

TAKE A BREAK!...OR MORE.

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# Take a break!...or more.\*

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## Abstract

I investigate the optimal timing and length of breaks in a model with fatigue. A break's length determines the worker's productivity once work is resumed. I show that all breaks should be identical, equally spaced and long enough to fully recover productivity. When taking breaks is costless, the higher the number of breaks the better. Otherwise, the optimal number is finite and those workers whose productivity falls more at the beginning of the day should take more breaks. Workaholics take their breaks too early and make them too short, from the employers' viewpoint. The opposite is true for leisure-oriented workers.

**JEL classification:** D13, J2.

**Keywords:** Fatigue, productivity, hours of work, breaks, leisure.

In the 1940s Los Wigwam Weavers, a necktie factory in Denver, lost its best young male workers to the war effort. The factory owner hired older men to replace the young ones, but they lacked the dexterity necessary to produce the ties to the firm's required standards. The old workers were then replaced with middle-aged women who had the required skills to produce high quality ties, but lack the stamina of the younger men and 'at the end of the workday their cheeks were sunken with fatigue..'(Sedgewick, 2020). To mitigate the effect of fatigue the women were allowed two 15-minute breaks, one in the morning and one in the afternoon, with coffee. The women who took breaks did 'as much work in six and a half hours as the older men had done in eight'. The factory owner, realizing that the time off resulted in a greater output, made the breaks compulsory. In those breaks the workers could do anything they wanted, so long as it wasn't work; what was important was to insure that the workers relaxed.<sup>1</sup>

In recent times the breaks at work have moved out of the coffee rooms into the modern napping spaces. A growing number of companies, including Huawei, Huffington post, Nike, NASA and Google, offer their employers facilities for resting. The rationale for the fancy "nap pods" and sofas in the office is, as with the traditional coffee rooms, the increase in the productivity and in the wellbeing of the workers.

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<sup>1</sup>The workers were not paid for the half an hour they were on break and the firm was sued by the Department of Labor. The ruling in favor of the firm was appealed and in 1956 a judge ruled that the breaks should be counted as work and be paid. The argument being that the increase production is 'one of the primary factors, if not the prime factor, which leads the employer to institute such break periods.' The paid coffee break was born.

Experimental evidence support the use of naps to boost workers' productivity (Florence (1924), Hamermesh (1990), Mednick et al. (2002), Mednick and Ehrman (2007)). For instance Mednick et al. (2002) show that taking a long nap increases performance. They let subjects repeat the same visual texture discrimination task four times a day for 60 minutes. Some subject were allowed to take naps of different length between the second and third repetition. While the performance of those subjects who weren't allowed to take a nap actually decreased over the course of the day, those who took the longest one were as alert as in the morning (when performance was at its highest).

Breaks from work not only eliminate fatigue but also help in what psychologists call the process of insight, namely passing in a sudden way from a state of not knowing to one of knowing. Archimedes' *eureka* moment is probably the most well-known example of insight. In the ample anecdotal evidence, the insight is produced while not working on the problem, either while doing something completely different, like Archimedes taking a bath, Poincare boarding a bus or while sleeping or daydreaming.

If breaks boost some workers' productivity/creativity it is in the employers' interest to allow those workers to take on-the-job breaks instead of forcing them to work all the time. Since neither are all workers equally affected by fatigue, nor are all tasks equally tiring, breaks benefit different workers and workers in different sectors to different extents. Fixed timetables, short, long with or without breaks, result in productive inefficiency for some workers. The same is true if breaks have intrinsic utility and workers are free to choose their timetables. In this case there will be some workers who fail to take breaks (if work is more enjoyable than leisure) when they should, others will take them when they should not (if leisure is more enjoyable than work) or take them too long from the employers' viewpoint.

In this paper I develop a simple model of fatigue to analyze the optimal timing and length of breaks. I assume that the length of the working day is fixed and that the worker has an instantaneous productivity function  $f$  which is non negative and decreasing in the time worked. The worker can take time-off in order to recover from fatigue. The break's length determines the starting productivity level once work is resumed. I assume that the productivity after the break cannot be larger than the worker's initial productivity  $f(0)$  and that the resting technology is linear in the break's length. In particular, a break of length  $b$  taken at time  $w$  resets the worker's productivity to the level it had at time  $\hat{t} = \max\{0, w - \gamma b\}$ , where  $\gamma$  is the effectiveness of the break. The larger  $\gamma$  the shorter the time needed for recovery.

The first result of the paper is that, conditional on taking breaks, the breaks should be long enough to guarantee full recovery. I also show that breaks will increase production when a very simple condition is satisfied: the ratio between the final and the initial productivities,  $f(T)/f(0)$ , cannot exceed  $\gamma/(1 + \gamma)$ . This condition will be more easily satisfied the stronger the effect of fatigue, the longer the working day and the better the recovery technology.

Throughout the paper I assume that the worker chooses the breaks to maximize total utility which includes not only what the worker produces but also the utility derived from leisure and or work. When only one break is allowed, the break will start in the first half of the working day and the worker will always work longer after the break. The starting time of the break is increasing in  $\gamma$  and in the utility of leisure  $u$ . Workers who like leisure more than they like working will start their breaks too late while workaholics will take them too early, from the employers' viewpoint. This result is driven by the full recovery result mentioned above. Those workers who enjoy leisure more than working will make the break longer by taking it later while workaholics will take it earlier to make it shorter and will work longer hours than their more leisure-oriented colleagues. Only when the utility of leisure is high enough, will the worker take non-productive time-off, namely she will extend the break beyond the length needed for full recovery. In this case the two working spells are of identical length and the worker no longer

works longer after the break. Making the working day longer does not make the worker work more; the worker will simply stay longer in the coffee room or lying on the sofa.

I introduce the possibility of taking multiple breaks and show that if the worker could choose the number of breaks and these were costless, she would take infinitely many breaks. Interestingly, a way to prevent the worker from taking nonproductive breaks is to let her take more breaks. If taking breaks results in some lost time, the optimal number will be finite. Those workers whose productivity falls more at the beginning of the day should be allowed more breaks. As before, the breaks will be identical and equally spaced. This resembles the so called polyphasic sleep adopted by Leonardo da Vinci who reportedly took 20-minute naps every four hours.

## 1 Model

Time is continuous and the worker can work in the time interval  $[0, T]$ , where  $T > 0$  is the length of the working day. The worker productivity is at its highest when she starts working and falls throughout the day. This is captured by the continuous and strictly decreasing function  $f : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ , where  $f(t)$  is the worker's instantaneous productivity after  $t$  hours of work. I assume that  $f(T) \geq 0$ .

### 1.1 One break

Suppose first that the worker can take one and only one break during her working day. Let  $w$  and  $b$  denote the time worked before the break and the break's length, respectively. Note that  $w$  is also the break's starting time. A break of length  $b$  taken at time  $t = w$  increases the worker's productivity to the level it had at  $\hat{t}(w, b)$ :

$$\hat{t}(w, b; \gamma) := \max\{0, w - \gamma b\}, \quad (1)$$

where  $\gamma$  measures the effectiveness of the break. The larger  $\gamma$  the smaller the time needed for recovery. Recovery cannot go beyond the worker's maximum instantaneous productivity  $f(0)$ .

The worker's instantaneous productivity when she resumes work is  $f(\hat{t}(w, b; \gamma))$  and will decline with the time worked according to  $f$ .

Let  $F(w)$  denote the production after  $w$  hours of work:

$$F(w) := \int_0^w f(t) dt. \quad (2)$$

A break of length  $b$  taken at time  $w$  leads to a total production equal to

$$F(w) + F(\hat{t}(w, b; \gamma) + T - w - b) - F(\hat{t}(w, b)), \quad (3)$$

where the first term corresponds to the production up to the break and the last two terms to the output produced after the break.

The worker derives utility from what she produces and from the time spent on and off work. The utility from the time at work and on breaks is

$$(T - b)v + b \cdot u = T \cdot v + b \cdot (u - v), \quad (4)$$

where  $v$  and  $u$  are the instantaneous working and leisure utilities, respectively.

From the decision making point of view the only relevant decision utility is the difference  $u - v$  which will be positive if the worker enjoys leisure more than she enjoys work and negative if the worker is a workaholic. Without loss of generality I assume hereafter that  $v = 0$  and allow  $u$  to take negative values. The worker optimization problem can be stated as choosing  $w$  and  $b$  to maximize,

$$F(w) + F(\hat{t}(w, b; \gamma) + T - w - b) - F(\hat{t}(w, b)) + b \cdot u, \quad (5)$$

subject to

$$w, b \geq 0, \quad (6)$$

and

$$w + b \leq T. \quad (7)$$

In order to have a meaningful problem we need to assume that  $f(0) > u$ . This guarantees that the worker will want to work at the beginning of the day.

The following lemma is key for the remainder of the paper.

**LEMMA 1.** *Assume  $(w^*, b^*) > (0, 0)$  solves (5) subject to (6)-(7). Then,*

$$\gamma \cdot b^* \geq w^*. \quad (8)$$

*Proof.* Assume that  $\gamma b^* < w^*$ . Consider a break of the same length  $b^*$  taken at  $w^* - \epsilon$  where  $\epsilon \in (0, w^* - \gamma b^*)$ . The gain after the break,

$$\int_0^\epsilon f(w^* - \epsilon - \gamma b^* + t) dt, \quad (9)$$

is larger than the loss suffered before the break,

$$\int_0^\epsilon f(w^* - \epsilon + t) dt. \quad (10)$$

Hence,  $\gamma b^* < w^*$  cannot maximize (5). Q.E.D.

Lemma 1 shows that, if the worker takes a break at  $t = w$ , the break will be of length  $w/\gamma$  or longer. Any time-off beyond  $w/\gamma$  will be pure leisure with no effect on productivity. This allows us to write  $b$  as,

$$b = \frac{w}{\gamma} + \ell, \quad (11)$$

where the first term is the productivity enhancing part of the break and the second is any time-off taken beyond that needed for full recovery. I shall refer to the latter as unproductive break or pure leisure and to the former as productive break. We can then write (5) as

$$\max_{w, \ell} F(w) + F(T - w - w/\gamma - \ell) + (w/\gamma + \ell)u, \quad (12)$$

subject to:

$$w, \ell \geq 0, \quad (13)$$

$$w + w/\gamma + \ell \leq T. \quad (14)$$

Let  $(w^*, \ell^*)$  solve (12) subject to (13) and (14). It follows from the Kuhn-Tucker conditions that

i) the worker will take a break whenever

$$f(T) - u < \gamma(f(0) - f(T)), \quad (15)$$

ii) if the break has an unproductive part ( $\ell^* > 0$ ),

$$f(w^*) = f(T - w^*(1 + 1/\gamma) - \ell^*) = u, \quad (16)$$

and,

iii) if all the break is productive,

$$f(T - (1 + 1/\gamma)w^*) - u = \gamma(f(w^*) - f(T - (1 + 1/\gamma)w^*)). \quad (17)$$

Note that condition (15) is always satisfied when  $u > f(T)$  since it is never optimal to work when the productivity is below the leisure utility  $u$ . This fact is behind condition (16): taking unproductive break prevents the productivity to fall below  $u$ . This will only happen when the leisure utility is positive and the working day is long enough,

$$T > f^{-1}(u)(2 + 1/\gamma). \quad (18)$$

If (18) is satisfied the worker has enough time to: i) work  $f^{-1}(u)$  hours, ii) fully recover from the accumulated fatigue, enjoy some non productive leisure and iii) work  $f^{-1}(u)$  hours more and call it a day. At the end of the two working spells the instantaneous productivity equals the leisure utility,  $u$ , and there are no gains from anticipating or postponing the break. When (18) is not satisfied the schedule above does not fit in the working day and the worker does not want to take any pure leisure. It is useful to note that (18) can be re-written as

$$u > u_f^L(T, \gamma) := f\left(\frac{T}{2 + 1/\gamma}\right). \quad (19)$$

The next results follows from (15)-(17).

**PROPOSITION 1.** *A break is productivity enhancing if*

$$\frac{f(T)}{f(0)} < \frac{\gamma}{1 + \gamma}. \quad (20)$$

*Proof.* Substituting  $u = 0$  in (15) and re-arranging gives (20). Q.E.D.

Condition (20) is more easily satisfied the more the productivity falls with previous work, the longer the working day, and the more effective the break, i.e: the larger  $\gamma$ . I turn now to the characterization of the optimal timing and length of the break.

**PROPOSITION 2.** *Assume that (15) holds and let  $(w^*, \ell^*)$  solve (12) subject to (13) and (14):*

$$w^* = f^{-1}(u), \quad (21)$$

and

$$\ell^* = T - f^{-1}(u)(2 + 1/\gamma), \quad (22)$$

when  $u > u_f^L(T, \gamma)$ . Otherwise,

$$\ell^* = 0, \tag{23}$$

and

$$w^* = w_f(T, u, \gamma), \tag{24}$$

where  $w_f(T, u, \gamma)$  is the unique  $w$  which solves (17). The starting time of the break,  $w_f(T, u, \gamma)$ , is increasing in  $T$ ,  $u$  and  $\gamma$ .

To see how  $u$  affects the willingness to take a break assume that a break is productivity enhancing, namely (20) holds. A worker with  $u = 0$  will want to take the break and by doing so will maximize total production. All workers with  $u > 0$  will also take a break since the break increases both production and leisure utility, but only those with  $u > u_f^L(T, \gamma)$  will take pure leisure ( $\ell^* > 0$ ). Some workaholics will take breaks because the gain in production more than compensates the loss in utility due to the shortening of the working time. Only those workers who are workaholic enough will fail to take the productivity enhancing break even though the break would increase production.

The following corollary follows.

**COROLLARY 1.** *When all the break is productive, the worker works shorter hours before than after the break. When the worker takes unproductive leisure, the two working spells are identical, total production is  $2F(f^{-1}(u))$ , and longer working days do not lead to more work since any increase  $T$  results in an equal increase in pure leisure.*

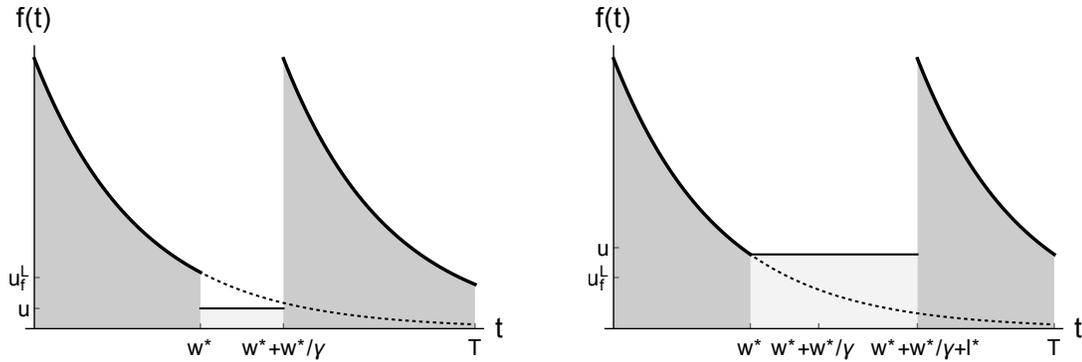


Figure 1:  $\ell^* = 0$  when  $u < u_f^L(T, \gamma)$  (left) and  $\ell^* > 0$  when  $u > u_f^L(T, \gamma)$  (right)

Figure 1 illustrates proposition 2. In each of the figures the area in dark grey are is the production; the area of clear grey rectangles is the leisure utility. In the figure on the right  $u > u_f^L(T, \gamma)$  and the worker takes unproductive time-off (pure leisure). The two working spells are identical and of length  $f^{-1}(u)$ . The figure on the left corresponds to the case in which the worker will only take productive time-off ( $u < u_f^L(T, \gamma)$ ). At the end of both working spells the productivity is higher than the leisure utility.

The actual length of the break is affected by  $u$  even if there is no unproductive time-off. When  $u = 0$ , the worker maximizes total production. When  $u > 0$ , the worker likes both production and

leisure and the only way to enjoy more leisure is to take a longer break. When all leisure is productive, the only way to enjoy a longer break is to take it later so that more time is needed for full recovery. The opposite occurs when  $u < 0$ . Those workaholics who actually take breaks will take them too early (relative to the product maximizing choice) so that the time required for full recovery will be shorter.

In the next section we show that it is in the worker's interests to take many breaks. We also characterize the optimal timing.

## 1.2 Multiple breaks

Assume now that the worker is allowed multiple breaks. Will she want to take them? The following lemma shows that one break can be improved upon.

**LEMMA 2.** *Assume that the worker wants to take a break. Then,  $n = 2, 3, \dots$  breaks are preferred to one break.*

*Proof.* Consider the optimal one-break solution  $(w^*, b^*)$  and take the time interval consisting of the first working spell and the productive part of the break:  $[0, w^* + w^*/\gamma]$ . Assume that the worker takes  $n$  identical breaks of length  $w^*/(\gamma n)$  at  $t = w^*/n, (2w^* + w^*/\gamma)/n, (3w^* + 2w^*/\gamma)/n, \dots, (nw^* + (n-1)w^*/\gamma)/n$ . Each of those breaks recovers productivity fully. The production with the  $n$  breaks is larger in the interval  $[0, w^* + w^*/\gamma]$  ( $nF(w^*/n) > F(w^*)$ ), and it is equal in the last working spell. Since the leisure/working utility remains unchanged,  $n = 2, 3, \dots$  breaks dominate the optimal one-break solution. Q.E.D.

The following corollary follows.

**COROLLARY 2.** *The larger the number of breaks the better.*

I next characterize the optimal  $n$ -break solution. Assume now the worker's objective is to maximize her utility with  $n$  breaks. As before I use lemma 1 and split the breaks into a productive and an unproductive part. Let  $w_i$  and  $\ell_i$  denote the lengths of the  $i$ -th working spell and of the non productive part of the  $i$ -th break. The workers objective function can be written as:

$$\max_{\{w_i, \ell_i\}_{i=1}^n} \sum_{i=1}^n F(w_i) + F\left(T - \sum_{i=1}^n w_i(1 + 1/\gamma) - \sum_{i=1}^n \ell_i\right) + u \sum_{i=1}^n (\ell_i + w_i/\gamma), \quad (25)$$

subject to

$$w_i, \ell_i \geq 0 \quad i = 1, 2, \dots, n, \quad (26)$$

$$\sum_{i=1}^n w_i(1 + 1/\gamma) + \sum_{i=1}^n \ell_i \leq T. \quad (27)$$

The worker will take unproductive breaks only when

$$T > f^{-1}(u)(1 + n + n/\gamma). \quad (28)$$

Condition (28) is the generalization of (18). The worker will work in  $n + 1$  identical working spells of length  $f^{-1}(u)$ , have enough time to recover and, in addition, enjoy some unproductive leisure. Total production will be  $(n + 1)F(u)$  and the time spent on breaks equal to  $T - (n + 1)f^{-1}(u)$ .

Since the RHS of (28) is increasing in  $n$ , a way to prevent unproductive leisure is to take a large enough number of breaks. In fact, the worker will not want to take unproductive breaks when  $n \geq n_f(T, u, \gamma)$ , where

$$n_f(T, u, \gamma) := \frac{\gamma}{1 + \gamma} \frac{T - f^{-1}(u)}{f^{-1}(u)}. \quad (29)$$

The following proposition focuses on the case with no unproductive leisure.

**PROPOSITION 3.** *Assume that (15) holds and  $n > n_f(T, u, \gamma)$ . Then,*

$$\ell_i = 0, \quad (30)$$

and

$$w_i^* = w_f(n; T, u, \gamma), \quad (31)$$

for all  $i = 1, 2, \dots, n$ , where  $w_f(n; T, u, \gamma)$  is the unique  $w$  which solves

$$f(T - (1 + 1/\gamma)n \cdot w) - u = \gamma(f(w) - f(T - (1 + 1/\gamma)n \cdot w)). \quad (32)$$

The following corollary follows from (32) and describes how the number of breaks affects the actual time spent at work.

**COROLLARY 3.** *Let  $w_f(n; T, u, \gamma)$  be the optimal length of each of the  $n$  identical working spells which are followed by a break.*

a)

$$w_f(n; T, u, \gamma) > w_f(n + 1; T, u, \gamma), \quad (33)$$

b)

$$n \cdot w_f(n; T, u, \gamma) < (n + 1) \cdot w_f(n; T, u, \gamma), \quad (34)$$

c)

$$\lim_{n \rightarrow \infty} T - n \cdot w_f(n; T, u, \gamma)(1 + 1/\gamma) = T_f^B(u, \gamma),$$

where

$$T_f^B(u, \gamma) := f^{-1} \left( \frac{\gamma f(0) + u}{1 + \gamma} \right), \quad (35)$$

is the largest working horizon, given  $u$  and  $\gamma$ , for which the worker does not want to take a break.

As  $n$  increases each working spell preceding a break becomes shorter but total time spent on early work increases. Since all breaks are productive and productivity is fully recovered, the total time spent on breaks will also increase. Those increases happen at the expense of the last working spell which approaches  $T_f^B(u, \gamma)$  as  $n$  goes to infinity.

Figure 2 illustrates this result for a linear productivity function,  $\gamma = 2$  (one hour of rest recovers two hours of work) and  $n = 1, 5$ . The dashed line on each figure is a working spell of length  $T_f^B(u, \gamma)$ . It is easy to see that the last working spell becomes shorter as  $n$  increases.

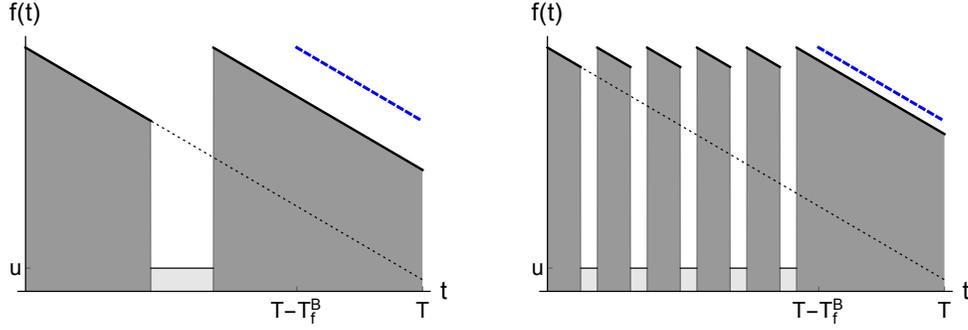


Figure 2: Solution with 1 and 5 breaks,  $f(t) = A - \alpha t$ ,  $\gamma = 2$ .

The sum of the areas in dark grey is the total production and the sum of the areas in clear grey is the utility derived from leisure. The sum of both give the indirect utility  $V_f(n; T, u, \gamma)$ :

$$V_f(n; T, u, \gamma) = nF(w_f(n; T, u, \gamma)) + F(T - n \cdot w_f(n; T, u, \gamma)(1 + 1/\gamma)) + n \cdot u \cdot w_f(n; T, u, \gamma)/\gamma. \quad (36)$$

It follows from Corollary 3 that

$$\lim_{n \rightarrow \infty} V_f(n; T, u, \gamma) = \frac{\gamma f(0) + u}{1 + \gamma} (T - T_f^B(u, \gamma)) + F(T_f^B(u, \gamma)), \quad (37)$$

where the second term is the production in the last working spell and the first is the total production earlier in the day.

To understand which workers benefit more from the breaks it is useful to consider the example in Figure 3. The three workers,  $f$ ,  $g$  and  $h$ , have the same initial and final productivities and would produce the same if they worked in the interval  $[0, T]$  without breaks:  $F(T) = G(T) = H(T)$ . The values of  $u$  and  $\gamma$  are such that  $T_i^B(u, \gamma) = T^B$  for  $i = f, g, h$  ( $u = 0$  and  $\gamma = 1$ ). The area under the curve in the top row is the total production without breaks.

The three figures in the middle row are the visual representation of the limit production (37). I have lumped together all the infinitely small working spells as well as all the infinitely small breaks. The area of the rectangle in each figure corresponds to the production before the last break (first term in (37)). This area is the same for the three workers. The area in clear grey is the production in the last working spell (second term in (37)). Note that this area is identical to the area in clear grey in the corresponding figure at the top. The gain from infinitely many breaks is the difference between the rectangle in the second row and the darker area in the corresponding figure at the top. The worker who benefits most from the breaks is  $g$ , the least productive in the last part of a working day without breaks.

The last row shows (37) (dotted line) as well as the indirect utilities for different number of breaks ( $V_i(n; \dots)$ ) (bullets). These figures illustrate that convergence to  $V_i^\infty$  is fastest for  $g$  and slowest for  $h$ , suggesting that even though  $g$  benefits more from the breaks, it is the worker whose productivity falls more at the beginning of the day who benefits the most from additional breaks.

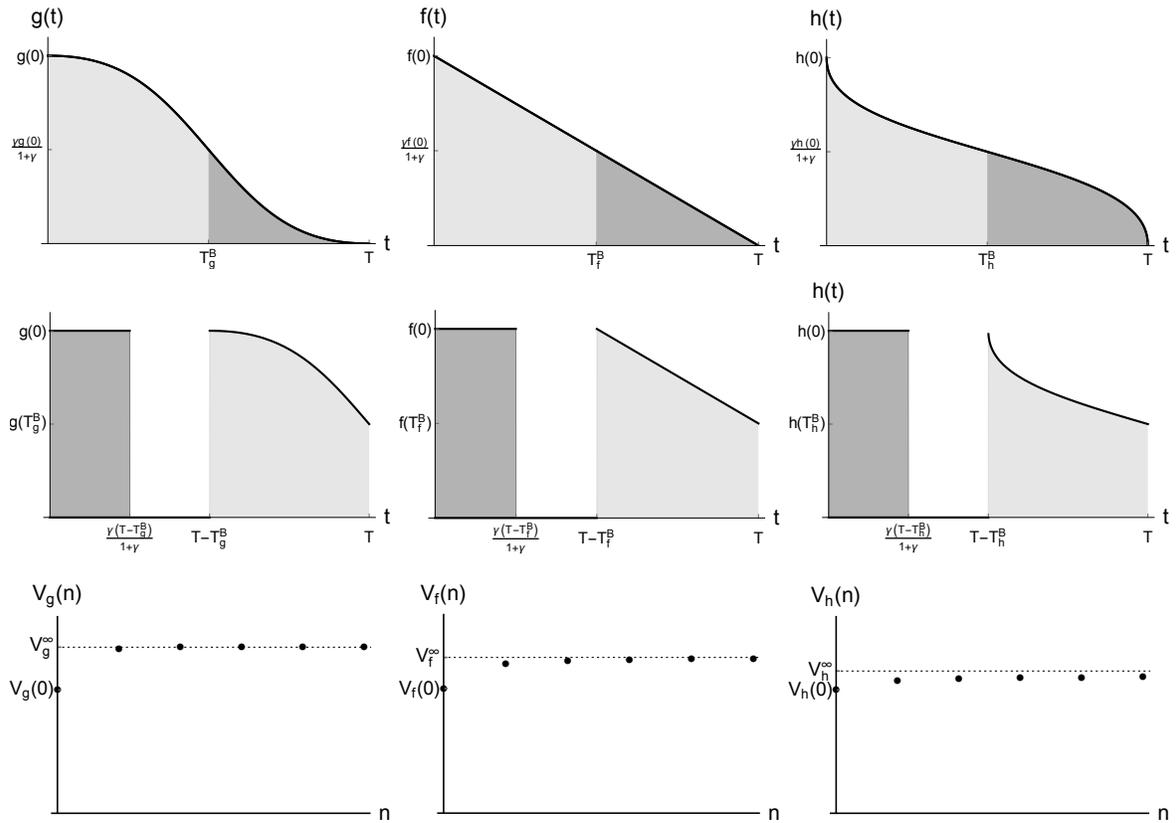


Figure 3: Productivities (top),  $V_i^\infty$  (middle),  $V_i(n, T\dots)$  (bottom)

## 2 Costly breaks

Corollary 2 hinges on the implicit assumption that taking breaks is costless, namely the to and fro between the loom and the coffee room does not result in any loss of time. I now relax this assumption and assume that each time the worker moves from the loom to the coffee room and back to the loom there is some wasted time which can only be avoided by reducing the number of breaks. This is related to one of the advantages of the division of labor mentioned by Adam Smith:

...the advantage which is gained by saving the time commonly lost in passing from one sort of work to another is much greater than we should at first view be apt to imagine it. A country weaver, who cultivates a small farm, must lose a good deal of time in passing from his loom to the field, and from the field to his loom...(Adam Smith, 1776)

Following Adam Smith's insight I assume that the worker performs two different tasks: proper work (at the loom) and proper resting (in the coffee room). I also assume that changing tasks is accompanied by lost time  $c > 0$ . During this time the worker neither recovers nor enjoys leisure. The effect of this cost of changing tasks is to reduce the total time available for work and recovery. In particular, a worker who takes  $n$  breaks will have now  $T - nc$  hours available for work and productive

rest. Corollary 2 is no longer true and the optimal number of breaks is finite. The intuition is simple, as the number of times the worker changes tasks increases, the time horizon available for work and recovery shrinks and the advantage of a further break will eventually disappear. The optimal number of breaks  $n_f^*$  solves

$$\max_n V_f(n, T - nc, \gamma, u), \quad (38)$$

subject to:

$$T - nc > T_f^B(u, \gamma). \quad (39)$$

To understand what are the determinants of the optimal number of breaks consider the example in Figure 4.

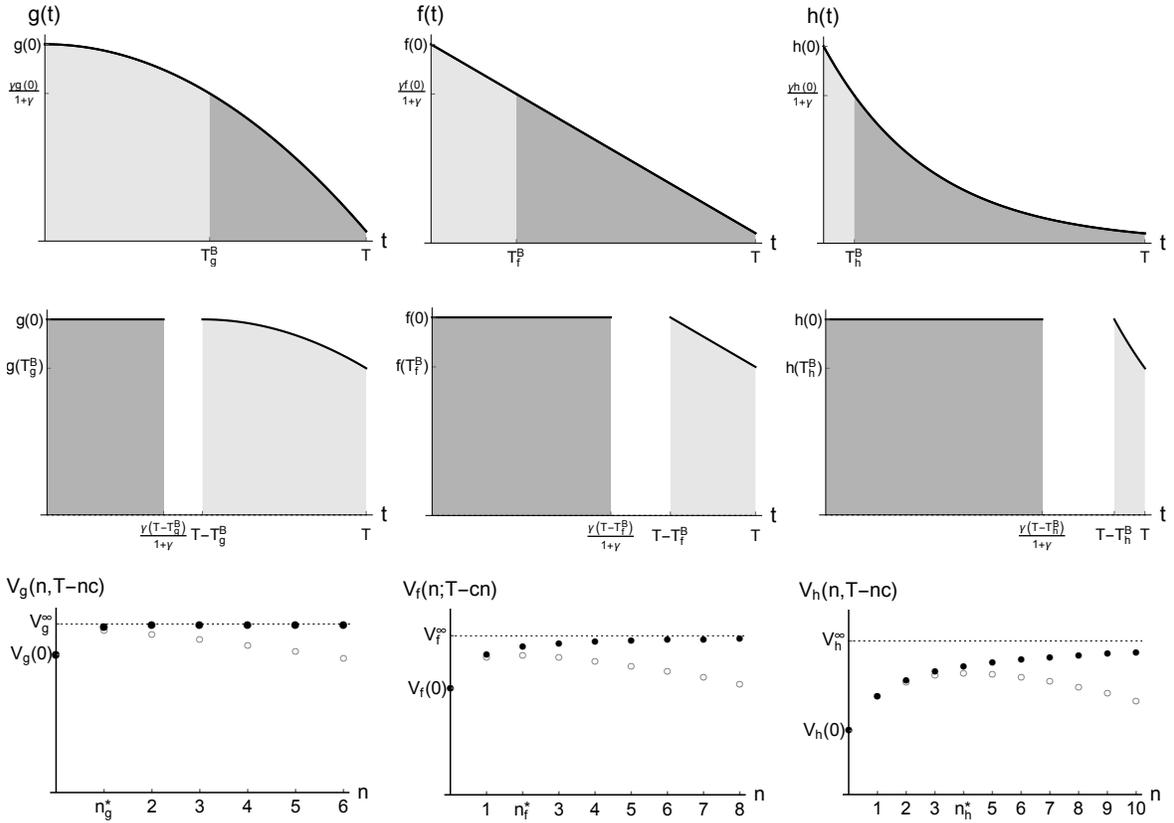


Figure 4: Productivities (top),  $V_i^\infty$  (middle),  $V_i(n, T - cn)$  (bottom)

As in the previous example there are three workers,  $f$ ,  $g$  and  $h$ , who have the same initial and final productivities and the same  $\gamma$  and  $u$  ( $u = 0$  and  $\gamma = 3$ ). Since now  $g(s) > f(s) > h(s)$  for all  $s \in (0, T)$ ,  $T_g^B > T_f^B > T_h^B$  and the first term in (37) is larger for  $h$  followed by  $f$  and  $g$ . The second term in (37) is the area below the productivity function for the interval  $[T_i^B, T]$ . In this example it is

largest for  $f$ , followed by  $g$  and  $h$ . The worker who benefits most from the breaks is  $h$  followed by  $f$  and  $g$ . This can be seen in the figures at the bottom row which show the values of  $V_i^\infty$  (dotted line) as well as the indirect utilities,  $V_i(n, \dots)$ , for  $n = 0, 1, 2, \dots$  (bullets). The bullets also illustrate that it is worker  $h$  the one who would need more breaks to reach her full potential. The circles in the figures in the last row show  $V_f(n, T - nc, \gamma, u)$ , the indirect utilities when a cost  $c > 0$  is included. It is easy to see that when breaks are costly,  $h$  should take more breaks than  $f$  and this more than  $g$ .

### 3 Conclusion

I have presented a simple model of fatigue and recovery which gives a rationale to having multiple breaks at work. Breaks should recover productivity fully so that when the worker resumes work after a break her productivity is at its maximum. If breaks are costly, the optimal number of breaks is finite and those workers whose productivity falls more at the beginning of the day should be allowed more breaks. The main take out of the paper is that breaks should fully recover productivity and that when multiple breaks are desirable, all the working spells before a break should be of identical length. The latter result relies on the assumption that the recovery parameter  $\gamma$  does not change throughout the day. The consumption of coffee or tea or other fatigue reducing substances as well as the natural circadian rhythm may affect the actual time needed for recovery and this may vary along the day.

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