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TRADE AND INFORMALITY IN THE PRESENCE OF LABOR MARKET FRICTIONS AND REGULATIONS*

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Abstract

We build an equilibrium model of a small open economy with labor market frictions and imperfectly enforced regulations. Heterogeneous firms sort into the formal or informal sector. We estimate the model using data from Brazil, and use counterfactual simulations to understand how trade affects economic outcomes in the presence of informality. We show that: (1) Trade openness unambiguously decreases informality in the tradable sector, but has ambiguous effects on aggregate informality. (2) The productivity gains from trade are understated when the informal sector is omitted. (3) Trade openness results in large welfare gains even when informality is repressed. (4) Repressing informality increases productivity, but at the expense of employment and welfare. (5) The effects of trade on wage inequality are reversed when the informal sector is incorporated in the analysis. (6) The informal sector works as an "unemployment," but not a "welfare buffer" in the event of negative economic shocks.

JEL codes: F14, F16, J46, O17

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1 Introduction

A substantial share of the labor force in many emerging and developing economies is employed informally; for example, in Latin America, informality ranges from 35 percent in Chile to 80 percent in Peru (Perry et al., 2007). Yet, the informal sector is near absent in theoretical and empirical work in trade. The few papers that have focused on the role of the informal sector during trade liberalization episodes suggest that shifts into and out of informal employment constitute important margins of labor market adjustment in response to trade shocks. These papers have relied on sectoral or regional variation to identify relative effects. For example, in the context of Vietnam, McCaig and Pavcnik (2018) exploit sectoral variation to show that the United States-Vietnam Bilateral Trade Agreement induced reallocation of labor from informal to formal manufacturing in the most affected sectors. On the other hand, studies that have focused on Latin America (e.g., Goldberg and Pavcnik (2003), Dix-Carneiro and Kovak (2019), Ponczek and Ulyssea (2020)) have found that trade liberalization often increases informal employment in the most impacted sectors or regions. Moreover, the results in Dix-Carneiro and Kovak (2019) and Ponczek and Ulyssea (2020) suggest that the informal sector may serve as a buffer to trade-displaced workers, and that in the absence of informality, the effects of foreign competition on unemployment might have been more severe. While the above studies point to a potentially important role of the informal sector during an economy's adjustment to trade or other economic shocks, they cannot—by design—speak to the aggregate implications of informality, and they do not permit welfare analysis. This paper aims to fill this gap by developing a general equilibrium framework that allows for such analysis.

On the worker side, one can broadly define informality in two ways: The first defines a worker as informal if she does not have permanent and stable employment associated with benefits such as health and social security. The second defines a worker as informal if, in addition to not receiving benefits, she is invisible to the tax authorities and her employer illegally evades labor market regulations (including minimum wages and firing rules). The first definition has become relevant even in developed countries in recent years with the emergence of the gig economy. The second definition applies primarily to developing countries where the tax evasion associated with informality is a first-order issue. On the firm side, informality implies that firms do not comply with taxes or relevant regulations (e.g., labor laws). This can be harmful for two main reasons. First, it may lead to substantial misallocation of resources and hamper growth, as unproductive firms that survive by evading taxes and avoiding compliance with labor market regulations prevent

the allocation of labor to more productive firms. Second, it implies tax evasion, hindering fiscal capacity and the provision of public goods. On the other hand, as suggested by Dix-Carneiro and Kovak (2019) and Ponczek and Ulyssea (2020), informality may provide *de facto* flexibility for firms and workers to cope with adverse shocks.

Building on Coşar et al. (2016), we develop a structural equilibrium model with heterogeneous firms that choose whether to operate in the formal or in the informal sector. The model features a rich institutional setting, where formal firms must comply with minimum wages, and are subject to firing costs as well as payroll and revenue taxes. However, taxes and labor market regulations are imperfectly enforced by the government, giving rise to incentives for some firms to be informal. Finally, the economy consists of tradable and non-tradable sectors that interact. Only formal firms that produce tradable goods are able to export.

We estimate the model using multiple data sources, including matched employer-employee data from formal and informal firms and workers in Brazil, as well as several other sources of firm- and worker-level data such as household surveys, manufacturing and services censuses, and customs data. Then, we conduct a series of counterfactual experiments to better understand the impact of trade shocks on an economy with a large informal sector. While the focus of the present paper is on trade, we note that the framework we develop can be applied to study the effects of several other policies, such as changes in payroll taxes, value added taxes, minimum wages, and unemployment benefits, either individually or jointly.

Brazil, with its excellent sources of data for the formal and the informal sectors, provides an excellent setting for our work. Nearly two thirds of businesses and 40 percent of GDP are informal (Ulyssea, 2018) and the labor regulations are both substantive and weakly enforced. Moreover, there is a clear definition of what constitutes informality: we define as informal workers those who do not hold a formal labor contract, clearly observable through the worker's booklet *carteira de trabalho*. Informal firms are those not registered with the tax authorities, which means that they do not possess the tax identification number required for Brazilian firms (*Cadastro Nacional de Pessoa Jurídica*—CNPJ) and which we are able to also observe.

Our estimated model rationalizes a number of findings reported in the empirical literature, while yielding new insights. We find that trade openness, induced by a reduction in iceberg trade costs, leads to large declines in informality in the tradable sector, an effect that is robust to the initial level of trade costs, the magnitude of their decline, and the regulatory environment. On the other hand, we find that the effects of trade openness on informality in the non-tradable sector

are more nuanced and context-dependent. As a result, the overall effect of trade on informality is ambiguous, and generally small. This result is consistent with the casual observation that the informal sector has not substantially shrunk in middle-income economies despite the large-scale liberalization episodes these experienced in the 1980s and 1990s (see, for example, World Bank, 2019).

Further, we find that trade openness is associated with substantive increases in productivity and welfare. Importantly, we show that the productivity gains from trade are severely understated in the tradable sector, if one leaves out the informal sector (as analyses of trade liberalization episodes typically do). For the non-tradable sector, our results point to a bias in the opposite direction, but in the aggregate, the tradable sector effect dominates. As a result, we conclude that the productivity gains from trade for the economy as a whole are understated in analyses focusing exclusively on the formal sector of the economy.

One of the main rationales for reducing informality is the wish to increase productivity. Our counterfactual analysis shows that indeed, reducing or eliminating informality through stricter monitoring and enforcement raises productivity. However, the productivity gains are achieved at the expense of employment and welfare. In contrast, trade liberalization achieves sizable productivity gains while raising aggregate welfare, and seems therefore a superior way to increase productivity.

Our analysis also has implications for wage inequality. We find that the inclusion of the informal sector reverses predictions on the effects of trade on inequality that is driven by firm heterogeneity. If we focus on the formal sector alone (again, as most earlier analyses have done), we observe that trade liberalization contributes to a rise in wage inequality. However, the effect in the informal sector goes in the opposite direction, while the distance between average formal and informal wages decreases. As a result, trade liberalization reduces aggregate wage inequality driven by differences across firms.

Finally, our results lend strong support to the view that the informal sector serves as an "unemployment buffer" during bad times: in the case of negative aggregate shocks, unemployment increases by considerably more if informality is repressed. However, this "unemployment buffer" role of informality does not translate into a "welfare buffer." We find that, in the event of a negative economic shock, welfare declines by less with lower informality. This somewhat counterintuitive result is due to a positive selection effect arising from the exit of inefficient, informal firms in that case—in other words, from a strong "creative destruction" effect.

The aforementioned results arise from the interaction of several mechanisms in operation, which

we discuss in the model section as well as in the presentation of the counterfactual experiments. In general, our Melitz-type framework implies several selection effects, as multiple entry and formalization thresholds in the tradable and non-tradable sectors shift in response to trade and domestic policies. These interact in our model with labor market frictions as well as with domestic regulations, especially minimum wages and enforcement of regulations, to produce the patterns we summarized above. Overall, our findings demonstrate the importance of incorporating the informal sector in analyses of trade policies.

Our paper is organized as follows. Section 2 outlines our model. Section 3 discusses the main regulations in place in the Brazilian economy, and section 4 describes the data we use to estimate the model. Section 5 details the estimation procedure, discusses identification and shows how the model fits key aspects of the data. Section 6 shows our counterfactual experiments and section 7 presents our main takeaways.

2 Model

We start by considering a closed-economy setup in sections 2.1 through 2.4 and extend the model to the open economy in section 2.5. Section 2.6 discusses the mechanisms through which trade affects informality and section 2.7 lists the equilibrium conditions.

2.1 Consumers

The economy is populated by homogeneous, infinitely-lived workers-consumers. Individuals derive utility from two composite goods, C and S, each combining tradable and non-tradable sector varieties, respectively.[1] Preferences are given by:

$$U = \sum_{t=1}^{\infty} \frac{C_t^{\zeta} S_t^{1-\zeta}}{(1+r)^t}, \tag{1}$$

where

$$C_t = \left(\int_0^{N_{Ct}} c_t(n)^{\frac{\sigma_C - 1}{\sigma_C}} dn \right)^{\frac{\sigma_C}{\sigma_C - 1}} \qquad S_t = \left(\int_0^{N_{St}} s_t(n)^{\frac{\sigma_S - 1}{\sigma_S}} dn \right)^{\frac{\sigma_S}{\sigma_S - 1}}, \tag{2}$$

and $\zeta \in (0,1)$ is the fraction of expenditure on tradable-sector goods, $\sigma_k > 1$ is the elasticity of substitution across varieties within sector $k \in \{C, S\}$, N_{kt} denotes the measure of varieties available in sector k at time t, $n \in (0, N_{kt})$ indexes varieties, and $\frac{1}{1+r}$ is the discount factor. As we will focus on steady-state equilibria, we henceforth drop the time subscript t for notational convenience.

[1] The terminology "tradable" and "non-tradable" is used to classify goods across sectors of the economy. Tradable refers primarily to manufacturing sectors, whereas non-tradable refers primarily to service sectors.

2.2 Firms

There is a continuum of firms with heterogeneous productivities in both the tradable and non-tradable sectors. Formal and informal firms coexist in both sectors, and each firm produces a unique variety $n \in (0, N_k)$, $k \in \{C, S\}$. Firms in each sector k produce using labor and intermediate inputs in a constant returns to scale Cobb-Douglas production function:

$$q_k(z, \ell, \iota_k) = z\ell^{\delta_k} \iota_k^{1-\delta_k}, \tag{3}$$

where z denotes the firm's total factor productivity, ℓ denotes the firm's employment size, ι_k denotes sector k's intermediate input usage, and $\delta_k \in (0, 1)$. Intermediate inputs ι_k in sector k are given by a combination of sector C and S goods:

$$\iota_k = im_C^{\lambda_k} im_S^{1-\lambda_k}, \tag{4}$$

where im_C and im_S are CES aggregates defined in exactly the same way and with the same parameters to those in equation (2,) and $\lambda_k \in (0, 1)$ is the share of sector k's intermediate input payments to sector C goods. Firms' idiosyncratic productivity z evolves over time following the AR(1) process below:

$$\ln z' = \rho_k \ln z + \sigma_k^z \varepsilon, \qquad \rho_k \in (0, 1), \qquad \varepsilon \sim N(0, 1), \tag{5}$$

where σ_k^z is the standard deviation of the shocks.[2]

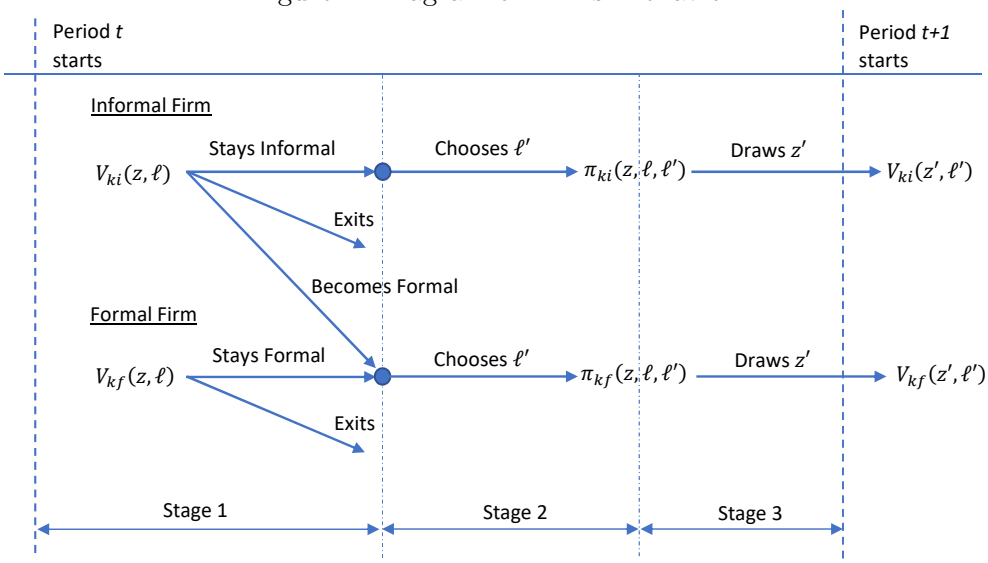
Monopolistic competition implies that gross revenues as a function of output q in sector $k \in \{C, S\}$ are given by:

$$\widetilde{R}_k(q) = \left(\frac{X_k}{P_k^{1-\sigma_k}} \right)^{\frac{1}{\sigma_k}} q^{\frac{\sigma_k - 1}{\sigma_k}}, \tag{6}$$

where X_k is total expenditure on sector k goods, $P_k = \left(\int_0^{N_k} p_k(n)^{1-\sigma_k} dn \right)^{\frac{1}{1-\sigma_k}}$ is the price index for sector $k \in \{C, S\}$, and $p_k(n)$ is the price charged by firm n in sector k. Aggregate expenditure on tradable-sector goods is given by $X_C = \zeta I + X_C^{int}$, where I is aggregate income and X_C^{int} is total expenditure, by firms, on intermediate goods from sector C. Similarly, for the non-tradable sector, aggregate expenditure is given by $X_S = (1 - \zeta) I + X_S^{int} + E_S$, where X_S^{int} is total expenditure, by firms, on intermediate goods from sector S and E_S represents expenditures on non-tradable-sector goods made by firms in order to cover entry, hiring, fixed and export costs (which we discuss below). Aggregate income is determined by total wages, government transfers and aggregate firms' profits.

[2] This process is imposed to be the same across formal and informal firms within tradable and non-tradable sectors. Unfortunately, we do not have longitudinal data on firms in the informal sector, so that this process cannot be separately identified for formal and informal firms.

Figure 1: Diagram of Firms' Behavior



Firms can freely adjust their intermediate input usage. Denote by $\iota_k(z,\ell)$ the optimal intermediate input usage of a firm in sector k with productivity z and ℓ workers. This firm's gross revenue can then be written as $R_k(z,\ell) \equiv \widetilde{R}_k\left(z\ell^{\delta_k}\iota_k(z,\ell)^{1-\delta_k}\right)$. It is easy to show that expenditures on intermediates are proportional to gross revenues, resulting in the following expression for firm-level value added:

$$VA_k(z,\ell) = \frac{\sigma_k - (1-\delta_k)(\sigma_k - 1)}{\sigma_k} R_k(z,\ell). \tag{7}$$

Timing

The timing of events is illustrated in Figure 1. Formal firms are indexed by f and informal firms are indexed by i. Consider an informal firm that starts period t with productivity z and employment level ℓ. In the first stage, this firm may: (i) stay informal; (ii) exit, either as a result of an endogenous decision or because of an exogenous shock that occurs with probability α_k; (iii) become formal. In the second stage, the firm decides whether to adjust its workforce (up or down) to ℓ' or not at all. Right after this decision the firm realizes profits and pays wages to its workers. In stage 3, the firm draws a new productivity value z' and starts period $t+1$ with state (z',ℓ'). The timing and sequence of events for formal firms is the same as that for informal ones, except that we do not allow them to become informal.

Hiring and Firing Costs

Both formal and informal firms in tradable and non-tradable sectors face hiring costs. We parameterize hiring costs as a function of employment levels ℓ and number of posted vacancies υ:

$$C_k^h(\ell, \upsilon) = \left(\frac{h_k}{\gamma_{k1}}\right)\left(\frac{\upsilon}{\ell^{\gamma_{k2}}}\right)^{\gamma_{k1}}, \tag{8}$$

where h_k, $\gamma_{k1} > 1$ and $\gamma_{k2} \in (0,1)$ are parameters to be estimated. If μ_{kj}^{υ} is the probability of filling a vacancy faced by a firm of type $j \in \{f, i\}$ in sector $k \in \{C, S\}$, then expanding from ℓ to ℓ' requires posting $\upsilon = \frac{\ell' - \ell}{\mu_{kj}^{\upsilon}}$ vacancies.[3] The cost of expanding from ℓ to ℓ' workers for a firm of type j in sector k is therefore given by:

$$H_{kj}(\ell, \ell') = \left(\mu_{kj}^{\upsilon}\right)^{-\gamma_{k1}}\left(\frac{h_k}{\gamma_{k1}}\right)\left(\frac{\ell' - \ell}{\ell^{\gamma_{k2}}}\right)^{\gamma_{k1}}. \tag{9}$$

The value of γ_{k2} controls the extent to which firm-level growth rates in employment decline with size, a stylized fact in the data that we discuss in section 4. The parameter γ_{k1} governs the convexity of the hiring function. Allowing for convexity is important for the model to be able to generate wage dispersion across firms. To build intuition for this fact, momentarily abstract from dynamic considerations. In this case, the wage determination process we discuss in section 2.4 implies that wages are proportional to value-added per worker, which is—by virtue of our assumptions—proportional to marginal value-added. Firms set marginal value added equal to the marginal cost of an additional worker. With linear hiring costs, the marginal cost is constant and equal across firms, so that wages will also be equalized across firms. In contrast, with convex hiring costs, the marginal cost of an additional worker is increasing in the growth of employment, so that expanding firms tend to pay higher wages.

Firing costs are entirely driven by regulations and affect formal firms only. They take the linear form:

$$F(\ell, \ell') = \kappa(\ell - \ell'), \tag{10}$$

where $\kappa > 0$ is the parameter governing the firing cost function. Consistent with the Brazilian labor market regulations, we assume that firing costs are equal across the C and S sectors. In our model, firing costs are collected by the government and are rebated back to consumers, while hiring costs are incurred in terms of the non-tradable-sector composite good.

[3] Note that the probability of filling a vacancy μ_{kj}^{υ} is an endogenous object that will depend on the aggregate number of vacancies in each sector $k \in \{C, S\}$ and firm type $j \in \{f, i\}$, as well as on the mass of unemployed workers.

Profit and Value Functions

Formal firms are subject to payroll and value added taxes, firing costs and the minimum wage regulation. The profit function of a formal firm in sector $k \in \{C, S\}$ is thus given by:

$$\pi_{kf}\left(z, \ell, \ell'\right) = (1 - \tau_y) VA_k\left(z, \ell'\right) - C_{kf}\left(z, \ell, \ell'\right) - \bar{c}_k, \tag{11}$$

where $\bar{c}_k$ denotes a per-period, fixed cost of operation, which we define in units of the non-tradable sector composite good; τ_y is a value-added tax, collected by the government and rebated to consumers. Due to hiring and firing costs, the total cost function for a formal firm adjusting from ℓ to ℓ' workers is given by the following expression:

$$C_{kf}\left(z, \ell, \ell'\right) = \begin{cases} (1 + \tau_w) \max\left\{w_{kf}\left(z, \ell'\right), \underline{w}\right\} \ell' + H_{kf}\left(\ell, \ell'\right) & \text{if } \ell' > \ell \\ (1 + \tau_w) \max\left\{w_{kf}\left(z, \ell'\right), \underline{w}\right\} \ell' + \kappa\left(\ell - \ell'\right) & \text{if } \ell' \leq \ell, \end{cases} \tag{12}$$

where the wage schedule $w_{kf}\left(z, \ell'\right)$ is the result of a bargaining problem between the firm and its workers that is detailed in section 2.4, $\underline{w}$ denotes the minimum wage and τ_w is the payroll tax, which is also assumed to be collected by the government and rebated to consumers.

Since formal firms have to choose whether to stay or leave their industry, their value function is given by:

$$V_{kf}\left(z, \ell\right) = (1 - \alpha_k) \max\left\{0, \max_{\ell'}\left\{\pi_{kf}\left(z, \ell, \ell'\right) + \frac{1}{1 + r} E_{z'|z} V_{kf}\left(z', \ell'\right)\right\}\right\}, \tag{13}$$

where α_k denotes the exogenous destruction probability that firms face every period for $k = C, S$. The solution of (13) leads to the employment policy function $\ell' = L_{kf}(z, \ell)$ and to the vacancy posting policy function $\upsilon_{kf}(z, \ell) = \frac{L_{kf}(z,\ell) - \ell}{\mu^{v}_{kf}} \times \mathcal{I}\left[L_{kf}(z, \ell) > \ell\right]$ (as well as to other policies such as exit and stay-active decisions).

While informal firms do not incur any of the regulatory costs (taxes, minimum wages, firing costs), they do face an expected cost of informality, which includes the probability of detection by the government and subsequent fines. It also includes a range of opportunity costs associated with informality such as scarce access to formal financial markets (e.g. credit lines), hampering the ability of firms to grow. As firms grow, they become more visible to the government and therefore are inspected with higher probability, which entails costs in the form of fines and bribes, or can lead to the firm shutting down its operations. Therefore, we allow the expected cost of informality as a fraction of revenues, p_{ki}, to depend on the firm's size ℓ'. Thus, the profit function of an informal firm is given by:

$$\pi_{ki}\left(z, \ell, \ell'\right) = VA_k\left(z, \ell'\right) - K^{inf}\left(z, \ell'\right) - C_{ki}\left(z, \ell, \ell'\right) - \bar{c}_k, \tag{14}$$

where $K^{inf}(z,\ell') \equiv p_{ki}(\ell') R_k(z,\ell')$ are the expected costs associated with informality, which we assume proportional to gross revenues. We work with the following simple specification:

$$p_{ki}(\ell') = \widetilde{a}_k \exp\left\{\widetilde{b}_k(\ell' - 1)\right\}. \tag{15}$$

Since informal firms are not subject to firing costs or other regulations, their cost function is given by:

$$C_{ki}(z,\ell,\ell') = \begin{cases} w_{ki}(z,\ell')\ell' + H_{ki}(\ell,\ell') & \text{if } \ell' > \ell \\ w_{ki}(z,\ell')\ell' & \text{if } \ell' \leq \ell, \end{cases} \tag{16}$$

where $w_{ki}(z,\ell')$ denotes the wage paid by an informal firm with productivity z and size ℓ'. The value functions of informal firms are similar to those of formal ones, except that they have the additional option to formalize their businesses:

$$V_{ki}(z,\ell) = (1-\alpha_k)\max\left\{\begin{array}{c} 0, \max\limits_{\ell'}\left\{\pi_{ki}(z,\ell,\ell') + \frac{1}{1+r}E_{z'|z}V_{ki}(z',\ell')\right\}, \\ \max\limits_{\ell'}\left\{\pi_{kf}(z,\ell,\ell') + \frac{1}{1+r}E_{z'|z}V_{kf}(z',\ell')\right\} \end{array}\right\}. \tag{17}$$

The solution of (17) leads to the employment policy function $\ell' = L_{ki}(z,\ell)$ and to the vacancy posting policy function $v_{ki}(z,\ell) = \frac{L_{ki}(z,\ell)-\ell}{\mu^v_{ki}} \times \mathcal{I}\left[L_{ki}(z,\ell) > \ell\right]$ (as well as to other policies such as exit, change to formal and stay informal decisions).

Entry

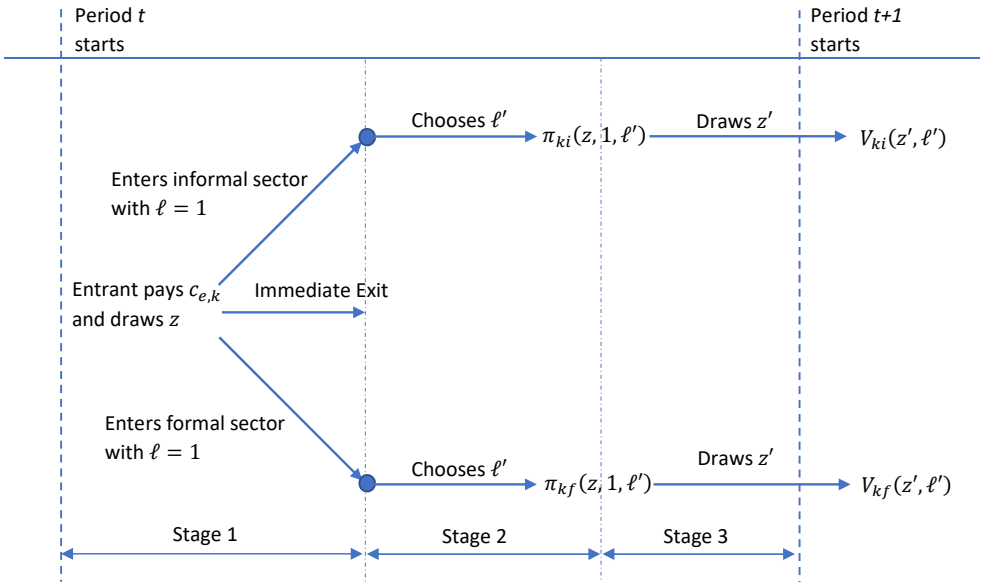
Firm entry is illustrated in Figure 2. Every period there is a mass M_k of entrants into the tradable and non-tradable sectors. In the first stage within the period, entrants observe their productivity z—drawn from the ergodic distribution g^e_k implied by (5)—after incurring a sunk cost $c_{e,k}$ of entry into sector k. Based on this productivity draw, the entering firm chooses to be formal or informal or to exit immediately. Formal and informal entrants start their first period with workforce $\ell = 1$, whose recruitment cost is subsumed in $c_{e,k}$. Following entry, in stage 2, the firm decides to adjust its labor force to ℓ' just before the production stage. It then behaves as an incumbent, drawing productivity z' for the next period right after production (stage 3). The value functions for entrants in either sector are given by:

$$V^e_{kj}(z) = \max_{\ell'}\left\{\pi_{kj}(z,1,\ell') + \frac{1}{1+r}E_{z'|z}V_{kj}(z',\ell')\right\}, \tag{18}$$

where $j = i, f$. The value at entry together with the entry conditions is defined by

$$V^e_k = E_z \max\left\{V^e_{ki}(z), V^e_{kf}(z), 0\right\}, \tag{19}$$

Figure 2: Diagram of Entry Behavior



whose solution leads to the entry policy functions. Assuming there are positive masses of entrants in each sector, free entry dictates that:

$$V_k^e = c_{e,k}. \tag{20}$$

2.3 Labor Market Frictions

Formal and informal labor markets are characterized by search and matching frictions, which prevent unemployed workers from immediately finding open vacancies and underlie part of hiring costs, as equation (9) highlights. We assume random search, and therefore all unemployed workers form a pool of individuals who randomly meet with formal or informal firms in one of the sectors $k = C, S$. Thus, formal and informal firms operating in tradable and non-tradable sectors compete for workers in the labor market. Given the total number of vacancies posted in each sector and type of firm $(\upsilon_{Cf}, \upsilon_{Ci}, \upsilon_{Sf}, \upsilon_{Si})$, and the mass of unemployed workers searching for jobs, L_u, the total number of matches that are formed is given by:

$$m\left(\upsilon_{Cf}, \upsilon_{Ci}, \upsilon_{Sf}, \upsilon_{Si}, L_u\right) = \phi \widetilde{\upsilon}^{\xi} L_u^{1-\xi}, \tag{21}$$

Where $\widetilde{\upsilon} = \upsilon_{Cf} + \upsilon_{Ci} + \upsilon_{Sf} + \upsilon_{Si}$ aggregates vacancies across sectors and types of firms, $\phi > 0$, and $0 < \xi < 1$. Matches are split across sectors and firm types in proportion to the number of vacancies posted, so that $m_{kj} = \frac{\upsilon_{kj}}{\widetilde{\upsilon}} \times m$ matches are formed with firms of type j in sector k. Thus, the probability of filling a vacancy $(\mu_{kj}^{\upsilon} = \frac{m_{kj}}{\upsilon_{kj}})$ is independent of sector and firm type. We denote

it by μ^{υ} and is given by:

$$\mu^{\upsilon} = \phi \left(\frac{L_u}{\widetilde{\upsilon}} \right)^{1-\xi}. \tag{22}$$

This expression highlights that formal firms directly compete with informal ones in the labor market. Finally, unemployed workers face job finding probabilities in each sector and firm type given by:[4]

$$\mu^e_{kj} \equiv \frac{m_{kj}}{L_u} = \frac{\upsilon_{kj}}{\widetilde{\upsilon}} \left(\frac{\phi}{(\mu^{\upsilon})^{\xi}} \right)^{\frac{1}{1-\xi}}. \tag{23}$$

2.4 Wages

Wage setting takes place after hiring and fixed costs have been sunk and matching has taken place. We assume that a union engages in collective bargaining with the employer on behalf of the workers over the surplus of the match, determining a wage $w_{kj}(z, \ell')$. The latter depends on the firm's size and productivity.

The surpluses of a formal firm in sector k (S^e_{kf}), and the union it faces (S^u_{kf}) are each defined as:

$$S^e_{kf}(z, \ell') = (1 - \tau_y) VA_k(z, \ell') - (1 + \tau_w) w_{kf}(z, \ell') \ell' + \frac{1}{1+r} E_{z'|z} V_{kf}(z', \ell'), \tag{24}$$

$$S^u_{kf}(z, \ell') = \left[w_{kf}(z, \ell') + \frac{1}{1+r} J^e_{kf}(z, \ell') - \left(b + b^u + \frac{1}{1+r} J^u \right) \right] \ell', \tag{25}$$

where b denotes the utility flow from being unemployed; b^u denotes the value of unemployment insurance benefits, which are only received by formal workers; J^u is the expected present value of search; and $J^e_{kf}(z, \ell')$ is the expected present value of a job in a formal-sector firm in sector k with current productivity z and workforce ℓ'—see Online Appendix I for its derivation.[5]

Let β be the parameter that drives workers' bargaining power. If the joint surplus of the firm and workers is positive, the outcome of bargaining is given by:

$$S^u_{kf}(z, \ell') = \beta \left(S^e_{kf}(z, \ell') + S^u_{kf}(z, \ell') \right). \tag{26}$$

Importantly, the overall surplus depends on the wage because of payroll taxes: in other words, the value of the surplus depends on how it is shared. This leads to a wage structure for formal workers

[4] μ^e_{kj} should be interpreted as the transition rate from unemployment to sector k and firm type j.

[5] We assume that if all workers leave, the firm exits, and that hiring costs and fixed operating costs are already sunk at the bargaining process.

defined by:

$$(1+\beta\tau_w)\, w^u_{kf}\left(z,\ell'\right) = (1-\beta)\left(b + b^u + \frac{1}{1+r}\left(J^u - J^e_{kf}\left(z,\ell'\right)\right)\right) + \beta\left((1-\tau_y)\frac{VA_k\left(z,\ell'\right)}{\ell'} + \frac{1}{1+r}E_{z'|z}\frac{V_{kf}\left(z',\ell'\right)}{\ell'}\right). \tag{27}$$

Thus, the bargained wage is proportional to a convex combination between the firm's value per worker and the worker's outside option net of the continuation value of the job to the worker, $J^e_{kf}\left(z,\ell'\right)$.[6] If $w^u_{kf}\left(z,\ell'\right)$ leads to a negative union surplus then we set the wage equal to its reservation value, that is, the wage $w^{res}_{kf}\left(z,\ell'\right)$ that solves $S^u_{kf}\left(z,\ell'\right)=0$. Therefore the outcome of the Nash bargaining process leads to wage schedule:

$$w_{kf}\left(z,\ell'\right) = \max\left\{w^u_{kf}\left(z,\ell'\right), w^{res}_{kf}\left(z,\ell'\right)\right\}. \tag{28}$$

As highlighted in equation (12), formal firms cannot pay below the minimum wage, so that wages effectively paid are given by $\max\left\{w_{kf}\left(z,\ell'\right),\underline{w}\right\}$.

Wages in the informal sector are determined in a similar way. However, in the informal sector unemployment insurance benefits are not offered, taxes are not paid and firms face an expected cost of informality. Thus, the wage informal firms pay to their workers is given by:

$$w_{ki}\left(z,\ell'\right) = \max\left\{w^u_{ki}\left(z,\ell'\right), w^{res}_{ki}\left(z,\ell'\right)\right\}, \tag{29}$$

where

$$w^u_{ki}\left(z,\ell'\right) = (1-\beta)\left(b + \frac{1}{1+r}\left(J^u - J^e_{ki}\left(z,\ell'\right)\right)\right) + \beta\left(\left(1-\frac{\sigma_k}{\sigma_k-(1-\delta_k)(\sigma_k-1)}\right)p_{ki}\left(\ell'\right)\frac{VA_k\left(z,\ell'\right)}{\ell'} + \frac{1}{1+r}E_{z'|z}\frac{V_{ki}\left(z',\ell'\right)}{\ell'}\right), \tag{30}$$

and $J^e_{ki}\left(z,\ell'\right)$ is the analogous to $J^e_{kf}\left(z,\ell'\right)$ in the informal sector, see Online Appendix I for details. Finally, the reservation wage, $w^{res}_{ki}\left(z,\ell'\right)$, solves $S^u_{ki}\left(z,\ell'\right)=0$, where S^u_{ki} is the surplus of the union in the informal sector. This places a lower bound on wages in the informal sector.

2.5 Open Economy

We now extend the model to the open economy case. We assume that the home country is small relative to the rest of the world and therefore foreign conditions do not react to its policies. In the following analysis, we drop the formal/informal qualifier in order to simplify notation, as we assume throughout that informal firms cannot export.[7]

[6] The factor of proportionality $1+\beta\tau_w$ highlights that workers also bear the cost of payroll taxes.

[7] This assumption comes from the fact that firms that are not registered cannot undertake the necessary legal and bureaucratic procedures to export.

Price Indices and Aggregate Variables

Let $N_{F,C}$ denote the measure of foreign varieties available to domestic consumers. Given the small open economy assumption, this variable is assumed to be fixed. Without loss of generality, we normalize the price index of free on board imports to be one, since foreign prices are exogenous to our model. Thus, the price index of imports in domestic currency becomes $P_{F,C} = \epsilon\tau_a\tau_c$, where ϵ is the exchange rate, $\tau_a - 1 > 0$ is the ad-valorem tariff and $\tau_c > 1$ the iceberg trade cost. The price index of domestically produced varieties $n \in (N_{F,C}, N_C]$ is given by $P_{H,C} = \left(\int_{N_{F,C}}^{N_C} p(n)^{1-\sigma_C} dn\right)^{\frac{1}{1-\sigma_C}}$, and the price index for the composite tradable sector good is given by $P_C = \left[P_{H,C}^{1-\sigma_C} + P_{F,C}^{1-\sigma_C}\right]^{\frac{1}{1-\sigma_C}}$.

The domestic demand for goods produced domestically is given by $Q_{H,C}(n) = D_{H,C} p(n)^{-\sigma_C}$, for $n \in (N_{F,C}, N_C]$ with $D_{H,C} \equiv \left(\frac{X_C}{P_C^{1-\sigma_C}}\right)$; and the domestic demand for foreign-produced goods is given by $Q_{H,C}(n) = D_{H,C}\left(\epsilon\tau_a\tau_c p^*(n)\right)^{-\sigma_C}$, for $n \in [0, N_{F,C}]$—where $p^*(n)$ is the price of foreign variety n in foreign currency. Finally, foreign demand for domestically produced goods is given by $Q_{F,C}(n) = D_F^*\left(p_x^*(n)\right)^{-\sigma_C}$, for $n \in (N_{F,C}, N_C]$, where $p_x^*(n)$ is the price of domestic variety n in the foreign country, denominated in foreign currency, and D_F^* is an exogenous foreign demand shifter. If $\mathcal{I}_C^x(n)$ denotes an indicator function that equals one if variety n is exported, we have that the value of aggregate imports (before import tariffs) and exports are given by the following expressions:

$$Imports = \frac{D_{H,C}\left(\epsilon\tau_a\tau_c\right)^{1-\sigma_C}}{\tau_a} \quad \text{and} \quad Exports = D_F^*\epsilon\int_{N_{F,C}}^{N_C} \mathcal{I}_C^x(n)\, p_x^*(n)^{1-\sigma_C} dn. \tag{31}$$

Exporters

Given the expression of foreign demand for home variety n just described, $Q_{F,C}(n)$, revenues from exports are given by $\epsilon D_F^{*\frac{1}{\sigma_C}}\left(q_x/\tau_c\right)^{\frac{\sigma_C-1}{\sigma_C}}$, where q_x is the total quantity exported. If a firm exports, it must decide which fraction η of its output to sell abroad. *Conditional on being an exporter*, total gross revenue for producing a total of q units and exporting a fraction η of this production is given by: $\widetilde{R}_C^x(q,\eta) = \exp\left(d_{H,C} + d_F(\eta)\right) q^{\frac{\sigma_C-1}{\sigma_C}}$, where $d_{H,C} = \ln\left(D_{H,C}^{\frac{1}{\sigma_C}}\right)$ and $d_F(\eta) = \ln\left((1-\eta)^{\frac{\sigma_C-1}{\sigma_C}} + \epsilon\left(\frac{D_F^*}{D_{H,C}}\right)^{\frac{1}{\sigma_C}}\left(\frac{\eta}{\tau_c}\right)^{\frac{\sigma_C-1}{\sigma_C}}\right)$. It is easy to verify that all exporters optimally decide to export the same fraction, η^o, of their production. In what follows, we will refer to $d_F(\eta^o)$ simply as d_F.[8] Empirically, d_F is directly related to the fraction of gross revenues coming

[8] When we substitute η^o into $d_F(\eta)$, we obtain $d_F \equiv d_F(\eta^o) = \log\left(\left(1 + \frac{D_F^*}{D_{H,C}}\epsilon^{\sigma_C}\tau_c^{1-\sigma_C}\right)^{\frac{1}{\sigma_C}}\right)$.

from exports among exporters, which is given by:

$$\frac{R_C^x(z,\ell') - R_C^{dom,x}(z,\ell')}{R_C^x(z,\ell')} = 1 - \exp\left(-\sigma_C \times d_F\right), \tag{32}$$

where $R_C^x(z,\ell') \equiv \widetilde{R}_C^x\left(q_C\left(z,\ell',\iota_C(z,\ell')\right),\eta^o\right)$ is the total gross revenue of an exporter with state (z,ℓ') and $R_C^{dom,x}(z,\ell')$ is the portion of an exporter's gross revenues coming from domestic sales.

The value-added function for exporters takes the form:

$$VA_C^x(z,\ell') = \exp\left(\frac{\sigma_C}{\sigma_C - 1}\Lambda_C \times d_F\right) VA_C^d(z,\ell'), \tag{33}$$

where $\Lambda_C \equiv \frac{\sigma_C - 1}{\sigma_C - (1-\delta_C)(\sigma_C - 1)}$ and VA_C^d is the value added function for non-exporters. It follows that the export decision is given by:

$$\mathcal{I}_C^x(z,\ell') = \begin{cases} 1 & \text{if } VA_C^x(z,\ell') - f_x > VA_C^d(z,\ell') \\ 0 & \text{otherwise,} \end{cases} \tag{34}$$

where $f_x > 0$ denotes the fixed cost of exporting, which is denominated in terms of the non-tradable composite good.

2.6 Trade and Informality: Discussion of Mechanisms

Our model includes several channels through which trade can impact informality, pushing the response to changes in trade openness in different directions. The first mechanism linking trade to informality is what we call "Melitz-type" mechanisms, which operate through various channels. Import tariffs $\tau_a - 1$ and iceberg trade costs τ_c directly affect expenditures X_C and price indices P_C, which determine the aggregate demand faced by domestic firms in sector C. Changes in trade barriers also affect the foreign demand shifter d_F, which in turn affects aggregate demand faced by exporters and the decision to export. Specifically, as trade barriers decline, exporters experience increases in aggregate demand, whereas purely domestic firms face declines in aggregate demand, as in Melitz (2003). These demand effects encourage exit of the least productive (purely domestic) formal firms. They are replaced by informal firms, which tends to increase informality. On the other hand, the same decline in aggregate demand faced by domestic firms pushes low-productivity informal firms out of the market, which tends to reduce informality.

In addition, tariffs and trade costs affect the price of the intermediate composites ι_k, effectively altering optimal intermediate input usage by firms and, therefore, their workers' productivity. This effect on intermediate inputs amplifies the impact of tariff or trade cost reductions and is similar in spirit to the "magnification" effect highlighted in the work of Fieler et al. (2018) and Coşar et al. (2016) among others. A change in trade policy or other trade costs hence directly affect

decisions of firms, including their decisions to enter, exit, and to produce as formal or informal. Specifically, a reduction in trade barriers tends to make all firms more productive, through access to cheaper intermediate goods. This encourages the most productive informal firms to grow and formalize. However, it can also lead to entry of lower productivity informal firms, which have now become profitable. Furthermore, trade openness induces a reallocation of resources toward larger and more productive firms that export. This will tend to further increase aggregate productivity and income, shifting aggregate demand up, generating incentives for productive informal firms to grow and formalize. The net effect of all these forces will depend on the values of the parameters we estimate.

The second mechanism linking openness to informality in our model is through its effect on unemployment. We show in section 4 that, in the Brazilian data, transitions from unemployment to informal employment are twice as large as transitions from unemployment to formal employment. Therefore, the channels in our model linking openness to unemployment have implications for the relative importance of the informal sector.

Accordingly, we now turn to the mechanisms linking trade to unemployment in our setup. All else equal, equation (33) shows that exporters' value added is magnified relative to that of non-exporters', since $\exp\left(\frac{\sigma_C}{\sigma_C-1}\Lambda_C \times d_F\right) > 1$. This implies that exporters' value added, and therefore their hiring and firing decisions, are more sensitive to productivity shocks z. Therefore, as in Coşar et al. (2016), reducing trade costs produces two opposing forces: (i) there is reallocation of workers toward larger and higher productivity firms, which tend to be more stable and have lower worker turnover (as they face larger costs of growing the workforce); (ii) due to the term $\exp\left(\frac{\sigma_C}{\sigma_C-1}\Lambda_C \times d_F\right)$, both new and old exporters become more sensitive to idiosyncratic shocks, which tends to increase turnover. We follow Coşar et al. (2016) and refer to these two forces as the "distribution effect" and "sensitivity effect," respectively. The bottom line is that increasing openness can increase or decrease labor turnover depending on which of the two effects dominates. Labor turnover is tightly linked to unemployment, as workers who are fired must spend at least one period in unemployment. As a result, increasing openness can lead to increases or decreases in unemployment. And, as explained in the previous paragraph, these unemployment changes will generate corresponding changes in the share of informality.

2.7 Equilibrium

We now summarize the equilibrium conditions below. Online Appendices A to I give further details.

1. Firms act optimally, make entry and exit decisions, and post vacancies according to equations (13), (17), (18) and (19). If entry is positive in sector k, the free entry condition (20) holds with equality.
2. Wage schedules solve the bargaining problem between workers and the firm, as in equations (28) and (29).
3. Labor markets clear, that is, the sum of employment levels across sectors and firms types and the number of unemployed workers must be equal to the total labor force $\overline{L}$.
4. Product markets clear. The sum of expenditures in each sector, including consumption, intermediate goods, costs of entry, hiring, and export costs must add up to revenues in the sector. Where relevant, this includes payment of tariffs.
5. Trade is balanced: $Imports = Exports$.
6. The government runs a balanced budget. All government revenues stemming from tax collection (including tariff revenues and fines on informal firms) and firing costs must exceed expenditures with unemployment benefits to all unemployed workers dismissed from formal employment. The budget surplus is directly rebated to consumers.
7. Aggregate income I is given by the sum of all wages and profits, plus the revenue from tariffs $(\tau_a - 1) \times Imports$, minus the total costs incurred by entering and hiring firms, fixed costs of operation and fixed costs of exporting.
8. We focus on steady state equilibria, where the distributions of states (z, ℓ), by sector and firm type, and all aggregate variables remain constant. In particular, no sector can be expanding or contracting, which implies that: (i) the flow of workers out of unemployment and into the formal/informal and tradable/non-tradable sectors must be the same as the flow out of these sectors and into unemployment; (ii) the mass of firms entering the informal sector must be equal to the mass of informal firms that decide to exit or to formalize their businesses in either sector $k \in \{C, S\}$; and (iii) the sum of the number of firms entering the formal sector and those formalizing their businesses must be equal to the mass of formal firms that decide to exit either sector $k \in \{C, S\}$.

3 Background: The cost of labor regulations in Brazil

The relevant laws and regulations that apply to formal labor relations in Brazil are contained in the Brazilian Labor Code (*Consolidação das Leis Trabalhistas*—CLT). According to the employment

index in Botero et al. (2004), the cost of labor regulations in Brazil is around 20 percent above the mean and median of 85 countries and more than 2.5 times larger than in the United States.

The main aspects of labor regulations in Brazil regarding their magnitude and potential impacts on the labor market, are: (i) the presence of a national minimum wage; (ii) unemployment insurance that is only available to formal workers; (iii) substantial firing costs; and (iv) sizable payroll taxes. Since these play an important role in our model and counterfactuals, we provide a brief background discussion of each of them individually.

The nominal value of the national minimum wage is determined by the federal government once a year and is typically binding for many firms. For instance, in 2003 (the year we use in our empirical analysis), the minimum wage corresponds to 49 percent of the national average wage and 81.3 percent of the national median wage.[9]

While the unemployment insurance (UI) rules are complex, in practice, most workers receive UI for 4 to 5 months and the value of the benefit depends on the worker's average wage in the three months before layoff. The replacement rate is 100 percent for individuals who earn one minimum wage, with an average replacement rate of 64 percent (all data come from Gerard and Gonzaga, forthcoming).[10]

As for the firing costs, the Brazilian labor regulation states that all formal workers with unjustified dismissal should receive a monetary compensation paid by the employer. In Brazil (and most Latin American countries), firms' outcomes (e.g. lack of businesses) are not considered a just cause for dismissal, thus any involuntary separation falls in this category (Heckman and Pagés, 2000). The magnitude of this compensation is determined as a fraction of the funds accumulated in the worker's *Fundo de Garantia por Tempo de Serviço* (FGTS), which is a job security fund proportional to job tenure and accumulates at a rate of roughly one monthly wage per year. Firms hand over additional severance payments to workers and a direct "penalty" to the government, which further increase the magnitude of the firing costs.[11]

Finally, Brazil has a burdensome tax system, which is characterized not only by high tax rates, but also by a complex structure that implies large compliance costs. For instance, the time required to comply with labor taxes in Brazil is almost 5 times higher than in the U.S. (491 and 100 hours,

[9] The mean and the median wages are computed using micro data from the National Household Survey (PNAD) and pooling together all formal and informal employees who are between 18 and 64 years old and work at least 20 hours per week.

[10] We focus on the rules in place before the 2015 reforms since our empirical analysis precedes them.

[11] Gonzaga et al. (2003) provide an in depth discussion of the legislation on dismissal costs in Brazil.

respectively).[12]

4 Data and Empirical Facts

We make use of seven datasets containing information on formal and informal firms and their workers. An overview of these datasets and their main features is provided in Table 1. A key source of information on formal-sector firms and workers is the *Relação Anual de Informações Sociais* (RAIS), which is a matched employer-employee dataset assembled by the Brazilian Ministry of Labor every year since 1976. RAIS is a high quality panel that contains the universe of formal firms and workers.[13] With these data, we can provide a detailed cross-sectional picture of the formal labor market in the C and S sectors, as well as generate important longitudinal statistics such as firm-level turnover and exit rates. We also make use of three firm-level surveys conducted by the Brazilian National Statistics Agency (*Instituto Brasileiro de Geografia e Estatística*, IBGE) which cover the *formal* manufacturing, retail and service sectors: *Pesquisa Industrial Anual* (PIA), *Pesquisa Anual de Comércio* (PAC), and *Pesquisa Anual de Serviços* (PAS), respectively. These surveys collect detailed information on firms' revenues and inputs, and combine a census of firms above a certain size threshold with a representative sample of smaller firms. Longitudinal statistics can be computed for firms surveyed in the census. We identify exporters in RAIS and PIA merging these datasets with administrative customs records from *Secretaria de Comércio Exterior* (SECEX).

These five datasets provide a comprehensive view of the formal sector, but omit the informal sector. Therefore, we use two additional data sources providing information on informal firms and workers. First, the *Pesquisa de Economia Informal Urbana* (ECINF) is a cross-sectional survey collected by IBGE in 2003, and was designed to be representative of the universe of *urban* firms with up to five employees (both formal and informal).[14] It is a matched employer-employee dataset that contains information on entrepreneurs, their businesses and employees. Firms are directly asked whether they are registered with the tax authorities and whether each of their workers has a formal labor contract. It is therefore possible to directly observe both firms' and workers' formal statuses. Given that the formality/informality statuses are self-reported, one could be concerned with measurement error and under-reporting. However, IBGE has a long tradition of accurately

[12] These data come from Doing Business (2007), which is the earliest report available on paying taxes in the Doing Business Initiative that provides comparability across a comprehensive set of countries.

[13] The RAIS dataset has been increasingly used in different applications. For recent examples see Dix-Carneiro (2014), Dix-Carneiro and Kovak (2017), Ulyssea (2018), among others.

[14] Although a few firms in the dataset have more than five employees, we restrict attention to those with five employees or less so that our sample is consistent with the population the survey targets.

Table 1: Summary of Datasets

Dataset	Source	Description
Relação Anual de Informações Sociais *RAIS* Years: 2003–2005	Ministry of Labor	Administrative matched employer-employee dataset. Covers all formal firms and workers. Detailed information on firms and workers, but no information on firm-level revenues, capital and expenditures with intermediate inputs.
Pesquisa Industrial Anual *PIA* Years: 2003, 2004	IBGE	Survey data on Manufacturing firms. Firm-level information such as revenues, capital, investment, expenditures with intermediate inputs, employment. Covers all firms with 30 employees or more; random sample of smaller firms.
Pesquisa Anual dos Serviços *PAS* Years: 2003, 2004	IBGE	Survey data on Service-sector firms. Firm-level information such as revenues, capital, investment, expenditures with intermediate inputs, employment. Covers all firms with 20 employees or more; random sample of smaller firms.
Pesquisa Anual do Comércio *PAC* Years: 2003, 2004	IBGE	Survey data on Retail and Commerce firms. Firm-level information such as revenues, capital, investment, expenditures with intermediate inputs, employment. Covers all firms with 20 employees or more; random sample of smaller firms.
Secretaria de Comércio Exterior *SECEX* Year: 2003	Ministry of Industry, Foreign Trade and Services	Administrative customs data. Export and import values at the firm level.
Economia Informal Urbana *ECINF* Year: 2003	IBGE	Survey data. Matched employer-employee dataset with detailed information on formal and informal firms and their workers. Representative sample of small businesses (firms with 5 employees or less). Information on formal status of the firm and its workers.
Pesquisa Mensal de Emprego *PME* Year: 2003	IBGE	Survey data. Rotating panel of households that covers the 6 main metropolitan areas in Brazil.

measuring labor informality, and has very strict confidentiality clauses. The information they collect cannot be used for auditing purposes by other government branches, in particular those responsible for enforcing the relevant laws and regulations. These characteristics, associated with the high levels of informality observed in the data, make us confident that respondents are not systematically underreporting their informality status.[15]

Finally, we draw from the Monthly Employment Survey (*Pesquisa Mensal de Emprego*, PME) to obtain information on worker allocations and labor market flows. This is a rotating panel with a similar design to the Current Population Survey in the United States. The survey covers the six main metropolitan areas in Brazil and contains detailed information on individuals' socio-demographic characteristics and labor market outcomes, including informal employment status. We define a worker to be informal if she does not hold a formal labor contract—that variable is explicitly recorded in PME. If a worker is self-employed, she is also treated as an informal worker.

[15] Additionally, Ulyssea (2018) shows that the ECINF survey reproduces very well the RAIS in all the dimensions that are common to both datasets (e.g., size and sectoral distributions), which is reassuring of ECINF's quality.

Similarly, we treat self-employed workers in ECINF as informal firms employing one worker. We exploit the panel structure of PME to estimate one-year labor market transitions between formal employment, informal employment (in both C and S sectors) and unemployment status.[16]

We need to impose some restrictions to make these seven datasets consistent with each other. First, because PME covers only six metropolitan regions, we restrict our samples to these regions whenever possible. Given the focus on metropolitan regions in PME and urban firms in ECINF, we remove firms and workers in agriculture, mining, coal, oil and gas industries. Second, we restrict attention to data from 2003, as ECINF is only available for that year. Whenever we need to compute dynamic statistics, we also employ data from 2004. Finally, we exclude firms in the public sector. Our Data Appendix provides additional details regarding data treatment—see Online Appendix J.

We now highlight five important facts on formal and informal firms and workers in Brazil, which we will target in our estimation procedure.[17]

Fact 1: Approximately 50% of the Brazilian labor force is informally employed. Transitions from unemployment to an informal job are at least twice as likely as transitions from unemployment to a formal job.

Table 2 uses data from PME to establish that 48.2% of all workers are informally employed. If we break down informality rates by sectors, we find that 35.6% of C-sector workers are informal, compared to 51.2% in the S sector. Moreover, transition rates from unemployment to informal jobs are twice as likely as those to formal jobs (45.3% compared to 21.1%).

Table 2: Employment Shares and Transition Rates

	Share of Workers	Transition Rates From Unemp.
Informal Tradable (Ci)	0.059	0.064
Formal Tradable (Cf)	0.106	0.050
Informal Non-Tradable (Si)	0.351	0.389
Formal Non-Tradable (Sf)	0.334	0.161
Unemployment	0.150	0.336
Share of Informal Employment	0.482	
Transition Rate from Unemp.		
to Informal Employment	0.453	
to Formal Employment	0.211	
Ratio	2.146	

Data source: 2003 PME.

Fact 2: The probability that a firm is informal declines sharply with its employment

[16] To minimize the effects of attrition in PME, we measure one-year transitions by annualizing four-month transitions.

[17] See Ulyssea (2020) for a more extensive discussion of stylized facts related to informality.

size.

We estimate, for each sector, regressions relating a firm-level informal status indicator to its number of employees. Table 3 uses data from ECINF to show that the fraction of informal firms rapidly declines with employment size.

Table 3: Firm-Level Informality Status vs. Firm-Level Employment

	Dep. Variable: $Informal\ Status\ Indicator_i$	
	C sector	S sector
Intercept	1.135	1.130
	(0.028)	(0.012)
ℓ_i	-0.179	-0.204
	(0.025)	(0.009)
Observations	1,194	7,273

Data source: 2003 ECINF. Standard errors in parentheses.

Fact 3: Informal firms are, on average, less productive than formal firms.

It has been widely documented that average productivity across firms in the informal sector is substantially lower than in the formal sector. Our datasets confirm this insight. Note that the sample of informal firms from ECINF cannot be directly compared with samples from PIA, PAS, and PAC, as ECINF is designed to cover firms with at most five employees. However, we can compare firm-level labor productivity (measured as revenue per worker) across formal and informal firms by estimating linear regressions relating firm-level revenues to their employment size. The idea is that once we condition on employment size, revenues of informal firms in ECINF can be compared to those of equal-sized formal firms in PIA, PAS, and PAC. The linear regression results reported in Panel A of Table 4 imply that, among C-sector firms of size one, formal firms are on average more productive than informal ones by 1.73 log-points. For C-sector firms of size five, this difference shrinks to 1.18 log-points. Similarly, for S-sector firms of size one, formal firms are, on average, 1.18 log-points more productive than informal ones. Finally, for S-sector firms of size five, this difference declines to 0.46 points.

Fact 4: The average informal worker is paid lower wages than the average formal worker.

It is a well documented fact that average wages in the informal sector are on average lower than those in the formal sector. Although RAIS provides information on firm-level wages for the population of formal firms, we do not have have firm-level wages for the population of informal

Table 4: Firm-Level log-Revenue per Worker and log-Wages *vs.* log-Employment

	A. Dep. Variable: $\log(Revenue_i/\ell_i)$				B. Dep. Variable: $\log(wage_i)$			
Sector / Firm Type	Cf	Sf	Ci	Si	Cf	Sf	Ci	Si
Intercept	10.118	10.004	8.391	8.825	8.509	8.436	8.013	8.417
	(0.013)	(0.005)	(0.037)	(0.015)	(0.006)	(0.002)	(0.033)	(0.014)
$\log(\ell_i)$	0.000	-0.128	0.342	0.321	0.117	0.105	0.292	0.231
	(0.005)	(0.003)	(0.114)	(0.050)	(0.003)	(0.001)	(0.103)	(0.048)
$Exporter_i$ (Dummy)	1.462				0.462			
	(0.021)				(0.014)			
Observations	16,986	43,861	1,070	6,202	20,075	145,981	1,071	6,205
Dataset	PIA + SECEX	PAS + PAC	ECINF	ECINF	RAIS + SECEX	RAIS	ECINF	ECINF

Standard errors in parentheses.

firms. ECINF contains wages of informal workers, but covers firms of size only up to five employees. We compare wages across datasets by regressing log wages on log size for each type of firm in the two sectors (C and S). Panel B of Table 4 shows that once we control for size, we see an important informal wage penalty in the C sector, but not in the S sector. Formal C-sector firms of size one tend to pay wages that are 0.5 log-points larger than their informal counterparts. For firms of size five this difference declines to 0.21 log-points. On the other hand, S-sector firms of size one pay roughly the same wages as their informal counterparts. Interestingly, if we condition on firms of size five, the second and fourth columns of Panel B of Table 4 imply that informal firms firms pay, on average, wages that are larger than their formal counterparts by 0.18 log-points. Still, because informal firms tend to be considerably smaller than formal ones, an *average* informal worker receives a lower wage than an average formal worker.

Fact 5: Firm-level labor turnover tends to decline with firm-level employment size. However, conditional on size, exporters tend to have higher turnover.

As shown in Coşar et al. (2016), labor turnover tends to decline with firm size. However, conditional on employment size, turnover tends to be larger for exporters. Table 5 reproduces these findings for Brazil. These relationships play a central role in our quantitative exercise as they gauge the importance of the redistribution and sensitivity effects highlighted in section 2.6 in response to trade shocks.

5 Estimation

We quantify the model outlined in section 2 in two steps. First, we fix a subset of parameters based on a combination of aggregate data, estimates from previous papers, and the statutory value

Table 5: Turnover, Firm Size and Export Status

	Dep. Variable: $Turnover_i$	
	C sector	S sector
Intercept	0.741	0.645
	(0.008)	(0.003)
$\log(\ell_i)$	-0.126	-0.096
	(0.003)	(0.002)
$Exporter_i$ (Dummy)	0.071	
	(0.019)	
Observations	20,342	147,936

Data Sources: 2003 and 2004 RAIS and 2003 SECEX. Turnover of firm i between 2003 and 2004 measured as $Turnover_i = \frac{|\ell_{i,2004} - \ell_{i,2003}|}{0.5 \times (\ell_{i,2004} + \ell_{i,2003})}$. Standard errors in parentheses.

of institutional parameters, such as value-added and payroll taxes. Next, we estimate the remaining parameters of the model using an Indirect Inference procedure, which allows us to combine information from the different data sources discussed in the previous section. Section 5.1 describes how we determine the parameters that are fixed throughout the estimation procedure. Section 5.2 discusses the estimation procedure, and section 5.3 addresses identification. Finally, section 5.4 presents the estimation results and discusses the fit of the model.

5.1 Fixed Parameters

As discussed in section 3, regulations are costly for employers, but they are complex. Therefore, we need to make simplifying assumptions to map them to our model's payroll tax (τ_w), value-added tax (τ_y), firing cost (κ), and unemployment insurance benefit (b_u). We follow Ulyssea (2018) and set τ_w so that it reflects the main taxes that are proportional to firms' wage bill, namely, the employers' social security contribution (20 percent), payroll tax (9 percent), and severance contributions to FGTS (8.5 percent). We combine two VAT-like taxes to calculate τ_y: *Imposto sobre Produtos Industrializados*, IPI (20 percent) and PIS/COFINS (9.25 percent).[18]

Firing costs are set based on Heckman and Pages (2000), who compute the cost of dismissing workers for several Latin American countries, including Brazil. Their calculation takes into account the specific features of dismissal costs we reviewed in section 3 and show that, on average, employers must pay approximately 1.9 months of wages to dismiss a worker. Considering that the *annual* average formal-sector wage in the 2003 RAIS data amounts to R$10,565, we obtain a firing cost κ of R$1,690 per worker. The minimum wage corresponds to the annualized value of the national

[18] We exclude state-level value-added taxes because these vary greatly across states and there is a complex system of tax substitution across the production chain, which would be impossible to properly capture.

monthly minimum wage in 2003: $\underline{w} = \text{R\$}2,880$. To compute unemployment insurance benefits, we assume that all workers receive the maximum duration of potential benefits (that is, 5 monthly payments). This figure is very close to both the mean and median of the duration of actually received benefits (Gerard and Gonzaga, forthcoming). Finally, we use the average monthly value of benefits paid in 2003, as reported by the Ministry of Labor: 1.37 times the minimum wage. This amounts to unemployment insurance benefits $b_u = 1.37 \times 5$ times the *monthly* minimum wage = R\$1,644.

The share of final expenditure in sector C goods, ζ, and the share of sector k's intermediate inputs payments to sector C goods, λ_k, are extracted from the 2000 and 2005 Brazilian National Accounts. We obtain $\zeta = 0.283$, $\lambda_C = 0.65$, and $\lambda_S = 0.29$, suggesting that tariffs and iceberg trade costs can have a substantial effect on labor productivity in both sectors. The iceberg trade cost $\tau_c = 2.5$ is obtained from Coşar et al. (2016), which is in turn based on estimates from Eaton and Kortum (2002), and the average import tariff $\tau_a - 1 = 0.12$ comes from 2003 data in UNCTAD TRAINS.

There are two sets of model parameters that are hard to identify given our data: the bargaining weight of workers β, and the matching function parameters (ϕ and ξ)—see Flinn (2006) for a discussion. We follow Mortensen and Pissarides (1999) and Ljungqvist and Sargent (2017) and impose symmetric bargaining, i.e. $\beta = 0.5$. We assume a matching function elasticity of $\xi = 0.5$, which is in the middle of the range surveyed by Petrongolo and Pissarides (2001), and $\phi = 0.576$, a choice we discuss in section 5.3. Table 6 summarizes the parameter values fixed throughout the estimation and their sources.

5.2 Estimation Procedure

We take the parameters described in Table 6 as given and estimate the remaining parameters using an Indirect Inference procedure. In this step, we estimate 27 parameters using 84 data moments and auxiliary model coefficients, ensuring that all equilibrium conditions listed in section 2.7 are met throughout the procedure. The estimation algorithm is described in detail in Supplementary Material I, but we highlight here a few important features of the procedure.

First, rewrite the value-added functions (7) and (33) as:

$$VA_k(z,\ell) = \Theta_k \Psi_k \left(\exp\left(\mathcal{I}_k^x(z,\ell)\, d_F\right)\right)^{\frac{\sigma_k}{\sigma_k - 1}\Lambda_k} \left(z\ell^{\delta_k}\right)^{\Lambda_k}, \tag{35}$$

Table 6: Fixed Parameters

Parameter	Description	Source	Value
τ_c	Iceberg Trade Cost	Coşar et al. (2016) and Eaton and Kortum (2002)	2.50
ζ	Share of final expend. on C	IBGE National Accounts (2000/2005)	0.283
λ_C	Prod. Function (Equation (4))	IBGE National Accounts (2000/2005)	0.645
λ_S	Prod. Function (Equation (4))	IBGE National Accounts (2000/2005)	0.291
r	Interest rate	Ulyssea (2010)	0.08
τ_y	Value Added Tax	Ulyssea (2018)	0.293
τ_w	Payroll Tax	Ulyssea (2018)	0.375
$\tau_a - 1$	Import Tariff	UNCTAD TRAINS	0.12
κ	Firing Costs (in R$)	Heckman and Pages (2000)	1,956.7
$\underline{w}$	Min. Wage (in R$)	Annualized 2003 value	2,880
b_u	Unemployment Benefit	$1.37 \times 5 = 6.85$ monthly Min. Wage	1,644
ξ	Matching Function	Petrongolo and Pissarides (2001)	0.5
ϕ	Matching Function	Own Calculations, see section 5.3	0.576
β	Workers' Bargaining Weight	Mortensen and Pissarides (1999)	0.5

Parameters based on the IBGE National Accounts employ simple averages between 2000 and 2005.

where $\Psi_k \equiv \left(P_k^m\right)^{-(1-\delta_k)\Lambda_k} \left(\exp\left(d_{H,k}\right)\right)^{\frac{\sigma_k}{\sigma_k-1}\Lambda_k}$ for $k = C, S$.[19] Also, define $\vartheta_{J_u} \equiv b + \frac{1}{1+r} J^u$, to be the expected discounted present value of unemployment. The estimation procedure treats the endogenous equilibrium objects Ψ_C, Ψ_S, and ϑ_{J_u} as "parameters" to be estimated.[20] Given a guess of these objects and of the remaining structural parameters, we are able to solve for the mass of entrants, the mass of active firms, the firm-level policy functions, the steady-state distribution of states, and the unemployment rate that are consistent with these guesses. To be precise, we minimize the loss function $\mathcal{L}$ given by: $\mathcal{L}(\Omega) = \sum_i W_i \left| m_i^{Model}(\Omega) - m_i^{Data} \right|$, where Ω denotes the set of parameters to be estimated, m_i^{Data} denotes the values of moments or auxiliay model coefficients measured in the data, $m_i^{Model}(\Omega)$ denotes the values of moments or auxiliary model coefficients generated by the model when the set of parameters is given by Ω, and W_i weighs the importance of moment i in the loss function. Ω includes the endogenous objects Ψ_C, Ψ_S, and ϑ_{J_u}, but excludes the structural parameters b and D_F^*, which are obtained after the minimization of $\mathcal{L}$ is complete. It also excludes parameters δ_k $(k = C, S)$, which are directly determined as functions of the elasticities of substitutions σ_k and the share of gross revenues devoted to intermediate goods payments using: $\delta_k = 1 - \frac{\sigma_k}{\sigma_k - 1} \left(\frac{Total\ Expenditures\ with\ Intermediates_k}{Total\ Gross\ Revenues_k} \right)_{Data}$, where the term in parenthesis is computed using the Brazilian National Accounts compiled by IBGE. Finally, Ω also omits the foreign demand

[19] $\Theta_k \equiv \left(\frac{1}{(1-\delta_k)\Lambda_k}\right)\left(\frac{(1-\delta_k)(\sigma_k-1)}{\sigma_k}\right)^{\frac{\sigma_k}{\sigma_k-1}\Lambda_k}$ and $\Lambda_k \equiv \frac{\sigma_k - 1}{\sigma_k - (1-\delta_k)(\sigma_k-1)}$ are sector-specific constants; P_k^m is the price of one unit of sector k's intermediate bundle; and $d_{H,k} \equiv \log\left(\left(\frac{X_k}{P_k^{1-\sigma_k}}\right)^{\frac{1}{\sigma_k}}\right)$ are domestic demand shifters.

[20] In counterfactual simulations, these are solved out under the new environment. Estimation relies on the assumption that the data comes from an equilibrium allocation.

shifter d_F. Given a guess of σ_C, it can be directly recovered using equation (32) and data on the average share of exporters' gross revenues that is actually exported (which is obtained from PIA and SECEX).

Once the minimization of $\mathcal{L}$ over Ω is concluded, the utility of unemployment is recovered using:

$$b = \vartheta_{J_u} - \frac{1}{1+r} J^u,$$

where J^u can be computed using equation (A.48) in the Online Appendix. In turn, D_F^* is recovered as:

$$D_F^* = \frac{\left(\exp\left(\sigma_C \times d_F\right) - 1\right) \left(P_C^m\right)^{(1-\delta_C)(\sigma_C - 1)} \Psi_C^{\frac{\sigma_C - 1}{\Lambda_C}}}{\bar{\epsilon}^{\sigma_C} \tau_c^{1-\sigma_C}},$$

where $\bar{\epsilon}$ is the exchange rate value that balances trade and P_C^m is the price of one unit of sector C's intermediate bundle.

We emphasize that the only differences between formal and informal firms are that the former are subject to regulations and taxes, and the latter face an expected cost of informality $K^{inf}(z, \ell')$. Otherwise, they have access to the same production and hiring technologies, and are subject to the same exogenous exit rates. However, these parameter restrictions do not significantly affect the final fit of the model.

5.3 Identification

To understand which moments in the data identify the parameters we estimate, first rewrite the hiring functions as: $H_k(\ell, \ell') = \widetilde{h}_k \left(\frac{\ell' - \ell}{\ell^{\gamma_{k2}}}\right)^{\gamma_{k1}}$, where $\widetilde{h}_k = (\mu^{\upsilon})^{-\gamma_{k1}} \left(\frac{h_k}{\gamma_{k1}}\right)$, for $k = C, S$. Our estimation procedure treats $\widetilde{h}_k$ as a "parameter" to be estimated. This term is identified based on the average level of turnover rates at the firm level as well as the unemployment rate, given that the two are closely related.[21] The auxiliary (linear) model relating firm-level turnover rates to log-employment and an export indicator gives information on the scale economies γ_{k2}. The auxiliary (linear) model relating log-wages to log-employment and an export indicator gives information on the convexity of hiring costs γ_{k1}, as it relates to wage dispersion across firms with different characteristics.

Note that $\widetilde{h}_k$ is a combination of a structural parameter h_k with an endogenous term μ^{υ}. Following the estimation of $\widetilde{h}_k$ and γ_{k1}, we recover h_k by imposing that $\mu^{\upsilon} = 0.5$ in the equilibrium we estimate.[22] In turn, we set ϕ so that we perfectly match the yearly transition rate from unem-

[21] In our model, worker separations are followed by a period of unemployment. This mechanically ties turnover rates to unemployment rates.

[22] Similarly to how we treat Ψ_C, Ψ_S, and ϑ_{J_u}, μ^{υ} is solved out in our subsequent counterfactual exercises.

ployment to employment in the data.[23]

The model needs the fixed costs of production $\bar{c}_k$ to match how the probability of firm-level exit rates depends on size. Exogenous destruction rates α_k are needed to match the average exit rates. Matching the relationship between firm-level revenues and firm size gives information on σ_k. The AR(1) process for productivity is pinned down by targeting two dynamic moments: the serial correlation in firm-level employment, and the serial correlation in firm-level revenues. The cross-sectional dispersions in firm-level employment size and revenues are also informative about the variance of shocks σ_k^z. The share of C-sector firms that export pins down the fixed cost of exporting f_x.

Finally, the cost of informality function $\widetilde{a}_k \exp\left\{\widetilde{b}_k\left(\ell-1\right)\right\}$ is identified by matching the firm-size distribution in the informal sector, the share of employment in the informal sector, and the fraction of informal firms conditional on employment size. All the moments and auxiliary models used in the estimation procedure (as well as the datasets we have used to compute each of them) are listed in Appendix K.

5.4 Estimation Results

5.4.1 Estimates

Table 7 presents our parameter estimates. Informality costs as a fraction of revenues, $p_{ki}(\ell)$, differ across sectors. In the C sector, the informality costs start at a relatively low value, but rapidly increase with size. On the other hand, the informality penalty starts at a larger value in the S sector, but increases with size at a slower pace. The different derivatives of the informality penalty function in the C and S sectors are intuitive, as one would expect large manufacturing firms to face increasing hurdles to remain invisible to the government.

Our hiring function estimates display convexity ($\gamma_C^1 = 2.1$, $\gamma_S^1 = 4.9$) and scale economies ($\gamma_C^2 = 0.14$, $\gamma_S^2 = 0.19$) in both sectors. These results are in the same ballpark as recent estimates from Coşar et al. (2016), who use data from Colombia. Similarly, our estimates of the elasticities of substitution in the C and S sectors, given respectively by $\sigma_C = 5.3$, and $\sigma_S = 3.3$, are within the range of earlier papers (see, for example, Coşar et al., 2016; De Loecker and Warzynski, 2012;

[23] Equation (23) implies that transition rates from unemployment to employment (in any sector), in the model, are given by: $\sum_{k,j}\mu_{kj}^e = \left(\frac{\phi}{(\mu^v)^\xi}\right)^{\frac{1}{1-\xi}}$. We choose ϕ so that our model perfectly matches the transition from unemployment to employment in the data, $Transition_{Data}^{U\to E}$, conditional on $\mu^v = 0.5$ and $\xi = 0.5$. That is, $\phi = \frac{(\mu^v)^\xi}{\left(Transition_{Data}^{U\to E}\right)^{\xi-1}} = 0.576$, as shown in Table 6.

Table 7: Parameter Estimates

Parameter	Description	$k = C$	$k = S$
$\widetilde{a}_k$	Cost of Informality, Intercept	0.161	0.373
$\widetilde{b}_k$	Cost of Informality, Convexity	0.131	0.013
h_k	Hiring Cost, Level	559.7	2348.9
γ_k^1	Hiring Cost, Convexity	2.067	4.896
γ_k^2	Hiring Cost, Scale Economies	0.139	0.192
σ_k	Elasticity of Substitution	5.321	3.281
ρ_k	Productivity AR(1) Process, Persistence Coeff.	0.978	0.977
σ_k^z	Productivity AR(1) Process, Variance of Shock	0.199	0.296
α_k	Exogenous Exit Probability	0.067	0.063
$\overline{c}_k$	Fixed Cost of Operation	23.071	27.047
δ_k	Labor Share in Production	0.266	0.54
c_k^e	Entry Cost	5,332.2	2,067.1
f_x	Fixed Cost of Exporting	55,856.9	
b	Utility Value of Unemployment	-8,662.5	
$(D_F^*)^{\frac{1}{\sigma_C}}$	Foreign Demand Shifter	969.2	

De Loecker et al., 2016; Broda and Weinstein, 2006). In particular, our estimates are consistent with Gervais and Jensen (2019) who find that services have elasticities of substitution about one-quarter smaller than manufacturing.

Finally, we estimate a negative value for b, implying a significant disutility of unemployment. Our estimate of -8,662.5 is equivalent to 3 times the 2003 minimum wage. As a benchmark, also in the context of a search model, Meghir et al. (2015) estimate a negative b for men in the São Paulo metropolitan region equal to 4.3 times the 2008 minimum wage.[24] As the search literature has shown, a negative value of unemployment is necessary to generate the magnitudes of wage dispersion typically found in the data (see, for example, Hornstein et al., 2011).

5.4.2 Model Fit

Tables A.2 through A.8 in Online Appendix K compare how our model-generated moments and auxiliary model coefficients compare to those obtained in the data. Overall, our model with 27 structural parameters fits our 84 target data moments or auxiliary model coefficients well. Importantly, Facts 1 through 5 highlighted in section 4 are all well matched. Specifically, Table A.2 shows that employment shares and transition rates from unemployment are on target, so that our model comfortably replicates Fact 1. The sharp relationship between informal status and size (Fact 2) is also well reproduced by our model, as shown in the lower panel of Table A.8. Tables A.6 and A.8

[24] Table 5 in their paper estimates a (monthly) flow value of unemployment of -1,308 for men in São Paulo. The average monthly minimum wage during the period they consider is of R$ 300.

show that average wages in the formal sector indeed exceed those in the informal sector, mirroring Fact 4. Fact 5, relating firm-level turnover to size and export status, is also well replicated as Table A.3 shows. Also noteworthy is the ability of the model to match how turnover rates relate to employment and export status conditional on both expansions and contractions. We do not directly target Fact 3, but we verify that the model implies that less productive firms tend to sort into informal status. Finally, Table A.6 shows that the model recreates the strong wage size and exporter premia found in the data, in both sectors. These are important moments to replicate, as they give us confidence in the wage inequality counterfactuals we conduct in section 6.1.5.

Having discussed the successes of the model, we now turn to the moments that were not as well matched, and discuss reasons for some of these mismatches. Although the model is able to replicate very well the dependence between firm-level turnover rates, firm-level employment, and export status, the *average* turnover rate is not as well replicated. There was a tension between matching the *level* of turnover rates and matching the unemployment rate.

Our model also tends to underestimate the dispersion of firm size in the formal sector, in both sectors C and S (see Table A.4). We hypothesize two reasons for this discrepancy. First, the data source we employ has a thicker than usual left tail of the firm-size distribution.[25] Second, the assumed normal distribution of productivity shocks naturally makes it harder to match the right tail of the size distribution. Although modelling firm-level dynamics according to equation (5) is standard in the literature (see Hopenhayn and Rogerson, 1993; Alessandria and Choi, 2014; Coşar et al., 2016, among many others), we would need a more flexible specification for the productivity process in order to better match the firm-size distribution in Brazil.[26] Relatedly, the dispersion in revenues in Table A.7 is also under-estimated.

Finally, we overestimate average productivity/revenues among informal C-sector firms. Although we have a fairly detailed grid of productivity, with 41 points, the grid tends to be sparse at low productivity levels, leading to jumps in average productivity in the informal sector in response to small shifts in parameters.[27] Still, our model-generated moments are consistent with the fact that informal firms are, on average, substantially smaller and less productive than formal firms.

[25] The 20^{th} and 40^{th} percentiles of firm size in the formal C sector are given by, respectively, 2 and 4. In the formal S sector, these are 1 and 2.

[26] Although standard in the literature, imposing log-normality can have important quantitative implications for the gains from integration, as discussed in Adão et al. (2020a).

[27] We use Tauchen's method (Tauchen, 1986) to discretize the Markov process described in equation(5).

6 Counterfactual Experiments

6.1 The Effects of Trade under Informality

We study the quantitative implications of our model for the relationship between openness and informality, unemployment, welfare, productivity and wage inequality. We focus on changes in iceberg trade costs τ_c, as the effects of tariffs τ_a were found to be relatively small (results are available upon request). The modest tariff effects are due to the fact that the initial trade costs ($\tau_c = 2.5$) in 2003 are high, while the initial tariff levels are relatively low (ca. 12%). Our counterfactual experiments consider four levels of τ_c: 1.5; 2; 2.5 (benchmark in 2003); and 6. The $\tau_c = 6$ case represents a near autarky scenario, while $\tau_c = 1.5$ represents a significant move toward international integration.[28] We solve the model setting the S sector composite good as the numeraire (that is, the price index P_S is fixed across counterfactuals). Therefore, all nominal parameters (including the minimum wage $\underline{w}$) are expressed in terms of the S sector composite good. All details of the simulation algorithm are available in Supplementary Material II.

6.1.1 Trade and Informality

We first investigate the effects of openness on the size of the informal sector. Figure 3 shows the effects of τ_c on the share of employment in the informal C and S sectors, as well as on the overall share of informal employment in the economy. We observe a strong negative relationship between openness and informality in the C sector. Specifically, if trade costs are reduced from $\tau_c = 2.5$ to $\tau_c = 1.5$, the share of employment in the informal C sector declines from 8.1% to 3.6%—a 55% reduction in the importance of the informal C sector for total employment. Conversely, if trade costs increased to near-autarky levels, the share of workers employed in the informal C sector would increase from 8.1% to 10%—a 21% increase. These responses of the informal sector to increased openness are broadly consistent with the empirical results of McCaig and Pavcnik (2018). They show that the Vietnamese manufacturing sectors that benefitted the most from tariff reductions resulting from the United States-Vietnam Bilateral Trade Agreement experienced substantial increases in formal-sector employment relative to other sectors.

What is behind this sharp relationship between openness and the size of the informal C sector? As openness increases, international competition in sector C intensifies, reducing demand for purely

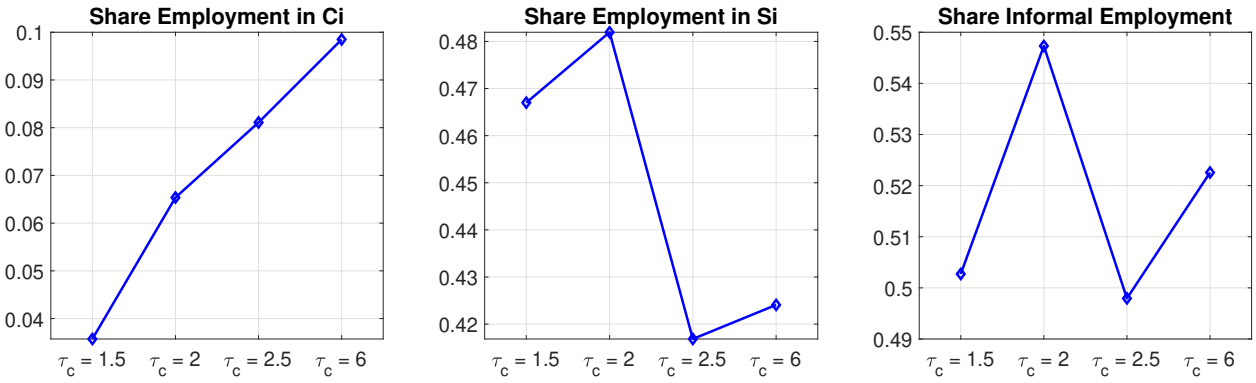
[28] Such a large decrease (ca. 40%) in trade costs is not implausible. Indeed, Dix-Carneiro et al. (2020) find that non-service goods import trade costs in Asia have declined by up to 40% between 2000 and 2014.

domestic C sector firms (which include all informal firms). This reduction in demand pushes low-productivity formal firms to informality, but it also drives low-productivity informal firms out of the market. The total effect of openness on informality depends on the relative magnitude of these two forces. Empirically, selection at the left tail of the distribution of informal firms dominates, leading to the strong negative relationship between openness and informality in the manufacturing sector.

Interestingly, the effect of openness on informality in the S sector is very different. Increases in openness from the benchmark lead to increases in the size of the informal S sector. The increase we observe as τ_c is reduced from 2.5 to 2 is quite substantial: the size of the informal S sector climbs from 41.7% to 48.2% of total employment—a 16% increase. Whereas openness reduces demand to purely domestic C-sector firms, it increases demand for S-sector firms for two reasons. First, real income increases with openness (as we show in section 6.1.3), resulting in an increase in demand for S sector goods for final consumption. Second, exporters and other productive firms in the C sector expand, leading to increased demand for intermediate goods produced by the S sector. This increase in demand encourages productive informal S-sector firms to expand and formalize, but it also encourages entry of low-productivity informal firms. Empirically, the latter effect dominates.

Figure 3 shows that, as we put the C and S sectors together, we find an inverse U-shaped relationship between the importance of informal employment and openness, as τ_c is reduced from 2.5 to 1.5. For moderate declines in τ_c, the positive effect of openness on informality in S dominates. However, for further declines in τ_c, the negative impact on informality in C tends to offset the former effect, so that the effect on informality as a whole is not economically significant. Finally, as the economy moves toward autarky, the positive effect on informality in the C sector dominates, and we observe a modest increase in the share of informal employment.

Figure 3: Trade and Informality



6.1.2 Trade and Unemployment

Next, we focus on the effect of openness on unemployment, which we display in the left panel of Figure 4. As τ_c is reduced from 2.5 to 1.5, the unemployment rate increases from 18.3% to 19.4%. This represents an increase of 6% in the total number of unemployed workers. A key driving force behind this increase in unemployment is the sensitivity effect discussed in section 2.6. As τ_c declines, we observe: (1) a shift in the size distribution in both C and S sectors toward larger firms; and (2) a shift in employment toward exporters, who experience increases in the demand they face. Effect (1) implies the distribution effect, whereby aggregate turnover should decline as larger firms tend to have lower turnover—see Fact 5 in section 4. Indeed, turnover in the S sector does decline according to our results. However, effect (2) implies the sensitivity effect, which can dominate the distribution effect if the increases in foreign demand and fraction of exporters are large enough. We find that, as a consequence of a strong sensitivity effect, turnover in the C sector increases with openness. Finally, note that the distribution effect in the S sector (which is over four times the size of the C sector in terms of total employment) prevents the effect of openness on total turnover and unemployment of being even larger. This observation highlights the importance of incorporating and carefully modeling the S sector when we study the aggregate implications of trade shocks.[29] To conclude, we note that unemployment increases as the economy moves toward autarky. In this case, the driving force behind the increase in unemployment is the decline in aggregate real income, and, therefore, in the demand faced by firms in both sectors. As we will see in section 6.3, the informal sector cushions the effect of the move to autarky on unemployment.

Figure 4: Trade, Unemployment and Welfare

Notes: Real Income refers to the real value of the sum of all wages and profits in the economy. Real Income 2 refers to the real value of the sum of all wages and profits in the economy including the disutility of unemployment $b \times L_u$. Real Income and Real Income 2 are both normalized at 1 for $\tau_c = 2.5$.

[29] Most studies of trade with heterogeneous firms carefully model the manufacturing sector, but abstract from firm heterogeneity and frictions in the services sector.

6.1.3 Trade and Welfare

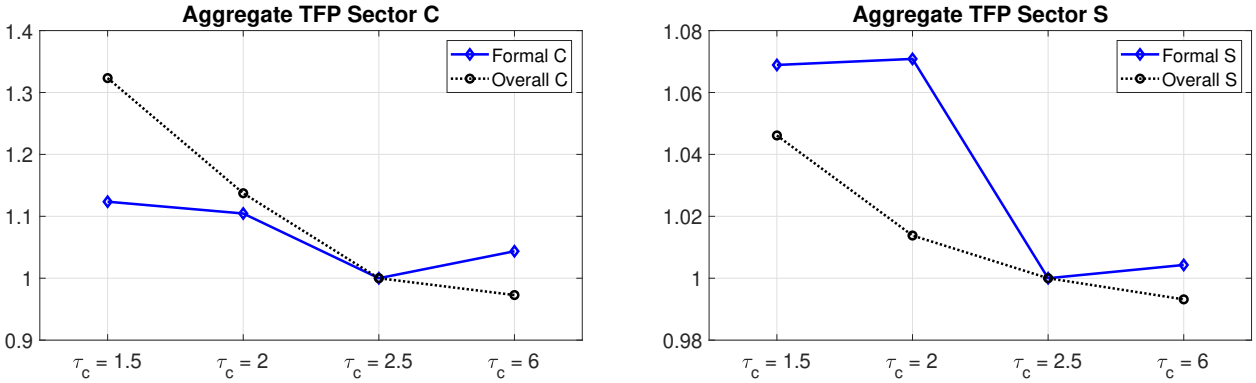
We now turn our attention to the impact of trade on welfare. We focus on two measures of welfare: Real Income, which is the indirect utility derived from equation (1), and real income minus the disutility associated with unemployment (we refer to the second measure as Real Income 2). The middle and right panels of Figure 4 present how these two measures of welfare depend on the level of openness. Note that the welfare measures are normalized at 1 for $\tau_c = 2.5$.

Regardless of the welfare measure we consider, we find that openness is positively related to welfare. Moving the economy toward autarky reduces Real Income by 2.5% and Real Income 2 by 3.9%. On the other hand, reducing trade costs to $\tau_c = 1.5$ leads to welfare increases between 21% and 23%. Large increases in productivity, through the reallocation of labor toward larger and more productive firms, drive these large welfare effects—despite the increase in unemployment.

6.1.4 Trade and Productivity

We turn to the effect of openness on aggregate Total Factor Productivity (TFP), and examine how it depends on incorporating the informal sector. We compute aggregate TFP within each sector as employment-weighted averages of firms' idiosyncratic productivities z. Figure 5 shows how aggregate TFP behaves in response to increasing openness within each sector. We normalize aggregate TFP to 1 at $\tau_c = 2.5$.

Figure 5: Trade and Aggregate TFP



Notes: Aggregate TFP is computed as the weighted average of the z's of all active firms—weights are given by firm-level employment. Aggregate TFP in each situation is normalized at 1 for $\tau_c = 2.5$.

As expected, we find that reductions in τ_c lead to increases in aggregate TFP in the formal C sector; a reduction of τ_c all the way to 1.5 results in a productivity increase of 12%. However, this increase in formal-sector aggregate productivity is considerably smaller than the increase in productivity in the C sector as a whole (including both formal and informal firms), which amounts

to 32%. This result suggests that studies focusing on the effect of trade on productivity using formal-sector data sources only (as most do) can significantly underestimate the aggregate productivity gains of trade. The main reason for this discrepancy is that trade drives highly unproductive informal C sector firms out of the market, freeing up resources to be reallocated to more productive formal ones.

The opposite pattern is found in the S sector, albeit at a much smaller scale. The effect of openness on overall S sector aggregate productivity is smaller than the effect on the formal S sector. As we noted before, as τ_c is reduced, demand faced by S sector firms increases, inducing highly unproductive informal S sector firms to enter. This entry mitigates the aggregate productivity gain we observe in the formal S sector. Overall, bundling the C and S sectors together, a reduction in τ_c from 2.5 to 1.5 leads to an increase in aggregate TFP of 7.6% in the formal sector, whereas this increase amounts to 9.6% once we incorporate the informal sector.

Interestingly, aggregate TFP among formal firms increases in both sectors as the economy moves toward autarky. As real income decreases with autarky, resources are partially reallocated toward less productive firms pushing aggregate TFP in the formal sector down. However, at the same time, low-productivity formal firms are driven out of the market, which drives aggregate TFP in the formal sector up through a selection effect. However, if we measure aggregate TFP across all firms, formal and informal, we observe a decline in aggregate TFP as resources move toward smaller and less productive firms.

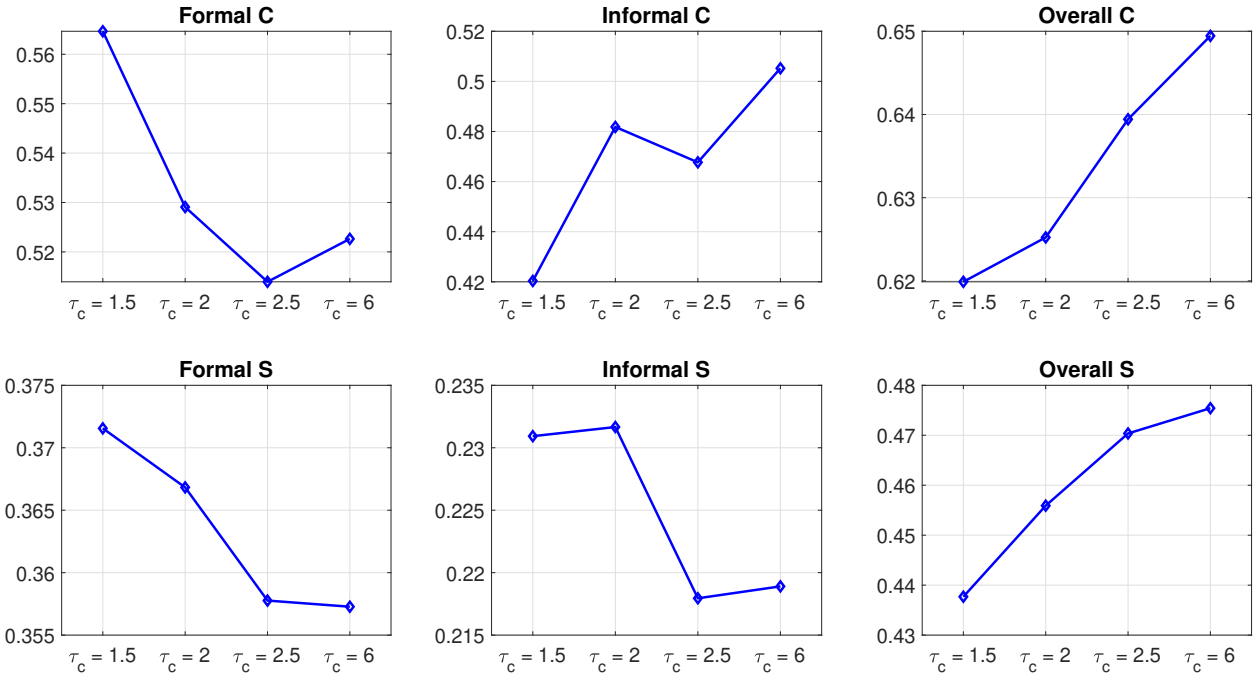
6.1.5 Trade and Wage Inequality

A large literature in International Trade has focused on the effect of openness on wage inequality. We revisit these impacts in the context of our model. Note that, in our model, wage inequality is driven by differences across firms, given that workers are homogeneous, and not by differential exposure of heterogeneous workers to trade. In this regard, we follow an important stream of the literature that includes, among others, Helpman et al. (2010) and Coşar et al. (2016). For studies relating trade to wage inequality driven by heterogeneous workers, see Costinot and Vogel (2010), Adão (2016), Galle et al. (2020), and Adão et al. (2020b) among others.

Figure 6 shows how the standard deviation of log-wages (across workers) behaves as we change τ_c. As we reduce τ_c from 2.5 to 1.5, our measure of wage inequality within the formal C sector increases by 10%. This increase in wage inequality within the formal sector occurs because the demand experienced by exporters—which tend to be larger and pay higher wages—increases, whereas

the demand faced by purely domestic firms decreases. This in turn raises the wage premium paid by exporters, increasing wage dispersion within the formal C sector.

Figure 6: Trade and the Std. Dev. of log-Wages Across Workers in the C and S sectors



The increase in inequality within the formal C sector in response to openness is qualitatively consistent with earlier findings (e.g., Helpman et al., 2016; Coşar et al., 2016, on Brazil and Colombia, respectively). However, if we focus on the informal sector, we observe a decline in wage inequality.[30] As we discussed in section 6.1.1, a trade cost reduction decreases demand for purely domestic firms. As a result, low-productivity firms that would otherwise be formal, become informal. This shift tends to amplify wage inequality within the informal sector as these firms pay relatively higher wages within that sector. However, low-paying informal firms at the bottom of the productivity distribution exit the market, compressing wage dispersion. The net effect of these forces is a reduction in wage inequality. Importantly, as we consider wage inequality across all workers (formal and informal) within the C sector, we obtain a different picture from the one documented in the literature. Reductions in τ_c are associated with *declines* in wage inequality: as τ_c is reduced from 2.5 to 1.5, wage inequality declines by ca. 4.6% within the C sector. As with our results on productivity, these findings highlight the importance of incorporating the informal sector in analyses of trade and wage inequality.

[30]This result seems to conflict with the findings in Adão et al. (2020b). However, Adão et al. (2020b) focus on a different measure of inequality, namely earnings inequality across heterogeneous workers. We remind the reader that wage inequality in our model is solely driven by differences across firms.

To understand these effects, it is helpful to use the law of total variance, and write:

$$Var\left(\log w|k\right) = \sum_{j\in\{f,i\}} p_{kj} Var\left(\log w|kj\right) + \sum_{j\in\{f,i\}} p_{kj}\left(E\left[\log w|kj\right] - E\left[\log w|k\right]\right)^2, \tag{36}$$

where $Var\left(\log w|k\right)$ is the variance of log-wages across all workers within sector k, and $Var\left(\log w|kj\right)$ is the variance of log-wages across workers employed by firms with formality status j in sector k. Similarly, $E\left[\log w|k\right]$ is the average of log-wages across all workers within sector k, and $E\left[\log w|kj\right]$ is the average of log-wages across workers employed by firms with formal status j in sector k. Finally, $p_{kj} \equiv \frac{L_{kj}}{L_{kf}+L_{ki}}$, where L_{kj} denotes total employment in firms of type j in sector k.

Appendix L shows how the different components of equation (36) behave as trade costs are reduced. Both the variance of log-wages within the formal C sector and its weight p_{Cf} increase (as Figure 6 and Figure 3 respectively show). On the other hand, the variance of log-wages within the informal C sector declines, but so does its weight p_{Ci}. On net, the first term on the right hand side of equation (36) slightly increases. However, the variance between the two groups—given by the second term of equation (36)—declines with τ_c, leading to a decrease in the total variance of log-wages within the C sector with openness (see Figure A.1 in the Appendix). This between-group effect occurs because the distance between $E\left[\log w|Cf\right]$ and $E\left[\log w|Ci\right]$ decreases. Furthermore, as workers are reallocated to the formal sector, $E\left[\log w|Cf\right] - E\left[\log w|C\right]$ declines, but its weight in (36), p_{Cf}, increases.

Figure 6 shows that wage inequality within the formal sector S sector also increases as we reduce τ_c. As trade costs are reduced, demand for S sector firms increases, leading to the formalization of the most productive informal firms and to entry into the formal sector at the left tail of formal-sector productivity (which often pays minimum wages). This reallocation results in more dispersion in wages in the formal sector. In the informal S sector, we have entry of unproductive, low-paying, informal firms and exit (to formality) of productive, (relatively) high-paying, informal firms. On net, this leads to an increase in wage inequality within the informal S sector as well. However, the between group component declines as the difference in average log-wages in the two sectors $E\left[\log w|Sf\right]$ and $E\left[\log w|Si\right]$ declines. This between-group effect dominates, so that overall wage inequality within the S sector declines. As the share of employment in the high-inequality C sector declines from 18% at $\tau_c = 2.5$ to 13.3% at $\tau_c = 1.5$, the resulting effect is that, as trade costs decline, wage inequality in the economy as a whole also declines.

6.1.6 Taking Stock

Incorporating the informal sector in the analysis of the effects of trade leads to several important insights. First, increasing openness strongly reduces informal employment in the tradable sector. However, the behavior of informality in the non-tradable sector is different. Given that employment in the non-tradable sector is more than four times larger than employment in the tradable sector, incorporating and carefully modeling the non-tradable sector is of first-order importance. We find that for small reductions in trade costs, the share of informal employment increases, but for further reductions, it reverts to a level close to the starting point. This result may explain why—despite the rapid integration of developing countries in world markets in the past three decades—informal employment has failed to substantially decline (see World Bank, 2019).

Second, our counterfactual analysis demonstrates that the sensitivity effect, whereby employment amongst exporters is more sensitive to shocks, is quantitatively important and drives increases in the unemployment rate as trade costs are reduced. As before, modeling the non-tradable sector is important for the quantitative results. In the non-tradable sector, turnover is reduced as resources are reallocated to larger and more stable firms when trade costs decline. This effect counteracts the increase in turnover in the tradable sector, and, consequently, attenuates the effect of trade on unemployment. Despite the increase in unemployment, our model predicts that a 40% reduction in trade costs leads to welfare gains of 21%.

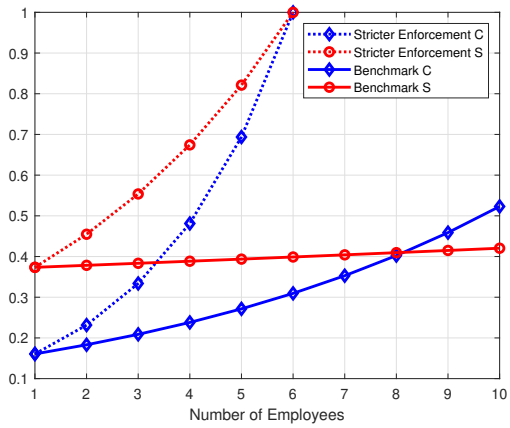
Third, we show that the total effect of openness on the tradable sector aggregate TFP can be up to 2.7 times larger if we incorporate the informal sector into the analysis. This result is important as the literature estimating the productivity gains from trade in developing countries invariably relies on information on formal firms only. Interestingly, we find the opposite pattern in the non-tradable sector: aggregate productivity gains in the formal non-tradable sector *overestimate* overall gains, but the magnitude of the discrepancy is much smaller than the one we find in the tradable sector.

Finally, our analysis corroborates previous results in the literature that openness in developing countries increases wage inequality across workers in the formal sector. However, we find that this effect is reversed once we take into account the informal sector. This suggests that research on the effects of trade on wage inequality in environments with large informal sectors (as in most developing economies) needs to explicitly model and account for informality.

6.2 Effects of Trade when the Informal Sector is Repressed

This section investigates the effects of trade on labor market outcomes and welfare when the informal sector is repressed. We consider two scenarios. First, we focus on a stricter enforcement policy where the convexity of the cost of informality is increased. As Meghir et al. (2015) discuss, this is equivalent to a monitoring policy where attention is disproportionally devoted to larger (and more visible) informal firms. Specifically, we choose $\widetilde{b}_k$ so that $p_{ki}(\ell = 6) = 1$ for $k = C, S$, while maintaining the $\widetilde{a}_k$ parameters at the estimated values in Table 7. This means that firms with six or more employees face an expected penalty equal to or larger than their current revenues.[31] The shifts in the cost of informality are illustrated in Figure 7. Second, we consider a scenario where no informal firm is allowed: a full informality ban. Although extreme and unlikely to be achievable in practice, this is a theoretically interesting paradigm to contemplate through the lens of our model.

Figure 7: Costs of Informality: Benchmark and Stricter Enforcement



Notes: $p_{Ci}(\ell)$ and $p_{Si}(\ell)$ are plotted against ℓ under the benchmark case and under the stricter enforcement policy.

Table 8 shows the effects of these policies conditional on the benchmark value of trade costs $\tau_c = 2.5$. First, the *Stricter Enforcement* policy substantially reduces the share of informal employment from 50% to 36%. The increase in the convexity of the cost of informality leads to the extiction of informal firms of size 3 or larger, but many informal firms of size 1 and 2 are still profitable. As informality is repressed, aggregate TFP in sector C increases by 8.5%, whereas aggregate TFP in sector S is roughly unchanged. Perhaps curiously, real value added per worker declines by 6% in the S sector and is essentially unchanged in sector C. As informality is repressed, the mass of firms operating in the C sector ($N_C = N_{Cf} + N_{Ci}$) decreases by almost 20%. Given our CES aggregators (see equation (2)), the price of the C composite good increases, despite higher aggregate TFP in

[31] We obtain $\widetilde{b}_C = 0.203$ and $\widetilde{b}_S = 0.109$ compared to the estimated benchmark values $\widetilde{b}_C = 0.131$ and $\widetilde{b}_S = 0.013$. We have experimented with other changes in the convexity of the cost of informality, leading to similar conclusions to those reported here.

the sector. Given the importance of C goods as intermediate inputs in both sectors, intermediate goods prices P_C^m and P_S^m increase in both sectors, subsequently reducing intermediate input usage and labor productivity. To conclude, we observe that the *Stricter Enforcement* policy has little effect on unemployment, but leads to a decline in real income of 5%.

Table 8: Effects of Increasing the Cost of Informality

	Benchmark	Stricter Enforcement	No Informality
Unemployment Rate	0.183	0.184	0.326
Share Emp. Ci	0.081	0.050	0
Share Emp. Cf	0.100	0.124	0.201
Share Emp. Si	0.417	0.313	0
Share Emp. Sf	0.402	0.514	0.799
Share Informal Emp.	0.498	0.362	0
$N_C = N_{Cf} + N_{Ci}$	1	0.813	0.268
$N_S = N_{Sf} + N_{Si}$	1	1.137	0.574
Aggregate TFP C	1	1.085	1.317
Real V.A. per worker C	1	0.988	0.856
Aggregate TFP S	1	0.993	1.397
Real V.A. per worker S	1	0.940	0.987
P_C^m	1	1.030	1.061
P_S^m	1	1.013	1.027
Real Income	1	0.950	0.787
Real Income 2	1	0.938	0.541

Notes: Real Income refers to the real value of the sum of all wages and profits in the economy. Real Income 2 refers to the real value of the sum of all wages and profits in the economy including the disutility of unemployment $b \times L_u$. Aggregate TFP is computed as the weighted average of the z's of all active firms—weights are given by firm-level employment. V.A. stands for value added. All variables below line 6 are normalized relative to Benchmark values.

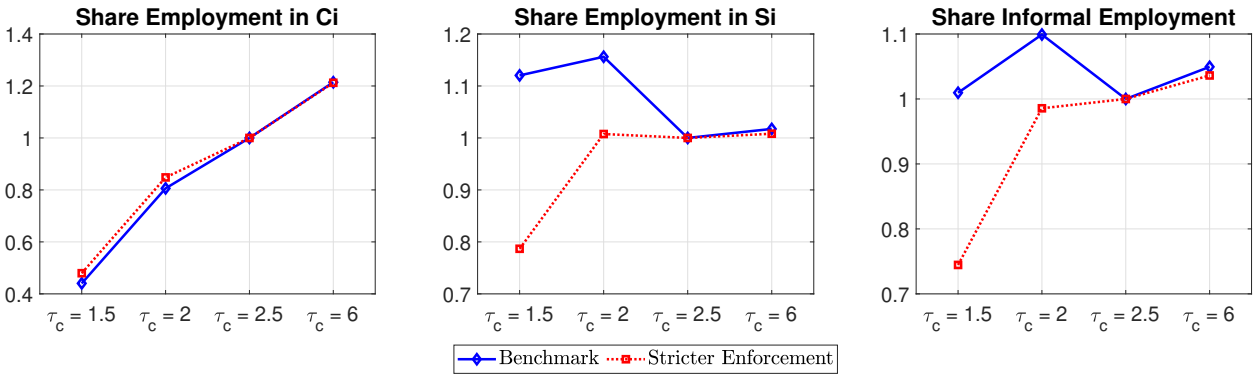
Turning to the *No Informality* policy, Table 8 shows that it substantially increases aggregate TFP, by over 30%, in both sectors. However, the unemployment rate shoots up, from 18.3% to 32.6%. As informality is no longer "allowed," previously low-productivity low-wage informal firms need to formalize to be able to operate. As these firms formalize, the minimum wage they are required to pay sharply reduces their profitability, leading to massive exit and a surge in the unemployment rate. Indeed, under the *No Informality* scenario, the minimum wage binds for over 50% of workers in both the C and S sectors (compared to 4% and 18.6% in the C and S sectors, respectively, in the benchmark scenario). As a consequence of this steep increase in the unemployment rate, welfare falls by 20% in response to this policy (and by ca. 50%, if one accounts for the disutility of unemployment).

One of the main justifications policy makers give for repressing informality is to increase aggregate productivity (Perry et al., 2007). The results in Table 8 demonstrate that this is indeed the case: lower informality implies higher TFP. But this happens at the expense of welfare. It is

interesting to contrast the effects of policies repressing informality to the effects of trade. As we discussed in the previous section, trade also increases productivity. But it does so while increasing welfare and reducing wage inequality. The trade effects are sizable: a 30% increase in TFP in the C sector requires a 40% reduction of trade costs (from 2.5 to 1.5), while welfare *increases* by ca. 20%. In contrast, a similar TFP increase through informality repression can only be achieved if welfare *declines* by ca. 20%.

We next investigate the effects of openness on informality, unemployment and welfare for each of the two scenarios considered in Table 8. Figure 8 shows the effect of τ_c on the size of the informal sector within C and S as well as on the overall share of informal employment. Regardless of the scenario we consider, increasing openness leads to sizable declines in informality in the C sector. However, the effects of openness on informality in the S sector are more nuanced. As previously explained, τ_c reductions cause: (a) an increase in the demand faced by exporters in C; (b) a decline in the demand faced by purely domestic firms in C; (c) an increase in the demand faced by firms in S. The increase in the demand faced by firms in S has in turn two effects: it encourages the formalization of relatively productive informal firms—which tends to reduce informality in the sector; and it encourages entry of unproductive informal firms at the bottom of the productivity distribution—which tends to increase informality in the sector. Figure 8 shows that the dominating force between these two mechanisms depends on the exact economic scenario we consider. The figure shows that as informality is repressed, the incentives for entry at the left tail of the productivity distribution are reduced.

Figure 8: Trade and Informality: Benchmark and Stricter Enforcement

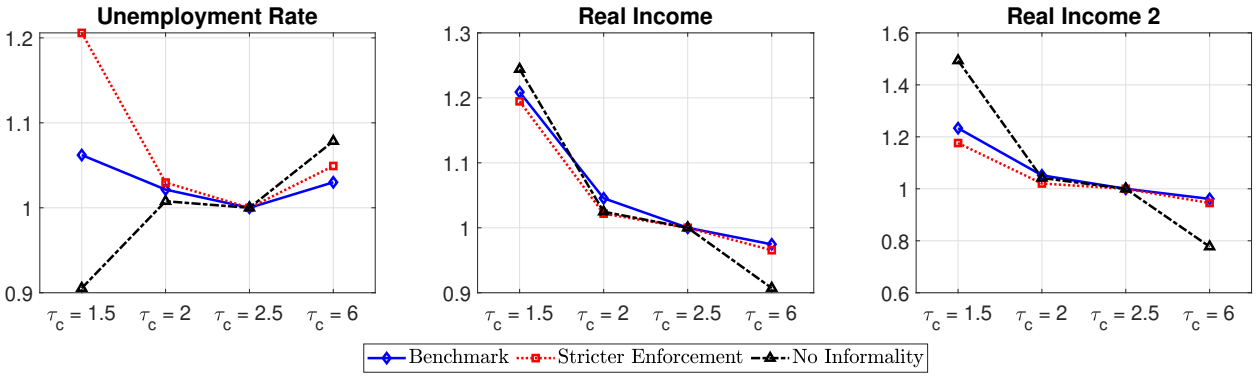


Notes: All variables are normalized relative to their values at $\tau_c = 2.5$.

Figure 9 investigates the implications of openness for unemployment across the scenarios we consider. Two distinct patterns are noticeable. First, increased openness leads to a substantial increase in unemployment when we focus on the *Stricter Enforcement* policy: the unemployment

rate jumps from 18.3% to 22.2%—or a 20% jump in the number of unemployed workers. As in the benchmark case, the sensitivity effect is the key driver. Turnover in the C sector increases, especially within the formal C sector. Given that the informal sector is repressed, transitions from unemployment to informality are much less frequent than under the benchmark, leading to an increase in the persistence in unemployment and, subsequently, to a larger effect on unemployment. Specifically, under *Stricter Enforcement*, the yearly job-finding rate from unemployment declines from 56.8% (with $\tau_c = 2.5$) to 48.1% (with $\tau_c = 1.5$). Second, openness leads to a decline in unemployment when we turn to the *No Informality* policy: the unemployment rate declines from 32.5% to 29.5%—or a 10% reduction in the number of unemployed workers. The sensitivity effect is still in action, but an opposing force dominates. As openness increases, real income and wages also rise, making the minimum wage less binding, thereupon leading to a decline in unemployment.[32]

Figure 9: Trade, Unemp., and Welfare: Benchmark, Stricter Enforcement and No Informality



Notes: All variables are normalized relative to their values at $\tau_c = 2.5$. Real Income refers to the real value of the sum of all wages and profits in the economy. Real Income 2 refers to the real value of the sum of all wages and profits in the economy including the disutility of unemployment $b \times L_u$.

Lastly, we investigate how the gains from trade depend on policies repressing the informal sector. Figure 9 reveals that, as τ_c is reduced from 2.5 to 2, the gains from trade under the *Benchmark* are 4.7%. These gains are approximately 100% larger than under the *Stricter Enforcement* or *No Informality* policies (which amount to 2.2% and 2.5%, respectively). As τ_c is further reduced from 2.5 to 1.5, the gains from trade under the benchmark reach 21%, and, under *Stricter Enforcement*, 19%. However, the gains under the *No Informality* policy jump to 24%. The main reason behind this larger jump is the strong decline in unemployment we see under this scenario, whereas unemployment tends to increase in the remaining scenarios.

Focusing on the other extreme, the losses from autarky are relatively modest in the scenarios we consider. The exception is the *No Informality* case, in which moving toward autarky leads to a

[32] Over 50% of all wages are binding with $\tau_c = 2.5$, but only 40% are binding with $\tau_c = 1.5$.

substantial increase in unemployment, resulting in a 9.3% decline in Real Income. The conclusion is that the gains from trade (and losses from autarky) significantly depend on the economic policies regarding informality. For small reductions in τ_c, gains can differ by as much as 100%. For large reductions in τ_c, the gains are more homogeneous, but still range between 19.4% and 24.4%.

6.3 The Informal Sector as a Buffer

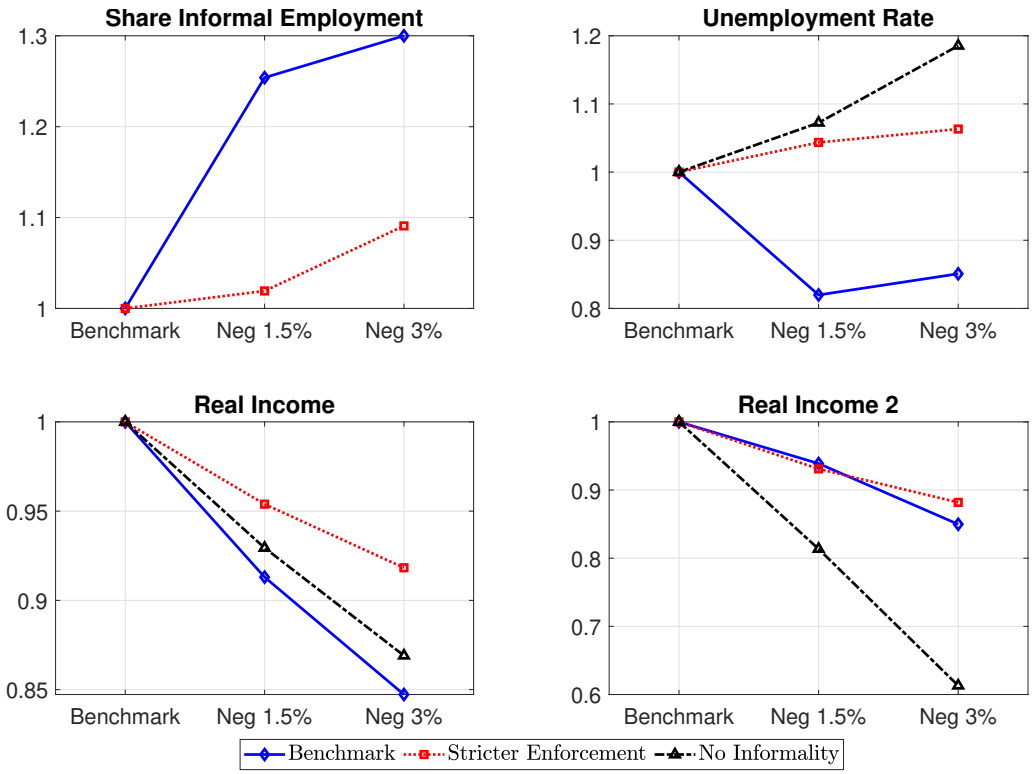
Dix-Carneiro and Kovak (2019) show that, as a result of the trade liberalization episode of the 1990s, Brazilian regions that were more exposed to foreign competition experienced increases in unemployment in the medium run. In the log run, the effect on unemployment was dissipated, but the informal sector increased in these regions relative to the national average. In light of these results, they hypothesized that the informal sector worked as an important shock absorber, and that, in the absence of the informal sector, the effect of import competition on unemployment would have persisted in the long run. Ponczek and Ulyssea (2020) further investigate this hypothesis and find that the effect of import competition on unemployment is larger (and the effect on informality smaller) in regions where informality is more tightly monitored (i.e., in regions where the cost of informality is higher).

Motivated by these empirical results, we examine the role of the informal sector as a buffer in the context of our model. Specifically, we study the response of the economy to negative productivity shocks under three scenarios: (a) Benchmark; (b) Stricter Enforcement Policy; and (c) No Informality.[33] We simulate negative shifts in the ergodic distribution of productivities z amounting to 1.5% and 3%. Figure 10 shows that negative productivity shocks do not lead to increases in unemployment in the benchmark economy, but do lead to substantial increases in informal employment—consistent with the long-run results documented by Dix-Carneiro and Kovak (2019). On the other hand, negative productivity shocks lead to significant increases in unemployment under the *No Informality* and *Stricter Enforcement* scenarios. In short, the stronger the repression of informality, the larger the increase of unemployment in response to negative shocks. This finding is consistent with the hypothesis that the informal sector serves as a shock absorber discussed in

[33]The research designs of Dix-Carneiro and Kovak (2019) and Ponczek and Ulyssea (2020) exploit variation across regions, so that the aggregate effects of trade cannot be identified. These aggregate, common to all regions, effects include the responses of exchange rates and trade imbalances. Simulations of tariff changes in our framework do not correspond to their research designs, as such simulations would generate exchange rate responses, whereas in the aforementioned papers, regions facing differential tariff exposures still face the same exchange rate. Given the common exposure to exchange rates and trade imbalances, the regional approach isolates the (adverse) competition effect induced by trade liberalization: some regions face a larger negative labor demand shock than others. One way to isolate this negative labor demand effect in the context of our model is by simulating aggregate labor demand shocks induced by aggregate productivity shocks.

Dix-Carneiro and Kovak (2019) and Ponczek and Ulyssea (2020).

Figure 10: Negative Productivity Shocks, Informality, Unemployment and Welfare



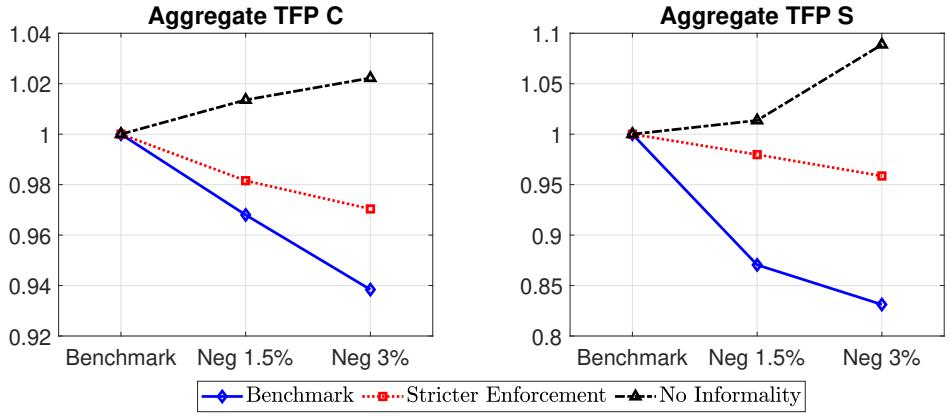
Notes: All variables are normalized relative to their values at $\tau_c = 2.5$. Real Income refers to the real value of the sum of all wages and profits in the economy. Real Income 2 refers to the real value of the sum of all wages and profits in the economy including the disutility of unemployment $b \times L_u$.

Next, we investigate how this buffer role of the informal sector translates into welfare effects. Figure 10 shows that, despite showing the weakest response of unemployment, the benchmark economy experiences the largest relative decline in real income in response to the negative shock. What explains this finding? Figure 11 shows that the negative shock leads to a reduction in aggregate TFP in the benchmark case (in excess of the original negative shock), as resources are shifted toward less productive informal firms. This effect on TFP is smaller in the *Stricter Enforcement* scenario, and goes in the opposite direction in the *No Informality* case—the negative productivity shock pushes lower productivity firms out of the market, fully offsetting the original shock. However, the large increase in unemployment in this case partially offsets the productivity gains. As a result, if we were to rank the three scenarios in terms of their effects on Real Income, the ranking would be: *Stricter Enforcement* > *No Informality* > *Benchmark*. If we focused on the welfare measure that includes the disutility of unemployment (Real Income 2), the corresponding ranking would become: *Stricter Enforcement* > *Benchmark* > *No Informality*. The important insight is that the "unemployment buffer effect" of the informal sector does not translate into a

"welfare buffer effect." This is because, a certain degree of informality repression reinforces the "creative destruction" aspect of a negative productivity shock, driving out of the market inefficient informal firms and increasing aggregate TFP.

We conclude noting that, as discussed earlier and shown in Table 8, in the absence of a productivity shock, the *Stricter Enforcement* policy lowers Real Income by 5% relative to the *Benchmark*. However, when the economy is hit with the 3% negative shock, Real Income ends up 3% higher under *Stricter Enforcement* compared to *Benchmark*. This suggests that welfare is actually less responsive to aggregate shocks when informality is somewhat repressed (but not completely eliminated).

Figure 11: Negative Productivity Shocks and Aggregate TFP



Notes: All variables are normalized relative to their values at $\tau_c = 2.5$. Aggregate TFP is computed as the weighted average of the z's of all active firms—weights are given by firm-level employment.

7 Conclusion

This paper developed a framework for evaluating the role of trade in an environment with a large informal sector, such as in developing economies. The framework generates several of the patterns documented by empirical reduced-form work on informality, while yielding new insights on the labor market, productivity, welfare, and wage inequality effects of trade under informality.

Specifically, our quantitative analysis shows that (1) Informality in the tradable sector is reduced as an economy opens up to trade; (2) In contrast, informality in the non-tradable sector may increase (depending on the starting point and the extent of the trade liberalization); (3) As a result, the total effect of trade openness on informality is ambiguous and may prove very small; and (4) The informal sector serves as an unemployment buffer when an economy faces a negative productivity shock. These are patterns that have been documented in the empirical literature and that our model rationalizes.

In addition, our analysis yields several new insights. First, we find that the effects of trade on productivity in the tradable sector are severely understated when the informal sector is left out; the same is true for the aggregate effects on productivity, though to a lesser extent. Second, we estimate large welfare gains from trade in the benchmark economy with informality. We find that these gains are robust to considering alternative setups in which informality is either completely or partially repressed. Third, we show that repressing informality increases productivity at the expense of welfare. In constrast, the same productivity gains can be achieved through trade liberalization while welfare increases. Fourth, we show that trade increases wage inequality in the formal tradable sector, however, this effect is reversed when the informal sector is incorporated in the analysis. Finally, we show that while the informal sector serves as an "employment buffer" in the event of negative productivity shocks, it does not serve as a "welfare buffer". Despite larger increases in unemployment, the welfare losses resulting from negative productivity shocks are lower when informality is partially (but not completely) repressed due to a "creative destruction" effect pushing inefficient informal firms out of the market in that case.

These results are the outcome of the interaction of the various mechanisms considered in our model. While we attempted to construct a model that captured the main features of a developing economy such as Brazil, feasibility constraints dictated a number of choices that could be the focus of future research. We conclude with highlighting some directions for such research. First, it would be desirable to incorporate worker heterogeneity in the model. Second, to allow for an intensive margin in informality (i.e., formal firms that hire informal workers). Third, to allow for more general preferences and substitution patterns. Finally, our analysis has focused on steady states. In the future, it would be interesting to explore transition dynamics associated with various policy changes.

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Online Appendix

A Steady-State Distribution of States

A.1 Informal Firms

Denote by $G_k(z'|z)$ the cumulative distribution function of z' conditional on z and $g_k(z'|z)$ its density. The period starts with N_{ki} informal firms and distribution of states $\psi_{ki}(z,\ell)$ at the very beginning of stage 1. After (endogenous and exogenous) exit, change in formal status, and entry, but before labor adjustment (end of stage 1 / beginning of stage 2) the distribution of states is:

$$\widetilde{\psi}_{ki}(z,\ell) \equiv \frac{\mathcal{I}[\ell=1] M_{ki}\psi^e_{ki}(z) + \mathcal{I}[\ell\geq 1](1-\alpha_k) N_{ki}\psi_{ki}(z,\ell) I^{stay}_{ki}(z,\ell)}{N_{ki}} \tag{A.1}$$

$$= \mathcal{I}[\ell=1]\frac{M_{ki}}{N_{ki}}\psi^e_{ki}(z) + \mathcal{I}[\ell\geq 1](1-\alpha_k)\psi_{ki}(z,\ell) I^{stay}_{ki}(z,\ell), \tag{A.2}$$

where M_{ki} is the mass of informal entrants into sector k, $\psi^e_{ki}(z)$ is the distribution of z productivities of entrants conditional on entry into the informal sector:

$$\psi^e_{ki}(z) \equiv \frac{g^e_k(z) I^{informal}_k(z)}{\int_{\widetilde{z}} g^e_k(\widetilde{z}) I^{informal}_k(\widetilde{z})\, d\widetilde{z}}. \tag{A.3}$$

In equation (A.1) the numerator is the total mass of firms with state (z,ℓ). The denominator is the total mass of firms at the stage we consider. In steady state, entrants replace firms who exit, so that there are N_{ki} firms at that stage.

After firms make adjustment decisions, and at the production stage (end of stage 2 / beginning of stage 3), the distribution of states is:

$$\widehat{\psi}_{ki}(z,\ell') \equiv \int_\ell \widetilde{\psi}_{ki}(z,\ell)\mathcal{I}\left[L_{ki}(z,\ell)=\ell'\right] d\ell. \tag{A.4}$$

At the end of the period, after production takes place, firms draw their productivity z' for the next period (stage 3). In steady state, the distribution of states at the very end of the period (end of stage 3) replicates the initial one (very beginning of stage 1):

$$\psi_{ki}(z',\ell') = \int_z \widehat{\psi}_{ki}(z,\ell') g_k(z'|z)\, dz. \tag{A.5}$$

To fix ideas, Table A.1 clarifies the notation for the distribution of states in different stages within a period.

A.2 Formal Firms

The period starts with N_{kf} formal firms and distribution of states $\psi_{kf}(z,\ell)$ at the very beginning of stage 1. After (endogenous and exogenous) exit, change in formal status, and entry, but before

Table A.1: Distributions of States at Different Stages

ψ_{kj}	Distribution of states at the very beginning, and at the very end of the period—very beginning of stage 1 and very end of stage 3
$\widetilde{\psi}_{kj}$	Distribution of states right after entry, exit, and change of formal status but **before** labor adjustment—very end of stage 1 / very beginning of stage 2
$\widehat{\psi}_{kj}$	Distribution of states after labor adjustment, at the production stage—end of stage 2

labor adjustment (end of stage 1 / beginning of stage 2) the distribution of states is:

$$\begin{aligned}\widetilde{\psi}_{kf}(z,\ell) \equiv= & \frac{\begin{array}{c}\mathcal{I}[\ell=1] M_{kf}\psi^{e}_{kf}(z) \\ +\mathcal{I}[\ell\geq 1](1-\alpha_k) N_{ki}\psi_{ki}(z,\ell) I^{change}_{ki}(z,\ell) \\ +\mathcal{I}[\ell\geq 1](1-\alpha_k) N_{kf}\psi_{kf}(z,\ell) I^{stay}_{kf}(z,\ell)\end{array}}{N_{kf}} \qquad (A.6)\\ = & \begin{array}{c}\mathcal{I}[\ell=1] \frac{M_{kf}}{N_{kf}}\psi^{e}_{kf}(z) \\ +\mathcal{I}[\ell\geq 1](1-\alpha_k) \frac{N_{ki}}{N_{kf}}\psi_{ki}(z,\ell) I^{change}_{ki}(z,\ell) \\ +\mathcal{I}[\ell\geq 1](1-\alpha_k) \psi_{kf}(z,\ell) I^{stay}_{kf}(z,\ell)\end{array}, \qquad (A.7)\end{aligned}$$

where M_{kf} is the mass of formal entrants into sector k, $\psi^{e}_{kf}(z)$ is the distribution of z productivities of entrants conditional on entry into the formal sector:

$$\psi^{e}_{kf}(z) \equiv \frac{g^{e}_{k}(z) I^{formal}_{k}(z)}{\int_{\widetilde{z}} g^{e}_{k}(\widetilde{z}) I^{formal}_{k}(\widetilde{z}) d\widetilde{z}}. \qquad (A.8)$$

The numerator in equation (A.6) is the total mass of firms with state (z,ℓ). The denominator is the total mass of firms at the stage we consider. In steady state, entrants replace firms who exit, so that there are N_{kf} firms at that stage. After firms make adjustment decisions, and at the production stage (end of stage 2 / beginning of stage 3), the distribution of states is:

$$\widehat{\psi}_{kf}(z,\ell') \equiv \int_{\ell} \widetilde{\psi}_{kf}(z,\ell) \mathcal{I}\left(L_{kf}(z,\ell)=\ell'\right) d\ell. \qquad (A.9)$$

At the end of the period, after production takes place, firms draw their productivity z' for the next period (stage 3). In steady state, the distribution of states at the very end of the period (very end of stage 3) replicates the initial one (very beginning of stage 1):

$$\psi_{kf}(z',\ell') = \int_{z} \widehat{\psi}_{kf}(z,\ell') g_{k}(z'|z) dz.$$

B Entry

Let M_k denote the mass of entrants in sector $k = C, S$. The fraction of entrants into the formal and informal sectors are given respectively by ω_{kf} and ω_{ki}:

$$\omega_{kf} \equiv \Pr\left(I_k^{formal}(z) = 1\right) = \int_z g_k^e(z)\, dz, \tag{A.10}$$

$$\omega_{ki} \equiv \Pr\left(I_k^{informal}(z) = 1\right) = \int_z g_k^e(z)\, dz. \tag{A.11}$$

Therefore, the masses of entrants in the formal and informal sectors are given by:

$$M_{ki} = \omega_{ki} M_k, \tag{A.12}$$

$$M_{kf} = \omega_{kf} M_k. \tag{A.13}$$

The masses of entrants into each sector, M_k, are pinned down by the free entry condition (assuming positive entry in both sectors):

$$c_{e,k} = V_k^e = \int_z \left[V_{ki}^e(z)\, I_k^{informal}(z) + V_{kf}^e(z)\, I_k^{formal}(z)\right] g_k^e(z)\, dz. \tag{A.14}$$

C Flow conditions for workers and firms

In order to write the labor market clearing conditions, we first define the following quantities.

- Number of workers at the **beginning** of the period in sector k (before entry, exit, change of formal status and labor adjustment), working in formal or informal firms (T stands for "total"):

$$W_{kj}^T = N_{kj} \underbrace{\int_z \int_\ell \ell \psi_{kj}(z, \ell)\, d\ell dz}_{\text{avg. \# of workers per firm}} = L_{kj} \tag{A.15}$$

  for $j = f, i$ and $k = C, S$.

- Number of workers in sector (k, j) who are fired because their firms receive a destruction shock:

$$W_{kj}^{DS} = \alpha_{kj} N_{kj} \int_z \int_\ell \ell \psi_{kj}(z, \ell)\, d\ell dz = \alpha_{kj} L_{kj} \tag{A.16}$$

- Number of workers in sector (k, j) who are fired due to **endogenous** firm exit:

$$W_{kj}^{EE} = (1 - \alpha_{kj})\, N_{kj} \times \int_z \int_\ell \ell \psi_{kj}(z, \ell)\, I_{kj}^{exit}(z, \ell)\, d\ell dz \tag{A.17}$$

  where $(1 - \alpha_{kj})\, N_{kj}$ is the mass of firms that survive after the destruction shock hits.

- Number (mass) of **surviving incumbent** firms in sector (k, j) in the interim period:

$$N'_{kj} \equiv (1 - \alpha_{kj})\, N_{kj} \int_z \int_\ell \psi_{kj}(z, \ell)\, I_{kj}^{stay}(z, \ell)\, d\ell dz \tag{A.18}$$

- Number of workers initially in sector (k,j) who are fired due to downsizing at the interim stage:

$$W_{kj}^{D} = N'_{kj} \int_{z} \int_{\ell} \widetilde{\psi}_{kj}^{incumbent}(z,\ell) \left(1 - I_{kj}^{hire}(z,\ell)\right) (\ell - L_{kj}(z,\ell)) \, d\ell dz \tag{A.19}$$

where $\widetilde{\psi}_{kj}^{incumbent}(z,\ell)$ is the distribution of states in the interim stage among **surviving incumbents**. Note that this is not the same distribution as $\widetilde{\psi}_{kj}(z,\ell)$ as it does not include entrants. It is obtained as follows:

$$\begin{aligned} \widetilde{\psi}_{kj}^{incumbent}(z,\ell) &\equiv \frac{(1-\alpha_{kj}) N_{kj} \psi_{kj}(z,\ell) I_{kj}^{stay}(z,\ell)}{N'_{kj}} \\ &= \frac{\psi_{kj}(z,\ell) I_{kj}^{stay}(z,\ell)}{\int_{\widetilde{z}} \int_{\widetilde{\ell}} \psi_{kj}\left(\widetilde{z},\widetilde{\ell}\right) I_{kj}^{stay}\left(\widetilde{z},\widetilde{\ell}\right) d\widetilde{\ell} d\widetilde{z}} \end{aligned} \tag{A.20}$$

- Total fraction of workers in the formal sector of sector k who are laid off, conditional on starting the period in a formal firm in sector k :

$$\chi_{kf}^{layoff} = \frac{W_{kf}^{DS} + W_{kf}^{EE} + W_{kf}^{D}}{W_{kf}^{T}} \tag{A.21}$$

$$= \alpha_k + \frac{\left(\begin{array}{c} (1-\alpha_k) \int_z \int_\ell \ell \psi_{kf}(z,\ell) I_{kf}^{exit}(z,\ell) \, d\ell dz \\ + (1-\alpha_k) \left(\int_z \int_\ell \psi_{kf}(z,\ell) I_{kf}^{stay}(z,\ell) \, d\ell dz \right) \times \\ \int_z \int_\ell \widetilde{\psi}_{kf}^{incumbent}(z,\ell) \left(1 - I_{kf}^{hire}(z,\ell)\right) (\ell - L_{kf}(z,\ell)) \, d\ell dz \end{array} \right)}{\int_z \int_\ell \ell \psi_{kf}(z,\ell) \, d\ell dz}$$

- Number of firms that start the period as informal firms, but end the period as formal firms (because they formalized).

$$N'_{ki \to f} \equiv (1-\alpha_k) N_{ki} \int_{z} \int_{\ell} \psi_{ki}(z,\ell) I_{ki}^{change}(z,\ell) \, d\ell dz, \tag{A.22}$$

where $(1-\alpha_k) N_{ki}$ is the mass of firms that survive after the destruction shock hits.

- Distribution of states among firms that switched from informal to formal, in the interim period—before adjusting the labor force.

$$\begin{aligned} \widetilde{\psi}_{ki \to f}(z,\ell) &\equiv \frac{(1-\alpha_k) N_{ki} \psi_{ki}(z,\ell) I_{ki}^{change}(z,\ell)}{N'_{ki \to f}} \\ &= \frac{\psi_{ki}(z,\ell) I_{ki}^{change}(z,\ell)}{\int_{\widetilde{z}} \int_{\widetilde{\ell}} \psi_{ki}\left(\widetilde{z},\widetilde{\ell}\right) I_{ki}^{change}\left(\widetilde{z},\widetilde{\ell}\right) d\widetilde{\ell} d\widetilde{z}}. \end{aligned} \tag{A.23}$$

- Number of workers who start the period in informal firms, but end the period in formal firms (their employers switched to formal, and they were not fired after the interim productivity

was realized):

$$W_{k,i\to f} = N'_{ki\to f}\int_z\int_\ell \widetilde{\psi}_{ki\to f}(z,\ell)\left(\begin{array}{c}\ell \times I^{hire}_{kf}(z,\ell)+\\ L_{kf}(z,\ell)\times\left(1-I^{hire}_{kf}(z,\ell)\right)\end{array}\right)d\ell dz \tag{A.24}$$

- Fraction of workers who start the period in informal firms, but end the period in formal firms:

$$\begin{aligned}\chi^{change}_{ki\to f} &= \frac{W_{k,i\to f}}{W^T_{ki}}\\ &= \frac{\left(\begin{array}{c}(1-\alpha_k)\left(\int_z\int_\ell \psi_{ki}(z,\ell)\, I^{change}_{ki}(z,\ell)\, d\ell dz\right)\times\\ \int_z\int_\ell \widetilde{\psi}_{ki\to f}(z,\ell)\left(\begin{array}{c}\ell\times I^{hire}_{kf}(z,\ell)+\\ L_{kf}(z,\ell)\times\left(1-I^{hire}_{kf}(z,\ell)\right)\end{array}\right)d\ell dz\end{array}\right)}{\int_z\int_\ell \ell\psi_{ki}(z,\ell)\,d\ell dz}\end{aligned} \tag{A.25}$$

- Number of workers who start the period in informal firms, but their employers switched to formal status:

$$W^{SF}_{ki} = (1-\alpha_k)\,N_{ki}\int_z\int_\ell \ell\psi_{ki}(z,\ell)\, I^{change}_{ki}(z,\ell)\,d\ell dz \tag{A.26}$$

- Fraction of workers who start employed in the informal sector and leave it in the interim period (became unemployed or employer switched to formal):

$$\begin{aligned}\chi^{leave}_{ki} &= \frac{W^{DS}_{ki}+W^{EE}_{ki}+W^{SF}_{ki}+W^{D}_{ki}}{W^T_{ki}}\\ &= \alpha_k + \frac{\left(\begin{array}{c}(1-\alpha_k)\int_z\int_\ell \ell\psi_{ki}(z,\ell)\, I^{exit}_{ki}(z,\ell)\,d\ell dz+\\ (1-\alpha_k)\int_z\int_\ell \ell\psi_{ki}(z,\ell)\, I^{change}_{ki}(z,\ell)\,d\ell dz+\\ (1-\alpha_k)\left(\int_z\int_\ell \psi_{ki}(z,\ell)\, I^{stay}_{ki}(z,\ell)\,d\ell dz\right)\times\\ \int_z\int_\ell \widetilde{\psi}^{incumbent}_{ki}(z,\ell)\left(1-I^{hire}_{ki}(z,\ell)\right)(\ell - L_{ki}(z,\ell))\,d\ell dz\end{array}\right)}{\int_z\int_\ell \ell\psi_{ki}(z,\ell)\,d\ell dz}\end{aligned} \tag{A.27}$$

With these objects, we can define the equilibrium conditions that refer to labor market flows:

$$\chi^{leave}_{ki} L_{ki} = L_u \mu^e_{ki} \tag{A.28}$$

$$\chi^{layoff}_{kf} L_{kf} = L_u \mu^e_{kf} + L_{ki}\chi^{change}_{ki\to f}. \tag{A.29}$$

These conditions state that the mass of workers in each sector (k,j) cannot be contracting or expanding in equilibrium (expressions (A.28) and (A.29)). Finally, the sum of unemployment and employment levels across sectors equals the total labor force $\overline{L}$:

$$L_{Cf} + L_{Ci} + L_{Sf} + L_{Si} + L_u = \overline{L}. \tag{A.30}$$

We can proceed in a similar way to define the equilibrium flow conditions for firms. The relevant objects follow.

- Fraction of formal firms exiting sector k:

$$\varrho_{kf}^{exit} = \alpha_k + (1-\alpha_k) \int_z \int_\ell I_{kf}^{exit}(z,\ell)\, \psi_{kf}(z,\ell)\, d\ell dz \tag{A.31}$$

- Fraction of informal firms exiting sector k:

$$\varrho_{ki}^{exit} = \alpha_k + (1-\alpha_k) \int_z \int_\ell \left(I_{ki}^{exit}(z,\ell) + I_{ki}^{change}(z,\ell) \right) \psi_{ki}(z,\ell)\, d\ell dz \tag{A.32}$$

- Fraction of informal firms changing status in sector k:

$$\varrho_{ki}^{change} = (1-\alpha_k) \int_z \int_\ell I_{ki}^{change}(z,\ell)\, \psi_{ki}(z,\ell)\, d\ell dz \tag{A.33}$$

Similarly to workers, the mass of firms in each sector (k,j) must be constant in steady state. This means that the inflow of firms must equal the outflow, which can be written as:

$$\varrho_{kf}^{exit} N_{kf} = M_{kf} + \varrho_{ki}^{change} N_{ki}, \tag{A.34}$$

$$\varrho_{ki}^{exit} N_{ki} = M_{ki}. \tag{A.35}$$

D Vacancies

Aggregate vacancies in sector kj are given by:

$$V_{kj} = N_{kj} \int_z \int_\ell v_{kj}(z,\ell)\, \widetilde{\psi}_{ki}(z,\ell)\, d\ell dz + \frac{M_{kj}}{\mu^v} \tag{A.36}$$

where $v_{kj}(z,\ell)$ is the number of vacancies a firm with productivity z and labor force ℓ posts and $\frac{M_{kj}}{\mu^v}$ is the number of vacancies posted at entry (and before adjustment in stage 2).

E Unemployment Benefits / Tax Collection / Transfers

Government Revenue is given by the sum of value-added taxes, payroll taxes, firing costs and import taxes:

$$\begin{aligned} G_{Rev} = & \sum_k N_{kf} \tau_y \int_z \int_{\ell'} VA_k(z,\ell')\, \widehat{\psi}_{kf}(z,\ell')\, d\ell' dz \\ & + \sum_k N_{kf} \tau_w \int_z \int_{\ell'} \max\left\{ w_{kf}(z,\ell'), \underline{w} \right\} \ell \widehat{\psi}_{kf}(z,\ell')\, d\ell' dz \\ & + \sum_k N_{kf} \kappa \int_z \int_\ell \widetilde{\psi}_{kf}(z,\ell) \left(\ell - L_{kf}(z,\ell) \right) \left(1 - I_{kf}^{hire}(z,\ell) \right) d\ell dz \\ & + (\tau_a - 1) \frac{D_{H,C} \left(\epsilon \tau_a \tau_c \right)^{1-\sigma_C}}{\tau_a}. \end{aligned} \tag{A.37}$$

Government spending with unemployment insurance is given by:

$$G_{UI} = b^u \times \underbrace{\sum_k \left(W_{kf}^{DS} + W_{kf}^{EE} + W_{kf}^{D}\right)}_{\text{mass of } \textbf{formal} \text{ workers who transition to unemployment}} \tag{A.38}$$

We impose that

$$T = G_{Rev} - G_{UI} \geq 0 \tag{A.39}$$

and that T is rebated to consumers.
Important note: part of the aggregate informality costs

$$\sum_k N_{ki} \int_z \int_{\ell'} p_{ki}\left(\ell'\right) R_k\left(z, \ell'\right) \widehat{\psi}_{ki}\left(z, \ell'\right) d\ell' dz \tag{A.40}$$

should be considered government revenue as these consist of fines. However, part of these costs should not, as they consist of opportunity costs associated with informality. Therefore, we do not add these costs to government revenue. However, the model redistributes these costs to consumers. One way to view this procedure is that these costs affect/distort the decisions of firms, but we do not consider these costs as wasted resources.

F Service Sector Market Clearing

Service sector goods are used for final consumption (consumers spend $(1-\zeta) I$ on it), intermediate inputs (firms spend X_S^{int} on it) and as inputs for hiring costs, fixed costs and entry costs (and fixed costs of exporting). The average (per firm) hiring costs in sector (k, j):

$$\overline{H}_{kj} = \int_z \int_\ell H_{kj}\left(\ell, L_{kj}\left(z, \ell\right)\right) I_{kj}^{hire}\left(z, \ell\right) \widetilde{\psi}_{kj}\left(z, \ell\right) d\ell dz, \tag{A.41}$$

and the fraction of tradable-sector goods firms that export is given by:

$$\mu_x = \int_z \int_{\ell'} \widehat{\psi}_{Cf}\left(z, \ell'\right) I_C^x\left(z, \ell'\right) d\ell' dz \tag{A.42}$$

Expenditure on entry and hiring costs, fixed costs of operations and export costs are given by:

$$E_S = \sum_{k=C,S\ ;\ j=i,f} N_{kj}\left(\overline{H}_{kj} + \overline{c}_{kj}\right) + N_{Cf}\mu_x f_x + \sum_{k=C,S} M_k c_{e,k} \tag{A.43}$$

G Aggregate Income

Aggregate income is given by total wages, government transfers and total profits:

$$\begin{aligned}
I = & \sum_k N_{ki} \int_z \int_{\ell'} w_{ki}\left(z,\ell'\right) \ell' \widehat{\psi}_{ki}\left(z,\ell'\right) d\ell' dz \\
& + \sum_k N_{kf} \int_z \int_{\ell'} \max\left\{w_{kf}\left(z,\ell'\right), \underline{w}\right\} \ell' \widehat{\psi}_{kf}\left(z,\ell'\right) d\ell' dz \\
& + \sum_k N_{ki} \int_z \int_{\ell'} \widetilde{\pi}_{ki}\left(z,\ell'\right) \widehat{\psi}_{ki}\left(z,\ell'\right) d\ell' dz \\
& + \sum_k N_{kf} \int_z \int_{\ell'} \widetilde{\pi}_{kf}\left(z,\ell'\right) \widehat{\psi}_{kf}\left(z,\ell'\right) d\ell' dz \\
& + G_{Rev} \\
& + \sum_k N_{ki} \int_z \int_{\ell'} p_{ki}\left(\ell'\right) R_k\left(z,\ell'\right) \widehat{\psi}_{ki}\left(z,\ell'\right) d\ell' dz \\
& - \sum_k N_{kf} \kappa \int_z \int_{\ell} \widetilde{\psi}_{kf}\left(z,\ell\right) \left(\ell - L_{kf}\left(z,\ell\right)\right) \left(1 - I_{kf}^{hire}\left(z,\ell\right)\right) d\ell dz \\
& - \sum_{k=C,S\ ;\ j=i,f} N_{kj} \overline{H}_{kj} \\
& - \sum_k M_k c_{e,k},
\end{aligned} \tag{A.44}$$

where profits $\widetilde{\pi}$ are computed **before** subtracting hiring costs.

H Trade Balance

Trade balance implies that total imports must equal total exports, which is given by:

$$\frac{D_{H,C}\left(\epsilon \tau_a \tau_c\right)^{1-\sigma_C}}{\tau_a} = Exports \tag{A.45}$$

I Worker Value Functions

Present value of a formal job at a firm with state (z,ℓ') at the production stage

$$\begin{aligned}
\overline{J}_{kf}^{e}\left(z,\ell'\right) = & \ w_{kf}\left(z,\ell'\right) \\
& + \frac{1-\alpha_k}{1+r} E_{z'|z} \left(\begin{array}{c}
\left(\frac{\alpha_k}{1-\alpha_k} + I_{kf}^{exit}\left(z',\ell'\right)\right) \times \left(b + b_u + \frac{1}{1+r} J^u\right) \\
+ I_{kf}^{stay}\left(z',\ell'\right) \times I_{kf}^{contract}\left(z',\ell'\right) \times p_{kf}^{fire}\left(z',\ell'\right) \times \left(b + b_u + \frac{1}{1+r} J^u\right) \\
+ I_{kf}^{stay}\left(z',\ell'\right) \times I_{kf}^{contract}\left(z',\ell'\right) \times \left(1 - p_{kf}^{fire}\left(z',\ell'\right)\right) \times \overline{J}_{kf}^{e}\left(z', L_{kf}\left(z',\ell'\right)\right) \\
+ I_{kf}^{stay}\left(z',\ell'\right) \times I_{kf}^{expand}\left(z',\ell'\right) \times \overline{J}_{kf}^{e}\left(z', L_{kf}\left(z',\ell'\right)\right)
\end{array} \right)
\end{aligned}$$

$$p_{kf}^{fire}\left(z',\ell'\right) \equiv \frac{\ell' - L_{kf}\left(z',\ell'\right)}{\ell'}$$
$$I_{kf}^{contract}\left(z',\ell'\right) \equiv I\left(L_{kf}\left(z',\ell'\right) < \ell'\right)$$
$$I_{kf}^{expand}\left(z',\ell'\right) \equiv I\left(L_{kf}\left(z',\ell'\right) \geq \ell'\right)$$

Rewriting:

$$\begin{aligned}
&\overline{J}_{kf}^{e}\left(z,\ell'\right) = w_{kf}\left(z,\ell'\right) \\
&+ \frac{1-\alpha_k}{1+r}\left(\frac{\alpha_k}{1-\alpha_k} + E_{z'|z}\left[I_{kf}^{exit}\left(z',\ell'\right)\right]\right) \times \left(b + b_u + \frac{1}{1+r}J^u\right) \\
&+ \frac{1-\alpha_k}{1+r}E_{z'|z}\left[I_{kf}^{stay}\left(z',\ell'\right) \times I_{kf}^{contract}\left(z',\ell'\right) \times p_{kf}^{fire}\left(z',\ell'\right)\right] \times \left(b + b_u + \frac{1}{1+r}J^u\right) \\
&+ \frac{1-\alpha_k}{1+r}E_{z'|z}\left[I_{kf}^{stay}\left(z',\ell'\right) \times I_{kf}^{contract}\left(z',\ell'\right) \times \left(1 - p_{kf}^{fire}\left(z',\ell'\right)\right) \times \overline{J}_{kf}^{e}\left(z', L_{kf}\left(z',\ell'\right)\right)\right] \\
&+ \frac{1-\alpha_k}{1+r}E_{z'|z}\left[I_{kf}^{stay}\left(z',\ell'\right) \times I_{kf}^{expand}\left(z',\ell'\right) \times \overline{J}_{kf}^{e}\left(z', L_{kf}\left(z',\ell'\right)\right)\right]
\end{aligned}$$

It will be convenient to work with:

$$J_{kf}^{e}\left(z,\ell'\right) \equiv (1+r)\left(\overline{J}_{kf}^{e}\left(z,\ell'\right) - w_{kf}\left(z,\ell'\right)\right) \tag{A.46}$$

Present value of an informal job at a firm with state (z,ℓ') at the production stage

$$\begin{aligned}
&\overline{J}_{ki}^{e}\left(z,\ell'\right) = w_{ki}\left(z,\ell'\right) \\
&+ \frac{(1-\alpha_k)}{1+r}E_{z'|z}\left(\begin{array}{c}
\left(\frac{\alpha_k}{(1-\alpha_k)} + I_{ki}^{exit}\left(z',\ell'\right)\right) \times \left(b + \frac{1}{1+r}J^u\right) \\
+I_{ki}^{stay}\left(z',\ell'\right) \times I_{ki}^{contract}\left(z',\ell'\right) \times p_{ki}^{fire}\left(z',\ell'\right) \times \left(b + \frac{1}{1+r}J^u\right) \\
+I_{ki}^{stay}\left(z',\ell'\right) \times I_{ki}^{contract}\left(z',\ell'\right) \times \left(1 - p_{ki}^{fire}\left(z',\ell'\right)\right) \times \overline{J}_{ki}^{e}\left(z', L_{ki}\left(z',\ell'\right)\right) \\
+I_{ki}^{stay}\left(z',\ell'\right) \times I_{ki}^{expand}\left(z',\ell'\right) \times \overline{J}_{ki}^{e}\left(z', L_{ki}\left(z',\ell'\right)\right) \\
+I_{ki}^{change}\left(z',\ell'\right) \times I_{kf}^{contract}\left(z',\ell'\right) \times p_{kf}^{fire}\left(z',\ell'\right) \times \left(b + b_u + \frac{1}{1+r}J^u\right) \\
+I_{ki}^{change}\left(z',\ell'\right) \times I_{kf}^{contract}\left(z',\ell'\right) \times \left(1 - p_{kf}^{fire}\left(z',\ell'\right)\right) \times \overline{J}_{kf}^{e}\left(z', L_{kf}\left(z',\ell'\right)\right) \\
+I_{ki}^{change}\left(z',\ell'\right) \times I_{kf}^{expand}\left(z',\ell'\right) \times \overline{J}_{kf}^{e}\left(z', L_{kf}\left(z',\ell'\right)\right)
\end{array}\right)
\end{aligned}$$

$$p_{ki}^{fire}\left(z',\ell'\right) \equiv \frac{\ell' - L_{ki}\left(z',\ell'\right)}{\ell'}$$
$$I_{ki}^{contract}\left(z',\ell'\right) \equiv I\left(L_{ki}\left(z',\ell'\right) < \ell'\right)$$
$$I_{ki}^{expand}\left(z',\ell'\right) \equiv I\left(L_{ki}\left(z',\ell'\right) \geq \ell'\right)$$

Rewriting

$$
\begin{aligned}
&\overline{J}^{e}_{ki}\left(z,\ell'\right) = w_{ki}\left(z,\ell'\right) \\
&+\frac{(1-\alpha_k)}{1+r}\left(\frac{\alpha_k}{(1-\alpha_k)}+E_{z'|z}\left[I^{exit}_{ki}\left(z',\ell'\right)\right]\right)\times\left(b+\frac{1}{1+r}J^u\right) \\
&+\frac{(1-\alpha_k)}{1+r}E_{z'|z}\left[I^{stay}_{ki}\left(z',\ell'\right)\times I^{contract}_{ki}\left(z',\ell'\right)\times p^{fire}_{ki}\left(z',\ell'\right)\right]\times\left(b+\frac{1}{1+r}J^u\right) \\
&+\frac{(1-\alpha_k)}{1+r}E_{z'|z}\left[I^{stay}_{ki}\left(z',\ell'\right)\times I^{contract}_{ki}\left(z',\ell'\right)\times\left(1-p^{fire}_{ki}\left(z',\ell'\right)\right)\times\overline{J}^{e}_{ki}\left(z',L_{ki}\left(z',\ell'\right)\right)\right] \\
&+\frac{(1-\alpha_k)}{1+r}E_{z'|z}\left[I^{stay}_{ki}\left(z',\ell'\right)\times I^{expand}_{ki}\left(z',\ell'\right)\times\overline{J}^{e}_{ki}\left(z',L_{ki}\left(z',\ell'\right)\right)\right] \\
&+\frac{(1-\alpha_k)}{1+r}E_{z'|z}\left[I^{change}_{ki}\left(z',\ell'\right)\times I^{contract}_{kf}\left(z',\ell'\right)\times p^{fire}_{kf}\left(z',\ell'\right)\right]\times\left(b+b_u+\frac{1}{1+r}J^u\right) \\
&+\frac{(1-\alpha_k)}{1+r}E_{z'|z}\left[I^{change}_{ki}\left(z',\ell'\right)\times I^{contract}_{kf}\left(z',\ell'\right)\times\left(1-p^{fire}_{kf}\left(z',\ell'\right)\right)\times\overline{J}^{e}_{kf}\left(z',L_{kf}\left(z',\ell'\right)\right)\right] \\
&+\frac{(1-\alpha_k)}{1+r}E_{z'|z}\left[I^{change}_{ki}\left(z',\ell'\right)\times I^{expand}_{kf}\left(z',\ell'\right)\times\overline{J}^{e}_{kf}\left(z',L_{kf}\left(z',\ell'\right)\right)\right]
\end{aligned}
$$

As before, it will be convenient to work with:

$$
J^{e}_{ki}\left(z,\ell'\right) \equiv (1+r)\left(\overline{J}^{e}_{ki}\left(z,\ell'\right)-w_{ki}\left(z,\ell'\right)\right) \tag{A.47}
$$

Value of Search

$$
J^u = \sum_{k,j}\mu^{e}_{kj}\int_{\ell}\int_{z'}\overline{J}^{e}_{kj}\left(z,L_{kj}\left(z,\ell\right)\right)g_{kj}\left(z,\ell\right)dzd\ell+\left(1-\sum_{k,j}\mu^{e}_{kj}\right)\left(b+\frac{1}{1+r}J^u\right) \tag{A.48}
$$

$$
\begin{aligned}
\widetilde{g}_{kj}\left(z,\ell\right) &= N_{kj}\widetilde{\psi}_{kj}\left(z,\ell\right)v_{kj}\left(z,\ell\right)+\mathcal{I}\left[\ell=1\right]\frac{M_{kj}}{\mu^{\upsilon}}\psi^{e}_{kj}\left(z\right) \\
g_{kj}\left(z,\ell\right) &= \frac{\widetilde{g}_{kj}\left(z,\ell\right)}{\int_z\int_{\ell}\widetilde{g}_{kj}\left(z,\ell\right)d\ell dz} = \frac{\widetilde{g}_{kj}\left(z,\ell\right)}{V_{kj}}
\end{aligned}
$$

J Data Appendix

We use six firm-level datasets containing information on formal and informal firms, as well on their workers. In addition to those, we use one worker-level dataset—*Pesquisa Mensal de Emprego* (PME)—which provides information on workers' allocations and labor market flows. We impose the following common filters across all datasets: we exclude firms and workers in the public sector, agriculture, mining, coal, oil and gas industries. 2003 is our reference year as the ECINF survey is only available for 1997 and 2003. All monetary values (e.g. revenues and wages) correspond to annual values. Finally, we rely on data from the 2000 and 2005 IBGE National Accounts to estimate utility and production function parameters. Sector C—the tradable sector—includes all manufacturing sectors (excluding mining, coal, oil and gas industries, as mentioned above). Sector S—the non-tradable sector—includes all services, commerce, construction, transportation, and utilities sectors. In the following sections, we describe the main variables we generate, as well as the moments and auxiliary models computed from each dataset.

J.1 RAIS and SECEX

RAIS (*Relação Anual de Informações Sociais*) is a matched employer-employee dataset assembled by the Brazilian Ministry of Labor every year since 1976. Establishments are identified by their *Cadastro Nacional de Pessoas Jurídicas* (CNPJ) number, which consists of 14 digits. To make RAIS data compatible with firm-level Census data (PIA, PAS, PAC), we aggregate establishments to the firm level using the first 8 digits of the CNPJ identifier. For multi-establishment firms featuring multiple 4-digit *CNAE* industry codes, we select the code accounting for the largest share of employment within the firm. A negligible share of firms (0.01 percent) have missing industry codes, so they are dropped from the analysis. Firm-level wages and employment are measured as of December of each year. December wages are subsequently annualized. We generate the following firm-level variables:

- *Exit indicator:* We pool RAIS data from 2003 through 2005 to create an exit indicator, which equals one if the firm operates in 2003 but is not found in the data in 2004 nor in 2005.
- *Firm-level employment:* the firm's number of employees, measured in December of each year. Let $\ell_{i,t}$ denote firm i's employment size in year t.
- *Average firm-level wage:* the firm's annual wage bill divided by number of employees, both measured in December of each year.
- *Firm-level Labor Turnover Rate:* for every firm i, we define

$$Turnover_i = \frac{|\ell_{i,2004} - \ell_{i,2003}|}{0.5 \times (\ell_{i,2004} + \ell_{i,2003})}.$$

SECEX (*Secretaria de Comércio Exterior*) is an administrative dataset from the federal government containing information on all export and import transactions. These transactions are

identified at the firm-level (through the 8 first digits of the CNPJ identifier) and can be merged to the firm-level RAIS data. This procedure allows us to compute exporter indicators for all C-sector firms. This dummy variable equals one if the firm reports any export transaction in 2003 and zero otherwise (i.e. the firm is found in RAIS but not in SECEX). Using RAIS and SECEX, we compute the following moments and auxiliary models.

Exit Rate (Formal Firms) – see Table A.3
Separately for C- and S-sector firms, we compute the mean of the exit dummy variable across all firms.

Exit Regressions (Formal Firms) – see Table A.3
We estimate the following regressions separately for C- and S-sector firms:

$$Exit_i = \alpha_k + \beta_k \log(\ell_i) + u_i$$

where i denotes a firm, $k = C, S$ denotes sector, u_i is the error term, and $Exit_i$ indicates whether firm i, active in 2003, exits the market in 2004.

Average Turnover (Formal Firms) – see Table A.3
We compute, separately for C- and S-sector firms, mean turnover rates across all firms.

Turnover Regressions – see Table A.3
We separately estimate the following regressions, conditional on C- and S-sector firms, respectively:

$$\begin{aligned} Turnover_i &= \alpha_C + \beta_C \log(\ell_{i,2003}) + \gamma_C Exporter_{i,2003} + u_i \\ Turnover_i &= \alpha_S + \beta_S \log(\ell_{i,2003}) + u_i \end{aligned}$$

where i denotes a firm, $Exporter_{i,2003}$ indicates if firm i exports in 2003, u_i is the error term and the remaining variables are defined as above. These regressions are also separately estimated conditional on expansions and contractions.

Log-Employment Serial Correlations (Formal Firms) – see Table A.3
We compute, separately for C- and S-sector firms, the serial correlations:

$$Corr(\log \ell_{i,2003}, \log \ell_{i,2004})_k \text{ for } k = C, S$$

Size Distribution of Formal Firms – see A.4
We compute, separately for C- and S-sector firms, the mean and standard deviation of log-employment across all firms, and the mean of log-employment across all C-sector exporters.

Fraction of Exporters – see Table A.5
We compute the share of all formal C-sector firms that export.

Log-Wages (Formal Firms) – see Table A.6
We compute, separately for C- and S-sector firms, the mean of log-wages across all formal firms.

Log-wage Regressions (Formal Firms) – see Table A.6
We estimate the following regressions, conditional on C- and S-sector firms, respectively (using data for 2003):

$$\begin{aligned} \log(w_i) &= \alpha_C + \beta_C \log(\ell_i) + \gamma_C Exporter_i + u_i \\ \log(w_i) &= \alpha_S + \beta_S \log(\ell_i) + u_i \end{aligned}$$

where i denote a firm, w_i is the (average) wage paid by firm i, u_i is the error term and the remaining variables are defined as above.

J.2 PIA, PAS and PAC (Firm-Level Surveys) and SECEX

Pesquisa Industrial Anual (PIA), *Pesquisa Anual de Comércio* (PAC), and *Pesquisa Anual de Serviços* (PAS) are firm-level surveys, covering the formal manufacturing, retail and service sectors, respectively. Conducted by the Brazilian Statistical Agency (IBGE), they contain detailed information on firms' inputs, output and revenues. They constitute a census for larger firms and a representative sample for smaller firms. In the manufacturing sector (PIA), all firms with at least 30 employees are part of the census and are surveyed every year, while firms with 5 to 29 employees are randomly sampled. The PAC (retail sector) and PAS (services) surveys have the same design, but have lower size thresholds for firms to be included in the census: firms with 20 employees or more are part of the census, while firms with up to 19 employees are randomly sampled. Finally, firms in PIA, PAS and PAC are also identified by their 8-digit CNPJ codes. Therefore, we are able to match SECEX with PIA to identify exporters. We use these datasets to obtain the following firm-level variables:

- *Annual gross revenues*
- *Export share:* for firm i, the share of revenues that comes from exports

$$Export\ Share_i = \frac{Value\ of\ Exports_i}{Revenues_i}$$

Using PIA, PAS, PAC and SECEX, we compute the following moments and auxiliary models.

Distribution of log-revenues – see Table A.7
We compute the mean and standard deviation of log-revenues across all firms in the C and S sectors.

Average Export Share
Average export share among all exporters, used to recover de value of d_F conditional on σ_C—see

Step 4 in section I.1 for details. We obtain that the average export share among exporters equals 0.264.

Fraction of Aggregate Revenues in the Formal C-Sector that is Exported – see Table A.5
Ratio between total exports and total revenues in the (formal) C sector.[34]

Serial Correlation of log-Revenues – see Table A.7
$Corr\left(\log Revenues_{i,2004}, \log Revenues_{i,2003}\right)$ separately for the C and S sectors. These moments are computed conditional on firms with at least 30 employees for PIA, and conditional on firms with at least 20 employees for PAS and PAC, so that they are part of the census and therefore surveyed in both years.

Log-Revenues Regressions – see Table A.7
We estimate the following regressions, conditional on C- and S sector firms (data from 2003):

$$\begin{aligned}\log\left(Revenues_i\right) &= \alpha_C + \beta_C \log\left(\ell_i\right) + \gamma_C Exporter_i + u_i \\ \log\left(Revenues_i\right) &= \alpha_S + \beta_S \log\left(\ell_i\right) + u_i\end{aligned}$$

where i denotes a firm, u_i is the error term and the remaining variables are defined as above.

J.3 ECINF (*Pesquisa de Economia Informal Urbana*)

ECINF was collected by IBGE in 1997 and 2003, and was designed to be representative of the universe of *urban* firms with up to five employees (both formal and informal). It is a matched employer-employee dataset that contains information on entrepreneurs, their businesses and employees. We use the same filters for industries we described above. Although a few firms in the dataset have more than five employees, we restrict attention to those with five employees or less so that our sample is consistent with the population the survey targets. We define as informal firms those that do not have a tax registration number, which means that they are not formally registered as a firm.

ECINF is comprised of two main files. The first contains information on businesses (these are small businesses, so there are no multi-establishment firms and we can use firm and establishment interchangeably) and the second contains information on workers. Before merging these data sources, we drop workers who are younger than 18 and older than 64 years old from the individual level data (only 890 observations are dropped). We then aggregate these data up to the firm level, providing us with information on firms' size and wage bill.[35] We merge this information with the first (firm-level) file using a unique firm identifier. Finally, we trim observations below the first

[34] The denominator comes from PIA's publication, Table 1.5 (pdf included in the replication folder). The two values used to compute the denominator correspond to the entries "Gross Revenues" and "Other Operational Revenues" of manufacturing firms (*Indústria da Transformação*).

[35] Thus, if a firm has employees older than 64 or younger than 18 years old they are not accounted for when we compute firm size.

percentile of the revenue distribution, which amounts to dropping firms with revenues very close to zero. We generate the following firm-level variables with ECINF:

- *Informality Indicator:* Dummy variable that equals one if the firm is not registered with the tax authorities.
- *Annual gross revenues*
- *Total number of employees*
- *Average wage*: firm's annual wage bill divided by number of people working at the firm. The wage bill includes the self-reported take-home earnings of the owner. For one-person firms, this is equal to the owner's take-home remuneration.

Using ECINF, we compute the following moments and auxiliary models.

Size Distribution (Informal Firms) – see Table A.8
We compute the following moments of firm-level log-employment separately for C- and S-sector informal firms: mean and standard deviation.

Distribution of Revenues (Informal Firms) – see Table A.8
We compute the mean of firms' log-revenues separately for C- and S-sector informal firms.

Log-Wages (Informal Firms) – see Table A.8
We compute the mean of firm-level log-wages separately for C- and S-sector firms.

Regression of Informal Status Indicator vs. Number of Employees – see Table A.8

$$Informal_i = \alpha_k + \beta_k \ell_i + u_i$$

where i denotes firms, $k = C, S$ denotes sector, and u_i is the error term.

J.4 PME

We use the *Pesquisa Mensal de Emprego* (PME) survey to obtain information on worker allocations and labor market flows. It is a rotating panel, in which individuals in a given household are interviewed for 4 consecutive months, they rest for 8 months and are then re-interviewed for additional 4 consecutive months, which implies a maximum panel length of 16 months. As in the firm-level data, we exclude those who are employed in the public sector, agriculture, mining, coal, oil and gas industries. As with ECINF, we only keep individuals who are between 18 and 64 years old. In addition, we exclude individuals who are out of the labor force, non-wage (unpaid) employees or employers. Finally, we restrict our attention to the years of 2003 and 2004. Thus, there are three possible states in our sample:

(i) Formal workers: those who have a formal labor contract, which in Brazil is defined by having a booklet (*carteira de trabalho*) that has been signed by her employer and that registers workers' entire employment history in the formal sector

(ii) Informal workers: those who do not have a signed booklet (without a formal contract), which includes self-employed workers

(iii) Unemployed: those who are not employed, but are actively searching for a job

We employ PME to generate the following moments:

Transition Matrix – see Table A.2
To obtain the annual transition matrix between states, we first estimate the 4-month transition matrix using information from the first and fourth interviews. Denote this 4-month transition matrix by M. We then estimate the annual transition matrix by computing M^3. This is preferable to using information from the first and fifth interviews—which are 12 months apart—given the high attrition rates between the fourth and fifth interviews, which are 8 months apart. This high attrition is common in panel surveys that have similar designs, as the survey unit is a particular address (e.g. an apartment) and individuals may move in and out during the 8-month rest period.

Workers' Allocations – see Table A.2
We use PME's sample weights to obtain the total number and the shares of individuals in each of the possible labor market statuses: (i) formal worker in the C sector; (ii) informal worker in the C sector; (iii) formal worker in the S sector; (iv) informal worker in the S sector; and (v) unemployment.

J.5 IBGE National Accounts

We employ information available from IBGE's 2000 and 2005 National Accounts to compute the share of final expenditures on sector C goods, ζ, sector k's fraction of intermediate expenditures on sector C goods, λ_k, and statistics relevant for the estimation of δ_k, which drives the importance of labor in sector k's production.

We compute ζ using final demand information, excluding Agriculture and Mining to be consistent with the filters we implemented in the datasets above. We obtain $\zeta = 0.296$, as is reported in Table 6.

To obtain information on δ_k (conditional on σ_k), we compute:

$$\frac{Total\ Expenditures\ with\ Intermediates_C}{Total\ Gross\ Revenues_C} = 0.596,$$
$$\frac{Total\ Expenditures\ with\ Intermediates_S}{Total\ Gross\ Revenues_S} = 0.320.$$

See Step 3 of section I.1 for details on how to use these statistics to obtain δ_C and δ_S.

Finally, we compute λ_k, for $k = C, S$, as:

$$\lambda_k = \frac{Total\ Expenditure\ with\ Sector\ C\ Intermediates}{Total\ Expenditure\ with\ Intermediates\ (across\ C\ and\ S)},$$

leading to $\lambda_C = 0.645$ and $\lambda_S = 0.291$, as is reported in Table 6.

K Model Fit: Moments Generated by the Model vs. Data

This section compares the moments generated by the model, using our estimates, with those computed from the data. Tables A.2 through A.8 shows that our model is able to replicate several salient features of the data.

Table A.2: Employment Shares and Transition Rates from Unemployment

Moment	Dataset	Model	Data
Share of Employment Ci	PME	0.067	0.059
Share of Employment Cf	PME	0.083	0.106
Share of Employment Si	PME	0.360	0.351
Share of Employment Sf	PME	0.315	0.334
Share Unemployment	PME	0.176	0.150
Share Informal Workers (Conditional on Working)	PME	0.518	0.482
Trans. Rate from Unemp. to Ci	PME	0.062	0.064
Trans. Rate from Unemp. to Cf	PME	0.051	0.050
Trans. Rate from Unemp. to Si	PME	0.383	0.389
Trans. Rate from Unemp. to Sf	PME	0.167	0.161
Trans. Rate from Unemp. to Unemp	PME	0.336	0.336
Ratio Trans. to Informal job / Trans. To Formal job	PME	2.042	2.146

Table A.3: Turnover-Related Moments and Auxiliary Models (Formal Sectors)

		C sector		S sector	
	Dataset	Model	Data	Model	Data
Exit Rate	RAIS	0.091	0.103	0.089	0.125
Average Firm-level Turnover	RAIS	0.231	0.505	0.198	0.525
$Corr(\log \ell_{t+1}, \log \ell_t)$	RAIS	0.947	0.929	0.942	0.914
$Exit_i = \alpha + \beta \log(\ell_i)$					
Intercept	RAIS	0.154	0.188	0.137	0.185
$log(\ell_i)$	RAIS	-0.028	-0.045	-0.040	-0.049
$Turnover_i = \alpha + \beta \log(\ell_i) + \gamma Exporter_i$					
Intercept	RAIS	0.435	0.741	0.315	0.645
$log(\ell_i)$	RAIS	-0.095	-0.126	-0.097	-0.096
$Exporter_i$	RAIS	0.071	0.071		
$Turnover_i = \alpha + \beta \log(\ell_i) + \gamma Exporter_i$, Conditional on Expansions					
Intercept	RAIS	0.410	0.692	0.278	0.690
$log(\ell_i)$	RAIS	-0.105	-0.138	-0.098	-0.150
$Exporter_i$	RAIS	0.119	0.116		
$Turnover_i = \alpha + \beta \log(\ell_i) + \gamma Exporter_i$, Conditional on Contractions					
Intercept	RAIS	0.456	0.744	0.335	0.624
$log(\ell_i)$	RAIS	-0.077	-0.101	-0.064	-0.064
$Exporter_i$	RAIS	0.056	0.056		

Table A.4: Firm-Size Distribution (Formal Sectors)

		C sector		S sector	
	Dataset	Model	Data	Model	Data
Avg. Firm-Level log-Employment	RAIS	2.249	1.918	1.213	1.237
Std Dev. Firm-Level log-Employment	RAIS	0.915	1.416	0.685	1.175
Avg. Exporter log-Employment	RAIS+SECEX	3.555	4.014		

Table A.5: Trade-Related Moments

	Dataset	Model	Data
Fraction of Exporters (among formal C firms)	RAIS + SECEX	0.129	0.073
Total Exports / (Total Formal Manufacturing Revenue)	SECEX + IBGE	0.133	0.134

Table A.6: Formal-Sector Wages

		C sector		S sector	
	Dataset	Model	Data	Model	Data
Avg. log-Wages	RAIS	8.635	8.769	8.413	8.567
$\log(w_i) = \alpha + \beta \log(\ell_i) + \gamma Exporter_i$					
Intercept	RAIS	8.301	8.509	8.288	8.436
$\log(\ell_i)$	RAIS	0.117	0.117	0.103	0.105
$Exporter_i$	RAIS	0.542	0.462		

Table A.7: Formal-Sector Revenues

		C sector		S sector	
	Dataset	Model	Data	Model	Data
Avg. log-Revenues	IBGE	12.652	12.726	10.898	10.814
Std. Dev. log-Revenues	IBGE	1.278	1.874	0.916	1.440
$Corr(\log Rev_t, \log Rev_{t+1})$	IBGE	0.727	0.929	0.630	0.845
$Rev_i = \alpha + \beta \log(\ell_i) + Exporter_i$					
Intercept	IBGE	9.995	10.118	9.500	10.004
$\log(\ell_i)$	IBGE	1.149	1.000	1.152	0.872
$Exporter_i$	IBGE	0.561	1.462		

Table A.8: Informal Sector Moments and Auxiliary Moments

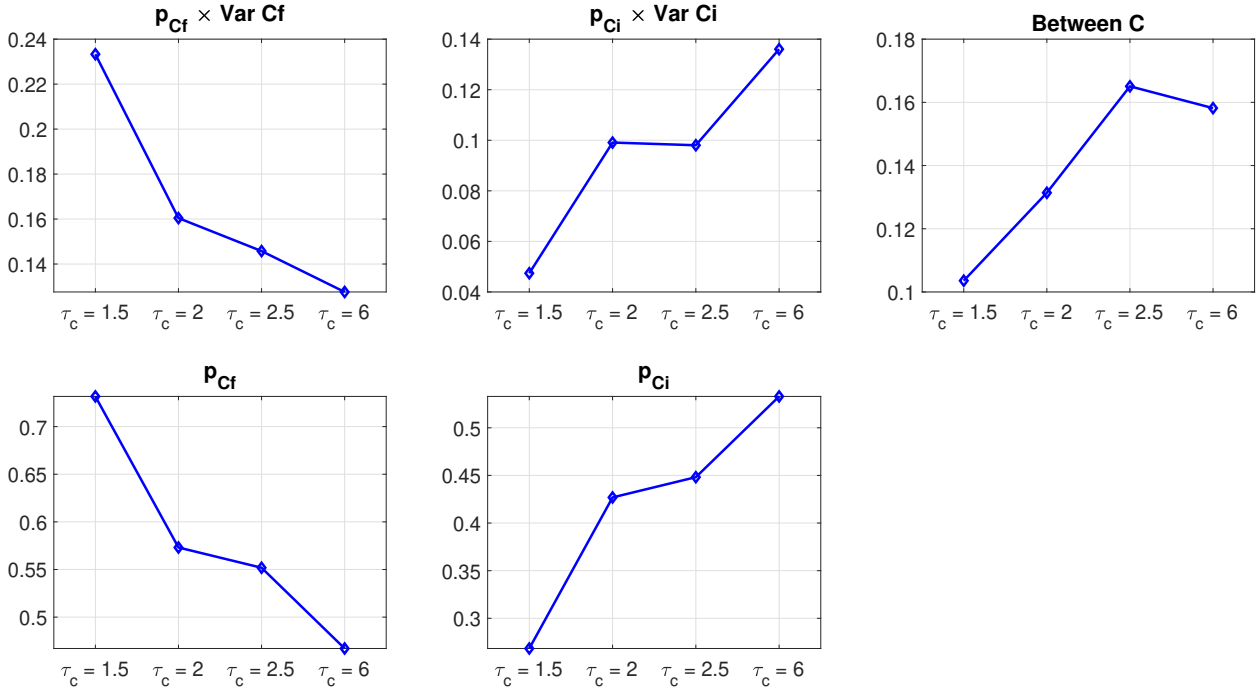
		C sector		S sector	
	Dataset	Model	Data	Model	Data
Average log-Employment	ECINF	0.189	0.105	0.244	0.097
Std. Dev. log-Employment	ECINF	0.316	0.303	0.355	0.274
Avg. log-Revenue	ECINF	9.596	8.531	9.253	8.953
Avg. log-Wages	ECINF	7.825	8.043	7.660	8.440
$Informal_i = \alpha + \beta \ell_i$					
Intercept	ECINF	1.308	1.135	1.212	1.130
ℓ_i	ECINF	-0.179	-0.179	-0.202	-0.204

Notes: All statistics are computed conditional on firms with five employees or less, both in the data and in the model.

L Trade and Wage Inequality: Additional Analyses

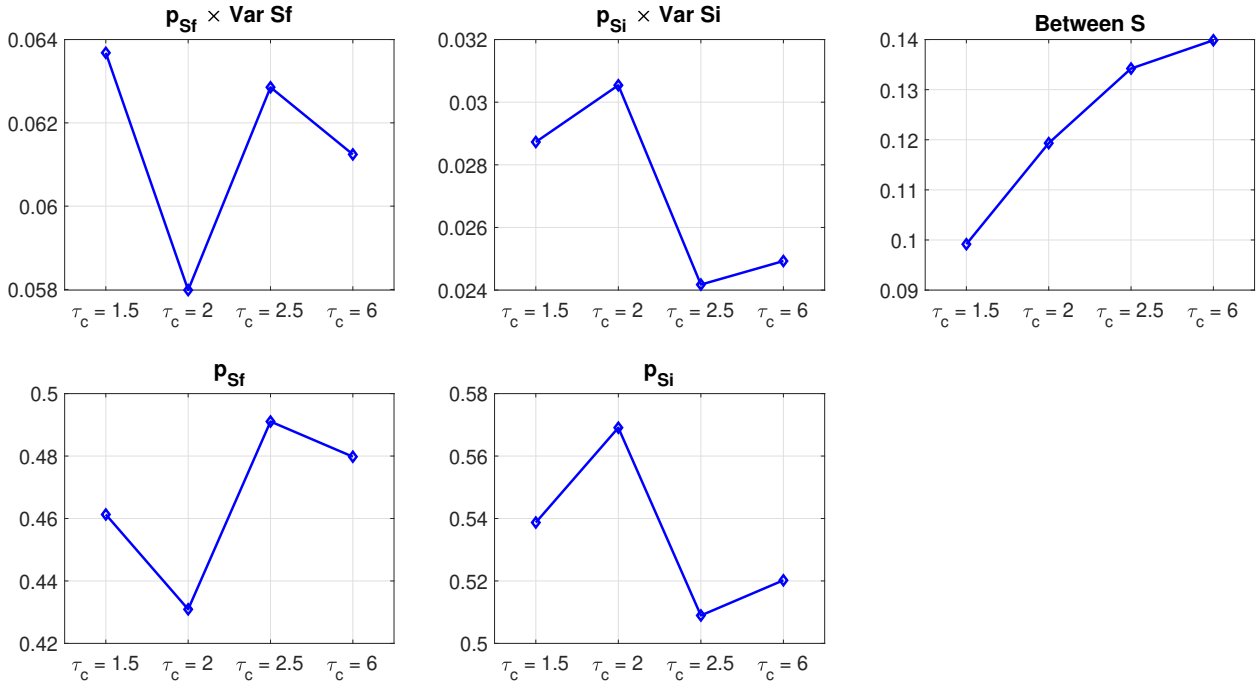
This section displays the response of the various terms of the variance decomposition (36) to trade costs, which are discussed in section 6.1.5.

Figure A.1: Trade and the Std. Dev. of log-Wages in the C Sector: Variance Decomposition



Notes: See variance decomposition (36). *Between* $C = \sum_{j \in \{f,i\}} p_{Cj} \left(E\left[\log w | Cj\right] - E\left[\log w | C\right]\right)^2$.

Figure A.2: Trade and the Std. Dev. of log-Wages in the S Sector: Variance Decomposition



Notes: See variance decomposition (36). *Between* $S = \sum_{j \in \{f,i\}} p_{Sj} \left(E\left[\log w | Sj\right] - E\left[\log w | S\right]\right)^2$.

Supplementary Material – Not for Publication

I Estimation Appendix

I.1 Estimation Algorithm

In this section we describe the estimation algorithm in detail, which we break down into several steps for expositional clarity.

Before we proceed, remember that value added for domestic producers in sector k is given by:

$$VA_k(z,\ell) = \Theta_k \left(P_k^m\right)^{-(1-\delta_k)\Lambda_k} \left(\exp\left(d_{H,k}\right)\right)^{\frac{\sigma_k}{\sigma_k-1}\Lambda_k} \left(z\ell^{\delta_k}\right)^{\Lambda_k},$$

where

$$P_k^m \equiv \frac{P_C^{\lambda_k} P_S^{1-\lambda_k}}{\lambda_k^{\lambda_k}\left(1-\lambda_k\right)^{1-\lambda_k}}, \tag{S.1}$$

$$\Theta_k \equiv \left(\frac{1}{\left(1-\delta_k\right)\Lambda_k}\right)\left(\frac{\left(1-\delta_k\right)\left(\sigma_k-1\right)}{\sigma_k}\right)^{\frac{\sigma_k}{\sigma_k-1}\Lambda_k},$$

and

$$\Lambda_k \equiv \frac{\sigma_k - 1}{\sigma_k - \left(1-\delta_k\right)\left(\sigma_k-1\right)}.$$

Rewrite value added for domestic producers as

$$VA_k(z,\ell) = \Theta_k \Psi_k \left(z\ell^{\delta_k}\right)^{\Lambda_k},$$

with

$$\Psi_k \equiv \left(P_k^m\right)^{-(1-\delta_k)\Lambda_k} \left(\exp\left(d_{H,k}\right)\right)^{\frac{\sigma_k}{\sigma_k-1}\Lambda_k}. \tag{S.2}$$

Note that Θ_k is a solely a function of model's parameters. On the other hand, Ψ_k is a function of model's parameters but also of equilibrium objects such as P_C, P_S and $d_{H,k}$. In turn, value added for exporters is given by:

$$VA_C(z,\ell) = \Theta_C \Psi_C \left(\exp\left(d_F\right)\right)^{\frac{\sigma_C}{\sigma_C-1}\Lambda_C} \left(z\ell^{\delta_C}\right)^{\Lambda_C}.$$

It will be convenient to define and work with

$$\vartheta_{J_u} \equiv b + \frac{1}{1+r} J^u.$$

Ψ_C, Ψ_S, ϑ_{J_u} are treated as parameters to be estimated along with the remaining ones, but these are all endogenous variables. The procedure below makes sure that the values guessed for Ψ_C and Ψ_S are equilibrium outcomes (see Step 9 for details). The number of entrants M_C and M_S will be set to match Ψ_C and Ψ_S. Given knowledge of ϑ_{J_u} and the remaining parameters, we can recover the flow utility of unemployment b and the value of unemployment J^u post-estimation.

Step 1a: λ_C and λ_S are obtained from input-output tables and fixed throughout.

Step 1b: Fix μ^{υ} and obtain ϕ using equation (23):

$$\phi = \left(\frac{\mu^{\upsilon}}{\left(Transition_{Data}^{U \to E}\right)^{\frac{\xi-1}{\xi}}} \right)^{\xi}$$

where $Transition_{Data}^{U \to E}$ is the transition rate from unemployment to employment in the data.

Step 2: Start with a parameter vector guess Ω, including values for Ψ_C, Ψ_S and ϑ_{J_u}.

Step 3: Obtain δ_k using $P_k^m \iota_k(z,\ell) = \frac{(1-\delta_k)(\sigma_k-1)}{\sigma_k} R_k(z,\ell)$:

$$\left(\frac{Total\ Expenditures\ with\ Intermediates_k}{Total\ Gross\ Revenues_k} \right)_{Data} = \frac{(1-\delta_k)(\sigma_k-1)}{\sigma_k}$$

$$\Rightarrow \delta_k = 1 - \frac{\sigma_k}{\sigma_k - 1} \left(\frac{Total\ Expenditures\ with\ Intermediates_k}{Total\ Gross\ Revenues_k} \right)_{Data}$$

$\left(\frac{Total\ Expenditures\ with\ Intermediates_k}{Total\ Gross\ Revenues_k} \right)_{Data}$ is obtained from input-output tables.

Step 4: Obtain d_F using equation (32):

$$E\left[Export\ Share | Exporter = 1\right]_{Data} = \left(1 - \exp\left(-\sigma_C \times d_F\right)\right)$$

$$\Rightarrow d_F = -\frac{1}{\sigma_C} \log\left(1 - E\left[Export\ Share | Exporter = 1\right]_{Data}\right)$$

$E\left[Export\ Share | Exporter = 1\right]_{Data}$ is the average share of exporters' gross revenues in sector C coming from exports, obtained from PIA and SECEX.

Step 5: This step solves for wage schedules $w_{kf}(z,\ell')$, $w_{ki}(z,\ell')$ as well as value functions $V_{kf}(z,\ell)$, $V_{ki}(z,\ell)$, $J_{kf}^e(z,\ell')$, $J_{ki}^e(z,\ell')$, and firms' policy functions.

Step 5a: Compute value added functions $VA_k(z,\ell)$.

Step 5b: Compute wage schedules $w_{kf}(z,\ell')$

- Guess a wage schedule $w_{kf}(z,\ell')$
- Compute the resulting $V_{kf}(z,\ell')$ using (13)
- Compute $J_{kf}^e(z,\ell')$ using (A.46)
- Compute $w_{kf}^u(z,\ell')$ using equation (27)
- Let $\widehat{w}_{kf}^u(z,\ell') = \omega_0 + \omega_1 \frac{VA_k(z,\ell')}{\ell'}$ be the linear projection of $w_{kf}^u(z,\ell')$ on $\left[1, \frac{VA_k(z,\ell')}{\ell'}\right]$

- Update $w_{kf}(z,\ell') = \max\left\{\widehat{w}^u_{kf}(z,\ell'), b_u + \vartheta_{J_u} - \frac{1}{1+r}J^e_{kf}(z,\ell'), \underline{w}\right\}$
- Restart until convergence

Step 5c: Compute wage schedules $w_{ki}(z,\ell')$

- Guess a wage schedule $w_{ki}(z,\ell')$
- Compute the resulting $V_{ki}(z,\ell')$ using (17)
- Compute $J^e_{ki}(z,\ell')$ using (A.47)
- Compute $w^u_{ki}(z,\ell')$ using equation (30)
- Let $\widehat{w}^u_{ki}(z,\ell') = \omega_0 + \omega_1\left(1 - \frac{\sigma_k}{\sigma_k-(1-\delta_k)(\sigma_k-1)}p_{ki}(\ell')\right)\frac{VA_k(z,\ell')}{\ell'}$ be the linear projection of $w^u_{ki}(z,\ell')$ on $\left[1, \left(1 - \frac{\sigma_k}{\sigma_k-(1-\delta_k)(\sigma_k-1)}p_{ki}(\ell')\right)\frac{VA_k(z,\ell')}{\ell'}\right]$
- Update $w_{ki}(z,\ell') = \max\left\{\widehat{w}^u_{ki}(z,\ell'), \vartheta_{J_u} - \frac{1}{1+r}J^e_{ki}(z,\ell')\right\}$
- Restart until convergence

Step 6: Solve for firms' entry decisions. Compute the fraction of entrants in the formal and informal sectors as follows:

$$\omega_{kf} \equiv \Pr\left(I^{formal}_k(z) = 1\right) = \int_z I^{formal}_k(z)\, g^e_k(z)\, dz$$

$$\omega_{ki} \equiv \Pr\left(I^{informal}_k(z) = 1\right) = \int_z I^{informal}_k(z)\, g^e_k(z)\, dz$$

Therefore, if M_k is the mass of entrants in sector k, the masses of formal and informal entrants in sector k are given by:

$$M_{ki} = \omega_{ki} M_k$$

$$M_{kf} = \omega_{kf} M_k$$

Finally, compute the distribution of z productivities among entrants, conditional on entry into sector kj.

$$\psi^e_{ki}(z) = \frac{g^e_k(z)\, I^{informal}_k(z)}{\int_{\widetilde{z}} g^e_k(\widetilde{z})\, I^{informal}_k(\widetilde{z})\, d\widetilde{z}},$$

$$\psi^e_{kf}(z) = \frac{g^e_k(z)\, I^{formal}_k(z)}{\int_{\widetilde{z}} g^e_k(\widetilde{z})\, I^{formal}_k(\widetilde{z})\, d\widetilde{z}}.$$

Step 7: Compute the steady-state distribution of states. For informal firms, start with a guess for ψ_{ki}. Then, compute

$$\varrho^{exit}_{ki} = \alpha_k + (1-\alpha_k)\int_z\int_\ell \left(I^{exit}_{ki}(z,\ell) + I^{change}_{ki}(z,\ell)\right)\psi_{ki}(z,\ell)\, d\ell dz.$$

In steady state $N_{ki} = \left(1 - \varrho_{ki}^{exit}\right) N_{ki} + M_{ki}$. Therefore, set $\frac{M_{ki}}{N_{ki}}$, the fraction of sector k informal firms that are entrants, to:

$$\boxed{\frac{M_{ki}}{N_{ki}} = \varrho_{ki}^{exit} = \frac{\omega_{ki} M_k}{N_{ki}}.}$$

Now, compute $\widetilde{\psi}_{ki}$:

$$\begin{aligned}\widetilde{\psi}_{ki}(z,\ell) = &\mathcal{I}[\ell = 1] \times \varrho_{ki}^{exit} \times \psi_{ki}^{e}(z) \\ &+ \mathcal{I}[\ell \geq 1] \times (1 - \alpha_k) \times \psi_{ki}(z,\ell)\, I_{ki}^{stay}(z,\ell),\end{aligned}$$

and $\widehat{\psi}_{ki}$:

$$\widehat{\psi}_{ki}\left(z,\ell'\right) = \int_{\ell} \widetilde{\psi}_{ki}(z,\ell)\, \mathcal{I}\left(L_{ki}(z,\ell) = \ell'\right) d\ell$$

Update ψ_{ki} with:

$$\psi_{ki}\left(z',\ell'\right) = \int_{z} \widehat{\psi}_{ki}\left(z,\ell'\right) g_k\left(z'|z\right) dz,$$

and repeat until convergence of ψ_{ki}. This converged value of ψ_{ki} will be used directly in the computation of ψ_{kf} below.

For formal firms, start with guess for ψ_{kf} and compute:

$$\begin{aligned}\varrho_{kf}^{exit} &= \alpha_k + (1 - \alpha_k) \int_z \int_\ell I_{kf}^{exit}(z,\ell)\, \psi_{kf}(z,\ell)\, d\ell dz, \\ \varrho_{ki}^{change} &= (1 - \alpha_k) \int_z \int_\ell I_{ki}^{change}(z,\ell)\, \psi_{ki}(z,\ell)\, d\ell dz.\end{aligned}$$

In steady state:

$$\begin{aligned}\varrho_{kf}^{exit} N_{kf} &= \varrho_{ki}^{change} \underbrace{N_{ki}}_{\frac{\omega_{ki} M_k}{\varrho_{ki}^{exit}}} + \omega_{kf} M_k \\ &= M_k \left(\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf} \right)\end{aligned}$$

So that:

$$\boxed{\frac{M_{kf}}{N_{kf}} = \frac{M_k \omega_{kf}}{N_{kf}} = \frac{\varrho_{kf}^{exit} \omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf}}}$$

Also, note that

$$\frac{M_{kf}}{N_{kf}} \times \frac{N_{ki}}{M_{ki}} = \frac{\varrho_{kf}^{exit} \omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf}} \frac{1}{\varrho_{ki}^{exit}} = \frac{\varrho_{kf}^{exit} \omega_{kf}}{\varrho_{ki}^{change} \omega_{ki} + \varrho_{ki}^{exit} \omega_{kf}}$$

and

$$\frac{M_{kf}}{N_{kf}} \times \frac{N_{ki}}{M_{ki}} = \frac{\omega_{kf}}{\omega_{ki}} \frac{N_{ki}}{N_{kf}}$$

Therefore,

$$\boxed{\frac{N_{ki}}{N_{kf}} = \frac{\varrho_{kf}^{exit}\omega_{ki}}{\varrho_{ki}^{change}\omega_{ki} + \varrho_{ki}^{exit}\omega_{kf}}}$$

Compute $\widetilde{\psi}_{kf}$ as:

$$\widetilde{\psi}_{kf}(z,\ell) = \mathcal{I}[\ell = 1] \times \underbrace{\frac{\varrho_{kf}^{exit}\omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}}\omega_{ki} + \omega_{kf}}}_{\frac{M_{kf}}{N_{kf}}} \psi_{kf}^{e}(z) + \mathcal{I}[\ell \geq 1] \times \left(\begin{array}{c} (1-\alpha_k)\,\psi_{kf}(z,\ell)\, I_{kf}^{stay}(z,\ell) \\ + (1-\alpha_k) \underbrace{\frac{\varrho_{kf}^{exit}\omega_{ki}}{\varrho_{ki}^{change}\omega_{ki} + \varrho_{ki}^{exit}\omega_{kf}}}_{\frac{N_{ki}}{N_{kf}}} \psi_{ki}(z,\ell)\, I_{ki}^{change}(z,\ell) \end{array} \right)$$

and $\widehat{\psi}_{kf}$ as:

$$\widehat{\psi}_{kf}(z,\ell') = \int_{\ell} \widetilde{\psi}_{kf}(z,\ell)\, \mathcal{I}\left(L_{kf}(z,\ell) = \ell'\right) d\ell.$$

Update ψ_{kf} with:

$$\psi_{kf}(z',\ell') = \int_{z} \widehat{\psi}_{kf}(z,\ell')\, g_k(z'|z)\, dz,$$

and repeat until convergence of ψ_{kf}.

At this point we have the following objects: ψ_{kj}, $\widetilde{\psi}_{kj}$, $\widehat{\psi}_{kj}$, ϱ_{ki}^{exit}, ϱ_{ki}^{change}, ϱ_{kf}^{exit}, $\chi_{ki\to f}^{change}$, χ_{kf}^{layoff}, and χ_{ki}^{leave} (see equations (A.21), (A.25) and (A.27)).

Step 8: Obtain the entry costs $c_{e,k}$ $(k = C, S)$:

$$c_{e,k} = V_k^e = \int_z \left[V_{ki}^e(z)\, I_k^{informal}(z) + V_{kf}^e(z)\, I_k^{formal}(z) \right] g_k^e(z)\, dz$$

These costs will be subtracted from aggregate income, and will be added to the expenditure on S-sector goods.

Step 9: This step solves for masses of entrants M_k's, masses of firms N_{kj}'s, aggregate vacancies V_{kj}'s and mass of unemployment L_u consistent with Ψ_C, Ψ_S, d_F and μ^v.

Step 9a: Write aggregate income I as a function of masses of entrants M_C and M_S.

Step 9b: Write P_C and P_S as functions of M_C and M_S.

Step 9c: Write X_C^{int} as a function of M_C and M_S.

Step 9d: Solve for $\frac{M_S}{M_C}$ that matches Ψ_C.

Step 9e: Separately pin down M_C and M_S using the labor market clearing equation $\overline{L} - L_u = \sum_{k=C,S,j=i,f} L_{kj}$. Express M_C and M_S as functions of L_u.

Step 9fe: Express masses of firms N_{kj} as functions of L_u.

Step 9g: Express aggregate posted vacancies V_{kj} as functions of L_u.

Step 9h: Use equation for μ^{υ} (and the value initially guessed in Step 1 for μ^{υ}) to obtain L_u consistent with Ψ_C, Ψ_S, d_F and μ^{υ}.

Step 9i: Go back and obtain masses of entrants M_k's, masses of firms N_{kj}'s, and aggregate vacancies V_{kj}'s.

Step 9j: Recover price indices P_C and P_S.

Step 9k: Compute deviation between government revenues and spending with unemployment insurance Dev_T.

Step 10: Obtain job finding rates μ^e_{kj} using aggregate vacancies V_{kj}'s and mass of unemployment L_u obtained in Step 9.

$$\mu^e_{kj} = \frac{m_{kj}}{L_u} = \phi \frac{V_{kj}}{\widetilde{V}} \left(\frac{\widetilde{V}}{L_u} \right)^{\xi}$$

Step 11: Use equations (A.28)-(A.29) to obtain allocations L_{Cf}, L_{Ci}, L_{Sf}, L_{Si}.

$$L_{Ci} = \frac{\mu^e_{Ci} L_u}{\chi^{leave}_{Ci}}$$

$$L_{Si} = \frac{\mu^e_{Si} L_u}{\chi^{leave}_{Si}}$$

$$L_{Cf} = \frac{\mu^e_{Cf} L_u + \chi^{change}_{Ci \to f} L_{Ci}}{\chi^{layoff}_{Cf}}$$

$$L_{Sf} = \frac{\mu^e_{Sf} L_u + \chi^{change}_{Si \to f} L_{Si}}{\chi^{layoff}_{Sf}}$$

Step 12: Compute deviation from the labor market clearing equation:

$$Dev_L = abs \left(\frac{\overline{L} - (L_{Cf} + L_{Ci} + L_{Sf} + L_{Si})}{\overline{L}} \right) ,$$

Step 13: Compute all moments to be matched with those in the data.

Step 14: Compute Loss Function. Add Model/Data deviations to equilibrium penalty $EQ_Penalty$. The objective function is therefore given by

$$L = L_{mom} + EQ_Penalty$$

Where L_{mom} penalizes deviations between moments in the data and $EQ_Penalty$ penalizes deviations from the labor market clearing condition:

$$EQ_Penalty = W_L Dev_L + W_T abs \left(\min \left\{ Dev_T, 0 \right\} \right)$$

With W_L and W_T denoting large weights and Dev_T is the relative deviation between government revenues and spending with unemployment insurance (see section I.2 for details). We highly penalize a negative Dev_T.

Step 15: Optimization routine picks new parameter vector Ω. Go back to Step 1 until convergence.

Step 16 (Post estimation): Obtain J^u using

$$J^u = \sum_{k,j} \mu^e_{kj} \int_\ell \int_z \overline{J}^e_{kj}\left(z, L_{kj}\left(z,\ell\right)\right) g_{kj}\left(z,\ell\right) dz d\ell + \left(1 - \sum_{k,j} \mu^e_{kj}\right) \vartheta_{J_u}.$$

Step 17 (Post estimation): At this point, we know J^u and can compute

$$b = \vartheta_{J_u} - \frac{1}{1+r} J^u,$$

Step 18 (Post-estimation): Obtain D^*_F (this is the parameter that we need for the counterfactuals as d_F is endogenous):

$$D^*_F = \frac{\left(\exp\left(\sigma_C \times d_F\right) - 1\right) \left(P^m_C\right)^{(1-\delta_C)(\sigma_C - 1)} \Psi_C^{\frac{\sigma_C - 1}{\Lambda_C}}}{\overline{\epsilon}^{\sigma_C} \tau_c^{1-\sigma_C}},$$

where $\overline{\epsilon}$ is the exchange rate value that balances trade:

$$\overline{\epsilon} = \frac{1}{\tau_a \tau_c} \left(P^m_C\right)^{(1-\delta_C)} \Psi_C^{\frac{1}{\Lambda_C}} \left(\tau_a Exports\right)^{\frac{1}{1-\sigma_C}}.$$

I.2 Estimation Algorithm – Further Details

This section details the steps within Step 9 of the estimation procedure.

Step 9: This step solves for masses of entrants M_k's, masses of firms N_{kj}'s, aggregate vacancies V_{kj}'s consistent with Ψ_C, Ψ_S and d_F.

We start with some definitions... Averages "per firm". All these quantities can be computed after Step 8, that is, after solving for the steady state distribution of states.

$$Avg_wbill_{ki} = \int_z \int_{\ell'} \left[w_{ki}\left(z,\ell'\right)\ell'\right] \widehat{\psi}_{ki}\left(z,\ell'\right) d\ell' dz \text{ for } k = C, S$$

$$Avg_wbill_{kf} = \int_z \int_{\ell'} \left[\max\left\{w_{kf}\left(z,\ell'\right), \underline{w}\right\}\ell'\right] \widehat{\psi}_{kf}\left(z,\ell'\right) d\ell' dz \text{ for } k = C, S$$

$$Avg_Firing_Costs_{kf} = \kappa \int_z \int_\ell \left[(\ell - L_{kf}(z,\ell)) \left(1 - I_{kf}^{hire}(z,\ell)\right)\right] \widetilde{\psi}_{kf}(z,\ell)\, d\ell dz \text{ for } k = C,S$$

$$Avg_Hiring_Costs_{kj} = \int_z \int_\ell \left[H_{kj}(\ell, L_{kj}(z,\ell))\, I_{kj}^{hire}(z,\ell)\right] \widetilde{\psi}_{kj}(z,\ell)\, d\ell dz \text{ for } k = C,S;\ j = i,f$$

$$Avg_Revenue_{kj} = \int_z \int_{\ell'} R_k(z,\ell')\, \widehat{\psi}_{kj}(z,\ell')\, d\ell' dz \text{ for } k = C,S;\ j = i,f$$

$$Avg_InfPenalty_{ki} = \int_z \int_{\ell'} \left[p_{ki}(\ell')\, R_k(z,\ell')\right] \widehat{\psi}_{ki}(z,\ell')\, d\ell' dz \text{ for } k = C,S$$

$$Avg_Vacancies_{kj} = \int_z \int_\ell v_{kj}(z,\ell)\, \widetilde{\psi}_{kj}(z,\ell)\, d\ell dz \text{ for } k = C,S;\ j = i,f$$

$$Avg_Exports_{Cf} = (1 - \exp(-\sigma_C \times d_F)) \int_z \int_{\ell'} \left[R_C(z,\ell')\, I_C^x(z,\ell')\right] \widehat{\psi}_{kf}(z,\ell')\, d\ell' dz$$

$$Fraction_Export_{Cf} = \int_z \int_{\ell'} I_C^x(z,\ell')\, \widehat{\psi}_{Cf}(z,\ell')\, d\ell' dz$$

$$Avg_size_{kj} = \int_z \int_\ell \ell \psi_{kj}(z,\ell)\, d\ell dz \text{ for } k = C,S;\ j = i,f$$

Now, define

$$\begin{aligned} Avg_Price_{kj} &= \int_z \int_{\ell'} p_{kj}(z,\ell')^{1-\sigma_k}\, \widehat{\psi}_{kj}(z,\ell')\, d\ell' dz \\ &= \int_z \int_{\ell'} \left(\frac{R_k(z,\ell')}{q_k(z,\ell',\iota_k(z,\ell'))} \right)^{1-\sigma_k} \widehat{\psi}_{kj}(z,\ell')\, d\ell' dz \text{ for } k = C,S;\ j = i,f. \end{aligned}$$

We cannot compute Avg_Price_{kj}—given Ω, Ψ_C and Ψ_S. However, note that:

$$\begin{aligned} Avg_Price_{kj} = &\left(\frac{(1-\delta_k)(\sigma_k - 1)}{\sigma_k} \right)^{(1-\delta_k)\Lambda_k} (P_k^m)^{(1-\sigma_k)(1-\delta_k)}\, \Psi_k^{(1-\sigma_k)\delta_k} \times \\ &\int_z \int_{\ell'} \left(z\left(\ell'\right)^{\delta_k} \right)^{\Lambda_k} \widehat{\psi}_{kj}(z,\ell')\, d\ell' dz, \end{aligned}$$

$$\begin{aligned} Avg_Price_{Cf} = &\left(\frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C} \right)^{(1-\delta_C)\Lambda_C} (P_C^m)^{(1-\sigma_C)(1-\delta_C)}\, \Psi_C^{(1-\sigma_C)\delta_C} \times \\ &\int_z \int_{\ell'} \left(z\left(\ell'\right)^{\delta_C} \right)^{\Lambda_C} \left(\exp\left(d_F \times I_C^x(z,\ell') \right) \right)^{-\delta_C \sigma_C \Lambda_C} \widehat{\psi}_{Cf}(z,\ell')\, d\ell' dz. \end{aligned}$$

So, given Ω, Ψ_C and Ψ_S we can compute:

$$\begin{aligned} \widetilde{Avg_Price}_{kj} &\equiv \left(\frac{(1-\delta_k)(\sigma_k - 1)}{\sigma_k} \right)^{(1-\delta_k)\Lambda_k} \Psi_k^{(1-\sigma_k)\delta_k} \int_z \int_{\ell'} \left(z\left(\ell'\right)^{\delta_k} \right)^{\Lambda_k} \widehat{\psi}_{kj}(z,\ell')\, d\ell' dz \\ &= (P_k^m)^{(\sigma_k - 1)(1-\delta_k)}\, Avg_Price_{kj}, \end{aligned}$$

$$\widetilde{Avg_Price}_{Cf} \equiv \left(\frac{(1-\delta_C)(\sigma_C-1)}{\sigma_C}\right)^{(1-\delta_C)\Lambda_C} \Psi_C^{(1-\sigma_C)\delta_C} \times$$
$$\int_z \int_{\ell'} \left(z\left(\ell'\right)^{\delta_C}\right)^{\Lambda_C} \left(\exp\left(d_F \times I_C^x\left(z,\ell'\right)\right)\right)^{-\delta_C\sigma_C\Lambda_C} \widehat{\psi}_{Cf}\left(z,\ell'\right) d\ell' dz$$
$$= \left(P_C^m\right)^{(\sigma_C-1)(1-\delta_C)} Avg_Price_{Cf}.$$

At this point, we can compute the following variables, as functions of M_C and M_S

$$N_{Ci} = \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C \tag{S.3}$$

$$N_{Si} = \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S \tag{S.4}$$

$$N_{Cf} = \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \tag{S.5}$$

$$N_{Sf} = \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \tag{S.6}$$

$$M_{Ci} = \omega_{Ci} M_C$$
$$M_{Si} = \omega_{Si} M_S$$
$$M_{Cf} = \omega_{Cf} M_C$$
$$M_{Sf} = \omega_{Sf} M_S$$

Firm-level expenditures with sector S goods (fixed operating costs, hiring costs, entry costs, fixed export costs)

$$\begin{aligned}
E_S = & \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \left(Avg_Hiring_Costs_{Cf} + \overline{c}_{Cf}\right) \\
& + \frac{\omega_{Ci} M_C}{\varrho_{Ci}^{exit}} \left(Avg_Hiring_Costs_{Ci} + \overline{c}_{Ci}\right) \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \left(Avg_Hiring_Costs_{Sf} + \overline{c}_{Sf}\right) \\
& + \frac{\omega_{Si} M_S}{\varrho_{Si}^{exit}} \left(Avg_Hiring_Costs_{Si} + \overline{c}_{Si}\right) \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C Fraction_Export_{Cf} f_x \\
& + M_C c_{e,C} \\
& + M_S c_{e,S}
\end{aligned}$$

Define c_C:

$$\begin{aligned} c_C \equiv \frac{E_{S,C}}{M_C} &= \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}}\left(Avg_Hiring_Costs_{Cf} + \overline{c}_{Cf}\right) \\ &+ \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}}\left(Avg_Hiring_Costs_{Ci} + \overline{c}_{Ci}\right) \\ &+ \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Fraction_Export_{Cf} f_x \\ &+ c_{e,C}, \end{aligned} \tag{S.7}$$

Where $E_{S,C}$ is firm-level expenditures with sector S goods (fixed costs, etc) coming from C-sector activity.

Define c_S:

$$\begin{aligned} c_S \equiv \frac{E_{S,S}}{M_S} &= \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}}\left(Avg_Hiring_Costs_{Sf} + \overline{c}_{Sf}\right) \\ &+ \frac{\omega_{Si}}{\varrho_{Si}^{exit}}\left(Avg_Hiring_Costs_{Si} + \overline{c}_{Si}\right) \\ &+ c_{e,S}, \end{aligned} \tag{S.8}$$

Where $E_{S,S}$ is firm-level expenditures with sector S goods (fixed costs, etc) coming from S-sector activity.

We can therefore write:

$$\begin{aligned} E_S &= E_{S,C} + E_{S,S} \\ &= c_C M_C + c_S M_S \end{aligned}$$

Market Clearing (C and S sectors)

Let I denote aggregate income. Then, market clearing in the C and S sectors must lead to:

$$\zeta I + X_C^{int} = Rev_C - Exports + \tau_a Imports$$

$$\left(1-\zeta\right) I + X_S^{int} + E_S = Rev_S$$

$$Imports = Exports$$

Note that expenditures on intermediates are proportional to gross revenues:

$$P_k^m \iota_k\left(z,\ell\right) = \frac{\left(1-\delta_k\right)\left(\sigma_k - 1\right)}{\sigma_k} R_k\left(z,\ell\right),$$

which leads to:

$$\begin{aligned} X_C^{int} &= \lambda_C \frac{(1-\delta_C)(\sigma_C-1)}{\sigma_C} Rev_C \\ &+ \lambda_S \frac{(1-\delta_S)(\sigma_S-1)}{\sigma_S} Rev_S \end{aligned}$$

$$\begin{aligned} X_S^{int} &= (1-\lambda_C) \frac{(1-\delta_C)(\sigma_C-1)}{\sigma_C} Rev_C \\ &+ (1-\lambda_S) \frac{(1-\delta_S)(\sigma_S-1)}{\sigma_S} Rev_S \end{aligned}$$

Where Rev_C and Rev_S are total gross revenues in sectors C and S respectively. Therefore:

$$\begin{aligned} I = &\left(1 - \frac{(1-\delta_C)(\sigma_C-1)}{\sigma_C}\right) Rev_C \\ &+ \left(1 - \frac{(1-\delta_S)(\sigma_S-1)}{\sigma_S}\right) Rev_S \\ &- E_S \\ &+ (\tau_a - 1)\, Exports \end{aligned}$$

Using

$$\begin{aligned} Rev_C &= Avg_Revenue_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C + Avg_Revenue_{Ci} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C \\ Rev_S &= Avg_Revenue_{Sf} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S + Avg_Revenue_{Si} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S \\ Exports &= Avg_Exports_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \\ E_S &= c_C M_C + c_S M_S \end{aligned}$$

Step 9a: Write aggregate income I as a function of masses of entrants M_C and M_S.

$$
\begin{aligned}
I = & \left(1 - \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C}\right) \left(\begin{array}{c} Avg_Revenue_{Cf} \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C \\ + Avg_Revenue_{Ci} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C \end{array} \right) \\
& + \left(1 - \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S}\right) \left(\begin{array}{c} Avg_Revenue_{Sf} \frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} M_S \\ + Avg_Revenue_{Si} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S \end{array} \right) \\
& - (c_C M_C + c_S M_S) \\
& + (\tau_a - 1) \left(Avg_Exports_{Cf} \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C \right)
\end{aligned}
$$

Therefore:

$$
I = a_C M_C + a_S M_S \tag{S.9}
$$

Where

$$
\begin{aligned}
a_C = & \left(1 - \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C}\right) \left(\begin{array}{c} Avg_Revenue_{Cf} \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} \\ + Avg_Revenue_{Ci} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \end{array} \right) \\
& + (\tau_a - 1) \left(Avg_Exports_{Cf} \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} \right) \\
& - c_C
\end{aligned}
$$

$$
\begin{aligned}
a_S = & \left(1 - \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S}\right) \left(\begin{array}{c} Avg_Revenue_{Sf} \frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} \\ + Avg_Revenue_{Si} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \end{array} \right) \\
& - c_S
\end{aligned}
$$

Step 9b: Write P_C and P_S as functions of M_C and M_S.

Price Index Sector C

$$
P_C^{1-\sigma_C} = P_{H,C}^{1-\sigma_C} + P_{F,C}^{1-\sigma_C}
$$

The domestic component is given by:

$$\begin{aligned}
P_{H,C}^{1-\sigma_C} &= N_{Cf} Avg_Price_{Cf} + N_{Ci} Avg_Price_{Ci} \\
&= \left(\begin{array}{c} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg_Price_{Cf} \\ + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg_Price_{Ci} \end{array} \right) M_C \\
&= \left(\begin{array}{c} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \widetilde{Avg_Price}_{Cf} \left(P_C^m\right)^{-(\sigma_C - 1)(1-\delta_C)} \\ + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \widetilde{Avg_Price}_{Ci} \left(P_C^m\right)^{-(\sigma_C - 1)(1-\delta_C)} \end{array} \right) M_C
\end{aligned}$$

We can therefore write $P_{C,H}$ as:

$$P_{H,C}^{1-\sigma_C} = b_C^1 \left(P_C^m\right)^{(1-\sigma_C)(1-\delta_C)} M_C,$$

Where

$$b_C^1 \equiv \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \widetilde{Avg_Price}_{Cf} + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \widetilde{Avg_Price}_{Ci}$$

The foreign component is given by:

$$P_{F,C}^{1-\sigma_C} = \left(\epsilon \tau_a \tau_c\right)^{1-\sigma_C}.$$

Under Trade Balance:

$$Exports = \frac{D_{H,C} \left(\epsilon \tau_a \tau_c\right)^{1-\sigma_C}}{\tau_a},$$

$$\begin{aligned}
&\Rightarrow \left(\epsilon \tau_a \tau_c\right)^{1-\sigma_C} = \frac{\tau_a \times Exports}{D_{H,C}} \\
&= \frac{\tau_a \times N_{Cf} Avg_Exports_{Cf}}{D_{H,C}} \\
&= \frac{\tau_a \times Avg_Exports_{Cf}}{\exp\left(\sigma_C \times d_{H,C}\right)} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \\
&= \left(P_C^m\right)^{-(\sigma_C - 1)(1-\delta_C)} \frac{\tau_a \times Avg_Exports_{Cf}}{\Psi_C^{\frac{\sigma-1}{\Lambda}}} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C.
\end{aligned}$$

Where we have used

$$\exp\left(\sigma_C \times d_{H,C}\right) = \left(\frac{\Psi_C}{\left(P_C^m\right)^{-(1-\delta_C)\Lambda_C}} \right)^{\frac{\sigma_C - 1}{\Lambda_C}}.$$

Therefore:

$$P_{F,C}^{1-\sigma_C} = b_C^2 \left(P_C^m\right)^{(1-\sigma_C)(1-\delta_C)} M_C,$$

Where

$$b_C^2 \equiv \frac{\tau_a \times Avg_Exports_{Cf}}{\Psi_C^{\frac{\sigma_C-1}{\Lambda_C}}} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}}.$$

Therefore:

$$\frac{P_C^{1-\sigma_C}}{\left(P_C^m\right)^{(1-\sigma_C)(1-\delta_C)}} = \underbrace{\left(b_C^1 + b_C^2\right)}_{b_C} M_C \tag{S.10}$$

Price Index Sector S

$$\begin{aligned}
P_S^{1-\sigma_S} &= N_{Sf} Avg_Price_{Sf} + N_{Si} Avg_Price_{Si} \\
&= \left(\begin{array}{c} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg_Price_{Sf} \\ + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg_Price_{Si} \end{array} \right) M_S \\
&= \left(\begin{array}{c} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \widetilde{Avg_Price}_{Sf} \left(P_S^m\right)^{-(\sigma_S-1)(1-\delta_S)} \\ + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \widetilde{Avg_Price}_{Si} \left(P_S^m\right)^{-(\sigma_S-1)(1-\delta_S)} \end{array} \right) M_S \\
&\Rightarrow P_S^{1-\sigma_S} = b_S \left(P_S^m\right)^{(1-\sigma_S)(1-\delta_S)} M_S
\end{aligned}$$

Where

$$b_S \equiv \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \widetilde{Avg_Price}_{Sf} + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \widetilde{Avg_Price}_{Si}$$

Therefore:

$$\frac{P_S^{1-\sigma_S}}{\left(P_S^m\right)^{(1-\sigma_S)(1-\delta_S)}} = b_S M_S. \tag{S.11}$$

Step 9c: Write X_C^{int} as a function of M_C and M_S.

$$\begin{aligned}
X_C^{int} &= \lambda_C \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C} \left(Avg_Revenue_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} + Avg_Revenue_{Ci} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \right) M_C \\
&+ \lambda_S \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S} \left(Avg_Revenue_{Sf} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} + Avg_Revenue_{Si} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \right) M_S \\
&= d_C M_C + d_S M_S
\end{aligned}$$

Where

$$d_C = \lambda_C \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C} \left(Avg_Revenue_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} + Avg_Revenue_{Ci} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \right)$$

$$d_S = \lambda_S \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S} \left(Avg_Revenue_{Sf} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} + Avg_Revenue_{Si} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \right)$$

Step 9d: Solve for $\frac{M_S}{M_C}$ that matches Ψ_C.
Remember that:

$$\exp\left(d_{H,C}\right) = \left(\frac{\zeta I + X_C^{int}}{P_C^{1-\sigma_C}}\right)^{\frac{1}{\sigma_C}}$$

Using (S.9), (S.10), (S.2) and manipulating, we obtain:

$$\Psi_C^{\frac{\sigma_C-1}{\Lambda_C}} = \left(\frac{\zeta a_C}{b_C} + \frac{d_C}{b_C}\right) + \frac{\zeta a_S + d_S}{b_C}\frac{M_S}{M_C}$$

$$\frac{M_S}{M_C} = \frac{b_C}{\zeta a_S + d_S}\left(\Psi_C^{\frac{\sigma_C-1}{\Lambda_C}} - \left(\frac{\zeta a_C}{b_C} + \frac{d_C}{b_C}\right)\right)$$

Step 9e: Separately pin down M_C and M_S using the labor market clearing equation $\overline{L} - L_u = \sum_{k=C,S,j=i,f} L_{kj}$. Express M_C and M_S as functions of L_u.

To separately pin down M_C and M_S, use the labor market clearig equation.

$$\begin{aligned}
\overline{L} - L_u &= N_{Cf}Avg_Size_{Cf} + N_{Ci}Avg_Size_{Ci} + N_{Sf}Avg_Size_{Sf} + N_{Si}Avg_Size_{Si} \\
&= \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C Avg_Size_{Cf} + \frac{\omega_{Ci}M_C}{\varrho_{Ci}^{exit}}Avg_Size_{Ci} + \\
&\frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S Avg_Size_{Sf} + \frac{\omega_{Si}M_S}{\varrho_{Si}^{exit}}Avg_Size_{Si} \\
&= \left(\frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg_Size_{Cf} + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}}Avg_Size_{Ci}\right) M_C + \\
&\left(\frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg_Size_{Sf} + \frac{\omega_{Si}}{\varrho_{Si}^{exit}}Avg_Size_{Si}\right) M_S
\end{aligned}$$

At this point, we can only express M_C and M_S as functions of L_u.
From now on write

$$\left(\frac{M_S}{M_C}\right)^* = \frac{b_C}{\zeta a_S + d_S}\left(\Psi_C^{\frac{\sigma_C-1}{\Lambda_C}} - \left(\frac{\zeta a_C}{b_C} + \frac{d_C}{b_C}\right)\right)$$

$$\Rightarrow M_S = \underbrace{\frac{b_C}{\zeta a_S + d_S}\left(\Psi_C^{\frac{\sigma_C-1}{\Lambda_C}} - \left(\frac{\zeta a_C}{b_C} + \frac{d_C}{b_C}\right)\right)}_{AA} M_C$$

Therefore:

$$M_S = AA \times M_C$$
$$AA = \frac{b_C}{\zeta a_S + d_S}\left(\Psi_C^{\frac{\sigma_C - 1}{\Lambda_C}} - \left(\frac{\zeta a_C}{b_C} + \frac{d_C}{b_C}\right)\right) \tag{S.12}$$

So that:

$$\begin{aligned}\overline{L} - L_u = &\left(\frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg_Size_{Cf} + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg_Size_{Ci}\right) M_C + \\ &\left(\frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg_Size_{Sf} + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg_Size_{Si}\right) AA \times M_C \\ &= BB \times M_C\end{aligned}$$

$$\begin{aligned}BB = &\left(\frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg_Size_{Cf} + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg_Size_{Ci}\right) + \\ &\left(\frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg_Size_{Sf} + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg_Size_{Si}\right) AA\end{aligned} \tag{S.13}$$

Finally:

$$M_C = \frac{\overline{L} - L_u}{BB} \tag{S.14}$$
$$M_S = \frac{AA}{BB}\left(\overline{L} - L_u\right) \tag{S.15}$$

Step 9f: Express masses of firms N_{kj} as functions of L_u.

Substituting (S.14) and (S.15) into (S.3)-(S.6) to obtain the masses of firms:

$$N_{Ci} = \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C = \underbrace{\frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \frac{1}{BB}}_{EE_C}\left(\overline{L} - L_u\right) = EE_C\left(\overline{L} - L_u\right)$$

$$N_{Si} = \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S = \underbrace{\frac{\omega_{Si}}{\varrho_{Si}^{exit}} \frac{AA}{BB}}_{EE_S}\left(\overline{L} - L_u\right) = EE_S\left(\overline{L} - L_u\right)$$

$$N_{Cf} = \underbrace{\frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \frac{1}{BB}}_{DD_C}\left(\overline{L} - L_u\right)$$

$$N_{Sf} = \underbrace{\frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}}\frac{AA}{BB}}_{DD_S}\left(\overline{L} - L_u\right)$$

Step 9g: Express aggregate posted vacancies V_{kj} as functions of L_u.

Now, substituting the expressions for the N_{kj}'s to obtain the number of vacancies in each sector as a function of L_u:

$$\begin{aligned} V_{Cf} &= N_{Cf}Avg_Vacancies_{Cf} + \frac{\omega_{Cf}M_C}{\mu^v} \\ &= Avg_Vacancies_{Cf} \times DD_C\left(\overline{L} - L_u\right) + \frac{\omega_{Cf}}{\mu^v}\frac{1}{BB}\left(\overline{L} - L_u\right) \\ &= \underbrace{\left(Avg_Vacancies_{Cf} \times DD_C + \frac{\omega_{Cf}}{\mu^v}\frac{1}{BB}\right)}_{FF_C}\left(\overline{L} - L_u\right) \\ &= FF_C \times \left(\overline{L} - L_u\right) \end{aligned} \tag{S.16}$$

$$\begin{aligned} V_{Ci} &= N_{Ci}Avg_Vacancies_{Ci} + \frac{\omega_{Ci}M_C}{\mu^v} \\ &= Avg_Vacancies_{Ci} \times EE_C\left(\overline{L} - L_u\right) + \frac{\omega_{Ci}}{\mu^v}\frac{1}{BB}\left(\overline{L} - L_u\right) \\ &= \underbrace{\left(Avg_Vacancies_{Ci} \times EE_C + \frac{\omega_{Ci}}{\mu^v}\frac{1}{BB}\right)}_{GG_C}\left(\overline{L} - L_u\right) \\ &= GG_C \times \left(\overline{L} - L_u\right) \end{aligned} \tag{S.17}$$

$$\begin{aligned} V_{Sf} &= N_{Sf}Avg_Vacancies_{Sf} + \frac{\omega_{Sf}M_S}{\mu^v} \\ &= \underbrace{\left(Avg_Vacancies_{Sf} \times DD_S + \frac{\omega_{Sf}}{\mu^v}\frac{AA}{BB}\right)}_{FF_S}\left(\overline{L} - L_u\right) \\ &= FF_S \times \left(\overline{L} - L_u\right) \end{aligned} \tag{S.18}$$

$$\begin{aligned} V_{Si} &= N_{Si}Avg_Vacancies_{Si} + \frac{\omega_{Si}M_S}{\mu^v} \\ &= \underbrace{\left(Avg_Vacancies_{Si} \times EE_S + \frac{\omega_{Si}}{\mu^v}\frac{AA}{BB}\right)}_{GG_S}\left(\overline{L} - L_u\right) \\ &= GG_S \times \left(\overline{L} - L_u\right) \end{aligned} \tag{S.19}$$

$$\begin{aligned}\widetilde{V} &= V_{Cf} + V_{Ci} + V_{Sf} + V_{Si} \\ &= \underbrace{\left(FF_C + GG_C + FF_S + GG_S\right)}_{JJ} \times \left(\overline{L} - L_u\right) \\ &= JJ \times \left(\overline{L} - L_u\right)\end{aligned}$$

Step 9h: Use equation for μ^v to obtain L_u.

We have written each V_{kj} in terms of L_u. Now, note that

$$\mu^v = \phi \left(\frac{L_u}{\widetilde{V}}\right)^{1-\xi}$$

We can invert this equation to obtain L_u.

$$\begin{aligned}\mu^v &= \phi \left(\frac{L_u}{JJ \times \left(\overline{L} - L_u\right)}\right)^{1-\xi} \\ \Rightarrow L_u^* &= \frac{\left(\mu^v\right)^{\frac{1}{1-\xi}} \times JJ \times \overline{L}}{\phi^{\frac{1}{1-\xi}} + \left(\mu^v\right)^{\frac{1}{1-\xi}} \times JJ}\end{aligned}$$

Step 9i: Go back and obtain masses of entrants M_k's (equations (S.14) and (S.15)), masses of firms N_{kj}'s (equations (S.3)-(S.6)), and aggregate vacancies V_{kj}'s (equations (S.16)-(S.19)). We are now able to compute transitions out of unemployment μ_{kj}^e (Step 8).

Step 9j: Recover price indices P_C and P_S.

Equations (S.1) and (S.10) lead to:

$$P_C = \left(\left(b_C M_C\right)^{\frac{1}{1-\sigma_C}} \left(\frac{1}{\lambda_C^{\lambda_C}\left(1-\lambda_C\right)^{1-\lambda_C}}\right)^{(1-\delta_C)}\right)^{\frac{1}{1-(1-\delta_C)\lambda_C}} P_S^{\frac{(1-\delta_C)(1-\lambda_C)}{1-(1-\delta_C)\lambda_C}}$$

Defining

$$\varpi_C = \left(\left(b_C M_C\right)^{\frac{1}{1-\sigma_C}} \left(\frac{1}{\lambda_C^{\lambda_C}\left(1-\lambda_C\right)^{1-\lambda_C}}\right)^{(1-\delta_C)}\right)^{\frac{1}{1-(1-\delta_C)\lambda_C}}$$

and

$$\varkappa_C = \frac{\left(1-\delta_C\right)\left(1-\lambda_C\right)}{1-\left(1-\delta_C\right)\lambda_C}$$

Allows us to write

$$P_C = \varpi_C P_S^{\varkappa_C}$$

Equations (S.1) and (S.11) lead to:

$$P_S = \left(\left(b_S M_S \right)^{\frac{1}{1-\sigma_S}} \left(\frac{1}{\lambda_S^{\lambda_S} \left(1-\lambda_S\right)^{1-\lambda_S}} \right)^{(1-\delta_S)} \right)^{\frac{1}{1-(1-\delta_S)(1-\lambda_S)}} P_C^{\frac{(1-\delta_S)\lambda_S}{1-(1-\delta_S)(1-\lambda_S)}}$$

Writing

$$\varpi_S = \left(\left(b_S M_S \right)^{\frac{1}{1-\sigma_S}} \left(\frac{1}{\lambda_S^{\lambda_S} \left(1-\lambda_S\right)^{1-\lambda_S}} \right)^{(1-\delta_S)} \right)^{\frac{1}{1-(1-\delta_S)(1-\lambda_S)}}$$

and

$$\varkappa_S = \frac{\left(1-\delta_S\right)\lambda_S}{1-\left(1-\delta_S\right)\left(1-\lambda_S\right)}$$

Allows us to write

$$P_S = \varpi_S P_C^{\varkappa_S}$$

Solving the system leads to:

$$P_C = \left(\varpi_C \left(\varpi_S \right)^{\varkappa_C} \right)^{\frac{1}{1-\varkappa_S \varkappa_C}}$$

Step 9k: Compute deviation between government revenues and spending with unemployment insurance Dev_T.

Government Revenue

$$\begin{aligned}
G_{Rev} = & \frac{\sigma_C - \left(1-\delta_C\right)\left(\sigma_C - 1\right)}{\sigma_C} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \tau_y Avg_Revenue_{Cf} \\
& + \frac{\sigma_S - \left(1-\delta_S\right)\left(\sigma_S - 1\right)}{\sigma_S} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \tau_y Avg_Revenue_{Sf} \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \tau_w Avg_wbill_{Cf} \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \tau_w Avg_wbill_{Sf} \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C Avg_Firing_Costs_{Cf} \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S Avg_Firing_Costs_{Sf} \\
& + \left(\tau_a - 1\right) \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C Avg_Exports_{Cf}
\end{aligned}$$

Government Spending with Unemployment Insurance

$$G_{UI} = \underbrace{\left(b^u \times \underbrace{\sum_k \left(W_{kf}^{DS} + W_{kf}^{EE} + W_{kf}^{D} \right)}_{\text{mass of } \textbf{formal} \text{ workers who transition to unemployment}} \right)}_{\text{Total Expenditure with Unemployment Benefits}}$$

Government Transfers

$$T = G_{Rev} - G_{UI}$$

We impose in the objective function that $Dev_T \geq 0$—in other words, we highly penalize $Dev_T < 0$

$$Dev_T = \frac{G_{Rev} - G_{UI}}{G_{Rev}}$$

When we compute aggregate income, we implicitly assumed that $G_{Rev} - G_{UI} \geq 0$.

II Simulation Appendix

II.1 Simulation Algorithm

Fix P_S at $\overline{P}_S$. Write the value added function as:

$$VA_k(z,\ell) = \Theta_k \left(\frac{\overline{P}_S^{1-\lambda_k}}{\lambda_k^{\lambda_k}(1-\lambda_k)^{1-\lambda_k}} \right)^{-(1-\delta_k)\Lambda_k} P_C^{-\lambda_k(1-\delta_k)\Lambda_k} \left(\exp\left(d_{H,k}\right)\right)^{\frac{\sigma_k}{\sigma_k-1}\Lambda_k} \left(z\ell^{\delta_k}\right)^{\Lambda_k}$$

Define

$$\Xi_k \equiv \Theta_k \left(\frac{\overline{P}_S^{1-\lambda_k}}{\lambda_k^{\lambda_k}(1-\lambda_k)^{1-\lambda_k}} \right)^{-(1-\delta_k)\Lambda_k},$$

and

$$\Phi_k \equiv P_C^{-\lambda_k(1-\delta_k)\Lambda_k} \left(\exp\left(d_{H,k}\right)\right)^{\frac{\sigma_k}{\sigma_k-1}\Lambda_k}.$$

Rewrite the value added function as:

$$VA_k(z,\ell) = \Xi_k \Phi_k \left(z\ell^{\delta_k}\right)^{\Lambda_k}.$$

μ^{υ}, ϑ_{J_u}, d_F, Φ_C, Φ_S are the endogenous variables to be determined in equilibrium. For a given value of these variables, Steps 1 through 11 below compute the deviations from equilibrium conditions given by $L_i\left(\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S\right)$ for $i = 1, ..., 5$. We then need to find values $\left(\mu^{\upsilon}\right)^*, \vartheta^*_{J_u}, d^*_F, \Phi^*_C, \Phi^*_S$ solving $L_i\left(\left(\mu^{\upsilon}\right)^*, \vartheta^*_{J_u}, d^*_F, \Phi^*_C, \Phi^*_S\right) = 0$ for all $i = 1, ..., 5$. We discuss potential solutions to this problem in Step 12.

We proceed by first imposing values for ϑ_{J_u}, μ^{υ}, d_F, Φ_C, Φ_S.

Step 1: This step solves for wage schedules $w_{kf}(z,\ell')$, $w_{ki}(z,\ell')$ as well as value functions $V_{kf}(z,\ell)$, $V_{ki}(z,\ell)$, $J^e_{kf}(z,\ell')$, $J^e_{ki}(z,\ell')$, and firms' policy functions.

Step 1a: Compute value added functions $VA_k(z,\ell)$.

Step 1b: Compute wage schedules $w_{kf}(z,\ell')$

- Guess a wage schedule $w_{kf}(z,\ell')$
- Compute the resulting $V_{kf}(z,\ell')$ using (13)
- Compute $J^e_{kf}(z,\ell')$ using (A.46)
- Compute $w^u_{kf}(z,\ell')$ using equation (27)
- Let $\widehat{w}^u_{kf}(z,\ell') = \omega_0 + \omega_1 \frac{VA_k(z,\ell')}{\ell'}$ be the linear projection of $w^u_{kf}(z,\ell')$ on $\left[1, \frac{VA_k(z,\ell')}{\ell'}\right]$

- Update $w_{kf}(z,\ell') = \max\left\{\widehat{w}^u_{kf}(z,\ell'), b_u + \vartheta_{J_u} - \frac{1}{1+r}J^e_{kf}(z,\ell'), \underline{w}\right\}$
- Restart until convergence

Step 1c: Compute wage schedules $w_{ki}(z,\ell')$

- Guess a wage schedule $w_{ki}(z,\ell')$
- Compute the resulting $V_{ki}(z,\ell')$ using (17)
- Compute $J^e_{ki}(z,\ell')$ using (A.47)
- Compute $w^u_{ki}(z,\ell')$ using equation (30)
- Let $\widehat{w}^u_{ki}(z,\ell') = \omega_0 + \omega_1\left(1 - \frac{\sigma_k}{\sigma_k-(1-\delta_k)(\sigma_k-1)}p_{ki}(\ell')\right)\frac{VA_k(z,\ell')}{\ell'}$ be the linear projection of $w^u_{ki}(z,\ell')$ on $\left[1, \left(1 - \frac{\sigma_k}{\sigma_k-(1-\delta_k)(\sigma_k-1)}p_{ki}(\ell')\right)\frac{VA_k(z,\ell')}{\ell'}\right]$
- Update $w_{ki}(z,\ell') = \max\left\{\widehat{w}^u_{ki}(z,\ell'), \vartheta_{J_u} - \frac{1}{1+r}J^e_{ki}(z,\ell')\right\}$
- Restart until convergence

Step 2: Solve for firms' entry decisions. Compute the fraction of entrants in the formal and informal sectors as follows:

$$\omega_{kf} \equiv \Pr\left(I^{formal}_k(z) = 1\right) = \int_z I^{formal}_k(z)\, g^e_k(z)\, dz$$

$$\omega_{ki} \equiv \Pr\left(I^{informal}_k(z) = 1\right) = \int_z I^{informal}_k(z)\, g^e_k(z)\, dz$$

Therefore, if M_k is the mass of entrants in sector k, the masses of formal and informal entrants in sector k are given by:

$$M_{ki} = \omega_{ki} M_k$$

$$M_{kf} = \omega_{kf} M_k$$

Finally, compute the distribution of z productivities among entrants, conditional on entry into sector kj.

$$\psi^e_{ki}(z) = \frac{g^e_k(z)\, I^{informal}_k(z)}{\int_{\widetilde{z}} g^e_k(\widetilde{z})\, I^{informal}_k(\widetilde{z})\, d\widetilde{z}},$$

$$\psi^e_{kf}(z) = \frac{g^e_k(z)\, I^{formal}_k(z)}{\int_{\widetilde{z}} g^e_k(\widetilde{z})\, I^{formal}_k(\widetilde{z})\, d\widetilde{z}}.$$

Step 3: Compute the steady-state distribution of states. For informal firms, start with a guess for ψ_{ki}. Then, compute

$$\varrho^{exit}_{ki} = \alpha_k + (1-\alpha_k)\int_z\int_\ell \left(I^{exit}_{ki}(z,\ell) + I^{change}_{ki}(z,\ell)\right)\psi_{ki}(z,\ell)\, d\ell dz.$$

In steady state $N_{ki} = \left(1 - \varrho_{ki}^{exit}\right) N_{ki} + M_{ki}$. Therefore, set $\frac{M_{ki}}{N_{ki}}$, the fraction of sector k informal firms that are entrants, to:

$$\boxed{\frac{M_{ki}}{N_{ki}} = \varrho_{ki}^{exit} = \frac{\omega_{ki} M_k}{N_{ki}}.}$$

Now, compute $\widetilde{\psi}_{ki}$:

$$\begin{aligned}\widetilde{\psi}_{ki}(z,\ell) = &\mathcal{I}[\ell = 1] \times \varrho_{ki}^{exit} \times \psi_{ki}^{e}(z) \\ &+ \mathcal{I}[\ell \geq 1] \times (1 - \alpha_k) \times \psi_{ki}(z,\ell)\, I_{ki}^{stay}(z,\ell),\end{aligned}$$

and $\widehat{\psi}_{ki}$:

$$\widehat{\psi}_{ki}\left(z,\ell'\right) = \int_{\ell} \widetilde{\psi}_{ki}(z,\ell)\, \mathcal{I}\left(L_{ki}(z,\ell) = \ell'\right) d\ell$$

Update ψ_{ki} with:

$$\psi_{ki}\left(z',\ell'\right) = \int_{z} \widehat{\psi}_{ki}\left(z,\ell'\right) g_k\left(z'|z\right) dz,$$

and repeat until convergence of ψ_{ki}. This converged value of ψ_{ki} will be used directly in the computation of ψ_{kf} below.

For formal firms, start with guess for ψ_{kf} and compute:

$$\begin{aligned}\varrho_{kf}^{exit} &= \alpha_k + (1 - \alpha_k) \int_z \int_\ell I_{kf}^{exit}(z,\ell)\, \psi_{kf}(z,\ell)\, d\ell dz, \\ \varrho_{ki}^{change} &= (1 - \alpha_k) \int_z \int_\ell I_{ki}^{change}(z,\ell)\, \psi_{ki}(z,\ell)\, d\ell dz.\end{aligned}$$

In steady state:

$$\begin{aligned}\varrho_{kf}^{exit} N_{kf} &= \varrho_{ki}^{change} \underbrace{N_{ki}}_{\frac{\omega_{ki} M_k}{\varrho_{ki}^{exit}}} + \omega_{kf} M_k \\ &= M_k \left(\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf} \right)\end{aligned}$$

So that:

$$\boxed{\frac{M_{kf}}{N_{kf}} = \frac{M_k \omega_{kf}}{N_{kf}} = \frac{\varrho_{kf}^{exit} \omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf}}}$$

Also, note that

$$\frac{M_{kf}}{N_{kf}} \times \frac{N_{ki}}{M_{ki}} = \frac{\varrho_{kf}^{exit} \omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf}} \frac{1}{\varrho_{ki}^{exit}} = \frac{\varrho_{kf}^{exit} \omega_{kf}}{\varrho_{ki}^{change} \omega_{ki} + \varrho_{ki}^{exit} \omega_{kf}}$$

and

$$\frac{M_{kf}}{N_{kf}} \times \frac{N_{ki}}{M_{ki}} = \frac{\omega_{kf}}{\omega_{ki}} \frac{N_{ki}}{N_{kf}}$$

Therefore,

$$\boxed{\frac{N_{ki}}{N_{kf}} = \frac{\varrho_{kf}^{exit}\omega_{ki}}{\varrho_{ki}^{change}\omega_{ki} + \varrho_{ki}^{exit}\omega_{kf}}}$$

Compute $\widetilde{\psi}_{kf}$ as:

$$\widetilde{\psi}_{kf}(z,\ell) = \mathcal{I}[\ell = 1] \times \underbrace{\frac{\varrho_{kf}^{exit}\omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}}\omega_{ki} + \omega_{kf}}}_{\frac{M_{kf}}{N_{kf}}} \psi_{kf}^{e}(z) + \mathcal{I}[\ell \geq 1] \times \left(\begin{array}{c} (1-\alpha_k)\,\psi_{kf}(z,\ell)\, I_{kf}^{stay}(z,\ell) \\ + (1-\alpha_k) \underbrace{\frac{\varrho_{kf}^{exit}\omega_{ki}}{\varrho_{ki}^{change}\omega_{ki} + \varrho_{ki}^{exit}\omega_{kf}}}_{\frac{N_{ki}}{N_{kf}}} \psi_{ki}(z,\ell)\, I_{ki}^{change}(z,\ell) \end{array} \right)$$

and $\widehat{\psi}_{kf}$ as:

$$\widehat{\psi}_{kf}(z,\ell') = \int_{\ell} \widetilde{\psi}_{kf}(z,\ell)\, \mathcal{I}\left(L_{kf}(z,\ell) = \ell'\right) d\ell.$$

Update ψ_{kf} with:

$$\psi_{kf}(z',\ell') = \int_{z} \widehat{\psi}_{kf}(z,\ell')\, g_k(z'|z)\, dz,$$

and repeat until convergence of ψ_{kf}.

At this point we have the following objects: ψ_{kj}, $\widetilde{\psi}_{kj}$, $\widehat{\psi}_{kj}$, ϱ_{ki}^{exit}, ϱ_{ki}^{change}, ϱ_{kf}^{exit}, $\chi_{ki\to f}^{change}$, χ_{kf}^{layoff}, and χ_{ki}^{leave} (see equations (A.21), (A.25) and (A.27)).

Step 4: Compute the values of entry V_k^e $(k = C, S)$:

$$V_k^e = \int_z \left[V_{ki}^e(z)\, I_k^{informal}(z) + V_{kf}^e(z)\, I_k^{formal}(z)\right] g_k^e(z)\, dz$$

and compute the deviations

$$L_5(\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S) = L_5(\vartheta_{J_u}, \mu^{\upsilon}, \Phi_S) = Dev_{entry,S} = \frac{V_S^e - c_{e,S}}{c_{e,S}}$$

$$L_4(\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S) = L_4(\vartheta_{J_u}, \mu^{\upsilon}, d_F, \Phi_C) = Dev_{entry,C} = \frac{V_C^e - c_{e,C}}{c_{e,C}}$$

Step 5: This step solves for masses of entrants M_k's, masses of firms N_{kj}'s, aggregate vacancies V_{kj}'s consistent with Φ_C, Φ_S, ϑ_{J_u}, d_F and μ^{υ}.
Step 5a: Write aggregate income I as a function of masses of entrants M_C and M_S.
Step 5b: Write M_C as a functions of P_C and M_S as a function of M_C and $\overline{P}_S$.
Step 5c: Write X_C^{int} as a function of M_C and M_S

Step 5d: Pin down M_C using the equation defining Φ_C, then obtain M_S.

Step 5e: Obtain masses of firms N_{kj}.

Step 5f: Obtain aggregate posted vacancies V_{kj} and $\widetilde{V}$.

Step 5g: Save the values for P_C and $P_{F,C}$ to be used in Step 9.

Step 6: Compute L_u

$$L_u = \left(\frac{\mu^{\upsilon}}{\phi}\right)^{\frac{1}{1-\xi}} \widetilde{V}$$

Step 7: Obtain job finding rates μ^e_{kj} using aggregate vacancies V_{kj}'s and mass of unemployment L_u obtained in Steps 5 and 6.

$$\mu^e_{kj} = \frac{m_{kj}}{L_u} = \phi \frac{V_{kj}}{\widetilde{V}} \left(\frac{\widetilde{V}}{L_u}\right)^{\xi}$$

Step 8: Use equations (A.28)-(A.29) to obtain allocations L_{Cf}, L_{Ci}, L_{Sf}, L_{Si}.

$$L_{Ci} = \frac{\mu^e_{Ci} L_u}{\chi^{leave}_{Ci}}$$

$$L_{Si} = \frac{\mu^e_{Si} L_u}{\chi^{leave}_{Si}}$$

$$L_{Cf} = \frac{\mu^e_{Cf} L_u + \chi^{change}_{Ci\to f} L_{Ci}}{\chi^{layoff}_{Cf}}$$

$$L_{Sf} = \frac{\mu^e_{Sf} L_u + \chi^{change}_{Si\to f} L_{Si}}{\chi^{layoff}_{Sf}}$$

Step 9: Compute

$$\bar{\epsilon} = \frac{P_{F,C}}{\tau_a \tau_c},$$

where $P_{F,C}$ was determined in Step 5.

Compute:

$$d'_F = \log\left(\left(1 + \frac{D^*_F}{\exp\left(\sigma_C \times d_{H,C}\right)} \bar{\epsilon}^{\sigma_C} \tau_c^{1-\sigma_C}\right)^{\frac{1}{\sigma_C}}\right),$$

where

$$\exp\left(\sigma_C \times d_{H,C}\right) = \Phi_C^{\frac{\sigma_C - 1}{\Lambda_C}} \left(P_C\right)^{\lambda_C(1-\delta_C)(\sigma_C - 1)},$$

and P_C was determined in Step 5. Compute the deviation

$$L_3\left(\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S\right) = Dev_{d_F} = \frac{d_F - d'_F}{d_F}$$

Step 10: Compute deviation from the labor market clearing equation:

$$L_1\left(\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S\right) = Dev_L = \frac{\overline{L} - \left(L_{Cf} + L_{Ci} + L_{Sf} + L_{Si} + L_u\right)}{\overline{L}}$$

Step 11: Compute the deviation

$$\begin{aligned} L_2\left(\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S\right) &= Dev_{J_u} \\ &= 1 - \frac{\left(\sum_{k,j} \mu^e_{kj} \int_\ell \int_z \overline{J}^e_{kj}\left(z, L_{kj}\left(z,\ell\right)\right) g_{kj}\left(z,\ell\right) dz d\ell + \left(1 - \sum_{k,j} \mu^e_{kj}\right) \vartheta_{J_u}\right)}{\left(1+r\right)\left(\vartheta_{J_u} - b\right)} \end{aligned}$$

Therefore, given $\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S$, we can compute deviations L_1, L_2, L_3, L_4, L_5.

Step 12: The equilibrium is given by $\left(\mu^{\upsilon}\right)^*, \vartheta^*_{J_u}, d^*_F, \Phi^*_C, \Phi^*_S$ solving

$$L_i\left(\left(\mu^{\upsilon}\right)^*, \vartheta^*_{J_u}, d^*_F, \Phi^*_C, \Phi^*_S\right) = 0 \text{ for all } i = 1, ..., 5$$

Step 13: Compute the price index for exports

$$P^*_X \equiv \left(\int_{N_{F,C}}^{N_C} \mathcal{I}^x_C\left(n\right) p^*_x\left(n\right)^{1-\sigma_C} dn\right)^{\frac{1}{1-\sigma_C}}$$

Note that

$$Exports = \epsilon D^*_F \left(P^*_X\right)^{1-\sigma_C}$$

So that:

$$P^*_X = \left(\frac{Exports}{\epsilon D^*_F}\right)^{\frac{1}{1-\sigma_C}}$$

A key difficulty is that, given the discrete approximations for the state space, the system above has discontinuities. We list a few solutions we implemented.

- Solve for the system using a sequential bisection method. This procedure has the drawback of being very slow.
- Solve for the system using an optimization routine minimizing the norm of the system. This procedure has the drawback of also being slow and to potentially be stuck in local minima.
- Our preferred solution is to approximate each function $L_i\left(\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S\right)$ with a third degree polynomial on the arguments. To do so, we draw a large number of values for $\left(\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S\right)$ and follow Steps 1 through 11 above to compute $L_i\left(\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S\right)$ at each of these points. We then fit third degree polynomials for each L_i function $i = 1, ..., 5$. Finally, we can use an out-of-the shelf solver to find the root of this approximated system.

II.2 Simulation Algorithm – Details

This section details the steps within Step 5 of the estimation procedure.

Step 5: This step solves for masses of entrants M_k's, masses of firms N_{kj}'s, aggregate vacancies V_{kj}'s consistent with Φ_C, Φ_S, ϑ_{J_u}, d_F, and μ^{υ}.

We start with some definitions... Averages "per firm". All these quantities can be computed after Step 4, that is, after solving for the steady state distribution of states.

$$Avg_wbill_{ki} = \int_z \int_{\ell'} \left[w_{ki}\left(z,\ell'\right)\ell' \right] \widehat{\psi}_{ki}\left(z,\ell'\right) d\ell' dz \text{ for } k = C,S$$

$$Avg_wbill_{kf} = \int_z \int_{\ell'} \left[\max\left\{ w_{kf}\left(z,\ell'\right), \underline{w} \right\}\ell' \right] \widehat{\psi}_{kf}\left(z,\ell'\right) d\ell' dz \text{ for } k = C,S$$

$$Avg_Firing_Costs_{kf} = \kappa \int_z \int_{\ell} \left[\left(\ell - L_{kf}\left(z,\ell\right)\right)\left(1 - I^{hire}_{kf}\left(z,\ell\right)\right) \right] \widetilde{\psi}_{kf}\left(z,\ell\right) d\ell dz \text{ for } k = C,S$$

$$Avg_Hiring_Costs_{kj} = \int_z \int_{\ell} \left[H_{kj}\left(\ell, L_{kj}\left(z,\ell\right)\right) I^{hire}_{kj}\left(z,\ell\right) \right] \widetilde{\psi}_{kj}\left(z,\ell\right) d\ell dz \text{ for } k = C,S;\ j = i,f$$

$$Avg_Revenue_{kj} = \int_z \int_{\ell'} R_k\left(z,\ell'\right) \widehat{\psi}_{kj}\left(z,\ell'\right) d\ell' dz \text{ for } k = C,S;\ j = i,f$$

$$Avg_InfPenalty_{ki} = \int_z \int_{\ell'} \left[p_{ki}\left(\ell'\right) R_k\left(z,\ell'\right) \right] \widehat{\psi}_{ki}\left(z,\ell'\right) d\ell' dz \text{ for } k = C,S$$

$$Avg_Vacancies_{kj} = \int_z \int_{\ell} v_{kj}\left(z,\ell\right) \widetilde{\psi}_{kj}\left(z,\ell\right) d\ell dz \text{ for } k = C,S;\ j = i,f$$

$$Avg_Exports_{Cf} = \left(1 - \exp\left(-\sigma_C \times d_F\right)\right) \int_z \int_{\ell'} \left[R_C\left(z,\ell'\right) I^x_C\left(z,\ell'\right) \right] \widehat{\psi}_{kf}\left(z,\ell'\right) d\ell' dz$$

$$Fraction_Export_{Cf} = \int_z \int_{\ell'} I^x_C\left(z,\ell'\right) \widehat{\psi}_{Cf}\left(z,\ell'\right) d\ell' dz$$

$$Avg_size_{kj} = \int_z \int_{\ell} \ell \psi_{kj}\left(z,\ell\right) d\ell dz \text{ for } k = C,S;\ j = i,f$$

Now, define

$$\begin{aligned} Avg_Price_{kj} &= \int_z \int_{\ell'} p_{kj}\left(z,\ell'\right)^{1-\sigma_k} \widehat{\psi}_{kj}\left(z,\ell'\right) d\ell' dz \\ &= \int_z \int_{\ell'} \left(\frac{R_k\left(z,\ell'\right)}{q_k\left(z,\ell',\iota_k\left(z,\ell'\right)\right)} \right)^{1-\sigma_k} \widehat{\psi}_{kj}\left(z,\ell'\right) d\ell' dz \text{ for } k = C,S;\ j = i,f. \end{aligned}$$

We cannot compute Avg_Price_{kj}—given Ω, Φ_C and Φ_S. However, note that:

$$Avg_Price_{kj} = \widetilde{\Xi}_k P_C^{(1-\sigma_k)(1-\delta_k)\lambda_k} \Phi_k^{\delta_k(1-\sigma_k)} \int\int \left(z\left(\ell'\right)^{\delta_k} \right)^{\Lambda_k} \widehat{\psi}_{kj}\left(z,\ell'\right) dz d\ell'$$

$$Avg_Price_{Cf} = \widetilde{\Xi}_C P_C^{(1-\sigma_C)(1-\delta_C)\lambda_C} \Phi_C^{\delta_C(1-\sigma_C)} \int\int \left(z\left(\ell'\right)^{\delta_C} \right)^{\Lambda_C} \left(\exp\left(d_F \times I^x_C\left(z,\ell'\right)\right) \right)^{-\delta_C\sigma_C\Lambda_C} \widehat{\psi}_{Cf}\left(z,\ell'\right) dz d\ell'$$

$$\widetilde{\Xi}_k = \left(\frac{\overline{P}_S^{1-\lambda_k}}{\lambda_k^{\lambda_k} (1-\lambda_k)^{1-\lambda_k}} \right)^{-(1-\delta_k)\Lambda_k} \left(\frac{(1-\delta_k)(\sigma_k - 1)}{\sigma_k} \right)^{(1-\delta_k)\Lambda_k}$$

So, given Ω, Φ_C and Φ_S we can compute:

$$\begin{aligned} \widetilde{Avg_Price}_{kj} &\equiv \widetilde{\Xi}_k \Phi_k^{\delta_k(1-\sigma_k)} \int \int \left(z \left(\ell' \right)^{\delta_k} \right)^{\Lambda_k} \widehat{\psi}_{kj} \left(z, \ell' \right) dz d\ell' \\ &= \frac{Avg_Price_{kj}}{P_C^{(1-\sigma_k)(1-\delta_k)\lambda_k}} \end{aligned}$$

$$\begin{aligned} \widetilde{Avg_Price}_{Cf} &\equiv \widetilde{\Xi}_C \Phi_C^{\delta_C(1-\sigma_C)} \int \int \left(z \left(\ell' \right)^{\delta_C} \right)^{\Lambda_C} \left(\exp \left(d_F \times I_C^x \left(z, \ell' \right) \right) \right)^{-\delta_C \sigma_C \Lambda_C} \widehat{\psi}_{Cf} \left(z, \ell' \right) dz d\ell' \\ &= \frac{Avg_Price_{Cf}}{P_C^{(1-\sigma_C)(1-\delta_C)\lambda_C}} \end{aligned}$$

At this point, we can compute the following variables, as functions of M_C and M_S:

$$N_{Ci} = \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C \tag{S.20}$$

$$N_{Si} = \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S \tag{S.21}$$

$$N_{Cf} = \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C \tag{S.22}$$

$$N_{Sf} = \frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} M_S \tag{S.23}$$

$$M_{Ci} = \omega_{Ci} M_C$$

$$M_{Si} = \omega_{Si} M_S$$

$$M_{Cf} = \omega_{Cf} M_C$$

$$M_{Sf} = \omega_{Sf} M_S$$

Firm-level expenditures with sector S goods (fixed operating costs, hiring costs, entry

costs, fixed export costs)

$$
\begin{aligned}
E_S = & \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \left(Avg_Hiring_Costs_{Cf} + \overline{c}_{Cf}\right) \\
& + \frac{\omega_{Ci} M_C}{\varrho_{Ci}^{exit}} \left(Avg_Hiring_Costs_{Ci} + \overline{c}_{Ci}\right) \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \left(Avg_Hiring_Costs_{Sf} + \overline{c}_{Sf}\right) \\
& + \frac{\omega_{Si} M_S}{\varrho_{Si}^{exit}} \left(Avg_Hiring_Costs_{Si} + \overline{c}_{Si}\right) \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C Fraction_Export_{Cf} f_x \\
& + M_C c_{e,C} \\
& + M_S c_{e,S}
\end{aligned}
$$

Define c_C:

$$
\begin{aligned}
c_C \equiv \frac{E_{S,C}}{M_C} = & \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \left(Avg_Hiring_Costs_{Cf} + \overline{c}_{Cf}\right) \\
& + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \left(Avg_Hiring_Costs_{Ci} + \overline{c}_{Ci}\right) \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Fraction_Export_{Cf} f_x \\
& + c_{e,C},
\end{aligned}
$$

Where $E_{S,C}$ is firm-level expenditures with sector S goods (fixed costs, etc) coming from C-sector activity.

Define c_S:

$$
\begin{aligned}
c_S \equiv \frac{E_{S,S}}{M_S} = & \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \left(Avg_Hiring_Costs_{Sf} + \overline{c}_{Sf}\right) \\
& + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \left(Avg_Hiring_Costs_{Si} + \overline{c}_{Si}\right) \\
& + c_{e,S},
\end{aligned}
$$

Where $E_{S,S}$ is firm-level expenditures with sector S goods (fixed costs, etc) coming from S-sector activity.

We can therefore write:

$$\begin{aligned} E_S &= E_{S,C} + E_{S,S} \\ &= c_C M_C + c_S M_S \end{aligned}$$

Market Clearing (C and S sectors)

Let I denote aggregate income. Then, market clearing in the C and S sectors must lead to:

$$\zeta I + X_C^{int} = Rev_C - Exports + \tau_a Imports$$

$$(1-\zeta) I + X_S^{int} + E_S = Rev_S$$

$$Imports = Exports$$

Note that expenditures on intermediates are proportional to gross revenues:

$$P_k^m \iota_k (z, \ell) = \frac{(1-\delta_k)(\sigma_k - 1)}{\sigma_k} R_k (z, \ell),$$

which leads to:

$$\begin{aligned} X_C^{int} = &\lambda_C \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C} Rev_C \\ &+ \lambda_S \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S} Rev_S \end{aligned}$$

$$\begin{aligned} X_S^{int} = &(1-\lambda_C) \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C} Rev_C \\ &+ (1-\lambda_S) \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S} Rev_S \end{aligned}$$

Where Rev_C and Rev_S are total gross revenues in sectors C and S respectively. Therefore:

$$\begin{aligned} I = &\left(1 - \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C}\right) Rev_C \\ &+ \left(1 - \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S}\right) Rev_S \\ &- E_S \\ &+ (\tau_a - 1) Exports \end{aligned}$$

Using

$$
\begin{aligned}
Rev_C &= Avg_Revenue_{Cf}\frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}}M_C + Avg_Revenue_{Ci}\frac{\omega_{Ci}}{\varrho_{Ci}^{exit}}M_C \\
Rev_S &= Avg_Revenue_{Sf}\frac{\varrho_{Si}^{change}\omega_{Si}+\varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}}M_S + Avg_Revenue_{Si}\frac{\omega_{Si}}{\varrho_{Si}^{exit}}M_S \\
Exports &= Avg_Exports_{Cf}\frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}}M_C \\
E_S &= c_C M_C + c_S M_S
\end{aligned}
$$

Step 5a: Write aggregate income I as a function of masses of entrants M_C and M_S.

$$
\begin{aligned}
I = &\left(1-\frac{(1-\delta_C)(\sigma_C-1)}{\sigma_C}\right)\left(\begin{array}{c} Avg_Revenue_{Cf}\frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}}M_C \\ +Avg_Revenue_{Ci}\frac{\omega_{Ci}}{\varrho_{Ci}^{exit}}M_C \end{array}\right) \\
&+\left(1-\frac{(1-\delta_S)(\sigma_S-1)}{\sigma_S}\right)\left(\begin{array}{c} Avg_Revenue_{Sf}\frac{\varrho_{Si}^{change}\omega_{Si}+\varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}}M_S \\ +Avg_Revenue_{Si}\frac{\omega_{Si}}{\varrho_{Si}^{exit}}M_S \end{array}\right) \\
&-(c_C M_C + c_S M_S) \\
&+(\tau_a-1)\left(Avg_Exports_{Cf}\frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}}M_C\right)
\end{aligned}
$$

Therefore:

$$
I = a_C M_C + a_S M_S \tag{S.24}
$$

Where

$$
\begin{aligned}
a_C = &\left(1-\frac{(1-\delta_C)(\sigma_C-1)}{\sigma_C}\right)\left(\begin{array}{c} Avg_Revenue_{Cf}\frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \\ +Avg_Revenue_{Ci}\frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \end{array}\right) \\
&+(\tau_a-1)\left(Avg_Exports_{Cf}\frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}}\right) \\
&-c_C
\end{aligned}
$$

$$
\begin{aligned}
a_S = &\left(1-\frac{(1-\delta_S)(\sigma_S-1)}{\sigma_S}\right)\left(\begin{array}{c} Avg_Revenue_{Sf}\frac{\varrho_{Si}^{change}\omega_{Si}+\varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \\ +Avg_Revenue_{Si}\frac{\omega_{Si}}{\varrho_{Si}^{exit}} \end{array}\right) \\
&-c_S
\end{aligned}
$$

Step 5b: Write M_C as a functions of P_C and M_S as a function of M_C and $\overline{P}_S$.

Price Index Sector C

$$P_C^{1-\sigma_C} = P_{H,C}^{1-\sigma_C} + P_{F,C}^{1-\sigma_C}$$

The domestic component is given by:

$$\begin{aligned} P_{C,H}^{1-\sigma_C} &= N_{Cf} Avg_Price_{Cf} + N_{Ci} Avg_Price_{Ci} \\ &= \left(\begin{array}{c} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg_Price_{Cf} \\ + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg_Price_{Ci} \end{array} \right) M_C \\ &= \left(\begin{array}{c} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \widetilde{Avg_Price}_{Cf} P_C^{(1-\sigma_C)(1-\delta_C)\lambda_C} \\ + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \widetilde{Avg_Price}_{Ci} P_C^{(1-\sigma_C)(1-\delta_C)\lambda_C} \end{array} \right) M_C \end{aligned}$$

We can therefore write $P_{C,H}$ as:

$$P_{C,H}^{1-\sigma_C} = P_C^{(1-\sigma_C)(1-\delta_C)\lambda_C} b_C^1 M_C,$$

Where

$$b_C^1 \equiv \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \widetilde{Avg_Price}_{Cf} + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \widetilde{Avg_Price}_{Ci}$$

The foreign component is given by:

$$P_{F,C}^{1-\sigma_C} = (\epsilon\tau_a\tau_c)^{1-\sigma_C}.$$

Under Trade Balance:

$$Exports = \frac{D_{H,C}\left(\epsilon\tau_a\tau_c\right)^{1-\sigma_C}}{\tau_a},$$

$$\begin{aligned} &\Rightarrow (\epsilon\tau_a\tau_c)^{1-\sigma_C} = \frac{\tau_a \times Exports}{D_{H,C}} \\ &= \frac{\tau_a \times N_{Cf} Avg_Exports_{Cf}}{D_{H,C}} \\ &= \frac{\tau_a \times Avg_Exports_{Cf}}{\exp\left(\sigma_C \times d_{H,C}\right)} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \\ &= (P_C)^{\lambda_C(1-\delta_C)(1-\sigma_C)} \frac{\tau_a \times Avg_Exports_{Cf}}{\Phi_C^{\frac{\sigma_C-1}{\Lambda_C}}} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C. \end{aligned}$$

Where we have used

$$\exp\left(\sigma_C \times d_{H,C}\right) = \Phi_C^{\frac{\sigma_C-1}{\Lambda_C}} \left(P_C\right)^{\lambda_C(1-\delta_C)(\sigma_C-1)}.$$

Therefore:

$$P_{F,C}^{1-\sigma_C} = (P_C)^{\lambda_C(1-\delta_C)(1-\sigma_C)} b_C^2 M_C,$$

Where

$$b_C^2 \equiv \frac{\tau_a \times Avg_Exports_{Cf}}{\Phi_C^{\frac{\sigma_C-1}{\Lambda_C}}} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}}.$$

Rewriting:

$$\begin{aligned} P_C^{1-\sigma_C} &= P_{C,H}^{1-\sigma_C} + P_{F,C}^{1-\sigma_C} \\ &= (P_C)^{\lambda_C(1-\delta_C)(1-\sigma_C)} b_C^1 M_C + (P_C)^{\lambda_C(1-\delta_C)(1-\sigma_C)} b_C^2 M_C \end{aligned}$$

So that:

$$P_C^{1-\sigma_C} = (b_C M_C)^{\frac{1}{(1-\lambda_C(1-\delta_C))}} \tag{S.25}$$

where

$$b_C \equiv b_C^1 + b_C^2.$$

Price Index Sector S

$$\begin{aligned}
P_S^{1-\sigma_S} &= N_{Sf} Avg_Price_{Sf} + N_{Si} Avg_Price_{Si} \\
&= \left(\begin{array}{c} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg_Price_{Sf} \\ + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg_Price_{Si} \end{array} \right) M_S \\
&= \left(\begin{array}{c} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \widetilde{Avg_Price}_{Sf} P_C^{(1-\sigma_S)(1-\delta_S)\lambda_S} \\ + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \widetilde{Avg_Price}_{Si} P_C^{(1-\sigma_S)(1-\delta_S)\lambda_S} \end{array} \right) M_S \\
&\Rightarrow P_S^{1-\sigma_S} = P_C^{(1-\sigma_S)(1-\delta_S)\lambda_S} b_S M_S
\end{aligned}$$

Where

$$b_S \equiv \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \widetilde{Avg_Price}_{Sf} + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \widetilde{Avg_Price}_{Si}$$

Given that $P_S = \overline{P}_S$ is fixed, we can also write M_S as a function of P_C and model parameters.

$$M_S = \frac{\overline{P}_S^{1-\sigma_S}}{b_S P_C^{(1-\sigma_S)(1-\delta_S)\lambda_S}}, \tag{S.26}$$

and using (S.25):

$$M_S = \frac{\overline{P}_S^{1-\sigma_S}}{b_S \left(b_C M_C\right)^{\frac{(1-\sigma_S)(1-\delta_S)\lambda_S}{(1-\sigma_C)(1-(1-\delta_C)\lambda_C)}}}. \tag{S.27}$$

Step 5c:Write X_C^{int} as a function of M_C and M_S.

$$\begin{aligned}
X_C^{int} &= \lambda_C \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C} \left(Avg_Revenue_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} + Avg_Revenue_{Ci} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \right) M_C \\
&+ \lambda_S \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S} \left(Avg_Revenue_{Sf} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} + Avg_Revenue_{Si} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \right) M_S \\
&= d_C M_C + d_S M_S
\end{aligned}$$

Where

$$d_C = \lambda_C \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C} \left(Avg_Revenue_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} + Avg_Revenue_{Ci} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \right)$$

$$d_S = \lambda_S \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S} \left(Avg_Revenue_{Sf} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} + Avg_Revenue_{Si} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \right)$$

Step 5d: Pin down P_C using the equation defining Φ_C, and obtain M_C and M_S.

We can now express aggregate income I as a function of P_C using equations (S.24), (S.25) and (S.26) and solve for P_C. Remember that:

$$\exp\left(d_{H,C}\right) = \left(\frac{\zeta I + X_C^{int}}{P_C^{1-\sigma_C}}\right)^{\frac{1}{\sigma_C}}$$

Using the formula defining Φ_C and manipulating, we obtain:

$$P_C^{-\lambda_C(1-\sigma_C)(1-\delta_C)}\Phi_C^{\frac{\sigma_C-1}{\lambda_C}} = \exp\left(\sigma_C \times d_{H,C}\right) = \frac{\zeta\left(a_C M_C + a_S M_S\right) + d_C M_C + d_S M_S}{P_C^{1-\sigma_C}},$$

which leads to

$$\Phi_C^{\frac{\sigma_C-1}{\lambda_C}} = \frac{\left(\zeta a_C + d_C\right) M_C + \left(\zeta a_S + d_S\right) M_S}{b_C M_C},$$

which allows us to solve for M_C

$$M_C = \frac{1}{b_C}\left(\frac{\left(\zeta a_S + d_S\right)\overline{P}_S^{1-\sigma_S}}{b_S\left(\Phi_C^{\frac{\sigma_C-1}{\lambda_C}} - \frac{\left(\zeta a_C + d_C\right)}{b_C}\right)}\right)^{\frac{1}{1+\frac{(1-\sigma_S)(1-\delta_S)\lambda_S}{(1-\sigma_C)(1-(1-\delta_C)\lambda_C)}}},$$

and then for M_S using (S.27).

Step 5e: Now that we have values of M_C and M_S, we obtain masses of firms N_{kj}.

$$N_{Ci} = \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C$$

$$N_{Si} = \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S$$

$$N_{Cf} = \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C$$

$$N_{Cf} = \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S$$

Step 5f: Obtain aggregate posted vacancies V_{kj}.
Now, substituting the expressions for the N_{kj}'s to obtain the number of vacancies in each sector as a function of L_u:

$$V_{Cf} = N_{Cf} Avg_Vacancies_{Cf} + \frac{\omega_{Cf} M_C}{\mu^v}$$

$$V_{Ci} = N_{Ci} Avg_Vacancies_{Ci} + \frac{\omega_{Ci} M_C}{\mu^v}$$

$$V_{Sf} = N_{Sf} Avg_Vacancies_{Sf} + \frac{\omega_{Sf} M_S}{\mu^v}$$

$$V_{Si} = N_{Si} Avg_Vacancies_{Si} + \frac{\omega_{Si} M_S}{\mu^v}$$

Step 5g: Save the values for P_C and $P_{F,C}$:

$$P_C^{1-\sigma_C} = (b_C M_C)^{\frac{1}{(1-(1-\delta_C)\lambda_C)}}$$

$$P_{F,C}^{1-\sigma_C} = (P_C)^{-\lambda_C(1-\delta_C)(\sigma_C-1)} b_C^2 M_C$$