NON-EXCLUSIVE INSURANCE WITH FREE ENTRY: A PEDAGOGICAL NOTE

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Non-Exclusive Insurance with Free Entry: A Pedagogical Note

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Abstract

We consider the Rothschild-Stiglitz model of insurance but without the exclusivity constraint. It turns out that there always exists a unique equilibrium, in which the reliable and unreliable consumers take out a primary insurance up to its quantity limit, and the unreliable take out further secondary insurance at a higher premium. We provide a simple proof of this result (extended to multiple types of consumers) with the hope that it may be pedagogically useful.

Key Words: non-exclusive insurance, free entry, adverse selection, primary-secondary insurance.

JEL Classification: D43, D82, D86

1 Introduction

Rothschild and Stiglitz ([8]) famously described a model of adverse selection in insurance markets in which firms could set quantity limits in order to hold down the number of unreliable clients they might have to face. They assumed that firms could demand exclusivity from their clients, and focused attention on a "separating" equilibrium in which reliable and unreliable clients took out completely different contracts. This separating equilibrium often did not exist, but was unique when it did.

Many different authors, starting with Jaynes ([7]) and including several others (see, e.g., [1], [2], [3], [5], [6], and the references therein) considered dropping the

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exclusivity constraint in specific economic contexts, and pointed to a different equilibrium in which both types take out a common primary insurance up to its quantity limit, and then the unreliable take out a further secondary insurance on their own at higher premium[1]. Two recent papers (see Attar et al [3] and Stiglitz et al [9]) show that this primary-secondary equilibrium always exists and is unique. The model in [3] is more general than that of Rothschild-Stiglitz (see Remark 5), while in [9] firms are permitted to use far more complex strategies than in Rothschild-Stiglitz.

In this paper, we go back to the basic model of Rothschild-Stiglitz, with just *one* amendment: contracts offered by firms are no longer exclusive, so that a consumer may take out contracts from as many firms as he likes. In all other respects, our model is no different from that of Rothschild-Stiglitz. This enables us to give a simple proof of the existence and uniqueness of primary-secondary insurance equilibrium (extended, in a natural manner, to any number of consumer-types, provided they can be ranked in terms of their reliability), which we hope will be of interest, at least from the pedagogical standpoint.

2 The Model

We describe the simplest model of insurance, in which each consumer begins with an endowment of one unit of the consumption good in her good (no-accident) state and nothing in her bad (accident) state. An insurance contract (π, Q), where[2] $\pi \in \mathcal{R}_{++}$ and $Q \in \mathcal{R}_{++}$, allows any consumer to choose any non-negative amount $z \leq Q$ of insurance, by paying a premium πz in both states in order to receive nothing in her good state and z in her bad state. We suppose a fixed interest rate, for convenience taken to be 0, so that in her good state the consumer ends up losing πz, but gains on net $(1-\pi)z$ in her bad state. If the consumer has probability p of an accident, then the insurance is actuarially fair (or, unfair) to her if $\pi = p$ (or, $\pi > p$).

[1]Indeed, we ourselves pointed to this equilibrium (see [4]), albeit in a different setting, where perfect competition substituted for free entry. To be precise, *all* conceivable contracts, most of which were *not* actively traded in equilibrium, were priced in [4] by the "invisible hand" of perfect competition (i.e., by the market forces of supply and demand). In effect, there was not just a continuum of consumer-clients in [4], but also a continuum of supplier-firms. There could be no question of an entrant firm upsetting the equilibrium, because its actions had no affect on market supply and it could not even be seen.

[2]By quoting exorbitantly high π and any $Q > 0$, a firm can ensure that no customer will come to it, and thus can abstain from participating in the market (i.e., the strict positivity of Q does not force participation). We assume $Q > 0$ because it makes for a cleaner presentation. (Allowing $Q = 0$ would not alter the final results, or add any analytical insights, but just necessitate more notation.)

Thus a consumer who takes out $z \leq Q$ units of the insurance contract (π, Q) and begins with goods $y = (y_G, y_B)$, will wind up consuming $y_G - \pi z$ in her good state, and $y_B + (1-\pi)z$ in her bad state. As z moves from 0 to Q, her consumption traverses the straight line segment with slope $-(1-\pi)/\pi$ from y to y' (where y' is obtained by setting $z = Q$). We denote this segment by $y \longrightarrow_{\pi,Q} y'$.

Let $S = \{(\pi, Q) : \pi \in \mathcal{R}_{++}, Q \in \pi \in \mathcal{R}_{++}\}$ denote the strategy set of each firm, consisting of contracts (π, Q). Insurance contracts are non-exclusive: each consumer may choose as many contracts as she wishes.

Given a strategy-profile $\sigma = (\pi^n, Q^n)_{n=1}^N \in S^N$ of firms $1, \ldots, N$ (where N is arbitrary), we define the consumption polygon $\Pi(\sigma)$ as follows. First let $\pi_1(\sigma) < \pi_2(\sigma) < \ldots < \pi_L(\sigma)$ denote the first-lowest, second-lowest, ..., L^{th}-lowest values among $\{\pi^1, \ldots, \pi^N\}$; let $N_l(\sigma) = \{n \in N : \pi^n = \pi_l(\sigma)\}$ be the set of firms quoting the l^{th}-rate, and let $Q_l(\sigma) = \sum_{n \in N_l(\sigma)} Q^n$ be the total quantity offered by them. Then $\Pi(\sigma)$ is the polygonal curve in $\mathcal{R}_+^2$ that starts at $y^0 = (1, 0)$ on the horizontal axis, goes up first by the amount $Q_1(\sigma)$ at the rate (slope) $-(1-\pi_1(\sigma))/\pi_1(\sigma)$, next by the amount $Q_2(\sigma)$ at the rate $-(1-\pi_2(\sigma))/\pi_2(\sigma)$, and so on, getting truncated at the point where it hits the vertical axis (*if* it happens to get that far by the L^{th} step). More precisely,

$$\Pi(\sigma) = y^0 \longrightarrow_{\pi_1(\sigma),Q_1(\sigma)} y^1 \longrightarrow_{\pi_2(\sigma),Q_2(\sigma)} y^2 \longrightarrow \ldots \longrightarrow_{\pi_L(\sigma),Q_L(\sigma)} y^L$$

Since premiums π_ℓ are increasing with ℓ, the polygonal curve is concave.

Let the consumer-types be denoted $1, \ldots, T$ whose population measures are $\mu_1, \ldots, \mu_T$, and whose bad states occur with probabilities $p_1, \ldots, p_T$ respectively, with $0 < p_1 < p_2 < \ldots < p_T$; i.e., type 1 is the most "reliable", and type T is least so. The utility function of type t is denoted $u_t : \mathcal{R}_+^2 \longrightarrow \mathcal{R}$ and is assumed to be continuous, strictly monotonic, and strictly concave. Furthermore the standard *single crossing property* is assumed, i.e., denoting consumptions in the good and bad states by x_G and x_B respectively, we have: if $t > t'$ then, at any point $(x_G, x_B) \in \mathcal{R}_+^2$, the indifference curve of u_t lies strictly below that of $u_{t'}$ to the left of the point and strictly above it to the right. A canonical example is given by $u_t(x_G, x_B) = (1-p_t)u(x_G) + p_t u(x_B)$ for some strictly monotonic and strictly concave utility u.

It is clear that

(i) on account of the strict concavity of u_t, any consumer of type t will maximize u_t on $\Pi(\sigma)$ at a unique point $x_t(\sigma) \in \Pi(\sigma)$ (in particular, all consumers of type t will consume the same bundle);

(ii) on account of the single crossing property, as t increases, $x_t(\sigma)$ will advance weakly on $\Pi(\sigma)$ further away from the initial endowment $(1, 0)$; more precisely, at any level l of $\Pi(\sigma)$, consumer-type t will take out strictly more insurance than the more reliable type $t' < t$, unless both have reached the quantity limit $Q_l(\sigma)$.

Given a polygon $\Pi(\sigma)$, a consumer of type t who takes out insurance $(z_{t1}(\sigma), ..., z_{tL}(\sigma))$ at each of the L levels, will end up consuming $x_t(\sigma) = (x_{tG}(\sigma), x_{tB}(\sigma))$, where

$$x_{tG}(\sigma) = 1 - \sum_{\ell=1}^{L} z_{tl}(\sigma)\pi_\ell(\sigma))$$

$$x_{tB}(\sigma) = \sum_{\ell=1}^{L} z_{tl}(\sigma)(1 - \pi_\ell(\sigma))$$

It is also clear that

(iii) the consumer will exhaust the better contracts before starting the lesser contracts, hence $z_\ell(\sigma) > 0$ implies that $z_j(\sigma) = Q_j(\sigma)$ for all $j < \ell$; and $z_\ell(\sigma) = 0$ implies that $z_j(\sigma) = 0$ for all $j > \ell$.

For ease of presentation[3], we suppose that if $z_{t\ell}(\sigma) < Q_\ell(\sigma)$, then the consumer takes out a pro rata share of the quantities offered on level ℓ, i.e. she takes out the amount

$$z_t^n(\sigma) = z_{t\ell}(\sigma)\frac{Q^n}{\sum_{j\in N_l(\sigma)} Q^j}$$

of insurance from each firm $n \in N_l(\sigma)$. It follows that not only is the final consumption $x_t(\sigma)$ a deterministic function of σ, but so are the amounts $z_t^n(\sigma)$ of insurance that any consumer type t takes out from each firm $n = 1, \ldots, N$ in order to get to $x_t(\sigma)$. Now, recalling that μ_t is the population measure of type t, the profit $F^n(\sigma)$ of firm n is

$$F^n(\sigma) = \pi^n \sum_{t=1}^{T} \mu_t z_t^n(\sigma) - \sum_{t=1}^{T} \mu_t z_t^n(\sigma) p_t = \text{Revenue - Cost}$$

It will be useful to speak of the unit cost $\sum_{t=1}^{T} \mu_t z_t^n(\sigma) p_t / \sum_{t=1}^{T} \mu_t z_t^n(\sigma)$, whenever the denominator is positive (i.e. level ℓ has active consumers), so that profit $F^n(\sigma) = \sum_{t=1}^{T} \mu_t z_t^n(\sigma)$[premium - unit cost]. Observe that, by the pro rata rule, every firm charging the same premium will incur the same unit cost.

Definition 1 ***(Equilibrium).*** *A strategy profile $\sigma = ((\pi^1, Q^1), \ldots, (\pi^N, Q^N))$ is called an* equilibrium *if: (a) no firm makes a loss, i.e, $F^n(\sigma) \geq 0$ for $n = 1, \ldots, N$;*

[3] And, *only* for ease of presentation. See Remark 1 for the case where we allow consumers to choose how much insurance to take out from firms offering the same rate.

(b) no entrant firm $N+1$ can make a profit, i.e., there is no (entrant) strategy $(\pi^{N+1}, Q^{N+1}) \in S$ such that, denoting $\sigma^* = ((\pi^1, Q^1), \ldots, (\pi^N, Q^N), (\pi^{N+1}, Q^{N+1}))$, we have

$$F^{N+1}(\sigma^*) = \pi^{N+1} \sum_{t=1}^{T} \mu_t z_t^{N+1}(\sigma^*) - \sum_{t=1}^{T} \mu_t z_t^{N+1}(\sigma^*) p_t > 0$$

Theorem 2 ***(Existence and Uniqueness)*** *There exists an equilibrium, and across all equilibria the consumptions of the consumers are invariant.*

We shall prove this in the next two sections, and give an explicit description of the unique equilibrium consumptions.

3 Existence of Equilibrium

Definition 3 ***(Canonical strategy profile)****. Let $\pi_i^* = \sum_{l=i}^{T} \mu_l p_l$, for $i = 1, \ldots, T$. Let Q_1^* be the amount of insurance that a consumer of type 1 would take out at the premium π_1^* if there were no quantity constraints on her. Define Q_i^* inductively as the amount of insurance that a consumer of type t would take out at the premium π_i^* if there were no quantity constraints on her and she had already taken out Q_1^* at π_1^*, Q_2^* at $\pi_2^*, \ldots,$and Q_{i-1}^* at π_{i-1}^*. Now take the set of firms to also be $1, \ldots, T$, i.e., one firm for each consumer-type; and define $\sigma^* = ((\pi_1^*, Q_1^*), \ldots, (\pi_T^*, Q_T^*))$ to be the* canonical strategy profile. (If $Q_i^* = 0$, *then firm i is understood to have been dropped.)*

It is worth noting that in the canonical strategy profile, the contract (π_i^*, Q_i^*) is not taken out by any type $t < i$, and is taken out until the quantity limit Q_i^* by all types $t \geq i$. Moreover, for each $Q_i^* > 0$, the premium π_i^* is set equal to its unit cost, so all contracts make zero profit.

Lemma 4 *The canonical strategy profile σ^* is an equilibrium.*

Proof. We need only verify that no new firm has incentive to enter.

Consider a potential entrant. If it quotes premium $\pi < \pi_1^*$, then (by (ii) and (iii)) all types will go to it, taking out nondecreasing amounts insurance as we descend the reliability ladder, so that the entrant's unit cost will be at least $\sum_{t=1}^{T} \mu_t p_t = \pi_1^*$, while its premium is strictly less, so it will make a loss. Next if it quotes $\pi = \pi_i^*$ for any i, then it gets the same mix of customers (by our pro-rata rule) as firm i did in σ^*, and therefore makes the same zero profit. Finally, if it quotes any π with

$\pi_i^* < \pi < \pi_{i+1}^*$, it will attract only types $i+1, \ldots, T$ so that its unit cost will be at least $\sum_{t=i+1}^{T} \mu_t p_t = \pi_{i+1}^*$, while its premium is strictly less, so it will make a loss. If it quotes $\pi > \pi_T$, it will attract no one and therefore make no profit. ■

4 Uniqueness of Equilibrium Consumptions

Lemma 5 *Let σ be any strategy profile at which there is a non-empty set $J = \{t : z_{t\ell}(\sigma) > 0\}$ of active consumer-types on level l of $\Pi(\sigma)$. Then, for small enough $\varepsilon > 0$, the strategy $(\pi_l(\sigma) - \varepsilon, \varepsilon)$ enables an entrant firm to come in and earn the premium $\pi_l(\sigma) - \varepsilon$ while incurring unit cost no more than $\sum_{t \in J} \mu_t p_t$, which in turn is no more than the common unit cost c of each active firm in σ at level ℓ. Furthermore, if J has at least two consumer types, and if the most reliable consumer in J is not taking out the full quantity $Q_\ell(\sigma)$ of insurance at level ℓ, then the entrant's unit cost is at most $\sum_{t \in J} \mu_t p_t < c$ for all small enough ε.*

Proof. Denote by $\Pi_\varepsilon = \Pi(\sigma, (\pi(\varepsilon), \varepsilon))$ the polygon formed after the entry, where $\pi(\varepsilon) = \pi_l(\sigma) - \varepsilon$. As $\varepsilon \longrightarrow 0$, the polygon Π_ε coverges to $\Pi(\sigma)$. Hence, by (i), each consumer-type's optimal choice on Π_ε converges to her optimal choice in $\Pi(\sigma)$. Since all the types in J take out positive insurance at premium $\pi_l(\sigma)$ in $\Pi(\sigma)$, they must be doing so in Π_ε (for small enough ε). However no one takes out insurance at premium $\pi_l(\sigma)$ in Π_ε before exhausting possibilities at the lower premium $\pi(\varepsilon)$ available in Π_ε. Hence all the consumer-types in J take out the *same* insurance up to the available limit ε at premium $\pi(\varepsilon)$ in Π_ε. This implies that the unit cost incurred by the entrant from J is $\sum_{t \in J} \mu_t p_t$. On the other hand, invoking (ii), the reliabilty mix of consumer types at level ℓ of $\Pi(\sigma)$ is no better than at $\pi(\varepsilon)$, thus $\sum_{t \in J} \mu_t p_t \leq c$. In addition, there may be some other consumer-types J' who also simultaneously take out insurance at premium $\pi(\varepsilon)$ in Π_ε *but then everyone in J' must be more reliable than the most reliable in J.* This further lowers the unit cost of the entrant and the desired conclusion hold with even more force. Finally, if the most reliable consumer j in J was not taking out full insurance in σ and other consumers are present in J, then (again by (ii)) these others strictly worsen the reliability mix in σ at premium $\pi_l(\sigma)$, compared to the mix at $\pi(\varepsilon)$, implying $\sum_{t \in J} \mu_t p_t < c$. ■

Lemma 6 *At any equilibrium, all firms make zero profits.*

Proof. Any firm i making a positive profit must have unit cost strictly below its premium. By Lemma 5, an entrant can charge an arbitrarily smaller premium and yet incur no higher unit cost, thus making a profit and contradicting equilibrium. ■

Lemma 7 *At any equilibrium, all consumptions are the same as in the canonical strategy profile σ^*.*

Proof. Let σ be any equilibrium. We shall show inductively that all consumer-types $t = 1, \ldots, T$ consume the same in σ as in σ^*, starting with type 1.

Assume that $1 < T$. (The case $1 = T$ will be considered later.) We claim that (a consumer of type) 1 takes out no insurance in σ if, and only if, she takes out no insurance in σ^*. For suppose she takes out positive insurance in σ^* but none in σ. This implies that the premium $\pi_1(\sigma) > \pi_1(\sigma^*) \equiv \pi_1^*$. Let an entrant come in at σ and offer $(\pi_1^* + \varepsilon, \varepsilon)$. For small enough ε, it follows from (i) and (ii), that all types will take this contract up to the limit ε, so that the unit cost of the entrant is $\sum_{t=1}^{T} \mu_t p_t \equiv \pi_1^*$ which is strictly less than its premium, so that the entrant makes a profit, a contradiction. Conversely suppose that type 1 takes out positive insurance in σ but none in σ^*. Then $\pi_1(\sigma) < \pi_1^*$. However, by (ii), the unit cost of firms quoting $\pi_1(\sigma)$ is *at least* $\sum_{t=1}^{T} \mu_t p_t \equiv \pi_1^*$, which is more than the premium $\pi_1(\sigma)$ they are charging, hence these incumbent firms make losses at σ, again a contradiction. This verifies our claim.

In view of the claim, it remains to consider the case where 1 takes out positive insurance in both σ and σ^*. We shall establish that $\pi_1(\sigma) = \pi_1^*$ and that all of $Q_1(\sigma)$ is taken out by 1, and that 1 does not proceed further onto the second level of $\Pi(\sigma)$.

Suppose that 1 takes out less than the full quantity $Q_1(\sigma)$ at premium $\pi_1(\sigma)$. Then, recalling $1 < T$, other less reliable consumer-types exist and, by (ii), these types take out strictly more insurance as 1 at $\pi_1(\sigma)$. So Lemma 5 can be applied, which shows that an entrant can charge an arbitrarily smaller premium than $\pi_1(\sigma)$, and yet incur unit cost bounded *strictly* below c, thus making a profit (since, by the definitiion of equilibrium, incumbent firms $N_1(\sigma)$ at level 1 in σ, whose unit costs were c, made no losses in the equilibrium σ prior to his entry), and contradicting that σ is an equilibrium. Thus 1 takes out $Q_1(\sigma)$ units of insurance. Then, by (ii), so do all other types, and the unit cost at level 1 of is $\sum_{t=1}^{T} \mu_t p_t$. By Lemma 6, $\pi_1(\sigma) = \sum_{t=1}^{T} \mu_t p_t \equiv \pi_1^*$.

Next suppose that 1 wants to take out more insurance than $Q_1(\sigma)$ at premium π_1^*. Let an entrant come in and offer $(\pi_1^* + \varepsilon, \varepsilon)$. For small enough ε, all types will take out full insurance (post-entry) on the first level $\pi_1(\sigma)$ and furthermore, by (ii) and the fact that $\pi_1^* + \varepsilon < \pi_2(\sigma)$, full insurance also from the entrant. The entrant will therefore incur unit cost $\sum_{t=1}^{T} \mu_t p_t \equiv \pi_1^*$, but earn the premium $\pi_1^* + \varepsilon$, making a profit, a contradiction. This proves that $Q_1(\sigma) = Q_1^*$.

Finally we turn to the case that $1 = T$, i.e., 1 is the solo type taking out insurance at σ. By Lemma 6, $\pi_1(\sigma) = \mu_1 p_1 \equiv \pi_1^*$. Clearly 1 will not take out more insurance than Q_1^*. If he takes out less (because less is offered, i.e., $Q_1(\sigma) < Q_1^*$) , an entrant can come in and offer $(\pi_1^* + \varepsilon, \varepsilon)$, get full subscription, with premium $\pi_1^*+\varepsilon$ exceeding unit cost $\mu_1 p_1$, and thus making a profit, a contradiction.

Now suppose, inductively, that $\pi_i(\sigma) = \pi_i^*$ and $Q_i(\sigma) = Q_i^*$ for types types $i = 1, \ldots, l$ and consider type $l+1 < L$. Repeat the argument given above, replacing type $1, \ldots, T$ by types $l+1, \ldots, T$ (i.e., just drop the types $1, \ldots, l$ from the picture and commence with type $l+1$ looking "ahead" on $\Pi(\sigma)$) to conclude that $\pi_{l+1}(\sigma) = \pi_{l+1}^*$ and $Q_{l+1}(\sigma) = Q_{l+1}^*$. ■

5 Remarks

(1) (**Beyond the Pro-Rata Rule)** When a consumer is indifferent between several firms $(\pi, Q^i)_{i\in I}$, and wants $z < \sum_{i\in I} Q^i$ units of insurance at the common premium π quoted by them, we could let *the consumer* choose how much insurance to take out from the different firms, instead of stipulating the pro rata rule that he take out $\left(Q^j / \sum_{i\in I} Q^i\right) z$ from $j \in I$. This would be tantamount to allowing the distribution of z among I to be arbitrary (subject to the upper bounds Q^i).However, in this case, we shall need to be careful in delineating the expectations that an entrant may entertain regarding the consumer-types it can attract. If its expectations are allowed to be too optimistic, in particular if it can cherry-pick amongst all those customers who like its contract at least as much as their current ones, then equilibrium will fail to exist. Some tempering of its optimism is needed in order to uphold equilibrium. One natural constraint is that the entrant should expect to get precisely those customers who *strictly* benefit by coming to it, i.e., it cannot cherry pick amongst them, but must accomodate *all*. At the same time impose the additional constraint that, on account of an implicit "cost of moving", those customers who are indifferent between the entrant and their current insurers will stay where they are. With this definition, all our results remain intact by the same arguments (with the obvious changes). The entrant could even turn pessimistic and make the assumption that, amongst those indifferent to it, only the most unreliable ones will present themselves at its door (alongside, of course, all those who strictly benefit). This, too, will leave our results intact.

(2) (**Pretending to be Many Firms, i.e. Menus)** We can allow each (incumbent or entrant) firm to issue as many elementary contracts (π, Q) as it likes, i.e., allow its strategy set to be $\cup_{n=1}^{\infty} S^n$. This will not alter equilibrium. To see this, consider a firm that issues $(\pi^1, Q^1), \ldots, (\pi^m, Q^m)$ with $\pi^1 < \ldots < \pi^m$ as part of a strategy profile σ. Let the firm unilaterally withdraw any one of its contracts (π^j, Q^j) . A moment's relection reveals that the clientale it will get at $(\pi^1, Q^1), \ldots, (\pi^{j-1}, Q^{j-1})$ will remain

unaffected, while the reliability mix of the clientale it will get at $(\pi^{j+1}, Q^{j+1}), \ldots, (\pi^m, Q^m)$ will not worsen, thus not worsening the profitability of every remaining contract. We conclude that an entrant can make a profit via a menu only if it can make a profit via a single contract. Moreover, if an incumbent makes a negative profit on one of its contracts, then it must make a positive profit on another one of its contracts (since it cannot make losses in equilibrium). But then by our standard argument, an entrant can come and make a positive profit as well.

(3) (**Mixing Exclusive and Non-Exclusive Contracts Destroys Equilibrium):** Suppose, for simplicity, that there are just two consumer-types. An entrant firm can always break our primary-secondary equilibrium σ^* by the strategem of issuing an *exclusive* contract that will lure *only* the reliable and enable them to consume above their σ^*-level. (The unreliable are kept at bay, because the quantity limit of the entrant is too restrictive for them). Moreover, as may be readily verified, the entrant's premium in this stratagem can be set higher than $\mu_1 p_1 + \mu_2 p_2$, hence it will make a profit. Conversely the separating equilibrium $\sigma'$of [8] can be broken whenever the consumption of the reliable at σ^* is better than her consumption at σ'. For, in this case, an entrant can intervene in σ' with the offer of the same quantity as the primary contract of σ^* and a premium that is *slightly* higher, get both types to fully subscribe to it and thereby make a profit.

(4) (**Welfare Comparisons)** Clearly the unreliable are always better off at the primary-secondary equilibrium σ^* than at the separating equilibrium σ'. If the reliable are sufficiently numerous compared to the unreliable, then it is easy to see that they too will prefer σ^* to σ', with the upshot that σ^* Pareto-dominates σ'. On the other hand, if the unreliable constitute the bulk of the population, one can *still* say that σ^* dominates σ' from the "majoritarian" point-of-view.

(5) **(General Adverse Selection with Ranked Consumers)** The arguments we have given do not depend on the particulars of the underlying insurance market. Let us suppose that firms sell quantities of a commodity to consumers. Firm n sets a price π_n and a quantity limit Q_n. Consumers can buy from as many firms as they want, up to the prescribed quantity limits. What is essential for our analysis to remain intact is that the consumer types can be ranked in the same order $1, ..., T$ in two senses. First, given any vector of firm offers, the quantities purchased at any price are weakly increasing in consumer type t, (and strictly increasing if the total quantity limit available at the price is not exhausted). Second, we need to suppose that the constant marginal cost c_t of producing the good for type is also increasing in t. See [3] for an excellent analysis, carried out in this general framework.

References

[1] Attar,A., T. Mariotti and F. Salanie (2014). Nonexclusive Competition under Adverse Selection, *Theoretical Economics,* 9 (1), 1-40.

[2] Attar,A., T. Mariotti and F. Salanie (2016a): Multiple Contracting in Insurance Markets. *Toulouse School of Economics Working Paper* 14-532

[3] Attar,A., T. Mariotti and F. Salanie (2017): Entry-Proofness and Market Breakdown under Adverse Selection. *Toulouse School of Economics Working Paper.*

[4] Dubey, P. and J. Geanakoplos (2001) Insurance Contracts Designed by Competitive Pooling. *Cowles Foundation Discussion Paper* 1315.

[5] Glosten, L.R. (1994). Is the Electronic Open Limit Order Book Inevitable ? *Journal of Finance*, 49 (4), 1127-1161.

[6] Hellwig, M.F. (1988). A Note on the Specification of Interfirm Communication in Insurance Markets with Adverse Selection. *Journal of Economic Theory,* 46 (1), 154-163.

[7] Jaynes, G.D. (1978). Equilibria in Monopolistically Competitive Insurance Markets. *Journal of Economic Theory,* 19 (2), 394-422.

[8] Rothschild, M. and J.E.Stiglitz (1976). Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Competition. *Quarterly Journal of Economics,* 90 (4), 629-649.

[9] Stiglitz, J.E., J.Yun, A. Kosenko (2017) Equilibrium in a Competitive Insurance Market under Adverse Selection with Endogenous Information. *NBER Working Paper* 23556