THE IMPACT OF CARBON TAXES ON THE VALUE OF FOSSIL FUEL RESERVES AND THE EFFICIENCY OF CLIMATE POLICY

By

William Nordhaus

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COWLES FOUNDATION FOR RESEARCH IN ECONOMICS
YALE UNIVERSITY
Box 208281
New Haven, Connecticut 06520-8281

http://cowles.yale.edu/

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Abstract

The present study analyzes the impact of carbon pricing along with other policies on the value of fossil fuel resources, CO_2 emissions, and economic welfare. It employs a model based on the Hotelling analysis of resource values and calibrates this approach to data on fossil resources, costs, demands, and CO_2 emissions. Total fossil-fuel resource rents are estimated to be $17 trillion (2021 US$) without carbon pricing. Oil and gas rents are unchanged for low carbon taxes but would decline by 40% with a $100/t$CO_2$ price. The losses in producer values would be only about 10% of the carbon tax revenues. The study also shows that other policies – such as ones involving ethical investing or subsidies for renewable energy – are very inefficient and poor substitutes for carbon pricing.

The Impact of Carbon Taxes on the Value of Fossil Fuel Reserves and the Efficiency of Climate Policy[1]

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What is the impact of climate-change policies such as carbon taxes on the value of fossil fuel reserves? How do carbon-tax revenues compare with producer-rent losses? How does carbon pricing compare with other policies?

On the question of the impact of carbon taxes on resource rents (the "carbon tax-resource rent impact"), the answer would seem to be obvious. Such policies would appear to reduce the value of reserves. The reason is that, since a carbon tax would reduce production (of coal or oil), the value of unproduced resources would decline. This effect would be a reason that companies with valuable fossil-fuel reserves (whether Exxon-Mobil or Saudi Arabia) would vigorously oppose climate-change policies.

The present study analyzes the quantitative impact of carbon taxes. It finds that the obvious is generally but not universally correct. Resource values disappear very quickly for coal, and rents approach zero at a carbon tax of \$250/tCO$_2$ for oil and gas.

However, depending upon the grade of the fuel and the substitution patterns, the impact of carbon taxes on resource rents may be either negative or positive. The results here indicate that there are anomalies at low carbon taxes, where carbon taxes raise oil and gas rents because of substitution. The exact pattern depends upon the details of supply and demand. The first section illustrates the impact using a simple Hotelling model of reserve pricing. The next sections deploy a simple model to test the impact of climate policies.

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[1] The author is grateful to Eugene Tan for discussions on the modeling approach. Tan provided helpful research assistance and has prepared a more complete multi-sector model of the energy sector (in process). The author is responsible for all errors and omissions. Version is Hotelling-0801122.docx.

The literature on the impact of climate policies on resource prices is surprisingly thin. An early study by Liski and Tahvonen (2004) used the Hotelling model to estimate the impact of carbon prices on OPEC revenues in a bilateral game between oil-importing and oil-exporting countries. They find that the impact of the carbon tax on monopoly rents of producers reinforces the economic benefit of the tax to consumers.

A study by Coulomb and Henriet (2013) analyzed the impact of caps on CO_2 concentrations in a simplified world with both high-carbon and low-carbon substitutes and showed that the impact was ambiguous. This analysis parallels the analysis in the next section in showing that the tax-rent impact depends on the carbon-intensity of the fuels that become substitutes as carbon taxes rise. The analysis is simplified relative to the full Hotelling model in their specification of the cost structure.

It seems likely that many energy companies have studied the question of the impact of policies on resource values. However, these studies are not in the public record, and attempts to extract them have been unsuccessful.

I. The Simple Analytics of the Carbon Tax-Rent Impact

Begin with a simple example of the pricing of energy resources based on the theory first developed by Hotelling (1931). For the example, I assume energy is produced by oil , $x^1(t)$, and a renewable fuel, $x^2(t)$. Oil is produced by limited oil-in-the ground ("resources"), which can be extracted over time, and the cost of extraction is constant at z^1. The initial resource base of oil is R_0. The renewable fuel has infinite supply at a constant cost of $z^2 > z^1$. (This example uses notation that is slightly simplified from the full model in section II and beyond for intuitive understanding.)

Demand is a constant of D per year. I assume that output is produced by n identical competitive firms which use the two fuels, $x_i^1(t)$ and $x_i^2(t)$. Each minimizes the discounted costs of meeting its share of demand, $d_i(t) = D/n$. The individual firm's problem then is:

(1) $\min_{\{x_i^1(t), x_i^2(t)\}} \quad \sum_{t=0}^{\infty} [x_i^1(t)z^1 + x_i^2(t)z^2](1+r)^{-t}, \; i = 1,\ldots,n$

subject to

(2) $d_i(t) = x_i^1(t) + x_i^2(t), \quad i = 1,\ldots,n, t = 0,1,\ldots$

(3) $\sum_{i=1}^{n} x_i^1(t) \le R_i, \quad i = 1,\ldots,n$

The market equilibrium is that the resource and demand conditions hold in the aggregate:

(4) $\sum_{i=1}^{n} d_i(t) \ge D, \qquad t = 0,1,\ldots$

(5) $\sum_{i=1}^{n} R_i \le R_0$

This problem is a straightforward linear programming (LP) problem that is easily solved. The solution produces optimal production and costs of each variable, where optimal values are shown as with a hat $(\hat{x})$. .The solution is intuitive and shows (for r > 0) that the firms produce only oil until it is exhausted and then renewables after that time. Given the parameters, oil is exhausted at time $\hat{T}$. When oil is exhausted, production switches to the perfectly substitutable renewable fuel.

The solution also calculates the shadow prices on demand and the resource. The key variable of interest for the present study is the royalty or Hotelling rent on the resource, $\hat{q}^1(t)$; the rent on the renewable fuel is zero because it is unlimited in supply, so $\hat{q}^2(t)$ = 0. The market price of the fuel is $\hat{p}(t)$. It is easily verified that the solution has the following trajectory of optimal rents, oil prices, and switching date as follows:

(6) $\hat{T} = R / D$

(7) $\hat{q}^1(t) = \hat{q}^1(\hat{T})e^{-r(\hat{T}-t)} = (z^2 - z^1)e^{-r(\hat{T}-t)}$ 、

(8) $\hat{p}(t) = z^1 + \hat{q}^1(t) = z^1 + \hat{q}^1(\hat{T})e^{-r(\hat{T}-t)}$

Next consider the impact of carbon taxes, assumed to be constant at tax rate τ per ton of CO_2 emissions. The two fuels have carbon content per unit of θ^1 for oil and θ^2 for the substitute fuel. This implies that the optimal prices and rents of the two fuels are the following:

$$(9) \qquad \hat{q}^1(\hat{T}) = (z^2 - z^1) + \tau(\theta^2 - \theta^1)$$

$$(10) \qquad \hat{q}^1(t) = [(z^2 - z^1) + \tau(\theta^2 - \theta^1)]e^{-r(\hat{T}-t)}$$

$$(11a) \qquad \hat{p}(t) = z^1 + \tau\theta^1 + [(z^2 - z^1) + \tau(\theta^2 - \theta^1)]e^{-r(\hat{T}-t)}, \quad t \leq \hat{T}$$

$$(11a) \qquad \hat{p}(t) = z^2 + \tau\theta^2, \quad t \geq \hat{T}$$

The key for understanding the impact of carbon prices on resource prices is equation (10). Rewriting this for first-period rent, we obtain $\hat{q}^1(0) = [(z^2 - z^1) + \tau(\theta^2 - \theta^1)]e^{-r\hat{T}}$. The impact of carbon taxes is seen in the second term to depend upon the carbon-intensity of the original fuel compared to that of the substitute fuel. The carbon tax-rent impact will have the intuitive negative sign (carbon taxes reduce the value of reserves) if the substitute is less carbon-intensive than the original fuel. However, the impact will be positive (carbon taxes raising the value of reserves) if the substitute is more carbon-intensive that the original fuel.

Here is a simple numerical example. In the case of the high-cost substitute, assume that the carbon-intensity is zero, $\theta^2 = 0$ (because it is a renewable fuel). Assume that the carbon tax is τ = \$50 per unit of fuel, the cost of the substitute is z^2 = \$100, the exhaustion time is 30 years, and the interest rate is r = 5% per year. Here, the rent rate falls by 63% with the carbon tax because the substitute fuel is less carbon-intensive.

An alternative would be where the substitute was a high-carbon fuel, such as the substitution of coal for oil or gas, or oil or gas from shales for conventional oil and gas. For this example, assume that the backstop is 1.5 times as carbon-intensive as the original fuel. Here, the rent rises by 31%. Table 1 shows a simple example of the impact of carbon taxes on rents for high- and low-carbon backstop substitutes.

There are many variants depending upon parameters. One important assumption concerns the price elasticity of demand. If demand is price-inelastic (with a price-elasticity of -0.5), in the example shown in Table 1, the signs are preserved. However, the impacts of the carbon tax are amplified. The carbon tax tends to raise energy prices, reduce demand, and lengthen the period until exhaustion. This effect will be to increase the size of the impact of the carbon tax on the rent.

An interesting and important example would be a transition from low-cost oil to high-cost oil. For example, suppose production moves from low-cost oil (perhaps from Saudi Arabia) to high-cost oil (say Arctic and deep-water oil). To a first approximation, there is no impact of carbon taxes on reserve prices because the carbon content of the fuels is identical. The impact would occur only at the “end of oil,” when another fuel (with either higher or lower carbon-intensity) began to meet the demand.

The bottom line on simple Hotelling models is that the impact of carbon taxes on the value of energy resources is ambiguous. The impact depends upon the relative carbon content of substitute fuels. If the substitute fuel is coal, which is more energy intensive, then the value of resources will rise. If the substitute is low-carbon renewables, then the value of resources will decline. We must therefore turn to realistic patterns of energy use and resource abundance and costs to find the answer.

	Carbon tax	Carbon intensity		Interest rate (per year)	Royalty (t = 0)
		Backstop	Fossil		
Carbon-free backstop					
No tax	0.00	0	1	5%	18.51
High tax	50.00	0	1	5%	6.94
Difference	50.00				-11.57
Carbon-intensive backstop					
No tax	0.00	1.5	1	5%	18.51
High tax	50.00	1.5	1	5%	24.29
Difference	50.00				5.78

Table 1. Example of impact of carbon tax

Example of impact of carbon tax on the price of fossil fuel reserves for two alternative backstop technologies, one carbon-free and the other carbon-intensive

II. A Realistic Hotelling Model of Rents and Climate Policy

a. Description of the Hotelling Model

Is it possible that future energy scenarios can provide information as to which of the two outcomes shown in Table 1 is more likely? The answer depends critically on the fine-grained details of the technology of substitutes for the major fossil fuels.

The present study develops and uses a Hotelling model of the energy market and CO_2 emissions to estimate the impact of climate policies on the value of energy resources. More specifically, we develop a moderately realistic approach by using a model of the energy market with production and consumption of the major fossil fuels and renewable substitutes, carbon dioxide emissions, and different carbon taxes and climate policies. The major outputs are the values of energy resources, which is calculated as the value of energy resources times their scarcity (Hotelling) rents. Using an optimization model that mimics a competitive market, we can estimate the Hotelling rents on different grades of different fuels. By changing policies, we then estimate the impact of carbon taxes on resource rents. Along the way, we can calculate energy consumption, energy prices, CO_2 emissions, and a measure of economic welfare.

The method used to estimate the market outcomes with and without carbon taxes relies on the "correspondence principle." The correspondence principle states that determining the prices and quantities in a general economic equilibrium is mathematically equivalent to solving the embedded mathematical optimization problem. We describe the reasoning briefly.

This deep result was first described by Samuelson (1949) and then elaborated rigorously by Negishi (1977). Negishi showed that a competitive equilibrium can be calculated by solving a mathematical programming (MP) problem. This mathematical program maximizes a social welfare function subject to the relevant production constraints. The complicated part of Negishi's theorem is that it requires finding the appropriate set of welfare weights, in which case the welfare optimizing solution is an Arrow-Debreu equilibrium. For an early approach, see Ginsburgh and van der Heyden (1988).

The interpretation of "market mechanisms as maximization or minimization devices" was developed by Tjalling Koopmans in his work on energy modeling as follows, "The use of optimization in [energy] models should be seen as a means of simulating, as a first approximation, the behavior of a system of interacting competitive markets" (Koopmans, 1978). This approach is widely used in integrated assessment models or IAMS of climate change (see for example the review of IAMs in Nordhaus 2015).

For concreteness, we denote the mathematical programming model of the energy sector as the "Hotelling model." For present purposes, the approach is a dynamic partial equilibrium, taking interest rates and other non-energy prices as given. Parametric inputs include the costs, demands, and other elements described in equations (1) through (7). The major difference between the Hotelling model and the simple one analyzed above is that the full problem has multiple fuels, grades, and time periods.

Here is the application of the correspondence principle to the problem of estimating the dynamics of a competitive energy market: If each firm in an industry is faced with the same market prices for its inputs and outputs, and if each firm chooses its activities so as to maximize the firm's discounted profits, then the outcome will be economically efficient. In more precise language, such an equilibrium will be economically efficient in the sense that (1) each firm will provide its output at minimum discounted cost; and (2) the requirements of the market will be met by producers in a manner that satisfies total demand at minimum discounted total cost to society.

Examining these two conditions, we see that our competitive equilibrium has indeed solved a maximization problem of sorts – it has found a way of providing the appropriate array of services at the lowest possible cost. More precisely, it has maximized the objective function embedded in the demand functions as the problem solved by a constrained mathematical programming (MP) problem. Consequently, we can mimic the outcome of the economic equilibrium by solving the MP problem that maximizes the same set of demand functions subject to the same set of technical constraints.

One interesting technical point is the role of "dual variables" in the calculations. The MP calculations provide both primal and dual variables in the optimal solution. The primal solutions are quantities such as production of oil or renewable energy, or CO_2 emissions, in each time period. The dual variables are price-type variables and represent the contribution of a variable or constraint to the objective, in this case to society's utility or welfare. Intuitively, the dual variable is the impact of increasing the variable by one unit on the objective function in the numeraire of the objective function (say discounted values).

For example, we might consider the impact of increasing by one unit the quantity of the highest grade of oil. In the base solution (with a zero carbon tax), the dual variable is $2.83/MJ, or approximately $17/barrel of oil. This is the 2019 current value of "oil in the ground" and represents the Hotelling rent on that grade of oil. It is a "current value" in the sense that it is the value paid on that date. For future payment (or markets), it grows at the appropriate discount rate.

As a last point, the oil price is equal to the production costs (extraction and so on) plus the Hotelling rent. So, suppose that the production cost of grade 1 of oil the oil was $8.00/MJ in 2019 and that grade 1 of oil is produced. Then the market price of oil would be $8.00/MJ + $2.83/MJ = $10.83/MJ.

We conclude this discussion of the correspondence principle by emphasizing the power and the limitation of the approach. Like stylized approaches, such as physics in a vacuum, the MP-as-market approach is a powerful tool for analyzing futures and policies, as we will see below. But it also has severe limitations, as we will discuss in the last section of the study.

b. ***Assumptions of the Model***

The model has a single energy service which can be met by a combination of four energy carriers or fuels. The four fuels are oil, gas, coal, and carbon-free renewables. There are reliable estimates of the production and wholesale prices of the four fuels at the wholesale level. They are assumed to be combined by a constant elasticity of substitution (CES) production function using the 2019 prices and quantities of the four fuels. After some testing, we found that the model best calibrates to data and existing models with an CES value of 2.

The key ingredients in the model are the estimates of the stocks of different exhaustible resources along with their extraction and production costs. Estimates of extraction costs and stocks of resources for 13 grades of each of the 4 energy carriers are taken from *Global Energy Assessment* (2015). These are updated to 2021 prices using the US GDP price index. Other energy values, such as fuel quantities and prices, are taken from British Petroleum, *Energy Statistics* (2022). The model is calibrated to 2019 prices and 2019 quantities as that was the last “normal” year before the pandemic and the Ukraine war disrupted energy markets. Details on the data assumptions are provided in the Appendix.

One prominent issue is the treatment of export taxes by OPEC and oil-exporting countries. For example, Saudi Arabia’s oil has production costs of about $10/barrel and sells at the market price. Suppose the difference is $50. If governments levy taxes as a percentage of the Hotelling rents, these taxes are non-distortionary and simply divide up the rents between the government and the owner. For Hotelling-type export taxes, therefore, we can ignore taxes and simply assume that some of the rents go to the government as taxes.

Production costs are calculated as the balancing item. They are determined so that the production plus extraction costs plus Hotelling rents (at a zero carbon price) equal the wholesale price in 2019. Total demand for energy (at a constant price) is assumed to grow at 2 percent per year, which is the growth rate of global primary energy consumption over the 2010-19 period. The price-elasticity of demand is assumed to be -0.5%. The model is calibrated in the GAMS framework to match 2019 estimates for all volumes and prices. The model then calculates the competitive equilibrium prices and quantities with annual estimates from 2019 to 2100.

A key assumption is the extraction costs of the fossil fuels. Figure 1 shows the cumulative extraction costs of different fossil fuels as of 2015 up to 35,000 EJ – exajoules (EJ = one quintillion or 10^{18} joules). Energy consumption was approximately 570 EJ in the initial year, 2019. For intuition, current consumption of 570 EJ is 948 trillion Btu. This is sufficient energy for the average person to boil 300 tons of water every year. Costs are measured in 2021 US$ per MJ (megajoules or million joules). For those used to Btus, one MJ is approximately 1000 Btu. Renewable fuels are estimated to cost $10 per MJ.

A central part of the Hotelling model is the abatement cost function. The is the response of emissions reductions to different carbon prices and is a standard feature of integrated assessment models (IAMs). We can test the abatement cost function from the Hotelling model in comparison with more complete models. For this purpose, I used the harmonized runs from the MUP (model uncertainty project), see Gillingham (2018). In these runs, each of six models used their standard model and identical carbon price trajectories. Figure 2 shows the estimated emissions control rate for the MUP median model and the Hotelling model (both in bold) and the six component models of the MUP study (in thin lines). The estimates are broadly consistent. However, the Hotelling model shows larger emissions reductions at low prices and smaller reductions at high prices. The two approaches are similar in impact at prices above \$50/tCO$_2$.

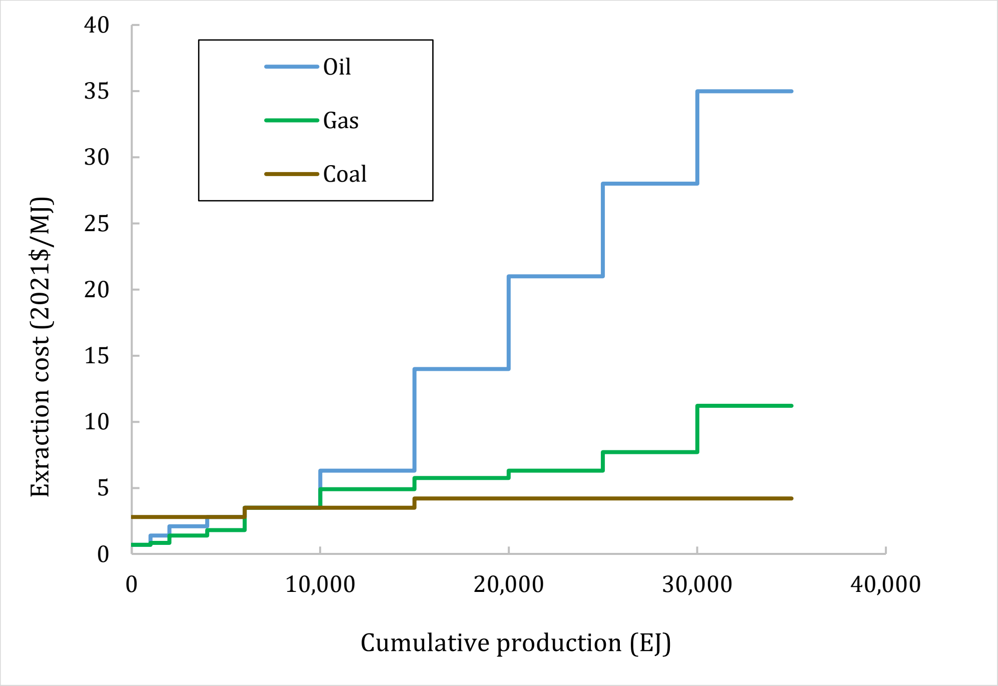


Figure 1. Extraction costs of fossil fuels

Cost of production are from GEA (2012). They are reflated to 2021 US$ using the US GDP price index.

The full Hotelling model starts with equations (1) though (5) with the added dimensions of fuels and grades. It then adds the objective function, which is an objective function that is the present value of utility using a constant-elasticity demand function; and adds a CES production function. Note that the objective function is that of the firms, not of society, because it includes taxes as a cost and excludes any externalities. The social welfare function is discussed below. The full equations are shown in the Appendix.

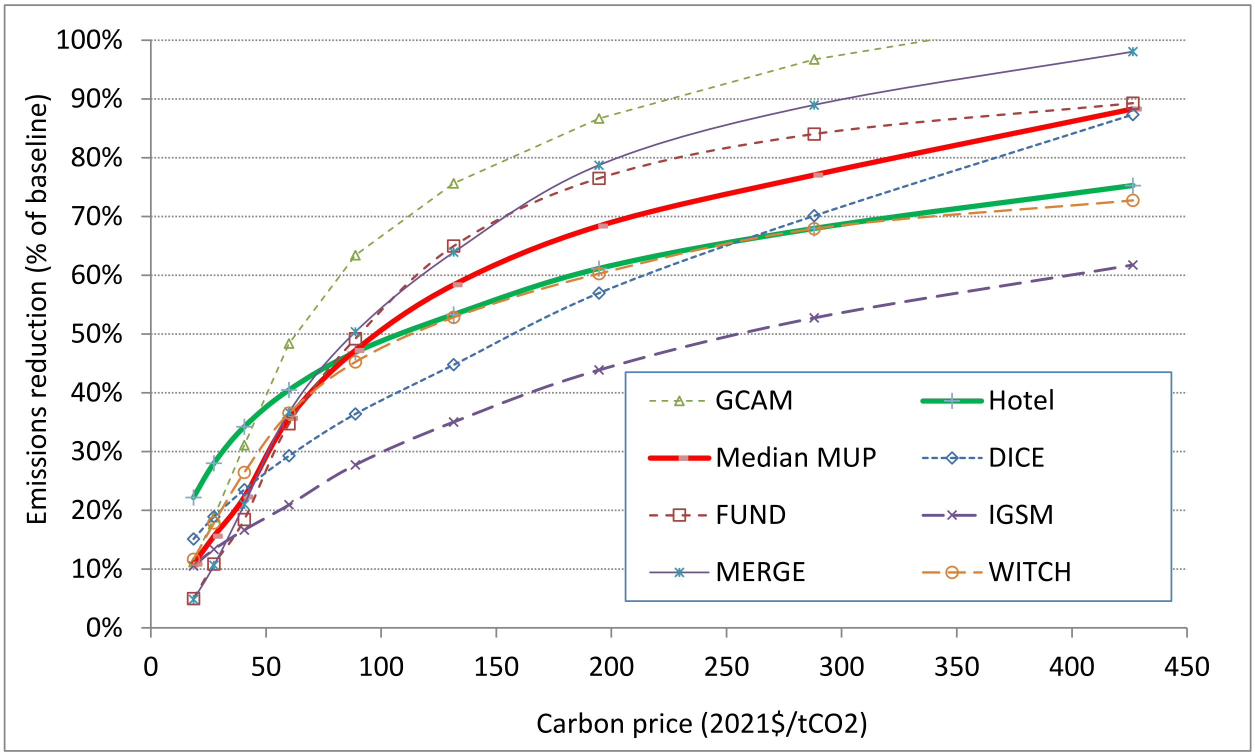


Figure 2. Comparison of emissions control/carbon price relationship for different models

[Source: Gillingham (2018)[a]]

The model is programmed and run in the GAMS modeling system. There are ten behavioral equations or identities per period, grade, and fuel for a total of 80 equations times the number of time periods. In total, the model has 5302 variables and 1038 constraints, but this is just a toy for modern mathematical programming software. The only complicated equations are the production and the demand functions. The balance of the model is accounting equations and constraints. The model for 2019 – 2100 with 51 tax rates takes 13 seconds to solve and is extremely stable.

III. Results of the Hotelling Model

a. *Magnitude of resource rents for fossil fuels*

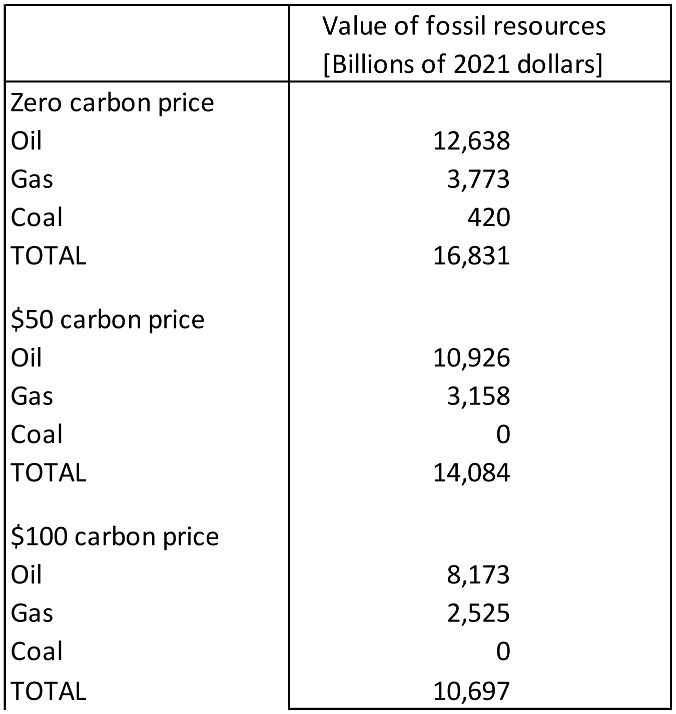
We can use the calculations from the Hotelling model to estimate the total resource rents from fossil fuels as shown in Table 2. "Resource rents" are the value of resources-in-the-ground as of 2019 valued at 2019 scarcity rents. These are calculated for carbon tax rates of \$0, \$50, and \$100 per ton of CO_2.

The total value of resource rents without a carbon tax is \$17 trillion. Oil rents are 75% of the total, and the remainder is virtually all from gas, with coal about 2% of rents. These proportions reflect the low extraction and transportation cost of oil and the high production costs and cost-elastic supply for coal. The impact of carbon taxes on rents is discussed in the next section.

There are no recent comprehensive estimates of the value of fossil rents. A useful comparison would be with Aramco, which owns and produces virtually all the oil and gas for Saudi Arabia. The oil reserves of Saudi Arabia are estimated to be 1600 EJ, and the estimated rents on the highest grade of oils are estimated in the present study to be about \$3/MJ, for a total of \$4.8 trillion. This compares with a market capitalization of Aramco of \$2.3 trillion in mid-2022. Since the corporation tax on oil companies in Saudi Arabia is 50%, these two values align reasonably closely.

The market cap of the largest 254 publicly owned oil and gas companies in mid-2022 was \$6.2 trillion. The estimated rents from the Hotelling model estimate rents that are slightly more than two times the market capitalization of the largest companies. While the difference between these two numbers is not easily decomposed, company values contain many elements other than Hotelling rents, such as returns on other assets, government taxes, overriding royalties, the diffuse ownership of the resources, and public ownership of most energy resources.

The market value of coal companies around the world is much smaller than that of oil and gas companies. The market capitalization of the largest coal companies in mid-2022 was $205 billion, which is about 50% of the estimated zero-carbo-price Hotelling rents.[b]Again, there are many reasons for the difference, but the closeness of the estimates provides some encouragement for the estimates provided here.



	Value of fossil resources [Billions of 2021 dollars]
Zero carbon price	
Oil	12,638
Gas	3,773
Coal	420
TOTAL	16,831
$50 carbon price	
Oil	10,926
Gas	3,158
Coal	0
TOTAL	14,084
$100 carbon price	
Oil	8,173
Gas	2,525
Coal	0
TOTAL	10,697

Table 2. Impact of carbon prices on value of rents

Table shows the impact of three different carbon tax rates on the value of fossil-fuel resources

[Source: Hotelling model] [c]

b. Impact of carbon prices on resource rents

The key question in this study is the impact of climate policies on Hotelling rents. For this question, we have varied carbon prices from \$0 to \$500 per ton of CO_2. Figure 3(a) shows the results for the first three grades of oil, while 3(b) shows the same for gas, and 3(c) for coal.

The general trajectory of the impact on fossil-resource rents with carbon prices is intuitive. Rents fall as carbon prices rise. For each resource, there are two general impacts. The first is the direct cost effect of the carbon tax on the cost of production, while the second is the substitution effect where the relative costs of different fuels change as carbon prices change. (Recall the discussion in the first section.)

The direct cost effect is the most important impact for all fuels. For oil and gas, the effects of taxes on royalties are roughly linear up to about \$250/tCO_2, as which point the rents virtually disappear for oil and gas. There is also a step decrease in oil and gas rents at around \$250/tCO_2. This discontinuity appears to be largely a substitution effect.

Note as well that the rents for oil are about three times those of gas. The reason is basically that the transportation costs of gas are so much higher than for oil that these low transport costs outweigh the low production costs and low carbon content of gas.

By contrast, the effect of carbon prices on coal rents is dramatically different from that on oil and gas. Coals rents are very low in any case and make up only about 1% of the wholesale price of coal. Coal rents disappear at around \$20/tCO_2. The coal share of primary energy declines by about 95% as carbon prices rise from \$0 to \$250/tCO_2. Coal production does not completely disappear because of the slightly limited substitution assumed in the model.

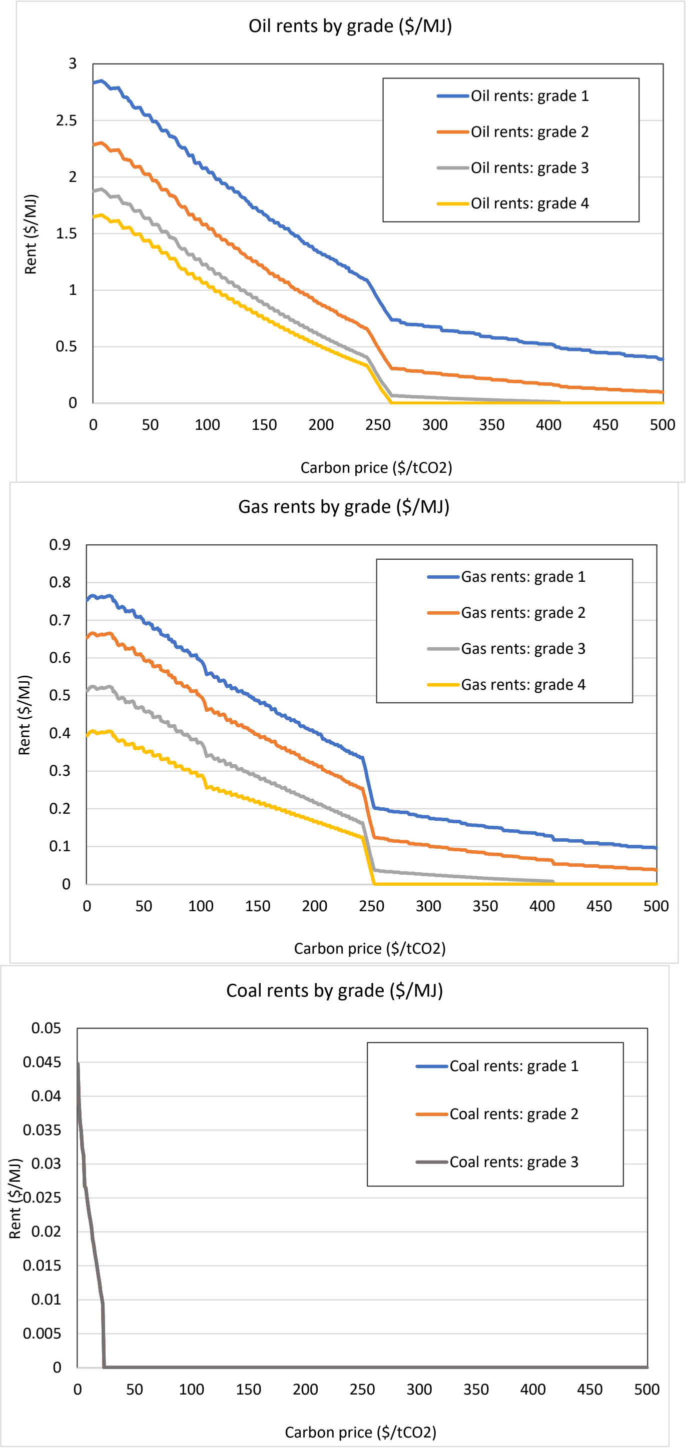


Figure 3. Impact of carbon prices on rents for (a) oil, (b) gas, and (c) coal

[Source: Hotelling model [d]]

The rents on coal are low to begin with (about one-tenth of gas and one-thirtieth of oil). Low rents are the effect of high abundance and relatively elastic supply, shown in Figure 1. More important is that coal rents disappear with low carbon taxes (hitting zero at about \$20/tCO$_2$) because of the high carbon content of coal per unit value of useful energy. Coal production continues at very low levels at higher carbon prices, but the production is so low that the lowest cost grade is not exhausted, meaning that rents are zero. [e]

Another finding is the impact of carbon prices on rents by grade. The results are intuitive for the impact on fossil fuel Hotelling rents. Examining the case of the \$50 per ton carbon tax, oil rents decline by 11 % in the top grade. Gas rents decline slightly less at 8%, reflecting the lower carbon content and higher value per unit energy. Coal rents decline by 50% at a \$20 per ton carbon price reflecting the high carbon content of the fuel.

c. Rising rents for low carbon prices

There is an interesting anomaly for oil and gas rents at low carbon prices. Resource rents rise for taxes up to \$20/tCO$_2$ for gas and for taxes up to \$10/tCO$_2$ for oil, as shown in Figure 4. This anomaly is the result of the substitution effect, where carbon taxes sharply reduce the share of coal production for the first \$20/tCO$_2$ while raising the shares of oil and gas. These changes in the mix increase the scarcity value of oil and gas and thereby increase oil and gas rents. Once coal production is sharply reduced, the substitution effect disappears, and higher carbon taxes reduce oil and gas rents.

An important finding, then, is that, for realistic energy supply conditions, a carbon tax has essentially no impact on resource rents and values for oil and gas up to \$20/tCO$_2$; furthermore, carbon prices have relatively small impacts on the value of high-grade oil and gas resources for carbon prices up to \$50/tCO$_2$. The value of oil and gas rents decline about 15% for a \$50 carbon tax. At the same time, the impact of carbon pricing is substantially higher for coal resources at even low carbon prices, with scarcity rents disappearing at a \$50/tCO2 price.

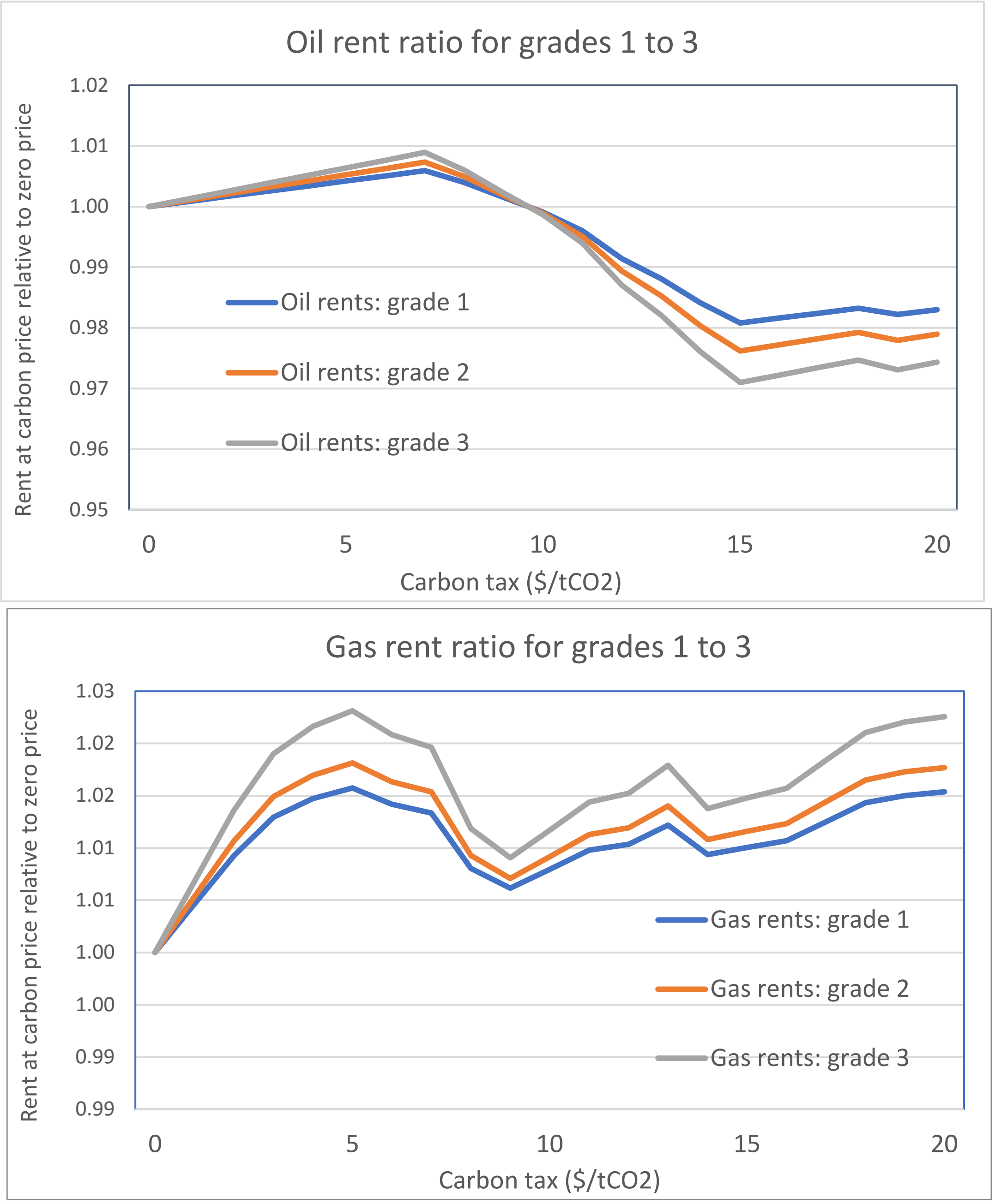


Figure 4. Rent ratio for oil and gas at low carbon prices

Figure shows the ratio of the rent at the carbon tax on the horizontal axis relative to the rent at a zero carbon tax. Note that the rents rise at low carbon prices. For reference, global carbon prices averaged about \$3/tCO_2 in 2021.

[Source: Hotelling model [f]]

d. Division of the surplus between resource rents and taxes

Owners of fossil fuels are likely to resist carbon pricing because high prices will reduce the value of their assets. At the same time, governments will earn substantial revenues from carbon taxes. What is the division of the rents under a carbon-tax regime?

Figure 5 shows the present value of tax revenues along with the total value of fossil resource rents at different carbon taxes. The present value of carbon tax revenues is much larger than the resource rents, reaching about \$110 trillion at \$500/tCO_2 compared to the maximum resource rents of \$17 trillion at a zero carbon price.

This finding parallels the finding of Bovenberg and Goulder (2001) and others that the fiscal gains from auctions or taxation on CO_2 emissions far outweigh the financial stakes of resource owners. For example, if the tax is set at an estimated social cost of carbon of \$50, the resource owners lose \$2.8 trillion, while the tax revenues are \$36.5 trillion. The ratio of rent losses to tax gains is 8%. Most of the impact of taxes is passed on to consumers rather than back to resource owners.

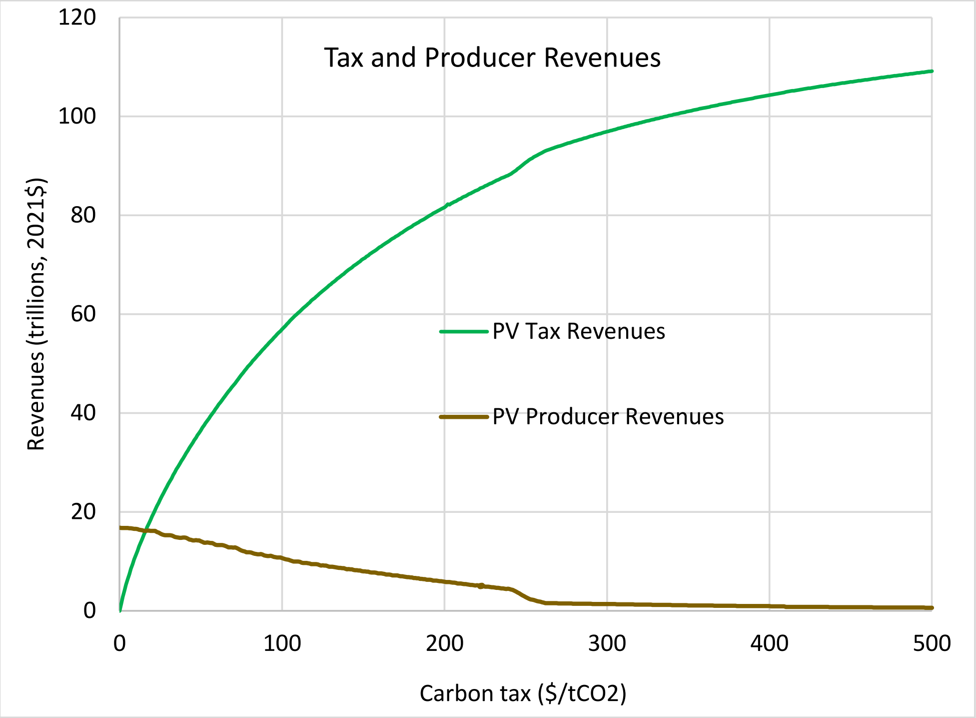


Figure 5. Present value carbon tax and resource rents.[g]

IV. Impacts of Alternative Policies

It is useful to remember that carbon pricing is just the tip of the iceberg of climate policies. Gaulin and Billon (2020) have complied a list of interventions by category, shown in Table 3. Divestment, blockades, and litigation are more important impediments to fossil fuel extraction that carbon pricing (by number if not dollar value). Yet, this list excludes the important categories of fuel taxes (on gasoline) and subsidies (such as those on renewable energy).

The analysis in the last section deals only with carbon pricing. Following the suggestions in Table 3, two alternative approaches to policy can be analyzed in the Hotelling approach and are presented in this section: renewable subsidies and "ethical" investments. This section also includes a discussion of the welfare measure of policies, which will differ from the objective function because of fiscal taxes and subsidies and externalities such as CO_2 emissions.

Category	Instruments	Number of countries	Number of cases
Financial initiatives			
	Emission trading schemes	37	40
	Divestment	26	701
	Carbon tax	17	19
	Subsidy removal	8	8
Physical initiatives			
	Blockading	81	325
	Moratoriums and bans	22	106
	Litigation	12	103

Table 3. Supply-constraint initiatives for climate policy

Source: Gaulin and Le Billon (2020).[h]

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a. ***Renewable subsidies***

Carbon taxes have proven a toxic mix with politics, although the toxicity varies across counties. Subsidies, by contrast, are catnip for politics. Among the most important examples are tax subsidies for renewable power (wind and solar) as well as implicit subsidies seen in regulations such as those that use feed-in tariffs.

In the present approach, we investigate the impact of subsidies to the fuel we label "renewables." This would involve lowering the price of renewable fuel below its market price. For concreteness, we investigate 10% and 50% subsidies. Since the estimated resource cost of renewable energy is \$10/MJ, this implies that the market prices would decline to \$9/MJ and \$5/MJ in the two cases.

The results, shown in Table 4, show the results of a combination of carbon taxes and renewable subsidies and present an interesting pattern. The impact of a carbon price alone up to \$100/tCO_2 show an impressive reduction in emissions of about 49% at \$100/tCO_2. By contrast, even substantial subsidies of 50% of renewables with a zero carbon price have an emissions reduction of only 17%.

Assumption	Emissions reduction	Welfare relative to base
	[% of no tax and no subsidy]	[Present value, billions 2021$]
No subsidy		
Ctax = $0 (base)	0%	0
Ctax = $50	38%	29,070
Ctax = $100	49%	25,498
Subsidy = 10% of renewable cost		
Ctax = $0	2%	1,375
Ctax = $50	39%	28,727
Ctax = $100	51%	24,067
Subsidy = 50% of renewable cost		
Ctax = $0	17%	874
Ctax = $50	50%	8,740
Ctax = $100	60%	(5,698)

Table 4. Impact of renewable subsidies on emissions reductions and welfare

[Source: Hotelling model [i]]

We also calculate the welfare costs of different policies. These represent the present value of true income, excluding taxes and including a correction for the impacts of emissions. Fiscal measures such as taxes and subsidies are excluded from welfare because they are not resource costs. Note, however, that a full discussion would include the distortionary impact of fiscal measures, which is omitted here. Additionally, a welfare measure would include an accounting for the cost of CO_2 emissions. This is done by subtracting CO_2 emissions time the social cost of carbon (SCC) in each period. For the calculations here, we use a value of $50/$tCO_2$ as the SCC, rising at 2% per year. (See the Appendix for a more detailed discussion.)

If we look at the welfare cost in the last column of Table 4, the results are even more powerful than the emissions results. In the absence of carbon pricing, subsidies have a positive welfare impact for subsidies of 10% but not for large subsidies of 50%. This is because – in the absence of carbon pricing – the subsidy acts as a second best (or at least positive) policy in tilting energy use away from carbon fuels. However, the subsidy is highly inefficient. The same emissions reductions can be attained with a relatively low carbon price and a much lower economic cost. Calculations find that a \$14/tCO_2 carbon price yields the same emissions reduction as a 50% renewable subsidy and has a welfare benefit of \$22 trillion compared to the \$0.8 trillion benefit of the 50% subsidy.

The inefficiency of the renewable subsidy is a fundamental problem with policies that do not treat the carbon conflict with precision. Subsidies encourage substitution away from *all* fossil fuels independent of their cost and carbon content. Indeed, when renewables exclude nuclear power, subsidies have no direct effect on carbon emissions. An efficient policy differentially reduces the fuels with high carbon content relative to their economic value.

If we look for example at the impact of a 10% subsidy on renewables, given the assumptions of the Hotelling model, this raises production of renewable energy by about 20%. The impact is to lower fossil fuel production by 1.5% (oil), 2.7% (gas), and 2.6% (coal). An efficient policy (with a carbon tax at \$10 and equal to the SCC), finds a completely different pattern of changes. With the efficient tax, oil and gas production are virtually unchanged (the share of oil declining 1.8% and gas increasing 0.3%). By contrast, the share of the high-carbon coal declines 32% while renewables' share increases by 10%. Figure 6 shows the comparative changes in different fuels for a renewable subsidy.

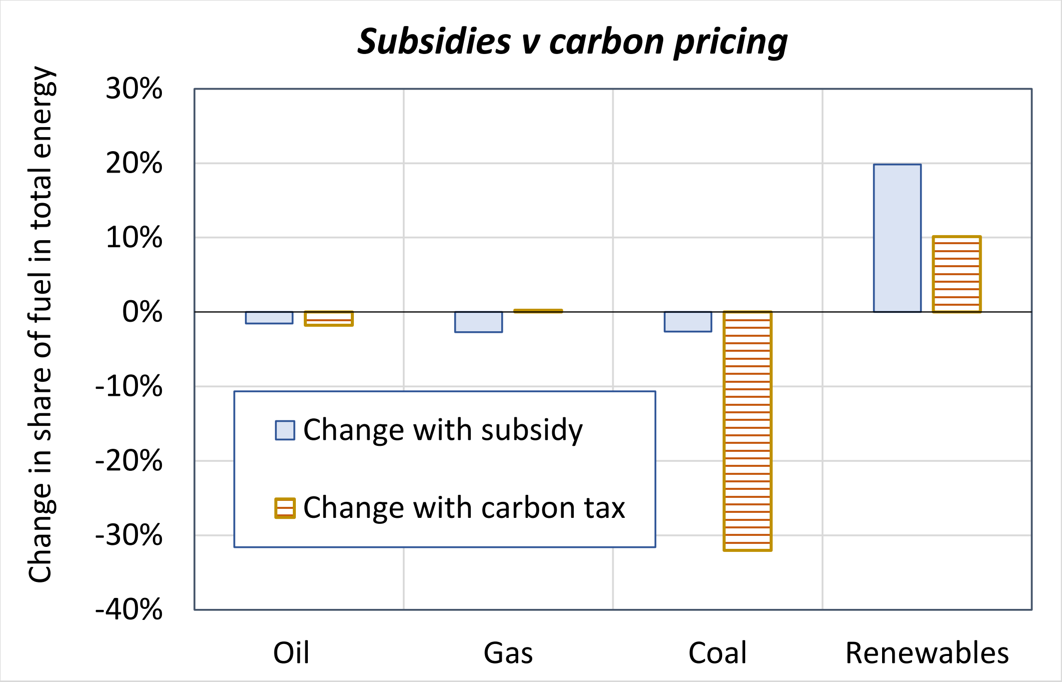

Figure 6. Impact on different fuels of renewable subsidy

Bars show the impact on the shares of different fuels for a 10% renewable subsidy and $10/tCO2 carbon tax. Note that the tax targets the high-carbon coal much more sharply than the other fossil fuels, thereby reducing emissions at lower costs.

[Source: Renewable version of Hoteling model. [j]]

This example captures the essential inefficiency of subsidies. They target “green” activities and will generally lower CO_2 emissions. But they do so in a manner that is not precisely targeted. For example, a subsidy for wind power might replace nuclear power (as was the plan in Germany), but that would have no impact on CO_2 emissions.

An analog to a renewable subsidy would be to subsidize “renewable consumption” activities. These might be defined as ones that have low carbon-intensity such as bicycles, walking shoes, mobile phones, and mass transit. However, since the renewable consumption activities are so extensive, they require vast expenses to subsidize them. And in the end, the impact is relatively small. Moving to a military analogy, unlike carbon taxes, renewable subsidies are dumb weapons that are too imprecise to reduce the most potent greenhouse gases.

The impact of subsidies on resource rents is shown in Table 5. Interestingly, the impact is to reduce the rents. The reason is that the substitution of renewables for fossil fuels lowers the demand for fossil fuels and thereby lowers their scarcity rents. The impact is roughly 1% decline in rents per 1% increase in the subsidy rate.

Total rents	Oil royalties grade 1	Oil royalties grade 2	Oil royalties grade 3	Gas royalties grade 1	Gas royalties grade 2	Gas royalties grade 3	Grades 1 and 2
	Billons of 2021$						
Subsidy = 0%	2,834	2,286	3,752	754	654	1,026	11,305
Subsidy = 10%	2,747	2,214	3,297	654	513	789	10,214
Subsidy = 50%	2,270	1,697	235	513	395	223	5,332

Table 5. Impact of renewable subsidies on resource rents by grade

[Source: Hotelling model [k]]

b. Ethical investment

Another important policy proposal is ethical investment. This usually falls under the rubric of ESG, or environmental, social, and governance policies.

While there are many versions of ESG on climate policy, one important one is to reduce the development of new oil and gas fields. For example, the International Energy Agency (IEA) wrote in 2021 that to achieve net zero emissions by 2050 would require a major change in the oil and gas industry. They wrote in *Net Zero by 2050: A Roadmap for the Global Energy Sector*, "Beyond projects already committed as of 2021, there are no new oil and gas fields approved for development in our pathway… No new oil and gas fields [can be] approved for development from 2021." This policy has been proposed on many occasions, such as to large university endowments, and would clearly be a major change in policies. Universities have been pressed on this issue. In 2021, Harvard University announced that it would not make investments in companies that explore for or develop further reserves of fossil fuels.

The general idea is that if new oil, gas, and coal fields are not developed, that will reduce the emissions of energy-based CO_2 emissions. To test this hypothesis, I examined two scenarios in which a fraction of oil and gas resources are removed from the resource base; the fractions are 25% and 50%. This might occur because they were sequestered by governments or perhaps put into conservation easements by private owners. (I ignore the possibility that divestment of fossil assets by one owner, such as a university endowment, might simply be bought by someone else without any impact on resource use.) For this analysis, I omitted coal resources because they are so abundant and widely dispersed.

Table 6 shows the results. The ethical investment policy of foreclosing future oil and gas production is even worse than the subsidy approach discussed above. With a $0 carbon price, the 25% resource reduction has virtually no impact on emissions (1% reduction), yet it reduces welfare by $5 trillion. The 50% reduction reduces emissions by 3% and costs $14 trillion. Note in line 2 of Table 6 that the $50/$tCO_2$ tax with no reduction of the resource base increases welfare substantially.

The impact on resource rents is, not surprisingly, to enrich the owners of fossil resources that are not affected by the shutdown. For the top three grades of oil and gas and a $0 carbon price, the total present value of rents rises from $11 trillion to $24 trillion. This is a result of the higher scarcity of oil and gas resources with the removal of a substantial fraction of the low-cost fossil resources.

None of this will be a surprise to those who have been watching the ill-designed Western energy sanctions on Russia. Restrictions on the purchase of Russian oil and gas drove up their price. Because of the highly price-inelastic short-run demand for these commodities, the effect has been to raise revenues for Russia, raise energy costs to the West, and trigger a painful bout of inflation. This episode should be a good lesson for advocates of strategies that would sequester low-cost fossil resources as a part of climate policy. Much, much better would be to tax the relevant “bad,” reduce demand, and thereby reduce emissions at very low welfare costs.

Assumption	Emissions reduction	Welfare relative to base
	[% of no tax and no subsidy]	[Present value, billions 2021$]
No change in resource base		
Ctax = $0	0%	0
Ctax = $50	38%	29,070
Ctax = $100	49%	25,498
Reduce resources 25%		
Ctax = $0	1%	(5,178)
Ctax = $50	39%	(52,395)
Ctax = $100	50%	(83,528)
Reduce resources 50%		
Ctax = $0	3%	(13,825)
Ctax = $50	41%	(59,501)
Ctax = $100	52%	(89,235)

Table 6. Impact of not developing oil and gas reserves
[Source: Hotelling model [l]]

V. Reservations about the Methods

As in any modeling study, the present one has several shortcomings that much be recognized. The first is the limitation of the modeling framework itself. While the “correspondence principle” is a powerful tool for emulating competitive markets, the energy market has many noncompetitive elements, as we have seen so vividly during various oil crises. Energy demand and supply are highly price-inelastic in the short run, whereas the model is a long-run approach. While the oil market is a global market, some countries (particularly oil-rich countries like Saudi Arabia) have market power and do not produce according to the Hotelling trajectory. Moreover, the assumption of a single demand category fails to capture the richness of the demand structure in energy markets. Finally, the discount rate is particularly complicated because of the presence of capital taxes that drive a wedge between pre-tax and post-tax returns, as well as appropriate corrections for risk.

A second concern lies in the data underlying the estimates. Much data in the energy sectors is of the highest quality, such as current prices and production. However, resource estimates have proven difficult to make with reasonable certainty. Particular uncertain are the unproved resources, which are based on many extrapolations or statistical sources. As an example, proven reserves of U.S. natural gas grew by a factor of 2.5 from 2000 to 2020. Even more uncertain are the estimates of costs of extraction. In general, these are proprietary, and the present figures are based on engineering estimates.

Particular parameters are ones that lend sensitivity to the estimates. The discount rate, much debated in many areas, has a major impact upon the rent estimates. If we use a risk-free discount rate of 1% per year, estimates of the rate of rent on low-cost oil and gas (and consequently the total rents) rise by a factor of three. Estimates of production costs other than extraction are a residual calculation and thus are model-based rather than data-based.

Key elasticity parameters do not appear to be major sources of sensitivity. The estimates of rents for $0 carbon prices are stable for alternative elasticities of demand or elasticities of substitution between different fuels. The impact of carbon pricing on emissions reductions is moderately sensitive to the elasticity of substitution (see Appendix Figure A-1).

Many estimates are firmly grounded. These include the energy content of fuels as well as the carbon emissions associated with fuels. The impact of other parameters, such as the estimates of resource bases, have little impact on the estimates of total resource rents. The estimate of the social cost of carbon only affects the welfare estimates and has no effect on other findings. Changing the terminal year has little effect; for example, moving the terminal year from 2100 to 2200 increases rent rates by only 1%.

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APPENDIX

I. Equations of the Hotelling model

The following are the full equations of the Hotelling model. Each run is for a given value of the carbon tax *(τ).*

$$(A.1) \quad \max_{\{x(i,j,k,\tau,t)\}} W(\tau) = \left\{ \sum_{t=0}^{\infty} U[D(\tau,t)](1+r)^{-t} - \sum_{t=0}^{\infty} C(\tau,t)(1+r)^{-t} \right\}$$

$$(A.2) \quad d(k,\tau,t) = \sum_{i,j} x(i,j,k,\tau,t)$$

$$(A.3) \quad \sum_{k} d(k,\tau,t) \geq D(\tau,t)$$

$$(A.4) \quad \sum_{t,\forall\tau} x(i,j,k,\tau,t) \leq R(i,j,k)$$

$$(A.5) \quad \sum_{k} R(i,j,k) \leq R_0(i,j)$$

$$(A.6) \quad CO_2(\tau,t) = \sum_{i,j,k} x(i,j,k,\tau,t)\,\theta(i,j)$$

$$(A.7) \quad C(\tau,t) = \sum_{i,j,k} [x(i,j,k,\tau,t) z(i,j,\tau)](1+r)^{-t}$$

The variables of the model are:

k = firm; j = fuel; i = grade; τ = carbon tax; t = time
$W(\tau)$ = welfare measure (2021$ discounted)
$U[D(\tau)]$ = utility function (2021$/yr)
$D(\tau,t)$ = demand for energy (EJ/yr)
r = discount rate (% per year)
$C(\tau,t) =$ total cost of fuels (2021$/yr)
$d(k,\tau,t) =$ demand met by firm k (EJ/yr)
$x(i,j,k,\tau,t) =$ output of fuel j of firm k for grade i for carbon tax τ (EJ/yr)
$z(i,j,\tau) =$ cost of fuel j of grade i (2021$)
$\theta(i,j) =$ CO_2 content of fuel j, grade i (GtCO$_2$/MJ)
$R(i,j,k) =$ Resource stock available to firm k (EJ)
$R_0(i,j) =$ Total resource stock of fuel j, grade i (EJ)
$CO_2(\tau,t) =$ Total CO_2 emissions in period t for carbon tax τ (GtCO$_2$/yr)

II.A Note on Welfare Measures

The model has two separate measures of economic output. The first is "utility," which is the variable that is optimized in the "market" solution. This measure includes fiscal taxes and subsidies (such as carbon taxes or renewable subsidies) but excludes non-market costs, such as those associated with CO_2 emissions. In the absence of any taxes or distortions, utility would represent the economic optimum.

A second measure is "welfare." This represents the appropriate target of economic policy. It excludes taxes and subsidies (which are assumed non-distortionary) and includes a correction for the impacts of emissions, measured as the social cost of carbon in a period times that period's CO_2 emissions.

The difference between utility and welfare can be seen in the GAMS program in section IV of this Appendix.

III. Supplemental Figures

The following compares the emissions control rate (ECR) for different CES functions and carbon prices. The ECR is higher for higher CES parameters.

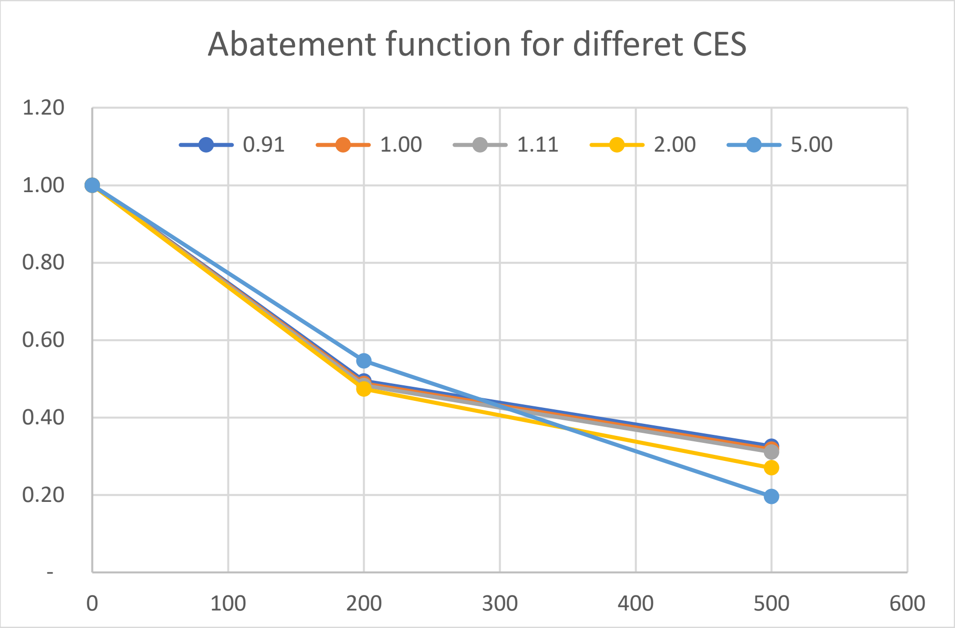

Figure A-1. Abatement cost function for different elasticities of substitution in production

[Source: Hotelling model ^{m}]

IV. Data for program

The following are the major inputs to the Hotelling program for resources:

```
----    313 PARAMETER ces  CES exponent
oil   1.660,    gas   1.350,    coal  0.478,    renew 0.870

----    313 PARAMETER R  reserves by grade
         oil        gas       coal      renew
g1    1000.000    1000.000    1000.000 1.000000E+7
g2    1000.000    1000.000    1000.000    1000.000
g3    2000.000    2000.000    2000.000    2000.000
g4    2000.000    2000.000    2000.000    2000.000
g5    4000.000    4000.000    4000.000    4000.000
g6    5000.000    5000.000    5000.000    5000.000
g7    5000.000    5000.000    5000.000    5000.000
g8    5000.000    5000.000    5000.000    5000.000
g9    5000.000    5000.000    5000.000    5000.000
g10    5000.000    5000.000    5000.000    5000.000
g11    5000.000    5000.000    5000.000    5000.000
g12   10000.000    5000.000    5000.000    5000.000
g13   10000.000   20000.000   20000.000   20000.000

----    313 PARAMETER Extcost  Extraction cost by grade 2021$
         oil        gas       coal      renew
g1       0.700      0.980      1.400      1.400
g2       1.400      1.120      1.400      1.400
g3       2.100      1.400      1.400      1.400
g4       2.800      1.820      1.400      1.400
g5       3.500      3.500      1.540      2.800
g6       6.300      4.900      1.540      2.800
g7      14.000      5.740      1.680      2.800
g8      21.000      6.300      1.680      4.200
g9      28.000      7.700      1.680      4.200
g10      35.000     11.200      1.680      4.200
g11      42.000     18.200      1.680      4.200
g12      49.000     21.000      2.800      4.200
g13      70.000     28.000      3.500      4.200

----    313 PARAMETER Prodcost  Production cost by fuel per MJ
         oil        gas       coal      renew
g1       8.610      9.800      2.380      8.400
g2       8.610      9.800      2.380      8.400
g3       8.610      9.800      2.380      8.400
g4       8.610      9.800      2.380      8.400
g5      17.220      9.800      2.380     11.200
g6      17.220      9.800      2.380     11.200
g7      17.220      9.800      2.380     11.200
g8      25.830      9.800      2.380     11.200
g9      25.830      9.800      3.570     14.000
g10      25.830      9.800      3.570     14.000
g11      25.830      9.800      3.570     14.000
g12      34.440      9.800      3.570     14.000
g13      34.440      9.800      3.570     14.000
```

V. Major GAMS statements

The following are the key equations for the estimates in GAMS language. The program is available from the author on request.

```
 Variable
          energy(t)                  'total energy production'
         x(i,j,t)                     'production of grade i fuel j year t'
         z(t)                        'cost'
         znotax(t)                   'cost without C tax'
         demand(t)                  'Total energy demand'
         reserveuse(i,j)            'Use of resources for fuels'
         fuel(j,t)                  'Total fuel production in year'
         energycons(t)               'Energy consumption'
         emco2(t)                   'Energy CO2 emissions'
         periodutil(t)              'Period utility to maximize'
         periodwelf(t)               'Welfare measure'
         utility                    'Utility to maximize'
         welfare                     'Welfare measure'

Positive Variable x, z, reserveuse, demand, energy;
Equation
         costeq(t)                          'Define objective function equation'
         costeqnotax(t)               'Define objective function without C tax equation'
         supplyeq(i,j)              'Observe supply limit for grade i equation'
         demandeq(t)                 'Satisfy demand for time t equation'
         demandeq(t)                'Quantity demanded period equation'
         energyeq(t)                         'Energy production function equation'
         fueleq(j,t)                'Fuel equation'
         energyconseq(t)            'Energy equation'
         emco2eq(t)                 'CO2 equation'
         utilityeq                  'Objective equation'
         periodutileq(t)            'Period utility function equation'
         periodwelfeq(t)             'Period welfare function equation'
         welfareeq                           'Welfare measure equation'
;
       energyeq(t)..                 energy(t)  =e= ces('oil')*fuel('oil',t)**gamces+ces('gas')*fuel('gas',t)**gamces
                                            +ces('coal')*fuel('coal',t)**gamces
                                              + ces('renew')*fuel('renew',t)**gamces)**(1/gamces);
       fueleq(j,t)..                fuel(j,t)  =e= sum(i, x(i, j, t));
       costeq(t)..                  z(t)  =e=  sum(i, sum(j, totcost(i,j,t)*x(i,j,t)));
       costeqnotax(t)..                     znotax(t)  =e=  sum(i, sum(j, totcost0(i,j,t)*x(i,j,t)));
       supplyeq(i,j)..               reserveuse(i,j) =e= sum(t, x(i, j, t));
       demandeq(t)..                  demand(t) =e= energy(t);
       periodutileq(t)..            periodutil(t)=e=scaleu*utilint*demand(t)**utilexp
                                            *(1+grdemand)**(t.val-2019)- z(t);
       periodwelfeq(t)..            periodwelf(t)=e=scaleu*utilint*demand(t)**utilexp
                                            *(1+grdemand)**(t.val-2019)- znotax(t)-scc(t)*emco2(t);
       energyconseq(t)..            energycons(t) =e= sum(i, sum(j, x(i,j,t)));
       emco2eq(t)..                 emco2(t) =e= sum(i, sum(j, co2(i,j)*x(i,j,t)));
       utilityeq..                  utility =e=  sum(t, periodutil(t)*rr(t)) + Scaleintutil;
       welfareeq..                  welfare =e=  sum(t, periodwelf(t)*rr(t)) + Scaleintwelf;
```