Inflation is Conflict*

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This paper isolates the role of conflict or disagreement on inflation in two ways. In the first part of the paper, we present a stylized model, kept purposefully away from traditional macro models. Inflation arises despite the complete absence of money, credit, interest rates, production, and employment. Inflation is due to conflict; it cannot be explained by monetary policy or departures from a natural rate of output or employment. In contrast, the second part of the paper develops a flexible framework that nests many traditional macroeconomic models. We include both goods and labor to study the interaction of price and wage inflation. Our main results provide a decomposition of inflation into "adjustment" and "conflict" inflation, highlighting the essential nature of the latter. Conflict should be viewed as the proximate cause of inflation, fed by other root causes. Our framework sits on top of a wide set of particular models that can endogenize conflict.

1 Introduction

Inflation is a messy phenomenon. Despite much experience and evidence, economists still debate its origins and precise mechanisms at play. Economic models can provide a lens to tell a more transparent story. This paper offers two lenses to explore and expand the perspective that the most proximate cause of inflation is "conflict"—defined below as a disagreement on relative prices.

Many economists confidently agree that extreme and persistent inflation episodes are understood as largely driven by the growth in money supply, often prompted by a need for seignorage. But how exactly does money transmit to inflation? The simplest idea is "too much money chasing too few goods." Formalizing this involves the quantity equation or more general forms of money demand. On closer inspection, one may still wonder

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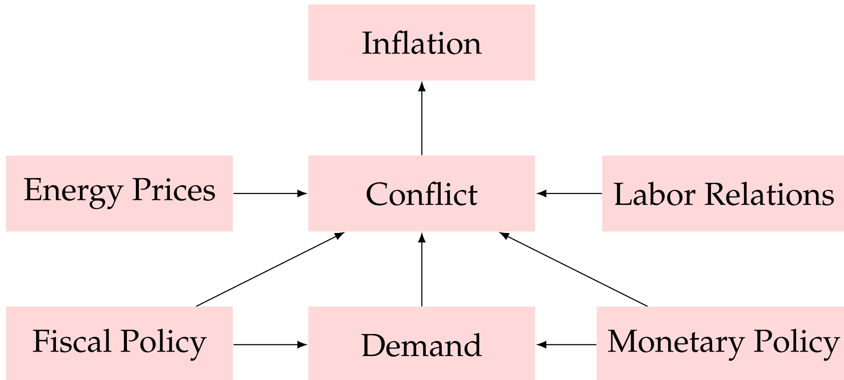

Figure 1: Inflation DAG: Conflict as Proximate with other Root Causes.

what "money chasing goods" means and how the prices in good markets adjusts to clear the money market. The story feels incomplete, as it requires out-of-equilibrium intuitions for a general-equilibrium macroeconomic situation—simple microeconomic ideas of supply and demand may not be a proper guide. Nevertheless, it is fair to say that this idea is very well rooted in economists' thinking and that money supply is central to all these stories.

For moderate and transitory inflations, things are less clear-cut and there is much less agreement. After all, money may chase goods and increase output instead of prices. Indeed, one important notion is that the transmission mechanism works from economic activity to inflation. Higher production and employment drive up costs and the real wage, leading firms to raise prices. According to these theories, inflation must be avoided by keeping the economy at the right "natural level" of output or employment, to avoid "too much spending chasing too few goods." Monetary policy, managed through interest rates shapes economic activity and inflation. Money supply is not central to this story, economic activity and its management through interest rates is.

Theories formalizing these ideas rely, among other things, on nominal rigidities. Indeed, the explicit modeling of agents setting prices greatly clarifies the mechanism generating inflation—a positive evolution relative to "money chasing goods". However, in these models, nominal rigidities are complemented with many additional assumptions to provide a complete but rather specific model and reach conclusions about natural output or natural interest rate.

To sum up, both traditional inflationary stories contain elements of truth and are not necessarily at odds with each other. In our view, these existing theories of inflation are either incomplete about the mechanism, or overly specific to cover the broad issues surrounding inflation. As such, they may describe the root causes of inflation in some situations, but fall short of isolating the more general and proximate cause of inflation. This leads to the question we address in this paper: What is the most minimal and general framework that spells out the mechanism for inflation and describes its most proximate causes?

This paper argues that the most proximate and general cause of inflation is conflict or disagreement. In this view, inflation results from incompatible goals over relative prices, with economic agents having only partial or intermittent control over nominal prices. Due to nominal rigidities, agents occasionally change a subset of prices that are under their control. Whenever they do, they adjust them to influence relative prices in their desired direction. When coupled with staggered prices this conflict manifests itself in a finite level of inflation: the conflict over relative prices is largely frustrated. Despite a stalemate in relative prices, the changes in prices motivated by this conflict gives rise to general and sustained inflation in all prices.

We argue that the conflict perspective is both insightful and general. First, we will present a situation where inflation cannot be easily understood using the traditional stories. Second, we will argue that most traditional stories of inflation are best viewed as special cases of the conflict perspective: they simply provide a theory endogenizing conflict, which then leads to inflation. Thus, nothing needs to be lost or tossed out. Instead, there are significant gains from the conflict perspective. Figure 1 illustrates our view, with conflict as the proximate cause of inflation; however, conflict is possibly affected, in turn, by many other forces; for example, conflict may be affected by the level of demand, which affects output and employment; demand, in turn may be affected by monetary and fiscal policy; thus, this would summarize very traditional stories or models of inflation. At the same time, other stories and models can fit under the conflict framework: conflict may be directly affected by labor market institutions, such as unions, or by outside shocks to the price of energy.

Our contribution consists of two separate but complementary parts. In the first part (Sections 2 and 3), we develop a stylized model that isolates conflict and helps develop intuition for the economic concept. We purposefully stay away from standard models, such as the New Keynesian model. This has a few advantages. One advantage is that it lends itself to a self-contained presentation based on basic microeconomic concepts, without requiring familiarity or adherence to particular standard macroeconomic models.

The other advantage of our stylized model is that it isolates the conflict perspective, leaving out traditional features: agents trade endowments of goods via barter using staggered prices, there is no money, no saving nor credit, no interest rates, and no way to affect the level of output. Inflation arises from the agents' desire to exercise market power, providing a clear illustration of the role of conflict in driving inflation. Since money is absent, inflation cannot result from "too much money chasing too few goods", since output cannot be affected by monetary policy, there is no natural level of output or natural interest rate to prevent inflation. We provide some extensions of our stylized model that emphasize the conflict perspective.

The contribution of this stylized model is to isolate the role of conflict in inflation. Indeed, it is meant as a shock to the system that may sow the seeds of doubt in economists, like ourselves, raised on the notion that to speak of inflation requires first and foremost a discussion of money and interest rates, complemented perhaps with the concepts of natural levels of output, employment or interest rates. The stylized model leaves no natural interpretation for inflation except for conflict.

The second part of the paper (Sections 4 and 5) is in some ways the polar opposite of the first part. We provide a general framework that helps bridge a conflict perspective with more traditional macro models. The framework is based on staggered price and wage setting as in standard New Keynesian models, but avoids adopting other special assumptions of these models. We provide an accounting exercise that decomposes wage and price inflation into an "adjustment" and a "conflict" component. Our results highlight that the adjustment component is rather limited, producing only transitory effects that are incapable of generating inflation in both wages and prices. In contrast, the conflict component is essential: it can generate more persistent inflation in both wages and prices.

Although this second part of the paper creates a bridge with standard models, it is also more general. Indeed, one benefit of the conflict perspective is that it offers a framework that sits on top of specific models, that can be seen as modeling the sources of conflict, while providing insights that are common to them.[1] For example, a conflict perspective can be made consistent with the simplest New Keynesian models of the labor market, where the marginal disutility of labor drives real wages. However, it can also easily accommodate other considerations, such as labor market institutions, search frictions, labor unions, behavioral biases. While each of these dimensions could be explicitly modeled, the conflict perspective acts as an overarching layer to think about these alternatives.

In our view, the conflict perspective should be viewed in this general and broad manner. Some may associate a conflict perspective, instead, with certain specific inflation episodes, but not others—such as during times of powerful labor unions and strained

[1] This is not unlike how economists have benefitted from thinking about growth accounting or isolating the mechanisms behind consumption smoothing choices. The economic insights gained by taking such broader perspectives become largely portable across a wide swath of models and questions.

labor relations. Likewise, some may associate it with an advocacy for income or price control policies, or as a critique of conventional interest rate policies. While these possibilities fit our general framework, none of them follow without adopting further specific modeling assumptions, which we do not undertake here. The perspective we offer in this paper is broader. Our goal is to provide a framework to think about conflict as the proximate cause of inflation. Specific models may then be thought of as endogenizing conflict in different ways, leading to different conclusions about the root causes of inflation, or about the effects of different policies.

The conflict view on inflation is by no means new, yet this perspective is largely unknown to most economists. It was developed and embraced by a relative minority associated with a Post-keynesian tradition. Rowthorn (1977) provided the seminal contribution, while Lavoie, 2022 contains an overview of the more recent literature. Our contribution extends the conflict perspective, providing new results and building bridges with traditional models. We hope our paper may help bring the conflict perspective to greater awareness among a broader set of economists.

Some work does not isolate a conflict perspective of inflation but gets close in spirit in discussions, in intuitions or in the nature of the exercises undertaken. We provide two examples. Blanchard (1986) models prices and wages rigidities and studies a permanent money supply shock. Although the analysis is carried out with a relatively standard neoclassical labor-supply framework, some of the discussions and intuitions transcend the boundaries of this territory: "attempts on both sides to maintain the same wage and price in the face of an adverse supply shock [...] lead to "cost push" inflation". Relatedly, Blanchard and Gali (2007) extend a standard New Keynesian model by adding an ad hoc real wage rigidity. In our view, this departure from the neoclassical labor-supply framework can be seen as an exploration of alternative real wage aspirations for workers from a conflict perspective.

In the second part of this paper, our paper builds on our own work on wage-price spirals Lorenzoni and Werning (2022). We adopt the staggered price and wage setting from Erceg et al. (2000), which has become a standard in the New Keynesian literature, as exemplified by Smets and Wouters (2007).

2 Inflation from Conflict without Money

In the first part of the paper we develop a stylized model that isolates inflation and conflict. The microeconomics involved is simple and fully spelled out. To ensure inflation is not driven by money we first assume that trade takes place through barter, with money and credit completely absent. To ensure inflation is not driven by high output, we assume fixed endowments. Abstracting from labor, we focus on prices only, instead of prices and wages.

2.1 Assumptions: Barter with Staggered Prices

Technology and Preferences. Consider two agents, A and B, and two goods, also labeled A and B (e.g., Apples and Bananas). Each period, agents have an endowment of their respective good—A owns A, B owns B. We normalize the endowment to one. Goods are perishable and must be consumed within each period. There is no storage technology or capital.

Preferences within a period are symmetric and given by the utility function

$$u(c, c')$$

where c denotes consumption of the own good and c' denotes consumption of the other good (e.g., for agent A, c is good A and c' is good B). The function u is concave and twice differentiable. Utility is the discounted sum

$$\sum_{t=0}^{\infty} \beta^t u(c_t, c'_t)$$

for some discount factor $\beta < 1$.

The symmetry of preferences across agents is not required for our main results, as we later show. Although in our baseline preferences are symmetric in terms of c and c', agents do not have the same preferences over goods A and B, unless we further impose symmetry across goods: $u(c, c') = u(c', c)$. For example, $u(c, c') \neq u(c', c)$ allows for "home bias" with agents preferring their own good.[2]

Regarding demographics, there are two interpretations of this baseline model. In the first, only two individuals exist in the economy. In the second, there may be many individuals of each type (possibly infinite), but each individual is permanently paired with an individual of the opposite type.[3] For concreteness, in our presentation we only refer to two individuals A and B.

[2]One may think of "home bias" in preferences as a stand in for the costs of exchange. For example, starting from a common utility over goods, if a fraction of the fruit that is exchanged becomes harmed, then in reduced form this induces preferences with home bias.

[3]Section 3 introduces a variant of the baseline model with an infinite number of individuals of each type that meet in random matches.

Competitive Equilibria and Edgeworth Box. Our baseline model is simply an exchange economy, repeating without change each period. Allocations can be pictured in an Edgeworth-box diagram.

The market arrangement in our model will feature staggered pricing and market power. However, competitive equilibria provides a benchmark, useful for comparison purposes. By symmetry of preferences and endowments, a symmetric equilibrium must exist, with a relative price of unity. To avoid trivial outcomes, we assume preferences are such that there is non-zero trade at this competitive equilibrium.[4]

Staggered Prices. Prices are set in a staggered fashion and remain unchanged for two periods: agent A sets prices in even periods and agent B in odd periods. Let P_t^* denote a newly reset price: it denotes the price of good A in even periods and the price of B in odd periods:

$$\begin{aligned} P_t^* &= P_t^A = P_{t+1}^A \qquad t = 0, 2, \ldots \\ P_t^* &= P_t^B = P_{t+1}^B \qquad t = 1, 3, \ldots \end{aligned}$$

The price of B at $t = 0$ is given. Using the above conditions, the sequence of reset prices $\{P_t^*\}$ determines all prices in this economy $\{P_t^A, P_t^B\}$. Thus, our goal is to solve for the equilibrium sequence $\{P_t^*\}$.

Prices are simply numbers expressed in a common unit of account. In other words, prices are quoted in "nominal" terms, best understood as expressed in terms of some physical or digital currency, perhaps due to convention. Importantly, however, we assume that agents have no access to such currencies in our baseline model. That is, they hold no currency and come into no contact with currency. Nor do they have access to any durable good or record-keeping device. Likewise, we will not consider trading strategies that depend on past trades,

That is, there is no money, no commodity money, no storage, and no way to save or borrow.

Trade by Barter. In our baseline model all trade takes place by barter using as terms of trade the ratio of the currently posted prices. In even periods $t = 0, 2, ...$ the relative price of A is

$$\frac{P_t^A}{P_t^B} = \frac{P_t^*}{P_{t-1}^*}$$

[4]Quantities at this symmetric equilibrium are symmetric, with both agents consuming $c = 1 - c' \geq 0$ for some $c' \geq 0$. We assume $c' > 0$. Note that the equilibrium does not generally have $c = c'$, unless utility is symmetric across goods: $u(c, c') = u(c', c)$.

while in odd periods $t = 1, 2, ...$ the relative price of A is

$$\frac{P_t^A}{P_t^B} = \frac{P_{t-1}^*}{P_t^*}$$

Quantities are determined by a take-it-or-leave-it offer proposed by one of the two agents. In our baseline, we assume that this is done by the agent who did not reset its price that period. We call this agent a "buyer" and call the agent setting the price a "seller", for that period.

In more detail, in even periods, after A has reset its price, agent B acts as the buyer and offers to buy c_t^A units of good A and pay a quantity of good B determined by the ratio of nominal prices,

$$\frac{P_t^A}{P_t^B} c_t^A = \frac{P_t^*}{P_{t-1}^*} c_t^A.$$

Upon receiving this offer, agent A can decide to accept or reject it. If the offer is accepted, they execute the barter exchange and consume; if the offer is rejected, both agents consume their endowment in that period.

In odd periods the trading protocol is identical, but reversing the roles of A and B.

2.2 Equilibrium Inflation

We now solve for an equilibrium, which turns out to be very simple. We solve backwards, starting with the buyer problem, then turning to the seller problem setting its price. We focus on Markov equilibria, that is, equilibria that depend on the relevant state variables.[5][6]

Buyer Problem. In a given period t, after prices are set, the buyer can buy the good of the seller at the relative price

$$p_t = \frac{P_t^*}{P_{t-1}^*}.$$

[5]The relevant state variables are the given nominal prices: P_{t-1}^* at the start of period t before the current price is set; (P_{t-1}^*, P_t^*) after the current price is set in period t.

[6]One could also study non-Markov, sub-game perfect equilibria, by allowing for strategies that depend on the history of past play. This gives a wider set of potential outcomes. Indeed, it may be possible to obtain the first-best with constant nominal prices $P_t^* = P_{-1}^*$ which gives the competitive equilibrium with a unitary relative price. This could be sustained by strategies that follow any deviation by reverting to the Markov equilibrium we study. For sufficiently high $\beta \to 1$ this punishment will dissuade deviations and sustain the first best.

However, we abstract from non-Markov equilibria in this paper because we find, in the present context, the Markov equilibrium concept more appealing. In addition, sustaining non-Markov equilibria becomes more involved in the extensions we develop with random matching (i.e., require greater monitoring).

Notice that if t is even, then p_t is the price of good A in terms of B, while if t is odd it is the price of good B in terms of A. Given symmetry, this notation allows us to characterize the buyer's and seller's problem in any period.

Dropping time subscripts, the buyer solves

$$V(p) = \max_{c,c'} u(c, c')$$

subject to

$$c = 1 - pc',$$

$$u(1 - c', pc') \geq u(1, 0).$$

The first constraint is simply the buyer's budget constraint; the second is a participation constraint to ensure that the seller is willing to accept the buyer's offer.[7] The solution can be written as a demand schedule

$$c' = D(p),$$

together with $c = 1 - pD(p)$. As the notation suggests, the buyer's problem is a classical consumer demand problem except for the presence of the participation constraint. However, in our baseline model this extra constraint never binds in equilibrium, as sellers choose prices that make themselves strictly better off. Indeed, the equilibrium in this baseline model is unchanged if we drop the participation constraint altogether.[8]

Seller Problem. Going backwards, consider now the seller who chooses P_t^* at the beginning of period t, taking as given the price P_{t-1}^* set by the other agent in the previous period. By varying P_t^* the seller is able to freely determine the relative price $p_t = P_t^* / P_{t-1}^*$ faced by the buyer.

Note that P_{t-1}^* has no direct impact on the set of relative prices available to the seller. In the Markovian equilibria we focus on, this implies that P_{t-1}^* will not affect the equilibrium relative price chosen by the seller, but simply scales the nominal price P_t^*. For the same reason, the seller in the current period anticipates that P_t^* will have no impact on p_{t+1}. This implies that the seller problem we study is static and given by

$$p^* \equiv \arg\max_p v(p), \tag{1}$$

[7] As usual, this is without loss of generality. A buyer can offer the autarky allocation with $c' = 0$ and $c = 1$, yielding the same outcome as a rejected offer. Thus, we can focus on equilibria where sellers accept all offers.

[8] This is no longer the case in some of our extensions. The participation constraint plays a more vital role in these extensions, even when it does not bind in equilibrium, by helping ensure that the price-setting problem is well defined.

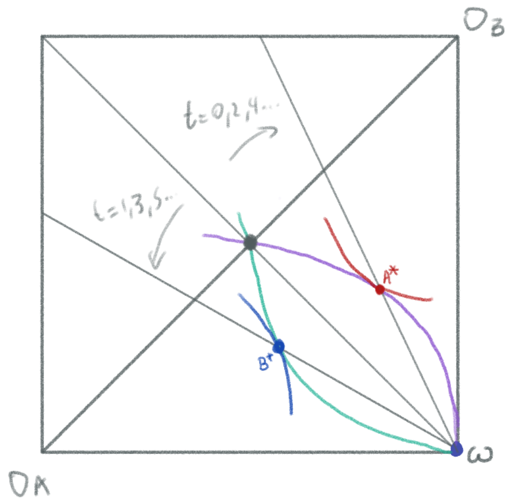

Figure 2: Edgeworth box diagram. The offer curve for A (green) and B (purple) intersect at the competitive equilibrium (assumed here with $c = c' = 1/2$). The optimum for seller A (red) and seller B (blue) feature a tangency between the offer curves and indifference curve. At the equilibrium, the relative price P_{At}/P_{Bt} cycles back and forth between these optima.

where

$$v(p) \equiv u(1 - D(p), pD(p)).$$

We assume an optimum p^* exists and is unique.[9] The first-order condition $v'(p^*) = 0$ gives

$$p^* = \frac{1}{1 - 1/\epsilon(p^*)} \cdot \frac{u_c(c, c')}{u_{c'}(c, c')}, \tag{2}$$

where $c = 1 - D(p)$, $c' = pD(p)$, and $\epsilon(p)$ denotes the local demand elasticity $-D'(p)p/D(p)$.[10] Just as for a standard monopolist, the relative price is set at a markup $\frac{1}{1-1/\epsilon}$ over the relevant marginal cost—which in this case is the marginal rate of substitution $u_c/u_{c'}$.

The equilibrium is illustrated in Figure , using an Edgeworth box diagram.

Equilibrium Inflation. Taking stock, we have seen that the rate of change in nominal prices is constant and entirely determined by preferences and endowments. Indeed, in this baseline model the solution is entirely driven by static considerations—it does not depend on the discount factor β nor on expected inflation. The next proposition collects these observation and shows that inflation is strictly positive.

[9] This is generically true, i.e., only in knife-edge specifications of u we have multiple global optima. Thus, it is relevant to focus on the uniqueness cases.

[10] As usual, a necessary condition for p^* to be optimal is $\epsilon(p^*) > 1$.

Proposition 1. *Inflation is constant and positive, unaffected by β and given by*

$$\frac{P_t^*}{P_{t-1}^*} > 1$$

where $\epsilon^* = \epsilon(P_t^*/P_{t-1}^*)$ is the local elasticity of demand.

The proof of strictly positive inflation is quite intuitive. It is natural to expect $p^* > 1$ but it cannot be read off directly from (2): the markup satisfies $\frac{1}{1-1/\epsilon} > 1$ but typically $u_c/u_{c'} < 1$. We first show that that seller always optimally chooses a relative price that is strictly above that of all competitive equilibria. Then, since there exists a symmetric competitive equilibrium with unity relative price, this implies that $P_t^*/P_{t-1}^* > 1$.[11]

Quasilinear Iso-elastic Example. To develop further intuition, we work out a simple example in closed form using the quasilinear iso-elastic utility function

$$u(c, c') = c + \bar{d}^{\frac{1}{\epsilon}} \frac{(c')^{1-\frac{1}{\epsilon}}}{1-\frac{1}{\epsilon}}$$

with $\epsilon > 1$ and $\bar{d} \in (0,1)$. This yields a familiar demand curve $D(p) = \bar{d}p^{-\epsilon}$ with constant elasticity over $p \geq 1$ (the participation constraint is not binding). [12] Condition (2) becomes

$$p^* = \frac{1}{1-1/\epsilon} \bar{d}^{-\frac{1}{\epsilon}} (p^* D(p^*))^{\frac{1}{\epsilon}}$$

which can be solved for[13]

$$p^* = \left(\frac{1}{1-1/\epsilon}\right)^{\frac{1}{2-1/\epsilon}} > 1.$$

As before, inflation is strictly positive. It follows that p^* is strictly decreasing in ϵ with $p^* \to \infty$ as $\epsilon \downarrow 1$ and $p^* \downarrow 1$ as $\epsilon \to \infty$. Thus, inflation can take any positive value as the elasticity varies.

Discussion: the Role of Conflict and Staggering. Inflation—a persistent and generalized rise in nominal prices—obtains in this model despite having abstracted from money, credit or savings, interest rates, production or employment. Standard culprits for inflation, or interpretations of the process, will just not do: inflation cannot be explained by an increase in money, nor the improper management of nominal interest rates (too low

[11] This does not assume that other, non-symmetric, equilibria do not exist.

[12] We restrict $\bar{d} < 1$ so that $c > 0$ for all $p \geq 1$, which is the relevant range of prices.

[13] This illustrates that $1 < p^* < \frac{1}{1-1/\epsilon}$ since x^θ for any $x > 1$ and $\theta < 1$ and $x = \frac{1}{1-1/\epsilon} > 1$ and $\theta = \frac{1}{2-1/\epsilon} \in (1/2, 1)$ for $\epsilon > 1$.

or too high), nor the level of demand, production or employment, or their stimulus by policy. By stripping almost everything away, the stylized model helps us see more clearly what remains.

Inflation results from the disagreement or conflict regarding relative prices in conjunction with the staggered setting of nominal prices. Both agents would like to enjoy a relative price that favors them, a higher price for the good they sell. Agents alternate attaining such conflicting aspirations, but these efforts lead to a constant rate of nominal price increases. Also, agents end up with an unfavorable relative price every other period. Indeed, on average over time relative prices are at a stalemate. The persist pursuit of conflicting aspirations over relative prices has no real winners and its energy gets canalized into a persistent increase in nominal prices.

As hinted above, disagreement or conflict over relative prices creates a force for inflation, but staggering is also crucial. Formally, it ensures an equilibrium with a finite level for inflation. To see this, consider a trading game similar to that of our baseline model, but where nominal prices are set simultaneously at the start of each period by both agents. No equilibrium exists for this game: the best response of each agent is to set a price higher than the other. Intuitively, staggering spreads the inflationary force of conflict over time, ensuring an equilibrium with finite inflation.

Particular models help see certain aspects of reality, but not others. Our model without money is purposefully stripped down, to isolate the conflict perspective, while preventing other interpretation. As such, it can be helpful to explore some questions, but not to answer others. One may ask: Is inflation necessarily always and everywhere a monetary phenomena? Is inflation necessarily due to excess demand or improper management of interest rates? The answers are 'no'. The stylized model forces one to look for other proximate causes of inflation.

However, this stylized model is certainly special and was not built to answer other questions. One should not conclude from this analysis that money or interest rates do not affect inflation, nor to deny an association between inflation and output or employment. In our view, many useful models and much empirical work supports such notions. However, we will argue that these models, which are also quite particular, are consistent with the conflict perspective.

Our view is that the more proximate cause of inflation is always conflict. However, conflict is typically endogenous. Many models may endogenize conflict in different ways. For example, monetary policy, via money supply or interest rates may have an influence on the economy and, thus, on conflict and inflation. The second part of our paper aims to provide a bridge between standard models and the conflict perspective in this light.

2.3 Two Simple Extensions

Here we discuss two very simple extensions. The next section provides a more significant variant of the model.

Random Roles for Buyer vs. Seller. We continue to consider two individuals A and B that are permanently matched. Agent A resets its price in even periods, while agent B does so in odd periods. Previously, we assumed that agent B acted as buyer in even periods and A did so in odd periods. That is, when agents reset their price they were seller. We now relax this assumption.

We now suppose that after an agent resets its price they will act as sellers with probability $\alpha \in (0,1]$ and as buyers with probability $1-\alpha \in [0,1)$ (across periods draws are independent). Thus, in even periods agent B acts as buyer with probability α; in odd periods A acts as buyer with probability α. Our baseline model amounts to the case with $\alpha = 1$; the case with $\alpha = 1/2$ is a simple case of interest since each agent has equal chances of acting as a buyer or seller in each period.

An agent resetting its price at any t now maximizes

$$\bar{v}(p) \equiv \alpha v(p) + (1-\alpha)V(1/p),$$

where $v(p)$ and $V(p)$ are the seller and buyer indirect utilities defined earlier. We assume that this problem is well posed with a finite and unique solution $p^* < \infty$. We note that this is more delicate than it was previously for the baseline model. Here the participation constraint of the seller imposed on the buyer plays a role in making the problem well posed.[14]

The thrust of Proposition 1 extends to this more general case. Once again there is positive inflation

$$\frac{P^*_{t+1}}{P^*_t} = p^* > 1.$$

We can now consider comparative statics on α. The solution must be in the range where $V(1/p)$ is increasing, it follows that p^* is decreasing in α. This implies that inflation is higher whenever $\alpha < 1$ than in the baseline model. Indeed, in the limit as $\alpha \to 0$ we obtain $p^* \to \infty$.

Non-Symmetric Preferences. For simplicity, we consider each extension separately, so let us return to the case with $\alpha = 1$. We now explore non-symmetric utility functions and

[14]To see this, note that if α is near 0 then $\bar{v}(p)$ approaches $V(1/p)$. Without the participation constraint this function is everywhere increasing and, thus, has no maxima; the participation constraint prevents this.

show that the thrust of Proposition 1 still goes through.

Agent A has utility $U_A(c_A, c_B)$ and agent B has $U_B(c_A, c_B)$. We no longer impose that $U_A(c, c') = U_B(c', c)$ as in the original model.

Take any competitive equilibrium relative price for the static endowment economy

$$\hat{p} = \frac{P_A}{P_B}.$$

Applying the same logic as in Proposition 1, when A resets its price, it will act as a monopolist and ensure a relative price that is higher than this competitive equilibrium

$$\frac{P_t^*}{P_{t-1}^*} = p_A^* > \hat{p}.$$

Symmetrically, when B resets its price

$$\frac{P_t^*}{P_{t-1}^*} = p_B^* > \frac{1}{\hat{p}}$$

Note that the comparison is to $1/\hat{p}$ because it represents the relevant relative price P_B/P_A whenever B is reseting its price as a seller. Combining the two inequalities gives

$$\frac{P_{t+1}^*}{P_t^*}\frac{P_t^*}{P_{t-1}^*} = \frac{P_{t+1}^*}{P_{t-1}^*} = p_A^* p_B^* > 1$$

for any $t = 0, 1, \ldots$ Thus, for each good, prices are always reset (every two periods) at a strictly higher price than they were previously in proportion $p_A^* p_B^* > 1$ for both goods.

3 Random Matching: Inflation Expectations and Money

We now explore a more significant extension, allowing for random matching, a common assumption in the search literature. The price-setting problem now becomes forward looking and creates a role for inflation expectations. Overall, our main results are unchanged: inflation emerges for the same reasons as it did earlier. Random matching leads to a higher equilibrium rate of inflation than the baseline model with fixed matches. However, inflation may rise or fall with inflation expectations.

We then extend the analysis by adding money. The nominal unit of account now represents an object that is explicitly included in the model. Will money affect inflation? Can a steadfast commitment to keep money supply fixed stop inflation, at least eventually? No: We show that conflict inflation prevails even when the nominal money supply is held constant. Indeed, nominal prices rise without bound and real money balances asymptotically vanish. Thus, along the equilibrium path money is always present and used, but over time we approach the cashless economy studied earlier.

3.1 Random Matching and Inflation Expectations

Previously we assumed just two agents, A and B, that were perpetually matched. This implied that the price setting problem was static and that expectations about the future did not play any role.

Next, we consider a variant with infinitely many individuals of each type, A and B. Agents meet in pairs to trade, but are matched at random each period against someone in the opposite type. Random matching is a more standard assumption in the macro-search literature. This extension makes the price reset problem dynamics and opens the door for expectations of future inflation to matter, impacting actual inflation today.

The timing is as follows. First, sellers reset prices. Then each seller is matched with a random buyer from the opposite type. Importantly, prices are reset without knowing the price of their trading partner. As before, type A agents are sellers in even periods and buyers in odd periods, and vice versa for B.

To simplify we focus on stationary equilibria, where prices are expected to increase at a constant rate: $P_t^* = \Pi P_{t-1}^*$ for some $\Pi > 0$. An agent resetting its price takes as given the nominal price set in the previous period P_{t-1}^* by the agent of the opposite type. Looking forward to their next period, they expect a nominal price $P_{t+1}^* = \Pi^2 P_{t-1}^*$. The seller chooses its price P_t^* in proportion to P_{t-1}^*

$$P_t^* = p^*(\Pi) P_{t-1}^*$$

where the optimal price increment $p^*(\Pi)$ is given by

$$p^*(\Pi) \equiv \arg\max_p \left\{ v(p) + \beta V(\Pi^2/p) \right\}$$

Intuitively, by resetting their price agents determine the relative price faced by their current buyers p and, thus, obtain $v(p)$ as sellers in the current period; further, they anticipate that next period the relative price they will face when acting as buyers is Π^2/p, obtaining utility $V(\Pi^2/p)$. That is, due to random matching price setters influence the terms of trade they face in the next period. Since V is strictly decreasing in the relevant range it follows that p^* is higher with random matching, compared to the baseline with permanent matches. Additionally, p^* is increasing in β and approaches the case with permanent matches as $\beta \to 0$.

In equilibrium, inflation is given as a solution to the fixed point

$$\Pi = p^*(\Pi).$$

In general, the function $p^*(\Pi)$ may be increasing or decreasing. In the quasilinear case (with $\epsilon > 1$) $p^*(\Pi)$ is decreasing.[15] Intuitively, for the current price setter, higher expected inflation Π, all things the same, implies that agents anticipate a higher price in the next period, when it is their turn to be buyers. This higher price may discourage purchases. This, in turn, may make them care less about influencing this future price using their current price setting choice.[16]

Now consider the model extension with $\alpha = 1/2$ so that being a buyer or seller is purely random. In this case we concluded that the optimal reset price maximizes $\bar{v}(p) = \frac{1}{2}v(p) + \frac{1}{2}V(1/p)$. However, with random matching the problem becomes

$$\max_p \{\bar{v}(p) + \beta\bar{v}(p/\Pi^2)\}.$$

Note that when there is no expected inflation so that $\Pi^e = 1$ then the solution is as before and maximizes $\bar{v}(p)$. It is easy to see that locally around $\Pi = 1$ then $p^*(\Pi)$ is increasing but less than one for one with Π.

For any α, a stationary equilibrium again solves $\Pi = p^*(\Pi)$. We have verified the existence and uniqueness of a stationary equilibrium in the quasilinear case; we suspect this conclusion holds for other utility functions of interest, although exploring the conditions is not our focus. One can also explore non-stationary rational expectations equilibria. For the quasilinear case we find that there are a continuum of equilibrium paths, indexed by the starting rate of change in prices. All these paths converge to the stationary one we have focused on.

[15] The first order condition is

$$v'(p) - \beta V'\left(\frac{\Pi^2}{p}\right)\frac{\Pi^2}{p^2} = 0$$

To see this note that $-V'(x)x = u_c D(x)x$ where $x = \Pi^2/p$ is the expected terms of trade. With quasilinear utility $u_c = 1$ and $D(x)x = x^{1-\epsilon}$ so that $V'(x)x$ falls with Π for fixed p. Assuming $v'' < 0$ the result then follows.

[16] On the other hand, there is a countervailing effect due to the concavity of the utility function: low purchases may imply high marginal utility, making them care more about improving their future terms of trade. This can be seen following the previous footnote and noting that $u_c(1 - D(x), xD(x))$ where $x = \Pi^2/p$ is not generally constant.

3.2 Adding Money

We now extend the random-matching model further to add money. Currency is held in the form of cash and used to facilitate some transactions. It also plays the role of unit of account: nominal prices are set in units of cash. Trade takes place using barter or cash, but cash has a distinct advantage.

Basics: Motivating Money. To motivate the use of money, we assume some transactions cannot be performed via barter. In particular, a fraction of the time agents suffer a shock and have no endowment to barter with.[17] This shock creates a need for liquidity that money can fill, as a medium of exchange.

There are many ways of modeling the details, and we chose them with an eye towards tractability. In particular, a well-known complication is keeping track of the evolution in the distribution of money across agents. To sidestep this issue we set up the model and focus on equilibria where money fully swaps hands each period: buyers fully spend all their cash balances, so that money travels back and forth each period between agents A and B.

Specifics: Barter and Money. To keep everything else as simple as possible, suppose $\alpha = 1$ so that agents resetting their prices play the role of sellers. Just as before, the price P_t^* is reset by the seller at the very beginning of the period and stays fixed in place for two periods.

The new aspects of the model are as follows. Each period is composed of a continuum of "instants" indexed by $s \in [0, 1)$. Agents are matched for an entire period, but meet each instant to trade and consume. Utility within a period is the simple integral over instants. Lifetime utility is thus

$$\sum_{t=0}^{\infty} \beta^t \int_0^1 u(c_t(s), c_t'(s))\, ds$$

where $(c_t(s), c_t'(s))$ denotes consumption at instant s within period t.

A fraction $1 - \delta$ of instants $s \in [0, 1 - \delta)$ are just as before and we call them "regular": both agents have their endowments. Buyers can trade by making take it or leave it offers and paying using the ratio of nominal prices P_t^* / P_{t-1}^* as the terms of trade.

[17] An equivalent interpretation is that a fraction of the time their endowment is intact, but cannot be used in exchange for the other good. One can tell many specific stories: that a fraction of the time the endowment cannot be transported, or the buyer "left the house without it", or the quality of the endowment (or its delivery) cannot be assured or verified by the counterparty, or the endowment is of a particular variety that the counterparty finds distasteful, etcetera, etcetera.

The remaining fraction δ of instants $s \in [1-\delta, 1)$ are "disasters": the buyer has no endowment so barter is impossible. However, the buyer can pay using cash. Only the seller price P_t^* is relevant in this case; the buyer price P_{t-1}^* is irrelevant.

We construct an equilibrium where buyers spend cash during disasters only and they spend all their cash balances. We proceed by conjecturing this is the case and then characterizing the seller pricing problem. We then return to verify the conjecture.

For simplicity, we assume additively separable utility

$$u(c, c') = F(c) + H(c')$$

with F and H concave and $-H''(c')c'/H'(c') < 1$ or equivalently that $H'(c')c'$ is increasing. The quasilinear iso-elastic case with $\epsilon > 1$ satisfies these conditions.

Seller Problem. The seller price is set for two periods. A fraction $1-\delta$ of transactions are regular barter ones; but a fraction δ involve cash and require further consideration. The seller takes the entire sequence of prices $\{P_t^*\}$ as given and choose $\tilde{P}_t^*$ to solve

$$(1-\delta)v(\tilde{p}_t) + \delta F(1 - m_t/\tilde{p}_t) + \beta\left((1-\delta)V\left(\frac{p_t p_{t+1}}{\tilde{p}_t}\right) + \delta H\left(\frac{m_t}{p_t p_{t+1}}\right)\right)$$

where $\tilde{p}_t = \tilde{P}_t^*/P_{t-1}^*$, $p_t = P_t^*/P_{t-1}^*$ and $m_t = M/P_{t-1}^*$. The first order condition for $\tilde{p}_t^*$ evaluated at the equilibrium $\tilde{p}_t = p_t$ gives

$$v'(p_t)p_t + m_{t+1}\frac{\delta}{1-\delta}u(1 - m_{t+1}, 0) = \beta V'(p_{t+1})p_{t+1}. \tag{3}$$

Together with the condition that $m_{t+1} = m_t/p_t$, this defines a dynamical system for $\{p_t, m_t\}$. The steady state of this system is $(p^*, 0)$, the steady-state of the moneyless model.

Buyer Cash-Management Problem. We now come back and check the conjecture that buyers spend their cash only during disasters and spend all of it then. This requires to possible deviations, providing two first-order conditions.

First, buyers can deviate from spending all their cash and hold on to a small amount, then spend it all two periods later during disasters. The first-order condition associated with this deviation is

$$H'(m_t) \geq \beta^2 \frac{1}{p_t p_{t+1}} H'(m_{t+2})$$

a standard intertemporal Euler that compares the marginal utility today to the discounted marginal utility times the real rate of return of money. Equivalently (multiplying both sides by m_t)

$$H'(m_t)m_t \geq \beta^2 m_{t+2} H'(m_{t+2}). \tag{4}$$

This condition will be automatically satisfied in the presence of positive inflation: we assume $H'(m)m$ is increasing and $m_{t+2} < m_t$.

Second, buyers can also deviate by also spending during regular instants $s \in [0, 1-\delta)$, not just during disasters. The first-order condition associated with this deviation is

$$G'(D(p_t)) \leq G'(m_t/p_t), \tag{5}$$

the marginal utility must be higher during a disaster. This condition simplifies to

$$D(p_t)p_t > m_t$$

which is satisfied whenever m_t is low enough. Intuitively, if buyer were sufficiently "cash rich" they would also want to spend this cash during regular instants.

Equilibrium with Money Swapping. A sequence $\{p_t, m_t\}_{t=0}^{\infty}$ is an equilibrium where money swaps hands each period if and only if (3) and (4) and (5) hold. We then have the following result.

Proposition 2. *For sufficiently small initial money balances $m_0 = M/P_{-1}$ there exist equilibria with strictly positive inflation. All these equilibria converge over time to the steady-state of the moneyless model.*

The point of this extension was to show that our results were not sensitive to a complete absence of money. They survive in an equilibrium where money exists and forever changing hands and used in transactions. In this model, money is valuable and, thus, demanded by agents. Even with a fixed supply of money, inflation from conflict prevails: the price level rises without bound, sending real money balances asymptotically towards zero.

How is this possible? Money is demanded for its real balances so pitting this demand against a constant money supply and rising prices should eventually create money scarcity that stops inflation in its tracks, or not?

Technically, in our model, note that money "demand" is not properly captured by any static condition such as "$M/P_t = L(\cdots)$". Thus, with fixed M, we cannot argue supposing L is constant (or bounded below) to conclude that P_t is pinned down (or bounded above). Instead, in our model, money holdings satisfy appropriate Euler equations, developed and checked above; these are dynamic relations, not static ones, and are fully consistent with shrinking real money balances.

To be sure, shrinking real money balances is unfortunate, it reduces liquidity and hurts trade. Intuitively, however, it does not create any obvious force for sellers to halt the rise in prices. Sellers still aspire to a favorable relative price, one above unity, and this generates inflation, as before. In our model, inflation continues to be generated by a disagreement or conflict in relative prices of goods; real money balances are simply a casualty that does not get in the way of these conflicting aspirations to stop inflation.

4 Inflation Mechanics and Conflict Accounting: Wages and Prices

The second part of our paper is in some ways the complete opposite of the first part. Earlier, we introduced a very specific, stylized and fully-specified model. The assumptions were not chosen for realism but to help isolate conflict as the driver of inflation. The model was purposefully kept distant from familiar benchmark models in macroeconomics. It featured barter in a setting with no money; there was no labor and output was fixed. The idea is that it is liberating to stray from familiar territory with well-established concepts and intuitions, e.g., inflation generated by monetary expansion or output gaps. We solved for the rational expectations equilibrium of this stylized model.

In contrast, the second part of our paper works with a framework that is fairly standard, and fully general, indeed, without the need of fully specifying all model components. The framework is general enough to fit many familiar models, such as New Keynesian one, but it is more general only adopting the staggered pricing assumptions, without committing to any further particulars (e.g. representative agent, technology, preferences, etc.). The analysis is also different: we characterizes properties of equilibria without fully solving for them.[18]

We begin by introducing a simple framework for the mechanics of inflation based on price and wage setting. We then discuss a few examples and provide general decompositions. Finally, we provide some results that lend meaning to our notion that inflation is driven by conflict. The analysis can be viewed as mechanical because we model in a relatively exogenous way the behavior of price and wage setters, by taking as given a sequence of "aspirations" for the real wages and profit margins. As shall be clear, this is a feature, not a bug. By virtue of being mechanical in this way, our analysis is fully general and applies to *any* path for these aspirations, including those endogenously generated by

[18] Fully solving is possible if one completes the model, but our analysis has the virtue of not requiring this.

a fully-specified models. We pursue this in a limited way in the next section. There we endogenize aspirations to adjust for inflation expectations. In other work we go further and use this general framework to interpret the workings of a fully-specified New Keynesian model (making choices for technology, preferences etc.).

4.1 Inflation and Aspirations

There is a continuum of firms and workers. Firms set the nominal price at which they are willing to sell goods, workers set the nominal wage at which they are willing to sell labor services. Both firms and workers are only allowed to reset prices and wages occasionally. Time is continuous and at each point in time firms are selected randomly to reset their price with Poisson arrival λ_p. The same happens for workers with arrival λ_w. The (log) price and wage index evolve according to price and wage inflation $\dot{p} = \pi$ and $\dot{w} = \pi^w$ given by[19]

$$\pi_t = \lambda_p \left(p_t^* - p_t\right),$$
$$\pi_t^w = \lambda_w \left(w_t^* - w_t\right),$$

where p_t^* and w_t^* are the (log) price and wage reset at time t. This staggered price and wage setting a la Calvo follows Erceg et al. (2000), but for now, without any of their general equilibrium specifics. Thus, our results apply more widely. [20]

Let us begin with the following decomposition of the reset price and wage:

$$p_t^* = w_t - f_t, \tag{6}$$
$$w_t^* = p_t + g_t. \tag{7}$$

The variables f_t and g_t capture the "aspirations" of firms and workers. Both aspirations are expressed in terms of a desired real wage. A high g_t represents a high real wage aspirations for workers, whereas a high f_t represents a low profit margin aspiration (high real wage aspiration) for firms. There is *conflict* whenever the two aspirations are incom-

[19] As is standard, we approximate by log-linearizing the price and wage indices around zero inflation. At any point in time t, the exact price index may be defined as some nonlinear, symmetric, homogenous of degree one function of all prices $\{p_{it}\}_i$. Due to symmetry, the first-order approximation for the price index p_t is the simple average $\int p_{it}\, di$. The same is true for the wage index.

[20] In fact, the analysis in this section applies more generally to any staggered time-dependent sticky price-wage model. For example, wages could be nominally rigid for a fixed amount of time a la Taylor-Phelps. All that is required is that the flow rate at which price and wage reset λ_p and λ_w are exogenous. The next section does make use of the Calvo pricing assumption, but the results could also be extended to a more general setting with some changes.

patible $f_t \neq g_t$. Substituting for p_t^* and w_t^* we obtain

$$\pi_t = \lambda_p \left(\omega_t - f_t\right), \tag{8}$$

$$\pi_t^w = \lambda_w \left(g_t - \omega_t\right), \tag{9}$$

where π_t^w and π_t denote wage and price inflation and ω_t denotes the real wage $w_t - p_t$. The dynamics of ω_t are simply given by

$$\dot{\omega}_t = \pi_t^w - \pi_t.$$

The form of (6) and (7) captures the notion that aspirations scale with the level of prices and wages, bygones are bygones, and everyone adjusts to the current level of prices and wages. For example, supposing $\{f_t, g_t\}$ to be exogenously given we can solve for the path of prices and wages $\{p_t, w_t\}$. Under the most straightforward interpretation, agents are naive and set p_t^* and w_t^* to achieve $w_t - p_t^* = f_t$ and $w_t^* - p_t = g_t$ without factoring in future inflation. Our analysis, however, will not be confined to this interpretation. Indeed, our analysis applies even when $\{f_t, g_t\}$ is endogenous to the path of $\{p_t, w_t\}$. One case, which we study explicitly in the next section, is when firms and workers care about future inflation.

In this section we work with any path $\{f_t, g_t\}$. This approach has the virtue that our analysis will apply to any way one models the aspirations. In the next section, we endogenize $\{f_t, g_t\}$ assuming agents have some targets $\{\phi_t, \gamma_t\}$ but choose $\{f_t, g_t\}$ taking into account future inflation.

4.2 Simple Examples: Conflict and Adjustment

To start with an extreme case, take the limit case with perfectly flexible wages, $\lambda_w \to \infty$. Then $\omega_t = g_t$ and equation (8) becomes

$$\pi_t = \lambda_p (g_t - f_t).$$

Inflation is proportional to conflict $g_t - f_t$. If there is no conflict $f_t = g_t$ and there is no inflation. When aspirations and, thus, the real wage, shift around the nominal wage performs all the adjustment needed and the price level remains fixed. This is clearly special to the flexible wage limit.[21]

Now go back to the general case ($\lambda_p < \infty$ and $\lambda_w < \infty$) and consider two simple paths

[21] Due to its simplicity, the perfectly flexible wage case is the standard benchmark in the simplest New Keynesian model. The symmetric case of sticky wages and perfectly flexible prices ($\lambda_p \to \infty$ and $\lambda_w < \infty$) leads to symmetric conclusions, with $\pi_t^w = \lambda_w (g_t - f_t)$.

for (f_t, g_t). Assume the initial real wage is $\omega_0 = 0$. We can interpret this as coming from a previous steady state with $f_t = g_t = 0$ for $t < 0$ that is disturbed by a shock at $t = 0$.

In the first example, $g_t = f_t = \bar{\omega} \neq 0$ for all $t \geq 0$, for some constant $\bar{\omega}$. The real wage is then $\omega_t = (1 - e^{-(\lambda_w+\lambda_p)t})\bar{\omega}$ which starts at $\omega_0 = 0$ and converges to $\omega_\infty = \bar{\omega}$. Price and wage inflation are

$$\pi_t^w = \lambda_w e^{-\left(\lambda_w+\lambda_p\right)t}\bar{\omega} \qquad\qquad \pi_t = -\lambda_p e^{-\left(\lambda_w+\lambda_p\right)t}\bar{\omega}.$$

If $\bar{\omega} < 0$ there is positive price inflation but negative wage inflation; the reverse is true if $\bar{\omega} > 0$. Wages and prices always move in opposite directions to contribute to the needed real wage adjustment from $\omega_0 = 0$ to $\omega_\infty = \bar{\omega}$. Who does most of this work? This is determined by their relative flexibility since $\pi_t^w/\pi_t = -\lambda_w/\lambda_p$, so that if wages are stickier than prices, then prices do most of the adjustment.

Now let us add conflict. Suppose $g_t = g$ and $f_t = f$ for all $t \geq 0$ with $g \neq f$. For concreteness, suppose $g = \omega_0 = 0$ and $f < 0$, i.e., worker aspirations are unchanged but firms aspire to reduce the real wage.[22] At date 0 price inflation is positive but wage inflation is zero

$$\pi_0 = \lambda_p\left(\omega_0 - f\right) = -\lambda_p f > 0 \qquad\qquad \pi_0^w = \lambda_w\left(g - \omega_0\right) = 0.$$

This pushes the real wage down initially, leading to $g > \omega_t$. Workers then start raising their nominal wages and the initial inflationary pressure in goods spills over to wages

$$\pi_t = \lambda_p\left(\omega_t - f\right) > 0 \qquad\qquad \pi_t^w = \lambda_w\left(g - \omega_t\right) > 0.$$

Since the real wage lies in between both aspirations $\omega_t \in (g, f)$ inflation rates remain positive for all $t \geq 0$. The real wage converges to a weighted average of firms' and workers' demands

$$\lim_{t\to\infty} \omega_t = \bar{\omega} = \frac{\lambda_w}{\lambda_w + \lambda_p} g + \frac{\lambda_p}{\lambda_w + \lambda_p} f,$$

with price and wage inflation converging to each other

$$\pi = \pi^w = \frac{\lambda_w \lambda_p}{\lambda_w + \lambda_p}\left(g - f\right) > 0.$$

Unlike the first example, here disagreement leads to positive inflation in both prices and wages. At first, price inflation is higher and the real wage falls, but eventually wage

[22]Lower real wages do not translate into higher profits necessarily. As becomes clearer later, the margin between p and w could reflect a desired higher markup, or a constant desired markup with a drop in productivity, or it could reflect a concern about future wage inflation.

inflation picks up and matches it. In the long run the real wage settles down at a level that can be seen as a compromise between the two aspirations. But this compromise is only apparent: the conflict is still present as both parties remain unsatisfied with the real wage $\bar{\omega}$, this provides price and wage pressures that are balanced and create a steady rate of inflation.

As a variant of the last example, suppose $g < 0$ and $f > 0$ with $\frac{\lambda_w}{\lambda_w+\lambda_p} g + \frac{\lambda_p}{\lambda_w+\lambda_p} f = 0$. Then we jump immediately to a steady state with constant and equal inflation in prices and wages. The real wage remains constant at its initial value $\omega_t = 0$. This case is one of pure conflict, without any transitional adjustment.

These examples suggest two sources of inflation: adjustment and conflict. Adjustments in the real wage inflation may call for inflation. But this type of inflation is transitional and does not generate inflation in both prices *and* wages. In contrast, conflict generates generalized inflation in both wages and prices and does so in a sustained manner. We argue next that these observations generalize.

4.3 General Decomposition: Conflict and Adjustment

Define

$$\tilde{\omega}_t \equiv \alpha f_t + (1-\alpha)\, g_t$$

a weighted average of worker and firm aspirations with weight $\alpha \equiv \frac{\lambda_p}{\lambda_w+\lambda_p} \in (0,1)$. Combining equations (8) and (9) gives the following decomposition.

Proposition 3. ***(Conflict and Adjustment Inflation)*** *For any path of $\{f_t, g_t\}$ price and wage inflation satisfies*

$$\pi_t = \Pi_t^C - \alpha \Pi_t^A,$$
$$\pi_t^w = \Pi_t^C + (1-\alpha)\, \Pi_t^A,$$

where the common "conflict" component

$$\Pi_t^C = (\lambda_p + \lambda_w)\alpha(1-\alpha)(g_t - f_t)$$

is driven by the difference of f_t and g_t and the "adjustment" component

$$\Pi_t^A = (\lambda_p + \lambda_w)(\tilde{\omega}_t - \omega_t)$$

is driven by the difference between the current real wage ω_t and the weighted average $\tilde{\omega}_t$.

The proposition provides a simple and general decomposition of both wage and price inflation. Let us discuss each component in turn.

The *conflict* component is proportional to the current disagreement $g_t - f_t$ and feeds into both price and wage inflation equally. Intuitively, this component of inflation can be thought of as a form of wasted energy, with workers and firms engaged in a tug of war at a stalemate: the conflict inflation Π_t^C does not contribute to any adjustment in the relative price ω_t, but simply adds to both price and wage inflation.

The conflict inflation rate has several properties. First, it is proportional to the disagreement $g_t - f_t$. Second, the coefficient $(\lambda_p + \lambda_w)\alpha(1-\alpha)$ is symmetric (if you swap λ_p for λ_w it is unchanged) and it it is constant-returns-to-scale in λ_w and λ_p (double both frequencies and inflation doubles). Third, for any given total frequency $\lambda_p + \lambda_w$, conflict inflation is zero whenever prices or wages are rigid, so that $\alpha = 0$ or $1 - \alpha = 0$. Conversely, conflict inflation is maximal when price and wage changes are equally frequent, $\alpha = 1/2$. Intuitively, this case maximizes the wage-price spiral: the feedback between price and wage inflation. Finally, conflict inflation is unaffected by the real wage ω_t.

The adjustment component is not related to the difference between f_t and g_t but instead to their levels summarized by the weighted average $\tilde{\omega}_t$ and the real wage ω_t. The adjustment component determines the dynamics of the real wage

$$\dot{\omega}_t = \pi_t^w - \pi_t^p = \Pi_t^A = (\lambda_p + \lambda_w)(\tilde{\omega}_t - \omega_t),$$

adjusting ω_t towards $\tilde{\omega}_t$ in proportion to its distance, at a speed that depends on the total frequency of price and wage adjustments. Nominal wages do part of the adjustment and nominal prices another part, in opposite directions. In our previous examples ω_t adjusted to a long-run value given by some constant weighted average $\tilde{\omega}$, but this need not be the case; we can allow $\tilde{\omega}_t$ to vary over time and never settle down.

Although adjustment inflation can produce inflation, we now provide two results that highlight the sense in which the common conflict component plays the starring role. Inflationary processes are often characterized as generalized increases in all prices (and wages) that are sustained over significant period of time. Our first observation is that at any point in time if there is no conflict $g_t - f_t = 0$ there can be no generalized inflation in both prices and wages.

Proposition 4. ***(Price and Wage Inflation Requires Conflict)*** *At all times conflict inflation is equal to the α-weighted average of π_t and π_t^w*

$$\Pi_t^C = \alpha \pi_t^w + (1-\alpha)\,\pi_t.$$

Thus, simultaneous price and wage inflation $\pi_t > 0$ and $\pi_t^w > 0$ requires conflict, i.e., $g_t - f_t >$

0.

Our second observation is that the adjustment component cannot sustain inflation over significant periods of time. To formalize this idea, define long-run averages

$$\bar{\pi} = \lim_{T\to\infty} \frac{1}{T}\int_0^T \pi_t \, dt \qquad \bar{\pi}^w = \lim_{T\to\infty} \frac{1}{T}\int_0^T \pi_t^w \, dt \qquad \bar{\Pi}^C = \lim_{T\to\infty} \frac{1}{T}\int_0^T \Pi_t^C \, dt.$$

We assume that $\{f_t, g_t\}$ are bounded to ensure these limits are well defined. Our next result links these three averages.

Proposition 5. ***(Time Averages of Inflation are Driven by Conflict)*** *The long-run average inflation rates satisfy*

$$\bar{\pi} = \bar{\pi}^w = \bar{\Pi}^C.$$

Thus, the long-run average of price and wage inflation equals the average conflict inflation.

This result says that prolonged episodes of inflation must be driven by conflict.[23] The intuition can be gleamed from our previous examples where f_t and g_t were constant: the adjustment force shifts the real wage in one direction, but this is only a temporary transition that does not produce sustained inflation. The result is more general and applies even when $\{f_t, g_t\}$ fluctuate to create never-ending fluctuations in real wages, without reaching a steady state. Intuitively, in these cases the adjustments component creates forces for both inflation *and* deflation that average out to zero over long periods of time.

To sum up, our mechanical decompositions show that broad notions of inflation require conflict. First, at any point in time inflation in both wages *and* prices requires conflict and conflict inflation is their common component. Second, over long periods of time average inflation is driven by conflict over that period. Without conflict there can be no generalized or persistent enough inflation episode.

5 Forward-Looking Aspirations

In the previous section, wage setters who reset w_t^* obtained their current real wage $w_t^* - p_t$ equal to their aspiration g_t; similarly, price setters obtained their current profit margin $p_t^* - w_t = f_t$. In this way, f_t and g_t capture the aspirations for the relative prices obtained *at that moment*. As such, these aspirations are immediately realized. However,

[23] Here we have taken time averages, but if we consider a situation with uncertainty and suppose $\{f_t, g_t\}$ are stationary stochastic processes, then the same can be said about the probabilistic average. That is, the unconditional expectation for inflation $\mathbb{E}[\pi_t]$ (or $\mathbb{E}[\pi_t^w]$) must be proportional to the unconditional expectation of conflict $\mathbb{E}[g_t - f_t]$.

with positive inflation these relative prices will not be sustained and will deteriorate over time. Indeed, as we saw in our examples the steady state with positive inflation had the real wage at a mid-point of f and g, so workers and firms are not obtaining g and f on average, they do so only when they reset.

One simple interpretation of the aspirations $\{f_t, g_t\}$ is that they represent naive goals from workers and firms that are frustrated by positive inflation. Workers set $w_t^* - p_t = g_t$ with the idea that this real wage will be maintained and then find themselves surprised when the real wage deteriorates; similarly for firms. This extreme interpretation is possible, but not at all necessary. As mentioned earlier, our previous decompositions hold even when $\{f_t, g_t\}$ are endogenous. In particular, they may not be naive and instead incorporate inflation expectations.

Thus, we take a first step in endogenizing f_t and g_t by considering forward looking-agents that have some desired (possibly moving) targets $\{\phi_t, \gamma_t\}$ for their relative price at time t and are aware of inflation. They adjust their immediate aspirations f_t and g_t to hit these target on average, over the time their wage or price is fixed. This requires adjusting f_t and g_t for their inflation expectations. We first provide formulas for these adjustments for any arbitrary expectations. We will then solve the (fixed-point) equilibrium outcome in the benchmark case under rational expectations.[24]

5.1 Aspirations Adjusted by Expected Inflation

The discussion above motivates the reset conditions[25]

$$p_t^* = (\rho + \lambda_p)\hat{\mathbb{E}}_t^p \int_0^\infty e^{-(\rho+\lambda_p)s}(w_{t+s} - \phi_{t+s})ds,$$
$$w_t^* = (\rho + \lambda_w)\hat{\mathbb{E}}_t^w \int_0^\infty e^{-(\rho+\lambda_w)s}(p_{t+s} + \gamma_{t+s})\, ds,$$

where $\rho > 0$ is a discount rate and $\hat{\mathbb{E}}_t^p$ and $\hat{\mathbb{E}}_t^w$ are the relevant subjective expectation held at time t by price and wage setters, respectively. These conditions reflect the condition that price and wage setters try to get, on average, the relative prices they target that can vary over time. This requires adjusting for their expectations of future prices and wages. Indeed, wage setters at t ensure that a weighted average of $w_t^* - p_{t+s} - \gamma_{t+s}$ equals zero; likewise price setters with $p_t^* - w_{t+s} - \phi_{t+s}$.

Importantly, these equation show a feedback between prices and wages that is the

[24]Since we abstract from aggregate uncertainty, this amounts to perfect foresight. However, the solution with uncertainty and rational expectations would be analogous.

[25]For now we take these reset equations as given. These linear reset rules can be derived as the optimizing a discounted expected quadratic objective that penalizes for deviations from the targets (ϕ, γ).

essence of the wage-price spiral. Wage setters care about current and future prices; price setters care about current and future wages. The complementarities work in "diagonal": from prices to wages, from wages to prices. The appendix generalizes the reset condition to allow for strategic complementarities within price setters and within wage setters.

The reset conditions can be recast in terms of aspirations as (see Appendix D)

$$f_t = \hat{\mathbb{E}}_t^p \int_0^\infty e^{-(\rho+\lambda_p)s}((\rho+\lambda_p)\phi_{t+s} - \pi_{t+s}^w)\, ds,$$
$$g_t = \hat{\mathbb{E}}_t^w \int_0^\infty e^{-(\rho+\lambda_w)s}((\rho+\lambda_w)\gamma_{t+s} + \pi_{t+s})\, ds.$$

which highlights the role of inflation expectations.[26] To see this more clearly take a simple case: consider a steady state with constant targets (ϕ, γ) and expectations $\hat{\pi} = \hat{\mathbb{E}}_t^w[\pi_{t+s}]$ and $\hat{\pi}^w = \hat{\mathbb{E}}_t^p[\pi_{t+s}^w]$. Then

$$f = \phi - \frac{\hat{\pi}^w}{\rho+\lambda_p} \qquad\qquad g = \gamma + \frac{\hat{\pi}^p}{\rho+\lambda_w},$$

and we see that inflation expectations contribute towards aspirations. Both workers and firms reset wages and prices more aggressively to protect themselves against the inflation they expect. Indeed, expected inflation contributes towards disagreement since

$$g - f = \gamma - \phi + \frac{\hat{\pi}^w}{\rho+\lambda_p} + \frac{\hat{\pi}^p}{\rho+\lambda_w} \tag{10}$$

so that disagreement is increased if $\hat{\pi}^w, \hat{\pi}^p > 0$.[27]

The adjustment for inflation disappears in two simple cases. First, if agents have $\hat{\mathbb{E}}_t^p \pi_{t+s}^w = 0$ or $\hat{\mathbb{E}}_t^w \pi_{t+s} = 0$ so that firms and workers do not anticipate inflation. The exogenous process for $\{\phi_t\}$ translates directly into $\{f_t\}$ and that for $\{\gamma_t\}$ into $\{g_t\}$ and

[26] Alternatively,

$$f_t = \hat{\mathbb{E}}_t^p \int_0^\infty e^{-(\rho+\lambda_p)s}((\rho+\lambda_p)(\phi_{t+s} - \omega_{t+s}) - \pi_{t+s})\, ds + \omega_t,$$
$$g_t = \hat{\mathbb{E}}_t^w \int_0^\infty e^{-(\rho+\lambda_w)s}((\rho+\lambda_w)(\gamma_{t+s} - \omega_{t+s}) + \pi_{t+s}^w)\, ds + \omega_t,$$

which incorporates the real wage. Here firms care about the gap between the real wage and their target, as well as price inflation; workers care about the gap between the real wage and their target, as well as wage inflation.

[27] It is worth mentioning that the form of adjustment for inflation expectations is dependent on the pricing model. The results in Werning (2022) show that the Calvo assumption adopted here implies a relatively high impact of inflation on p_t^* and w_t^*. In contrast with nominal rigidities of fixed length a la Taylor, this "passthrough" is approximately half that of Calvo. On the other hand, away from Calvo pricing past inflation matters because those resetting prices are not randomly selected and may older prices that (with positive inflation) may be further behind and need to catch up more.

we can simply take $\{f_t, g_t\}$ as exogenous. One possibility is that expectations of inflation are not rational and "well anchored" around low (or zero) inflation.[28] A variant of this idea is that agents have non-zero expectations but may not act on them at low inflation rates (Rowthorn, 1977; Werning, 2022). Second, even with full rationality, in the myopic limit obtains when $\rho \to \infty$ where agents only care about the immediate present, then $f_t \to \phi_t$ and $g_t \to \gamma_t$, unaffected by inflation expectations.

5.2 Inflation and Conflict Under Rational Expectations

Next, we impose rational expectations so that firm and worker expectations coincide with the objective expectation implied by the model, $\hat{\mathbb{E}}_t^p = \hat{\mathbb{E}}_t^w = \mathbb{E}_t$. We will abstract from uncertainty so this amounts to assuming agents have perfect foresight for the future paths for their targets ϕ_t and γ_t as well as for the endogenous paths for wages and prices w_t and p_t.

Combining the reset price and wage equations with the laws of motions for p_t and w_t and differentiating gives (Appendix E)

$$\rho \pi_t = \Lambda_p \left(\omega_t - \phi_t\right) + \dot{\pi}_t, \tag{11}$$

$$\rho \pi_t^w = \Lambda_w \left(\gamma_t - \omega_t\right) + \dot{\pi}_t^w, \tag{12}$$

where again $\omega_t = w_t - p_t$, with coefficients

$$\Lambda_p \equiv \lambda_p(\rho + \lambda_p) \qquad \text{and} \qquad \Lambda_w = \lambda_w(\rho + \lambda_w).$$

Solving the differential equation for price and wage inflation[29]

$$\pi_t = \Lambda_p \int_0^\infty e^{-\rho s}(\omega_{t+s} - \phi_{t+s})\, ds,$$
$$\pi_t^w = \Lambda_w \int_0^\infty e^{-\rho s}(\gamma_{t+s} - \omega_{t+s})\, ds.$$

each proportional to the present value of future differences between the real wage ω_t and their respective targets. These are the forward looking analogs of equations (8) and (9) in the previous section.

Given that these equations depend on future values of the real wage and that the

[28] The issue is somewhat symmetric: in countries experiencing high inflation and undergoing a stabilization program to lower inflation, there is concern that g_t will not fall, as workers want to maintain the real wage peaks they have obtained in the past, even if these peaks were not representative of the average real wage, due to high inflation.

[29] This is the unique solution satisfying $e^{-\rho t}\pi_t \to 0$ and $e^{-\rho t}\pi_t^w \to 0$.

dynamic of the real wage is still given by $\dot{\omega}_t = \pi_t^w - \pi_t$ the solution of the model for a given path $\{\phi_t, \gamma_t\}$ is no longer purely backward looking and is not yet complete. The next section does this by solving the implied second order differential equation in ω_t.

For now we provide a decomposition that is analogous to the one we derived in Section (4), but now for a given $\{\phi_t, \gamma_t\}$ instead of $\{f_t, g_t\}$.

Define the the weights $\alpha^f \equiv \Lambda_p/(\Lambda_p + \Lambda_w)$. We then have the following result.

Proposition 6. ***(Conflict and Adjustment Inflation Reprise)*** *Price and wage inflation can be decomposed in two components as follows*

$$\pi_t = \Pi_t^{C,f} - \alpha^f \Pi_t^{A,f},$$
$$\pi_t^w = \Pi_t^{C,f} + (1-\alpha^f)\Pi_t^{A,f},$$

where $\Pi_t^{C,f}$ is the forward-looking conflict component

$$\Pi_t^{C,f} = (\Lambda_p + \Lambda_w)\alpha^f(1-\alpha^f)\int_0^\infty e^{-\rho s}(\gamma_{t+s} - \phi_{t+s})\, ds$$

driven by the present value of differences between ϕ_t and γ_t, while $\Pi_t^{A,f}$ is the forward-looking adjustment component

$$\Pi_t^{A,f} = (\Lambda_p + \Lambda_w)\int_0^\infty e^{-\rho s}(\tilde{\omega}_{t+s} - \omega_{t+s})\, ds,$$

driven by the present value of differences between the real wage ω_t and the weighted-average target $\tilde{\omega}_t^f = \alpha^f \phi_t + (1-\alpha^f)\gamma_t$.

This result is reminiscent of the result obtained in the previous section using the reduced-form aspirations $\{f_t, g_t\}$. One difference is in its derivation: the present result relies on further equilibrium conditions: it solves the rational expectations fixed-point consistency condition between expectations and inflation.

To get some intuition for the connection between the decomposition in Sections 4 and 5, consider a steady state. In steady state, we must have

$$\pi = \pi^w = \Pi^{C,f} = \Pi^C.$$

Proposition 6 gives us this expression for conflict inflation

$$\Pi^{C,f} = (\Lambda_p + \Lambda_w)\alpha^f(1-\alpha^f)\frac{1}{\rho}(\gamma - \phi).$$

Whereas Proposition 3 combined with (10) gives

$$\begin{aligned}\Pi^C &= (\lambda_p + \lambda_w)\alpha(1-\alpha)(g-f) \\ &= (\lambda_p + \lambda_w)\alpha(1-\alpha)\left(\gamma - \phi + \frac{\Pi^C}{\rho+\lambda_p} + \frac{\Pi^C}{\rho+\lambda_w}\right).\end{aligned}$$

This last equation features Π^C on both sides because inflation contributes to increase the conflict $g-f$. When one solves for Π^C one obtains the expression for $\Pi^{C,f}$ in the previous equation. If $\gamma - \phi > 0$ then $g - f > \gamma - \phi > 0$ due to the inflation-driven conflict (from (10)). At the same time, the coefficients in the previous equations satisfy

$$(\Lambda_p + \Lambda_w)\alpha^f(1-\alpha^f)\frac{1}{\rho} > (\lambda_p + \lambda_w)\alpha(1-\alpha).$$

Both expressions produce the same level of inflation: the expression for $\Pi^{C,f}$ with a smaller conflict in targets $\gamma - \phi$ but stronger amplification, the expression for Π^C with a larger conflict $g - f$ but weaker amplification. The greater coefficient on $\gamma - \phi$ incorporates both the direct conflict in targets as well as the indirect conflict generated by inflation.

One can also state results analogous to Propositions 4 and 5. In particular

$$\alpha^f \pi_t + (1-\alpha^f)\pi_t^w = \Pi_t^{C,f},$$

so that a weighted average of price and wage inflation rates equals the forward-conflict inflation. In particular, this implies that generalized price and wage inflation requires $\gamma_t > \phi_t$. This weighted average condition is not identical to that of Proposition 4 because $\alpha^f \neq \alpha$ unless $\alpha = 1/2$. It follows that whenever $\pi_t \neq \pi_t^w$ (away from steady states) it must be that $\Pi_t^{C,f} \neq \Pi_t^C$. Intuitively, even without underlying conflicts in targets, the adjustment inflation $\Pi_t^A = \Pi_t^{A,f}$ creates inflation and deflation in prices and wages. These, in turn, creates conflict in aspirations $g_t - f_t$ as workers and firms try to protect themselves against inflation.[30]

[30]Suppose $\gamma_t = \phi_t$ for all $t \geq 0$ then we have

$$g_t - f_t = \int_0^\infty e^{-(\rho+\lambda_w)s}\pi_{t+s}\,ds + \int_0^\infty e^{-(\rho+\lambda_p)s}\pi_{t+s}^w\,ds$$

and since $\pi_t = -\alpha^f \Pi_t^A$ and $\pi_t^w = (1-\alpha^f)\Pi_t^A$

$$g_t - f_t = -\alpha^f \int_0^\infty e^{-(\rho+\lambda_w)s}\Pi_{t+s}^A\,ds + (1-\alpha^f)\int_0^\infty e^{-(\rho+\lambda_p)s}\Pi_{t+s}^A\,ds$$

which vanishes when $\lambda_w = \lambda_p$ but is otherwise generally non-zero.

6 Conclusion

This paper aimed to provides a fresh perspective on the role of conflict in inflation—defined as a disagreement on relative prices. We extended existing ideas by isolating conflict and by creating a bridge with current macroeconomic models, highlighting its role.

In our view, traditional ideas and models of inflation have been very useful, but are either incomplete about the mechanism or unnecessarily special. The broad phenomena of inflation deserves a wider and more adaptable framework, much in the same way as growth accounting is useful and transcends particular models of growth. The conflict view offers exactly this, a framework and concept that sits on top of most models. Specific fully specified models can provide different stories for the root causes, as opposed to proximate causes, of inflation by endogenizing conflict. Conflict is the most general and proximate cause for inflation.

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Appendix

A Proof of Proposition 1

The seller cannot choose quantities. If they could they would obtain higher utility

$$\tilde{v}(p) \equiv \max_{c,c'} u(c, c') \text{ s.t. } c' = p\,(1-c)\,.$$

Since $(c, c') = (1 - D(p), pD(p))$ is feasible we have

$$v(p) = u(1 - D(p), pD(p)) \leq \tilde{v}(p) \text{ for all } p > 0$$

with strict inequality as long as $(c, c') = (1 - D(p), pD(p))$ is not optimal for $\tilde{v}(p)$. Notice that $p = 1$ is a competitive equilibrium price and at a competitive equilibrium price, when both agents are price takers they choose quantities consistent with market clearing. This implies that

$$v(1) = \tilde{v}(1).$$

The function $\tilde{v}\,(p)$ is strictly increasing in p since $c' > 0$ and thus $1 - c > 0$. We conclude that for any $p < 1$, we have

$$v(p) \leq \tilde{v}(p) < \tilde{v}(1) = v(1),$$

so $p < 1$ is not optimal in problem (1). Moreover, by the Envelope theorem $v'(1) = \tilde{v}'(1) > 0$ which implies that $p = 1$ cannot be optimal. Therefore, the optimum must satisfy $p^* > 1$.

B Proof of Proposition 2

For m_0 small enough and p_0 close enough to the moneyless steady state $\bar{p} > 1$ the sequence satisfying 3 is close to the moneyless one with $m_t = 0$, which implies that $p_t > 1$, so that $m_t \downarrow 0$ and $p_t \to \bar{p}$. For small enough m_0, the condition (5) is satisfied since $m_t \leq m_0$. The condition 4 becomes

$$H'(m_t)m_t \geq \beta^2 H'(m_{t+2})m_{t+2}$$

which is satisfied since $\beta < 1$, $m_{t+2} < m_t$ and $H'(m)m$ is increasing. Thus, all the conditions for an equilibrium are satisfied.

C Proof of Proposition 5

Solving the ODE for ω_t gives

$$\omega_t = \omega_0 e^{-\left(\lambda_p+\lambda_w\right)t} + \left(\lambda_p + \lambda_w\right) \int_0^t e^{-\left(\lambda_p+\lambda_w\right)(t-s)} \tilde{\omega}_s ds,$$

so the boundedness of f_t and g_t implies that ω_t is bounded. The fact that $\dot{\omega}_t = \Pi_t^A$ implies

$$\frac{1}{T} \int_0^T \Pi_t^A dt = \frac{\omega_T - \omega_0}{T}.$$

Since the numerator is bounded this implies $\lim_{T\to\infty} \frac{1}{T} \int_0^T \Pi_t^A dt = 0$. The result follows.

D Derivation of Aspirations with Expected Inflation

Starts with

$$\begin{aligned} f_t &= (\rho + \lambda_p) \hat{\mathbb{E}}_t^p \int_0^\infty e^{-(\rho+\lambda_p)s} (\phi_{t+s} - (w_{t+s} - w_t))\, ds \\ &= (\rho + \lambda_p) \hat{\mathbb{E}}_t^p \int_0^\infty e^{-(\rho+\lambda_p)s} (\phi_{t+s} - \int_0^s \pi_{t+z}^w dz)\, ds \end{aligned}$$

Integrating the second term by parts yields the desired result. Analogous calculations apply for g_t.

E Proof of Equations (11) and (12)

We start with a generalized version of the resetting equations that allows for strategic complementarities. Under perfect foresight

$$\begin{aligned} p_t^* &= (\rho + \lambda_p) \int_t^\infty e^{-(\rho+\lambda_p)(s-t)} (\eta w_s + (1-\eta) p_s - \eta \phi_s) ds \\ w_t^* &= (\rho + \lambda_w) \int_t^\infty e^{-(\rho+\lambda_w)(s-t)} (\psi p_s + (1-\psi) w_s + \psi \gamma_s)\, ds \end{aligned}$$

where the main text assumes $\eta = \psi = 1$. Here $\eta < 1$ captures a situation with strategic complementarities where firms wish to have their prices close to that of other producers (e.g. their competitors or suppliers of an input). Likewise $\psi < 1$ captures strategic complementarities among wage setters.

This implies

$$\pi_t = \lambda_p(\rho + \lambda_p) \int_t^\infty e^{-(\rho+\lambda_p)(s-\tau)} (\eta(\omega_s - \phi_s) + \frac{1}{\rho + \lambda_p} \pi_s) \, ds$$
$$\pi_t^w = \lambda_w(\rho + \lambda_w) \int_t^\infty e^{-(\rho+\lambda_w)(s-t)} (\psi(\gamma_s - \omega_s) + \frac{1}{\rho + \lambda_w} \pi_s^w) ds$$

Differentiating and canceling terms gives the desired result:

$$\rho\pi_t = \eta\Lambda_p(\omega_t - \phi_t) + \dot{\pi}_t$$
$$\rho\pi_t^w = \psi\Lambda_w(\gamma_t - \omega_t) + \dot{\pi}_t^w.$$