Heterogeneous Downward Nominal Wage Rigidity: Foundations of a Nonlinear Phillips Curve*

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Abstract

We propose a model with heterogeneous downward nominal wage rigidity for individual labor varieties. The model delivers a nonlinear wage Phillips curve that is relatively steep at high levels of inflation and flat at low levels of inflation, implying a low cost of reducing high levels of inflation. At the same time, for regular size fluctuations in inflation, we show that the model preserves the dynamic properties of the new-Keynesian model. The predicted nonlinear Phillips curve matches well the pattern of wage inflation and unemployment observed in the United States over the past 40 years. Although the equilibrium features occasionally binding constraints for individual labor types, there are no such constraints in the aggregate making the model amenable to perturbation analysis.

Keywords: Downward nominal wage rigidity, nonlinear wage Phillips curve, heterogeneity.
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1 Introduction

The resilience of the labor market in the midst of the post-Covid-19 monetary tightening cycle has spurred renewed interest in whether the Phillips curve is steeper at high levels of inflation. In its original formulation, Phillips (1958) documented a contemporaneous negative empirical relationship between wage inflation and unemployment. Phillips emphasized the nonlinearity of his estimated relationship. He found that at high levels of inflation the Phillips curve is steeper than at low levels of inflation suggesting that the costs of fighting inflation are relatively low when unemployment is low. He conjectured that this type of nonlinearity was the consequence of downward nominal wage rigidity.

This paper proposes a model of a nonlinear wage Phillips curve due to heterogeneous downward nominal wage rigidity. Specifically, wage rigidity is assumed to vary in intensity across a continuum of labor varieties. The nominal wage of each labor variety is bounded below by the average wage prevailing in the previous period times a variety-specific scalar. In all respects other than the heterogeneity of downward nominal wage rigidity, the model economy is standard; households and firms operate in competitive markets and are rational and forward looking.

In equilibrium the model delivers a nonlinear wage Phillips curve. An increase in wage inflation raises the fraction of labor varieties that are not constrained by the wage lower bound. As a result, the fraction of the labor force suffering involuntary unemployment falls. These effects imply a negative relationship between current wage inflation and current unemployment. Importantly, the sensitivity of the implied relationship between unemployment and wage inflation changes at different levels of aggregate activity. For low levels of inflation, a large measure of workers is stuck at their wage lower bound. As a result, an increase in inflation, by lowering the real value of the wage lower bound, raises employment for a large number of workers. Thus, equilibrium unemployment is relatively sensitive to changes in inflation. By contrast, for high levels of inflation, the mass of workers with a binding wage constraint is small, so an increase in inflation stimulates employment, but only for a small group of workers, rendering unemployment relatively insensitive to changes in inflation.

To calibrate the Phillips curve predicted by the model, it is sufficient to have three pieces of information: a point on the Phillips curve, the slope at that point, and the elasticity of substitution across different labor varieties. For the first piece of information we use average unemployment and average wage inflation in the United States. Values for the second and third pieces of information are taken from studies that predate the present paper. Given this information the global curvature of the Phillips curve is endogenously determined. We find that the calibrated Phillips curve captures relatively well the nonlinear relationship between unemployment and nominal wage growth observed in the U.S. economy over the past four decades.

The contemporaneous relationship between unemployment and wage inflation implied by the present model is in line with Phillips' empirical formulation, but departs from the Phillips curve induced by the new-Keynesian model. Specifically, the new-Keynesian model implies a forward-looking Phillips curve that relates unemployment not only to current wage inflation but also to future expected wage inflation. The reason why the new-Keynesian model generates an expectations augmented wage Phillips curve is that it assumes that workers have market power. This assumption together with the assumption of nominal wage rigidity implies that the wage setting decision is forward looking, as today's nominal wage choice impacts the entire expected future path of the worker's real wage. The assumption that workers have market power can be justified in economies with a strong presence of labor unions, but is less tenable in economies, like the United States, in which secularly a small fraction of the labor force is unionized. For this reason in the present paper we do away with the assumption that workers have market power.

In spite of the aforementioned differences with the new-Keynesian framework, for regular fluctuations of inflation around the intended target, under plausible calibrations, the proposed model delivers equilibrium dynamics that are quantitatively similar to those associated with the standard new-Keynesian model with wage rigidity. An implication of this result is that the assumption that workers have market power does not appear to play a crucial role, at least for standard calibrations of the model considered in business-cycle analysis.

In sum, the proposed model globally delivers a nonlinear Phillips curve, but locally preserves the dynamic properties of the new-Keynesian model. The global property provides theoretical support for the relatively low cost in terms of employment resulting from the stabilization efforts in the aftermath of the Covid-19 inflation spike. The local property provides support, just like the standard new-Keynesian model does, for the use of conventional monetary stabilization policy during regular short-run fluctuations.

Finally, the paper makes a methodological contribution. One impediment that has limited a more widespread adoption of models with downward nominal wage rigidity in monetary analysis in spite of their empirical appeal, is the difficulty to approximate their equilibrium conditions due to the presence of occasionally binding constraints. This is most relevant for medium scale models used for policy analysis. This paper contributes to overcoming this impediment. Unlike standard models with homogeneous downward nominal wage rigidity, the proposed model is amenable to perturbation analysis, which is the standard method used to approximate and estimate equilibrium dynamics in models with two-sided (downward and upward) nominal rigidity. Although in the present formulation there are occasionally binding constraints at the level of individual labor varieties, in the aggregate the equilibrium conditions do not feature such restrictions, thereby allowing for the differentiation of the aggregate equilibrium conditions around the deterministic steady-state.

This paper is related to a large literature on the role of nominal wage rigidity for macroeconomic adjustment. As mentioned earlier, the starting point is the empirical estimate by Phillips (1958) of a negative nonlinear relation between wage inflation and unemployment. In the context of the new-Keynesian framework, sticky wages à la Calvo was introduced by Erceg, Henderson, and Levin (2000). The derivation of a wage Phillips curve in the context of that model is presented in Galí (2011) and Casares (2010). Kim and Ruge-Murcia (2009) study a model with nominal wage rigidity à la Rotemberg but with an asymmetric wage adjustment cost function. They estimate the parameters of this cost function and find that wage cuts are more costly than wage increases. Elsby (2009) studies downward nominal wage rigidity in the context of a model in which firms have monopsony power in the labor market. Benigno and Ricci (2011) also study downward nominal wage rigidity but in a model in which workers have monopoly power. Unlike the present study, the papers cited above are not concerned with the global nonlinearity of the short-run wage Phillips curve.

Schmitt-Grohé and Uribe (2016, 2017) investigate the implications of downward nominal wage rigidity for macroeconomic adjustment in dynamic general equilibrium models of open and closed economies. In contrast to the present formulation, our earlier studies maintain a constant lower bound on nominal wages across labor varieties. This class of models yields a limiting case of nonlinearity, characterized by a horizontal Phillips curve at all levels of unemployment and a vertical curve at full employment, unless one introduces an ad hoc assumption that the wage lower bound decreases with the unemployment rate. The framework presented here nests the model with a constant homogeneous wage lower bound as a special case. Building upon this model class, we demonstrate that introducing heterogeneity in wage rigidity across labor varieties results in an empirically relevant nonlinear Phillips curve, without the need to assume a direct link between unemployment and the wage lower bound. Another distinction between the heterogeneous and homogeneous versions of the downward nominal wage rigidity model is that the latter is not amenable to perturbation analysis due to the occasionally binding constraint in its aggregated equilibrium conditions.

There is also a literature combining labor search frictions and nominal rigidities including Faia (2008), Gertler, Sala, and Trigari (2008), and Dupraz, Nakamura, and Steinsson (2022). Relative to this literature the present paper does not consider search frictions. Instead the source of involuntary unemployment is a labor variety specific form of downward nominal wage rigidity. Benigno and Eggertsson (2023) add downward nominal wage rigidity to a new-Keynesian model with labor search frictions and find that the predicted Phillips curve, relating price inflation to labor market tightness (the ratio of vacancies to unemployed workers) has a kink. This study shares with the present paper the finding that downward nominal wage rigidity can give rise to a nonlinear Phillips curve. The present study and this paper differ in the root cause of nonlinearity. In their formulation nonlinearity occurs because wages are assumed to be flexible when the tightness ratio is less than one and downwardly rigid when it is greater than one, whereas in the present model nonlinearity emerges endogenously as a result of heterogeneity in downward nominal wage rigidity.

The empirical relevance of downward nominal wage rigidity has been extensively documented by, among others, Card and Hyslop (1996), Kahn (1997), Gottschalk (2005), Barattieri, Basu, and Gottschalk (2014), Daly and Hobijn (2014), Schmitt-Grohé and Uribe (2016), and Jo (2022). Finally, empirical estimates of the wage Phillips curve are presented in Galí (2011) and Galí and Gambetti (2019). We use the latter of these two papers to discipline our quantitative analysis.

The remainder of the paper is organized as follows. Section 2 presents the model with heterogeneous downward nominal wage rigidity. This section also shows that the equilibrium conditions do not include occasionally binding constraints in the aggregate, allowing for a characterization of the equilibrium using perturbation methods. Section 3 shows that the model implies a wage Phillips curve that is globally nonlinear. Section 4 shows that for standard calibrations in a neighborhood around the steady state the equilibrium dynamics implied by the model with heterogeneous downward nominal wage rigidity are similar to those of the new-Keynesian model of wage rigidity. Section 5 concludes.

2 The Model

The model features firms that use a variety of labor inputs and standard consumers. Nominal wages are downwardly rigid and the degree of rigidity varies across labor varieties. The presence of downward nominal wage rigidity causes the labor market to function in a non-Walrasian fashion.

2.1 Firms

Firms are price takers. They use labor as the sole input to produce a final good. Profits are given by

$$P_t z_t F(h_t) - W_t h_t,$$

where P_t denotes the product price level, h_t denotes labor, W_t denotes the nominal wage rate, z_t is an exogenous productivity shock, and $F(\cdot)$ is an increasing and concave production function. The optimality condition determining the demand for labor is

$$z_t F'(h_t) = \frac{W_t}{P_t}, \tag{1}$$

which equates the marginal product of labor to the real wage.

The labor input h_t is assumed to be a composite of a continuum of labor varieties h_{jt} for $j \in [0, 1]$. The aggregation technology is of the form

$$h_t = \left[\int_0^1 h_{jt}^{1-\frac{1}{\eta}} dj \right]^{\frac{1}{1-\frac{1}{\eta}}}, \tag{2}$$

where $\eta > 0$ is the elasticity of substitution across labor varieties. The firm chooses the quantity of each labor variety h_{jt} to minimize its total labor cost, $\int_0^1 W_{jt} h_{jt} dj$, subject to the aggregation technology (2), given its desired amount of the labor composite h_t and taking as given the wage of each variety of labor, denoted W_{jt}. This cost minimization problem yields the demand for labor of type j

$$h_{jt} = \left(\frac{W_{jt}}{W_t} \right)^{-\eta} h_t, \tag{3}$$

where

$$W_t = \left[\int_0^1 W_{jt}^{1-\eta} dj \right]^{\frac{1}{1-\eta}} \tag{4}$$

is the cost-minimizing price of one unit of aggregate labor, that is, when h_{jt} is chosen optimally for all j, the aggregate wage rate W_t satisfies $W_t h_t = \int_0^1 W_{jt} h_{jt} dj$. Firms are assumed to be always on their demand schedules for labor varieties.

2.2 Households

The representative household has preferences over streams of consumption, denoted c_t, described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t),$$

where $\beta \in (0, 1)$ is a subjective discount factor, and $U(\cdot)$ is an increasing and concave period utility function. The household supplies inelastically $\bar{h}$ units of labor of each variety $j \in [0, 1]$. Section 4 endogenizes the supply of labor.

The economy faces an exogenous natural rate of unemployment denoted u_t^n. The natural rate of unemployment reflects frictions in the labor market unrelated to nominal rigidity

(Friedman, 1968). The effective supply of labor variety j is then given by

$$h_{jt} \leq \bar{h}(1 - u_t^n). \tag{5}$$

Employment of each variety of labor is demand determined, so the household takes h_{jt} as given. Sometimes the household will not be able to sell all the units of labor it supplies. In these circumstances, it will suffer involuntary unemployment above the natural rate.

Each period $t \geq 0$, households can trade a nominally risk free discount bond denoted B_t that pays the interest rate i_t when held between periods t and $t+1$. In addition, each period the household pays real lump-sum taxes in the amount τ_t and receives profits from the ownership of firms in the amount ϕ_t. Its sequential budget constraint is then given by

$$c_t + \frac{B_t/P_t}{1+i_t} + \tau_t = \int_0^1 \frac{W_{jt}}{P_t} h_{jt} dj + \frac{B_{t-1}/P_{t-1}}{1+\pi_t} + \phi_t,$$

where

$$\pi_t \equiv \frac{P_t}{P_{t-1}} - 1 \tag{6}$$

denotes the inflation rate. The household chooses contingent plans for bond holdings and consumption to maximize its lifetime utility subject to its sequential budget constraint and some no-Ponzi game borrowing limit. The optimality conditions associated with consumption and bond holdings give rise to the Euler equation

$$U'(c_t) = \beta(1+i_t) E_t \frac{U'(c_{t+1})}{1+\pi_{t+1}}. \tag{7}$$

We now turn to a description of the proposed form of nominal rigidity, which is the novel element of the model.

2.3 Heterogeneous Downward Nominal Wage Rigidity

Each period $t \geq 0$, the nominal wage of every variety $j \in [0,1]$ is assumed to be subject to a lower bound constraint of the form

$$W_{jt} \geq \gamma(j) W_{t-1}, \tag{8}$$

where $\gamma(j)$ is a positive and increasing function governing the degree of downward nominal wage rigidity of labor variety j. This formulation of downward nominal wage rigidity nests the homogeneous case studied in Schmitt-Grohé and Uribe (2016), which obtains when the function $\gamma(j)$ is independent of j. The wage lower bound is assumed to depend on the past average wage rate, W_{t-1}, instead of on the past variety-specific wage rate, W_{jt-1}, to facilitate aggregation.

The labor market closes with a slackness condition imposed at the level of each labor variety,

$$[\bar{h}(1-u_t^n) - h_{jt}][W_{jt} - \gamma(j)W_{t-1}] = 0. \tag{9}$$

According to this condition, when an occupation suffers unemployment above the natural rate, the wage rate must be stuck at its lower bound. The slackness condition also says that if in a given occupation the wage rate is above its lower bound, then the occupation must display full employment, defined as an unemployment rate equal to the natural rate.

2.4 The Government

The central bank sets the nominal interest rate according to a Taylor rule of the form

$$1 + i_t = \frac{1+\pi^*}{\beta}\left(\frac{1+\pi_t}{1+\pi^*}\right)^{\alpha_\pi}\left(\frac{y_t}{y}\right)^{\alpha_y}\mu_t, \tag{10}$$

where π^* denotes the central bank's inflation target, y_t denotes aggregate output, y denotes the steady-state value of y_t, α_π and α_y are parameters, and μ_t is an exogenous and stochastic monetary shock.

We assume that fiscal policy is passive in the sense that government solvency is satisfied independently of the path of the price level.

2.5 Equilibrium

In equilibrium, aggregate output is given by

$$y_t = z_t F(h_t). \tag{11}$$

Market clearing in the goods market requires that consumption equal output,

$$c_t = y_t. \tag{12}$$

We are now ready to define a competitive equilibrium.

Definition 1 (Competitive Equilibrium) *A competitive equilibrium is a set of processes c_t, y_t, h_t, h_{jt}, W_t, W_{jt}, P_t, π_t, and i_t satisfying* (1) *and* (3)-(12) *for all $j \in [0,1]$ and $t \geq 0$, given the initial wage W_{-1} and the exogenous disturbances z_t, μ_t, and u_t^n.*

Next, we show that the equilibrium conditions can be written in terms of a single labor variety.

2.6 Equilibrium in j^* Form

We consider an equilibrium in which for every $t \geq 0$ there exists a cut-off labor variety denoted $j_t^* \in (0,1)$ that operates at full employment, $h_{jt} = \bar{h}(1-u_t^n)$ for $j = j_t^*$, and for which the wage lower bound holds with equality, $W_{j_t^* t} = \gamma(j_t^*)W_{t-1}$. Evaluating the labor demand (3) at $j = j_t^*$, yields the condition

$$\bar{h}(1-u_t^n) = \left(\frac{\gamma(j_t^*)}{1+\pi_t^W}\right)^{-\eta} h_t, \tag{13}$$

where

$$\pi_t^W \equiv \frac{W_t}{W_{t-1}} - 1 \tag{14}$$

denotes wage inflation in period t.

Because $\gamma(j)$ is strictly increasing, it follows that all varieties $j < j_t^*$ must also pay the wage $\gamma(j_t^*)W_{t-1}$, and thus operate at full employment. To see this, let $W_t^* \equiv \gamma(j_t^*)W_{t-1}$ and suppose first, contrary to the claim, that $W_{jt} < W_t^*$ for some $j < j_t^*$. Then, by (3) we have that $h_{jt} = (W_{jt}/W_t)^{-\eta}h_t > (W_t^*/W_t)^{-\eta}h_t = \bar{h}(1-u_t^n)$, which violates the time constraint (5). Intuitively, since at W_t^* there is full employment, a wage lower than W_t^* would induce a demand for labor in excess of full employment, which is impossible. Suppose now that, contrary to the claim, $W_{jt} > W_t^*$ for some $j < j_t^*$. Then by the same logic $h_{jt} < \bar{h}(1-u_t^n)$. Further, $W_{jt} > W_t^* = \gamma(j_t^*)W_{t-1} > \gamma(j)W_{t-1}$. So we have that in this case $\bar{h}(1-u_t^n) - h_{jt} > 0$ and $W_{jt} - \gamma(j)W_{t-1} > 0$, which violates the slackness condition (9).

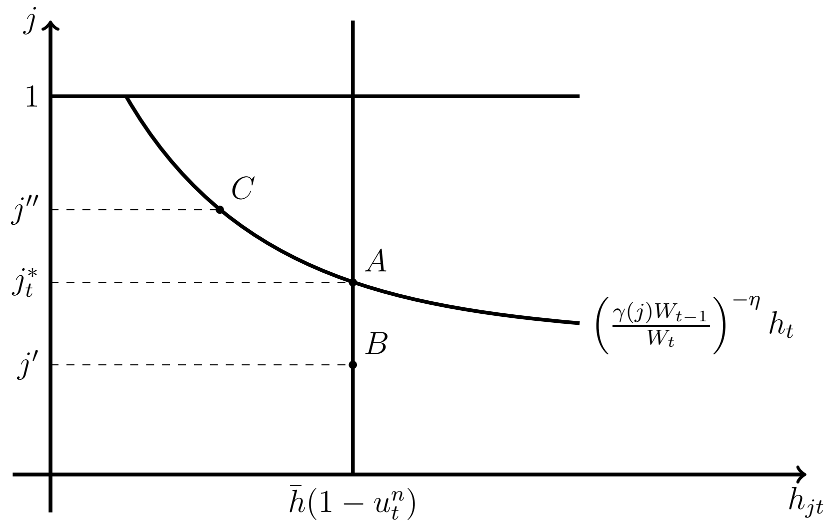
It also follows that all labor varieties $j > j_t^*$ are stuck at their wage lower bound and suffer involuntary unemployment. To see this, use (3) and (8) to write, for any $j > j_t^*$, $h_{jt} = (W_{jt}/W_t)^{-\eta}h_t \leq (\gamma(j)W_{t-1}/W_t)^{-\eta}h_t < (\gamma(j_t^*)W_{t-1}/W_t)^{-\eta}h_t = \bar{h}(1-u_t^n)$. This shows that all labor varieties $j > j_t^*$ suffer involuntary unemployment above the natural rate. It then follows immediately from the slackness condition (9) that $W_{jt} = \gamma(j)W_{t-1}$, that is, wages of all labor varieties $j > j_t^*$ are stuck at their lower bounds.

Summing up, in the equilibrium we are considering, we have that

$$\begin{cases} h_{jt} = \bar{h}(1-u_t^n) \text{ and } W_{jt} = \gamma(j_t^*)W_{t-1} & \text{for } j \leq j_t^* \\ h_{jt} < \bar{h}(1-u_t^n) \text{ and } W_{jt} = \gamma(j)W_{t-1} & \text{for } j > j_t^* \end{cases}. \tag{15}$$

The cut-off variety j_t^* is an important object in this model because it governs the extensive

Figure 1: Determination of Wages and Employment Across Labor Varieties



Notes. The downward slopping line depicts the demand for labor when the wage constraint is binding in the space (j, h_{jt}), where $j \in [0, 1]$ indexes labor varieties and h_{jt} denotes the quantity of labor of variety j demanded by firms. The vertical line depicts the supply of labor net of natural unemployment as a function of the labor variety j.

margin of unemployment, that is, how many occupations will operate below potential.

Figure 1 provides a graphical explanation of the determination of wages and employment across labor varieties. The downward sloping curve represents the demand for labor of each variety, $(h_{jt}$, as a function of j when the variety-spacific wage equals its lower bound, $W_{ij} = \gamma(j)W_{t-1}$. The vertical line represents the labor supply net of natural unemployment, $\bar{h}(1-u_n)$ as a function of j. The intersection of the two lines at point A determines the cut-off variety j_t^*. This is because at point A there is full employment and the wage constraint exactly binds, which are the two conditions defining j_t^*. Points located above and to the right of the downward sloping line are infeasible, because they imply that $W_{jt} < \gamma(j)W_{t-1}$, which violates the wage lower bound. Points located to the right of the vertical line are also unfeasible because they violate the resource constraint $h_{jt} \leq \bar{h}(1-u_t^n)$. Points to the left of both the downward sloping line and the vertical line are also infeasible because they imply that the wage lower bound is slack, $W_{jt} > \gamma(j)W_{t-1}$ and that there is involuntary unemployment, $h_{jt} < \bar{h}(1-u_t^n)$, which violates the slackness condition $(W_{jt} - \gamma(j)W_{t-1})(h_{jt} - \bar{h}(1-u_t^n) = 0$.

Thus, continuing with Figure 1, for any $j < j_t^*$, the equilibrium pair (j, h_{jt}) lies on the downward sloping line, and for any $j < j_t^*$, it lies on the vertical line. For example, for variety $j' < j_t^*$, the equilibrium is at point B, where there is full employment and the wage is unconstrained (i.e., workers receive a wage strictly above its lower bound). By contrast, for variety $j'' > j_t^*$, the equilibrium is at point C, where there is involuntary unemployment and the wage lower bound is binding (i.e., the wage is stuck at its lower bound). Aggregate unemployment, u_t, is given by the surface of the triangular area located above the downward sloping line, to the left of the vertical line, and below 1.

Next, we analyze the determination of j_t^* in general equilibrium. To this end, write the wage aggregation equation (4) as

$$\begin{aligned} W_t^{1-\eta} &= \int_0^1 W_{jt}^{1-\eta} dj \\ &= \int_0^{j_t^*} [\gamma(j_t^*)W_{t-1}]^{1-\eta} dj + \int_{j_t^*}^1 [\gamma(j)W_{t-1}]^{1-\eta}\, dj \\ &= W_{t-1}^{1-\eta}\left[j_t^*\gamma(j_t^*)^{1-\eta} + \int_{j_t^*}^1 \gamma(j)^{1-\eta} dj \right]. \end{aligned}$$

The second equality follows from the results summarized in (15). Using the definition of wage inflation given in (14) and rearranging gives

$$(1+\pi_t^W)^{1-\eta} = j_t^*\gamma(j_t^*)^{1-\eta} + \int_{j_t^*}^1 \gamma(j)^{1-\eta} dj. \tag{16}$$

According to this expression, wage inflation is increasing in the cut-off labor variety j_t^*. To understand why, suppose that the cut-off variety increases from $j_t^{*'}$ to $j_t^{*''} > j_t^{*'}$. Then, all varieties from 0 to $j_t^{*'}$ are unconstrained before and after the increase in j_t^*. As a result, their wages increase from $\gamma(j_t^{*'})W_{t-1}$ to $\gamma(j_t^{*''})W_{t-1}$. Varieties j between $j_t^{*'}$ and $j_t^{*''}$ were constrained before the change and become unconstrained after. For these workers, the wage rate increases from $\gamma(j)W_{t-1} < \gamma(j_t^{*''})W_{t-1}$ to $\gamma(j_t^{*''})W_{t-1}$. Finally labor varieties $j > j_t^{*''}$ are constrained before and after the change in j_t^*, so their wages remain unchanged. Since for every variety j the nominal wage either increases or stays constant, it follows that the average wage increases.

We are now ready to define the competitive equilibrium in j_t^* form.

Definition 2 (Competitive Equilibrium in j^* Form) *A competitive equilibrium is a set of processes j_t^*, y_t, h_t, $w_t \equiv W_t/P_t$, i_t, π_t, and π_t^W, satisfying*

$$y_t = z_t F(h_t), \tag{17}$$

$$U'(y_t) = \beta(1+i_t)E_t \frac{U'(y_{t+1})}{1+\pi_{t+1}}, \tag{18}$$

$$z_t F'(h_t) = w_t, \tag{19}$$

$$1 + i_t = \frac{1+\pi^*}{\beta}\left(\frac{1+\pi_t}{1+\pi^*}\right)^{\alpha_\pi}\left(\frac{y_t}{y}\right)^{\alpha_y}\mu_t, \tag{20}$$

$$1 + \pi_t^W = \frac{w_t}{w_{t-1}}(1+\pi_t), \tag{21}$$

$$\bar{h}(1-u_t^n) = \left(\frac{\gamma(j_t^*)}{1+\pi_t^W}\right)^{-\eta} h_t, \tag{22}$$

and

$$(1+\pi_t^W)^{1-\eta} = j_t^*\gamma(j_t^*)^{1-\eta} + \int_{j_t^*}^1 \gamma(j)^{1-\eta}dj, \tag{23}$$

given the initial condition w_{-1} and the stochastic processes z_t, μ_t, and u_t^n.

Equilibrium conditions (17)–(21) are standard components of optimizing monetary models, with or without nominal rigidity. The Keynesian features of the model appear in the last two equilibrium conditions. Equation (22) says that there is one labor variety, j_t^*, for which there is full employment and the wage constraint just binds. Equation (23) says that wage inflation is a weighted average of the wage increase across varieties relative to the average wage prevailing the previous period. For equation (23) to hold with equality at all times it must be the case that in equilibrium wage inflation be neither too high nor too low so as to rule out the corner solutions $j_t^* = 0$ and $j_t^* = 1$.[1]

In this model, monetary disturbances have real effects. To see this, it suffices to consider, as an example, a situation in which the economy is initially in steady state and in period 0 experiences an unexpected purely transitory fall in the monetary disturbance μ_t. Suppose that after the shock there is perfect foresight. Suppose, contrary to the claim, that the fall in μ_t does not affect the real allocation (y_t or h_t for any $t \geq 0$). Then, by the Euler equation (18) and the Taylor rule (20), we have that the inflation rate π_t must change either at $t = 0$ or at $t = 1$ or both. Also, by the labor demand (19), the real wage w_t must stay constant, otherwise h_t would move. Then, by (21), wage inflation, π_t^W, must change either at $t = 0$ or at $t = 1$ or both. In turn, by (22), j_t^* must change either at $t = 0$ or at $t = 1$ or both, but in such a way as to keep constant the ratio $\gamma(j_t^*)/(1+\pi_t^W)$, otherwise h_t would be affected. But, according to (23), the ratio $\gamma(j_t^*)/(1+\pi_t^W)$ can stay constant only if $\gamma'(j) = 0$, which is a contradiction.

[1] Formally, for equilibria displaying small fluctuations around the steady state, an interior solution is guaranteed if $\left[\int_0^1 \gamma(j)^{1-\eta}dj\right]^{1/(1-\eta)} < 1 + \pi^* < \gamma(1)$.

3 The Wage Phillips Curve

The aggregate unemployment rate, denoted u_t, is given by the integral of the unemployment rates across all labor varieties. Formally,

$$
\begin{aligned}
u_t &\equiv \int_0^1 \left(\frac{\bar{h}-h_{jt}}{\bar{h}}\right) dj \\
&= u_t^n j_t^* + \int_{j_t^*}^1 \left(\frac{\bar{h}-h_{jt}}{\bar{h}}\right) dj \\
&= u_t^n j_t^* + (1-j_t^*) - \frac{h_t}{\bar{h}} \int_{j_t^*}^1 \left(\frac{W_{jt}}{W_t}\right)^{-\eta} dj \\
&= u_t^n j_t^* + (1-j_t^*) - \left(\frac{W_{t-1}}{W_t}\right)^{-\eta} \frac{h_t}{\bar{h}} \int_{j_t^*}^1 \gamma(j)^{-\eta} dj.
\end{aligned}
$$

The second and fourth equalities follow from (15) and the third from (3). Using the definition of wage inflation given in (14) and equilibrium condition (22) to eliminate $(W_{t-1}/W_t)^{-\eta}\, h_t$, we can write

$$
u_t = u_t^n + (1-u_t^n)\left[(1-j_t^*) - \int_{j_t^*}^1 \left(\frac{\gamma(j)}{\gamma(j_t^*)}\right)^{-\eta} dj\right]. \tag{24}
$$

The right hand side of equation (24) is decreasing in j_t^*. It follows that as j_t^* increases, the unemployment rate falls. This is intuitive because all activities below the cut-off threshold j_t^* operate at full employment, so the higher the cut-off threshold is, the smaller the set of activities displaying involuntary unemployment above the natural rate will be.

Given the natural rate of unemployment, u_t^n, equations (23) and (24) implicitly represent a contemporaneous relationship involving only unemployment and wage inflation (u_t and π_t^W). Further, u_t and $\pi_t^W$are negatively related. To see this, recall that equation (23) implies that π_t^W is increasing in j_t^* and that equation (24) implies that u_t is decreasing in j_t^*. Thus, the model's implied relationship between unemployment and wage inflation represents a downward sloping wage Phillips curve. This relationship captures the idea, often used in the early empirical literature on downward nominal wage rigidity (e.g., Card and Hyslop, 1997), that inflation greases the wheels of the labor market.[2]

We note that the wage Phillips curve implied by the model is static. In particular, it does not feature future expected inflation. In this sense, the present model departs from the new-Keynesian framework in which the wage Phillips curve is forward looking (Erceg, Henderson, and Levin, 2000; Galí, 2011). In both models, households and firms are rational, optimizing, and forward looking. The reason why the new-Keynesian model produces a forward-looking

[2]The phrase is often attributed to Tobin (1972), although that paper does not explicitly use it.

Phillips curve is its assumption that workers have monopoly power. By contrast, in the heterogeneous downward nominal wage rigidity model proposed here, households and firms are assumed to be price takers in the labor market. In this way, the present model provides microfoundations to Phillips's original formulation of a static wage Phillips curve (Phillips, 1958). The following proposition summarizes this result.

Proposition 1 (Phillips's Phillips Curve) *The model with heterogeneous downward nominal wage rigidity implies a static negative relation between wage inflation, π_t^W, and the unemployment rate, u_t.*

We now turn to the characterization of the wage Phillips curve in the short and long runs.

3.1 The Short-Run Wage Phillips Curve

The short-run wage Phillips curve is the locus of points (u_t, π_t^W) satisfying equations (23) and (24) for a given value of the natural rate of unemployment u_t^n.

To illustrate the properties of the short-run wage Phillips curve implied by the model, we consider a linear functional form for $\gamma(j)$ and calibrate its parameters. Specifically, assume that

$$\gamma(j) = (1 + \pi^*)^\delta (\Gamma_0 + \Gamma_1 j). \tag{25}$$

Here, the parameter $\delta \in [0, 1]$ captures the degree of wage indexation to long-run inflation, and the parameters $\Gamma_0, \Gamma_1 > 0$ govern the degree of downward nominal wage rigidity. The time unit is a quarter. We set $\Gamma_0 = 0.978$ and $\Gamma_1 = 0.031$ to match the slope of the wage Phillips curve at a particular wage-inflation unemployment pair. Specifically, we target a slope of -0.74 if π_t^W is expressed in percent per year (or -0.74/4 if π_t^W is expressed in percent per quarter), which is consistent with the estimate presented in Galí and Gambetti (2019) for the United States over the period 1986 to 2007. Also, we target a steady-state rate of unemployment of 6 percent and a steady-state rate of inflation of 3 percent per year to match the median values observed in the United States over the period 1986 to 2007 (the sample period in Galí and Gambetti, 2019). That is, we assume that when π_t^W is equal to 3 percent per year, then u_t is equal to 6 percent and the slope of the wage Phillips curve is -0.74. We set the steady-state natural rate of unemployment at 4 percent ($u^n = 0.04$) and fix u_t^n at u^n. We assume full indexation of wages ($\delta = 1$), as in much of the related literature. For example, Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) assume that the weights on steady-state inflation and lagged inflation in the indexation scheme add up to one. Section 3.2 shows that when $\delta = 1$, the steady-state real allocation (y_t, h_t, and u_t) is independent of the inflation rate. Finally, we set the elasticity of substitution across labor

Table 1: Parameter Values

Parameter	Value	Description
Γ_0	0.978	Parameter of the $\gamma(j)$ function
Γ_1	0.031	Parameter of the $\gamma(j)$ function
δ	1	Wage indexation parameter of the $\gamma(j)$ function
π^*	$1.03^{1/4} - 1$	Steady state inflation rate
u^n	0.04	Natural rate of unemployment
η	11	Elasticity of substitution across labor varieties
β	0.99	Subjective discount factor
σ	1	Inverse of intertemporal elasticity of substitution
θ	5	Inverse of Frisch elasticity of labor supply
α	0.75	Labor elasticity of output
α_π	1.5	Inflation coefficient of Taylor rule
α_y	0.125	Output coefficient of Taylor rule
ρ_μ	0.5	Persistence of monetary shock
ρ_z	0.9	Persistence of technology shock

Note. The time unit is a quarter.

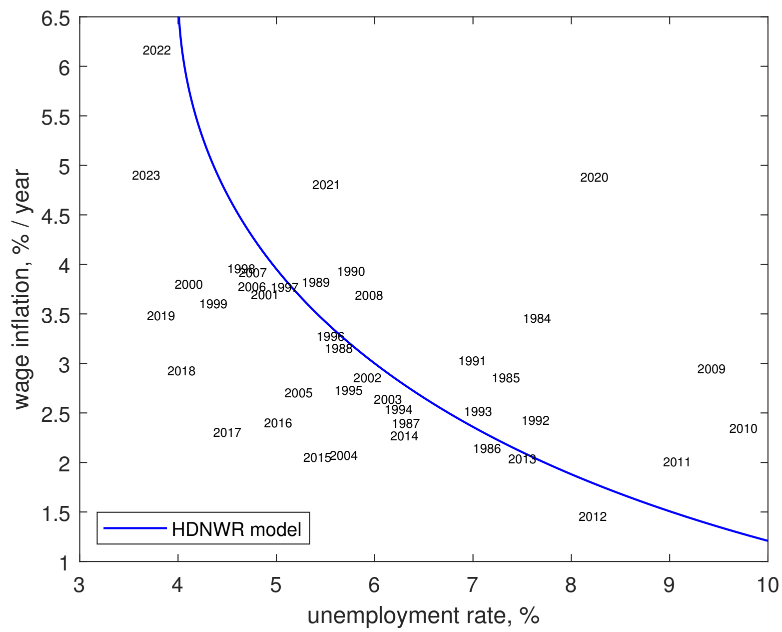
varieties to 11 ($\eta = 11$). This number is an average of the values used in Erceg, Henderson, and Levin (2000), Christiano, Eichenbaum, and Evans (2005), and Galí (2015). The top panel of Table 1 summarizes the parameter values used in the computation of the Phillips curve. (The bottom panel of this table is discussed in section 4.)

Figure 2 shows with a solid line the short-run wage Phillips curve predicted by the calibrated heterogeneous downward nominal wage rigidity model in the space (u_t, π_t^W). By construction, when the unemployment rate is 6 percent, the annual wage inflation rate is 3 percent. Also by construction, at that point, the slope of the Phillips curve is equal to -0.74. The predicted wage Phillips curve is convex, relatively steep at high levels of inflation and relatively flat at low levels of inflation implying that the costs in terms of unemployment of reducing high inflation are relatively small. For example, lowering inflation from 6 to 5 percent would increase the unemployment rate by only 0.3 percentage points, whereas lowering inflation from 3 to 2 percent would increase the unemployment rate by 1.7 percentage points.

Figure 2 also shows annual U.S. unemployment and wage inflation data for the period 1984 to 2023.[3] The nonlinearity of the Phillips curve, which was not targeted in the

[3] Annual wage inflation is computed as the average of year-over-year monthly wage inflation. The measure of monthly nominal wages is Average Hourly Earnings of Production and Nonsupervisory Employees, FRED series AHETPI. The annualized unemployment rate is the arithmetic mean of monthly unemployment rates, FRED series UNRATE. The observation labeled 2023 in the figure refers to unemployment and wage inflation in the first six months of 2023.

Figure 2: The Short-Run Wage Phillips Curve



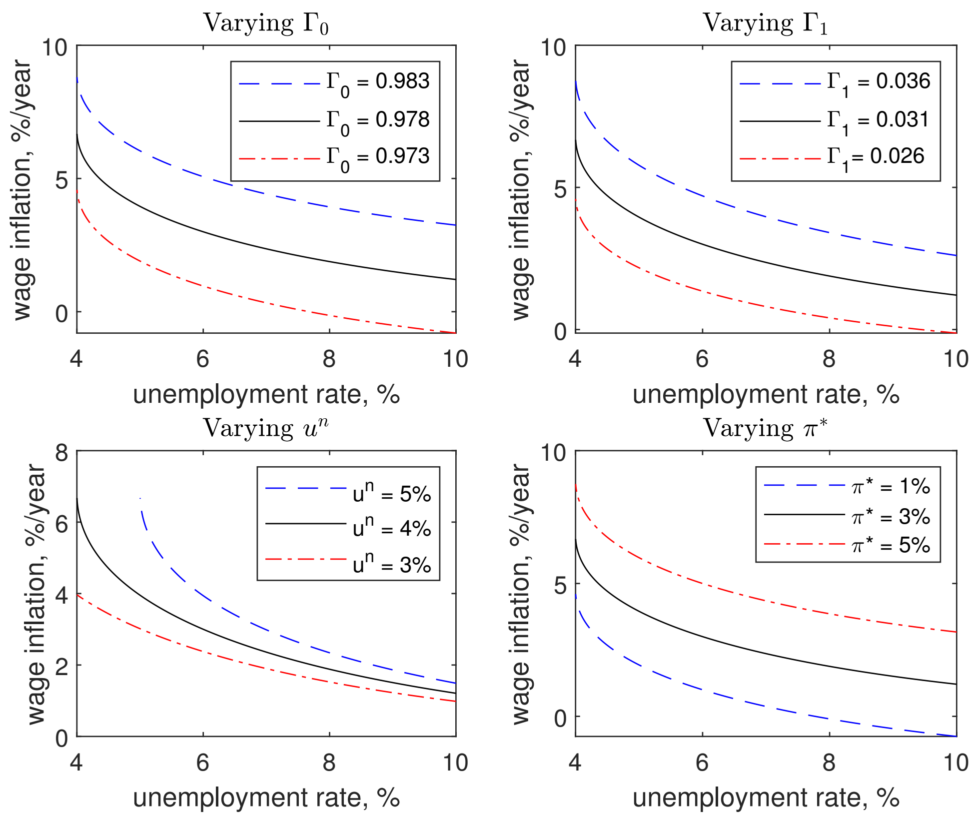
Notes. The figure shows with a solid line the short-run wage Phillips curve implied by the calibrated heterogeneous downward nominal wage rigidity (HDNWR) model. The figure also shows the (u_t, π_t^W) pairs observed in annual U.S. data over the period 1984 to 2023.

calibration—recall that the calibration targets only one point along the Phillips curve and the slope at that point—captures relatively well the overall shape of the observed cloud of unemployment and wage inflation pairs. In particular, the post Covid-19 observations (2022 and 2023) characterized by high inflation and low unemployment fall reasonably close to the steep portion of the Phillips curve implied by the calibrated model.

Taken together, the results presented in this section suggest that the heterogeneous downward nominal wage rigidity model captures the empirical regularity first documented by Phillips, namely, that the relationship between unemployment and wage inflation is nonlinear and, in particular, convex. Phillips hypothesized that the reason for the observed nonlinearity was downward nominal wage rigidity, but he did not present a theoretical model with that feature. A contribution of the present paper is therefore to show that downward nominal wage rigidity can indeed give rise to an empirically plausible nonlinear short-run wage Phillips curve.

Figure 3 displays how changes in key structural parameters of the model shift the short-run wage Phillips curve. A given level of unemployment requires a higher wage inflation rate the more downwardly rigid nominal wages are (the higher Γ_0 and Γ_1 are) and the higher the inflation target is (the higher π^* is). That is, an increase in any of these three parameters

Figure 3: Shifters of the Short-Run Wage Phillips Curve



Notes. Solid lines correspond to the baseline calibration. The parameter π^* is the inflation target. The parameters Γ_0 and Γ_1 pertain to the wage lower bound function $\gamma(j) = (1+\pi^*)^\delta(\Gamma_0 + \Gamma_1 j)$ (equation 25). The parameter u^n represents the natural rate of unemployment. The figure shows that the short-run wage Phillips curve shifts up and to the right when the degree of downward nominal wage stickiness increases (Γ_0 or Γ_1 increase), when the natural rate of unemployment rises (u^n increases), or when the inflation target increases (π^* increases).

shifts the short-run Phillips curve up and to the right. The intuition behind these effects is as follows. Wage inflation acts as a lubricant of the labor market because the higher wage inflation is, the larger the number of activities that are not constrained by the wage lower bound will be. An increase in Γ_0, Γ_1, or π^* raises the wage lower bound. Thus, the economy needs more lubricant to maintain the same level of unemployment. For the same reason, an increase in the degree of wage indexation, δ, (not shown in Figure 3) moves the Phillips curve up and to the right. Finally, an increase in the natural rate of unemployment shifts the short-run Phillips curve to the right, but not one for one. Intuitively, the reason is that for varieties that display Keynesian unemployment ($u > u^n$), an increase in the natural rate of unemployment simply changes the unempolyment type of some workers from Keynesian to natural. But for labor varieties displaying full employment ($u = u^n$, or zero Keynesian unemployment), an increase in the natural rate of unemployment raises the unemployment rate one for one.

3.2 The Long-Run Wage Phillips Curve

The long-run wage Phillips curve is the locus of points $(u_t, \pi_t^W) = (u, \pi^W)$ satisfying equations (23) and (24) for $u_t^n = u^n$, where variables without a time subscript denote steady-state values. The difference between the short- and long-run Phillips curves is that in the long run wage inflation and price inflation are both equal to the inflation target. Specifically, because output is constant in the steady state, the Euler equation (18) implies the long-run Fisher relationship

$$i = \frac{1+\pi}{\beta} - 1.$$

This expression and the Taylor rule (20) imply that in the steady state inflation must be at its target level,

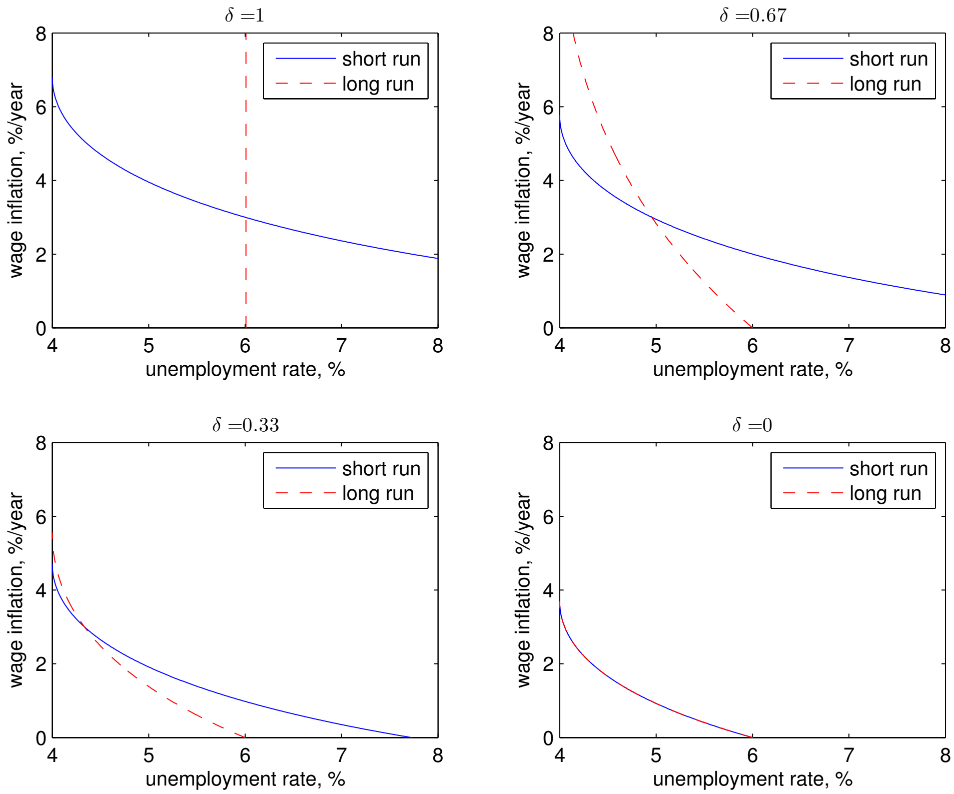
$$\pi = \pi^*.$$

Since in the steady state the real wage is constant, equilibrium condition (21) implies that wage inflation equals product-price inflation,

$$\pi^W = \pi^*.$$

Equilibrium conditions (23) and (24) evaluated at $u_t = u$, $\pi_t^W = \pi^*$, and $u_t^n = u^n$ constitute a relationship between inflation and unemployment in the steady state, which we call the long-run wage Phillips curve. It follows immediately that in the absence of wage indexation ($\delta = 0$), that is, when the function $\gamma(\cdot)$ is independent of π^*, the short- and long-run Phillips curves coincide. But this ceases to be the case when wages are indexed to steady-state

Figure 4: The Long-Run Wage Phillips Curve



Notes. The parameter δ pertains to the wage lower bound function $\gamma(j) = (1+\pi^*)^{\delta}(\Gamma_0 + \Gamma_1 j)$ (equation 25). The figure shows that the long-run wage Phillips curve is in general downward sloping and steeper than its short-run counterpart. The long-run wage Phillips curve is vertical when $\delta = 1$ and identical to the short-run wage Phillips curve when $\delta = 0$.

inflation. To see this, consider again the linear functional form for $\gamma(\cdot)$ given in equation (25). In this case, equilibrium conditions (23) and (24) evaluated at $u_t = u$, $\pi_t^W = \pi^*$, and $u_t^n = u^n$, become

$$(1+\pi^W)^{(1-\eta)(1-\delta)} = j^* \widetilde{\gamma}(j^*)^{1-\eta} + \int_{j^*}^{1} \widetilde{\gamma}(j)^{1-\eta} dj, \tag{26}$$

$$u = u^n + (1-u^n)\left[(1-j^*) - \int_{j^*}^{1} \left(\frac{\widetilde{\gamma}(j)}{\widetilde{\gamma}(j^*)}\right)^{-\eta} dj\right], \tag{27}$$

where $\widetilde{\gamma}(j) \equiv \Gamma_0 + \Gamma_1 j$.

It is clear from (26) and (27) that under full wage indexation ($\delta = 1$) the long-run wage Phillips curve is perfectly vertical in the space (u, π^W). This is intuitive. Under full indexation, an increase in inflation fails to inject grease in the labor market in the long run, as indexation soaks it up one for one. By contrast, under imperfect indexation ($\delta < 1$), only a fraction δ of an increase in inflation is absorbed by indexation and the rest is grease to the labor market.

To see more precisely what happens for intermediate degrees of wage indexation ($\delta \in (0,1)$), Figure 4 displays the long-run wage Phillips curve for four different degrees of wage indexation and compares it to its short-run counterpart. The figure illustrates that absent full indexation the long-run wage Phillips curve is downward sloping and that as the degree of wage indexation goes down the slope of the long-run wage Phillips curve falls. In fact, the long-run wage Phillips curve rotates around the point $(u, \pi^W) = (0.06, 0)$ counterclockwise as δ declines. To see why this is so, recall that the calibration targets an unemployment rate of 6 percent and assumes full indexation. Therefore, the left-hand side of (26) is equal to 1, regardless of the value of π^W. This uniquely pins down the steady value of j^* and by equation (27) also the steady state value of u. When $\delta < 1$ and inflation is zero ($\pi^W = 0$), then the left-hand side of equation (26) is also equal to 1, regardless of the value of δ. Thus, the long-run wage Phillips curve must contain the point $(u, \pi^W) = (0.06, 0)$ for any value of δ.

Comparing the long-run and the short-run Phillips curves, the figure shows that for positive degrees of wage indexation $\delta \in (0, 1]$, the long-run Phillips curve is steeper than its short-run counterpart. The intuition why the wage Phillips curve is steeper in the long run is as follows. In the short run, movements in the inflation rate are not accompanied by movements in the long-run rate of inflation, so they grease the labor market one for one. By contrast, to the extent that δ is greater than zero, only a fraction $(1 - \delta)$ of an increase in inflation greases the labor market in the long run.

4 Regular Dynamics

We have established that the heterogeneous downward nominal wage rigidity (HDNWR) model predicts that the unemployment costs of reducing inflation are much lower at high inflation rates than at low inflation rates. This result concerns the global properties of the model. A natural question is whether for regular fluctuations in a neighborhood around the intended inflation target, the HDNWR model produces equilibrium dynamics that are similar to those induced by standard new-Keynesian models of nominal wage rigidity. The answer is that this is indeed the case. This result is of interest because the wage Phillips curve implied by the HDNWR model is static whereas the one implied by existing new-Keynesian models of wage stickiness is forward looking. The claim in this section is not that the similarity in the dynamics predicted by the two models is necessarily valid for all possible calibrations and shock specifications. Rather the claim is that it is valid for calibrations and shocks typically used in the related literature.

To derive this result the section characterizes the equilibrium dynamics of the HDNWR model and compares them to those implied by the new-Keynesian (NK) model with Calvo-type wage stickiness. Because an endogenous labor supply is a necessary feature of the latter model, to facilitate comparison, the section begins by endogenizing the labor choice in the HDNWR model.

4.1 The HDNWR Model with Endogenous Labor Supply

Suppose now that the representative household derives disutility from supplying labor. Specifically, replace the lifetime utility function considered thus far with the function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U(c_t) - \int_0^1 V(h_{jt}^s) dj \right], \tag{28}$$

where h_{jt}^s denotes the amount of labor of type j supplied in period t, and $V(\cdot)$ is a convex labor disutility function. To facilitate aggregation, we use the functional form

$$V(h) = \frac{h^{1+\theta}}{1+\theta}, \tag{29}$$

which is often used in the related literature (e.g., Galí, 2015). As before, there is rationing in the labor market: for each labor type j, at the going wage W_{jt} households may not be able to sell all the units of labor they offer. The household sets its desired supply of labor of variety j to equate the marginal rate of substitution between labor and consumption to the variety-specific real wage. Formally, the supply of labor of type j is given by

$$\frac{V'(h_{jt}^s)}{U'(c_t)} = w_{jt}, \tag{30}$$

where $w_{jt} \equiv W_{jt}/P_t$. We continue to assume that there is an exogenous amount of involuntary unemployment unrelated to wage stickiness, embodied in the variable u_t^n denoting the natural rate of unemployment. The restriction that employment is voluntary now takes the form

$$h_{jt} \leq h_{jt}^s (1 - u_t^n). \tag{31}$$

This expression says that the household is not willing to have more members employed than the ones it voluntarily supplies to the market net of the ones that are naturally unemployed.

The household's budget constraint and the optimality conditions associated with consumption and bond holdings are unchanged. The firm's demand for labor of variety $j \in [0, 1]$, given by equation (3), is also unchanged.

As before, we consider an equilibrium in which each period $t \geq 0$ there is a cut-off labor variety, j_t^*, that operates at full employment,

$$h_{j_t^* t} = h_{j_t^* t}^s (1 - u_t^n), \tag{32}$$

and for which the wage constraint holds with equality,

$$W_{j_t^* t} = \gamma(j_t^*) W_{t-1}. \tag{33}$$

Combining these two conditions with the labor demand (3) and the labor supply (30) yields

$$\frac{V'\left(\frac{h_t}{1-u_t^n}\left(\frac{\gamma(j_t^*)}{1+\pi_t^W}\right)^{-\eta}\right)}{U'(c_t)} = \frac{\gamma(j_t^*) w_{t-1}}{1+\pi_t}. \tag{34}$$

It can be shown that, as in the case of an inelastic labor supply, all labor varieties $j < j_t^*$ operate at full employment and are paid the same wage as variety j_t^*. Also, all varieties $j > j_t^*$ are constrained by the wage lower bound and suffer unemployment above the natural rate.

The definition of a competitive equilibrium with an endogenous labor supply is then identical to that given in Definition 2, except that equation (22) is replaced by equation (34).

With an endogenous labor supply, the unemployment rate is the ratio of unemployed labor to the total labor supply. Formally,

$$u_t = \frac{\int_0^1 (h_{jt}^s - h_{jt}) dj}{\int_0^1 h_{jt}^s dj}.$$

Using the functional form (29) for the disutility of labor and equations (3), (30), (32), and (33), we can rewrite the unemployment rate as

$$u_t = u_t^n + (1 - u_t^n) \frac{\int_{j_t^*}^1 \left[\left(\frac{\gamma(j)}{\gamma(j_t^*)}\right)^{\frac{1}{\theta}} - \left(\frac{\gamma(j)}{\gamma(j_t^*)}\right)^{-\eta}\right] dj}{j_t^* + \int_{j_t^*}^1 \left(\frac{\gamma(j)}{\gamma(j_t^*)}\right)^{\frac{1}{\theta}} dj}. \tag{35}$$

Note that as the elasticity of labor supply approaches zero ($\theta \to \infty$), equations (34) and (35) converge to equations (22) and (24) with $\bar{h}$ normalized to 1, and the model becomes the one with inelastic labor supply studied in sections 2 and 3.

The following definition summarizes the equilibrium with endogenous labor supply.

Definition 3 (Competitive Equilibrium with Endogenous Labor Supply) *A competitive equilibrium in the economy with endogenous labor supply is a set of processes j_t^*, y_t, h_t, u_t, w_t, i_t, π_t, and π_t^W, satisfying* (17)-(21), (23), (34), *and* (35), *given the initial condition w_{-1} and the stochastic processes z_t, μ_t, and u_t^n.*

As in the case of an inelastic labor supply, the model features a wage Phillips curve implicitly given by equations (23) and (35) linking current unemployment, u_t, and current wage inflation, π_t^W.

In spite of the fact that the model features occasionally binding constraints at the level of individual varieties of labor, the complete set of equilibrium conditions given in Definition 3 does not. This means that the model is amenable to a characterization of the equilibrium dynamics using perturbation methods. We summarize this result in the following proposition:

Proposition 2 (HDNWR and Perturbation) *The equilibrium dynamics of the HDNWR model with inelastic or elastic labor supply described in Definitions 2 and 3, respectively, can be approximated using perturbation techniques.*

Thus, to obtain the implied impulse responses of the model to exogenous shocks we can follow the customary approach of linearizing the equilibrium conditions around the nonstochastic steady state.

The quantitative analysis that follows adopts this approach. The calibration of the model is summarized in Table 1. The parameters appearing in the top panel of the table were already discussed in section 3. Because the model now features an endogenous labor supply, the parameters Γ_0 and Γ_1 were recalibrated using the same targets for the slope of the Phillips curve and steady-state unemployment. The implied values are $\Gamma_0 = 0.9781$ and $\Gamma_1 = 0.0310$, which are the same as those associated with the HDNWR model with inelastic labor supply up to the third significant digit.

We assume a period consumption subutility function of the form $U(c) = (c^{1-\sigma} - 1)/(1-\sigma)$ and a production function of the form $F(h) = h^{\alpha}$. Following Galí (2015), we set $\sigma = 1$ $\alpha = 0.75$, $\beta = 0.99$, $\theta = 5$, $\alpha_\pi = 1.5$, and $\alpha_y = 0.5/4$.

4.2 Response to a Monetary Shock

We assume that the monetary shock follows an autoregressive process of order one

$$\ln \mu_t = \rho_\mu \ln \mu_{t-1} + \epsilon_t^\mu, \tag{36}$$

where ϵ_t^μ is a mean zero i.i.d. innovation, and $\rho_\mu \in [0, 1)$ is a parameter. Following Galí (2015), we set $\rho_\mu = 0.5$.

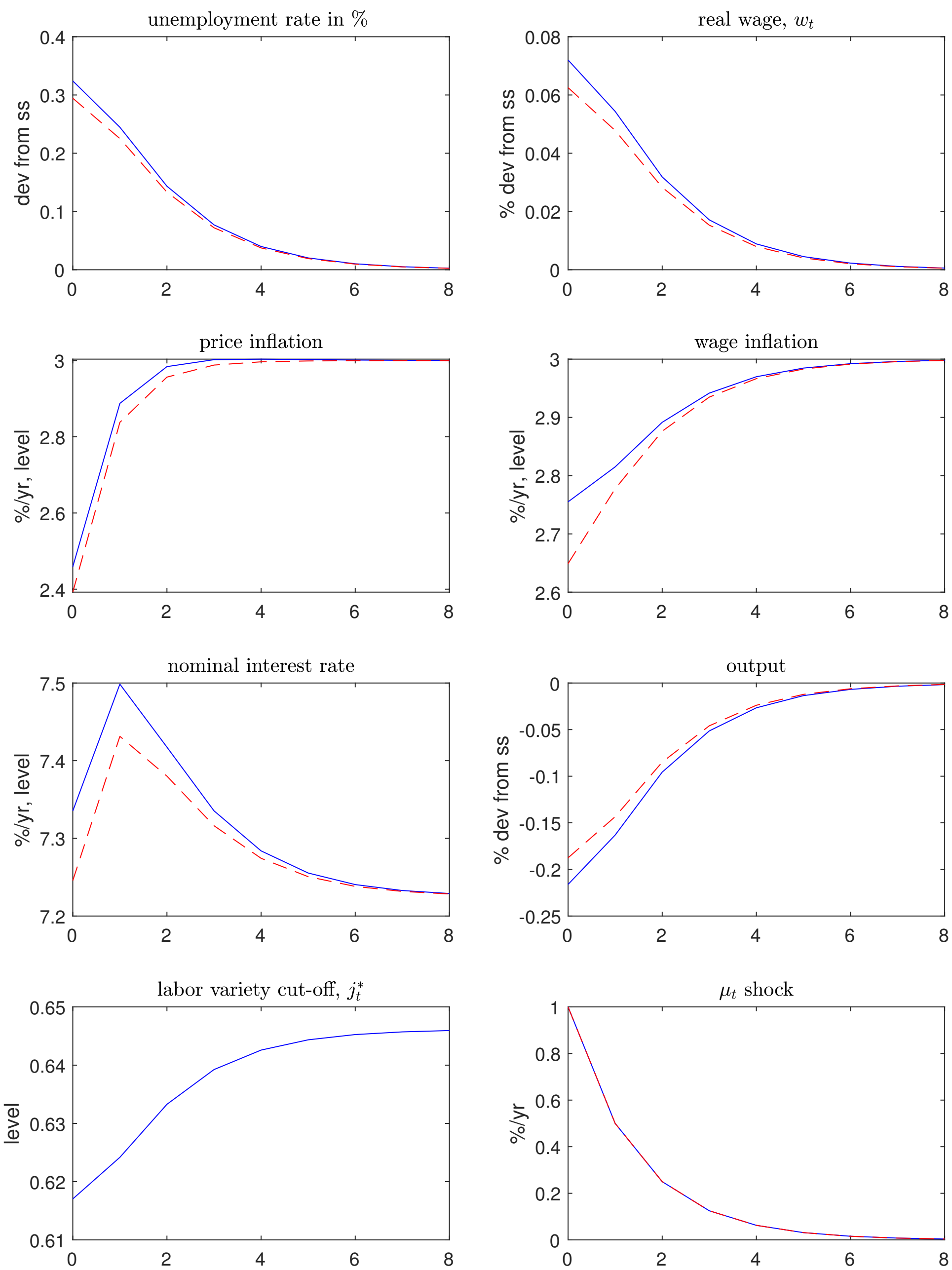
Figure 5 displays with solid lines the impulse response to a one percent annualized increase in μ_t. In equilibrium this monetary contraction results in a 0.11 percentage point increase in the policy interest rate (from its steady-state value of 7.23 percent to 7.34 percent). The increase in the interest rate is smaller than the increase in μ_t because of the contemporaneous adjustment of the endogenous variables that enter the Taylor rule, π_t and y_t. The HDNWR model predicts that the tightening in monetary conditions is deflationary. An efficient adjustment of the labor market would require a fall in nominal wages large enough to perfectly offset the fall in prices. However, due to the presence of downward nominal wage rigidity, the decline in nominal wages is insufficient. That is, a larger number of job varieties become constrained by the lower bound on nominal wages. This frictional adjustment is reflected in a decline in the labor variety cutoff j_t^*. In turn, the fact that the real wage is inefficiently high for more labor varieties causes an increase in involuntary unemployment and hence a decline in output and consumption.

For comparison, we consider a canonical NK sticky-wage model that departs from the HDNWR model only in its wage setting module. Specifically, we assume that wages are set in a Calvo fashion as in Erceg, Henderson, and Levin (2000) and define the unemployment rate as in Galí (2011). A detailed derivation of the NK model we use here can be found in a technical appendix (Schmitt-Grohé and Uribe, 2022). All parameters of the NK model that are common to the HDNWR model are assigned the same values, namely, those given in Table 1. The common parameters are π^*, η, β, σ, θ, α, α_π, α_y, and ρ_μ. As in the HDNWR model, in the NK model we assume full indexation of wages to steady-state inflation.

It remains to explain how we calibrate the degree of nominal wage rigidity in the NK model. We cannot directly adopt the strategy used to calibrate the HDNWR model, namely, to match the slope of the implied wage Phillips curve to the one estimated by Galí and Gambetti (2019). The reason is that the empirical Phillips curve estimated by Galí and Gambetti is not forward looking and therefore does not have a natural theoretical counterpart in the NK model. Instead, we assume that the fraction of types of labor that cannot reoptimize wages in any given period in the NK model is equal to the steady-state fraction of types of labor that are stuck at the wage lower bound in the HDNWR model. Formally, letting θ_w denote the fraction of wages that are not set optimally in any given period in the NK model, we impose $\theta_w = 1 - j^*$, where j^* is the deterministic steady-state value of j_t^*. The resulting value of θ_w is 0.35. This value is low relative to those typically used in the NK literature. For example, Galí (2015) sets θ_w to 0.75. To address this issue, we also consider a calibration in which $\theta_w = 1 - j^* = 0.75$.

Figure 5 displays with dashed lines the response of the NK model to a 1 percent per annum increase in the monetary shock μ_t when $\theta_w = 1 - j^* = 0.35$. The figure shows that

Figure 5: Impulse Responses to a Monetary Tightening



Notes. Solid lines correspond to the HDNWR model and dashed lines to the NK model with Calvo wage stickiness. The size of the monetary shock is 1 percent per annum and its serial correlation is 0.5. The horizontal axes measure quarters after the shock.

for most variables the responses predicted by the NK and HDNWR models are quite close. Figure 6 compares the impulse responses of the two models when $\theta_w = 1 - j^* = 0.75$. In this case, to preserve comparability, we recalibrate the HDNWR model. Specially, we drop the requirement that it matches the slope of the Galí-Gambetti wage Phillips curve and instead target a value of 0.75 for $1 - j^*$. The resulting values of Γ_0 and Γ_1 are 0.9908 and 0.0175. Understandably, because now wages are more rigid, both models predict a more subdued response of wage inflation and a larger response of unemployment. The important point for the purpose of the present discussion, however, is that both models deliver quite similar dynamics.

4.3 Response to a Technology Shock

Figure 7 displays the response of the HDNWR model to a 1-percent positive productivity shock (a 1 percent innovation in z_t). The shock is assumed to follow a first-order autoregressive process of the form

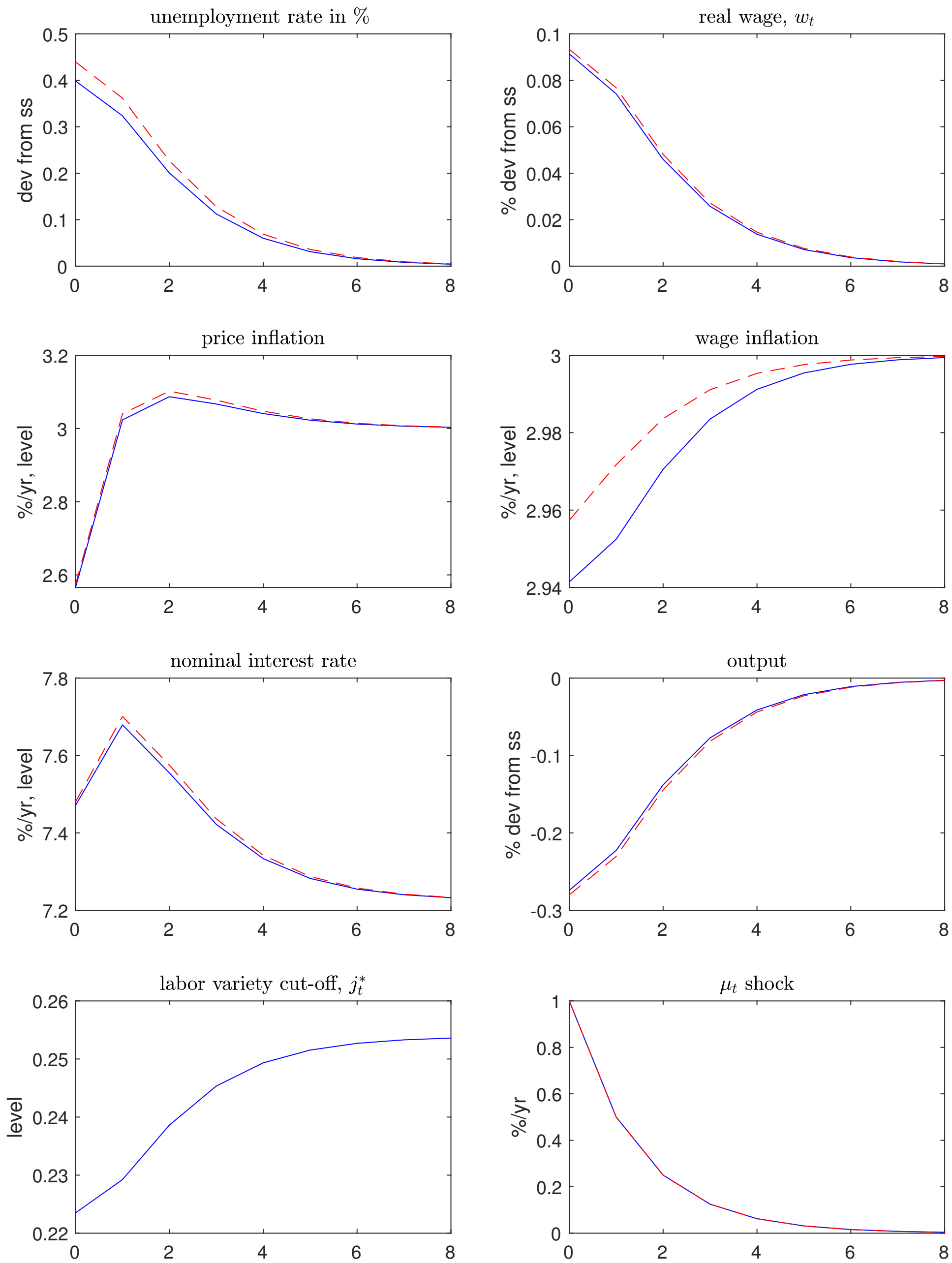
$$\ln z_t = \rho_z \ln z_{t-1} + \epsilon_t^z,$$

where ϵ_t^z is a mean-zero i.i.d. disturbance and ρ_z is a parameter. Following Galí (2015), we set ρ_z equal to 0.9.

The increase in output following the positive technology shock puts downward pressure on product-price inflation. The increase in labor productivity following the technological improvement pushes nominal wages up. This relaxes the wage constraint for some wage varieties (j_t^* goes up on impact), inducing a fall in unemployment in the initial period. As the technology shock begins to return to its stationary position, real wages fall. However, due to the presence of wage rigidity, they fall at a slower pace than the one consistent with full employment. As a result, unemployment rises and remains above steady state throughout the transition.

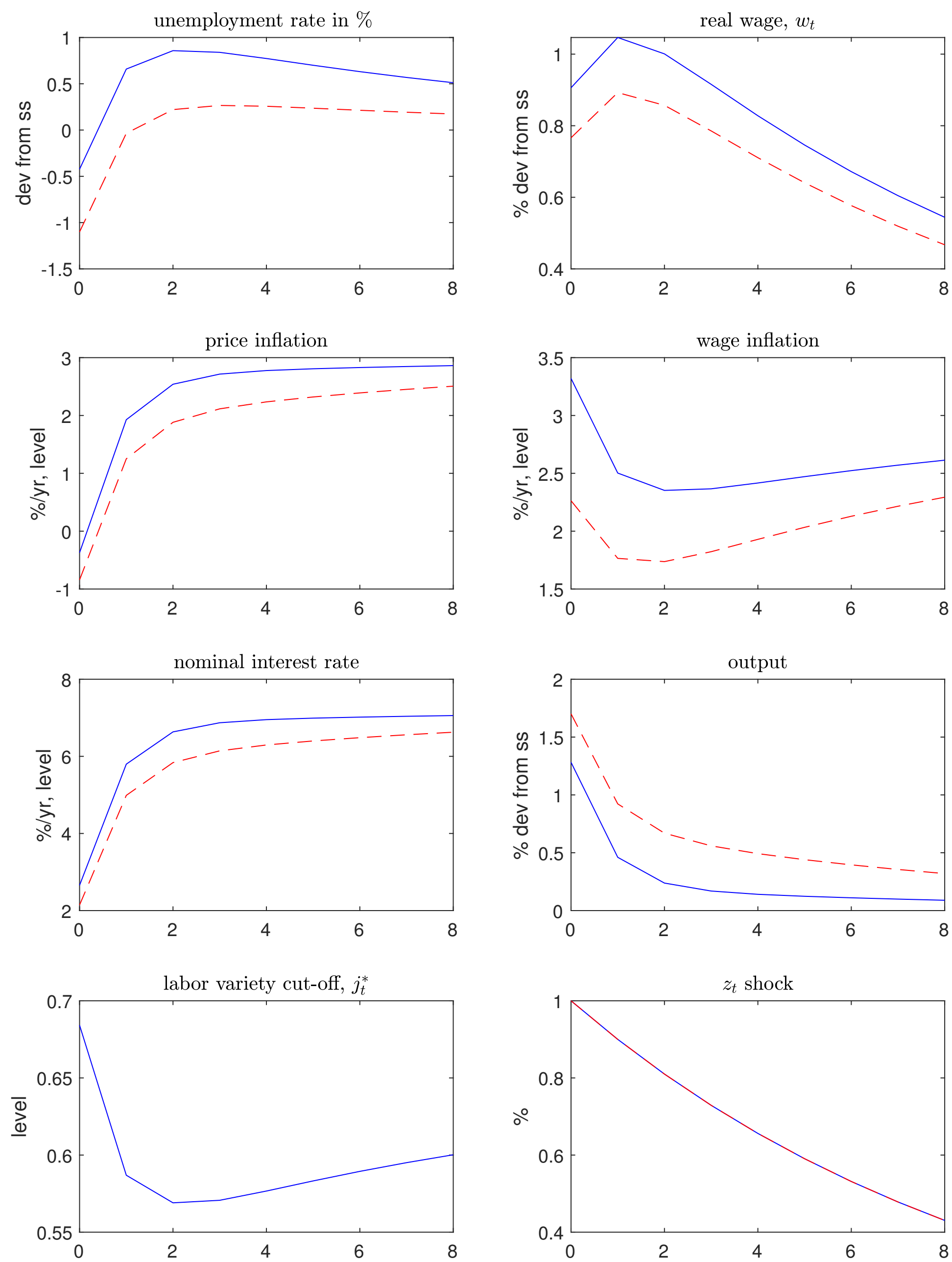
The response of the NK model to the positive productivity shock, shown with dashed lines in Figure 7, is similar. The main difference is that under the present calibration in the NK model unemployment experiences a larger decline on impact and a smaller subsequent increase. This difference in the response of unemployment is due to the relatively low value picked for the degree of wage rigidity ($\theta_w = 0.35$). When we set θ_w to the more conventional value of 0.75 and recalibrate the HDNWR model accordingly, then the response of unemployment to the technological improvement is almost the same in both models. This result is shown in Figure 8. The two models produce virtually the same responses to the positive productivity shock not only for unemployment but also for most of the other endogenous variables displayed in the figure.

Figure 6: Impulse Responses to a Monetary Tightening with a Higher Degree of Wage Rigidity ($\theta_w = 1 - j^* = 0.75$)



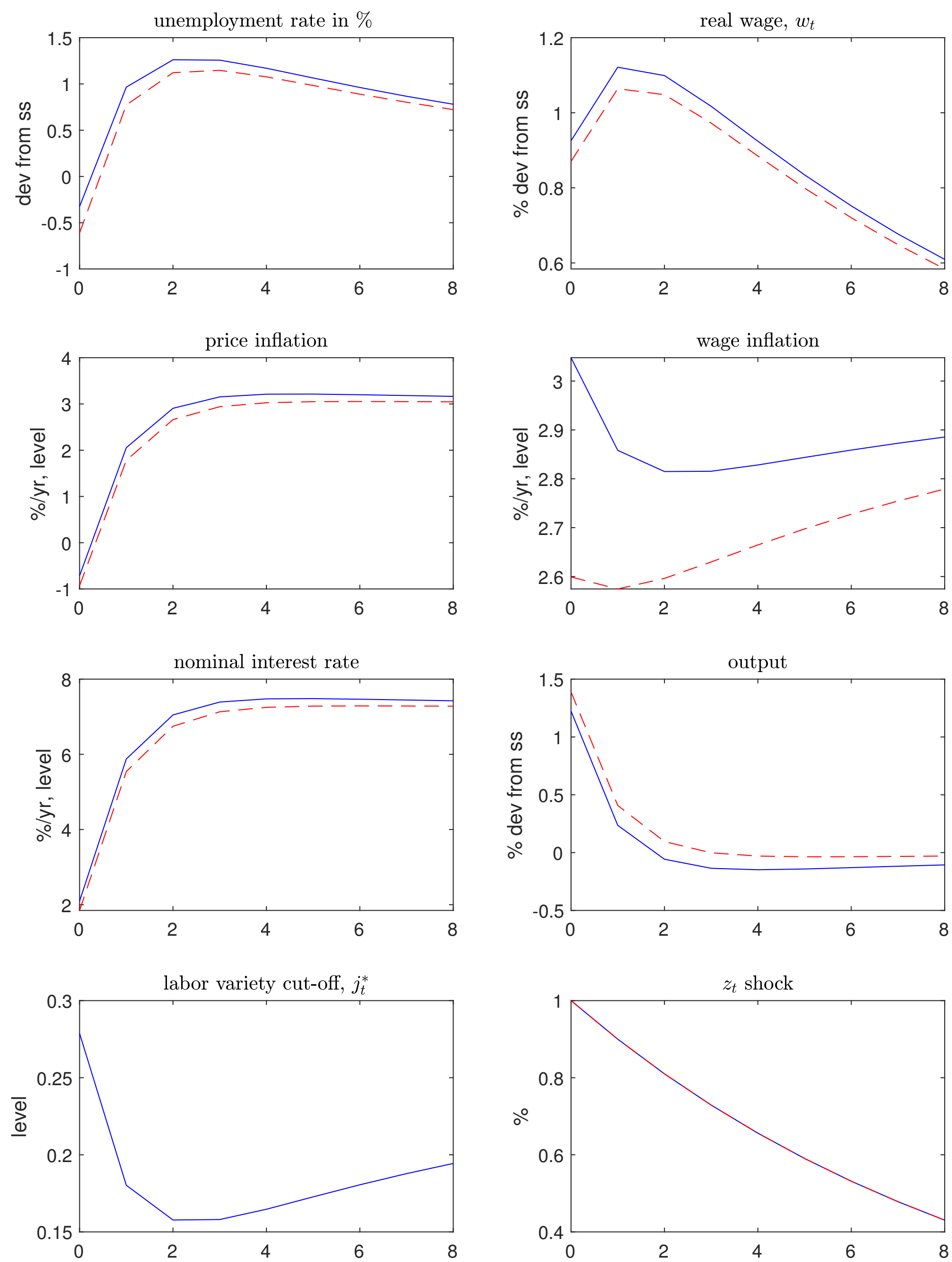
Notes. Solid lines correspond to the HDNWR model and dashed lines to the NK model with Calvo wage stickiness. The size of the monetary shock is 1 percent per annum and its serial correlation is 0.5. The horizontal axes measure quarters after the shock.

Figure 7: Impulse Responses to a Productivity Shock



Notes. Solid lines correspond to the HDNWR model and dashed lines to the NK model with Calvo wage stickiness. The size of the shock is 1 percent and its serial correlation is 0.9.

Figure 8: Impulse Responses to a Productivity Shock with a Higher Degree of Wage Rigidity ($\theta_w = 1 - j^* = 0.75$)



Notes. Solid lines correspond to the HDNWR model and dashed lines to the NK model with Calvo wage stickiness. The size of the shock is 1 percent and its serial correlation is 0.9.

Overall, the results of the present section suggest that for regular fluctuations in a neighborhood around the steady state, at least for the calibrations considered here, the forward-looking nature of the wage Phillips curve in the NK framework, which up to first order is the key difference between the HDNWR model and the NK model, does not appear to play a crucial role for the predicted response to monetary and productivity disturbances.

To obtain some intuition for this result, consider the linear versions of the wage Phillips curves in the HDNWR and NK models, which can be written, respectively, as

$$\hat{\pi}_t^W = \kappa_1 \hat{u}_t$$

and

$$\hat{\pi}_t^W = \beta E_t \hat{\pi}_{t+1}^W + \kappa_2 \hat{u}_t,$$

where a $\hat{}$ superscript denotes deviations from the nonstochastic steady state. Iterating the NK Phillips curve forward, yields

$$\hat{\pi}_t^W = \sum_{t=0}^{\infty} \beta^j \kappa_2 E_t \hat{u}_{t+j}.$$

Assume for simplicity that in equilibrium unemployment follows an AR(1) law of motion of the form $E_t \hat{u}_{t+j} = \lambda(\rho, \chi)^j \hat{u}_t$, where the persistence parameter $\lambda(\rho, \chi)$ is an endogenous object that depends not only on the persistence of the exogenous shock in question, $\rho = \rho_\mu, \rho_z$, but also on the vector χ containing other structural parameters of the NK model that influence the endogenous persistence in unemployment.[4] Then, we have that

$$\hat{\pi}_t^W = \frac{\kappa_2}{1 - \lambda(\rho, \chi)\beta} \hat{u}_t.$$

The two models will deliver more similar dynamics the more similar are κ_1 and $\kappa_2/(1 - \lambda(\rho, \chi)\beta)$. More importantly, the similarity of the two models will not be much affected by changes in the persistence of the exogenous shock, ρ, if $\lambda(\rho, \chi)$ is relatively insensitive to ρ. As it turns out, for the two shocks considered, λ is not too sensitive to changes in ρ for values of $\rho = \rho_\mu, \rho_z$ between 0 and the respective calibrated values. For this range, the endogenous persistence in unemployment built in the NK model dominates the persistence inherited from the exogenous shocks. As the exogenous shocks became highly serially correlated, their persistence might dominate the persistence of unemployment. In this range, the dynamics of the HDNWR and NK models can be quantitatively quite dissimilar.

[4]In the NK model with wage stickiness and a Taylor rule endogenous persistence arises from past real wages being a state variable.

5 Conclusion

This paper contributes to understanding nonlinearities in the trade off between inflation and unemployment. To this end, it proposes a model with heterogeneous downward nominal wage rigidity across labor varieties. This innovation results in a convex wage Phillips curve, where the relationship between wage inflation and unemployment varies depending on the level of inflation. At high inflation levels, the Phillips curve is relatively steep, indicating that the costs of reducing high inflation can be low in terms of employment. Conversely, at low inflation levels, the curve is relatively flat, suggesting that inflation has a more significant impact on employment.

The wage Phillips curve predicted by the model captures relatively well the observed pattern of wage inflation and unemployment in the United States since the mid 1980s. In particular, it predicts that the inflation-unemployment pairs corresponding to the post-Covid-19 inflation spike lie in a steep portion of the curve. The predicted nonlinearity in the Phillips curve provides an explanation for the observed robustness of the labor market during the significant monetary tightening triggered by the post-pandemic inflation episode in the United States and elsewhere.

For regular fluctuations in a neighborhood of the intended inflation target, the dynamic properties of the model are qualitatively and quantitatively comparable to those of standard new-Keynesian models for typical calibrations of the structural parameters, including the persistence of the underlying driving forces. The similarity in the dynamics of the two models for regular economic fluctuations arises in spite of the fact that the wage Phillips curve is static in the heterogeneous downward nominal wage rigidity model and forward-looking in the new-Keynesian model. The reason for this equivalence is that for levels of serial correlationof shocks typically assumed in macro models, the equilibrium persistence of unemployment is not too sensitive to the serial correlation of the driving forces. For highly persistent shocks, however, the similarity of the two models may break down.

Methodologically, the paper contributes to macroeconomic modeling by providing a model with downward nominal wage rigidity that is amenable to analysis using perturbation techniques. This property arises because, although the model features occasionally binding constraints at the micro level, no such constraints appear at the aggregate level. This is not the case in the homogeneous version of the model, which, in spite of its empirical relevance, has impeded the adoption of downward nominal wage rigidity in the formulation, computation, and estimation of medium scale models for policy evaluation. We hope that the proposed modification will significantly lower this entry barrier for models with this relevant type of nominal rigidity.

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