

Stabilization with Fiscal Policy

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Abstract

I reconsider the long-standing consensus view that macroeconomic stabilization should rely on monetary policy, not fiscal policy. I use an analytically tractable heterogeneous agent New Keynesian (HANK) model that is parameterized so as to admit a bubble in public debt. In this context, I show that it is possible to stabilize either inflation or output in response to aggregate shocks by varying only fiscal policy (that is, lump-sum uniform transfers). In contrast, when the public debt bubble is large, it is impossible to stabilize either inflation or output by varying only interest rates (monetary policy).

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1 Introduction

This paper reconsiders the long-standing consensus view that macroeconomic stabilization should rely primarily on monetary policy, not fiscal policy. I use an analytically tractable heterogeneous agent New Keynesian (HANK) model that is parameterized so that the real interest rate is perpetually less than the growth rate (of zero). This restriction is readily justified via reference to data from the prior thirty years or even before.¹ Following Brunnermeier, Merkel, and Sannikov (2021), I refer to this situation as being one in which there is a public debt bubble.²

In this context, I show that it is possible to stabilize either inflation or output in the face of a wide range of aggregate shocks by varying uniform lump-sum transfers (fiscal policy) while keeping interest rates (monetary policy) unchanged.³ In contrast, if the level of public debt is sufficiently large, it is impossible to stabilize either inflation or output by varying monetary policy (interest rates) while keeping uniform lump-sum transfers (fiscal policy) unchanged. The analysis implies that, in the presence of a large public debt bubble, adjustments to fiscal policy (specifically, transfers) are a more reliable stabilization tool than adjustments to monetary policy (interest rates).

Why does the existence of a public debt bubble enhance the effectiveness of fiscal policy? Without a bubble, an increase in current transfers needs to be financed using reductions in

¹See Blanchard (2019) among others.

²In the models considered in the paper, all debt, including public debt, matures in one period. So, I am not using the word “bubble” to refer to a situation in which agents hold an unbacked durable asset simply because they believe that it will be valued by others in the future. The words “public debt bubble” mean instead that two conditions are satisfied in equilibrium:

- the real interest rate is perpetually less than the growth rate (which is set to zero throughout).
- the primary surplus is perpetually negative.

It is immediate from these conditions that the intertemporal government budget constraint fails (somewhat dramatically) to be satisfied.

³The economic line between fiscal and monetary policy is not a clear one. In this paper, I refer to a uniform (indexed) lump-sum transfer as being a form of fiscal policy. But certainly similar kinds of transfers have been seen as a form of monetary policy (Friedman (1969)). I would argue that my nomenclature is consistent with institutional features in the US and other advanced economies: statutorily, it is not possible for unelected monetary policymakers to make direct transfers to households. (In a non-bubbly setting, Wolf (2021) adopts a similar division between monetary and non-monetary instruments.)

future spending or increases in future taxes. As a result, current stimulus can give rise to a future fiscal drag. In contrast, in the presence of a bubble, an increase in transfers can be financed solely through the issue of new debt, and does not imply a need for additional future taxes. Since the new public debt generates more asset income for its holders, current fiscal stimulus has an unambiguously positive effect on both current and future demand.

What goes wrong with monetary policy? Monetary policy is, in an intertemporal sense, self-defeating. Current stimulus, accomplished through a reduction in interest rates, lowers future asset income. That creates a drag on future demand which needs to be offset anew using monetary policy easing. If the size of the public debt is sufficiently large, then this linkage means that macroeconomic stabilization can only be accomplished via an ever-increasing - that is, unsustainable - sequence of interest rate cuts.⁴

The results in this paper stand in stark contrast to the implications of representative agent New Keynesian (RANK) models (see, for example, Gali (2015)). In these models, fiscal policy (variations in uniform lump-sum transfers) has no effect on inflation or output because of Ricardian equivalence. Instead, monetary policy plays the lead role in stabilization, as the central bank uses interest rate cuts (increases) to offset the effects of adverse (positive) demand shocks. A fiscal policy response (via adjustment of lump-sum taxes/transfers) is necessary, but only in order to ensure that the government's intertemporal budget constraint is satisfied.

The RANK model is, at least implicitly, based on the assumption that households are fully insured against idiosyncratic shocks. As is done in this paper, this premise of full insurance has been abandoned by a fast-growing literature on monetary policy in HANK models.⁵ Its focus is on how the inclusion of ex-post heterogeneity (created by incomplete asset markets) affects the impact and transmission of monetary policy. The contribution of this paper to

⁴Mian, Straub, and Sufi (2021) and McKay and Wieland (2021) also show how currently stimulative monetary policy can be a drag on future aggregate demand. My contribution is to show, in a simple HANK formulation, that this intertemporal effect can be sufficiently strong to make inflation-stabilizing or output-stabilizing monetary stimulus infeasible.

⁵See, among others, Kaplan, et al. (2018), Auclert (2019), Broer, et al. (2020), and Acharya and Dogra (2020).

that literature is to incorporate public debt bubbles into HANK models, and to show (as discussed above) how that feature makes fiscal policy a superior tool for stabilization policy.

There is also a literature on monetary policy in models with *private sector* bubbles.⁶ It focuses primarily on how monetary policy should respond to variations in the size of the bubble. In contrast, this paper abstracts from private sector asset bubbles, and instead studies how the existence of a public debt bubble should affect macroeconomic policy.

Finally, the analysis in this paper provides a point of contact between standard Dynamic Stochastic General Equilibrium (DSGE) macro modeling and the so-called heterodox modern monetary theory (MMT) described in, among other sources, Mitchell, Wray, and Watts (2019) and Mankiw (2020).⁷ A main starting point for MMT is the presumption is that governments can always pay off its liabilities using the issue of fresh liabilities, and so increases in spending do not give rise to a need to raise future taxes (or to cut future spending). This premise aligns with the parametric restriction in this paper that there is a public debt bubble. It is then perhaps not all that surprising that the major policy conclusion of MMT is similar to that in this paper: governments can use adjustments in taxes/transfers to stabilize the macroeconomy, without any variation in interest rates.

2 Model

In this section, I present the model, its steady-state, and a log-linearized approximation to its near-steady-state dynamics. From a technical perspective, the model essentially is an embedding of standard New Keynesian elements into a simplified version of the setup analyzed by Kocherlakota (2021).

⁶See, among others, Gali (2014), Allen, et al. (2018), Asiryan, et al. (2021), and Dong, Miao, and Wang (2020).

⁷Michau (2021) provides an alternative modeling connection by assuming that a representative household derives utility from its real wealth.

2.1 Description

There is a unit measure of agents. Their individual states have two possible outcomes $\{H, L\}$. The states' dynamic evolution is stochastically independent across individuals and governed by a common Markov chain with transition matrix:

$$\begin{aligned} P_{HH} &= (1 - p) & P_{HL} &= p \\ P_{LH} &= \rho & P_{LL} &= (1 - \rho) \end{aligned}$$

where $0 < \rho, p < 1$. The stationary density for this matrix is given by:

$$P_H = \frac{\rho}{(p + \rho)}, P_L = \frac{p}{(p + \rho)}.$$

I assume that in period 1, the fractions of agents in each of the two states is given by the stationary density, and so these fractions remain constant over time.

The agents live forever and maximize the expectation of a time-separable utility function:

$$\sum_{t=1}^{\infty} \beta^{t-1} u(c_t, n_t, s_t), 0 < \beta < 1.$$

Here, c_t is consumption in period t , n_t is labor in period t , and s_t is the individual's state in period t . I assume that in period t :

$$\begin{aligned} u(c_t, n_t, H) &= \ln(c_t) - n_t^{\psi+1}/(\psi + 1) \\ u(c_t, n_t, L) &= \bar{v}_t c_t \end{aligned}$$

In state H , all agents are endowed with N^{max} units of time, and can, in period t , produce $A_t x$ units of consumption using x units of time, for any $x \in [0, N^{max}]$. Note that both A_t and \bar{v}_t are common across all agents in state H in period t .

In what follows, I always assume that N^{max} is sufficiently high that this upper bound never binds. The logarithmic utility assumption is purely for notational convenience; all of

the results in the paper generalize to the case in which marginal utility has a more general power form.

Agents in state L are not endowed with any time and so are unable to produce any goods. In this state, they experience a high urgency to consume, which is captured by linearity and high values of marginal utility. This high urgency to consume can be seen as capturing a variety of contingencies, including the impact of health shocks.

Wages are determined in a competitive labor market, and so the real wage equals the marginal rate of substitution between labor and consumption for the (productive) agents in state H . Hence, the real wage W_t satisfies the restriction:

$$W_t = \gamma \left(\frac{Y_t}{A_t P_H} \right)^\psi C_{Ht} \quad (1)$$

where Y_t is output in period t and C_{Ht} is the consumption of agents in state H in period t . Note that the restriction (1) assumes that all agents in state H have the same consumption in period t . We shall see later that this assumption is satisfied in equilibrium.

At each date, there is a unit measure of firms. I assume that their pricing behavior is such that the inflation rate Π_t from period $(t - 1)$ to period t satisfies the restriction:

$$\begin{aligned} \ln(1 + \Pi_t) &= \ln(\gamma) + \kappa \ln(W_t/A_t) + \beta_F \ln(1 + \Pi_{t+1}) \\ \kappa &\in (0, \infty), \beta_F \in (0, 1), \gamma > 0 \end{aligned}$$

This is the standard (log-linear) New Keynesian Phillips curve. I assume that all⁸ firm profits⁹ accrue to agents in state H .

There are two forms of macroeconomic policy in the model. In terms of monetary policy,

⁸It is possible to generalize the results in the paper to allow agents in state L to receive at least some of the firms' profits. However, it would be nontrivial to include a market for firm shares (as in Kaplan, et al. (2018)) while also incorporating a public debt bubble.

⁹I do not impose any restriction on the firm discount factor β_F . However, it can be shown that, since profits only accrue to agents in state H , the firm's discount factor in a steady-state is given by $\beta(1 - p)$ (that is, the discount factor applied by a household in state H in period t to its consumption in period $(t + 1)$ if it is in state H). Thus, the firm discounts profits at a positive rate, even when the interest rate is negative.

the government commits to a sequence of one-period nominal interest rates $(R_t)_{t=1}^{\infty}$. In terms of fiscal policy, the government commits to a sequence of *real* (indexed) transfers $(\tau_t)_{t=1}^{\infty}$ that are uniform across agents.

In period t , households can buy one-period government bond¹⁰ that pay the specified (nominal) interest rate R_t . They can also borrow and lend among themselves at the given interest rate R_t . In period t , households face a short-sales/borrowing limit:

$$b_t(1 + r_t) \geq -B_{t+1}^{max}$$

Here, b_t is the real (consumption) value of the household's bondholdings and includes both their holdings of public debt and net holdings of private debt. The term r_t represents the real interest rate from period t to period $(t + 1)$. Hence,

$$(1 + r_t) = \frac{(1 + R_t)}{(1 + \Pi_{t+1})},$$

where Π_{t+1} is the inflation rate from period t to period $(t + 1)$. (More generally, Π_{t+1} should represent inflation expectations. However, I focus on perfect foresight outcomes throughout the paper.) The borrowing limit B_{t+1}^{max} is also subject to aggregate shocks.

The per-capita real (consumption) payoff of government bonds in period t is denoted $B_t^{pub}/(1 + \Pi_t)$. Hence, B_t^{pub} represents the nominal per-capita payoff in period t , normalized by the period $(t - 1)$ price level. The per-capita period $(t - 1)$ value of the government bonds is:

$$\frac{B_t^{pub}}{(1 + \Pi_t)(1 + r_{t-1})} = \frac{B_t^{pub}}{(1 + R_{t-1})}$$

units of consumption. The flow government budget constraint is then given by:

$$\frac{B_{t+1}^{pub}}{(1 + R_t)} = \frac{B_t^{pub}}{(1 + \Pi_t)} + \tau_t$$

¹⁰This public liability is isomorphic to interest-bearing reserves held by commercial banks at the central bank, as long as the reserve requirement is not binding.

as the government finances its period t transfer τ_t and real bond payoff $\frac{B_t^{pub}}{(1+\Pi_t)}$ using its period t issuance of one-period bonds.

2.2 Household Consumptions

In this subsection, I provide a characterization of optimal household consumptions, under the presumption that $r_t < 0$ for all $t \geq 1$ and the borrowing limit B_t^{max} is constant over time.

Suppose that households consume a strictly positive amount in any state L (we verify this claim shortly). Their Euler equation in state H leaves them indifferent between consuming more today or saving via government debt:

$$\frac{1}{c_{Ht}} = \beta(1 + r_t) \left\{ \frac{(1 - p)}{c_{H,t+1}} + p\bar{v} \right\}$$

This Euler equation implies¹¹ that, in period t , all households consume the same amount C_{Ht}^* in state H , regardless of their bondholdings:

$$\frac{1}{C_{Ht}^*} = p\bar{v} \sum_{s=0}^{\infty} \beta^{s+1} (1 - p)^s \prod_{n=0}^s (1 + r_{t+n}) \quad (2)$$

This common consumption satisfies:

$$\frac{1}{C_{Ht}^*} < \bar{v}$$

since $p > 0$ and $\beta(1 + r_{t+s}) < \beta < 1$ for all $s \geq 1$.

Next, consider a household that enters state L in period t with bonds that pay off $b_t (\geq -B^{max}/(1 + r_t))$ units of consumption. (If $b_t < 0$, then the household owes consumption

¹¹The Euler equation is a linear difference equation that has a family of solutions:

$$\frac{1}{C_{Ht}} = \frac{1}{C_{Ht}^*} + \varepsilon_t,$$

where $\varepsilon_{t+1} = \beta^{-1}(1 + r_t)^{-1}(1 - p)^{-1}\varepsilon_t$ and ε_1 can take on any positive value. However, it is readily shown that, for any $\varepsilon_1 > 0$, the resulting (falling) sequence of state H consumptions implies that, eventually, it is optimal for consumption in state L to also equal zero. But this plan in which consumption converges to zero along almost every sample path violates the transversality condition.

to lenders.) In state L , the household prefers to borrow as much as is possible because its current marginal utility from consumption exceeds its expected future marginal utility of consumption:

$$\begin{aligned} \bar{v} - \beta(1 + r_t) \left[\frac{\rho}{C_{H,t+1}^*} + (1 - \rho)\bar{v} \right] \\ > \bar{v} - \beta(1 + r_t) [\rho\bar{v} + (1 - \rho)\bar{v}] \\ > 0 \end{aligned}$$

where the last inequality is implied by $\beta(1 + r_t) < 1$. Hence, their consumptions satisfy:

$$c_{Lt} = b_t + B^{max} / (1 + r_t) + \tau_t \quad (3)$$

where b_t is the payoff of any bonds held at the beginning of the period, and τ_t is the (positive) uniform real transfer. Note that, since $r_t < 0$, $c_{Lt} > 0$ for any $b_t \geq -B^{max}$.

Agents in state L consume different amounts depending on their bondholdings at the beginning of the period. But this heterogeneity is irrelevant for the determination of aggregate outcomes. For this reason, it is useful to keep track of *per-capita consumption* for agents in state L . That, in turn, depends on their per-capita bondholdings at the beginning of period t (that is, the cross-sectional average of the term b_t in (3)).

To calculate this average, note that agents in state H buy all of the public bonds and do all of the lending to agents in state L . Hence, the per-capita period t real bond payoff of agents who were in state H in period $(t - 1)$ is given by:

$$\frac{B_t^{pub}}{P_H(1 + \Pi_t)} + \frac{B_t^{max} P_L}{P_H}.$$

In contrast, the per capita period t real bond payoff of agents who were in state L in period $(t - 1)$ is given by:

$$-B_t^{max}$$

because these agents borrow as much as possible. As a result, the per capita real bond payoff of agents in state L in period t is given by the weighted average:

$$\begin{aligned}
& \frac{pP_H}{P_L} \left(\frac{B_t^{pub}}{P_H(1+\Pi_t)} + \frac{B_t^{max}P_L}{P_H} \right) + (-B_t^{max}) \frac{(P_L - P_H p)}{P_L} \\
&= \frac{pB_t^{pub}}{P_L(1+\Pi_t)} + pB_t^{max} - B_t^{max} + B_t^{max}P_H p / P_L \\
&= \frac{pB_t^{pub}}{P_L(1+\Pi_t)} - (1-p-\rho)B_t^{max}.
\end{aligned}$$

If we plug this expression into (3), we obtain the following expression for per-capita consumption for agents in state L :

$$\bar{C}_{Lt} = \frac{pB_t^{pub}}{P_L(1+\Pi_t)} - (1-p-\rho)B_t^{max} + B_t^{max}/(1+r) + \tau_t.$$

2.3 Bubbly Stationary Equilibrium

In this subsection, I characterize, and establish the existence of, *bubbly stationary equilibria*.

Suppose that the exogenous parameters $(\nu_t, A_t, B_{t+1}^{max})$ are constant over time. Suppose too that policy is set so that the nominal interest rate equals \bar{R} for all dates, and the uniform real transfer equals $\bar{\tau}$ for all dates. In this economy, a bubbly stationary equilibrium is a specification of per capita consumptions (C_H, \bar{C}_L) , per capita output Y , the real wage W , public debt B^{pub} , the inflation rate Π , and a negative real interest rate r that satisfy the seven restrictions:

$$\begin{aligned}
\frac{1}{C_H} &= \beta(1+r)\left(\frac{1-p}{C_H} + p\bar{\nu}\right) \\
Y &= P_H C_H + P_L \bar{C}_L \\
\bar{C}_L &= \frac{pB^{pub}/(1+\Pi)}{P_L} + B^{max}/(1+r) - B^{max}(1-p-\rho) + \bar{\tau} \\
W &= \left(\frac{Y}{AP_H}\right)^\psi C_H \\
(1+\Pi) &= \gamma(W/A)^{\frac{\kappa}{1-\beta_F}} \\
B^{pub} &= -\frac{\bar{\tau}(1+\bar{R})}{r} \\
(1+\bar{R}) &= (1+\Pi)(1+r).
\end{aligned}$$

The following proposition establishes conditions for the existence of a bubbly stationary equilibrium.

Proposition 1. *Consider any real interest rate $r \in (-1, 0)$ and any transfer $\bar{\tau} > 0$. Assuming that N^{max} is sufficiently large, there is a nominal interest rate \bar{R} such that $(C_H, \bar{C}_L, Y, W, \Pi, B^{pub}, r)$ is a bubbly stationary equilibrium given $(\bar{R}, \bar{\tau})$.*

Proof. Consider any $r \in (-1, 0)$. The consumption of agents in state H satisfies their Euler equation:

$$C_H = \frac{(1 - \beta(1+r)(1-p))}{\beta p \bar{\nu}(1+r)}$$

Then, the real debt satisfies the government's flow budget constraint:

$$\frac{B^{pub}}{(1+\Pi)} = \frac{-\bar{\tau}(1+r)}{r}$$

Then solve for \bar{C}_L such that:

$$\begin{aligned}\bar{C}_L &= \frac{pB^{pub}}{P_L(1+\Pi)} + B^{max}/(1+r) - B^{max}(1-p-\rho) + \bar{\tau} \\ &= \frac{-p\bar{\tau}(1+r)}{rP_L} + B^{max}/(1+r) - B^{max}(1-p-\rho) + \bar{\tau}\end{aligned}$$

We can then solve for output Y as:

$$Y = P_H C_H + P_L C_L$$

Then, the real wage W satisfies:

$$W = \left(\frac{Y}{AP_H}\right)^\psi C_H$$

and the households in state H set labor equal to:

$$N_H = \frac{Y}{AP_H}$$

as long as N^{max} is larger than $\frac{Y}{AP_H}$.

Next, find Π so that it satisfies the NKPC:

$$\begin{aligned}(1+\Pi)^{1-\beta_F} &= \gamma(W/A)^\kappa \\ &= \gamma\left(\frac{1}{A^{\psi+1}}\left(\frac{Y}{P_H}\right)^\psi C_H\right)^\kappa\end{aligned}$$

Then the nominal interest $(1+\bar{R}) = (1+\Pi)(1+r)$ and the constant level of public debt $B^{pub} = -\frac{\tau(1+\bar{R})}{r}$.

Finally, check that the household's Euler inequality is satisfied in state L :

$$\begin{aligned}
& \bar{\nu} - \beta(1+r)[\rho/C_H + (1-\rho)\bar{\nu}] \\
= & \bar{\nu} - \beta(1+r)\left[\rho\frac{\beta p\bar{\nu}(1+r)}{1-\beta(1+r)(1-p)} + (1-\rho)\bar{\nu}\right] \\
& > \bar{\nu} - \left[\rho\frac{p\bar{\nu}}{1-(1-p)} + (1-\rho)\bar{\nu}\right] \\
& = 0
\end{aligned}$$

where the inequality is implied by the observation that the expression is strictly decreasing in $(1+r)$ and $1/\beta > 1 > (1+r)$. \square

2.4 Log-Linearization

In this subsection, I provide a first-order approximation to the dynamics of the economy around a bubbly steady-state. For any $t \geq 1$, define:

$$\begin{aligned}
& (\hat{c}_{Ht}, \hat{c}_{Lt}, \hat{y}_t, \hat{w}_t, \hat{b}_t^{pub}, \hat{\tau}_t) \\
= & \left(\ln\left(\frac{C_{Ht}}{C_H}\right), \ln\left(\frac{\bar{C}_{Lt}}{\bar{C}_L}\right), \ln\left(\frac{Y_t}{Y}\right), \ln\left(\frac{W_t}{W}\right), \ln\left(\frac{B_t^{pub}}{B^{pub}}\right), \ln\left(\frac{\tau_t}{\bar{\tau}}\right)\right)
\end{aligned}$$

to be logged deviations from steady-state and define:

$$(\hat{r}_t, \hat{\pi}_t, \hat{R}_t) = \left(\ln\left(\frac{1+r_t}{1+r}\right), \ln\left(\frac{1+\Pi_t}{1+\Pi}\right), \ln\left(\frac{1+R_t}{1+\bar{R}}\right)\right).$$

I allow the state L marginal utility ν , the borrowing limit B^{max} , and productivity A to depend on time $t \geq 1$ according to:

$$\begin{aligned}
\nu_t &= \exp(\hat{\nu}_t)\bar{\nu} \\
B_t^{max} &= \exp(\hat{b}_t^{max})B^{max}, \hat{b}_1^{max} = 0 \\
A_t &= \exp(\hat{a}_t)A
\end{aligned}$$

Suppose that the policy choices $(\hat{R}_t, \hat{\tau}_t)_{t=1}^\infty$ and the exogenous parameters $(\hat{\nu}_t, \hat{b}_t^{max}, \hat{a}_t)_{t=1}^\infty$ are in a (sufficiently small) neighborhood¹² of zero. Then, the first-order Taylor series approximation to the seven time-dependent equilibrium conditions (written as functions of the hatted variables) takes the form:

$$-\hat{c}_{Ht} = \hat{r}_t + \beta(1+r)((1-p)(-\hat{c}_{H,t+1}) + p\bar{\nu}\hat{\nu}_{t+1}C_H), t \geq 1 \quad (4)$$

$$\hat{y}_t = \frac{P_H C_H}{Y} \hat{c}_{Ht} + \frac{P_L C_L}{Y} \hat{c}_{Lt}, t \geq 1 \quad (5)$$

$$\bar{C}_L \hat{c}_{Lt} = \frac{pB^{pub}}{P_L(1+\Pi)} (\hat{b}_t^{pub} - \hat{\pi}_t) - \frac{B^{max}}{(1+r)} \hat{r}_t + \bar{\tau} \hat{\tau}_t \quad (6)$$

$$+ \frac{B^{max}}{(1+r)} \hat{b}_{t+1}^{max} - B^{max}(1-p-\rho) \hat{b}_t^{max}, t \geq 1 \quad (7)$$

$$\hat{w}_t = \psi(\hat{y}_t - \hat{a}_t) + \hat{c}_{Ht}, t \geq 1 \quad (8)$$

$$\hat{\pi}_t = \kappa(\hat{w}_t - \hat{a}_t) + \beta_F \hat{\pi}_{t+1}, t \geq 1 \quad (9)$$

$$\hat{b}_{t+1}^{pub} = (1+r)(\hat{b}_t^{pub} - \hat{\pi}_t) + \hat{R}_t + \hat{\tau}_t \bar{\tau} (1+R)/B^{pub}, t \geq 1 \quad (10)$$

$$\hat{r}_t = \hat{R}_t - \hat{\pi}_{t+1}, t \geq 1 \quad (11)$$

$$\hat{b}_1^{pub} = 0 \quad (12)$$

The first equation (4) is the approximate Euler equation of agents in state H . The second equation (5) is the approximate aggregate resource constraint. The third equation (6) is the approximate consumption of agents in state L . The fourth equation (8) is the approximate labor optimality condition (for state H agents). The fifth equation (9) is the New Keynesian Phillips curve. The sixth equation (10) is the approximate law of motion of public debt. The seventh equation (11) is the approximate Fisher equation. I refer to uniformly bounded (in absolute value) solutions to these equations as being *log-linearized equilibria*.

There are at least four aspects of interest for these various restrictions.

- Because $r < 0$, given any sequence $(\hat{R}_t, \hat{\pi}_t, \hat{\tau}_t)_{t=1}^\infty$ that is bounded from above and below, the public debt sequence $(\hat{b}_t^{pub})_{t=1}^\infty$ implied by the government's flow budget constraint

¹²As defined by the sup-norm.

(10) and the initial restriction that $\hat{b}_1^{pub} = 0$ is also bounded from above and below. The presence of a public debt bubble eliminates the connection between monetary and fiscal policy imposed by an intertemporal government budget constraint.

- Equation (4) implies that (as in the RANK model) state H consumptions are functions only of the future (and present). In contrast, equation (6) implies that state L consumptions are functions only of the past (and present).
- As in McKay, Nakamura, and Steinsson (2017), the logged Euler equation (4) features a discount factor $(\beta(1+r)(1-p))$ which is absent from the RANK logged Euler equation.
- The distribution of consumption affects the location of the NKPC. Thus, suppose that $\hat{a}_t = 0$ (so that there are no productivity shocks). If we combine (8) and (9), we obtain:

$$\hat{\pi}_t = \kappa(\psi\hat{y}_t + \hat{c}_{Ht}) + \beta_F\hat{\pi}_{t+1}.$$

Then, even if $\hat{\pi}_t = 0$ for all $t \geq 1$, it may not be true that $\hat{y}_t = 0$ for all $t \geq 1$. The consumption of state H agents acts as an endogenous shifter to the NKPC when it is written in terms of \hat{y}_t . Intuitively, when state H agents are consuming a lot, they are relatively unwilling to work, and the real wage is higher than would be predicted from output alone.

3 Results

In this section, I consider the following policy question. Suppose that the economy is in a steady-state and is then hit by a previously wholly unanticipated (“MIT”) shock to the future time path of:

- ν (the urgency of consumption in state L)
- A (labor productivity)

- B^{max} (the borrowing limit)

Given this shock, I ask first whether it is possible to keep inflation or output at its steady-state level in a log-linearized equilibrium using fiscal policy and then whether it is possible to keep inflation or output at its steady-state level in a log-linearized equilibrium using monetary policy. The answer to the former question is yes and the answer to the latter question is no.

I provide no welfare-based justification for the pursuit of stable inflation or stable output.¹³ However, it is worth noting that many central banks (including the Federal Reserve in the United States) purport to conduct policy so as to achieve these objectives.

In the next subsection, I use a simple example to explain the economics behind the results. I then present the results more formally in Sections 3.2 and 3.3. I relegate proofs to an appendix.

3.1 Results in an Example Economy

In this subsection, I illustrate the main results using a simplified version of the (log-linearized) model.¹⁴ I set $p = \rho = 1$, and $P_H = P_L = 0.5$. This specification implies that households switch deterministically between state H and state L , with an equal share in each state. I suppose also that $\kappa = 0$, so that prices are fixed over time, and that $B_{max} = 0$, so there is no borrowing.

In this setting, I suppose that there is a one-period shock to the marginal utility of consumption in state L :

$$\hat{v}_2 > 0, \hat{v}_t = 0, t \neq 2.$$

I consider three kinds of policy responses: laissez-faire, purely fiscal, and purely monetary. I ask whether it is possible to stabilize output at its steady-state level in a log-linearized

¹³There is a divine coincidence (Blanchard and Gali (2007)) in this model, in the sense that stable inflation ($\hat{\pi}_t = 0$ for all $t \geq 1$) is equivalent to ensuring that productivity tracks the marginal rate of substitution between consumption and leisure ($\hat{a}_t = \psi \hat{y}_t + \hat{c}_{Ht}$) for the (productive) agents in state H . It follows that, if $A^{1+\psi} = (Y/P_H)^\psi C_H$ in the steady-state (which is certainly not guaranteed), then inflation stabilization delivers productive efficiency.

¹⁴I thank a referee for suggesting this great example.

economy using these policy responses.

It will be useful to keep in mind the following steady-state values, when the steady state transfer is given by $\bar{\tau}$ and the steady-state real (nominal) interest rate¹⁵ is given by $r < 0$:

$$\begin{aligned} C_H &= \frac{1}{\beta(1+r)\bar{\nu}} \\ \bar{C}_L &= 2B^{pub} + \bar{\tau} \\ Y &= 0.5\bar{C}_H + 0.5\bar{C}_L \\ B^{pub} &= -\frac{\bar{\tau}(1+r)}{r} \end{aligned}$$

3.1.1 Laissez-Faire

In a laissez-faire response, both interest rates and transfers remain at their steady-state levels:

$(\hat{R}_t)_{t=1}^\infty = (\hat{\tau}_t)_{t=1}^\infty = 0$. We can then solve for $(\hat{y}_t)_{t=1}^\infty$ as follows:

$$\begin{aligned} \hat{c}_{H1} &= -\hat{\nu}_2; \hat{c}_{Ht} = 0, t > 1 \\ \hat{y}_1 &= 0.5\hat{c}_{H1}\frac{C_H}{Y}; \hat{y}_t = 0, t > 1 \end{aligned}$$

There is a one-period fall in output, as households in state H in period 1 cut back on their spending.

3.1.2 Purely Fiscal Response

Suppose again that $\hat{R}_t = 0, t \geq 1$, so that there is no monetary policy response. We shall see that, because $r < 0$, it is possible to choose a bounded sequence $(\hat{\tau}_t)_{t=1}^\infty$ of transfers that stabilizes output - that is, induces a log-linearized equilibrium in which $\hat{y}_t = 0, t \geq 1$.

Since $\hat{R}_t = 0, t \geq 1$, the households in state H consume as follows:

$$\hat{c}_{H1} = -\hat{\nu}_2; \hat{c}_{Ht} = 0, t \geq 1.$$

¹⁵Since inflation is zero, the nominal interest rate is also negative - but it need only be slightly so.

In order to generate a log-linearized equilibrium in which $\hat{y}_t = 0, t \geq 1$, we need the households in state L to consume:

$$\hat{c}_{L1} = \frac{C_H \hat{\nu}_2}{C_L}; \hat{c}_{Lt} = 0, t > 1.$$

To achieve this outcome, we set transfers and bond supplies as follows:

$$\begin{aligned} C_H \hat{\nu}_2 &= \hat{\tau}_1 \bar{\tau} \\ \hat{\tau}_t \bar{\tau} &= -2 \hat{b}_t^{pub} B^{pub}, t \geq 2 \\ B^{pub} \hat{b}_{t+1}^{pub} &= B^{pub} \hat{b}_t^{pub} (1+r) + \hat{\tau}_t \bar{\tau} (1+r), t \geq 1 \\ \hat{b}_1^{pub} &= 0 \end{aligned}$$

Substituting out, we get:

$$\begin{aligned} \hat{b}_2^{pub} &= \frac{C_H \hat{\nu}_2}{B^{pub}} (1+r) \\ \hat{b}_{t+1}^{pub} &= -\hat{b}_t^{pub} (1+r), t \geq 2. \\ \hat{\tau}_t &= -2 \frac{\hat{b}_t^{pub} (1+r)}{r}, t \geq 1 \end{aligned}$$

In period 1, there is an increase in transfers to offset the decline in spending from the households in state H . That stimulus is financed by an increase in public debt that leads to more spending by households in state L in period 2. This intertemporal linkage implies that, to keep aggregate demand constant, transfers need to cycle around the steady-state. But the key observation here is that since $r < 0$, this cycle is bounded: its magnitude shrinks to zero exponentially over time.

3.1.3 Purely Monetary Response

Now suppose that $\hat{\tau}_t = 0, t \geq 1$, so that there is no fiscal policy response. We shall see that if $r < 0$ and $|r|$ is sufficiently small, then it is impossible to choose a bounded sequence $(\hat{R}_t)_{t=1}^{\infty}$ of interest rates that stabilizes output - that is, induces a log-linearized equilibrium in which

$$\hat{y}_t = 0, t \geq 1.$$

Without fiscal tools, it is impossible to affect the consumptions of state L households in period 1. Hence, to stabilize output, the interest rate in period 1 needs to offset the shock to the demand of state H households:

$$\hat{R}_1 = -\hat{\nu}_2.$$

The interest rate cut depresses the asset income of the state L households in period 2, and reduces their spending:

$$\begin{aligned}\hat{b}_2^{pub} &= \hat{R}_1 \\ \hat{c}_{L2}\bar{C}_L &= 2B^{pub}\hat{b}_2^{pub}\end{aligned}$$

To replace this lost demand with spending from state H households, it is necessary to cut interest rates again:

$$\begin{aligned}\hat{R}_2 &= \frac{\hat{c}_{L2}\bar{C}_L}{C_H} \\ &= \frac{2B^{pub}\hat{b}_2^{pub}}{C_H}\end{aligned}$$

This interest rate cut in period 2 lowers the level of period 3 public debt. The percentage fall is proportional to the percentage fall in period 2 public debt:

$$\begin{aligned}\hat{b}_3^{pub} &= \hat{b}_2^{pub}(1+r) + \hat{R}_2 \\ &= \hat{b}_2^{pub}\left(1+r + \frac{2B^{pub}}{C_H}\right)\end{aligned}$$

The key to the argument is that, when $2B^{pub}/C_H + r > 0$, the percentage fall in period 3 public debt is larger than that of period 2 public debt.

Formally, when iterated forwards, this logic gives rise to the following chain of interven-

tions:

$$\begin{aligned}
\hat{R}_1 &= -\hat{\nu}_2 \\
\hat{b}_2^{pub} &= \hat{R}_1 \\
\hat{c}_{Lt}\bar{C}_L &= 2B^{pub}\hat{b}_t^{pub}, t \geq 2 \\
\hat{R}_t &= \hat{c}_{Lt}\bar{C}_L/C_H, t \geq 2 \\
\hat{b}_{t+1}^{pub} &= \hat{b}_t^{pub}(1+r) + \hat{R}_t, t \geq 2.
\end{aligned}$$

The resultant sequence of interest rate cuts takes the form:

$$\begin{aligned}
\hat{R}_{t+1} &= \left(1+r + \frac{2B^{pub}}{C_H}\right)\hat{R}_t \\
&= \left(1+r - \beta\bar{\nu}\frac{2\bar{\tau}(1+r)^2}{r}\right)\hat{R}_t, t \geq 1 \\
\hat{R}_1 &= -\hat{\nu}_2
\end{aligned}$$

The sequence is explosive as long as:

$$\begin{aligned}
0 &< \left(r + \frac{2B^{pub}}{C_H}\right) \\
&= \left(r - \beta\bar{\nu}\frac{2\bar{\tau}(1+r)^2}{r}\right)
\end{aligned}$$

This is satisfied if $r < 0$ and:

$$\frac{r^2}{(1+r)^2} < 2\beta\bar{\nu}\bar{\tau}.$$

Intuitively, when $|r|$ is near zero, the level of public debt B^{pub} is large. A small interest rate cut of, say, Δ in period t then gives rise to a large fall in the asset income of, and in spending by, state L households. To offset this huge decline in demand through the state H households' intertemporal substitution channel, the period $(t+1)$ interest rate has to be cut by more than Δ .

3.2 Effectiveness of Fiscal Policy

In this subsection, I prove through two theorems that it is possible to stabilize the macroeconomy through an appropriate choice of fiscal policy (transfers), while keeping monetary policy (interest rates) unchanged. The first theorem concerns the stabilization of inflation in response to any shock.

Theorem 1. *Let $(\hat{a}_t, \hat{b}_{t+1}^{max}, \hat{\nu}_{t+1})_{t=1}^{\infty}$ be any sequence of real 3-tuples that is uniformly bounded from above and below. If $\hat{R}_t = \hat{\pi}_t = 0$ for all $t \geq 1$, there exists a unique specification $(\hat{\tau}_t)_{t=1}^{\infty}$ of fiscal policy such that there is a log-linearized equilibrium.*

Proof. In Appendix. □

The proof follows the logic of Section 3.1.2: it shows that, in order to stabilize inflation, transfers rise in response to an increase in productivity \hat{a}_t , an increase in the marginal utility $\hat{\nu}_t$ of consumption in state L , or a fall in the borrowing limit \hat{b}_t^{max} . The higher transfers lead agents in state L to consume more, and on to an increase in marginal costs that spills into higher inflation. The public debt bubble plays a key role: The government is able to self-finance these higher transfers because $r < 0$.

Even though the specification of fiscal policy in Theorem 1 stabilizes inflation, it does not necessarily stabilize output. For example, if the productivity innovation sequence $(\hat{a}_t)_{t=1}^{\infty}$ is positive, then it is efficient for the productive agents (in state H) to generate more output through their labor. That extra output is entirely consumed by the unproductive agents (in state L) because they receive a larger transfer. If the shock to idiosyncratic risk $\hat{\nu}_2$ is high, then (without a monetary policy response) the productive agents necessarily save more and consume less. They are then more willing to work to generate output for the unproductive agents, who receive larger transfers to buy the extra consumption.

The next theorem focuses on output stabilization when there are demand shocks (to ν and B_{max}).

Theorem 2. Let $(\hat{b}_{t+1}^{max}, \hat{\nu}_{t+1})_{t=1}^{\infty}$ be any sequence of real 2-tuples that is uniformly bounded from above and below. If $\hat{R}_t = \hat{y}_t = 0$ for all $t \geq 1$, there exists a specification $(\hat{\tau}_t)_{t=1}^{\infty}$ of fiscal policy such that there is a log-linearized equilibrium.

Proof. In Appendix. □

In the proof, state H consumption \hat{c}_H and inflation $\hat{\pi}$ are jointly determined by the NKPC and the Euler equation as a response to $\hat{\nu}$. Then, to stabilize output, transfers are used to offset the state H consumption deviations \hat{c}_{Ht} , given the impact of changes in inflation $\hat{\pi}$ and the borrowing constraint \hat{b}^{max} on state L consumption.

This logic is similar to that of the proof of Theorem 1. However, there is one subtlety: there is a range of values for the slope of the NKPC such that the equilibrium \hat{c}_H and $\hat{\pi}$ are not uniquely pinned down by the marginal utility process $\hat{\nu}$. In this situation, in order to stabilize output, the fiscal policy must be formulated in terms of the endogenous variables $(\hat{c}_H, \hat{\pi})$, rather than the exogenous variable $\hat{\nu}$.

3.3 Ineffectiveness of Monetary Policy

In this subsection, I prove that, if r is sufficiently close to (but less than) zero, it is impossible to stabilize the macroeconomy through an appropriate choice of monetary policy (interest rates), while keeping fiscal policy (transfers) unchanged. The first theorem concerns the inability to stabilize inflation in response to any shock. In its statement, it is useful to define:

$$\hat{\theta}_{t+1}^{max} \equiv \frac{B^{max}}{(1+r)} \hat{b}_{t+1}^{max} - B^{max}(1-p-\rho) \hat{b}_t^{max}$$

to be the change in per-capita consumption that agents in state L receive from the sequence of shocks to their borrowing limit.¹⁶

Theorem 3. Let $\lambda \in (0, 1)$, and suppose $\hat{a} = (\lambda^{t-1} \hat{a}_1)_{t=1}^{\infty}$, $\hat{\theta}^{max} = (\lambda^{t-1} \hat{\theta}_2^{max})_{t=1}^{\infty}$ and $\hat{\nu} =$

¹⁶The proofs of Theorems 3 and 4 rely on somewhat complicated algebra. They can be simplified considerably by assuming that $B^{max} = 0$.

$(\lambda^{t-1}\hat{\nu}_1)_{t=1}^\infty$, where exactly one element of $\{\hat{a}_1, \hat{\theta}_2^{max}, \hat{\nu}_1\}$ is non-zero. Suppose too that $\hat{\tau}_t = \hat{\pi}_t = 0$ for all $t \geq 1$. There exists a cutoff $r_{cut} < 0$ which is independent of λ such that if $r \geq r_{cut}$, the log-linearized equilibrium conditions are not satisfied for any $\hat{R} = (\hat{R}_t)_{t=1}^\infty$.

Proof. In Appendix. □

The crux of the proof is that if $|r|$ is sufficiently small, the level of public debt in steady state is large relative to the consumptions of the state H households. Just as in the example in Section 3.1.3, this creates a need for an ever-increasing sequence of interest rate cuts to stabilize inflation.

The following theorem applies this same logic to output stabilization in response to demand shocks (to ν and B^{max}).

Theorem 4. *Let $\lambda \in (0, 1)$, and suppose $\hat{\theta}^{max} = (\lambda^{t-1}\hat{\theta}_2^{max})_{t=1}^\infty$ and $\hat{\nu} = (\lambda^{t-1}\hat{\nu}_1)_{t=1}^\infty$, where exactly one element of $\{\hat{\theta}_2^{max}, \hat{\nu}_1\}$ is non-zero. Suppose too that $\hat{\tau}_t = \hat{y}_t = 0$ for all $t \geq 1$. There exists a cutoff $r'_{cut} < 0$ which is independent of λ such that if $r \geq r'_{cut}$, the log-linearized equilibrium conditions are not satisfied for any $\hat{R} = (\hat{R}_t)_{t=1}^\infty$.*

Proof. In Appendix. □

4 Additional Considerations

In this section, I explore three additional issues: indeterminacy, the simultaneous stabilization of output and inflation, and the potential consequences of log-linearization.

4.1 Indeterminacy

Models with bubbles and/or money often exhibit indeterminacy of equilibria. In this paper, this possibility is constrained by the stickiness of prices.¹⁷ Thus, as the example in Section

¹⁷The possibility of indeterminacy is also constrained by the “discounted” Euler equation (McKay, Nakamura, and Steinsson (2017)) implied by the incompleteness of markets. Intuitively, the feedback from the future to the present that is essential for indeterminacy is limited because the choices of households in state H are independent of outcomes in future realizations of state L .

3.1 illustrates, there is a unique log-linearized equilibrium when prices are rigid ($\kappa = 0$). However in the next proposition, I show that if prices are sufficiently flexible, then (for fixed monetary and fiscal policy) there exists a continuum of log-linearized equilibria. To simplify the proof, I assume $\beta_F = 0$ (so that the NKPC is not forward-looking) and that $B_{max} = 0$ (so that there is no borrowing).

Proposition 2. *Let $\beta_F = B_{max} = 0$. Suppose $(\hat{R}_t, \hat{\tau}_t)_{t=1}^{\infty}$ equals zero. Then, for κ sufficiently large (so that prices are sufficiently flexible), there exists a continuum of (log-linearized) equilibria in which inflation $(\hat{\pi}_t)_{t=1}^{\infty}$ is non-zero in all periods.*

Proof. In Appendix. □

This kind of indeterminacy is, as noted above, common in models with bubbles and/or money. Gali (2015) describes how the indeterminacy (of bounded equilibria) can be eliminated in RANK models using monetary policy rules. However, a theme of this paper is that it is possible to achieve macroeconomic stability using fiscal policy alone. Along these lines, it is important to note that the government can achieve uniqueness of *bounded* log-linear equilibria using a simple fiscal policy rule, in which transfers depend on real debt.¹⁸ The fiscal policy rule takes the form:

$$\hat{\tau}_t \bar{r}(1 + R)/B = \Psi(\hat{b}_t^{pub} - \hat{\pi}_t), t \geq 2,$$

where $(\Psi + (1 + r)) > 1$. This ensures that, if $\hat{\pi}_1 \neq 0$, then the future evolution of the public debt is inconsistent with a bounded equilibrium because:

$$\begin{aligned} (\hat{b}_{t+1}^{pub} - \hat{\pi}_{t+1}) &= (1 + r)(\hat{b}_t^{pub} - \hat{\pi}_t) + \Psi(\hat{b}_t^{pub} - \hat{\pi}_t), t \geq 2 \\ (\hat{b}_2^{pub} - \hat{\pi}_2) &= (1 + r)(-\hat{\pi}_1) \neq 0. \end{aligned}$$

¹⁸Asiryanyan, et. al. (2021) describe another approach to controlling equilibrium indeterminacy in a context with asset bubbles.

4.2 “Conventional” Policy

As discussed in the introduction, the conventional macroeconomic policy response to shocks involves a combination of fiscal and monetary elements. The following proposition shows that, given a persistent shock to the marginal utility parameter ν , it is possible to use this more coordinated policy approach to stabilize output and inflation simultaneously.

Proposition 3. *Suppose $\hat{\nu} = (\lambda^{t-1}\hat{\nu}_1)_{t=1}^{\infty}$ for $\lambda \in (0, 1)$ and $\hat{\nu}_1$ non-zero. Suppose too that $\hat{y}_t = \hat{\pi}_t = 0$ for all $t \geq 1$. Then there exists a unique specification $(\hat{R}_t, \hat{\tau}_t)_{t=1}^{\infty}$ of monetary-fiscal policy such that there is a (log-linearized) equilibrium. The specification of monetary policy satisfies the restriction:*

$$\hat{R}_t = -\beta(1+r)p\bar{\nu}\lambda^t\hat{\nu}_1 C_H, t \geq 1$$

Proof. In Appendix. □

A positive shock to the marginal utility parameter ν is similar to a downward shock to the natural real rate of interest in a standard New Keynesian model. The monetary policy described in Proposition 3 can thus be seen as equivalent to the standard result (Gali (2015, p. 103)) that it is optimal for the nominal interest rate to track the natural real rate of interest.

However, the economic logic underlying this characterization is different in this model from that in the standard RANK framework. In the RANK model, monetary policy is being used to select the desired point on a (policy-invariant) Phillips curve. In Proposition 3, monetary policy pins down c_{Ht} and so determines the location of the Phillips curve. Fiscal policy is then used to pick a desired point on the Phillips curve.

4.3 Log-Linearization

As is common in monetary economics, the results about fiscal and monetary policy in this paper are characterizations only of log-linearized equilibria. What, if anything, do they imply about equilibria of the original (unapproximated) economy?

In terms of the effectiveness of fiscal policy, Theorems 1 and 2 are stated for shocks of any size. More generally, though, the log-linear approximation should only be expected to work for sufficiently small shocks. With that (important) caveat, I would expect Theorems 1 and 2 to be applicable in the original (unapproximated) model economy.

The situation is different in terms of the ineffectiveness of monetary policy. Theorems 3 and 4 show that the dynamics of output-stabilizing or inflation-stabilizing monetary policy are explosive near the original (high public debt) steady-state. The theorems are silent about whether these dynamics remain explosive once the economy migrates away from that steady-state. Indeed, it is possible to show that for any bubbly stationary equilibrium with a real interest rate r , there is necessarily another bubbly stationary equilibrium with that same level of steady-state output, but with a distinct real interest rate r' . Given this multiplicity, Theorems 3 and 4 do not rule out the possibility that output-stabilizing monetary policy will, in fact, simply track a path from one steady-state to another steady-state.¹⁹

Even in that case, though, I see Theorems 3 and 4 as being relevant because of (unmodelled) bounds on the policy variables. For example, suppose that there is a lower bound (from the existence of currency or other considerations) R_{LB} on the nominal interest rate which is strictly less than the steady-state nominal interest rate \bar{R} . Then, as long as the dynamics of output-stabilizing (or inflation-stabilizing) monetary policy are explosive in this interval (R_{LB}, \bar{R}) , monetary policy will not be effective at responding to adverse shocks because it will get “trapped” by the lower bound.

With positive demand shocks, Theorems 3 and 4 imply that stabilizing monetary policy requires the public debt to grow away from the original steady-state. But in this instance, monetary policy would be ineffective as a stabilization tool as long as there is some (sufficiently tight) upper bound on public debt.

¹⁹In the unapproximated version of the example model in Section 3.1 with rigid prices, this possibility can be shown to be reality: output-stabilizing monetary policy simply leads the economy away from a stationary equilibrium in which \bar{R} is slightly negative to another stationary equilibrium in which the nominal interest rate is (potentially much) more negative. The details are available upon request. (I thank a referee for pointing this out to me.)

5 Conclusion

The conventional approach to macroeconomic stabilization is centered on monetary policy: raise interest rates to lower spending/output/inflation and reduce interest rates to spur spending/output/inflation. I show that, within a simple analytical HANK model, this approach is ineffective if the real interest rate is less than, but close to, the growth rate. The problem is a dynamic one: a current interest rate cut reduces future asset income and so gives rise to a need for a future interest rate cut. When the public debt is sufficiently large (that is, the interest rate is close to the growth rate), this intertemporal linkage produces an unsustainable ever-increasing chain of monetary stimulus.

Instead, this paper finds that, in the presence of a public debt bubble, macroeconomic stabilization is best accomplished through fiscal policy, as the government can increase transfers to spur spending/output/inflation and lower transfers to reduce spending/output/inflation. A key aspect of this finding is that a debt bubble clarifies the demand effects of fiscal stimulus, because it is possible to finance increases in current transfers without any change in future taxes/transfers. The result can be seen as providing theoretical support for the temporary increases in (near-)universal base income used to confront the adverse macroeconomic conditions of 2020-21. More generally, the analysis provides a formal theoretical foundation for regularizing a fiscal approach to macroeconomic support.

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Appendix

In this appendix, I gather the proofs of Theorems 1-4 and Propositions 2-3. I use the standard notation L and L^{-1} to refer to the lag operator and its inverse. In context, there is no ambiguity between this notation and the labelling of the two household states.

Proof of Theorem 1

Let $\Delta_{t+1} = \beta(1+r)C_H p \bar{\nu} \hat{\nu}_{t+1}$ for all $t \geq 1$. Because $\hat{\pi}_t = 0$ for all $t \geq 1$, the Euler equation implies:

$$\hat{c}_{Ht} = \alpha \hat{c}_{H,t+1} - \Delta_{t+1}, t \geq 1,$$

where $\alpha = \beta(1+r)(1-p)$. Hence:

$$\hat{c}_{Ht} = - \sum_{s=1}^{\infty} \alpha^{s-1} \Delta_{t+s}, t \geq 1$$

is uniformly bounded.

Since $\hat{\pi}_t = 0$ for all $t \geq 1$, the Phillips curve implies that:

$$\begin{aligned} \hat{c}_{Ht} &= \hat{a}_t - \psi \hat{y}_t \\ &= \hat{a}_t - \psi (P_L \bar{C}_L \hat{c}_{Lt} / Y + P_H C_H \hat{c}_{Ht} / Y) \end{aligned}$$

there exists a positive constant

$$\xi' = \frac{(Y + \psi P_H C_H)}{\psi P_L} > 0$$

such that:

$$\bar{C}_L \hat{c}_{Lt} = \frac{\hat{a}_t Y}{\psi P_L} - \xi' \hat{c}_{Ht}$$

This implies that the transfers and bond supplies satisfy the state L budget constraint with $\hat{\pi}_t = 0$:

$$\hat{\tau}_t \bar{\tau} = \frac{\hat{a}_t Y}{\psi P_L} - \xi' \hat{c}_{Ht} - \frac{p B^{pub}}{P_L(1 + \Pi)} \hat{b}_t^{pub} - \frac{B^{max}}{(1 + r)} \hat{b}_{t+1}^{max} + B^{max}(1 - p - \rho) \hat{b}_t^{max}, t \geq 1$$

and the government's flow budget constraint with $\hat{\pi}_t = 0$:

$$\hat{b}_{t+1}^{pub} = \hat{b}_t^{pub}(1 + r) + \bar{\tau} \hat{\tau}_t(1 + R)/B^{pub}.$$

(Note that $\hat{b}_1^{max} = 0$.)

Substituting the former budget constraint into the latter, we obtain a first-order difference equation in \hat{b} :

$$\hat{b}_{t+1}^{pub} = \hat{b}_t^{pub}(1 + r)\left(1 - \frac{p}{P_L}\right) + \frac{(1 + R)}{B^{pub}} \phi_t, \hat{b}_1^{pub} = 0, t \geq 1$$

where:

$$\phi_t = \frac{\hat{a}_t Y}{\psi P_L} - \xi' \hat{c}_{Ht} - \left(\frac{B^{max}}{(1 + r)} \hat{b}_{t+1}^{max} - B^{max}(1 - p - \rho) \hat{b}_t^{max}\right), t \geq 1$$

Since $|\phi_t|$ is bounded and:

$$|(1 - p/P_L)| = |(1 - \rho - p)| < 1,$$

the public debt sequence implied by this difference equation is also bounded.

The transfer sequence can then be chosen using either of the budget constraints, so that:

$$\hat{\tau}_t \bar{\tau} = \frac{\hat{a}_t Y}{\psi P_L} - \xi' \hat{c}_{Ht} - \frac{p B^{pub}}{P_L(1 + \Pi)} \hat{b}_t^{pub} - \left(\frac{B^{max}}{(1 + r)} \hat{b}_{t+1}^{max} - B^{max}(1 - p - \rho) \hat{b}_t^{max}\right), t \geq 1.$$

The real wage \hat{w}_t in period t is equal to \hat{a}_t . Output is given by:

$$\hat{y}_t = (\hat{a}_t - \hat{c}_{Ht})/\psi.$$

Proof of Theorem 2

Because $\hat{y}_t = 0$, the NKPC implies that:

$$\hat{\pi}_t = \kappa \hat{c}_{Ht} + \beta_F \hat{\pi}_{t+1}, t \geq 1.$$

Hence, we can write:

$$\hat{\pi}_t = \kappa(1 - \beta_F L^{-1})^{-1} \hat{c}_{Ht}$$

Let $\Delta_{t+1} = \beta(1+r)C_H p \bar{\nu} \hat{\nu}_{t+1}$. Then, we can write the Euler equation as:

$$\hat{c}_{Ht} = \kappa(1 - \beta_F L^{-1})^{-1} \hat{c}_{H,t+1} + \alpha \hat{c}_{H,t+1} - \Delta_{t+1}, t \geq 1,$$

We can rewrite as:

$$((1 - \beta_F L^{-1})(1 - \alpha L^{-1}) - \kappa L^{-1}) \hat{c}_{Ht} = -(1 - \beta_F L^{-1}) \Delta_{t+1}, t \geq 1.$$

We can factor the quadratic on the left-hand side as:

$$(1 - r_1 L^{-1})(1 - r_2 L^{-1}) \hat{c}_{Ht} = -(1 - \beta_F L^{-1}) \Delta_{t+1}$$

where $0 < r_2 < \beta_F$. Hence, this expression can be rewritten as:

$$(1 - r_1 L^{-1}) \hat{c}_{Ht} = -(1 - \beta_F L^{-1}) \sum_{s=0}^{\infty} r_2^s \Delta_{t+s+1}.$$

There are then three cases. In the first case, the NKPC is sufficiently near-flat that

$(1 - \alpha)(1 - \beta_F) > \kappa$. Under this restriction, $(1/\beta_F) > (1/r_1) > 1$ and there is a unique uniformly bounded solution for \hat{c}_H :

$$\hat{c}_{Ht} = (1 - r_1 L^{-1})^{-1}(1 - r_2 L^{-1})^{-1}(\Delta_{t+1} - \beta_F \Delta_{t+2}), t \geq 1.$$

In the second case, the NKPC is sufficiently near-vertical that $\kappa > (1 + \alpha)(1 + \beta_F)$. Then, $(1/r_1) < -1$. Again, there is a unique uniformly bounded solution for \hat{c}_H :

$$\hat{c}_{Ht} = (1 - r_1 L^{-1})^{-1}(1 - r_2 L^{-1})^{-1}(\Delta_{t+1} - \beta_F \Delta_{t+2}), t \geq 1.$$

Finally, in the third case, the slope of the Phillips curve is intermediate, so that $(1 - \alpha)(1 - \beta_F) \leq \kappa \leq (1 + \alpha)(1 + \beta_F)$. In this case, the root r_1 satisfies $1 \geq 1/r_1 \geq -1$. There is a family of uniformly bounded solutions for \hat{c}_H , indexed by \hat{c}_{H1} :

$$\hat{c}_{H,t+1} = (1/r_1)\hat{c}_{Ht} + (1 - \beta_F L^{-1}) \sum_{s=0}^{\infty} r_2^s \Delta_{t+s+1}/r_1, t \geq 1$$

\hat{c}_{H1} given.

In what follows, we choose one of the possible solutions for \hat{c}_H and then solve for the transfers required to stabilize output given that selection. As noted above, the inflation sequence is given by:

$$\hat{\pi}_t = \kappa \sum_{s=0}^{\infty} \beta_F^s \hat{c}_{H,t+s}$$

The consumptions in state L are given by:

$$\bar{C}_L \hat{c}_{Lt} = \frac{-\hat{c}_{Ht} C_H P_H}{P_L}.$$

This implies that the transfers and bond supplies satisfy the state L budget constraint (6)

for all $t \geq 1$:

$$\begin{aligned} \hat{\tau}_t \bar{\tau} = & -\frac{P_H C_H \hat{c}_{Ht}}{P_L} - \frac{p B^{pub}}{P_L (1 + \Pi)} (\hat{b}_t^{pub} - \hat{\pi}_t) - \frac{B^{max}}{(1 + r)} \hat{\pi}_{t+1}, \\ & - \left(\frac{B^{max}}{(1 + r)} \hat{b}_{t+1}^{max} - B^{max} (1 - p - \rho) \hat{b}_t^{max} \right) \end{aligned}$$

and the government's flow budget constraint (10):

$$\hat{b}_{t+1}^{pub} = (\hat{b}_t^{pub} - \hat{\pi}_t)(1 + r) + \bar{\tau} \hat{\tau}_t (1 + R) / B^{pub}.$$

(Note that $\hat{b}_1^{max} = 0$.)

Substituting the former budget constraint into the latter, we obtain a first-order difference equation in \hat{b}^{pub} :

$$\hat{b}_{t+1}^{pub} = \hat{b}_t^{pub} (1 + r) \left(1 - \frac{p}{P_L}\right) + \phi'_t, \hat{b}_1^{pub} = 0,$$

where:

$$\begin{aligned} \phi'_t = & -\hat{\pi}_t (1 + r) \left(1 - \frac{p}{P_L}\right) - \frac{(1 + R) P_H C_H \hat{c}_{Ht}}{B^{pub} P_L} - \frac{(1 + R) B^{max}}{B^{pub} (1 + r)} \hat{\pi}_{t+1} \\ & - \frac{(1 + R)}{B^{pub}} \left(\frac{B^{max}}{(1 + r)} \hat{b}_{t+1}^{max} - B^{max} (1 - p - \rho) \hat{b}_t^{max} \right) \end{aligned}$$

Since:

$$|(1 - p/P_L)| = |(1 - \rho - p)| < 1,$$

and $(\hat{c}_{Ht}, \hat{\pi}_t, \hat{b}_{t+1}^{max})_{t=1}^{\infty}$ is uniformly bounded, the public debt sequence implied by this difference equation is also bounded.

The transfer sequence can then be chosen using either of the budget constraints, so that:

$$\begin{aligned}\hat{\tau}_t \bar{\tau} &= -\frac{P_H C_H \hat{c}_{Ht}}{P_L} - \frac{p B^{pub}}{P_L(1+\Pi)}(\hat{b}_t^{pub} - \hat{\pi}_t) - \frac{B^{max}}{(1+r)}\hat{\pi}_{t+1} \\ &\quad - \left(\frac{B^{max}}{(1+r)}\hat{b}_{t+1}^{max} - B^{max}(1-p-\rho)\hat{b}_t^{max}\right), t \geq 1\end{aligned}$$

We can complete the specification of the equilibrium by noting that $\hat{w}_t = \hat{c}_{Ht}$ and $\hat{r}_t = -\hat{\pi}_t$.

Proof of Theorem 3

Define:

$$\begin{aligned}C_H(r) &= \frac{(1 - \beta(1+r)(1-p))}{\beta(1+r)p\bar{v}} \\ Y(r) &= P_L \left(\frac{p}{P_L} \frac{\bar{\tau}(1+r)}{-r} + B^{max}/(1+r) - B^{max}(1-p-\rho) + \bar{\tau} \right) + P_H C_H(r)\end{aligned}$$

and:

$$\bar{C}_L(r) = \frac{(Y - P_H C_H(r))}{P_L}.$$

These are the steady-state levels of state H consumption, per-capita output, and per-capita state L consumption for any given real interest rate $r < 0$.

Since $\hat{\pi}_t = 0$ for all t , the Phillips curve implies that:

$$\hat{a}_t = \psi \hat{y}_t + \hat{c}_{Ht}, t \geq 1$$

and so:

$$\begin{aligned}\hat{c}_{Ht} &= \hat{a}_t - \psi \hat{y}_t \\ &= \hat{a}_t - \psi (P_L \bar{C}_L(r) \hat{c}_{Lt} / Y(r) + P_H C_H(r) \hat{c}_{Ht} / Y(r))\end{aligned}$$

Hence, there exists:

$$\xi^*(r) = \frac{(Y(r) + \psi P_H C_H(r))}{\psi P_L} > 0$$

such that:

$$\bar{C}_L(r)\hat{c}_{Lt} = \frac{Y(r)}{\psi P_L}\hat{a}_t - \xi^*(r)\hat{c}_{Ht} \quad (13)$$

for all $t \geq 1$. The left-hand side can be rewritten as:

$$\bar{C}_L(r)\hat{c}_{Lt} = \frac{p}{P_L} \frac{(1+r)\bar{\tau}}{-r} \hat{b}_t^{pub} - \frac{B^{max}}{(1+r)} \hat{R}_t + \hat{\theta}_{t+1}^{max}, t \geq 1 \quad (14)$$

where I have used the restriction that:

$$\hat{\tau}_t = 0, t \geq 1.$$

Note from the government's flow budget constraint that:

$$\hat{R}_t = \hat{b}_{t+1}^{pub} - (1+r)\hat{b}_t^{pub} \quad (15)$$

given the restrictions that $\hat{\tau}_t = 0$ and $\hat{\pi}_t = 0$ for all $t \geq 1$. Define:

$$z_{t+1} = \hat{b}_{t+1}^{pub} - \lambda \hat{b}_t^{pub}$$

Since $\hat{a}_{t+1} - \lambda \hat{a}_t = \hat{\theta}_{t+2}^{max} - \lambda \hat{\theta}_{t+1}^{max} = 0$ for all $t \geq 1$, the restrictions (13), (14), and (15) imply that

$$\hat{c}_{H,t+1} - \lambda \hat{c}_{Ht} = A_1^*(r)z_{t+1} + A_2^*(r)z_{t+2}, t \geq 1$$

where:

$$\begin{aligned}
A_1^*(r) &= \frac{-B^{max} + \frac{p}{P_L} \frac{\bar{\tau}(1+r)}{r}}{\xi^*(r)} < 0 \\
A_2^*(r) &= \frac{B^{max}/(1+r)}{\xi^*(r)} > 0 \\
A_1^*(r) + A_2^*(r) &= \frac{\frac{p}{P_L} \frac{\bar{\tau}(1+r)}{r} - \frac{rB^{max}}{(1+r)}}{\xi^*(r)}
\end{aligned}$$

Since $\hat{\pi}_t = 0$, the log-linearized Euler equation can be written

$$\hat{c}_{Ht} = -\hat{R}_t + \beta(1+r)(1-p)(\hat{c}_{H,t+1}) - \beta(1+r)p\bar{\nu}\lambda^t \hat{\nu}_1 C_H, t \geq 1 \quad (16)$$

By quasi-differencing (16), we obtain:

$$A_1^*(r)z_{t+1} + A_2^*(r)z_{t+2} + z_{t+2} - (1+r)z_{t+1} - A_1^*(r)\alpha(r)z_{t+2} - A_2^*(r)\alpha(r)z_{t+3} = 0, t \geq 2$$

where $\alpha(r) = \beta(1+r)(1-p) < 1$. The characteristic function of this second-order difference equation is:

$$\Phi^*(L) = (A_1^*(r) - (1+r))L^2 + (A_2^*(r) - A_1^*(r)\alpha(r) + 1)L - A_2^*(r)\alpha(r).$$

We can evaluate Φ^* to find that:

$$\begin{aligned}
\Phi^*(0) &= -A_2^*(r)\alpha(r) < 0 \\
\Phi^*(\alpha(r)) &= \alpha(r) - (1+r)\alpha(r)^2 > 0 \\
\Phi^*(1) &= (A_1^*(r) + A_2^*(r))(1 - \alpha(r)) - r \\
&= (1 - \alpha(r)) \frac{\frac{p}{P_L} \frac{\bar{\tau}(1+r)}{r} - \frac{rB^{max}}{(1+r)}}{\xi^*(r)} - r
\end{aligned}$$

If r is sufficiently close to zero, then:

$$\Phi^*(1) \approx -\psi(1 - \beta(1 - p)) < 0$$

In this case, the roots of the characteristic equation are both positive and less than one. The only stable solution to the second-order difference equation is to set $z_{t+1} = 0$ for all $t \geq 1$. This implies that:

$$\begin{aligned} \hat{b}_{t+1}^{pub} &= \lambda^{t-1} \hat{b}_1^{pub}, t \geq 1 \\ &= 0 \end{aligned}$$

But this solution leads to a contradiction. Since $\hat{b}^{pub} = 0$, $\hat{R} = 0$, and the Euler equation (16) implies that

$$\hat{c}_{Ht} = -\frac{\lambda^t \beta(1+r)p\bar{\nu}_1 C_H(r)}{(1-\alpha(r))}, t \geq 1. \quad (17)$$

At the same time, (13) and (14) imply that:

$$-\xi^*(r)\hat{c}_{Ht} = -\frac{Y(r)}{\psi P_L} \lambda^{t-1} \hat{a}_1 + \lambda^{t-1} \hat{\theta}_2^{max} \quad (18)$$

Together, (17) and (18) violate the assumption that exactly one component of $(\hat{a}_1, \hat{b}_2^{max}, \hat{\nu}_1)$ is non-zero.

Proof of Theorem 4

Let $C_H(r)$ and $\bar{C}_L(r)$ be state H and state L consumptions, in steady states indexed by the real interest rate $r < 0$ (as defined in the proof of Theorem 3). Define: $\xi(r) = \frac{P_H C_H(r)}{P_L} = \frac{P_H(1-\beta(1+r)(1-p))}{P_L \beta p \bar{\nu}(1+r)} > 0$.

Since $\hat{y}_t = 0$, we know that:

$$\bar{C}_L(r)\hat{c}_{Lt} = -\xi(r)\hat{c}_{Ht}. \quad (19)$$

The left-hand side can be rewritten as:

$$\bar{C}_L(r)\hat{c}_{Lt} = \frac{p}{P_L} \frac{\bar{\tau}(1+r)}{-r} (\hat{b}_t^{pub} - \hat{\pi}_t) - \frac{B^{max}}{(1+r)} \hat{r}_t + \hat{\theta}_{t+1}^{max}, t \geq 1 \quad (20)$$

where I have used the zero restrictions on $\hat{\tau}$ and \hat{y} . Define:

$$x_t = (\hat{b}_t^{pub} - \hat{\pi}_t), t \geq 1$$

where $\hat{b}_1^{pub} = 0$. Then (20) implies that:

$$\hat{c}_{Ht} = A_1(r)x_t + A_2(r)x_{t+1} + A_3(r)\hat{\theta}_{t+1}^{max} \quad (21)$$

where:

$$\begin{aligned} A_1(r) &= \frac{-B^{max} + \frac{p}{P_L} \frac{\bar{\tau}(1+r)}{r}}{\xi(r)} < 0 \\ A_2(r) &= \frac{B^{max}/(1+r)}{\xi(r)} > 0 \\ A_3(r) &= -\frac{1}{\xi(r)} \end{aligned}$$

Define:

$$z_{t+1} = x_{t+1} - \lambda x_t.$$

It follows that:

$$\hat{c}_{H,t+1} - \lambda \hat{c}_{Ht} = A_1(r)z_{t+1} + A_2(r)z_{t+2}$$

The government's flow budget constraint implies that:

$$\hat{r}_t = x_{t+1} - (1+r)x_t$$

where we again use the restriction that $\hat{\tau}_t = 0$. Recall the approximate Euler equation (4):

$$\hat{c}_{Ht} = -\hat{r}_t + \beta(1+r)(1-p)(\hat{c}_{H,t+1}) - \beta(1+r)p\bar{\nu}\lambda^t\hat{\nu}_1 C_H, t \geq 1 \quad (22)$$

Then by quasi-differencing (22), we obtain

$$A_1(r)z_{t+1} + A_2(r)z_{t+2} + z_{t+2} - (1+r)z_{t+1} - A_1(r)\alpha(r)z_{t+2} - A_2(r)\alpha(r)z_{t+3} = 0, t \geq 1$$

where $\alpha(r) = \beta(1+r)(1-p) < 1$. The characteristic function of this second-order difference equation is:

$$\Phi(L) = (A_1(r) - (1+r))L^2 + (A_2(r) - A_1(r)\alpha(r) + 1)L - A_2(r)\alpha(r).$$

It is readily seen that:

$$\begin{aligned} \Phi(0) &= -A_2(r)\alpha(r) < 0 \\ \Phi(\alpha(r)) &= \alpha(r) - (1+r)\alpha(r)^2 > 0 \\ \Phi(1) &= (A_1(r) + A_2(r))(1 - \alpha(r)) - r \\ &= (1 - \alpha(r)) \frac{\frac{p}{P_L} \frac{\bar{\tau}(1+r)}{r} - \frac{rB_{max}}{(1+r)}}{\xi(r)} - r \end{aligned}$$

The sign of $\Phi(1)$ is negative if r is sufficiently close to zero (while still being negative). In this case, the roots of the characteristic equation are both positive and less than one. The only stable solution to the second-order difference equation is to set $z_{t+1} = 0$ for all $t \geq 1$.

This implies that:

$$\begin{aligned} x_{t+1} &= \lambda^t x_1 \\ &= \lambda^t(-\hat{\pi}_1), t \geq 1 \end{aligned}$$

The log-linearized Phillips curve provides another connection between inflation and consumption:

$$\begin{aligned}\hat{\pi}_1 &= \kappa \hat{c}_{H1} + \beta_F \hat{\pi}_2 \\ &= \kappa \hat{c}_{H1} / (1 - \beta_F \lambda) \text{ (by solving forward).}\end{aligned}$$

This expression leads to a contradiction, given that exactly one of $\hat{\theta}_2^{max}$ or $\hat{\nu}_1$ is non-zero. Suppose first that $\hat{\nu}_1$ is zero. Then the Euler equation implies:

$$\hat{c}_{H1} = \frac{-\hat{\pi}_1(1+r) + \hat{\pi}_1\lambda}{(1 - \beta(1+r)(1-p)\lambda)}.$$

Suppose that r is sufficiently near zero that:

$$\frac{\kappa}{(1 - \beta_F)} < \frac{(1 - \beta(1+r)(1-p))}{-r}.$$

Then, for any $\lambda \in (0, 1)$:

$$\frac{\kappa}{(1 - \beta_F \lambda)} \frac{\lambda - (1+r)}{(1 - \beta(1+r)(1-p)\lambda)} \neq 1$$

and so:

$$\begin{aligned}\hat{\pi}_1 &= \frac{\kappa}{(1 - \beta_F \lambda)} \hat{c}_{H1} \\ &= \frac{\kappa}{(1 - \beta_F \lambda)} \frac{\lambda - (1+r)}{(1 - \beta(1+r)(1-p)\lambda)} \hat{\pi}_1\end{aligned}$$

implies $\hat{\pi}_1 = \hat{c}_{H1} = 0$. But this is a contradiction of (21), since $\hat{\theta}_2^{max}$ is non-zero.

Now suppose instead that $\hat{\theta}_2^{max}$ is zero. Then (21) implies that:

$$\begin{aligned}\hat{c}_{H1} &= -A_1(r)\hat{\pi}_1 - A_2(r)\lambda\hat{\pi}_1 \\ &= (-A_2(r)\lambda - A_1(r))\frac{(1 - \beta_F\lambda)}{\kappa}\hat{c}_{H1}.\end{aligned}$$

Suppose r is sufficiently near zero that:

$$(-A_2(r) - A_1(r))\frac{(1 - \beta_F)}{\kappa} > 1.$$

Then:

$$(-A_2(r)\lambda - A_1(r))\frac{(1 - \beta_F\lambda)}{\kappa} \neq 1$$

for all $\lambda \in (0, 1)$ and so $\hat{c}_{H1} = \hat{\pi}_1 = 0$. But this is a violation of the assumption that \hat{v}_1 is non-zero.

Proof of Proposition 2

Pick an arbitrary $\hat{\pi}_1$. Recall the flow budget constraint for agents in state L is:

$$\bar{C}_L\hat{c}_{Lt} = \frac{pB^{pub}}{P_L(1 + \Pi)}(\hat{b}_t^{pub} - \hat{\pi}_t), t \geq 1$$

and the government flow budget constraint is:

$$\begin{aligned}(\hat{b}_{t+1}^{pub} - \hat{\pi}_{t+1}) &= (1 + r)(\hat{b}_t^{pub} - \hat{\pi}_t), t \geq 1 \\ \hat{b}_1^{pub} &= 0\end{aligned}$$

It follows that:

$$\bar{C}_L\hat{c}_{Lt} = \frac{pB^{pub}}{P_L(1 + \Pi)}(1 + r)^{t-1}(-\hat{\pi}_1), t \geq 1$$

In period 1, we can use the NKPC and the resource constraint to solve for $(\hat{c}_{H1}, \hat{y}_1)$:

$$\hat{y}_1 Y = \frac{pB^{pub}}{(1 + \Pi)}(-\hat{\pi}_1) + P_H \bar{C}_H \hat{c}_{H1}$$

$$\hat{\pi}_1 = \kappa(\psi \hat{y}_1 + \hat{c}_{H1})$$

The solution to these two equations in two unknowns can be written as:

$$\hat{y}_1 = A_{Y1} \hat{\pi}_1$$

$$\hat{c}_1 = A_{H1} \hat{\pi}_1$$

where A_{Y1} and A_{H1} are constants. We now have the NKPC, the Euler equation, and the aggregate resource constraint to solve for $(\hat{\pi}_t, \hat{c}_{Ht}, \hat{y}_t)_{t=2}^{\infty}$:

$$\hat{\pi}_t = \kappa(\psi \hat{y}_t + \hat{c}_{Ht}), t \geq 2$$

$$\hat{c}_{Ht} = \hat{\pi}_{t+1} + \alpha \hat{c}_{H,t+1}, t \geq 1$$

$$\hat{y}_t = -\frac{pB^{pub}}{Y(1 + \Pi)}(\hat{\pi}_1)(1 + r)^{t-1} + \frac{P_H \bar{C}_H}{Y} \hat{c}_{Ht}, t \geq 2$$

Substituting the resource constraint into the NKPC, we get:

$$\hat{\pi}_t = \kappa(\psi(-\frac{pB^{pub}(1 + r)^{t-1} \hat{\pi}_1}{Y(1 + \Pi)} + \frac{P_H \bar{C}_H}{Y} \hat{c}_{Ht}) + \hat{c}_{Ht}), t \geq 2$$

and then into the Euler equation:

$$\hat{c}_{Ht} = \kappa \Gamma_0 (1 + r)^t \hat{\pi}_1 + \kappa \Gamma_1 \hat{c}_{H,t+1} + \alpha \hat{c}_{H,t+1}, t \geq 1$$

Here:

$$\begin{aligned}\Gamma_0 &= \frac{-\psi B^{pub} p}{Y(1 + \Pi)} \\ \Gamma_1 &= \frac{\psi P_H \bar{C}_H}{Y} + 1 \\ \hat{c}_{H1} &= A_{H1} \hat{\pi}_1\end{aligned}$$

Suppose κ is large enough that:

$$\frac{1}{(\alpha + \kappa \Gamma_1)} < 1$$

Then, for any $\hat{\pi}_1$,

$$\begin{aligned}\hat{c}_{H,t+1} &= \frac{\hat{c}_{Ht}}{(\alpha + \kappa \Gamma_1)} - \frac{(1+r)^t \Gamma_0 \hat{\pi}_1}{(\alpha + \kappa \Gamma_1)}, t \geq 1 \\ \hat{c}_{H1} &= A_{H1} \hat{\pi}_1\end{aligned}$$

is a bounded solution.

We can then solve for the other equilibrium elements as:

$$\begin{aligned}\hat{\pi}_t &= \kappa \left(\psi \left(-\frac{P_L (1+r)^{t-1} \hat{\pi}_1}{Y} + \frac{P_H \bar{C}_H}{Y} \hat{c}_{Ht} \right) + \hat{c}_{Ht} \right), t \geq 2 \\ \hat{y}_t &= -\frac{P_L (1+r)^{t-1} \hat{\pi}_1}{Y} + \frac{P_H \bar{C}_H}{Y} \hat{c}_{Ht}, t \geq 2 \\ \hat{c}_{Lt} &= \frac{p B^{pub}}{\bar{C}_L P_L (1 + \Pi)} (1+r)^{t-1} (-\hat{\pi}_1), t \geq 1 \\ \hat{b}_{t+1}^{pub} &= (1+r)^t (-\hat{\pi}_1) + \hat{\pi}_{t+1}.\end{aligned}$$

Proof of Proposition 3

If $\hat{y}_t = \hat{\pi}_t = 0$, then $\hat{c}_{Ht} = 0$. From the Euler equation, it follows that:

$$\hat{R}_t = -\Delta_{t+1}, t \geq 1$$

where $\Delta_{t+1} = \beta(1+r)p\bar{v}_{t+1}C_H$. Given this choice of interest rate policy, we can solve for transfers and debt levels to satisfy the budget constraint of agents in state L :

$$\hat{\tau}_t\bar{\tau} = -\frac{pB^{pub}}{P_L(1+\Pi)}\hat{b}_t^{pub} + \frac{B^{max}}{(1+r)}\hat{R}_t, t \geq 1$$

and the government's flow budget constraint:

$$\hat{b}_{t+1}^{pub} = (1+r)\hat{b}_t^{pub} + \hat{R}_t + \hat{\tau}_t\bar{\tau}(1+R)/B^{pub}, t \geq 1$$

By substituting the former budget constraint into the latter, we obtain:

$$\begin{aligned}\hat{b}_{t+1}^{pub} &= (1+r)(1-p/P_L)\hat{b}_t^{pub} + \hat{R}_t\left(1 + \frac{B^{max}(1+R)}{(1+r)B^{pub}}\right), t \geq 1 \\ \hat{b}_1^{pub} &= 0\end{aligned}$$

Since $|1-p/P_L| < 1$, the resulting bond sequence is bounded. We can then solve for transfers as:

$$\hat{\tau}_t\bar{\tau} = -\frac{pB^{pub}}{P_L(1+\Pi)}\hat{b}_t^{pub} + \frac{B^{max}}{(1+r)}\hat{R}_t.$$