

# IMPROVED INFORMATION IN SEARCH MARKETS

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# Improved Information in Search Markets\*

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## Abstract

This paper studies how an improved information environment affects consumer search and firm competition. We find conditions for information improvement to have unambiguous impacts on search duration, price and consumer welfare. In many cases consumers benefit from information improvement regardless of how it affects the market price, but there are also cases where information improvement raises price significantly so that consumers suffer from it. Our model provides a unified way to consider the market implications of various types of information improvement such as search advertising, personalized recommendations, filtering, and VR shopping technology.

**Key words:** consumer search, price competition, information improvement

**JEL classification:** D43, D83, L13

## 1 Introduction

Over the past two decades consumers have experienced a significantly improved information environment in their shopping process. For example, they often use online platforms to gather product information such as search engines (e.g. Google), product comparison websites (e.g. Expedia), and e-commerce marketplaces (e.g. Amazon).

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These platforms not only help consumers save on the cost of finding sellers, but also often guide consumers towards better and more relevant products. For instance, personalized recommendations or filtering enables consumers to encounter and consider more relevant products first; using a better display technology or offering customer reviews makes the inspection and comparison of products more informative. A related trend is that sellers make use of consumer data (e.g. via data brokers or social media) to target their ads and sometimes even offer personalized products. This also makes consumers face a search pool with more relevant products.

Do consumers search more or less among more relevant products? With a better search pool, they become choosier and aim to find a better matched product before stopping their search, but it also becomes more likely for them to encounter a well-matched product at each step. Do consumers always benefit from improved product information in a search environment? This should be the case if product prices remain unchanged (and if consumers have no intrinsic privacy concerns). However, sellers usually have incentives to adjust their prices given that consumer search behavior is influenced by the information environment. How might sellers change their price? Is it possible that improved information inflates the market price so that consumers end up worse off? These questions are also relevant for platform design. For example, if a platform such as a search engine makes profit from per-click fees, it will have an incentive to make consumers search longer. If a platform is a product comparison website and its profit is from percentage commission fees, its interest will be more aligned with the sellers'. If a platform faces strong competition from other platforms, it will put more weight on consumer surplus.

Given the importance of these questions, it is not surprising that some research has been done on this topic. They include, for example, Eliaz and Spiegler (2011), de Corniere (2016), and Zhong (2020). As we will discuss in detail later, a common message from these works is that an improved search pool will induce consumers to search less and at the same time intensify price competition among sellers (whenever consumer search remains active). However, a profit-maximizing platform may choose to degrade the search pool, or the fee it charges to sellers may inflate their cost and so the final market price.

In this paper we revisit the problem of how improved information affects consumer search and market performance. Our main contribution is to consider a more general form of information improvement and show that its impact is generally ambiguous. In particular, a better information environment can cause higher market prices and harm consumers even without a strategic platform. We adopt the search framework developed in Wolinsky (1986) and Anderson and Renault (1999), and consider a large number of sellers, each supplying a horizontally differentiated product. When a consumer visits a seller, she discovers both its product price and how well matched its product is. The match utility is a random draw from some distribution. As we will discuss more later, this framework has become a standard setup for studying many consumer search related economic issues. We model information improvement by assuming that consumers face a new match utility distribution which is greater than the original one in the sense of “increasing convex order”. Two leading cases are when the match utility distribution becomes higher in the sense of first-order stochastic dominance (FOSD) (e.g. when consumers face more relevant products), or more dispersed in the sense of mean-preserving spread (MPS) (e.g. when the inspection of each product becomes more informative). We do not explicitly model strategic platforms in this paper. Instead we focus on an exogenous change of the information environment and aim to understand its market implications more deeply. Considering strategic platforms is certainly important, but the additional insight from doing so also often depends on the modelling details such as the platform market structure, the fee structure, and whether sellers and consumers multihome or not.

We introduce the model and some preliminaries in Section 2. In Section 3 we examine how improved information affects consumer search behavior. We show that consumers search longer if the distribution becomes more dispersed such that the expected benefit from one more search becomes greater for a given level of the best match utility so far in terms of *percentile*. This defines “excess wealth order” in the stochastic order literature, a requirement stronger than MPS when the mean remains unchanged. The information improvement modelled in the aforementioned existing works leads to a less dispersed match utility distribution and so shorter searches, but this is not always true for a general information improvement.

In Section 4 we study how information improvement influences market price. We show that firms price lower if the new distribution is such that the expected benefit from one more search becomes greater for a given level of the best match utility so far in terms of *hazard rate*. A change of the match utility distribution usually has two opposite effects on pricing. For example, if the reservation match utility in the consumer search rule were fixed, firms would like to raise their prices if the distribution becomes higher in terms of hazard rate, but actually consumers set a higher reservation match utility when the search pool is better, and this makes firms want to lower their price. In the existing works, a clear-cut price comparison result is available because information improvement is modelled in a particular way so that the hazard rate of the match utility distribution remains unchanged.

In Section 5 we investigate how improved information affects consumer welfare. Consumers benefit from information improvement in our model if it lowers market prices. More generally, we show that information improvement benefits consumers if the new distribution is such that the expected benefit from one more search becomes greater for a given level of the best match utility so far in terms of *virtual value* (which is the match utility minus the reciprocal of the hazard rate). However, it is possible that information improvement inflates the market price and consumers suffer from that.

Simpler conditions for unambiguous impacts of information improvement are available in some special cases. For instance, when the search friction is sufficiently small, we show that if information improvement does not change the maximum possible match utility, then in both cases of FOSD and MPS the market price drops and consumers both search less and are better off. Also, when the new distribution is a truncation of the original one from below, information improvement benefits consumers regardless of its impact on price (provided that the search market remains active).

Our model is built on the search framework developed in Wolinsky (1986) and Anderson and Renault (1999).<sup>1</sup> This framework has now become the workhorse model for many recent works on consumer search. They include, for example, prominence

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<sup>1</sup>The other well-known branch of the consumer search literature considers homogeneous products and focuses on price search. The classic works include Diamond (1971), Varian (1980), Burdett and Judd (1983), and Stahl (1989). However, the frameworks there are not suitable for studying product information improvement which motivates this paper.

and ordered search (e.g. Armstrong et al., 2009), attention-grabbing advertising (e.g. Haan and Moraga-Gonzalez, 2011), product design and the long-tail phenomenon (e.g. Bar-Isaac et al., 2012), multiproduct search and retail market structure (e.g. Zhou, 2014, and Rhodes and Zhou, 2019), price directed search (e.g. Choi et al., 2018), and paid recommendation by intermediaries (e.g. Teh and Wright, 2020). Our paper offers a comparative static analysis of this classic setup with respect to the match utility distribution, a question which is important in many applications but which has not been studied systematically in the literature.<sup>2</sup>

As pointed out before, there are existing related works on information quality and search markets. Eliaz and Spiegler (2011) study how a profit-maximizing search engine controls the quality of the search pool. It is shown that the search engine has an incentive to encourage the entry of low-relevance firms by using a low per-click fee. This leads to longer searches and higher market prices. de Corniere (2016) investigates how sellers or a search engine choose the degree of targeting in the context of search advertising. He shows that targeting or an improved search pool makes consumers search less but intensifies price competition among sellers. The final market price, though, may become higher because the per-click fee inflates sellers' cost. Zhong (2020) studies how an online retail platform chooses the match precision in personalized recommendation in a search environment. He shows a similar result that a higher match precision leads to more intense price competition among sellers (whenever consumer search remains active).<sup>3</sup> All these works are built on the Wolinsky framework or a close variant, but they focus on a particular type of information quality change as detailed below.

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<sup>2</sup>Section 4 in Anderson and Renault (1999) discusses how the degree of product differentiation affects search and price competition. They consider the case with a finite number of firms and capture the degree of product differentiation by a multiplicative parameter in front of the match utility random variable. Given their full-market coverage assumption, the change of product differentiation is a special case of the MPS relationship.

<sup>3</sup>If the precision is too high so that consumers never search beyond the first encountered seller, then monopoly pricing will arise, similar to the Diamond (1971) paradox.

## 2 The model

There is a continuum of firms, each supplying a differentiated product at a constant marginal cost normalized to zero. There is a continuum of consumers, each having at most a unit demand for one of the products. We normalize the measure of consumers per firm to one. Both firms and consumers are risk neutral, and each consumer has a zero outside option. In the benchmark case, a product's match utility for a consumer, denoted by  $X_F$ , is a random draw from a distribution with CDF  $F(x)$  and support  $[\underline{x}_F, \bar{x}_F]$ . The realization of  $X_F$  is assumed to be i.i.d. across consumers and products. This implies that firms are *ex ante* symmetric and consumers have idiosyncratic preferences.

In the case with improved information consumers face a new match utility distribution with CDF  $G(x)$ . Let  $X_G$  denote the associated new random variable, and let  $[\underline{x}_G, \bar{x}_G]$  be its support.

**Assumption 1**  $X_G$  is greater than  $X_F$  in the increasing convex order, i.e.,  $\mathbb{E}[\psi(X_G)] \geq \mathbb{E}[\psi(X_F)]$  for any increasing and convex function  $\psi$  whenever the expectations exist.

This essentially assumes that  $X_G$  is higher or more dispersed than  $X_F$ . Two leading cases are:  $X_G$  is an FOSD of  $X_F$  (denoted by  $X_G \geq_{\text{FOSD}} X_F$ ), or  $X_G$  is an MPS of  $X_F$  (denoted by  $X_G \geq_{\text{MPS}} X_F$ ).<sup>4</sup> FOSD captures the scenario when the products in a consumer's search pool become more relevant to the consumer. MPS captures the scenario when the inspection of each product becomes more informative so that the distribution of the estimated match utility becomes more dispersed. Suppose both  $F$  and  $G$  are differentiable, and their associated densities are  $f$  and  $g$ , respectively.

An implicit assumption in our model is that the improved search pool (even after some less relevant products are removed, for example) still has many products, and the products still appear symmetric *ex ante* to consumers.

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<sup>4</sup>See, for example, section 4.A in Shaked and Shanthikumar (2007) for a comprehensive discussion of the increasing convex order. It implies that a risk-seeking decision maker prefers  $X_G$  over  $X_F$ . An alternative definition is that there exists a random variable  $Y$  such that  $X_G \geq_{\text{FOSD}} Y \geq_{\text{MPS}} X_F$  or  $X_G \geq_{\text{MPS}} Y \geq_{\text{FOSD}} X_F$ . Note that  $X_G \geq_{\text{FOSD}} X_F$  if  $G(x) \leq F(x)$  for all  $x$ , and  $X_G \geq_{\text{MPS}} X_F$  if  $\int_{-\infty}^x G(\tilde{x})d\tilde{x} \geq \int_{-\infty}^x F(\tilde{x})d\tilde{x}$  for all  $x$  and the equality holds at  $x = \max\{\bar{x}_F, \bar{x}_G\}$ .

In either case, consumers initially have imperfect information about each product’s match utility and price. They can, however, search sequentially to gather information: by incurring a cost  $s > 0$  a consumer can visit a firm and discover both its match utility and price. During the search process, consumers know the common match utility distribution across products and hold a rational belief of firms’ pricing strategy. Since there are no common shocks across firms, we assume that upon observing an off-equilibrium price in a firm, consumers believe that the other firms still adopt their equilibrium pricing strategy. Firms set their prices simultaneously to maximize their own profit given their rational expectation of consumer search behavior, and consumers search optimally given the match utility distribution and their rational expectation of firms’ pricing strategy. In either case we look for a symmetric equilibrium where all firms charge the same price and consumers search actively and randomly. Note that in our setup with a continuum of symmetric firms, consumers will never return to retrieve a product inspected before, and so whether consumers have free recall or not does not matter. We aim to investigate how an improved search pool with a new distribution  $G$  affects consumer search behavior, market price and consumer welfare.

In the following, we give a few examples which help connect our model with some existing works.

(i) *Quality control by search engines.* Eliaz and Spiegler (2011) consider a variant of the Wolinsky model where each product is either a match or not for a consumer, and conditional on being a match its match utility is a random draw from a common continuous distribution with CDF, say,  $\Phi(x)$ . Products differ in their quality, denoted by  $q$ , in terms of their chance of being a match for a consumer, and the quality is unobservable to consumers.<sup>5</sup> The trade can take place only via a search engine which can control the quality of firms displayed to consumers by setting a per-click fee. Since a higher-quality firm is more willing to join, only the products with a quality above a certain threshold, say,  $\hat{q}$  will join and so be displayed to consumers. Consumers search in this pool sequentially and randomly, and when they encounter a matched product they also immediately discover its match utility. This model differs from ours

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<sup>5</sup>This approach of modelling firm quality heterogeneity in a search framework is from Chen and He (2011). A similar framework has also been used to study, for example, targeted search and product design in Yang (2013), and search and quality investment in Chen, Li, and Zhang (2020).



as it has *ex ante* firm heterogeneity, but its feature of binary match outcomes ensures symmetric pricing across firms, so that it is essentially the same as our model with  $F(x) = \mathbb{E}[1 - q + q\Phi(x)]$  and  $G(x) = \mathbb{E}[1 - q + q\Phi(x)|q \geq \hat{q}]$ , where the expectation is taken over  $q$ . Clearly here  $G$  is an FOSD of  $F$ .

(ii) *Targeted search advertising.* de Corniere (2016) considers a Salop circular model where both a continuum of firms/advertisers and a continuum of consumers are uniformly (but independently) distributed on the circle. In the benchmark, when a consumer enters a query which reveals her taste location, the search engine displays all the firms randomly to her, and the consumer then conducts a sequential search in a random order. This is a spatial version of the Wolinsky model (and it was developed in Wolinsky, 1983). More precisely, since the disutility of buying a non-ideal product is assumed to be weakly convex in the distance between the consumer's taste location and the product location, the model is equivalent to the Wolinsky model with a weakly increasing match utility density function  $f$  (so that  $1 - F$  is concave). de Corniere is interested in the scenario where either the firms or a search engine can control the match precision. In particular, if a firm chooses a match broadness  $d$ , it will appear in a consumer's search pool only if it is within the distance of  $d$  from the consumer's location. This is the same as the Wolinsky model with a truncated distribution where a consumer sees a firm only if its match utility is above a threshold, say,  $\hat{x}$ . If all firms choose the same threshold or the search engine sets the same threshold for all firms, consumers infer that all the firms in their search pool have a match utility distribution with CDF

$$G(x) = \frac{F(x) - F(\hat{x})}{1 - F(\hat{x})}.$$

Here  $G$  is also an FOSD of  $F$ .<sup>6</sup>

(iii) *Filtering and elimination by aspects.* Suppose each product has two attributes and the match utility of a product for a consumer is  $X = X_1 + X_2$ , where  $X_j$  is attribute

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<sup>6</sup>Zhong (2020) studies a similar search design problem in the Wolinsky model. He assumes that personalized recommendations lead to a truncated distribution. More generally we can consider a targeting or recommendation technology by which the platform sees a signal of each product's match utility for a consumer and only displays to the consumer the products with a signal above a certain threshold. When the signal has the standard monotone likelihood ratio property, the distribution of the expected match utility of the displayed products is an FOSD of the original distribution.

$j$ 's match utility. Suppose  $X_1$  and  $X_2$  are independent of each other, and let  $F_j$  be the CDF of  $X_j$ . A popular heuristic decision rule studied in psychology and behavioral economics is “elimination by aspects” (e.g. Tversky, 1972). Suppose consumers are able to filter products (e.g. via a product comparison website) according to attribute 1 and only consider those with  $X_1 > \hat{x}_1$ . Then all the products in the consumer's search pool have a match utility distribution

$$G(x) = \int \frac{F_1(x - x_2) - F_1(\hat{x}_1)}{1 - F_1(\hat{x}_1)} dF_2(x_2) = \frac{F(x) - F_1(\hat{x}_1)}{1 - F_1(\hat{x}_1)} .$$

Here again  $G$  is an FOSD of  $F$ .

(iv) *More informative product inspection.* Suppose that when a consumer inspects a product, she learns a signal of the true match utility which is distributed according to a CDF  $\Phi(x)$ . The signal perfectly reveals the true match utility with probability  $\theta$  and is a pure noise (e.g. a random draw from  $\Phi(x)$  independent of the true match utility) with probability  $1 - \theta$ . Then conditional on a signal realization  $\tilde{s}$ , the consumer's estimate of the match utility is  $\theta\tilde{s} + (1 - \theta)\mu$ , where  $\mu$  is the mean of the true match utility. Suppose the inspection becomes more informative in the sense that the signal precision rises from  $\theta_F$  to  $\theta_G > \theta_F$  (e.g. because a platform starts offering customer reviews or introducing VR shopping). This fits our model with

$$F(x) = \Phi\left(\frac{x - (1 - \theta_F)\mu}{\theta_F}\right) \quad \text{and} \quad G(x) = \Phi\left(\frac{x - (1 - \theta_G)\mu}{\theta_G}\right) .$$

In this example  $G$  is an MPS of  $F$ . Note that this MPS relationship remains true more generally whenever the signal becomes more informative in the Blackwell sense.

In sum, in examples (i)-(iii) information improvement leads to a more “selective” (random) search pool, and in example (iv) information improvement leads to a more “informative” (random) search pool.

For convenience, for a random variable  $X$  with CDF  $H(x)$ , we write

$$\mathbb{E}[(X - u)_+] \equiv \int_{\underline{x}}^{\bar{x}} \max\{0, x - u\} dH(x) = \int_u^{\bar{x}} [1 - H(x)] dx , \quad (1)$$

where the second equality is from integration by parts. This expression captures the expected benefit from an additional search when the match utility distribution is  $H$  and the best match utility so far is  $u$ . We call the price which maximizes  $p[1 - H(p)]$  the standard monopoly price associated with the match utility distribution  $H$ .

For our analysis below we make the following technical assumptions:

**Assumption 2** Both  $1-F$  and  $1-G$  are  $-1$ -concave (i.e., both  $1/(1-F)$  and  $1/(1-G)$  are convex), and the search cost  $s$  is less than  $\min_{i=F,G} \mathbb{E}[(X_i - p_i^M)_+]$ , where  $p_i^M$  is the standard monopoly price associated with distribution  $i$ .

It is ready to check that the assumption of  $-1$ -concavity is equivalent to both  $x - \frac{1-F(x)}{f(x)}$  and  $x - \frac{1-G(x)}{g(x)}$  being increasing functions. As we will see, this ensures that the equilibrium price in each case is determined by the first-order condition. Notice also that the  $-1$ -concavity condition is weaker than the often assumed condition in the literature that  $1-F$  and  $1-G$  are log-concave (or equivalently their hazard rate functions are increasing),<sup>7</sup> and it is satisfied by many often-used distributions. As we will explain later, the search cost condition ensures an active search market in each case.

## 2.1 Some preliminaries

We now characterize the equilibrium in the case of distribution  $F$ . (The analysis for the case of  $G$  is analogous.) Let  $p_F$  denote the symmetric equilibrium price, and let  $r_F$  denote the *reservation match utility* which uniquely solves

$$\mathbb{E}[(X_F - r_F)_+] = \int_{r_F}^{\bar{x}_F} [1 - F(x)] dx = s . \quad (2)$$

When firms charge the same price, a consumer will then cease her search if and only if the best match utility so far is greater than  $r_F$ . Note that  $r_F$  is interior (i.e.  $r_F > \underline{x}_F$ ) under our search-cost assumption, and so some consumers will search beyond the first encountered firm.

It is convenient to denote by

$$\sigma_F \equiv F(r_F)$$

the probability that in equilibrium a consumer will continue to search after visiting a firm. We call it the “search propensity”. By changing the variable in (2) from  $x$  to

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<sup>7</sup>Log-concavity is 0-concavity, and  $\rho$ -concavity is more stringent than  $\rho'$ -concavity when  $\rho > \rho'$ . See, e.g., Caplin and Nalebuff (1991) for the concept of  $\rho$ -concavity.

$t = F(x)$ , we can also define the search propensity as the solution to

$$\int_{\sigma_F}^1 \frac{1-t}{f(F^{-1}(t))} dt = s. \quad (3)$$

Suppose now that a firm unilaterally deviates to price  $p$ . If a consumer comes to visit it, she will stop searching and buy its product immediately if its match utility is such that  $X_F - p > r_F - p_F$ , where the latter is the continuation surplus if the consumer chooses to search on (which is also the equilibrium consumer surplus). Hence, the firm's deviation profit will be proportional to  $p[1 - F(r_F - p_F + p)]$ . In equilibrium the firm should have no incentive to deviate, which requires

$$p_F = \frac{1 - F(r_F)}{f(r_F)}. \quad (4)$$

This first-order condition is also sufficient for defining the equilibrium price when  $1 - F$  is  $-1$ -concave.<sup>8</sup> We can also express  $p_F$  as a function of search propensity:

$$p_F = \frac{1 - \sigma_F}{f(F^{-1}(\sigma_F))}. \quad (5)$$

Both expressions (4) and (5) for  $p_F$  will be useful in the subsequent analysis.

Consumers are willing to participate into the market if  $r_F - p_F > 0$ , or equivalently if  $r_F - \frac{1-F(r_F)}{f(r_F)} > 0$ . Since the standard monopoly price  $p_F^M$  solves  $p = \frac{1-F(p)}{f(p)}$ , this is equivalent to  $r_F > p_F^M$  given the  $-1$ -concavity assumption. Therefore, from the definition of  $r_F$  we know that the primitive condition for an active market is  $s < \mathbb{E}[(X_F - p_F^M)_+]$ , where the right-hand side is the consumer surplus in the monopoly case.<sup>9</sup> This is the search cost condition stated in Assumption 2. In this range of search costs, when  $s$  increases, the reservation match utility  $r_F$  becomes smaller and so does the search propensity. This increases the price if the hazard rate function  $\frac{f}{1-F}$  is increasing (or if  $1 - F$  is log-concave), but decreases the price if the hazard rate function is decreasing (or if  $1 - F$  is log-convex). Under the  $-1$ -concavity condition, however, an increase of  $s$  always lowers consumer surplus  $r_F - p_F$ , regardless of how price varies.

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<sup>8</sup>When  $1 - F$  is  $-1$ -concave,  $x - \frac{1-F(x)}{f(x)}$  is an increasing function, and then it is easy to check that a firm's profit is single-peaked at  $p = p_F$ .

<sup>9</sup>When  $s$  is above this threshold, there will be no equilibrium with an active market. One way to avoid that uninteresting outcome is to assume that the first search is free. In that case, consumers will always buy from the first firm they encounter and each firm charges the monopoly price  $p_F^M$ .

An analogous analysis for the case of  $G$  applies when  $s < \mathbb{E}[(X_G - p_G^M)_+]$ . In particular, the reservation match utility  $r_G$  in the new case solves  $\mathbb{E}[(X_G - r_G)_+] = s$  and the search propensity is  $\sigma_G \equiv G(r_G)$ . Then the new market price is

$$p_G = \frac{1 - G(r_G)}{g(r_G)} = \frac{1 - \sigma_G}{g(G^{-1}(\sigma_G))}. \quad (6)$$

### 3 Consumer search behavior

We first examine how information improvement affects consumer search behavior and in particular consumer search duration. Search duration is often the only observable related to consumer search behavior, and is also relevant to platform design if a platform, say, a search engine makes money from charging firms per-click fees.

Given  $X_G$  is greater than  $X_F$  in the increasing convex order, we have

$$\mathbb{E}[(X_G - u)_+] \geq \mathbb{E}[(X_F - u)_+] \text{ for any } u \quad (7)$$

since  $(X - u)_+$  is an increasing and convex function of  $X$ .<sup>10</sup> That is, for any given best match utility so far, the expected benefit from one more search is greater in the case of  $G$  than in the case of  $F$ . From the definition of  $r_F$  and  $r_G$ , it is immediate that consumers become choosier and set a higher reservation match utility in the case of  $G$  (i.e.,  $r_F \leq r_G$ ).

This, however, does *not* mean that consumers necessarily search longer in the case of  $G$  since the distribution changes at the same time. For example, when  $G$  is higher than  $F$  in the sense of FOSD, consumers are also more able to find a high match utility at each firm. More precisely, the (expected) consumer search duration is determined by the search propensity:

$$l_F \equiv \frac{1}{1 - \sigma_F} \quad \text{and} \quad l_G \equiv \frac{1}{1 - \sigma_G}, \quad (8)$$

but how an information improvement affects the search propensity is not clear. The following result reports conditions for a clear-cut comparison of search duration.

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<sup>10</sup>In fact (7) is an alternative definition of the increasing convex order, as any increasing convex function can be approximated by a linear combination of  $(X - u)_+$  with different  $u$ 's.

**Proposition 1** (i) Consumers search longer in search pool  $G$  (i.e.  $l_F \leq l_G$ ) if  $X_G$  is greater than  $X_F$  in the “excess wealth order”, i.e. if

$$\mathbb{E}[(X_G - G^{-1}(\sigma))_+] \geq \mathbb{E}[(X_F - F^{-1}(\sigma))_+] \text{ for any } \sigma \in (0, 1) . \quad (9)$$

(ii) Suppose both  $f(\bar{x}_F)$  and  $g(\bar{x}_G)$  are strictly positive. Then there exists  $\hat{s}$  such that for  $s < \hat{s}$ , consumers search longer in search pool  $G$  if and only if  $g(\bar{x}_G) < f(\bar{x}_F)$ .

The first result is ready to see from the definition of search propensity. From (3) and its counterpart for  $G$ , we have

$$\int_{\sigma_F}^1 \frac{1-t}{f(F^{-1}(t))} dt = \int_{\sigma_G}^1 \frac{1-t}{g(G^{-1}(t))} dt .$$

Then  $\sigma_F \leq \sigma_G$  (i.e. consumers search longer in the case of  $G$ ) if

$$\int_{\sigma}^1 \frac{1-t}{f(F^{-1}(t))} dt \leq \int_{\sigma}^1 \frac{1-t}{g(G^{-1}(t))} dt \quad (10)$$

for any  $\sigma \in (0, 1)$ . This is equivalent to (9) by changing variable from  $x$  to  $t = F(x)$  or  $G(x)$ . The second result is proved in the appendix.

A result similar to result (i) has been shown in Chateauneuf, Cohen and Meilijson (2004) (see its section 2.3.4). Excess wealth order is one way to compare the degree of variability of two random variables. It is location-free as only percentiles matter.<sup>11</sup> Notice that  $\mathbb{E}[(X_F - F^{-1}(\sigma))_+]$  is the expected benefit from one more search in the case of  $F$  when the best match utility so far has reached the  $100\sigma_{\text{th}}$  percentile. So (9) means that when the best match utility so far has reached a given percentile, the consumer has a higher incentive to search in the case of  $G$  than in the case of  $F$ . When  $X_F$  and  $X_G$  have the same mean, excess wealth order implies MPS, the more familiar concept for comparing dispersion.<sup>12</sup> But MPS is not sufficient for a clear-cut comparison result

<sup>11</sup>See Section 3.C in Shaked and Shanthikumar (2007) for a comprehensive discussion of the excess wealth order.

<sup>12</sup>When two random variables have the same mean, excess wealth order therefore implies increasing convex order. This, however, may not be true if they have different means. For example, suppose  $X_F$  is uniform on  $[0, 1]$  and  $X_G$  is uniform on  $[k, 1]$  with  $k \in (0, 1)$ . It is easy to verify that  $X_F$  is greater than  $X_G$  in the excess wealth order (i.e.  $X_F$  is more dispersed), but  $X_G$  is clearly greater than  $X_F$  in the increasing convex order since  $X_G \geq_{\text{FOSD}} X_F$ .

concerning search duration.<sup>13</sup>

Result (ii) is intuitive to understand. When  $s$  is close to zero, consumers will cease their search only if they find a match utility close to the upper bound of the distribution. Then basically the density of match utility at the upper bound determines the likelihood of ceasing search. If  $F$  and  $G$  share the same upper bound  $\bar{x}$ , both FOSD and MPS imply that  $G(x) \leq F(x)$  for  $x$  close to  $\bar{x}$  and so  $g(\bar{x}) \geq f(\bar{x})$ . Therefore, result (ii) implies that  $G$  will induce consumers to search less when the search friction is small.

A sufficient condition for (9) or (10) is  $f(F^{-1}(t)) \geq g(G^{-1}(t))$ . One can check this is equivalent to that  $G^{-1}(t) - F^{-1}(t)$  increases in  $t$ , i.e., the quantile difference between  $X_G$  and  $X_F$  increases in  $t$ . This is the definition of  $X_G$  being greater than  $X_F$  in the “dispersive order”.<sup>14</sup> Then we have the following result:

**Corollary 1** *Consumers search longer in search pool  $G$  (i.e.  $l_F \leq l_G$ ) if  $X_G$  is greater than  $X_F$  in the “dispersive order” (i.e. if  $G^{-1}(t) - F^{-1}(t)$  increases in  $t$ ).*

A similar result is also shown in Choi and Smith (2019). Dispersive order is another way to compare dispersion. It is a stronger requirement than excess wealth order but is also easier to check. One special case of dispersive order is that  $\alpha X + \beta$  is greater than  $X$  in dispersive order for any constant  $\beta$  whenever  $\alpha \geq 1$ . This implies, for example, that if both  $F$  and  $G$  are normal distributions, consumers search longer in  $G$  if it has a greater variance. (The way how Anderson and Renault, 1999, model product differentiation also belongs to this dispersive order relationship.)

We now discuss whether consumers search longer or shorter in the four examples introduced before. In the first three examples, it can be shown that  $G$  is smaller (greater) than  $F$  in the dispersive order if  $1 - F$  is log-concave (log-convex), while in

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<sup>13</sup>Here is one counterexample: Suppose  $F$  has a triangle density on  $[0, 1]$  (i.e.  $F(x) = 2x^2$  for  $x \in [0, \frac{1}{2}]$  and  $1 - 2(1 - x)^2$  for  $x \in [\frac{1}{2}, 1]$ ). Suppose  $G$  is the uniform distribution on  $[0, 1]$  and so it is an MPS of  $F$ . When  $s < \frac{1}{12}$ , one can check that  $r_F = 1 - (\frac{3}{2}s)^{1/3}$  and  $r_G = 1 - (2s)^{1/2}$ . Then  $l_F = [2(1 - r_F)^2]^{-1} < l_G = [1 - r_G]^{-1}$  if and only if  $s > \frac{1}{81}$ . In other words, whether consumers search longer or not in the case of  $G$  depends on the magnitude of the search cost.

<sup>14</sup>Dispersive order has been used to study various economics problems. See, for example, Ganuza and Penalva (2010) for its application in information disclosure in auctions, Zhou (2017) and Choi, Dai, and Kim (2018) for its application in oligopolistic competition.

the fourth example,  $G$  is always greater than  $F$  in the dispersive order. Hence, we have the following result. (All omitted proofs can be found in the appendix.)

**Corollary 2** *In examples (i)-(iii), consumers search shorter in search pool  $G$  if  $1 - F$  is log-concave and longer if  $1 - F$  is log-convex. In example (iv), consumers always search longer in search pool  $G$ .*

One implication of Corollary 2 is that if a platform such as a search engine aims to maximize consumer search duration (e.g. because it makes money from per-click fees), it has an incentive to make the search pool more “informative” as in example (iv), but not more “selective” as in examples (i)-(iii) if the match utility distribution is regular in terms of log-concavity.

## 4 Market price

We now examine how information improvement affects the market price. (Profit comparison is the same as price comparison since profit is proportional to price in our model given all consumers buy in equilibrium.) In our model each firm acts as a local monopolist facing consumers who regard the continuation value of search as their outside option. As we have seen, the equilibrium price is the reciprocal hazard rate of the match utility distribution (which reflects the demand composition) evaluated at the reservation match utility (which captures consumer search incentive). A change of the match utility distribution often has opposite effects on these two components. For example, suppose information improvement leads to a higher distribution in terms of hazard rate (i.e.  $\frac{g(x)}{1-G(x)} \leq \frac{f(x)}{1-F(x)}$ ). From (4) it is ready to see that this increases the market price for a given reservation match utility. However, as this improvement is a case of FOSD, we also have  $r_G \geq r_F$  and this is an opposite force to lower the market price if the hazard rate functions are increasing. The price expression in (5) helps illustrate a similar trade-off when information improvement leads to a more dispersed distribution in terms of dispersive order (i.e.  $g(G^{-1}(t)) \leq f(F^{-1}(t))$ ). This increases the price for a given search propensity, but as this change induces a greater search propensity as shown in Corollary 1, there is also an opposite force to lower the price if



the hazard rate functions are increasing.<sup>15</sup> For this reason, it is often hard to obtain a clear-cut result on how a change of the match utility distribution affects price in the Wolinsky model.

The following result reports conditions for an unambiguous price comparison result. For convenience, let

$$\tau_F(x) \equiv \frac{1 - F(x)}{f(x)} \quad \text{and} \quad \tau_G(x) \equiv \frac{1 - G(x)}{g(x)}$$

be the reciprocal hazard rate in the case of  $F$  and  $G$ , respectively. They are decreasing (increasing) functions if and only if  $1 - F$  and  $1 - G$  are log-concave (log-convex).

**Proposition 2** (i) *Suppose both of the reciprocal hazard rates  $\tau_F$  and  $\tau_G$  are monotonic. Price is higher in search pool  $G$  (i.e.  $p_F \leq p_G$ ) if*

$$\mathbb{E}[(X_G - \tau_G^{-1}(p))_+] \leq \mathbb{E}[(X_F - \tau_F^{-1}(p))_+] \quad \text{for any } p \quad (11)$$

*and at least one of the reciprocal hazard rates is decreasing. The opposite is true if one of the two conditions is reversed.*

(ii) *Suppose both  $f(\bar{x}_F)$  and  $g(\bar{x}_G)$  are strictly positive. Then there exists  $\hat{s}$  such that for  $s < \hat{s}$ , price is higher in search pool  $G$  if and only if  $g(\bar{x}_G) < f(\bar{x}_F)$ .*

Notice that  $\mathbb{E}[(X_F - \tau_F^{-1}(p))_+]$  is the expected benefit from one more search in the case of  $F$  when the best match utility so far has reached a certain level in terms of hazard rate. There are no existing stochastic order concepts which imply (11). A simple case where the opposite of (11) holds is when information improvement does not change the hazard rate of the match utility distribution (which is true in examples (i)-(iii) as we show below). The second result for a small  $s$  is intuitive. When the search friction is small, consumers will not stop searching until finding an almost perfect match. In other words, for each firm their marginal consumers have a match utility close to the upper bound. The density of these marginal consumers essentially determines firms' pricing incentive.

The second result has two implications: First, together with result (ii) in Proposition 1, we can conclude that when  $s$  is small, search duration and price move in the *same*

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<sup>15</sup>In these two cases, if the hazard rate functions are decreasing, then the two forces work in the same direction and the market price will increase.

direction, which is opposite to the usual intuition from search models that price is higher (lower) when consumers search less (more). Intuitively, when the match utility distribution becomes, for example, more concentrated around the upper bound, it is as if products become less differentiated. This induces consumers to search less, but at the same time price competition is intensified.<sup>16</sup> (This is not always true if the search cost is not small.) Second, if  $F$  and  $G$  share the same upper bound  $\bar{x}$ , both FOSD and MPS imply  $g(\bar{x}) \geq f(\bar{x})$ , and so when  $s$  is small,  $G$  must induce a lower market price. However, when  $F$  and  $G$  have different upper bounds (e.g. in example (iv)), the outcome can be reversed as shown in the corollary below.

**Corollary 3** *In examples (i)-(iii), price and profit are lower in search pool  $G$  if  $1 - F$  is log-concave and higher if  $1 - F$  is log-convex. In example (iv), price and profit are higher in search pool  $G$  at least when the search cost is sufficiently small.*

In examples (i)-(iii),  $G(x) = kF(x) + 1 - k$  for  $x$  in the support of  $G$ , where  $k > 1$  is a constant. Then

$$\frac{1 - G(x)}{g(x)} = \frac{k(1 - F(x))}{kf(x)} = \frac{1 - F(x)}{f(x)},$$

i.e. the two distributions have the same hazard rate. This special property leads to a clear-cut price and profit comparison result given  $r_G \geq r_F$ .<sup>17</sup> When this property does not hold, however, information improvement with  $X_G \geq_{\text{FOSD}} X_F$  can induce a higher market price even in the regular case with log-concavity. For example, consider  $F(x) = x$  and  $G(x) = x^{10}$ . One can check, for instance, when  $s = 0.1$ , we have  $p_G \approx 0.52 > p_F \approx 0.45$ . (But given  $g(1) > f(1) > 0$  in this example, price must go down with  $G$  if the search cost is sufficiently small.)

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<sup>16</sup>A related observation is that if  $G$  induces a higher price for any permitted  $s$ , it must also induce consumers to search longer. To see that, notice that differentiating both (3) and its counterpart for  $\sigma_G$  with respect to  $s$  yields  $p_F(-\frac{d\sigma_F}{ds}) = p_G(-\frac{d\sigma_G}{ds}) = 1$ , and so  $p_F \leq p_G$  implies  $-\frac{d\sigma_G}{ds} \leq -\frac{d\sigma_F}{ds}$ . Since  $\sigma_F = \sigma_G = 1$  at  $s = 0$  and both decrease in  $s$ , this leads to  $\sigma_G \geq \sigma_F$  and so search duration is longer in the case of  $G$  for any permitted  $s$ .

<sup>17</sup>Note that when  $1 - F$  is log-convex, it must be the case that  $f(\bar{x}_F) = 0$  and so result (ii) in Proposition 2 does not apply. Thus, the result that price rises in the case of  $G$  in the first three examples does not contradict with Proposition 2.

In example (iv),  $G$  is an MPS of  $F$  with a broader support. We have  $\bar{x}_i = \theta_i \bar{x} + (1 - \theta_i)\mu$ , and so  $g(\bar{x}_G) = \phi(\bar{x})/\theta_G < f(\bar{x}_F) = \phi(\bar{x})/\theta_F$ , where  $\phi$  is the density function of  $\Phi$ . The result when  $s$  is small then follows from result (ii) in Proposition 2. (This is opposite to the case when  $G$  is an MPS of  $F$  and they share the same upper bound.) The same result can hold even for a larger search cost. Consider the uniform example with  $\Phi(x) = x$ . Then when the signal precision is  $\theta$ , the CDF is  $\frac{1}{\theta} \left(x - \frac{1-\theta}{2}\right)$  and it has support  $[\frac{1-\theta}{2}, \frac{1+\theta}{2}]$ . One can check that the reservation match utility is  $\frac{1}{2}(1 + \theta) - \sqrt{2\theta s}$  and the equilibrium price is  $\sqrt{2\theta s}$  when  $s < \min\{\frac{\theta}{2}, \frac{(1+\theta)^2}{32\theta}\}$  (which is required by Assumption 1). Therefore, in this example price always increases as information improves as long as the search market is active.

If a platform such as a product comparison website aims to maximize industry profit (e.g. because it earns a percentage of sellers' profit by charging commission fees), it has no incentive to make the search pool more "selective" as in examples (i)-(iii) when the distribution is regular,<sup>18</sup> but often has an incentive to make the search pool more "informative" as in example (iv).

## 5 Consumer surplus

In our setup total welfare is simply the reservation match utility. Hence, information improvement must enhance total welfare when  $G$  is greater than  $F$  in the increasing convex order. Given consumer surplus equals the reservation match utility minus price, information improvement must also benefit consumers if it induces a lower price.

More generally, let us define two "virtual value" functions:

$$\eta_F(x) \equiv x - \frac{1 - F(x)}{f(x)} \quad \text{and} \quad \eta_G(x) \equiv x - \frac{1 - G(x)}{g(x)}.$$

Then consumer surplus is  $\eta_F(r_F)$  and  $\eta_G(r_G)$ , respectively. (Recall that both of the  $\eta$  functions are increasing given the  $-1$ -concavity assumption.) Similar results as in Proposition 2 follow if we replace the  $\tau$  functions there by the  $\eta$  functions.

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<sup>18</sup>Notice, however, that the market is fully covered in our model due to the existence of an infinite number of firms, so the potential effect of information improvement on consumer participation is ignored.

**Proposition 3** (i) Consumers are better off in search pool  $G$  if

$$\mathbb{E}[(X_G - \eta_G^{-1}(v))_+] \geq \mathbb{E}[(X_F - \eta_F^{-1}(v))_+] \text{ for any } v. \quad (12)$$

(ii) Suppose both  $f(\bar{x}_F)$  and  $g(\bar{x}_G)$  are strictly positive. Then there exists  $\hat{s}$  such that for  $s < \hat{s}$ , consumers are better off in search pool  $G$  if and only if

$$\bar{x}_G - 2\sqrt{\frac{2s}{g(\bar{x}_G)}} > \bar{x}_F - 2\sqrt{\frac{2s}{f(\bar{x}_F)}}.$$

Note that  $\mathbb{E}[(X_F - \eta_F^{-1}(v))_+]$  is the expected benefit from one more search in the case of  $F$  when the best match utility so far has reached a given level in terms of the virtual value  $\eta(x)$ . There are no existing stochastic order concepts which imply (12). As in the case of price comparison, a simple case where (12) holds is when information improvement does not change the hazard rate. Result (ii) follows immediately from the proofs of result (ii) in both Proposition 1 and Proposition 2. It implies that when the search cost is small and both  $F$  and  $G$  have the same upper bound, consumer surplus comparison is simply the reverse of price comparison. However, the outcome can be very different if  $F$  and  $G$  have different upper bounds. For instance, in example (iv) information improvement can enhance both profit and consumer surplus.

**Corollary 4** In examples (i)-(iii), consumers are better off in search pool  $G$ . In example (iv), consumers are better off in search pool  $G$  at least when the search cost is sufficiently small.

In examples (i)-(iii), we have known that  $\frac{1-F(x)}{f(x)} = \frac{1-G(x)}{g(x)}$ . Then  $\eta_F(x) = \eta_G(x)$  and so consumers must benefit from information improvement given  $r_F \leq r_G$  and  $\eta_F(x)$  is increasing. This is true regardless of how information improvement affects the price.

In example (iv), when  $s \approx 0$  one can check that consumer surplus with signal precision  $\theta$  is approximately  $\theta\bar{x} + (1-\theta)\mu - 2\sqrt{2s\theta\phi(\bar{x})}$ . This is increasing in  $\theta$  when  $s$  is small. This result, however, is not robust to a larger  $s$ . Consider the uniform example with  $\Phi(x) = x$ . Following the analysis before, one can check that consumer surplus in this example, when the signal precision is  $\theta$ , is  $\frac{1}{2}(1+\theta) - 2\sqrt{2\theta s}$ , which increases in  $\theta$  if and only if  $s < \frac{\theta}{8}$ . Recall that our solution is valid when  $s < \min\{\frac{\theta}{2}, \frac{(1+\theta)^2}{32\theta}\}$ , and this search cost cap is greater than  $\frac{\theta}{8}$  for any  $\theta$ . Therefore, in this example, starting

from any  $\theta$  improving information benefits consumers if  $s \leq \frac{\theta}{8}$  but harms consumers if  $\frac{\theta}{8} < s < \min\{\frac{\theta}{2}, \frac{(1+\theta)^2}{32\theta}\}$ . That is, the negative price effect dominates when the search cost is relatively high.

One implication of Corollary 4 is that if a platform aims to improve consumer surplus (e.g. because it faces strong competition from other platforms), it has an incentive to make the search pool more “selective” as in examples (i)-(iii) and also more “informative” as in example (iv) at least when the search cost is small.

## 6 Conclusion

This paper has studied how product information improvement affects consumer search duration, market price, and consumer welfare. Although in general the impact on each variable can go either direction, we have derived conditions for an unambiguous assessment. In particular, we show that when the search friction is small, search duration and market price tend to move in the same direction, and information improvement benefits consumers if it does not change the maximum possible match utility. Our setup also provides a unified perspective to consider various types of information improvement which have been separately studied in the literature. This paper regards the information improvement as exogenous. In practice, however, information improvement is often strategically chosen by firms or information platforms. This lack of endogenous information in our model clearly limits the relevance of our welfare assessments.

There are other possible ways to model improved information in a search market. For example, Anderson and Renault (2000) model information improvement by assuming that in the Wolinsky model some consumers become informed (i.e. they know the match utilities of all the products) before search. Since these consumers have no incentives to search beyond the best matched product which they already know, their presence relaxes price competition and harms other uninformed consumers. This is similar as making some consumers informed of their best matched products (e.g. due to a perfect personalized recommendation). If all consumers are informed of their best matched products, then they will not search beyond the recommended product, and due to Diamond (1971)’s argument each firm will act as a monopolist conditional on being the

best matched supplier.<sup>19</sup> A more general approach is to assume that consumers are informed of several top matched products (but without their ranking). Since the top matched products have an improved conditional match utility distribution, the situation is similar to the FOSD case in this paper. There is an extra complication, however, when the total number of firms is finite: the (conditional) match utilities of the top products are correlated and this causes significant complexity in the demand analysis.<sup>20</sup> It is also possible to consider information design in a search market. For example, Dogan and Hu (2019) consider the same search framework and study how informative the inspection of each product should be if we want to maximize consumer surplus. This is related to example (iv) in Section 2 if we allow for a general signal structure. More broadly, information design in a search market can consider not only the informativeness of each product inspection, but also the disclosure of relative valuations across products, and even the control of which sellers to display to consumers in the spirit of Eliaz and Spiegel (2011) and de Corniere (2016).

## Appendix

*Proof of Proposition 1:* Here we prove result (ii). It suffices to show the result when  $s \approx 0$ . Recall that  $\sigma_F$  solves  $\int_{\sigma_F}^1 \frac{1-t}{f(F^{-1}(t))} dt = s$ . When  $s$  is close to zero,  $\sigma_F$  is close to 1. Using the (second-order) Taylor expansion and  $f(F^{-1}(1)) = f(\bar{x}_F) > 0$ , we can approximate the integral term on the left-hand side as  $\frac{1}{2}(\sigma_F - 1)^2 / f(\bar{x}_F)$ . Then

$$1 - \sigma_F \approx \sqrt{2sf(\bar{x}_F)}. \tag{13}$$

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<sup>19</sup>More precisely each firm will act a multiproduct monopolist which sells all the products in the market. With an infinite number of firms, this will lead to a price equal to the maximum match utility and so the market will collapse unless the first search is free. The outcome, however, will be very different if each consumer is informed of the product with the highest surplus (i.e. match utility minus price). In that case the outcome will be the same as in the perfect information case. See, e.g., Teh and Wright (2020) for a model of recommendation with consumer search in this vein.

<sup>20</sup>A tractable case is studied in Burguet and Petrikaite (2019). They consider targeted advertising in the Wolinsky model with a finite number of firms. They assume that each firm sends their ads only to the consumers who regard their product as one of the top *two* products, and consumers search only among the products from which they receive ads.

Similarly, one can derive  $1 - \sigma_G \approx \sqrt{2sg(\bar{x}_G)}$  when  $s$  is close to zero. Then the desired result follows immediately.

*Proof of Corollary 2:* We use the dispersive order result in Corollary 1 to prove this result. In examples (i)-(iii) we have

$$G(x) = kF(x) + 1 - k \quad (14)$$

for  $x$  in the support of  $G$ , where  $k > 1$  is a constant. Then

$$G^{-1}(t) = F^{-1}\left(1 - \frac{1-t}{k}\right) \quad (15)$$

for any  $t \in (0, 1)$ . One can check that  $F^{-1}(t) - G^{-1}(t)$  increases in  $t$  if  $kf(G^{-1}(t)) > f(F^{-1}(t))$  for  $k > 1$ , and otherwise decreases in  $t$ . Notice that  $kf(G^{-1}(t)) = f(F^{-1}(t))$  at  $k = 1$ , and one can check that the derivative of  $kf(G^{-1}(t))$  with respect to  $k$  is

$$f(z) + k \frac{f'(z)}{f(z)} \frac{1-t}{k^2} = f(z) + \frac{f'(z)}{f(z)} [1 - F(z)] , \quad (16)$$

where  $z = G^{-1}(t)$  and the equality used (15). When  $1 - F$  is log-concave, we have  $f^2 + (1 - F)f' \geq 0$  and so (16) is positive. Then  $F^{-1}(t) - G^{-1}(t)$  increases in  $t$  and so  $F$  is greater than  $G$  in the dispersive order. The opposite is true if  $1 - F$  is log-convex.<sup>21</sup>

In example (iv),  $X_G$  can be written as  $kX_F + (1-k)\mu$ , where  $k = \mathbb{E}[\theta | \theta \geq \hat{\theta}] / \mathbb{E}[\theta] > 1$ . Since dispersive order is location-free,  $X_G$  is greater than  $X_F$  in dispersive order and so consumers search longer in the case of  $G$ .

*Proof of Proposition 2:* (i) It is more convenient to prove this result by using the price expression with the reservation match utility. Suppose  $\tau_F(x)$  is decreasing. Since  $p_F = \tau_F(r_F)$  and  $p_G = \tau_G(r_G)$  and both  $\tau_F(\cdot)$  and  $\tau_G(\cdot)$  are monotonic functions, the definitions of  $r_F$  and  $r_G$  imply that

$$\mathbb{E}[(X_G - \tau_G^{-1}(p_G))_+] = \mathbb{E}[(X_F - \tau_F^{-1}(p_F))_+] .$$

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<sup>21</sup>One may wonder, given  $F^{-1}(t) - G^{-1}(t) < 0$  in the examples (i)-(iii) and  $F^{-1}(1) - G^{-1}(1) = 0$  (if the upper bound of the distribution support is finite), how  $F^{-1}(t) - G^{-1}(t)$  can be decreasing in  $t$ . Notice, however, that the log-convexity of  $1 - F$  requires the support of the distribution have an infinite upper bound, in which case  $\lim_{t \rightarrow 1} [F^{-1}(t) - G^{-1}(t)]$  should be  $-\infty$ .

On the other hand, letting  $p = p_G$  in (11) yields

$$\mathbb{E}[(X_G - \tau_G^{-1}(p_G))_+] \leq \mathbb{E}[(X_F - \tau_F^{-1}(p_G))_+].$$

Then we have

$$\mathbb{E}[(X_F - \tau_F^{-1}(p_F))_+] \leq \mathbb{E}[(X_F - \tau_F^{-1}(p_G))_+]$$

or equivalently  $\tau_F^{-1}(p_F) \geq \tau_F^{-1}(p_G)$ . This implies  $p_F \leq p_G$  given  $\tau_F(x)$  is decreasing. (If  $\tau_G(x)$  is decreasing, a similar argument applies by letting  $p = p_F$  in (11).)

(ii) It suffices to prove the result when  $s \approx 0$ . Notice that when  $\sigma$  is close to 1 and  $f(\bar{x}_F) > 0$ , we have

$$\frac{1 - \sigma}{f(F^{-1}(\sigma))} \approx \frac{1 - \sigma}{f(\bar{x}_F)}$$

by using the Taylor expansion. This, together with (13), implies that when  $s$  is close to zero, we have

$$p_F = \frac{1 - \sigma_F}{f(F^{-1}(\sigma_F))} \approx \sqrt{\frac{2s}{f(\bar{x}_F)}}.$$

Similarly,

$$p_G \approx \sqrt{\frac{2s}{g(\bar{x}_G)}}.$$

Then the desired result follows.

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