AGGREGATE FLUCTUATIONS IN ADAPTIVE PRODUCTION NETWORKS

By

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Aggregate fluctuations in adaptive production networks

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To counteract the adverse effects of shocks, such as the global pandemic, on the economy, governments have discussed policies to improve the resilience of supply chains by reducing dependence on foreign suppliers. In this paper, we develop and quantify an adaptive production network model to study network resilience and the consequences of reshoring of supply chains. In our model, firms exit due to exogenous shocks or the propagation of shocks through the network, while firms can replace suppliers they have lost due to exit subject to switching costs and search frictions. Applying our model to a large international firm-level production network dataset, we find that restricting buyer–supplier links via reshoring policies reduces output and increases volatility and that volatility can be amplified through network adaptivity.

production networks | supply chains | shocks | aggregate fluctuations | resilience

Significance

Economic production is organized in supply chain networks. The resilience of supply chains is, therefore, crucial for robust economic growth. Governments have discussed policies to improve the resilience of supply chains, including the notion of reshoring parts of the supply chains to reduce dependence on foreign suppliers. A calibrated version of our theory predicts that restricting buyer–supplier links via such policies reduces output and increases economic fluctuations and that economic instability can be amplified through network adaptivity.

Following a series of adverse shocks—the pandemic, the ongoing shortage of semiconductors, the stranding of the container ship *Ever Given* in the Suez Canal—the world has become acutely aware of the vulnerabilities in global supply chains (1, 2). As a result, multiple governments have initiated systematic reviews of their countries' supply chains in key sectors with the aim of improving "resilience." Prominent among these discussions is the notion of reshoring at least some parts of the supply chains to reduce dependence on foreign suppliers.

Early analyses of the impact of reshoring on supply chain vulnerability do not show systematic benefits of reshoring in cushioning the impact of a large shock, such as the pandemic. However, existing work uses models in which the production network is exogenously fixed either to the observed equilibrium or to a hypothetical reshored equilibrium. Exogenous network models provide an incomplete picture of network resilience as they do not incorporate the notion that the production network itself might adjust following an adverse event, and reshoring supply chains may have an impact on those adjustments.

This paper develops and quantifies a tractable adaptive network model to study network resilience and the consequences of reshoring of supply chains. We consider a large cohort of firms connected to each other via input–output linkages (3). Firms' production recipes can be described by a set of input and output characteristics, and input–output links are formed only if the output characteristics of a seller match the input characteristics of a buyer (*cf* refs. 4 –6). At a single point in time, the production equilibrium is characterized by the textbook network model of refs. 7 and 8. The network dynamics are endogenously generated (*cf* refs. 9 –14). Firms match with each other and experience a variety of shocks. A firm may lose a supplier or a customer. In that case, it can find others, subject to a matching friction. Firms are hit by shocks, which may lead them to exit (*cf* refs. 15 –18), and these shocks may propagate to their customers, also leading them to exit. Firms can also replace existing suppliers with new suppliers subject to search frictions or switching costs.

We provide an analytic characterization of the time evolution of the number of firms, the degree (customers) distribution, and the firms' life span distribution in the form of a system of stochastic differential equations (SDEs). Moreover, we characterize the solution to these SDEs and their asymptotic behavior. We show that the degree distribution follows a power law, and a firm's life span is exponentially distributed. Both observations are consistent with the data.

We then use a rich, little explored firm-level dataset to estimate the key parameters of the model. Our main source of information is the FactSet Revere Business Relationships database. It contains information about the production network between more than 60,000 firms across many sectors and countries. We structurally estimate the parameters governing switching costs and search frictions that determine the adaptivity of the production network.

Our main counterfactual exercise examines the implications of reshoring supply chains on aggregate volatility. We simulate breaking supply links between three large economies (the United States, East Asia, and the European Union) and compute the aggregate volatility for each of these economies in the new equilibrium. Crucially in our exercise,

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the counterfactual includes all the margins of adjustment of the network. When supply chain links are severed, the domestic network in all three economies reorganizes itself as firms replace the unavailable foreign suppliers with domestic ones. The network is shown to adjust itself following shocks in both the baseline and the counterfactual scenarios. We show that network adaptivity does not significantly help cushion the impact of shocks in the reshored scenario. On the contrary, it amplifies aggregate fluctuations.

The analysis of the propagation of shocks in supply networks is related to the theories of information dynamics and epidemic spread in social networks. This literature has identified a threshold below which information or epidemics cannot spread widely. This threshold is typically related to topological properties of the network (19, 20). Ignoring network endogeneity, however, limits the robustness and applicability of these results. An exception is ref. 21, which analyzes epidemic spreading on an adaptive network where susceptible nodes rewire links away from infected neighbors. Ref. 21 shows that rewiring can lead to the formation of highly connected susceptible clusters, in which the infection can spread more easily. This means that while rewiring can help to isolate infected individuals, the aggregate topological effects can lower the threshold and amplify the spread of infections. [Similarly, reducing modularity in ecological systems can increase their instability (22).] In this paper, rewiring of the production network can lead to the formation of densely connected clusters in which firm-level economic shocks can propagate more easily. While rewiring links from lost suppliers might have a positive local effect on an individual firm's survival probability, the aggregate effect of rewiring on the network topology can lead to amplified aggregate fluctuations.

The Model

Consider an economy populated by N_t firms indexed by $i = 1, \ldots, N_t$. Time is continuous and indexed by $t \in \mathbb{R}_+$. Let the network at time t be denoted by $G_t \in \mathcal{G}(N_t)$, with $\mathcal{G}(N_t)$ the class of all directed networks with N_t firms.

Fig. 1 depicts the different events taking place during the evolution of the production network. The formation of the production network with dynamic entry and exit is described by a Markov chain.

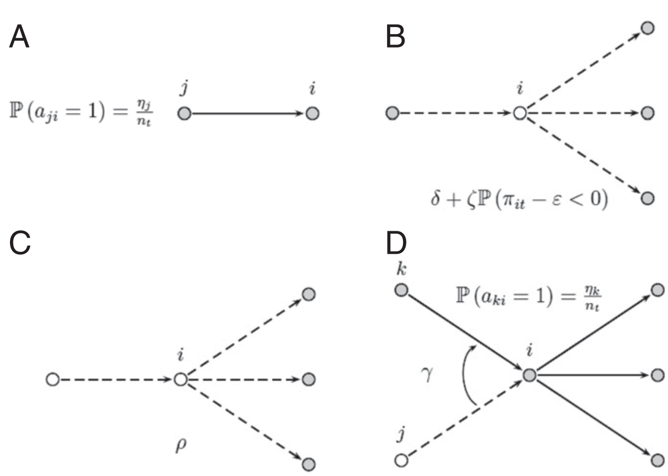

Fig. 1. The different events that happen during the time evolution of the Markov chain introduced in *Definition*: (*A*) entry, (*B*) exit due to a large or small shock, (*C*) shock propagation, and (*D*) replacement of a supplier after exit. Filled circles indicate firms that have not exited, while empty circles indicate firms that have exited.

***Definition*:** We consider a continuous time Markov chain $(G_t)_{t \geq 0}$. Starting from an initial state G_0, in a small time interval t to $t + \Delta t$, the following events happen with corresponding rates.

Entry. With rate $\Omega > 0$, a new firm is born, and its number of input goods ℓ is drawn from a distribution $h(\ell)$ with compact support $\ell \in \{1, 2, \ldots, L\}$, $L \geq 1$. The characteristics of each input good are drawn independently with probability $p \in [0, 1]$, and the characteristics of its output good are drawn with probability $q \in [0, 1]$. Then, for each input good, an incumbent firm whose output characteristics match the entrant's input characteristics becomes the supplier to the entrant.

Exit.

1) Large shocks. With rate $\delta \geq 0$, a firm exits.
2) Small shocks. With rate $\zeta \geq 0$, a firm is hit by an additive profit shock, $\varepsilon \geq 0$, following a $\mathrm{Pareto}(\bar{\varepsilon}, 1)$ distribution. If the firm's after-shock profit is negative, it exits.
3) Shock propagation. With rate $\rho \geq 0$, a firm that has lost all its suppliers exits.

Rewiring. If a firm loses a supplier due to exit, with rate $\gamma \geq 0$ it attempts to replace it with a firm whose output characteristics match the firm's input characteristics.

Upon creation, a firm is characterized by 1) a number of suppliers ℓ, drawn from a distribution $h(\ell)$ with compact support $\ell \in \{1, 2, \ldots, L\}$ and $L \geq 1$, and 2) a tuple $\mathbf{h}_i = (\mathbf{h}_i^-, \mathbf{h}_i^+)$ of a zero–one vector of input characteristics $\mathbf{h}_i^- \in \{0, 1\}^K$ and output characteristics $\mathbf{h}_i^+ \in \{0, 1\}^K$ of length K. These firm attributes are not time varying, and all ℓ suppliers to the firm must satisfy the firm's input characteristics.

We further assume that each input characteristic h_{ik}^- is one with probability $p \in [0, 1]$ and zero otherwise (4–6). Similarly, each output characteristic h_{ik}^+ is one with probability $q \in [0, 1]$ and zero otherwise for all $k = 1, \ldots, K$. Under this assumption of independently drawn input–output characteristics, the stock of input characteristics of a firm i, denoted by $|\mathsf{S}(\mathbf{h}_i^-)|$,* follows a $\mathrm{Binomial}(p, K)$ distribution, while the stock of output characteristics, $|\mathsf{S}(\mathbf{h}_i^+)|$, follows a $\mathrm{Binomial}(q, K)$ distribution.

When a potential buyer–supplier pair meets, the buyer will only use the supplier if the match has positive productivity. The productivity is only positive if the output characteristics of the supplier satisfy the input characteristics of the buyer. For the buyer and supplier characteristics to match, it has to be that the supplier has a one in each of the K elements of the buyer's input characteristic vector $\mathbf{h}_i^-$ that has a one. That is, we can think of a value of one in a buyer's input characteristic vector as signifying that the buyer "requires" that characteristic. For a successful match, it must be that the supplier also has a one in that entry, and so, the supplier's input "satisfies" that requirement. On the other hand, when the entry is zero in the buyer's vector, the supplier can have either zero or one in that entry.

Thus, a firm i with a large stock of output characteristics, $|\mathsf{S}(\mathbf{h}_i^+)|$, produces a good that can be used by many firms as an input. These goods can be thought of as "general purpose technologies" (23, 24). Under the above assumptions, as shown in *SI Appendix*, a firm's propensity to become selected as a supplier is

*Let the support of $\mathbf{h}$ be given by $\mathsf{S}(\mathbf{h})$ and its cardinality given by $|\mathsf{S}(\mathbf{h})|$.

proportional to a random variable $\eta \geq 0$ that follows a lognormal distribution with density

$$f(\eta) = \frac{1}{\sqrt{2\pi\tilde{\sigma}^2}} \frac{1}{\eta} e^{-\frac{(\ln\eta - \tilde{\mu})^2}{2\tilde{\sigma}^2}}, \quad [1]$$

SD $\tilde{\sigma} = \sqrt{Kq(1-q)} \ln(1 + \frac{p}{q(1-p)})$, and mean $\tilde{\mu} = Kq\ln(1 + \frac{p}{q(1-p)}) + K\ln(1-p)$ given parameters $p, q \in [0,1]$. In the following, we will refer to η as the attractiveness of a firm as a supplier.

When a firm is instantiated, it is matched with all ℓ of its suppliers, assuming the suppliers with the firm's required input characteristics exist in the economy.[†] Initially, the firm does not sell to other firms, only to the final consumers. Over time, the firm will acquire other firm customers, as new firms with matching input characteristics are born. Moreover, we assume that there exists an outside sector producing a homogeneous good that can be used as an input by firms without an intermediate goods–producing supplier (10).

As in ref. 25, we distinguish between small and large shocks. If a firm is hit by a large shock, it exits (independently of firm size). The assumption regarding small shocks in our model is as follows; a firm i hit by a cost shock ε exits if its current profits are lower than the size of the shock: $\pi_{it} < \varepsilon$. This assumption is the simplest one that allows us to introduce a size-dependent probability of exit, such that the larger and more profitable firms are less likely to fail. This formulation rules out dynamic considerations, whereby the firm will stay in operation despite negative profits if it expects positive profits in the future. This can be rationalized either through liquidity constraints or by assuming that once the firm receives a shock ε that is greater than its current profits, the shock becomes permanent.

The probability that firm i exits conditional on receiving a shock ε is given by

$$\mathbb{P}(\pi_{it} - \varepsilon < 0) = \left(\frac{\pi_{it}}{\bar{\varepsilon}}\right)^{-1}, \quad [2]$$

where we have made use of the Pareto assumption. As shown in part 2 of *Proposition 2* in *Materials and Methods*, the profit of firm i is an increasing function of the weighted number of customers of i:

$$\pi_{it} \propto 1 + \alpha \sum_{j=1}^{N_t} a_{ijt} \frac{1}{d_{jt}^-}. \quad [3]$$

With an appropriate normalization of parameters, the exit probability in Eq. **2** can then be written as[‡]

$$\mathbb{P}(\pi_{it} - \varepsilon < 0) = \frac{1}{1 + \alpha \sum_{j=1}^{N_t} a_{ijt} \frac{1}{d_{jt}^-}}. \quad [4]$$

In the special case when firms have only a single supplier (*cf* ref. 10), this becomes

$$\mathbb{P}(\pi_{it} - \varepsilon < 0) = \frac{1}{1 + \alpha d_{it}^+}. \quad [5]$$

As larger firms have a lower probability of exiting, we call this preferential survival (17). This is consistent with the empirical evidence (13, 26, 27).

A firm that has lost all its suppliers and is unable to replace them will experience a high probability of exiting as well in future periods (shock propagation). Once a firm has lost its suppliers, it can continue to produce by buying from an outside sector. However, the provision of the inputs for production from this outside sector is highly unreliable or not compatible with the firm's production technology and breaks down at a rate ρ. When this happens, the firm is also forced to shut down production and exits. Further, note that we assume that links are removed among incumbent firms because of firms exiting. This is consistent with the empirical evidence presented in ref. 13, where it is documented that in the US production network, about 70% of all link destructions are due to a firm exiting.

The parameter γ is a measure of the adaptivity of the network. It is similar to macroeconomic models of price stickiness, where the opportunity for firms to reset their prices in any particular period is a random event and the probability that they are unable to do so is known as the "Calvo probability" (28). Once again, we assume that when a supplier with fitting characteristics exists, the firm successfully finds it and forms a match. If there are multiple such suppliers, the firm flips a coin between them.

Theoretical Analysis

It is possible to make analytical predictions about the dynamical behavior of the production network by making a heterogeneous mean-field assumption (29)[§] and modeling the network as a Markov jump process (35). We keep track of the number $N_k^i(\eta)$ of firms with i suppliers, k customers, and attractiveness η and how this quantity changes over time due to entry, exit, or rewiring (as specified in *Definition*). As shown in *Proposition 3* in *Materials and Methods*, we can derive a system of SDEs (Eq. **11**) that allows us to determine $N_k^i(\eta)$ at any point in time starting from any given initial conditions. To make this challenging task more tractable, in this section we further focus on the case of firms having at most one supplier such that the probability of exit of a firm is given by Eq. **5**. Moreover, we consider the case of high entry rates when the network becomes large as this is the empirically relevant case.

The following proposition characterizes the asymptotic degree distribution: that is, the distribution over the number of buyers across firms, the number of firms, and a firm's lifetime distribution. In order to derive these asymptotic distributions, we solve for the fixed points of the SDEs (Eq. **11**) that govern the system dynamics. This gives us a set of recursive equations that can be solved using the Poincaré–Perron theorem (36) (the proof of *Proposition 1* in *SI Appendix* has further details).

Proposition 1. *Let the probability of exit of a firm be given by* Eq. **5**. *Further, denote by* $\tau = \frac{n^0}{n}$ *the fraction of firms with in-degree zero, and let* $\kappa = \sum_\ell \frac{1}{1+\alpha\ell} \frac{n_\ell}{n}$ *be the average shifted inverse out-degree.*

1) *The expected number of firms at time t is given by*

$$n_t = \frac{1 - (1 - (\delta + \zeta\kappa + \rho\tau))e^{-t(\delta+\zeta\kappa+\rho\tau)}}{\delta + \zeta\kappa + \rho\tau}, \quad [6]$$

[†]In the theoretical analysis, we assume that the network is large enough such that this is always the case. When the model is simulated numerically, if the fitting supplier does not exist, the firm remains unmatched and therefore, has fewer than ℓ suppliers upon birth. If there are multiple suppliers that fit the firm's characteristics, the firm flips a coin between them.

[‡]We make the normalizing assumption $(1-\alpha-\beta)(1-\alpha)h/(m\beta\bar{\varepsilon}) = 1$, and with the profit function stated in part 2 of *Proposition 2*, we obtain the exit probability in Eq. **4**.

[§]Introduced in ref. 30, this approximation scheme has become a mainstay of network analysis, particularly in the literature studying contagion processes in scale-free networks (*cf* refs. 31–34).

with the limit

$$\lim_{t\to\infty} n_t = \frac{1}{\delta + \zeta\kappa + \rho\tau}. \quad [7]$$

2) *The asymptotic out-degree distribution is given by*

$$P^{+}(k) \simeq \frac{1}{(k-1)!}\int_0^{\eta_{\max}} k^{-\psi(\eta)}\, d\eta, \quad [8]$$

where in Eq. ***8*** *we have defined by* $\psi(\eta) = ((1-\eta)\kappa\zeta - \tau(\eta(\gamma+\rho)+\rho) - \delta\eta)/(\delta+\kappa\zeta)$.

3) *The firm's lifetime* (T) *distribution is exponential and given by*

$$\mathbb{P}(T > t) \simeq \int_0^{\eta_{\max}} e^{-\chi(\eta)t}\, d\eta, \quad [9]$$

where in Eq. ***9*** *we have defined by* $\chi(\eta) = \delta + (r(\delta + \kappa r))/(\delta + \kappa\zeta + ((\delta + \rho\tau + \kappa\zeta)(1+\gamma\langle k\rangle))\eta) + \rho$.

The asymptotic number of firms in Eq. **7** is decreasing with the exit rates, δ, ζ, and ρ. The out-degree distribution $P^{+}(k)$ in Eq. **8** is a power law decaying function with a variance that is increasing with the number of characteristics K (*SI Appendix*, Fig. S5). Such heavy-tailed degree distributions are consistent with the empirical evidence. The lifetime distribution, $\mathbb{P}(T > t)$ in Eq. **9**, is asymptotically exponential, also confirming previous empirical studies (37). (The empirical lifetime and degree distributions together with the theoretical predictions of the model can be seen in *SI Appendix*, Fig. S13.)

Proposition 4 in *Materials and Methods* further analyzes fluctuations around the stationary state. In particular, we find that the variance of the number of firms in the network is increasing over a broad range of the rewiring rate γ until reaching a saturation point. This shows that while rewiring can increase the expected number of firms in the stationary state, the fluctuations around the stationary state may also increase. We will analyze similar fluctuations in total output in the counterfactual scenario studied in the following section.

Empirical Strategy and Results

Data. We obtain information about firm-level buyer–supplier relationships from the FactSet Revere Business Relationships database. Our sample consists of 64,467 firms with 614,154 links and supply chain information ranging from 2003 to 2015. The data come with information about the identities of the firms involved in a buyer–supplier link and the date of establishment of the link. The data have also been analyzed recently in ref. 13 for the US economy. Fig. 2 illustrates the largest (weakly) connected component[¶] in the firm-level production network among US firms in the year 2010. The network is strongly heterogeneous and is characterized by a few nodes (hubs) taking central positions in the network. Additional information about the buyer–supplier network data can be found in *SI Appendix*.

To obtain information about firms entering or exiting the network, we combine information on the year of establishment and the year of exit from Compustat Segments (38, 39) and Capital IQ. We find 6,656 firms that exited over the years from 1976 to 2012. For the remaining firms, we use the first and last years of observed sales of a firm to determine their entry and exit times, respectively. Additional information about the entry and exit data can be found in *SI Appendix*.

[¶]A weakly connected component is a maximal subgraph of the original graph where all nodes are connected to each other by some path, ignoring the direction of links.

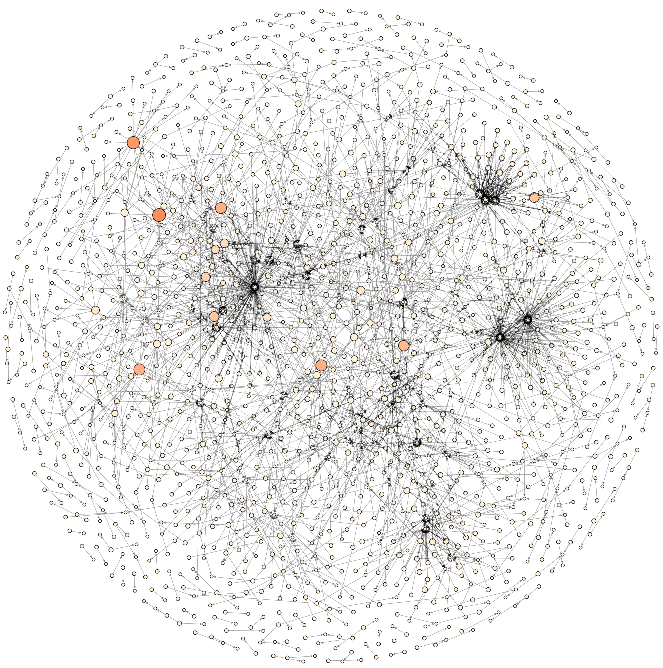

Fig. 2. The firm-level production network (largest weakly connected component) in the year 2010 among US firms. Different shades of the nodes represent varying out-degrees.

Estimation. We use our microdata on buyer–supplier relationships and observed firm exits to estimate the parameters of our model. The number of product characteristics K is set to 300 following ref. 40. (Ref. 40 uses a 300-industries classification scheme as it is most analogous to popular alternatives, including three-digit Standard Industrial Classification (SIC) codes and four-digit North American Industry Classification System codes, which have 274 and 331 industries, respectively.) The distribution $h(\ell)$ of the number of suppliers $\ell \in \{1, 2, \ldots, L\}$ can be taken from the empirical degree distribution. Similarly, from the observed firm exits, we can estimate the exit rate parameters δ, ζ, and ρ using a survival model (41). From the events where an incumbent firm replaces a lost supplier with another incumbent firm, we can learn the rewiring probability γ. The remaining parameters can be obtained from matching the aggregate distributions for the lifetimes and out-degrees of the firms between the data and the model (*cf* Eqs. **8** and **9**). The details of the estimation procedure can be found in *Materials and Methods*.

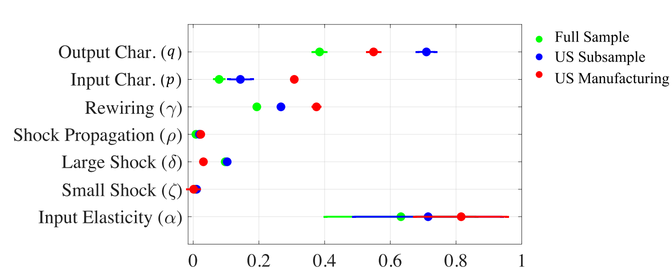

Fig. 3. Parameter estimates (filled circles) with SEs (bars) for the elasticity (α), small shock (ζ), large shock (δ), shock propagation (ρ), rewiring (γ), input characteristic (p), and output characteristic (q) for different samples (full sample and US and US manufacturing subsamples). The parameter estimation procedure is explained in *Materials and Methods*, where model parameter α is estimated from Eq. **15**, parameters (δ, ζ, ρ) are the MLE of Eq. **18**, γ is the MLE of Eq. **19**, and (p, q) are being estimated from Eq. **20** using a Metropolis Hastings Markov Chain Monte Carlo – Pseudo Maximum Likelihood Estimation (MCMC-PMLE) algorithm (42). Financial (SIC 6,000 to 6,799) and public (SIC 9,100 to 9,729) institutions have been dropped from the estimation sample. Char., characteristic.

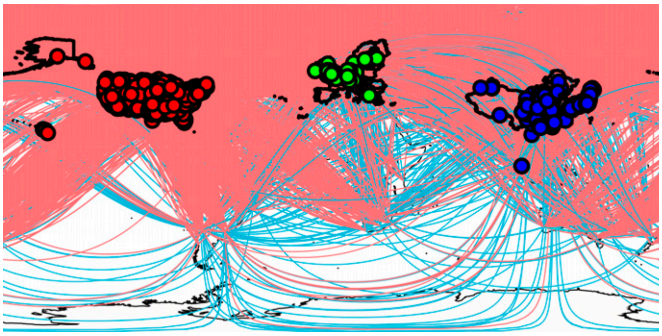

Fig. 4. Buyer–supplier links between US and non-US firms, European Union and non–European Union firms, and East Asian (China, Japan, South Korea, Hong Kong, and Singapore) and non–East Asian firms (indicated in red). Firm locations are indicated with circles.

Fig. 3 shows the estimated parameters of the model. All parameters have the expected signs and are statistically significant (at the 1% level; the only exception is the estimate for ζ, which turns out not to be significant in the full sample but significant and of similar magnitude in the subsamples considered), with similar estimates across different subsamples. *SI Appendix*, Fig. S13 shows a good fit between the data and the predictions of the model regarding the lifetime, in-degree, and out-degree distributions.

Results. After having estimated the parameters of the model, we are now able to evaluate counterfactual scenarios in which we simulate the impact of different policy interventions. In particular, in the following we test the effectiveness of reshoring of supply chains in the production network. We consider a counterfactual in which only buyer–supplier links among domestic firms in the United States, the European Union, and East Asia (China, Japan, South Korea, Hong Kong, and Singapore) are permitted. An illustration of the affected buyer–supplier links can be seen in Fig. 4.

For the counterfactual simulations, we initialize the network at the observed network in the last year in the sample and iterate the network over a predefined time window. Further, we draw the attractiveness values η_i of the firms in the network (*cf* Eq. **1**) using the observed cases in which a firm i became a supplier to an entrant.[#]

Fig. 5 shows the impulse response functions of the relative change in total output for firms in the United States, the European Union, and East Asia comparing the benchmark (no restrictions) with the counterfactual over varying time horizons (ranging from 1/2 to 1, 2, and 4 years in the top row panels to the bottom row panels). The impact of reshoring supply chains is drastic. We observe a short-run output loss of 22% for the United States, 60% for the European Union, and 70% for Asia, with a long-term recovery for all countries to reach the benchmark output level.

To investigate the robustness of these results, we analyze the effect of changes in network adaptivity (that is, the rewiring rate γ) to absorb shocks from policy interventions. If firms can replace lost suppliers more easily (by increasing γ), then this might mitigate the output loss incurred from restricting supply chains domestically. However, we do not find support for this hypothesis. Fig. 6 shows that the decline in total output in the counterfactual is not significantly reduced when considering higher rewiring rates, γ. Moreover, we find that the economy becomes more unstable. The sales coefficient of variation [the same result is obtained when considering the SD of log total sales as a measure of volatility as in ref. 7 since the SD of the logarithm is invariant under a multiplicative change (43)] is significantly higher for US, East Asian, and European firms in the counterfactual and is increasing with γ. Thus, we find that volatility increases with the renationalization of supply chains and that network adaptivity is increasing volatility.

We can further analyze how the network structure changes by comparing the counterfactual with the benchmark and how these structural differences are affected by network adaptivity. We find that the average size of the largest weakly connected component (a weakly connected component is a maximal subgraph of the original graph where all nodes are connected to each other by some path, ignoring the direction of links) is increasing and that the number of weakly connected components is decreasing in the counterfactual, where firms can form links only among a smaller set of national suppliers and thus, form more densely connected (national) clusters. Moreover, this effect is increasing with higher rewiring rates, γ. This means that with increasing network adaptivity, the network becomes more densely connected and less compartmentalized (22). The increased connectivity facilitates the propagation of shocks, similar to the spread of a virus in a densely connected social network (21), and this is increasing output volatility (as indicated by the sales coefficient of variation). Hence, we find that restricting buyer–supplier links reduces output and increases volatility and that volatility can be amplified through network adaptivity.[‖]

Conclusion

In this paper, we study a fully endogenous model of production networks in which firms can replace suppliers they have lost due to exit subject to switching costs and search frictions. Our model and data enable us to study the impact of policy interventions on aggregate output in an adaptive network. We show that restricting buyer–supplier links via reshoring policies reduces output and increases volatility. Moreover, we show that, contrary to conventional wisdom, greater network adaptivity does not necessarily make it more stable but in contrast, can amplify volatility. Our framework can be used to assess the impact of various other inventions or shocks from political, epidemic, or climate origins. At the same time, we acknowledge that the baseline analysis does not incorporate several forces that would determine the consequences of renationalizing supply chains. For example, international linkages could be more fragile because of disruptions in ocean shipping or geopolitical events, such as tariff changes, embargoes, or sanctions. Allowing for the possibility that international linkages are inherently different from domestic ones could qualify our results in important ways and remains a fruitful avenue for future research.

[#] Using the fact that the probability that an incumbent firm i becomes the supplier to an entrant indexed with t can be written as $\frac{\eta_i}{N_t}$, the estimated attachment kernels $\hat{\eta}_i$ can be obtained from maximizing the log likelihood $\log \mathscr{L}(\eta_i) = \sum_{t=t_i^{\text{in}}}^{t_i^{\text{out}}} \left[a_{ti,t} \log\left(\frac{\eta_i}{N_t}\right) + (1 - a_{ti,t}) \log\left(1 - \frac{\eta_i}{N_t}\right)\right]$, where $a_{ti,t} \in \{0, 1\}$ is an indicator variable indicating whether the entrant t selected firm i as a supplier at time t, t_i^{in} denotes the entry time of firm i, and t_i^{out} denotes the exit time of firm i. As expected, the estimated values for η_i are highly correlated with the average out-degree d_i^+ of a firm i across time.

[‖] In *SI Appendix*, we show that our findings remain largely unchanged when incorporating additional sources of heterogeneity, such as an increased volatility of international links as compared with domestic buyer–supplier links. Note that in response to the shutting down of international links, the domestic firms will use domestic links more intensively if measured by the input expenditure shares. However, since conditional on the number of suppliers, the input expenditure share on all suppliers is the same, and therefore all domestic supplier expenditure shares rise proportionally. It may be that in the real world, some domestic expenditure shares would rise by more than others. We do not have sufficient data to model and discipline uneven adjustment of the domestic expenditure shares, as our data do not have comprehensive coverage of input expenditure shares. Enriching the model to allow for this possibility remains a fruitful avenue for future research.

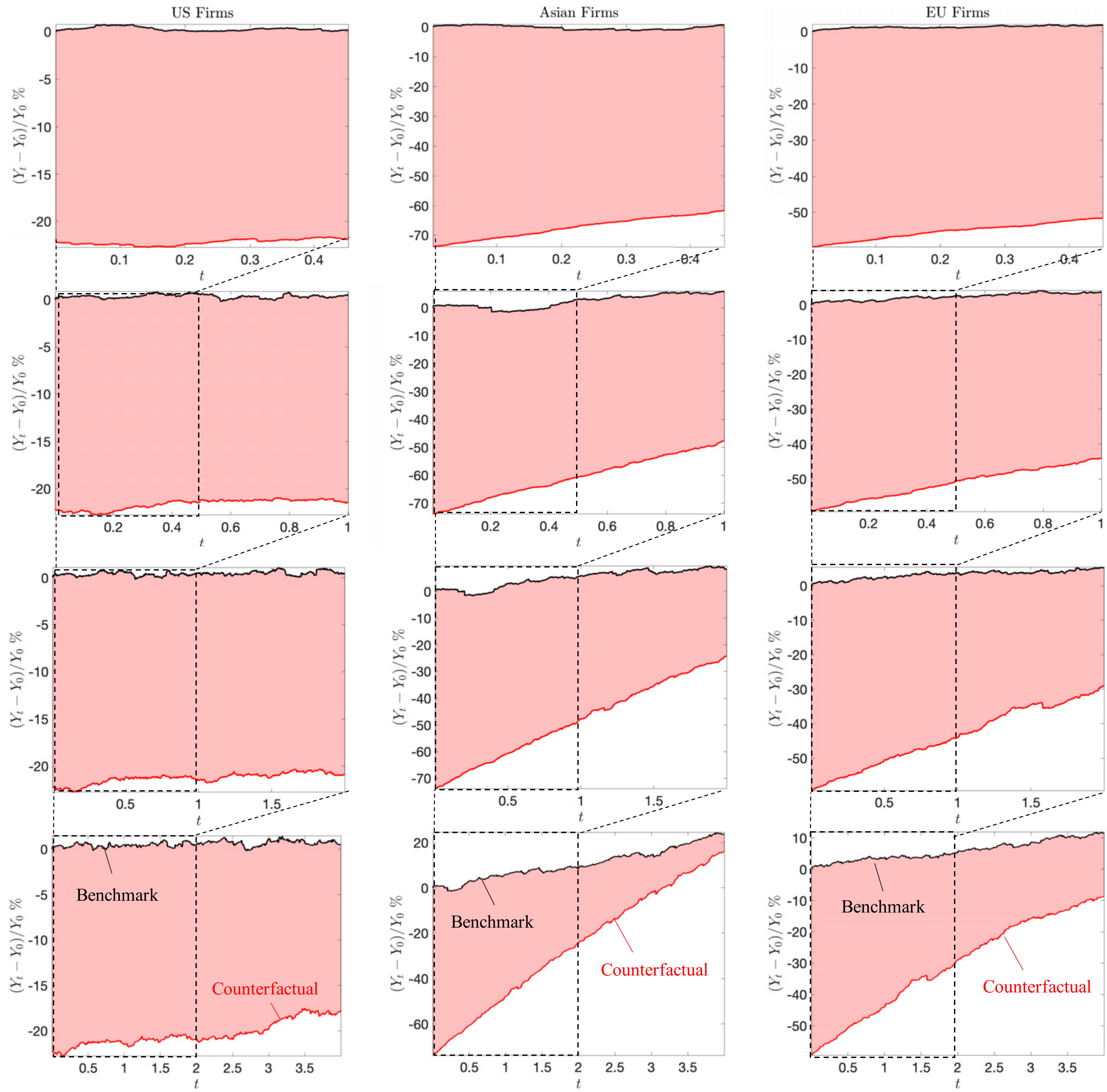

Fig. 5. Impulse response functions of the relative change in total output for the United States, the European Union, and East Asia (China, Japan, South Korea, Hong Kong, and Singapore) of typical trajectories comparing the benchmark with the counterfactual over varying time horizons (ranging from 1/2 to 1, 2, and 4 years in the top row panels to the bottom row panels).

Materials and Methods

Preferences and Technology. Labor is the only primary factor of production, with the aggregate endowment normalized to one. A representative household owns the firms, supplies labor, and maximizes utility

$$u_t = A \prod_{i=1}^{m} c_{it}^{\frac{1}{m}},$$

subject to the budget constraint

$$\sum_{i=1}^{m} c_{it} p_{it} = h_t + \sum_{i=1}^{N_t} \pi_{it},$$

where c_{it} is the quantity of firm i's output in final consumption, p_{it} is the price of i's output, h_t is the wage, and π_{it} is firm i's profit. Note that not all firms' output needs to be valued by consumers; that is, m can be smaller than N_t.

Firm i combines labor and inputs to produce output using a Cobb–Douglas production function**

$$x_{it} = z_{it} l_{it}^{\beta} \left(\prod_{j=1}^{N_t} x_{jit}^{w_{jit}} \right)^{\alpha}, \qquad [10]$$

where z_{it} is the firm's productivity, l_{it} is the labor input, and x_{jit} is the quantity of intermediate inputs produced by firm j and sold to firm i. Output elasticities of labor and intermediate inputs, $\alpha, \beta \in (0, 1)$, respectively, satisfy $0 < \alpha +$

**The production function in Eq. 10 and economic environment considered here for a fixed network structure can be found in refs. 7 and 44. The special case in which a firm has only a single supplier is analyzed in ref. 10. Further, assuming that there exists a small cost for establishing a link, the use of a single supplier will generically be optimal. We will analyze this special case in more detail in the next section.

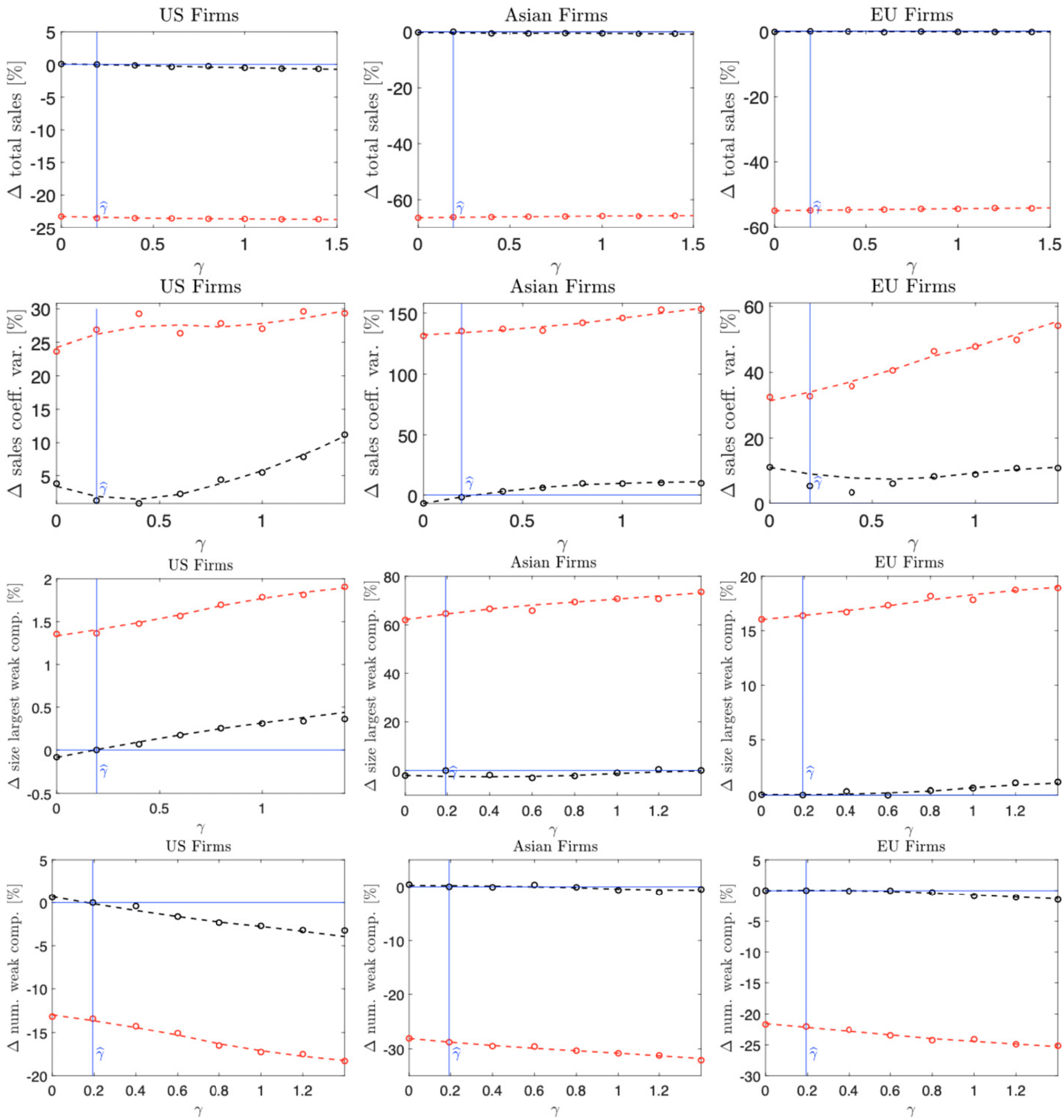

Fig. 6. The relative change in total sales, the sales coefficient of variation (that is, the SD divided by the mean), the size of the largest weakly connected component, and the number of weakly connected components for different values of γ compared with the estimated $\hat{\gamma}$ indicated with vertical lines for US, East Asian, and European firms. The estimates for the remaining parameters of the model are taken from Fig. 3 (full sample). Symbols indicate the average across a time series of a simulation of the stochastic process as introduced in *Definition*, with a time window corresponding to half a year prediction horizon. The dashed lines indicate a smoothed fit. The counterfactual (indicated in red) corresponds to the case in which only domestic links between firms in the United States, East Asia (China, Japan, South Korea, Hong Kong, and Singapore), and the European Union are allowed (*cf* Fig. 4).

$\beta < 1$. Let $w_{jit} \in [0, 1]$ be the intensity with which firm i uses intermediate input x_{jit} for production, such that $\sum_{j=1}^{N_t} w_{jit} = 1$. These weights are indexed by t because they can change over time as firms add or drop input suppliers.

Market Structure and Equilibrium. We assume that firms are price takers. This can be justified by assuming that there are many (potential) firms that can produce good i. The setup is equivalent to a competitive model in which firms operate a constant returns technology with labor, intermediates, and a firm-specific factor, as in the Lucas span-of-control formulation (45). The goods and market clearing conditions are $x_{it} = \sum_{j=1}^{N_t} x_{ijt} + c_{it}$, $\forall i = 1, \ldots, N_t$, and $1 = \sum_{i=1}^{N_t} l_{it}$.

Conditional on a fixed network, the model corresponds to the textbook network model of ref. 7 but with diminishing rather than constant returns to scale in production. We further assume that weights are uniform across existing suppliers and denote by $a_{jit} \in \{0, 1\}$ the indicator for whether firm i buys from j.

The following proposition describes the firms' profits in equilibrium conditional on network structure G_t and relates them to a well-known measure of centrality in the social network analysis literature (3, 46, 47).

Proposition 2. *Consider the production economy described above for a given network $G_t \in \mathcal{G}(N_t)$ representing the input–output relationships between firms. Let $d_{it}^{-} = \sum_{j=1}^{N_t} a_{jit}$ denote the in-degree of firm i, and let $d_{it}^{+} = \sum_{j=1}^{N_t} a_{ijt}$ denote its out-degree. Denote by $\mathbf{W}_t$ the $N_t \times N_t$ matrix of weights $w_{jit} = a_{jit}/d_{it}^{-}$ at time t.*

1) *The equilibrium profits are given by $\pi_{it} = \frac{(1-\alpha-\beta)(1-\alpha)h_t}{m\beta} b_i(\mathbf{W}_t, \alpha)$, where $\mathbf{b}(\mathbf{W}_t, \alpha) \equiv (\mathbf{I}_{N_t} - \alpha \mathbf{W}_t)^{-1}\mathbf{u}$ is the Katz–Bonacich centrality of $\mathbf{W}_t$ with parameter α and $\mathbf{u}$ a vector of ones.*
2) *In the limit of small α and when firms have only a single supplier, profits of firm i are to a first order a linear function of its out-degree d_{it}^{+} [i.e., $\pi_{it} = \frac{(1-\alpha-\beta)(1-\alpha)h_t}{m\beta}\left(1 + \alpha d_{it}^{+}\right) + O\left(\alpha^2\right)$].*

A special case of interest is when α is small (while this limit is useful for analytical model characterization, in the quantitative section we calibrate α based on data) and the first-order Taylor approximation in part 2 of *Proposition 2* holds. The main implication of this approximation is that the profit of firm i is an increasing function of the number d_i^{+} of customers of i. This resembles similar models in the literature, such as ref. 27, in which firm profits are proportional to the number of products it sells.

System Dynamics. The state of the stochastic heterogenous mean-field model at time t is specified by the number $N_k^i(\eta)$ of firms with i suppliers, k customers, and attractiveness η (omitting time indices for brevity). In the limit of large but finite Ω, a scaling parameter of the entry rate, the network grows large, and the random variables $N_k^i(\eta)$ converge to a continuous stochastic process. We then can provide a complete characterization for the time evolution of the degree distribution in the form of a system of SDEs as it is stated in the following proposition (48).

Proposition 3. *In the limit of small α, $L = 1$, and $\Omega \to \infty$, we have convergence in law $N_k^i(\eta)/\Omega \xrightarrow{d} n_k^i(\eta)$, where the $n_k^i(\eta)$ obey the SDEs*

$$\frac{d}{dt}n_k^1(\eta) = F_k^1(\eta) + \frac{1}{\sqrt{\Omega}}\xi_k^1(\eta),$$
$$\frac{d}{dt}n_k^0(\eta) = F_k^0(\eta) + \frac{1}{\sqrt{\Omega}}\xi_k^0(\eta), \quad \textbf{[11]}$$

with the drift functions

$$\begin{aligned}
F_k^1(\eta) =\ & \delta_{k,0}f(\eta) + \frac{\eta(n_{k-1}^1(\eta) - n_k^1(\eta))}{n\langle\eta\rangle} - \left(\delta + \frac{\zeta}{1+\alpha k}\right)n_k^1(\eta) \\
& + \frac{(k+1)n_{k+1}^1(\eta) - kn_k^1(\eta)}{n\langle k\rangle}\sum_{k'}\left(\delta + \frac{\zeta}{1+\alpha k'}\right)n_{k'}^1 \\
& - \frac{n_k^1(\eta)}{n^1}\sum_{k'}k'\left[\left(\delta + \frac{\zeta}{1+\alpha k'}\right)n_{k'}^1 + \left(\rho + \delta + \frac{\zeta}{1+\alpha k'}\right)n_{k'}^0\right] \\
& + \gamma n_k^0(\eta) + \gamma n^0\eta\frac{n_{k-1}^1(\eta) - n_k^1(\eta)}{n\langle\eta\rangle}, \\
F_k^0(\eta) =\ & \eta\frac{n_{k-1}^0(\eta) - n_k^0(\eta)}{n\langle\eta\rangle} - \left(\rho + \delta + \frac{\zeta}{1+\alpha k}\right)n_k^0(\eta) \\
& + \frac{(k+1)n_{k+1}^0(\eta) - kn_k^0(\eta)}{n\langle k\rangle}\sum_{k'}\left(\delta + \frac{\zeta}{1+\alpha k'}\right)n_{k'}^1 \\
& + \frac{n_k^1(\eta)}{n^1}\sum_{k'}k'\left[\left(\delta + \frac{\zeta}{1+\alpha k'}\right)n_{k'}^1 + \left(\rho + \delta + \frac{\zeta}{1+\alpha k'}\right)n_{k'}^0\right] \\
& - \gamma n_k^0(\eta) + \gamma n^0\eta\frac{n_{k-1}^0(\eta) - n_k^0(\eta)}{n\langle\eta\rangle} \quad \textbf{[12]}
\end{aligned}$$

where $\langle\cdot\rangle$ denotes the expectation, $\xi_k^i(\eta)$ are white noise variables with variance–covariance matrix $\langle\xi_k^i(\eta), \xi_\ell^j(\eta')\rangle = \Sigma_{k,\ell}^{i,j}(\eta,\eta')(\mathbf{n})$ defined by

$$\Sigma_{k,\ell}^{i,j}(\eta,\eta')(\mathbf{n}) = \sum_e (\boldsymbol{s}_e)_k^i(\eta)(\boldsymbol{s}_e)_\ell^j(\eta')r_e(\Omega\mathbf{n}), \quad \textbf{[13]}$$

and $\mathbf{n}$ denotes the stacked vector of the $n_k^i(\eta)$.

The drift functions $F_k^i(\eta)$ in Eq. **12** keep track of the expected changes in the number $N_k^i(\eta)$ of firms with i suppliers, k customers, and attractiveness η. For example, the first term $\delta_{k,0}f(\eta)$ in $F_k^i(\eta)$ in Eq. **12** captures the contribution of entering firms to $n_0^1(\eta)$; the second term $\frac{\eta(n_{k-1}^1(\eta)-n_k^1(\eta))}{n\langle\eta\rangle}$ is the selection of incumbent firms as a supplier to the entrant; the third term $\left(\delta + \frac{\zeta}{1+\alpha k}\right)n_k^1(\eta)$ is the exit of firms with one supplier, k customers, and attractiveness η; the fourth term $\frac{(k+1)n_{k+1}^1(\eta)-kn_k^1(\eta)}{n\langle k\rangle}\sum_{k'}\left(\delta + \frac{\zeta}{1+\alpha k'}\right)n_{k'}^1$ is the exit of buyers of $n_{k+1}^1(\eta)$ or $n_k^1(\eta)$ firms; the fifth term $\frac{n_k^1(\eta)}{n^1}\sum_{k'}k'\left[\left(\delta + \frac{\zeta}{1+\alpha k'}\right)n_{k'}^1 + \left(\rho + \delta + \frac{\zeta}{1+\alpha k'}\right)n_{k'}^0\right]$ is the exit of suppliers of $n_k^1(\eta)$ firms; and the last two terms, $\gamma n_k^0(\eta)$ and $\gamma n^0\eta\frac{n_{k-1}^1(\eta)-n_k^1(\eta)}{n\langle\eta\rangle}$, capture the contributions from rewiring when an $n_k^0(\eta)$ becomes an $n_k^1(\eta)$ firm or when $n_{k-1}^1(\eta)$ or $n_k^1(\eta)$ firms become the new suppliers after rewiring, respectively.

From Eq. **11**, we can then compute the out-degree distribution as $P^{+}(k) = \frac{n_k^1 + n_k^0}{n} = \frac{1}{n}\int(n_k^1(\eta) + n_k^0(\eta))d\eta$, for all $k = 0, 1, \ldots$.[††] Moreover, the average shifted inverse out-degree, $\kappa = \sum_\ell \frac{1}{1+\alpha\ell}\frac{n_\ell}{n}$, in *Proposition 1* can be computed from the fixed point of Eq. **11**.

Aggregate Fluctuations and Adaptivity. In the following, we analyze the role of network adaptivity in generating fluctuations. The fluctuations around the stationary state of the SDEs in [**11**] are derived in the proposition below.

Proposition 4. *Let the assumptions of Proposition 3 hold, and let $\mathbf{n}$ denote the stacked vector of the expected number $n_k^i(\eta)$ of firms with i suppliers, k customers, and attractiveness η. Then, the stochastic dynamical system in Eq. **11** has the following properties.*

1) *In long times, the expected system state converges*

$$\lim_{t\to\infty}\mathbb{E}[\mathbf{n}] = \mathbf{n}^\star + O(\Omega^{-1}),$$

where for each i, k, η, the vector $\mathbf{n}^\star$ satisfies

$$F_k^i(\eta)\Big|_{\mathbf{n}=\mathbf{n}^\star} = 0,$$

*with the drift functions $F_k^i(\eta)$ defined in Eq. **12**.*

2) *Fluctuations in system states are Gaussian to leading order:*

$$\lim_{t\to\infty}\mathbb{E}[\Omega(\mathbf{n}-\mathbf{n}^\star)(\mathbf{n}-\mathbf{n}^\star)^\top] = \boldsymbol{\Xi} + O(\Omega^{-1}),$$

where $\boldsymbol{\Xi}$ solves the matrix Lyapunov equation $\mathbf{A}\boldsymbol{\Xi} + \boldsymbol{\Xi}\mathbf{A}^\top + \mathbf{B} = 0$, in which matrices $\mathbf{A}$ and $\mathbf{B}$ have entries

$$A_{k,\ell}^{i,j}(\eta,\zeta) = \frac{\partial}{\partial n_\ell^j(\zeta)}F_k^i(\eta)\Big|_{\mathbf{n}=\mathbf{n}^\star}, B_{k,\ell}^{i,j}(\eta,\zeta) = \Sigma_{k,\ell}^{i,j}(\eta,\zeta)(\mathbf{n}^\star). \quad \textbf{[14]}$$

In particular, the total number of firms is asymptotically characterized as a Gaussian random variable with mean $n^\star = \sum_{i,k,\eta}[\mathbf{n}^\star]_k^i(\eta)$ and variance $\sigma_n^2 = \mathrm{tr}(\boldsymbol{\Xi})$.

[††] Note that it is possible to generalize Eq. 11 by allowing an entrant to select ℓ suppliers from the incumbent firms, where ℓ follows a distribution $h(\ell)$ with compact support $\ell \in \{1, \ldots, L\}$. This is shown in *SI Appendix*. These equations will be useful for the efficient estimation of the parameters of the model.

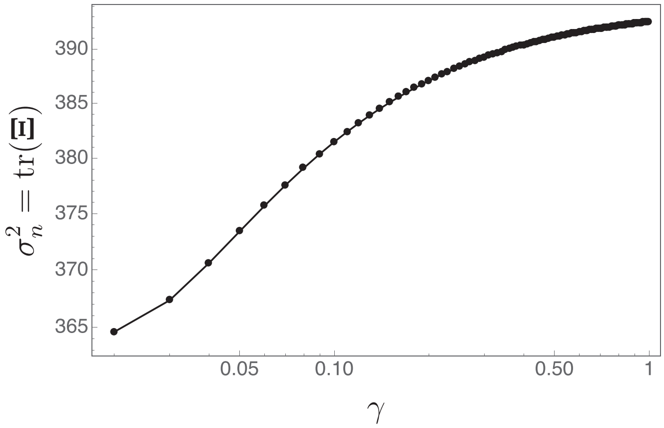

Fig. 7. The asymptotic variance of the number of firms, $\sigma_n^2 = \mathrm{tr}(\Xi)$, from *Proposition 4* as a function of the rewiring rate γ. The parameters used are $\delta = \zeta = \rho = 0.001$.

For the numerical implementation of *Proposition 4*, we make the following assumptions. First, as we cannot represent infinite-dimensional matrices or vectors, we impose a maximum degree large enough that all entries in the support of the degree distribution are represented. Second, we solve for the fixed point $\mathbf{n}^*$ of Eq. **11** numerically by integrating with forward Euler iterations. Third, we determine the Jacobian in Eq. **14** using symbolic algebra. Finally, as the number of variables is too large to compute all entries of the noise correlator $\mathbf{B}$ in full, we approximate it by its diagonal matrix. We have observed in small-scale examples that the diagonal is dominant. Based on this procedure, Fig. 7 shows the asymptotic variance of the number of firms, $\sigma_n^2 = \mathrm{tr}(\Xi)$, from *Proposition 4* as a function of the rewiring rate γ. We find that the variance is increasing over a broad range of γ until reaching a saturation point.

It is also possible to consider an extension of the model in which we allow firms to replace existing suppliers at a rate that is increasing with the supplier's propensity to exit. The model and corresponding SDEs for this case are derived in *SI Appendix*. There, we show that this extension can amplify the effect of a higher network adaptivity, leading to greater fluctuations.

Parameter Estimation. We use our firm-level data with buyer–supplier relationship information and observed firm exits to estimate the parameters $\boldsymbol{\theta} = (K, L, \alpha, \rho, \delta, \zeta, \gamma, p, q) \in \boldsymbol{\Theta}$ of our model. The input elasticity parameter α can be obtained from the first-order condition of the firm's intermediate input demand (*SI Appendix*, Eq. **30**)

$$\alpha = \frac{\sum_{j=1}^{N} p_j x_{ji}}{p_i x_i}. \qquad \textbf{[15]}$$

We measure total costs for intermediate input goods, $\sum_{j=1}^{N} p_j x_{ji}$, by total sales minus value added, where value-added and total sales are taken from the balance sheet statement of the firm. (For the value-added information, we have complemented our data sample with information from Bureau van Dijk's Orbis database.)

From the observed firm exits, we can estimate the exit rate parameters δ, ζ, and ρ using a survival model (41). The corresponding hazard function $\lambda_i(t)$ for the exit of firm i at time t conditional on not having exited before is given by

$$\lambda_i(t) = \delta + \frac{\zeta}{b_i(\mathbf{W}, \alpha)} + \rho \mathbb{1}_{\{d_{it}^- = 0\}}, \qquad \textbf{[16]}$$

while the survival function is defined as

$$S_i(t) = \exp\left\{-\int_0^t \lambda_i(s) ds\right\}. \qquad \textbf{[17]}$$

Note that when t_i denotes the exit time of firm i, then the survival function is defined as $S_i(t) = \mathbb{P}(t_i > t)$, and the hazard function is defined as $\lambda_i(t) = \lim_{\Delta t \downarrow 0} \mathbb{P}\left(t_i \in [t, t+\Delta t) \middle| t_i \geq t\right)$.

With the (noninformative) right-censoring indicator variable $c_i \in \{0, 1\}$,[‡‡] the log likelihood associated with firm exit can be written as (41)

$$\ln \mathscr{L}_E(\delta, \zeta, \rho) = \sum_{i=1}^{N} \left((1 - c_i) \ln \lambda_i(t_i) - \Lambda_i(t_i)\right), \qquad \textbf{[18]}$$

where $0 \leq t_1 \leq t_2 \leq \ldots \leq t_C$ denote the observed lifetimes of the firms in the data sample composed of n firms, and the cumulative hazard function is defined as $\Lambda_i(t) = \int_0^t \lambda_i(s) ds = -\ln S_i(t)$.[§§]

From the events where an incumbent firm replaces a lost supplier with another incumbent firm, we can learn the rewiring probability γ. For each firm I, let $\mathcal{S}_i$ denote the directed input links from firm k supplying to firm i that have been lost due to the exit of firm k. Then, the hazard function that a link ki gets removed from $\mathcal{S}_i$ because firm i was able to find another supplier k' is given by $\lambda_{ki}(t) = \gamma$. The survivor function is given by $S_{ki}(t) = \exp\{-\int_0^t \gamma ds\} = e^{-\gamma t}$, and the cumulative hazard function is defined as $\Lambda_{ki}(t) = \int_0^t \gamma ds = \gamma t = -\ln S_{ki}(t)$.

With the (noninformative) right-censoring indicator variable $c_{ki} \in \{0, 1\}$, the log likelihood associated with rewiring can be written as[¶¶]

$$\ln \mathscr{L}(\gamma) = \sum_{i=1}^{N} \sum_{ki \in \mathcal{S}_i} \left((1 - c_{ki}) \ln \gamma - \gamma \tau_{ki}\right), \qquad \textbf{[19]}$$

where τ_{ki} is the observed duration (until replacement) of the lost link ki. The average duration is 4.33 y, and the number of observations is 25,093.

The remaining parameters of the model can be obtained from the aggregate distributions for the lifetimes and out-degrees of the firms. The pseudolog likelihood (PLL) is given by[##]

$$\log \mathscr{L}(\boldsymbol{\theta}) = \sum_{k=0}^{\infty} \left(P_{\mathrm{obs}}^+(k) \log P^+(k) + P_{\mathrm{obs}}(k) \log P(k)\right), \qquad \textbf{[20]}$$

where $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ denotes the vector of parameters, $(P_{\mathrm{obs}}^+(k))_{k \geq 0}$ is the observed empirical out-degree distribution, $(P_{\mathrm{obs}}(k))_{k \geq 0}$ is the observed empirical lifetime distribution, and $P^+(k)$ and $P(k)$ are the corresponding asymptotic theoretical predictions from the solution to Eqs. **8** and **9**, respectively. (The extension to multiple input suppliers is discussed in *SI Appendix*.) To match the number of firms in the observed network, we use Eq. **7** as a parameter constraint on $\boldsymbol{\theta}$ in the maximization routine to obtain the maximum likelihood estimation (MLE) estimates from maximizing the PLL of Eq. **20**. (Ref. 50 has shown that the PLL is asymptotically consistent in a general class of network formation processes.)

Data, Materials, and Software Availability. In this paper, we make use of data on buyer–supplier relationships between private and publicly listed companies that are part of the FactSet Supply Chain Relationships database. As this database is proprietary, we cannot make it publicly available. However, it can be accessed by any researcher through a subscription to FactSet (or the Wharton Research Data Services), and many universities actually have subscriptions to this database for their students, faculty, and staff. We explain in detail how the data from FactSet can be obtained so that other investigators can obtain the

‡‡ Because the data sample has a finite duration, we observe the lifetimes of firms from time 0 until a censoring time t_C. Some lifetimes end by this time t_C (when exit happens before t_C), while others will still be alive at time t_C and will end at some time in the interval (t_C, ∞).

§§ Note that we might also have firms that are left truncated at the time $t_L > 0$, the starting point of the observation period. These firms contribute $\lambda_i(t + t_L) S_i(t + t_L)/S_i(t_L) = \lambda_i(t + t_L) \exp\{-\int_{t_L}^t \lambda_i(s) ds\}$ to the likelihood (41, 49), where we have used the homogeneity property of the exponential function in Eq. 17. Shifting time t by t_L then allows us to write the log likelihood as in Eq. 18.

¶¶ Denote by $T = \sum_{i=1}^{n} \sum_{ki \in \mathcal{S}_i} \tau_{ki}$ the total rewiring time and by $C = \sum_{i=1}^{N} \sum_{ki \in \mathcal{S}_i} (1 - c_{ki})$ the total number of rewired links. The log likelihood can then be written as $\ln \mathscr{L}(\gamma) = C \ln \gamma - \gamma T$. The first order condition yields $\frac{C}{\gamma} = T$, and the MLE is, therefore, given by $\widehat{\gamma} = C/T$ with an SD from the Hessian given by $\widehat{\sigma}_\gamma = C/T^2$.

Maximizing the PLL is equivalent to minimizing the Kullback–Leibler divergence from P_{obs} to P, which is defined to be $D_{\mathrm{KL}}(P_{\mathrm{obs}} \| P) = -\sum_{k=0}^{\infty} P_{\mathrm{obs}}(k) \log\left(\frac{P(k)}{P_{\mathrm{obs}}(k)}\right)$.

data independently and replicate our results. Moreover, we provide a copy of the programs used to create the final results and a detailed instruction for how to use these programs together with the data from FactSet on the Harvard Dataverse (51).

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