Online Appendix of
Informational Intermediation, Market Feedback, and Welfare Losses

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OA.1 Formal Definition of Histories and Strategies

For any period $t \geq 0$, the history of the seller at period t consists of all the prices charged by the seller until period $t-1$, all the disclosure policies adopted by the intermediary until period $t-1$, and all tie-breaking rules used by the tie-breaker until period $t-1$. Specifically, a history for the seller at period t is denoted by $\tilde{h}^t = \{p_s, D_s, q_s\}_{s=0}^{t-1}$, where $p_s \geq 0$ is the price charged by the seller in period s, $D_s \in \mathcal{D}$ is the disclosure policy adopted by the intermediary in period s, and $q_s \in [D_s(p_s^+), D_s(p_s)]$ is the tie-breaking rule adopted by the tie-breaker in period s. For any $t \geq 1$, let $\widetilde{\mathcal{H}}^t$ be the collection of sequences $\{p_s, D_s, q_s\}_{s=1}^{t-1}$ such that $p_s \geq 0$, $D_s \in \mathcal{D}$, and $q_s \in [D_s(p_s^+), D_s(p_s)]$ for all $s \in \{0, \ldots, t-1\}$. Also, let $\widetilde{\mathcal{H}}^0 := \emptyset$. Finally, let $\widetilde{\mathcal{H}} := \cup_{t=0}^{\infty} \widetilde{\mathcal{H}}^t$.

In the meantime, in any period $t \geq 0$, the history of the intermediary is the history of the seller and the price charged by the seller in period t. That is, a history for the intermediary in period $t \geq 0$ denoted by $h^t = (\tilde{h}^t, p_t)$, for some $\tilde{h}^t \in \widetilde{H}^t$ and some $p_t \geq 0$. Let $\mathcal{H}^t$ be defined as $\widetilde{\mathcal{H}}^t \times \mathbb{R}_+$ for all $t \geq 0$, and let $\mathcal{H} := \cup_{t=0}^{\infty} \mathcal{H}^t$.

Furthermore, in any period $t \geq 0$, the history of the tie-breaker is the history of the seller joint with the price charged by the seller and the disclosure policy adopted by the intermediary in period t. That is, a history for the tie-breaker in period $t \geq 0$ denoted by $\hat{h}^t = (\tilde{h}^t, p_t, D_t)$, for some $\tilde{h}^t \in \widetilde{\mathcal{H}}^t$, some $p_t \geq 0$, and some $D_t \in \mathcal{D}$. Let $\widehat{\mathcal{H}}^t$ be defined as $\widetilde{\mathcal{H}}^t \times \mathbb{R}_+ \times \mathcal{D}$ for all $t \geq 0$, and let $\widehat{\mathcal{H}} := \cup_{t=0}^{\infty} \widehat{\mathcal{H}}^t$.

With this definition, the seller's strategy can be formally defined as a measurable function from $\widetilde{H}$ to $\mathbb{R}_+$; the intermediary's strategy can be defined as a measurable function from $\mathcal{H}$ to $\mathcal{D}$; and the tie-breaker's strategy can be defined as a measurable function from $\widehat{H}$ to $[0,1]$ such that the value of this function must be in $[D_t(p_t^+), D_t(p_t)]$ for any $(\tilde{h}^t, p_t, D_t) \in \widehat{\mathcal{H}}^t$, for all t. Finally, for any histories $h^t, h^s \in \mathcal{H}$ for the intermediary, we say that h^t is a predecessor of h^s if $s > t$ and $h^s = (h^t, \{D_\tau, q_\tau, p_{\tau+1}\}_{\tau=t}^{s-1})$, and we say that $\sigma|_{h^t}$ is a continuation strategy of the intermediary if there exists a strategy σ of the intermediary such that $\sigma(h^s) = \sigma|_{h^t}(h^s)$ if h^t is a predecessor of h^s.

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OA.2 Equilibria with Infinite Continuation Values

The main text restricts attention to equilibrium outcomes in which the intermediary's continuation payoff at every histroy is finite. These equilibria are arguably more natural since the intermediary would be indifferent among many choices—especially off the equilibrium path—when there are history at which the continuation diverges. This in turn allows the intermediary to punish the seller's deviation severely and generates other equilibrium outcomes. For completeness, however, we characterize these equilibria as well. For the ease of exposition, we refer to equilibria in which the intermediary's continuation value at every history as being *finite*. Any equilibrium that is not finite is called *infinite*.

Recall that r^* is the revenue guarantee for the seller. Let

$$\widetilde{\beta} := \frac{1-\gamma\delta}{\delta(\mathbb{E}[v]-r^*)}.$$

Note that $\underline{\beta} < \widetilde{\beta} < \overline{\beta}$.

Proposition OA.1 (Inefficiency of High Feedback–Infinite Stationary). *For any $\beta < \widetilde{\beta}$, there exists a unique stationary equilibrium outcome $\mathbf{z}^{\mathrm{s}}(\beta)$ and any stationary equilibrium that induces outcome $\mathbf{z}^{\mathrm{s}}(\beta)$ must be finite. Furthermore, $\mathbf{z}^{\mathrm{s}}(\beta)$ dominates $\mathbf{z}^{\mathrm{s}}(\beta')$ for all β, β' such that $\underline{\beta} < \beta < \beta' < \widetilde{\beta}$.*

The welfare implication of Proposition OA.1 is essentially the same as that of Proposition 1. The only differences are (i) the upper bound below which a unique stationary equilibrium outcomes exists becomes lower when infinite stationary equilibria are allowed, and (ii) the range of market feedback where unintended welfare loss is caused by higher market feedback becomes smaller.

Similar to Proposition 1, we prove Proposition OA.1 by characterizing every infinite stationary equilibria. To state this characterization, for any $q \in [0,1]$, recall that

$$S(q) = \int_0^q \overline{D}^{-1}(z)\,\mathrm{d}z.$$

Note that $S(\overline{D}(p)) = p\overline{D}(p) + \int_p^\infty \overline{D}(v)\,\mathrm{d}v$ is the sum of consumer surplus and sales revenue when the price is p and when the demand is $\overline{D}$. With this additional notation, we can now state the characterization.

Theorem OA.1 (Stationary Equilibrium Outcomes). *For any $\beta \geq 0$, an infinite stationary equilibrium exists if and only if $\beta \geq \widetilde{\beta}$. Moreover, for any stationary equilibrium outcome $\mathbf{z}^{\mathrm{s}} = (r^{\mathrm{s}}, \sigma^{\mathrm{s}}, \omega^{\mathrm{s}}, p^{\mathrm{s}}, \{m_t^{\mathrm{s}}\})$, exactly one of the following is true.*

1. *$\mathbf{z}^{\mathrm{s}}$ is a finite stationary equilibrium outcome.*
2. *$\omega^{\mathrm{s}} = \infty$,*

$$\begin{aligned}\frac{1-\gamma\delta}{\delta\beta} \leq{}& \sigma^{\mathrm{s}} \leq \mathbb{E}[v] - r^* \\ \mathbb{E}[v] - \sigma^{\mathrm{s}} \leq{}& p^{\mathrm{s}} \leq \frac{r^{\mathrm{s}}}{S^{-1}(r^{\mathrm{s}}+\sigma^{\mathrm{s}})}, \\ \max\left\{S(\overline{D}(\zeta(\mathbb{E}[v]-\sigma^{\mathrm{s}}))) - \sigma^{\mathrm{s}}, r^*\right\} \leq{}& r^{\mathrm{s}} \leq \mathbb{E}[v] - \sigma^{\mathrm{s}}\end{aligned}$$

and $m_t^{\mathrm{s}} = (\gamma + \beta\sigma^{\mathrm{s}})^t$ for all $t \geq 1$.

Proof. We first show that any stationary equilibrium must be finite whenever $\beta < \widetilde{\beta}$. Suppose that $\mathbf{z}^{\mathrm{s}} = (r^{\mathrm{s}}, \sigma^{\mathrm{s}}, \omega^{\mathrm{s}}, p^{\mathrm{s}}, \{m_t^{\mathrm{s}}\})$ is a stationary equilibrium outcome, and suppose that, by way of contradiction, $\omega^{\mathrm{s}} = \infty$. Then it must be that

$$\delta(\gamma + \beta\sigma^{\mathrm{s}}) \geq 1,$$

which in turn implies that

$$\sigma^{\mathrm{s}} \geq \int_{p^\beta}^{\infty} \overline{D}(v)\,\mathrm{d}v.$$

Meanwhile, since the total surplus is at most $\mathbb{E}[v]$ and since the seller's revenue r^{s} must be at least r^* in any subgame perfect equilibrium, it must be that

$$\sigma^{\mathrm{s}} \leq \mathbb{E}[v] - r^*.$$

Together, we have

$$\int_{p^\beta}^{\infty} \overline{D}(v)\,\mathrm{d}v \leq \mathbb{E}[v] - r^*,$$

which is equivalent to $\beta \geq \tilde{\beta}$, a contradiction.

Now suppose that $\beta \geq \widetilde{\beta}$. We first show that any infinite stationary equilibrium outcome $\mathbf{z}^{\mathrm{s}} = (r^{\mathrm{s}}, \sigma^{\mathrm{s}}, \omega^{\mathrm{s}}, p^{\mathrm{s}}, \{m_t^{\mathrm{s}}\})$ must satisfy condition 2 of the theorem. Indeed, by definition, $\omega^{\mathrm{s}} = \infty$ and $m_t = (\gamma + \beta\sigma^{\mathrm{s}})^t$ for all t, which in turn implies that $\delta(\gamma + \beta\sigma^{\mathrm{s}}) \geq 1$. Rearranging, we have

$$\sigma^{\mathrm{s}} \geq \frac{1 - \gamma\delta}{\delta\beta}.$$

Meanwhile, since the seller's revenue in every period must be at least r^* in any subgame perfect equilibrium, $r^{\mathrm{s}} \geq r^*$. Therefore, since the efficient surplus is $\mathbb{E}[v]$, we have

$$\sigma^{\mathrm{s}} \leq \mathbb{E}[v] - r^{\mathrm{s}} \leq \mathbb{E}[v] - r^*.$$

Furthermore, notice that since $\sigma^{\mathrm{s}} = \int_{p^{\mathrm{s}}}^{\infty} D(v)\,\mathrm{d}v$ for some $D \in \mathcal{D}$, it must be that $\int_{p^{\mathrm{s}}}^{\infty} \overline{D}(v)\,\mathrm{d}v \geq \sigma^{\mathrm{s}}$. Given any σ and any p such that $\int_{p}^{\infty} \overline{D}(v)\,\mathrm{d}v \geq \sigma$, notice that the function

$$q \mapsto S(q) - pq$$

is quasi-concave and hence the equation

$$S(q) - pq = \sigma$$

has at most two solutions, denoted as $\underline{q}(p,\sigma) \leq \overline{q}(p,\sigma)$. It then follows that

$$\underline{q}(p,\sigma) \leq D(p) \leq \overline{q}(p,\sigma)$$

for all $D \in \mathcal{D}$ such that $\int_{p}^{\infty} D(v)\,\mathrm{d}v = \sigma$. Moreover, since $q \mapsto S(q) - pq$ is increasing, $\underline{q}(\cdot,\sigma)$ is decreasing in p. As a result, for any $D \in \mathcal{D}$ and for any $p \geq 0$ such that $\int_{p}^{\infty} D(v)\,\mathrm{d}v = \sigma$,

$$pD(p) \geq p\underline{q}(p,\sigma) \geq (\mathbb{E}[v] - \sigma)\underline{q}(\mathbb{E}[v] - \sigma, \sigma) = S(\underline{q}(\mathbb{E}[v] - \sigma, \sigma)) - \sigma.$$

Lastly, notice that by the definition of $\underline{q}(p,\sigma)$ and $\zeta(p)$, we have $\underline{q}(\mathbb{E}[v] - \sigma, \sigma) = \overline{D}(\zeta(\mathbb{E}[v] - \sigma))$. Together, it must be that

$$\max\left\{S(\overline{D}(\zeta(\mathbb{E}[v] - \sigma^{\mathrm{s}}))) - \sigma^{\mathrm{s}}, r^*\right\} \leq r^{\mathrm{s}} \leq \mathbb{E}[v] - \sigma^{\mathrm{s}}.$$

In the meantime, since $\int_{p^{\mathrm{s}}}^{\infty} D(v)\,\mathrm{d} = \sigma^{\mathrm{s}}$ and $p^{\mathrm{s}} D(p^{\mathrm{s}}) = r^{\mathrm{s}}$ for some $D \in \mathcal{D}$, it must be that $p^{\mathrm{s}} \geq \mathbb{E}[v] - \sigma^{\mathrm{s}}$ and that

$$S\left(\frac{r^{\mathrm{s}}}{p^{\mathrm{s}}}\right) - r^{\mathrm{s}} \geq \sigma^{\mathrm{s}},$$

which is equivalent to

$$\mathbb{E}[v] - \sigma^{\mathrm{s}} \leq p^{\mathrm{s}} \leq \frac{r^{\mathrm{s}}}{S^{-1}(r^{\mathrm{s}} + \sigma^{\mathrm{s}})},$$

as desired.

Conversely, suppose that $\mathbf{z}^{\mathrm{s}} = (r^{\mathrm{s}}, \sigma^{\mathrm{s}}, \omega^{\mathrm{s}}, p^{\mathrm{s}}, \{m_t^{\mathrm{s}}\})$ satisfies condition 2 of the theorem. We now construct a stationary equilibrium whose outcome is $\mathbf{z}^{\mathrm{s}}$. To this end, let the seller's strategy be p^{s} for all periods and for all histories. Consider a strategy for the intermediary as follows: For any history, if the seller charges $p \neq p^{\mathrm{s}}$ in the same period, choose a solution of

$$\min_{D \in \mathcal{D}} pD(p^{+}).$$

Otherwise, if the seller charges p^{s} choose $D \in \mathcal{D}$ such that

$$D(p^{\mathrm{s}}) = \frac{r^{\mathrm{s}}}{p^{\mathrm{s}}}$$

and that

$$\int_{p^{\mathrm{s}}}^{\infty} D(v)\,\mathrm{d}v = \sigma^{\mathrm{s}}.$$

Using the same arguments as above, since

$$\begin{aligned}
\frac{1-\gamma\delta}{\delta\beta} &\leq \sigma^{\mathrm{s}} \leq \mathbb{E}[v] - r^{*} \\
\mathbb{E}[v] - \sigma^{\mathrm{s}} &\leq p^{\mathrm{s}} \leq \frac{r^{\mathrm{s}}}{S^{-1}(r^{\mathrm{s}} + \sigma^{\mathrm{s}})}, \\
\max\left\{S(\overline{D}(\zeta(\mathbb{E}[v] - \sigma^{\mathrm{s}}))) - \sigma^{\mathrm{s}}, r^{*}\right\} &\leq r^{\mathrm{s}} \leq \mathbb{E}[v] - \sigma^{\mathrm{s}},
\end{aligned}$$

such $D \in \mathcal{D}$ exists. Finally, consider the following strategy for the tie-breaker: For any history, if the seller chooses $p \neq p^{\mathrm{s}}$ and the intermediary chooses any $D \in \mathcal{D}$ in the same period, then chooses $q = D(p^{+})$. If the seller chooses p^{s} and the intermediary chooses any $D \in \mathcal{D}$ in the same period, then choose $q = D(p)$.

By construction, the above strategy profile is stationary-Markov. Moreover, since $r^{\mathrm{s}} \geq r^{*}$ and since the seller can get at most r^{*} if he deviates given the intermediary's and the tie-breaker's strategies, the seller would never deviate. As for the intermediary, when the price is p^{s} and the market size is m, by construction, the market size in the next period is $(\gamma + \beta\sigma^{\mathrm{s}})m$. Since $\sigma^{\mathrm{s}} \geq (1-\gamma\delta)/\delta\beta$, the intermediary's payoff is $\omega^{\mathrm{s}} = \infty$ given the seller always charges p^{s} and given the tie-breaker's strategy. Meanwhile, if the seller charges $p \neq p^{\mathrm{s}}$, then given that the seller charges p^{s} in all future periods, choosing the solution of

$$\min_{D \in \mathcal{D}} pD(p^{+})$$

and then return to the aforementioned strategy in future periods still gives a present discounted profit of ∞. Together, the intermediary would not deviate given the seller's and the tie-breaker's strategies. Thus, this strategy profile is indeed a stationary equilibrium with outcome $\mathbf{z}^{\mathrm{s}}$.

Finally, since

$$\frac{1-\gamma\delta}{\delta\beta} \leq \mathbb{E}[v] - r^*$$

there exists $\mathbf{z}^{\mathrm{s}}$ that satisfies condition 2 of the theorem whenever $\beta \geq \widetilde{\beta}$. Together with the proofs above, it then follows that there exists an infinite stationary equilibrium whenever $\beta \geq \widetilde{\beta}$. This completes the proof. ∎

We now discuss infinite subgame perfect equilibria. From Theorem OA.1, it follows immediately that an infinite subgame perfect equilibrium exists whenever $\beta \geq \widetilde{\beta}$. In fact, the converse is also true: An infinite subgame perfect equilibrium exists only if $\beta \geq \widetilde{\beta}$. When an infinite subgame perfect equilibrium exists, the intermediary can always be incentivized to use the most severe punishment in the event of a deviation, and hence the seller can be incentivized to charge any price in $[r^*, p^*]$. As a result, any feasible payoff can be supported by a subgame perfect equilibrium, as summarized below. To this end, for any $\beta \geq 0$, let $\Omega(\beta)$ denote the set of intermediary's payoffs among all subgame perfect equilibria.

Theorem OA.2 (Subgame Perfect Equilibrium Payoffs). *An infinite subgame perfect equilibrium exists if and only if $\beta \geq \widetilde{\beta}$. Furthermore, for any $\beta \geq 0$,*

$$\Omega(\beta) = \begin{cases} \Omega^*(\beta), & \text{if } \beta < \widetilde{\beta} \\ [\underline{\omega}, \infty], & \text{if } \beta \geq \widetilde{\beta} \end{cases}.$$

Proof. By Theorem OA.1, since any stationary equilibrium is a subgame perfect equilibrium, an infinite subgame perfect equilibrium exists if $\beta \geq \widetilde{\beta}$. Conversely, suppose that $\beta < \widetilde{\beta}$. By way of contradiction, suppose that there exists a history at which the intermediary's payoff in some period $T \in \mathbb{N}$ is $\omega_T = \infty$. Then there must exist some $t > T$ such that

$$\delta(\gamma + \beta\sigma_t) \geq 1,$$

where σ_t is the consumer surplus in period t of this history. Rearranging, we have

$$\sigma_t \geq \int_{p^\beta}^{\infty} \overline{D}(v)\,\mathrm{d}v.$$

Meanwhile, since the total surplus in period t of this history must be at most $\mathbb{E}[v]$ and since the sales revenue r_t must be no less than r^* (otherwise the seller can has a profitable deviation in period t of this history). Thus, it must be that

$$\sigma_t \leq \mathbb{E}[v] - r^*.$$

Together, we have

$$\int_{p^\beta}^{\infty} \overline{D}(v)\,\mathrm{d}v \leq \mathbb{E}[v] - r^*,$$

which is equivalent to $\beta \geq \widetilde{\beta}$, a contradiction.

As a result, for any $\beta < \widetilde{\beta}$, $\Omega(\beta) = \Omega^*(\beta)$. Now suppose that $\beta \geq \widetilde{\beta}$. To show that $\Omega(\beta) = [\underline{\omega}, \infty]$, by Theorem OA.1 and Theorem 2, it suffices to show that for any $\beta \in [\widetilde{\beta}, \widehat{\beta})$ and for any $\hat{\omega} \in [\omega^*, \underline{\boldsymbol{\omega}}^*(\beta)]$, there exists a subgame perfect equilibrium in which the intermediary's payoff is $\hat{\omega}$. To this end, fix any such $\hat{\omega}$. Since $\hat{\omega} \leq \underline{\boldsymbol{\omega}}^*(\beta) \leq \mathbb{E}[v]$, there exists $\hat{p}$ such that $\hat{p}\overline{D}(v^{-1}(\hat{p}))$. As shown in the proof of Theorem 2 (see

Case 2), $\hat{\omega} \leq \underline{\boldsymbol{\omega}}^*(\beta) \leq \omega^\beta$ implies that $\xi(\hat{p}|\hat{\omega}) = v^{-1}(\hat{p})$. Moreover, for any $p' \geq 0$, let $D_{p'}$ be any solution of $\min_{D \in \mathcal{D}} p' D(p'^+)$.

Now consider the following strategy profile:

- Start by playing regime $\hat{p}$-MYOPIC. If the seller deviates to $p' \neq \hat{p}$, then enter regime $D_{p'}$-PUNISH immediately. Otherwise, stay in the same regime.

- Under regime $D_{p'}$-PUNISH. If the intermediary deviates, then move to regime $\hat{p}$-MYOPIC. Otherwise, play an infinite stationary equilibrium in the next period.

It remains to show that this strategy profile is a subgame perfect equilibrium. To see this, notice that except for the history at which an infinite stationary equilibrium is played, the intermediary's continuation value is finite. Therefore, by Lemma 1, it suffices to verify that the seller and the intermediary do not have incentives to deviate under each regime. Indeed, under regime $\hat{p}$-MYOPIC, the sales revenue after any deviation of the seller is at most r^*. Therefore, the seller does not have any incentive to deviate. Meanwhile, given price $\hat{p}$, since $\xi(\hat{p}|\hat{\omega}) = v^{-1}(\hat{p})$, the intermediary's best response is indeed myopic. Under regime $D_{p'}$-PUNISH. If the intermediary follows the strategy, then her payoff would be ∞. If she deviates, then her payoff would be at most $\mathbf{W}(p'|\omega^*) < \infty$, and hence the intermediary would not deviate. As a result, the strategy profile above constitutes a subgame perfect equilibrium, and the intermediary's payoff in this equilibrium is $\hat{\omega}$. This completes the proof. ■

In the meantime, for any $\beta \geq 0$, let $\mathbf{Z}(\beta)$ be defined as

$$\mathbf{Z}(\beta) := \begin{cases} \mathbf{Z}^*(\beta), & \text{if } \beta < \widetilde{\beta} \\ \left\{ (r, \sigma, p) \in \mathbb{R}^3_+ \,\middle|\, \begin{array}{c} r^* \leq r \leq p \\ (\mathbb{E}[v] - p)^+ \leq \sigma \leq S\left(\frac{r}{p}\right) - r \end{array} \right\}, & \text{if } \beta \geq \widetilde{\beta} \end{cases}.$$

Then the subgame perfect equilibrium outcomes can be characterized as well.

Corollary OA.1 (Subgame Perfect Equilibrium Outcomes). *For any $\beta \geq 0$ and for any subgame perfect equilibrium outcome $\mathbf{z} = \{r_t, \sigma_t, \omega_t, p_t, m_t\}$, $(r_t, \sigma_t, p_t) \in \mathbf{Z}(\beta)$ for all $t \geq 0$. Furthermore, for any $\beta \geq 0$, for any $T \geq 0$, and for any $(r, \sigma, p) \in \mathbf{Z}(\beta)$, there exists a subgame perfect equilibrium outcome $\mathbf{z} = \{r_t, \sigma_t, \omega_t, p_t, m_t\}$ such that $r_T = r$, $\sigma_T = \sigma$, and $p_T = p$.*

Proof. For any $\beta < \tilde{\beta}$, by Theorem OA.2, since every subgame perfect equilibrium is finite, Corollary 1 ensures any subgame perfect equilibrium outcome $\mathbf{z} = \{r_t, \sigma_t, \omega_t, p_t, m_t\}$ must be such that $(r_t, \sigma_t, p_t) \in \mathbf{Z}^*(\beta) = \mathbf{Z}(\beta)$. Moreover, for any $(r, \sigma, p) \in \mathbf{Z}(\beta) = \mathbf{Z}^*(\beta)$ and for any $T \geq 0$, Corollary 1 ensures that there exists a subgame perfect equilibrium outcome $\mathbf{z} = \{r_t, \sigma_t, \omega_t, p_t, m_t\}$ such that $r_T = r$, $\sigma_T = \sigma$, and $p_T = p$.

Now consider any $\beta \geq \widetilde{\beta}$. First consider any subgame perfect equilibrium outcome $\mathbf{z} = \{r_t, \sigma_t, \omega_t, p_t, m_t\}$. For any $t \geq 0$, clearly $r^* \leq r_t \leq p_t$. Furthermore, let D_t be the disclosure policy chosen by the intermediary on the equilibrium path in period t, it then follow that

$$\sigma_t = \int_{p_t}^{\infty} D_t(v)\, \mathrm{d}v.$$

Since $D_t \in \mathcal{D}$, $\sigma_t \geq (\mathbb{E}[v] - p_t)^+$. Moreover, since $D_t \in \mathcal{D}$ is nonincreasing, it must be that

$$\int_{p_t}^{\infty} \overline{D}(v)\,\mathrm{d}v \geq \int_{p_t}^{\infty} D_t(v)\,\mathrm{d}v \geq \sigma_t - (p - p_t)D_t(p_t),$$

for all $p \geq 0$. As a result,

$$\sigma_t \leq \min_{p \geq 0} \left[\int_p^{\infty} \overline{D}(v) + (p - p_t)D_t(p_t) \right] = S\left(\frac{r_t}{p_r}\right) - r_t,$$

where the equality follows from the first order condition of the minimization problem, which implies that at the solution $\hat{p}_t$, $\overline{D}(\hat{p}_t) = D_t(p_t) = r_t/p_t$. Together, we have

$$(\mathbb{E}[v] - p_t)^+ \leq \sigma_t \leq S\left(\frac{r_t}{p_r}\right) - r_t,$$

for all $t \geq 0$.

Conversely, consider any $(r, \sigma, p) \in \mathbf{Z}(\beta)$ and any $T \geq 0$. As in the proof of Corollary 1, it suffices to find a subgame perfect equilibrium with outcome $\mathbf{z} = \{r_t, \sigma_t, \omega_t, p_t, m_t\}$ such that $r_0 = r$, $\sigma_0 = \sigma$, and $p_0 = p$. As shown in the proof of Corollary 1, since $(\mathbb{E}[v] - p)^+ \leq \sigma \leq S(r/p) - r$, there exists $D_0 \in \mathcal{D}$ such that $pD_0(p) = r_0$ and $\int_p^{\infty} D_0(v)\,\mathrm{d}v = \sigma$.

Now consider the following strategy profile: In period 0, the seller charges price p; the intermediary chooses $D_0 \in \mathcal{D}$ if the seller charges p, and chooses any solution of $\min_{D \in \mathcal{D}} p' D(^{\prime +})$ if the seller charges $p' \neq p$; and the tie-breaker breaks tie in favor of the seller when the seller charges p, and against the seller when he charges $p' \neq p$. From period 1 onward, if the seller charges p in period 0 and if the intermediary chooses D_0, or if the seller charges any $p' \neq p$ and the intermediary chooses a solution of $\min_{D \in \mathcal{D}} p' D(^{\prime +})$, then all players play an infinite stationary equilibrium. Otherwise, the play the subgame perfect equilibrium that gives the intermediary equilibrium payoff ω^*.

We claim that this strategy profile constitutes a subgame perfect equilibtium. To see this, first notice that it suffices to verify that both the seller and the intermediary do not have any incentive to deviate in period 0. For the seller, given the intermediary's strategy, charging price p gives payoff $(1 - \alpha)r$, while the largest possible revenue from deviation is $(1-\alpha)r^* \leq (1-\alpha)r$. Thus, the seller does not have any incentive to deviate. For the intermediary, if she follows the strategy, then the continuation play is an infinite stationary equilibrium and hence her payoff would be ∞, whereas if she deviates after seeing any price $p' \geq 0$, her normalized continuation payoff would be $\omega^* < \infty$. Therefore, the intermediary does not have any incentive to deviate either. This completes the proof. ■

Together with Corollary OA.1, the implication of Proposition 2 can be extended even when allowing for infinite subgame perfect equilibria, as summarized below.

Proposition OA.2 (Inefficiency of High Feedback—Subgame Perfect)**.** *For any $\beta \in [0, \overline{\beta}]$, there exists a subgame perfect equilibrium outcome $\mathbf{z}^*(\beta)$ that is dominated by any other subgame perfect equilibrium outcomes. Furthermore, for any γ, δ such that $\gamma\delta \leq 1/2$, there exists $\widetilde{\beta}(\gamma, \delta) \in (0, \overline{\beta}]$ such that for any $0 < \beta < \beta' < \widetilde{\beta}(\gamma, \delta)$, $\mathbf{z}^*(\beta)$ dominates $\mathbf{z}^*(\beta')$.*

Lastly, we show that the parametric restrictions for Theorem 2 ($\beta < \beta^*$) imposed in the main text, when focusing on finite subgame perfect equilibrium, is tight.

Proposition OA.3. *A finite subgame perfect equilibrium exists if and only if* $\beta < \beta^*$

Proof. The proof of Theorem 2 implies that a finite subgame perfect equilibrium exists whenever $\beta < \beta^*$. It remains to show that finite subgame perfect equilibria do not exist whenever $\beta \geq \beta^*$. To see this, notice that $\beta \geq \beta^*$ implies $h(\omega) > \omega$ for all $\omega \geq 0$. Now let $\omega_0 := \omega^*$ and define $\{\omega_n\}$ recursively as

$$\omega_n := h(\omega_{n-1}) = \delta\left(\gamma + \beta\int_{\left(1-\frac{\alpha}{\delta\beta\omega_{n-1}}\right)p^*}^{\infty} \overline{D}(v)\,\mathrm{d}v\right)\omega_{n-1},$$

for all $n \in \mathbb{N}$. By the same arguments as in the proof of Lemma 5, the intermediary's continuation payoff at any history in any subgame perfect equilibrium must be at least ω_n for all n. However, since $h(\omega) > \omega$ for all $\omega \geq 0$, $\liminf_{n\to\infty}\{\omega_n\} = \infty$ and hence there is no subgame perfect equilibria . ■

OA.3 Omitted Proofs for Section 6

OA.3.1 Proof of Proposition 3

Consider the strategy profile where the seller charges $\overline{p}$, the intermediary chooses $\overline{D}$ after observing any prices, and the tie breaker breaks ties in favor of the seller regardless in every period. Clearly this strategy profile is stationary-Markov. Moreover, given the seller's strategy at any history, choosing $\overline{D}$ is always a best response for the intermediary since $\overline{D}$ maximizes the subscription fee and the market growth rate at the same time. Lastly, given that the intermediary always chooses $\overline{D}$, $\overline{p}$ is the unique best response for the seller at any history. Therefore, this strategy profile is indeed a stationary equilibrium.

Furthermore, when $\beta < \overline{\beta}$, we have

$$\delta\left(\gamma + \beta\int_{\overline{p}}^{\infty} \overline{D}(v)\,\mathrm{d}v\right) < 1$$

and hence the intermediary's payoff in the stationary equilibrium described above is

$$\rho^{\mathrm{s}} = \frac{\widetilde{\alpha}\int_{\overline{p}}^{\infty}\overline{D}(v)\,\mathrm{d}v}{1-\delta\left(\gamma+\beta\int_{\overline{p}}^{\infty}\overline{D}(v)\,\mathrm{d}v\right)} < \infty.$$

Therefore, the intermediary's continuation value is finite at every history.

To see that this stationary equilibrium induces a unique outcome whenever $\beta < \overline{\beta}$, consider any other stationary equilibrium in which the intermediary's continuation paypoff is finite at every history. Let $\mathbf{y} = (r, \sigma, \omega, p, \{m_t\})$ denote its outcome. Since $\omega < \infty$, by the same arguments as the proof of Lemma 1, it follows that the intermediary must choose $D \in \mathcal{D}$ that attains

$$\omega = \sup_{D\in\mathcal{D}}\left[\widetilde{\alpha}\int_p^{\infty} D(v)\,\mathrm{d}v + \delta\left(\gamma+\beta\int_p^{\infty} D(v)\,\mathrm{d}v\right)\omega\right].$$

As a result, it follows that $\omega = \rho^{\mathrm{s}}$, $\sigma = \int_{\overline{p}}^{\infty}\overline{D}(v)\,\mathrm{d}v$, $p = \overline{p}$, and $r = \overline{p}\overline{D}(\overline{p})$.

Lastly, for any β, β' such that $0 < \beta < \beta' < \overline{\beta}$, the intermediary's equilibrium payoffs is higher under β', and the market growth rate is also higher under β'. Thus, it must be that $\mathbf{y}^{\mathrm{s}}(\beta')$ dominates $\mathbf{y}^{\mathrm{s}}(\beta)$. This completes the proof. ■

OA.3.2 Proof of Proposition 4

Notice that by Proposition 1 and Proposition 2, ω^{s} is nonincreasing on $[0, \overline{\beta})$ and ρ^{s} is nondecreasing on $[0, \overline{\beta})$. Moreover, if

$$\frac{\widetilde{\alpha}}{\alpha} < \frac{\mathbb{E}[v]}{\int_{\overline{p}}^{\infty} \overline{D}(v)\,\mathrm{d}v},$$

then

$$\rho^{\mathrm{s}}(0) = \frac{\widetilde{\alpha}\int_{\overline{p}}^{\infty} \overline{D}(v)\,\mathrm{d}v}{1-\gamma\delta} < \frac{\alpha\mathbb{E}[v]}{1-\gamma\delta} = \omega^{\mathrm{s}}(0).$$

Thus, there exists $\beta^0 > 0$ such that $\rho^{\mathrm{s}}(\beta) < \omega^{\mathrm{s}}(\beta)$ for all $\beta < \beta^0$.

In the meantime, if

$$\frac{\widetilde{\alpha}}{\alpha} + 1 < \frac{\mathbb{E}[v]}{\int_{\overline{p}}^{\infty} \overline{D}(v)\,\mathrm{d}v},$$

then

$$\rho^{\mathrm{s}}(\underline{\beta}) = \frac{\widetilde{\alpha}\int_{\overline{p}}^{\infty} \overline{D}(v)\,\mathrm{d}v}{1-\delta\left(\gamma + \frac{1-\gamma\delta}{\delta\mathbb{E}[v]}\int_{\overline{p}}^{\infty} \overline{D}(v)\,\mathrm{d}v\right)} < \frac{\alpha\mathbb{E}[v]}{1-\gamma\delta} = \omega^{\mathrm{s}}(\underline{\beta}).$$

Hence, there exists $\beta^0 > \underline{\beta}$ such that $\rho^{\mathrm{s}}(\beta) < \omega^{\mathrm{s}}(\beta)$ for all $\beta < \beta^0$. This completes the proof. ■

OA.4 Omitted Proofs for Section 7

OA.4.1 Proof of Proposition 5

First, notice that by similar arguments, an analogous version of Lemma 1 can be established. As a result, $\tilde{\mathbf{z}}^{\mathrm{s}} = (r^{\mathrm{s}}, \sigma^{\mathrm{s}}, \omega^{\mathrm{s}}, \pi^{\mathrm{s}}, p^{\mathrm{s}}, \{m_t^{\mathrm{s}}\})$ is a (finite) stationary equilibrium outcome if and only if there exists $\mathbf{D}^{\mathrm{s}} : \mathbb{R}_+ \to \mathcal{D}$ such that[1]

$$\omega^{\mathrm{s}} = \sup_{D\in\mathcal{D}} \left[\alpha p^{\mathrm{s}} D(p^{\mathrm{s}}) + \delta\left(\gamma + \beta\int_{p^{\mathrm{s}}}^{\infty} D(v)\,\mathrm{d}v\right)\omega^{\mathrm{s}}\right]; \tag{OA.1}$$

that

$$\mathbf{D}(\cdot|p) \in \Delta(p|\omega^{\mathrm{s}}),\ \forall p \geq 0; \tag{OA.2}$$

and that

$$\begin{aligned}\pi^{\mathrm{s}} =& \sup_{p\geq 0}\left[(1-\alpha)p\mathbf{D}^{\mathrm{s}}(p|p) + \rho\left(\gamma + \beta\int_{p}^{\infty} \mathbf{D}^{\mathrm{s}}(v|p)\,\mathrm{d}v\right)\pi^{\mathrm{s}}\right]\\ =&(1-\alpha)p^{\mathrm{s}}\mathbf{D}(p^{\mathrm{s}}|p^{\mathrm{s}}) + \rho\left(\gamma + \beta\int_{p^{\mathrm{s}}}^{\infty} \mathbf{D}^{\mathrm{s}}(v|p^{\mathrm{s}})\,\mathrm{d}v\right)\pi^{\mathrm{s}}\end{aligned} \tag{OA.3}$$

By Lemma 3, it then follows that

$$\omega^{\mathrm{s}} = \alpha p^{\mathrm{s}}\overline{D}(\xi(p^{\mathrm{s}}|\omega^{\mathrm{s}})) + \delta\left(\gamma + \beta\left(\int_{\xi(p^{\mathrm{s}}|\omega^{\mathrm{s}})}^{\infty} \overline{D}(v)\,\mathrm{d}v - (p^{\mathrm{s}} - \xi(p^{\mathrm{s}}|\omega^{\mathrm{s}}))\overline{D}(\xi(p^{\mathrm{s}}|\omega^{\mathrm{s}}))\right)\right)\omega^{\mathrm{s}}$$

[1] π^{s} denotes the seller's equilibrium payoff

and that

$$\begin{aligned}\pi^{\mathrm{s}} &= \max_{p\geq 0} \Pi(p|\omega^{\mathrm{s}}, \pi^{\mathrm{s}}) \\ &:= \max_{p\geq 0}\left[(1-\alpha)p\overline{D}(\xi(p|\omega^{\mathrm{s}})) + \rho\left(\gamma + \beta\left(\int_{\xi(p|\omega^{\mathrm{s}})}^{\infty} \overline{D}(v)\,\mathrm{d}v - (p - \xi(p|\omega^{\mathrm{s}}))\overline{D}(\xi(p|\omega^{\mathrm{s}}))\right)\right)\pi^{\mathrm{s}}\right] \\ &= (1-\alpha)p^{\mathrm{s}}\overline{D}(\xi(p^{\mathrm{s}}|\omega^{\mathrm{s}})) + \rho\left(\gamma + \beta\left(\int_{\xi(p^{\mathrm{s}}|\omega^{\mathrm{s}})}^{\infty} \overline{D}(v)\,\mathrm{d}v - (p^{\mathrm{s}} - \xi(p^{\mathrm{s}}|\omega^{\mathrm{s}}))\overline{D}(\xi(p^{\mathrm{s}}|\omega^{\mathrm{s}}))\right)\right)\pi^{\mathrm{s}}\end{aligned}$$

We now claim that in any (finite) stationary equilibrium, it must be that

$$\xi(p^{\mathrm{s}}|\omega^{\mathrm{s}}) = \left(1 - \frac{\alpha}{\delta\beta\omega^{\mathrm{s}}}\right)^{+} p^{\mathrm{s}}.$$

To see this, suppose the contrary, that $v^{-1}(p^{\mathrm{s}}) > (1-\alpha/\delta\beta\omega^{\mathrm{s}})^{+}p^{\mathrm{s}}$. If $\omega^{\mathrm{s}} \leq \alpha/\delta\beta$, then $v^{-1}(p^{\mathrm{s}}) > 0$ and $\Pi(p|\omega^{\mathrm{s}}, \pi^{\mathrm{s}}) = (1-\alpha)p\overline{D}(v^{-1}(p))$ for all $p \geq \mathbb{E}[v]$. This then implies that $(1-\alpha)\mathbb{E}[v] = (1-\alpha)\mathbb{E}[v]\overline{D}(v^{-1}(\mathbb{E})) > (1-\alpha)p^{\mathrm{s}}\overline{D}(v^{-1}(p^{\mathrm{s}}))$, a contradiction. Meanwhile, if $\omega^{\mathrm{s}} > \alpha/\delta\beta$, then $p^{\mathrm{s}} > 0$ and hence there exists $\varepsilon > 0$ such that $v^{-1}(p) > (1-\alpha/\delta\beta\omega^{\mathrm{s}})p$ for all $p \in (p^{\mathrm{s}}-\varepsilon, p^{\mathrm{s}})$. This then implies that $\Pi(p|\omega^{\mathrm{s}}, \pi^{\mathrm{s}}) = (1-\alpha)p\overline{D}(v^{-1}(p)) = (1-\alpha)p^{\mathrm{s}}\overline{D}(v^{-1}(p^{\mathrm{s}})) = \Pi(p^{\mathrm{s}}|\omega^{\mathrm{s}}, \pi^{\mathrm{s}})$, a contradiction.

As a result, it must be that

$$\omega^{\mathrm{s}} = \delta\left(\gamma + \beta\int_{\left(1-\frac{\alpha}{\delta\beta\omega^{\mathrm{s}}}\right)^{+}}^{\infty} \overline{D}(v)\,\mathrm{d}v\right)\omega^{\mathrm{s}} \iff \left(1 - \frac{\alpha}{\delta\beta\omega^{\mathrm{s}}}\right)^{+} p^{\mathrm{s}} = p^{\beta}. \tag{OA.4}$$

Thus, if $\beta \leq \underline{\beta}$, then $(1-\alpha/\delta\beta\omega^{\mathrm{s}})^{+}p^{\mathrm{s}} = 0$. Since $p^{\mathrm{s}} > 0$ by optimality and by $\pi^{\mathrm{s}} < \infty$, it must be that $\omega^{\mathrm{s}} \leq \alpha/\delta\beta$. This then implies that $\Pi(p|\omega^{\mathrm{s}}, \pi^{\mathrm{s}}) = (1-\alpha)p\overline{D}(v^{-1}(p))$ for all $p \geq 0$, and hence $p^{\mathrm{s}} = \mathbb{E}[v]$. As a result, $\omega^{s} = \alpha\mathbb{E}[v]/(1-\gamma\delta)$ and $\pi^{\mathrm{s}} = (1-\alpha)\mathbb{E}[v]/(1-\gamma\delta)$.

Conversely, if $\omega^{\mathrm{s}} > \alpha/\delta\beta$, then it must be that $\beta > \underline{\beta}$ and that $(1-\alpha/\delta\beta\omega^{\mathrm{s}})p^{\mathrm{s}} = p^{\beta}$. In this case, we may write π^{s} as

$$\pi^{\mathrm{s}} = \frac{1-\alpha}{\alpha}\omega^{\mathrm{s}}\frac{1-\delta(\gamma+\beta\sigma^{\mathrm{s}})}{1-\rho(\gamma+\beta\sigma^{\mathrm{s}})},$$

where

$$\sigma^{\mathrm{s}} = \int_{p^{\beta}}^{\infty} \overline{D}(v)\,\mathrm{d}v - \frac{\alpha}{\delta\beta\omega^{\mathrm{s}} - \alpha}p^{\beta}\overline{D}(p^{\beta}).$$

Using this, we may write $\Pi(p|\omega^{\mathrm{s}}, \pi^{\mathrm{s}})$ as

$$\Pi(p|\omega^{\mathrm{s}}, \pi^{\mathrm{s}}) = \widetilde{\Pi}\left(\left(1 - \frac{\alpha}{\delta\beta\omega^{\mathrm{s}}}\right)p\middle|\omega^{\mathrm{s}}\right),$$

for all $p \geq 0$, where

$$\begin{aligned}\widetilde{\Pi}(\tilde{p}|\omega^{\mathrm{s}}) &:= (1-\alpha)\frac{\delta\beta\omega^{\mathrm{s}}}{\delta\beta\omega^{\mathrm{s}} - \alpha}\tilde{p}\overline{D}(\tilde{p}) + \rho\left(\gamma + \beta\left(\int_{\tilde{p}}^{\infty} \overline{D}(v)\,\mathrm{d}v - \frac{\alpha}{\delta\beta\omega^{\mathrm{s}} - \alpha}\tilde{p}\overline{D}(\tilde{p})\right)\right)\frac{1-\alpha}{\alpha}\omega^{\mathrm{s}}\frac{1-\delta(\gamma+\beta\sigma^{\mathrm{s}})}{1-\rho(\gamma+\beta\sigma^{\mathrm{s}})} \\ &= \omega^{\mathrm{s}}\frac{1-\alpha}{\alpha}\left[\frac{\alpha\beta}{\delta\beta\omega^{\mathrm{s}} - \alpha}\left(\delta - \rho\frac{1-\delta(\gamma+\beta\sigma^{\mathrm{s}})}{1-\rho(\gamma+\beta\sigma^{\mathrm{s}})}\right)\tilde{p}\overline{D}(\tilde{p}) + \rho\left(\gamma + \beta\int_{\tilde{p}}^{\infty} \overline{D}(v)\,\mathrm{d}v\right)\omega^{\mathrm{s}}\frac{1-\delta(\gamma+\beta\sigma^{\mathrm{s}})}{1-\rho(\gamma+\beta\sigma^{\mathrm{s}})}\right],\end{aligned}$$

for all $\tilde{p} \geq 0$. As a result, for any $\omega^{\mathrm{s}} \in [0, \infty)$, maximizing Π by choosing $p \geq 0$ is equivalent to maximizing $\widetilde{\Pi}$ by choosing $\tilde{p} \geq 0$, and hence, by (OA.4), $\widetilde{\Pi}(p^{\beta}|\omega^{\mathrm{s}}) \geq \widetilde{\Pi}(\tilde{p}|\omega^{\mathrm{s}})$ for all $\tilde{p} \geq 0$. Moreover, since $\xi(p^{\mathrm{s}}|\omega^{\mathrm{s}}) =$

$(1-\alpha/\delta\beta\omega^{\mathrm{s}})p^{\mathrm{s}}$, it must be that $(1-\alpha/\delta\beta\omega^{\mathrm{s}})p^{\mathrm{s}} \geq v^{-1}(p^{\mathrm{s}})$. This then implies that either $\widetilde{\Pi}'(p^{\beta}|\omega^{\mathrm{s}}) = 0$ and $g^{\beta}(p^{\beta}) \leq \omega^{\mathrm{s}}$ or $\widetilde{\Pi}'(p^{\beta}|\omega^{\mathrm{s}}) \geq 0$ and $g^{\beta}(p^{\beta}) = \omega^{\mathrm{s}}$.

Meanwhile, note that

$$\widetilde{\Pi}(p^{\beta}|\omega^{\mathrm{s}}) = \omega^{\mathrm{s}}\frac{1-\alpha}{\alpha}\left[\frac{\alpha\beta}{\delta\beta\omega^{\mathrm{s}}-\alpha}\left(\delta-\rho\frac{1-\delta(\gamma+\beta\sigma^{\mathrm{s}})}{1-\rho(\gamma+\beta\sigma^{\mathrm{s}})}\right)(\overline{D}(p^{\beta})+p^{\beta}\overline{D}'(p^{\beta}))-\rho\beta\overline{D}(p^{\beta})\frac{1-\delta(\gamma+\beta\sigma^{\mathrm{s}})}{1-\rho(\gamma+\beta\sigma^{\mathrm{s}})}\right].$$

As a result, since $p^{\beta} \to 0$ as $\beta \to \underline{\beta}$ and since the equilibrium price p^{s} must be bounded away from 0 whenever $\beta < \overline{\beta}$, (OA.4) implies that $\delta\beta\omega^{\mathrm{s}} - \alpha \to 0$ as $\beta \to \underline{\beta}$. Thus, since $p^{\beta} \leq \overline{p}$ whenever $\beta < \overline{\beta}$ and since $p \mapsto p\overline{D}(p)$ is quasi-concave, there exists $\boldsymbol{\beta}(\rho)$ such that $\widetilde{\Pi}(p^{\beta}|\omega^{\mathrm{s}}) \geq 0$ for all $\beta \in (\underline{\beta}, \boldsymbol{\beta}(\rho))$. Together with $p^{\overline{\beta}} = \overline{p}$ and $\overline{D}(\overline{p}) - \overline{p}\overline{D}'(\overline{p}) = 0$, it follows that $\boldsymbol{\beta}(\rho) \to \underline{\beta}$ as $\rho \uparrow \delta$. Consequently, there exists a continuously decreasing function $\boldsymbol{\beta}$, with $\boldsymbol{\beta}(0) = \overline{\beta}$ and $\lim_{\rho\uparrow\delta}\boldsymbol{\beta}(\rho) = \underline{\beta}$ such that for any $\rho \in [0, \delta)$ and for any $\beta \in (\underline{\beta}, \boldsymbol{\beta}(\rho))$, if $\tilde{\mathbf{z}}^{\mathrm{s}} = (r^{\mathrm{s}}, \sigma^{\mathrm{s}}, \omega^{\mathrm{s}}, \pi^{\mathrm{s}}, p^{\mathrm{s}}, \{m_t^{\mathrm{s}}\})$ is a (finite) stationary equilibrium outcome, then $g^{\beta}(p^{\beta}) = \omega^{\mathrm{s}}$, which in turn implies that $\sigma^{\mathrm{s}} = 0$, $p^{\mathrm{s}} = v(p^{\beta})$, $r^{\mathrm{s}} = (1-\gamma\delta)/\alpha g^{\beta}(p^{\beta})$, and $\pi^{\mathrm{s}} = (1-\alpha)r^{\mathrm{s}}/(1-\gamma\rho)$.

Conversely, for any ρ and for any $\beta \in (\underline{\beta}, \boldsymbol{\beta}(\rho))$, let r^{s}, ω^{s}, p^{s}, σ^{s}, and $\{m_t^{\mathrm{s}}\}$ be defined as in Theorem 1, and let $\pi^{\mathrm{s}} := (1-\alpha)r^{\mathrm{s}}/(1-\gamma\rho)$. Then there exists $\mathbf{D}^{\mathrm{s}} : \mathbb{R}_+ \to \mathcal{D}$ such that satisfies (OA.1), (OA.2), and (OA.3), which in turn implies that $(r^{\mathrm{s}}, \sigma^{\mathrm{s}}, \omega^{\mathrm{s}}, \pi^{\mathrm{s}}, \{m_t^{\mathrm{s}}\})$ is a finite stationary equilibrium outcome, where $m_t^{\mathrm{s}} = \gamma^t$ for all t. This completes the proof. ■

OA.4.2 Proof of Proposition 6

Similar to the baseline model, the one-shot deviation principle holds in this setting, and an analog of Lemma 2 can be established as in Lemma OA.1.

Lemma OA.1. *A finite stationary equilibrium is characterized by a tuple $(\omega^{\mathrm{s}}, p^{\mathrm{s}}, \mathbf{D}^{\mathrm{s}})$ with $\omega^{\mathrm{s}}, p^{\mathrm{s}} \in [0, \infty)$ and $\mathbf{D}^{\mathrm{s}} : \mathbb{R}_+ \to \mathcal{D}$ that satisfy the following conditions:*

$$\omega^{\mathrm{s}} = \sup_{D\in\mathcal{D}}\left[\alpha p^{\mathrm{s}} D(p^{\mathrm{s}}) + \delta\omega^{\mathrm{s}} \cdot f\left(\int_{p^{\mathrm{s}}}^{\infty} D(v)\,\mathrm{d}v\right)\right], \tag{OA.5}$$

$$p^{\mathrm{s}}\mathbf{D}^{\mathrm{s}}(p^{\mathrm{s}}|p^{\mathrm{s}}) \geq p\mathbf{D}^{\mathrm{s}}(p|p), \tag{OA.6}$$

for all $p \geq 0$,

$$\alpha p\mathbf{D}^{\mathrm{s}}(p|p) + \delta\omega^{\mathrm{s}} \cdot f\left(\int_p^{\infty} \mathbf{D}^{\mathrm{s}}(v|p)\,\mathrm{d}v\right) \geq \alpha p D(p) + \delta\omega^{\mathrm{s}} \cdot f\left(\int_p^{\infty} D(v)\,\mathrm{d}v\right), \tag{OA.7}$$

for all $p \geq 0$ and for all $D \in \mathcal{D}$. Furthermore, for any stationary equilibrium $(\omega^{\mathrm{s}}, p^{\mathrm{s}}, \mathbf{D}^{\mathrm{s}})$, its outcome is constant over time and is given by $(r^{\mathrm{s}}, p^{\mathrm{s}}, \sigma^{\mathrm{s}}, \omega^{\mathrm{s}})$, where $r^{\mathrm{s}} := p^{\mathrm{s}}\mathbf{D}^{\mathrm{s}}(p^{\mathrm{s}}|p^{\mathrm{s}})$, $\sigma^{\mathrm{s}} := \int_{p^{\mathrm{s}}}^{\infty} \mathbf{D}^{\mathrm{s}}(v|p^{\mathrm{s}})\,\mathrm{d}v$.

Let $\tilde{\xi}(p|\omega)$ be the solution of

$$\alpha p = \delta\omega(p-\xi)f'\left(\int_{\xi}^{\infty}\overline{D}(v)\,\mathrm{d}v - (p-\xi)\overline{D}(\xi)\right),$$

if the solution exists, and let $\tilde{\xi}(p|\omega) = 0$ otherwise.

Lemma OA.2. *For any $\alpha, \delta \in (0,1), \omega \geq 0$, any twice differentiable, increasing and concave function $f : \mathbb{R}_+ \to \mathbb{R}_+$, and for any $p \geq 0$,*

$$\Delta^*(p|\omega) := \operatorname*{argmax}_{D \in \mathcal{D}} \left[\alpha p D(p) + \delta\omega \cdot f\left(\int_p^\infty D(v)\,\mathrm{d}v \right) \right]$$

is nonempty. Moreover, for any $D \in \Delta^*(p|\omega)$,

$$D(v) = \overline{D}(\xi(p|\omega)), \tag{OA.8}$$

for all $v \in [\xi(p|\omega), p]$ and

$$\int_{\xi(p|\omega)}^\infty D(v)\,\mathrm{d}v = \int_{\xi(p|\omega)}^\infty \overline{D}(v)\,\mathrm{d}v, \tag{OA.9}$$

where $\xi(p|\omega) := \max\left\{ \tilde{\xi}(p|\omega), v^{-1}(p) \right\}$.

Proof. Arguments similar to the proof of Lemma 2 imply that the intermediary's problem can be simplified to

$$\max_{\xi \in [v^{-1}(p), p]} \alpha p \overline{D}(\xi) + \delta\omega f\left(\int_\xi^\infty \overline{D}(v)\,\mathrm{d}v - (p - \xi)\overline{D}(\xi) \right), \tag{OA.10}$$

which, by continuity of $\overline{D}$, has a solution. This implies that $\Delta^*(p|\omega)$ is nonempty. Moreover, the derivative of the objective function in (OA.10) is

$$\left[\alpha p - \delta\omega(p - \xi) f'\left(\int_\xi^\infty \overline{D}(v)\,\mathrm{d}v - (p - \xi)\overline{D}(\xi) \right) \right] \overline{D}'(\xi).$$

Notice that $\alpha p - \delta\omega(p - \xi) f'\left(\int_\xi^\infty \overline{D}(v)\,\mathrm{d}v - (p - \xi)\overline{D}(\xi) \right)$ is increasing in ξ and is positive when $\xi = p$. Also, $\overline{D}'(\xi) < 0$ for all ξ. Thus, the intermediary's problem is concave in ξ and the solution $\xi(p|\omega)$ of (OA.10) is given by $\xi(p|\omega) := \max\left\{ \tilde{\xi}(p|\omega), v^{-1}(p) \right\}$. This in turn implies that any $\widehat{D} \in \mathcal{D}$ satisfying the condition given by the lemma must be in $\Delta^*(p|\omega)$. ■

It is convenient to define

$$\tilde{\beta}(p|\omega) := f'\left(\int_{\xi(p|\omega)}^\infty \overline{D}(v)\,\mathrm{d}v - (p - \xi(p|\omega))\overline{D}(\xi(p|\omega)) \right).$$

Lemma OA.3. *Given any $\alpha, \delta \in (0,1)$, $\omega \geq 0$ and $\beta \geq 0$, for any $\eta > 0$, let*

$$\mathcal{P}(\eta) := \left\{ p = \operatorname*{argmax}_{p \geq 0} p\mathbf{D}^*(p|p) : \ \mathbf{D}^* \textit{ is a selection of } \Delta^*(\cdot|\omega) \textit{ for some } f \in \mathcal{F}_1 \bigcup \mathcal{F}_2(\beta, \eta) \right\}.$$

Then, $\lim_{\eta \to 0} \sup_{p \in \mathcal{P}(\eta)} |p - \tilde{p}| = 0$, where $\tilde{p}$ is the unique value such that

$$v^{-1}(\tilde{p}) \leq \left(1 - \frac{\alpha}{\delta\beta\omega} \right)^+ \tilde{p} \leq \overline{p}, \tag{OA.11}$$

with at least one inequality binding. Furthermore, if $\delta\beta\omega \leq \alpha$, we have $\mathcal{P}(\eta) = \{\tilde{p}\}$ for any η; if $\delta\beta\omega > \alpha$ and $v^{-1}(\tilde{p}) = (1 - \frac{\alpha}{\delta\beta\omega})\tilde{p} < \overline{p}$, there exists $\overline{\eta} > 0$ such that $\mathcal{P}(\overline{\eta}) = \{\tilde{p}\}$, $\tilde{\beta}(\tilde{p}|\omega) = f'(0)$ and $\int_{\tilde{p}}^\infty \mathbf{D}^*(v|\tilde{p})\,\mathrm{d}v = 0$ for any $f \in \mathcal{F}_1 \bigcup \mathcal{F}_2(\beta, \overline{\eta})$ and any selection $\mathbf{D}^*$ of $\Delta^*(\cdot|\omega)$.

Proof. Given any f, by Lemma OA.2, for any selection $\mathbf{D}^*$ of $\Delta^*(\cdot|\omega)$,

$$p\mathbf{D}^*(p|p) = p\overline{D}(\xi(p|\omega)) = p\overline{D}\left(\max\left\{\left(1-\frac{\alpha}{\delta\tilde{\beta}(p|\omega)\omega}\right)^+ p, v^{-1}(p)\right\}\right)$$
$$= \min\left\{p\overline{D}\left(\left(1-\frac{\alpha}{\delta\tilde{\beta}(p|\omega)\omega}\right)^+ p\right), p\overline{D}(v^{-1}(p))\right\},$$

where the last equality follows from the fact that $\overline{D}$ is strictly decreasing.

Notice that by definition, we have $\tilde{\beta}(p|\omega) \in [f'(0), f'(\mathbb{E}(v)]$. Therefore, if $\|f''\| < \eta$ and as η goes to 0, we must have $\tilde{\beta}(p|\omega)$ converge to $f'(0) = \beta$ for all p. This implies that as η goes to 0, the seller's optimal price must converge to the price $\tilde{p}$ that solves

$$\min\left\{p\overline{D}\left(\left(1-\frac{\alpha}{\delta\beta\omega}\right)^+ p\right), p\overline{D}(v^{-1}(p))\right\}, \tag{OA.12}$$

which has exactly the same form as the seller's problem in the baseline model with a market feedback level at β. Thus, the properties of $\tilde{p}$ stated in the lemma follows directly from Lemma 4.

Furthermore, when $\delta\beta\omega \leq \alpha$, since $f'' < 0$, we must have $\delta\tilde{\beta}(p|\omega)\omega \leq \alpha$ for any p. Hence, the seller's problem is to maximize

$$p\overline{D}(v^{-1}(p)),$$

which is the same as the problem in the baseline model. So the optimal price must be $\tilde{p}$ for any f with $f'(0) = \beta$. Thus, $\mathcal{P}(\eta) = \{\tilde{p}\}$ for any $\eta > 0$. Lemma OA.2 implies that $\int_{\tilde{p}}^{\infty} \mathbf{D}^*(v|\tilde{p})\, dv = 0$ for any $\mathbf{D}^*$ that is a selection of $\Delta^*(\cdot|\omega)$ and $\tilde{\beta}(\tilde{p}|\omega) = f'(0)$.

On the other hand, if $\delta\beta\omega > \alpha$ and $v^{-1}(\tilde{p}) = (1-\frac{\alpha}{\delta\beta\omega})^+\tilde{p} < \overline{p}$, for η small enough, we have $\delta\tilde{\beta}(p|\omega)\omega > \alpha$ for all p, so

$$p\overline{D}\left(\left(1-\frac{\alpha}{\delta\tilde{\beta}(p|\omega)\omega}\right)^+ p\right) = p\overline{D}\left(\left(1-\frac{\alpha}{\delta\tilde{\beta}(p|\omega)\omega}\right) p\right). \tag{OA.13}$$

Its derivative with respect to p is

$$\overline{D}\left(\left(1-\frac{\alpha}{\delta\tilde{\beta}(p|\omega)\omega}\right) p\right) + \left(1-\frac{\alpha}{\delta\tilde{\beta}(p|\omega)\omega}\right) p\overline{D}'\left(\left(1-\frac{\alpha}{\delta\tilde{\beta}(p|\omega)\omega}\right) p\right)$$
$$+ p^2\overline{D}'\left(\left(1-\frac{\alpha}{\delta\tilde{\beta}(p|\omega)\omega}\right) p\right)\left(1-\frac{\alpha}{\delta\tilde{\beta}(p|\omega)\omega}\right)', \tag{OA.14}$$

which, as η goes to 0, converges to

$$\overline{D}\left(\left(1-\frac{\alpha}{\delta\beta\omega}\right) p\right) + \left(1-\frac{\alpha}{\delta\beta\omega}\right) p\overline{D}'\left(\left(1-\frac{\alpha}{\delta\beta\omega}\right) p\right). \tag{OA.15}$$

Since $p\overline{D}(p)$ is strictly concave, (OA.15) must be strictly decreasing in p, positive for $p < \frac{\delta\beta\omega}{\delta\beta\omega-\alpha}\overline{p}$, and negative for $p > \frac{\delta\beta\omega}{\delta\beta\omega-\alpha}\overline{p}$. Hence, given any $\epsilon > 0$, for η small enough, (OA.14) is positive for $p < \frac{\delta\beta\omega}{\delta\beta\omega-\alpha}\overline{p} - \epsilon$ and negative for $p > \frac{\delta\beta\omega}{\delta\beta\omega-\alpha}\overline{p} + \epsilon$. Since $v^{-1}(\tilde{p}) = (1-\frac{\alpha}{\delta\beta\omega})^+\tilde{p} < \overline{p}$, for η small enough, the seller's revenue function equals (OA.13) and must be strictly increasing for $p < \tilde{p}$, and it equals $p\overline{D}(v^{-1}(p))$ and is strictly decreasing for $p > \tilde{p}$. Therefore, the optimal price is $\tilde{p}$, i.e., $\mathcal{P}(\eta) = \{\tilde{p}\}$ for η small enough. According to Lemma OA.2, we have $\int_{\tilde{p}}^{\infty} \mathbf{D}^*(v|\tilde{p})\,\mathrm{d}v = 0$ for any $\mathbf{D}^*$ that is a selection of $\Delta^*(\cdot|\omega)$, which also implies that $\tilde{\beta}(\tilde{p}|\omega) = f'(0)$. ■

To describe the equilibrium outcomes in this setting, we define

$$g^*(p) := \frac{\alpha}{\delta f'(0)}\left(1 + \frac{p\overline{D}(p)}{\int_p^{\infty} \overline{D}(v)\,\mathrm{d}v}\right),$$

for all $p \in [0, \overline{p}]$, and let

$$p^* := \inf\left\{p \geq 0 \middle| \delta\left(f(0) + f'(0)\int_p^{\infty} \overline{D}(v)\,\mathrm{d}v\right) \geq 1\right\}.$$

Theorem OA.3. *There exists a continuously decreasing function $h : (\underline{\beta}, \overline{\beta}) \to \mathbb{R}_+$ such that every $f \in \mathcal{F}_1 \cup [\bigcup_{\beta \in (\underline{\beta},\overline{\beta})} \mathcal{F}_2(\beta, h(\beta))]$ induces a unique finite stationary equilibrium outcome. Furthermore, the following are equivalent:*

1. *$\mathbf{z}^{\mathrm{s}} = (r^{\mathrm{s}}, \sigma^{\mathrm{s}}, \omega^{\mathrm{s}}, p^{\mathrm{s}}, \{m_t^{\mathrm{s}}\})$ is a finite stationary equilibrium outcome.*

2. $$p^{\mathrm{s}} = \begin{cases} \mathbb{E}[v], & \text{if } f \in \mathcal{F}_1 \\ v(p^*), & \text{if } f \in [\bigcup_{\beta \in (\underline{\beta},\overline{\beta})} \mathcal{F}_2(\beta, h(\beta))] \end{cases};$$

$$\omega^{\mathrm{s}} = \begin{cases} \frac{\alpha \mathbb{E}[v]}{1-\gamma\delta}, & \text{if } f \in \mathcal{F}_1 \\ g^*(p^*), & \text{if } f \in [\bigcup_{\beta \in (\underline{\beta},\overline{\beta})} \mathcal{F}_2(\beta, h(\beta))] \end{cases}; \quad r^{\mathrm{s}} = \begin{cases} \mathbb{E}[v], & \text{if } f \in \mathcal{F}_1 \\ \frac{(1-\gamma\delta)}{\alpha} g^*(p^*), & \text{if } f \in [\bigcup_{\beta \in (\underline{\beta},\overline{\beta})} \mathcal{F}_2(\beta, h(\beta))] \end{cases};$$

$\sigma^{\mathrm{s}} = 0$, and $m_t^{\mathrm{s}} = f(0)^t$, for all $t \geq 1$.

Proof. We first show that given any $f'(0) < \overline{\beta}$, the equilibrium outcome $(r^{\mathrm{s}}, p^{\mathrm{s}}, \sigma^{\mathrm{s}}, \omega^{\mathrm{s}}, \{m_t^{\mathrm{s}}\})$ described in the statement of the theorem is indeed a finite stationary equilibrium if either $f'(0) \in [0, \underline{\beta}]$ or $f \in \mathcal{F}_2(f'(0), \eta)$ for $\eta > 0$ small enough. To this end, we will show that for any such tuple, there exists $\mathbf{D}^{\mathrm{s}} : \mathbb{R}_+ \to \mathcal{D}$ such that $(\omega^{\mathrm{s}}, p^{\mathrm{s}}, \mathbf{D}^{\mathrm{s}})$ satisfies the conditions of Lemma OA.1.

Case 1: $f \in \mathcal{F}_1$.
In this case, we have $f'(0) \leq \underline{\beta}$. Consider any selection $\mathbf{D}^{\mathrm{s}}$ of $\Delta^*(\cdot|\omega^{\mathrm{s}})$. Since $p^{\mathrm{s}} = \mathbb{E}[v]$ and thus $v^{-1}(p^{\mathrm{s}}) = 0$, Lemma OA.3 implies that $p^{\mathrm{s}} \in \operatorname{argmax}_p p\mathbf{D}^{\mathrm{s}}(p|p)$, which establishes (OA.6). Meanwhile, Lemma OA.3 also implies that that

$$\int_{p^{\mathrm{s}}}^{\infty} \mathbf{D}^{\mathrm{s}}(v|p^{\mathrm{s}})\,\mathrm{d}v = 0.$$

Moreover, given $p^{\mathrm{s}} = \mathbb{E}[v]$, Lemma OA.2 implies that $\mathbf{D}^{\mathrm{s}}(p^{\mathrm{s}}|p^{\mathrm{s}}) = \overline{D}(0) = 1$. Together,

$$\begin{aligned}\sup_{D \in \mathcal{D}}\left[\alpha p^{\mathrm{s}} D(p^{\mathrm{s}}) + \delta f\left(\int_{p^{\mathrm{s}}}^{\infty} D(v)\,\mathrm{d}v\right)\omega^{\mathrm{s}}\right] =& \alpha p^{\mathrm{s}} \mathbf{D}^{\mathrm{s}}(p^{\mathrm{s}}|p^{\mathrm{s}}) + \delta f\left(\int_{p^{\mathrm{s}}}^{\infty} \mathbf{D}^{\mathrm{s}}(v|p^{\mathrm{s}})\,\mathrm{d}v\right)\omega^{\mathrm{s}} \\ =& \alpha\mathbb{E}[v] + \delta f(0)\omega^{\mathrm{s}} \\ =& \omega^{\mathrm{s}} \\ =& \frac{\alpha p^{\mathrm{s}} \mathbf{D}^{\mathrm{s}}(p^{\mathrm{s}}|p^{\mathrm{s}})}{1 - \delta f\left(\int_{p^{\mathrm{s}}}^{\infty} \mathbf{D}^{\mathrm{s}}(v|p)\,\mathrm{d}v\right)}\end{aligned}$$

which establishes (OA.5) and (OA.7).

Case 2: $f \in \mathcal{F}_2(\beta, \eta)$ for some $\eta > 0$.
In this case, we have $f'(0) \in (\underline{\beta}, \overline{\beta})$. Take any selection $\mathbf{D}^{\mathrm{s}}$ of $\Delta^*(\cdot|\omega^{\mathrm{s}})$. Hence,

$$\left(1 - \frac{\alpha}{\delta f'(0)\omega^{\mathrm{s}}}\right)^+ p^{\mathrm{s}} = \left(1 - \frac{\alpha}{\delta f'(0)\omega^{\mathrm{s}}}\right) p^{\mathrm{s}} = v^{-1}(p^{\mathrm{s}}) = p^* < \overline{p},$$

which in turn implies that, by Lemma OA.3, for η small enough,

$$\int_{p^{\mathrm{s}}}^{\infty} \mathbf{D}^{\mathrm{s}}(v|p^{\mathrm{s}})\, \mathrm{d}v = 0$$

and therefore,

$$\omega^{\mathrm{s}} = \alpha p^{\mathrm{s}} \overline{D}(\xi(p^{\mathrm{s}}|\omega^{\mathrm{s}})) + f(0)\delta\omega^{\mathrm{s}} = \alpha p^{\mathrm{s}} \mathbf{D}^{\mathrm{s}}(p^{\mathrm{s}}|p^{\mathrm{s}}) + \delta f\left(\int_{p^{\mathrm{s}}}^{\infty} \mathbf{D}^{\mathrm{s}}(v|p^{\mathrm{s}})\, \mathrm{d}v\right)\omega^{\mathrm{s}},$$

which establishes (OA.5). Furthermore, since $p^* < \overline{p}$, $p^{\mathrm{s}} < \delta f'(0)\omega^{\mathrm{s}}\overline{p}/(\delta f'(0)\omega^{\mathrm{s}} - \alpha)$ and hence, for η small enough, p^{s} is the unique maximizer of $p\overline{D}(\xi(p|\omega^{\mathrm{s}}))$ according to Lemma OA.3. Thus, by Lemma OA.2, $(\omega^{\mathrm{s}}, p^{\mathrm{s}}, \mathbf{D}^{\mathrm{s}})$ satisfies (OA.6) and (OA.7). We define $h(\beta)$ to be the supermum of the values of η such that the above arguments are valid as being required in Lemma OA.3.

We now show that for any finite stationary equilibrium, its outcome $(r^{\mathrm{s}}, p^{\mathrm{s}}, \sigma^{\mathrm{s}}, \omega^{\mathrm{s}}, \{m_t^{\mathrm{s}}\})$ must satisfy the conditions given by Theorem OA.3. By Lemma OA.1, if $(\omega^{\mathrm{s}}, p^{\mathrm{s}}, \mathbf{D}^{\mathrm{s}})$ satisfy (OA.5), (OA.6), and (OA.7) such that $r^{\mathrm{s}} = p^{\mathrm{s}}\mathbf{D}^{\mathrm{s}}(p^{\mathrm{s}}|p^{\mathrm{s}})$, $\sigma^{\mathrm{s}} = \int_{p^{\mathrm{s}}}^{\infty} \mathbf{D}^{\mathrm{s}}(v|p^{\mathrm{s}})\, \mathrm{d}v$ and $m_t^{\mathrm{s}} = f(\sigma^{\mathrm{s}})^t$. It follows immediately that $r^{\mathrm{s}}, \sigma^{\mathrm{s}}, \{m_t^{\mathrm{s}}\}$ satisfy the condition given by Theorem OA.3 if ω^{s} and p^{s} satisfy these conditions. Thus, it suffices to show that $\omega^{\mathrm{s}}, p^{\mathrm{s}}$ satisfy these conditions. Now consider three cases separately.

Case 1: $\omega^{\mathrm{s}} \leq \alpha/\delta f'(0)$.
In this case, by Lemma OA.3, it immediately follows that

$$\left(1 - \frac{\alpha}{\delta f'(0)\omega^{\mathrm{s}}}\right)^+ p^{\mathrm{s}} = 0 = v^{-1}(p^{\mathrm{s}})$$

and hence $p^{\mathrm{s}} = \mathbb{E}[v]$, which in turn, by (OA.5), implies that $\omega^{\mathrm{s}} = \alpha\mathbb{E}[v]/(1 - f'(0)\delta)$. For this to be consistent with $\omega^{\mathrm{s}} \leq \alpha/\delta f'(0)$, it must be that $f'(0) \leq \underline{\beta}$, i.e., $f \in \mathcal{F}_1$.

Case 2: $\omega^{\mathrm{s}} > \alpha/\delta f'(0)$ and

$$\left(1 - \frac{\alpha}{\delta f'(0)\omega^{\mathrm{s}}}\right) p^{\mathrm{s}} = v^{-1}(p^{\mathrm{s}}) < \overline{p}. \tag{OA.16}$$

In this case, Lemma OA.3 implies that, there exists $\overline{\eta}$ such that, if $\|f''\| < \overline{\eta}$, p^{s} is the unique optimal price for the seller, i.e., it satisfies (OA.6). Then, Lemma OA.2 implies that

$$\omega^{\mathrm{s}} = \delta\left(f(0) + f'(0)\int_{\left(1 - \frac{\alpha}{\delta f'(0)\omega^{\mathrm{s}}}\right)p^{\mathrm{s}}}^{\infty} \overline{D}(v)\, \mathrm{d}v\right)\omega^{\mathrm{s}},$$

and hence, together with (OA.11), it must be that $f'(0) \in [\underline{\beta}, \overline{\beta}]$ and

$$\left(1 - \frac{\alpha}{\delta f'(0)\omega^{\mathrm{s}}}\right) p^{\mathrm{s}} = p^*.$$

Meanwhile, since (OA.16) is equivalent to

$$\omega^{\mathrm{s}} = g^*\left(\left(1 - \frac{\alpha}{\delta f'(0)\omega^{\mathrm{s}}}\right) p^{\mathrm{s}}\right),$$

it must be that $\omega^{\mathrm{s}} = g^*(p^*)$ and hence $p^{\mathrm{s}} = v(p^*)$.

Case 3: $\omega^{\mathrm{s}} > \alpha/\delta f'(0)$ and (OA.16) is violated. In this case, Lemma OA.3 implies that, as $\|f''\|$ goes to zero

$$p^{\mathrm{s}} \to \frac{\delta f'(0)\omega^{\mathrm{s}}}{\delta f'(0)\omega^{\mathrm{s}} - \alpha}\overline{p} \quad \text{and} \quad \xi(p|\omega^{\mathrm{s}}) \to \overline{p}. \tag{OA.17}$$

In this case, Lemma OA.1 and Lemma OA.2 implies that

$$\delta\left[f'\left(\int_{\overline{p}}^{\infty} \overline{D}(v)\,\mathrm{d}v - (p^{\mathrm{s}} - \overline{p})\overline{D}(\overline{p})\right)(p^{\mathrm{s}} - \overline{p})\overline{D}(\overline{p}) + f\left(\int_{\overline{p}}^{\infty} \overline{D}(v)\,\mathrm{d}v - (p^{\mathrm{s}} - \overline{p})\overline{D}(\overline{p})\right)\right] \to 1,$$

or equivalently,

$$\delta\left[f'\left(\sigma(p^{\mathrm{s}})\right)(p^{\mathrm{s}} - \overline{p})\overline{D}(\overline{p}) + f\left(\sigma(p^{\mathrm{s}})\right)\right] \to 1, \tag{OA.18}$$

where $\sigma(p^{\mathrm{s}}) = \int_{\overline{p}}^{\infty} \overline{D}(v)\,\mathrm{d}v - (p^{\mathrm{s}} - \overline{p})\overline{D}(\overline{p})$.

Notice that as $\|f''\|$ goes to zero, $f'(\sigma^{\mathrm{s}}) \to f'(0)$ and $f(\sigma^{\mathrm{s}}) \to f(0) + f'(0) \cdot \sigma^{\mathrm{s}}$. Hence, if $f'(0) < \overline{\beta}$,

$$\begin{aligned}
\delta\left[f'(\sigma^{\mathrm{s}})(p^{\mathrm{s}} - \overline{p})\overline{D}(\overline{p}) + f(\sigma^{\mathrm{s}})\right] \to & \delta\left[f'(0)(p^{\mathrm{s}} - \overline{p})\overline{D}(\overline{p}) + f(0) + f'(0) \cdot \sigma^{\mathrm{s}}\right] \\
< & \delta\left(f(0) + \overline{\beta}\int_{\overline{p}}^{\infty} \overline{D}(v)\right) \\
= & 1
\end{aligned} \tag{OA.19}$$

which contradicts with equation (OA.18). Therefore, for $\|f''\|$ small enough, this case can only occur when $f'(0) \geq \overline{\beta}$.

Therefore, there exists a function $h : (\underline{\beta}, \overline{\beta}) \to \mathbb{R}_+$ such that every $f \in \mathcal{F}_1 \cup [\bigcup_{\beta \in (\underline{\beta}, \overline{\beta})} \mathcal{F}_2(\beta, h(\beta))]$ induces a unique finite stationary equilibrium outcome as described in this lemma. Finally, notice that the upper bound of $\|f''\|$ for the preceding arguments to be valid is bounded away from zero except for β arbitrarily close to $\overline{\beta}$. Hence, the function $h : (\underline{\beta}, \overline{\beta}) \to \mathbb{R}_+$ can be chosen to be continuously decreasing.

■

Proof of Proposition 6. This proposition immediately follows from Theorem OA.3. ■