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Strategic Incentives for Innovations and Market Competition

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Abstract

We consider a principal-agent model to examine the relationship between risk and incentives of firms who invest in cost-reducing R&D and compete in the product market. We show that a change in risk may trigger opposite responses of rivals in the same industry: lower risk may induce some firms to strengthen, while other firms to weaken the incentives provided to their agents. This result holds regardless of the mode of competition in the product market, Cournot or Bertrand, as long as the rivals' R&D decisions are strategic substitutes. Our model can generate new empirical implications and can provide an explanation for the lack of strong empirical support in the literature for a negative relationship between risk and incentives. It also has policy implications about the effect of risk on the incentives to innovate and welfare.

Keywords: Risk, Power of incentives, Product market competition, Innovation.
JEL: D82, L13, O30

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1 Introduction

We consider a principal-agent model with moral hazard to address the question of how changes in risk affect the compensation contracts offered by firms who invest in cost-reducing R&D and compete in the product market. Do all firms adjust their contracts qualitatively the same way when risk increases? We argue that higher risk triggers asymmetric (and opposite) responses of product market rivals: if rivals' R&D decisions are strategic substitutes, a negative and a positive relationship between risk and incentives can coexist for firms active in the same industry.

The relationship between risk and incentives provided by pay-for-performance compensation contracts has received significant attention in contract theory. The conventional wisdom in models with moral hazard, originating from Holmström (1979) and Holmström & Milgrom (1987), is that the optimal contract balances an increase in risk with weaker incentives for effort due to risk-sharing between the owner of the firm (the principal) and the manager (the agent). For higher risk, an agent requires more insurance, implying that incentives optimally decrease. However, the empirical support for this prediction is mixed (Prendergast (2002)). In particular, empirical analysis of existing contractual arrangements in many uncertain environments - e.g., executive payments in knowledge based industries and in financial sector, sales or franchise agreements - have, in many cases, unveiled a positive link between risk and incentives.[1] This paper shows that the latter result can also hold in a theoretical framework where rival managers, who have different exposure to risk, compete in the product market.

We consider two risk-neutral firms that, prior to competition in the product market (Cournot or Bertrand), hire risk-averse agents to conduct cost-reducing R&D. The R&D outcome of each firm depends on its agent's unobservable effort and the realization of a project-specific shock. The shocks that hit the rivals' R&D productions are correlated. Thus, the principal offers to her agent a risk-sharing contract (Holmström & Milgrom (1987)) that specifies the payment as a function of both rivals' actual cost reductions.[2] We allow for two kinds of asymmetries which are crucial for our results: either (i) firms are exposed to the same amount of risk (measured by the variance of a common shock) while the agents have different degrees of risk aversion, or (ii) agents with same degrees of risk aversion are appointed by firms that are subject to different levels of risk (measured by the variance of idiosyncratic shocks).

If we shut down the strategic interactions between firms, by assuming that they operate in different industries, higher risk implies weaker incentives, resulting in a smaller decrease of expected marginal cost and lower sales. If firms are asymmetric with respect to the cost of incentivizing their agents, the expected equilibrium reductions in marginal costs will also be asymmetric. We show that this negative risk-incentives relationship may cease to hold when we allow firms to compete for consumers. The firm with the lower cost of incentivizing its agent (either because its agent is less risk averse or its idiosyncratic risk is lower) will experience a relatively smaller reduction in marginal cost and so it will benefit in terms of market share. Higher market share introduces a new effect on incentives, named the *business stealing* effect. If rivals' R&D decisions are strategic substitutes, this effect is positive for the firm with the lower cost of providing incentives and works against the standard (negative) *insurance* effect. When the business stealing effect is strong enough, it can generate a positive relationship between risk and incentives for the firm that gains market share. Hence, a change in risk may have opposite effects on the

[1] More recent theoretical works on moral hazard attempted to generate a positive relationship by considering, for instance, the role of input monitoring or endogenous matching - they are discussed in the literature review later in this section. However, the risk-incentives relationship is always negative in models based on Holmström (1979).

[2] Prendergast (1999) provides a review of the principal-agent literature.

equilibrium R&D incentives of firms in the same industry.

Competition between firms and asymmetric responses to an increase in risk are necessary conditions for this result. In addition, incentives must be strategic substitutes so as a higher risk that diminishes a firm's R&D effort and thus shrinks its business in the product market will induce the winning firm in terms of market share to provide stronger incentives. In the linear demand case, incentives are strategic substitutes in both Bertrand and Cournot settings: the nature of strategic interactions in the R&D stage does not depend on the mode of competition in the product market. When demand is nonlinear, we discuss the additional conditions about the curvature of the demand that need to hold.[3]

The negative effects of the moral hazard problem on the equilibrium incentives of product market competitors have been analyzed by Hart (1983), Hermalin (1992), Schmidt (1997), Raith (2003) and Piccolo et al. (2008), among others.[4,5] In a similar framework as ours but with identical firms, Chalioti (2015) argues that, due to product market competition, rivals exert such high levels of R&D that they burn up their profits.[6] In the presence of moral hazard, underprovision of R&D incentives due to risk-sharing can generate considerable cost-savings, implying higher profits for both rivals. Thus, higher risk can make firms better off, because it decreases agents' equilibrium effort. However, this paper derives a positive relationship between risk and profits (not incentives).

The existing literature that derives a positive relationship between risk and incentives considers different settings. Lafontaine & Slade (2002) argue that it is the stronger incentives that cause a higher profit volatility. So, the positive relationship is the outcome of a reverse causation. Ackerberg & Botticini (2002), Serfes (2005) and Serfes (2008), among others, highlight the mechanism of endogenous matching between principals and agents. The latter models consider that principals compete for 'high quality' agents, but assume away any direct competition among contracts, once principals have matched with agents. When the (stable) matching is negative assortative, low risk-averse agents match with high risk projects (principals). Low risk aversion implies that the agent can tolerate stronger incentives, and so in equilibrium, they can derive a positive correlation between risk and incentives. With endogenous matching (but no direct competition among contracts), this positive correlation is observed only *across* all principal-agent pairs (due to endogenous sorting). Prendergast (2002) departs from the standard risk-sharing model and highlights the role of monitoring in contractual agreements. In risky environments, monitoring of the agent's actions is more difficult. As a result, the principal gives more discretion to the agent and the contract entails high powered incentives. Raith (2003) considers an endogenous number of symmetric firms that compete in prices along a Salop circle. When the degree of product substitutability increases more firms will enter the market. This has two effects: (i) incentives decrease and (ii) the (*endogenous*) variance of profits decrease. Thus, although exogenous risk and incentives are still negatively related, there exists a positive correlation between the variance of profits and incentives.

[3] When incentives are strategic complements, the relationship between risk and incentives is always negative for all firms.

[4] Raith (2003) examines the effect of competition on incentives, when there are changes in the number of competitors, the market size, the transportation cost or the cost of entry. Nickell (1996) and Vickers (1995) review the existing works on the relationship between competition and incentives, while Vives (2008) provides a survey of the existing literature on the effect of competition on innovation. Amir et al. (2000) examine firms' R&D strategic interactions when Cournot competitors innovate simultaneously or sequentially in the presence of R&D spillovers. Technology adoption incentives of market rivals are analyzed in Milliou & Petrakis (2011).

[5] Aghion et al. (2005), among others, study empirically the effect of competition on incentives. Griffith (2001) and Baggs & De Bettignies (2007) examine the relationship between competition and agency cost.

[6] Chiu et al. (2012) study the behavior of the relative and partial risk-aversion measures. Mirrlees & Raimondo (2013) analyze strategies in a continuous-time principal-agent model.

The underlying mechanism and the thrust of the results in existing works are very different from ours. We focus on the risk faced by agents and highlight the role of competition among them in shaping the contract characteristics. Risk is *exogenous* and affects asymmetrically the rivals' equilibrium incentives. We show that a positive relationship between risk and incentives can arise in a *given* firm (principal-agent pair). This paper is also related with the literature on cost-reducing investments, where the notion of strategic substitutability at the R&D stage (with possible counter effects from spillovers) is typically the case, e.g., Bagwell & Staiger (1994), Leahy & Neary (1997), Athey & Schmutzler (2001) and Schmutzler (2013). It is in line with the studies that have examined the question of whether static strategic complementarities/substitutabilities translate into dynamic ones in multi-stage games (Vives (2009)). Amir & Wooders (2000) analyze a two-stage game with cost-reducing R&D and product market competition à la Cournot/Bertrand, but when there is no moral hazard.

There are interesting policy implications of this model. Public policies whose goal is to decrease market risk in order to encourage all firms to innovate more may have the opposite result. As expected, firms with highly risk-averse agents will respond heavily to a decrease in risk, providing stronger incentives and investing more in R&D. However, firms with less risk-averse agents whose agency cost is small will respond less strongly. In fact, as the former firms exert more effort and further decrease their production cost, they will extend their business at the expense of their rivals who have hired less risk-averse agents. The latter firms may invest less as risk decreases. Thus, we argue that policies aiming in risk reduction may unexpectedly induce some firms in the industry to innovate less, defeating the purpose of such practices. One can also verify that our results will continue to hold in environments that do not necessarily encompass product market competition - for instance, in political campaigns - as long as actions in the first stage of the game are strategic substitutes (perhaps for other reasons), and the parties compete for "market" share.

Beyond the obvious importance of our analysis for the relationship between risk and power of incentives, it can also shed new light on other aspects of the firm's internal organizational structure, such as firm boundaries (Aghion et al. (2004), Alonso et al. (2008) and Hart & Holmström (2010), among others). In particular, as risk increases, the moral hazard risk-sharing model predicts that the incentives for vertical integration should also increase (Lafontaine & Slade (2007)). This is because high-powered incentives that typically exist outside a firm become more 'costly' and thus each firm wishes to rely less on the market. Based on our arguments, such a positive relationship between risk and vertical integration need not hold. Firms that gain in market share as risk increases and their cost of providing insurance is small enough, can operate under vertical separation. Therefore, we provide an alternative explanation for a negative risk-integration relationship that has also found empirical support (Lafontaine & Slade (2007)).

The paper is organized as follows. Section 2 presents the model. It discusses the R&D technology and the compensation contracts. Section 3 solves the game and examines the relationship between risk and R&D incentives when firms are involved in Cournot competition in the product market. It first performs the analysis with general demand and cost-of-effort functions, and then it discusses the linear demand case. In section 4, we focus on the R&D motives of Bertrand rivals and discuss whether the mode of competition in the product or R&D stages influences the rivals' R&D responses to an increase in risk. Section 5 concludes and discusses empirical implications of this model.

2 The model

The market consists of two firms 1 and 2, indexed by i and j where $i \neq j$. Each firm (it) is run by a risk-neutral principal (she) who appoints a risk-averse agent (he) to run the R&D department of the firm. We will be using the terms principal and firm interchangeably. The main goal of the R&D department is to achieve a lower marginal cost of production for the firm. The parties participate in a three-stage game. In stage 1, each principal offers a contract that stipulates a piece-rate pay based on (observable) R&D outcomes in order to encourage her agent to exert cost-reducing effort. In stage 2, if the agent accepts the offer, he chooses an effort level, which is unobservable to the principal. In stage 3, after the R&D outcomes have become common knowledge, the agents receive their compensation and then the principals (firms) compete in the product market, either in quantities or in prices.

2.1 R&D technology

The product market is populated by a continuum of identical consumers with mass equal to 1. We assume that each firm's initial marginal cost is $\bar{c} > 0$. This cost decreases with a firm's R&D output, $y_i \equiv e_i + \varepsilon_i$, where e_i is its agent's effort and ε_i is a project-specific shock. Thus, after the completion of the R&D process, firm i's marginal cost is $c_i = \bar{c} - y_i$. The random term ε_i is drawn from a bivariate normal distribution with zero mean and variance σ_i^2. The shocks, ε_i and ε_j, are correlated, where $\sigma_{ij} = \operatorname{cov}(\varepsilon_i, \varepsilon_j)$ is their covariance and $\rho \equiv \frac{\sigma_{ij}}{\sigma_i \sigma_j}$ denotes the correlation coefficient, $|\rho| \leq 1$.[7,8] Firm i's net profit for any realization of the marginal cost c_i is $\pi_i - w_i$, where π_i is the Cournot or Bertrand profit, depending on the mode of product market competition, and w_i is the agent's realized compensation.

2.2 Researchers' objectives and incentive contracts

To conduct R&D, agent i incurs disutility $g(e_i)$. This function is twice continuously differentiable and convex, implying that there are diminishing returns to scale in the R&D production process. We also assume that $g(0) = 0$, $g'(0) = 0$ and $\lim_{e_i \to \infty} g'(e_i) = \infty$. Agent i receives the reward w_i and has constant absolute risk-averse (CARA) preferences. He derives utility

$$V_i(w_i) = -e^{-r_i[w_i - g(e_i)]}, \tag{1}$$

where r_i is the Arrow-Pratt measure of risk aversion. Throughout this analysis, we assume $r_j \geq r_i > 0$, implying that firm j hires a more risk-averse agent than firm i.

Holmström & Milgrom (1987) establish that in a model much like ours, the optimal contracts are linear. In particular, agent i's compensation depends linearly on both agents' R&D-outputs due to the correlation of the market shocks. Relative performance evaluation schemes exploit all available information and allow each principal to better perceive her agent's effort by comparing both researchers' R&D

[7] As is usual in models that employ the normal distribution for the error term, e.g. Raith (2003), we assume that the intercept of the inverse demand and $\bar{c}$ are such that, relative to the standard deviation of the error term, so that the probabilities of a negative or very high marginal costs are practically zero. We discuss it further when we analyze the linear Cournot model.

[8] The type of correlation (positive or negative) may depend on whether the agents use similar or different R&D technologies. For instance, firms that produce hard disks may hire researchers that use either magnetic or holographic technologies. In this case, a market shock may affect the output of the projects that are based on these two technologies in a different way.

outcomes. Agent i's contract takes the form

$$w_i = \alpha_i + \beta_i y_i + \gamma_i y_j, \tag{2}$$

where α_i denotes the fixed salary component and β_i, γ_i are the pay-for-own and pay-for-rival performance parameters, respectively. If the agent rejects the offer, he picks the outside option which yields zero utility.

3 Managerial contracts and Cournot competition

We recursively solve the game where Cournot rivals make their decisions in each stage simultaneously and independently. We begin the analysis by considering a market with general demand and cost-of-effort functions. Then, we consider linear demands and quadratic cost functions.

3.1 R&D incentives with general market demand

We show that higher risk can increase incentives as long as rivals' R&D decisions are strategic substitutes. Let us assume that $\sigma_i^2 = \sigma_j^2 = \sigma^2$. Increases in σ^2 will influence both firms' R&D best responses. A general utility function $U(q_i, q_j)$ will generate the inverse demand system $p_i = d_i(q_i, q_j)$, where q_i is firm i's output and p_i denotes its price. This function is downward sloping, $\frac{\partial d_i}{\partial q_i} < 0$, and the cross derivatives are negative, $\frac{\partial d_i}{\partial q_j} < 0$, implying that goods are substitutes. An increase in firm i's output has also a stronger impact on its own market price than on its rival's: $\left|\frac{\partial d_i}{\partial q_i}\right| > \left|\frac{\partial d_j}{\partial q_i}\right|$. Thus, for a given realization of the marginal cost, firm i's Cournot profit is $\pi_i^c = (d_i - c_i)q_i$. The superscript c indicates the values in a setting with general demand functions and Cournot competition in the downstream market. The following assumptions on the profit functions also hold.

$(C.1)$ Each firm's profit function is strictly quasi-concave in its own output.

$(C.2)$ $\frac{\partial^2 \pi_i^c}{\partial q_i^2} + \left|\frac{\partial^2 \pi_i^c}{\partial q_i \partial q_j}\right| < 0$ for any i, j.

$(C.3)$ $\frac{\partial^2 \pi_i^c}{\partial q_i \partial q_j} < 0$ for any i, j.

Assumption $(C.2)$ guarantees that firms' reaction functions in the product market are well-behaved and their slopes are less than one. In turn, it ensures that in the production stage, there exists a unique interior Nash equilibrium in quantities. Assumption $(C.3)$ guarantees that rivals' quantity decisions are strategic substitutes.

Definition 1 (Degree of substitutability of products under Cournot competition) *Firms' products exhibit decreasing (increasing) substitutability, if an increase in a rival's production diminishes a firm's profit at a decreasing (increasing) rate:* $\frac{\partial^2 \pi_i^c}{\partial q_j^2} > (<) 0$.

According to Definition 1, for decreasing substitutability, the demand for firm i's product needs to be a convex function of q_j: $\frac{\partial^2 d_i}{\partial q_j^2} > 0$. As q_j increases, the two products become weaker substitutes; i.e., the negative effect of q_j on d_i (the price of good i) becomes smaller. For increasing substitutability, the demand function of firm i in its rival's output is required to be concave: $\frac{\partial^2 d_i}{\partial q_j^2} < 0$.

In the last stage, firms observe the realization of the marginal costs and compete for consumers. The equilibrium quantities are $q_i^c(\beta_i, \beta_j)$ and $q_j^c(\beta_i, \beta_j)$. In the first two stages, all decisions are taken uncertainty. Each principal is the residual claimant on firm's net profits, which are equal to the expected Cournot profits net of her agent's compensation, $\Pi_i^c = E\{\pi_i^c - w_i\}$, where the operator E signifies integration over the bivariate normal distribution of the two shocks. To conduct R&D, given the beliefs about firm j's effort (denoted by $\widehat{e}_j$) as a response to firm i's level of incentives, each principal i offers a contract that maximizes her net expected profits, preserving agent i's participation and incentives to perform. Thus, each principal i solves the following problem:

$$\max_{\{\alpha_i, \beta_i, \gamma_i, e_i\}} \Pi_i(\alpha_i, \beta_i, \gamma_i, e_i; \widehat{e}_j) = E\{\pi_i - w_i\}$$

$$\text{subject to } \; e_i^* = \arg\max_{e_i} CE_i \qquad (IC_i)$$

$$CE_i \geq 0 \qquad (IR_i)$$

The incentive compatibility constraint (IC_i) demonstrates that agent i will choose the R&D effort level that maximizes the certainty equivalence of his utility,

$$CE_i = \alpha_i + \beta_i e_i + \gamma_i \widehat{e}_j - \frac{r_i \sigma^2}{2}\left(\beta_i^2 + \gamma_i^2 + 2\beta_i\gamma_i\rho\right) - g\left(e_i\right).$$

Thus, the optimal effort must satisfy the first order condition,

$$g'\left(e_i\right) = \beta_i. \tag{3}$$

The individual rationality constraint (IR_i) serves to guarantee that agent i will stay in the firm and conduct R&D only if by doing so, his expected utility exceeds his reservation utility of zero.

Equation (3) indicates that the optimal pay-for-own performance parameter, β_i^*, will be positive. An agent's higher R&D output will be rewarded with a higher payment. Following Itoh (1991), the assumptions of agents' CARA preferences and the normality of the random terms as well as the concavity of V_i in e_i allow us to use the first-order approach.[9] Hence, equation (3) can replace the IC_i constraint in principal i's problem. The IR_i constraint binds at the optimum, implying that the base payment, α_i, guarantees agent i's participation. Principal i has complete bargaining power and appropriates all the surplus.[10] Using equation (3), we solve $CE_i = 0$ with respect to α_i.

For any value of β_i, the optimal pay-for-rival performance parameter is

$$\gamma_i^* = -\rho\beta_i. \tag{4}$$

[9] Itoh (1991) states that in a multi-agent model, the first-order approach requires further assumptions in addition to the monotone likelihood ratio property and the convexity of the distribution function condition (CDFC). In particular, we need to use a generalized CDFC for the joint probability distribution of shocks, the wage schemes must be nondecreasing and the coefficient of absolute risk-aversion must not decline too quickly. In our model with agents' CARA preferences, normally distributed random terms, linear contracts and R&D production functions, the above requirements are satisfied.

[10] We assume away competition among the principals for the less risk averse agent. Given that principals are ex-ante identical, if principals were competing for the 'more efficient' agent that would increase the rent that agent receives via a higher base salary. All the other results would not change.

If the R&D output shocks are positively correlated, $\rho > 0$, the optimal γ_i is negative. The principal perceives that the researchers perform in a 'favorable' environment and by setting γ_i^* negative, she is able to filter out the common shock from her agent's payment. In fact, the principal penalizes her agent when the rival researcher does better. If $\rho < 0$, by setting $\gamma_i^* > 0$, the principal allows her agent to suffer less from a bad outcome and encourages him to perform. The optimal γ_i is chosen so that agent i's payment is no longer sensitive to agent j's R&D output. Using equations (3) and (4), we now need to obtain the optimal pay-for-own performance parameters in both agents' contracts. Lemma 1 highlights the effects of the R&D incentives, β_i and β_j, on firm i's equilibrium outputs.

Lemma 1 (Effects of R&D on optimal outputs under Cournot Competition) *Firm i's equilibrium output is increasing in its own agent's R&D incentives and decreasing in its rival's incentives:* $\frac{\partial q_i^c}{\partial \beta_i} > 0$ *and* $\frac{\partial q_i^c}{\partial \beta_j} < 0$. *Moreover,* q_i^c *and* q_j^c *are linear functions and additively separable in* β_i *and* β_j, $\frac{\partial^2 q_i^c}{\partial \beta_i \partial \beta_j} = 0$.

Proof In appendix A.1. ■

We simultaneously solve both principals' problems and derive the optimal values of β_i and β_j. In equilibrium, the level of R&D conducted by the firms depends on the strategic properties of agents' R&D incentives. Thus, we need to specify the conditions under which incentives are strategic substitutes or complements.

3.2 Relationship between risk and R&D incentives

We begin by examining firm i's first order condition with respect to β_i. Using the envelope theorem, β_i affects expected profits through q_j, the marginal cost c_i and the agent's compensation:

$$\frac{\partial \Pi_i^c}{\partial \beta_i} = E\left[\frac{\partial \pi_i^c}{\partial q_j}\frac{\partial q_j^c}{\partial \beta_i} + \frac{\partial \pi_i^c}{\partial c_i}\frac{\partial c_i}{\partial \beta_i} - \frac{dw_i}{d\beta_i}\right] = 0. \tag{5}$$

The first term is the strategic (indirect) effect of β_i, while the second and third terms capture the direct effects on firm i's profits that arises even in the absence of product market competition (Fudenberg & Tirole (1984)). The strategic effect is positive because higher β_i (and hence higher q_i) makes the innovator tougher: given that rivals' outputs are strategic substitutes, an increase in firm i's incentive and production forces firm j to produce less, increasing the profits of the innovative firm i. The second term is also positive because more R&D lower the expected marginal cost which benefits the innovator. The third term is negative because stronger incentives are provided through a higher expected wage paid by the principal.

In the innovation-contracting stage, agent i's R&D incentive β_i responds to a change in β_j as follows[11]

[11] We also assume that $\left|\frac{d\beta_i}{d\beta_j}\right| < 1$ to obtain a unique equilibrium $\left(\beta_i^c, \beta_j^c\right)$.

$$\frac{d\beta_i}{d\beta_j} = -\frac{E[H_i]}{E[\Theta_i^c]}, \text{ where } H_i \equiv \underbrace{\frac{\partial^2 \pi_i^c}{\partial q_j^2}}_{\substack{>(<)0 \\ \text{Decr. (incr.)} \\ \text{substitutability}}} \underbrace{\frac{\partial q_j^c}{\partial \beta_j}\frac{\partial q_j^c}{\partial \beta_i}}_{\substack{<0 \\ \text{Lemma 1}}} + \underbrace{\frac{\partial^2 \pi_i^c}{\partial q_j \partial q_i}}_{<0} \underbrace{\frac{\partial q_i^c}{\partial \beta_j}\frac{\partial q_j^c}{\partial \beta_i}}_{\substack{>0 \\ \text{Lemma 1}}} + \underbrace{\frac{\partial^2 \pi_i^c}{\partial c_i \partial q_i}}_{<0} \underbrace{\frac{\partial q_i^c}{\partial \beta_j}\frac{\partial c_i}{\partial \beta_i}}_{>0}, \quad (6)$$

where $E[\Theta_i^c] < 0$ follows from the second-order condition.[12] There are three (possibly) opposing effects that are present and determine the nature of rivals' strategic interactions in R&D.

The first term in (6) depends on the second derivative of q_j on the price of good i. Under decreasing substitutability, the benefit of a higher β_i for firm i, which is generated from the positive indirect effect in (5), is smaller the higher the β_j (and hence the higher the q_j). Firm i is more reluctant to raise its β_i when it expects β_j to increase. This is a source of strategic substitutability. On the other hand, increasing substitutability is a source of strategic complementarity. This term is zero when demand is linear and additively separable.

The other two terms also arise in the linear demand case and are sources of strategic substitutability. A higher β_j (and thus a higher q_j) induces firm i to lower its own β_i for two reasons: (i) q_i decreases and so does the benefit from a cost reduction and (ii) the best-response of firm i in the product market when q_j increases is to become less aggressive by lowering its own q_i (given that quantities are strategic substitutes). Thus, weaker incentives from firm i is the profit-maximizing response to an increasing β_j. However, the presence of the first effect can change the nature of rivals' strategic interactions. Lemma 2 specifies the conditions under which incentives are strategic substitutes.[13]

Lemma 2 (Strategic Interactions in R&D under Cournot Competition) *Researchers' R&D incentives, β_i and β_j, are strategic substitutes if demand functions exhibit decreasing or weakly increasing substitutability: $\frac{d\beta_i}{d\beta_j} < 0$ if and only if*

$$\frac{\partial^2 \pi_i^c}{\partial {q_j}^2} > -\frac{\partial q_i}{\partial \beta_j}\left(\frac{\partial q_j}{\partial \beta_j}\frac{\partial q_j}{\partial \beta_i}\right)^{-1}\left(\frac{\partial^2 \pi_i^c}{\partial q_j \partial q_i}\frac{\partial q_j}{\partial \beta_i} + \frac{\partial^2 \pi_i^c}{\partial c_i \partial q_i}\frac{\partial c_i}{\partial \beta_i}\right).$$

Proof In appendix A.2. ■

We can now establish that, in equilibrium, as σ^2 increases, the negative relationship between incentives and risk can be reversed for the firm with the less risk-averse agent, provided that firms' R&D decisions are strategic substitutes.[14] We need to analyze the underlying effects of σ^2 on rivals' optimal incentives. We differentiate both rivals' first-order conditions at the incentives-setting stage (equation

[12] The form of Θ_i^c is given in Section (A.2).

[13] The result in Lemma 2 is reminiscent of Vives (2009). In Section 3.1.2 of his paper, he presents a deterministic model of capacity investments in a Cournot duopoly. One of the sufficient conditions for cost-reducing investments to be strategic substitutes is what we call in our paper 'decreasing substitutability'. The function of the optimal output $q_i^*(\beta_i, \beta_j)$ needs also to be submodular in (β_i, β_j), which is also true in our model (Lemma 1 states that the cross partial derivative is zero).

[14] There is one-to-one relationship between incentives, β_i^*, and optimal effort, e_i^*, as well as the relative performance parameter, γ_i^*, that is used to filter out the common uncertainty from researchers' R&D performances. Thus, indirectly, our discussion sheds also insights on the changes of the optimal efforts.

(5)), with respect to σ^2 using equations (3) and (4). In particular, we take $\frac{d(\partial\Pi_i^c/\partial\beta_i)}{d\sigma^2} = 0$, that implies $\frac{\partial(\partial\Pi_i^c/\partial\beta_i)}{\partial\sigma^2} + \frac{\partial^2\Pi_i^c}{\partial\beta_i^2}\frac{d\beta_i^*}{d\sigma^2} = 0$. The decomposition of the effects of σ^2 on both firms' optimal R&D incentives gives

$$\underbrace{E[H_j]\frac{d\beta_j^*}{d\sigma^2}}_{\text{business stealing effect}} - \underbrace{r_i\left(1-\rho^2\right)\beta_i^* g''\left(e_i\left(\beta_i^*\right)\right)}_{\text{insurance effect}} + \underbrace{E[\Theta_i^c]}_{<0 \text{ by the SOC}}\frac{d\beta_i^*}{d\sigma^2} = 0, \tag{7}$$

$$\underbrace{E[H_i]\frac{d\beta_i^*}{d\sigma^2}}_{\text{business stealing effect}} - \underbrace{r_j\left(1-\rho^2\right)\beta_j^* g''\left(e_j\left(\beta_j^*\right)\right)}_{\text{insurance effect}} + \underbrace{E[\Theta_j^c]}_{<0 \text{ by the SOC}}\frac{d\beta_j^*}{d\sigma^2} = 0. \tag{8}$$

Equation (7) shows the effects of σ^2 on firm i's optimal incentives, while equation (8) decomposes the effect on β_j^*.

Let us first suppose that the business stealing effect is absent because each firm is a monopoly. A higher σ^2 affects both β_i^* and β_j^* negatively. Then, allow for strategic interactions with respect to incentives under the assumption that rivals' R&D decisions are strategic substitutes - $H_i < 0$ and $H_j < 0$. Upon inspection of (7) and (8), we infer that only one of the following two possibilities can arise: (i) either risk affects incentives negatively in both firms, or (ii) positively in one firm and negatively in the other. When the asymmetry across firms with respect to the degrees of risk aversion of their agents is significant - i.e., r_j is high enough relative to r_i - the insurance effect is small in firm i and large in firm j. Let us assume that firm i's insurance effect is arbitrarily close to zero. In equation (7), the derivatives $\frac{d\beta_i^*}{d\sigma^2}$ and $\frac{d\beta_j^*}{d\sigma^2}$ cannot have the same sign. If the insurance effect in firm j is strong enough, from equation (8), we have $\frac{d\beta_j^*}{d\sigma^2} < 0$, which implies that $\frac{d\beta_i^*}{d\sigma^2} > 0$.

In the regime where R&D decisions are strategic complements, $H_i > 0$ and $H_j > 0$, risk always decreases the incentives for both rivals, as is standard in the literature. Given that $\frac{d\beta_j^*}{d\sigma^2} < 0$, the business stealing effect in equation (7) is always negative. Both effects move to the same directions leading firm i also to underprovide R&D incentives as σ^2 increases, $\frac{d\beta_i^*}{d\sigma^2} < 0$. Equation (8) is also satisfied only when higher σ^2 decreases both β_i^* and β_j^*. Proposition 1 establishes that risk and incentives can be positively related for the firm that is less exposed to risk, as long as rivals' R&D decisions are strategic substitutes and business stealing incentives are strong.[15]

Proposition 1 (Opposing effects of risk on optimal R&D incentives) *Suppose that $r_i < r_j$. If rivals' R&D decisions are strategic substitutes, a higher σ^2 weakens the optimal R&D incentives of firm j, whose agent is more risk-averse, $\frac{d\beta_j^*}{d\sigma^2} < 0$, while it strengthens the optimal R&D incentives of firm i, whose agent is less risk-averse, $\frac{d\beta_i^*}{d\sigma^2} > 0$, as long as firm i's cost of incentivizing its agent is much lower than that of firm j.*

Proof In appendix A.3. ■

[15] A positive relationship between effort provision and insurance holds when product market competition is stiff. Thus, this relationship is weakened as we are considering firms whose products are less differentiated. In the polar case of monopolies, risk always decreases effort in equilibrium.

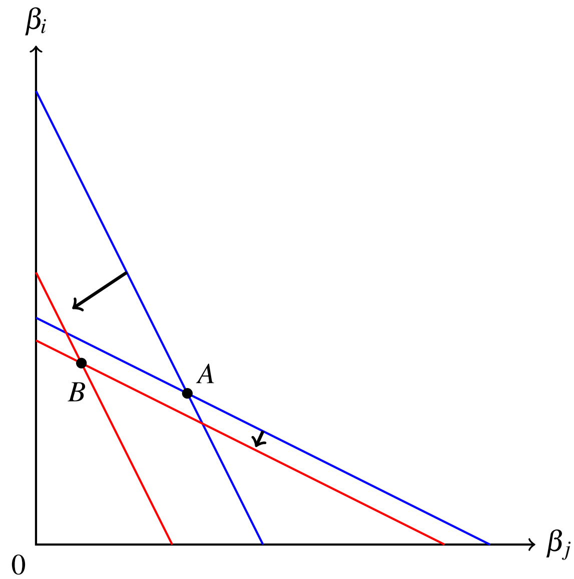

Figure 1: Risk σ increases and both best responses shift in. The initial equilibrium is at A and the new equilibrium at B. The best response of firm j shift more than that of firm i. As a result firm i offers stronger incentives, while firm j offers weaker incentives.

The intuition of Proposition 1 is best captured by Figure 1. Assume that incentives between the two firms are strategic substitutes (and the best-response curves are linear). Higher risk shifts both reaction curves inwards, implying that when we hold the incentives of the rival fixed, the firm in question optimally weakens incentives (insurance effect). But risk shifts the reaction curves in different magnitudes across the two firms. The firm with the more risk-averse agent, firm j, experiences the bigger shift, thereby lowering its incentives significantly. This is a commitment to decrease output in the next stage, which presents an opportunity for firm i, whose cost of incentivizing its own agent is relatively low, to strengthen its incentives in order to gain market share (business stealing effect).

This model establishes that managerial incentives respond to a change in the corporate environment common to all firms in the industry - e.g., systemic risk - in a fashion that dispels traditional agency theory. This can also happen as managerial incentives respond to a change in a firm's specific corporate environment - e.g., idiosyncratic risk. When only the risk to which one firm is exposed changes (say σ_i^2), the effect on incentives must be asymmetric across the two firms: the equilibrium incentives move in opposite directions. This is because only firm i's best-response curve shifts inwards. As σ_i^2 increases, firm j will *always* provide higher-power incentives to its agent, for any r_i, r_j, ρ and σ_j^2. Thus, if market changes affect only one firm (or the change in the other firm is significantly weaker), the standard result in the literature never holds. This is true even when $r_i = r_j = r$ - i.e., the agents are homogeneous - as long as there exists asymmetry in the variance of the idiosyncratic risks, σ_i^2 and σ_j^2.

Changes in $|\rho|$ can also manifest changes in the level of risk. Suppose that $r_i < r_j$ and $\sigma_i^2 = \sigma_i^2$, while $|\rho|$ increases. For higher $|\rho|$, the variance of wages decreases and so does the risk to which agents are exposed. Thus, less insurance is required, decreasing the cost of exerting effort. Equations (7) and (8) show that for higher $|\rho|$, the business stealing effect is strengthened, and is more likely to dominate the insurance effect for the firm with the less risk averse agent.

This analysis boils down to the following: in industries where competition is intense and thus business stealing incentives are strong – such as in microelectronics-based industries, pharmaceuticals, or even in the financial sector – as firms are exposed to higher risk, they may not adjust their pay-for-performance R&D incentives in the same direction. Asymmetries on the part of agents' preferences towards risk or on the variance of R&D production shocks play a key role. What drives the result is the strategic benefit of the firms compared to the cost of providing insurance to their agents.

3.3 Equilibrium with linear market demand

Following Singh & Vives (1984), the representative consumer's utility is

$$U(q_i, q_j) = a(q_i + q_j) - \left[\frac{1}{2}\left(q_i^2 + q_j^2\right) + bq_iq_j\right] + m,$$

implying that firms i's inverse demand is $p_i = a - q_i - bq_j$, where a stands for the maximum willingness to pay, $a > \bar{c}$, $b \in [0,1]$ measures the degree of product substitutability and m is the numeraire good. For simplicity, we set $b = 1$.[16] In the downstream market, after the realization of the marginal costs, firms compete in quantities and maximize $\pi_i = [a - q_i - q_j - c_i]\, q_i$. The equilibrium output is $q_i^* = \frac{1}{3}(a - 2c_i + c_j)$ and the Cournot profit is $\pi_i^* = (q_i^*)^2$.[17]

We also assume that the cost-of-effort functions are quadratic of the form $g(e_i) = \frac{k}{2}e_i^2$, where higher k indicates lower efficiency of R&D technology. In the contracting stage, using equations (3), (4) and that the IR_i constraint is binding, principal i's constrained maximization problem reduces in maximizing

$$\Pi_i^c = \frac{1}{9}\left[a - \bar{c} + 2\frac{\beta_i}{k} - \frac{\beta_j}{k}\right]^2 + \frac{5 - 4\rho}{9}\sigma^2 - \frac{\nu_i}{2k}\beta_i^2, \tag{9}$$

where $\nu_i \equiv 1 + kr_i\sigma^2\left(1 - \rho^2\right)$. It can be easily verified that incentives are strategic substitutes; i.e., they inherit the properties of the variables in the product market. Solving for the equilibrium incentives, we derive

$$\beta_i^* = \frac{4(a - \bar{c})[3k\nu_j - 4]\, k}{16 + 3k[9k\nu_i\nu_j - 8(\nu_i + \nu_j)]}. \tag{10}$$

We make the assumption $\nu_i > \frac{4}{3k}$ to guarantee that $\beta_i^* > 0$. If the agents have the same degree of risk aversion, implying that $\nu_i = \nu_j = \nu$, the equilibrium piece-rate pay reduces to

$$\beta_i^* = \beta_j^* = \frac{4(a - \bar{c})\, k}{9k\nu - 4}.$$

Clearly, higher risk, measured by an increase in σ^2 (or a decrease in the degree of correlation $|\rho|$), will induce both principals to provide weaker incentives in equilibrium. If agents have heterogeneous degrees

[16] We will allow for $b < 1$ in some of our numerical exercises where we examine the roles of: (i) product substitutability and (ii) the mode of competition.

[17] Agent i's compensation w_i can be a function of the profit realizations instead of the cost realizations, without affecting the results, given that there is a one-to-one relationship from (c_i, c_j) to (π_i, π_j). See also Raith (2003) for a similar argument.

of risk aversion, implying that $v_i < v_j$, the relationship between risk and incentives for firm i becomes positive if

$$r_i < \frac{4r_j(3k-4)}{32-12k(6v_j-1)+27k^2v_j^2}.$$

For example, when $r_i = 0.2$, $r_j = 1$, $k = 1.35$, $a = 100$, $\bar{c} = 45$ and $\rho = 0.1$, the incentives in firm i are increasing in $\sigma \in [0, 0.3]$. Moreover, the highest value of the equilibrium effort is about 40. This suggests that the probability of a negative realized marginal cost (or a marginal cost higher than a) is practically zero, given that σ is, say, less than one.

One can argue that a policy maker who cares about aggregate welfare may adopt the wrong policy by assuming that incentives are always increasing when risk decreases. Risk, measured by σ, has two effects on expected welfare, which is the sum of expected consumer, producer and agents' surpluses: (i) a direct effect since expected welfare is a function of σ and (ii) an indirect effect through the change of incentives and effort. A social planner may attempt to lower the risk (or more broadly the cost of incentivizing the agents) that surrounds the R&D process in order to boost incentives and effort and consequently welfare. For example, consider policies that encourage R&D cooperation among firms (e.g., Leahy & Neary (1997)). These policies can increase the correlation coefficient $|\rho|$, because firms' R&D approaches and inputs become more similar. In our model, this is equivalent to a reduction in the common standard deviation, since the cost of incentivizing an agent depends on $\sigma^2(1-\rho^2)$.

We compute expected aggregate welfare and examine how it changes with respect to $\rho > 0$, using the equilibrium incentives. We consider the following numerical analysis using the linear Cournot model. When firms are symmetric (e.g., $r_i = r_j = 0.6$ and $\sigma = 0.4$), the relationship between ρ and incentives is positive. In this case, expected welfare increases with a higher ρ, which indicates lower risk. Hence, a policy that encourages R&D cooperation, and assuming that this results in a higher correlation, has a positive effect on expected welfare. Holding all other parameter values constant, let us now assume that firms are asymmetric - by considering a mean-preserving spread in r's ($r_i = 0.2$ and $r_j = 1$) which insures that the relationship between risk and incentives in the most efficient firm is positive. For high degrees of correlation - i.e., higher than (approximately) 0.75 - expected welfare now decreases as ρ increases.[18]

4 Managerial contracts and Bertrand competition

We now analyze the rivals' strategic decisions and the role of insurance provision when firms compete à la Bertrand in the product market. We show that a positive risk-incentives relationship can also be obtained under Bertrand competition. The representative consumer's maximization problem gives rise to a general demand system $q_i = D_i(p_i, p_j)$. The direct demand functions are downward sloping, $\frac{\partial D_i}{\partial p_i} < 0$, and the cross-derivatives are positive, $\frac{\partial D_i}{\partial p_j} > 0$. The own-price effect, $\left|\frac{\partial D_i}{\partial p_i}\right|$, is also larger than the cross-price effect, $\frac{\partial D_i}{\partial p_j}$. Firm i's realized profit is given by $\pi_i^b - w_i$, where $\pi_i^b \equiv (p_i - c_i)D_i$. The superscript

[18]In our model, firms begin with the same marginal cost and use the same R&D process, while they appoint asymmetric agents in terms of their degree of risk aversion. Instead, one could consider firms with identical agents but different initial marginal costs; i.e., $\bar{c}_i < \bar{c}_j$ and $r_i = r_j$. In this case, firms will always innovate less in equilibrium as risk increases, $\frac{\partial \beta_i^*}{\partial \sigma^2} < 0$ and $\frac{\partial \beta_j^*}{\partial \sigma^2} < 0$.

b denotes the choices of Bertrand rivals. The following assumptions on the profit functions are also in order.

$(B.1)$ The profit function is quasi-concave in own price.

$(B.2)$ $\frac{\partial^2 \pi_i^b}{\partial p_i^2} + \left| \frac{\partial^2 \pi_i^b}{\partial p_i \partial p_j} \right| < 0$ for any i, j.

$(B.3)$ $\frac{\partial^2 \pi_i^b}{\partial p_i \partial p_j} > 0$ for any i, j.

Similarly to the Cournot case, assumptions $(B.2)$ and $(B.3)$ guarantee the interiority and uniqueness of the equilibrium in prices.

Definition 2 (Degree of substitutability of products under Bertrand competition) *Firms' products exhibit increasing (decreasing) substitutability, if an increase in a rival's price raises a firm's profit at an increasing (decreasing) rate:* $\frac{\partial^2 \pi_i^b}{\partial p_j^2} > (<) 0$ *for any i, j.*

For increasing substitutability, the demand for firm i's product needs to be convex in its rival's price, $\frac{\partial^2 D_i}{\partial p_j^2} > 0$, while a concave demand function in p_j is required for decreasing substitutability between firms' products, $\frac{\partial^2 D_i}{\partial p_j^2} < 0$.

We derive the equilibrium prices $p_i^b(\beta_i, \beta_j)$ and $p_j^b(\beta_i, \beta_j)$. In the R&D and contract stages, firm i chooses the R&D incentives that maximize its expected Bertrand profit net its agent's expected compensation. Using the envelope theorem, β_i affects expected profits through p_j, the marginal cost c_i and the agent's expected compensation:

$$\frac{\partial \Pi_i^b}{\partial \beta_i} = E\left[\frac{\partial \pi_i^b}{\partial p_j}\frac{\partial p_j^b}{\partial \beta_i} + \frac{\partial \pi_i^b}{\partial c_i}\frac{\partial c_i}{\partial \beta_i} - \frac{dw_i}{d\beta_i}\right] = 0. \tag{11}$$

The first term of (11) is the strategic effect and, unlike the Cournot model, is negative. This is because cost-reducing R&D allows the innovator to set a lower price, which triggers its rival to cut its own price as well, resulting in lower profits for the innovator. Thus, competition among Bertrand rivals gives rise only to detrimental effects on their profits. The other two terms are similar to those in the Cournot model. Lemma 3 establishes the effect of agent i's R&D incentives on its own firm's and its rival's equilibrium prices as well as the cross effects.

Lemma 3 (Incentives and optimal prices under Bertrand Competition) *Firm i's equilibrium price is decreasing in both (own and rival) agents' R&D incentives:* $\frac{\partial p_i^b}{\partial \beta_i} < 0$ *and* $\frac{\partial p_i^b}{\partial \beta_j} < 0$ *for any i and j. Moreover, the cross-effect is non-zero,* $\frac{\partial^2 p_j^b}{\partial \beta_i \partial \beta_j} \neq 0$.

Proof In appendix A.4. ∎

The modularity of $p_i^b(\beta_i, \beta_j)$ depends on the curvature of the demand function: the cross-partial derivative of the demand with respect to prices. Using equation (11), we examine the strategic nature of incentives by considering $\frac{d\beta_i}{d\beta_j} = -\frac{E[M_i]}{E[\Theta_i^b]}$, where

$$M_i \equiv \underbrace{\frac{\partial \pi_i^b}{\partial p_j}\frac{\partial^2 p_j^b}{\partial \beta_i \partial \beta_j}}_{\substack{<(>)\,0\\ \text{Submodular}\\ \text{(supermodular)}\\ \text{equil. price}}} + \underbrace{\frac{\partial^2 \pi_i^b}{\partial p_j^2}}_{\substack{>(<)\,0\\ \text{Incr. (decr.)}\\ \text{substitutability}}} \underbrace{\frac{\partial p_j^b}{\partial \beta_j}\frac{\partial p_j^b}{\partial \beta_i}}_{\substack{>0\\ \text{Lemma 3}}} + \underbrace{\frac{\partial^2 \pi_i^b}{\partial p_j \partial p_i}}_{>0} \underbrace{\frac{\partial p_i^b}{\partial \beta_j}\frac{\partial p_j^b}{\partial \beta_i}}_{\substack{>0\\ \text{Lemma 3}}} + \underbrace{\frac{\partial^2 \pi_i^b}{\partial c_i \partial q_i}\frac{\partial q_i^b}{\partial \beta_j}\frac{\partial c_i}{\partial \beta_i}}_{<0}. \tag{12}$$

From the second order condition, we have $E[\Theta_i^b] < 0$.[19] The last three terms in (12) are analogous to the terms in equation (6) in the Cournot model and the intuition is similar. Thus, the first term in equation (12) arises only in the Bertrand setting.[20] A higher β_i triggers a lower p_j which hurts the innovator. When the equilibrium price is submodular, this negative strategic effect on firm i's profit becomes even more negative as p_j decreases. Thus, as firm j innovates more to lower its price, firm i should lower the level of β_i to counteract the negative impact on its profits of a lower p_j. This is a source of strategic substitutability. The reverse is true when the equilibrium price is supermodular. Note also that the first and second terms in (12) only arise when the demand is non-linear.

Lemma 4 (Strategic interactions in R&D under Bertrand competition) *Researchers' R&D incentives are strategic substitutes, $\frac{d\beta_i}{d\beta_j} < 0$, if and only if*

$$\frac{\partial^2 p_j^b}{\partial \beta_i \partial \beta_j} < -\left(\frac{\partial \pi_i^b}{\partial p_j}\right)^{-1}\left[\left(\frac{\partial^2 \pi_i^b}{\partial p_j^2}\frac{\partial p_j^b}{\partial \beta_j} + \frac{\partial^2 \pi_i^b}{\partial p_j \partial p_i}\frac{\partial p_i^b}{\partial \beta_j}\right)\frac{\partial p_j^b}{\partial \beta_i} + \frac{\partial^2 \pi_i^b}{\partial c_i \partial q_i}\frac{\partial q_i^b}{\partial \beta_j}\frac{\partial c_i}{\partial \beta_i}\right].$$

Proof In appendix A.5. ■

We argue that if the condition stated in Lemma 4 is satisfied so that rivals' R&D decisions are strategic substitutes, as risk increases, the relationship between incentives and risk can be positive for the firm with a less risk-averse agent. A result similar to the one stated in Proposition 1 holds. The proof and intuition are the same as in the Cournot case and hence they are omitted.

In a Bertrand model with a linear demand, only the last two effects in equation (12) arise.[21] In addition, the third effect *always* dominates the forth effect, implying that rivals' R&D decisions are strategic substitutes. Thus, firms' decisions in the R&D stage do not inherit the properties of the strategic variables (prices) being utilized in the product market.

In numerical exercises, we have also investigated the role of the intensity of product market competition (and hence the strength of the business stealing effect) on the relationship between risk and incentives. First, we allowed the product differentiation parameter b to take any value in $[0,1]$ and examined its role, both in the Cournot and in the Bertrand linear models. Second, we analyzed the impact

[19] See Section A.5 for the form of Θ_i^b.

[20] The additional term in (12) arizes because in the Cournot model the marginal cost is c_i, while in the Bertrand model, it is $c_i\frac{\partial q_i}{\partial p_i}$. Thus, how $\frac{\partial q_i}{\partial p_i}$ depends on p_i matters when demand is non-linear.

[21] The linear demand has the form $q_i = \frac{a}{1+b} - \frac{1}{1-b^2}p_i + \frac{b}{1-b^2}p_j$.

of a change in the mode of competition from Cournot to Bertrand for a given b. For low values of b, the relationship is negative for both firms. If b exceeds a threshold, the firm with the less risk averse agent can experience a positive risk-incentives relationship. The threshold in the Cournot model is also higher than in the Bertrand model, which is consistent with the fact that the business stealing effect in Bertrand is stronger.

5 Concluding remarks

This paper introduces asymmetry between two risk-averse agents, who are appointed by product market rivals, in a standard moral hazard principal-agent model. Agents conduct cost-reducing innovation prior to competition in quantities or prices. The asymmetry in this model is stemming from the different degrees of agents' risk aversion or from different idiosyncratic risks. Each firm offers to its agent a contract that entails a fixed salary and a variable pay (incentives) that depends on the realized R&D outcomes of both firms. Thus, we allow for relative performance evaluations. In this setting, we examine the strategic properties of pay-for-performance compensation as well as the relationship between risk and the power of incentives. When demand is linear, firms' R&D decisions are always strategic substitutes irrespective of the mode of product market competition. We also state the conditions under which rival's R&D decisions are strategic substitutes even if demand is non-linear. Next, we show that the standard negative relationship between risk and incentives may not hold. In particular, the response of the firms can be asymmetric and opposite to changes in risk. The firm that has hired the less risk-averse manager, or is exposed to lower idiosyncratic risk, may even strengthen its agent's incentives as risk increases.

Our analysis and results have interesting public policy implications, when considering policies and regulations that attempt to reduce the uncertainty (broadly defined) surrounding innovative activities in an industry. Given that the conventional wisdom is that uncertainty impedes innovation, such policies aim at boosting the R&D level. We show that lower risk can create opposing reactions across firms in the same industry, in terms of how strongly the principals incentivize their agents to conduct innovation. The firms whose R&D incentives are weakened will invest less in innovation. Hence, such policy interventions can have unintended adverse consequences.

Our model suggests new avenues for future empirical research. One can examine the relationship between managerial compensation schemes and the corporate environment faced by asymmetric firms. The model predicts that, if firms are heterogeneous and managerial incentives are strategic substitutes, an increase in risk may strengthen the incentives offered by one set of firms and weaken the incentives offered by another set. It is important to emphasize that this result can hold true across firms in the *same* industry. Many empirical studies have failed to find strong support for the negative relationship between risk and incentives for firms in the same market and this model may explain why. It suggests that this relationship will be negative when firms operate in different markets, or when the intensity of strategic interactions among firms in the same market is weak. However, when strategic interactions are strong and business stealing is an important issue, higher risk may strengthen the equilibrium R&D incentives of a group of firms active in the same industry. Therefore, future empirical work, in testing the risk/pay-for-performance relationship, should group the firms (observations) according to the cost of incentivizing the agents, i.e., in a 'low cost' group and in a 'high cost' group. The estimate of the risk coefficient for the low cost group can be positive. Our analysis can also offer an alternative explanation about why a positive estimate for the risk-incentives relationship can be obtained even when all firms are grouped together and it has to do with the problem of sample selection. It may very well be the case that the sample of observations, from the population of firms, comes disproportionately from the low cost group. This is because the firms in this group are bigger and hence more likely to be selected.

We also derive the conditions under which firms' R&D decisions are strategic substitutes or complements to highlight the importance of the strategic nature of firms' reactions to changes in risk in an industry. The strategic nature of managerial incentives and thus of compensation schemes is itself empirically testable. Provided that firms typically sell products in many industries with different degrees of competition and attempt to strengthen their strategic position by investing in cost-reducing technologies, observed changes in compensation schemes reflect the agglomeration of the business stealing effects. In some industries, the business stealing effects may be negative, depending on rivals' response to increases in risk, while in some others, these effects may be positive.

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A APPENDIX

A.1 Proof of Lemma 1

We take the first-order conditions in the product market, $\frac{\partial \pi_i}{\partial q_i} = (d_i - c_i) + \frac{\partial d_i}{\partial q_i} q_i = 0$ and $\frac{\partial \pi_j}{\partial q_j} = (d_j - c_j) + \frac{\partial d_j}{\partial q_j} q_j = 0$, and differentiate them with respect to β_i (which affects c_i which in turn affects equilibrium prices). We get, respectively,

$$\frac{\partial^2 \pi_i}{\partial q_i^2}\frac{\partial q_i}{\partial \beta_i} + \frac{\partial^2 \pi_i}{\partial q_i \partial q_j}\frac{\partial q_j}{\partial \beta_i} = -1 \text{ and } \frac{\partial^2 \pi_j}{\partial q_j \partial q_i}\frac{\partial q_i}{\partial \beta_i} + \frac{\partial^2 \pi_j}{\partial q_j^2}\frac{\partial q_j}{\partial \beta_i} = 0.$$

We solve them and obtain the derivatives

$$\begin{aligned}
\frac{\partial q_i}{\partial \beta_i} &= -\frac{1}{\Lambda_c}\frac{\partial^2 \pi_j}{\partial q_j^2} = -\frac{1}{\Lambda_c}\left(\frac{\partial^2 d_j}{\partial q_j^2} q_j + 2\frac{\partial d_j}{\partial q_j}\right) > 0 \\
\frac{\partial q_j}{\partial \beta_i} &= \frac{1}{\Lambda_c}\frac{\partial^2 \pi_j}{\partial q_j \partial q_i} = \frac{1}{\Lambda_c}\left(\frac{\partial^2 d_j}{\partial q_j \partial q_i} q_j + \frac{\partial d_j}{\partial q_i}\right) < 0,
\end{aligned}$$

where $\Lambda_c \equiv \frac{\partial^2 \pi_i}{\partial q_i^2}\frac{\partial^2 \pi_j}{\partial q_j^2} - \frac{\partial^2 \pi_j}{\partial q_j \partial q_i}\frac{\partial^2 \pi_i}{\partial q_i \partial q_j} > 0$ is implied from the stability condition. The signs of the above derivatives follow from the assumptions $(C.1)$–$(C.3)$. Finally, $\frac{\partial q_i}{\partial \beta_i}$ is not a function of c_j and hence is not affected by β_j, implying $\frac{\partial^2 q_i}{\partial \beta_i \partial \beta_j} = 0$.

A.2 Proof of Lemma 2

To derive the slope of firm i's R&D best-response curve when firms compete in quantities, we totally differentiate equation (5) and obtain:

$$\left\{\frac{\partial^2\pi_i^c}{\partial q_j^2}\left(\frac{\partial q_j}{\partial\beta_i}\right)^2+\frac{\partial^2\pi_i^c}{\partial q_j\partial q_i}\frac{\partial q_i}{\partial\beta_i}\frac{\partial q_j}{\partial\beta_i}+\frac{\partial\pi_i^c}{\partial q_j}\frac{\partial^2 q_j}{\partial\beta_i^2}+\frac{\partial^2\pi_i^c}{\partial c_i\partial q_i}\frac{\partial q_i}{\partial\beta_i}\frac{\partial c_i}{\partial\beta_i}-\frac{\partial w'(\beta_i)}{\partial\beta_i}\right\}d\beta_i+$$
$$\left\{\frac{\partial^2\pi_i^c}{\partial {q_j}^2}\frac{\partial q_j}{\partial\beta_j}\frac{\partial q_j}{\partial\beta_i}+\frac{\partial^2\pi_i^c}{\partial q_j\partial q_i}\frac{\partial q_i}{\partial\beta_j}\frac{\partial q_j}{\partial\beta_i}+\frac{\partial\pi_i^c}{\partial q_j}\frac{\partial^2 q_j}{\partial\beta_i\partial\beta_j}+\frac{\partial^2\pi_i^c}{\partial c_i\partial q_i}\frac{\partial q_i}{\partial\beta_j}\frac{\partial c_i}{\partial\beta_i}\right\}d\beta_j=0.$$

Given that $\frac{\partial^2 q_j}{\partial\beta_i^2}=0$ and $\frac{\partial^2 q_j}{\partial\beta_i\partial\beta_j}=0$, the above expression reduces to equation (6). The coefficient of $d\beta_i$, denoted by Θ_i^c, is negative from the second order condition. Therefore, $\frac{d\beta_i}{d\beta_j}<0$ if and only if

$$\frac{\partial^2\pi_i^c}{\partial q_j^2}\frac{\partial q_j}{\partial\beta_j}\frac{\partial q_j}{\partial\beta_i}>-\frac{\partial q_i}{\partial\beta_j}\left(\frac{\partial^2\pi_i^c}{\partial q_j\partial q_i}\frac{\partial q_j}{\partial\beta_i}+\frac{\partial^2\pi_i^c}{\partial c_i\partial q_i}\frac{\partial c_i}{\partial\beta_i}\right),$$

implying the condition in Lemma 2.

A.3 Proof of Proposition 1

Solving equations (7) and (8), we get (to reduce the length of the expressions, we have omitted the E operators in front of the H's and Θ's)

$$\begin{aligned}\frac{d\beta_j^*}{d\sigma^2}&=\frac{2\Theta_i^c\beta_j^*r_j\left(1-\rho^2\right)g''\left(e_j\left(\beta_j^*\right)\right)-H_i\beta_i^*r_i\left(1-\rho^2\right)g''\left(e_i\left(\beta_i^*\right)\right)}{4\Theta_i^c\Theta_j-H_iH_j},\\ \frac{d\beta_i^*}{d\sigma^2}&=\frac{2\Theta_j^c\beta_i^*r_i\left(1-\rho^2\right)g''\left(e_i\left(\beta_i^*\right)\right)-H_j\beta_j^*r_j\left(1-\rho^2\right)g''\left(e_j\left(\beta_j^*\right)\right)}{4\Theta_i^c\Theta_j^c-H_iH_j},\end{aligned}$$

where $4\Theta_i^c\Theta_j^c-H_iH_j>0$. Recall that the signs of H_i and H_j - given by equation (6) - determine the slope of firms' R&D best-response curves. If firms' R&D decisions are strategic substitutes, both H_i and H_j are negative. In this regime, let higher σ^2 decrease β_j, $\frac{d\beta_j^*}{d\sigma^2}<0$, implying

$$R\equiv\frac{\beta_i^*r_i\left(1-\rho^2\right)g''\left(e_i\left(\beta_i^*\right)\right)}{\beta_j^*r_j\left(1-\rho^2\right)g''\left(e_j\left(\beta_j^*\right)\right)}<\frac{2\Theta_i^c}{H_i}.$$

Thus, higher σ^2 will increase firm i's optimal R&D incentives, $\frac{d\beta_i^*}{d\sigma^2}>0$, only if

$$R<\frac{H_j}{2\Theta_j^c}.$$

Notice that $\frac{H_j}{2\Theta_j^c}<\frac{2\Theta_i^c}{H_i}$, since $4\Theta_i^c\Theta_j^c-H_iH_j>0$. Thus, if $R<\frac{H_j}{2\Theta_j^c}$, we have $\frac{d\beta_i}{d\sigma^2}>0$ and $\frac{d\beta_j}{d\sigma^2}<0$, as specified in Proposition 1. If $\frac{H_j}{2\Theta_j^c}<R<\frac{2\Theta_i^c}{H_i}$, we have $\frac{d\beta_i^*}{d\sigma^2}<0$ and $\frac{d\beta_j^*}{d\sigma^2}<0$. Notice that R cannot exceed

$\frac{2\Theta_i^c}{H_i}$, because in this case, both $\frac{d\beta_i^*}{d\sigma^2}$ and $\frac{d\beta_j^*}{d\sigma^2}$ would be positive. This is not possible since an increase in σ^2 shifts both firms' R&D best-response curves inwards. Additionally, we cannot have $\frac{d\beta_j^*}{d\sigma^2} > 0$ and $\frac{d\beta_i^*}{d\sigma^2} < 0$ under the assumption that $r_j > r_i$. It requires higher σ^2 to decrease the optimal R&D incentives of the firm with the lower risk-averse agent (firm i) to a greater extend, which cannot hold.

A.4 Proof of Lemma 3

To examine the effect of agent i's R&D incentives on both firms' optimal prices, we take the first-order conditions in the product market, $\frac{\partial \pi_i^b}{\partial p_i} = D_i + (p_i - c_i)\frac{\partial D_i}{\partial p_i} = 0$ and $\frac{\partial \pi_j^b}{\partial p_j} = D_j + (p_j - c_j)\frac{\partial D_j}{\partial p_j} = 0$. Differentiating them with respect to β_i gives, respectively,

$$\frac{\partial^2 \pi_i^b}{\partial p_i^2}\frac{\partial p_i}{\partial \beta_i} + \frac{\partial^2 \pi_i^b}{\partial p_i \partial p_j}\frac{\partial p_j}{\partial \beta_i} = -\frac{\partial D_i}{\partial p_i} \text{ and } \frac{\partial^2 \pi_j^b}{\partial p_j \partial p_i}\frac{\partial p_i}{\partial \beta_i} + \frac{\partial^2 \pi_j^b}{\partial p_j^2}\frac{\partial p_j}{\partial \beta_i} = 0.$$

We solve them and obtain

$$\frac{\partial p_i}{\partial \beta_i} = -\frac{1}{\Lambda_b}\frac{\partial^2 \pi_j^b}{\partial p_j^2}\frac{\partial D_i}{\partial p_i} < 0 \text{ and } \frac{\partial p_j}{\partial \beta_i} = \frac{1}{\Lambda_b}\frac{\partial^2 \pi_j^b}{\partial p_j \partial p_i}\frac{\partial D_i}{\partial p_i} < 0,$$

where $\Lambda_b \equiv \frac{\partial^2 \pi_i^b}{\partial p_i^2}\frac{\partial^2 \pi_j^b}{\partial p_j^2} - \frac{\partial^2 \pi_j^b}{\partial p_j \partial p_i}\frac{\partial^2 \pi_i^b}{\partial p_i \partial p_j} > 0$. The signs of the above derivatives follow from the assumptions $(B.1)$–$(B.3)$; i.e., prices are strategic complements and a firm's demand is downward sloping in its own price. Finally, the effect of β_j on $\frac{\partial p_i}{\partial \beta_i}$ comes through c_j which appears both in Λ_b and in $\frac{\partial^2 \pi_j^b}{\partial p_j^2}$. With general demand functions, we have $\frac{\partial^2 p_i}{\partial \beta_j \partial \beta_j} \neq 0$.

A.5 Proof of Lemma 4

We totally differentiate (11) with respect to β_i and β_j to obtain

$$\left\{\frac{\partial^2 \pi_i^b}{\partial p_j^2}\left(\frac{\partial p_j}{\partial \beta_i}\right)^2 + \frac{\partial^2 \pi_i^b}{\partial p_j \partial p_i}\frac{\partial p_j^b}{\partial \beta_i}\frac{\partial p_i^b}{\partial \beta_i} + \frac{\partial \pi_i^b}{\partial p_j}\frac{\partial^2 p_j}{\partial \beta_i^2} + \left(\frac{\partial^2 \pi_i^b}{\partial c_i \partial p_i}\frac{\partial p_i^b}{\partial \beta_i} + \frac{\partial^2 \pi_i^b}{\partial c_i \partial p_j}\frac{\partial p_j^b}{\partial \beta_i}\right)\frac{\partial c_i}{\partial \beta_i} - \frac{\partial w'(\beta_i)}{\partial \beta_i}\right\} d\beta_i^b +$$
$$\left\{\frac{\partial^2 \pi_i^b}{\partial p_j^2}\frac{\partial p_j^b}{\partial \beta_j}\frac{\partial p_j^b}{\partial \beta_i} + \frac{\partial^2 \pi_i^b}{\partial p_j \partial p_i}\frac{\partial p_i^b}{\partial \beta_j}\frac{\partial p_j^b}{\partial \beta_i} + \frac{\partial \pi_i^b}{\partial p_j}\frac{\partial^2 p_j^b}{\partial \beta_i \partial \beta_j} + \left(\frac{\partial^2 \pi_i^b}{\partial c_i \partial p_i}\frac{\partial p_i^b}{\partial \beta_j} + \frac{\partial^2 \pi_i^b}{\partial c_i \partial p_j}\frac{\partial p_j^b}{\partial \beta_j}\right)\frac{\partial c_i}{\partial \beta_i}\right\} d\beta_j^b = 0.$$

Given that $\frac{\partial^2 \pi_i^b}{\partial c_i \partial p_i}\frac{\partial p_i^b}{\partial \beta_i} + \frac{\partial^2 \pi_i^b}{\partial c_i \partial p_j}\frac{\partial p_j^b}{\partial \beta_i} = \frac{\partial^2 \pi_i^b}{\partial c_i \partial q_i}\frac{\partial q_i^b}{\partial \beta_i}$ and $\frac{\partial^2 \pi_i^b}{\partial c_i \partial p_i}\frac{\partial p_i^b}{\partial \beta_j} + \frac{\partial^2 \pi_i^b}{\partial c_i \partial p_j}\frac{\partial p_j^b}{\partial \beta_j} = \frac{\partial^2 \pi_i^b}{\partial c_i \partial q_i}\frac{\partial q_i^b}{\partial \beta_j}$, the above expression reduces to equation (12), where the coefficient of $d\beta_i^b$, denoted by Θ_i^b, is negative from the second order condition. Thus, we have $\frac{d\beta_i^b}{d\beta_j^b} < 0$ if and only if

$$\frac{\partial \pi_i^b}{\partial p_j}\frac{\partial^2 p_j^b}{\partial \beta_i \partial \beta_j} < -\left[\left(\frac{\partial^2 \pi_i^b}{\partial p_j^2}\frac{\partial p_j^b}{\partial \beta_j} + \frac{\partial^2 \pi_i^b}{\partial p_j \partial p_i}\frac{\partial p_i^b}{\partial \beta_j}\right)\frac{\partial p_j^b}{\partial \beta_i} + \frac{\partial^2 \pi_i^b}{\partial c_i \partial q_i}\frac{\partial q_i^b}{\partial \beta_j}\frac{\partial c_i}{\partial \beta_i}\right],$$

implying the condition in Lemma 4.