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Forecasting the Effects of Government Fiscal Policy

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(This report leans heavily on discussions with S. Cohn, G.T. Gunn, S.E. Hagen, H.G. Nelson, and A. Smithies of the U.S. Bureau of the Budget, T.M. Brown of the Department of Reconstruction and Supply, Canada, and J. Marschak, Cowles Commission.)

1. In whatever time is left over from work on the Federal budget proper, the Fiscal Analysis Division of the Bureau of the Budget investigates, partly on its own behalf and partly on behalf of the Council of Economic Advisers, the effect of various possible alternative government fiscal policies on national income in one of its various forms. It is also interested directly in the forecasting of various tax yields. Concern is divided between the national income effects of government expenditure policies such as the Marshall Plan and the national income and government revenue effects of tax rates.

To answer these questions, the usual Keynesian model, based on consumption as a function of disposable income, is used. The principal novelty is the considerably refined method of handling taxes. In many models, total tax collections (more precisely, accrued tax liabilities) have simply been taken as exogenous. As the government fixes tax rates and not total tax return, the assumption that the latter rather than the former are exogenous is clearly inadmissible, on theoretical grounds. Even for practical forecasting considerations, the forecast cannot be made by treating tax returns as exogenous, since one would not know what level to assume for them without knowing the national income to be forecast.

2. Detailed Analysis of the Budget Bureau Model.

The basic equations, (as in any Keynesian analysis) are the consumption relation

$$(2.1) \quad C = a_1 Y_1 + b_1,$$

and the identity

$$(2.2) \quad C + I + \Delta H + F + G_1 = Y_2.$$

where C = personal consumption expenditures,

Y_1 = disposable personal income,

Y_2 = gross national product,

I = expenditures on producers' durable equipment and new construction

H = stock of inventories,

F = net foreign investment,

G_1 = government purchases of goods and services,

and $\Delta H = H - H_{-1}.$

There are two equations in seven unknowns. Two of the five degrees of freedom are eliminated by taking F and G_1 as exogenous, or, better, predetermined with respect to a certain quasi-complete model. F and G_1 are certainly not independent of the rest of the economic system in the long run. This viewpoint will not be challenged here, but it is to be remarked that forecasting these variables may still involve difficulties. G_1 can presumably be determined from a detailed knowledge of the projects of the government. A question I should like to raise is whether it is the value of G_1 in constant or in current dollars which should be taken as given. The first means that the government's policy is to decide on a fixed program and spend whatever money is required at current prices; the second means that the program is increased or decreased

according as the price level falls or rises in such a way as to spend the preassigned sum of money. With the present scale of government expenditures, relatively small (percentagewise) errors in forecasting G_1 may give rise to appreciable errors in forecasting national income via the multiplier effect. Of course, this is not a major source of error.

If a definite policy of contracyclical variation in government expenditures on goods and services is adopted, G_1 can no longer be taken as an exogenous variable. Instead, the equation embodying the policy must be added to the system.

Forecasting foreign trade is much more difficult, in principle, than forecasting government spending. In the prewar period, F was probably a function of certain aspects of the domestic economic situation, so that assuming F to be exogenous may involve an error in fitting a model to prewar data. For the present, forecasting F by assuming that foreign countries will buy as much here as their dollar supply permits is surely satisfactory. Note that in this case, it is the value of F in current dollars and not that in constant dollars which can be taken as exogenous.

Even taking G_1 and F as exogenous, we are shy three equations of a complete system. The Budget Bureau has concentrated attention on analysis of the difference $Y_1 - Y_2$, which is largely, though not entirely, composed of tax collections, which in turn are related to Y_1 or Y_2. The three major kinds of taxes are indirect business tax and nontax liabilities, which are related to gross national product, corporate profits tax liability, which are related to corporate profits before inventory revaluation, and personal tax and non-tax payments, which are related to personal income.

(Note that there is implied in this classification a theory
of varying behavior on the part of businessmen as against
consumers. The former are supposed to react to tax <u>liabilities,</u>
the latter to tax <u>payments.</u>) Other relevant variables are
business reserves, dividends, transfer payments, and contributions
to social insurance funds. The Budget Bureau's "basic table"
may be represented by the following relations.

Let G_2 = Compensation of employees by government
  and government enterprises

Y_3 = private gross national product

Y_4 = national income

Y_5 = personal income

R = capital consumption allowances

P_1 = corporate profits and inventory valuation
 adjustment

P_2 = corporate inventory profits (the negative of
 corporate inventory valuation adjustment)

T_1 = indirect business tax and nontax liability
 + business transfer payments - subsidies
  current surplus of government enterprises

T_2 = net interest paid by government + government
 transfer payments + business transfer payments
 - contributions for social insurance - excess
 of wage accruals over disbursements

T_3 = corporate profits tax liability

T_4 = personal tax and nontax payments

S = undistributed corporate profits.

All concepts are taken from the National Income Supplement,
<u>Survey of Current Business,</u> July, 1947 (referred to as National
Income).

Identities:

(2.3) $\qquad Y_3 = Y_2 - G_2,$

(2.4) $\qquad Y_4 = Y_2 - T_1 - R,$

(2.5) $\qquad Y_5 = Y_4 + T_2 - S - T_3 + P_2,$

(2.6) $\qquad Y_1 = Y_5 - T_4$

Government Tax Policy Equations:

(2.7) $\qquad T_1 = \alpha_1 Y_3 + \beta_1,$

(2.8) $\qquad T_2 = \alpha_2 Y_4 + \beta_2,$

(2.9) $\qquad T_3 = \alpha_3 (P_1 + P_2) + \beta_3,$

(2.10) $\qquad T_4 = \alpha_4 Y_5 + \beta_4.$

Economic Behavior Equations:

(2.11) $\qquad R = a_2 Y_3 + b_2,$

(2.12) $\qquad P_1 = a_3 Y_3 + b_3,$

(2.13) $\qquad S = a_4(P_1 + P_2 - T_3) + b_4.$

To these we may add the assumption that G_2 is exogenous. Even with this assumption, we have added twelve equations and twelve variables and are thus three equations short of a complete system.

The number of equations and variables can be reduced by eliminating the identities and government tax policy equations. Eliminating Y_2, Y_3, Y_4, Y_5, T_1, T_2, T_3 and T_4 from equations (2.2) - (2.10), we have

(2.14) $\qquad C + I + \Delta H + F + G_1$

$$= \gamma_1 Y_1 + \gamma_2 G_2 + \gamma_3 P_1 + \gamma_4 P_2 + \gamma_5 R + \gamma_6 S + \gamma_7,$$

where the γ's are functions of the α's and β's.

The economic behavior equations (2.11) - (2.13) become

(2.15) $\quad R = a_2 \left[\gamma_1 Y_1 + (\gamma_2 - 1) G_2 + \gamma_3 P_1 + \gamma_4 P_2 + \gamma_5 R + \gamma_6 S + \gamma_7 \right] + b_2,$

(2.16) $\quad P_1 = a_3 \left[\gamma_1 Y_1 + (\gamma_2 - 1) G_2 + \gamma_3 P_1 + \gamma_4 P_2 + \gamma_5 R + \gamma_6 S + \gamma_7 \right] + b_3,$

(2.17) $\quad S = a_4 \left[(1 - \gamma_3)(P_1 + P_2) - \beta_3 \right] + b_4.$

To recapitulate, the set of equations implied in the "basic table", together with the Keynesian consumption equation and income identity, is contained in equations (2.1) and (2.14) - (2.17). The endogenous variables are C, Y_1, I, $\triangle H$, P_1, P_2, S and R; the exogenous variables are F, G_1, G_2, and the tax rates reflected in the variables γ_1, ...,γ_7 and α_3 (these variables are not independent).

3. Some Comments on the Budget Bureau Model

One question that arises is whether the various quantities appearing in the above relations are real or monetary. From the viewpoint of the tax relations, it is clear that they must be monetary except in (2.7) where the prevalence of specific excise taxes suggests that Y_3 should be taken as real. The various identities can be interpreted either way; in (2.2) and its equivalent (2.14), however, $\triangle H$ should be replaced by the value of the change in inventories if monetary quantities are used. From now on, this term will be written as $p\triangle H$, where $\triangle H$ is the change in real inventories and p is a suitable general price level (perhaps the

B.L.S. wholesale price index). The economic relations (2.11 -
2.13) and their equivalents (2.15) - (2.17) can be about as
plausibly defended on either interpretation of the symbols.
The consumption relation (2.1) presents difficulties. Usually, it
has been assumed to hold for real variables, on the basis of
theoretical reasoning as to the rational behavior of the consumer.
If the true relation is between real consumption and real income,
we have

$$\frac{C}{P} = a_1 \frac{Y_1}{p} + b_1$$

or

$$C = a_1 Y_1 + b_1 p$$

Note that the marginal propensity for real consumption out of
real income is a_1, while the marginal propensity for monetary
consumption out of monetary income (measured by time derivatives)
is $a_1 + b_1 \frac{dp}{dY}$. As $\frac{dp}{dY}$ may be taken greater than zero, the latter

marginal propensity is usually greater than the former. To take
account of a money illusion, it would perhaps be best to rewrite
the consumption equation as

(3.1) $$C = a_1 Y + b_1 p + c_1.$$

With the change suggested above, the identity (2.14)
is rewritten, letting all variables be monetary,

(3.2) $$C + I + p \Delta H + F + G_1$$
$$= \delta_1 Y_1 + \delta_2 G_2 + \delta_3 P_1 + \delta_4 P_2 + \delta_5 R + \delta_6 S + \delta_7.$$

We have now seemingly introduced an additional endogenous
variable, p, but actually, we can now write down a definition of

inventory profits, so we also have an additional relation,

(3.3) $$P_2 = (p - p_{-1})H_{-1}$$

Needless to say, I have ignored a number of problems here. For example, I have identified the price appropriate to consumption with that appropriate to inventories. Also, there may be some questions as to the relation between P_2 as defined in (3.3) and inventory revaluation adjustment as defined by the Commerce Department.

There are some plausible alternatives to the relations (2.11) - (2.13) on which (2.15) - (2.17) are based. (2.11) might be generalized by including K_{-1}, the stock of durable producers' goods at the beginning of the year as an additional variable.

(3.4) $$R = a_2 \left[\gamma_1 Y_1 + (\gamma_1 - 1)6_1 + \gamma_3 P_1 + \gamma_4 P_2 + \gamma_6 S + \gamma_7 \right] + k_2 p K_{-1} + c_2,$$

(3.5) $$K = K_{-1} + \frac{I - R}{p} .$$

(2.12) offers the greatest difficulties. If we assume that corporate behavior is not significantly different from non-corporate and that the desire of an enterprise to assume an incorporated form is based on exogenous institutional factors, we might assume corporate operating profits (i.e., exclusive of inventory profits) form a proportion of national income other than wages and salaries (P_3), the proportion being itself subject to a trend.

(3.6) $$P_1 = a_3 P_3 + b_3 t + c_3,$$

where t is time (in years). This introduces a new endogenous variable P_3. Let W be labor income from private business (compen-

sation of employees other than military and government civilian wages and salaries). Then

$$P_3 = W + G_2 + Y_4$$

For W, Klein on theoretical grounds suggested that W depends on national income other than military and government civilian wages. The same must therefore be true of P_3. Hence

(3.7) $\quad P_3 = a_5(Y_4 - G_2) + b_5(Y_4 - G_2)_{-1} + c_5 t + d_5.$

The lagged value of $Y_4 - G_2$ reflects the role of anticipations in business decisions; t reflects the growing strength of labor unions.

(3.8) $\quad Y_4 = \delta_1 Y_1 + \delta_2 P_1 + \delta_3 P_2 + \delta_4 S + \delta_5)$

where $\delta_1, \ldots, \delta_5$ are functions of $\alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_3, \beta_4,$

(3.7) is essentially a profit maximization theory.

(3.7) has been shown to have a close fit. It is equivalent to Klein's (3.3.1a) in his monograph or (3.4) in his paper The use of econometric models ...

This theory does not preclude that profits per se may effect corporations' pricing policies. It merely assumes that once prices are set, the corporations adjust output and demand for labor as to maximize profits. Discussion of price-adjusting equations will be deferred to the section on equations needed to complete the system.

(2.13) might be modified in accordance with Tinbergen's equation for dividends. Let Z_{-1} be cumulated corporate surplus at the beginning of the year.

(3.9) $\quad S = a_4 [(1 - \alpha_2)(P_1 + P_2) - \beta_2] + b_4 Z_{-1} + c_4,$

(3.10) $\quad Z = Z_{-1} + S.$

It has also been suggested that higher personal tax
rates may stimulate corporate savings. This would suggest
introducing α_4 and β_4 as variables in (3.9), but the exact
method of introducing them is not clear.

4. Equation to Complete the System.

Three equations are needed to complete the system:
an investment relation, an explanation of inventory accumulation,
and a price adjustment relation.

For the investment relation, Klein's hypothesis that
I is a function of profits, current and lagged, and the initial
stock of capital seems reasonable.

$$(4.1) \quad I = a_6 \, P_3 + b_6 \, (P_3)_{-1} + c_6 \, p K_{-1} + d_6.$$

For the explanation of inventory accumulation, Klein's
relation again seems reasonable and has fitted well.

$$(4.2) \quad H = a_7 \, p + b_7 \, p_{-1} + c_7 \left(\frac{Y_3}{p} - \Delta H \right) + d_7 \, H_{-1} + e_7.$$

I have modified Klein's expression to include gross rather than
net output in the transaction demand term. This seems reasonable
since sales are gross rather than net of depreciation.

The price-adjustment equation is the most difficult of
all, since the theory is so little known. The Walrasian law of
supply and demand in aggregative form suggests that prices move
opposite to some kind of inventories. This suggests using the
stock of inventories rather than the transaction demand as
governing the changes of prices. It has also been suggested that
large corporations (e.g., steel and automobile) charge lower
prices than they might otherwise when their profits are high
(presumably in the interests of a long run profit maximization).

$$(4.3) \quad \Delta p = a_8 \left[H - c_7 \left(\frac{Y_3}{p} - \Delta H \right) \right] + b_8 \left(P_1 + P_2 \right) + c_8$$

Apart from introducing additional variables in (4.3), other variants may be suggested. Klein, in an unpublished model for Canada, dropped the price-adjustment equation, and introduced the wage rate as a new endogenous variable and two new equations, one a wage adjustment equation like (4.3) and one a production function. The economic as opposed to the mathematical meaning of these variations is not clear.