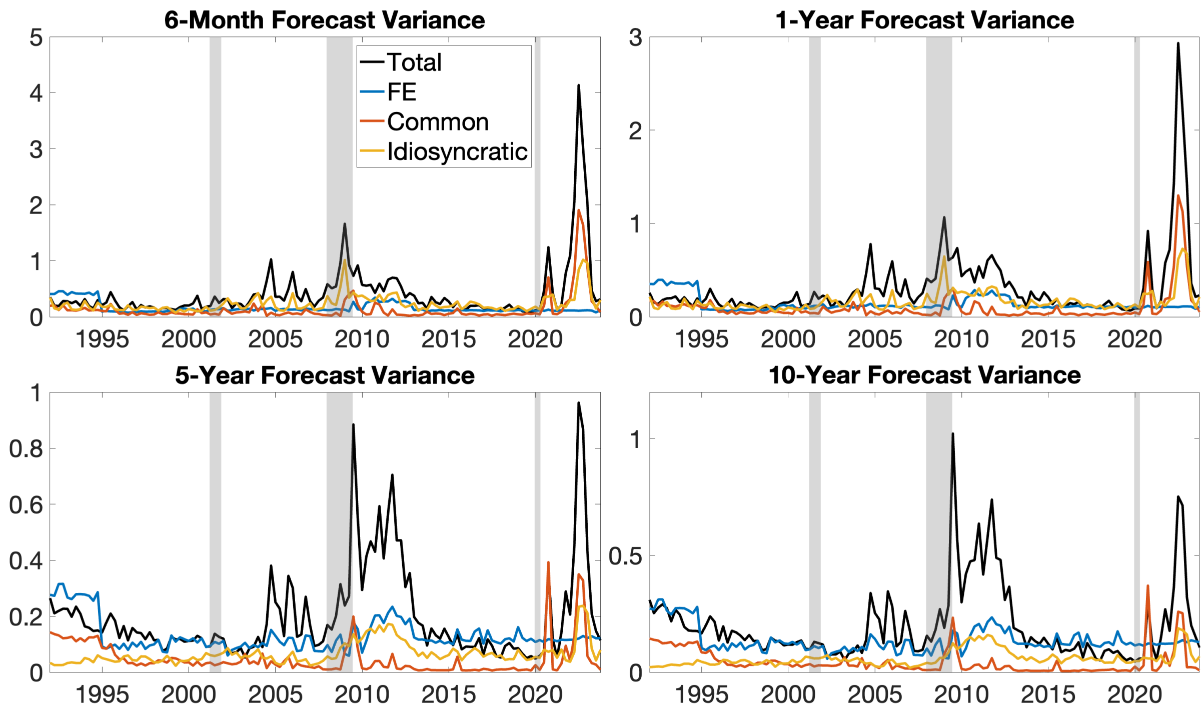
Figure 9 reports the decomposition result of Equation (25) by the forecast horizon. The upper panels display the decomposition for short-term inflation expectations (6-month and 1-year ahead). The bottom panels report the decomposition for longer-term inflation forecasts (5- and 10-years ahead). The disagreement about short-term inflation forecasts is mainly explained by idiosyncratic information (yellow line). Meanwhile, individual prior beliefs (blue line) play a larger role in the disagreement about long-run inflation expectations than idiosyncratic information. The contribution from common information (red line) is the smallest on average, regardless of the forecast horizon. For long-run expectations disagreement, common information is an important driver of increased disagreement in times of high uncertainty, such as the Great Recession and the Covid-19 pandemic. The pandemic is distinguished from the previous recessions in that common information is the main driver behind the dramatic increase in disagreement about both short- and long-run forecasts.

Figure 10 reports the share of common information in the variance of forecast at each point in time. The contribution of common information to the disagreement changes dramatically over time. The share exhibits countercyclicality, increasing in times of large economic shocks, regardless of the forecasting horizon. This observation suggests that forecasters pay more attention to public information during an economic downturn or the period of inflation uncertainty, but translate the public information into their forecasts in different manners.

Furthermore, common information has different effects on the short-run and long-run forecasts and over the recession episodes. Figure 11 shows the share of common information in the disagreement over the forecast horizon. Before the Great Recession, the role of common information in the disagreement is low over the forecast horizon. The share is larger in the short run than in the long run. Quite differently, at the peak of the Great Recession, common information plays a larger role than before the economic downturn. In particular, the share is larger in the long-run forecasts than in the short-run forecasts (middle panel). The Covid pandemic is unique in that common information was the main driver of increased disagreement

[8] In this context, the dispersion due to permanent heterogeneity is interpreted as the natural level of disagreement.

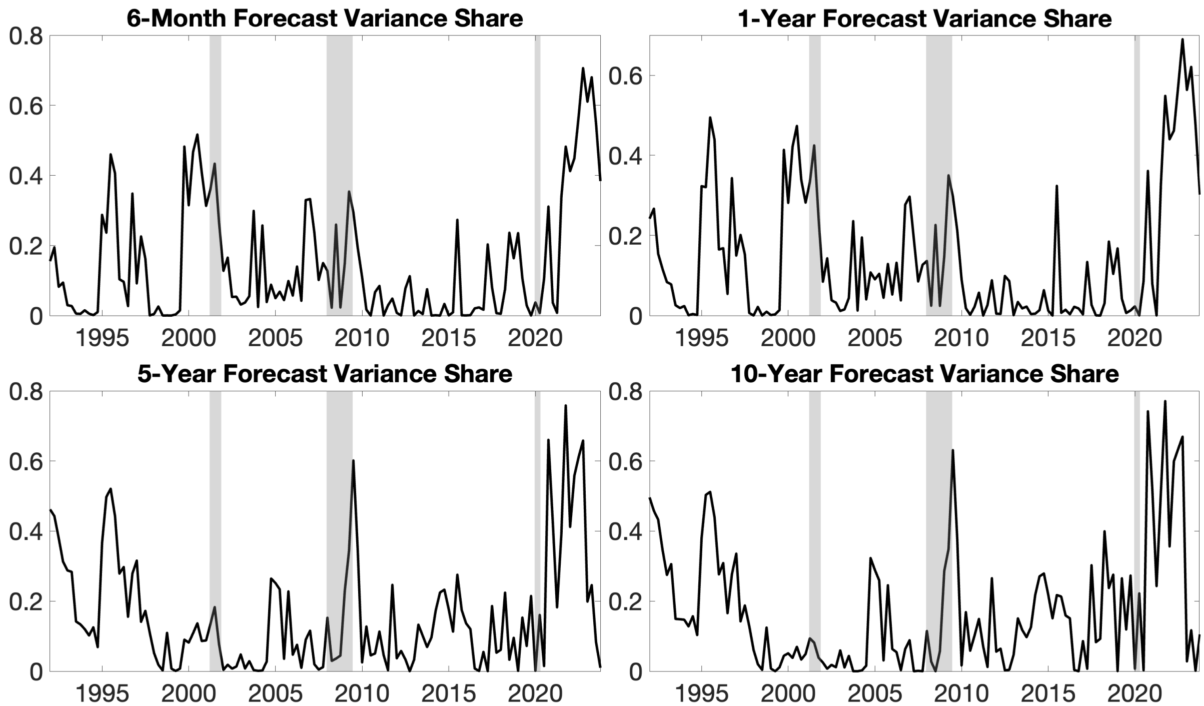
Figure 9: FORECAST VARIANCE DECOMPOSITION



Notes: The figure shows the decomposition of the overall disagreement about inflation expectations (black line) into portions driven by individual prior beliefs (denoted by FE, blue line), heterogeneous responses to common information (denoted by Common, red line), and idiosyncratic information (denoted by Idiosyncratic, yellow line). The gray areas denote the NBER recessions.

Sources: Authors' calculation

Figure 10: Forecast Variance Share of Common Component



Notes: The figure shows the fraction of overall disagreement about inflation expectations driven by heterogeneous responses to common information. The gray areas denote the NBER recessions.
Sources: Authors' calculation

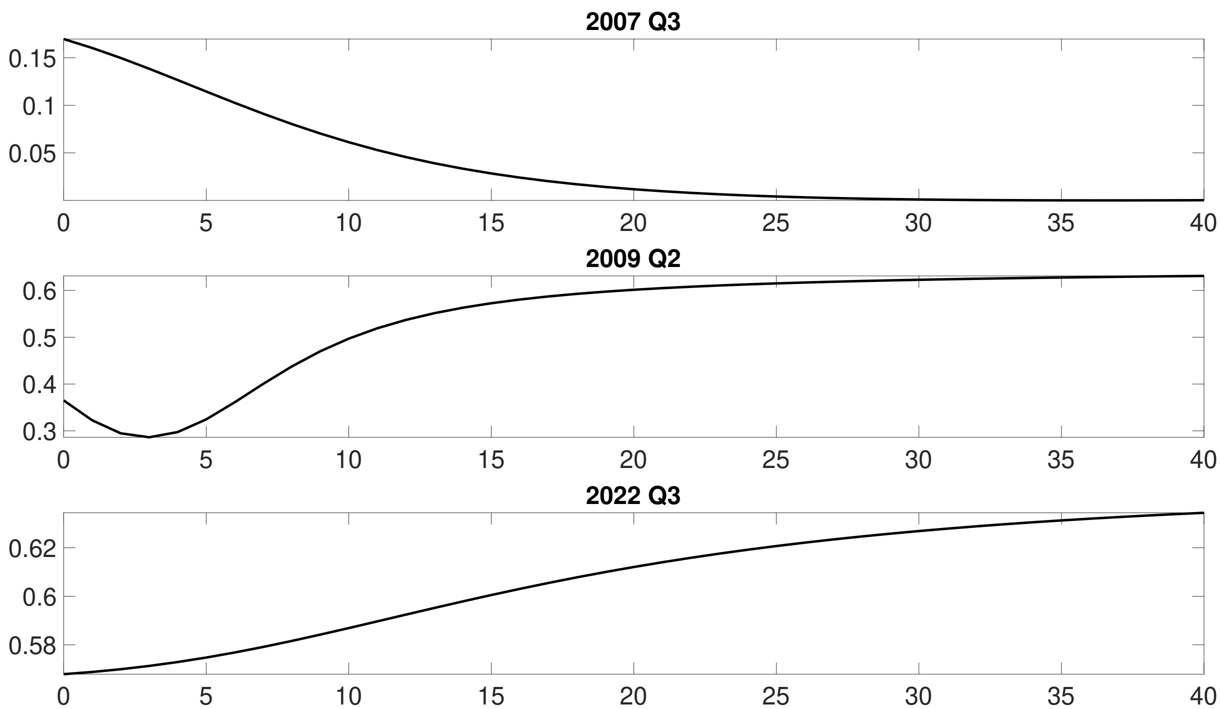
across all forecasting horizons (bottom panel). The share is greater than 0.5 across all forecast horizons. In the presence of shocks that are unprecedented in nature and magnitude and the consequent policy responses, forecasters paid attention to public information during the pandemic. However, due to the high uncertainty, forecasters had different interpretations about the public information and produced quite different inflation projects across the forecasting horizons.

The importance of common information in long-term disagreement in times of high economic uncertainty suggests that monetary policy may be able to anchor long-horizon expectations effectively with clearer communication.

6 Implications for monetary policy

We examine the extent to which the informational content of monetary policy can effectively reduce disagreement about inflation expectations.

Figure 11: TERM STRUCTURE OF VARIANCE SHARES AT THREE DATES



Notes: The figure shows the fraction of overall disagreement about inflation expectations driven by heterogeneous responses to common information over the forecast horizon at three different points in time. The top panel reports the estimates as of 2007:Q3, the middle panel as of 2009:Q2, and the bottom panel as of 2022:Q3.

Sources: Authors' calculation

6.1 Model: SVAR-IV

For this analysis, we employ a structural vector autoregression model with external instrument variables (SVAR-IV, henceforth). The baseline SVAR-IV is the specification from Stock and Watson (2018). As an instrument for the informational content of monetary policy, we consider the following two shocks : Fed reactions to economic news Bauer and Swanson (2023) and forward guidance shock from Swanson (2021). Bauer and Swanson (2023, 2022) adjust the high-frequency monetary policy surprises by removing the portions reflecting recent macroeconomic and financial news. Bauer and Swanson call this effect *"Fed's response to news."* Since our goal is to estimate the effect of monetary policy communication on the disagreement among forecasters, we use this purged portion as a monetary-policy information shock and hence the instrument for the SVAR model. We also consider the forward guidance shock from Swanson (2021) as another type of surprise in the monetary policy communication for robustness checks.

Our VAR model includes the following quarterly macroeconomic variables: the log of industrial production, the log of the consumer price index, the excess bond premium from Gilchrist and Zakrajšek (2012), the two-year treasury yield, and two disagreement estimates.[9] The two disagreement estimates include disagreement about 6-month ahead inflation expectations driven by common and idiosyncratic information, respectively. Replacing the short-term disagreement estimates, we consider disagreement about 10-year-ahead inflation expectations driven by common and idiosyncratic information. We consider short-run and long-run disagreement separately, because VAR estimates, those from a VAR with a large vector in particular, can be sensitive to model specification.[10] Nevertheless, we also extend the baseline VAR to include the four disagreement estimates altogether in one vector.

We stack the six variables in a vector Y_t and estimate a reduced-form VAR as follows:

$$Y_t = \alpha + B(L)Y_{t-1} + u_t \tag{26}$$

where α is a constant, $B(L)$ is a matrix of polynomial in the lag operator, and u_t is a 6×1 vector of serially uncorrelated residuals, with $Var(u_t) = I$. We estimate Equation (26) from 1990:Q1-2019:Q4 using ordinary least squares with 4 lags.

As in the literature on SVAR-IV with the external instrument, we assume that the reduced

[9]Our model is similar to Bauer and Swanson (2022).

[10]Graves et al. (2023) employed a similar approach.

form shock u_t is expressed into a linear combination of structural shocks as follows:

$$u_t = S\epsilon_t \tag{27}$$

where ϵ_t is a vector of serially uncorrelated structural shocks. Given our research question, we assume that one of the structural shocks is a monetary policy shock. Let ϵ_t^{mp} denote the monetary policy shock and order it first in the vector ϵ_t. The first column of S, denoted by s_1, captures the impact of ϵ_t^{mp} on Y_t.

We use the high-frequency identification to estimate the impact of ϵ_t^{mp}. Let z_t denote the high-frequency shock of our interest—Fed's reaction to news or forward guidance—converted to a quarterly series by summing over all the high-frequency surprises within each quarter. As well known in the literature, for z_t to be a valid instrument of ϵ_t^{mp}, the relevance condition should be satisfied:

$$E[z_t\epsilon_t^{mp}] \neq 0, \tag{28}$$

and the following exogeneity condition should be satisfied as well:

$$E[z_t\epsilon_t^{-mp}] = 0, \tag{29}$$

where ϵ_t^{-mp} denotes elements in ϵ_t other than the first Stock and Watson (2018).

With z_t, we estimate the impact effect in the SVAR-IV as in Stock and Watson (2018). Specifically, we order the two-year Treasury yield first in Y_t and denote it by Y_t^{2y}. We then arrive at our empirical model:

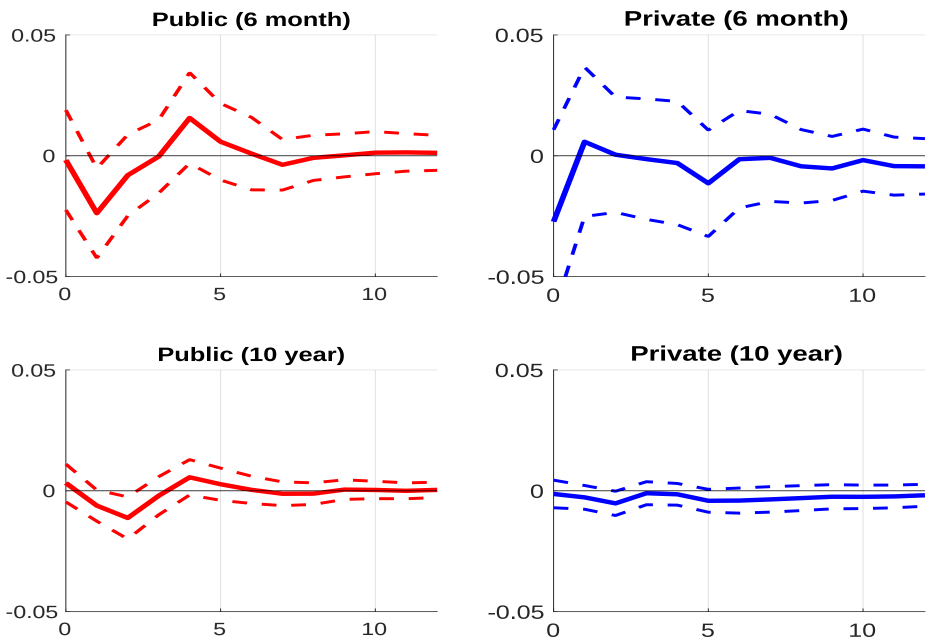
$$Y_t = \tilde{\alpha} + \tilde{B}(L)Y_{t-1} + s_1 Y_t^{2y} + \tilde{u}_t.$$

This regression model is estimated with two-stage least squares with z_t as the instrument for Y_t^{2y}. Note that the first element of s_1 is unity for normalization. In the impulse responses, we report the impact effect of 25 basis points rather than 1 percentage point. Given the estimated s_1 and $B(L)$, we can calculate the impulse response function for each variable.

6.2 Estimation result

Figure 12 reports the response of disagreement about short-term (6-month ahead) and long-term (10-year ahead) inflation expectations to Fed's contractionary reactions to economic news.

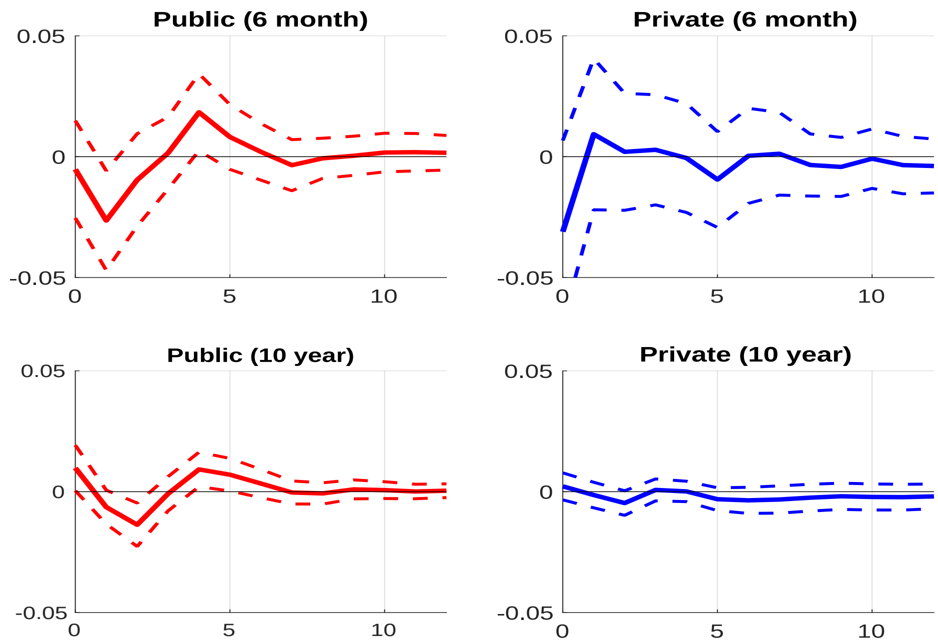
Figure 12: RESPONSES OF DISAGREEMENT TO FED'S REACTION TO NEWS



Notes: The figure shows the responses of disagreement about inflation expectations driven by public and private information to a monetary policy information shock from Bauer and Swanson (2023). The upper panels show the responses of disagreement about 6-month ahead inflation expectations. The bottom panels show the responses of disagreement about 10-year ahead inflation expectations. The dashed line captures the 95 percent confidence intervals.

Sources: Authors' calculation

Figure 13: RESPONSES OF DISAGREEMENT TO FORWARD GUIDANCE



Notes: The figure shows the responses of disagreement about inflation expectations driven by public and private information to a forward guidance shock from Bauer and Swanson (2023). The upper panels show the responses of disagreement about 6-month ahead inflation expectations. The bottom panels show the responses of disagreement about 10-year ahead inflation expectations. The dashed line captures the 95 percent confidence intervals.
Sources: Authors' calculation

Note that the Fed's contractionary reaction implies downside inflation stabilization. The left column reports the responses of disagreement driven by common information, and the right column reports those driven by idiosyncratic information. Fed's positive take of economic news reduces disagreement about both short-run and long-run inflation forecasts one quarter after the announcement, and the effects are statistically significant. Fed's reaction equivalent of the 25 basis point rise in the policy rate reduces the commonality-driven short-term disagreement by 0.025 percentage point and the commonality-driven long-run disagreement by 0.01 percentage point. These estimates are about one-tenth of the short-run and long-run variance on average. Meanwhile, the effects are not statistically significant in disagreement driven by idiosyncratic information.

We further analyze the effects of the forward-guidance shocks. Figure 13 reports the responses of the disagreement with the forward guidance. Similarly to the result of Fed's reactions to news, forward guidance is effective in reducing disagreement driven by common information both in the short-run and long-run, but the effect is not statistically significant on disagreement driven by private information. All told, our result suggests that disagreement driven by public information is the margin on which monetary policy can affect to anchor inflation expectations.

7 Implications for theory

This section discusses the implications of our empirical findings on theoretical models. We pay particular attention to the disagreement about long-run inflation forecasts. The gist of our finding is summarized as follows: First, individual prior beliefs and the idiosyncratic information are the main factors creating the long-run disagreement; Second, common and idiosyncratic information drives changes in the disagreement; Third, heterogeneous responses to common information is the key driver of increased disagreement about long-term inflation forecasts in times of economic downturns or high inflation uncertainty. We analyze what features of existing theoretical models can capture these empirical findings and then identify the missing piece in the literature to draw implications for the next step in research.

We review the following models: the sticky information model; the noisy information model; disagreement about means and long-run priors. Table 2 summarizes the main features of each model in the context of our empirical findings. Note that there is no scope for disagreement in the full-information rational expectation (FIRE) model, where economic agents are ex ante identical and efficiently process available information.

To begin with, the importance of individual prior beliefs indicates that *heterogeneity in beliefs about long-run means* Patton and Timmermann (2010) is the key to understanding the disagreement between forecasters and is consistent with the recent finding of Farmer et al. (2021) that the priors of individual forecasters play a key role in long-term macroeconomic forecasts. Forecasters may have different priors on the long-term underlying inflation. Note that our statistical model captures the forecaster-specific long-run prior with the individual fixed effect. This individual fixed effect is different from the idiosyncratic information that a forecaster has, in that the long-run prior does not change over time unlike the idiosyncratic information. Therefore, disagreement induced by individual prior beliefs does not respond particularly well to macroeconomic shocks. This margin accounts for the bulk of disagreement about long-run inflation forecasts, with a smaller contribution to disagreement about short-run inflation expectations.

Disagreement driven by idiosyncratic information can be accounted for by the noisy information model. The noisy information model Woodford (2001) can generate disagreement among forecasters, but is limited to characterize countercyclical disagreement or increased disagreement to a large economic shock. In the model, forecasters are ex-ante identical with the time-invariant information precision that are the same across forecasters, but they face the realization of signals with idiosyncratic errors at any point in time. This feature generates disagreement that does not vary over the business cycle. In this context, the noisy information model provides an economic interpretation of the disagreement involving idiosyncratic information in our model.

Disagreement driven by common information is explained by the noisy information model with heterogeneity and the sticky information model. The noisy information model with heterogeneous information precision across forecasters can generate increased disagreement to economic shocks Coibion and Gorodnichenko (2012), while the model without heterogeneity is limited to do so. Notice that the factor loadings of the level and slope in our statistical model may have economic interpretation of the heterogeneous information precision, in that the factor loadings different across forecasters create disagreement from common information. Meanwhile, the sticky information model can also generate disagreement at all times and capture increased disagreement in times of large shocks if a certain condition is satisfied. In this model, economic agents update their information set periodically due to the cost of information acquisition Mankiw and Reis (2002). Thus, the disagreement arises because only a fraction of forecasters update their forecasts in response to macroeconomic news, while others do not. The model can capture that increased disagreement in response to macroeconomic news. However, in the model, the increased disagreement dissipates over time as more forecasters update their information set. This feature of the model matches well with the little role of common information in normal times, but its increased importance during an economic downturn and the normalization after the recession. Meanwhile, if the frequency of economic shocks is higher than the frequency of information update, the model can also generate disagreement at all times.

However, existing models are limited in characterizing the different importance of permanent heterogeneity and idiosyncratic and common information in short- and long-term inflation forecasts. In particular, common information plays a larger role in the countercyclical disagreement in the long-run forecasts than in the short run. Notice that our statistical model captures the heterogeneity across the forecast horizons with the distinct factor loadings for the level

Table 2: DISAGREEMENT IN THE MODELS OF EXPECTATION FORMATION

	FIRE	Sticky Information	Noisy info. (Same)	Noisy info. (Different)	Disagreement about means
Scope of disagreement	X	✓	✓	✓	✓
Permanent heterogeneity	X	X	X	X	✓
Changing idiosyncratic disagreement	X	X	✓	X	X
Countercyclical common disagreement	X	✓	X	✓	X
Forecast-horizon differences	X	X	X	X	X

and the slope. Considering the factor loadings capture heterogeneous responses to common information, the two sets of loadings suggest the need of two-dimensional heterogeneity in information precision, the forecast-horizon specific precision, and the precision heterogeneous across forecasters. In the context of information friction model, heterogeneity over the forecst horizons may be captured with the frequency of information update different across forecasting horizons. To the best of our knowledge, no such model that can characterize the two-dimensional heterogeneity in disagreement is available. Overall, our empirical findings shed light on the future path for modeling expectation formation and information acquisition.

8 Discussions

This section discusses an alternative model and methodology. Section 8.1 considers a time-varying parameter model. Section 8.2 considers a non-parametric approach as an alternative of our parametric model.

8.1 Time-varying parameter model

A. Fixed factor loading

In our model, the factor loadings of each forecaster are fixed. One might argue that our model is limited to capture the changing responsiveness of forecasters to common information. The time-invariant factor loadings are a restriction, but it is not a too strong restriction considering the limited number of observations for each forecaster. An individual forecaster stays in the sample for 27 quarters on average, which is a bit too short to allow regime changes in the factor loadings for each individual.

Note that our model allows each forecaster to have his or her own unique loadings, although the loadings are constant for a forecaster. Therefore, in our model, two similar forecasters observed at two different points in time have different loadings, reflecting increased or decreased attention to potentially similar policy changes, for instance.

Our goal is to parse out the portion of cross-sectional variance attributed to common information. In other words, as long as the common component—the product of common factor and factor loading—is distinguished from the idiosyncrasy, the distinction of factor loading from common factor is not necessary. For instance, if the common component does not change in spite of an increase in the factor loading, the portion of disagreement driven by common information does not change and hence the increased factor loading does not matter.

B. Stochastic Volatility

We do not allow changing variance for the dynamics of idiosyncratic component and common factors. However, we assume that each forecaster has a unique variance for his or her idiosyncratic component. Since an individual forecaster stays in the sample for a relatively short period of time (average 26 quarters), the fixed volatility for each forecaster is not a too strong assumption. Considering the churns of forecasters, the forecast-specific variance for idiosyncrasy can be thought of allowing stochastic volatility in the aggregate sense. However, one caveat is that for those who stay in the sample for a long enough period of time, stochastic volatility may be needed to adequately estimate the variance for the idiosyncratic component.

The resilience of the model estimates to the Covid-19 shock is a practical concern. If the pandemic observations dramatically alter the parameter estimates, the pre-pandemic inference changes dramatically with the inclusion of a handful of pandemic observations. In this case, including stochastic volatility may robustify the inference, as it discounts the pandemic volatility and largely prevents the model from carrying backward the Covid shock for the pre-pandemic inference. To check how reliable the estimates are to the Covid shock, we compare the model estimates through 2019:Q4 with those through 2023:Q4. In particular, the estimates and the main conclusion are robust to the inclusion of the pandemic observations for the period prior to the COVID era. This observation suggests that our result is fairly robust even without the absence of stochastic volatility.

8.2 Evidence from a non-parameteric model

Our baseline model is a highly parameterized model with a large number of parameters. Since the model is estimated with maximum likelihood, the numerical optimization is costly and timeconsuming. One may argue that our conclusion may be sensitive to the particular parametric assumptions that we impose and that the parameter estimates may be unstable because of the estimation difficulty.

For robustness checks along this margin, we consider an alternative two-step approach. The first step is to construct a nonparametric model that describes the individual term structure of inflation expectations. In this model, we use Legendre polynomials to fit the near-term variations in inflation forecasts and a log function to fit the long-term inflation projection beyond one year ahead. We fit each forecaster's observed inflation projections with the polynomials and the log-function with least squares. From this step, the level and slope factor for each individual is estimated. In the second step, we estimate a dynamic factor model for each factor to parse out the common and idiosyncratic components. We estimate the factor model with the algorithm of Banbura and Modugno (2014), since the data sets for level and slope are in an incomplete panel structure. Finally, we recover the portions of individual term structure attributed to common and idiosyncratic information and the portions reflecting permanent heterogeneity. Further details about this model are documented in the Appendix.

Compared to the baseline model, the alternative model is less costly to estimate. In addition, we can allow for more than one common factor for the level or slope without much estimation cost. However, this convenience comes with cost. The log function is not flexible enough to capture observed forecasts beyond one year out and hence produce unrealistic long-end estimates—a noticeable decline in the long end— during the Covid-19 pandemic. This problem is not observed in our baseline model. However, the overall conclusion is robust regardless.

9 Conclusion

We develop a new parametric model for individual inflation expectations at different forecasting horizons, an individual term structure of inflation expectations. We estimate our model with individual-level data in the Survey of Professional Forecasters and uncover rich heterogeneity in the dynamic pattern of the term structure between forecasters and over time. Our new statistical model allows us to analyze disagreement in a comprehensive way. We decompose the disagreement about inflation expectations at each forecasting horizon into a component driven by individual prior beliefs and private and public information, which is entirely new in the literature.

We find that the disagreement about future inflation is mainly driven by forecasters' private information and long-run priors with the former's contribution larger in the short run than in the long run and vice versa for the latter. However, in times of high inflation uncertainty or economic recessions, heterogeneous reactions to public information play a key role in the increased disagreement about long-term inflation expectations. The Covid-19 pandemic is a unique period of time in which public information led to disagreement on both short-term and long-term inflation projections.

Our empirical findings have important implications for policy and theory. Disagreement about inflation expectations serves as an independent measure of how well anchored inflation expectations are. Since monetary policy is public information, disagreement driven by heterogeneous responses to public information captures the degree to which communication on monetary policy affects the disagreement. Our result suggests that clear communication about monetary policy can be particularly effective in reducing disagreement and anchoring inflation expectations in times of great economic uncertainty. Furthermore, our empirical result points to the area of new research. Different dynamic patterns of disagreement over the term structure suggest the need to consider heterogeneity over the forecasting horizon in the model of information acquisition.

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Appendix A Detailed Notation for Nelson-Siegel Model and Estimation

This section provides full expression for the lengthy notation adoped in the model.

The observation vector in any period, y_t is given by

$$y_t := \begin{bmatrix} \pi_{1,t\to t+1|t} \\ \pi_{1,t+1\to t+2|t} \\ \pi_{1,t+2\to t+3|t} \\ \pi_{1,t+3\to t+4|t} \\ \pi_{1,t+3\to t+7|t} \\ \pi_{1,t+2\to t+6|t} \\ \pi_{1,t+1\to t+5|t} \\ \pi_{1,t\to t+4|t} \\ \pi_{1,t\to t+7|t+11} \\ \pi_{1,t+6\to t+10|t} \\ \pi_{1,t+5\to t+9|t} \\ \pi_{1,t+4\to t+8|t} \\ \pi_{1,t\to t+19|t} \\ \pi_{1,t\to t+18|t} \\ \pi_{1,t\to t+17|t} \\ \pi_{1,t\to t+16|t} \\ \pi_{1,t\to t+39|t} \\ \pi_{1,t\to t+38|t} \\ \pi_{1,t\to t+37|t} \\ \pi_{1,t\to t+36|t} \\ \vdots \\ \pi_{n,t\to t+37|t} \\ \pi_{n,t\to t+36|t} \end{bmatrix} \tag{A1}$$

The first four elements of y_t correspond to fixed horizon forecasts of one to four quarters ahead and are typically observed every period. Only four of the final sixteen elements of y_t observed in any given quarter. These final sixteen elements correspond to fixed event forecasts, where each group of four correspond to the fixed event correctly mapped to the right forecast for the given quarter in which the survey is being conducted.

For the final eight elements, which correspond to forecasts of average inflation over five and ten year periods including the current calendar year, we must adjust them to directly with our model. Specifically

- In Q1, we define

$$\pi_{i,t\to t+19|t} := \frac{4}{19}\left(5\pi_{i,t-1\to t+19|t} - \frac{1}{4}\pi_{i,t-1\to t|t}\right)$$
$$\pi_{i,t\to t+39|t} := \frac{4}{39}\left(10\pi_{i,t-1\to t+19|t} - \frac{1}{4}\pi_{i,t-1\to t|t}\right)$$

- In Q2, we define

$$\pi_{i,t\to t+18|t} := \frac{4}{18}\left(5\pi_{i,t-1\to t+19|t} - \frac{1}{4}\pi_{i,t-1\to t|t} - \frac{1}{4}\pi_{i,t-2\to t-1|t}\right)$$
$$\pi_{i,t\to t+38|t} := \frac{4}{38}\left(10\pi_{i,t-1\to t+19|t} - \frac{1}{4}\pi_{i,t-1\to t|t} - \frac{1}{4}\pi_{i,t-2\to t-1|t}\right)$$

- In Q3, we define

$$\pi_{i,t\to t+17|t} := \frac{4}{17}\left(5\pi_{i,t-1\to t+19|t} - \frac{1}{4}\pi_{i,t-1\to t|t} - \frac{1}{4}\pi_{i,t-2\to t-1|t} - \frac{1}{4}\pi_{i,t-3\to t-2|t}\right)$$
$$\pi_{i,t\to t+37|t} := \frac{4}{37}\left(10\pi_{i,t-1\to t+19|t} - \frac{1}{4}\pi_{i,t-1\to t|t} - \frac{1}{4}\pi_{i,t-2\to t-1|t} - \frac{1}{4}\pi_{i,t-3\to t-2|t}\right)$$

- In Q4, we define

$$\pi_{i,t\to t+16|t} := \frac{4}{16}\left(5\pi_{i,t-1\to t+19|t} - \frac{1}{4}\pi_{i,t-1\to t|t} - \frac{1}{4}\pi_{i,t-2\to t-1|t} - \frac{1}{4}\pi_{i,t-3\to t-2|t} - \frac{1}{4}\pi_{i,t-4\to t-3|t}\right)$$

$$\pi_{i,t\to t+36|t} \coloneqq \frac{4}{36}\left(10\pi_{i,t-1\to t+19|t} - \frac{1}{4}\pi_{i,t-1\to t|t} - \frac{1}{4}\pi_{i,t-2\to t-1|t} - \frac{1}{4}\pi_{i,t-3\to t-2|t} - \frac{1}{4}\pi_{i,t-4\to t-3|t}\right)$$

For the nowcasts $\pi_{i,t-1\to t|t}$ and backcasts $\pi_{i,t-2\to t-1|t}$, we use the reported values from the SPF. For the two and three period backcasts $\pi_{i,t-3\to t-2|t}$ and $\pi_{i,t-4\to t-3|t}$, we use the most recently available vintage of the CPI at the time that the forecast was made.

Define the measurement equation mapping matrix H as

$$H := \begin{bmatrix}
\beta_{1,L} & -\beta_{1,S} f_S(0,1) & 1 & f_S(0,1) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S} f_S(1,2) & 1 & f_S(1,2) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S} f_S(2,3) & 1 & f_S(2,3) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S} f_S(3,4) & 1 & f_S(3,4) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S} f_S(3,7) & 1 & f_S(3,7) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S} f_S(2,6) & 1 & f_S(2,6) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S} f_S(1,5) & 1 & f_S(1,5) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S} f_S(0,4) & 1 & f_S(0,4) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S} f_S(7,11) & 1 & f_S(7,11) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S} f_S(6,10) & 1 & f_S(6,10) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S} f_S(5,9) & 1 & f_S(5,9) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S} f_S(4,8) & 1 & f_S(4,8) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S} f_S(0,19) & 1 & f_S(0,19) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S} f_S(0,18) & 1 & f_S(0,18) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S} f_S(0,17) & 1 & f_S(0,17) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S} f_S(0,16) & 1 & f_S(0,16) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S} f_S(0,39) & 1 & f_S(0,39) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S} f_S(0,38) & 1 & f_S(0,38) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S} f_S(0,37) & 1 & f_S(0,37) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S} f_S(0,36) & 1 & f_S(0,36) & \dots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\beta_{n,L} & -\beta_{n,S} f_S(0,37) & 0 & 0 & \dots & 1 & f_S(0,37) \\
\beta_{n,L} & -\beta_{n,S} f_S(0,36) & 0 & 0 & \dots & 1 & f_S(0,36)
\end{bmatrix} \tag{A2}$$

Define the constant vector in the measurement equation, μ_y, as

$$\mu_y := \begin{bmatrix} \alpha_L - \alpha_S f_S(0,1) \\ \alpha_L - \alpha_S f_S(1,2) \\ \alpha_L - \alpha_S f_S(2,3) \\ \alpha_L - \alpha_S f_S(3,4) \\ \alpha_L - \alpha_S f_S(3,7) \\ \alpha_L - \alpha_S f_S(2,6) \\ \alpha_L - \alpha_S f_S(1,5) \\ \alpha_L - \alpha_S f_S(0,4) \\ \alpha_L - \alpha_S f_S(7,11) \\ \alpha_L - \alpha_S f_S(6,10) \\ \alpha_L - \alpha_S f_S(5,9) \\ \alpha_L - \alpha_S f_S(4,8) \\ \alpha_L - \alpha_S f_S(0,19) \\ \alpha_L - \alpha_S f_S(0,18) \\ \alpha_L - \alpha_S f_S(0,17) \\ \alpha_L - \alpha_S f_S(0,16) \\ \alpha_L - \alpha_S f_S(0,39) \\ \alpha_L - \alpha_S f_S(0,38) \\ \alpha_L - \alpha_S f_S(0,37) \\ \alpha_L - \alpha_S f_S(0,36) \\ \vdots \\ \alpha_L - \alpha_S f_S(0,37) \\ \alpha_L - \alpha_S f_S(0,36) \end{bmatrix} \tag{A3}$$

Define the vector measurement errors v_t as

$$v_t \coloneqq \begin{bmatrix} v_{1,1,t} \\ v_{1,2,t} \\ v_{1,3,t} \\ v_{1,4,t} \\ v_{1,5,t} \\ v_{1,6,t} \\ v_{1,7,t} \\ v_{1,8,t} \\ v_{1,9,t} \\ v_{1,10,t} \\ v_{1,11,t} \\ v_{1,12,t} \\ v_{1,13,t} \\ v_{1,14,t} \\ v_{1,15,t} \\ v_{1,16,t} \\ v_{1,17,t} \\ v_{1,18,t} \\ v_{1,19,t} \\ v_{1,20,t} \\ \vdots \\ v_{n,19,t} \\ v_{n,20,t} \end{bmatrix} \tag{A4}$$

The covariance matrix of the the measurement error vector R is given by

$$R := \operatorname{diag}\left(\begin{bmatrix} \sigma^2_{v,1} \\ \sigma^2_{v,2} \\ \sigma^2_{v,3} \\ \sigma^2_{v,4} \\ \sigma^2_{v,5} \\ \sigma^2_{v,6} \\ \sigma^2_{v,7} \\ \sigma^2_{v,8} \\ \sigma^2_{v,9} \\ \sigma^2_{v,10} \\ \sigma^2_{v,11} \\ \sigma^2_{v,12} \\ \sigma^2_{v,13} \\ \sigma^2_{v,14} \\ \sigma^2_{v,15} \\ \sigma^2_{v,16} \\ \sigma^2_{v,17} \\ \sigma^2_{v,18} \\ \sigma^2_{v,19} \\ \sigma^2_{v,20} \\ \cdots \\ \sigma^2_{v,19} \\ \sigma^2_{v,20} \end{bmatrix}\right) \tag{A5}$$

Appendix B Gibbs Sampler Details

1. Sample $\boldsymbol{\theta}_1 = \left\{\alpha_{i,L}, \alpha_{i,S}, \beta_{i,L}, \beta_{i,S}\right\}_{i=1}^{N}$ conditional on remaining parameters.

   Since all of the shocks are assumed to be independent, this can be treated as a separate regression model for each forecaster i. We assume independent, multivariate normal priors for each group of four parameters $\left[\alpha_{i,L}, \alpha_{i,S}, \beta_{i,L}, \beta_{i,S}\right]'$ across each foracaster i, where

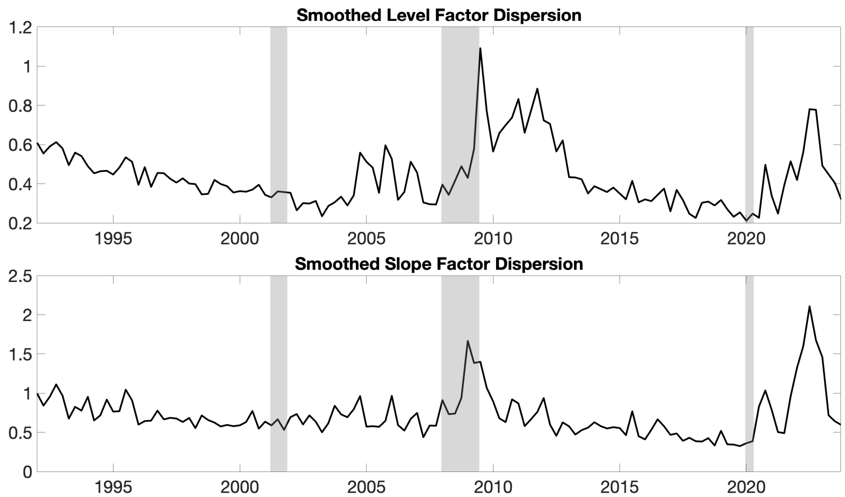
$$\begin{bmatrix} \alpha_{i,L} \\ \alpha_{i,S} \\ \beta_{i,L} \\ \beta_{i,S} \end{bmatrix} \sim N\left(\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i\right)$$

2. Sample $\boldsymbol{\theta}_2 = \text{vec}(\boldsymbol{A})$

Appendix C Other figures

This section shows figures that are not included in the main text.

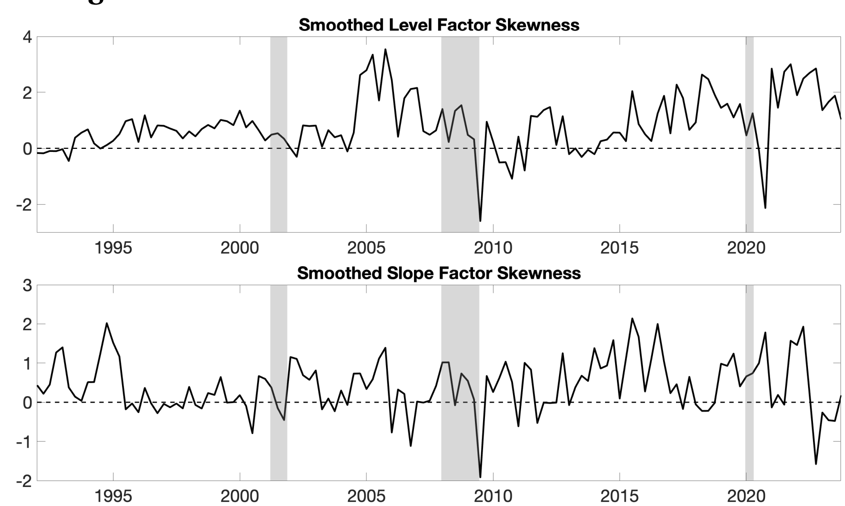
Figure C1: SMOOTHED IDIOSYNCRATIC FACTOR DISPERSION



Notes: The shaded areas denote the NBER recessions.

Sources: Authors' calculation

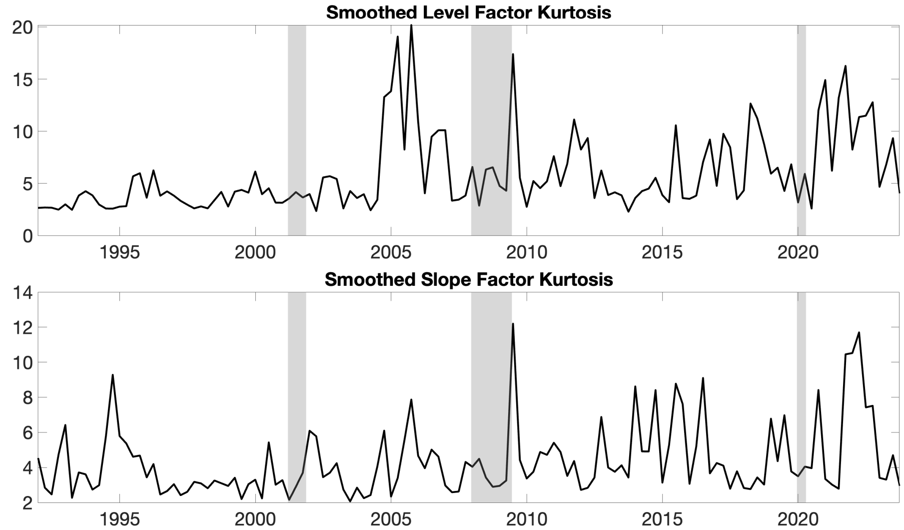
Figure C2: SMOOTHED IDIOSYNCRATIC FACTOR SKEWNESS



Notes: The shaded areas denote the NBER recessions.

Sources: Authors' calculation

Figure C3: Smoothed Idiosyncratic Factor Kurtosis



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Notes: The shaded areas denote the NBER recessions.

Sources: Authors' calculation

Appendix D Nonparametric representation

In this section, we build and estimate an alternative non-parametric model which is estimable with least squares. This non-parametric model also recovers the complete path of individual inflation forecasts over a 10-year horizon at each point in time. We find that the empirical result from the alternative model is broadly consistent with our baseline result, confirming the robustness of our baseline conclusion.

D.1 Model

The term structure is modeled as the sum of two different components, short-run and long-run. The short-run model characterizes the forecasts $\pi_{t-1\to t|t}$ to $\pi_{t+3\to t+4|t}$, while the long-run model characterizes the forecasts $\pi_{t+3\to t+4|t}$ to $\pi_{t+38\to t+39|t}$. Short-run forecasts are characterized by level, slope, and curvature factors while long-run forecasts are characterized by a long-term slope. Since we have more observations of individual forecasts at the short-end than the long-end, we are able to use a more flexible short-run econometric model.

D.1.1 Short-run Forecasts

For horizons 0 to 4 quarters ahead, forecasts are modeled as follows

$$
\begin{aligned}
\pi_{t+h-1\to t+h|t} &= l_t + s^s_t(h) + c_t(h) \\
&= l_t + s^s_t \cdot \frac{h}{4} + c_{1,t} L_2\left(\frac{2h}{4} - 1\right) + c_{2,t} L_4\left(\frac{2h}{4} - 1\right)
\end{aligned}
\tag{D6}
$$

where l_t is a level factor, s^s_t is a slope factor (with an s superscript to distinguish it from the long-run slope factor), and $c_{1,t}$ and $c_{2,t}$ are curvature factors. The curvature $c_t(h)$ is a weighted sum of the 2nd and 4th Legendre polynomials, L_2 and L_4 respectively.

The level factor l_t is determined as the nowcast of the current QoQ inflation rate $\pi_{t-1\to t|t}$. The short-run slope factor s^s_t is measured as $\pi_{t+3\to t+4|t} - \pi_{t-1\to t|t}$, the difference between the 4-quarter ahead forecast and the nowcast. The short-term curvature $c_t(h)$ captures the residual variation in inflation forecasts that are not accounted for by the short-term level and slope.

D.1.2 Long-run Forecasts

The path of inflation forecasts between $\pi_{t+3\to t+4|t}$ and $\pi_{t+38\to t+39|t}$ is determined a single long-run slope parameter. We only consider a one-parameter model because it is difficult to identify curvature from the observed averages of annual forecasts.[11] To take into account possible non-linearity in long-run inflation expectations, we adopt a log function to characterize the long-run slope:

$$
\pi_{t+h-1\to t+h|t} = \pi_{t+3\to t+4|t} + s^l_t(h) \quad \text{for} \quad h \geq 5.
$$

$s^l_t(h)$ is the slope of h-period-ahead projection:

$$
s^l_t(h) = \lambda_t \cdot \frac{\ln(h-4)}{\ln(39-4)} \quad \text{for} \quad h = 5, \ldots, 39
$$

[11] In particular, the only information we have for forecasts of inflation between 5 and 10 years out is from the 10-year average forecast.

where λ_t is the overall change in inflation expectations between 1 and 10 years out. It follows that

$$\pi_{t+38\to t+39|t} = \pi_{t+3\to t+4|t} + \lambda_t.$$

In this model, the only unknown is λ_t.[12]

D.1.3 Full Model

Combining the short-term and long-term forecasting models we can write the model for a general forecast

$$\pi_{t\to t+h|t} = l_t + \mathbb{1}\{h \leq 4\}\left[s_t^s(h) + c_t(h)\right] + \mathbb{1}\{h > 5\}\left[s_t^s + s_t^l(h)\right]$$

for $h = 0, \ldots, 39$. For the 1Q backcast, $\pi_{t-2\to t-1|t}$, we use the value provided in an individual forecaster's survey response. For any further backcasts, $\pi_{t-h|t-h+1|t}$ for $h > 2$, the forecaster is assumed to use the latest available vintage of inflation data.

D.2 Estimation of the Model

The level factor l_t is identified as the nowcast $\pi_{t-1\to t|t}$

$$\hat{l}_t = \pi_{t-1\to t|t} \tag{D7}$$

The short-run slope factor is identified as the difference between the four-quarter ahead forecast $\pi_{t+3\to t+4|t}$ and the nowcast $\pi_{t-1\to t|t}$

$$\hat{s}_t^s = \pi_{t+3\to t+4|t} - \pi_{t-1\to t|t} \tag{D8}$$

[12] Note that our functional form for the long-run implies forecasts are monotonically increasing or decreasing from one to ten years out. We also tried fitting a more conventional Nelson-Siegel model with curvature to the long-run but found that we could not precisely estimate the parameters given the limited information we have for medium and long-run forecasts.

Collect the deviations of the forecasts $\pi_{t\to t+1|t}$, $\pi_{t+1\to t+2|t}$, and $\pi_{t+2\to t+3|t}$ from the values predicted by the term structure in the vector $\hat{\varepsilon}_{s,t}$:

$$\hat{\varepsilon}_{s,t} = \begin{bmatrix} \pi_{t\to t+1|t} - \hat{l}_t - \hat{s}^s_t \cdot \frac{1}{4} \\ \pi_{t+1\to t+2|t} - \hat{l}_t - \hat{s}^s_t \cdot \frac{2}{4} \\ \pi_{t+2\to t+3|t} - \hat{l}_t - \hat{s}^s_t \cdot \frac{3}{4} \end{bmatrix} \tag{D9}$$

The curvature factors $c_{1,t}$ and $c_{2,t}$ are estimated by OLS from a regression of $\hat{\varepsilon}_{s,t}$ on

$$L_t = \begin{bmatrix} L_2\left(\frac{2}{4}-1\right) & L_4\left(\frac{2}{4}-1\right) \\ L_2\left(\frac{4}{4}-1\right) & L_4\left(\frac{4}{4}-1\right) \\ L_2\left(\frac{6}{4}-1\right) & L_4\left(\frac{6}{4}-1\right) \end{bmatrix}$$

which gives

$$\begin{bmatrix} \hat{c}_{1,t} \\ \hat{c}_{2,t} \end{bmatrix} = \left(L_t' L_t\right)^{-1} L_t' \hat{\varepsilon}_{s,t} \tag{D10}$$

Note that we need not use $\pi_{t-1\to t|t}$ and $\pi_{t+3\to t+4|t}$ to estimate the curvature factors since the curvature $c_t(h)$ is equal to 0 by construction at those values for all $c_{1,t}$ and $c_{2,t}$.

For long-run forecasts, the estimation depends on the quarter in which the survey is conducted since the forecasts are always relative to the first quarter of the year. Collect the deviations of the one to three-year ahead forecasts and the five and ten-year averages of Q4 over Q4 forecasts from their model implied counterparts in a vector $\hat{\varepsilon}_{l,t}$:

$$\hat{\varepsilon}_{l,t} = \begin{bmatrix} \pi_{q_{a-1}(t)\to q_{a-1}(t)+3|t} - \hat{\pi}_{q_{a-1}(t)\to q_{a-1}(t)+3|t} \\ \pi_{q_{a-1}(t)+4\to q_{a-1}(t)+7|t} - \hat{\pi}_{q_{a-1}(t)+4\to q_{a-1}(t)+7|t} \\ \pi_{q_{a-1}(t)+8\to q_{a-1}(t)+11|t} - \hat{\pi}_{q_{a-1}(t)+8\to q_{a-1}(t)+11|t} \\ \frac{1}{5}\pi_{q_{a-1}(t)\to q_{a-1}(t)+19|t} - \frac{1}{5}\hat{\pi}_{q_{a-1}(t)\to q_{a-1}(t)+19|t} \\ \frac{1}{10}\pi_{q_{a-1}(t)\to q_{a-1}(t)+39|t} - \frac{1}{10}\hat{\pi}_{q_{a-1}(t)\to q_{a-1}(t)+39|t} \end{bmatrix} \tag{D11}$$

Define h_1^* such that $\pi_{q_{a-1}(t)+h_1^*-1\to q_{a-1}(t)+h_1^*|t} = \pi_{t+3\to t+4|t}$. That is, h_1^* is the forecast horizon relative to the final quarter of the year prior to when the survey was conducted that corresponds to the 4Q-ahead forecast in the SPF. For any forecast horizon $h > 0$ such that $t + h \le 39$, define $h_1 \equiv \min(h, h_1^*)$. Finally, define $h_2 \equiv h - h_1$. This allows us to define

$$\begin{aligned}\hat{\pi}_{q_{a-1}(t)\to q_{a-1}(t)+h|t} &= \sum_{s=1}^{h_1-1} \pi_{q_{a-1}(t)+s-1\to q_{a-1}(t)+s|t} + \sum_{s=h_1}^{h_1+h_2} \left(\pi_{t+3\to t+4|t} + s_t^l(s-h_1+5)\right) \\ &= \sum_{s=1}^{h_1-1} \pi_{q_{a-1}(t)+s-1\to q_{a-1}(t)+s|t} + (h_2+1) + \lambda_t \sum_{s=h_1}^{h_1+h_2} \frac{\ln(s-h_1+1)}{\ln(35)}\end{aligned}$$

The long-run slope λ_t is estimated by minimizing the sum of squared residuals $\hat{\varepsilon}_{l,t}$

$$\hat{\lambda}_t = \underset{\lambda_t}{\operatorname{argmin}}\, \hat{\varepsilon}_{l,t}' \hat{\varepsilon}_{l,t} \tag{D12}$$

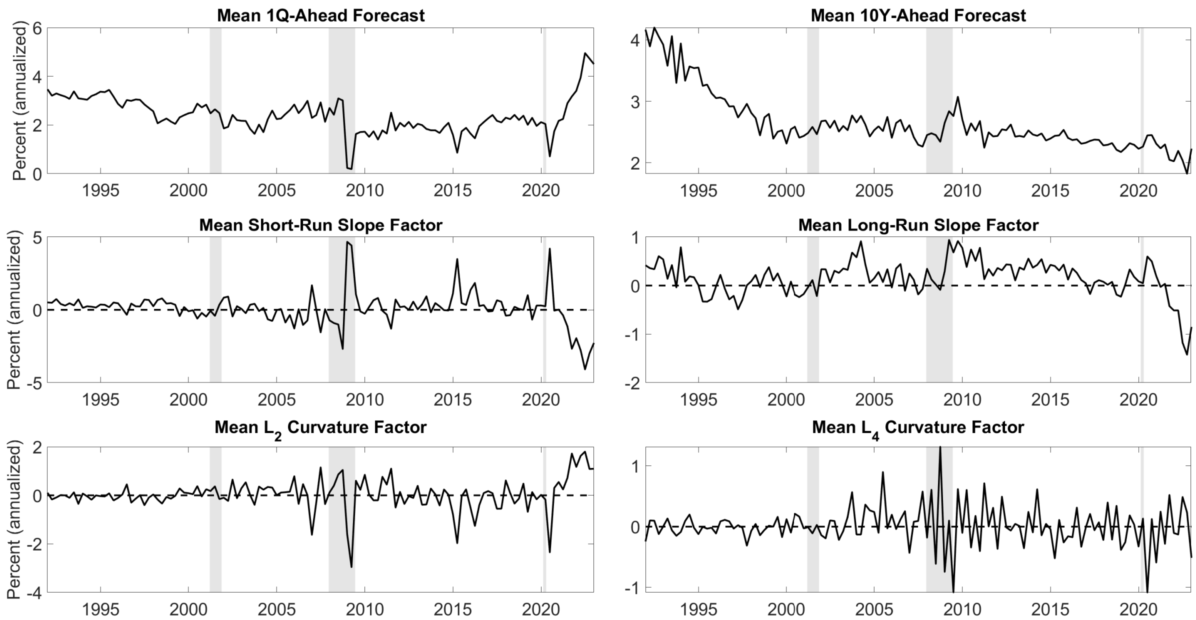
D.3 Estimation Results

This section reports and discusses our results from the estimation of the term-structure model. We estimate our model for every forecaster and every time period available in the SPF for whom we have sufficient data.[13]

Figure D4 plots the cross-sectional means of the estimated factors from our model over time. The top left panel plots the mean level factor, which tracks the consensus inflation nowcast (and thus realized inflation to a large extent) by construction. The top right panel plots the implied consensus forecast of QoQ inflation 10 years ahead, i.e. the forecast of inflation between 38 and 39 quarters out, $\pi_{t+38\to t+39|t}$. This is a summary measure of long-run inflation expectations, which unlike forecasts of 10-year average inflation, contains only information about long-run beliefs. Long-run inflation expectations exhibited a sharp downward trend in the 1990s, stabilizing around 2.5% by the early 2000s. In the later part of the sample they have drifted down even further nearing 2% during the COVID pandemic.

[13] So long as we observe non-missing values for $\pi_{t-1\to t|t}$, $\pi_{t+3\to t+4|t}$, and so long as two elements of both $\hat{\varepsilon}_{s,t}$ and $\hat{\varepsilon}_{l,t}$ are well-defined for forecaster i at time t, we estimate an individual-level term structure of inflation expectations for forecaster i at time t.

Figure D4: MEAN OF ESTIMATED FACTORS

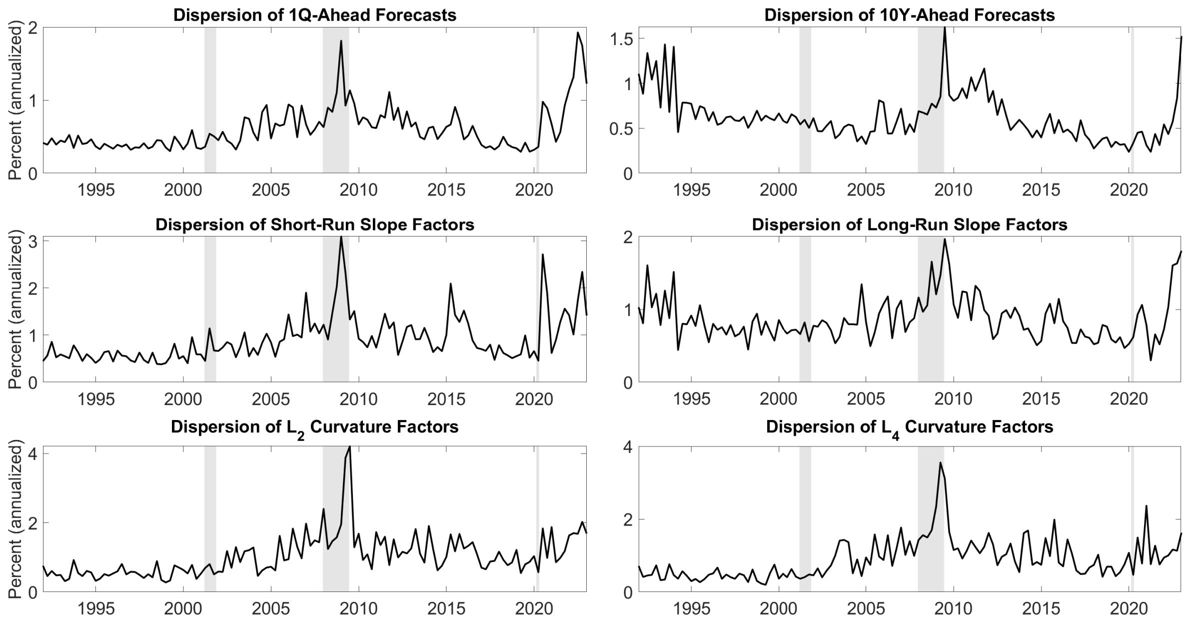


Notes: The shaded areas denote the NBER recessions.
Sources: Authors' calculation

The middle left panel plots the mean of the short-run slope factor, which tracks the expected path of inflation over the year following the survey. The middle right panel plots the mean of the long-run slope factor, which tracks the expected path of inflation between one and ten years after the survey. The term structure of inflation expectations is typically upwards sloping, with the short-run slope factor being positive for about 62% of the sample and the long-run slope factor being positive for about 76% of the sample. The steepest positive slopes occur in the years following the dotcom bubble and in the Great Recession. The long-run slope has systematically declined over the 2010s and is about -0.5% as of 2022Q1. The short-run and long-run slope factors have a correlation of 0.38 and thus still exhibit a large amount of independent variation. The bottom panels plot the short-run curvature factors $c_{1,t}$ (left) and $c_{2,t}$ (right). The estimates are noisy, reflecting the short-term noisiness.

Figure D5 plots the cross-sectional standard deviations, i.e. dispersions, of the estimated factors from our model over time. Each panel corresponds to the same factor as in Figure D4. Long-run inflation factors were more dispersed than short-run expectations early in our

Figure D5: Dispersion of Estimated Factors



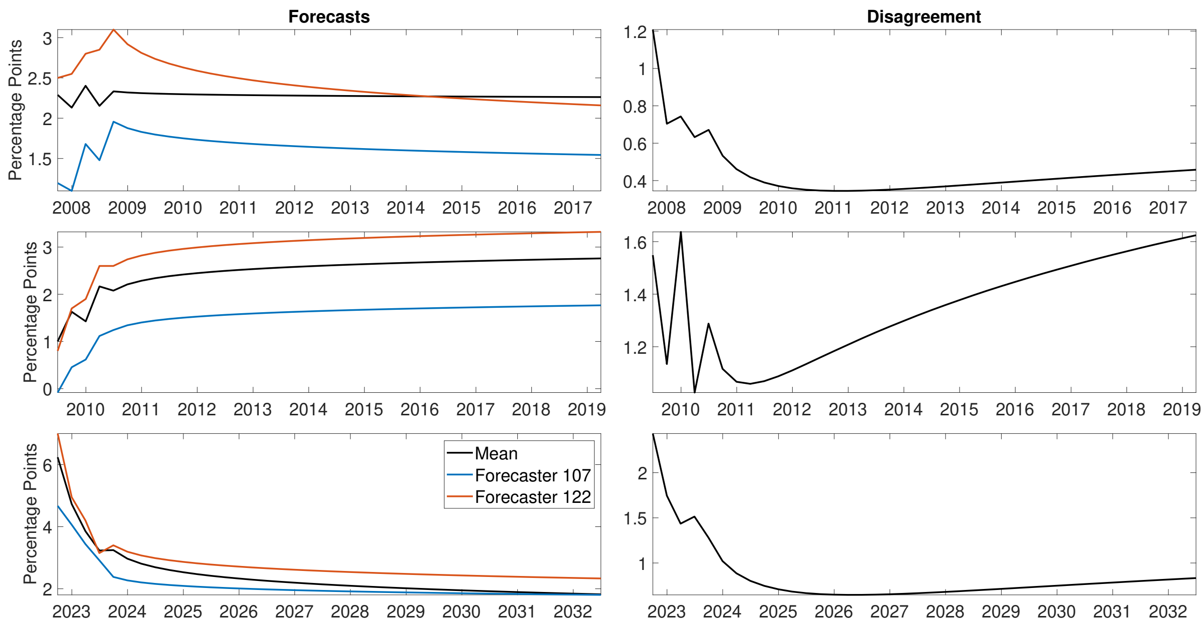
Notes: The shaded areas denote the NBER recessions.
Sources: Authors' calculation

sample but since around the Great Recession, that pattern has flipped and short-run factors have tended to be more dispersed. The dispersion of all factors spiked during the Great Recession but interestingly long-run factors have remained relatively stable during the COVID-19 pandemic, suggesting strong credibility of central bank policy and communication. In the most recent couple of quarters there has been an increase in long-run dispersion which may be an early warning sign of this pattern reversing.

Figure D6 plots the estimated term structure of inflation expectations at different points in time. The columns correspond to 2007:Q3, 2009:Q2, and 2022:Q1 respectively. The rows correspond to particular forecasters, the consensus forecast, and the overall dispersion respectively. The dates are chosen to showcase the flexibility of the term structure model in fitting different patterns.

First we consider 2007:Q3. This was the final survey conducted before the start of the Great Recession. The consensus term structure is relatively flat, with the nowcast and 10-year ahead inflation expectations being around 2.25%. The estimates imply some nonlinear expected short-

Figure D6: Estimated Term Structure of Inflation Expectations at Three Dates



Sources: Authors' calculation

run variation in inflation, displaying oscillation between 2.1 and 2.4% over the year following the survey before steadily declining over the one to ten-year ahead horizons. The two forecasters' trajectories are quite different from the consensus. Dispersion is high in the short-run and low in the medium and long-run.

Next consider 2009:Q2, in the middle of the Great Recession. The term structures are upward sloping for both forecasters and the consensus. The forecasters expect a short-run increase and decline from 1%, before a steady increase over the one to ten-year ahead horizons to about 1.6%. The consensus forecasts increase mostly monotonically from 1% to about 2.7%. The forecasters show significantly lower long-run inflation than the consensus whereas they were aligned in 2007:Q3. Dispersion is high in the short and long-run but lower in the medium run.

Finally we consider 2022:Q1. Inflation expectations are mostly monotonically decreasing for both forecasters and the consensus, with most of the decline occurring over the first year out and then a slow decline from one to ten years ahead. Dispersion is highest at the 3 and 4Q ahead horizons, and declines rapidly at medium and long horizons.

D.4 Dynamic decomposition of disagreement

This section investigates the role of individual prior beliefs and public and private information in the disagreement. We employ a dynamic factor model to assess the role of public and private information in changes of individual i's j-period-ahead inflation projection.

D.4.1 Dynamic factor model for the term-structure of inflation expectations

Consider forecaster i's j-period-ahead inflation projection denoted by z_{it}^j. The variable z_{it}^j is composed of the portion that comoves with other forecasters' j-period-ahead projection and the portion that does not. The mean of z_{it}^j over t is denoted by c_i^j. Let x_{it}^j denote the former that reflects public information and e_{it}^j denote the latter that is the outcome of forecaster i's private information. Note that x_{it}^j and e_{it}^j have zero means. In this environment, z_{it}^j is the sum of c_i^j, and x_{it}^j and e_{it}^j.

$$z_{it}^j = c_i^j + x_{it}^j + e_{it}^j. \tag{D13}$$

Suppose that there are total N forecasters. We collect z_{it}^j of all forecasters into a vector $z_t^j = [z_{1t}^j, z_{2t}^j, \cdots, z_{Nt}^j]'$, the common components in the vector $x_t^j = [x_{1t}^j, x_{2t}^j, \cdots, x_{Nt}^j]'$, and the idiosyncratic components in the vector $e_t^j = [e_{1t}^j, e_{2t}^j, \cdots, e_{Nt}^j]'$. Equation (D13) is rewritten into the following:

$$z_t^j = c^j + x_t^j + e_t^j. \tag{D14}$$

The vectors, x_t^j and e_t^j, are estimated with a dynamic factor model. We first standardize z_t^j so that z_t^j has a zero mean and a unit variance. Let $y_t^j = [y_{1t}^j, y_{2t}^j, \cdots, y_{Nt}^j]'$ denote the vector of standardized level factors. The model postulates:

$$y_t^j = \Lambda^j f_t^j + r_t^j, \qquad r_t^j \sim i.i.d. N(0, R^j), \tag{D15}$$

where f_t^j is an $p^j \times 1$ vector of common factors and Λ^j denotes an $(N \times p^j)$ matrix of factor loadings. The vector of idiosyncratic disturbances is captured by $r_t^j = [r_{1t}^j, r_{2t}^j, \cdots, r_{Nt}^j]\prime$.[14] The idiosyncratic components r_t^j is assumed to be uncorrelated with f_t^j at all leads and lags. The vector r_t^j is drawn from a normal distribution with mean zero and covariance matrix R^j, where R^j is a diagonal matrix.

The vector of common components, f_t^j, follows a VAR process of order q^j:

$$f_t^j = C_1^j f_{t-1}^j + C_2^j f_{t-2}^j + \cdots + C_{q^j} f_{t-q^j}^j + u_t^j, \qquad u_t^j \sim i.i.d.N(0, Q^j). \tag{D16}$$

In the above equation, $C_1^j, C_2^j, \cdots, C_{q^j}^j$ denote the $p^j \times p^j$ matrices of VAR coefficients. We assume that the vector of innovations u_t^j is drawn from a normal distribution with zero means and covariance matrix Q^j.[15]

Note that forecasters join the survey and leave the sample at a different point in time. Therefore, the dataset has an unbalanced panel structure due to missing observations and ragged edges. Banbura and Modugno (2014) develop an algorithm that estimates equations (D15)-(D16) in such data environment. Banbura-Modugno's estimation algorithm is summarized as follows. First, the data are demeaned and standardized. The unbalanced panel dataset is transformed into a balanced one by filling in missing values and ragged edges with the interpolated values of each forecaster's j-period-ahead projection.[16] Second, principal components are estimated from the constructed balanced dataset. We use the estimated principal components as the starting factors for the iteration of EM algorithm. The smoothed estimates of factors are obtained from the *E-step* and the parameters are estimated via linear projections in the *M-step*.[17]

The number of factors are determined based on IC_1, IC_2, and IC_3 from Bai and Ng (2002) with the balanced dataset obtained from the second step of Banbura-Modugno's algorithm.[18]

[14] We also consider the case where r_t^j follows an AR(1) process for robustness checks. The result is robust as documented in the appendix.

[15] Since our sample period begins from the mid 1990s, non-stationarity or I(1) component in the idiosyncratic component are less of an issue.

[16] See the online appendix for more details about Banbura and Modugno (2014).

[17] See the appendix for more details on the algorithm.

[18] We set the maximum number of factors to be 5. All three statistics often do not agree about the optimal number of factors, as it is well known. In this case, we check the robustness of estimates given the number of factors

After this step, we choose the number of lags in Equation (D16) based upon Akaike Information Criterion (henceforth, AIC).

The common component, x^j_{it}, is estimated from the following. Notice that both the estimated factor and the idiosyncratic component have zero means and unit variances, as they are estimated from the standardized data. Therefore, we scale the factors multiplied by the factor loadings by the standard deviation of forecaster i's j-period-ahead projection (σ^j_i) and add the mean of forecaster i's j-period-ahead projection (μ^j_i):

$$x^j_{it} = \Lambda^j_i f^j_t \times \sigma^j_i,$$

where Λ^j_i is the elements in ith row of Λ^j. The portion of private information in forecaster i's j-period-ahead projection, e_{it}, is then computed from

$$e^j_{it} = z^j_{it} - x^j_{it} - c^j_i.$$

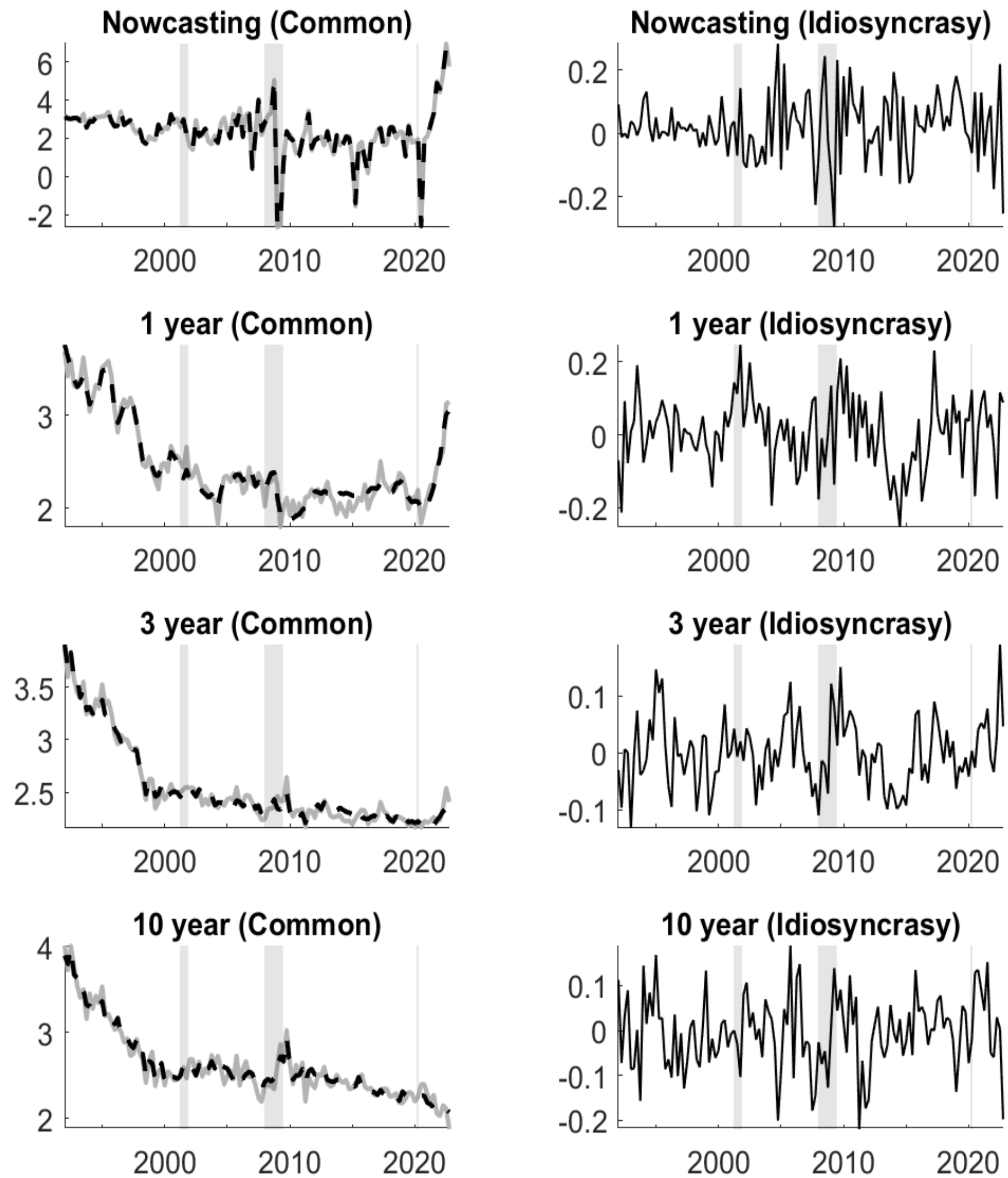
We adopt essentially the same model for all j. However, the model specification is allowed to change according to the information criteria for the number of factors and that of lags for the factor dynamics.

D.5 The fit of model

This section checks the fit of model to examine whether we have reliable estimates or not. Figure D7 displays the mean of $(\hat{x}^j_{it} + \hat{c}^j_i)$ and that of $\hat{e}^j_{it}$ by the forecast horizon of inflation expectations. The left panels show the mean component (grey line) and the portion of mean component attributed to by public information or the common factors (black dashed line). Overall, the dynamic factor model portrayed in the previous section fits the mean of inflation forecasts fairly well. The right panels display the portion of mean forecasts attributed to private information. This portion is noisy, satisfying the assumption in Equation (D15).

suggested by each information criterion.

Figure D7: MEAN OF INDIVIDUAL FORECASTERS' PROJECTION BY FORECASTING HORIZON (PERCENTAGE POINT)



Notes: The left panel displays the mean common component and the right panel displays the mean idiosyncratic component. Shaded areas denote the NBER recessions.

Sources: Authors' calculation

D.6 Dynamic Decomposition of Disagreement on Inflation Forecasts

This section explores the degree to which public and private information accounts for changes in the disagreement about inflation forecasts at each horizon. The disagreement is measured by cross-sectional dispersion of inflation forecasts.

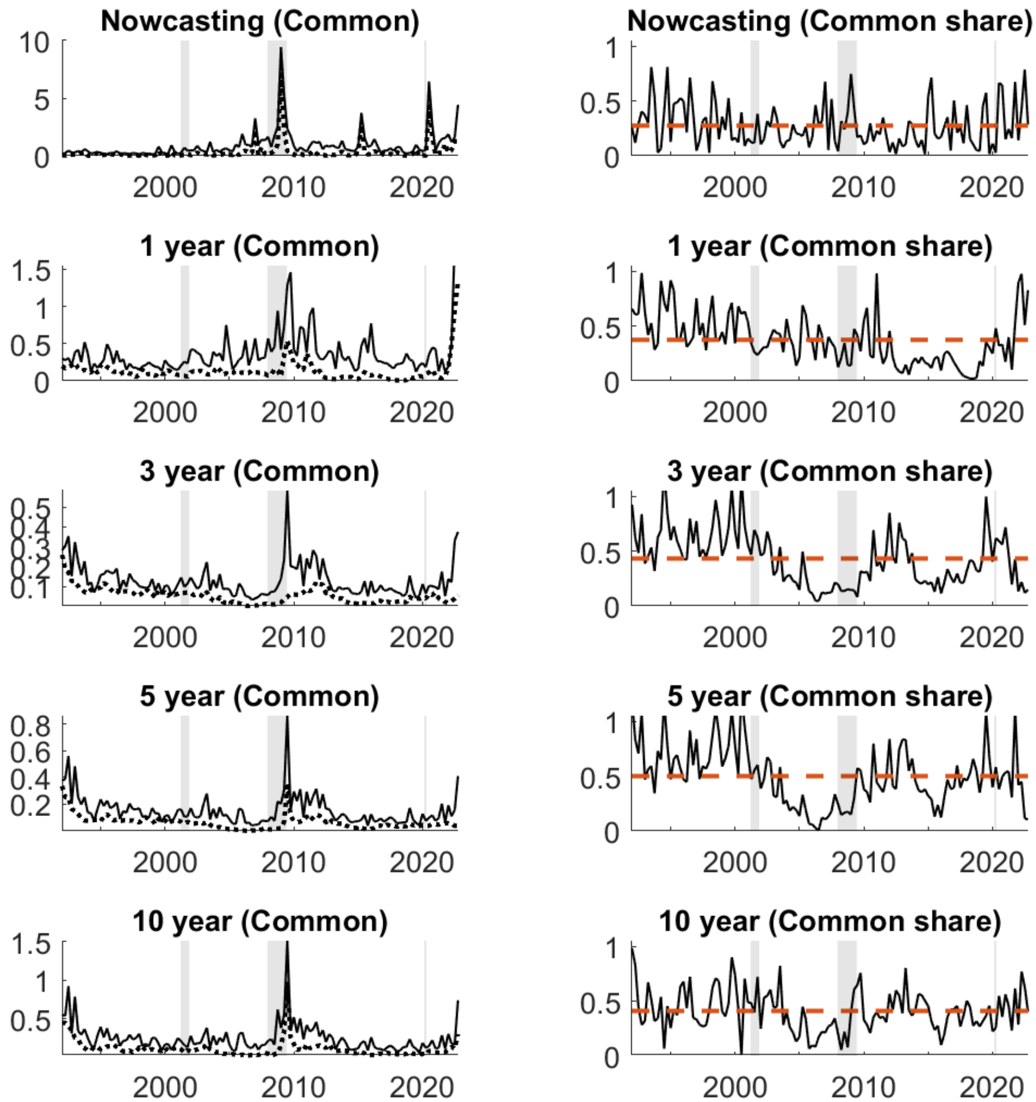
We decompose the dispersion of j-period-ahead forecasts into the portions attributed to prior beliefs and public and private information. We measure the disagreement with the variance of each component in (D13).

$$var_i(\tilde{z}_{it}^j) \approx var_i(\tilde{c}_i^j) + var_i(\tilde{x}_{it}^j) + var_i(\tilde{e}_{it}^j). \tag{D17}$$

Figure D8 reports the decomposition result of Equation (D17). The left panels display $var(\tilde{x}_{it}^j)$ (dotted line) along with $var(\tilde{z}_{it}^j)$ (solid line). The right panels display the share of $var(\tilde{x}_{it}^j)$ out of $var(\tilde{z}_{it}^j)$ (solid line) and the average of the sample period (dashed line).

There are three notable features. First, the role of public information is more important in the disagreement about longer-horizon forecasts than in that about shorter-horizon forecasts. Specifically, public information accounts for about a quarter of the dispersion in nowcasting, 30% in the 1-year-ahead dispersion, and a half of the 10-year-ahead dispersion. This suggests that the disagreement on long-term expectations is importantly accounted for by heterogeneous reactions of forecasters to public information. Meanwhile, the disagreement about short-term forecasts is driven by private information given that the forecasters are fairly homogeneous about interpreting public information for their near-term forecasts.

Figure D8: DECOMPOSITION OF THE DISAGREEMENT ABOUT j-PERIOD-AHEAD INFLATION (PERCENTAGE POINT)



Notes: The left panel displays the mean common component and the right panel displays the mean idiosyncratic component. Shaded areas denote the NBER recessions.

Sources: Authors' calculation