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SOME ECONOMIC EFFECTS OF TECHNOLOGICAL CHANGE
by Herbert A. Simon

Some estimates of the magnitude of the direct effect upon national income of the introduction of cheap atomic energy or other technological changes can be obtained by studying simple economic models. The change in energy costs will be introduced as a change in the production function for energy. In the first section, a rather general model will be examined; this will be reduced to a macromodel of aggregates in a subsequent section; and finally a special case of the macromodel will be examined.

I.

Consider an economy in which a consumption good (y) is produced by the use of certain factors of production ($x_\lambda : \lambda=1, \dots, n$). The supply of x_1 (labor and land) will be considered fixed, while the supply of the other factors will be determined by production functions involving these factors.

Thus we have

$$(1.1) \quad y = \phi(x_{11}, x_{21}, \dots, x_{n1})$$

$$(1.2) \quad x_\lambda = \psi^{(\lambda)}(x_{1\lambda}, x_{2\lambda}, \dots, x_{n\lambda}; \alpha_\lambda), \quad (\lambda=2, \dots, n),$$

$$(1.3) \quad \sum_{i=1}^n x_{i1} = x_1 = \text{constant},$$

$$(1.4) \quad \sum_{i=1}^n x_{i\lambda} = x_\lambda, \quad (\lambda=2, \dots, n).$$

The $x_{\lambda i}$ are components of the x_λ devoted to different types of production. The α_λ are parameters describing the state of technology in each industry. (It can easily be shown that the results which follow hold even if some of the x_{ij} are equal to zero.)

From the usual assumption of profit maximization under competition we get additional relations:

$$(1.5) \quad \phi_1 = \phi_\lambda \psi_1^{(\lambda)}, \quad (\lambda=2, \dots, n),$$

$$(1.6) \quad \psi_1^{(\lambda)} = \psi_0^{(\lambda)} \psi_1^{(\rho)}, \quad (\lambda, \rho=2, \dots, n),$$

where the subscripts designate partial differentiation.

Equations (1.1) - (1.6) give us (n^2+n) independent relations among the (n^2+n) variables $y, x_\lambda, x_{ij}^{i,j=1, \dots, n}$ ($\lambda=2, \dots, n$). Hence we can consider these variables as functions of the α_λ , and in particular, we can solve for dy as a function of the $d\alpha_\lambda$. Taking total differentials of (1.1) - (1.2), we have:

$$(1.7) \quad dy = \sum_{i=1}^n \phi_i dx_{i1},$$

$$(1.8) \quad dx_\lambda = \sum_{i=1}^n \psi_i^{(\lambda)} dx_{i\lambda} + \psi_\alpha^{(\lambda)} d\alpha_\lambda.$$

Substituting (1.8) in (1.7), it can easily be shown^{1.} that the resulting expression reduces to:

$$(1.9) \quad dy = \sum_{\lambda=2}^n \phi_\lambda \psi_\alpha^{(\lambda)} d\alpha_\lambda.$$

$$(1.10) \quad \frac{dy}{y} = \sum_{\lambda=2}^n \frac{p_\lambda x_\lambda}{p_y x} \frac{\psi_\alpha^{(\lambda)}}{\psi^{(\lambda)}} d\alpha_\lambda = \sum_{\lambda=2}^n \frac{U_\lambda}{U_y} \frac{\psi_\alpha^{(\lambda)}}{\psi^{(\lambda)}} d\alpha_\lambda,$$

where $p_\lambda = \tau \phi_\lambda$; $p_y = \tau$; $U_\lambda = p_\lambda x_\lambda$. The interpretation of (1.10) is simplified if we introduce $\delta \psi^{(\lambda)} = \psi_\alpha^{(\lambda)} d\alpha_\lambda$. Then we have:

$$(1.11) \quad \frac{dy}{y} = \sum_{\lambda=2}^n \frac{U_\lambda}{U_y} \frac{\delta \psi^{(\lambda)}}{\psi^{(\lambda)}}.$$

^{1.} See Kenneth May, "The Aggregation Problem for a One-Industry Model," Econometrica, 14:289-291 (October, 1946). Professor May's notation differs from that used here, but his model is essentially the same.

But $\frac{\delta\psi(\lambda)}{\psi(\lambda)}$ is the relative change in x_λ brought about by the technological change in its production function, for given quantities of the factors of production. Hence, from (1.11) we read that the relative change in y is the sum of the products of U_λ/U_y by the relative change in productivity of the λ th industry.

If we specialize to the case where $\frac{\delta\psi(\lambda)}{\psi(\lambda)} = k$ for λ in some set Ω ; $\frac{\delta\psi(\lambda)}{\psi(\lambda)} = 0$ for λ not in Ω , we get:

$$(1.12) \quad \frac{dy}{y} = \frac{k}{U_y} \sum_{\lambda \in \Omega} U_\lambda$$

It will be useful, at this point, to examine the economic significance of our results. We are considering, of course, only first-order effects, which will be dominant if the dd_λ are small.

1. From (1.9) we see that the increase in y is that which would be achieved if the quantities of factors used in producing x_λ were held constant, while the increment in x_λ resulting from the change in technology were applied directly to the production of y . The absence of more complex substitution effects (or rather, their elimination in reaching equation (1.9)) is due to the fact that decreasing or increasing returns produce only second-order effects, and hence do not come into the picture.

2. From (1.10), we see that the effects of technological changes in different industries are additive, which follows from the fact that we are considering only first-order effects. From (1.11) we see that the relative change in y is proportional to the ratio of the value of the factor produced (and not merely that fraction of it used in producing y) to the value of y ; and proportional also to the relative change in the production function of the factor. From (1.12) we see further that if several factors are increased in productivity by the same proportion, they can be treated as a single factor.

5. In a previous study of this problem, Kenneth May has warned that ignoring the interrelations of industries may lead to an underestimate of aggregate effects. From his analysis he argues: "it appears that technological change has a cumulative effect in the aggregate and that extreme care must be exercised in drawing conclusions about over-all effects from data on particular firms."².

How can the possibility, which May foresees, of cumulative effects, be reconciled with our result that the effects are additive? The contradiction is apparent rather than real, and largely hinges on the terminological question of what one means by "cumulative." In the first place, May considers in his model the possibility of technological change in the consumption good industry, hence his equation corresponding to (3.11) contains the additional

term $\frac{U_y}{U_y} \frac{\delta \phi}{\phi'} = \frac{\delta \phi}{\phi'}$ and equation (1.12) becomes:

$$(1.12^0) \quad \frac{dy}{y} = k \left\{ \frac{\sum_{\lambda \in \Omega} U_{\lambda}}{U_y} + 1 \right\};$$

and we have $\frac{dy}{y} > k$.

This is a rather inconsequential point, since, in our model it is perfectly possible to have $\sum_{\lambda \in \Omega} U_{\lambda} > U_y$ and hence $\frac{dy}{y} > k$. The real issue is whether technological change in an economy of n industries can be estimated correctly from a model in which some of these industries are represented collectively only as aggregates.

². Kenneth May, "Technological Change and Aggregation," Econometrica, 15:51 - 63. (January, 1947).

We must recall that the U_λ in (1.12) are gross values of the various factors of production and not simply net "value added by manufacture." Moreover, the U_λ in (1.12) refer, not to the quantities of the factors employed directly in the production of y , but to the total quantities of the factors produced. It is for this reason that $\sum_{\lambda \in \Omega} U_\lambda$ will in general be much greater than U_y . Hence, if the set Ω is sufficiently large we will have $\sum_{\lambda \in \Omega} U_\lambda > U_y$.

Hence, if the magnitude of a (small) technological improvement affecting one or several firms is known, the total effect upon the production of consumption goods in the economy can be correctly estimated from a knowledge of the U_λ of the firms in question and of U_y , by the use of equation (1.12).

4. The question may still be asked whether our result is invariant with respect to the definition of a "firm". It would be highly unsatisfactory if the result obtained for a firm carrying on an integrated series of production processes would be altered by breaking up the production function for the firm into the several production functions for the separate processes. That this is not the case can easily be shown.

The difficulty involved in vertical integration is that integration will in general reduce $\sum_{\lambda \in \Omega} U_\lambda$ by eliminating some of the intermediate factors.

Consider two firms (or industries), the product of the first being used entirely as a factor in the production of the second, then $\sum U$ for these two firms will be $U_a + U_b$; while if the production function were written for a vertically integrated firm combining both production processes, we would have $U = U_b$.

The appearance of paradox is removed when it is recognized that substituting in our model the production function of the integrated firm for the two original production functions not only changes $\sum U$, but produces also a compensating change in the k of equation (1.12).

A more concrete understanding of these results may be obtained by applying them to the specific case of a technological change in the electric power industry. We may consider the production function for (a) electric energy at the central generating station, (b) electric energy transmitted to the plant, (c) electric energy applied through a power tool and (d) a commodity produced by this tool -- each of these being taken as a distinct product.

Suppose we have a technological improvement in (a) of magnitude k . This means that a given quantity of factors of production will produce $(k + 1)$ times as much energy at the central station as before. If our consumption good be represented by y , we have:

$$(1.13) \quad \frac{dy}{y} = \frac{V_a}{V_y} k.$$

If we consider now a power industry integrating steps (a) and (b), we will have:

$$(1.14) \quad \frac{dy}{y} = \frac{V_a}{V_y} k = \frac{V_b}{V_y} \left(\frac{V_a}{V_b} k \right) = \frac{V_a}{V_y} k^{\theta}$$

$$\text{where } k^{\theta} = \frac{V_a}{V_b} k$$

In other words, we can interpret our change in productivity as resulting from a technological change of magnitude k^{θ} in an industry with product valued at V_b . Similarly:

$$(1.15) \quad \frac{dy}{y} = \frac{V_a}{V_y} k = \frac{V_c}{V_y} \left(\frac{V_a}{V_c} k \right) = \frac{V_a}{V_y} k^{\eta}$$

Here k^{θ} and k^{η} are properly interpreted as the relative changes in productivity in the integrated industries, however defined, corresponding

to a change in productivity k in one of the component productive processes.

Similar adjustments can be made where technological change of magnitude k takes place in several of the productive processes in the integrated industry -- say in power generation and in power transmission.

Consider the case of a three-firm economy with the functions:

$$(1.1A) \quad y = \phi(x_{11}, x_2),$$

$$(1.2A) \quad x_2 = \psi(x_{12}, x_3, \alpha_2),$$

$$(1.3A) \quad x_3 = \xi(x_{13}, \alpha_3),$$

$$(1.4A) \quad x_{11} + x_{12} + x_{13} = x = \text{constant}.$$

We obtain:

$$(1.10A) \quad \frac{dy}{y} = \frac{v_2}{v_y} \frac{\psi_{\alpha_2}}{\psi} d\alpha_2 + \frac{v_3}{v_y} \frac{\xi_{\alpha_3}}{\xi} d\alpha_3,$$

$$\text{and, if } \frac{\psi_{\alpha_2}}{\psi} d\alpha_2 = \frac{\xi_{\alpha_3}}{\xi} d\alpha_3 = k, \text{ i.e. } \frac{\delta\psi}{\psi} = \frac{\delta\xi}{\xi} = k,$$

we have:

$$(1.12A) \quad \frac{dy}{y} = \frac{v_2 + v_3}{v_y} k.$$

Now consider a macromodel in the variables $y, x_{11}, x_{14} = x_{12} + x_{13}$, and x_2

$$(1.1A^0) \quad y = \phi(x_{11}, x_2),$$

$$(1.2A^0) \quad x_2 = \psi^1(x_{14}, \alpha_2, \alpha_3) = \psi(x_{12}, \xi(x_{13}, \alpha_3), \alpha_2),$$

$$(1.3A^0) \quad x_{11} + x_{14} = x = \text{constant}.$$

Then:

$$(1.10A^0) \quad \frac{dy}{y} = \frac{v_2}{v_y} \left\{ \frac{\psi^1_{\alpha_2}}{\psi^1} d\alpha_2 + \frac{\psi^1_{\alpha_3}}{\psi^1} d\alpha_3 \right\}$$

$$= \frac{v_2}{v_y} \left\{ \frac{\psi_{\alpha_2}}{\psi} d\alpha_2 + \frac{\psi_3 \xi_{\alpha_3}}{\psi} d\alpha_3 \right\}.$$

But since $\psi_3 = \frac{p_3}{p_2} \cdot \frac{\psi_3 \xi}{\psi} = \frac{\sqrt{3}}{\sqrt{2}}$, and we get

$$\begin{aligned}
 (1.12B^E) \quad \frac{dy}{y} &= \frac{\sqrt{2}}{\sqrt{y}} \left\{ \frac{\psi_2}{\psi} d\alpha_2 + \frac{\sqrt{3}}{\sqrt{2}} \frac{\xi}{\xi} d\alpha_3 \right\} \\
 &= \frac{\sqrt{2}}{\sqrt{y}} \left\{ 1 + \frac{\sqrt{3}}{\sqrt{2}} \right\} k = \frac{\sqrt{2}}{\sqrt{y}} k^0.
 \end{aligned}$$

Hence, we see that a technological change of magnitude k in two firms in the micromodel corresponds to a technological change of magnitude $k^0 = (1 + \frac{\sqrt{3}}{\sqrt{2}}) k$ in the macromodel.

In terms of our power-industry example, this means that a technological improvement of magnitude k in power production, and in power generation, corresponds to a change of magnitude $k^0 = (1 + \frac{\sqrt{3}}{\sqrt{2}}) k$ in the integrated industry.

II

From these results it will be interesting to derive actual estimates of the increase in production of consumption goods that might result from technological improvements in the electric power industry (or any other industry). Table I gives values of $\frac{dy}{y}$ calculated from equation (1.12) for various values of k and V_Ω / σ_y , where $V_\Omega = \sum_{\lambda \in \Omega} \sigma_\lambda$.

For the power industry (including transmission of electric energy) in the United States, V_Ω / σ_y is approximately .05. Hence an increase of 5 per cent in the productivity of the power industry might increase production of consumption goods by one-quarter of one per cent; while an increase of productivity of 100 per cent might increase consumption goods by 5 per cent.

The estimates in the table are reliable only for the smaller values of k since second-order effects have been ignored.

Table I.
Values of $\frac{dy}{y}$

$\sqrt{\alpha}/\nu$ \ k	.05	.10	.20	.50	1	9
.05	.0025	.005	.01	.025	.05	.45
.10	.005	.01	.02	.05	.10	.90
.20	.010	.02	.04	.10	.20	1.80
.30	.015	.03	.06	.15	.30	2.70
.40	.020	.04	.08	.20	.40	3.60
.50	.025	.05	.10	.25	.50	4.50

The estimates of Table I can also be obtained from a two-industry macromodel ³ -- a result which is not surprising in view of our previous discussion of integration. Let:

- (2.1) $Y = \eta(X_a, E),$
- (2.2) $E = \xi(X_b, \alpha),$
- (2.3) $X_a + X_b = X = \text{constant},$
- (2.4) $\eta_x = \eta_E \xi_x$

Taking total differentials, we find that

$$(2.5) \quad dY = \eta_E \xi_x d\alpha.$$

This equation is of the same form as (1.10) when all the α_j 's except

³ This is essentially the model employed by May, op. cit. in footnote 2, pp. 54-58.

one are taken as constant. Then (2.1) - (2.4) can be shown to be a macromodel of (1.1) - (1.6) if we take $Y = y; \gamma = \phi; X_a = x_{11}; X_b = x_1 - x_{11}; E = x_n;$
 $(\alpha = \alpha_n; \alpha_\lambda = 0 (\lambda \neq n));$
 $\xi = \psi^{(n)}; \gamma_x = \phi_1; \gamma_E = \phi_n; \xi_x = \psi_1^{(n)}.$ 4.

Defining again $\delta \xi = \xi_\alpha d\alpha$, we get

$$(2.6) \quad \frac{\delta Y}{Y} = \frac{P_E}{P_Y} \frac{E}{Y} \frac{\delta \xi}{\xi} = \frac{V_E}{V_Y} \frac{\delta \xi}{\xi} = \frac{V_E}{V_Y} k.$$

III

In the case of decreasing returns, we may expect the values of $\frac{dy}{y}$ to be smaller than those shown in Table I for large values of k. To obtain some estimate of the effect of decreasing returns consider a one-industry macro-model for the special case of the Cobb-Douglas functions. 5.

$$(3.1) \quad Y = AX_a^s E^t$$

$$(3.2) \quad E = \alpha X_b^r$$

$$(3.3) \quad X_a + X_b = X = \text{constant},$$

$$(3.4) \quad \frac{X_b}{X_a} = \frac{rt}{s}$$

Here, as before, E must be considered the quantity of power employed in the entire economy, not simply the quantity employed in the production of Y.

4. Cf. May, op. cit. in footnote 1, pp. 289-291.

5. It should be pointed out that this macromodel can not be deduced from a micromodel in which the production functions for the individual firms are Cobb-Douglas functions.

If we consider a change, $\Delta\alpha$ in the parameter we will obtain new equilibrium values, \bar{Y} , \bar{X}_a , \bar{X}_b , \bar{E} , for our variables. But, since

$$(3.5) \quad \frac{\bar{X}_b}{\bar{X}_a} = \frac{rt}{s} \text{ and } \bar{X}_a + \bar{X}_b = X = X_a + X_b, \text{ we have:}$$

$$(3.6) \quad \bar{X}_a = X_a; \bar{X}_b = X_b. \quad \text{Hence,}$$

$$(3.7) \quad \bar{Y} = Y + \Delta Y = AX_a^s X_b^{rt} (\alpha + \Delta\alpha)^t;$$

$$(3.8) \quad \frac{Y + \Delta Y}{Y} = \frac{(\alpha + \Delta\alpha)^t}{\alpha^t} = \left(1 + \frac{\Delta\alpha}{\alpha}\right)^t.$$

If as previously, we set $\frac{\Delta\alpha}{\alpha} = k$, we find

$$(3.9) \quad \log \frac{Y + \Delta Y}{Y} = t \log (1 + k).$$

But $t = \frac{\sigma_E}{\sigma_Y}$, and we have:

$$(3.10) \quad \log \frac{Y + \Delta Y}{Y} = \frac{\sigma_E}{\sigma_Y} \log (1 + k)$$

Hence, if the ratio of total energy costs to total value of consumption goods is known, $\frac{\Delta Y}{Y}$ can be calculated for various values of k , as shown in the following table:

Table II
Values of $\frac{\Delta Y}{Y}$

$\frac{\sigma_E}{\sigma_Y} k$	1	9	19	49	99
.02	.014	.05	.06	.08	.10
.05	.035	.12	.16	.22	.26
.10	.07	.26	.35	.48	.59
.20	.15	.59	.82	1.19	1.51
.30	.23	1.00	1.45	2.24	2.98

It can be seen from the table that if the value of energy produced in the economy is only two percent of the value of consumption goods, a 100% increase in productivity of the power industry ($k = 1$) will increase production of consumption goods 1.4%; a 900% increase ($k = 9$) will increase production 5%.

In computing the increased productivity of the power industry our previous analysis of aggregation shows that it does not matter whether we consider cost of energy at the generating plant, cost at the factory, or cost including power appliances. In each case we will get different values for σ_E , but correspondingly different values for k ; hence, our estimates of ΔY will be identical.

Finally, it is interesting to compare the results obtained in this section and shown in Table II with the first-order effects tabulated in Table I. It will be seen that the values of $\frac{\Delta Y}{Y}$ in Table I are slightly too large for $k = 1$, and very much too large for $k = 9$. We conclude that the first order effects predominate when $k \leq 1$.