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Trade, Leakage, and the Design of a Carbon Tax*

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Abstract

Climate policies vary widely across countries, with some countries imposing stringent emissions policies and others doing very little. When climate policies vary across countries, energy-intensive industries have an incentive to relocate to places with few or no emissions restrictions, an effect known as leakage. Relocated industries would continue to pollute but would be operating in a less desirable location. We consider solutions to the leakage problem in a simple setting where one region of the world imposes a climate policy and the rest of the world is passive. We solve the model analytically and also calibrate and simulate the model. Our model and analysis imply: (1) optimal climate policies tax both the supply of fossil fuels and the demand for fossil fuels; (2) on the demand side, absent administrative costs, optimal policies would tax both the use of fossil fuels in domestic production and the domestic consumption of goods created with fossil fuels, but with the tax rate on production lower due to leakage; (3) taxing only production (on the demand side), however, would be substantially simpler, and almost as effective as taxing both production and consumption, because it would avoid the need for border adjustments on imports of goods; (4) the effectiveness of the latter strategy depends on a low foreign elasticity of energy supply, which means that forming a taxing coalition to ensure a low foreign elasticity of energy supply can act as a substitute for border adjustments on goods.

Keywords: climate change, carbon taxes, leakage, border adjustments
JEL Codes: F18, H23, Q54

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1 Introduction

If nations adopt different prices on greenhouse gases, industries have an incentive to relocate where carbon prices are low. The result, known as leakage, is an increase in emissions in low-tax countries, undermining the efficacy of climate change policies while at the same time distorting the location of production. Concerns about leakage have been central to the design of carbon policies in the United States, the European Union, and other regions of the world.

The most common response to leakage is to impose what are known as carbon border adjustments or more simply border adjustments. Border adjustments combine taxes on the emissions associated with imports and rebates of prior taxes paid for exports. They shift the tax downstream, for example, from emissions from domestic production to emissions associated with domestic consumption. They are thought to help insulate the tax from leakage because, with border adjustments, the tax would be the same regardless of the location of production. Every carbon tax bill introduced in the current Congress includes border adjustments. The European Union has proposed a version for its cap and trade system. Border adjustments have also been subject to significant study. (For a recent review of the literature, see Böhringer et al (2022).)

Notwithstanding their prominence, it is still not clear whether, or the extent to which, border adjustments are effective, and how they compare to alternative approaches. To answer this question, we consider the design of a carbon tax in a simple setting where one region of the world imposes a carbon policy and the rest of the world does not. The taxing region sets policies to address climate change while taking into account the possibility of leakage. We solve the model to find the optimal (i.e., most efficient) choices for the taxing region, constraining those choices to fit with commonly proposed policies.[1] We also calibrate the model and simulate various policies to get a sense of the size of the effects.

We get the following results.

(1) *The most efficient policy imposes the tax on both the supply and the demand for fossil fuels.* The usual result in taxation is that in the absence of avoidance, or

[1] The approach builds on but simplifies the analysis in Kortum and Weisbach (2021). Two key differences are: (1) this paper restricts the set of policies that the taxing region can impose to those that are similar to existing or proposed policies while Kortum and Weisbach find the unrestricted optimal policy and (2) because we restrict choices to simpler policies, some restrictions on the model in Kortum and Weisbach (2021) can be relaxed.

evasion, the legal incidence of a tax doesn't alter its economic effects. As a result, in the absence of trade (or if the tax were global), a carbon tax could be imposed entirely upstream on extractors to minimize administrative costs, as suggested by Metcalf and Weisbach (2009). With trade and the possibility of leakage, this result no longer holds.

In particular, carbon taxes are commonly imposed on the use of fossil fuels in production or on the implicit consumption of fossil fuels embodied in goods, but in both cases, on the demand for fossil fuels. Taxes on the demand for fossil fuels lower their global price, inducing an increase in their use or consumption abroad. Taxes on the extraction of fossil fuels, that is, on their supply, by contrast, raise their global price, inducing an increase in extraction abroad. Our first result is that the optimal policy combines taxes on supply and demand, so that these effects offset, allowing the taxing region to control responses in the rest of the world.[2]

Incorporating this principle into the design of carbon taxes involves an almost trivial adjustment to current proposals yet offers potentially enormous gains in terms of the effectiveness of the tax. Many current carbon tax bills impose the tax nominally on extraction. They then impose border adjustments on energy (that is taxes on the imports of fossil fuels and rebate of taxes paid on exports of fossil fuels) at the same tax rate, to shift the tax downstream to domestic production.[3] If the border adjustments on energy were imposed at a lower rate than the nominal extraction tax, a portion of the tax would remain on extraction instead of being shifted downstream. Our simulations here show that this minor

[2]This result was implicit in the seminal paper of Markusen (1975). Contributions by Hoel (1994), Keen and Kotsogiannis (2014), and Balistreri, Kaffine, and Yonezawa (2019) develop the idea in the context of carbon taxes and carbon border adjustments. Nonetheless, it does not appear to have been incorporated into the design of carbon taxes. In fact, we are not aware of any carbon taxes (or cap and trade systems) that incorporate this principle. One possible explanation is that the models in those papers were restrictive. In particular, those papers assumed an economy with extraction and direct consumption of fossil fuels, such as for transportation or residential heating. They did not include a manufacturing or production sector of the economy. Leakage concerns, however, are largely focused on the production of goods. We show that the result applies in a more general economy with production and the possibility of leakage due to shifts in the location of production.

[3]H.R. 2307, The Energy Innovation and Carbon Dividend Act of 2021, is a typical example. It imposes a tax on oil refineries, coal mines, and any entity entering natural gas into the natural gas transmission system. In addition, importers of oil, coal, or natural gas must pay an import tariff, and exporters of these fuels receive a rebate of prior taxes paid. Both the import tariff and the export rebate are at the same rate as the underlying tax.

change has the potential to dramatically improve the effectiveness of the tax in reducing global emissions.

While this hybrid policy—combining a tax on extraction and a demand-side tax—is always desirable to maximize the efficiency of the tax, there remains the question of how to impose the demand-side tax. Should it be on production, consumption, or some combination?

(2) *On the demand side, to maximize efficiency, impose taxes on both production and consumption.* Our second result is that in the absence of administrative costs, imposing the tax both on emissions from domestic production and on emissions associated with domestic consumption maximizes the efficiency of unilateral carbon taxes when there is trade. The tax rate on production, however, should be lower than the tax rate on consumption to address concerns about leakage. If leakage is zero, the tax rate on production should equal the tax rate on consumption. If leakage is 100%, the tax rate on production should be zero.

This result answers the widely posed question of whether border adjustments should include export rebates in addition to import tariffs.[4] In particular, to implement this set of taxes, the taxing region starts with a nominal extraction tax and shifts part of it downstream to production by imposing border adjustments on imports and exports of energy (at a lower rate than the nominal extraction tax). To shift the tax further downstream to consumption, the taxing region imposes border taxes on imports of goods at the same rate as the border adjustments on energy. The corresponding rebate on exports of goods, which removes taxes on domestic production for goods sold abroad, is lower than the import tariff, leaving some part of the tax on domestic production. The rebate on export is not typically zero, however, because of the possibility of leakage. We show that the rebate on exports in fact scales linearly with leakage. If leakage were zero there would be no rebate on exports while if leakage were 100 percent there would be a full rebate (i.e., at the same rate as the border adjustment on imports).

(3) *Administrative costs may make border adjustments on goods prohibitively*

[4]The European Union's proposed Carbon Border Adjustment Mechanism (CBAM) would require importers of carbon intensive goods to purchase emission permits, but wouldn't rebate the permit cost for production of goods for export. In our analysis this policy (no rebates on exports) would be justified if there is little leakage due to customers outside of the European Union substituting away from EU exports. In contrast, H.R. 2307 (described in footnote 3) would provide full export rebates on carbon-intensive goods subject to border adjustments. In our analysis, this policy (full rebates on exports) would be justified if there is 100% leakage due to customers outside of the United States substituting away from US exports.

expensive. Administrative costs may outweigh the efficiency benefits of imposing taxes on both production and consumption. The key reason is that to impose taxes on emissions associated with domestic consumption, the taxing region must impose border adjustments on imports of goods. As discussed in Kortum and Weisbach (2017), doing so will be complex and expensive. Imposing a tax only on domestic production only requires border adjustments on imports and exports of energy, not goods. Border adjustments on energy are simple to impose. As a result, a tax on extraction and emissions from production is much simpler to impose than a tax on extraction and emissions associated with consumption.

Our simulations show that a combination of a tax on extraction and production often performs nearly as well as a tax on extraction and consumption. The gains from imposing border adjustments on goods is small. The key reason our simulations differ from those in the prior literature, which show modest but noticeable gains from border adjustments on goods, is that we simulate taxes on production and consumption as hybrid taxes that also include a tax on extraction (point (1) above) while the prior literature does not simulate hybrid systems. If the benefits from imposing border adjustments on goods is small, combining just a tax on extraction and a tax on domestic production may be the best policy.

The key parameter in this comparison is the foreign elasticity of energy supply. If this parameter is low, the combination of an extraction and production tax performs almost as well as taxes that also fall on consumption. If, however, the foreign elasticity of energy supply is high, the simpler combination of an extraction and production tax no longer performs well. In this case, shifting a portion of the tax downstream to consumption, via border adjustments on goods, may be worth consideration notwithstanding the administrative costs.

(4) *Ensuring that countries with a high elasticity of energy supply are in the taxing coalition may allow the use of a simpler tax system with fewer efficiency losses.* Building on point (3), one way to improve the effectiveness of the tax without having to impose border adjustments on goods is to ensure that the foreign elasticity of energy supply is low. To do this, the taxing coalition can work to include countries with a high elasticity of energy supply. In effect, this strategy—including countries with a high elasticity of energy supply in the taxing coalition—acts as a substitute for border adjustments.

We develop these results in four parts. Section 2 presents a model where individuals directly consume fossil fuels (for example, for transportation and residential heating), to illustrate the logic of combining taxes on supply and demand. Section 3 introduces trade in goods to allow us to study leakage. It shows that the results from Section 2 carry over to this more realistic setting and shows how the various demand-side policies compare to one another. Section 4 provides our numerical simulations. Section 5 discusses the results and concludes.

2 Trade in Energy but Not Goods

We start by reviewing and extending the theory of optimal carbon policy in a two-region world where energy is used directly in consumption, such as for transportation or residential heating, but is not embodied in traded goods. This case provides intuition for why efficient carbon policies act on both the supply and the demand side of the energy market. The same intuition carries over to the more general case. The setting is similar to Hoel (1994). In Section 3 we introduce traded goods that are produced in either country with energy as an input, which allows us to consider leakage.

2.1 Graphical Intuitions

To develop intuitions, we use a graphical illustration of how domestic taxes affect trade. We assume that there are two regions of the world, Home and Foreign, that extract fossil fuel energy in quantities Q_e and Q_e^* and directly consume quantities C_e and C_e^* (a * indicates Foreign). Home imposes a carbon policy while Foreign is passive. (In the figures that follow, we draw supply and demand intersecting at the same price for Home and Foreign.)

The left hand panel of Figure 1 shows the conventional supply and demand diagram for a good, here fossil fuel energy, and a tax, t_c, imposed on consumers. The interpretation is, equivalently, that the taxing region is the entire world or that there is no trade between the taxing region and the rest of the world (autarky). The tax creates a wedge between the amount consumers pay, $p_e + t_c$, and the amount sellers (here extractors of energy) receive, p_e. The equilibrium sets $Q_e = C_e$ given the wedge between extractors and consumers. As is conventional, in autarky it doesn't matter if the tax is imposed on extractors or consumers because the wedge between the two would be the same regardless.

If there is trade in energy, illustrated by the right hand panel of Figure 1, we can see that p_e can't be an equilibrium. If the price of energy goes down from p_0 to p_e, Foreign extractors would extract less energy while Foreign consumers would demand more, generating a net demand for Home exports, a demand which can't be met if $Q_e = C_e$.

Figure 1: Autarky

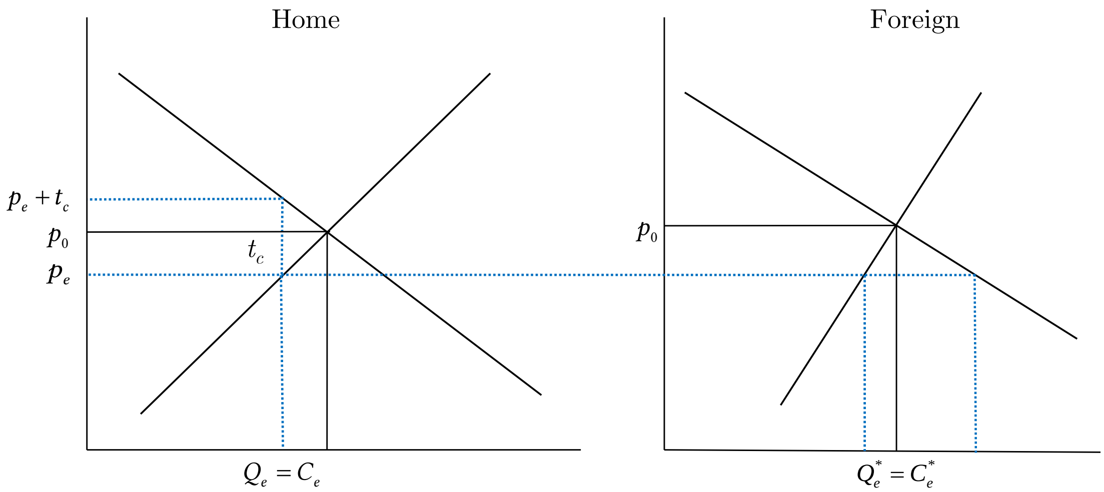


Figure 2 shows the equilibrium that would arise if Home taxes the consumption of energy and trades with Foreign. The price of energy, p_e, would still go down relative to the price without a tax, but it would go down less than it would in autarky. The lower price of energy would induce excess demand, $X_e = C_e^* - Q_e^*$, in Foreign (though less than illustrated in Figure 1), but Home would now have excess supply $C_e < Q_e$ at the equilibrium price. The price of energy would go down just enough that Home's excess supply, $X_e = Q_e - C_e$, matches Foreign's excess demand. At that price, global supply, $Q_e + Q_e^*$, equals global demand, $C_e + C_e^*$.

Figure 3 shows the equilibrium if Home instead chooses to tax extractors, imposing a tax of t_e instead of t_c at the same rate. The logic is the same as with the consumption tax except now the price of energy seen by Foreign extractors goes up. Foreign consumers demand less energy while Foreign extractors produce more, resulting in excess supply in Foreign. To be in equilibrium, the price of energy goes up less than it would in autarky, inducing excess demand in Home ($C_e > Q_e$). In equilibrium, the price of energy adjusts so that Home's excess demand equals Foreign's excess supply.

Figure 2: Trade with a consumption tax

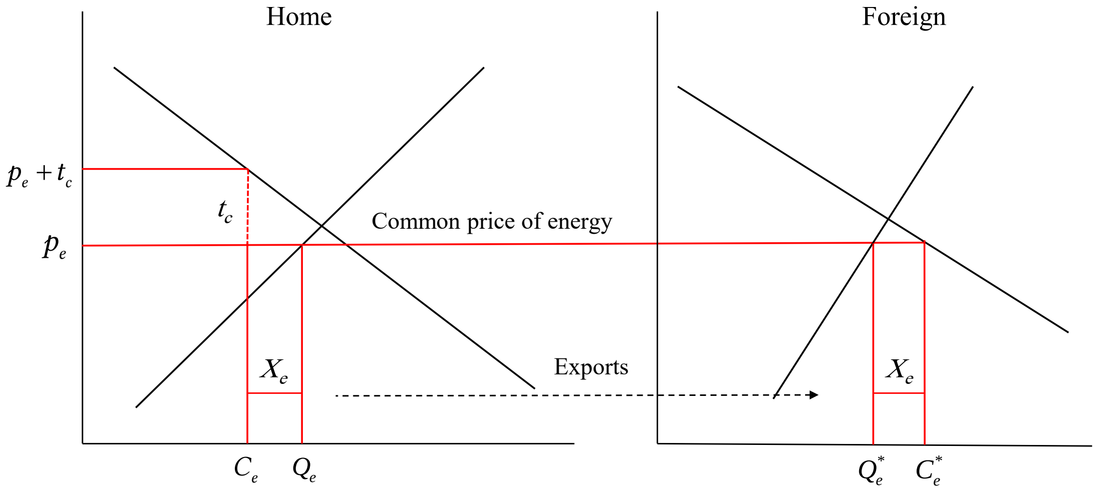
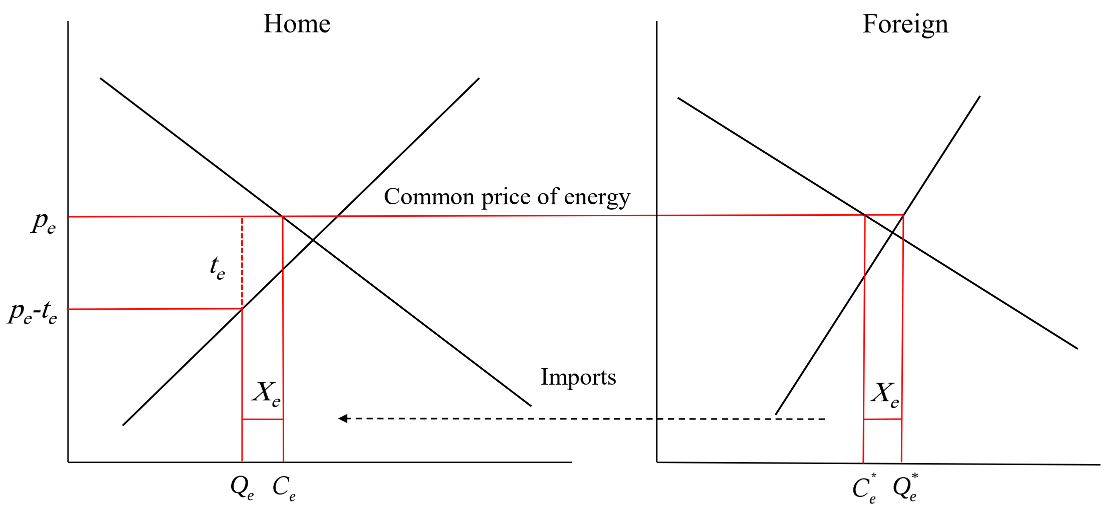


Figure 3: Trade with an extraction tax



The question, which we address immediately below, is how Home optimizes in this situation. As we will show, rather than choosing either a pure consumption tax or a pure extraction tax, Home mixes the two, which allows it to better control responses in Foreign.

2.2 Basic Model

To formalize the problem illustrated in Section 2.1, continue to assume that there are two regions, Home, which implements a carbon policy, and Foreign, which is passive. Home and Foreign are endowed with labor, L and L^*. They both extract carbon-based energy and trade it at price p_e. The labor required to extract a quantity of energy Q_e in Home is $c(Q_e)$ while to extract Q_e^* in Foreign requires $c^*(Q_e^*)$. Both c and c^* are strictly increasing, convex, and differentiable functions. A numeraire good, which we call services, is produced one-for-one with labor and is traded at price 1. Consumption of services in the two regions is constrained by the labor available to produce them, $C_s + C_s^* = L + L^* - c(Q_e) - c^*(Q_e^*)$. Consumption of energy is constrained by global extraction of energy, $C_e + C_e^* = Q_e + Q_e^* = Q_e^W$. We choose units so that global carbon emissions equal global extraction, $E = Q_e^W$.

Welfare in the two regions, U and U^*, depends positively on consumption of goods and services and negatively on global emissions. To keep the analysis transparent we assume that welfare is additively separable:

$$U = C_s + u(C_e) - \varphi E$$
$$U^* = C_s^* + u^*(C_e^*) - \varphi^* E,$$

where u and u^* are strictly increasing, concave, and differentiable functions. (Appendix A.2 shows that our key result, equation (2) below, holds without the assumption of additive separability.) We treat $\varphi^W = \varphi + \varphi^*$ as the marginal global social cost of carbon.

Foreign's energy supply curve, $Q_e^*(p_e)$, satisfies $c^{*\prime}(Q_e^*(p_e)) = p_e$, with slope $Q_e^{*\prime} > 0$. Foreign's energy demand curve, $C_e^*(p_e)$, satisfies $u^{*\prime}(C_e^*(p_e)) = p_e$, with slope $C_e^{*\prime} < 0$. (Derivatives appear as $f' = df/dx$.) Thus if p_e increases, Foreign extraction rises and Foreign consumption falls. Home indirectly influences Foreign extraction and consumption by manipulating the global price of energy through its carbon policy. If Home reduces C_e the energy price declines while if it reduces Q_e the energy price rises. We can think of Home as choosing p_e rather than choosing Q_e and C_e.

Following Keen and Kotsogiannis (2014), we assume that Home can't adopt policies that make Foreign worse off. All policies must be Pareto improvements. This approach eliminates terms-of-trade considerations and, in addition, helps motivate the assumption that Foreign remains passive. Within the model, it requires that Home transfer services to keep Foreign welfare at a threshold $\bar{U}^*$. With that transfer Foreign can consume services:

$$C_s^*(p_e, E) = \bar{U}^* + \varphi^* E - u^*(C_e^*(p_e)).$$

The particular value of $\bar{U}^*$ doesn't enter into our formulas for efficient policies. (Appendix B shows what changes if we replace this constraint on Foreign welfare with a trade-balance constraint.)

We break the problem into two parts. First, Home chooses its carbon policy to meet an arbitrary global emissions goal, $\bar{E}$. Later, Home optimizes its choice of $\bar{E}$. Home focuses on global emissions rather than domestic emissions because the harm is the same regardless of the source of emissions. Because of this focus it takes leakage into account, as we will see in the following section.[5]

For a fixed global emissions goal, Home's optimal policy is the solution to:

$$\max_{p_e} C_s + u(C_e) - \varphi \bar{E},$$

subject to labor market clearing and energy market clearing:

$$\begin{aligned} C_s &= L + L^* - c(\bar{E} - Q_e^*(p_e)) - c^*(Q_e^*(p_e)) - C_s^*(p_e, \bar{E}) \\ C_e &= \bar{E} - C_e^*(p_e). \end{aligned}$$

The first-order condition implies:[6]

$$(p_e - c')Q_e^{*\prime} = (u' - p_e)|C_e^{*\prime}|. \tag{1}$$

(The absolute value on the slope of Foreign demand makes all terms positive.)

To interpret this equation, define the extraction wedge as the difference between

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[5]In the Paris Agreement, nations set domestic emissions goals rather than global goals, but the joint aim was to produce a global goal. One of the problems with the Paris structure is that it creates an incentive to offshore emissions because doing so makes it easier for a nation to meet is stated emissions goal.

[6]Substituting the two constraints into Home's objective function, along with the expression for $C_s^*(p_e, \bar{E})$, and then differentiating with respect to p_e, the first-order condition is:

$$c' Q_e^{*\prime} - c^{*\prime} Q_e^{*\prime} + u^{*\prime} C_e^{*\prime} - u' C_e^{*\prime} = 0.$$

Applying the competitive-market conditions in Foreign, $c^{*\prime} = u^{*\prime} = p_e$, the first-order condition reduces to (1).

the marginal cost of extracting energy in Foreign and Home, $p_e - c'$, and the consumption wedge as the difference between the marginal value of consuming energy in Home and Foreign, $u' - p_e$. A higher extraction wedge, corresponding to lower Q_e, raises the energy price while a higher consumption wedge, corresponding to lower C_e, reduces the energy price. Either wedge represents a global inefficiency. The optimal balance is for Home to equate the product of the price response of Foreign extraction and the extraction wedge to the product of the price response of Foreign consumption (in absolute value) and the consumption wedge. In this way Home minimizes the global inefficiency due to its inability to separately set Foreign extraction and consumption. Crucially, both wedges must be positive since Foreign supply is increasing and Foreign demand is decreasing in the energy price.

This condition will be satisfied with a combination of taxes in Home: an extraction tax equal to the extraction wedge and a consumption tax equal to the consumption wedge. Since both wedges are positive so are both taxes: it is optimal for Home to tax both the demand side and the supply side of the energy market. Rearranging equation (1), the relative tax rates satisfy:

$$\frac{t_e}{t_c} = \frac{|C_e^{*\prime}|}{Q_e^{*\prime}}. \tag{2}$$

Equation (2) has a standard elasticity-type explanation, which is that Home wants to avoid taxes on highly responsive items. A tax on the the demand for energy, t_c, lowers the energy price seen in Foreign, causing Foreign demand to go up. The more responsive Foreign demand is to the price of energy, the lower the tax on domestic consumption. Similarly, a tax on domestic extraction increases the price of energy in Foreign, causing an increase in extraction there. The more responsive Foreign supply is to the price of energy, the lower the tax on domestic extraction. The optimal ratio of the taxes balances these concerns.[7]

[7] Variations on the figures from the previous subsection illustrate the advantage of following this principle in two extreme cases. The first case is a vertical Foreign demand curve ($|C_e^{*\prime}| = 0$), so that Foreign consumption C_e^* is fixed. Equation (2) implies that a pure consumption tax is optimal in this case. Sure enough, the after-tax price is higher in Figure 2 than in Figure 1, resulting in lower C_e and hence lower global emissions with trade (under this optimal policy) than in autarky. The second case is a vertical Foreign supply curve ($Q_e^{*\prime} = 0$), so that Foreign extraction Q_e^* is fixed. Equation (2) implies that a pure extraction tax is optimal in this case. Sure enough, the after-tax price is lower in Figure 3 than in Figure 1, resulting in lower Q_e and hence lower global emissions with trade (under this optimal policy) than in autarky. It is

If Home optimizes $\bar{E}$, the sum of the taxes equals the marginal global social cost of carbon: $t_e + t_c = \varphi^W$.[8] Home's taxes satisfy its part of the optimality condition for a globally harmonized tax (see Appendix A.1) even though Foreign doesn't tax carbon. The individual taxes are then:

$$
\begin{aligned}
t_e &= \varphi^W \frac{|C_e^{*\prime}|}{Q_e^{*\prime} + |C_e^{*\prime}|} \\
t_c &= \varphi^W \frac{Q_e^{*\prime}}{Q_e^{*\prime} + |C_e^{*\prime}|},
\end{aligned}
\tag{3}
$$

with their sum equal to the global social cost of carbon φ^W and their ratio satisfying (2). The intuitions for these values are the same as for Equation (2). Looking at the expression for t_e, the higher $Q_e^{*\prime}$, the lower the value of t_e. Similarly, the higher the value of $|C_e^{*\prime}|$, the lower the value of t_c.[9]

Figure 4 illustrates. Equation (2) requires that the consumption tax multiplied by the slope of Foreign's demand curve equal the extraction tax multiplied by the slope of Foreign's supply curve. The height of each rectangle is the tax. The ratio of the widths is equal to the ratio of the slopes of the supply and demand curves, $Q_e^{*\prime}$ and $|C_e^{\prime}|$ (with p_e on the y axis, slopes are read off the x axis).[10] At the optimum, the mix of t_e and t_c is set so that the size of the two rectangles are the same, as shown in Figure 4. Because we drew supply steeper than demand, i.e. $Q_e^{*\prime} < |C_e^{\prime}|$, the optimal extraction tax in this illustration exceeds the optimal consumption tax.

easy to see that contradicting (2), by applying a pure extraction tax in the first case or a pure consumption tax in the second, would yield smaller reductions in global emissions (even smaller than than under autarky.

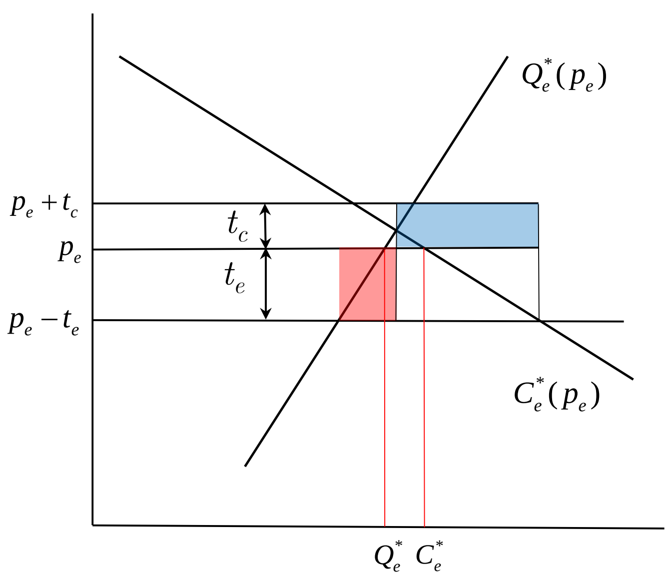
[8]Substituting the two constraints into Home's objective function and then differentiating with respect to $\bar{E}$, the first-order condition is:

$$-c' - \varphi^* + u' - \phi = (p_e - c') + (u' - p_e) - \varphi^W = 0.$$

[9]These results change in two ways if we replace the Foreign welfare constraint with trade balance (see Appendix B). First, equation (2) becomes $t_e Q_e^{*\prime} = t_c|C_e^{*\prime}| + X_e$. When Home's net exports of energy, $X_e = Q_e - C_e$, are higher, it relies more on the extraction tax in order to improve its terms of trade. Second, Home's optimal emissions goal now implies that $t_e + t_c$ equals φ rather than φ^W. Home no longer considers the marginal social cost of emissions in Foreign.

[10]This is because the widths are the base of the triangles underneath the supply and demand curves, above $p_e - t_e$, and centered on the intersection of supply and demand.

Figure 4: Optimal extraction and consumption taxes



2.3 Policy Implementation

The taxes described in (2) and (3) are *effective* taxes. As noted, current carbon tax bills in the United States often begin with a *nominal* tax τ on domestic extraction. They then impose taxes on US energy imports and rebate prior taxes paid on US energy exports, which we call border adjustments on energy and denote by β_e. Border adjustments on energy shift the nominal tax τ on extraction downstream. In the present model, with no manufacturing sector, border adjustments on energy shift the tax all the way downstream to consumption.[11]

Current carbon tax bills set $\beta_e = \tau$ so that they shift the entire tax downstream, setting the effective tax on extraction to zero. The basic model here says that's not optimal. To get to the optimal policy Home should impose the border adjustments on energy at a lower rate than the underlying extraction tax, i.e. $\beta_e < \tau$. A partial border adjustment shifts only a portion of the tax downstream to consumption. To implement the optimal effective taxes t_e and t_c in (3) Home would impose a nominal extraction tax at rate $\tau = t_e + t_c = \varphi^W$ and border adjustments at rate

[11]If we add goods production, as we do in Section 3, border adjustments on energy only shift the tax to producers. As we discuss in Section 3.5, in that case border adjustments on goods are needed to shift the tax to consumption.

$\beta_e = t_c$ on energy imports and exports. This strategy of a nominal tax and border adjustments leaves the optimal effective tax on extraction at rate $t_e = \tau - \beta_e$.

2.4 Policy Coordination

Up to this point we have treated the Foreign region as passive. It has no carbon taxes and doesn't consider introducing any when Home imposes them. We now consider the optimal policy for Home if Foreign already has carbon taxes. We continue to assume that Foreign is strategically passive; it doesn't adjust its tax rates in response to Home's policy.

Let t_e^* be Foreign's extraction tax, t_c^* its consumption tax, and $\tilde{\varphi}^*$ the sum of the two. Home's optimal policy, including its optimal emissions goal, turns out to be a simple generalization of (3):[12]

$$\begin{aligned} t_e &= t_e^* + (\varphi^W - \tilde{\varphi}^*)\frac{|C_e^{*\prime}|}{Q_e^{*\prime} + |C_e^{*\prime}|} \\ t_c &= t_c^* + (\varphi^W - \tilde{\varphi}^*)\frac{Q_e^{*\prime}}{Q_e^{*\prime} + |C_e^{*\prime}|}. \end{aligned} \tag{4}$$

Home's optimal tax rates mimic the tax rates in Foreign with an adjustment based on the differences in the overall level of carbon taxation in Home and Foreign, $\varphi^W - \tilde{\varphi}^*$. This adjustment is optimally split between Home's extraction tax and its consumption tax, in the same ratio as (2). If it happens that Foreign chooses $\tilde{\varphi}^* = \varphi^W$ then Home simply matches the tax rates of Foreign and the global optimum, with harmonized taxes, is obtained (as in Appendix A.1).

Implementing these policies, whether harmonized or not, is simple. Foreign imposes a nominal tax on extraction at rate $\tilde{\varphi}^*$ with a border adjustment on its imports and exports of energy at rate $\beta_e^* = t_c^*$. Likewise, Home imposes a nominal tax on extraction at rate φ^W with a border adjustment on its imports and exports of energy at rate $\beta_e = t_c$, where t_c is given by the second equation in (4).

[12]The first-order condition for Home's optimal global energy price, from footnote 6, is unchanged. But now the competitive-market conditions in Foreign are $c^{*\prime} = p_e - t_e^*$ and $u^{*\prime} = p_e + t_c^*$. Combining these results with Home's competitive-market conditions ($c' = p_e - t_e$ and $u' = p_e + t_c$) gives:

$$(t_e^* - t_e)Q_e^{*\prime} + (t_c^* - t_c)C_e^{*\prime} = 0.$$

The first-order condition for Home's optimal emissions goal, from footnote 8, is also unchanged so that $t_e + t_c = \varphi^W$ (while $t_e^* + t_c^* = \tilde{\varphi}^*$). Substituting in these results, the equation above can be solved for t_e and t_c as in (4).

3 Trade in Energy and Goods

A key concern for unilateral carbon taxes is how those taxes affect the location of production. In particular, a unilateral carbon tax on production might cause production, and the resulting emissions, to shift offshore, an effect known as leakage. The basic model in Part 2, however, had only extraction and consumption of energy. It did not include the use of energy in production of traded goods.

We now extend the model to include production in both regions. The production sector in each region manufactures tradable final goods using carbon-based energy. Goods are produced with varying levels of efficiency in different locations using a combination of labor and energy. They are traded based on Ricardian comparative advantage. Taxes on production alter the regions' comparative advantage, generating leakage.

In Kortum and Weisbach (2021) we derive the optimal carbon policy for Home in such a setting, without restricting the choices available to Home. Here, in order to connect directly with current policy proposals, we restrict Home to particular combinations of taxes: (i) the optimal combination of an extraction and consumption tax, (ii) the optimal combination of an extraction and production tax, and (iii) the optimal combination of all three. Some of the structure in our earlier analysis isn't relevant for these simpler policies, so we leave it out. (Appendix D brings back the structural assumptions, which are used in the numerical illustrations of Section 4.)

3.1 Model Structure

We retain the welfare expressions from the basic model, but replace utility from consuming carbon-based energy with utility from consuming goods, both domestically produced and imported.[13] These goods embody the energy used in their production in either Home or Foreign.

To trace and possibly tax emissions from production and the implicit emissions associated with consumption, we denote the implicit consumption of energy embodied in goods as C_e with a superscript denoting the source of the good and the location of consumption: C_e^d is energy in goods produced domestically

[13]To keep this extended model tractable we drop the direct consumption of energy underlying the basic model. Retaining energy consumed directly (together with energy used to produce tradable goods) would be a useful extension to the quantitative illustrations in Section 4.

Table 1: Carbon Matrix, OECD, 2015

	Home	Foreign	Total
Home	$C_e^d = 11.3$	$C_e^m = 2.5$	$C_e = 13.8$
Foreign	$C_e^x = 0.9$	$C_e^f = 17.6$	$C_e^* = 18.5$
Total	$G_e = 12.2$	$G_e^* = 20.1$	$C_e^W = 32.3$
Extraction	$Q_e = 8.6$	$Q_e^* =$23.7	$Q_e^W = 32.3$

Units: gigatons of CO_2.

and consumed domestically, C_e^m is energy in goods Home imports, C_e^x is energy in goods Home exports, and C_e^f is energy in goods Foreign both produces and consumes. The total quantity of energy associated with goods consumed in Home is $C_e = C_e^d + C_e^m$. Similarly $C_e^* = C_e^f + C_e^x$. We can also account for all energy used in producing goods in Home, $G_e = C_e^d + C_e^x$ and in Foreign $G_e^* = C_e^f + C_e^m$.

Table 1 shows how these values relate to one another, with rows showing emissions by location of consumption and columns by location of production. Table 1 shows the values for the year 2015 under the assumption that Home is the OECD.[14] As we will discuss in Section 4, we use these values to calibrate our model for simulation. Global emissions in 2015 were 32.3 $GtCO_2$ and of that, the OECD emitted 12.2 $GtCO_2$. Most of that, 11.3 $GtCO_2$ was consumed domestically. The OECD imported 2.5 $GtCO_2$ so that it consumed 13.8 $GtCO_2$.

In the basic model of Section 2.2 we started with a planner in Home setting quantities (implicitly via its choice of p_e and explicitly via its choice of $\bar{E}$). Here we directly model a competitive market economy with a policy maker choosing tax rates. In addition to an extraction tax, t_e, we will need to consider three demand-side taxes corresponding to the three sources of demand that Home can influence through its taxes: (i) a tax t_d on the energy C_e^d used to produce goods in Home for the domestic market, (ii) a tax t_m on the energy C_e^m used to produce the goods Home imports, and (iii) a tax t_x on the energy C_e^x used to produce Home exports. The consumption tax considered in Section 3.2 restricts $t_d = t_m = t_c$ and $t_x = 0$. The production tax considered in Section 3.3 restricts $t_d = t_x = t_p$

[14]Table 1 corresponds to Table 5 in Kortum and Weisbach (2021). The source is the Trade in Embodied CO_2 database made available by the OECD (2019).

and $t_m = 0$. The combination of all three taxes considered in Section 3.4 removes these restrictions, allowing arbitrary combinations of production and consumption taxes.

Note that these taxes are effective taxes. While effective taxes are unique, there are a number of different ways to implement them. In particular, instead of directly imposing the effective taxes, Home could start with a nominal extraction tax and impose border adjustments on imports and exports of energy and of goods. Various combinations of border adjustments produce each of the policies we consider. We defer the discussion of implementation to Section 3.5, and here work with effective taxes.

Because we are working with prices and taxes, it is convenient to use indirect utility functions, which give the maximum welfare that a region can attain given spending and prices. Those prices are the effective cost of the energy embedded in the goods that are consumed. They are given by $p_e^d = p_e + t_d$, $p_e^m = p_e + t_m$, $p_e^x = p_e + t_x$, and $p_e^f = p_e$.[15] Production and trade in services means wages (and the price of services) are 1 in both regions.

Exploiting the separability assumptions of the basic model, welfare becomes:

$$
\begin{aligned}
U &= Y + \tilde{u}(p_e^d, p_e^m) - \varphi E \\
U^* &= Y^* + \tilde{u}^*(p_e^f, p_e^x) - \varphi^* E.
\end{aligned}
$$

The tilde on $\tilde{u}$ and $\tilde{u}^*$ distinguishes indirect utility from direct utility u and u^* in the basic model. Here Y and Y^* represent the levels of spending in Home and Foreign.

Spending in each region comes from labor income, rents to the energy sector, tax revenue, and transfers (from Home to Foreign): $Y = L + R_e + R_t - T$ and $Y^* = L^* + R_e^* + R_t^* + T$. Home's tax revenue is $R_t = t_e Q_e + t_d C_e^d + t_m C_e^m + t_x C_e^x$. We usually assume that Foreign has no carbon policy so gets no tax revenue, $R_t^* = 0$. (We relax that assumption in Section 3.6.) Rents to the energy sector in Home are $R_e = (p_e - t_e)Q_e - c(Q_e)$ while in Foreign $R_e^* = p_e Q_e^* - c^*(Q_e^*)$. As in the basic model, we assume that the level of transfers keep Foreign welfare at $\bar{U}^*$, so $T = \bar{U}^* + \varphi^* E - \tilde{u}^*(p_e^f, p_e^x) - L^* - R_e^*$.

[15] The carbon tax that Home imposes on imports indirectly raises the cost of energy for producers in Foreign serving consumers in Home. Foreign producers purchase energy at price p_e, but they anticipate that to sell goods in Home they will pay the tax t_m for each unit of energy they use. Their effective cost of energy is thus $p_e^m = p_e + t_m$.

Substituting these sources of spending into Home welfare and dropping constants, Home's objective is to choose taxes that maximize the objective:

$$\mathcal{L} = R_e + R_e^* + R_t + \tilde{u}(p_e^d, p_e^m) + \tilde{u}^*(p_e^f, p_e^x) - \varphi^W E. \tag{5}$$

Recall that global emissions equal global extraction, $E = Q_e^W$.[16] In solving this maximization problem, the policy maker accounts for how its choice of taxes affects the energy price and quantities of energy supplied and demanded in the global energy market. (When there is no ambiguity, we denote the response of any variable y to the energy price by $y' = \partial y / \partial p_e$.)

3.2 Taxing Extraction and Consumption

Our first application of this model is to solve for the optimal combination of an extraction tax t_e and a consumption tax t_c. Under a consumption tax $p_e^d = p_e^m = p_e + t_c$ for goods consumed in Home and $p_e^f = p_e^x = p_e$ for goods consumed in Foreign, no matter where they are produced.

Home maximizes the objective $\mathcal{L}$ in (5) by choosing t_e and t_c (the full derivation is in Appendix C.1). Taking the first-order conditions yields $t_e + t_c = \varphi^W$ and equation (2) from the basic model. Together they imply (3) from the basic model. Adding trade in goods that embody carbon-based energy doesn't matter when we limit the policy to consist of an extraction tax and a consumption tax. Home still uses both taxes, with their ratio being the relative price sensitivity of implicit energy demand to energy supply in Foreign. The sum of the tax rates remains equal to the Pigouvian global externality.

While the bottom line looks like the solution to the basic model, there is a key distinction. In Section 2 we found that the combination of an extraction tax and a consumption tax was optimal. Here that's not necessarily true since we *imposed* a carbon policy consisting of only those two taxes (and then found a condition for their optimal magnitudes). In what follows we explore other taxes, and how they might replace or be combined with an extraction and consumption tax.

[16] If we were to introduce an arbitrary global emissions goal, as in the basic model, we would replace (5) with a Lagrangian incorporating a constraint $\bar{E}$ on emissions. In the formulas that follow φ^W would be replaced with the Lagrange multiplier on the constraint.

3.3 Taxing Extraction and Production

Suppose that instead of an extraction and consumption tax Home is restricted to an extraction tax and a production tax, t_p. Under a production tax $p_e^d = p_e^x = p_e + t_p$ for goods produced in Home and $p_e^f = p_e^x = p_e$ for goods produced in Foreign, no matter where they are consumed.

While we didn't need to consider leakage in the combination of an extraction tax and a consumption tax, with a production tax we do. Unlike with a tax on consumption, a tax on Home's production reduces its comparative advantage, causing a shift in the location of production and hence leakage. Leakage is conventionally defined as the increase in Foreign emissions relative to the decrease in domestic emissions, for a given change in t_p:

$$\Lambda = -\frac{\partial G_e^*/\partial t_p}{\partial G_e/\partial t_p} > 0. \tag{6}$$

Note that there are two sources of leakage captured by Λ.[17] Foreign can increase its use of energy to serve its own consumers: C_e^f might go up relative to C_e^x in response to an increase in t_p. In addition, Home can increase its imports from Foreign: C_e^m might go up relative to C_e^d in response to an increase in t_p. With only a production tax, Home is subject to both sources of leakage. As we will see, if Home is also able to tax consumption (i.e. taxing imports), it can eliminate the latter source, leaving only the increase in C_e^f relative to C_e^x, or what we will call "Foreign leakage."

Home maximizes the objective $\mathcal{L}$ in (5) by choosing t_e and t_p (the full derivation is in Appendix C.2). Evaluating the first-order conditions yields the analog of equation (2), now for the optimal ratio of an extraction tax to a production tax:

$$\frac{t_e}{t_p} = \frac{|G_e^{*\prime}| + \Lambda |G_e^{\prime}|}{(1-\Lambda) Q_e^{*\prime}}. \tag{7}$$

The energy-price sensitivity of Foreign production, $|G_e^{*\prime}|$, tilts the optimum toward an extraction tax in (7), similar to how $|C_e^{*\prime}|$ does so in (2). Furthermore, greater leakage, as measured by Λ, makes it optimal to tax extraction at a higher rate relative to production. The reason is that with more leakage, the production tax

[17] Our use of a Greek letter does not imply that the leakage rate is an invariant constant. It will typically vary with the production tax rate, for example.

becomes less effective in lowering global emissions.

The first order conditions also imply:

$$t_e + \frac{t_p}{1-\Lambda} = \varphi^W. \tag{8}$$

As leakage goes up the (unweighted) sum of the two taxes goes down. The policy becomes less effective with greater leakage, and Home responds by taxing less. The optimal extraction-production tax loses the feature that the sum of the taxes imposed by Home—here the wedge it creates between the after-tax price paid by its producers and the after-tax price received by its extractors—is equal to the global social cost of carbon, as in the extraction-consumption policy. Leakage limits Home's willingness to tax carbon.

Combining (7) and (8) gives the analog of (3) for an extraction-production tax:

$$\begin{aligned} t_e &= \varphi^W \frac{|G_e^{*\prime}| + \Lambda |G_e'|}{Q_e^{*\prime} + |G_e^{*\prime}| + \Lambda |G_e'|} \\ t_p &= \varphi^W \frac{(1-\Lambda) Q_e^{*\prime}}{Q_e^{*\prime} + |G_e^{*\prime}| + \Lambda |G_e'|}. \end{aligned} \tag{9}$$

Greater leakage not only tilts taxes toward extraction, it actually raises the extraction tax rate (while lowering the production tax rate by even more).

3.4 Taxing Extraction, Consumption, and Production

Finally, suppose Home is free to choose t_d, t_x, and t_m independently (together with an extraction tax, t_e).[18] The effective cost of energy is $p_e + t_d$ for Home producers supplying the domestic market, $p_e + t_m$ for Foreign producers supplying imports to Home, and $p_e + t_x$ for Home exporters. Home maximizes the objective $\mathcal{L}$ in (5) by choosing t_e, t_d, t_m, and t_x (the full derivation is in Appendix C.3).

Although Home has the flexibility to tax imports differently than domestically produced goods, it chooses not to. The first-order conditions for t_m and t_d imply $t_d = t_m$. Home acts as if it's choosing a consumption tax: $t_d = t_m = t_c$.

Because this policy involves elements of a production tax as well, in the form of t_x, we need to introduce leakage again. Due to the consumption-tax element

[18] This freedom still does not allow Home to reach the optimal policy found in Kortum and Weisbach (2021). That policy also includes per-unit export subsidies for exported goods. We ignore that feature of an optimal policy in this paper.

that we just derived, however, there is no leakage in serving Home consumers – producers in both Home and Foreign face the same price of energy when selling in Home. If $t_x > 0$, however, Foreign producers still have an advantage relative to Home producers when serving Foreign consumers, resulting in Foreign leakage (denoted with a *). Foreign leakage is the increase in Foreign production to serve Foreign consumers relative to the decrease in Home production to serve Foreign consumers, both for a given change in t_x:

$$\Lambda^* = -\frac{\partial C_e^f / \partial t_x}{\partial C_e^x / \partial t_x} > 0. \tag{10}$$

The first-order conditions for t_e, t_c, and t_x yield two tax ratios. The first is the analog of that for the extraction-production policy (7):

$$\frac{t_e}{t_x} = \frac{|C_e^{f\prime}| + \Lambda^* |C_e^{x\prime}|}{(1 - \Lambda^*) Q_e^{*\prime}}.$$

The second is the analog of that for the extraction-consumption policy (2):

$$\frac{t_e}{t_c} = \frac{|C_e^{f\prime}| + \Lambda^* |C_e^{x\prime}|}{Q_e^{*\prime}}. \tag{11}$$

The numerator of (11) is less than $|C_e^{*\prime}|$ as long as $\Lambda^* < 1$. With Foreign leakage below 100% it is optimal to raise the consumption tax relative to the extraction tax, compared to the case for an extraction-consumption tax (2). Keeping a tax on Home's exports means that Foreign consumers are taxed, reducing the need for Home to use the extraction tax to lower the tax on Home consumers.

These first-order conditions also imply $t_e + t_c = \varphi^W$ as for an extraction-consumption tax. Combining all these results, the optimal policy is:

$$\begin{aligned} t_e &= \varphi^W \frac{|C_e^{f\prime}| + \Lambda^* |C_e^{x\prime}|}{Q_e^{*\prime} + |C_e^{f\prime}| + \Lambda^* |C_e^{x\prime}|} \\ t_c &= \varphi^W \frac{Q_e^{*\prime}}{Q_e^{*\prime} + |C_e^{f\prime}| + \Lambda^* |C_e^{x\prime}|} \\ t_x &= (1 - \Lambda^*) t_c. \end{aligned} \tag{12}$$

While we refer to it as an extraction-production-consumption tax, the production component is only present in the tax on exports, t_x.

Two restricted versions of this policy are insightful. The first is to simply set $t_x = 0$, ignoring the corresponding first-order condition. The resulting problem is equivalent to taxing only extraction and consumption, as in Section 3.2. It emerges as optimal here if $\Lambda^* = 1$. If Foreign leakage is 100%, taxing exports doesn't reduce global emissions so it is best to set $t_x = 0$. The second is to set $t_x = t_c$. This condition would be optimal if $\Lambda^* = 0$. If there were no Foreign leakage, there would be no reason to lower the tax on exports relative to the tax on domestic consumption.[19]

3.5 Policy Implementation

In Section 2.3 we noted that if we start with a nominal extraction tax of τ, adding partial border adjustments $0 < \beta_e < \tau$ on the imports and exports of energy shifts a portion of the tax downstream. In the basic model (i.e., without manufacturing) these border adjustments shift β_e of the tax all the way downstream to consumers of energy leaving an effective tax $t_e = \tau - \beta_e$ on extraction.

When we add manufacturing and trade in goods, border adjustments on energy shift the tax to producers who use energy to manufacture goods. Home needs additional border adjustments on the imports and exports of goods to shift the tax to the implicit consumption of carbon. Because the extraction-production-consumption policy treats imports and exports of goods differently

[19]This second policy might also arise because of legal or policy constraints. For example, trade law might require exports to be taxed at the same rate as domestic consumption. Such considerations likely influenced the design of the EU's proposed CBAM (see footnote 4). If Home is constrained to set $t_x = t_c$, it should optimize over $\tilde{t}_c$ (a combined consumption-production tax, so that $t_x = t_c = \tilde{t}_c$) and t_e. To solve the resulting first-order conditions requires introducing yet a third measure of leakage:

$$\tilde{\Lambda}^* = -\frac{\partial C_e^f / \partial \tilde{t}_c}{\partial C_e / \partial \tilde{t}_c + \partial C_e^x / \partial \tilde{t}_c},$$

so that $\tilde{\Lambda}^* < \Lambda^*$. The solution for optimal tax rates is:

$$t_e = \varphi^W \frac{|C_e^{f\prime}| + \tilde{\Lambda}^* |C_e' + C_e^{x\prime}|}{Q_e^{*\prime} + |C_e^{f\prime}| + \tilde{\Lambda}^* |C_e' + C_e^{x\prime}|}$$
$$\tilde{t}_c = \varphi^W \frac{(1 - \tilde{\Lambda}^*) Q_e^{*\prime}}{Q_e^{*\prime} + |C_e^{f\prime}| + \tilde{\Lambda}^* |C_e' + C_e^{x\prime}|}.$$

The form of these expressions is familiar from the solution for taxing extraction and production, in Section 3.3.

(that is, $t_p \neq t_c$), Home needs separate border adjustments to implement this policy: a border adjustment on the energy content of imports of goods (β_m), and a border adjustment on the energy content of exports of goods (β_x). With these three border adjustments (β_e, β_m, and β_x) and a nominal tax on the extraction of energy, (τ), Home can implement any of the three hybrids considered in this paper. Table 2 shows the mapping, specific to each policy, from effective tax rates to a nominal tax on extraction together with border adjustments, that achieves the same outcome.

Table 2: Policy Implementation with Border Adjustments

Taxes on:	τ	β_e	β_m	β_x
extraction and production	$t_e + t_p < \varphi^W$	t_p	0	0
extraction and consumption	$t_e + t_c = \varphi^W$	t_c	t_c	t_c
extraction, production, and consumption	$t_e + t_c = \varphi^W$	t_c	t_c	$t_c - t_x$

τ is the nominal extraction tax, β_e is the border adjustment on energy, β_m (imports) and β_x (exports) are border adjustments on goods.

To implement the extraction-production hybrid in expression (9), the first row of Table 2 shows that Home would impose a nominal extraction tax of $\tau = t_e + t_p$ and border adjustments on imports and exports of energy at a lower rate of $\beta_e = t_p$. This shifts t_p downstream to production, leaving $\tau - t_p$ on extraction. Because this border adjustment is only on energy, it would be simple to implement – energy imports and exports are already highly regulated and monitored. It would, moreover, only require a slight rewording of existing legislative proposals, namely reducing the magnitude of the border adjustment on energy from τ to β_e (as well as eliminating any border adjustments on goods found in the legislation).

To implement the extraction-consumption hybrid in expression (3), Home would impose a nominal extraction tax of $\tau = t_e + t_c$ and border adjustments on imports of energy at a lower rate of $\beta_e = t_c$, much like for the extraction-production case. To shift the tax downstream to consumption, however, Home would also have to impose border adjustments on imports and exports of goods (β_m and β_x respectively) also at rate t_c. This leaves a tax of $\tau - t_c$ on extraction, and no tax on production. As we discuss in Kortum and Weisbach (2017), computing accurate border adjustments on goods is expensive and complex because there is no straightforward way to determine the implicit energy content of imports (or even exports). Any resulting border adjustments are likely to be inaccurate. Whether it is worth to incur these costs to impose border adjustments on goods depends on whether, and if so by how much, the extraction-consumption hybrid outperforms the extraction-production hybrid, an issue we explore in our quantitative illustrations of Section 4.

Finally, to implement the combination of all three taxes (12), Home would again impose a nominal extraction tax of $\tau = t_e + t_c$. The border adjustment on energy and on imports of goods is $\beta_e = \beta_m = t_c$. Unlike with the extraction-consumption tax, however, there is an even lower border adjustment on the export of goods, $\beta_x = t_c - t_x$. That is, to tax exports at an effective rate of t_x under this implementation, producers of goods would receive an export rebate of $\beta_x = \Lambda^* t_c$. The tax on production is proportional to Foreign leakage. If Λ^* is zero, there should be no rebate on exports. As Foreign leakage goes up, so does the export rebate. With $\Lambda^* = 1$, the rebate on exports of goods would equal the tax on imports of goods. There would be no tax on production, and the combined policy would be an extraction-consumption tax. That is, the value of Foreign leakage, Λ^*, gives us the answer to the commonly-posed policy question of whether border adjustments should include rebates of prior taxes paid for exports of goods. While implementing this three-way hybrid involves all the difficulties associated with computing the carbon content of goods, it would be no more difficult to administer than the extraction-consumption tax.

3.6 Policy Coordination

How does Home's optimal policy adjust if Foreign also implements carbon taxes? Using notation similar to that for Home, suppose Foreign taxes its extraction at rate t_e^*, its production for domestic consumption at rate t_f, its imports (Home's exports) at rate t_x^*, and its exports (Home's imports) at rate t_m^*. Home then sets its policy taking these rates as given.

Consider the case from Section 3.4, where Home can tax extraction, consumption, and production. In this case, Home's basic policy remains the same: $t_e + t_c = \varphi^W$ and $t_x = (1 - \Lambda^*)t_c$.[20] Home, however, adjusts the policy along two dimensions. (The full derivation is in Appendix C.4.)

[20]While the expressions for Home's policy are the same, the value of Λ^* may not be because Foreign's taxes will affect the benefits of shifting production to Foreign.

First, Home reduces its carbon tax on imported goods so that the overall tax on imports is $t_m + t_m^* = t_c$. The logic is to keep Home consumption from being distorted by the tax on imports, hence crediting the tax imposed by Foreign. This feature appears in proposed border adjustments in US carbon-tax bills (see footnote 3) and in the EU's CBAM (see footnote 4), both of which would credit carbon prices already paid on imported goods that are subject to the border adjustment.

Second, the mix of t_e and t_c changes to reflect Foreign's mix of taxes. In particular, Home adds t_a to its extraction tax in (12) and subtracts t_a from its consumption tax, where t_a depends on a combination of Foreign taxes:

$$t_a = \frac{t_e^* Q_e^{*\prime} - t_f |C_e^{f\prime}| - t_x^* |C_e^{x\prime}|}{Q_e^{*\prime} + |C_e^{f\prime}| + \Lambda^* |C_e^{x\prime}|}. \tag{13}$$

The logic is that Home shifts its taxes to align more closely with Foreign taxes on extraction and consumption, e.g. raising t_e (and lowering t_c) if t_e^* increases.

To illustrate, suppose that Foreign chose to impose its tax entirely on extraction, setting $t_f = t_x^* = t_m^* = 0$. Furthermore, suppose that the level of Foreign's carbon tax is the same as Home's, so that $t_e^* = \varphi^W$. In this case, applying (13), Home would also choose to tax only extraction because doing so would eliminate distortions on the supply side and avoid introducing distortions on the demand side. Roughly the same logic would hold if Foreign chose to tax only the demand side, although the plethora of demand-side taxes makes it messy.

We get a sharp result if, in parallel to Home's policy, Foreign's tax on its imports doesn't distort its own consumption, i.e. $t_x^* = t_f - t_x$. In this case we can can think of t_f as Foreign's consumption tax, t_c^*. This policy requires, akin to a Nash equilibrium, that Foreign anticipate Home's optimal tax on exports, $t_x = (1 - \Lambda^*) t_c$. In this setting, Home's optimal extraction and consumption taxes satisfy (4), with $\tilde{\varphi}^* = t_e^* + t_c^*$. Home thus mimics the extraction and consumption taxes in Foreign, then adjusts the overall level of the two according to how Foreign supply and demand respond to the energy price, as in (2).

4 Quantitative Illustrations

To get a sense of the size of the economic benefits from the various types of hybrid taxes, we calibrate and simulate the model described in Part 3. Our sufficient-statistic formulas for optimal taxes in Sections 2 and 3 give intuitions, but they don't allow us to compute numerical values or to compare welfare across all policies. To do this, we need to add structure to the model, including functional forms for extraction and production and for the efficiency of production of goods in each region. We follow the approach taken in Kortum and Weisbach (2021), which is fully described there. Appendix D shows that the analysis here is compatible with the structure imposed in Kortum and Weisbach (2021).

We calibrate the business as usual competitive equilibrium of the model to the data on global carbon flows from Table 1. Then, following the approach in Dekle, Eaton, and Kortum (2007), we can compute the effects of various policies relative to this baseline. Calibrating the model this way subsumes transport costs for goods, which Kortum and Weisbach (2021) model as iceberg costs.

In addition to our calibration to the CO_2 matrix, we also need values for several elasticities. As we will discuss, a key parameter is Foreign's elasticity of energy supply, $\epsilon_S^* = p_e Q_e^{*\prime}/Q_e^*$. Our baseline value is $\epsilon_S^* = 0.5$ but because of uncertainty in this value, we also show simulations for $\epsilon_S^* = 2$.[21]

Our figures show what we call "policy frontiers" for various combinations of taxes. Along the x-axis of the frontier is the cost of the policy, measured as the decline in services consumption as a percent of the business-as-usual level of spending on goods consumption (ignoring the benefits of emissions reductions). The y-axis shows the resulting global emissions reductions as a percent of their business-as-usual level (which with no policy is 32.3 gigatons of CO_2). The frontier for a given policy (when optimized) is traced out by ranging over values of φ^W, so that each point on the line shows the emissions reductions that Home's policy would achieve for a given social cost of carbon. The red x on each line shows the policy that Home would choose in each case when $\varphi^W = 2$, which means that the global social cost of a unit of carbon is twice the value of energy containing a unit of carbon.

Figure 6 compares the three hybrid policies and the two standard approaches to carbon taxes, a tax on domestic production and that same tax with border adjustments on goods (which shifts it to domestic consumption). As can be seen, with this calibration, all three hybrid policies perform similarly and substantially

[21] We also set the share of energy in production equal to 0.15, the Foreign demand elasticity ($\epsilon_D^* = p_e C_e^{*\prime}/C_e^*$) equal to 1, and the trade elasticity equal to 4. These parameters are described briefly in Appendix D, with more details on the calibration explained in Kortum and Weisbach (2021).

outperform the two standard approaches. For example, adding an extraction-tax component to a production tax nearly doubles the global emissions reductions the policy would achieve at any given cost.

In this calibration, there is almost no advantage to adding border adjustments on goods. The emissions reductions that are achievable with a simpler tax – the combination of an extraction and production tax – are about the same. Given the complexities of imposing border adjustments on goods, the modest additional emissions reductions are unlikely to be worth the costs.

Figure 6 explores the robustness of these results to Foreign's energy supply elasticity by setting $\epsilon_S^* = 2$ instead of 0.5. The extraction-production tax now performs less well. The reason is that with a high value of ϵ_S^*, the extraction component of the various hybrid policies, which raise the global energy price, induce a significant positive response by Foreign extractors. The policies must, as a result, rely more on demand-side taxes, and the leakage costs of the production tax therefore play a larger role. In this case, all of the policies that use a consumption tax as the demand-side tax (including a pure consumption tax) outperform the policies that rely on a production tax. Because a demand-side tax on consumption does not cause leakage, policies that impose the demand side tax on consumption are more robust to the value of ϵ_s^*.

Whether the gains from imposing border adjustments on goods (to shift the tax downstream to consumption) are worth the costs depends primarily on (1) the risk of a high value of ϵ_S^*, (2) the costs of imposing border adjustments on goods, and (3) the size of the taxing coalition.

To explore the role of coalition size, particularly as measured by production, we include China in the taxing region. To do so we recalibrate the model to the values of embodied CO_2 shown in Table 3.[22] The table shows that moving China to the region called Home (in Table 1 China was in the region called Foreign) nearly doubles the baseline amount of CO_2 emitted in production by the coalition (Home), with a somewhat smaller increase in implicit consumption of CO_2.

Figures 7 and 8 illustrate the effects of adding China to the taxing coalition. As expected, under all policies, adding China to the taxing coalition dramatically increases the possible global emissions reductions. Once again, the hybrid policies substantially outperform the traditional approaches, indicating that the benefits

[22]Table 3 corresponds to Table 9 in Kortum and Weisbach (2021). As for Table 1 above, the source is the Trade in Embodied CO_2 database made available by the OECD (2019).

Figure 5: Policy frontiers of OECD with low Foreign elasticity

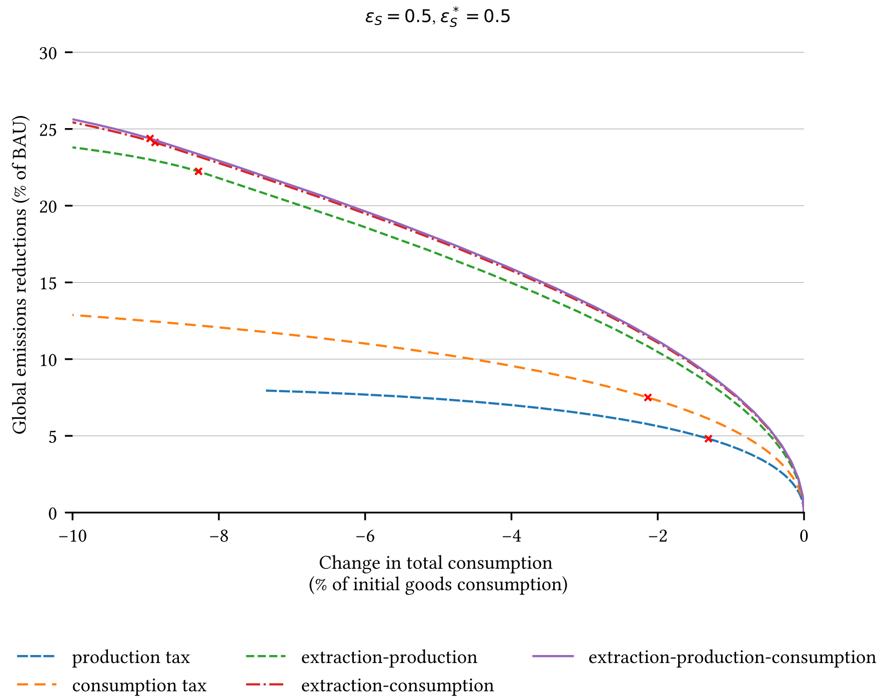


Figure 6: Policy frontiers of OECD with high Foreign elasticity

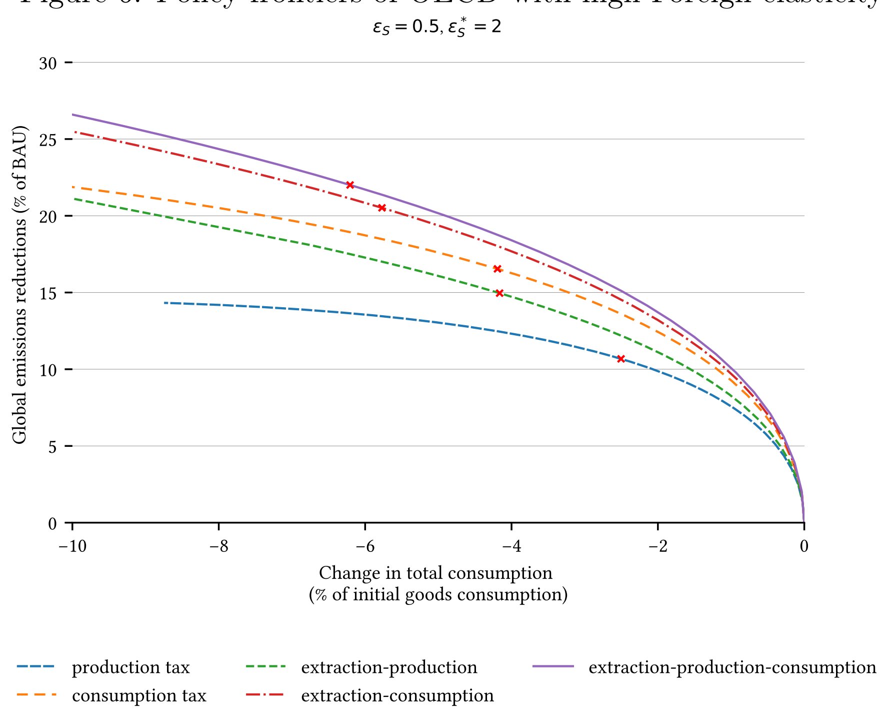


Table 3: Calibration for the OECD plus China

	Home	Foreign	Total
Home	$C_e^d = 20.1$	$C_e^m = 1.7$	$C_e = 21.8$
Foreign	$C_e^x = 1.4$	$C_e^f = 9.1$	$C_e^* = 10.5$
Total	$G_e = 21.5$	$G_e^* = 10.8$	$C_e^W = 32.3$
Extraction	$Q_e = 16.24$	$Q_e^* =16.1$	$Q_e^W = 32.3$

of the hybrid policies continue even with the larger taxing coalition.[23]

Another effect of adding China to the taxing coalition is that now the extraction-production hybrid is more robust to the value of ϵ_S^*. Since the coalition now represents two-thirds of the CO_2 emitted in production, there are fewer opportunities for leakage with China in the taxing coalition (Λ declines). As a consequence, the production tax performs relatively better than with the smaller taxing coalition.

We suspect that this result is general, in the sense that the choice of the taxing coalition affects the relative performance of the various taxes. Because the extraction-production tax is so much simpler to implement, a promising strategy is to form a taxing coalition for which this tax performs well. In particular, including countries with a substantial base of production and a high elasticity of energy supply in the taxing coalition is a promising strategy because doing so lowers both Λ and ϵ_S^*, allowing the taxing region to use the simpler extraction-production hybrid, thereby avoiding border adjustments on goods.[24]

5 Discussion and Conclusion

We can summarize our finding as follows:

- When there is trade and the possibility of leakage, carbon taxes are most efficient if they are imposed on both sides of the market, that is, on both

[23]At the limit, however, where the taxing coalition is the entire world, all the taxes would perform the same. Therefore, at some point, the simple taxes should perform about as well as the hybrid policies.

[24]This strategy, as it relates to the extraction elasticity, has similarities to Harstad (2012).

Figure 7: Policy frontiers of OECD plus China with low Foreign elasticity

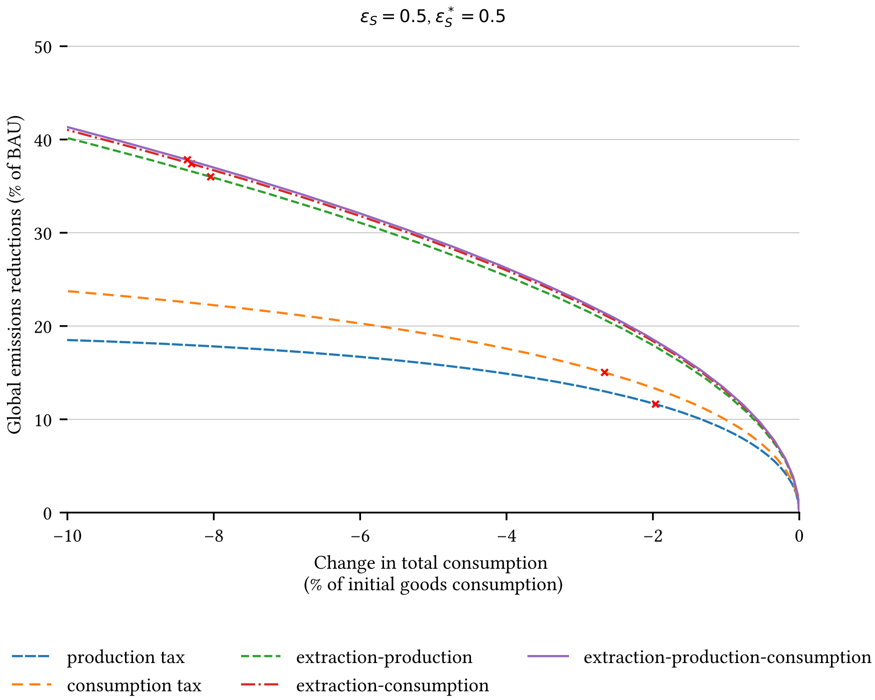
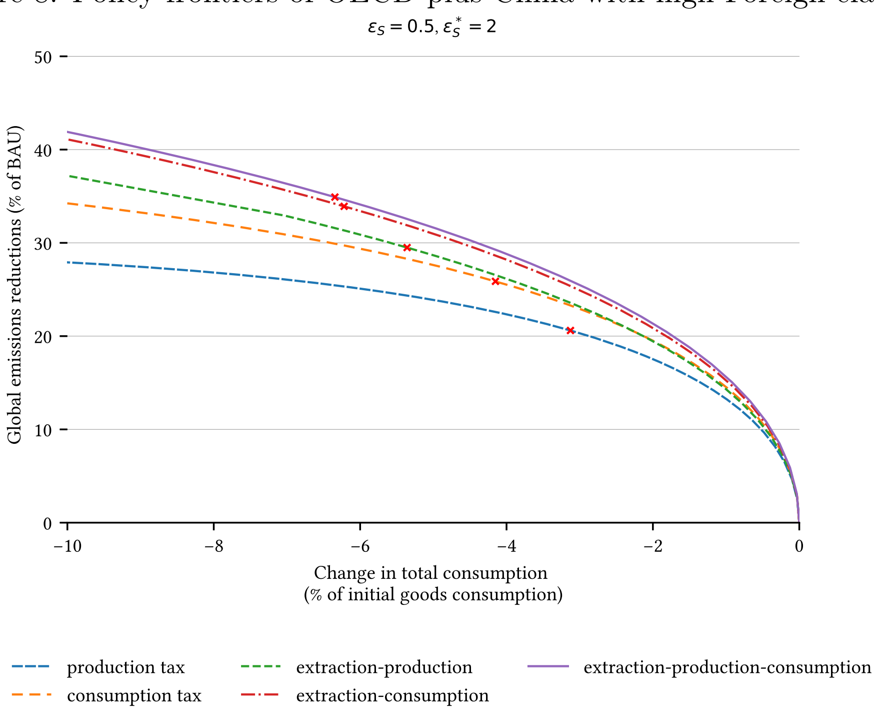


Figure 8: Policy frontiers of OECD plus China with high Foreign elasticity



the extraction or supply of fossil fuels and the use or demand for fossil fuels. There are potentially large gains from this strategy. This point, which was known as early as Markusen (1975) (writing in a related context), appears not to be widely appreciated. It involves a simple change to current proposals, and there seems to be no reason not to pursue this approach to improving the functioning of carbon taxes.

- The relative portion of the tax that should stay on extraction to maximize efficiency depends on the foreign reaction to the different taxes, as measured by the slope of foreign's supply and demand curves. The core idea is similar to familiar notions in the design of taxes more generally, which is that we should not impose high taxes on highly responsive items. Here the response (to the resulting change in the global energy price) is measured by the slope of the supply and demand curves for energy in non-taxing regions.

- If we do not take administrative costs into account, the taxing region maximizes efficiency by taxing fossil fuels at all stages of their use as they flow through the economy: extraction, production, and consumption. The production component of the tax, however, is muted by leakage. If leakage were zero, the production tax would be at the same rate as the consumption tax. If leakage is 100%, the production tax should be zero, with the tax in that case falling only on extraction and consumption.

- To implement this policy, the taxing region can impose a nominal tax on extraction, (at the optimum at the global social cost of carbon). It then shifts a portion of the tax downstream to production via border adjustments on energy at a lower rate (with the relative rates on extraction and on production determined as just discussed). In addition, the taxing region further shifts the tax to consumption by imposing border adjustments on imports of goods at the same rate as the border adjustments on energy. Finally, to lower the production tax to account for leakage, it rebates a portion of the tax on exports of goods.

- This analysis explains how to set the rebate, if any, on exports of goods, a common problem in carbon tax design. Absent concerns about leakage, to maximize efficiency, there should be no rebate for exports, leaving the tax on domestic production equal to the tax on domestic consumption. With leakage, however, the taxing region should remove part of the tax on exports, and if leakage were 100%, the rebate would be of the entire tax previously paid.

- The administrative costs, however, of imposing a tax on consumption would be high because there is no straightforward way to observe the emissions associated with imports of goods. Moreover, in our baseline simulation, the gains from imposing border adjustments on the imports of goods, relative to the simpler extraction-production combination, are small. As a result, the extraction-production tax may be a superior instrument, when taking both efficiency and administrative costs into account. It could be implemented simply and accurately by imposing a nominal tax on extraction and border adjustments on the imports and exports of energy at a lower rate.

- This latter conclusion depends on the foreign elasticity of energy supply. Our baseline calibration, which assumed that the taxing region was the OECD and the rest of the world did not impose a tax, set the foreign elasticity of energy supply at 0.5. As this value goes up, the effectiveness of the extraction-production tax goes down relative to combinations that include a consumption tax. The reason is that the extraction tax component becomes less effective when foreign extraction is more sensitive to the price of energy. As a result, efficient policies have to rely more on demand-side taxes, and the demand-side tax in the extraction-production combination is subject to leakage. The demand-side tax in the extraction-consumption combination (i.e., with border adjustments on goods) is not subject to leakage. Therefore, as the foreign elasticity of energy supply goes up, border adjustments on goods become more desirable.

- Finally, one way to make the tax more effective and simpler is to include high elasticity of supply countries in the taxing coalition. This makes the extraction production tax more effective and, therefore, reduces the need to rely on border adjustments for goods. That is, the make-up of the taxing coalition and the design of the tax interact, and a well-chosen taxing coalition may allow a simpler tax system.

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Appendix

A Extensions of the Basic Model

We drop the linearly separable assumption on welfare in this appendix to consider both the global optimum and the unilateral optimum. Welfare in either country is here a general differentiable function of its three arguments, increasing in the first and second, while decreasing in the third:

$$U = u(C_s, C_e, E)$$
$$U^* = u^*(C_s^*, C_e^*, E).$$

The marginal social costs of carbon for Home and Foreign, in terms of the numeraire, are:

$$\varphi = -(\partial u/\partial E)/(\partial u/\partial C_s) = -u_3/u_1$$
$$\varphi^* = -(\partial u^*/\partial E)/(\partial u^*/\partial C_s^*) = -u_3^*/u_1^*,$$

with $\varphi^W = \varphi + \varphi^*$. (For a function f of a vector x, we denote $f_i = \partial f/\partial x_i$.)

A.1 Global Optimum

Suppose that Home can dictate a policy for Foreign as long as it transfers services T_s to keep Foreign welfare at a threshold, $\bar{U}^*$. The optimal policy is the solution to a Lagrangian (with a Lagrange multiplier, μ, on the Foreign welfare constraint):

$$\max_{\{T_s, C_e^*, Q_e^*\}} u(C_s, C_e, \bar{E}) + \mu\left[u^*(C_s^*, C_e^*, \bar{E}) - \bar{U}^*\right],$$

subject to:

$$C_s = L - c(\bar{E} - Q_e^*) - T_s$$
$$C_e = \bar{E} - C_e^*$$
$$C_s^* = L^* - c^*(Q_e^*) + T_s.$$

The first order conditions are: $u_1 = \mu u_1^*$, $u_2 = \mu u_2^*$, and $u_1 c' = \mu u_1^* c'^*$, which can be distilled down to: $u_2/u_1 = u_2^*/u_1^*$ and $c' = c^{*\prime}$. These two conditions rule out a wedge between Home and Foreign either in the marginal value of energy consumption (in terms of the numeraire) or in the marginal cost of extracting energy. But, they admit a wedge, common to both countries, between the marginal cost of extracting energy and it's marginal value, $u_2/u_1 - c' = u_2^*/u_1^* - c^{*\prime} \neq 0$.

The level of this wedge is determined by taking the first-order condition for the emissions goal itself, $\bar{E}$:

$$-u_1 c' + u_2 = -u_3 - \mu u_3^*.$$

Substituting in $\mu = u_1/u_1^*$ and dividing through by u_1, we get that the wedge equals the global externality:

$$u_2/u_1 - c' = -(u_3/u_1 + u_3^*/u_1^*) = \varphi^W.$$

A key point is that there is no need to distinguish between the consumption wedge, $u_2/u_1 - p_e$, and the extraction wedge, $p_e - c'$, in this global optimum.

These conditions will hold in a competitive equilibrium with taxes. With a consumption tax of t_c consumers equate their marginal rate of substitution between energy and services to $p_e + t_c$ while with an extraction tax of t_e extractors equate their marginal extraction costs to $p_e - t_e$. The first optimality condition says that a consumption tax must be harmonized between Home and Foreign, $t_c = t_c^*$, while the second says that an extraction tax must be harmonized, $t_e = t_e^*$. The third condition says that taxes on extraction and consumption must sum to the global externality:

$$t_c + t_e = \varphi^W.$$

Conditional on their sum, the allocation of the tax across consumption and extraction is arbitrary. Any combination adding to the marginal global social cost of carbon attains the global optimum.

A.2 Unilateral Optimum

We now turn to the problem of Section 2, the optimal policy when Home can only indirectly influence Foreign extraction and consumption by choosing the price of energy. We will continue to assume that Home uses transfers of services, T_s, to keep Foreign welfare above a threshold of $\bar{U}^*$.

To solve this problem we follow Keen and Kotsogiannis (2014) and employ

Foreign's expenditure function, defined as:

$$e^*(p_e, \bar{U}^*, \bar{E}) = \min\left\{C_s^* + p_e C_e^* \;\mid\; u^*(C_s^*, C_e^*, \bar{E}) = \bar{U}^*\right\}$$

Two key properties of the expenditure function are:

$$\begin{aligned} e_1^* = \partial e^*/\partial p_e &= C_e^*(p_e, \bar{U}^*, \bar{E}) \\ e_3^* = \partial e^*/\partial \bar{E} &= -u_3^*/u_1^*. \end{aligned}$$

Here $C_e^*(p_e, \bar{U}^*, \bar{E})$ is simply Foreign's compensated demand for energy. Its partial derivative with respect to the global emissions goal is denoted by $C_{e,3}^*$. We treat the slope of this energy demand curve (the partial derivative with respect to the energy price) as strictly negative, $C_e^{*\prime} < 0$ (the notation $C_e^{*\prime}$, instead of $C_{e,1}^*$, facilitates comparison to other results in the paper).

Foreign obtains income from labor, L^*, and rents from the energy sector, $p_e Q_e^* - c(Q_e^*)$. It also gets transfers of services, T_s, and net energy imports, valued at $p_e X_e$, from Home. (Home's net exports of energy are $X_e = Q_e - C_e$.) Foreign expenditure, Y^*, is the sum of income, transfers of services, and the value net energy imports:

$$Y^* = L^* + (p_e Q_e^* - c(Q_e^*)) + T_s + p_e X_e = L^* + p_e C_e^* - c(Q_e^*) + T_s.$$

If Foreign expenditure is $Y^* = e^*(p_e, \bar{U}^*, \bar{E})$ it can achieve welfare of $\bar{U}^*$ when the energy price is p_e and the global emissions goal is $\bar{E}$.

Foreign's energy supply curve, $Q_e^*(p_e)$, satisfies $c^{*\prime}(Q_e^*(p_e)) = p_e$. The slope of this energy supply curve is $Q_e^{*\prime} > 0$.

Home's optimal policy is then the solution to the Lagrangian:

$$\max_{\{T_s, p_e\}} u(C_s, C_e, \bar{E}) + \mu\left[L^* + p_e C_e^* - c^*(Q_e^*) + T_s - e^*(p_e, \bar{U}^*, \bar{E})\right],$$

subject to:

$$\begin{aligned} Q_e^* &= Q_e^*(p_e) \\ C_e^* &= C_e^*(p_e, \bar{U}^*, \bar{E}) \\ C_s &= L - c(\bar{E} - Q_e^*(p_e)) - T_s \\ C_e &= \bar{E} - C_e^*(p_e, \bar{U}^*, \bar{E}). \end{aligned}$$

Here, to control outcomes in Foreign, we maximize over p_e whereas in the first-best problem we maximized separately over C_e^* and Q_e^*.

The first-order conditions are $u_1 = \mu$ and:

$$u_1 c' Q_e^{*\prime} - u_2 C_e^{*\prime} = \mu \left(-C_e^* - p_e C_e^{*\prime} + c^{*\prime} Q_e^{*\prime} + e_1^*\right).$$

Applying $e_1^* = C_e^*$ and $c^{*\prime} = p_e$ they reduce to:

$$(p_e - c')Q_e^{*\prime} = (u_2/u_1 - p_e)|C_e^{*\prime}|.$$

Since u_2/u_1 has the same interpretation as u' in Section 2, this result shows that equation (1) in the paper is robust to welfare being non-separable in it's arguments.

The first-order condition for the emissions goal is:

$$-u_1 c' + u_2 - u_2 C_{e,3}^* + u_3 = \mu \left(-p_e C_{e,3}^* + e_3^*\right)$$

and hence the overall wedge is:

$$u_2/u_1 - c' = \left(u_2/u_1 - p_e\right) C_{e,3}^* + \varphi^W.$$

This conditions looks like that for the global optimum, but with an additional term. Suppose $C_{e,3}^* > 0$ so that Foreign's compensated demand for energy is increasing in $\bar{E}$.[25] This term gives an added reason for Home to lower emissions, as doing so shifts energy consumption away from Foreign. Such a shift is beneficial because the value of energy consumption is higher in Home then in Foreign, $u_e/u_s > p_e$, as dictated by the second condition. For linearly separable Foreign welfare $C_{e,3}^* = 0$ and this first condition collapses to the corresponding condition for the global optimum.

[25]For example, if Foreign welfare is $u^*(C_s^*, C_e^*, \bar{E}) = (C_s^*)^\gamma (C_e^*)^{1-\gamma} (\bar{E})^{-\phi}$ then its compensated demand for energy is:

$$C_e^*(p_e, \bar{U}^*, \bar{E}) = ((1-\gamma)/\gamma)^\gamma p_e^{-\gamma} (\bar{E})^\phi \bar{U}^*,$$

and $C_{e,3}^* = \phi C_e^*/\bar{E} > 0$. If instead Foreign welfare is linearly separable, $u^*(C_s^*, C_e^*, \bar{E}) = C_s + u^*(C_e^*) - \varphi^* \bar{E}$ (with $u^{*\prime} > 0$ and $u^{*\prime\prime} < 0$), then its compensated demand for energy depends only on the energy price and $C_{e,3}^* = 0$.

Solving for the individual taxes:

$$t_c = \varphi^W \frac{Q_e^{*\prime}}{Q_e^{*\prime} + |C_e^{*\prime}| - C_{e,3}^* Q_e^{*\prime}}$$
$$t_e = \varphi^W \frac{|C_e^{*\prime}|}{Q_e^{*\prime} + |C_e^{*\prime}| - C_{e,3}^* Q_e^{*\prime}}.$$

These expressions are the same as (3) in the paper except that each has an additional term $C_{e,3}^* Q_e^{*\prime}$ in the denominator.

B Trade Balance

Suppose that we impose trade balance, $C_s - Q_s = p_e(Q_e - C_e) = p_e X_e$ (hence $T_s + p_e X_e = 0$) while removing the constraint that Home maintain Foreign welfare $\bar{U}^*$. The planner's problem in Section 2 is unchanged except that the first constraint becomes:

$$C_s = L - c(\bar{E} - Q_e^*(p_e)) + p_e(C_e^*(p_e) - Q_e^*(p_e)).$$

The first-order condition for p_e gives:

$$(p_e - c')Q_e^{*\prime} - X_e = (u' - p_e)|C_e^{*\prime}| \tag{14}$$

Replacing each wedge with the corresponding tax rate, it follows that:

$$t_e Q_e^{*\prime} = t_c |C_e^{*\prime}| + X_e.$$

If Home is a net exporter of energy it improves its terms of trade by relying more on the extraction tax rather than the consumption tax.

The first-order condition for the emissions goal is $u' - c' = \varphi$, which implies

$$t_e + t_c = \varphi.$$

When Home is not forced to choose a Pareto improving policy, it ignores the social cost of carbon in Foreign, φ^*. Combining results, we get the analog of (3):

$$\begin{aligned} t_e &= \frac{\varphi|C_e^{*\prime}| + X_e}{Q_e^{*\prime} + |C_e^{*\prime}|} \\ t_c &= \frac{\varphi Q_e^{*\prime} - X_e}{Q_e^{*\prime} + |C_e^{*\prime}|}. \end{aligned} \tag{15}$$

The two differences are that φ replaces φ^W and that positive net exports of energy from Home tilt the policy towards an extraction tax and away from a consumption tax.

So far we have assumed that Foreign does not tax carbon. Section 2.4 shows that, under a Foreign welfare constraint, Home's optimal policy would adapt to carbon taxes in Foreign. Here, with the constraint on Foreign welfare removed, the expression for the first-order condition (14) is invariant to carbon taxes in Foreign.[26] The reason for this invariance is that Home is no longer concerned with global wedges, the gap in the marginal cost of extraction between Home and Foreign and the gap in the marginal utility of consumption between Home and Foreign. Instead, Home is only concerned with its own wedges, the gap between the marginal cost of extraction as well as the marginal utility of consumption and the global price at which it can buy or sell energy.

C Trade in Energy and Goods

Here we provide derivations of the optimality conditions in Section 3. Each case involves optimizing the objective (5) by choosing tax rates. The price of energy responds endogenously to the choice of taxes so as to clear the energy market, which we denote by dp_e/dt_i for $i \in \{e, d, m, x\}$. (We denote $\partial x_e/\partial p_e$ by x_e' for any quantity of extraction or consumption of energy x_e.)

To simplify the first-order conditions that follow we exploit envelope conditions. Roy's identity gives: $\tilde{u}_1 = -C_e^d$, $\tilde{u}_2 = -C_e^m$, $\tilde{u}_1^* = -C_e^f$, and $\tilde{u}_2^* = -C_e^x$. Hotelling's lemma gives: $\partial R_e/\partial p_e = Q_e$ (hence $\partial R_e/\partial t_e = -Q_e$) and $\partial R_e^*/\partial p_e = Q_e^*$. In combination, these results eliminate a term that would otherwise appear in each

[26] Of course carbon taxes in Foreign would alter equilibrium outcomes so that all the elements of the first-order condition would need to be evaluated at the new equilibrium. Furthermore $Q_e^{*\prime}$ would be evaluated at $p_e - t_e^*$ and $C_e^{*\prime}$ at $p_e + t_c^*$ (rather than both being evaluated at p_e), where t_e^* is the extraction tax and t_c^* the consumption tax imposed by Foreign.

of the first-order conditions, since:

$$\left(\frac{\partial R_e}{\partial p_e} + \frac{\partial R_e^*}{\partial p_e} + \tilde{u}_1 + \tilde{u}_2 + \tilde{u}_1^* + \tilde{u}_2^*\right)\frac{dp_e}{dt_i} = \left(Q_e^W - C_e^W\right)\frac{dp_e}{dt_i} = 0, \tag{16}$$

for $i \in \{e, d, m, x\}$.

The derivatives of Home tax revenue are: $\partial R_t/\partial p_e = t_e Q_e' + t_d C_e^{d\prime} + t_m C_e^{m\prime} + t_x C_e^{x\prime}$, $\partial R_t/\partial t_e = Q_e + t_e \partial Q_e/\partial t_e$, and $\partial R_t/\partial t_i = C_e^i + t_i \partial C_e^i/\partial t_i$, for $i \in \{d, m, x\}$. We now apply these results to the specific cases considered in Section 3.

C.1 Taxing Extraction and Consumption

A consumption tax sets $t_d = t_m = t_c$ and $t_x = 0$. The first-order conditions for maximizing (5) with respect to t_e and t_c, after applying (16), are:

$$\begin{aligned}\frac{\partial R_e}{\partial t_e} + \frac{\partial R_t}{\partial t_e} + \frac{\partial R_t}{\partial p_e}\frac{dp_e}{dt_e} &= \varphi^W\left(\frac{\partial Q_e}{\partial t_e} + Q_e^{W\prime}\frac{\partial p_e}{\partial t_e}\right)\\ \frac{\partial R_t}{\partial t_c} + \frac{\partial R_t}{\partial p_e}\frac{dp_e}{dt_c} + \tilde{u}_1 + \tilde{u}_2 &= \varphi^W Q_e^{W\prime}\frac{\partial p_e}{\partial t_c}\end{aligned}$$

Applying the other envelope results and canceling terms, we get:

$$\begin{aligned}t_e\frac{\partial Q_e}{\partial t_e} + \left(t_e Q_e' + t_c C_e'\right)\frac{dp_e}{dt_e} &= \varphi^W\left(\frac{\partial Q_e}{\partial t_e} + Q_e^{W\prime}\frac{dp_e}{dt_e}\right)\\ t_c\frac{\partial C_e}{\partial t_c} + \left(t_e Q_e' + t_c C_e'\right)\frac{dp_e}{dt_c} &= \varphi^W Q_e^{W\prime}\frac{dp_e}{dt_c}.\end{aligned}$$

Energy market-clearing implies:

$$\begin{aligned}\frac{dp_e}{dt_e} &= \left(\frac{-1}{Q_e^{W\prime} - C_e^{W\prime}}\right)\frac{\partial Q_e}{\partial t_e}\\ \frac{dp_e}{dt_c} &= \left(\frac{1}{Q_e^{W\prime} - C_e^{W\prime}}\right)\frac{\partial C_e}{\partial t_c}.\end{aligned}$$

Substituting these price derivatives into the first-order conditions, canceling $\partial Q_e/\partial t_e$ from the first, and canceling $\partial C_e/\partial t_c$ from the second, we arrive at:

$$\begin{aligned}t_e\left(C_e^{W\prime} - Q_e^{W\prime}\right) + t_e Q_e' + t_c C_e' &= \varphi^W C_e^{W\prime}\\ t_c\left(Q_e^{W\prime} - C_e^{W\prime}\right) + t_e Q_e' + t_c C_e' &= \varphi^W Q_e^{W\prime}.\end{aligned}$$

Subtracting the second from the first yields $t_e + t_c = \varphi^W$. Substituting this expression for φ^W back into the first gives:

$$\frac{t_e}{t_c} = \frac{|C_e^{*\prime}|}{Q_e^{*\prime}}.$$

Combining the two gives (3), which is the solution to the problem in Section 3.2.

C.2 Taxing Extraction and Production

A production tax sets $t_d = t_x = t_p$ and $t_m = 0$. The first-order conditions for maximizing (5) with respect to t_e and t_p, after applying (16), are:

$$\frac{\partial R_e}{\partial t_e} + \frac{\partial R_t}{\partial t_e} + \frac{\partial R_t}{\partial p_e}\frac{dp_e}{dt_e} = \varphi^W \left(\frac{\partial Q_e}{\partial t_e} + Q_e^{W\prime}\frac{\partial p_e}{\partial t_e}\right)$$
$$\frac{\partial R_t}{\partial t_p} + \frac{\partial R_t}{\partial p_e}\frac{dp_e}{dt_p} + \tilde{u}_1 + \tilde{u}_2^* = \varphi^W Q_e^{W\prime}\frac{\partial p_e}{\partial t_p}$$

Applying the other envelope results and canceling terms, we get:

$$t_e\frac{\partial Q_e}{\partial t_e} + (t_e Q_e' + t_p G_e')\frac{dp_e}{dt_e} = \varphi^W \left(\frac{\partial Q_e}{\partial t_e} + Q_e^{W\prime}\frac{dp_e}{dt_e}\right)$$
$$t_p\frac{\partial G_e}{\partial t_p} + (t_e Q_e' + t_p G_e')\frac{dp_e}{dt_p} = \varphi^W Q_e^{W\prime}\frac{dp_e}{dt_p}.$$

Energy market clearing implies:

$$\frac{dp_e}{dt_e} = \left(\frac{-1}{Q_e^{W\prime} - G_e^{W\prime}}\right)\frac{\partial Q_e}{\partial t_e}$$
$$\frac{dp_e}{dt_p} = \left(\frac{1}{Q_e^{W\prime} - G_e^{W\prime}}\right)\frac{\partial G_e^W}{\partial t_p}.$$

In the second market clearing equation we can substitute in the formula for leakage (6), in the form $\partial G_e^W/\partial t_p = (1-\Lambda)\partial G_e/\partial t_p$.

Substituting each of the two market-clearing conditions into the corresponding first-order condition, canceling $\partial Q_e/\partial t_e$ from the first, and canceling $\partial G_e/\partial t_p$

from the second, we get:

$$t_e \left(G_e^{W\prime} - Q_e^{W\prime}\right) + t_e Q_e' + t_p G_e' = \varphi^W G_e^{W\prime}$$
$$\frac{t_p}{1-\Lambda}\left(Q_e^{W\prime} - G_e^{W\prime}\right) + t_e Q_e' + t_p G_e' = \varphi^W Q_e^{W\prime}.$$

Subtracting the second from the first yields (8), $t_e + t_p/(1-\Lambda) = \varphi^W$. Substituting this expression for φ^W back into the first equation we get the analog of (7) and hence (9).

C.3 Taxing Extraction, Consumption, and Production

In this most general case tax rates are unconstrained, resulting in four first-order conditions for maximizing the objective (5) with respect to t_e, t_d, t_m, and t_x.

The first order conditions for t_d and t_m, after applying (16), are:

$$\frac{\partial R_t}{\partial t_d} + \frac{\partial R_t}{\partial p_e}\frac{\partial p_e}{\partial t_d} + \tilde{u}_1 = \varphi^W Q_e^{W\prime}\frac{dp_e}{dt_d}$$
$$\frac{\partial R_t}{\partial t_m} + \frac{\partial R_t}{\partial p_e}\frac{\partial p_e}{\partial t_m} + \tilde{u}_2 = \varphi^W Q_e^{W\prime}\frac{dp_e}{dt_m}.$$

Applying the other envelope results and canceling terms, we get:

$$t_d\frac{\partial C_e^d}{\partial t_d} + \left(t_d C_e^{d\prime} + t_m C_e^{m\prime} + t_x C_e^{x\prime} + t_e Q_e'\right)\frac{dp_e}{dt_d} = \varphi^W Q_e^{W\prime}\frac{dp_e}{dt_d}$$
$$t_m\frac{\partial C_e^m}{\partial t_m} + \left(t_d C_e^{d\prime} + t_m C_e^{m\prime} + t_x C_e^{x\prime} + t_e Q_e'\right)\frac{dp_e}{dt_m} = \varphi^W Q_e^{W\prime}\frac{dp_e}{dt_m}.$$

Energy market clearing implies:

$$\frac{dp_e}{dt_d} = \left(\frac{1}{Q_e^{W\prime} - C_e^{W\prime}}\right)\frac{\partial C_e^d}{\partial t_d}$$
$$\frac{dp_e}{dt_m} = \left(\frac{1}{Q_e^{W\prime} - C_e^{W\prime}}\right)\frac{\partial C_e^m}{\partial t_m},$$

Substituting in these price derivatives, the first-order conditions reduce to $t_d = t_m$.

It is optimal to tax energy embodied in Home's consumption at the same rate, $t_c = t_d = t_m$, whether the goods are produced domestically or imported. Applying this condition, we can add C_e^d and C_e^m to form a single first-order condition for t_c.

Following the same procedures as above, the first order conditions for t_e, t_c, and t_x can now be reduced to:

$$\begin{aligned} t_e \frac{\partial Q_e}{\partial t_e} + \left(t_e Q_e' + t_c C_e' + t_x C_e^{x\prime}\right) \frac{dp_e}{dt_e} &= \varphi^W \left(\frac{\partial Q_e}{\partial t_e} + Q_e^{W\prime} \frac{dp_e}{dt_e} \right) \\ t_c \frac{\partial C_e}{\partial t_c} + \left(t_e Q_e' + t_c C_e' + t_x C_e^{x\prime}\right) \frac{dp_e}{dt_c} &= \varphi^W Q_e^{W\prime} \frac{dp_e}{dt_c} \\ t_x \frac{\partial C_e^x}{\partial t_x} + \left(t_e Q_e' + t_c C_e' + t_x C_e^{x\prime}\right) \frac{dp_e}{dt_x} &= \varphi^W Q_e^{W\prime} \frac{dp_e}{dt_x}. \end{aligned}$$

The corresponding market-clearing conditions are:

$$\begin{aligned} \frac{dp_e}{dt_e} &= \left(\frac{-1}{Q_e^{W\prime} - C_e^{W\prime}} \right) \frac{\partial Q_e}{\partial t_e} \\ \frac{dp_e}{dt_c} &= \left(\frac{1}{Q_e^{W\prime} - C_e^{W\prime}} \right) \frac{\partial C_e}{\partial t_c} \\ \frac{dp_e}{dt_x} &= \left(\frac{1}{Q_e^{W\prime} - C_e^{W\prime}} \right) \frac{\partial C_e^*}{\partial t_x}. \end{aligned}$$

In the last of these market-clearing conditions we can substitute in the formula for Foreign leakage (10), in the form $\partial C_e^*/\partial t_x = (1 - \Lambda^*)\partial C_e^x/\partial t_x$.

Substituting each of the three market-clearing conditions into the corresponding first-order condition, canceling $\partial Q_e/\partial t_e$ from the first, canceling $\partial C_e/\partial t_c$ from the second, and canceling $\partial C_e^x/\partial t_x$ from the third, we get a simple three-equation system:

$$\begin{aligned} t_e \left(C_e^{W\prime} - Q_e^{W\prime} \right) + \left(t_e Q_e' + t_c C_e' + t_x C_e^{x\prime}\right) &= \varphi^W C_e^{W\prime} \\ t_c \left(Q_e^{W\prime} - C_e^{W\prime} \right) + \left(t_e Q_e' + t_c C_e' + t_x C_e^{x\prime}\right) &= \varphi^W Q_e^{W\prime} \\ \frac{t_x}{1 - \Lambda^*} \left(Q_e^{W\prime} - C_e^{W\prime} \right) + \left(t_e Q_e' + t_c C_e' + t_x C_e^{x\prime}\right) &= \varphi^W Q_e^{W\prime}. \end{aligned}$$

Subtracting the third equation from the first, much like for the extraction-production tax, gives $t_e + t_x/(1 - \Lambda^*) = \varphi^W$. Subtracting the second from the first, we get $t_e + t_c = \varphi^W$ as in the extraction-consumption case. Together these two results imply $t_x = (1 - \Lambda^*)t_c$. Substituting these results for φ^W and t_x back into the first equation we get the ratio of the extraction to consumption tax rate, given in (11). Together these results yield expressions for all three taxes, given in (12).

C.4 Policy Coordination

For the extraction-consumption-production case, we now allow for the possibility that Foreign taxes carbon at rates t_e^*, t_f, t_m^*, and t_x^*. Foreign is not strategic, so we treat these tax rates as unchanging parameters. Here t_m^* is the tax rate that Foreign applies to carbon embodied in Home's imports of goods (hence Foreign's exports) and t_x^* is the tax rate that Foreign applies to carbon embodied in Home's exports of goods (hence Foreign's imports). Thus the flows of carbon C_e^m and C_e^x may be taxed both by Home and Foreign, at rate $t_m + t_m^*$ and $t_x + t_x^*$, respectively.

Foreign tax revenue, R_t^*, which now enters the objective function (5), is given by:

$$R_t^* = t_e^* Q_e^* + t_f C_e^f + t_m^* C_e^m + t_x^* C_e^x.$$

The derivatives of Foreign tax revenue are: $\partial R_t^*/\partial p_e = t_e^* Q_e^{*\prime} + t_f C_e^{f\prime} + t_m^* C_e^{m\prime} + t_x^* C_e^{x\prime}$, $\partial R_t^*/\partial t_m = t_m^* \partial C_e^m/\partial t_m$, and $\partial R_t^*/\partial t_x = t_x^* \partial C_e^x/\partial t_x$. Note that $\partial R_t^*/\partial t_e = \partial R_t^*/\partial t_d = 0$.

The objective (5) becomes:

$$\mathcal{L} = R_e + R_e^* + R_t + R_t^* + \tilde{u}(p_e^d, p_e^m) + \tilde{u}^*(p_e^f, p_e^x) - \varphi^W E.$$

There are four first-order conditions for maximizing it with respect to t_e, t_d, t_m, and t_x. The first order conditions for t_d and t_m, after applying (16), are:

$$\frac{\partial R_t}{\partial t_d} + \frac{\partial R_t}{\partial p_e}\frac{\partial p_e}{\partial t_d} + \frac{\partial R_t^*}{\partial p_e}\frac{\partial p_e}{\partial t_d} + \tilde{u}_1 = \varphi^W Q_e^{W\prime}\frac{dp_e}{dt_d}$$
$$\frac{\partial R_t}{\partial t_m} + \frac{\partial R_t^*}{\partial t_m} + \frac{\partial R_t}{\partial p_e}\frac{\partial p_e}{\partial t_m} + \frac{\partial R_t^*}{\partial p_e}\frac{\partial p_e}{\partial t_m} + \tilde{u}_2 = \varphi^W Q_e^{W\prime}\frac{dp_e}{dt_m}.$$

Applying the other envelope results and canceling terms, we get:

$$t_d\frac{\partial C_e^d}{\partial t_d} + \left(\frac{\partial R_t}{\partial p_e} + \frac{\partial R_t^*}{\partial p_e}\right)\frac{dp_e}{dt_d} = \varphi^W Q_e^{W\prime}\frac{dp_e}{dt_d}$$
$$(t_m + t_m^*)\frac{\partial C_e^m}{\partial t_m} + \left(\frac{\partial R_t}{\partial p_e} + \frac{\partial R_t^*}{\partial p_e}\right)\frac{dp_e}{dt_m} = \varphi^W Q_e^{W\prime}\frac{dp_e}{dt_m}.$$

Energy market clearing implies:

$$\frac{dp_e}{dt_d} = \left(\frac{1}{Q_e^{W\prime} - C_e^{W\prime}}\right)\frac{\partial C_e^d}{\partial t_d}$$
$$\frac{dp_e}{dt_m} = \left(\frac{1}{Q_e^{W\prime} - C_e^{W\prime}}\right)\frac{\partial C_e^m}{\partial t_m},$$

Substituting in these price derivatives, the first-order conditions reduce to $t_d = t_m + t_m^*$.

It is optimal to tax energy embodied in Home's consumption at the same rate, $t_c = t_d = t_m + t_m^*$, whether the goods are produced domestically or imported, while ignoring whether the tax is applied by Home, by Foreign, or by both. If Foreign taxes Home's imports at a higher rate, Home reduces its tax on imports to keep the overall tax, $t_m + t_m^*$, equal to t_d. Applying this condition and noting that $\partial C_e^d/\partial t_d + \partial C_e^m/\partial t_m = \partial C_e/\partial t_c$, we can add these two first-order conditions to form a first-order condition for t_c.

The first order conditions for t_e, t_c, and t_x reduce to:

$$t_e\frac{\partial Q_e}{\partial t_e} + \left(t_eQ_e' + t_e^*Q_e^{*\prime} + t_cC_e' + t_fC_e^{f\prime} + (t_x+t_x^*)C_e^{x\prime}\right)\frac{dp_e}{dt_e} = \varphi^W\left(\frac{\partial Q_e}{\partial t_e} + Q_e^{W\prime}\frac{dp_e}{dt_e}\right)$$
$$t_c\frac{\partial C_e}{\partial t_c} + \left(t_eQ_e' + t_e^*Q_e^{*\prime} + t_cC_e' + t_fC_e^{f\prime} + (t_x+t_x^*)C_e^{x\prime}\right)\frac{dp_e}{dt_c} = \varphi^W Q_e^{W\prime}\frac{dp_e}{dt_c}$$
$$t_x\frac{\partial C_e^x}{\partial t_x} + \left(t_eQ_e' + t_e^*Q_e^{*\prime} + t_cC_e' + t_fC_e^{f\prime} + (t_x+t_x^*)C_e^{x\prime}\right)\frac{dp_e}{dt_x} = \varphi^W Q_e^{W\prime}\frac{dp_e}{dt_x}.$$

The corresponding market-clearing conditions are:

$$\frac{dp_e}{dt_e} = \left(\frac{-1}{Q_e^{W\prime} - C_e^{W\prime}}\right)\frac{\partial Q_e}{\partial t_e}$$
$$\frac{dp_e}{dt_c} = \left(\frac{1}{Q_e^{W\prime} - C_e^{W\prime}}\right)\frac{\partial C_e}{\partial t_c}$$
$$\frac{dp_e}{dt_x} = \left(\frac{1}{Q_e^{W\prime} - C_e^{W\prime}}\right)\frac{\partial C_e^*}{\partial t_x}.$$

In the last of these market-clearing conditions we can substitute in the formula for Foreign leakage (10), in the form $\partial C_e^*/\partial t_x = (1-\Lambda^*)\partial C_e^x/\partial t_x$.

Substituting each of the three market-clearing conditions into the corresponding first-order condition, canceling $\partial Q_e/\partial t_e$ from the first, canceling $\partial C_e/\partial t_c$ from the second, and canceling $\partial C_e^x/\partial t_x$ from the third, we get:

$$t_e\left(C_e^{W\prime} - Q_e^{W\prime}\right) + \left(t_e Q_e' + t_e^* Q_e^{*\prime} + t_c C_e' + t_f C_e^{f\prime} + (t_x + t_x^*) C_e^{x\prime}\right) = \varphi^W C_e^{W\prime}$$
$$t_c\left(Q_e^{W\prime} - C_e^{W\prime}\right) + \left(t_e Q_e' + t_e^* Q_e^{*\prime} + t_c C_e' + t_f C_e^{f\prime} + (t_x + t_x^*) C_e^{x\prime}\right) = \varphi^W Q_e^{W\prime}$$
$$\frac{t_x}{1-\Lambda^*}\left(Q_e^{W\prime} - C_e^{W\prime}\right) + \left(t_e Q_e' + t_e^* Q_e^{*\prime} + t_c C_e' + t_f C_e^{f\prime} + (t_x + t_x^*) C_e^{x\prime}\right) = \varphi^W Q_e^{W\prime}.$$

Subtracting the third from the first, much like for the extraction-production tax, gives $t_e + t_x/(1-\Lambda^*) = \varphi^W$. Subtracting the second from the first, we get $t_e + t_c = \varphi^W$ as in the extraction-consumption case. Together these two results imply $t_x = (1-\Lambda^*)t_c$. Substituting the result for $t_c = \varphi^W - t_e$ and for $t_x = (1-\Lambda^*)(\varphi^W - t_e)$ back into the first equation yields:

$$t_e = \varphi^W \frac{|C_e^{f\prime}| + \Lambda^*|C_e^{x\prime}|}{Q_e^{*\prime} + |C_e^{f\prime}| + \Lambda^*|C_e^{x\prime}|} + \frac{t_e^* Q_e^{*\prime} - t_f|C_e^{f\prime}| - t_x^*|C_e^{x\prime}|}{Q_e^{*\prime} + |C_e^{f\prime}| + \Lambda^*|C_e^{x\prime}|}. \tag{17}$$

The first term on the right-hand side is the same as in (12) while the second term is the value of the adjustment, t_a, from (13).

To obtain a simpler and more intuitive result, suppose Foreign sets t_x^* to avoid distorting its consumption decisions (given the value of t_x set by Home) so that $t_x^* = t_f - t_x$. As in Section 2.4 we define $\tilde{\varphi}^* = t_e^* + t_c^*$, where $t_c^* = t_f = t_x + t_x^*$ is Foreign's implicit consumption tax. Using this notation and the restriction on t_x^*, we have $t_f = t_c^* = \tilde{\varphi}^* - t_e^*$ and

$$t_x^* = \tilde{\varphi}^* - t_e^* - (1-\Lambda^*)(\varphi^W - t_e).$$

Substituting these two expressions into (17) we get:

$$t_e = t_e^* + \left(\varphi^W - \tilde{\varphi}^*\right)\frac{|C_e^{*\prime}|}{Q_e^{*\prime} + |C_e^{*\prime}|},$$

which is the expression for Home's extraction tax in the first equation of (4). Since $t_c = \varphi^W - t_e$ and $t_c^* = \tilde{\varphi}^* - t_e^*$ we get the second equation of (4) as well.

D Structure for Quantitative Illustrations

Our quantitative illustrations are from Kortum and Weisbach (2021), which explicitly follows the Dornbusch, Fisher, and Samuelson (1977) Ricardian model of trade with a unit continuum of goods (DFS). Here we show that our analysis in this paper is fully compatible with that DFS structure. We also introduce the functional forms that we impose for the quantitative results.

The relative efficiency of producing good $j \in [0, 1]$ in Home is $a_j^*/a_j = F(j)$, where a_j is Home's total input requirement and a_j^* is Foreign's. The function F is assumed to be continuous and strictly decreasing in j. For the quantitative results, we parameterize the comparative advantage function as:

$$F(j) = \left(\frac{A}{A^*}\frac{1-j}{j}\right)^{1/\theta},$$

where A and A^* capture absolute advantage in goods production in Home and Foreign while θ is the trade elasticity.

Producers combine inputs of labor and energy in a constant-returns-to-scale production function to produce any good in any region. The wage is 1 and the relevant after-tax energy price is p, which in Home is either p_e^d (to serve the domestic market) or p_e^x (to export). The associated unit cost function for Home producers to supply good j is $f_j(p) = a_j f(p)$. By Shepard's lemma, the unit energy requirement for good j is $e_j = a_j f'(p)$. The same holds for Foreign producers, with a_j replaced by a_j^*, taking account of the energy price they face, p_e^f (to serve local consumers) or p_e^m (for goods imported to Home). The quantitative illustrations parameterize the comparative advantage function as $f(p) = p^{1-\alpha}$, where α is the labor share and $1 - \alpha$ is the energy share in goods production.

Consider goods produced for the Home market (the argument will carry over in an obvious way to the Foreign market). Effective prices of energy are $p_e^d = p_e + t_d$ and $p_e^m = p_e + t_m$. At these after-tax prices, Home producers can supply good j at cost $a_j f(p_e^d)$ while the cost to Foreign producers supplying Home is $\tau^* a_j^* f(p_e^m)$. Here τ^* is the iceberg trade cost. Home consumers will purchase j from the cheapest supplier. Thus they buy goods $j \in [0, \bar{j}_m)$ from domestic producers and goods $j \in (\bar{j}_m, 1]$ from Foreign producers. The threshold satisfies:

$$F(\bar{j}_m) = \frac{1}{\tau^*}\frac{f(p_e^d)}{f(p_e^m)},$$

making Home consumers indifferent about where they buy good $\bar{j}_m$ (since it costs the same from either source). Consumers in Home, buying from the low-cost supplier, face prices:

$$p_j = a_j f(p_e^d) \quad j \le \bar{j}_m$$
$$p_j = \tau^* a_j^* f(p_e^m) \quad j \ge \bar{j}_m.$$

Kortum and Weisbach (2021) assume CES preferences over the unit continuum of goods:

$$C_g = \left(\int_0^1 c_j^{(\sigma-1)/\sigma} dj \right)^{\sigma/(\sigma-1)},$$

where σ is the demand elasticity. Welfare of a representative consumer in Home is assumed to be:

$$U = C_s + \frac{\sigma}{\sigma - 1} \eta^{1/\sigma} \left(C_g^{(\sigma-1)/\sigma} - 1 \right) - \varphi E,$$

where E is global carbon emissions. Facing prices p_j, the utility maximizing consumption of good j is $c_j = \eta p_j^{-\sigma}$.

The price index P_g associated with C_g is:

$$P_g = \left(\int_0^1 p_j^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}.$$

Expressed as a function of the after-tax prices of energy in Home and Foreign:

$$P_g(p_e^d, p_e^m) = \left(\int_0^{\bar{j}_m} \left(a_j f(p_e^d) \right)^{1-\sigma} dj + \int_{\bar{j}_m}^1 \left(\tau^* a_j^* f(p_e^m) \right)^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}.$$

The threshold $\bar{j}_m$ is also a function of these after-tax prices of energy but, by the envelope theorem, that doesn't matter for the derivatives of P_g (since the cost of sourcing the threshold good from Home or Foreign producers is the same). For example:

$$
\begin{aligned}
\frac{\partial P_g}{\partial p_e^d} &= \frac{1}{1-\sigma} P_g^\sigma \left\{ \int_0^{\bar{j}_m} (1-\sigma) p_j^{-\sigma} a_j f'(p_e^d) dj \right. \\
&\left. + \left(\left(a_{\bar{j}_m} f(p_e^d) \right)^{1-\sigma} - \left(\tau^* a^*_{\bar{j}_m} f(p_e^m) \right)^{1-\sigma} \right) \partial \bar{j}_m / \partial p_e^d \right\} \\
&= P_g^\sigma \int_0^{\bar{j}_m} p_j^{-\sigma} a_j f'(p_e^d) dj.
\end{aligned}
$$

In Section 3 we express Home welfare in terms of indirect utility:

$$
U = Y + \tilde{u}(p_e^d, p_e^m) - \varphi E.
$$

Using the DFS structure, with CES preferences, we have:

$$
\tilde{u}(p_e^d, p_e^m) = \frac{\eta}{\sigma - 1} P_g(p_e^d, p_e^m)^{-(\sigma-1)} - \frac{\eta^{1/\sigma}\sigma}{\sigma - 1}, \tag{18}
$$

which is an explicit formula for the term in the objective (5). The derivations of the results in Section 3 apply Roy's identity:

$$
\begin{aligned}
\partial \tilde{u} / \partial p_e^d = \tilde{u}_1 = -C_e^d \\
\partial \tilde{u} / \partial p_e^m = \tilde{u}_2 = -C_e^m.
\end{aligned}
$$

As a reality check, we can differentiate (18) to get:

$$
\tilde{u}_1 = -\eta P_g^{-\sigma} \partial P_g / \partial p_e^d = -\int_0^{\bar{j}_m} a_j f'(p_e^d) \eta p_j^{-\sigma} dj = -\int_0^{\bar{j}_m} e_j c_j dj = -C_e^d.
$$

Roy's identity is confirmed, in spite of $\bar{j}_m$ governing the extensive margin of trade.

Energy embodied in Home consumption is:

$$
\begin{aligned}
C_e = C_e^d + C_e^m &= \int_0^{\bar{j}_m} e_j c_j dj + \int_{\bar{j}_m}^1 \tau^* e_j^* c_j dj \\
&= \int_0^{\bar{j}_m} a_j f'(p_e^d) \eta \left(a_j f(p_e^d) \right)^{-\sigma} dj + \int_{\bar{j}_m}^1 \tau^* a_j^* f'(p_e^m) \eta \left(\tau^* a_j^* f(p_e^m) \right)^{-\sigma} dj.
\end{aligned}
$$

Applying the functional form for $f(p)$ we have:

$$f'(p)\eta f(p)^{-\sigma} = (1-\alpha)p^{-\epsilon_D},$$

where $\epsilon_D = \alpha + (1-\alpha)\sigma$ is the energy demand elasticity. Substituting in this functional form:

$$C_e = (1-\alpha)\eta \left(p_e^d\right)^{-\epsilon_D} \int_0^{\bar{j}_m} a_j^{1-\sigma} dj + (1-\alpha)\eta \left(p_e^m\right)^{-\epsilon_D} \int_{\bar{j}_m}^1 \left(\tau^* a_j^*\right)^{1-\sigma} dj.$$

On the supply side, we restrict the cost functions for extraction to yield constant supply elasticities. Hence, we take:

$$c(Q_e) = c \times Q_e^{(\epsilon_S+1)/\epsilon_S}$$

$$c^*(Q_e^*) = c^* \times (Q_e^*)^{(\epsilon_S^*+1)/\epsilon_S^*},$$

where c and c^* are constants, which are subsumed in calibrating to data on extraction in each region, while ϵ_S and ϵ_S^* are the energy supply elasticities.