IRVING FISHER, RAGNAR FRISCH AND THE ELUSIVE QUEST FOR MEASURABLE UTILITY

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Abstract: Commitment to the behaviorist approach to utility theory, to the usefulness of mathematics in economic analysis and to equalization of the marginal utility of income as a principle of just taxation brought Irving Fisher and Ragnar Frisch to attempt to measure the marginal utility of income, and led them to collaborate in forming the Econometric Society and sponsoring the establishment of the Cowles Commission, institutions advancing economic theory in connection to mathematics and statistics.

To be presented at a symposium at the University of Oslo, December 3, 2019, honoring the 50th anniversary of the award to Ragnar Frisch of the Royal Bank of Sweden Prize in Economic Science in Memory of Alfred Nobel and the 30th anniversary of the award of that prize to Trygve Haavelmo.

Introduction

Ragnar Frisch acclaimed Irving Fisher (1892) as the pioneer of the behaviorist approach to consumer

choice theory and ordinal utility, preceding Pareto and differing from Edgeworth's hope to measure

sensations. While preference ordering suffices for analysis of individual choice of goods, Frisch (1926,

1932) and Fisher (1927) sought to measure marginal utility by seeking some commodity class whose

marginal utility was independent of the consumption of other classes of commodities. Fisher (1927)

linked this goal to Edgeworth's principle that just taxation should equalize the utility of taxes paid across

people. Frisch and Fisher were unsuccessful in identifying such a commodity class and further research

on cardinal utility, such as by Fisher's protégé William Vickrey (1945), looked instead to comparison of

risky situations, using probability weights to measure the marginal utility of wealth, while researchers

such as Wald (1940) and Wallis and Friedman (1942) looked to determination of indifference surfaces

rather than utility functions. Nonetheless the attempts by Frisch and Fisher to measure the marginal

utility of money had important consequences, beyond the stimulus their writings gave to Vickrey (1945).

Frisch (1926), followed up by Franz Alt (1936), pioneered the axiomatic approach to utility theory, and

Frisch drew attention to Fisher's priority over Pareto in the history of choice theory. The shared concern

of Frisch and Fisher with measurement of marginal utility brought Frisch to Yale as a visiting professor in

1930-31, leading to their collaboration in founding the Econometric Society at the end of 1930 and then

in late 1931 and early 1932 to persuade the Econometric Society to accept Alfred Cowles's offer to

finance both a journal (*Econometrica*) and a research foundation affiliated with the society (the Cowles

Commission for Research in Economics). Yale's offer to Frisch to remain as a full professor at the end of his visiting position led the University of Oslo to appoint Frisch as a full professor, just five years after his doctorate, and to establish a research laboratory for him.

Irving Fisher on Measurable Utility

Irving Fisher (1927, 157-58) regretted that "The basic importance of this concept [marginal utility][1] has been partially lost sight of because of the growth of statistical economics and the lack hitherto of any method of showing that such a purely psychical magnitude is at least capable of being measured, granted the necessary data… If so-called 'marginal utility' of anything, (or, as I prefer to say, if the want-for-one-more unit of anything) is a true mathematical quantity, should not the marginal want be measurable?" Fisher added that "In my first economic publication [1892, pp. 11-24, 86-89], I endeavored to show that this magnitude is measurable – in theory, at least. My object here is to go one step further and to show that even the problem of statistically measuring it should not be considered impossible." In a footnote he stated that "So far as I know this is the only attempt (other than Edgeworth's therein cited [in Fisher 1892]) of treating 'utility' or 'want' as a definite mathematical quantity."

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[1] Fisher (1927, p. 157), writing in honor of John Bates Clark, praised Clark for "his discovery, independently of Jevons, Menger, and Walras or their anticipators, of the concept of 'Marginal Utility' … He is the only American who has that honor." Although Fisher (1927) cited his dissertation (Fisher 1892) on the same page, he did not draw an explicit parallel to his own independent rediscovery of general equilibrium analysis, managing to obtain the books of Walras and Edgeworth only as he was about to submit his completed thesis: "The equations in Chapter I, §10, were found by me two years, when I had read no mathematical economist except Jevons" (Fisher 1892, p. 4).

Fisher there drew attention to a remarkable aspect of his and Edgeworth's works. Jevons, Carl

Menger and Léon Walras, the marginalist pioneers of the 1870s, had reasoned in terms of cardinal

utility. In contrast, Edgeworth (1881) and Fisher (1892) pioneered an ordinal approach to preferences

and choice, drawing indifference curves (before Vilfredo Pareto) and, for more two commodities,

indifference surfaces and showing that choice theory did not require knowing the unobservable

marginal utility of any good, but only the ratio between the marginal utilities of any two goods, the

marginal rate of substitution represented by the slope of the indifference curve, a curve given (in

principle) by observable choices of an individual. All that needed to be observed was whether an

individual preferred bundle A to bundle B, not how many utils of pleasure or desire or want the

individual would derive from either bundle. And yet, having shown that cardinally measurable utility was

not needed to analyze choice by an individual among bundles of goods, Edgeworth (1881, 1887) and

Fisher (1892, pp. 11-24, 86-89) concerned themselves with how to measure marginal utility whereas

Jevons, Menger and Walras did not (see Ellsberg 1954 on the history of concepts of utility). While

ordinal utility, without interpersonal comparisons of utility, sufficed for understanding individual

demand for goods, Edgeworth (1919) and Fisher (1927) later advanced interpersonal equalization of the

utility of taxes paid as the criterion for just taxation. In the case of Fisher (1892), concern with measuring

utility, even if cardinally measurable utility was not needed to analyze individual choice, was consistent

with his engineering background as a student of the physicist Josiah Willard Gibbs, which contributed to

Fisher's focus on concrete results. For his 1891 dissertation and 1892 book, Fisher did not merely

rediscover the Walrasian system of simultaneous equations for general equilibrium but constructed a

hydraulic mechanism to simulate the determination of equilibrium prices and quantities, anticipating

the much later development of computable general equilibrium (Scarf 1967, Dimand 2019, Chapter 2).

Edgeworth (1881, 1887) appealed to experimental psychology, equating a util to the smallest

perceptible increment in pleasure or sense of well-being. Fisher (1892, p. 5) protested that "This foisting

of Psychology upon Economics seems to be inappropriate and vicious. Others besides Prof. Edgeworth

have done it. Gossen and Jevons appeared to regard the 'calculus of Pleasure and Pain' as part of the

profundity of their theory … The result has been that 'mathematics' has been blamed [by J. K. Ingram]

for 'restoring the metaphysical entities previously discarded.'" Edgeworth took no offense at the

allegation of viciousness, reviewing Fisher's thesis enthusiastically in the *Economic Journal*, inviting

Fisher to Oxford, visiting Fisher in New Haven and, as editor of the *Economic Journal*, publishing Fisher's

first four journal articles (see Allen 1993, pp. 55, 63-64, 67, 104, 112, 133, 170 on the long friendship and

intellectual exchange between Edgeworth and Fisher). In contrast to Edgeworth's psychological

approach, Fisher (1892) took what a great admirer of his dissertation, Ragnar Frisch, termed "the

behavioristic approach to the quantitative definition of economic utility … the choice point of view in the

analysis of utility is perhaps best known in connection with Pareto's name. However, the theory of

choice was, in fact, first introduced and developed with great consistency by Fisher" (Frisch's lectures at

Yale in 1930[2], in Bjerkholt and Qin 2010, pp.83-84).

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[2] Although Frisch said this while visiting Fisher's university, Yale, he had made a similar statement of Fisher's priority over Pareto in Frisch (1926).

Fisher and Frisch at Yale in 1930-31

Fisher and Frisch first met when Frisch was in the United States on a Rockefeller Fellowship in 1927-28, at which time Frisch told Fisher about Frisch (1926) if Fisher had not previously heard of it. Their shared commitment to "the choice point of view in the analysis" led Fisher (1927) and Frisch (1926, 1932; Bjerkholt 1995, Part I) to attempt empirical measurement of marginal utility (as did Fisher's last student, William Vickrey 1945) and, together with their shared support of the applicability of mathematics to economics (e.g. Fisher 1930), initiated a conversation between them that brought Frisch to Yale, where he lectured on "A Dynamic Approach to Economic Theory" from February to December 1930 (Bjerkholt and Qin 2010), a visiting professorship that resulted in Frisch and Fisher collaborating in the creation of the Econometric Society (Bjerkholt 1998, 2017) and in Yale offering Frisch a professorship. That offer provoked the University of Oslo to promote him to full professor from July 1931 even though, as Bjerkholt and Qin (2010, p. 11) observe that as a largely self-taught mathematician and statistician "his [formal] educational background was unimpressive – only a two-year economics program at the University of Oslo," although he later defended a doctoral dissertation in statistics in Oslo in 1926.

Frisch (1926) and Fisher (1927) both approached measurement of marginal utility of money by seeking some class of commodities whose marginal utility was independent of the consumption of other classes of commodities. In the absence of such a commodity class, Fisher and Frisch, who were both

index number theorists (Fisher 1922, Frisch 1930, 1936), each favored an approximation based on knowing that the constant-utility index of the cost of living is bounded by the Paasche and Laspeyres indexes. Bjerkholt and Qin (2010, p. 21) report that "After Frisch's arrival at Yale in 1930, [Fisher and Frisch] set out to write jointly a monograph on utility measurement. Their plan somehow aborted; instead Frisch completed the manuscript of a monograph entitled *New Methods for Measuring Marginal Utility* while at Yale in 1931 and had it published in 1932." Frisch (1947) recalled that he and Fisher quarreled over Fisher's refusal to adopt the concise mathematical style and axiomatic approach of Frisch (1926), even though, as Bjerkholt and Qin (2010, p. 21) observe, Frisch (1932) "does not pursue axiomatization or dynamization of the utility concept."

While Frisch was at Yale, he and Fisher collaborated with Charles Roos of Cornell and Joseph Schumpeter on a circular letter in June 1930 soliciting interest in creating an international society for the advancement of economic theory in connection with mathematics and statistics, and then in late November on a letter of invitation to the organizing meeting of the Econometric Society at the end of December. The following year, when Alfred Cowles 3rd wrote to Fisher (the founding president of the Econometric Society) offering to finance what became *Econometrica* and the Cowles Commission for Research in Economics, Fisher's acquaintance with the Cowles family (having been a Yale undergraduate at the same time as Cowles's father and uncle), the trust between Fisher and Frisch, and a visit by Frisch to Cowles in Colorado Springs combined to reassure the European members of the society's council, who were understandably skeptical of windfall offers of funding in the depths of the Depression

(Bjerkholt 1998, 2017, Dimand 2019). Fisher and Frisch were the most active members of the

commission's advisory council, as well as speaking at Cowles summer research conferences in Colorado.

Frisch on Measurement of Utility

"Intermediate between mathematics, statistics and economics, we find a new discipline which, for

lack of a better name, may be called *econometrics*," announced Ragnar Frisch (1926, p. 386).

"Econometrics has as its aim to subject abstract laws of theoretical political economy or 'pure'

economics to experimental and numerical verification, and thus to turn pure economics, as far as it is

possible, into a science in the strict sense of the word. The econometric study that I shall present is an

attempt to realize the dream of [William Stanley] Jevons[3]: to measure the variation in the marginal

utility of economic goods. I shall give special attention to the variation in the marginal utility of

money"—discarding as an evasion of the problem the assumption by Alfred Marshall (and, at least

implicitly, by Jules Dupuit half a century before Marshall) of a constant marginal utility of money for

sufficiently small changes in consumption bundles. Although Frisch published his essay in a Norwegian

series of mathematical publications, he did so in French and stated that "The study was made during a

stay in Paris, 1923", when he was studying mathematics there. Frisch was already familiar with at least

some of Fisher's work when writing his 1926 study, mentioning Fisher's preference for Charles Gide's

term desirability rather than utility (Frisch 1926, p. 395, perhaps with Fisher 1918 in mind) and

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[3] Frisch cited the paragraph "Numerical Determination of the Laws of Utility" by Jevons ([1871] 1911, p. 146).

discussing the basis on which "Mr. Fisher and Mr. Pareto after him have studied the static equilibrium of exchange" (Frisch 1926, p. 391, clearly with reference to Fisher 1892 since Fisher was stated to precede Pareto). Bjerkholt and Dupont-Kiefer (2009) report that Frisch had already studied Fisher (1892) while in Paris.

The Paris connection was important to Frisch's involvement with Fisher, the formation of the Econometric Society and measurement of marginal utility. Upon publication, Frisch sent a copy of Frisch (1926) to François Divisia (still famous for the Divisia index), initiating an exchange of fourteen letters leading to the creation at the end of 1930 of the Econometric Society, of which Frisch was the moving spirit (and from 1933 founding editor of *Econometrica*), Divisia the first vice-president and second president, and Fisher the first president (Bjerkholt 1998, 2017). In 1933 Frisch gave a series of lectures on econometrics in Paris, in the prestigious Poincaré series noted, for example, for Bruno De Finetti's lectures on subjective probability (De Finetti 1937, see also De Finetti 1935 on utility measurement). While *New Methods of Measuring Marginal Utility* (Frisch 1932), drafted at Yale, was forthcoming, Frisch published related articles in *Revue d'Économie Politique* (Frisch and Moret 1931, Moret and Frisch 1932) with Jacques Moret, a follower of the Lausanne school of general equilibrium (Moret 1915) who had translated Fisher's dissertation (Fisher 1917) and who was, along with Divisia, one of the four French economists among 26 recipients of a letter from Fisher, Frisch, Charles Roos and Joseph Schumpeter about organizing an Econometric Society (letter of invitation, 17 June 1930, and list of invitees on the Econometric Society website). Moret shared both Frisch's admiration for Fisher's

dissertation as the origin of the choice-theoretic approach to utility and Fisher's and Frisch's interest in trying to measure marginal utility.

Where Fisher (1927) began with a policy-relevant problem, that of justifying a progressive income tax by Edgeworth's principle of equal sacrifice (equalizing marginal utility of income across persons, Edgeworth 1919), the first numbered section (after the unnumbered introduction) of Frisch (1926) presented two sets of three axioms (axioms of choice, transitivity and addition), first for an individual at a given position choosing between two displacements and then for an individual who knows that on two different occasions he will be in two different positions, choosing among displacements from those positions. Independently of the merits of his method to measure marginal utility, Frisch (1926) is thus important as a pioneering axiomatization of utility theory, an approach which he did not take further in Frisch (1932) but did discuss in Yale lectures (Bjerkholt and Qin 2010, pp. 93-94) and which was carried further, with reference to Frisch and in a presentation in Oslo, by Franz Alt (1936, 1937). The axiomatization of utility theory, continued by John von Neumann and Oskar Morgenstern in the 1940s for maximization of expected utility and culminating in the work of Gerard Debreu (1959)[4], who had spent the spring of 1950 with Frisch in Oslo, paralleled the axiomatization of probability theory, notably by Andrei N. Kolmogorov in the 1930s. Frisch's axiomatic presentation was a major difference between

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[4] However, Debreu (1959, 1983) never cited Frisch or Fisher or Franz Alt in any context. Werner Hildenbrand, in his introduction to Debreu (1983, p. 3) acknowledges that "Indeed there had already been a few axiomatic treatments of economic theory prior to Debreu" but, apart from papers by Abraham Wald about the existence of general equilibrium in the proceedings of Karl Menger's Vienna mathematical colloquium in the 1930s, mentions only von Neumann and Morgenstern (1944) before Cowles monographs of Tjalling Koopmans and Kenneth Arrow in the early 1950s, omitting Frisch (1926) and Alt (1936) as pioneering axiomatizations of choice and utility theory.

his and Fisher's essays on measuring marginal utility, leading to what Frisch (1947) termed "quite an argument" between them.

Based on his axioms, Frisch (1926, pp. 401-402) ruled out the famous function of "moral expectation" or marginal utility proposed by Daniel Bernouilli ([1738] 1954)[5], which in Frisch's notation expressed the function g(r) as a constant divided by (r – a) as well as the utility function proposed by Charles Jordan (1924) equating g(r) to a constant divided by the square of (r – a). According to Frisch, the simplest function for the marginal utility of money consistent with his axioms would be a constant divided by (log r – log a).

Where Fisher (1927) only proposed a method for measuring marginal utility, if it was possible to define a class of commodities such that its marginal utility was independent of the consumption of other classes of commodities, Frisch (1926) attempted to actually perform such a calculation, using for illustration monthly data from June 1920 to December 1922 for French real income, sales of sugar at the Union des Coopérateurs, and the price of sugar divided by the cost of living index. Frisch used more extensive data in his 1932 monograph. As Fisher remarked in his 1942 book on income taxation (reprinted in Fisher 1997, Vol. 12, p. 264), Frisch's 1932 statistical results "were, however, startlingly different from what many people would have expected. According to his results (very tentative, of course), if a man's income is halved, his marginal want for one more dollar is not *more* than doubled, as commonly supposed – not even *doubled* – but *less* than doubled. Or conversely if his income is doubled

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[5] Frisch cited a German translation of Bernoulli, published in Leipzig in 1896.

his desire for one more dollar, although reduced, is reduced by less than half" so that Edgeworth's

principle of equal sacrifice would imply "not progressive, but *regressive* rates" (Fisher's italics).

Further Contributions to the Fisher-Frisch Investigation of Measurable Utility

   A few authors followed up Frisch's studies. Frederick Waugh (1935, p. 376) credited Fisher (1927) and

Frisch (1926, 1932) with having "shown that under certain conditions it is possible to measure

quantitatively the variation in the marginal utility of money from one period of time to another or from

one group of persons to another" although he cautioned that the results of Fisher and Frisch applied to

the marginal utility of money of selected groups of consumers, not necessarily representative of "the

whole populations of France and the United States." Like Fisher, Waugh cited the principle of "equal

sacrifice" in taxation as a motivation for his study. Waugh, at the US Bureau of Agricultural Economics

when his article appeared, studied with Frisch in 1932-33, on a one-year fellowship from the Social

Science Research Council, and applied Frisch's methods to US data on income and on the prices and

consumption of sugar, coffee, meat and butter: "With the kind assistance of Professor Frisch and his

assistants at the Universitets Økonomiske Institutt in Oslo, these data were studied both by graphic and

by mathematical-statistical methods." Waugh (1935, p. 385) reached "the conclusion that *in the period*

1916 to 1921 and again in the period 1922 to 1932 the per capita consumption of foods was almost

constant from year to year … if the average diet in a given period was constant, we can assume that the

marginal utility of that diet was very nearly constant and thus we have a convenient and reliable base

with which to compare other utilities" (italics in original). Later writers did not, however, find such a convenient simplification offered by the data.

In contrast to Waugh's enthusiasm for Frisch's approach, R. G. D. Allen (1933, p. 187) rejected "the validity of an 'approximation' adopted at the very outset [of Frisch 1932] … taking the marginal utility of money as a function, not of the individual's income and the prices of all the various consumers' goods on the market, but of two variables only, i.e. the income and the price of living … not an approximation at all; it can only be regarded as an extra, and most serious, assumption. Further, the making of the assumption seems to take from the notion of money marginal utility most of its essential and distinctive meaning." But, contrary to Allen's insistence on the full set of variables that theory states should be arguments of a utility function, Fisher and Frisch, apart from the exact representation possibility if the marginal utility of one commodity class was independent of the consumption of other commodity classes, sought to approximate empirical reality by making use of the true constant-utility index of the cost of living being bounded by the Paasche and Laspeyres indexes. The true constant-utility index (the "price of living") was not observable but might be approximated by an index within those bounds. Fisher's candidate for such an index would be the Fisher ideal index, the geometric mean of the Paasche and Laspeyres indexes (Fisher 1922), which satisfied more of Fisher's seven "statistical tests" or criteria for a desirable index than any other index formula (Frisch 1930, 1936, was to show that no index could satisfy more than six of the seven criteria, see Dimand 2019 and my introduction and afterword to Fisher 1997, Vol. 7).

Frisch's axiomatic approach to measurement of marginal utility (which contrasted to Fisher's more discursive, non-axiomatic presentation) was taken up by the mathematician and physicist Franz Alt, who, in addition to publishing in German in the Austrian journal *Zeitschrift für Nationalökonomie* (Alt [1936] 1971), came to Oslo to present his results at the International Congress of Mathematicians in 1936 (summarized in English as Alt 1937), placing his research in a discourse with Frisch. Chipman, Hurwicz, Richter and Sonneschein (1971, pp. 327-329) noted that Alt's first three axioms, regarding transitivity and connexity of binary preference relations, corresponded to the axioms stated by Frisch but Alt went further to established necessary and sufficient conditions for solvability, making use of the topological property of connectedness used two decades later by Gerard Debreu: "To a large extent many of the most essential features of contemporary models of measurable utility were anticipated in this pioneering study by Franz Alt" (p. 329).

William Vickrey's "Measuring Marginal Utility by Reactions to Risk" (1945) transformed the discussion of measuring marginal utility by appealing to the treatment of expected utility by Von Neumann and Morgenstern (1944), recognizing the importance of that analysis even before the axiomatization of rationality as expected utility maximization in an appendix to the 1947 second edition. Fisher (1927), cited in Vickrey's first and last footnotes, and Frisch (1932), cited in Vickrey's second footnote, found a utility function defined up to a positive linear transformation by assuming that commodity classes could be defined so that the marginal utility of one commodity class is independent of the consumption of another class. Vickrey showed that comparing risky alternatives allowed deriving such a utility function

without the Fisher-Frisch assumption: the probability weights attached to two risky alternatives to make

an individual indifferent between the gamble and a sure thing served as an index of utility. Vickrey, a

Nobel laureate who was president of the American Economic Association 75 years after Fisher, was

Fisher's last and most distinguished student. Vickrey graduated in engineering in mathematics from Yale

and went on to doctoral study in economics at Columbia in 1935, the year Fisher retired from Yale.

Fisher, whose PhD was jointly in mathematics and political economy and who began his teaching career

as an assistant professor of mathematics, was the only Yale economist of that era likely to attract a Yale

mathematics major to become an economist. Fisher thanked Vickrey in a 1939 *American Economic*

Review on double taxation of savings for "kindness and patience in reading and criticizing sundry

preliminary drafts of this article" and in the preface of a 1942 book on income taxation or reading most

of the manuscript and making important suggestions (Fisher 1997, Vol. 12, pp. 118, 161) while Vickrey's

dissertation and first book devoted a chapter to Fisher's income concept and expenditure tax proposal

(Dimand and Koehn 2002).

Conclusion

   Ragnar Frisch (1947) spoke of "the crucial contribution" and "monumental importance" of Irving

Fisher's *Mathematical Investigations in the Theory of Value and Price* (1892): "I remember the intensity

with which, in my younger days, I dug into Fisher's dissertation, and the same can undoubtedly be said

about many other economists of our generation. Later, this work was followed up by his paper on the

Measurement of Marginal Utility. When we are speaking not about the ideas that cause the shorter

swings, or even the sub-secular swings of the thinking in economics, but about those that are

responsible for the really long-time trend of our science, then it will be hard to find any single work that

has been more influential than Fisher's dissertation. It will be standing there as a mile-stone long after

our great-grandchildren are dead and forgotten." Just as Frisch (1926) has received scanty notice in

accounts of the history of axiomatic approaches to economic theory, Frisch's recognition of the

"monumental importance" of Fisher (1892), notably its presentation of a behaviorist approach to ordinal

utility and choice before Pareto, has not been widely shared: the sole mention of Fisher (1892) in

Weintraub (*General Equilibrium Analysis*, 1985, p. 37) states only that "Irving Fisher has a photograph of

a complicated piece of hydraulic machinery as a model in his book on value theory" without indicating

that Fisher's book was about general equilibrium (or citing the book). But there can be no doubt of the

strength of the book's impact on Frisch, or of the importance of the collaboration between Fisher and

Frisch for the establishment of the Econometric Society and the Cowles Commission.

William Barber reported that "Fisher [1927] was breaking new ground here. He subsequently learned,

however, that Ragnar Frisch had anticipated him in an essay published in French in 1926. When

distributing reprints of his own article, Fisher thereafter enclosed a mimeographed statement

acknowledging Frisch's priority in utility measurement" (Fisher 1997, Vol. 12, p. 2n)[6]. Fisher (1937, p.

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[6] Allowing for the publication lag of the volume honoring Clark, Fisher likely wrote his essay before Frisch published
(Bjerkholt and Qin 2010, p. 21, suggest that Fisher 1927 was written in 1925) but after Frisch wrote the first version
of his 1926 study in Paris in 1923.

31), in an *Econometrica* article on income taxation that had been a series of four lectures to the 1936

Cowles Commission summer conference, held that "we need not wholly despair of someday seeing

econometric science conquer, to some extent, the problem of measuring even the Y's [psychological

satisfactions translated into money]. In fact, an attempt has been made by me to take a first step in that

direction; and Professor Ragnar Frisch had independently done the same in a different way. He has since

made still more notable contributions to this elusive subject" with three footnotes citing Fisher (1927),

Frisch (1926) and Frisch (1932). Going beyond Fisher's acknowledgement of Frisch having published first,

Olav Bjerkholt and Duo Qin (2010, p. 21) state that Fisher came "to recognize that Frisch [1926] had

chosen a better approach." Frisch (1947, p. 3) did not mention anything about Fisher recognizing Frisch's

approach as better when he told the American Statistical Association and Econometric Society dinner

honoring Fisher's 80th birthday:

> Fisher and I had quite an argument over the way in which he presented his theory in that paper
> on utility measurement. I said that he should have condensed it to a few pages by a free use of a
> compact mathematical symbolism. That, I claimed, would have saved many of us a lot of time.
> But he would not give in to this argument. Then, some years later, I gave a series of lectures in
> Oslo, on utility axiomatics and the application of utility theory to the theory of demand. As the
> weeks passed by, I was greatly grieved by discovering that the audience dwindled down to
> nothing, when it finally became apparent that the students didn't understand a word of what I
> had been trying to tell them. Then one day one of the students came along and told me that in
> my collection of reprints – which is always at the disposal of the students – he had picked up
> Fisher's paper on utility measurement, and then it had suddenly dawned upon him what I had
> been trying to say. You may imagine the joy I got out of writing to friend Fisher and telling him
> of this story.

Frisch added that "I think that this is much more than a good story. It is symbolic of Fisher's ability to

see clearly, and to stick to it in the midst of both friendly and adverse criticism." Frisch's telling of this

story in his tribute to Fisher was also symbolic, of his generous appreciation of a fellow scientist and of

17

the longstanding influence on Frisch's thought of what Frisch termed Fisher's 1892 development of "the choice point of view in the analysis of utility."

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