THE POSSIBILITY OF A SOCIAL WELFARE INDEX
Abstract and Notation

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1. Abstract.

It is generally agreed that the doctrine of consumers' sovereignty
in the determination of social welfare represents a value judgment.
Originally, this doctrine was applied by assuming the possibility of
comparing the intensities of feeling of different individuals (Marshall,
Edgeworth) through summing their utilities. More recent writers have
dropped this assumption but still express consumers' sovereignty by say-
ing that society attaches a preassigned social importance to various
individuals. This idea is implicit in much of modern welfare economics
and explicit in the work of Bergson, Lange, and Samuelson; it is also
implied in the process of voting. The chief result of the present
paper is that we cannot accomplish this last aim in general; if the possi-
bility of interpersonal comparison of feelings is denied, then there
cannot be devised any method of passing from individual to social desires
which will yield for any set of individual desires a consistent social
preference scale.

The result can be stated more precisely, as follows: By a "preference
scale" will be meant an ordering of alternative social states which is
consistent in the sense that if one state is preferred to another and the
latter to a third, then the first state is preferred to the third. Five
natural conditions are imposed on the process of passing from individual

to social states: (1) For each set of individual preference scales
(one scale for each individual), there shall correspond a social
preference scale; (2) if one state is preferred to another by society
when a certain set of individual tastes prevails and if the individual
tastes then alter in such a way that the first state rises in some
individual preference scales and does not fall in any, while the
relative position in the scales of all other states remains unchanged,
then the first state is still preferred to the second; (3) the choice
made by society from any given set of alternatives depends only on the
preferences of the individuals in society as among the alternatives in
that set; (4) given any two alternatives, there is some set of indi-
vidual preference scales such that society will prefer the first alter-
native to the second (i.e., no choices are dictated by convention);
(5) there is no individual such that society automatically prefers one
alternative to another whenever he does, regardless of the desires of
other individuals (i.e., there is no dictator). Then it can be shown
that there is no method of passing from individual to social preference
scales which will satisfy all five conditions.

It seems to me that the most reasonable condition to weaken is the
one that requires the method to work for all possible individual prefer-
ence scales. Instead, we must consider what individual desires in regard
to social decisions are likely to be in view of the socio-ethical norm
of the particular culture complex in question and frame a social welfare
index which will satisfy the various conditions prescribed above only
for such limited ranges of individual preference.

2. Notation

x, y, z	Alternative social states
x R y	x is preferred or indifferent to y by society
x P y	x is preferred to y (i.e., xRy and not yRx,) by society
x I y	x is indifferent to y (i.e., X R y and y R x), by society.

The subscript $\underline{i}$ in the letters P, P, or AI means that the relation in question holds in the preference scale of individual $\underline{i}$.

S	A set of alternative social states
C(S)	The alternative (s) which society chooses from S, i.e., the set of elements x in S such that x R y for all y in S.

The letters R, R_i by themselves denote the whole preference scales of society and of individual $\underline{i}$ respectively.

Definitions: 1. A social welfare function is a rule which yields a social preference scale for each possible set of individual preference scales.

2. A social welfare function is said to be dictatorial if the rule is such that for some individual i and every x and y, xPy, whenever x P y, regardless of the tastes of all individuals other than ___ .

3. A social welfare function is said to be conventional if for some x and y, x Ry regardless of individual tastes.

3. Symbolic Statement of Axioms, Conditions, and Results

I. For all x and y, either xRy or yRx.

II. For all x, y, and z, if xRy and yRz, then xRz.

I and II are assumed to hold for each R_i as well as for the social preference scale R.

Conditions on forming a Social Preference Scale from Individual Preference Scales.

1. For each R_i...., R_n, an R shall be formed which shall satisfy I and II.

2. Consider two alternative sets of individual tastes: the first set of tastes is expressed by the individual rankings R_{11}...,R_n; the second set by the rankings R_1;..., R^1_n. Suppose the second set R^1_i; has the properties (a) that the relative ranking of all alternatives excluding a certain alternative x, is the same as under R; and (b) that the position of the alternative x, relative to every other alternative is not lower than under R_i. Then, if the social welfare function is such that, under the set of tastes R_i, it prescribes that x shall be preferred to y, we require that under the set of tastes R^1_i, the social welfare function shall also prescribe that x is preferred to y. [1]

3. If, for all i and all x and y in some set of alternatives, x R y if and only if xR^1_i y then the alternative or alternatives chosen by society from S when tastes R_1 ..,R_n prevail are the same as those chosen when tastes $_1$,...., $R_n$prevail.

Theorem. If there are more than two alternatives, the only social welfare functions satisfying conditions 1-3 are either conventional or dictatorial.

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[1]The wording of this condition is due to F. Modigliani.