

A Monetary Policy Asset Pricing Model

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Abstract

We propose a model where monetary policy is the key determinant of aggregate asset prices (financial conditions). Spending decisions are made by a group of agents (“households”) that respond to aggregate asset prices, but with noise, delays, and inertia. Asset pricing is determined by a different group of forward-looking agents (“the market”). The central bank (“the Fed”) targets asset prices to close the output and inflation gaps. Our model explains several facts, including why the Fed stabilizes asset price fluctuations driven by financial market shocks (“the Fed put/call”), but *destabilizes* asset prices in response to aggregate demand or supply shocks that induce positive output gaps and inflation (as in the late stages of the Covid-19 recovery). When the market and the Fed have different beliefs, the market perceives monetary policy “mistakes” that induce a policy risk premium. Belief disagreements may also generate a “behind the curve” phenomenon and provide a microfoundation for monetary policy shocks driven by the Fed’s belief surprises.

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1. Introduction

Financial markets and monetary policy are in a love-hate relationship. For much of the recent decades, and particularly during the recovery from the Covid-19 recession, monetary policy has been a stabilizing force for financial markets (“the Fed put”). This changed in early 2022, when the anticipation of a rapid Fed tightening led to a sharp drop in asset prices. Why does monetary policy stabilize markets in some periods but not in others? How do these policy decisions and switches affect asset prices and risk premia?

Abrupt policy changes (or their perception) often take place in the midst of substantial uncertainty about the underlying state of the economy and the appropriate policy response. In this context, market participants routinely fear that central banks may make “mistakes” or be “behind-the-curve.” How do these disagreements between the market and policymakers affect asset prices and their premia, and monetary policy itself?

Addressing these questions requires a model in which monetary policy closely interacts with financial markets to achieve its macroeconomic stabilization goals. Most macroeconomic models do not attribute a large role to financial markets—beyond treating them as a potential source of macroeconomic shocks. Monetary policy works by controlling interest rates, but its implications for aggregate asset prices and financial markets often remain unclear. Conversely, most asset pricing models do not feature monetary policy or its macroeconomic objectives. They also do not feature important macroeconomic frictions, such as policy transmission lags, that generate enormous complexity for real-world monetary policy and are a source of disagreements between policymakers and market participants.

In this paper, we fill some of this gap by developing *a monetary policy asset pricing model*. We envision a *two-speed economy*: a slow and unsophisticated macroeconomic side, and a fast and sophisticated financial market side. Spending decisions are made by a group of agents (“households”) that respond to aggregate asset prices (financial conditions), but with noise, delays, inertia, and possibly other behavioral frictions. Asset pricing is determined by a different group of agents (“the market”), who are forward looking, and immediately react to economic shocks and the (likely) monetary policy response to those shocks. The central bank (“the Fed”) intermediates between these two sides of the economy to achieve macroeconomic balance: it “controls” aggregate asset prices to steer the spending decisions of households and align aggregate demand with aggregate supply. In particular, the Fed *wants* to influence the behavior of households, but it *needs* to operate through the market. The market and the Fed have their own sets of beliefs about future macroeconomic conditions and the appropriate policy. Therefore, the Fed

needs to closely monitor and “cooperate” with the market to achieve its objectives. In our baseline setup, nominal prices are fully sticky and the Fed focuses on closing the output gap (we relax this assumption in an extension).

Our analysis revolves around one idea: When the Fed is unconstrained and acts optimally, *monetary policy* becomes the key driver of *aggregate* asset prices. The Fed adjusts its policy tools (e.g., the interest rate) to keep aggregate asset prices where it would like them to be to close the output gap. In this context, the traditional mechanisms that drive asset valuations (e.g., expectations, risk premia, and so on) become the drivers of the *optimal interest rate*. We investigate the implications of this idea in several variants of our model that differ in the degree of sophistication of households’ spending behavior and belief disagreements between the Fed and the market.

We start with a benchmark case in which households follow the optimal consumption rule with log utility: they respond to aggregate wealth immediately, with a constant marginal propensity to consume (MPC) out of wealth. This benchmark illustrates the logic of our model and explains why the Fed tends to stabilize asset price fluctuations driven by risk premia or beliefs (“the Fed put/call”).

We then consider a scenario where households make noisy deviations from the optimal rule. We refer to these deviations as (non-financial) *aggregate demand shocks*. These shocks induce *opposite* fluctuations in aggregate asset prices. When there is a negative demand shock, the Fed *increases* asset prices to offset the negative output gap the shock would otherwise induce. Conversely, when there is a positive demand shock, the Fed decreases asset prices to offset the positive output gap (and the inflationary pressure). This creates the appearance of “excess” volatility in aggregate asset prices. However, this policy-induced volatility plays a useful role and shields the economy from shocks that would otherwise exacerbate business cycles.

We then turn to a more realistic setting that includes *transmission delays*: in addition to acting with noise, households are inertial and respond to asset prices with a lag. These lags are empirically well documented and they make monetary policy difficult. The Fed needs to forecast future macroeconomic conditions because it effectively sets policy for a future period. Consequently, the Fed’s *beliefs* matter for aggregate asset prices. When the Fed expects aggregate supply to increase (as in the Covid-19 recovery) or aggregate demand to decrease, it targets higher asset prices. Conversely, when the Fed expects higher demand or lower supply, it sets lower asset prices. In this context, *macroeconomic news* (about demand) affects aggregate asset prices and induces asset price volatility. The news shifts the Fed’s beliefs and its target asset price. With more precise news, the model

starts to resemble the case without transmission lags. The Fed becomes more “activist” and preempts future macroeconomic conditions more aggressively. This makes output less volatile, but increases asset price volatility.

We then add *aggregate demand inertia*: households partly repeat their own *past* spending behavior and respond to (past) asset prices more gradually. This type of inertia naturally follows from realistic microeconomic frictions such as adjustment costs or habit formation. With inertia, current conditions persist into the future, even if the driving shocks are not persistent. The Fed then targets asset prices that neutralize the future effects of current conditions. When output is low (below its potential), asset prices are high (above their potential)—a phenomenon that we call *asset price overshooting*. This provides an explanation for why asset prices in the Covid-19 recovery overshot their pre-Covid levels, and why they declined abruptly once the output gap turned positive.

Next, we introduce our final key ingredient: the Fed and the market can have belief disagreements about future demand (or supply). With belief disagreements, the Fed still implements the aggregate asset price that is appropriate under its own belief. However, disagreements can affect the risk premium and the policy interest rate the Fed needs to set to achieve its target asset price. In particular, when the market has different beliefs than the Fed, the market perceives *policy “mistakes”* and demands a *policy-risk-premium*. The Fed acts optimally under its belief, but the market thinks the Fed is making a “mistake” and is targeting the wrong asset price. With recurring belief disagreements, the market anticipates excessive policy-induced volatility and demands a policy risk premium, which is especially high at times of macroeconomic uncertainty and disagreements.

Belief disagreements also provide an explanation for why the market sometimes thinks the Fed is “*behind-the-curve*” and will eventually reverse course. The market thinks the Fed will fail to achieve macroeconomic balance: for instance, a *demand-optimistic* market (that expects higher aggregate demand than the Fed) thinks the Fed will induce a positive output gap (and inflation). In view of inertia, the demand-optimistic market further thinks that once the output gap becomes positive, the Fed will have to reduce (overshoot) asset prices. In terms of interest rates, the market thinks the Fed will switch from setting rates “too low” to higher-than-usual rates.

Finally, belief disagreements provide a theory of *endogenous* monetary policy shocks. In particular, surprise changes in the Fed’s beliefs drive interest rates and aggregate asset prices. When the Fed is revealed to be more demand-optimistic than the market expected, aggregate asset prices decline and interest rates increase—providing a microfoundation for “monetary policy shocks” driven by policy announcements or speeches. Moreover, in view

of the “behind-the-curve” phenomenon, a surprise increase in the Fed’s demand optimism can *reduce* the long-term interest rates (absent a change in the forward risk premium), while raising the short-term rates, contributing to the yield curve inversion.

For simplicity, in most of the paper we assume fully sticky goods’ prices. In the final part of the paper, we endogenize inflation via a standard New Keynesian Phillips Curve (NKPC). In this context, the same logic behind our earlier results implies that *positive inflation surprises are bad news for real asset prices*—as long as inflation is driven by demand shocks or somewhat persistent supply shocks. A positive demand shock increases aggregate demand and inflation today and in the future (via aggregate demand inertia), which induces the Fed to target lower aggregate asset prices (overshooting). Likewise, a negative supply shock increases inflation and induces the Fed to target lower asset prices as long as the shock is somewhat persistent. It follows that both demand shocks and persistent supply shocks induce a *negative* covariance between inflation and real aggregate asset prices.

Literature review. This paper continues our investigation of the interaction between monetary policy, financial markets, and business cycles. Our earlier work focused on spillover effects from financial markets to macroeconomic outcomes. When monetary policy is constrained, financial market shocks or frictions—such as time-varying risk premia or financial speculation—can cause aggregate demand recessions and motivate prudential policies (see, e.g., Caballero and Simsek (2020, 2021c); Pflueger et al. (2020); Caballero and Farhi (2018)). Likewise, policy constraints and financial frictions can amplify supply shocks and motivate unconventional monetary policy (see Caballero and Simsek (2021a)). This paper uses a similar framework but focuses on the *spillback* effects from the *needs* of the macroeconomy to financial markets. To make these needs realistic, we enrich the macroeconomics side of our earlier model with ingredients such as demand shocks, transmission delays, and demand inertia. We focus on the asset pricing implications of a monetary policy framework aimed at stabilizing this richer economy by influencing financial conditions.

In terms of the specific modeling ingredients, we build on some of the insights in our recent work. In Caballero and Simsek (2021b), we showed that aggregate demand inertia induces the Fed to generate a temporary disconnect between the real economy and asset prices. In Caballero and Simsek (2022a), we began our exploration of the consequences of disagreements between the Fed and the market for optimal monetary policy. The former paper studies a one-off shock, while the latter paper’s analysis is conducted within a standard log-linearized New Keynesian model. This paper integrates the monetary policy

insights of both papers into a proper asset pricing model with risk and risk-premia. This integration enables us to obtain several new results that have no counterparts in our earlier work. Among other results, we show that *the Fed’s beliefs drive aggregate asset prices*, and its disagreements with the market generate a *policy risk premium* and a *behind-the-curve* phenomenon.

The idea that asset prices are influenced by *macroeconomic* conditions is familiar from consumption-based asset pricing models (e.g., Lucas (1978)). Relative to this literature, our model has two distinctive features. First, we assume output is determined by *aggregate demand* (due to nominal rigidities). This feature creates a central role for the Fed: in our model, asset prices are driven by macroeconomic conditions *filtered through the Fed’s beliefs*. Second, we assume aggregate consumption reacts to asset prices with noise, delays, and inertia. This feature allows for richer dynamics between asset prices and consumption than typically emphasized in the literature.¹

The connection between the Fed and asset prices is also present in an emergent New Keynesian literature with explicit risk markets (Caballero and Simsek (2020); Kekre and Lenel (2021); Pflueger and Rinaldi (2020)). That literature focuses on risk-market shocks or monetary policy shocks, whereas we focus on *macroeconomic shocks* and highlight how they can *spill back* to risk markets through the Fed’s response to these shocks. Also, in that literature the Fed is often embedded in a Taylor-type rule, rather than being an optimizing agent with its own set of beliefs.

Closer to our paper and contemporaneously, Bianchi et al. (2022a,b) build and estimate models in which asset prices are determined by forward-looking agents (“investors”), whereas the macroeconomic dynamics are driven by less sophisticated agents with inertial beliefs (“households”). They emphasize that investors’ beliefs about monetary policy regimes affect asset prices. While we share some of these ingredients, our model has the key difference that macroeconomic outcomes are affected by asset prices (financial conditions). This channel drives our results, as it provides the rationale for the Fed to target aggregate asset prices.

There is an extensive finance literature documenting “excess” volatility in aggregate asset prices, such as the stock market (see, e.g., Shiller (2014)). The literature has emphasized a number of *financial-market* shocks that could induce aggregate asset price volatility, e.g., time-varying risk premia, time-varying beliefs, or supply-demand effects (see, e.g., Cochrane (2011); Campbell (2014); Gabaix and Koijen (2021)). We complement

¹In recent work, Anderson (2021) shows that allowing for consumption mistakes can improve the empirical success of the consumption CAPM, but he does not analyze nominal rigidities or monetary policy (see also Lynch (1996); Marshall and Parekh (1999); Gabaix and Laibson (2001)).

this literature by showing that *macroeconomic shocks*, along with the *optimal monetary policy response* to these shocks, can also cause the appearance of “excess” volatility in aggregate asset prices. In our model, an activist Fed trying to stabilize the economy in response to (non-financial) aggregate demand shocks will deliberately generate asset price fluctuations not linked to underlying productivity.

Our model is also consistent with the “excess” volatility in long-term bonds observed by Van Binsbergen (2020). In our model, monetary policy works through risk-free interest rates. Therefore, interest rates and bond valuations are the ultimate “shock absorbers”—they respond to financial-market shocks *and to macroeconomic shocks*. When there is a financial-market shock, the Fed changes interest rates to insulate aggregate asset prices from this shock (the Fed put). When there is a non-financial aggregate demand shock, the Fed once again changes interest rates—this time to influence aggregate asset prices to insulate the economy from the demand shock. The two types of shocks have different effects on the covariance of bond and stock prices (when stock prices are viewed as a levered claim on aggregate output). A negative financial shock reduces stock prices and raises bond prices, whereas a positive aggregate demand shock reduces both stock and bond prices. Assuming the composition of these shocks changes over time, our model can also speak to the changes in the covariance between bond and stock returns observed in the data (see, e.g., Pflueger and Viceira (2011); Campbell et al. (2009, 2020)).

Finally, the idea that monetary policy affects and operates through asset prices is well supported empirically by Jensen et al. (1996), Thorbecke (1997), Jensen and Mercer (2002), Rigobon and Sack (2004), Ehrmann and Fratzscher (2004), Bernanke and Kuttner (2005), Bauer and Swanson (2020), among others. Moreover, Cieslak and Vissing-Jorgensen (2020) conduct a textual analysis of FOMC documents and find strong support for the idea that the Fed pays attention to stock prices and cuts interest rates after stock price declines (“the Fed put”). We build upon these insights and turn them into an asset pricing framework. If monetary policy operates through financial markets, then the Fed would ideally like aggregate asset prices (financial conditions) to be consistent with monetary policy objectives. These objectives depend on the nature of the shocks that hit the economy and on the macroeconomic frictions. In our model, the Fed provides a put (and a call) for the market’s belief (valuation) shocks, and “uses” the market to offset non-financial aggregate demand shocks.

The rest of the paper is organized as follows. Section 2 introduces our baseline model without transmission delays or inertia. This section illustrates the central idea that monetary policy objectives determine aggregate asset prices and derives our results about

“the Fed put/call.” Section 3 adds (non-financial) demand shocks and shows how they can lead to “excess” asset price volatility. Section 4 introduces policy transmission lags and shows that the Fed’s beliefs drive asset prices. This section also shows that macroeconomic news can increase asset price volatility. Section 5 introduces inertia and shows that it leads to asset price overshooting and a Wall/Main Street disconnect. Section 6 introduces disagreements between the market and the Fed and shows that disagreements can generate a policy risk premium, induce a “behind the curve” phenomenon, and provide a microfoundation for monetary policy shocks. Section 7 endogenizes inflation via a standard New Keynesian Phillips Curve (NKPC). Section 8 provides final remarks. The appendix contains the omitted derivations and proofs.

2. The baseline model

In this section, we describe a baseline version of our model. In subsequent sections, we enrich the model by adding frictions to the *macroeconomic* side of the economy, such as transmission lags and aggregate demand inertia. We keep the financial market side the same throughout.

2.1. Environment

The economy is set in discrete time $t \in \{0, 1, \dots\}$. There are four types of agents: “asset-holding households” (the households), “hand-to-mouth agents,” “portfolio managers (the market),” and “the central bank (the Fed).” Hand-to-mouth agents do not play an important role beyond decoupling the households’ consumption behavior from the labor supply. Households make consumption-saving decisions (possibly with frictions) that drive aggregate demand. The market makes a portfolio choice decision on behalf of the households and determines asset prices. The Fed sets monetary policy to close the output gap.

Supply side and nominal rigidities. The supply side features a competitive final goods sector and monopolistically competitive intermediate goods firms that produce according to

$$Y_t = \left(\int_0^1 Y_t(\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad \text{where } Y_t(\nu) = A_t L_t(\nu)^{1-\alpha}.$$

For now, the intermediate good firms have fully sticky nominal prices (we endogenize inflation in Section 7). Since these firms operate with a markup, they find it optimal to

meet the demand for their good (for relatively small demand shocks, which we assume). Therefore, output is determined by aggregate demand, which depends on the consumption of households, C_t^H , and hand-to-mouth agents, C_t^{HM} :

$$Y_t = C_t^H + C_t^{HM}. \quad (1)$$

Labor is supplied by the hand-to-mouth agents. They have the per-period utility function

$$\log C_t^{HM} - \chi \frac{L_t^{1+\varphi}}{1+\varphi},$$

which leads to a standard labor supply curve (see Appendix A.1).

With these production technologies, if the model was fully competitive, labor's share of output would be constant and given by $(1 - \alpha)Y_t$. However, since the intermediate good firms have monopoly power and make pure profits, labor's share is smaller than $(1 - \alpha)Y_t$. To simplify the exposition, we assume the government taxes part of the firms' profits (lump-sum) and redistributes to workers (lump-sum), so that labor's share is as in the fully competitive case (see Appendix A.1 for details). This implies the spending of hand-to-mouth agents (who supply all labor) is

$$C_t^{HM} = (1 - \alpha)Y_t. \quad (2)$$

Combining Eqs. (1) and (2) yields

$$Y_t = \frac{C_t^H}{\alpha}. \quad (3)$$

Hand-to-mouth agents create a Keynesian multiplier effect, but output is ultimately determined by (asset-holding) *households'* spending, C_t^H .

Potential output and aggregate supply shocks. Consider a flexible-price benchmark economy without nominal rigidities (the same setup except the intermediate good firms have fully flexible prices). In this benchmark, the equilibrium labor supply is constant and solves $\chi(L^*)^{1+\varphi} = \frac{\varepsilon-1}{\varepsilon}$ (see Appendix A.1). Output is given by $Y_t^* = A_t(L^*)^{1-\alpha}$. We refer to Y_t^* as *potential* output. Log potential output, $y_t^* = \log Y_t^*$, is driven by A_t and evolves according to

$$y_{t+1}^* = y_t^* + z_{t+1}, \quad \text{where } z_{t+1} \sim N(0, \sigma_z^2). \quad (4)$$

For simplicity, supply shocks are permanent and follow a log-normal distribution.

In our model with sticky prices, output is given by (3) and can deviate from its potential. We let $y_t = \log Y_t$ denote log output and $\tilde{y}_t = y_t - y_t^*$ denote the *output gap*.

Financial assets. There are two assets. There is a market portfolio, which is a claim on firms' profits αY_t (the firms' share of output). We let P_t denote the ex-dividend price of the market portfolio. Its gross return is given by

$$R_{t+1} = \frac{\alpha Y_{t+1} + P_{t+1}}{P_t}. \quad (5)$$

There is also a risk-free asset in zero net supply. Its gross return R_t^f is set by the Fed, as we describe subsequently.

Households' preferences and consumption-saving decisions. Households have standard preferences:

$$E_t \left[\sum_{h=0}^{\infty} \beta^{t+h} \log C_{t+h}^H \right], \quad (6)$$

along with the budget constraint

$$\begin{aligned} W_{t+1} + C_{t+1}^H &= W_t \left((1 - \omega_t) R_t^f + \omega_t R_{t+1} \right) \\ &= D_{t+1} + K_{t+1}, \end{aligned} \quad (7)$$

where $D_{t+1} = W_t \left[(1 - \omega_t) (R_t^f - 1) + \omega_t \frac{\alpha Y_{t+1}}{P_t} \right]$

and $K_{t+1} = W_t \left[1 - \omega_t + \omega_t \frac{P_{t+1}}{P_t} \right].$

W_t denotes the end-of-period wealth and ω_t denotes the market portfolio weight in period t . The term $W_t \left((1 - \omega_t) R_t^f + \omega_t R_{t+1} \right)$ is the beginning-of-period wealth in period $t + 1$. The second line breaks this term into a component that captures the interest and dividend income (D_{t+1}) and a residual component that captures the capital (K_{t+1}). This distinction will facilitate our exposition in subsequent sections.

Households make a consumption-savings decision. However, they do not *necessarily* make an optimal decision. Rather, we assume households follow consumption rules. In the baseline model, we assume households follow the optimal consumption rule with the preferences in (6), which is given by

$$C_t^H = (1 - \beta) (D_t + K_t). \quad (8)$$

Households spend a fraction of their beginning-of-period wealth. In subsequent sections, we consider empirically-grounded deviations from this rule and investigate the implications for asset prices.

The portfolio managers (the market) and the portfolio allocation. Households delegate their portfolio choice to portfolio managers (the market), who invest on their behalf. The market makes a portfolio allocation to maximize expected log household *wealth*,

$$\max_{\omega_t} E_t^M \left[\log \left(W_t \left(R_t^f + \omega_t \left(R_{t+1} - R_{t+1}^f \right) \right) \right) \right]. \quad (9)$$

We formulate the portfolio problem in terms of wealth, rather than consumption, to allow for consumption inertia. With inertia, consumption in a period might not provide an accurate representation of investors' welfare. In contrast, wealth is forward looking and captures the *ideal* consumption a household could choose if she was not inertial. We assume portfolio managers maximize log-wealth in line with the households' preferences in (6). In the baseline model (absent inertia), problem (9) results in portfolio allocations that maximize the households' utility. The superscript M captures the market's belief.

Problem (9) implies a standard optimality condition,

$$E_t^M \left[\left(R_{t+1} - R_t^f \right) \frac{1}{R_t^f + \omega_t \left(R_{t+1} - R_t^f \right)} \right] = 0. \quad (10)$$

Asset market clearing and equilibrium returns. Financial markets are in equilibrium when the households hold the market portfolio, both before and after the portfolio allocation:

$$W_t = P_t \quad \text{and} \quad \omega_t = 1. \quad (11)$$

Substituting $\omega_t = 1$ into the optimality condition (10), we obtain $E_t^M \left[\frac{R_t^f}{R_{t+1}} \right] = 1$. Assuming R_{t+1} is (approximately) log-normally distributed, this implies a *risk balance condition*,

$$E_t^M [r_{t+1}] + \frac{1}{2} \text{var}_t^M [r_{t+1}] - i_t = r p_t \equiv \text{var}_t^M [r_{t+1}]. \quad (12)$$

We use lower case letters to represent the log of the corresponding variable and $i_t = \log R_t^f$ to denote the log risk-free interest rate. In equilibrium, the expected excess return on the market portfolio is equal to the required risk premium, which is determined by the variance of the aggregate return.

The central bank (the Fed) and monetary policy. In each period, the Fed sets the risk-free rate R_t^f (without commitment) to minimize the discounted sum of quadratic log output gaps:

$$\max_{R_t^f} -\frac{1}{2} E_t^F \left[\sum_{h=0}^{\infty} \beta^h \tilde{y}_{t+h}^2 \right]. \quad (13)$$

The superscript F captures the Fed's belief. In the baseline model, the solution to problem (13) is simple: the Fed always sets the interest rate that closes the output gap,

$$Y_t = Y_t^*, \quad \text{which implies } \tilde{y}_t = y_t - y_t^* = 0. \quad (14)$$

When we change the consumption rule in (8), it will not be feasible to set the current output gap to zero. We will modify the optimality condition (14) accordingly.

2.2. Equilibrium

We next find the equilibrium in the baseline model and illustrate the main idea that applies throughout the paper: *monetary policy is the key determinant of the aggregate asset price*. In this section, optimal monetary policy implies a unique asset price that does not depend on beliefs. In subsequent sections, when we introduce transmission delays, optimal monetary policy *under the Fed's beliefs* drives the aggregate asset price.

Combining Eqs. (7) and (11), we obtain $D_t = \alpha Y_t$, $K_t = P_t$. In equilibrium, dividends are equal to the firms' share of output. Capital is equal to the (ex-dividend) value of the market portfolio. Substituting these observations into the consumption rule in (8), we obtain

$$C_t^H = (1 - \beta) (\alpha Y_t + P_t).$$

Substituting Eq. (3) ($C_t^H = \alpha Y_t$) into this expression yields *an output-asset price relation*

$$\begin{aligned} Y_t &= \frac{1}{\alpha\beta} (1 - \beta) P_t \\ \implies y_t &= m + p_t, \quad \text{where } m \equiv \log \left(\frac{1 - \beta}{\alpha\beta} \right). \end{aligned} \quad (15)$$

Output depends on aggregate wealth, P_t , the MPC out of wealth $(1 - \beta)$, and the Keynesian multiplier $1/(\alpha\beta)$. The second line describes the relation in logs. The constant m is the log of the MPC times the multiplier.

The output-asset price relation in (15) (and its variants) plays a key role in our analysis. In this section, monetary policy ensures that output is equal to its potential at all times,

$y_t = y_t^*$ (see (14)). Therefore, the output-asset price relation implies a close association between asset prices and *potential* output,

$$y_t^* = y_t = m + p_t.$$

We can “invert” this equation to solve for the (log) asset price

$$p_t = y_t^* - m. \tag{16}$$

In this baseline model without frictions, the Fed targets an asset price such that, given the MPC and the Keynesian multiplier (captured by m), households spend just enough to ensure that aggregate demand is equal to aggregate supply.

There is a remaining question of *how* the Fed achieves this asset price. This depends on the financial market side of the model. Specifically, we can combine (5) with (14) and (16) to calculate

$$R_{t+1} = \frac{1}{\beta} \frac{Y_{t+1}^*}{Y_t^*} \implies r_{t+1} = \rho + z_{t+1}. \tag{17}$$

Here, $\rho = \log \frac{1}{\beta}$ is a constant. The returns are log-normal and driven by permanent productivity shocks. Substituting this into (12), we calculate

$$i_t = \rho - \frac{1}{2} r p_t \quad \text{and} \quad r p_t = \sigma_z^2. \tag{18}$$

In this model, the risk premium is driven by the volatility of *supply* shocks. The level of the risk premium does not affect asset prices, but it affects the risk-free interest rate. The Fed optimally adjusts the risk-free interest rate to achieve the desired asset price in (16).

While the baseline model is simple, it sheds light on an important phenomenon: the central banks’ tendency to stabilize asset price fluctuations driven by beliefs or the risk premium.² To formalize this idea, consider the same setup with the addition of *market*

²The baseline model also provides a natural explanation for the secular increase in asset valuations in recent decades. In this model, a decline in the MPC ($1 - \beta$) *increases* the aggregate asset price given potential output. With a lower MPC, the central bank must induce a higher asset price to ensure that aggregate demand is aligned with aggregate supply. This observation, along with the trends in wealth inequality, provides one explanation for the rising asset valuations in recent decades. As Straub (2021) documents, wealthy households tend to have a lower MPC *out of wealth* compared to poorer households. Thus, rising wealth inequality—as we have seen in recent decades—reduces the households’ *average* MPC out of wealth. As the average MPC declines, the Fed is forced to increase asset prices to induce the same amount of spending. Other factors, such as aging demographics, might have further contributed to the decline in the MPC and exacerbated the rise in valuations.

belief shocks. Specifically, suppose the market thinks the supply shocks are drawn from

$$z_{t+1} \sim N(b_t, \sigma_z^2), \quad \text{where } b_t \sim N(0, \sigma_b^2). \quad (19)$$

Here, b_t denotes the expected belief for next period’s productivity, which itself is a random variable drawn from an i.i.d. distribution. The previous analysis remains unchanged: the asset price is still given by (16) and it *does not* reflect the belief shock. Using (17), we calculate $E_t^M[r_{t+1}] = \rho + b_t$ and $\text{var}_t^M[r_{t+1}] = \sigma_z^2$. This in turn implies

$$i_t = \rho + b_t - \frac{1}{2}\sigma_z^2 \quad \text{and} \quad rp_t = \sigma_z^2. \quad (20)$$

Result 1 (The Fed put/call). *In the baseline model with market belief shocks, the aggregate asset price does not depend on the market’s realized belief. Belief shocks (or risk premium shocks) affect the interest rate, but they do not increase aggregate asset price volatility or the risk premium.*

When the market becomes more pessimistic (low b_t), the Fed reduces the interest rate to keep the aggregate asset price unchanged—providing an explanation for the *Fed put* (see (20)). Intuitively, the Fed stabilizes financial market shocks to prevent them from damaging the real side of the economy. This result provides a sharp contrast with our analysis of (non-financial) aggregate demand shocks, which we turn to next.

Remark 1 (Output-asset price relation). *The output-asset price relation (15) can also be interpreted as a reduced form for various channels that link asset prices and aggregate demand. For example, in Caballero and Simsek (2020) we show that adding investment also leaves the output-asset price relation qualitatively unchanged (due to a Q-theory mechanism).*

3. Aggregate demand shocks and “excess” volatility

In the rest of the paper, we analyze the asset pricing implications of empirically-motivated deviations from the optimal consumption rule. We start by introducing (non-financial) demand shocks: households deviate from the optimal rule in a random fashion. These shocks induce fluctuations in the aggregate asset price that are *seemingly* unrelated to the real economy. While these fluctuations appear to be excessive, they play an important economic function, since they buffer the real economy from demand shocks that would otherwise induce business cycles. In addition to illustrating “excess volatility,” the analysis

in this section provides a stepping-stone into the policy transmission lags and aggregate demand inertia that we analyze in the rest of the paper.

Formally, suppose households follow

$$C_t^H = (1 - \beta) (D_t + K_t \exp(\delta_t)), \quad \text{where } \delta_t \sim N(0, \sigma_\delta^2). \quad (21)$$

Here, δ_t captures aggregate demand shocks: All else equal, higher δ_t means households spend more than predicted by the optimal rule. This is a simple modeling device to capture a variety of shocks that affect aggregate demand in practice, e.g., consumer sentiment shocks or fiscal policy shocks. The exact functional form does not play an important role beyond simplifying the expressions. We assume the demand shocks are transitory, although our analysis is flexible and can accommodate more persistent shocks.

Following the same steps as before, we obtain the output-asset price relation

$$Y_t = \frac{1 - \beta}{\alpha\beta} P_t \exp(\delta_t) \implies y_t = m + p_t + \delta_t, \quad (22)$$

where recall that $m = \log\left(\frac{1-\beta}{\alpha\beta}\right)$. There is still a one-to-one relation between output and asset prices. However, there is also a noise term driving aggregate demand and output that the Fed needs to address. Combining the output-asset price relation with the policy rule yields

$$y_t = y_t^* = m + p_t + \delta_t, \quad (23)$$

which implies

$$p_t = y_t^* - \delta_t - m. \quad (24)$$

As before, asset prices are proportional to supply shocks. However, they are also *inversely* proportional to demand shocks. All else equal, higher consumer demand implies that the economy needs *lower* asset prices to achieve its potential output.

Next, we describe how demand shocks affect the aggregate risk premium and the interest rate. Unlike in the previous section, the return on the market portfolio no longer follows a log-normal distribution. We introduce a log-linear approximation to returns that we use in the rest of the paper. Absent shocks, the dividend price ratio is constant and given by $\alpha Y_t^*/P_t^* = \frac{1-\beta}{\beta}$ (see (16)). In Appendix A.3, we log-linearize (5) around this

ratio to obtain

$$r_{t+1} = \kappa + (1 - \beta) y_{t+1} + \beta p_{t+1} - p_t, \quad (25)$$

where $\kappa = -\beta \log \beta - (1 - \beta) \log \left(\frac{1 - \beta}{\alpha} \right)$.

This is the Campbell-Shiller approximation applied to our model (see Campbell (2017)).

Combining Eqs. (23 – 25) (and simplifying the constants), we obtain

$$\begin{aligned} r_{t+1} &= \rho + (1 - \beta) y_{t+1}^* + \beta (y_{t+1}^* - \delta_{t+1}) - (y_t^* - \delta_t) \\ &= \rho + \delta_t + z_{t+1} - \beta \delta_{t+1}. \end{aligned} \quad (26)$$

Returns are affected by supply *and* demand shocks. A positive *future* demand shock δ_{t+1} reduces the realized return. Using (12), the interest rate and the risk premium are given by

$$i_t = \rho + \delta_t - \frac{1}{2} r p_t \quad \text{and} \quad r p_t = \sigma_z^2 + \beta^2 \sigma_\delta^2. \quad (27)$$

The Fed achieves its target asset price in (24) by adjusting the interest rate in response to the aggregate demand shock. The required risk premium is *greater* than in the baseline model, because aggregate demand shocks generate additional volatility in asset prices. The following result summarizes this discussion.

Result 2 (Demand shocks and policy-induced “excess” volatility). *When monetary policy is unconstrained, a positive demand shock reduces the aggregate asset price (and vice versa for a negative demand shock). Therefore, the Fed successfully mitigates the output effect of demand shocks, but in doing so it increases asset price volatility and the aggregate risk premium.*

This result provides a sharp contrast to “the Fed put” we analyzed in the previous section. When shocks create an imbalance in *the real economy*, as opposed to financial markets, the Fed *destabilizes* asset prices. Intuitively, the Fed *uses* asset prices to counter the demand shocks that would otherwise induce business cycles. To an outside observer, asset prices might appear to be “excessively” volatile, but this volatility plays a useful role. This is the first illustration of how demand shocks can create a seeming “disconnect” between the performance of financial markets and the real economy. We obtain a stronger disconnect result in subsequent sections, when we introduce policy lags and aggregate demand inertia.

4. Asset pricing with policy transmission lags

So far, we have assumed that monetary policy affects asset prices instantaneously and that asset prices affect aggregate demand instantaneously. These assumptions imply that monetary policy is very powerful: it can set output to its potential at all times and states. In practice, monetary policy has much less control over the output gap. An important constraint is that aggregate demand has *inertia* and responds to asset prices (financial conditions) with substantial lags (see Woodford (2005), Chapter 5). These lags imply that some output gaps are unavoidable. They also imply that the Fed must forecast future aggregate demand and supply. Thus, the Fed's beliefs drive monetary policy decisions *and the* aggregate asset price. We next introduce transmission lags and analyze *how* the Fed's beliefs affect asset prices and interest rates.

4.1. The Fed's beliefs and asset prices

Suppose households follow a modified version of the rule in (21):

$$C_t^H = (1 - \beta) (D_t + K_{t-1} \exp(\delta_t)), \quad (28)$$

where $\delta_t \sim N(0, \sigma_\delta^2)$ as before. That is, households respond to the *lagged* value of the capital portion of their wealth. To simplify the equations, we assume households respond to dividend and interest income immediately.

Following the same steps as before, we obtain the output-asset price relation

$$Y_t = \frac{1 - \beta}{\alpha\beta} P_{t-1} \exp(\delta_t) \implies y_t = m + p_{t-1} + \delta_t. \quad (29)$$

Asset prices affect output as before, but the effects operate with a lag.

An immediate implication of Eq. (29) is that output gaps can no longer be zero at all times and states. To see this, consider the equilibrium in period t . Since p_{t-1} is predetermined, output fluctuates with demand shocks δ_t . However, potential output still evolves according to (4) and fluctuates according to supply shocks z_t . Since δ_t and z_t are uncorrelated (by assumption), the output gap is non-zero except for a measure zero set of events. Because output responds to asset prices with a lag, *both supply and demand shocks* lead to output gaps, which the Fed cannot offset.

In this case, the Fed minimizes the same quadratic objective function (13) as before, but subject to the constraint (29). In every period, the Fed sets policy *without commitment*: it takes its future policy decisions as given. It is then easy to show that the optimal

policy implies

$$E_t^F [y_{t+1}] = E_t^F [y_{t+1}^*]. \quad (30)$$

The Fed sets *expected* demand equal to *expected* supply, *under its belief*.

Combining the policy rule in (30) with (29) and (4) yields

$$E_t^F [y_t^* + z_{t+1}] = E_t^F [y_{t+1}] = E_t^F [m + p_t + \delta_{t+1}].$$

Expected supply depends on the Fed's expectation for potential output. Expected demand depends on the current asset price and the Fed's expectation for the demand shock. As before, we invert this equation to solve for the equilibrium asset price

$$p_t = y_t^* - E_t^F [\tilde{\delta}_{t+1}] - m, \quad (31)$$

where $\tilde{\delta}_{t+1} \equiv \delta_{t+1} - z_{t+1}$ is *the net demand shock*. In contrast to the previous sections, the asset price now depends on the Fed's expectation about future macroeconomic conditions.

Result 3 (Fed's beliefs and asset prices). *In the model with transmission lags, asset prices are decreasing in the Fed's beliefs about future net aggregate demand. The Fed implements higher asset prices (looser financial conditions) when it expects lower future demand or higher future supply. Conversely, the Fed implements lower asset prices (tighter financial conditions) when it expects higher future demand or lower future supply.*

Substituting Eq. (31) back into (29), we solve for future output and its gap:

$$\begin{aligned} y_{t+1} &= y_t^* + \delta_{t+1} - E_t^F [\tilde{\delta}_{t+1}] \\ \tilde{y}_{t+1} &= \tilde{\delta}_{t+1} - E_t^F [\tilde{\delta}_{t+1}]. \end{aligned} \quad (32)$$

The first equation says that output is driven by *demand* shocks relative to the Fed's forecast of *net* demand, while supply shocks do not affect output contemporaneously. The second equation says that the output gap is driven by the unforecastable component of net demand shocks. If demand is realized to be higher than (or supply is realized to be lower than) what the Fed forecasted, then the output gap is positive.

Remark 2 (Quantifying transmission lags). *We capture transmission lags by assuming that spending responds to asset prices with a delay of one period. How should we think of the length of a period? We envision the period length as the planning horizon of the Fed: a period is sufficiently long that the Fed can expect its current decisions to*

have a meaningful impact on the real economic activity in the next period. For a rough “calibration,” consider Romer and Romer (2004), who analyze the effects of monetary policy shocks on economic activity. They find that the effects on output gradually build up over time and the maximum impact is obtained after about two years. Chodorow-Reich et al. (2021) find very similar lags for the stock market wealth effect. Based on these studies, we might think of a period in this section to be about two years. Alternatively, we can consider somewhat shorter periods (six months or one year) but introduce aggregate demand inertia, as we do in the next section, so that asset prices have a meaningful effect in each period that grows over time (see Remark 3 for further discussion of transmission lags).

4.2. Aggregate demand news and asset price volatility

Since the Fed’s beliefs affect asset prices, *changes in* the Fed’s beliefs can cause asset price volatility. We next investigate this mechanism in a benchmark setting in which the Fed and the market have common beliefs. We consider the implications of disagreements between the Fed and the market in Section 6. Throughout the paper, when agents have common beliefs, we drop the superscript on the expectations and the variance operators.

The Fed’s beliefs can be a source of volatility even when the Fed and the market share common beliefs, because beliefs shift in response to *news* about future macroeconomic imbalances. To capture news, suppose the agents receive an informative signal about future aggregate demand—news about future supply leads to similar results. Specifically, the agents receive a signal of the next period’s demand:

$$s_t = \delta_{t+1} + e_t, \quad \text{where } e_t \sim N(0, \sigma_e^2).$$

For now, the Fed and the market agree on the interpretation of the signal. Recall that demand shocks are drawn from the i.i.d. distribution, $N(0, \sigma_\delta^2)$. Therefore, after observing s_t , the Fed and the market have common posterior beliefs given by

$$\begin{aligned} \delta_{t+1} &\sim N(\gamma s_t, \sigma_\delta^2) & \text{where} & & (33) \\ \gamma &= \frac{1/\sigma_e^2}{1/\sigma_e^2 + 1/\sigma_\delta^2} & \text{and} & & \frac{1}{\sigma_\delta^2} = \frac{1}{\sigma_e^2} + \frac{1}{\sigma_\delta^2}. \end{aligned}$$

The posterior mean is a dampened version of the signal, and the posterior variance is smaller than the prior variance.

With this setup, agents’ common belief for the expected net demand in the next period

is $E_t [\tilde{\delta}_{t+1}] = E_t [\delta_{t+1}] = \gamma s_t$ (since $E_t [z_{t+1}] = 0$). Combining this with Eqs. (31 – 32) yields

$$\begin{aligned} p_t &= y_t^* - \gamma s_t - m \\ y_{t+1} &= y_t^* + \delta_{t+1} - \gamma s_t. \end{aligned} \tag{34}$$

Positive news about future demand *reduces* the aggregate asset price (negative news about supply would have a similar effect). Conversely, negative demand news *increases* the aggregate asset price.

Eq. (34) implies that demand news has a different effect on the conditional volatility of output and asset prices,

$$\text{var}_t (y_{t+1}) = \sigma_{\delta}^2 \quad \text{and} \quad \text{var}_t (p_{t+1}) = \sigma_z^2 + (\sigma_{\delta}^2 - \sigma_{\delta}^2). \tag{35}$$

Output volatility depends on the *unforecastable* demand variance, $\text{var}_t (\delta_{t+1} - \gamma s_t) = \sigma_{\delta}^2$, whereas asset price volatility depends on the *forecastable* demand variance, $\text{var}_t (\gamma s_{t+1}) = \sigma_{\delta}^2 - \sigma_e^2$ (as well as the supply variance σ_z^2). Note from (33) that a more precise signal (lower σ_e^2) reduces the unforecastable demand variance (lower σ_{δ}^2). Therefore, more precise signals about demand reduce macroeconomic volatility at the expense of increasing asset price volatility.

Next consider the return volatility and risk premium. Recall from (25) that the aggregate return is given by

$$r_{t+1} = \kappa + (1 - \beta) y_{t+1} + \beta p_{t+1} - p_t.$$

Combining this with Eq. (35), we calculate the risk premium

$$rp_t = \text{var}_t (r_{t+1}) = (1 - \beta)^2 \sigma_{\delta}^2 + \beta^2 (\sigma_z^2 + \sigma_{\delta}^2 - \sigma_{\delta}^2). \tag{36}$$

When $\beta > 1 - \beta$ (which holds for reasonable calibrations), more precise signals about demand also *increase* the return volatility and the risk premium. More precise signals reduce the volatility of cash flows, since they improve macroeconomic stability, but they increase the volatility of capital gains, since they make asset prices more volatile. Since asset prices matter more for conditional asset returns, the second force dominates.

Finally, using (12), (25), (34), we calculate the interest rate as

$$i_t = E_t [r_{t+1}] - \frac{1}{2} rp_t, \quad \text{where } E_t [r_{t+1}] = \rho + \gamma s_t. \tag{37}$$

After positive demand news, the Fed reduces the aggregate asset price by *increasing* the interest rate. The following result summarizes this discussion.

Result 4 (Aggregate demand news and volatility). *After positive aggregate demand news ($s_t > 0$), the Fed increases the policy interest rate (i_t) and decreases the aggregate asset price (p_t). More precise news (lower σ_e^2 and $\sigma_{\frac{2}{3}}^2$) reduces the conditional volatility of output but increases the conditional volatility of asset prices. When $\beta > 1 - \beta$, more precise news also increases the return volatility and the risk premium.*

With more precise signals, the Fed becomes more “activist” and preemptively responds to demand shocks. This makes output less volatile but asset prices more volatile. The model starts to resemble the case without transmission lags analyzed in Section 3.

5. Asset pricing with aggregate demand inertia

So far, we have focused on the lagged response of aggregate demand to financial conditions. In practice, aggregate demand has its own inertia. All else equal, strong current spending implies strong spending in the future (and vice versa for weak spending). Inertia naturally follows from realistic microeconomic frictions, such as various types of adjustment costs or habit formation (see Caballero and Simsek (2021b, 2022b) for further discussion). Quantitative New-Keynesian models typically assume this type of inertia, because it helps match the observed delayed response of aggregate demand to a variety of shocks. We next adjust the consumption rule to capture aggregate demand inertia and derive the implications for asset prices.

5.1. Asset price overshooting

Suppose households follow a modified version of the rule in (21),

$$C_t^H = (1 - \beta) D_t + \beta \left[\eta C_{t-1}^H + (1 - \eta) \frac{1 - \beta}{\beta} K_{t-1} \right] \exp(\delta_t). \quad (38)$$

For simplicity, we keep the response to dividend income unchanged. We change the remaining part of the consumption function so that households respond to a weighted-average of their past spending and lagged aggregate wealth. The parameter η captures the extent of inertia. We also multiply the coefficient on lagged spending by β , which ensures that the equation holds in a steady state. In Caballero and Simsek (2021b), we derive a version of this equation by assuming that in every period only a fraction $1 - \eta$

of agents adjust their spending. Here, we simply assume the equation as an aggregate “rule” and derive its implications for asset prices.

Following the same steps as before, we obtain the output-asset price relation

$$Y_t = \left(\eta Y_{t-1} + (1 - \eta) \frac{1 - \beta}{\alpha \beta} P_{t-1} \right) \exp(\delta_t).$$

In Appendix A.5, we approximate this relation (around the steady state for Y_t/P_t) to obtain

$$y_t = (1 - \eta) m + \eta y_{t-1} + (1 - \eta) p_{t-1} + \delta_t. \quad (39)$$

When $\eta = 0$, the relation is the same as (29): there are policy lags, but no aggregate demand inertia. When $\eta > 0$, there is also aggregate demand inertia. Note that aggregate demand inertia creates *endogenous persistence*: aggregate demand persists over time, even though aggregate demand shocks are transitory.

As before, we assume the Fed sets i_t to minimize the objective function in (13) subject to the output dynamics in (39) (without commitment). It is easy to check that this leads to the same optimality condition as before (see (30)),

$$E_t^F [y_{t+1}] = E_t^F [y_{t+1}^*].$$

The Fed sets the asset price to target a zero output gap in the next period. Combining this with Eqs. (4) and (39), we obtain

$$E_t^F [y_t^* + z_{t+1}] = E_t^F [y_{t+1}] = E_t^F [(1 - \eta) m + \eta y_t + (1 - \eta) p_t + \delta_{t+1}].$$

As before, we invert this equation to solve for the equilibrium asset price

$$p_t = y_t^* - \frac{\eta}{1 - \eta} \tilde{y}_t - \frac{E_t^F [\tilde{\delta}_{t+1}]}{1 - \eta} - m, \quad \text{where } \tilde{\delta}_{t+1} \equiv \delta_{t+1} - z_{t+1}. \quad (40)$$

Compared to Eq. (31), the asset price has two differences that we note in the following result.

Result 5 (Asset price overshooting and amplification). *Aggregate demand inertia ($\eta > 0$) leads to asset price overshooting: when the output gap is low (below potential), asset prices are high (above potential). In addition, greater inertia η induces the Fed to amplify its response to the current output gap and to its net demand forecast.*

Since current aggregate demand persists over time, the Fed responds to the *current*

output gap. When the output gap is low, the Fed targets a higher asset price to neutralize the future effects of current weakness. For the amplification part, note that inertia reduces the MPC out of wealth in a given period (controlling for the cumulative impact). The Fed “tunes up” the signal to compensate for inertia and induce a faster recovery.

Substituting Eq. (40) back into (39), we solve for future output and its gap:

$$y_{t+1} = y_t^* + \delta_{t+1} - E_t^F [\tilde{\delta}_{t+1}] \quad (41)$$

$$\tilde{y}_{t+1} = \tilde{\delta}_{t+1} - E_t^F [\tilde{\delta}_{t+1}]. \quad (42)$$

These expressions are the same as before [see (32)]. Since the Fed responds aggressively to neutralize the effects of current output gap, future output and its gap are driven by unforecastable shocks, as before.

5.2. Wall/Main street disconnect

Eqs. (40 – 41) imply that asset price overshooting also leads to a *Wall/Main Street disconnect*. To formalize this disconnect, consider the setup in Section 4.2 in which agents receive a signal about aggregate demand and agree on its interpretation. Specifically, agents’ common expectation for the demand shock is given by $E_t[\delta_{t+1}] = \gamma s_t$, and their expected supply shock is zero, $E_t[z_{t+1}] = 0$. Then, Eqs. (40 – 42) (for period t) imply

$$p_t = y_{t-1}^* + z_t - \frac{\eta}{1-\eta} (\delta_t - \gamma s_{t-1} - z_t) - \frac{\gamma s_t}{1-\eta} - m \quad (43)$$

$$y_t = y_{t-1}^* + \delta_t - \gamma s_{t-1} \quad (44)$$

$$\tilde{y}_t = \delta_t - \gamma s_{t-1} - z_t. \quad (45)$$

A negative demand shock ($\delta_t < \gamma s_{t-1}$) induces output and the aggregate asset price to move in *opposite* directions. Output falls below its past potential, $y_t < y_{t-1}^*$, whereas the aggregate asset price (on average) rises above its past potential, $p_t > y_{t-1}^*$. Thus, demand shocks generate a *disconnect* between the real economy and financial markets—a more extreme version of the “excess” volatility result we saw in Section 3. The following result formalizes this disconnect and completes the characterization of equilibrium.

Result 6 (Wall/Main Street disconnect). *With aggregate demand inertia ($\eta > 0$), demand shocks induce a negative conditional covariance between output and the asset price:*

$$\text{cov}_{t-1}(y_t, p_t) = - \left(\frac{\eta}{1-\eta} \right)^2 \sigma_{\tilde{\delta}}^2,$$

where $\sigma_{\delta}^2 = \text{var}_{t-1}(\delta_t - \gamma s_{t-1})$ is the unforecastable demand variance [see (33)]. The equilibrium return r_t , risk premium rp_t , and the interest rate i_t are given by Eqs. (A.19 – A.21) in the appendix.

Remark 3 (The role of transmission lags). *Results 5 and 6 echo our findings in Caballero and Simsek (2021b). There, we assumed aggregate demand inertia but no transmission lags. In that environment, if there is no cost to overshooting asset prices, then the Fed closes the output gaps immediately by increasing (and subsequently decreasing) asset prices by an infinite amount to compensate for inertia. However, once we introduce realistic costs to asset price overshooting, we recover the analogues of Results 5 and 6. Hence, transmission delays can also be viewed as capturing unmodeled costs to asset price overshooting. While the Fed might be able to shorten transmission lags by increasing asset price overshooting, there are natural limits to this alternative policy.*

6. Asset pricing with Fed-market disagreements

In practice, market participants are opinionated and have their own views of macroeconomic conditions and appropriate policy (see Caballero and Simsek (2022a)). We next derive the asset pricing implications belief disagreements between the market and the Fed. In this context, the Fed still implements the asset price that is appropriate under its *own* belief (therefore, our earlier results mostly still apply). However, disagreements affect the risk premium: the market perceives additional asset price volatility driven by policy “mistakes”. Disagreements also induce a “behind-the-curve” phenomenon and affect the policy interest rate the Fed needs to set to achieve its target asset price. Finally, disagreements provide a microfoundation for monetary policy shocks driven by the Fed’s belief surprises.

We introduce belief disagreements by modifying the signal environment from Section 4.2. As before, agents receive a public signal about aggregate demand. Unlike before, the Fed and the market disagree about the interpretation of this signal. After observing the public signal, each agent $j \in \{F, M\}$ forms an idiosyncratic interpretation, μ_t^j . Given this interpretation, the agent believes the public signal is drawn from

$$s_t =^j \delta_{t+1} - \mu_t^j + e_t, \quad \text{where } e_t \sim N(0, \sigma_e^2).$$

The noise term e_t is i.i.d. across periods and independent from other random variables. The notation $=^j$ captures that the equality holds under agent j ’s belief. Given their

interpretations, agents form posterior mean-beliefs:

$$E_t^F [\delta_{t+1}] = \gamma (s_t + \mu_t^F) \quad \text{and} \quad E_t^M [\delta_{t+1}] = \gamma (s_t + \mu_t^M), \quad (46)$$

where γ is the same as before (see (33)). Each agent thinks its interpretation is correct. Hence, when agents interpret the signal differently, they develop belief disagreements about the future aggregate demand shock. For now, we assume agents observe each others' interpretations (and beliefs).

We also assume that agents' interpretations follow a joint Normal distribution that is i.i.d. across periods (and both agents know this distribution)

$$\mu_t^F, \mu_t^M \sim N(0, \sigma_\mu^2) \quad \text{and} \quad \text{corr}(\mu_t^F, \mu_t^M) = 1 - \frac{D}{2} \text{ with } D \in [0, 2]. \quad (47)$$

The parameter D captures the scope of disagreement. When $D = 0$, interpretations are the same and there are no disagreements. Eq. (47) also implies:

$$E_t^j [\mu_{t+1}^F - \mu_{t+1}^M] = 0 \quad \text{and} \quad \text{var}_t^j [\mu_{t+1}^F - \mu_{t+1}^M] = D\sigma_\mu^2. \quad (48)$$

Agents think interpretation differences have mean zero and variance increasing with D .

A key implication of this setup is that each agent thinks the other agent's posterior belief is a "noisy" version of her own belief. To see this, consider the Fed's posterior belief

$$\gamma (s_{t+1} + \mu_{t+1}^F) = \gamma (s_{t+1} + \mu_{t+1}^M) + \gamma (\mu_{t+1}^F - \mu_{t+1}^M). \quad (49)$$

The market thinks its own belief, $\gamma (s_{t+1} + \mu_{t+1}^M)$, is correct. Therefore, the market thinks the Fed's belief is a noisier version of its own belief. Specifically, the market's perceived variance of the Fed's future belief is the sum of the forecastable demand variance ($\sigma_\delta^2 - \sigma_{\bar{\delta}}^2$) and a noise term that increases with the scope of disagreement,

$$\text{var}_t^M (\gamma (s_{t+1} + \mu_{t+1}^F)) = (\sigma_\delta^2 - \sigma_{\bar{\delta}}^2) + \gamma^2 D \sigma_\mu^2. \quad (50)$$

We next turn to the characterization of equilibrium. With disagreements, the equilibrium price, output, and output gap still satisfy (40 – 42) in Section 5.1. In particular, these outcomes are determined by *the Fed's* beliefs. The Fed's expected demand is given

by $E_t^F [\tilde{\delta}_{t+1}] = E_t^F [\delta_{t+1}] = \gamma (s_t + \mu_t^F)$. Combining these observations, we obtain

$$p_t = y_t^* - \frac{\eta}{1-\eta} \tilde{y}_t - \frac{\gamma (s_t + \mu_t^F)}{1-\eta} - m \quad (51)$$

$$y_t = y_{t-1}^* + \delta_t - \gamma (s_{t-1} + \mu_{t-1}^F) \quad (52)$$

$$\tilde{y}_t = \delta_t - \gamma (s_{t-1} + \mu_{t-1}^F) - z_t. \quad (53)$$

Eq. (A.22) in the appendix characterizes the equilibrium return r_{t+1} .

6.1. Policy risk premium

Eq. (51) illustrates that the Fed still shields the economy from forecasted demand shocks *under its belief*. However, the market has different beliefs and thinks the Fed should be targeting a different price. Therefore, the market thinks the Fed is making a policy “mistake.”

The anticipation of future “mistakes” increases the market’s perceived asset price volatility. To see this, note that the price in the next period depends on the Fed’s belief in the next period,

$$p_{t+1} = y_{t+1}^* - \frac{\eta}{1-\eta} \tilde{y}_{t+1} - \frac{\gamma (s_{t+1} + \mu_{t+1}^F)}{1-\eta} - m.$$

Combining this expression with (52 – 53), we obtain

$$\begin{aligned} \text{var}_t^M (p_{t+1}) &= \text{var}_t^{\text{com}} (p_{t+1}) + \gamma^2 D \sigma_\mu^2 \\ \text{where } \text{var}_t^{\text{com}} (p_{t+1}) &= \left(\frac{\eta}{1-\eta} \right)^2 \sigma_\delta^2 + \left(\frac{1}{1-\eta} \right)^2 (\sigma_z^2 + \sigma_\delta^2 - \sigma_\delta^2). \end{aligned}$$

Here, $\text{var}_t^{\text{com}} (p_{t+1})$ denotes the asset price volatility that would obtain if the beliefs were common. Compared to this benchmark, the market thinks asset prices will be more volatile. The market’s perceived price volatility is increasing in the scope of disagreement, D .

The market’s perceived asset price volatility also increases the risk premium (see Appendix A.4 for a derivation)

$$\begin{aligned} \text{var}_t^M [r_{t+1}] = rp_t &= rp_t^{\text{com}} + \beta^2 \gamma^2 D \sigma_\mu^2 \\ \text{where } rp_t^{\text{com}} &= \left(\frac{1-\eta-\beta}{1-\eta} \right)^2 \sigma_\delta^2 + \left(\frac{\beta}{1-\eta} \right)^2 (\sigma_z^2 + \sigma_\delta^2 - \sigma_\delta^2). \end{aligned} \quad (54)$$

The following result summarizes this discussion.

Result 7 (Disagreements and the policy-risk-premium). *When there is a greater scope of disagreement between the Fed and the market (higher D), the market thinks the Fed’s belief will be “noisier.” The market perceives a greater asset price volatility and demands a greater risk premium (higher rp_t).*

6.2. “Behind-the-curve”

So far, we illustrated how the market’s anticipation of *future* disagreements ($\mu_{t+1}^F - \mu_{t+1}^M$) induces a risk premium. We next describe the effect of *current* disagreements ($\mu_t^F - \mu_t^M$). We show that these disagreements induce a phenomenon that we call *behind-the-curve*. A market that disagrees with the Fed thinks the Fed will not be able to stabilize future output gaps. With inertia, the market also thinks the Fed will have to reverse course and make a large policy adjustment to address the future output gaps that its “mistake” will induce. These perceptions of “mistakes” and “behind-the-curve” also affect the policy interest rate.

First consider the market’s expectation for the future output gap, $\tilde{y}_{t+1} = y_{t+1} - y_{t+1}^*$. Eq. (53) shows that the future output gap depends on the future demand shock relative to relative to *the Fed’s* current posterior belief, $\delta_{t+1} - \gamma (s_t + \mu_t^F)$ (along with an unforecastable supply shock, z_{t+1}). Consequently, the market’s expected output gap is given by

$$\begin{aligned} E_t^M [\tilde{y}_{t+1}] &= E_t^M [\delta_{t+1} - \gamma (s_t + \mu_t^F)] \\ &= E_t^M [\delta_{t+1} - \gamma (s_t + \mu_t^M)] + \gamma (\mu_t^M - \mu_t^F) \\ &= \gamma (\mu_t^M - \mu_t^F). \end{aligned} \tag{55}$$

The second line uses (49) applied to period t , and the last line uses the fact that $E_t^M [\delta_{t+1} - \gamma (s_t + \mu_t^M)] = 0$ (the market thinks its belief is unbiased).

Eq. (55) says that the market thinks the Fed is making a “mistake” and will not be able to achieve its target output gap on average (recall that the Fed targets a zero output gap, $E_t^F [\tilde{y}_{t+1}] = 0$). A demand-optimistic market that expects greater aggregate demand than the Fed ($\mu_t^M > \mu_t^F$) thinks the Fed’s policy is “too loose” and will induce positive output gaps. Conversely, a demand-pessimistic market the Fed policy is “too tight” and will induce negative output gaps.

Next consider the market’s expectation for the future asset price, p_{t+1} . Using Eq. (51)

(for $t + 1$) and Eq. (55), we obtain

$$\begin{aligned} E_t^M [p_{t+1}] &= y_t^* - \frac{\eta E_t^M [\tilde{y}_{t+1}]}{1 - \eta} - m \\ &= y_t^* - \frac{\eta \gamma (\mu_t^M - \mu_t^F)}{1 - \eta} - m. \end{aligned} \quad (56)$$

This expression illustrates *behind-the-curve*: A demand-optimistic market ($\mu_t^M > \mu_t^F$) expects relatively *low* future asset prices, $E_t^M [p_{t+1}] < y_t^* - m$. Intuitively, the market thinks, once the positive output gap develops, the Fed will realize its “mistake” and will have to reverse course: inducing overshooting of asset prices in the downward direction. Conversely, a demand-pessimistic market expects relatively *high* asset prices: it thinks, once the negative output gap develops, the Fed will overshoot asset prices in the upward direction.

Eqs. (55 – 56) also show that “behind-the-curve” induces competing effects on the market’s expected return. On the one hand, a demand-optimistic market expects relatively high cash-flows (driven by the output boom). On the other hand, the market also expects relatively low asset prices. In Appendix A.5, we fully characterize the equilibrium and calculate the expected return as

$$E_t^M [r_{t+1}] = \rho + \frac{\eta \tilde{y}_t + \gamma (s_t + \mu_t^F)}{1 - \eta} + \left[(1 - \beta) - \frac{\beta \eta}{1 - \eta} \right] \gamma (\mu_t^M - \mu_t^F). \quad (57)$$

The first term in the square bracket captures the cash-flow effect of disagreements and the second term captures the asset-price effect through the “behind-the-curve” effect. When $\eta < 1 - \beta$ (inertia is not sufficiently large), the cash flow effect dominates and a more demand-optimistic market (greater μ_t^M) expects a higher return. When $\eta > 1 - \beta$ (inertia is sufficiently large), the asset-price effect dominates and a more demand-optimistic market expects a *lower* return.

In either case, disagreements affect the interest rate the Fed needs to set to achieve its policy target. To see this, note that Eqs. (12) and (57) imply

$$i_t = \rho + \frac{\eta \tilde{y}_t + \gamma s_t}{1 - \eta} + (\beta + \eta) \frac{\gamma \mu_t^F}{1 - \eta} + (1 - \beta - \eta) \frac{\gamma \mu_t^M}{1 - \eta} - \frac{rp_t}{2}, \quad (58)$$

where $rp_t = \text{var}_t^M (r_{t+1})$ is given by Eq. (54). The equilibrium interest rate depends on a *weighted average* of the Fed’s and the market’s beliefs. In particular, the Fed’s policy decision incorporates the market’s belief except for the knife edge case $\eta = 1 - \beta$. When inertia is relatively low $\eta < 1 - \beta$. The Fed *partially accommodates* the market’s belief even

though it does not agree with the market—a point that we emphasize in Caballero and Simsek (2022a). On the other hand, when inertia is relatively high, the Fed *overweights* its own belief relative to the case without disagreements (see (A.21)).

Why does the Fed react to the market’s belief? Intuitively, the market’s perception that the Fed is making a “mistake” and is “behind the curve” affects its asset valuation. When inertia is low, a demand-optimistic market expects a high return via the anticipation of high cash flows. This induces a demand-pessimistic Fed to set a relatively high interest rate that reflects the market’s view. When inertia is high, a demand-optimistic market expects a relatively low return via the anticipation of low asset prices. This induces a demand-optimistic Fed to cut the rate more aggressively to implement its view. In either case, by setting the appropriate interest rate, the Fed still achieves its desired asset price. The following result summarizes this discussion.

Result 8 (Behind-the-curve and the policy interest rate). *Current disagreements between the Fed and the market affect the market’s expected future output gap and asset price according to (55 – 56) and the policy interest rate according to (58). A demand-optimistic market ($\mu_t^M > \mu_t^F$) thinks the Fed is “behind-the-curve” and will induce a positive output gap, $E_t^M [\tilde{y}_{t+1}] > 0$; after which it will have to reverse course and overshoot asset prices in the downward direction, $E_t^M [p_{t+1}] < y_t^* - m$. When inertia is relatively low ($\eta < 1 - \beta$), a demand optimistic market also induces the (demand-pessimistic) Fed to set a higher interest rate that partially accommodates the market’s belief. Conversely, when inertia is relatively high ($\eta > 1 - \beta$), a demand optimistic market induces the (demand-pessimistic) Fed to set a lower interest rate that overweights the Fed’s own belief.*

6.3. Fed belief surprises and monetary policy shocks

In an environment with disagreements, the Fed’s belief can change without a corresponding change in the market’s belief. The later stages of the Covid-19 recovery illustrated how these types of *Fed belief surprises* can have a large effect on the aggregate asset price. We next analyze the price impact of the Fed’s belief surprises and show that these surprises provide a theory of *endogenous* monetary policy shocks.

To capture Fed belief surprises, suppose each period has two phases. Initially, the market does not know the Fed’s interpretation μ_t^F . Later in the period, the market learns μ_t^F (before portfolio and consumption decisions). Our goal is to understand how the revelation of the Fed’s interpretation to the market affects asset prices. For simplicity, suppose the Fed knows the market’s interpretation μ_t^M throughout.

Initially, the market does not know the Fed’s interpretation and needs to form an expectation about it. Using (47), the market thinks

$$\mu_t^F = \tilde{\beta}\mu_t^M + \tilde{\varepsilon}_t^F, \quad (59)$$

where $\tilde{\beta} = \text{corr}(\mu_t^F, \mu_t^M) = 1 - \frac{D}{2}$ and $\tilde{\varepsilon}_t^F$ has a zero mean. Given μ_t^M , the market expects the Fed’s interpretation to be $\tilde{E}_t^M[\mu_t^F] = \tilde{\beta}\mu_t^M$. Here, we use $\tilde{E}_t^M[\cdot]$ to denote the expectations operator before the revelation of the Fed’s actual belief μ_t^F . Therefore, the market also expects the aggregate asset price to be [see (51)]

$$\tilde{E}_t^M[p_t] = y_t^* - \frac{\eta}{1-\eta}\tilde{y}_t - \frac{\gamma(s_t + \tilde{\beta}\mu_t^M)}{1-\eta} - m.$$

Later in the period, the market learns μ_t^F and the aggregate price is realized to be

$$p_t = y_t^* - \frac{\eta}{1-\eta}\tilde{y}_t - \frac{\gamma(s_t + \mu_t^F)}{1-\eta} - m.$$

Combining these observations, we obtain

$$p_t - \tilde{E}_t^M[p_t] = -\frac{\gamma(\mu_t^F - \tilde{\beta}\mu_t^M)}{1-\eta} = -\frac{\gamma\tilde{\varepsilon}_t^F}{1-\eta}. \quad (60)$$

The *surprise change in the Fed’s belief* (driven by its residual interpretation given the market’s interpretation) affects asset prices. When the Fed is revealed to be more demand-optimistic than the market expected, asset prices decline. Conversely, when the Fed is revealed to be more demand-pessimistic than expected, asset prices increase.

Using (58), it is also easy to check that the revelation of the Fed’s belief affects the interest rate:

$$i_t - \tilde{E}_t^M[i_t] = \frac{\beta + \eta}{1-\eta}\gamma\tilde{\varepsilon}_t^F. \quad (61)$$

This surprise increase in the interest rate (partly) drives the valuation decline in (60).

Result 9 (Fed belief surprises and asset prices). *When the Fed is revealed to be more demand-optimistic than the market expected, $\mu_t^F > \tilde{E}_t^M[\mu_t^F] = \tilde{\beta}\mu_t^M$, the interest rate increases and the aggregate asset price declines.*

In practice, the Fed’s beliefs are usually revealed to the market during monetary policy announcements or speeches. In Caballero and Simsek (2022a), we use this observation to develop a theory of “monetary policy shocks.” When the Fed announces a higher interest

rate than the market expected, this decision reveals its belief surprise, as illustrated by (61). This in turn drives aggregate asset prices, as illustrated by (60).³

One caveat is that, in the data, monetary policy shocks seem to affect stock prices through the risk premium (see Bernanke and Kuttner (2005)). In our model, monetary policy shocks operate via the traditional interest rate channel. However, the logic of the model suggests that the impact on asset prices comes *before* the impact on the interest rate: When the market learns the Fed is more demand-optimistic, it also learns that it will achieve a lower asset price, one way or another. In the current model, the Fed achieves this outcome by adjusting the interest rate. However, one can imagine richer versions of this model (e.g., with heterogeneous interactions among market participants), in which the Fed has the same impact on asset prices but the channel shifts from the interest rate to the risk premium.

It is also worth noting that the model features a policy-risk-premium that is tied to beliefs (see (54)). If the policy announcement provides information about the scope of new disagreements (D), then it can also affect the policy risk premium. In other words, during policy events, the market may not only learn *what* the Fed thinks, but also *how* the Fed thinks—and how much it is likely to deviate from its own view in future periods.

Fed belief surprises and behind-the-curve. We next show that the *interaction* of the Fed belief surprises and the “behind-the-curve” phenomenon implies that monetary policy shocks (driven by the Fed’s beliefs) have the *opposite* effect on the short-term rate and the *market’s* expected *future* short-term rates.

To see this, first observe that agents disagree about the expected future short-term rate. Consider Eq. (58) for period $t + 1$, which describes the future short-term rate. Taking the expectation under the Fed’s belief in period t , we obtain

$$\begin{aligned} E_t^F [i_{t+1}] &= \rho + \frac{\eta E_t^F [\tilde{y}_{t+1}]}{1 - \eta} - \frac{rp_{t+1}}{2} \\ &= \rho - \frac{rp_{t+1}}{2}. \end{aligned}$$

The Fed expects future output gaps to be zero. Thus, the Fed expects the future interest rate to be centered around its long-run level, with an adjustment for the risk premium. Taking the expectation under the market’s belief in period t (after the revelation of μ_t^F),

³We also show that it is optimal for the Fed to set the rate in (58) and reveal its belief.

we instead obtain

$$\begin{aligned} E_t^M [i_{t+1}] &= \rho + \frac{\eta E_t^M [\tilde{y}_{t+1}]}{1 - \eta} - \frac{rp_{t+1}}{2} \\ &= \rho + \frac{\eta\gamma(\mu_t^M - \mu_t^F)}{1 - \eta} - \frac{rp_{t+1}}{2}. \end{aligned} \quad (62)$$

Since the market expects future output gaps to be non-zero, it also expects the future interest rate to react to these output gaps. In particular, a demand-optimistic market ($\mu_t^M > \mu_t^F$) expects the Fed to induce a positive output gap, which will then force the Fed to aggressively raise the interest rate.

Next note that a positive Fed belief surprise ($\tilde{\varepsilon}_t^F > 0$) *reduces* the market's expected future short-term rate. In particular, using (62) and (59), we obtain

$$E_t^M [i_{t+1}] - \tilde{E}_t^M [i_t] = -\frac{\eta}{1 - \eta} \gamma \tilde{\varepsilon}_t^F. \quad (63)$$

Recall also that a positive Fed belief surprise *increases* the current short-term rate (see (61)). This leads to the following result.

Result 10 (Fed belief surprises and behind-the-curve). *When the Fed is revealed to be more demand-optimistic than the market expected, $\mu_t^F > \tilde{E}_t^M [\mu_t^F] = \tilde{\beta}\mu_t^M$, the short-term rate i_t increases but the market's expected future short-term rate $E_t^M [i_{t+1}]$ decreases.*

When the Fed becomes more pessimistic about demand, it cuts the short-term interest rate to implement its view. However, the market thinks these cuts are “mistaken” and will induce positive output gaps, which will then induce the Fed to hike the *future* interest rates above their long-run levels.

This result implies that monetary policy shocks *can* have the *opposite* effect on short-term rates and forward interest rates. Consider the one-period-ahead forward rate, $F_{t,1}$: the rate that an investor can lock in at time t for a risk-free investment at time $t + 1$. Let $f_{t,1} = \log F_{t,1}$ denote the log forward rate. In this model, the log forward rate is equal to *the market's* expected log interest rate, $E_t^M [i_{t+1}]$, plus a term premium that remains constant over time. Hence, a corollary of Result 10 is that a positive Fed belief surprise raises the short-term rate i_t but reduces the forward interest rate $f_{t,1}$. In practice, unlike in our model, monetary policy shocks seem to affect the risk premium in forward rates (see Hanson and Stein (2015)). Therefore, Result 10 makes more robust predictions for the market's expected future short-term rates rather than the forward rates.

7. Asset pricing with inflation

In this section we extend our setup to allow for partially flexible prices and inflation. We focus on the textbook setup in which inflation is determined by a New-Keynesian Phillips Curve (NKPC). In this context, we show that demand shocks and (somewhat persistent) supply shocks induce a *negative* covariance between inflation and the real aggregate asset price.

Consider the model with transmission lags and inertia, with the difference that intermediate firms' nominal prices are not fully sticky. We adopt the standard Calvo setup: at each instant a randomly selected fraction of firms reset their nominal price, with a constant hazard. This price remains unchanged until the firm gets to adjust again. This leads to the standard NKPC (see Galí (2015) for a derivation):

$$\pi_t = \kappa \tilde{y}_t + \beta E_t^P [\pi_{t+1}]. \quad (64)$$

Here, π_t denotes the log-deviation of the nominal price level from its steady-state level. The parameter, κ , is a composite price flexibility parameter that depends on the rate of price adjustment along with other parameters. Here, the superscript P denotes the price-setters' beliefs.

We also adjust the Fed's problem to incorporate the costs of inflation gaps [cf. (13)]

$$\max_{R_t^f} -\frac{1}{2} E_t^F \left[\sum_{h=0}^{\infty} \beta^h (\tilde{y}_{t+h}^2 + \psi \pi_{t+h}^2) \right] \quad (65)$$

Here, ψ denotes the relative welfare weight for the inflation gaps. We normalize the inflation target to zero so the inflation gap is equal to inflation. As before, the Fed sets policy without commitment.

Finally, suppose all agents (the firm, the market, and the price setters) have common beliefs. In Caballero and Simsek (2022a), we show that disagreements between the Fed and the price setters affect the market's expected inflation and induce a policy trade-off similar to "cost-push" shocks. Here, we abstract from these effects to focus on other drivers of inflation.

With these assumptions, Appendix A.7 shows that there is an equilibrium in which the Fed's optimality condition implies [see Eqs. (A.25) and (A.26)]

$$E_t [\tilde{y}_{t+1}] = 0 \quad \text{and} \quad E_t [\pi_{t+1}] = 0.$$

In particular, the Fed still targets a zero *expected* output gap. By doing this, the Fed also achieves zero *expected* inflation. That is, in this setup, the “divine coincidence” applies in *expectation*: the Fed does not face a trade-off between stabilizing the output gap and inflation.

Since the Fed still targets a zero output gap on average, $E_t [\tilde{y}_{t+1}] = 0$, the equilibrium is the same as in the previous section. In particular, the output gap and the aggregate asset price are given by [see (40 – 42)],

$$\begin{aligned}\tilde{y}_t &= \tilde{\delta}_t - E_{t-1} [\tilde{\delta}_t] \\ p_t &= y_t^* - \frac{\eta}{1-\eta} \tilde{y}_t - \frac{E_t [\tilde{\delta}_{t+1}]}{1-\eta} - m.\end{aligned}$$

As before, output gaps are driven by net demand shocks, $\tilde{\delta}_t \equiv \delta_t - z_t$. Combining this observation with NKPC (64), inflation is given by

$$\pi_t = \kappa \tilde{y}_t = \kappa \left(\tilde{\delta}_t - E_{t-1} [\tilde{\delta}_t] \right).$$

Since expected inflation is zero, inflation tracks the current output gap. Therefore, inflation is also driven by net demand shocks, $\tilde{\delta}_t$.

We next characterize the covariance between stock prices and inflation. To this end, consider the signal environment from Section 4.2, in which the agents receive a signal about future demand and agree on its interpretation. In particular, agents’ expected demand shock is given by $E_t [\delta_{t+1}] = \gamma s_t$, and their expected supply shock is zero, $E_t [z_{t+1}] = 0$. Then, we obtain

$$\begin{aligned}\tilde{y}_t &= \delta_t - \gamma s_{t-1} - z_t \\ \pi_t &= \kappa (\delta_t - \gamma s_{t-1} - z_t) \\ p_t &= y_{t-1}^* + z_t - \frac{\eta}{1-\eta} (\delta_t - \gamma s_{t-1} - z_t) - \frac{\gamma s_t}{1-\eta} - m.\end{aligned}$$

These expressions illustrate that both (persistent) supply shocks and demand shocks induce a negative covariance between inflation and the real aggregate asset price. That is, *positive inflation surprises are bad news for asset prices*.

For intuition, first consider a negative supply shock, $z_t < 0$. This shock drives up inflation π_t . It also reduces the aggregate asset price p_t . Since the shock is persistent, the Fed targets a lower asset price to align future demand with the lower level of future

supply.⁴ Next consider a positive demand shock, $\delta_t > \gamma s_{t-1}$. This also drives up inflation π_t , while increasing current output and its gap, y_t and \tilde{y}_t . Since aggregate demand has inertia ($\eta > 0$) high current output persists into the future. Therefore, the Fed targets a lower asset price to reduce future output and align it with the future supply. The following result summarizes this discussion.

8. Final Remarks

Summary. In this paper, we developed a framework to analyze the impact of monetary policy on asset prices. The central idea is that when the Fed is unconstrained and acts optimally, monetary policy becomes the key driver of the *aggregate* asset price. The Fed adjusts its policy tools to keep aggregate asset prices where it would like them to be to close the output gap. We investigate the implications of this idea in a two-speed economy where macroeconomic agents (“households”) are slow and unsophisticated, and financial market agents (“the market”) are fast and sophisticated but endowed with their own set of beliefs. The Fed intermediates between these two sides to achieve macroeconomic balance. We analyzed several versions of the model that differ in the households’ spending behavior.

Our model highlights several forces that drive asset prices and interest rates. First, purely financial-market shocks (such as time-varying beliefs or risk premia) do not affect aggregate asset prices—they are absorbed by the interest rate. The Fed stabilizes the asset price impact of these shocks to prevent excessive macroeconomic fluctuations.

Second, non-financial aggregate demand shocks induce the *opposite* fluctuations in aggregate asset prices, creating the appearance of “excess” asset price volatility. The Fed *uses* aggregate asset prices to insulate the economy from demand shocks. With more precise macroeconomic signals, the Fed more aggressively preempts future aggregate demand shocks, which improves macroeconomic stability but increases asset price volatility. When aggregate demand has inertia, the Fed overshoots asset prices to neutralize the recessions or booms caused by demand shocks, which leads to a Wall/Main Street disconnect.

Third, *disagreements* between the Fed and the market also drive the aggregate risk premium and the interest rate. The market anticipates excessive policy-induced volatility and demands a policy risk premium, which is especially high at times of macroeconomic uncertainty and disagreements. Moreover, the market thinks the Fed is making a policy

⁴This result applies as long as the supply shock is somewhat persistent. If the z_t shock was transitory, it would drive up current inflation but it would not affect the asset price since the Fed would “overlook” the shock.

“mistake” and will eventually reverse course—a phenomenon that we refer to as “*behind-the-curve*.” The market’s perceptions of “mistakes” and “behind-the-curve” affect the policy interest rate the Fed needs to set to achieve its target asset price. In this environment, the revelation of the Fed’s beliefs to the market—the Fed belief surprises—drives aggregate asset prices and provides a microfoundation for monetary policy shocks. Also, since the market thinks the Fed is “behind-the-curve,” monetary policy shocks driven by the Fed’s beliefs have the *opposite* effect on the short-term interest rate and the market’s expectations for the future short-term interest rates.

Future work. A general theme of our paper is that the Fed targets *aggregate asset prices* (financial conditions), rather than the policy interest rate. The policy interest rate is simply the tool the Fed uses to achieve its target asset price. This observation has two implications. First, our model makes stronger predictions for aggregate asset prices than for the policy rate. Asset prices are driven by the Fed’s perception of macroeconomic imbalances. In contrast, the policy interest rate is driven by subtle details of the model, such as disagreements between the Fed and the market, the extent of aggregate demand inertia, and various forces that drive the risk premium.

Second, formulating policy rules in terms of aggregate asset prices (financial conditions), rather than in terms of the policy rate, could be helpful. Our model supports *Taylor-like rules in terms of aggregate asset prices*. For instance, Eq. (40) from Section 5 describes the aggregate asset valuations, $p_t - y_t^*$ (the ratio of asset price to potential output), as a function of the current output gap, \tilde{y}_t (and a second term that incorporates the Fed’s beliefs about future macroeconomic conditions). In richer extensions of our model, where different asset prices might have a different impact on aggregate demand, the policy would want to target a *financial conditions index (FCI)* that weights different asset valuations (or interest rates) according to *their impact on aggregate demand*. We leave the analysis of the optimal FCI for future work.

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A. Appendix: Omitted Derivations

This appendix presents the analytical derivations and proofs omitted from the main text.

A.1. Microfoundations for the supply side

In this section, we describe the details of the supply side that we describe in Section 2.1 and use throughout the paper.

The supply side is the same as in Caballero and Simsek (2021b), with the difference that here we allow for shocks to potential output. In particular, the real side of the economy features two types of agents: “asset-holding households” (the households) denoted by superscript $i = H$, and “hand-to-mouth agents” denoted by superscript $i = HM$. There is a single factor, labor.

Hand-to-mouth agents supply labor according to standard intra-period preferences. They do not hold financial assets and spend all of their income. We write the hand-to-mouth agents’ problem as,

$$\begin{aligned} \max_{L_t} \log C_t^{HM} - \chi \frac{L_t^{1+\varphi}}{1+\varphi} \\ Q_t C_t^{HM} = W_t L_t + T_t. \end{aligned} \tag{A.1}$$

Here, φ denotes the Frisch elasticity of labor supply, Q_t denotes the nominal price for the final good, W_t denotes the nominal wage, and T_t denotes lump-sum transfers to labor (described subsequently). Using the optimality condition for problem (A.1), we obtain a standard labor supply curve

$$\frac{W_t}{Q_t} = \chi L_t^\varphi C_t^{HM}. \tag{A.2}$$

Households own and spend out of the market portfolio and they supply no labor.

Production is otherwise similar to the standard New Keynesian model. There is a continuum of monopolistically competitive firms, denoted by $\nu \in [0, 1]$. These firms produce differentiated intermediate goods, $Y_t(\nu)$, subject to the Cobb-Douglas technology,

$$Y_t(\nu) = A_t L_t(\nu)^{1-\alpha}. \tag{A.3}$$

Here, $1 - \alpha$ denotes the share of labor in production and A_t the total factor productivity. We allow A_t to change over time to capture supply shocks [see (4)].

A competitive final goods producer combines the intermediate goods according to the

CES technology,

$$Y_t = \left(\int_0^1 Y_t(\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu \right)^{\varepsilon/(\varepsilon-1)}, \quad (\text{A.4})$$

for some $\varepsilon > 1$. This implies the price of the final consumption good is determined by the ideal price index,

$$Q_t = \left(\int_0^1 Q_t(\nu)^{1-\varepsilon} d\nu \right)^{1/(1-\varepsilon)}, \quad (\text{A.5})$$

and the demand for intermediate good firms satisfies,

$$Y_t(\nu) \leq \left(\frac{Q_t(\nu)}{Q_t} \right)^{-\varepsilon} Y_t. \quad (\text{A.6})$$

Here, $Q_t(\nu)$ denotes the nominal price set by the intermediate good firm ν .

The labor market clearing condition is

$$\int_0^1 L_t(\nu) d\nu = L_t. \quad (\text{A.7})$$

The goods market clearing condition is

$$Y_t = C_t^H + C_t^{HM}. \quad (\text{A.8})$$

Finally, to simplify the distribution of output across factors, we assume the government taxes part of the profits lump-sum and redistributes to workers to ensure they receive their production share of output. Specifically, each intermediate firm pays lump-sum taxes determined as follows:

$$T_t = (1 - \alpha) Q_t Y_t - W_t L_t. \quad (\text{A.9})$$

This ensures that in equilibrium hand-to-mouth agents receive and spend their production share of output, $(1 - \alpha) Q_t Y_t$, and consume [see (A.1)]

$$C_t^{HM} = (1 - \alpha) Y_t. \quad (\text{A.10})$$

Households receive the total profits from the intermediate good firms, which amount to the residual share of output, $\Pi_t \equiv \int_0^1 \Pi_t(\nu) d\nu = \alpha Q_t Y_t$.

Flexible-price benchmark and potential output. To characterize the equilibrium, it is useful to start with a benchmark setting without nominal rigidities. In this bench-

mark, an intermediate good firm ν solves the following problem,

$$\Pi = \max_{Q,L} QY - W_t L - T_t \quad (\text{A.11})$$

$$\text{where } Y = A_t L^{1-\alpha} = \left(\frac{Q}{Q_t}\right)^{-\varepsilon} Y_t.$$

The firm takes as given the aggregate price, wage, and output, Q_t, W_t, Y_t , and chooses its price, labor input, and output Q, L, Y .

The optimal price is given by

$$Q = \frac{\varepsilon}{\varepsilon - 1} W_t \frac{1}{(1 - \alpha) A_t L^{-\alpha}}. \quad (\text{A.12})$$

The firm sets an optimal markup over the marginal cost, where the marginal cost depends on the wage and (inversely) on the marginal product of labor.

In equilibrium, all firms choose the same prices and allocations, $Q_t = Q$ and $L_t = L$. Substituting this into (A.12), we obtain a labor demand equation,

$$\frac{W_t}{Q_t} = \frac{\varepsilon - 1}{\varepsilon} (1 - \alpha) A_t L_t^{-\alpha}. \quad (\text{A.13})$$

Combining this with the labor supply equation (A.2), and substituting the hand-to-mouth consumption (A.10), we obtain the equilibrium labor as the solution to,

$$\chi (L^*)^\varphi (1 - \alpha) Y_t^* = \frac{\varepsilon - 1}{\varepsilon} (1 - \alpha) A_t (L^*)^{-\alpha}.$$

In equilibrium, output is given by $Y_t^* = A_t (L^*)^{1-\alpha}$. Therefore, the equilibrium condition simplifies to,

$$\chi (L^*)^{1+\varphi} = \frac{\varepsilon - 1}{\varepsilon}.$$

We refer to L^* as the potential labor supply and $Y^* = A_t (L^*)^{1-\alpha}$ as the potential output.

Fully sticky prices. We next describe the equilibrium with nominal rigidities. For simplicity, we focus on the case with full price stickiness. In particular, intermediate good firms have a preset nominal price that remains fixed over time, $Q_t(\nu) = Q^*$. This implies the nominal price for the final good is also fixed and given by $Q_t = Q^*$ [see (A.5)]. Then,

each intermediate good firm ν at time t solves the following version of problem (A.11),

$$\begin{aligned} \Pi &= \max_L Q^*Y - W_tL - T_t \\ &\text{where } Y = AL^{1-\alpha} \leq Y_t. \end{aligned} \tag{A.14}$$

For small aggregate demand shocks (which we assume) each firm optimally chooses to meet the demand for its goods, $Y = AL^{1-\alpha} = Y_t$. Therefore, each firm's output is determined by aggregate demand, which is equal to spending by households and hand-to-mouth agents [see (A.8)],

$$Y_t = C_t^H + C_t^{HM}.$$

This establishes Eq. (1) in the main text.

Finally, recall that hand-to-mouth agents' spending is given by $C_t^{HM} = (1 - \alpha)Y_t$ [see Eq. (A.10)]. Combining this with $Y_t = C_t^H + C_t^{HM}$, the aggregate demand for goods is determined by the households' spending,

$$Y_t = \frac{C_t^H}{\alpha}.$$

This establishes Eq. (3) in the main text.

A.2. Omitted derivations in Section 2

Proof of Result 1. Presented in the main text. □

A.3. Omitted derivations in Section 3

Consider the model with demand shocks. The equilibrium is characterized in the main text. Here, we present the details of the Campbell-Shiller decomposition in (25) and complete the proof of Result 2.

To derive (25), first note that Eq. (5) implies

$$\begin{aligned} r_{t+1} &= \log \left(\frac{\alpha Y_{t+1}}{P_{t+1}} \frac{P_{t+1}}{P_t} + \frac{P_{t+1}}{P_t} \right) \\ &= \log \left(\frac{\alpha Y_{t+1}}{P_{t+1}} + 1 \right) + \log \left(\frac{P_{t+1}}{P_t} \right) \\ &= \log(1 + X_{t+1}) + p_{t+1} - p_t. \end{aligned} \tag{A.15}$$

Here, we have defined the dividend price ratio, $X_t = \alpha Y_t / P_t$. Absent shocks, this ratio is

constant and given by $X^* = \alpha Y_t^*/P_t^* = \frac{1-\beta}{\beta}$ (see (16)). Let $x_t = \log(X_t/X^*)$ denote the log deviation of this ratio from its steady-state level. Consider the term, $\log(1 + X_{t+1}) = \log(1 + X^* \exp(x_{t+1}))$. Using a Taylor approximation around $x_{t+1} = 0$, we obtain

$$\begin{aligned} \log(1 + X_{t+1}) &\approx \log(1 + X^*) + \frac{X^*}{1 + X^*} x_{t+1} \\ &\approx \log\left(\frac{1}{\beta}\right) + (1 - \beta) \left(\log\left(\frac{\alpha Y_{t+1}}{P_{t+1}}\right) - \log\left(\frac{1 - \beta}{\beta}\right) \right). \end{aligned}$$

Substituting this approximation into (A.15) and collecting the constant terms, we obtain (25) in the main text.

Proof of Result 2. The proof is mostly presented in Section 3. Here, we complete the characterization of equilibrium. Using Eq. (22) along with $y_t = y_t^*$, we obtain

$$p_t = y_t^* - \delta_t - m \text{ where } m = \log\left(\frac{1 - \beta}{\alpha\beta}\right).$$

This proves (24) in the main text. Substituting this along with $y_{t+1} = y_{t+1}^*$ into (25), we obtain

$$\begin{aligned} r_{t+1} &= \kappa + (1 - \beta) y_{t+1}^* + \beta p_{t+1} - p_t \\ &= \kappa + (\beta - 1) \log\left(\frac{\alpha\beta}{1 - \beta}\right) + (1 - \beta) y_{t+1}^* + \beta (y_{t+1}^* - \delta_{t+1}) - (y_t^* - \delta_t) \\ &= \rho + (1 - \beta) y_{t+1}^* + \beta (y_{t+1}^* - \delta_{t+1}) - (y_t^* - \delta_t) \\ &= \rho + \delta_t + z_{t+1} - \beta \delta_{t+1}. \end{aligned}$$

The third line substitutes κ from (25) and $\rho = -\log\beta$ to calculate the constant term. This last line substitutes $y_{t+1}^* = y_t^* + z_{t+1}$ to describe the return in terms of the shocks. This establishes (26) in the main text and completes the proof. \square

A.4. Omitted derivations in Section 4

Consider the model with transmission lags analyzed in Section 4. The equilibrium is mostly characterized in the main text. Here, we derive the exact versions of Eqs. (29) and (31) (including the constant terms). We also complete the proofs of Results 3 – 9.

We have the modified version of the consumption rule

$$C_t^H = (1 - \beta) (D_t + K_{t-1} \exp(\delta_t)).$$

Substituting $D_t = \alpha Y_t$ and $K_{t-1} = P_{t-1}$ and $C_t^H = \alpha Y_t$, we obtain

$$Y_t = \frac{1 - \beta}{\alpha \beta} P_{t-1} \exp(\delta_t).$$

Taking logs, we obtain (29),

$$y_t = m + p_{t-1} + \delta_t \text{ where } m = \log\left(\frac{1 - \beta}{\alpha \beta}\right).$$

Note that the Fed ensures (see (30))

$$E_t^F [y_{t+1}] = E_t^F [y_{t+1}^*].$$

Combining this with (29) and (4), we obtain

$$m + p_t + E_t^F [\delta_{t+1}] = y_t^* + E_t^F [z_{t+1}].$$

Solving for the asset price, we obtain (31),

$$p_t = y_t^* - E_t^F [\tilde{\delta}_{t+1}] - m \text{ where } \tilde{\delta}_{t+1} \equiv \delta_{t+1} - z_{t+1}.$$

This completes the characterization of equilibrium.

Proof of Result 3. Presented in the main text and completed above. □

Proof of Result 4. Most of the proof is presented in the main text. To calculate the volatility induced by news, note that γ_{s_t} and $\delta_{t+1} - \gamma_{s_t}$ capture the *forecastable* and the *unforecastable* components of aggregate demand shocks. These components are uncorrelated with one another and have variance given by

$$\text{var}_t(\delta_{t+1} - \gamma_{s_t}) = \sigma_{\tilde{\delta}}^2 \quad \text{and} \quad \text{var}_t(\gamma_{s_{t+1}}) = \sigma_{\delta}^2 - \sigma_{\tilde{\delta}}^2. \quad (\text{A.16})$$

Combining this expression with Eq. (34) establishes Eq. (35) in the main text.

To calculate the risk premium and the interest rate, note that Eq. (25) implies

$$\begin{aligned} r_{t+1} &= \kappa + (1 - \beta) y_{t+1} + \beta p_{t+1} - p_t \\ &= \rho + (1 - \beta) (y_t^* + \delta_{t+1} - \gamma_{s_t}) + \beta (y_{t+1}^* - \gamma_{s_{t+1}}) - (y_t^* - \gamma_{s_t}) \\ &= \rho + \gamma_{s_t} + (1 - \beta) (\delta_{t+1} - \gamma_{s_t}) + \beta (z_{t+1} - \gamma_{s_{t+1}}). \end{aligned} \quad (\text{A.17})$$

Here, the second line substitutes y_{t+1}, p_{t+1}, p_t using (34) and simplifies the constant terms (similar to the proof of Result 2). The last line substitutes $y_{t+1}^* = y_t^* + z_{t+1}$ and simplifies the expression. Combining this expression with (A.16), we obtain Eq. (36) in the main text. Combining the expression with (12), we also obtain (37), completing the proof. \square

A.5. Omitted derivations in Section 5

Consider the model with aggregate demand inertia analyzed in Section 5. The equilibrium is mostly characterized in the main text. Here, we derive the versions of (39) and (40) that include the constant terms. We also complete the proofs of Results 5 – 10.

We have the modified version of the consumption rule

$$C_t^H = (1 - \beta) D_t + \beta \left[\eta C_{t-1}^H + (1 - \eta) \frac{1 - \beta}{\beta} K_{t-1} \right] \exp(\delta_t).$$

Substituting $D_t = \alpha Y_t$ and $K_{t-1} = P_{t-1}$ and $C_t^H = \alpha Y_t$, we obtain

$$Y_t = \left(\eta Y_{t-1} + (1 - \eta) \frac{1 - \beta}{\alpha \beta} P_{t-1} \right) \exp(\delta_t).$$

Dividing by P_{t-1} and taking logs, we obtain

$$\begin{aligned} y_t &= \log \left(\eta \frac{Y_{t-1}}{P_{t-1}} + (1 - \eta) \frac{1 - \beta}{\alpha \beta} \right) + p_{t-1} + \delta_t \\ &= \log \left(\eta Z_{t-1} + (1 - \eta) Z^* \right) + p_{t-1} + \delta_t \\ &= \log \left(1 + \eta \left(\frac{Z_{t-1}}{Z^*} - 1 \right) \right) + \log Z^* + p_{t-1} + \delta_t. \end{aligned} \tag{A.18}$$

Here, the second line substitutes the output price ratio, $Z_t = Y_t/P_t$, and its steady-state level, $Z^* = Y_t^*/P_t^* = \frac{1-\beta}{\alpha\beta}$ (see (16)).

Next, let $z_{t-1} = \log(Z_{t-1}/Z^*)$ denote the log deviation of the output price ratio from its steady-state level. Note that

$$\log \left(1 + \eta \left(\frac{Z_{t-1}}{Z^*} - 1 \right) \right) = \log(1 + \eta(\exp(z_{t-1}) - 1)) \approx \eta z_{t-1}.$$

Here, the last line applies a Taylor approximation around $z_{t-1} = 0$. Substituting this into

(A.18), we obtain

$$\begin{aligned}
y_t &= \eta z_{t-1} + \log Z^* + p_{t-1} + \delta_t \\
&= (1 - \eta) \log Z^* + \eta \log Z_{t-1} + p_{t-1} + \delta_t \\
&= (1 - \eta) m + \eta (y_{t-1} - p_{t-1}) + p_{t-1} + \delta_t \\
&= (1 - \eta) m + \eta y_{t-1} + (1 - \eta) p_{t-1} + \delta_t.
\end{aligned}$$

Here, the second line substitutes $z_{t-1} = \log(Z_{t-1}/Z^*)$. The third line substitutes $Z_{t-1} = Y_{t-1}/P_{t-1}$ and $m = \log Z^* = \log\left(\frac{1-\beta}{\alpha\beta}\right)$ (see (15)). The last line establishes Eq. (39).

Next consider the equilibrium asset price. Recall that the Fed ensures (see (30))

$$E_t^F [y_{t+1}] = E_t^F [y_{t+1}^*].$$

Combining this with the exact version of (39) and (4), we obtain

$$(1 - \eta) m + \eta y_t + (1 - \eta) p_t + E_t^F [\delta_{t+1}] = y_t^* + E_t^F [z_{t+1}].$$

Solving for the asset price, we obtain

$$\begin{aligned}
p_t &= \frac{y_t^* - E_t^F [\tilde{\delta}_{t+1}] - \eta y_t}{1 - \eta} - m \\
&= y_t^* - \frac{\eta}{1 - \eta} \tilde{y}_t - \frac{E_t^F [\tilde{\delta}_{t+1}]}{1 - \eta} - m.
\end{aligned}$$

As before, we define $\tilde{\delta}_{t+1} \equiv \delta_{t+1} - z_{t+1}$ as the net demand shock. The second line substitutes $y_t = y_t^* + \tilde{y}_t$ and rearranges terms. This establishes (31) and completes the characterization of the equilibrium.

Proof of Result 5. Presented in the main text and completed above. \square

Proof of Result 6. Note that the unforecastable component of the demand shock, $\delta_t - \gamma s_{t-1}$, is uncorrelated with the supply shock, z_t . It is also uncorrelated with the signal for the next period's demand, s_t (since the demand shocks are i.i.d.). Combining these observations with (43) implies $cov_{t-1}(y_t, p_t) = -\left(\frac{\eta}{1-\eta}\right)^2 \sigma_\delta^2$ where $\sigma_\delta^2 = var_{t-1}(\delta_t - \gamma s_{t-1})$.

We next characterize the equilibrium return r_{t+1} , the risk premium rp_t , and the interest rate i_t . Recall from (25) that the aggregate return is given by

$$r_{t+1} = \kappa + (1 - \beta) y_{t+1} + \beta p_{t+1} - p_t.$$

Combining this with Eqs. (43) and (44), we obtain

$$\begin{aligned}
r_{t+1} &= \kappa + (1 - \beta)(y_t^* + \delta_{t+1} - \gamma s_t) \\
&\quad + \beta \left(y_{t+1}^* - \frac{\eta \tilde{y}_{t+1} + \gamma s_{t+1}}{1 - \eta} - m \right) - \left(y_t^* - \frac{\eta \tilde{y}_t + \gamma s_t}{1 - \eta} - m \right) \\
&= \rho + (1 - \beta)(y_t^* + \delta_{t+1} - \gamma s_t) \\
&\quad + \beta \left(y_t^* + z_{t+1} - \frac{\eta(\delta_{t+1} - \gamma s_t - z_{t+1}) + \gamma s_{t+1}}{1 - \eta} \right) \\
&\quad - \left(y_t^* - \frac{\eta(\delta_t - \gamma s_{t-1} - z_t) + \gamma s_t}{1 - \eta} \right) \\
&= \rho + \frac{\gamma s_t}{1 - \eta} + \frac{\eta}{1 - \eta}(\delta_t - \gamma s_{t-1} - z_t) \\
&\quad + \left((1 - \beta) - \beta \frac{\eta}{1 - \eta} \right) (\delta_{t+1} - \gamma s_t) + \frac{\beta}{1 - \eta} (z_{t+1} - \gamma s_{t+1}). \quad (\text{A.19})
\end{aligned}$$

Here, the second equality simplifies the constant terms and substitutes $\tilde{y}_{t+1} = \delta_{t+1} - \gamma s_t - z_{t+1}$ (see (45)) and $y_{t+1}^* = y_t^* + z_{t+1}$. The last equality collects similar terms together. The equilibrium return depends on the future demand shock relative to expectations, $\delta_{t+1} - \gamma s_t$, the future supply shock z_{t+1} , and the realization of the future demand signal, s_{t+1} .

Combining (A.19) with (A.16), we calculate

$$rp_t = \text{var}_t(r_{t+1}) = \left(\frac{1 - \eta - \beta}{1 - \eta} \right)^2 \sigma_\delta^2 + \left(\frac{\beta}{1 - \eta} \right)^2 (\sigma_z^2 + \sigma_\delta^2 - \sigma_\delta^2). \quad (\text{A.20})$$

Combining (A.19) with (12), we further obtain

$$\begin{aligned}
i_t &= E_t[r_{t+1}] - \frac{1}{2}rp_t \\
\text{where } E_t[r_{t+1}] &= \rho + \frac{\gamma s_t}{1 - \eta} + \frac{\eta}{1 - \eta}(\delta_t - \gamma s_{t-1} - z_t). \quad (\text{A.21})
\end{aligned}$$

This completes the proof. \square

A.6. Omitted derivations in Section 6

Eqs. (51 – 53) in the main text characterizes the asset price, the output, and the output gap. To facilitate the proofs in this section, we also characterize the return r_{t+1} . Using (25), the return is given by

$$r_{t+1} = \kappa + (1 - \beta)y_{t+1} + \beta p_{t+1} - p_t.$$

Substituting for the equilibrium output and the price from (51 – 53), we obtain

$$\begin{aligned}
r_{t+1} &= \kappa + (1 - \beta) (y_t^* + \delta_{t+1} - \gamma (s_t + \mu_t^F)) \\
&\quad + \beta \left(y_{t+1}^* - \frac{\eta \tilde{y}_{t+1} + \gamma (s_{t+1} + \mu_{t+1}^F)}{1 - \eta} - m \right) - \left(y_t^* - \frac{\eta \tilde{y}_t + \gamma (s_t + \mu_t^F)}{1 - \eta} - m \right) \\
&= \rho + \frac{\eta \tilde{y}_t + \gamma (s_t + \mu_t^F)}{1 - \eta} + (1 - \beta) (\delta_{t+1} - \gamma (s_t + \mu_t^F)) \\
&\quad + \beta \left(\frac{z_{t+1}}{1 - \eta} - \frac{\eta (\delta_{t+1} - \gamma (s_{t+1} + \mu_{t+1}^F))}{1 - \eta} - \frac{\gamma (s_{t+1} + \mu_{t+1}^F)}{1 - \eta} \right) \\
&= \rho + \frac{\eta \tilde{y}_t + \gamma (s_t + \mu_t^F)}{1 - \eta} \\
&\quad + \frac{1 - \eta - \beta}{1 - \eta} (\delta_{t+1} - \gamma (s_t + \mu_t^F)) + \frac{\beta}{1 - \eta} (z_{t+1} - \gamma (s_{t+1} + \mu_{t+1}^F)). \tag{A.22}
\end{aligned}$$

Here, the second equality simplifies the constant terms and substitutes $\tilde{y}_{t+1} = \delta_{t+1} - \gamma (s_t + \mu_t^F) - z_{t+1}$ and $y_{t+1}^* = y_t^* + z_{t+1}$. The last equality collects similar terms together. The realized return depends on the future demand shock relative to the Fed's expectations, $\delta_{t+1} - \gamma (s_t + \mu_t^F)$, the future supply shock z_{t+1} , and the realization of the Fed's future signal, $s_{t+1} + \mu_{t+1}^F$.

Proof of Result 7. Most of the proof is presented in the main text. It remains to characterize the risk premium. Using (A.22), we obtain

$$\begin{aligned}
rp_t &= \text{var}_t^M [r_{t+1}] \\
&= \text{var}_t^M \left[\frac{1 - \eta - \beta}{1 - \eta} \delta_{t+1} + \frac{\beta}{1 - \eta} (z_{t+1} - \gamma (s_{t+1} + \mu_{t+1}^F)) \right] \\
&= \left(\frac{1 - \eta - \beta}{1 - \eta} \right)^2 \sigma_\delta^2 + \left(\frac{\beta}{1 - \eta} \right)^2 [\sigma_z^2 + \sigma_\delta^2 - \sigma_\delta^2 + \gamma^2 D \sigma_\mu^2]. \tag{A.23}
\end{aligned}$$

Here, we have used (50) and the analogue of (A.16). Combining this with (36), we obtain Eq. (54) in the main text. \square

Proof of Result 8. It remains to characterize the equilibrium interest rate. Taking the expectation of (A.22) under the *market's* belief, we obtain

$$\begin{aligned}
E_t^M [r_{t+1}] &= \rho + \frac{\eta \tilde{y}_t + \gamma (s_t + \mu_t^F)}{1 - \eta} + \frac{1 - \eta - \beta}{1 - \eta} E_t^M [\delta_{t+1} - \gamma (s_t + \mu_t^F)] \\
&= \rho + \frac{\eta \tilde{y}_t + \gamma (s_t + \mu_t^F)}{1 - \eta} + \frac{1 - \eta - \beta}{1 - \eta} \gamma (\mu_t^M - \mu_t^F)
\end{aligned}$$

Here, the first line uses $E_t^M [z_{t+1}] = 0$ and $E_t^M [s_{t+1} + \mu_{t+1}^F] = 0$ (the market thinks the Fed's future signal will be unbiased on average). The second line substitutes $E_t^M [\delta_{t+1} - \gamma (s_t + \mu_t^F)] = \gamma (\mu_t^M - \mu_t^F)$, which follows from (55). Combining this expression with (12) and rearranging terms, we obtain (58),

$$i_t = \rho + \frac{\eta \tilde{y}_t + \gamma s_t}{1 - \eta} + (\beta + \eta) \frac{\gamma \mu_t^F}{1 - \eta} + (1 - \beta - \eta) \frac{\gamma \mu_t^M}{1 - \eta} - \frac{r p_t}{2}.$$

Here, $r p_t$ is given by (54). This completes the proof. \square

Proof of Result 9. Presented in the main text. \square

Proof of Result 10. Presented in the main text. \square

A.7. Omitted derivations in Section 7

Consider the model with inflation presented in Section 7. The analysis is mostly presented in the main text. Here, we show that, absent disagreements, there is an equilibrium in which the Fed targets a zero output gap on average as before $E_t [\tilde{y}_{t+1}] = 0$ (“divine coincidence” in expectations).

As before, the Fed effectively controls the aggregate asset price p_t . Therefore, we write the Fed's problem as:

$$\begin{aligned} \max_{p_t} & -\frac{1}{2} E_t^F \left[\sum_{h=0}^{\infty} \beta^h (\tilde{y}_{t+h}^2 + \psi \pi_{t+h}^2) \right] & (A.24) \\ y_t & = (1 - \eta) m + \eta y_{t-1} + (1 - \eta) p_{t-1} + \delta_t \\ \pi_t & = \kappa \tilde{y}_t + \beta E_t^M [\pi_{t+1}]. \end{aligned}$$

Here, the last two lines follow from Eqs. (39) and (64), respectively.

Suppose the agents have common beliefs, $E_t^F = E_t^M \equiv E_t$. Then, we conjecture an equilibrium in which the expected inflation is zero, $E_t [\pi_{t+1}] = 0$, and the output gap is given by (42) in Section 5,

$$\tilde{y}_{t+1} = \tilde{\delta}_{t+1} - E_t [\tilde{\delta}_{t+1}].$$

These conjectures also imply that inflation tracks the output gap,

$$\pi_{t+1} = \kappa \tilde{y}_{t+1} = \kappa \left(\tilde{\delta}_{t+1} - E_t [\tilde{\delta}_{t+1}] \right).$$

Using these conjectures, the Fed's objective function (A.24) becomes

$$-\frac{1}{2} (1 + \psi\kappa^2) \left[\tilde{y}_t + E_t [\tilde{y}_{t+1}^2] + E_t \left[\sum_{h=2}^{\infty} \beta^h \tilde{y}_{t+h}^2 \right] \right].$$

The current output gap \tilde{y}_t is predetermined and not influenced by the current Fed decision. The future output gaps $\{\tilde{y}_{t+2}, \tilde{y}_{t+3}..\}$ are driven by unforecastable future shocks and therefore they are also not influenced by the current Fed decision. Using these observations, the optimality condition for problem (A.24) implies

$$E_t [\tilde{y}_{t+1}] = 0. \tag{A.25}$$

That is, the Fed targets a zero output gap on average as before. Consequently, the equilibrium is the same as in Section 5, which verifies our conjecture that the output gap is given by (42).

We next verify our conjecture that the expected inflation is zero, $E_t [\pi_{t+1}] = 0$. First we take period t expectations of the NKPC Eq. (64) for period $t + 1$ to obtain

$$E_t [\pi_{t+1}] = \kappa E_t [\tilde{y}_{t+1}] + \beta E_t [\pi_{t+2}].$$

We then solve this equation forward (and assume inflation remains bounded in the limit) to obtain

$$E_t [\pi_{t+1}] = \kappa \sum_{h=1}^{\infty} \beta^h E_{t+h-1} [\tilde{y}_{t+h}] = \kappa E_t \left[\sum_{h=1}^{\infty} \beta^h E_{t+h-1} [\tilde{y}_{t+h}] \right] = 0. \tag{A.26}$$

Here, the second equality uses the law of iterated expectations and the last equality substitutes (A.25). This verifies $E_t [\pi_{t+1}] = 0$ and completes the characterization of equilibrium.