SPATIAL LINKAGES, GLOBAL SHOCKS, AND LOCAL LABOR MARKETS: THEORY AND EVIDENCE

By

Rodrigo Adão, Costas Arkolakis, and Federico Esposito

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COWLES FOUNDATION FOR RESEARCH IN ECONOMICS
YALE UNIVERSITY
Box 208281
New Haven, Connecticut 06520-8281

http://cowles.yale.edu/

Spatial Linkages, Global Shocks, and Local Labor Markets: Theory and Evidence*

Rodrigo Adão
Chicago Booth

Costas Arkolakis
Yale

Federico Esposito
Tufts

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Abstract

How do shocks to economic fundamentals in the world economy affect local labor markets? In a framework with a flexible structure of spatial linkages, we characterize the model-consistent shock exposure of a local market as the exogenous shift in its production revenues and consumption costs. In general equilibrium, labor outcomes in any market respond directly to the market's own shock exposure, and indirectly to other markets shocks exposures. We show how spatial linkages control the size and the heterogeneity of these indirect effects. We then develop a new estimation methodology – the Model-implied Optimal IV (MOIV) – that exploits quasi-experimental variation in economic shocks to estimate spatial linkages and evaluate their counterfactual implications. Applying our methodology to US Commuting Zones, we find that difference-in-difference designs based on model-consistent measures of local shock exposure approximate well the differential effect of international trade shocks across CZs, but miss around half of the aggregate effect, partly due to the offsetting action of indirect effects.

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1 Introduction

How large and heterogeneous are the gains and losses in a country following an aggregate economic shock, such as a change in trade policy or a productivity boom in a foreign country? The lack of exogenous variation in such aggregate shocks makes it challenging to answer these questions in general equilibrium (see Nakamura and Steinsson (2018)). To circumvent this identification problem, economists have followed two different approaches. On the one hand, there is a growing literature that exploits quasi-experimental variation in cross-regional exposure to aggregate economic shocks to credibly estimate their differential impact on regional economic outcomes.[1] However, extrapolating counterfactual predictions in general equilibrium from the estimated differential responses is not straightforward (Kline and Moretti, 2014; Beraja, Hurst, and Ospina, 2016; Muendler, 2017), a problem particularly acute in the presence of spatial links that generate endogenous correlation in regional outcomes (Moretti, 2011). On the other hand, a growing literature relies on general equilibrium quantitative analysis that enables the computation of the aggregate effect of counterfactual changes in trade costs and productivity.[2] This literature has not clearly evaluated how the aggregate counterfactual predictions are connected to the evidence on differential regional responses to exogenous shocks and how robust they are to specifications of the model's general equilibrium forces.

Our paper provides a bridge between these two approaches by formalizing a methodology that exploits quasi-experimental variation to credibly estimate cross-regional differential responses to various economics shocks, and simultaneously enables to robustly aggregate the counterfactual implications of these estimates in general equilibrium. We start by providing a theoretical characterization of how spatial links determine the impact of changes in trade costs and productivity in the world economy on local labor markets that is robust within a class of general equilibrium spatial models. Using this characterization, we derive measures of the local exposure to economic shocks that are directly connected to the reduced-form predictions of the model. We establish that the aggregation problem is equivalent to the measurement of the indirect effect of a market's shock exposure on other markets, which themselves depend on the strength of spatial links in the economy. We then propose a

[1]There is a vast literature studying the differential effect of international trade shocks across regional economies – e.g., see Topalova (2010), Kovak (2013), Autor, Dorn, and Hanson (2013), Acemoglu et al. (2016), Dix-Carneiro and Kovak (2017), Pierce and Schott (2016b). Several papers have also documented a significant effect of local exposure to technological shocks on regional labor market outcomes – e.g., see Autor and Dorn (2013), Bustos, Caprettini, and Ponticelli (2016), and Acemoglu and Restrepo (2017). Finally, an emerging literature evaluates the effect of regional exposure to macroeconomic shocks on regional outcomes – e.g., see Mian and Sufi (2014), Nakamura and Steinsson (2014) and Beraja, Hurst, and Ospina (2016).

[2]For reviews of these quantitative frameworks, see Costinot and Rodriguez-Clare (2013) for gravity trade models and Redding and Rossi-Hansberg (2016) for spatial models.

methodology to estimate spatial links that combines quasi-experimental variation in economic shocks and our model-consistent exposure measures. In our empirical application to US Commuting Zones, we quantify the importance of such spatial links in shaping the general equilibrium responses to the recent China's productivity boom.

The first part of the paper examines the role of spatial linkages in shaping the impact of economic shocks on labor markets outcomes across regions. The starting point of our analysis is a multi-region economy with a single freely-traded homogeneous good. We introduce two functions that govern spatial links in terms of endogenous labor supply and labor productivity. The combination of these two functions into a single "labor module" controls the magnitude of the reduced-form response of regional employment and wages to productivity shocks in the own local market – the direct shock effect – and in other local markets in the economy – the indirect shock effects. Thus, these indirect general equilibrium effects are part of the total impact of the economic shock on any region, affecting both the differential and the aggregate responses of regional outcomes.

We provide three results to clarify this point using a first-order approximation of the model's general equilibrium predictions. We first show that, if spatial links are not present (i.e., markets are segmented), the differential response of local outcomes to changes in local productivity yields both the differential and the aggregate effects of productivity shocks in general equilibrium. These effects are increasing in the combined magnitude of the own-market elasticity of labor supply and labor productivity. Second, adding spatial linkages that are homogeneous across markets implies a reduced-form response that entails an "endogenous" time fixed-effect, containing the sum of the symmetric indirect spillovers of productivity shocks in all regions. Conditional on the spatial distribution of the shock, the magnitude of the fixed-effect is increasing in the combined strength of the symmetric spatial links in the economy. Finally, if linkages are heterogeneous across markets, then indirect spillovers are also heterogeneous and, therefore, are not absorbed by a fixed-effect. However, weak conditions guaranteeing the uniqueness of the spatial equilibrium imply that indirect effects are bounded by the magnitude of the spatial links in the "labor module."

We then extend our analysis to a Generalized Spatial Economy that features differentiated goods produced by multiple local labor markets, i.e. sector-region pairs. This specification gives rise to an additional source of spatial links that takes the form of a bilateral trade demand function. These trade links also generate indirect effects across markets, which are absent only in the special case of an economic shock to a set of markets effectively forming a small open economy. By properly specifying the shape of spatial links, we show that our model is general enough to replicate the labor market predictions of a wide class of trade and geography models. Accordingly, our results are robust to the particular assumptions of various models regulating the strength of connections among local labor markets in general equilibrium.[3]

We use this environment to propose a generalized way to measure exposure of local markets to economic shocks in the world economy. This connects the first part of our paper, the theoretical characterization, to the second part, the measurement and estimation. In our general equilibrium model, we characterize the direct and indirect effects on local outcomes created by two model-consistent measures of shock exposure of different markets. These measures capture the shock to both sides of a market's terms-of-trade: the shift in (i) the world demand for local labor, which we name "revenue exposure," and (ii) the local cost of world products, which we name "consumption exposure." The theory illustrates that these two exposure measures have opposite effects on local employment. In the special case of gravity trade links, they are effectively the partial equilibrium impact of the shock on the measures of firm and consumer market access introduced by Redding and Venables (2004). These exposure measures allow the empirical evaluation of the impact of exogenous economic shocks on local labor markets in a model-consistent way. In fact, the indirect effect of these measures build the link between estimated differential effects of local shock exposure and the model's counterfactual predictions in general equilibrium. This is in contrast with quantitative exercises that have enough degrees of freedom to match entirely the cross-regional data, but do not evaluate the causal implications of the model following exogenous economics shocks.

In light of these results, the second part of the paper develops a methodology to estimate spatial links within a country. First, we outline a data generating process in which markets experience changes in productivity and amenities, and spatial links depend on an unknown vector of "deep" structural parameters. Conditional on this vector, we establish conditions for the recovery of unobservable shocks in each market's productivity and amenities using observed changes in labor outcomes across markets.[4] Second, given these structural residuals, the parameter vector can be estimated using a class of moment conditions that combine the recovered local shocks and arbitrary functions of exogenous foreign cost shocks. This step exploits quasi-experimental variation in shock exposure across markets, as in Topalova (2010), Kovak (2013) and Autor, Dorn, and Hanson (2013). Finally, we follow the Optimal IV

[3]These frameworks include: (i) Neoclassical trade theories with external economies of scale (as in Ethier (1982a,b)) and without (as in Anderson (1979) and Eaton and Kortum (2002)); (ii) New trade theory models with homogeneous firms (as in Krugman (1980)); (iii) New economic geography models (as in Krugman (1991) and Allen and Arkolakis (2014)); (iv) Roy spatial assignment models (as in Galle, Rodriguez-Clare, and Yi (2017) and Adão (2015)). Finally, the model specified without trade frictions yields predictions equivalent to the framework in Rosen (1979)-Roback (1982) and, more recently, in Kline and Moretti (2014).

[4]This step is distinct from other works in spatial economics, that invert structural residuals using the entire general equilibrium structure of the model, as in Allen and Arkolakis (2014), Monte, Redding, and Rossi-Hansberg (2018), and Faber and Gaubert (2016).

approach of Chamberlain (1987) to characterize the "optimal" variance-minimizing estimator within this class and its Two-Step GMM feasible implementation. In particular, we show that such an estimator uses the impact of *observed* cost shocks on the *endogenous* variables predicted by our general equilibrium model – the Model-implied Optimal IV (MOIV).[5] Intuitively, our methodology is a generalization of shift-share instrumental variables applied in many empirical papers since Bartik (1991) and Altonji and Card (1991). Rather than using an ad-hoc local exposure "share" to the aggregate "shift," the MOIV relies on the general equilibrium model to dictate the heterogeneity in the impact of the "shift" on different local labor markets.

The last part of the paper applies our methodology to quantify the importance of spatial links for the impact of international trade shocks on employment across Commuting Zones (CZs) in the United States. Specifically, we parametrize the Generalized Spatial Economy by using a multiple-industry gravity trade structure with parameters controlling links in labor supply and productivity across sectors and regions. Our empirical application entails 722 CZs in the US, 58 foreign countries, 31 manufacturing industries, one non-manufacturing sector, and non-employment through a home sector. In the estimation of the model's structural parameters, we exploit quasi-experimental variation induced by industry-level Chinese export growth between 1997 and 2007, as in Autor, Dorn, and Hanson (2013). To evaluate the robustness of our results, we also report estimates based on alternative measures of China's competitiveness shock used in the literature – e.g., the removal of tariff uncertainty on Chinese imports (Pierce and Schott, 2016a), and changes in Chinese firms productivity (Hsieh and Ossa, 2016).

The implementation of our empirical methodology yields estimates of the structural parameters governing cross-market links in our model. Specifically, we estimate an elasticity of employment to real wage of 0.1 in the manufacturing sector and 0.3 in the non-manufacturing sector. Our estimates also suggest that the between-region elasticity of labor supply is one-third of the between-sector elasticity. Moreover, using a specification similar to the one proposed in Ahlfeldt et al. (2015), we estimate a local agglomeration force with a similar magnitude as that in a Krugman model, as well as productivity spillovers that decline with distance. To assess our model's counterfactual implications, we compare the cross-regional patterns of actual changes in manufacturing employment with those predicted by our model

[5] The approach of Chamberlain (1987) has been used in partial equilibrium models by Berry, Levinsohn, and Pakes (1999), Petrin (2002), and Reynaert and Verboven (2014). In general equilibrium, model-implied instruments have been also used recently in Monte, Redding, and Rossi-Hansberg (2018), Faber and Gaubert (2016), Eckert and Peters (2018), and Allen and Donaldson (2018). Our work is most close to the approach of Allen, Arkolakis, and Takahashi (2018) in a gravity setup. Our contribution is, for a general class of spatial models, to formally establish a class of consistent estimators and among them characterize the optimal estimator, effectively an aggregation function over an exogenous shock.

in response to the Chinese competition shock. We find a positive and statistically significant relationship that holds even conditional on the CZ's initial labor market and demographic conditions.

Finally, we decompose the model's predicted changes in labor outcomes in each CZ into the direct and the indirect general equilibrium effects of the shock exposure of all CZs. We obtain three main results. First, stronger import competition from China reduces the world demand for labor in the manufacturing sector for the majority of CZs. Cross-regional variation in the manufacturing "revenue exposure" is a strong predictor of the cross-regional variation in manufacturing employment and wage losses in our general equilibrium model. Second, the positive impact of lower consumption costs is fairly homogeneous across CZs and compensates half of the average negative impact of higher foreign import competition. Third, our estimates of the spatial links in the US yield compensating indirect effects from other CZs, which further offset the negative impact on the local manufacturing sector. Interestingly, these general equilibrium indirect effects do not substantially affect the cross-regional variation due to the direct exposure, but shift the employment responses up by a magnitude equivalent to half of the average negative impact of the shock to manufacturing revenues.

The use of quasi-experimental variation in regional shock exposure has become a powerful tool in the investigation of the labor market consequences of a variety of economic shocks since the seminal papers by Altonji and Card (1991), Bartik (1991) and Blanchard and Katz (1992). More recently, empirical specifications point towards strong *differential* cross-regional and cross-sectoral effects of exposure to international shocks on employment and wages (e.g., Topalova (2010), Kovak (2013), Autor, Dorn, and Hanson (2013), Autor et al. (2014), Pierce and Schott (2016a)). We contribute to this literature by establishing theoretically and empirically the role of spatial links in generating indirect spillover effects of local shock exposure on other markets. In a sense, as in Heckman, Lance, and Taber (1998), we characterize the effect of "treated" markets on "non-treated" markets that is part of the response of local outcomes in general equilibrium. In addition, we show that trade links affect the measurement of the exposure of any particular market to an economic shock in the world economy – that is, a market's "treatment" intensity in terms of production revenues and consumption costs.[6]

Our paper is also related to a growing literature that relies on general equilibrium quantitative analysis to compute the *aggregate* effect of counterfactual changes in trade costs

[6]Heterogeneity in spatial links also gives rise to heterogeneity in the direct effect of the own market's shock exposure – i.e., heterogeneous treatment effects across regions. This heterogeneity has been emphasized recently by Monte, Redding, and Rossi-Hansberg (2018) in a model with commuting flows across regions. Instead, we focus on the implication of spatial links for the measurement of shock exposure and the magnitude of indirect effects.

and productivity, as in Allen and Arkolakis (2014) and Caliendo et al. (2018b). Recently, several papers have focused on quantifying the labor market consequences of the integration of China into the world economy (e.g., Galle, Rodriguez-Clare, and Yi (2017), Lee (2015), Hsieh and Ossa (2016), and Caliendo, Dvorkin, and Parro (2018a)). Our theoretical results establish that the reduced-form predictions of this class of quantitative models depend on three functions governing spatial links, which can be estimated from the impact of economic shocks on trade and labor outcomes across local markets. Given estimates of these functions, the model's predictions are robust to any specific micro-foundations. In this sense, our analysis characterizes "deep" functions governing spatial links in the model (see e.g. Rosenzweig and Wolpin (2000) and Wolpin (2013)) and generalizes previous work where spatial linkages take a constant elasticity functional form – as in Arkolakis, Costinot, and Rodríguez-Clare (2012), Donaldson and Hornbeck (2016), Allen, Arkolakis, and Takahashi (2018), and Bartelme (2018) – or do not feature externalities and regional mobility – as in Adao, Costinot, and Donaldson (2017).

In this unified environment, we propose a new model-consistent way of measuring regional exposure to various economic shocks. These measures interact the observed shock with market-specific weights that use information on bilateral trade links. They are related to measures of "market access" in Redding and Venables (2004) and Donaldson and Hornbeck (2016), but have the benefit of being robust across models and not requiring solving for the model's general equilibrium.[7]

The rest of the paper is structured as follows. Section 2 describes a simplified spatial model featuring a flexible structure of spatial links in labor productivity and labor supply, which gives rise to indirect effects across markets. Section 3 extends the analysis to a Generalized Spatial Economy that features bilateral trade as an additional source of spatial links. Section 4 describes our empirical methodology, which we then implement in Section 5 using data on Commuting Zones in the United States. Using the theoretical model and the empirical estimates, Section 6 conducts a number of counterfactual exercises. Section 7 concludes.

2 Simplified Spatial Model

We begin by proposing a model with multiple local labor markets featuring a flexible structure of spatial links in productivity and labor supply. In general equilibrium, these links imply that changes in productivity in any particular "treated" market percolate across space, giving

[7] In a recent related paper, Hornbeck and Moretti (2018) provide reduced-form evidence on migration responses to a regional productivity shock over long time horizons. They then measure the indirect effect of a local productivity shock stemming from the population loss in other regions of the country.

rise to indirect reduced-form effects on other "non-treated" local markets in the economy. If quantitatively important, these indirect spillovers affect the measurement of both the differential and the aggregate effects of economic shocks across local labor markets. We also show that the strength of spatial links determines the existence of a unique spatial equilibrium, as well as the magnitude and heterogeneity of indirect reduced-form effects.

2.1 Environment

We assume that the world economy is constituted of countries, c, each a collection of regions, $r \in \mathcal{R}_c$. We denote origin markets as i, and destination markets as j. In the rest of the paper, we use bold variables to denote stacked vector of market-level outcomes, $\boldsymbol{x} \equiv [x_i]_i$, and bar bold variables to denote matrices with bilateral market-to-market variables, $\bar{\boldsymbol{x}} \equiv [x_{ij}]_{i,j}$.

To focus on domestic links in labor supply and agglomeration, we assume that there is single sector with a freely traded homogeneous good. The next section introduces a richer structure of domestic and foreign trade links between markets.

Representative Household. In each country, there exists a representative household with preferences over consumption and labor supply in different markets. The representative agent's utility function is given by

$$U_c\left(\boldsymbol{C}, \boldsymbol{L}\right), \tag{1}$$

where $\boldsymbol{C} \equiv \{C_i\}_i$ and $\boldsymbol{L} \equiv \{L_i\}_i$ are respectively vectors of consumption and labor supply in all markets. We assume that $U_c(\cdot)$ is twice differentiable, increasing in $\boldsymbol{C}$, and quasi-concave in $(\boldsymbol{C}, \boldsymbol{L})$.

We impose that, in each market j, expenditures must be equal to labor income. Specifically, the representative agent faces the following budget constraint:

$$PC_j = w_j L_j \quad \forall j, \tag{2}$$

where w_j is the wage rate per unit of labor and P is the price of the homogeneous good.[8]

We consider a competitive environment where, in deciding consumption and labor supply, the representative agent takes as given prices and wages. Thus, the utility maximization problem yields the labor supply in market j as a function of the vector of real wages, $\omega_j \equiv w_j/P$, in all regions of the country:

$$L_j \in \Phi_j\left(\boldsymbol{\omega}\right), \tag{3}$$

[8] To simplify the exposition of our results, we do not allow for transfers across markets in the budget constraint (2). All results hold in the presence of exogenous transfers per unit of labor income ρ_j, in which case consumption in market j must be equal to $\rho_j w_j L_j$.

where, for each real wage vector, $\Phi_j(\boldsymbol{\omega})$ is a convex set.

The shape of $\Phi_j(\cdot)$ flexibly captures spatial links in labor supply. To see this, consider the case in which $U_c(\cdot)$ is strictly quasi-concave, so that $\Phi_j(\cdot)$ is a differentiable function with elasticity matrix $\bar{\boldsymbol{\phi}} \equiv [\phi_{ij}]_{i,j}$ where $\phi_{ij} \equiv \frac{\partial \log \Phi_i}{\partial \log \omega_j}$. The cross-market elasticity ϕ_{ij} summarizes the strength of migration responses to changes in real wages. The special case without regional labor mobility corresponds to $\phi_{ij} = 0$ for all $i \neq j$. Similarly, the own-market elasticity ϕ_{ii} regulates local labor supply responses to local real wages, incorporating the intensive and extensive margins of employment adjustment.[9]

Production. In each market, there exists a representative firm that operates under perfect competition. Production requires only labor and it is subject to external economies of scale. Market i's production function is

$$Y_i = \tau_i \Psi_i(\boldsymbol{L}) L_i, \tag{4}$$

where $\Psi_i(\cdot)$ is a strictly positive real function.

In equilibrium, the profit maximization problem implies that

$$\omega_i \geq \tau_i \Psi_i(\boldsymbol{L}) \quad \text{with equality if } L_i > 0. \tag{5}$$

The term $\tau_i \Psi_i(\cdot)$ has two labor productivity components: τ_i is an exogenous shifter, and $\Psi_i(\cdot)$ is a function of employment in *all* markets. This endogenous productivity term governs the strength of agglomeration and congestion forces in our model, as summarized by the elasticity matrix $\bar{\boldsymbol{\psi}} \equiv [\psi_{ij}]_{i,j}$ with $\psi_{ij} = \frac{\partial \log \Psi_i}{\partial \log L_j}$. The cross-market elasticity ψ_{ij} regulates spatial spillovers in production costs that can be positive, as in the case of spatial knowledge diffusion, or negative, as in the case of capital mobility described in the Online Appendix D.2.6. The own-market elasticity ψ_{ii} controls the sensitivity of productivity to changes in local employment that arise, for example, from Marshallian external economies of scale.[10]

Equilibrium. The equilibrium is defined as vectors of real wage, $\boldsymbol{\omega} \equiv \{\omega_i\}_i$, and employment, $\boldsymbol{L} \equiv \{L_i\}_i$, that satisfy conditions (3) and (5) for every market i.

[9]The utility function in (1) does not explicitly impose that the sum of labor supply across markets is constant, allowing our model to capture endogenous changes in aggregate labor supply in the country. The assumption of constant aggregate labor supply is common in the geography literature – see, for example, Allen and Arkolakis (2014); Allen, Arkolakis, and Takahashi (2018); Bartelme (2018). As discussed below, such restriction can be imposed directly in the specification of the utility function in (1). More generally, Appendix D establishes that, by specifying the shape of $\Phi_j(\cdot)$, our model is observationally equivalent to various micro-founded models with different degrees of worker mobility across regions and sectors.

[10]More generally, Appendix D formally establishes that, through the proper specification of the matrix $\bar{\boldsymbol{\psi}}$, our model is observationally equivalent to different micro-founded models with agglomeration and congestion forces.

2.2 Direct and Indirect Effects of Global Shocks

We now turn to the counterfactual predictions of our model regarding changes in wages and employment following exogenous shocks in productivity across markets. Our objective is to trace down the implications of these shocks as they are propagated through spatial linkages, and assess their effects on different local labor markets.

To simplify the exposition, we focus on the particular case in which $\{\Psi_i(\cdot), \Phi_i(\cdot)\}_i$ are differentiable functions, and we henceforth assume that the equilibrium entails positive employment in every market. In this case, conditions (3) and (5) hold with equality in equilibrium, implying that the vector of real wages solves the following system that we refer to as the "local labor market module:"

$$\mathbf{\Lambda}(\boldsymbol{\omega}) \equiv \log \boldsymbol{\omega} - \log \mathbf{\Psi}\left(\mathbf{\Phi}\left(\boldsymbol{\omega}\right)\right) = \log \boldsymbol{\tau}. \tag{6}$$

The function $\mathbf{\Lambda}(\boldsymbol{\omega})$ summarizes how the equilibrium conditions depend on the endogenous vector of real wages and, therefore, plays a central role in shaping the properties of the economy's equilibrium. Accordingly, restrictions on the Jacobian matrix of $\mathbf{\Lambda}(\boldsymbol{\omega})$ guarantee that there is a unique equilibrium with positive employment everywhere.

Assumption 1. *Assume that $\Psi_i(\cdot)$ and $\Phi_i(\cdot)$ are continuously differentiable functions and $\Psi_i(\cdot)$ is bounded above and $\lim_{w_i \to 0} \frac{\Psi_i(\mathbf{\Phi}(\boldsymbol{\omega}))}{w_i} = \infty$. Denote the Jacobian matrix of the system (6) by $\bar{\boldsymbol{\lambda}}(\boldsymbol{\omega}) \equiv [\lambda_{ij}(\boldsymbol{\omega})]_{i,j}$ with $\lambda_{ij}(\boldsymbol{\omega}) \equiv \frac{\partial \Lambda_i(\boldsymbol{\omega})}{\partial \log \omega_j}$. For every $\boldsymbol{\omega}$, assume that (i) the Jacobian's diagonals are positive $\lambda_{ii}(\boldsymbol{\omega}) > 0$, and (ii) there is a vector $\{h_i(\boldsymbol{\omega})\}_i \gg 0$ such that the Jacobian's off-diagonals are bounded, $h_i(\boldsymbol{\omega})\lambda_{ii}(\boldsymbol{\omega}) > \sum_{j \neq i} |\lambda_{ij}(\boldsymbol{\omega})| h_j(\boldsymbol{\omega})$.*

Assumption 1 effectively imposes restrictions on the response of market i's production cost to changes in i's real wage, ω_i, relative to the response of market i's production cost to real wage changes in other markets, ω_j for $j \neq i$. Specifically, it requires a weighted sum of the cross-market effects embedded in $\lambda_{ij}(\boldsymbol{\omega})$ for $j \neq i$ to be lower than the own-market effect in $\lambda_{ii}(\boldsymbol{\omega})$. Notice that this condition does not impose restrictions on the sign of the off-diagonal effects that can be either positive or negative for different markets and initial conditions.[11] In our context, Assumption 1 holds trivially whenever labor supply or productivity are exogenous – i.e., $\bar{\boldsymbol{\phi}} = 0$ or $\bar{\boldsymbol{\psi}} = 0$.

Under Assumption 1, we study how spatial links affect the counterfactual predictions of our model using a first-order approximation of the log-change in real wages following changes

[11] As shown in Proposition 1 below, this "diagonal dominance condition" is sufficient to have uniqueness of the equilibrium. Note that this is a much weaker requirement for uniqueness than (weak) gross substitution, as discussed in Arrow and Hahn (1971) p. 233-p.234.

in the vector of exogenous productivity shifters, $\boldsymbol{\tau}$. We use $\hat{x}_j = x'_j/x_j$ to denote the ratio between the variables in the new and the initial equilibria, and use a superscript 0 to denote variables in the initial equilibrium. Immediately, the "local labor market module" in (6) implies that

$$\bar{\boldsymbol{\lambda}} \log \hat{\boldsymbol{\omega}} = \log \hat{\boldsymbol{\tau}} \quad \text{with} \quad \bar{\boldsymbol{\lambda}} \equiv \bar{\boldsymbol{I}} - \bar{\boldsymbol{\psi}}^0 \bar{\boldsymbol{\phi}}^0, \tag{7}$$

where by $\bar{I}$ we denote the identity matrix. Notice that $\bar{\boldsymbol{\lambda}} = \bar{\boldsymbol{\lambda}}(\boldsymbol{\omega}^0)$ but, to simplify the exposition, we will suppress this notation for much of the remaining analysis. The bounds in Assumption 1 imply that there exists at most one solution to the system in (6). Following exogenous changes in productivity, the unique solution of the log-linear system in (7) approximates the changes in real wages across markets.

Proposition 1. *Suppose Assumption 1 holds. There is a unique vector of real wages, $\boldsymbol{\omega} \equiv \{\omega_i\}_i$, that solves the local labor market module in (6) for any given $\boldsymbol{\tau} \equiv \{\tau_i\}_i$. Up to a first-order approximation, the reduced-form impact of productivity shocks on labor market outcomes is*

$$\begin{aligned} \log \hat{\boldsymbol{\omega}} &= \bar{\boldsymbol{\beta}} \log \hat{\boldsymbol{\tau}} \\ \log \hat{\boldsymbol{L}} &= \bar{\boldsymbol{\phi}}^0 \bar{\boldsymbol{\beta}} \log \hat{\boldsymbol{\tau}} \end{aligned} \tag{8}$$

where $\bar{\boldsymbol{\beta}} \equiv \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\psi}}^0 \bar{\boldsymbol{\phi}}^0\right)^{-1}$.

Proof. Appendix A.1.

Proposition 1 states that equation (8) approximates the endogenous changes in employment and real wage, $\hat{\boldsymbol{\omega}}$, as a function of the exogenous shocks to productivity, $\hat{\boldsymbol{\tau}}$. Such a response is the unique solution of the log-linear version of the system of simultaneous equations in (7). Accordingly, as defined in chapter 9 of Wooldridge (2010), equation (8) is the *reduced-form effect* of productivity shocks on local labor markets in general equilibrium. The matrix $\bar{\boldsymbol{\beta}}$ represents the *reduced-form elasticity* of local wages to productivity shocks. This inverse matrix can be represented as a series expansion of powers of the product matrix of the structural elasticities, $\bar{\boldsymbol{\psi}}^0 \bar{\boldsymbol{\phi}}^0$, which indicates the amplification mechanism present due to spatial linkages.

Thus, the *reduced-form* system in (8) outlines the role of spatial links in shaping the effect of economic shocks on local labor markets in general equilibrium. In fact, it implies that the response in wages takes the following form:

$$\log \hat{\omega}_i = \underbrace{\beta_{ii} \log \hat{\tau}_i}_{\text{direct effect}} + \underbrace{\sum_{j \neq i} \beta_{ij} \log \hat{\tau}_j}_{\text{indirect effect}}, \tag{9}$$

where β_{ij} is the (i, j) entry of $\bar{\boldsymbol{\beta}}$.

Equation (9) implies that, in general equilibrium, any shock to a particular region j percolates in space through cross-market linkages in productivity and labor supply. More formally, any local shock $\hat{\tau}_j$ has a direct effect of β_{jj} on the "treated" market j, and indirect effects on other "non-treated" markets with magnitude given by the cross-market elasticity β_{ij}. As pointed out by Heckman, Lance, and Taber (1998), this effect of "treated" markets on "non-treated" markets is part of the general equilibrium impact of changes in economic fundamentals. Whenever these indirect spillovers are large and heterogeneous, they can be an important part of both the differential and the aggregate effects of productivity shocks. In contrast, whenever these effects are small, the response of local outcomes to local shocks approximates the general equilibrium impact of economic shocks.

We now formally establish that the strength of spatial links in $\bar{\boldsymbol{\psi}}^0\bar{\boldsymbol{\phi}}^0$ determines the importance of indirect spillovers in general equilibrium.

Theorem 1. *Suppose Assumption 1 holds. Consider the impact of changes in productivity across markets, $\hat{\boldsymbol{\tau}} \equiv \{\hat{\tau}_i\}_i$.*
1) In a set of segmented markets, $\lambda_{ij} = \lambda 1_{[i=j]}$, the real wage response is

$$\log \hat{\omega}_i = \frac{1}{\lambda} \log \hat{\tau}_i.$$

2) In a set of markets with symmetric links, $\lambda_{ij} = \lambda 1_{[i=j]} - \tilde{\lambda}_j$, the real wage response is

$$\log \hat{\omega}_i = \frac{1}{\lambda} \log \hat{\tau}_i + \frac{1}{\lambda} \sum_j \frac{\tilde{\lambda}_j}{\lambda - \sum_d \tilde{\lambda}_d} \log \hat{\tau}_j.$$

3) In a set of generic markets, the real wage response is given by equation (9) such that

$$\frac{|\beta_{ij}|}{|\beta_{jj}|} < \frac{\sum_{k \neq i} |\lambda_{ik}(\boldsymbol{\omega}^0)| h_k(\boldsymbol{\omega}^0)}{\lambda_{ii}(\boldsymbol{\omega}^0) h_j(\boldsymbol{\omega}^0)} < \frac{h_i(\boldsymbol{\omega}^0)}{h_j(\boldsymbol{\omega}^0)}.$$

Proof. Appendix A.2.

The first part of Theorem 1 shows that, in absence of spatial links, reduced-form responses do not entail indirect effects. In this case, regions are isolated economies that are only affected by local productivity shifters, and thus the response of local wages to changes in local productivity yields both the differential and the aggregate effect of productivity shocks in general equilibrium. In addition, notice that, by the definition in (7), $1/\lambda$ is increasing in the local elasticity of agglomeration and labor supply. Thus, as the combined effect of these forces increases, the response of local labor market outcomes to local productivity shocks also increases.

The second part of the Theorem outlines the reduced-form responses in the presence of symmetric spatial links. That is, wage changes in market j have a symmetric impact on the production cost of any other market $i \neq j$, as in the case of Logit functions of labor supply and productivity analyzed in Kline and Moretti (2014) and Allen, Arkolakis, and Takahashi (2018). This symmetry implies that the reduced-form response entails an "endogenous" fixed-effect that contains the sum of the symmetric indirect spillover effects of productivity shocks in the economy. Conditional on the fixed effect, differential variation in local productivity shocks is associated with differential changes in the real wage across markets. However, these differential effects do not correspond to the complete effect of the shock on local markets, because the fixed effect contains the component of the response that is identical to all markets.[12]

This expression sheds light on how spatial links affect the sign and the magnitude of the indirect effects. The denominator $\lambda - \sum_o \tilde{\lambda}_o$ captures the general equilibrium feedback (direct and indirect) effects and is, in general, constrained to be positive given Assumption 1. This implies that the sign of the indirect effect is the same as that of the structural spatial links in $\{\tilde{\lambda}_j\}_j$. Specifically, whenever $\tilde{\lambda}_j < 0$, the indirect effect of market j on any other market is also negative. In this case, the indirect spillover effects of shocks in other markets attenuate the direct impact of local shocks in general equilibrium.[13]

This expression also shows that the importance of the indirect effects generated by any market j is proportional to the magnitude of its structural spatial links, $\tilde{\lambda}_j$. To see this, consider the ratio between the absolute values of the direct effect and the indirect effect of a shock in market j, $|\beta_{ij}|/|\beta_{jj}|$. Using the expression in the second part of Theorem 1, this ratio is

$$\frac{|\beta_{ij}|}{|\beta_{jj}|} = \frac{|\tilde{\lambda}_j|}{|\lambda - \sum_{d \neq j} \tilde{\lambda}_d|},$$

which is increasing in the absolute value of $\tilde{\lambda}_j$.

While the indirect effect of each individual market might be negligible, the combination of the indirect effect generated by *all* markets can be quantitatively large. To see this, consider the special case of identical spatial links such that $\tilde{\lambda}_j = \tilde{\lambda}/N$, where N is the number of markets. In this case, whenever the number of markets is large, $\beta_{ij} \approx 0$ is small for $i \neq j$, but

[12] In order to include all indirect effects into an "endogenous" fixed-effect, similar symmetry assumptions are routinely maintained in empirical papers in macroeconomics, development, and urban economics – for example, see Kline and Moretti (2014), Nakamura and Steinsson (2014), Beraja, Hurst, and Ospina (2016) and Acemoglu and Restrepo (2017).

[13] This arises in an economy with log-linear local agglomeration, $\psi_{ij} = \psi 1_{[i=j]}$ with $\psi > 0$, and a Logit function of labor supply, $\phi_{ij} = \phi 1_{[i=j]} - \tilde{\phi}_j$ with $\tilde{\phi}_j > 0$. Under these assumptions, $\lambda = 1 - \psi\phi$ and $\tilde{\lambda}_j = -\psi\tilde{\phi}_j$.

$\sum_{j \neq i} \beta_{ij} \propto \tilde{\lambda}/(\lambda - \tilde{\lambda})$. This implies that the magnitude of the indirect effects also depends on the spatial pattern of the shock. A fully correlated shock across all markets, $\hat{\tau}_i = \hat{\tau}$, has the potential to generate sizable indirect effects, despite small indirect effects of any individual region.

This intuition carries through also in the case of asymmetric spatial links. In the last part of the Theorem, we show that the magnitude of indirect spillovers is bounded by the magnitude of structural spatial links in the initial equilibrium. However, in this case, heterogeneity in labor supply and productivity responses, $\bar{\boldsymbol{\psi}}^0 \bar{\boldsymbol{\phi}}^0$, translates into heterogeneity in the reduced-form elasticity across markets: β_{ij} varies across both i and j. Such a heterogeneity implies that indirect spillover effects are not absorbed by a fixed-effect across markets.

Taken together, these results indicate that any investigation of the effect of economic shocks on local labor markets requires a careful assessment of the structural spatial links in the economy. They determine the direct and indirect reduced-form effects of shocks to economic fundamentals. We now discuss a generalized spatial framework that allow us to measure exposure to foreign shocks and estimate spatial links.

3 Generalized Spatial Model

We now present a generalized spatial model with a rich structure of bilateral trade across markets and show that the main insights of the previous section carry through. In this setup, we establish theoretically consistent measures of exposure to foreign shocks that take into account the asymmetric exposure of different local markets in terms of consumption costs and production revenues.

3.1 Environment

We consider a multiple sectors extension of the model in Section 2, in which each region-sector pair produces a potentially differentiated good, as in Armington (1969) and Anderson (1979). Throughout the rest of the paper, we define a region-sector pair a ‘‘local market’’, and, as in Section 2, denote origin markets as i, and destination markets as j. For this reason, we use the more general term ‘‘cross-market’’ links to specify the connections across markets in the economy.[14]

[14]Notice that the definition of a market may vary depending on the empirical application of our model. In Section 5, a market corresponds to either a manufacturing or non-manufacturing sector in a U.S. Commuting Zone. Different assumptions on the degree of mobility of labor across sectors and regions imply different labor supply functions, affecting wage differentials across markets in equilibrium. We get back to this point in Section 3.3.

Representative Household. As in Section 2, equation (1) is the utility function of the representative agent over consumption and labor supply in different markets. We assume that C_j is an index that aggregates quantities consumed of the differentiated goods produced in all origin markets,

$$C_j \equiv V_j\left(\boldsymbol{c}_j\right), \tag{10}$$

where $\boldsymbol{c}_j \equiv \{c_{ij}\}_i$ with c_{ij} denoting the consumption in market j of the good produced in market i. We assume that the function $V_j(\cdot)$ is twice differentiable, increasing, and quasi-concave in all arguments. Importantly, we also restrict $V_j(\cdot)$ to be homogeneous of degree one, so that we can separate the problem of allocating spending shares across origin markets from the problem of determining labor supply across markets in the country.

Let p_{ij} be the price of the good produced in market i supplied to market j. The budget constraint in market j is given by

$$\sum_i c_{ij} p_{ij} = w_j L_j. \tag{11}$$

The homogeneity of $V_j(\cdot)$ implies that, conditional on prices, the solution of the cost minimization problem yields the price index in market j:

$$P_j = P_j\left(\boldsymbol{p}_j\right) \equiv \min_{\boldsymbol{c}_j} \sum_o p_{oj} c_{oj} \quad \text{s.t.} \quad V_j\left(\boldsymbol{c}_j\right) = 1, \tag{12}$$

with the associated spending share on goods from origin i given by

$$x_{ij} \in X_{ij}\left(\boldsymbol{p}_j\right). \tag{13}$$

The price index and spending share functions inherit the usual properties of demand implied by utility maximization. The price index $P_j(\cdot)$ is homogeneous of degree one, concave, and differentiable. In addition, $X_{ij}(\cdot)$ is a convex set, with a single element if $V_j(\cdot)$ is strictly quasi-concave.

In our model, the trade demand $X_{ij}(\cdot)$ regulates the strength of cross-market links in bilateral trade flows. When goods produced in different markets are homogeneous, as in Section 2, any spending share vector is attainable as long as prices of all suppliers are identical. More generally, cross-market links depend on the sensitivity of $X_{ij}(\cdot)$ to changes in bilateral prices. With differentiated products, such links are summarized by the elasticity structure of the trade demand function: $\chi_{oij} \equiv \frac{\partial \log X_{ij}}{\partial \log p_{oj}}$. As discussed below, the flexibility of $X_{ij}(\cdot)$ allows our model to replicate the predictions of a variety of trade models.[15]

[15] The particular case of a Logit trade demand function, $\chi_{oij} = -\chi\left(1_{[i=o]} - x_{oj}\right)$, is especially important, as it covers the popular class of gravity trade models analyzed in Arkolakis, Costinot, and Rodríguez-Clare (2012); Costinot and Rodriguez-Clare (2013).

As in Section 2, given the vector of real wages, the utility maximization problem of the representative agent yields the labor supply in any market j:

$$L_j \in \Phi_j\left(\boldsymbol{\omega}\right), \tag{14}$$

with $\omega_i \equiv w_i/P_i$.

Production. In each market, we assume that the production function takes the form in (4) and impose that there are iceberg trade costs to ship goods between markets. In this environment, the profit maximization problem of competitive firms implies that

$$p_{ij} = \tau_{ij} p_i, \tag{15}$$

where τ_{ij} is the iceberg trade cost of delivering a good produced in i to j, and p_i is the endogenous production cost that, in equilibrium, must satisfy

$$\omega_i \geq \frac{p_i}{P_i}\Psi_i\left(\boldsymbol{L}\right) \quad \text{with equality if } L_i > 0. \tag{16}$$

In the special case of a single freely homogeneous product of Section 2, prices are equalized everywhere, so that $p_i/P_i = \tau_i$ and this condition is equivalent to (5). More generally, p_i/P_i is an endogenous variable that measures the competitiveness of market i relative to competitors in the local market. By revealed preferences, it is directly related to spending shares in market i. We exploit this property to connect equation (16) to the data on bilateral trade flows in Section 4.

Market Clearing. To close the model, we specify the labor market clearing condition. In each market, total labor payments must be equal to total revenues,

$$w_i L_i = \sum_j x_{ij} w_j L_j. \tag{17}$$

Equilibrium. We define the competitive equilibrium as $\{p_i, P_i, L_i, \omega_i\}_i$ such that conditions (12)–(17) hold given the normalization that $p_m \equiv 1$ for an arbitrary market m.

As in Section 2, we assume that the mappings $\{\Psi_i(\cdot), \Phi_i(\cdot), X_{ij}(\cdot)\}_{i,j}$ are differentiable functions, and focus on equilibria with positive employment in every market. Since iceberg trade costs identical to all markets act as productivity shifters, we simplify the notation by normalizing $\tau_i \equiv 1$. It is useful to represent the equilibrium conditions more compactly with two sets of equations in terms of the vectors of real wages, $\boldsymbol{\omega} \equiv \{\omega_i\}_i$, and production costs, $\boldsymbol{p} \equiv \{p_i\}_i$. The combination of (14) and (16) implies that, conditional on p_i, the equilibrium must satisfy the "labor market module":

$$\omega_i = \frac{p_i}{P_i}\Psi_i\left(\mathbf{\Phi}\left(\boldsymbol{\omega}\right)\right), \tag{18}$$

where, by equations (12) and (15), $P_j = P_j\left(\left\{\tau_{oj}p_o\right\}_o\right)$.

In addition, the equilibrium must satisfy the "trade module":

$$p_i\Psi_i\left(\mathbf{\Phi}\left(\boldsymbol{\omega}\right)\right)\Phi_i\left(\boldsymbol{\omega}\right) = \sum_j x_{ij}p_j\Psi_j\left(\mathbf{\Phi}\left(\boldsymbol{\omega}\right)\right)\Phi_j\left(\boldsymbol{\omega}\right). \tag{19}$$

where, by equations (13) and (15), $x_{ij} = X_{ij}\left(\left\{\tau_{oj}p_o\right\}_o\right)$.

We turn next to analyze the direct and indirect effects of exogenous changes in fundamentals on local labor markets using a first-order approximation of equations (18)–(19).[16]

3.2 Measurement of Local Exposure to Shocks

To measure the local exposure to cost shocks we propose a new, generalized, measure of exposure that is theoretically consistent and is linked to previous structural and reduced form approaches of measuring local shock exposure. This depends on the shape of trade links embedded in

$$\bar{\boldsymbol{\chi}} \equiv \left[\sum_d y_{id}\chi_{jid}\right]_{i,j}, \quad \bar{\boldsymbol{y}} \equiv \left[y_{ij}\right]_{i,j}, \quad \bar{\boldsymbol{x}} \equiv \left[x_{ji}\right]_{i,j}, \tag{20}$$

where y_{ij} denotes the share of market i's revenue from sales to market j, and x_{ji} denotes the share of market i's expenditure on goods from market j. The element $\bar{\chi}_{ij}$ of the matrix $\bar{\boldsymbol{\chi}}$ represents the elasticity of market i's revenue to changes in the production costs of market j. For market i, $\bar{\chi}_{ij}$ captures the cross-market links arising from changes in the competitive environment in all destination markets triggered by changes in the endogenous production costs of competitor j. The matrices $\bar{\boldsymbol{y}}$ and $\bar{\boldsymbol{x}}$ capture, respectively, the overall revenue and spending shares across markets.

We then define two measures of market i's direct exposure to cost shocks in the world economy: i's consumption exposure,

$$\log\hat{\eta}_i^C\left(\hat{\boldsymbol{\tau}}\right) \equiv \sum_j x_{ji}^0 \log\hat{\tau}_{ji}, \tag{21}$$

[16] In Online Appendix D.1, we show that knowledge of the cross-market links in $\{\Psi_i(\cdot), \Phi_i(\cdot), X_{ij}(\cdot)\}_{i,j}$ is sufficient to uniquely determine the effect of changes in economic fundamentals as a solution of the non-linear system of equilibrium conditions in (18)–(19).

and i's revenue exposure,

$$\log \hat{\eta}_i^R(\hat{\boldsymbol{\tau}}) \equiv \sum_j \sum_o y_{ij}^0 \chi_{oij} \log \hat{\tau}_{oj}. \tag{22}$$

Specifically, $\hat{\eta}_i^R$, is the change in market i's revenue triggered by the cost shock of competitors, $\hat{\tau}_{oj}$. The weight of each destination-competitor is given by the interaction between i's initial revenue exposure to the destination market, y_{ij}^0, and the demand sensitivity of that market to the competitor's cost, χ_{oij}. In addition, $\hat{\eta}_i^C$ captures the change in the cost of the consumption bundle in market i triggered by the shock, with weights equal to market i's initial import shares, x_{ji}^0. Jointly $\hat{\eta}_i^R$ and $\hat{\eta}_i^C$ capture the combined effects of shocks to economic fundamentals on a market's terms-of-trade through both production and consumption.[17]

While capturing forces present in spatial models in a succinct way, the additional advantage of these measures is that they can be immediately computed from any exogenous cost shocks and cross-market trade links observed in the initial equilibrium. For instance, with gravity trade links, we have that $\chi_{oij} = -\chi \left(1_{[i=o]} - x_{oj}^0\right)$ and the expressions in (21) and (22) only depend on bilateral trade flows in the initial equilibrium (up to the constant trade elasticity). In such particular case, $\hat{\eta}_i^C$ and $\hat{\eta}_i^R$ correspond to the partial equilibrium (holding wages and employment constant) impact of cost shocks on *changes* in consumer and firm market access, introduced in Redding and Venables (2004) and used henceforth in a large literature studying the impact of trade shocks (see for example Redding and Sturm (2008), Donaldson and Hornbeck (2016), and Bartelme (2018)).

Notice that expressions in (21) and (22) interact market-specific initial conditions and cost shocks in the world economy. These exposure measures have a structure similar to that of shift-share exposure measures used in a variety of empirical analysis since Bartik (1991), including Autor, Dorn, and Hanson (2013), Kovak (2013) and Acemoglu et al. (2016). In Section 5, we make this connection explicit in the context of the parametric version of the model used in our empirical application.

Given these definitions, the log-linear version of the labor market module in (18) is

$$\bar{\boldsymbol{\lambda}} \log \hat{\boldsymbol{\omega}} = \log \hat{\boldsymbol{p}} - \log \hat{\boldsymbol{P}}. \tag{23}$$

As in Section 2, $\bar{\boldsymbol{\lambda}}$ is invertible under Assumption 1. Therefore, by log-linearizing equation

[17] In the Online Appendix D.3.2, we show that input-output linkages yield an additional measure of local shock exposure in terms of input costs: the exogenous shock to market i's producer price index (analogous to the shock in the consumer price index in equation (21)).

(12), we determine the change in real wages:

$$\log \hat{\boldsymbol{\omega}} = \bar{\boldsymbol{\beta}} \left[\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}} \right) \log \hat{\boldsymbol{p}} - \log \hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) \right],$$

where $\bar{\boldsymbol{\beta}} \equiv \bar{\boldsymbol{\lambda}}^{-1} = \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\psi}}^0 \bar{\boldsymbol{\phi}}^0 \right)^{-1}$. Combining this with the log-linearized version of the trade module in (19), the vector of changes in production cost solves the following system:

$$\bar{\boldsymbol{\gamma}} \log \hat{\boldsymbol{p}} = \log \hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\mu}} \bar{\boldsymbol{\beta}} \log \hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}), \tag{24}$$

with $\bar{\boldsymbol{\mu}} \equiv \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}} \right) \left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{\psi}} \right) \bar{\boldsymbol{\phi}}$ and $\bar{\boldsymbol{\gamma}} \equiv \bar{\boldsymbol{I}} - \bar{\boldsymbol{\chi}} - \bar{\boldsymbol{y}} + \bar{\boldsymbol{\mu}} \bar{\boldsymbol{\beta}} \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}} \right)$.

The Jacobian matrix $\bar{\boldsymbol{\gamma}}$ of the log-linear system in (24) combines two well known general equilibrium forces in trade and geography models. As in gravity trade models, changes in production costs trigger responses in spending shares across markets – i.e., the trade substitution effect in $X_{ij}(\cdot)$, whose elasticity matrix is $\bar{\boldsymbol{\chi}}$. As in spatial models, cost changes affect the spatial allocation of labor and, therefore, the relative size of destination markets – i.e., the revenue effect in $w_i L_i$, whose elasticity matrix is $\bar{\boldsymbol{\mu}} \bar{\boldsymbol{\beta}} \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}} \right)$. As in Section 2, by bounding the off-diagonals of the equilibrium system's Jacobian, we establish the uniqueness of the model's equilibrium and its counterfactual predictions.

Assumption 2. *For every equilibrium price vector $\boldsymbol{p}$, assume that (i) $\gamma_{ii}(\boldsymbol{p}) > 0$, and (ii) there is a vector $\{h_i(\boldsymbol{p})\}_i \gg 0$ such that $h_i(\boldsymbol{p})\gamma_{ii}(\boldsymbol{p}) > \sum_{j \neq i,m} |\gamma_{ij}(\boldsymbol{p})| h_j(\boldsymbol{p})$, given any reference market m. Assume also $\Phi_i(\cdot)$ is bounded above and that $\lim_{p_i \to 0} \frac{X_{ij}\Phi_j\Psi_j}{p_i} = \infty$ for some j.*

Under Assumption 2, we establish the following result.

Proposition 2. *Suppose that Assumptions 1 and 2 hold. There exist unique vectors of real wages, $\boldsymbol{\omega} \equiv \{\omega\}_i$, and production costs, $\boldsymbol{p} \equiv \{p\}_i$, that solve (18)–(19). Up to a first-order approximation, the impact of cost shocks $\hat{\boldsymbol{\tau}}$ on labor market outcomes is given by*

$$\begin{array}{rcl} \log \hat{\boldsymbol{\omega}} & = & \bar{\boldsymbol{\gamma}}^R \log \hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) - \bar{\boldsymbol{\gamma}}^C \log \hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) \\ \log \hat{\boldsymbol{L}} & = & \bar{\boldsymbol{\phi}} \left[\bar{\boldsymbol{\gamma}}^R \log \hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) - \bar{\boldsymbol{\gamma}}^C \log \hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) \right] \end{array} \tag{25}$$

where $\bar{\boldsymbol{\gamma}}^R \equiv \bar{\boldsymbol{\beta}} \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}} \right) \bar{\boldsymbol{\alpha}}$, $\bar{\boldsymbol{\gamma}}^C \equiv \bar{\boldsymbol{\beta}} \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\gamma}}^R \bar{\boldsymbol{\mu}} \bar{\boldsymbol{\beta}} \right)$, $\bar{\boldsymbol{\alpha}} \equiv \bar{\boldsymbol{M}}' \left(\bar{\boldsymbol{M}} \bar{\boldsymbol{\gamma}} \bar{\boldsymbol{M}}' \right)^{-1} \bar{\boldsymbol{M}}$, and $\bar{\boldsymbol{M}}$ is the matrix obtained from deleting the m-th row from the identity matrix.

Proof. Appendix A.3.

The expressions in (25) outline the *reduced-form* transmission channels of cost shocks across local labor markets in our model. In this more general model, the magnitude of indirect spillover effects still depends on local labor market linkages through the reduced-form matrix $\bar{\boldsymbol{\beta}}$. However, the presence of trade linkages creates additional transmission channels through cross-market effects on trade demand and consumption costs summarized by the definitions of matrices $\bar{\boldsymbol{\gamma}}^R$ and $\bar{\boldsymbol{\gamma}}^C$. The indirect spillover effects that correspond to the off-diagonal elements of these matrices arise from the fact that, in response to direct shock exposure, good market clearing requires changes in production costs, which trigger further endogenous responses in trade flows due to the substitution and the market size effects in trade demand.

To gain intuition for these forces, we now consider the implications of assuming constant elasticity cross-market links, commonly referred to as "gravity":

Assumption 3. *Assume that, for all i and j, (i) labor market linkages are $\phi_{ij} = \phi 1_{[i=j]} - \tilde{\phi}_j$ and $\psi_{ij} = \psi 1_{[i=j]} - \tilde{\psi}_j$, and (ii) the trade elasticity is $\chi_{oij} = \chi 1_{[i=o]} - \chi x_{oj}$.*

The following result characterizes the magnitude of direct and indirect effects under different assumptions regarding the nature of trade linkages across markets.

Proposition 3. *Suppose Assumptions 1-3 hold. Consider the impact of cost shocks $\hat{\boldsymbol{\tau}}$.*
1) Assume that markets are segmented, i.e. $\tilde{\lambda}_j = 0$. Consider a set of small open economies $i \in I$ such that, for all $j \neq i$, $y^0_{ji} = x^0_{ij} \approx 0$. If $\hat{\tau}_{oj} = \hat{\tau}_{jo} = 0$ for all $o, j \notin I$, then

$$\log \hat{\omega}_i = \begin{cases} \gamma^R_{ii} \log \hat{\eta}^R_i(\hat{\boldsymbol{\tau}}) - \gamma^C_{ii} \log \hat{\eta}^C_i(\hat{\boldsymbol{\tau}}) & \text{if} \quad i \in I \\ 0 & \text{if} \quad i \notin I \end{cases}$$

2) Assume an initial equilibrium with symmetric trade costs i.e., $\tau_{ij} = \tau_i$ for all j. Then,

$$\log \hat{\omega}_i = \gamma^R \log \hat{\eta}^R_i(\hat{\boldsymbol{\tau}}) - \gamma^C \log \hat{\eta}^C_i(\hat{\boldsymbol{\tau}}) + \sum_j \left(\tilde{\gamma}^R_j \log \hat{\eta}^R_j(\hat{\boldsymbol{\tau}}) - \tilde{\gamma}^C_j \log \hat{\eta}^C_j(\hat{\boldsymbol{\tau}}) \right).$$

Proof. Appendix A.4.

Proposition 3 generalizes the first two parts of Proposition 1 in the presence of the trade module, constant trade elasticities, and for shocks to overall revenue and consumer exposure, $\hat{\eta}^R_i, \hat{\eta}^C_i$. The first part considers a shock to a set of small open economies with segmented markets, which is the parallel to the first part of Proposition 3. In this case, the shock does not generate indirect effect. This follows from two assumptions. First, for any small open economy $i \in I$, wages can freely adjust without affecting labor demand in the rest of the country: $\gamma^R_{ji} = \gamma^C_{ji} \approx 0$ for all $j \neq i$. Second, small open economies do not trigger changes in the consumption cost and revenue of other markets: $\log \hat{\eta}^R_j = \log \hat{\eta}^C_j = 0$ for all $j \neq i$.

This is the main intuition behind the multiple-sector models that motivate the empirical specifications in Kovak (2013) and Autor, Dorn, and Hanson (2013).

The second part of Proposition 1 considers the special case of gravity trade demand without trade costs in the initial equilibrium ($\tau_{ij} = \tau_i$ for all j). In this case, we show that the symmetric trade linkages generate symmetric indirect spillover effects in equilibrium, giving rise to an "endogenous" fixed-effect in the reduced-form impact of cost shocks on labor market outcomes.

An important force incorporated in the system of equations (25), due to the presence of the trade module, is the direct compensating effect due to shocks to the consumption exposure $\hat{\eta}_i^C$. A shock to productivity or trade costs that may shrink the revenues of a market could be compensated at the same time by the effect of lower local prices. In fact, in the special case of exogenous labor supply ($\bar{\boldsymbol{\phi}} = 0$), we have that $\bar{\boldsymbol{\mu}} = 0$ and $\bar{\boldsymbol{\gamma}}^C = \bar{\boldsymbol{I}}$, which implies that, for any market, an increase in consumption cost triggers an identical reduction in the local real wage.

3.3 Discussion: Equivalences and Extensions

We now discuss how our theoretical environment unifies a number of existing frameworks in the trade and geography literature, through the shape of the aggregate mappings $\left\{\{X_{ij}(\cdot)\}_i, \Phi_j(\cdot), \Psi_j(\cdot)\right\}_j$. By properly specifying these mappings, our generalized spatial model generates counterfactual predictions for changes in labor market outcomes that are observationally equivalent to those implied by a wide range of spatial models. Thus, in the class of models covered by our model, the researcher does not need to take a stance on the underlying microeconomic assumptions that generate the unobservable shocks, as this matters only insofar it affects the shape of the macroeconomic mappings. We organize spatial models into four broad categories using different assumptions on these mappings and the Online Appendix D.2 formally establishes these equivalence results.

The first category includes trade models featuring workers that permanently reside in each location. Such models entail different mappings of bilateral trade flows $X_{ij}(\cdot)$, with $\Psi_i(\cdot) = \Phi_i(\cdot) = 1$ for all i. The main representative from this class is the trade gravity model with its various micro-foundations. Examples of such class include the Armington trade model (Anderson (1979)), variations of the heterogeneous technology Ricardian model, as in Eaton and Kortum (2002), and other forms of competition and demand structures discussed in Arkolakis, Costinot, and Rodríguez-Clare (2012), Costinot and Rodriguez-Clare (2013), and Arkolakis et al. (2017). In addition, our generalized trade demand is observationally equivalent to the one studied by Adao, Costinot, and Donaldson (2017) in the context of a one-factor neoclassical economy.

The second category includes models with agglomeration economies, in which the micro-foundation determines the mapping $\Psi_i(\cdot)$. In Krugman (1980), firm entry and increasing returns to scale in production give rise to agglomeration forces that depend on employment in each market – in this case, $\psi_{ii} = 1/(\sigma - 1)$ and $\psi_{ij} = 0$ for $i \neq j$, where σ is the elasticity of substitution across varieties. In the same spirit, the specification in Allen and Arkolakis (2014) corresponds to the case of $\psi_{ii} = \alpha$ and $\psi_{ij} = 0$ for $i \neq j$. Marshallian external economies of scale, as introduced in Ethier (1982b), correspond to similar specifications, with i denoting a sector in a region. Moreover, the cross-market productivity effects in ψ_{ij} incorporate congestion forces arising from other factors of production, such as land, as in Allen and Arkolakis (2014) and Caliendo et al. (2018b), and agglomeration spillovers arising from technology diffusion, as in Fujita, Krugman, and Venables (1999) and Lucas and Rossi-Hansberg (2003).[18]

Third, through the shape of the labor supply mapping $\Phi_i(\cdot)$, our model replicates predictions of trade and geography models featuring different degrees of labor mobility and amenity externalities across sectors and regions. This is the case for "New Economic Geography" models, as in Krugman (1991) and Helpman (1998), where there may be mobility across regions but not across sectors. Similarly, this is the case for traditional trade models, such as the neoclassical setup and more recent quantitative multiple-sector gravity setups surveyed by Costinot and Rodriguez-Clare (2013), where there is mobility across sectors but not across regions. More recently, a series of papers introduces a constant-elasticity gravity structure on labor supply stemming from heterogeneity in location and sector-specific preferences or efficiency – e.g., Allen and Arkolakis (2014), Redding (2016), Allen, Arkolakis, and Takahashi (2018), Bartelme (2018), Bryan and Morten (2015), and Galle, Rodriguez-Clare, and Yi (2017). More generally, the unrestricted function $\Phi_j(\cdot)$ implies that our models is observationally equivalent to a generalized Roy model, as in Adão (2015).[19]

Fourth, combined restrictions on $(X_{ij}(\cdot), \Phi_j(\cdot), \Psi_j(\cdot))$ imply that our model is observationally equivalent to some existing quantitative spatial models reviewed by Redding and Rossi-Hansberg (2016). The formal equivalence requires potentially different transfer rules

[18] Technology levels that are proportional to population are also postulated in Kortum (1997); Eaton and Kortum (2001). Models with spatial diffusion of knowledge specify cross-location spillovers of knowledge so that $\psi_{ij} > 0$ even if $i \neq j$. Our environment also accommodates models with multiple sectors that differ in terms of market structure and strength of economies of scale – e.g., Krugman and Venables (1995), Balistreri, Hillberry, and Rutherford (2010), Kucheryavyy, Lyn, and Rodríguez-Clare (2016).

[19] In Appendix D.3 we also extend our model to allow markets to be region-sector-occupation triples. With this more general definition of a market, the model is observationally equivalent to environments with gravity-like labor supply function across occupations, as in Burstein, Morales, and Vogel (2016) and Lee (2015).

that specify how income of non-labor factors are allocated across markets. For instance, Allen and Arkolakis (2014) impose a local transfer that is proportional to the income of the residents in a location, and Caliendo et al. (2018b) allow for the possibility that rental income is concentrated in a national portfolio and then split equally to the residents of each location.

In the Online Appendix D.3, we extend the model to incorporate input-output linkages in production, as in Eaton and Kortum (2002) and Caliendo and Parro (2014). Whenever the bilateral trade demand for final and intermediate goods are identical, input-output linkages do not affect the counterfactual predictions of the model (conditional on aggregate mappings). This point is similar to the one made in Allen, Arkolakis, and Takahashi (2018). In contrast, differences in the trade demand for final and intermediate goods give rise to an additional measure of local shock exposure. Such measure captures the shift in the cost of inputs triggered by the economic shock and is closely related to the measures used in Acemoglu, Autor, Dorn, Hanson, and Price (2016) and Wang, Wei, Yu, and Zhu (2018).

We also extend our framework to incorporate additional features highlighted by the trade and geography literature. As we describe in the Online Appendix D.3, we extend the model to allow for: i) workers commuting across regions, as in Ahlfeldt, Redding, Sturm, and Wolf (2015), Monte, Redding, and Rossi-Hansberg (2018) and Allen, Arkolakis, and Li (2015); and ii) multiple worker groups in production, as in Cravino and Sotelo (2017). Counterfactual predictions in the extended models require additional data and mappings compared to our baseline framework. These extensions elucidate how generalizations of our framework may affect the measures of exposure and the theoretical implications for the indirect effects.

4 Econometric Methodology

Our theoretical results establish the importance of cross-market links in shaping the impact of economic shocks on local labor markets in general equilibrium. We now tackle the problem of estimating these links. In particular, we focus on the estimation of the elasticity structure of labor supply, $\Phi_i(\cdot)$, and agglomeration, $\Psi_i(\cdot)$. Throughout our analysis, we assume that the elasticity for bilateral trade flows, $X_{ij}(.)$, is known, since its estimation has been the goal of an extensive literature in international trade.[20]

[20]In single-sector gravity models, the demand for bilateral trade flows only depends on the trade elasticity that has been studied by an extensive empirical literature – for a review, see Head and Mayer (2013). In addition, Caliendo and Parro (2014) and Costinot, Donaldson, and Komunjer (2011) consider multiple-sector gravity models where these functions only depend on the sector-level trade elasticity that is estimated using sector-level bilateral trade flows. More recently, Adao, Costinot, and Donaldson (2017) consider the problem of non-parametrically identifying the functions controlling bilateral trade flows in a competitive environment. It is possible to show that a similar argument holds in our environment, leading to the non-parametric identification of $X_{ij}(\cdot)$.

We develop our methodology in three steps. First, we describe the data generating process that imposes that $\Phi_i(\cdot)$ and $\Psi_i(\cdot)$ depend on a vector of unknown "deep" parameters. In this parametric model, we establish that, conditional on $\Phi_i(\cdot)$ and $\Psi_i(\cdot)$, local unobserved shocks in productivity and labor supply are identified from observable data on trade and labor outcomes. Second, we construct a class of moment conditions using observed foreign cost shifters that are orthogonal to local shocks in productivity and labor supply. Finally, we use our general equilibrium model to show that the "optimal" instrument in this class is the impact of *observed* foreign cost shocks on the *endogenous* variables predicted by our general equilibrium model – i.e., the Model-implied Optimal IV (MOIV).

4.1 Estimating Equations

In every period t, we assume that the world economy is generated by the model of Section 3. In equilibrium, trade and labor outcomes are endogenously determined by the solution of (12)–(17). Let y^t denote the value of variable y in period t, and $\hat{y}^t = y^t/y^0$ denote its change between a base period 0 and period t, with $\Delta \log y^t \equiv \log \hat{y}^t$.

We start by restricting the functions controlling labor supply and agglomeration forces.

Assumption 3a. *Assume that* $\Phi_j^t(\{\omega_i\}_i) = \Phi_j(\{\nu_i^t \omega_i\}_i|\boldsymbol{\theta})$ *and* $\Psi_j^t(\{L_i\}_i) = \Psi_j(\{L_i\}_i|\boldsymbol{\theta})$ *in every period* t*. The functions* $\Phi_j(\cdot|\boldsymbol{\theta})$ *and* $\Psi_j(\cdot|\boldsymbol{\theta})$ *are known differentiable functions of a vector of unknown parameters* $\boldsymbol{\theta} \in \mathbb{R}^s$*.*

This assumption imposes that the shape of the labor supply and productivity functions are known, except for a vector of unknown "deep" parameters $\boldsymbol{\theta}$. The rest of this section outlines a methodology to consistently estimate $\boldsymbol{\theta}$ using our general equilibrium model. To this end, we impose additional restrictions that allow the recovery of changes in unobserved local shifters, $\{\hat{\nu}_i^t, \hat{\tau}_{ii}^t\}$, from the observed changes in trade and labor outcomes, $\left\{\hat{x}_{ij}^t, \hat{\omega}_i^t, \hat{L}_i^t\right\}_i$, and the observed equilibrium variables in the base period, $\boldsymbol{W}^0 \equiv \left\{X_{ij}^0, L_j^0\right\}_{i,j}$. We consider the following invertibility assumption.

Assumption 3b. $\Phi_j(\cdot)$ *and* $X_{ij}(\cdot)$ *are functions such that (i)* $\{L_j\}_j = \{\Phi_j(\boldsymbol{\omega}|\boldsymbol{\theta})\}_j$ *is invertible, and (ii)* $\{x_{ij}\}_i = \left\{X_{ij}\left(\{p_{oj}\}_o\right)\right\}_i$ *is invertible (up to a scalar) for all* j*.*

The first part of Assumption 3b imposes that the system $\{L_j\}_j = \{\Phi_j(\boldsymbol{\omega}|\boldsymbol{\theta})\}_j$ is invertible and can be written as $\omega_j = \Phi_j^{-1}(\boldsymbol{L}|\boldsymbol{\theta})$.[21] The combination of this restriction with the labor

[21]The invertibility of the structural residuals is a crucial step in many empirical structural frameworks – see Berry (1994); Berry and Haile (2014). In our model, invertibility is guaranteed if the utility function has the following separable form: $U_c = \sum_j \nu_j C_j + U(\{L_j\})$, with $U(\cdot)$ strictly quasi-concave. In this case, the labor

supply equation in (14) yields the change in labor supply shifters:

$$\Delta \log \nu_j^t = -\Delta \log \omega_j^t + \Delta \log \Phi_j^{-1}\left(\boldsymbol{L}^t|\boldsymbol{\theta}\right). \tag{26}$$

To recover the productivity shifter, notice that the combination of $p_{ii}^t = \tau_{ii}^t p_i^t$ and the zero profit condition in (16) yields

$$\Delta \log \tau_{jj}^t = \Delta \log p_{jj}^t/P_j^t - \Delta \log \omega_j^t + \Delta \log \Psi_j\left(\boldsymbol{L}^t|\boldsymbol{\theta}\right). \tag{27}$$

The implementation of equation (27) requires a measure of p_{ii}^t/P_i^t. This implies either taking an explicit stance on the price data to measure p_{ii}^t/P_i^t, or using trade data to invert p_{ii}^t/P_i^t with the function $X_{ij}(\cdot)$. Since our model yields equivalent counterfactual outcomes to a wide class of existing frameworks, one should be cautious in using the first method. In fact, different theories about competition and firm behavior may lead to different dis-aggregated prices, as argued by Arkolakis, Costinot, and Rodríguez-Clare (2012) and Simonovska and Waugh (2014), and thus different measured aggregate prices. Thus, in the second part of Assumption 3b, we effectively impose that it is possible to recover relative prices from observed trade flows: $p_{oj}^t/p_{jj}^t = X_{oj}^{-1}\left(\boldsymbol{x}_j^t\right)$ where $\boldsymbol{x}_j^t \equiv \left\{x_{ij}^t\right\}_i$.[22]

Recalling that the price index function $P_j(\cdot)$ is homogeneous of degree one,

$$p_{jj}^t/P_j^t = \left[P_j\left(\left\{p_{ij}^t/p_{jj}^t\right\}_i\right)\right]^{-1} = \left[P_j\left(\left\{X_{ij}^{-1}\left(\boldsymbol{x}_j^t\right)\right\}_i\right)\right]^{-1}.$$

Notice that the integrability of the demand function $X_{ij}(\cdot)$ yields the price function $P_j(\cdot)$. So, p_{jj}^t/P_j^t is effectively a function of the bilateral trade demand function and the observed equilibrium vector of spending shares. Given the parameter vector $\boldsymbol{\theta}$, expressions (26)–(27) relate changes in the unobserved shifters, $\left\{\nu_j^t, \tau_{jj}^t\right\}$, to changes in observed variables, $\left\{\boldsymbol{x}_j^t, \omega_j^t, L_j^t\right\}$. We summarize this relationship in the following expression:

$$\begin{bmatrix} \Delta \log \nu_j^t \\ \Delta \log \tau_{jj}^t \end{bmatrix} = \begin{bmatrix} -\Delta \log \omega_j^t \\ \Delta \log p_{jj}^t/P_j^t - \Delta \log \omega_j^t \end{bmatrix} + \begin{bmatrix} \Delta \log \Phi_j^{-1}\left(\boldsymbol{L}^t|\boldsymbol{\theta}\right) \\ \Delta \log \Psi_j\left(\boldsymbol{L}^t|\boldsymbol{\theta}\right) \end{bmatrix}. \tag{28}$$

The system in (28) contains our main estimating equations, which rely on two premises.

supply function is the unique solution of the system of first-order conditions: $\nu_j \omega_j = \frac{\partial U}{\partial L_j}$. Whenever the labor supply function is homogeneous of degree zero, it is possible to relax Assumption 3b by imposing that $\Phi_j(\cdot|\boldsymbol{\theta})$ is invertible up to a scalar. This case arises in spatial models with constant aggregate employment.

[22] The demand function $X_{ij}(\cdot)$ is invertible (up to scalar) if it satisfies the connected substitutes property – see Berry, Gandhi, and Haile (2013). Adao, Costinot, and Donaldson (2017) show that the invertibility of $X_{ij}(\cdot)$ is a central property for its non-parametric identification, and that it is guaranteed in a generalized class of Ricardian economies.

First, because our approach hinges on the specification of "deep" parameters, it requires the correct functional forms of $\Phi_j(\cdot|\boldsymbol{\theta})$ and $\Psi_j(\cdot|\boldsymbol{\theta})$. Second, since expression (28) follows directly from the equilibrium conditions (14) and (16), these conditions must specify correctly the labor supply and production decisions in the economy. Violations of these premises introduce additional endogenous variables in (28) that give rise to usual concerns regarding omitted variable bias in the estimation of $\boldsymbol{\theta}$.[23] However, and more importantly, expression (28) is robust to misspecification in other equilibrium conditions of the model and, therefore, relies on weaker assumptions than those necessary to invert local unobservable shocks using the economy's full general equilibrium structure – as in Monte, Redding, and Rossi-Hansberg (2018); Faber and Gaubert (2016); Allen and Arkolakis (2014); Allen, Arkolakis, and Takahashi (2018).

To estimate $\boldsymbol{\theta}$ using expression (28), notice that, in general equilibrium, trade and labor outcomes are correlated with the unobserved shifters: $\left\{\boldsymbol{x}_j^t, \omega_j^t, L_j^t\right\}$ are endogenous variables that depend on the changes in *all* exogenous shifters, $\left\{\left\{\hat{\tau}_{ij}^t\right\}_i, \hat{\nu}_j^t\right\}$. For this reason, we develop a methodology that exploits the structure of our model to construct moment conditions for the consistent estimation of $\boldsymbol{\theta}$.

4.2 Model-implied Optimal IV

We now derive moment conditions for the estimation of $\boldsymbol{\theta}$ using the recovered error terms in (28) and observed cost shifters for a set of regions $i \in I$. To this end, assume that we have an observable variable, $\hat{\boldsymbol{z}}^t \equiv \left\{\hat{z}_{ij}^t\right\}_{i\in I}$, that satisfies the following conditions.

Assumption 3c. *There exists a trade cost shifter, $\hat{\boldsymbol{z}}^t \equiv \left\{\hat{z}_{ij}^t\right\}_{i\in I}$, such that*

1. $E\left[\Delta \log \tau_{ij}^t | \hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right] = \kappa \Delta \log z_{ij}^t$ *for* $i \in I$,
2. $E\left[\Delta \log \nu_j^t | \hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right] = E\left[\Delta \log \tau_{jj}^t | \hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right] = 0$ *for* $j \notin I$.

Assumption 3c imposes that changes in trade costs in the model, $\hat{\tau}_{ij}^t$, are log-linearly related to changes in the observable shifter, $\hat{z}_{ij}^t$. This log-linearity restriction significantly simplifies the conditions for optimality of our methodology, but it is not necessary to compute

[23] Any parametric empirical approach is subject to similar concerns. In our case, the separability of local unobserved shocks in equation (28) allows the non-parametric identification of the functions $(\Psi_j(\cdot), \Phi_j(\cdot))$ as long as instrumental variables satisfy the completeness condition proposed by Newey and Powell (2003) – for similar strategies, see Berry and Haile (2014), and Adao, Costinot, and Donaldson (2017). Expression (28) is subject to misspecification of the channels determining labor supply and production costs in conditions (14) and (16). For instance, as discussed in Appendix D.3, extensions of our model introduce extra variables into these conditions and, therefore, imply modified versions of (28).

our instrument. In addition, the second part of Assumption 3c states that, conditional on the initial vector of endogenous variables $\boldsymbol{W}^0$, the cost shifter $\hat{\boldsymbol{z}}^t$ for origin market $i \in I$ is mean-independent from local shocks to productivity and labor supply in any market $j \notin I$.

Notice that Assumption 3c is similar to the assumption required by empirical papers investigating the labor market consequences of foreign trade shocks, such as Topalova (2010), Kovak (2013), Autor, Dorn, and Hanson (2013), and Pierce and Schott (2016a). To see this, consider an economy with two countries such that $i \in I$ denote markets in the foreign country and $j \notin I$ denote markets in the domestic country. In this case, $\hat{z}_{ij}^t$ is an observed shock affecting the cost of foreign goods in the domestic market. Examples include changes in tariff and non-tariff applied by the domestic country on foreign goods, or changes in productivity in different sectors of the foreign country. In this setting, Part 1 states that the observed shock must indeed affect the cost of foreign goods in the domestic markets, and Part 2 states that the observed foreign cost shock is mean-independent from unobserved shocks to local labor markets in the domestic country.

To construct moment conditions, we introduce a function capturing the exposure of each market to the observable trade cost shock: $H_i\left(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right)$. By the law of iterated expectations, Assumption 3c immediately implies that, for any function $H_i(\cdot)$,

$$E\left[H_i\left(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right)\epsilon_i^t\right] = 0. \tag{29}$$

where $\epsilon_i^t \equiv \left(\Delta \log \nu_j^t, \Delta \log \tau_{jj}^t\right)'$.

The moment condition in (29) yields the following class of GMM estimators.

Definition 1. *Let $H_i\left(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right)$ be a $S \times 2$ matrix of functions. The GMM estimator is*

$$\hat{\boldsymbol{\theta}}_H \equiv \arg min_{\boldsymbol{\theta}} \left[\sum_{i,t} H_i(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0) e_i^t\left(\boldsymbol{\theta}\right)\right]' \left[\sum_{i,t} H_i(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0) e_i^t\left(\boldsymbol{\theta}\right)\right] \tag{30}$$

where $e_i^t\left(\boldsymbol{\theta}\right)$ is the vector ϵ_i^t recovered with (28).

Definition 1 outlines a standard GMM estimator based on the moment condition in (29). Notice that the dimension of $H_i(\cdot)$ is such that the number of moments is equal to the number of parameter in $\boldsymbol{\theta}$. This does not imply any loss of generality, since it is always possible to define $H_i(\cdot)$ to include an optimal weighting matrix in case the exposure function has dimensionality higher than S. Under standard regularity conditions, $\hat{\boldsymbol{\theta}}_H$ converges in probability to the true parameter vector, $\boldsymbol{\theta}$, and has an asymptotically normal distribution.[24]

[24] Newey and McFadden (1994) provide regularity conditions for consistency and normality of GMM

In order to implement the GMM estimator and estimate the vector of parameters $\boldsymbol{\theta}$, one has to specify the desired exposure function $H_i(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0)$ to the cost shock $\hat{\boldsymbol{z}}^t$. One common approach in the literature is the use of exposure functions constructed from shift-share instrumental variables – that is, the interaction of the cost shock, $\hat{\boldsymbol{z}}^t$, with local conditions of market i – e.g., the market's share of employment in different industries or firms. While this approach is intuitive and parsimonious, we take a step further and select the "efficient" exposure function $H_i(\cdot)$. Specifically, note that although any exposure function $H_i(\cdot)$ yields a consistent estimator of $\boldsymbol{\theta}$, different functions vary in terms of asymptotic variance — that is, the estimators differ in precision. To choose the exposure function $H_i(\cdot)$, we follow the approach in Chamberlain (1987) and select the one minimizing the asymptotic variance of the estimator. Applying the result in Chamberlain (1987), we show in Appendix A.5 that the most efficient estimator in the class of estimators in Definition 1 is

$$H_i^*(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0) \equiv E\left[\nabla_{\boldsymbol{\theta}} e_i^t(\boldsymbol{\theta}) | \hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right] \left(\Omega_i^t\right)^{-1} \tag{31}$$

where $\Omega_i^t \equiv E\left[e_i^t(\boldsymbol{\theta}) e_i^t(\boldsymbol{\theta})' | \hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right]$. The matrix Ω_i^t in (31) implies that the optimal IV attributes higher weight to observations with a lower variance of local unobserved shocks. In the case of homoskedastic independent shocks, $\Omega_i^t = \Omega^t$ is the GMM optimal moment weight matrix.

The efficient exposure function also depends on $E\left[\nabla_{\boldsymbol{\theta}} e_i^t(\boldsymbol{\theta}) | \hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right]$: the expected response of the endogenous variables associated with $\boldsymbol{\theta}$ induced by the exogenous trade cost shifter $\hat{\boldsymbol{z}}^t$. Up to a first-order approximation, the expression of $e_i^t(\boldsymbol{\theta})$ in (28) implies that

$$E\left[\nabla_{\boldsymbol{\theta}} e_i^t(\boldsymbol{\theta}) | \hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right] \approx \nabla_{\boldsymbol{\theta}} \boldsymbol{B}_i(\boldsymbol{\theta})\, E\left[\Delta \log \boldsymbol{L}^t | \hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right], \tag{32}$$

where $\boldsymbol{B}_i(\boldsymbol{\theta})$ is the i-th row of $[\bar{\boldsymbol{\phi}}^{-1}(\boldsymbol{\theta}),\ \bar{\boldsymbol{\psi}}(\boldsymbol{\theta})]$. The function $\boldsymbol{B}_i(\boldsymbol{\theta})$ controls how the parameter $\boldsymbol{\theta}$ affects the elasticity structure of labor supply and agglomeration and, therefore, how the trade shock $\hat{\boldsymbol{z}}^t$ affects $e_i^t(\boldsymbol{\theta})$ through its expected effect on employment, $E\left[\Delta \log \boldsymbol{L}^t | \hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right]$.

We construct a log-linear approximation for the optimal instrument in (31) with the predicted impact of trade cost shocks on employment in our general equilibrium model. Specifically, the combination of equations (31)–(32) and Proposition 2 yields the following proposition.

Proposition 4. *Suppose the world economy is generated by the model in Section 3, satisfying*

estimators of the form in Definition 1 – for consistency, see Theorems 2.6–2.7 and, for normality, see Theorem 3.4. Such regularity conditions require $\boldsymbol{\theta}$ to be the unique solution of the moment condition in (29). This is implied by usual rank conditions establishing identification in GMM estimators. For instance, if $\Delta \log \nu_i^t(\boldsymbol{\theta})$ and $\Delta \log \tau_{ii}^t(\boldsymbol{\theta})$ are linear in $\boldsymbol{\theta}$, then uniqueness requires $H_i(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0)$ to be correlated with the endogenous variables multiplying $\boldsymbol{\theta}$.

Assumptions 3a–3c. The function $H_i^*(\cdot)$ in (31) determines the estimator with the minimum asymptotic variance in the class of estimators in Definition 1, and is approximately given by

$$H_i^{MOIV}(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0|\boldsymbol{\theta}) = \kappa \nabla_{\boldsymbol{\theta}} \boldsymbol{B}_i\left(\boldsymbol{\theta}, \boldsymbol{W}^0\right) \Delta \log \boldsymbol{L}\left(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0|\boldsymbol{\theta}\right) \left(\Omega_i^t\right)^{-1}, \tag{33}$$

where

$$\Delta \log \boldsymbol{L}\left(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0|\boldsymbol{\theta}\right) \equiv \bar{\boldsymbol{\gamma}}^R(\boldsymbol{\theta}) \log \hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{z}}^t|\boldsymbol{W}^0) - \bar{\boldsymbol{\gamma}}^C(\boldsymbol{\theta}) \log \hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{z}}^t|\boldsymbol{W}^0). \tag{34}$$

Proof. *Appendix A.6.*

The MOIV estimator uses the general equilibrium model to approximate $E\left[\Delta \log \boldsymbol{L}^t|\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right]$. Intuitively, through the lens of the model, this is the best predictor of how trade cost shocks affect employment across markets, leading to the most precise estimates. In other words, whenever the general equilibrium model is well specified, it provides the most accurate measure of the impact of the cost shock across local markets.[25]

Notice that the MOIV estimator in (33) depends on the unknown parameter $\boldsymbol{\theta}$. To avoid a cumbersome computation of the estimator, we now characterize an asymptotically equivalent two-step estimator. Using a guess of the structural parameters, we compute the predicted changes in endogenous variables, $\Delta \log \boldsymbol{L}\left(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0|\boldsymbol{\theta}_0\right)$, and the instrumental variable, $H_i^{MOIV}(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0|\boldsymbol{\theta}_0)$. In the first-step, we use this instrumental variable to obtain $\hat{\boldsymbol{\theta}}_1$ with the estimator in (30). Since the instrument is a function of $\left(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right)$, the first-step estimator is a consistent estimator of $\boldsymbol{\theta}$, but it is not optimal because it was computed using an arbitrary guess of the parameter vector. Thus, in the second-step, we use the consistent estimate $\hat{\boldsymbol{\theta}}_1$ to compute the instrumental variable $H_i^{MOIV}(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0|\hat{\boldsymbol{\theta}}_1)$ and use it to obtain $\hat{\boldsymbol{\theta}}_2^{MOIV}$ with the estimator in (30).

Proposition 5. *The Model-implied Optimal IV estimator is asymptotically equivalent to the estimator obtained from the following two-step procedure.*

Step 1. *Using an initial guess $\boldsymbol{\theta}_0$, compute $H_i^{MOIV}(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0|\boldsymbol{\theta}_0)$ and estimate $\hat{\boldsymbol{\theta}}_1$ with (30).*
Step 2. *Using $\hat{\boldsymbol{\theta}}_1$, compute $H_i^{MOIV}(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0|\hat{\boldsymbol{\theta}}_1)$ and estimate $\hat{\boldsymbol{\theta}}_2^{MOIV}$ with (30).*

Proof. Appendix A.7.

[25] To gain intuition for this result, assume that $\Omega_i^t = \sigma I$, $\Delta \log \Psi_i\left(\boldsymbol{L}^t|\boldsymbol{\theta}\right) = \theta_1 \Delta \log L_i^t$ and $\Delta \log \Phi_i^{-1}\left(\boldsymbol{L}^t|\boldsymbol{\theta}\right) = \theta_2 \Delta \log L_i^t$. In this case, $H_i^{MOIV}(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0|\boldsymbol{\theta}) = \kappa\sigma \Delta \log L_i\left(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0|\boldsymbol{\theta}\right)$ and, therefore, MOIV is the model's predicted employment response to the trade cost shock. More generally, the vector $\nabla_{\boldsymbol{\theta}} \boldsymbol{B}_i\left(\boldsymbol{\theta}, \boldsymbol{W}^0\right)$ in (33) yields a linear combination of the predicted employment responses in different markets to capture their effect on residuals through the structural parameters to be estimated.

5 Estimation of Cross-Market Linkages in the US

We now use the theoretical results developed in the previous section to estimate the cross-market linkages across Commuting Zones (CZs) in the United States. We start by proposing a parametric multi-industry version of the model introduced in Section 3 that entails a flexible structure of cross-market linkages, yet with a parsimonious set of structural parameters. We then estimate these parameters with the methodology proposed in Section 4, using as exogenous cost shifter a measure of industry-level Chinese productivity growth between 1997 and 2007. We show that our model's predicted employment responses are consistent with the observed cross-region patterns of employment changes in the period.

5.1 Multiple-Sector Spatial Model

The first step of our methodology is the parametrization of the Generalized Spatial Economy presented in Section 3. Overall, while we focus on flexibly modeling spatial links, we try to strike a balance between the tractability of our empirical application and the generality of our theoretical environment. To achieve such a balance, we rely on existing frameworks in the literature to guide our parametric choices of the functional forms governing cross-market links in labor supply, productivity, and trade flows. Appendix B.1 outlines the utility and production functions generating our parametric model.

In each period t, we assume that the equilibrium of our model generates outcomes in the world economy. Each country c has multiple regions, $r \in \mathcal{R}_c$, and multiple industries, $n = 1, ..., N$. Industries are divided into two sectors: manufacturing, $s = M$, and non-manufacturing, $s = N$. In our empirical application, we assume that a market is a sector-region pair such that endogenous production costs are identical in all industries in the same sector and region. This is guaranteed by two restrictions in the model. First, labor in all industries within a market is perfect substitutable in the representative household's preferences, so that there is a single wage rate for each market. Second, agglomeration forces are identical in all industries in a market, and only depend on employment across all sector-region pairs. In the rest of the section, we use k, s to denote sectors, and r, d to denote regions.

Bilateral Trade Flows. We follow the extensive literature on quantitative gravity trade models by imposing nested CES preferences for goods produced in different sectors and regions. Specifically, conditional on bilateral good prices, we assume that the industry-level spending share of market sd on goods produced in industry n of market kr is

$$\bar{x}^t_{n,kr,sd} = \left(\frac{p^t_{n,kr,sd}}{P^t_{n,k,sd}}\right)^{-\bar{\chi}_n}, \tag{35}$$

where $P^t_{n,k,sd} = \left[\sum_r \left(p^t_{nr,sd}\right)^{1-\bar{\chi}_n}\right]^{\frac{1}{1-\bar{\chi}_n}}$ is the industry-level price index, and $\bar{\chi}_n$ is the between-origin elasticity of substitution, which we allow to vary across industries within manufacturing, as in Costinot and Rodriguez-Clare (2013) and Caliendo and Parro (2014).

In addition, we assume that, in market sd, the spending share on industry n of sector k is

$$\tilde{x}^t_{n,k,sd} = \alpha_{n,k,sd}\left(\frac{P^t_{k,sd}}{P^t_{sd}}\right)^{-\chi}, \tag{36}$$

where $P^t_{sd} = \left[\sum_k \left(P^t_{k,sd}\right)^{-\chi}\right]^{-\frac{1}{\chi}}$ is the price index in market sd. The parameter χ corresponds to the elasticity of substitution between manufacturing and non-manufacturing sectors, while $\alpha_{n,k,sd}$ is the constant spending share on goods from industry n. A similar nested gravity structure of trade flows, featuring a Cobb-Douglas structure across industries within a sector and CES across aggregate sectors, has been used recently in Costinot and Rodriguez-Clare (2013) and Cravino and Sotelo (2017).

The combination of these two expressions yields the spending share of market sd on goods produced in market kr:

$$x^t_{kr,sd} = \sum_{n\in k} \bar{x}^t_{n,kr,sd}\tilde{x}^t_{n,k,sd}. \tag{37}$$

Labor Supply. We assume that the representative household maximizes a nested utility function over the allocation of labor between sectors in each region, and between regions in each country. Specifically, we assume that the employment share in sector s of region r is

$$\bar{L}^t_{kr} = \left(\frac{\nu^t_{sr}\omega^t_{sr}}{W^t_r}\right)^{\phi_e}, \tag{38}$$

where $W^t_r \equiv \left[\sum_s \left(\nu^t_{sr}\omega^t_{sr}\right)^{\phi_e}\right]^{\frac{1}{\phi_e}}$ is the wage index in region r. In the model, the parameter ϕ_e regulates the between-sector labor supply elasticity.

We incorporate non-employment in the model by introducing an outside home sector, $s = H$, that yields an exogenous payoff given by $\nu^t_{Hr}\omega^t_{Hr} \equiv \upsilon^t_{Hr}$. Importantly, this assumption implies that global cost shocks do not affect the payoff of being non-employed.[26] In addition,

[26]This assumption is similar to the one recently adopted by Kim and Vogel (2018). Note that it implies a direct effect of trade shocks on employment through changes in the price index, since it affects the real wage in manufacturing and non-manufacturing but not the payoff of being non-employed. Such an effect has the

we assume that population of region r is

$$\tilde{L}_r^t = \frac{\left(\nu_r^t W_r^t\right)^{\phi_m}}{\sum_{d \in \mathcal{R}_c} \left(\nu_d^t W_d^t\right)^{\phi_m}}, \tag{39}$$

where the parameter ϕ_m controls the sensitivity of the extensive margin of employment across regions to the regional wage index. The combination of these two expressions determines labor supply in market kr:

$$L_{kr}^t \equiv \bar{L}_{kr}^t \tilde{L}_r^t.$$

The labor supply structure in (38)–(39) is related to recent quantitative trade and geography models featuring Logit functions of labor supply across sectors and regions. Whenever $\phi_m = 0$, our labor supply structure is isomorphic to that implied by Roy models with a Frechet distribution of sector-specific efficiency and preferences – such as Galle, Rodriguez-Clare, and Yi (2017). In addition, if $\phi_m = \phi_e$, our labor supply structure is equivalent to a static version of the model in Caliendo, Dvorkin, and Parro (2018a), which impose the same elasticity of relative employment across sectors and regions. Finally, by imposing $\phi_e \to \infty$, the model yields a single wage rate in each region, with a Logit function of labor supply across regions, as in Allen, Arkolakis, and Takahashi (2018). More generally, it is easy to show that our labor supply structure can be micro-founded by a model where heterogeneous individuals draw idiosyncratic preferences for sectors and regions from a Generalized Extreme Value distribution, as in McFadden (1980).

Technology. We assume that, for all industries in sector k of region r, the endogenous productivity term is

$$\Psi_{kr}\left(\boldsymbol{L}^t\right) = \Pi_d (L_{kd}^t)^{\psi \pi_{rd}} \quad \text{with} \quad \pi_{rd} = \frac{D_{rd}^{-1} L_{kd}^0}{\sum_{o \in \mathcal{R}_c} D_{ro}^{-1} L_{ko}^0}, \tag{40}$$

where D_{rd} is the distance between r and d, and L_{kd}^0 is employment in a base period.

In this specification, the parameter ψ controls the strength of agglomeration and congestion forces. This specification imposes that cross-market productivity spillovers are inversely related to the distance between markets. It is important to notice that equation (40) approximates the functional form introduced by Ahlfeldt et al. (2015) to specify the impact of changes in employment in a market on the productivity of other markets. By allowing

potential to partially offset disruptive effects of international competition on local labor markets due to the reduction in consumption costs triggered by cheaper imported goods. In order to relax this assumption, one needs to explicitly incorporate a role for unemployment and disability transfers across regions. Exploring the consequences of this assumption for employment responses is an interesting avenue for future research, but beyond the scope of this paper.

for cross-market spillovers in sectoral productivity, equation (40) is a generalization of applications that rely solely on local spillovers with $\pi_{rd} = 0$ for $r \neq d$ – e.g., see Bartelme (2018) and Allen, Arkolakis, and Takahashi (2018).[27]

Market Clearing. To close the model, we specify the labor market clearing condition. In each market, labor income equals revenue plus an exogenous transfer:

$$w^t_{kr} L^t_{kr} = \sum_{sd} x^t_{kr,sd} (w^t_{sd} L^t_{sd} + T^t_{sd}),$$

where, as in Dekle, Eaton, and Kortum (2007), T^t_{sd} is constant in terms of world production.

Estimating Equations. We now derive the equations for the estimation of the structural parameters in our model. As described below, we use literature estimates to parametrize the elasticities governing the bilateral trade demand. Using equations (35)–(36), we can write the relative competitiveness of a market, $Q^t_{kr} \equiv p^t_{kr,kr} / P^t_{kr}$, in terms of observed spending shares:

$$\log Q^t_{kr} = -\frac{1}{\chi} \log \tilde{x}^t_{k,kr} - \sum_{n \in k} \frac{\alpha_{n,k,kr}}{\bar{\chi}_n} \log \left(\bar{x}^t_{n,kr,kr} \right), \quad (41)$$

where, in market kr, $\tilde{x}^t_{k,kr}$ is share of sector k in total spending and $\alpha_{n,k,kr}$ is the share of industry n in spending on sector k.[28]

By plugging expression (40) into (27), we obtain the equation for the estimation of the parameter controlling productivity spillovers in the model,

$$\Delta \log \omega^t_{kr} - \Delta \log Q^t_{kr} = \psi \sum_d \pi_{rd} \Delta \log L^t_{kd} + \Delta \log \tau^t_{kr,kr}, \quad (42)$$

with Q^t_{kr} given by (41).

Turning to the labor supply equations, the assumption that the home sector's payoff is identical in all regions implies that the employment share in the home sector is given by

[27] Ahlfeldt et al. (2015) impose that $\Psi_r \left(\boldsymbol{L}^t \right) \equiv \left(\sum_d e^{-\delta \tau_{rd}} L^t_d \right)^{\psi}$, where τ_{rd} is the travel time between r and d. Under the assumption that travel time is proportional to distance ($\tau = \log D_{rd}$), this specification generates, up to a first-order approximation, productivity responses identical to those in our parametric model: $\Delta \log \Psi_r \left(\boldsymbol{L}^t \right) \approx \psi \sum_d \frac{D^{-\delta}_{rd} L^0_{kd}}{\sum_{o \in \mathcal{R}_c} D^{-\delta}_{ro} L^0_{ko}} \Delta \log \left(L^t_d \right)$. We set $\delta = 1$ in our baseline specification, and investigate alternative specifications of the decay rate in Appendix B.3.

[28] Cobb-Douglas preferences across industries in sector k yields $\log P^t_{k,kr} = \sum_{n \in k} \alpha_{n,k,kr} P^t_{n,k,kr}$ and, thus,

$$\log Q^t_{kr} = \log \frac{P^t_{k,kr}}{P^t_{kr}} \frac{p^t_{kr,kr}}{P^t_{k,kr}} = \log \frac{P^t_{k,kr}}{P^t_{kr}} + \sum_{n \in k} \alpha_{n,k,kr} \log \left(\frac{p^t_{kr,kr}}{P^t_{n,k,kr}} \right).$$

We obtain expression (41) by replacing $\log(p^t_{kr,kr} / P^t_{n,k,kr}) = -(1/\bar{\chi}_n) \log(\bar{x}^t_{n,kr,kr})$ using (35) and $\log P^t_{k,kr} / P^t_{kr} = -(1/\chi) \log \tilde{x}^t_{n,k,kr}$ using (36).

$\bar{L}^t_{Hr} = (W^t_r)^{-\phi_e}$. Thus, for any region r, the log-ratio of equation (38) for sector k and the home sector implies

$$\Delta \log \omega^t_{kr} = \frac{1}{\phi_e} \Delta \log \left(\bar{L}^t_{kr} / \bar{L}^t_{Hr} \right) - \frac{1}{\phi_e} \Delta \log \left(\nu^t_{kr} / \upsilon^t_{Hr} \right). \tag{43}$$

Similarly, the combination of $\bar{L}^t_{Hr} = (\tilde{\nu}^t_{Hr} / W^t_r)^{\phi_e}$ and equation (39) yields

$$\Delta \log \tilde{L}^t_r = -\frac{\phi_m}{\phi_e} \Delta \log \left(\bar{L}^t_{Hr} \right) + \delta^t + \Delta \log \upsilon^t_r, \tag{44}$$

where $\delta^t \equiv -\log \sum_d \left(\nu^t_d W^t_d \right)^{\phi_m}$ and $\Delta \log \upsilon^t_r \equiv \phi_m \Delta \log \nu^t_r \upsilon^t_{Hr}$. Equations (42)-(44) constitute our estimating equations for the vector of structural parameters $\boldsymbol{\theta} \equiv (\phi_e, \phi_m, \psi)$.

5.2 Data

To apply our methodology, we combine several datasets to construct trade and labor outcomes for regional markets in the United States between 1997 and 2007. We now describe the main variables in our analysis, and discuss the details of the data construction in Appendix C.

Labor Market Data. Our geographical units of analysis are the Commuting Zones (CZs), introduced by Tolbert and Sizer (1996) and recently used in several empirical papers – e.g., Autor, Dorn, and Hanson (2013), Autor and Dorn (2013) and Acemoglu and Restrepo (2017). For each CZ in mainland United States, we use the county-level data from County Business Pattern (CBP) to construct employment and average wage in the manufacturing and non-manufacturing sectors. We obtain data on county-level working-age population from the Census U.S. Intercensal County Population Data. Finally, we construct the price index in each CZ using the Cost of Living Index of urban areas published by the Council for Community and Economic Research (C2ER).[29] Our final sample contains labor market outcomes for 722 CZs in 1997 and 2007.

Trade Data. We combine data on US domestic shipments from the Commodity Flow Survey (CFS) and international trade data from UN Comtrade to construct a matrix of trade flows

[29] This is a well-known source of living cost differentials among cities in the United States – e.g., see Moretti (2013). We use the cost of living index for 292 MSAs to construct changes in price indices across CZs between 1997 and 2007. For the CZs without a matched set of MSAs, we assign the cost of living of the state with the majority of the CZ's population in 2000. As highlighted by Feenstra (1994), our price index does not fully capture the effect of new varieties in the cost of living. One possible solution is to use the local prices in the Nielsen Homescan Dataset to construct variety-adjusted price indices, as in Handbury and Weinstein (2014). However, this would significantly reduce our sample, since bar-code price data is only available for a subset of regions after 2004.

for 31 manufacturing industries and one non-manufacturing sector, between 722 CZs in the mainland United States, the states of Alaska and Hawaii, and 58 foreign countries.[30] We first use the UN Comtrade data to construct a country-to-country matrix of trade flows in each of the 32 industries. We then use information on shipments between US states in the CFS to estimate industry-level gravity equations, which we use to impute trade flows between CZs in the United States in 1997 and 2007. Finally, we merge these two intermediate datasets by proportionally splitting industry-level US trade flows across CZs using the CZ's share of national employment in each industry. We assume that the cost of shipping goods from any industry n in market kr to *any* sector of region d is the same, $\tau^t_{n,ko,sd} = \tau^t_{n,ko,d} \quad \forall s$, so that the bilateral trade demand above yields identical spending shares for *any* sector s of region d, $x^t_{ko,sd} = x^t_{ko,d}$ for all s. Appendix B describes the details of the trade data construction.

Trade Demand Parametrization. We parametrize the trade demand structure using estimates in the literature. In particular, we use the estimates in Caliendo and Parro (2014) to calibrate the industry-level trade elasticity – see Table 5 in Appendix C. In addition, we set the elasticity between manufacturing and non-manufacturing goods to $\chi = -0.3$, which is within the range of available estimates in the literature.[31]

5.3 Exposure to Chinese Import Competition

Our methodology requires cost shifters that are orthogonal to local shocks in productivity and labor supply affecting regional markets in the United States. In the spirit of Autor, Dorn, and Hanson (2013) and Autor, Dorn, Hanson, and Song (2014), we exploit the increase in Chinese exports between 1997 and 2007 by estimating the following regression:

$$\log X^{2007}_{n,c\tilde{c}} - \log X^{1997}_{n,c\tilde{c}} = \delta_{n,c} + \zeta_{n,\tilde{c}} + \epsilon_{n,c,\tilde{c}}, \tag{45}$$

where c and $\tilde{c}$ denote countries in our sample (excluding the United States).

Our measure of the Chinese export shock in industry n is the estimated exporter fixed-effect for China in that industry, $\hat{z}_n = -\hat{\delta}_{n,China}$. This source of cross-industry variation in China's export competitiveness is similar to the one used in Autor, Dorn, and Hanson (2013). It is a measure of the cross-industry variation in Chinese exports to world regions outside the

[30] Tables 12 and 5 in Appendix C display, respectively, the list of countries and industries used in our empirical application. Our industry classification is an aggregated version of the 42 commodity groups (SCTG) in the public CFS data.

[31] The between-sector elasticity of $\chi = -0.3$ is close to the estimates in Comin et al. (2015) and Cravino and Sotelo (2017). It is somewhat between the numbers found in Sposi (2018) ($\chi = -0.6$) and Herrendorf, Rogerson, and Valentinyi (2013) ($\chi = -0.15$).

US between 1997 and 2007. As such, it is mainly driven by China's accession to the WTO and fast productivity growth.[32] Table 5 in Appendix B.3 shows the estimated shock in each of the 31 manufacturing industries in our sample.

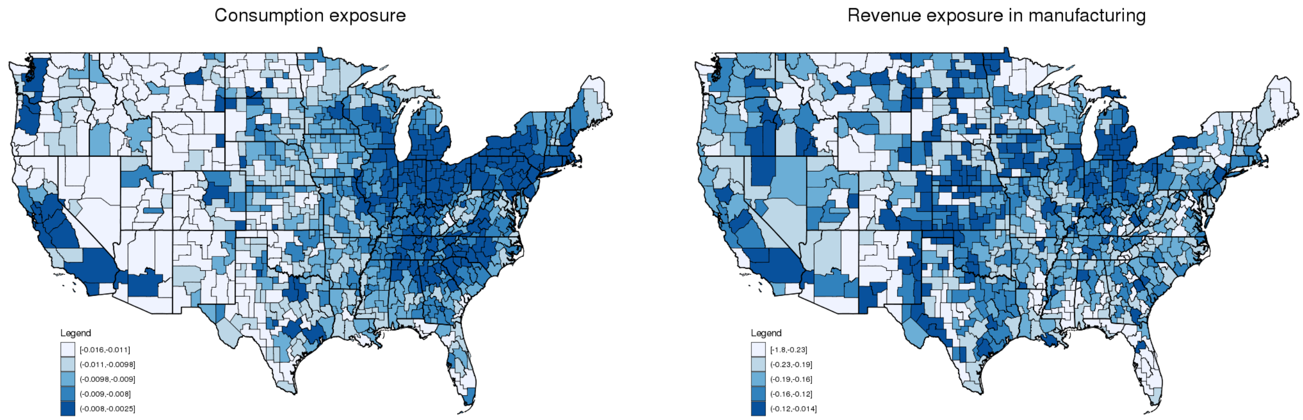
Notice that the gravity structure of our model provides a structural interpretation for $\hat{\delta}_{n,China}$. It combines the endogenous change in Chinese production costs, $\bar{\chi}_n \Delta \log p_{MChina}$, and the exogenous change in Chinese productivity and export costs, $\bar{\chi}_n \Delta \log \tau_{n,China}$. This raises two potential concerns about our cost shock measure. First, productivity and labor supply shocks in US CZs may affect the estimated export shock through the endogenous response of wages in China. Second, our structural model implies that the industry-level shock should be adjusted by the industry's trade elasticity $\bar{\chi}_n$. In Appendix B.3, we address these concerns by estimating our model with alternative configurations for the industry-level shock used in the extensive literature investigating the rise of China in the world economy. Specifically, we consider (i) the export shock adjusted by the industry's trade elasticity, $\{\hat{z}_n/\bar{\chi}_n\}_n$, (ii) the average growth in firm-level productivity, from Hsieh and Ossa (2016), (iii) the removal of the uncertainty on the US revoking NTR tariffs to Chinese goods – i.e., the so-called NTR gaps used by Pierce and Schott (2016a) and Handley and Limão (2017), (iv) the change in the bilateral trade costs between China and the US CZs computed with the procedure in Head and Ries (2001), as in Adao, Costinot, and Donaldson (2017). In all cases, we obtain qualitatively similar results as those presented below.

Figure 1 reports the exposure of US CZs to the increase in Chinese exports in terms of manufacturing revenues and consumption costs. These measures interact the industry-level Chinese export shock, $\hat{z}_n$, with the initial spending and revenue shares in each CZ. The map on the left shows the consumption exposure in (21). The shock triggered similar reductions in consumption costs across CZs. The average reduction was 1%, with a standard deviation of 0.2%. In our data, the CZ's manufacturing employment and spending explain 25% of the cross-regional variation in consumption exposure.

The map on the right plots the manufacturing revenue exposure from (22). On average, CZs experience a 18% exogenous decline in their manufacturing revenues. The cross-regional variation in exposure depends on both the CZ's initial revenues composition across industries as well as the spending share on Chinese goods in each industry by the CZ's trade partners. For example, Tampa in Florida, one of the worst hit CZs, is specialized in the production of fertilizers, which is the industry with the largest increase in Chinese import competition. More generally, the regions most exposed to the China shock are in Florida, West Coast

[32] One may be concerned that China's export growth was stronger in sectors in which the US experienced slow productivity growth and, therefore, losses in international market shares. However, Figure 5 in Appendix B.3 shows that our shock measure has only a positive weak correlation with the US exporter fixed-effect obtained from the estimation of 45 in a sample with the US.

Figure 1: Exposure to Chinese export growth, 1997-2007



(especially the Mountain Division) and northern New England.[33]

To gain intuition about the source of cross-regional variation embedded in the revenue exposure $\Delta \log \eta_i^R(\hat{\boldsymbol{\tau}})$, Appendix B.2 shows that it is effectively a shift-share exposure measure, in which the "shift" is the demand-adjusted cost shock in each destination-industry, and the "share" is the share of each destination-industry in the market's revenue. In the special case of no bilateral trade costs (i.e. $\tau_{n,kr,sd} = \tau_{n,kr,kr}$ for all sd), $\Delta \log \eta_i^R(\hat{\boldsymbol{\tau}})$ is proportional to a shift-share exposure measure where the "shift" is the industry-level shock, $\hat{\delta}_{n,China}$, and the "share" is the share of industry n in the CZ's total manufacturing employment. This special case is related to the exposure measures used in the empirical specification of recent papers – e.g., Topalova (2010), Kovak (2013) and Autor, Dorn, and Hanson (2013). In Figure 4 of Appendix B.2, we show that this commonly used shift-share measure and our model's manufacturing revenue exposure have a correlation of 0.3.

5.4 Structural Parameters Estimates

We now implement the two-step procedure described in Section 4 to estimate the structural parameters $\boldsymbol{\theta} \equiv (\phi_e, \phi_m, \psi)$. In each step, we recover the structural errors using equations (42)–(44), and compute instrumental variables using the model's predicted impact of the China manufacturing productivity shock on CZ's labor market outcomes.

Table 1 reports our baseline estimates, along with the standard errors clustered at the state-level. Panel A presents the estimates obtained from the first-step of our procedure

[33] In the non-manufacturing sector, the effect on the revenue exposure is instead positive but very small. Since $\chi = -0.3$, manufacturing and non-manufacturing goods are complements, and thus cheaper manufacturing goods from China trigger an increase in the demand of the domestic non-manufacturing sector.

and Panel B the estimates from the second-step.[34] In line with the asymptotic properties shown in Section 4, the slightly higher F statistic in Panel B compared to that in Panel A indicates that the two-step procedure entails efficiency gains, but these gains are small in our application. Appendix B.3 shows that there are no further efficiency gains from a third-step estimation where the instrument is computed with the estimates of Panel B.

Columns (1)–(2) present the parameters governing labor supply responses in the model. In column (1), our estimate yields a between-sector elasticity of $\hat{\phi}_e \approx 1.1$. That is, a 1% increase in the sector's relative wage triggers a 1.1% increase in the sector's relative employment. In 1997, this implies an average extensive margin elasticity of 0.10% and 0.27% for the manufacturing and non-manufacturing sectors respectively.[35] Interestingly, our estimates suggest that the between-sector elasticity is higher than the between-region elasticity of labor supply. The point estimate in column (2) yields an elasticity of the CZ's population to its wage index of 0.4. However, as in Autor, Dorn, and Hanson (2013), our migration responses are not precisely estimated, resulting in the non statistically significant in column (2).

Finally, consider the parameter governing productivity spillovers in column (3). Given the specification in 40, an increase in the CZ's manufacturing employment of 1% yields an increase in the CZ's manufacturing productivity of $\psi\pi_{rr}$%. On average across CZs, our point estimate indicates that such a productivity response is 0.35%. Thus, our estimate of the local agglomeration elasticity is similar to the one implied by a Krugman model (i.e., the inverse of the trade elasticity).[36] In addition, our specification also yields productivity spillovers across regions. If all other CZs experience a 1% increase in manufacturing employment, the local productivity in a CZ increases by 0.27%, giving rise to potentially large cross-market productivity spillovers. To our knowledge, there are no available estimates in the literature that we can use for comparison.

[34] To compute the instrumental variable in the first step, we calibrate our model to approximate the predictions of a benchmark model without agglomeration and labor supply spillovers across regions – i.e., $\psi^0 = 0$ and $\phi_m^0 = 0$. In addition, we calibrate the model with between-sector employment elasticity of $\phi_e^0 = 0.5$. In the second-step of our procedure, we use the first-step estimates in Panel A to re-compute the instrumental variable.

[35] The labor supply function in (38) implies that, if the real wage in sector k increases by 1%, the employment share in sector k increases by $\phi_e \bar{L}_{ko}^0(1 - \bar{L}_{ko}^0)$. When combining both sectors, this expression yields an average uncompensated elasticity of labor supply of 0.26, which is within the range of estimates reviewed by Chetty et al. (2013). Note that Chetty et al. (2013) review estimates of the compensated elasticity of labor supply, but argue that, due to small income effects on labor supply, uncompensated and compensated elasticities are typically very similar.

[36] Our estimates are consistent with estimates presented in recent papers. Kline and Moretti (2014) estimate an elasticity of county productivity with respect to manufacturing density of 0.4, and Bartelme et al. (2017) estimate a median industry-level agglomeration elasticity that is around half of the elasticity implied by the Krugman model. In contrast, Ciccone (2002) finds that the elasticity of local productivity to local population is, for European regions, around 20% of our estimate.

Table 1: Estimates of Structural Parameters

	$-\phi_m/\phi_e$	$1/\phi_e$	ψ
	(1)	(2)	(3)
Panel A: First-Step			
	0.860***	-0.821	0.385**
S.E.	(0.106)	(1.064)	(0.166)
F Stat.	14.4	1.6	8.7
Panel B: Second-Step			
	0.917***	-0.395	0.635***
S.E.	(0.121)	(0.467)	(0.177)
F Stat.	16.5	5.9	9.2
N	722	1444	1444

Notes: Sample of 722 Commuting Zones and 2 Sectors in 1997-2007. Models are weighted by start of period CZ share of national population. Instrumental variable computed with $\phi_m^0 = \psi^0 = 0$ and $\phi_e^0 = 0.5$ in Panel A, and with First-Step estimates in Panel B. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

5.5 Model Fit

We now use the estimated structural parameters to investigate our model's fit in terms of cross-regional responses in sectoral employment. We use the estimated Chinese export growth between 1997 and 2007 as the only source of model variation. Such a comparison is essential to evaluate our model's ability to replicate the observed cross-market variation in local labor markets. This can be seen as test of the cross-regional predictions of our model and, therefore, provides support for the model's counterfactual implications.

We estimate the following linear model:

$$\Delta \log L_{Mr}^t = \kappa \Delta \log L_{Mr}^{pred}\left(\{\hat{z}_n\}_n\right) + \boldsymbol{X}_r^t \gamma + \epsilon_r^t, \tag{46}$$

where $\Delta \log L_{Mr}^t$ is the observed log-change in manufacturing employment in CZ r between 1997 and 2007, and $\Delta \log L_{Mr}^{pred}\left(\{\hat{z}_n\}_n\right)$ is the predicted change in employment of our (linearized) model given the exposure vector of the CZ's manufacturing sector to the 1997-2007 productivity shock in China, conditionally on fitting the initial equilibrium of 1997. In addition, the vector $\boldsymbol{X}_r^t$ contains a set of controls for CZs' initial characteristics that might independently affect manufacturing employment while being correlated with the CZ's exposure to Chinese productivity growth.

Table 2 reports the results of the estimation of the linear regression in (46) using different control sets of CZs' initial characteristics. Without any controls, column (1) reports a

Table 2: Cross-Market Model Fit: Manufacturing Employment

Dependent variable: Log-change in manufacturing employment, 1997-2007				
	(1)	(2)	(3)	(4)
Predicted log-change in manuf. employment	7.52**	7.16**	6.44***	6.84***
	(3.54)	(2.99)	(2.34)	(2.08)
R^2	0.05	0.17	0.22	0.27
Sector composition controls	No	Yes	No	Yes
Demographic controls	No	No	Yes	Yes

Notes: Sample of 722 Commuting Zones in 1997-2007. Models are weighted by start of period CZ share of national population. Sector composition controls in 1997: share of working-age population employed in manufacturing and share of spending on manufacturing goods. Demographic controls in 1990: the population share with a college education, the foreign born population share, the employment rate of working-age women, and employmeny share in routine-intensive occupations. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

positive and statistically significant relationship between actual and predicted changes in manufacturing employment across US CZs. This indicates that, in response to the Chinese productivity shock, our model's general equilibrium predictions are consistent with the observed cross-region pattern of manufacturing employment changes.

In columns (2)–(4), we evaluate the robustness of this relationship to a set of demographic and labor force measures that potentially eliminate confound effects. Column (2) reports a similar coefficient when controlling for the CZ's sector employment and spending composition in 1997. This specification eases concerns that the China exposure variable may in part be picking up an overall trend decline in US manufacturing. In column (3), we control for the following set of CZ's demographic characteristics in 1990: the population share with a college education, the foreign born population share, the employment rate of working-age women, and the employment share in routine intensive jobs (as defined in Autor and Dorn (2013)). This specification shows that our results are robust to employment trends associated with shocks emphasized by recent empirical papers, including immigration and technological shocks. Finally, column (4) presents the estimation of the regression with the entire set of controls. It is important to notice that this set of controls accounts for a significant share of the cross-region variation in manufacturing employment growth in the period, as the R2 increases from 0.05 in column (1) to 0.27 in column (4).[37]

The methodology in Section 4 provides an interpretation for the estimated coefficient in Table 2. Assumption 3c implies that it corresponds to the pass-through from the measured productivity shock to the structural cost shocks in the model and, therefore, should be used to adjust the shocks magnitude in the model's predicted effect. Using this adjustment and

[37] Table 6 in Appendix B.3 shows that the structural estimates of Table 1 vary within their 95% with the inclusion of this additional set of controls.

the specification in column (4), we find that the difference between the CZs at the 25th and the 75th percentiles of predicted response of manufacturing employment is 5.1 log-points. Thus, our model's differential effect has a similar magnitude to the one estimated by Autor, Dorn, and Hanson (2013): in their preferred specification, the CZ at the 75th percentile of import exposure experienced a manufacturing employment decline 4.5 log-points stronger than the CZ in the 25th percentile.[38]

Moving beyond manufacturing employment, Panel A of Table 3 reports the estimation of a version of the regression (46) with different labor market outcomes. Column (2) shows that the predicted employment responses in non-manufacturing are also positively correlated with the observed changes in non-manufacturing employment across CZs. Columns (3)–(4) investigate the model's cross-regional fit in terms of real wages. While we find a positive and statistically significant relationship for the change in the manufacturing real wage, we only obtain an imprecise relationship between the predicted and actual changes in non-manufacturing real wage across CZs. Under the interpretation that the estimated coefficient is the shock pass-through, the estimates in columns (1)–(4) of Panel A should be identical. In fact, despite the difference in the point estimates, the large standard errors of the estimates for the non-manufacturing sector imply that the estimated coefficients in Panel A are not statistically different from each other at usual significance levels.

Finally, Panel B of Table 3 investigates the importance of different components of the model's predicted response to the trade shock. In particular, we analyze the differential responses to the CZ's exposure in terms of manufacturing revenue, $\log \hat{\eta}^R_{Mr}$, and consumption, $\log \hat{\eta}^C_r$. For both employment and real wage, we find that the impact of a manufacturing revenue shock is positive and statistically significant in the manufacturing sector but non-significant in the non-manufacturing sector. The exogenous increase in the CZ's price index has a negative impact on employment in both the manufacturing and the non-manufacturing sectors. However, the coefficients are imprecisely estimated due to the lack of variation in consumption exposure across US CZs. The next section investigates the contribution each of these components to the model's predicted responses in general equilibrium.[39]

[38] To compute the differential impact of import competition on manufacturing employment, we use the estimated coefficient of 4.231 reported in column (1) of Table 5.A in Autor, Dorn, and Hanson (2013). The difference of import exposure of the regions at the 75th and 25th percentiles in Autor, Dorn, and Hanson (2013) is 1.06, which multiplied by the 4.231 yields a predicted differential impact of 4.5 log-points. In the Online Appendix B.3.3, we show that we closely replicate the estimated cross-regional employment response in Autor, Dorn, and Hanson (2013) using their shift-share exposure measure to Chinese import competition in our sample of 31 manufacturing industries.

[39] The Online Appendix B.3.3 investigate cross-regional employment responses to alternative shift-share measures of the CZ's exposure to Chinese import competition. In particular, we find that these measures are negatively correlated with manufacturing employment growth across CZs.

Table 3: Cross-Market Model Fit: Sector-level Outcomes

Dependent variable: Log-change in sector outcome, 1997-2007	Employment		Real Wage	
	Manuf.	Non-manuf.	Manuf.	Non-manuf.
	(1)	(2)	(3)	(4)
Panel A: Model's general equilibrium prediction				
Predicted log-change	6.84***	16.03**	8.86**	-4.18
	(2.08)	(6.94)	(3.35)	(12.49)
Panel B: CZ 's shock exposure				
Manufacturing revenue	1.05***	-0.17	1.64***	0.12
	(0.31)	(0.11)	(0.36)	(0.16)
Consumption cost	-22.79	-9.12**	-27.05	-2.76
	(13.73)	(3.95)	(18.68)	(3.71)

Notes: Sample of 722 Commuting Zones in 1997-2007. Models are weighted by start of period CZ share of national population. All specifications include the set of baseline controls in column (4) of Table 2. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

6 Quantifying the Effects of Global Shocks on Local Labor Markets

Having estimated the deep elasticities that regulate cross-market links in our model, we now investigate the general equilibrium responses of labor markets outcomes to global shocks. To this end, we first present the reduced-form elasticity of local outcomes to the shock exposure of different CZs. Then, we interact these elasticities with each CZ's exposure to global shocks and compute the predicted responses in general equilibrium. We focus on a first-order approximation of these predicted effects to perform a decomposition into the direct effect of local shock exposure as well as indirect effect of the exposure of other local labor markets. More generally, besides these two effects, the model's general equilibrium predicted changes also entail a residual of the first-order approximation. We focus on the first-order approximation of the models predictions to precisely measure the relative importance of the direct and indirect effects.[40]

[40] The Online Appendix D.1 outlines the system of nonlinear equations that delivers the exact predicted changes in labor market outcomes in the model. Using this system, we found that the residual of the first-order approximation was small relative to the predicted effects in many variations of our empirical exercise.

6.1 Reduced-Form Elasticities of Local Employment to Shock Exposure

In this section, we use equation (25) to compute the reduced-form elasticities of local employment to manufacturing shock exposure across US CZs. We report the reduced-form elasticities of real wages in Table 11 in Appendix B.3. Specifically, we use the structural parameters in Panel B of Table 1 to compute $\gamma^R_{kr,Md}$ and $\gamma^C_{kr,Md}$ in the observed equilibrium of 1997. Table 4 presents the average of these reduced-form elasticities for the 722 CZs in our sample.

Columns (1) and (2) report, respectively, the average effect of a shock in a CZ's manufacturing sector on that CZ's employment in the manufacturing and non-manufacturing sectors. Panel A shows that an exogenous decline of 1% in local manufacturing revenue triggers an average decline of 0.18% in manufacturing employment and an average increase of 0.02% in non-manufacturing employment. Thus, in response to negative shocks to the local manufacturing sector, our model predicts manufacturing employment losses that are not fully compensated by non-manufacturing employment growth. This prediction is in line with the employment responses to Chinese import competition presented in Autor, Dorn, and Hanson (2013).

Panel B shows that exogenous shocks to consumption costs also generate substantial changes in employment in our model. If the cost of the consumption bundle of manufacturing workers falls by 1%, then employment in manufacturing increases by 1.19%. These quantitatively important employment responses arise from two margins. First, because we normalize the payoff of being in the home sector to one in every region, the exogenous change in the price index has a first-order impact in the relative payoff of being non-employed in each CZ. Second, a lower price index makes the CZ more attractive to workers from other CZs in the country.

Turning to the cross-region indirect effects of shock exposure, column (3) evaluates the average indirect impact of a shock in a CZ's manufacturing sector on manufacturing employment in all other CZs. Specifically, column (3) reports the averages $\sum_{d\neq r} l_{Md}\gamma^R_{Md,Mr}$ and $\sum_{d\neq r} l_{Md}\gamma^C_{Md,Mr}$, where $l_{Md} \equiv L_{Md}/\sum_{d'} L_{Md}$ is the CZ's share in national manufacturing employment.[41] The small coefficients in column (3) indicate that the indirect effect of any specific CZ is very close to zero. In other words, CZs are approximately "small open economies": local shock exposure does not trigger quantitatively large effects elsewhere.[42]

[41] Formally, the effect of a revenue shock in region r on aggregate manufacturing employment in other regions is $\frac{\partial \log \sum_{d\neq r} L_{Md}}{\partial \log \eta^R_r} = \sum_{d\neq r} \frac{L_{Md}}{\sum_{d'\neq r} L_{Md'}} \frac{\partial \log L_{Md}}{\partial \log \eta^R_r} = \sum_{d\neq r} l_{Md}\gamma^R_{Md,Mr}$.

[42] This feature of indirect effects being small is somewhat consistent with the findings in Caliendo, Parro, Rossi-Hansberg, and Sarte (2018b). In a quantitative spatial model featuring free mobility of workers across

Table 4: Reduced-Form Elasticities of Employment to Local Manufacturing Shock Exposure

	Own region		Other regions	
	$\gamma_{Mr,Mr}$	$\gamma_{Nr,Mr}$	$\sum_{d\neq r} l_{Md}\gamma_{Md,Mr}$	$\sum_{d\neq r} \gamma_{Mr,Md}$
	(1)	(2)	(3)	(4)
Panel A: Employment elasticity to revenue exposure ($\gamma^R_{Mr,Md}$)				
Avg.	0.1807	-0.0181	0.0002	-0.1730
Panel B: Employment elasticity to consumption exposure ($\gamma^C_{Mr,Md}$)				
Avg.	1.1923	0.1255	-0.0006	-0.5296

Notes: Average reduced-form elasticity computed with equation (25) using the estimates in Panel B of Table 1 and the observed equilibrium in 1997. M denotes the manufacturing sector and N denotes the non-manufacturing sector.

In contrast, column (4) shows the average impact on a CZ's manufacturing employment of a common manufacturing shock to *all other* CZ's – that is, the cross-region average of $\sum_{d\neq r}\gamma^R_{Mr,Md}$ and $\sum_{d\neq r}\gamma^C_{Mr,Md}$. Column (4) in Panel A shows that if other CZs experience a correlated shock that raises their revenue by 1%, then local employment in a typical CZ falls by 0.17%. This negative reduced-form indirect effect follows from the positive impact of higher revenue on the nominal wage of other CZs, which lowers local employment due to out-migration and higher consumption costs.

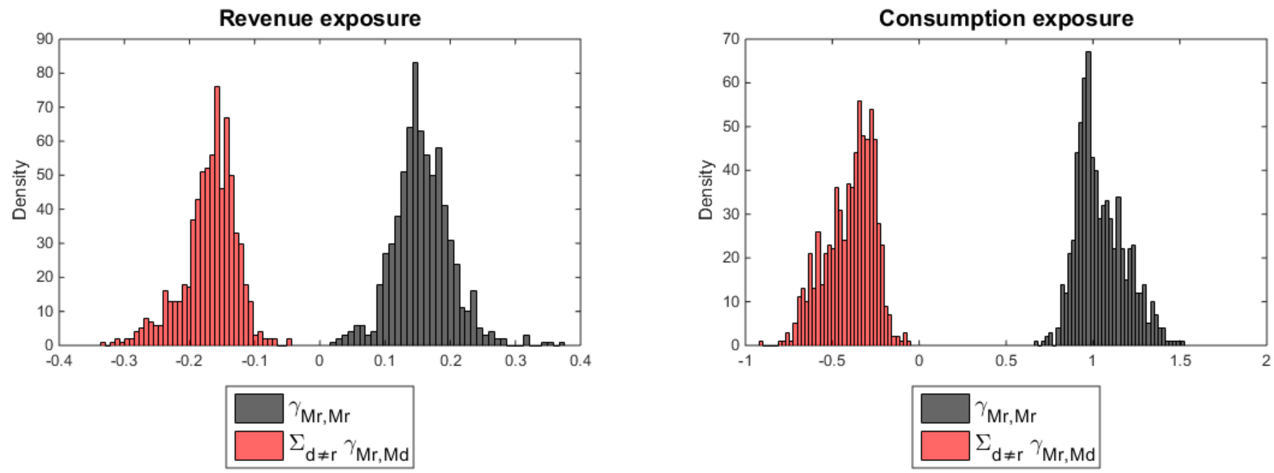
The similarity between the magnitudes of the effects in columns (1) and (4) of Panel A implies that indirect effects can partially offset local direct employment losses triggered by Chinese import competition. These findings are directly connected to the theoretical results shown in Theorem 1: a fully correlated shock across all markets has the potential to generate sizable indirect effects, and thus offset the direct effect, despite small indirect effects of any individual region.

Moreover, another compensating effect is given by the response of employment to consumption exposure. In fact, the indirect effects in column (4) of Panel B are smaller than the direct effect in column (1), suggesting that the *direct* consumption exposure is an important force that offsets the direct revenue exposure. We will explore this relationship further in Section 6.2.

Lastly, Figure 2 reports the entire distribution of the implied responses of local employment to local shocks, for both revenue and consumption exposures. It illustrates that our model yields heterogeneous reduced-form effects across US CZs – for instance, the standard deviation

regions and input-output linkages, they show that, following a state-specific productivity shock, the indirect effect on other states is one order of magnitude smaller than the direct effect of the shock.

Figure 2: Distribution of reduced-form elasticities, Manufacturing Employment



Notes: Average reduced-form elasticity computed with equation (25) using the estimates in Panel B of Table 1 and the observed equilibrium in 1997. M denotes the manufacturing sector and N denotes the non-manufacturing sector.

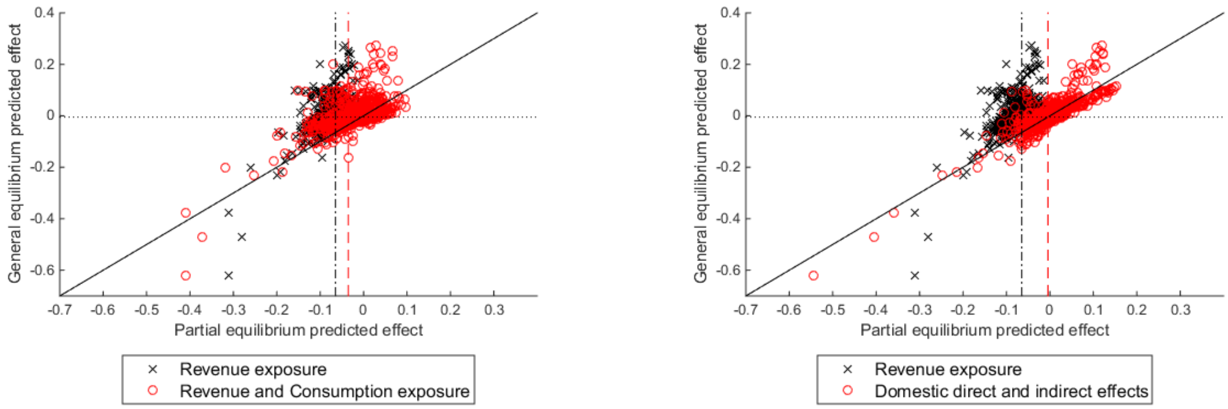
of the direct revenue elasticity $\gamma^R_{Mr,Mr}$ is 0.045. Our framework also features rich heterogeneity in the indirect effects of a local shocks on other regions. In fact, the distribution of indirect effects in Figure 2 suggests that indirect effects across CZs are as heterogeneous as the direct effects – for instance, the standard deviation of the indirect revenue elasticity is 0.042. In the following section, we will further explore the implications of such heterogeneity for the impact of global shocks on US labor markets.[43]

6.2 Impact of Global Shocks on US Local Labor Markets

We conclude this section with a quantification of the differential and aggregate effects of global shocks on sectoral employment across US CZs. We decompose these effects into components related to the direct response to local shock exposure and the indirect response to the exposure of other CZs in the country. We first evaluate the impact on CZs in the US of the same China export shock we have used in the estimation, and then we conduct a number of robustness exercises.

[43]Monte, Redding, and Rossi-Hansberg (2018) show that commuting flows across counties generates variation in the reduced-form response of local employment to local shocks. However, our focus is on the magnitude and dispersion of the *indirect* reduced-form responses of local employment to shocks in other regions of the country.

Figure 3: Predicted Change in Manufacturing Employment, China shock



Notes: The figure on the left reports the scatter plot of the general equilibrium response of manufacturing employment against the predicted revenue exposure and against the sum of the predicted revenue and consumption exposures. It also displays the average (across CZs) of the general equilibrium response (horizontal line), revenue exposure (black line), revenue and consumption exposure (red line). The figure on the right reports the scatter plot of the general equilibrium response against the predicted revenue exposure, and against the sum of predicted (domestic) direct and indirect effects from revenue and consumption exposures, computed using Proposition . It also displays the average (across CZs) of the general equilibrium response (horizontal line), revenue exposure (black line), domestic direct and indirect effects (red line).

6.2.1 China Export Shock

We quantify the importance of local shock exposure in revenue and consumption for the model's predicted employment responses. To this end, we plot the general equilibrium employment response following the China export shock against alternative partial equilibrium exposure measures.[44] The graph on the left of Figure 3 reports the relationship between the general equilibrium predictions of our model, $\Delta \log L_{kr}^{pred}(\hat{z})$, and the revenue exposure in manufacturing, i.e. $\Delta \log \eta_{Mr}^{R}(\hat{z})$, and the consumption exposure, i.e. $\Delta \log \eta_{Mr}^{C}(\hat{z})$. The graph on the right, instead, displays the (domestic) direct and indirect effects.

We first take a look at the general equilibrium predicted responses of manufacturing employment. Figure 3 shows that there is a large degree of heterogeneity in the employment responses of US CZs. For the majority of the CZs, which account for 66% of the US population, manufacturing employment falls due to Chinese import competition. In these

[44]Specifically, we denote each exposure measure by m, with $m = GE$ representing our baseline general equilibrium model. For each m, we compare predictions using the following linear regression:

$$\Delta \log L_{Mr}^{GE}(\hat{\tau}) = \alpha^{m} + \rho^{m} \Delta \log L_{Mr}^{m}(\hat{\tau}) + e_{Mr}^{m},$$

where $\Delta \log L_{Mr}^{m}(\hat{\tau})$ is the shock exposure in manufacturing employment according to measure m. We then compute the partial-equilibrium employment responses of exposure m as $\hat{\rho}^{m} \Delta \log L_{Mr}^{m}(\hat{\tau})$, with $\hat{\rho}^{m}$ being the coefficient estimated in the equation above.

CZs, the average decline in manufacturing real wage is 1.16%. In contrast, 33% of the population lives in CZs that experience an expansion in manufacturing employment, with an average real wage gain of 1.09%. By combining winning and losing CZs, we obtain a small aggregate impact of rising Chinese import competition: on average, manufacturing employment falls by only 0.51%, while real wages decline by 0.064%. These small aggregate effects of the China shock are similar to the findings reported in a number of recent structural quantitative papers - e.g. Hsieh and Ossa (2016), Lee (2015), Galle, Rodriguez-Clare, and Yi (2017), Adao, Costinot, and Donaldson (2017), and Caliendo, Dvorkin, and Parro (2018a).

Interestingly, non-manufacturing employment absorbs part of displaced workers due to Chinese import competition, due to the reallocation of workers across sectors and the positive employment impact of lower good prices. Overall, non-manufacturing employment falls only in 13% of the CZs, and aggregate non-manufacturing employment increases by 0.9%.

A striking feature of the scatter plots in Figure 3 is the tight fit of the cross-regional predictions of the partial-equilibrium measures. The revenue exposure alone, in particular, is able to capture a large variation in the predicted employment effect across US CZs, and on average it entails a decline in manufacturing employment of 6.47% due to the loss in competitiveness from the China shock. The consumption exposure, on the other hand, typically shifts upward the responses of CZs employment, due to the lower consumption costs of Chinese goods. The consumption exposure accounts for half the difference between the "average" effect of revenue exposure and the general equilibrium predicted effect. Also, note that the consumption exposure does not have substantial cross-regional variation, and this explains the imprecise estimates in Table 3.

Another feature of the scatter plots is that most points are above the 45^o degree line: the partial-equilibrium predicted employment declines tend to be stronger than the total general equilibrium responses. This arises because the partial-equilibrium measures ignore the offsetting indirect effects triggered by the common component of the shock exposure – i.e., the combined effect in column (4) of Table 4. In fact, the scatter plots on the right of Figure 3 highlight that the indirect effects coming from other US commuting zones shift up manufacturing employment, due to cheaper goods available from other regions negatively affected by the China shock. On average, indirect effects offset about half of the negative response of the revenue exposure.

In sum, while the revenue exposure accounts for most of the cross-regional variation in the manufacturing employment responses, the consumption exposure offsets around half of the average effect of $\Delta \log \eta_{Mr}^{R}$, which implies, together with the offsetting impact of the indirect effects from other CZs, an aggregate effect of the China shock of approximately zero.

6.2.2 Robustness

In this section we investigate the robustness of our empirical results to different specifications of the model. We first study how the relationship between the shock exposure and general equilibrium effect of the China shock is affected by labor linkages across markets. Figure 7 in Appendix B.4 shows that shutting down the migration and agglomeration elasticities does not substantially alter the fit of the local exposures to the general equilibrium response of manufacturing employment. It does, however, affect their magnitudes. In fact, while the average predicted effect is still close to zero, the average revenue and consumption exposures are smaller than in the baseline. This is consistent with Theorem 1, which shows that labor linkages across markets amplify the response of employment to shocks.

Second, we investigate how our main findings are affected by our assumption about the unemployment margin. In particular, with the same parameters as in the baseline, we feed the China export shock in a version of our empirical model that does not feature the home sector. Figure 8 shows that, even without this margin of labor reallocation, the relationship between local exposures and general equilibrium responses is very similar to the baseline model.

Finally, we examine the response of local labor markets following a bilateral trade cost shock. Specifically, starting from the labor market equilibrium in 2007, we revert US *import* trade costs with Mexico and Canada back to what they were in 1993, the year before NAFTA was implemented. We use the gravity structure of our model and the assumption that trade costs are symmetric, as in Head and Ries (2001), to recover the change in trade costs from observed changes in trade shares.[45] Figure 9 shows that, despite the different nature of the shock, the fit of the revenue and consumption exposure is similar to the one for the China export shock, although the magnitudes are typically smaller than in the baseline.

7 Conclusions

We propose an integrated treatment for estimating the effect of economic shocks in the world economy on local labor markets. Our analysis offers a bridge between quantitative general equilibrium models and estimates of differential responses across markets. As in

[45] Imposing symmetric trade costs, our gravity model implies that

$$\hat{\tau}_{n,Mi,kU} = \left(\frac{\hat{\bar{x}}_{n,Mi,U}}{\hat{\bar{x}}_{n,Mi,i}} \cdot \frac{\hat{\bar{x}}_{n,MU,i}}{\hat{\bar{x}}_{n,MU,U}} \right)^{-\frac{1}{2\chi_n}},$$

where $i \in (\text{Canada}, \text{Mexico})$, U stands for US, and n is each of the 31 industries within the manufacturing sector M. We use country-sector-level trade data from UN Comtrade for 1993 and 2007 to compute the trade shares in the equation above.

quantitative structural approaches, we evaluate the general equilibrium effect of economic shocks on each region. We show that this response has two components: the direct impact of local shock exposure and the indirect effect of the shock exposure of other markets. By providing a thorough theoretical characterization of these effects and a model-consistent way to measure local shock exposure, we can exploit credible quasi-experimental variation, as in recent empirical papers. We use such variation to estimate the deep elasticities governing cross-market links and evaluate the model's reduced-form predictions regarding changes in outcomes across local labor markets. Our results quantify the role of spatial links in shaping the factual and counterfactual responses of labor markets to international trade shocks.

Several interesting avenues for future research emerge from our study. Extensions of our Generalized Spatial Model offer different measures of shock exposure that can be computed using richer data. Furthermore, as our approach is flexible in specifying functional forms for the cross-market links, finer micro-data can leverage on the MOIV methodology to improve on the precision of estimated parameters. More importantly, our approach, easily implementable with a simple GMM procedure, can be used for the estimation of structural parameters in a wide class of general equilibrium models.

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A Proofs

A.1 Proof of Proposition 1

To establish the existence of equilibrium, consider the following system of equilibrium equations:

$$F_i(\{p_j^*\}_j) = 0.$$

We use the following two results regarding existence and uniqueness of equilibrium prices.

Lemma 1. *[Mas-Colell, Whinston, and Green (1995) 17.C.1] Suppose that $F(\cdot)$ is a function defined for every $\boldsymbol{p} \in \mathbb{R}^N_{++}$ such that $F(\cdot)$ is (i) differentiable, (ii) homogeneous of degree 0, (iii) satisfies Walras' law, $\sum_{i=1}^N p_i F_i(\boldsymbol{p}) = 0$ for all $\boldsymbol{p}$, and (iv) there exists a s such that $F_i(\boldsymbol{p}) < s$ for every $\boldsymbol{p}$,and (v) if $\boldsymbol{p}^n \to \boldsymbol{p}$ with $p_j = 0$ for some j, $\min_i\{F_i(\boldsymbol{p}^n)\} \to -\infty$. Then, $\boldsymbol{p}^* \in \mathbb{R}^N_{++}$ exists.*

Lemma 2. *[Arrow and Hahn (1971) T.9.12 (p. 234)] Suppose that $F(\cdot)$ satisfies the conditions in Lemma 1, and denote $f_{ij}(\boldsymbol{p}) \equiv \frac{\partial F_i(\boldsymbol{p})}{\partial p_j}$. Assume that, for all $\boldsymbol{p}^* \in \mathbb{R}^N_+$ with $F(\boldsymbol{p}^*) = 0$, (i) $f_{ij}(\boldsymbol{p}^*) > 0$ and (ii) $\exists\{h_i(\boldsymbol{p}^*)\}_{i=1}^N \gg 0$ such that $h_i(\boldsymbol{p}^*) f_{ii}(\boldsymbol{p}^*) > \sum_{j \neq i,m} |f_{ij}(\boldsymbol{p}^*)| h_j(\boldsymbol{p}^*)$ for all $i = 1, ..., N$. Then, $\boldsymbol{p}^* \in \mathbb{R}^N_{++}$ is unique.*

To establish the result, notice that $(\boldsymbol{w}^*, p^*)$ is an equilibrium with positive employment everywhere if, and only if,

$$\tilde{\Lambda}_i(\boldsymbol{w}^*, p) = 0$$

where

$$\tilde{\Lambda}_i(\boldsymbol{w}, p) \equiv 1 - \frac{p}{w_i}\tau_i \Psi_i(\boldsymbol{\Phi}(\boldsymbol{w}, p)) \quad \forall i = 1, ..., N$$

$$\tilde{\Lambda}_{N+1}(\boldsymbol{w}, p) = \sum_{i=1}^N [\tau_i \Psi_i(\boldsymbol{\Phi}(\boldsymbol{w}, p)) - \frac{w_i}{p}].$$

To establish uniqueness, we verify the conditions of Lemmas 1 and 2. Notice that our function is an excess supply function, i.e. the negative of an excess demand function considered by Mas-Colell, Whinston, and Green (1995)

1. Continuity: By assumption, $\Psi_i(\Phi(\boldsymbol{w}, p))$ is differentiable, so $\tilde{\Lambda}_i(\boldsymbol{w}, p)$ is also continuous.

2. Homogeneous of Degree Zero: Consider $(\boldsymbol{w}, p)$ and $(b\boldsymbol{w}, bp)$ with $b > 0$. Both vectors imply the same vector of real wages, $\omega_i = w_i/p = bw_i/bp$, so $\boldsymbol{\Phi}(b\boldsymbol{w}, bp) = \boldsymbol{\Phi}(\boldsymbol{w}, p)$ and $\tilde{\Lambda}_i(b\boldsymbol{w}, bp) = \tilde{\Lambda}_i(\boldsymbol{w}, p)$.

3. Walras' law: For any $(\boldsymbol{w}, p)$,

$$\sum_{i=1}^N w_i \tilde{\Lambda}_i(\boldsymbol{w}, p) + p\tilde{\Lambda}_{N+1}(\boldsymbol{w}, p) = \sum_{i=1}^N [w_i - p\tau_i \Psi_i(\boldsymbol{\Phi}(\boldsymbol{w}, p))] + \sum_{i=1}^N [p\tau_i \Psi_i(\boldsymbol{\Phi}(\boldsymbol{w}, p)) - w_i] = 0,$$

   where the last equality follows from the zero profit condition.

4. Bounded from above: Because $\Psi_i(.) \geq 0$, $\tilde{\Lambda}_i(\boldsymbol{w}, p) \leq 1$ for every $i = 1, ..., N$. Given Assumption 1 the function $\Psi_i(.)$ is bounded above, $\Psi_i(.) < \bar{\Psi}$ and $\tilde{\Lambda}_{N+1}(\boldsymbol{w}, p) < \sum_i \tau_i \bar{\Psi}$.

5. Limit: Consider a sequence $(\boldsymbol{w}^n, p^n) \to (\boldsymbol{w}, p)$. If $\lim_{n\to\infty} w_i^n = 0$ for some i, then for every n

$$\min_{i'} \left\{ \tilde{\Lambda}_{i'}(\boldsymbol{w}^n, p^n) \right\} \leq 1 - \frac{p}{w_i} \tau_i \Psi_i(\boldsymbol{\Phi}(\boldsymbol{w}^n, p^n)).$$

We assume that $\lim_{n\to\infty} \frac{\Psi_i}{w_i^n} = \infty$ for all i if $\lim_{n\to\infty} w_i^n = 0$. These two combined imply that $\lim_{n\to\infty} \min \left\{ \tilde{\Lambda}_i(\boldsymbol{w}^n, p^n) \right\} = -\infty$.

6. Conditions on the Jacobian: Notice that

$$\frac{\partial \tilde{\Lambda}_i}{\partial w_i} = \frac{p}{w_i^2} \tau_i \Psi_i(\boldsymbol{\Phi}(\boldsymbol{w}, p)) \left(1 - \frac{\partial \Psi_i(\boldsymbol{\Phi}(\boldsymbol{w}, p))}{\partial w_i} \frac{w_i}{\Psi_i(\boldsymbol{\Phi}(\boldsymbol{w}, p))} \right) = \frac{p}{w_i^2} \tau_i \Psi_i(\boldsymbol{\Phi}(\boldsymbol{w}, p)) \lambda_{ii}(\boldsymbol{\omega}).$$

$$\frac{\partial \tilde{\Lambda}_i}{\partial w_j} = \frac{p}{w_i w_j} \tau_i \Psi_i(\boldsymbol{\Phi}(\boldsymbol{w}, p)) \left(- \frac{\partial \Psi_i(\boldsymbol{\Phi}(\boldsymbol{w}, p))}{\partial w_j} \frac{w_j}{\Psi_i(\boldsymbol{\Phi}(\boldsymbol{w}, p))} \right) = \frac{p}{w_i w_j} \tau_i \Psi_i(\boldsymbol{\Phi}(\boldsymbol{w}, p)) \lambda_{ij}(\boldsymbol{\omega})$$

Immediately, Assumption 1(i) implies that $\frac{\partial \tilde{\Lambda}_i}{\partial w_i} > 0$. Assumption 1(ii) implies that there exists $h_i \equiv h_i(\boldsymbol{\omega})$ such that

$$h_i \lambda_{ii} > \sum_{j \neq i} |\lambda_{ij}| h_j,$$

which is equivalent to

$$(h_i w_i) \frac{p}{w_i^2} \tau_i \Psi_i(\boldsymbol{\Phi}(\boldsymbol{w}, p)) \lambda_{ii} > \sum_{j \neq i} \left| \frac{p}{w_i w_j} \tau_i \Psi_i(\boldsymbol{\Phi}(\boldsymbol{w}, p)) \lambda_{ij} \right| (h_j w_j)$$

By defining $\tilde{h}_i \equiv h_i w_i$, we have that $\tilde{h}_i \frac{\partial \tilde{\Lambda}_i}{\partial w_i} > \sum_{j \neq i} |\frac{\partial \tilde{\Lambda}_i}{\partial w_i}| \tilde{h}_j$.

A.2 Proof of Theorem 1

Part 1. $\lambda_{ij}(\boldsymbol{\omega}^0) = \lambda 1_{[i=j]}$ implies that $\bar{\boldsymbol{\lambda}}(\boldsymbol{\omega}^0) = \lambda \bar{\boldsymbol{I}}$ and, therefore, $\bar{\boldsymbol{\beta}} = \lambda^{-1} \bar{\boldsymbol{I}}$.

Part 2. $\lambda_{ij}(\boldsymbol{\omega}^0) = \lambda 1_{[i=j]} - \tilde{\lambda}_j$ implies that $\bar{\boldsymbol{\lambda}}(\boldsymbol{\omega}^0) = \lambda \bar{\boldsymbol{I}} - \mathbb{I} \tilde{\boldsymbol{\lambda}}'$, where $\mathbb{I}$ is a vector of ones and $\tilde{\boldsymbol{\lambda}} \equiv \{\tilde{\lambda}_j\}_j$. We guess and verify that $\bar{\boldsymbol{\lambda}}(\boldsymbol{\omega}^0)^{-1} = \lambda^{-1} \bar{\boldsymbol{I}} + \left[\lambda \left(\lambda - \sum_j \tilde{\lambda}_j \right) \right]^{-1} \mathbb{I} \tilde{\boldsymbol{\lambda}}'$.

$$\begin{aligned}
\bar{\boldsymbol{\lambda}}(\boldsymbol{\omega}^0) \bar{\boldsymbol{\lambda}}(\boldsymbol{\omega}^0)^{-1} &= \bar{\boldsymbol{I}} + \lambda \left[\lambda \left(\lambda - \textstyle\sum_j \tilde{\lambda}_j \right) \right]^{-1} \mathbb{I} \tilde{\boldsymbol{\lambda}}' - \lambda^{-1} \mathbb{I} \tilde{\boldsymbol{\lambda}}' - \left[\lambda \left(\lambda - \textstyle\sum_j \tilde{\lambda}_j \right) \right]^{-1} \mathbb{I} \left(\tilde{\boldsymbol{\lambda}}' \mathbb{I} \right) \tilde{\boldsymbol{\lambda}}' \\
&= \bar{\boldsymbol{I}} + \lambda \left[\lambda \left(\lambda - \textstyle\sum_j \tilde{\lambda}_j \right) \right]^{-1} \mathbb{I} \tilde{\boldsymbol{\lambda}}' - \lambda^{-1} \mathbb{I} \tilde{\boldsymbol{\lambda}}' - \left[\lambda \left(\lambda - \textstyle\sum_j \tilde{\lambda}_j \right) \right]^{-1} \left(\textstyle\sum_j \tilde{\lambda}_j \right) \mathbb{I} \tilde{\boldsymbol{\lambda}}' \\
&= \bar{\boldsymbol{I}} - \left[\lambda^{-1} - \left[\lambda \left(\lambda - \textstyle\sum_j \tilde{\lambda}_j \right) \right]^{-1} \left(\lambda - \textstyle\sum_j \tilde{\lambda}_j \right) \right] \mathbb{I} \tilde{\boldsymbol{\lambda}}' = \bar{\boldsymbol{I}}
\end{aligned}$$

where the second equality follows from $\tilde{\boldsymbol{\lambda}}' \mathbb{I} = \sum_j \tilde{\lambda}_j$..

Part 3. This is an extension of Lemma 2.1 in Li, Chen, and Wang (2009). We first show that $|\beta_{kj}|/h_k < |\beta_{jj}|/h_j$ for all k and j. Suppose that there is $p \neq j$ such that $|\beta_{pj}|/h_p \geq |\beta_{kj}|/h_k$ for all k. By definition of inverse matrix, we have that $\sum_k \lambda_{pk} \beta_{kj} = 0$ for $p \neq j$, which implies that $\lambda_{pp} \beta_{pj} = -\sum_{k \neq p} \lambda_{pk} \beta_{kj}$. Thus,

$$\lambda_{pp}|\beta_{pj}| = |\sum_{k\neq p} \lambda_{pk}\beta_{kj}| \leq \sum_{k\neq p} |\lambda_{pk}||\beta_{kj}| \leq \sum_{k\neq p} |\lambda_{pk}||\beta_{pj}|\frac{h_k}{h_p} \quad \Rightarrow \quad h_p\lambda_{pp} \leq \sum_{j\neq p} |\lambda_{pk}|h_k,$$

which contradicts Assumption 1(ii).

To establish the result, notice that, by definition, $\lambda_{ii}\beta_{ij} = -\sum_{k\neq i}\lambda_{ik}\beta_{kj}$ for every $j \neq i$. Thus,

$$\lambda_{ii}|\beta_{ij}| \leq \sum_{k\neq i} |\lambda_{ik}||\beta_{kj}| < \sum_{k\neq i} |\lambda_{ik}||\beta_{jj}|\frac{h_k}{h_j} \quad \Rightarrow \quad \frac{|\beta_{ij}|}{|\beta_{jj}|} < \frac{\sum_{k\neq i}|\lambda_{ik}|h_k}{\lambda_{ii}h_j}. \blacksquare$$

A.3 Proof of Proposition 2

We start by replicating the argument in Proposition 1 to establish that, for each $\{p_i/P_i\}$, there exists a unique real wage vector satisfying labor market module in (18). To this end, $\boldsymbol{\omega}^*$ satisfies (18) if, and only if, $\tilde{\boldsymbol{\omega}}^* \equiv \tilde{p}^*\boldsymbol{\omega}^*$ and $\tilde{p}^*$ solve the following system,

$$\tilde{\Lambda}_i(\tilde{\boldsymbol{\omega}}^*, \tilde{p}^*) = 0$$

where

$$\tilde{\Lambda}_i(\tilde{\boldsymbol{\omega}}, \tilde{p}) \equiv 1 - \frac{\tilde{p}}{\tilde{\omega}_i}\frac{p_i}{P_i}\Psi_i\left(\boldsymbol{\Phi}\left(\frac{\tilde{\boldsymbol{\omega}}}{\tilde{p}}\right)\right) \qquad \forall i = 1, ..., N$$

$$\tilde{\Lambda}_{N+1}(\tilde{\boldsymbol{\omega}}, \tilde{p}) = \sum_{i=1}^{N}\left[\frac{p_i}{P_i}\Psi_i\left(\boldsymbol{\Phi}\left(\frac{\tilde{\boldsymbol{\omega}}}{\tilde{p}}\right)\right) - \frac{\tilde{\omega}_i}{\tilde{p}}\right].$$

Under Assumption 1, Proposition 1 implies that the solution $(\tilde{\boldsymbol{\omega}}^*, \tilde{p}^*)$ is a unique solution up to a scalar. Thus, $\boldsymbol{\omega}^* = \tilde{\boldsymbol{\omega}}^*/\tilde{p}$ is unique for each vector of relative competitiveness, $\{p_i/P_i\}_i$. Thus, the local labor market module in (18) defines the real wage vector as function of the relative competitiveness vector and, because $P_i = P_i(\{\tau_{oi}p_o\}_o)$, of the vector of production costs,$\boldsymbol{p}$. Thus, we can write

$$\boldsymbol{\omega} = \boldsymbol{\Omega}(\boldsymbol{p}),$$

where, by the implicit function theorem,

$$\left[\frac{\partial \log \omega_i}{\partial \log p_j}\right]_{i,j} \equiv \bar{\boldsymbol{\Omega}} = \bar{\boldsymbol{\beta}}(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}).$$

We then substitute $\boldsymbol{\omega} = \boldsymbol{\Omega}(\boldsymbol{p})$ into the trade module in (19) to write the equilibrium as the solution of

$$\Gamma_i(\boldsymbol{p}) = 0 \quad \forall i$$

where $\boldsymbol{p}_j \equiv \{\tau_{oj}p_o\}_o$ and

$$\Gamma_i(\boldsymbol{p}) \equiv \Psi_i\left(\boldsymbol{\Phi}\left(\boldsymbol{\Omega}(\boldsymbol{p})\right)\right)\Phi_i\left(\boldsymbol{\Omega}(\boldsymbol{p})\right) - \frac{1}{p_i}\sum_j X_{ij}\left(\boldsymbol{p}_j\right)\Psi_j\left(\boldsymbol{\Phi}\left(\boldsymbol{\Omega}(\boldsymbol{p})\right)\right)\Phi_j\left(\boldsymbol{\Omega}(\boldsymbol{p})\right)p_j.$$

To establish uniqueness, we verify the conditions of Lemmas 1 and 2 as in the proof of Proposition 1.

1. The function $\Gamma_i(\boldsymbol{p})$ is differentiable because all functions are differentiable by assumption.

2. To show that the system is homogeneous of degree zero, recall that $P_i(\cdot)$ is homogeneous of degree one, which implies that $\kappa p_i/P_i(\{\tau_{oi}\kappa p_o\}_o) = p_i/P_i(\{\tau_{oi}p_o\}_o)$ and $\boldsymbol{\Omega}(\kappa\boldsymbol{p}) = \boldsymbol{\Omega}(\boldsymbol{p})$. Since $X_{ij}(\cdot)$ is homogeneous of degree zero, it is straight forward to verify that $\Gamma_i(\kappa\boldsymbol{p}) = \Gamma_i(\boldsymbol{p})$.

3. To verify that Walras' law holds, note that

$$\begin{array}{rcc} \sum_i p_i\Gamma_i(\boldsymbol{p}) & = & \sum_i p_i\Psi_i\left(\boldsymbol{\Phi}\left(\boldsymbol{\Omega}(\boldsymbol{p})\right)\right)\Phi_i\left(\boldsymbol{\Omega}(\boldsymbol{p})\right) - \sum_j\left[\sum_i X_{ij}\left(\boldsymbol{p}_j\right)\right]\Psi_j\left(\boldsymbol{\Phi}\left(\boldsymbol{\Omega}(\boldsymbol{p})\right)\right)\Phi_j\left(\boldsymbol{\Omega}(\boldsymbol{p})\right)p_j \\ & = & \sum_i p_i\Psi_i\left(\boldsymbol{\Phi}\left(\boldsymbol{\Omega}(\boldsymbol{p})\right)\right)\Phi_i\left(\boldsymbol{\Omega}(\boldsymbol{p})\right) - \sum_j p_j\Psi_j\left(\boldsymbol{\Phi}\left(\boldsymbol{\Omega}(\boldsymbol{p})\right)\right)\Phi_j\left(\boldsymbol{\Omega}(\boldsymbol{p})\right) \\ & = & 0 \end{array}$$

4. Given Assumptions 1 and 2 the excess supply system has the following upper bound:

$$\Gamma_i(\boldsymbol{p}) < \Psi_i\left(\boldsymbol{\Phi}\left(\boldsymbol{\Omega}(\boldsymbol{p})\right)\right)\Phi_i\left(\boldsymbol{\Omega}(\boldsymbol{p})\right) < \bar{\Psi}\bar{\Phi}$$

5. Let $\bar{\boldsymbol{p}}$ be a real price vector with $p_i = 0$ for some i. Notice that because of the upper bounds conditions on Assumptions 1 and 2 we have

$$0 \leq \lim_{\boldsymbol{p}^n \to \bar{\boldsymbol{p}}} \Psi_i\left(\boldsymbol{\Phi}\left(\boldsymbol{\Omega}(\boldsymbol{p})\right)\right)\Phi_i\left(\boldsymbol{\Omega}(\boldsymbol{p})\right) \leq \bar{\Psi}\bar{\Phi}.$$

Making use of the limiting conditions in Assumption 2 if $\lim_{n\to\infty} p_i^n = 0$, then for every n also notice that

$$\min_{i'}\{\Gamma_{i'}(\boldsymbol{p})\} \leq N\bar{\Psi}\bar{\Phi} - \frac{1}{p_i}\sum_j X_{ij}\left(\boldsymbol{p}_j\right)\Psi_j\left(\boldsymbol{\Phi}\left(\boldsymbol{\Omega}(\boldsymbol{p})\right)\right)\Phi_j\left(\boldsymbol{\Omega}(\boldsymbol{p})\right)p_j.$$

Then, $\lim_{n\to+\infty}\frac{X_{ij}\Phi_j\Psi_j}{p_i^n} = \infty$ for some j and thus $\lim_{\boldsymbol{p}^n\to\bar{\boldsymbol{p}}}\Gamma_i(\boldsymbol{p}) = -\infty$.

6. We now establish diagonal dominance of the excess supply system at any equilibrium price vector. To this end, notice that

$$\begin{array}{rcc} \frac{\partial\Gamma_i(\boldsymbol{p})}{\partial\log p_o} & = & \Psi_i\left(\boldsymbol{\Phi}\left(\boldsymbol{\Omega}(\boldsymbol{p})\right)\right)\Phi_i\left(\boldsymbol{\Omega}(\boldsymbol{p})\right)\left[\sum_d\psi_{id}\sum_{\tilde{d}}\phi_{d\tilde{d}}\Omega_{\tilde{d}o} + \sum_{\tilde{d}}\phi_{i\tilde{d}}\Omega_{\tilde{d}o}\right] \\ & + & \frac{1}{p_i}\sum_j X_{ij}\left(\{\tau_{oj}p_o\}_o\right)\Psi_j\left(\boldsymbol{\Phi}\left(\boldsymbol{\Omega}(\boldsymbol{p})\right)\right)\Phi_j\left(\boldsymbol{\Omega}(\boldsymbol{p})\right)p_j\left(1[i=o]-1[j=o]\right) \\ & - & \frac{1}{p_i}\sum_j X_{ij}\left(\boldsymbol{p}_j\right)\Psi_j\left(\boldsymbol{\Phi}\left(\boldsymbol{\Omega}(\boldsymbol{p})\right)\right)\Phi_j\left(\boldsymbol{\Omega}(\boldsymbol{p})\right)p_j\left(\chi_{oij} + \sum_d\psi_{id}\sum_{\tilde{d}}\phi_{d\tilde{d}}\Omega_{\tilde{d}o} + \sum_{\tilde{d}}\phi_{j\tilde{d}}\Omega_{\tilde{d}o}\right) \end{array}$$

Define $\tilde{Y}_i \equiv \Psi_i\left(\boldsymbol{\Phi}\left(\boldsymbol{\Omega}(\boldsymbol{p})\right)\right)\Phi_i\left(\boldsymbol{\Omega}(\boldsymbol{p})\right)$ and $\tilde{y}_{ij} \equiv \frac{1}{p_i\tilde{Y}_i}X_{ij}\left(\{\tau_{oj}p_o\}_o\right)\Psi_j\left(\boldsymbol{\Phi}\left(\boldsymbol{\Omega}(\boldsymbol{p})\right)\right)\Phi_j\left(\boldsymbol{\Omega}(\boldsymbol{p})\right)p_j$ Thus,

$$\begin{array}{rcc} \frac{1}{\tilde{Y}_i}\frac{\partial(\Gamma_i(\boldsymbol{p}))}{\partial\log p_o} & = & \sum_d\psi_{id}\sum_{\tilde{d}}\phi_{d\tilde{d}}\frac{\partial\log\omega_{\tilde{d}}}{\partial\log p_o} + \sum_{\tilde{d}}\phi_{j\tilde{d}}\frac{\partial\log\omega_{\tilde{d}}}{\partial\log p_o} + \sum_j\tilde{y}_{ij}\left(1[i=o]-1[j=o]\right) \\ & - & \sum_j\tilde{y}_{ij}\left[\chi_{oij} + \sum_d\psi_{id}\sum_{\tilde{d}}\phi_{d\tilde{d}}\frac{\partial\log\omega_{\tilde{d}}}{\partial\log p_o} + \sum_{\tilde{d}}\phi_{j\tilde{d}}\frac{\partial\log\omega_{\tilde{d}}}{\partial\log p_o}\right]. \end{array}$$

Notice that $\sum_j\tilde{y}_{ij} = 1$ for every equilibrium price. Hence,

$$\begin{array}{rcc} \left[\frac{1}{\tilde{Y}_i}\frac{\partial\Gamma_i(\boldsymbol{p})}{\partial\log p_o}\right]_{i,o} & = & \bar{\boldsymbol{\psi}}\bar{\boldsymbol{\phi}}\bar{\boldsymbol{\Omega}} + \bar{\boldsymbol{\phi}}\bar{\boldsymbol{\Omega}} + \bar{\boldsymbol{I}} - \bar{\boldsymbol{y}} - \bar{\boldsymbol{\chi}} - \bar{\boldsymbol{y}}\left[\bar{\boldsymbol{\psi}}\bar{\boldsymbol{\phi}}\bar{\boldsymbol{\Omega}} + \bar{\boldsymbol{\phi}}\bar{\boldsymbol{\Omega}}\right] \\ & = & \bar{\boldsymbol{I}} - \bar{\boldsymbol{\chi}} - \bar{\boldsymbol{y}} + \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}}\right)\left(\bar{\boldsymbol{\psi}} + \bar{\boldsymbol{I}}\right)\bar{\boldsymbol{\phi}}\bar{\boldsymbol{\Omega}} \\ & = & \bar{\boldsymbol{I}} - \bar{\boldsymbol{\chi}} - \bar{\boldsymbol{y}} + \bar{\boldsymbol{\mu}}\bar{\boldsymbol{\Omega}} \\ & = & \bar{\boldsymbol{I}} - \bar{\boldsymbol{\chi}} - \bar{\boldsymbol{y}} + \bar{\boldsymbol{\mu}}\bar{\boldsymbol{\beta}}(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}) \\ & = & \bar{\boldsymbol{\gamma}}(\boldsymbol{p}) \end{array}$$

This implies that $\frac{\partial \Gamma_i(\boldsymbol{p})}{\partial p_o} = \frac{\tilde{Y}_i}{p_o}\gamma_{io}$. Immediately, by Assumption 2(i), $\frac{\partial \Gamma_i(\boldsymbol{p})}{\partial p_i} = \frac{\tilde{Y}_i}{p_i}\gamma_{ii} > 0$. Assumption 2(ii) implies that there exists $h_i \equiv h_i(\boldsymbol{p})$ such that

$$h_i \gamma_{ii} > \sum_{j \neq i} |\gamma_{ij}| h_j,$$

which is equivalent to

$$(h_i p_i) \frac{\tilde{Y}_i}{p_i} \gamma_{ii} > \sum_{j \neq i} |\frac{\tilde{Y}_i}{p_j} \gamma_{ij}| (p_j h_j).$$

By defining $\tilde{h}_i \equiv h_i p_i$, we have that $\tilde{h}_i \frac{\partial \Gamma_i(\boldsymbol{p})}{\partial p_i} > \sum_{j \neq i} |\frac{\partial \Gamma_i(\boldsymbol{p})}{\partial p_j}| \tilde{h}_j$.■

A.4 Proof of Proposition 3

In this proof, we simplify the notation by writing $\log \hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}) = \log \hat{\boldsymbol{\eta}}^R$ and $\log \hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}) = \log \hat{\boldsymbol{\eta}}^C$.

Part 1: Small Open Economy. Under the assumption of segmented markets, the equilibrium satisfies the following system of equations:

$$\log \hat{L}_i = \phi \left(-\log \hat{\eta}_i^C + \log \hat{p}_i - \sum_o x_{oi}^0 \log p_o + \psi \log \hat{L}_i \right),$$

$$\sum_o \left(1_{[o=i]} - y_{io}^0 - \bar{\chi}_{io}\right) \log \hat{p}_o = \log \hat{\eta}_i^R - (1+\psi) \sum_j \left(1_{[j=i]} - y_{ij}^0\right) \log \hat{L}_j.$$

Consider a shock that only affects a set of small open economies $d \in I$: $\log \hat{\tau}_{ij} = 0$ and $\log \zeta_i$ if $i \neq d$ and $j \neq d$. By the definition, we have that $x_{di}^0 = y_{id}^0 = 0$ all $i \neq d$, which implies that

$$\log \hat{\eta}_i^C \equiv x_{di}^0 \log \hat{\tau}_{di} = 0, \text{ and } \log \hat{\eta}_i^R \equiv \chi \sum_d y_{jd}^0 \left(x_{id}^0 \log \hat{\tau}_{id}\right) = 0.$$

Thus, for all $i \neq d$, $\log \hat{\eta}_i^C = \log \hat{\eta}_i^R = 0$ and the system above becomes

$$\log \hat{L}_i = \phi \left(\log \hat{p}_i - \sum_o x_{oi}^0 \log p_o + \psi \log \hat{L}_i \right),$$

$$\sum_o \left(1_{[o=i]} - y_{io}^0 - \bar{\chi}_{io}\right) \log \hat{p}_o = -(1+\psi) \sum_j \left(1_{[j=i]} - y_{ij}^0\right) \log \hat{L}_j.$$

This implies that we can solve the problem recursively. The system above is satisfied if $\log p_i = \log \hat{L}_i = 0$ for all $i \neq d$. For each small open economy d, we have to solve the following system:

$$\log \hat{L}_d = \phi \left(-\log \hat{\eta}_d^C + (1 - x_{dd}) \log \hat{p}_d + \psi \log \hat{L}_d \right)$$

$$\left(1 - y_{dd}^0 - \bar{\chi}_{dd}\right) \log \hat{p}_d = \log \hat{\eta}_d^R - \left(1 - y_{dd}^0\right)(1+\psi) \log \hat{L}_d.$$

Define $\beta \equiv (1 - \phi\psi)^{-1}$. Rearranging the first expression in this system, we get

$$\log \hat{L}_d = \beta\phi \left(-\log \hat{\eta}_d^C + \left(1 - x_{dd}^0\right) \log \hat{p}_d \right), \tag{47}$$

Substituting (47) into the second equation in the system above, we get that

$$\left(1-y_{dd}^{0}-\bar{\chi}_{dd}\right)\log\hat{p}_{d}=\log\hat{\eta}_{d}^{R}-\left(1-y_{dd}^{0}\right)(1+\psi)\beta\phi\left(-\log\hat{\eta}_{d}^{C}+\left(1-x_{dd}^{0}\right)\log\hat{p}_{d}\right)$$

Define $\alpha_{dd}\equiv\left(1-y_{dd}^{0}-\bar{\chi}_{dd}+(1+\psi)\beta\phi\left(1-y_{dd}^{0}\right)\left(1-x_{dd}^{0}\right)\right)^{-1}$. Thus

$$\log\hat{p}_{d}=\alpha_{dd}\left(\log\hat{\eta}_{d}^{R}+\left(1-y_{dd}^{0}\right)(1+\psi)\beta\phi\log\hat{\eta}_{d}^{C}\right). \tag{48}$$

By combining expressions (47) and (48),

$$\log\hat{\omega}_{d}=\beta\alpha_{dd}^{R}\log\hat{\eta}_{d}^{R}-\beta\alpha_{dd}^{C}\log\hat{\eta}_{d}^{C}$$

where $\alpha_{dd}^{R}\equiv\left(1-x_{dd}^{0}\right)\alpha_{dd}$ and $\alpha_{dd}^{C}\equiv\left[1-\alpha_{dd}^{R}\left(1-y_{dd}^{0}\right)(1+\psi)\beta\phi\right]$. Notice that if the small open economies are symmetric with $x_{dd}=x$ and $y_{dd}=y$, then $\alpha_{dd}^{R}=\alpha^{R}$ and $\alpha_{dd}^{C}=\alpha^{C}$.

Part 2. We start by establishing the following lemma.

Lemma. *Take any two matrices $\bar{\boldsymbol{a}}$ and $\bar{\boldsymbol{b}}$ such that $a_{ij}=a1_{[i=j]}-\tilde{a}_{j}$ and $b_{ij}=b1_{[i=j]}-\tilde{b}_{j}$. Then, $\bar{\boldsymbol{c}}\equiv\bar{\boldsymbol{a}}\bar{\boldsymbol{b}}$ has entries with $c_{ij}=ab1_{[i=j]}-b\tilde{a}_{j}-a\tilde{b}_{j}+\tilde{b}_{j}\sum_{d}\tilde{a}_{d}=c1_{[i=j]}-\tilde{c}_{j}$.*

Proof.

$$c_{ij}=\sum_{d}a_{id}b_{dj}=\sum_{d}\left(a1_{[i=d]}-\tilde{a}_{d}\right)\left(b1_{[d=j]}-\tilde{b}_{j}\right)=ab1_{[i=j]}-\left[a\tilde{b}_{j}+\sum_{d}\tilde{a}_{d}\left(b1_{[d=j]}-\tilde{b}_{j}\right)\right]\quad\blacksquare$$

Using this lemma, notice that, under Assumption 3,$\phi_{ij}=\phi1_{[i=j]}-\tilde{\phi}_{j}$ and $\psi_{ij}=\psi1_{[i=j]}-\tilde{\psi}_{j}$, which implies that $\lambda_{ij}=\lambda1_{[i=j]}-\tilde{\lambda}_{j}$. Thus, Part 2 of Proposition 1 immediately implies that

$$\log\hat{\boldsymbol{\omega}}=\bar{\boldsymbol{\beta}}\left(\log\hat{\boldsymbol{p}}-\log\hat{\boldsymbol{P}}\right), \tag{49}$$

where

$$\beta_{ij}=\lambda^{-1}1_{[i=j]}+\bar{\lambda}^{-1}\tilde{\lambda}_{j}\quad\text{with}\quad\bar{\lambda}\equiv(\lambda-\sum_{j}\tilde{\lambda}_{j})\lambda.$$

We now characterize the matrix $\bar{\boldsymbol{\gamma}}$, which is defined as

$$\bar{\boldsymbol{\gamma}}\equiv\bar{\boldsymbol{I}}-\bar{\boldsymbol{\chi}}-\bar{\boldsymbol{y}}+\left(\bar{\boldsymbol{I}}-\bar{\boldsymbol{y}}\right)\left(\bar{\boldsymbol{I}}+\bar{\boldsymbol{\psi}}\right)\bar{\boldsymbol{\phi}}\bar{\boldsymbol{\beta}}\left(\bar{\boldsymbol{I}}-\bar{\boldsymbol{x}}\right).$$

With frictionless trade ($\tau_{ij}=1$ for all i and j), $p_{ij}=\tau_{i}p_{i}$, $x_{ij}=x_{i}=X_{i}(\{\tau_{o}p_{o}\}_{o})$ and $y_{ji}=y_{i}=w_{j}L_{j}/(\sum_{d}w_{d}L_{d})$ for all j. Since trade balance holds at the world economy, $w_{i}L_{i}=x_{i}\sum_{j}w_{j}L_{j}$ and $x_{i}=y_{i}$ for all i. The gravity trade structure in Assumption 3 implies that $\chi_{oij}=-\chi(1_{[o=i]}-x_{o})$ and, therefore,

$$\bar{\chi}_{ij}\equiv\sum_{d}y_{id}\chi_{jid}=-\chi1_{[i=j]}+\chi\sum_{d}y_{id}x_{j}=-\chi1_{[i=j]}+\chi x_{j}$$

Thus, $\bar{\boldsymbol{\chi}}=-\chi(\bar{\boldsymbol{I}}-\bar{\boldsymbol{x}})$, implying that

$$\bar{\boldsymbol{\gamma}}\equiv(1+\chi)\left(\bar{\boldsymbol{I}}-\bar{\boldsymbol{x}}\right)+\bar{\boldsymbol{\kappa}}\left(\bar{\boldsymbol{I}}-\bar{\boldsymbol{x}}\right)$$

where

$$\bar{\boldsymbol{\kappa}}\equiv\left(\bar{\boldsymbol{I}}-\bar{\boldsymbol{x}}\right)\left(\bar{\boldsymbol{I}}+\bar{\boldsymbol{\psi}}\right)\bar{\boldsymbol{\phi}}\bar{\boldsymbol{\beta}}.$$

Let us define $\bar{\boldsymbol{a}} \equiv \bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}$ and $\bar{\boldsymbol{b}} \equiv \left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{\psi}}\right) \bar{\boldsymbol{\phi}} \bar{\boldsymbol{\beta}}$ such that $\bar{\boldsymbol{\kappa}} \equiv \bar{\boldsymbol{a}}\bar{\boldsymbol{b}}$. By the lemma above, we can write $b_{ij} \equiv \kappa 1[i=j] - \tilde{b}_j$. This implies that $\kappa_{ij} = \kappa\left(1[i=j] - x_j\right) - \tilde{b}_j + \sum_d x_d \tilde{b}_j = \kappa\left(1[i=j] - x_j\right)$. Hence,

$$\bar{\boldsymbol{\kappa}} = \kappa\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}\right)$$

and

$$\bar{\boldsymbol{\gamma}} \equiv (1+\chi)\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}\right) + \kappa\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}\right)\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}\right).$$

Since $\sum_d \left(1_{[i=d]} - x_d\right)\left(1_{[d=j]} - x_j\right) = 1_{[i=j]} - x_j - x_j + x_j \sum_d x_d = 1_{[i=j]} - x_j$, we have that $\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}\right)\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}\right) = \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}\right)$ and

$$\bar{\boldsymbol{\gamma}} \equiv (1+\chi+\kappa)\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}\right). \tag{50}$$

By defining $\gamma \equiv 1+\chi+\kappa$ and $\tilde{\gamma}_j = \gamma x_j$, we have that $\gamma_{ij} = \gamma 1[i=j] - \tilde{\gamma}_j$. Thus, Part 2 of Proposition 1 implies that, for every $i \neq m$,

$$\log \hat{p}_i = \frac{1}{\gamma}\hat{h}_i + \frac{1}{\gamma}\sum_{j\neq m}\frac{x_j}{x_m}\hat{h}_j$$

with $\hat{\boldsymbol{h}} = \log \hat{\boldsymbol{\eta}}^R + \bar{\boldsymbol{\kappa}} \log \hat{\boldsymbol{\eta}}^C$ and, therefore,

$$\hat{h}_i = \log \hat{\eta}_i^R + \kappa \log \hat{\eta}_i^C - \kappa \sum_j x_j \log \hat{\eta}_j^C.$$

For $i \neq m$,

$$\log \hat{p}_i = \frac{1}{\gamma}\left(\log \hat{\eta}_i^R + \kappa \log \hat{\eta}_i^C\right) + \frac{1}{\gamma}\sum_{j\neq m}\frac{x_j}{x_m}\left(\log \hat{\eta}_j^R + \kappa \log \hat{\eta}_j^C\right) - \frac{1}{x_m}\frac{\kappa}{\gamma}\sum_j x_j \log \hat{\eta}_j^C \tag{51}$$

To write an equivalent expression for $i = m$ with $\log \hat{p}_m = 0$, notice that

$$\begin{aligned}\textstyle\sum_j x_j \log \hat{\eta}_j^R &= -\chi \textstyle\sum_j x_j \left[\sum_d y_d \left(\log \hat{\tau}_{jd} - \log \hat{\eta}_d^C\right)\right] \\ &\quad -\chi \textstyle\sum_d y_d \left[\sum_j x_j \log \hat{\tau}_{jd} - \log \hat{\eta}_d^C\right] \\ &\quad -\chi \textstyle\sum_d y_d \left[\log \hat{\eta}_d^C - \log \hat{\eta}_d^C\right] = 0\end{aligned}$$

This implies that $\log \hat{p}_m = 0$ can be written as

$$\log \hat{p}_m = \frac{1}{\gamma}\left(\log \hat{\eta}_m^R + \kappa \log \hat{\eta}_m^C\right) + \frac{1}{\gamma}\sum_{j\neq m}\frac{x_j}{x_m}\left(\log \hat{\eta}_j^R + \kappa \log \hat{\eta}_j^C\right) - \frac{1}{x_m}\frac{\kappa}{\gamma}\sum_j x_j \log \hat{\eta}_j^C. \tag{52}$$

Finally, the change in the price index is $\log \hat{P}_i = \log \hat{\eta}_i^C + \sum_j x_j \log \hat{p}_j$, which in combination with (51) and (52) yields

$$\log \hat{P}_i = \log \hat{\eta}_i^C + \frac{1}{\gamma}\sum_{j\neq m}\frac{x_j}{x_m}\left(\log \hat{\eta}_j^R + \kappa \log \hat{\eta}_j^C\right) - \frac{1-x_m}{x_m}\frac{\kappa}{\gamma}\sum_j x_j \log \hat{\eta}_j^C$$

and, therefore,

$$\log \hat{p}_i - \log \hat{P}_i = \frac{1}{\gamma} \log \hat{\eta}_i^R - \left(1 - \frac{\kappa}{\gamma}\right) \log \hat{\eta}_i^C - \frac{\kappa}{\gamma} \sum_j x_j \log \hat{\eta}_j^C.$$

Finally, the combination of this expression and equation (49) yields

$$\begin{aligned} \log \hat{\omega}_i &= \frac{1}{\gamma} \left[\frac{1}{\lambda} \log \hat{\eta}_i^R + \frac{1}{\lambda} \textstyle\sum_j \tilde{\lambda}_j \log \hat{\eta}_j^R \right] \\ &- \left[\frac{1}{\lambda} \left(1 - \frac{\kappa}{\gamma}\right) \log \hat{\eta}_i^C + \frac{1}{\lambda} \textstyle\sum_j \left(\left(1 - \frac{\kappa}{\gamma}\right) \tilde{\lambda}_j + \frac{\kappa}{\gamma} \lambda x_j \right) \log \hat{\eta}_j^C \right], \end{aligned}$$

where $\bar{\lambda} \equiv (\lambda - \sum_j \tilde{\lambda}_j)\lambda$. ■

A.5 Proof of Expression (31)

The asymptotic variance of the GMM estimator for any function $H_i(\cdot)$ is

$$V(H) = \left(E\left[H_i\left(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right) G_i\right]\right)^{-1} \left(E\left[H_i\left(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right) e_i e_i{}' H_i\left(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right)'\right]\right) \left(E\left[H_i\left(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right) G_i\right]\right)^{-1\prime} \tag{53}$$

where $G_i \equiv E\left[\nabla_{\boldsymbol{\theta}} e_i(\boldsymbol{\theta}) | \hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right]$.

The asymptotic variance of the Optimal IV estimator in (31) is

$$V(H^*) = E\left[G_i{}' \left(\Omega\left(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right)\right)^{-1} G_i\right] \tag{54}$$

with $\Omega\left(\hat{\boldsymbol{z}}^{\boldsymbol{t}}, W^0\right) = E\left[e_i(\boldsymbol{\theta}) e_i(\boldsymbol{\theta})') | \hat{\boldsymbol{z}}^{\boldsymbol{t}}, W^0\right]$.

We now show that $V(H) - V(H^*)$ is positive semi-definite for any $H_i(\cdot)$:

$$\begin{aligned} V(H) - V(H^*) &= \left(E\left[H_i^t G_i^t\right]\right)^{-1} \left(E\left[\left(H_i^t e_i^t\right)\left(H_i^t e_i^t\right)'\right]\right) \left(E\left[H_i^t G_i^t\right]\right)^{-1\prime} - \left(E\left[G_i^{t\prime} \Omega^{-1} G_i^t\right]\right)^{-1} \\ &= \left(E\left[H_i^t G_i^t\right]\right)^{-1} \left(E\left[\left(H_i^t e_i^t\right)\left(H_i^t e_i^t\right)'\right] - E\left[H_i^t G_i^t\right] \left(E\left[G_i^{t\prime} \Omega^{-1} G_i^t\right]\right)^{-1} E\left[H_i^t G_i^t\right]'\right) \left(E\left[H_i^t G_i^t\right]\right)^{-1}. \end{aligned}$$

Let us define

$$U_i^t \equiv H_i^t e_i^t - E\left[\left(H_i^t e_i^t\right)\left(G_i^{t\prime} \Omega^{-1} e_i^t\right)'\right] \left(E\left[G_i^{t\prime} \Omega^{-1} G_i^t\right]\right)^{-1} G_i^{t\prime} \Omega^{-1} e_i^t,$$

which implies that

$$E\left[U_i^t U_i^{t\prime}\right] = E\left[\left(H_i^t e_i^t\right)\left(H_i^t e_i^t\right)'\right] - E[H_i^t G_i^t] \left(E\left[G_i^t \Omega^{-1} G_i^t\right]\right)^{-1} E[H_i^t G_i^t]'.$$

Therefore,

$$V(H) - V(H^*) = \left(E\left[H_i^t G_i^t\right]\right)^{-1} \left(E\left[U_i^t U_i^{t\prime}\right]\right) \left(E\left[H_i^t G_i^t\right]\right)^{-1\prime}.$$

Since $E\left[U_i^t U_i^{t\prime}\right]$ is positive semi-definite, $V(H) - V(H^*)$ is also positive semi-definite. Therefore, the asymptotic variance is minimized at H^*.■

A.6 Proof of Proposition 4

From Proposition 2,

$$E\left[\Delta \log \boldsymbol{L}^t | \hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right] \approx \bar{\boldsymbol{\gamma}}^R(\boldsymbol{\theta}) E\left[\log \hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}^t) | \hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right] - \bar{\boldsymbol{\gamma}}^C(\boldsymbol{\theta}) E\left[\log \hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}^t) | \hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right]. \tag{55}$$

Because of Assumption 3c, equation (21) implies that

$$\begin{array}{rcl} E\left[\log \hat{\eta}_i^C(\hat{\boldsymbol{\tau}})|\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right] & = & \sum_j x_{ji}^0 E\left[\log \hat{\tau}_{ji}|\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right] \\ & = & \kappa \sum_j x_{ji}^0 \log \hat{z}_{ji} \\ & = & \kappa \log \hat{\eta}_i^C(\hat{\boldsymbol{z}}|\boldsymbol{W}^0), \end{array}$$

and equation (22) implies that

$$\begin{array}{rcl} E\left[\log \hat{\eta}_i^R(\hat{\boldsymbol{\tau}})|\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right] & = & \sum_j \sum_o y_{ij}^0 \chi_{oij} E\left[\log \hat{\tau}_{oi}|\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right] \\ & = & \kappa \sum_j \sum_o y_{ij}^0 \chi_{oij} \log \hat{z}_{oi} \\ & = & \kappa \log \hat{\eta}_i^R(\hat{\boldsymbol{z}}|\boldsymbol{W}^0). \end{array}$$

By plugging these two expressions into equation (55), we have that

$$E\left[\Delta \log \boldsymbol{L}^t|\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right] \approx \Delta \log \boldsymbol{L}\left(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0|\boldsymbol{\theta}\right). \tag{56}$$

The combination of (32) and (56) yields the first-order approximation in (33) for the optimal IV in (31).■

A.7 Proof of Proposition 5

We use the strategy in Section 6.1 of Newey and McFadden (1994) to establish asymptotic properties of two-step estimators. To this end, we define the joint moment equation for the two estimating steps:

$$\left(\hat{\boldsymbol{\theta}}_2, \hat{\boldsymbol{\theta}}_1\right) \equiv \arg\min_{\theta_2,\theta_1} \left(\sum_{i,t} v_i^t\left(\boldsymbol{\theta}_2, \boldsymbol{\theta}_1\right)\right)' \left(\sum_{i,t} v_i^t\left(\boldsymbol{\theta}_2, \boldsymbol{\theta}_1\right)\right) \tag{57}$$

where

$$v_i^t\left(\boldsymbol{\theta}_2, \boldsymbol{\theta}_1\right) \equiv \left[\begin{array}{cc} H_i^{MOIV}(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0|\boldsymbol{\theta}_1) e_i^t(\boldsymbol{\theta}_2) & H_i^{MOIV}(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0|\boldsymbol{\theta}_0) e_i^t(\boldsymbol{\theta}_1) \end{array}\right]$$

We have that $\left(\hat{\boldsymbol{\theta}}_2, \hat{\boldsymbol{\theta}}_1\right) \xrightarrow{p} (\boldsymbol{\theta}, \boldsymbol{\theta})$, with an asymptotic variance given by

$$Var\left(\hat{\boldsymbol{\theta}}_2, \hat{\boldsymbol{\theta}}_1\right) = \left(\tilde{G}'\tilde{\Omega}^{-1}\tilde{G}\right)^{-1}$$

where $\tilde{G} \equiv \left[\nabla_{(\boldsymbol{\theta}_2,\boldsymbol{\theta}_1)} v_i^t\left(\boldsymbol{\theta}_2, \boldsymbol{\theta}_1\right)\right]$ and $\tilde{\Omega} \equiv E\left[\left(v_i^t\left(\boldsymbol{\theta}_2, \boldsymbol{\theta}_1\right)\right)\left(v_i^t\left(\boldsymbol{\theta}_2, \boldsymbol{\theta}_1\right)\right)'\right]$.

Define $h_i^t \equiv H_i^{MOIV}(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0|\boldsymbol{\theta}) e_i^t(\boldsymbol{\theta})$ and $\bar{h}_i^t \equiv H_i^{MOIV}(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0|\boldsymbol{\theta}_0) e_i^t(\boldsymbol{\theta})$. Thus, $\tilde{G}$ and $\tilde{\Omega}$ are given by

$$\tilde{\Omega} = E\left[\begin{array}{cc} h_i^t h_i^{t\prime} & h_i^t \bar{h}_i^{t\prime} \\ \bar{h}_i^t h_i^{t\prime} & \bar{h}_i^t \bar{h}_i^{t\prime} \end{array}\right] \quad \text{and} \quad \tilde{G} = \left[\begin{array}{cc} G & G_1 \\ 0 & G_2 \end{array}\right]$$

where

$$G \equiv E\left[H_i^{MOIV}(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0|\boldsymbol{\theta}) \nabla_{\boldsymbol{\theta}} e_i^t(\boldsymbol{\theta})\right]$$

$$G_1 \equiv E\left[\nabla_{\boldsymbol{\theta}} H_i^{MOIV}(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0|\boldsymbol{\theta}) e_i^t(\boldsymbol{\theta})\right]$$

$$G_2 \equiv E\left[H_i^{MOIV}(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0|\boldsymbol{\theta}_0)\nabla_{\boldsymbol{\theta}} e_i^t(\boldsymbol{\theta})\right].$$

By Assumption 3c, any function of $\left(\hat{\boldsymbol{z}}^t, \boldsymbol{W}^0\right)$ is orthogonal to $e_i^t(\boldsymbol{\theta})$, which implies that $G_1 = 0$. Thus, $\left(\tilde{G}'\tilde{\Omega}^{-1}\tilde{G}\right)^{-1}$ is block diagonal and the marginal distribution of $\hat{\boldsymbol{\theta}}_2$ is asymptotically normal with variance

$$Var\left(\hat{\boldsymbol{\theta}}_2\right) = \left(G'\Omega^{-1}G\right)^{-1},$$

which is equivalent to the asymptotic distribution of the Optimal IV in (54). ■

B Online Appendix: Empirical Application

B.1 Representative household utility in model of Section 5.1

Labor Supply. Assume that the representative household preferences have the following nested structure, if $\sum_{kr} L_{kr} = 1$,

$$U_c = \left[\sum_d \nu_d \left(\sum_s (\nu_{sd} C_{sd}) (L_{sd})^{-\frac{1}{\phi_e}} \right)^{\frac{\phi_m - 1}{\phi_m} \frac{\phi_e}{\phi_e - 1}} \right]^{\frac{\phi_m}{\phi_m - 1}}$$

and $U_c = -\infty$ whenever $\sum_{kr} L_{kr} \neq 1$.

In sector s of region d, the real consumption is $C_{sd} = \omega_{sd} L_{sd}$. Thus, the representative household solves the following second-stage problem:

$$\max_{\{L_{sd}\}_{sd}} \left[\sum_d \nu_d \left((L_{Hd})^{\frac{\phi_e - 1}{\phi_e}} + \sum_s (\nu_{sd} \omega_{sd}) (L_{sd})^{\frac{\phi_e - 1}{\phi_e}} \right)^{\frac{\phi_m - 1}{\phi_m} \frac{\phi_e}{\phi_e - 1}} \right]^{\frac{\phi_m}{\phi_m - 1}} \quad \text{s.t.} \quad \sum_{kr} L_{kr} = 1.$$

Let μ be Lagrange multiplier of the constraint. The first-order condition for L_{sd} is

$$\kappa_c \tilde{w}_d (L_{Hd})^{-\frac{1}{\phi_e}} = \mu$$

$$\kappa_c \tilde{w}_d (\nu_{sd} \omega_{sd}) (L_{sd})^{-\frac{1}{\phi_e}} = \mu$$

where

$$\kappa_c \equiv \left[\sum_d \nu_d \left((L_{Hd})^{\frac{\phi_e - 1}{\phi_e}} + \sum_s (\nu_{sd} \omega_{sd}) (L_{sd})^{\frac{\phi_e - 1}{\phi_e}} \right)^{\frac{\phi_m - 1}{\phi_m} \frac{\phi_e}{\phi_e - 1}} \right]^{\frac{\phi_m}{\phi_m - 1} - 1}$$

$$\tilde{w}_d \equiv \nu_d \left((L_{Hd})^{\frac{\phi_e - 1}{\phi_e}} + \sum_s (\nu_{sd} \omega_{sd}) (L_{sd})^{\frac{\phi_e - 1}{\phi_e}} \right)^{\frac{\phi_m - 1}{\phi_m} \frac{\phi_e}{\phi_e - 1} - 1}.$$

Thus,

$$\frac{\bar{L}_{sd}}{\bar{L}_{Hd}} = (\nu_{sd} \omega_{sd})^{\phi_e}$$

and

$$\bar{L}_{Hd} = \frac{L_{Hd}}{L_{Hd} + \sum_s L_{sd}} = \frac{1}{1 + \sum_s (\nu_{sd} \omega_{sd})^{\phi_e}}.$$

The first-order condition for L_{sd} also implies that

$$\frac{\tilde{w}_d}{\tilde{w}_0} = \left(\frac{L_{Hd}}{L_{H0}} \right)^{\frac{1}{\phi_e}}$$

such that

$$\tilde{w}_d = \nu_d \left((L_{Hd})^{-\frac{1}{\phi_e}} \left(L_{Hd} + \sum_s L_{sd} \right) \right)^{\frac{\phi_m - 1}{\phi_m} \frac{\phi_e}{\phi_e - 1} - 1}.$$

Defining $\tilde{L}_d \equiv L_{Hd} + \sum_s L_{sd}$,

$$\frac{\nu_d}{\nu_0} \left(\left(\frac{L_{Hd}}{L_{H0}} \right)^{-\frac{1}{\phi_e}} \left(\frac{\tilde{L}_d}{\tilde{L}_o} \right) \right)^{-\frac{\phi_e - \phi_m}{\phi_m(\phi_e - 1)}} = \left(\frac{L_{Hd}}{L_{H0}} \right)^{\frac{1}{\phi_e}}$$

and finally

$$\frac{\tilde{L}_d}{\tilde{L}_o} = \left(\frac{\nu_d}{\nu_0} \right)^{\phi_m} \left(\frac{\bar{L}_{Hd}}{\bar{L}_{H0}} \right)^{-\frac{\phi_m - 1}{\phi_e - 1}}. \tag{58}$$

B.2 Shift-Share representation of regional shock exposure in model of Section 5.1

Consider our parametric model of Section 5.1. To obtain a shift-share representation, we simplify the model by imposing Cobb-Douglas preferences between manufacturing and non-manufacturing. That is, for simplicity, we assume that $\chi = 0$. In this case, the revenue exposure is the impact of the shock on a market's revenue holding constant wages and employment everywhere:

$$\log \hat{\eta}^R_{kr}(\hat{\boldsymbol{\tau}}) \equiv \sum_{n,o,sd} \frac{\partial \log Y^0_{kr}}{\partial \log \tau^t_{n,ko,sd}} d \log \tau^t_{n,ko,sd}$$

$$= \sum_{n,o,sd} \frac{x^0_{kr,sd} \left(w^0_{sd} L^0_{sd} \right)}{Y^0_{kr}} \frac{\partial \log x^0_{kr,sd}}{\partial \log \tau^t_{n,ko,sd}} d \log \tau^t_{n,ko,sd}$$

By combining this expression with equation (35),

$$\frac{\partial \log x^0_{kr,sd}}{\partial \log \tau^t_{n,ko,sd}} = \frac{x^0_{n,kr,sd}}{x^0_{kr,sd}} \frac{\partial \log x^0_{n,kr,sd}}{\partial \log \tau^t_{n,ko,sd}} = -\bar{\chi}_n \frac{x^0_{n,kr,sd}}{x^0_{kr,sd}} \left(1[r = o] - \bar{x}_{n,ko,sd} \right)$$

where $\bar{x}_{n,ko,sd}$ is the share of spending on goods from region r in industry n by sd.

Let$X^t_{n,kr,sd} \equiv x^t_{n,kr,sd} \left(w^t_{sd} L^t_{sd} \right)$ be the total sales of industry n of market kr to region sd. So,

$$\log \hat{\eta}^R_{kr}(\hat{\boldsymbol{\tau}}) \equiv - \sum_{n,o,sd} \frac{X^0_{n,kr,sd}}{Y^0_{kr}} \left(1[r = o] - \bar{x}_{n,ko,sd} \right) \bar{\chi}_n d \log \tau^t_{n,ko,sd}.$$

As in Section 5.3, we consider a shock to the productivity of a foreign country $\bar{o}$ such that, for all destination sd, $d\log\tau^t_{n,kr,sd} = 0$ if $r \neq \bar{o}$ and $d\log\tau^t_{n,k\bar{o},sd} = d\log\tau^t_n$. Thus,

$$\log\hat{\eta}^R_{kr}(\hat{\boldsymbol{\tau}}) \equiv \sum_n \left(\sum_{sd} \frac{X^0_{n,kr,sd}}{Y^0_{kr}} \bar{x}_{n,k\bar{o},sd}\right) \bar{\chi}_n d\log\tau^t_n.$$

This expression clearly outlines that, in our empirical application, the revenue exposure has a shift-share structure where the industry shock is $\bar{\chi}_n d\log\tau^t_n$ and industry-market exposure is $\sum_{sd}(X^0_{n,kr,sd}\bar{x}_{n,k\bar{o},sd})/Y^0_{kr}$. This shift-share expression entails two adjustments. First, our model implies that the magnitude of the industry-level shock must be adjusted by the the industry's trade elasticity. Intuitively, conditional on the same exogenous productivity change, the demand response is larger in industries with a higher demand elasticity. Second, the industry-region exposure adjusts the share of industry n in market kr revenue by the importance of country $\bar{o}$ across destination markets sd. Because of the gravity-trade structure, the demand response in market sd is proportional to the initial spending share of that market on goods from $\bar{o}$.

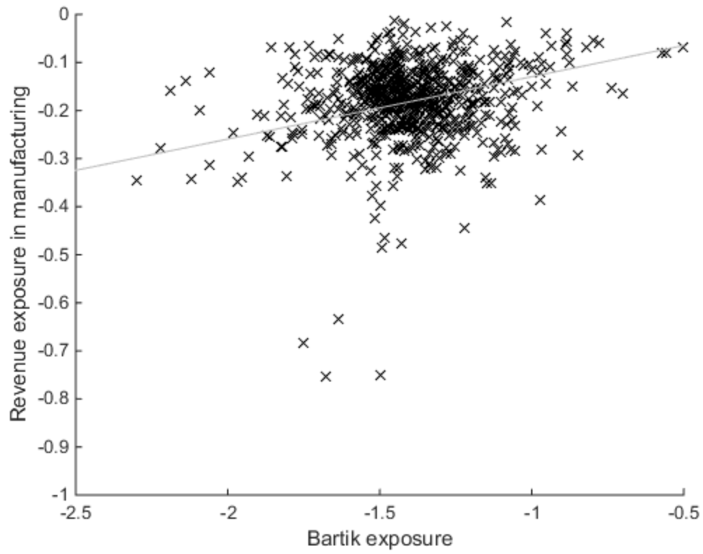
Whenever these two sources of heterogeneity are shut down, the revenue exposure is proportional to a shift-share specification based on industry-region employment shares and the industry shocks. To see this, assume that all destination markets are have identical industry-level spending share on country $\bar{o}$ (i.e., $\bar{x}_{n,k\bar{o},sd} = \bar{x}_{k\bar{o}}$ for all sd and n), and the trade elasticity is identical in all industries ($\bar{\chi}_n = \bar{\chi}$). In this special case,

$$\log\hat{\eta}^R_{kr}(\hat{\boldsymbol{\tau}}) = (\bar{\chi}\bar{x}_{k\bar{o}}) \sum_n l^0_{n,kr} \left(d\log\tau^t_n\right),$$

which $l^0_{n,kr}$ is the share of industry n in the total employment of sector k of region r in the initial equilibrium.

To evaluate the importance of these adjustments in practice, Figure 4 reports the relation between the revenue exposure in manufacturing in our baseline empirical model and the shift-share exposure measure, $\sum_n l^{1997}_{n,kr}\hat{\delta}_{n,China}$. The two measures have a correlation of 0.3. This indicates that they rely on different sources of cross-regional variation.

Figure 4: Manufacturing revenue exposure and shift-share exposure



Notes: Scatter plot of the revenue exposure in manufacturing in the baseline empirical model against the shift-share exposure measure. A least-square best fit line is reported.

B.3 Additional Results: Empirical Specification

B.3.1 Chinese export growth shock

In this section, we present our measure of the shock to Chinese exports between 1997 and 2007. Table 5 presents the list of industries in our sample, along with the calibrated trade elasticity and the various sources of industry-level Chinese cost shock. As explained in the main text, we obtain the estimates of the trade elasticity from Caliendo and Parro (2014). The adjusted export shock is our baseline shock divided by the trade elasticity, $\hat{\delta}_{n,China}/\bar{\chi}_n$. To obtain the inverted bilateral trade cost, we implement the procedure in Head and Ries (2001) for China and each CZ r (that is, $\hat{\tau}_{n,rC} = \hat{\tau}_{n,Cr} = (\hat{x}_{n,rC}\hat{x}_{n,Cr}/\hat{x}_{n,rr}\hat{x}_{n,CC})^{-1/\bar{\chi}_n}$. For the NTR gap, we use the data in Pierce and Schott (2016a) to compute the change in the trade cost between each CZ and China by taking the simple average of the NTR Gaps among the HS6 goods in the corresponding SCTG. Finally, we computed the firm-level productivity growth in 1997-2007 using the unadjusted annual measured productivity growth in column (3) of Table 6 in Hsieh and Ossa (2016).

There are two striking features in the table. First, there is great cross-industry variation in the magnitude of the cost shock, which we exploit in estimation. Second, the different measures of industry-level shocks are only imperfectly correlated, providing us with different sources of variation for estimation.

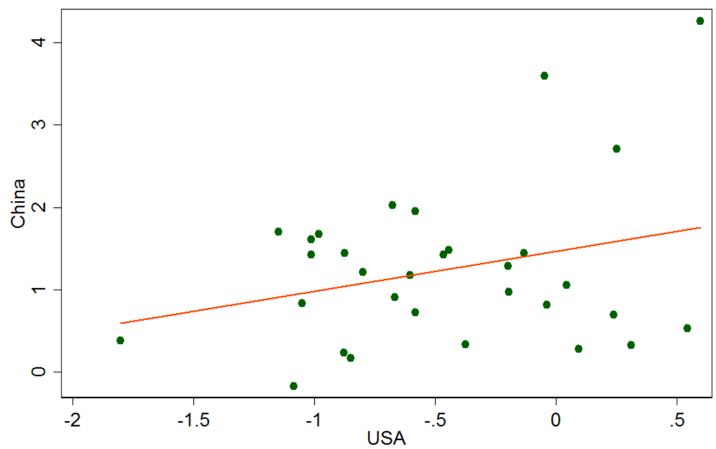
Before proceeding, Figure 5 investigates the cross-industry correlation between the exporter fixed-effects of China and the US. To this end, we obtain $\hat{\delta}_{n,US}$ by estimating equation (45) with the US in the sample. The figure presents a scatter plot of $\hat{\delta}_{n,China}$ and $\hat{\delta}_{n,US}$ for the 31 manufacturing industries in our sample. We can see that they have a weak positive correlation.

Table 5: Industries: Parameters and Shocks

Industry	SCTG	Trade Elast.	Chinese cost shock: Export (baseline)	Export (adj)	Inverted cost	NTR Gap	Prod. Hsieh-Ossa
Animals, cereals	1-2	8.59	0.33	0.04	-0.18	0.04	1.06
Other agriculture	3	8.59	0.23	0.03	-0.04	0.11	1.06
Animal origin goods	4	8.59	0.17	0.02	-0.09	0.07	1.06
Meat, fish, seafood	5	8.59	0.90	0.11	-0.13	0.09	1.06
Grain products	6	2.83	0.83	0.29	-0.31	0.10	1.16
Other prepared food	7	2.83	0.72	0.26	-0.17	0.13	1.16
Alcoholic beverages	8	2.83	-0.17	-0.06	-0.10	0.34	1.16
Tobacco products	9	8.59	0.38	0.04	-0.03	0.22	1.16
Mining	10	14.83	0.28	0.02	-0.02	0.12	1.06
Oil products	15-19	69.31	0.52	0.01	0.02	0.04	0.72
Basic chemicals	20	3.64	1.05	0.29	-0.16	0.14	1.29
Pharmaceutical	21	3.64	0.70	0.19	-0.18	0.17	1.29
Fertilizers	22	3.64	4.26	1.17	-0.28	0.00	1.29
Chemical products	23	3.64	1.28	0.35	-0.29	0.21	1.29
Plastics and rubber	24	0.88	1.17	1.33	-0.95	0.29	0.92
Logs and other wood	25	10.19	0.33	0.03	-0.08	0.00	1.02
Wood products	26	10.19	1.61	0.16	-0.13	0.21	1.02
Pulp, paper	27	8.32	3.60	0.43	-0.21	0.20	0.89
Paper articles	28	8.32	1.70	0.20	-0.10	0.29	0.89
Printed products	29	8.32	1.48	0.18	-0.18	0.14	0.89
Textiles and leather	30	5.99	1.22	0.20	-0.16	0.42	0.65
Nonmetallic mineral	31	3.38	1.45	0.43	-0.16	0.32	1.13
Base metals	32	6.58	2.71	0.41	-0.14	0.17	1.17
Articles of base metal	33	5.03	1.43	0.28	-0.18	0.32	1.17
Machinery	34	2.87	1.95	0.68	-0.39	0.31	1.18
Electronic equip.	35	11.02	1.68	0.15	-0.09	0.32	1.23
Vehicles	36	0.49	1.44	2.94	-2.12	0.18	1.06
Transportation equip.	37	0.9	2.03	2.25	-0.88	0.25	1.06
Precision instruments	38	4.95	0.97	0.20	0.00	0.36	0.70
Furniture	39	4.95	1.42	0.29	-0.12	0.40	0.70
Miscellaneous	40-43	4.95	0.82	0.17	0.01	0.38	0.70
Services	NA	5	-	-	-	-	-
Median		5.02	1.11	0.20	-0.15	0.19	1.06
Average		7.89	1.20	0.41	-0.25	0.20	1.01
St. Dev.		11.51	0.96	0.64	0.40	0.12	0.26
Correl. w/ baseline		-0.17	1.00	0.42	-0.21	0.11	0.22

Notes: The inverted trade shocks are, for each industry, the average change across US CZs in the cost of imports from China.

Figure 5: Change in Exporter Fixed-Effect of China and US: 31 manufacturing industries, 1997-2007



Notes: Scatter plot of the estimated industry-level exporter fixed effect for China against the corresponding fixed effect for US. A least-square best fit line is reported.

B.3.2 Estimation of Structural Parameters: Robustness

This section investigates the robustness of the results reported in Table 1. In every specification, we compute the predicted changes in CZ-level outcomes using the first-step estimates of the structural parameters reported in Panel A of Table 1.

We start by reporting, in Table 6, the results obtained with alternative sets of controls. We can see that the additional controls do not affect significantly the estimates of ϕ_e and ψ reported in Panels B and C. However, the estimate of the migration elasticity is sensitive to the control set: as we sequentially include controls, the first-stage becomes weaker and the estimate more imprecise.

Table 7 reports the estimates of the structural parameters using the model's predicted response of labor market outcomes with alternative parameter estimates and shock sources. Column (2) shows that we obtain similar estimates when MOIV is computed with the second-step estimates reported in Panel B of Table 1. This suggests that there are small efficiency gains of moving beyond the two-step feasible implementation of the MOIV estimator, as indeed suggested by Proposition 5.

Columns (3)–(7) report estimates obtained with the alternative configuration of the industry-level shock described above. Relative to the baseline estimates, the estimated elasticities of labor supply remain similar but the agglomeration elasticity is can be larger. In fact, column (8) reports the results of estimation of the structural parameters with the predicted responses with all sources of cost shocks. The p-value of the over-identification test is low, which suggests that either the model is not well specified or the exogeneity restriction is not valid for one of the shocks.

Finally, Table 8 reports the estimated agglomeration elasticity under different parametrization of the function controlling how local productivity depends on employment of other regions. Specifically, column (2) reports the estimation of our model under the assumption of $\pi_{rr} = 1$

and $\pi_{rd} = 0$ for all $r \neq d$. In this case, productivity only depends on the own-market employment level, as in Krugman (1980) and Allen and Arkolakis (2014). The estimated parameter of 0.60 indicates strong local agglomeration forces. Alternatively, in column (3), we estimate the model under the assumption that the decay of π_{ij} on distance is 0.35 – the estimate reported in columns (3) of Table 5 of Ahlfeldt et al. (2015). In this case, the estimated parameter suggests even stronger productivity spillovers across markets.

Table 6: Structural Parameter Estimates: Alternative Control Set

	(1)	(2)	(3)	(4)
Panel A: $-\phi_m/\phi_e$				
	-0.395	0.320	-1.576	-0.262
S.E.	(0.467)	(0.462)	(2.311)	(1.724)
F Stat.	5.94	3.67	1.14	0.61
Panel B: $1/\phi_e$				
	0.917***	0.876***	0.823***	0.796***
S.E.	(0.121)	(0.098)	(0.107)	(0.089)
F Stat.	16.46	16.85	17.79	15.65
Panel C: ψ				
	0.635***	0.588***	0.506***	0.479**
S.E.	(0.177)	(0.161)	(0.195)	(0.204)
F Stat.	9.22	9.57	8.94	9.23
Sector composition controls:	No	Yes	No	Yes
Demographic controls:	No	No	Yes	Yes

Notes: Sample of 722 Commuting Zones and 2 Sectors in 1997-2007. Models are weighted by start of period CZ share of national population. Control sets defined in Table 2. Instrumental variable computed with First-Step estimates of Table 1A. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Table 7: Structural Parameter Estimates: Alternative Instrumental Variables

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel A: $-\phi_m/\phi_e$							
	-0.390	-0.300	-1.230	0.390	-0.82	-1.83	-0.33
S.E.	(0.467)	(0.374)	(0.936)	(0.35)	(0.91)	(1.63)	(0.45)
F Stat.	5.94	8.32	2.96	7.30	2.38	1.88	2.99
J-test (p-value)							0.003
Panel B: $1/\phi_e$							
	0.92***	0.90***	0.71***	1.18***	1.25***	1.41***	1.19***
S.E.	(0.121)	(0.122)	(0.117)	(0.09)	(0.19)	(0.24)	(0.11)
F Stat.	16.46	16.21	7.90	120.23	40.84	9.77	51.88
J-test (p-value)							0.052
Panel C: ψ							
	0.64***	0.63***	0.260	0.55***	1.12***	1.08***	0.70***
S.E.	(0.177)	(0.195)	(0.188)	(0.15)	(0.20)	(0.23)	(0.15)
F Stat.	9.22	8.81	5.02	76.75	30.12	15.37	37.95
J-test (p-value)							0.016
Cost Shock:							
Export (baseline)	Yes	Yes	No	No	No	No	Yes
Export (adjsuted)	No	No	Yes	No	No	No	Yes
Firm productivity	No	No	No	Yes	No	No	Yes
NTR Gap	No	No	No	No	Yes	No	Yes
Inverted trade cost	No	No	No	No	No	Yes	Yes
MOIV parameters:	1st	2nd	2nd	2nd	2nd	2nd	2nd

Notes: Sample of 722 Commuting Zones and 2 Sectors in 1997-2007. Models are weighted by start of period CZ share of national population. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Table 8: Structural Parameter Estimates: Alternative Agglomeration Specification

	(1)	(2)	(3)
	0.635***	0.604***	0.871***
S.E.	(0.177)	(0.169)	(0.270)
F Stat.	9.22	9.35	8.07
Distance decay:	$\delta = 1$	$\delta = \infty$	$\delta = 0.35$

Notes: Sample of 722 Commuting Zones and 2 Sectors in 1997-2007. Models are weighted by start of period CZ share of national population. Instrumental variable computed with estimates Table 1A. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

B.3.3 Model Fit: Robustness

This section investigates the differential responses in manufacturing employment to alternative measures of the CZ's exposure to Chinese import competition. Specifically, We present the estimation of equation (46) using alternative measures of the Chinese export shock and alternative shift-share exposure measures. All specifications include the full set of controls in column (4) of Table 2.

Table 9 investigates the cross-regional employment effects obtained with shift-share exposure measures. Column (1) replicates the results in Autor, Dorn, and Hanson (2013). To this end, the industry-level "shift" is the change in Chinese imports of other developed countries normalized by the 1990 employment in the US, $(X^{2007}_{n,China,j} - X^{1997}_{n,China,j})/L^{1990}_{n,US}$, and the "share" is the share of industry n in the CZ's total employment. Notice that Autor, Dorn, and Hanson (2013) multiply the log-change in manufacturing employment by 100. So, in order to compare our estimates to theirs, we need to multiply the estimated coefficient in Table 10 by 100. In this case, our estimated cross-regional effect is 6.5, which is similar to the estimated effect of 4.2 in Table 5 of Autor, Dorn, and Hanson (2013).

Column (2) reports the differential employment effect of a similar shift-share exposure where the "shift" is Chinese export shock $\{\hat{\delta}_{n,Chna}\}_n$ (as described in Section 5). The estimated coefficient indicates that CZs more exposed to the Chinese import competition experienced a statistically significant lower relative growth in manufacturing employment. Finally, column (3) reports the cross-regional impact of the shift-share measure where the "share" is the share of industry n in manufacturing employment. In this case, the point estimate is negative, but it is not statistically significant.

Column (1) replicates the baseline results of Table 2. In column (2), we adjust the Chinese export shock by the industry's trade elasticity: the cost shock is $z_n = \hat{\delta}_{n,China}/\bar{\chi}_n$ using the $\bar{\chi}_n$ reported in Table 5. Despite the fact that the average magnitude of this adjusted shock measure is 30% of the average baseline shock, the estimated coefficient in column (2) is only 50% higher than the coefficient in column (1). This indicates that the cross-regional variation in predicted changes in manufacturing employment is mainly driven by industries with a low trade elasticity – in fact, the cross-industry correlation between $\hat{\delta}_{n,China}$ and $\bar{\chi}_n$ is -0.2.

Column (3) shows that the estimated coefficient is higher when the cost shock measure is the firm-level productivity growth of Hsieh and Ossa (2016). This is partially driven by the lower cross-regional variation in exposure to the measured productivity shock due to its lower cross-industry variation.

Columns (4) and (5) show that the estimated coefficients are much higher when we consider the impact of removing NTR gaps and changes in bilateral trade costs. In all these cases, the cross-regional correlation between predicted employment responses to the baseline and the alternative cost shocks is above 0.4. So, the smaller effects of the trade cost shocks in columns (4) and (5) are partially capturing the larger impact of changes in Chinese productivity.

Table 9: Model Fit: Manufacturing Employment - Alternative Specifications

Dependent variable: Log-change in manufacturing employment, 1997-2007

	(1)	(2)	(3)
Shift-share exposure	-0.065***	-0.768***	-0.084
	(0.017)	(0.237)	(0.088)
R^2	0.270	0.234	0.268
Industry-level shock			
Baseline:	No	Yes	Yes
Normalized import change (ADH):	Yes	No	No
CZ's industry employment share in			
Total employment:	Yes	Yes	No
Manufacturing employment:	No	No	Yes

Notes: Sample of 722 Commuting Zones in 1997-2007. Models are weighted by start of period CZ share of national population. All specifications include the set of baseline controls in column (4) of Table 2. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Table 10: Model Fit: Manufacturing Employment - Alternative Industry-level Shocks

Dependent variable: Log-change in manufacturing employment, 1997-2007

	(1)	(2)	(3)	(4)	(5)
Predicted manuf. log-change in employment	6.84***	9.53**	12.99**	43.25***	50.69**
	(2.079)	(3.883)	(5.232)	(7.87)	(24.51)
R^2	0.27	0.23	0.24	0.34	0.27
Cost Shock:					
Export (baseline)	Yes	No	No	No	No
Export (adjsuted)	No	Yes	No	No	No
Firm productivity	No	No	Yes	No	No
NTR Gap	No	No	No	Yes	No
Inverted trade cost	No	No	No	No	Yes

Notes: Sample of 722 Commuting Zones in 1997-2007. Models are weighted by start of period CZ share of national population. All specifications include the set of baseline controls in column (4) of Table 2. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

B.4 Additional Results: Counterfactual Analysis

B.4.1 Reduced-form elasticities for real wages

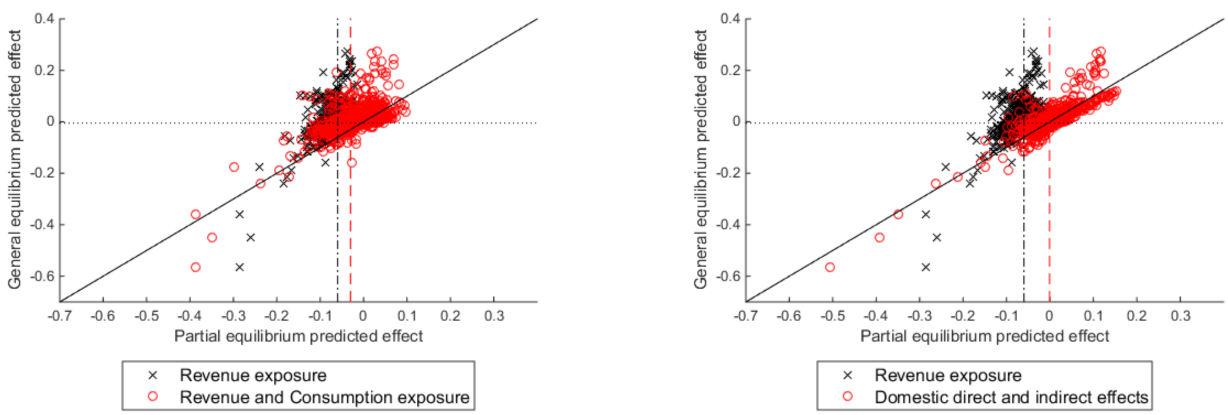
Table 11: Reduced-Form Elasticities of Employment to Local Manufacturing Shock Exposure

	Own region		Other regions	
	$\gamma_{Mr,Mr}$	$\gamma_{Nr,Mr}$	$\sum_{d\neq r} l_{Md}\gamma_{Md,Mr}$	$\sum_{d\neq r} \gamma_{Mr,Md}$
	(1)	(2)	(3)	(4)
Panel A: Employment elasticity to revenue exposure ($\gamma^{R}_{Mr,Md}$)				
Avg.	0.1741	-0.0082	0.0004	-0.1435
Panel B: Employment elasticity to consumption exposure ($\gamma^{C}_{Mr,Md}$)				
Avg.	1.2096	0.2313	0.0008	-0.3625

Notes: Average reduced-form elasticity computed using the estimates in Panel B of Table 1 and the observed equilibrium in 1997. M denotes the manufacturing sector and N denotes the non-manufacturing sector.

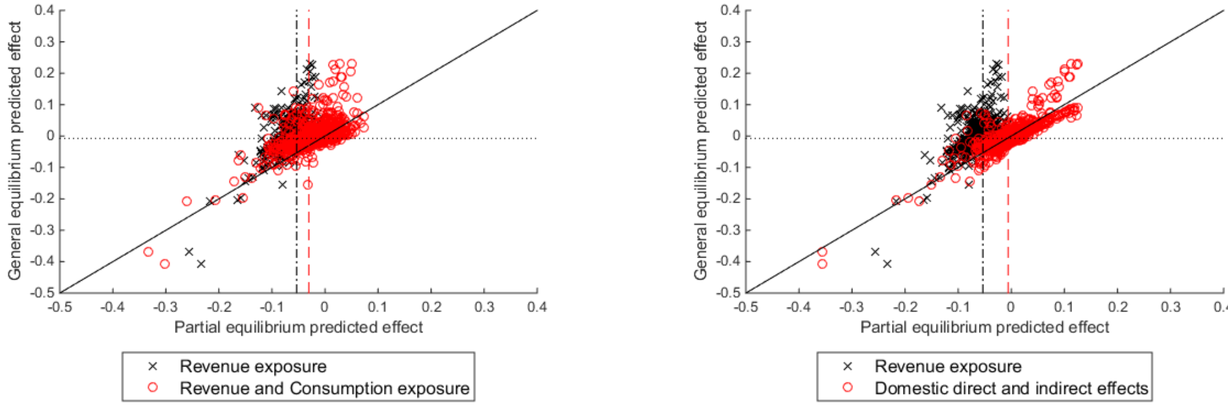
B.4.2 Robustness

Figure 6: Predicted Change in Manufacturing Real Wage, China shock



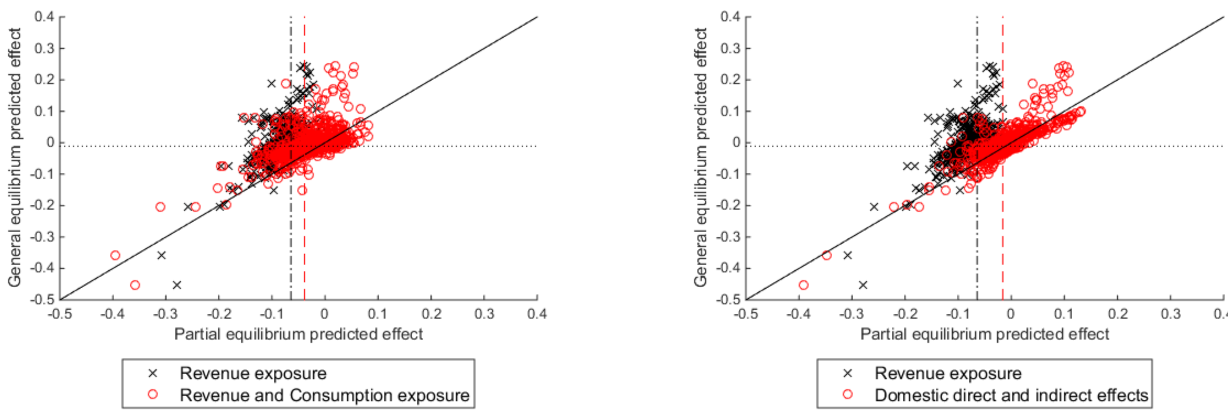
Notes: The figure on the left reports the scatter plot of the general equilibrium response of manufacturing real wage against the predicted revenue exposure and against the sum of the predicted revenue and consumption exposures. It also displays the average (across CZs) of the general equilibrium response (horizontal line), revenue exposure (black line), revenue and consumption exposure (red line). The figure on the right reports the scatter plot of the general equilibrium response against the predicted revenue exposure, and against the sum of predicted (domestic) direct and indirect effects from revenue and consumption exposures, computed using Proposition . It also displays the average (across CZs) of the general equilibrium response (horizontal line), revenue exposure (black line), domestic direct and indirect effects (red line).

Figure 7: Predicted Change in Manufacturing Employment, no labor links



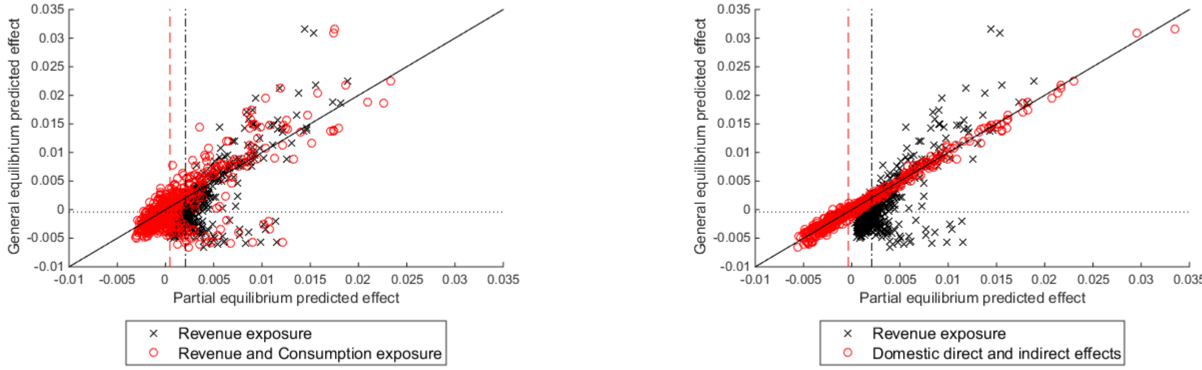
Notes: The figure on the left reports the scatter plot of the general equilibrium response of manufacturing employment in a model without agglomeration and migration across regions, against the predicted revenue exposure and against the sum of the predicted revenue and consumption exposures. It also displays the average (across CZs) of the general equilibrium response (horizontal line), revenue exposure (black line), revenue and consumption exposure (red line). The figure on the right reports the scatter plot of the general equilibrium response against the predicted revenue exposure, and against the sum of predicted (domestic) direct and indirect effects from revenue and consumption exposures, computed using Proposition . It also displays the average (across CZs) of the general equilibrium response (horizontal line), revenue exposure (black line), domestic direct and indirect effects (red line).

Figure 8: Predicted Change in Manufacturing Employment, no home sector



Notes: The figure on the left reports the scatter plot of the general equilibrium response of manufacturing employment in a model without the home sector, against the predicted revenue exposure and against the sum of the predicted revenue and consumption exposures. It also displays the average (across CZs) of the general equilibrium response (horizontal line), revenue exposure (black line), revenue and consumption exposure (red line). The figure on the right reports the scatter plot of the general equilibrium response against the predicted revenue exposure, and against the sum of predicted (domestic) direct and indirect effects from revenue and consumption exposures, computed using Proposition . It also displays the average (across CZs) of the general equilibrium response (horizontal line), revenue exposure (black line), domestic direct and indirect effects (red line).

Figure 9: Predicted Change in Manufacturing Employment, NAFTA shock



Notes: The figure on the left reports the scatter plot of the general equilibrium response of manufacturing employment after a NAFTA shock, against the predicted revenue exposure and against the sum of the predicted revenue and consumption exposures. It also displays the average (across CZs) of the general equilibrium response (horizontal line), revenue exposure (black line), revenue and consumption exposure (red line). The figure on the right reports the scatter plot of the general equilibrium response against the predicted revenue exposure, and against the sum of predicted (domestic) direct and indirect effects from revenue and consumption exposures, computed using Proposition . It also displays the average (across CZs) of the general equilibrium response (horizontal line), revenue exposure (black line), domestic direct and indirect effects (red line).

C Online Appendix: Data Construction

Our sample consists of 722 US commuting zones, 58 foreign countries plus Alaska and Hawaii. Table 12 lists the foreign countries. We divide the manufacturing sector in 31 industries, listed in Table 5.

Table 12: Sample of Countries

Argentina	Kazakhistan
Australia	South Korea
Austria	Mexico
Baltic Republics	Malaysia
Bulgaria	North Africa
Belarus	Netherlands
Benelux	Norway
Brazil	New Zealand
Canada	Pakistan
Switzerland	Philippines
Chile	Poland
China	Portugal
Colombia	Rest of Africa
Czech Republic	Rest of Asia
Germany	Rest of Central America
Denmark	Rest of Europe
Spain	Rest of South America
Finland	Romania
France	Russia
United Kingdom	Saudi Arabia
Greece	Singapore
Croatia	Slovakia
Hungary	Sweden
Indonesia	Thailand
India	Taiwan
Ireland	Ukraine
Iran	Uruguay
Italy	Venezuela
Japan	South Africa

Notes: Baltic Republics includes Estonia, Lithuania and Latvia; North Africa includes Algeria, Egypt, Ethiopia, Morocco, Tunisia; countries named "Rest of X" include all the remaining countries of continent X not included in the table.

C.1 World Trade Matrix

We construct a matrix of bilateral industry-level trade flows among 722 US Commuting Zones, Alaska, Hawaii and 58 foreign countries for 1997 and 2007.

1. We create country-to-country matrix of trade flows at the 2-digit SCTG classification used in the CFS. To this end, we use the BACI trade dataset from UN Comtrade at the HS6 level. We use it to construct trade flows for the 31 industries in Table 5 between the USA and the 58 countries in Table 12. We merge this data with the Eora MRIO dataset to obtain the domestic spending share in each industry. Since the EORA dataset uses a more aggregated industry classification, we assign identical spending shares to all SCTG industries in the EORA sectors. We obtain trade flows in non-manufacturing directly from EORA.

2. We then create a trade matrix between US states and foreign countries at the SCTG-level. We use state-to-state shipments data at the SCTG level from the Commodity Flow Survey

released by the US Census in 1997 and 2007. One issue is that in the CFS dataset some shipment values are suppressed or missing. We use a gravity-based approach to impute these suppressed values, that we describe in the sub-section C.1.1 below. Finally, we convert shipment flows into trade flows as follows.

(a) Let $\left(Z_{dj}^{kt}, Z_{jd}^{kt}\right)$ denote the trade flows between each of the 40 US custom districts, d, and foreign country, j, by sector k and year t. We obtain $\left(Z_{dj}^{kt}, Z_{jd}^{kt}\right)$ from the US Merchandise Trade Files released annually by the US Census between 1990 and 2016. The exports and imports of state i to foreign country j are

$$X_{ij}^{kt} = \sum_{d} a_{i}^{dj,kt} \cdot Z_{dj}^{kt}$$

$$X_{ji}^{kt} = \sum_{d} b_{i}^{dj,kt} \cdot Z_{jd}^{kt}$$

where $a_{i}^{dj,kt}$ and $b_{i}^{dj,kt}$ correspond to the share of total exports and imports in district d whose respective origin and destination are state i.

i. We construct bilateral trade flows between US states for each sector and year. Let $\tilde{X}_{ir}^{kt}$ denote the value of shipments from state i to state r of goods in sector k at year t. The trade flows between state i to state r are services:

$$X_{ir}^{kt} = \tilde{X}_{ir}^{kt} - \sum_{d,j} \left(\tilde{a}_{ir}^{dj,kt} \cdot Z_{dj}^{kt} + \tilde{b}_{ir}^{dj,kt} \cdot Z_{jd}^{kt}\right)$$

where $\tilde{a}_{ir}^{dj,kt}$ and $\tilde{b}_{ir}^{dj,kt}$ correspond respectively to the share of total exports and imports in district d transiting between states i and r. To compute the variables above, we assume that the transit route is the same for all export and import of all sectors with identical state of origin/destination, port of exit/entry, and foreign country of origin/destination. Using the US Census data on state of origin exports by port and destination, we compute the following variables:

$$a_{i}^{dj,kt} = b_{i}^{dj,kt} = \frac{\text{exports}_{i}^{dj,t}}{\sum_{l} \text{exports}_{l}^{dj,t}} \quad \text{and} \quad \tilde{a}_{ir}^{dj,kt} = \tilde{b}_{ir}^{dj,kt} = \frac{\text{exports}_{ij}^{dj,t}}{\sum_{r,l} \text{exports}_{rl}^{dj,t}}$$

ii. We adjust domestic sales of the residual sector to include local spending in

$$X_{ii}^{NT,t} = \left(\sum_{k \neq NT} \sum_{r} X_{ri}^{kt}\right) e_{i}^{t}$$

where e_{i}^{t} is the expenditure ratio between non-tradeable and tradeable goods of state i at year t obtained from the BEA state-level accounts.

(b) We merge the trade bilateral trade flows of US states with the bilateral trade flows of the US and other countries in the BACI database. To this end, we use US domestic sales in BACI to normalize total expenditures of US states on goods produced from other US states. We also distribute the bilateral trade flows of the US in the BACI among US states using each state share in total trade flows to/from other foreign countries obtained in the previous step. The final output consists of $X_{ij,k}$: trade flow from i to j in SCTG sector k, where i, j are US states or foreign countries.

3. The final step is to use the trade matrix with US states and foreign countries to construct trade flows for US Commuting Zones at the SCTG-level. To this end, we construct the participation of each CZ r in its state $i(r)$ production and consumption. The production share is the CZ's share in the state's total employment in industry n, $R^t_{n,r}(j) \equiv L^t_{n,r}/(\sum_{r' \in i(r)} L^t_{n,r'})$, and the spending share is the CZ's total employment share in the state, $E^t_{n,r}(i) \equiv \bar{L}^t_r/(\sum_{r' \in i(r)} \bar{L}^t_{r'})$ with $\bar{L}^t_r \equiv \sum_{n'} L^t_{r,n'}$. For each CZ, export value to country j is $X^t_{cj,n} = R^t_{n,r} X_{i(c)j,n}$ and import value from country j is $X^t_{ir,n} = E^t_{n,r} X_{ij(c),n}$. Finally, we impute trade flows between CZs using a gravity procedure. We first use state-to-state trade flows computed in the previous step to estimate a gravity regression for each industry n:

$$\log X^t_{ij,n} = \beta^t_0 + \beta^t_1 1[i = j] + \beta^t_2 \ln d_{ij} + \beta_3 \ln Y^t_{i,n} + \beta_4 \ln E^t_{j,n} + e^t_{ij,k}$$

where d_{ij} is the bilateral distance between state i and j, $Y^t_{i,n}$ is the total production in state i, $E^t_{j,n}$ is the total expenditure in state j. We then use the estimated coefficients to compute the predicted flows between CZ r and d

$$\log \hat{x}^t_{rd,k} = \hat{\beta}^t_0 + \hat{\beta}^t_1 1[i = j] + \hat{\beta}^t_2 \ln d_{ij} + \hat{\beta}_3 \ln R^t_{i,n} + \hat{\beta}_4 \ln E^t_{j,n}$$

where $R^t_{n,r}$ are employment shares and $E^t_{n,k}$ are expenditure shares computed in the previous step. We rescale these predicted values by the corresponding share of national flows coming from US domestic sales from the BACI dataset. In the non-manufacturing sector, we use a similar procedure using a higher value for the distance elasticities, $\hat{\beta}_1$ and $\hat{\beta}_2$. In particular, we follow Eckert (2018) by adjusting these parameters to be 50% higher than the estimates for the manufacturing sector.

C.1.1 Methodology to replace suppressed values in the CFS

We implement the imputation procedure separately for each of the 31 industries in Table 5. To simplify the notation, we drop the industry subscript. Using observed data on bilateral shipments between US states in the tradeable sector, we estimate the following gravity equation, for every year t:

$$\log \tilde{X}_{ij} = \beta_0 + \beta_1 \ln d_{ij} + \beta_2 \ln Y_i + \beta_2 \ln E_j + e_{ij}$$

where d_{ij} is the bilateral distance between state i and j, Y_i is the total production in state i, E_j is the total expenditure in state j, and e_{ij} is the econometric error. Then we obtain the predicted values

$$\log \hat{X}_{ij} = \hat{\beta}_0 + \hat{\beta}_1 \ln d_{ij} + \hat{\beta}_2 \ln X_i + \hat{\beta}_2 \ln X_j.$$

We compute the residual outflows for each state as $\bar{Y}_i = Y_i - \sum_j \tilde{X}_{ij}$, and the residual inflows as $\bar{E}_j = E_j - \sum_i \tilde{X}_{ij}$. For suppressed values, we assume that the true trade flow equals:

$$\tilde{X}_{ij} = \hat{X}_{ij} \xi_i \gamma_j.$$

We must have that the summation of predicted flows across destinations for each origin has to be equal to total production: $\sum_j \tilde{X}_{ij} = \bar{Y}_i$. Also, the summation of predicted flows across origins for each destination has to be equal to total expenditures: $\sum_i \tilde{X}_{ij} = \bar{E}_j$. To compute ξ_i and γ_j, we use the following algorithm. For state i, consider the vector of exports to all states $\tilde{X}_{ij}$ and the imports $\tilde{X}_{ji}$. Then, we compute the following ratios: $\xi_i = \sum_j \tilde{X}_{ij}/\bar{Y}_i$ and $\gamma_i = \sum_j \tilde{X}_{ji}/\bar{E}_j$. We then adjust $\hat{X}_{ij} = \tilde{X}_{ij}/\xi_i$ and $\hat{X}_{ji} = \tilde{X}_{ji}/\gamma_i$. For state $j+1$, repeat the same procedure, but keeping constant the exports and imports of the previous adjusted states 1 to j, and adjusting the total expenditures and production. Finally, we use these predicted (and consistent with the aggregates) values to fill the suppressed shipments.

D Online Appendix: Equivalences and Extensions

This online appendix has three parts. First, we establish the general solution of our model in the non-linear system of equilibrium conditions. Second, we formally establish the equivalence of our model's counterfactual predictions to those implied by a number of existing trade and geography models. Finally, we extend our model to account for other sources of cross-market links.

D.1 Non-Linear DEK Expressions

Consider the solution of the non-linear system of equilibrium equations following changes in economic fundamentals. Consider an equilibrium with positive production in all markets. The labor market module in (14)–(16) imply written in changes:

$$L_i^0 \hat{L}_i = \Phi_i \left(\left\{ \omega_j^0 \hat{\omega}_j \right\}_j \right) \tag{59}$$

$$\hat{w}_i = \hat{p}_i \frac{\Psi_i \left(\left\{ L_j^0 \hat{L}_j \right\}_j \right)}{\Psi_i \left(\left\{ L_j^0 \right\}_j \right)}. \tag{60}$$

In addition, the market clearing condition in (17) yields

$$w_i^0 L_i^0 \left(\hat{w}_i \hat{L}_i \right) = \sum_j x_{ij}^0 \hat{x}_{ij} w_j^0 L_j^0 \left(\hat{w}_j \hat{L}_j \right), \tag{61}$$

where the changes in spending shares and price indices in (12)–(13) are given by

$$x_{ij}^0 \hat{x}_{ij} = X_{ij} \left(\left\{ \frac{\tau_{oj}^0 p_o^0}{P_j^0} \hat{\tau}_{oj} \hat{p}_o \right\}_o \right) \quad \text{and} \quad \hat{P}_j = P_j \left(\left\{ \frac{\tau_{oj}^0 p_o^0}{P_j^0} \hat{\tau}_{oj} \hat{p}_o \right\}_o \right). \tag{62}$$

The system (59)–(62) determines the changes in endogenous variables, $\{\hat{p}_i, \hat{P}_i, \hat{L}_i, \hat{\omega}_i\}_i$, implied by any combination of shocks, $\{\hat{\tau}_{ij}\}_{i,j}$. It depends on the aggregate mappings $\{\{X_{ij}(\cdot)\}_j, \Phi_i(\cdot), \Psi_i(\cdot)\}_i$ as well as initial outcomes, $\{\{x_{ij}^0\}_j, w_i^0, L_i^0\}_i$, and initial prices and shifters, $\{\{\tau_{ij}^0\}_j, p_i^0, P_i^0\}_i$. Notice however that our model – and thus a large number of spatial models – is over-identified: there are multiple degrees of freedom to match observed labor and trade outcomes in the initial equilibrium. We show that it is always possible to choose the location of the preference and productivity shifters in $\{\{X_{ij}(\cdot)\}_j, \Phi_i(\cdot), \Psi_i(\cdot)\}_i$ to replicate the initial levels of trade flows and labor market outcomes across markets, while normalizing shifters of trade costs, and productivity in the initial equilibrium. The normalization of bilateral effective prices in the initial equilibrium is analogous to that imposed in neoclassical economies by Adao, Costinot, and Donaldson (2017).

Thus, in equations (59)–(62), we choose initial shifters such that

$$\tau_{ij}^0 p_i^0 \equiv 1, \quad P_j^0 \equiv 1, \quad \Psi_i \left(\left\{ L_j^0 \right\}_j \right) \equiv 1 \quad \forall i, j. \tag{63}$$

Given the normalization in (63), we can use the system in (59)–(62) to characterize the counterfactual predictions of our model.

Proposition 6. *Consider the Generalized Spatial Economy satisfying Assumptions 1 and 2. Conditional on initial levels of endogenous variables* $\{\{x_{ij}^0\}_j, w_i^0, L_i^0\}_i$, *the mappings* $\{\{X_{ij}(\cdot)\}_j, \Phi_i(\cdot), \Psi_i(\cdot)\}_i$

are sufficient to uniquely characterize counterfactual changes in endogenous outcomes, $\{\hat{p}_i, \hat{P}_i, \hat{L}_i, \hat{\omega}_i\}_i$*, implied by any combination of shocks,* $\{\hat{\tau}_{ij}\}_{i,j}$*, as a solution of (59)–(62).*

Proof. Proposition 2 immediately guarantees that there is a unique equilibrium for the initial and the final set of shifters. So, we only need to show that, by specifying preferences and technology, we obtain an equilibrium with identical trade and labor outcomes as the initial equilibrium under the normalization in (63).

Initial Equilibrium. Consider an initial equilibrium such that

$$L_j^0 = \Phi_j\left(\left\{\frac{w_i^0}{P_i^0}\right\}_i\right)$$

$$p_i^0 = \frac{w_i^0}{P_i^0}\frac{1}{\Psi_i\left(\left\{L_j^0\right\}_j\right)}$$

$$w_i^0 L_i^0 = \sum_j x_{ij}^0 w_j^0 L_j^0$$

$$x_{ij}^0 = X_{ij}\left(\left\{\tau_{oj}^0 p_o^0\right\}_o\right) \quad \text{and} \quad P_j^0 = P_j\left(\left\{\tau_{oj}^0 p_o^0\right\}_o\right).$$

Alternative Economy. Denote $\Psi_i^0 \equiv \Psi_i\left(\left\{L_j^0\right\}_j\right)$. Let us construct an alternative economy without trade costs ($\tilde{\tau}_{ij} \equiv 1$), where technology is given by

$$\tilde{\Psi}_i\left(\{L_j\}_j\right) \equiv \frac{1}{\Psi_i^0}\Psi_i\left(\{L_j\}_j\right),$$

and preferences are given by

$$\tilde{U}_c\left(\{C_j\}_j, \{L_j\}_j\right) \equiv U_c\left(\left\{\frac{1}{P_j^0}C_j\right\}_j, \{L_j\}_j\right)$$

$$\tilde{V}_j\left(\{c_{ij}\}_i\right) \equiv V_j\left(\left\{c_{ij}\frac{\Psi_i^0 P_j^0}{\tau_{ij}^0}\right\}_i\right).$$

In this case, we immediately get that

$$\tilde{\Phi}_j\left(\{\omega_i\}_i\right) = \Phi_j\left(\left\{\frac{1}{P_i^0}\omega_i\right\}_i\right)$$

and

$$\tilde{X}_{ij}\left(\{p_{oj}\}_o\right) = X_{ij}\left(\left\{\frac{\tau_{oj}^0}{P_j^0\Psi_i^0}p_{oj}\right\}_o\right) \quad \text{and} \quad \tilde{P}_j\left(\{p_{oj}\}_o\right) = P_j\left(\left\{\frac{\tau_{oj}^0}{P_j^0\Psi_i^0}p_{oj}\right\}_o\right).$$

Equilibrium of Alternative Economy. In this economy, the equilibrium entails $\tilde{w}_i = w_i^0$, $\tilde{L}_i = L_i^0$,$\tilde{x}_{ij} = x_{ij}^0$, and $\tilde{p}_{ij} = \tilde{P}_i = 1$. To see this, notice that

$$\tilde{p}_i = \frac{\tilde{w}_i}{\tilde{\Psi}_i\left(\left\{L_j^0\right\}_j\right)} = w_i^0,$$

$$\tilde{\Phi}_j\left(\left\{\frac{\tilde{w}_i}{\tilde{P}_i}\right\}_i\right) = \Phi_j\left(\left\{\frac{1}{P_i^0}w_i^0\right\}_i\right) = L_j^0,$$

$$\tilde{x}_{ij} = \tilde{X}_{ij}\left(\{\tilde{\tau}_{oj}\tilde{p}_o\}_o\right) = X_{ij}\left(\left\{\frac{\tau_{oj}^0 p_o^0}{P_j^0}\right\}_o\right) = x_{ij}^0,$$

$$\tilde{P}_j = \tilde{P}_j\left(\{\tilde{\tau}_{oj}\tilde{p}_o\}_o\right) = P_j\left(\left\{\frac{\tau_{oj}^0 p_o^0}{P_j^0}\right\}_o\right) = 1$$

Finally, the labor market clearing condition holds:

$$\tilde{w}_i\tilde{L}_i = w_i^0 L_i^0 = \sum_j x_{ij}^0 w_j^0 L_j^0 = \sum_j \tilde{x}_{ij}\tilde{w}_j\tilde{L}_j. \quad \blacksquare$$

D.2 Equivalences

We now discuss how our theoretical environment unifies a number of existing frameworks in spatial economics. We show that the shape of the mappings $\{\{X_{ij}(\cdot)\}_i, \Phi_j(\cdot), \Psi_j(\cdot)\}_j$ encompasses the central forces in a wide range of spatial and trade models. We start by introducing a formal definition of the models for which the equilibrium outcomes of the Generalized Spatial Model of Section 3 are observationally equivalent to.

Definition 2. The Generalized Spatial Model of Section 3 is observationally equivalent to Economy N with respect to the shifters $\{\tau_{ij}\}_{i,j}$ if

1. There exist *unique* mappings $\{\{X_{ij}^N(\cdot)\}_i, \Phi_j^N(\cdot), \Psi_j^N(\cdot)\}_j$ such that the equilibrium of Economy N is characterized by conditions (12)–(17) for *any* levels of $\{\tau_{ij}\}_{i,j}$;

2. There exist preferences, (1) and (10), and technology, (4), that imply $\{\{X_{ij}^N(\cdot)\}_i, \Phi_j^N(\cdot), \Psi_j^N(\cdot)\}_j$.

This definition requires that, independent of the levels of the exogenous shifters, Economy N must satisfy the equilibrium conditions (12)–(17) for unique mappings $\{\{X_{ij}^N(\cdot)\}_i, \Phi_j^N(\cdot), \Psi_j^N(\cdot)\}_j$. This implies that any combination of shocks to the shifters $\{\tau_{ij}\}_{i,j}$ yields identical counterfactual outcomes in labor markets. We use Definition 2 to establish that our model is observationally equivalent to several existing frameworks under specific parametric restrictions on the shape of $\{\{X_{ij}(\cdot)\}_i, \Phi_j(\cdot), \Psi_j(\cdot)\}_j$. In particular, we show the equivalence with, respectively: (i) Neoclassical models with economies of scale, (ii) New trade theory models, (iii) New economic geography models, (iv) Spatial assignment models, and (v) Spatial assignment models with other factors of production.

D.2.1 Neoclassical Economy

Environment. Consider a neoclassical economy with a single factor of production. We denote all the variables of this economy that are potentially different from the Generalized Spatial Economy with a superscript N. The proofs follows the logic of the proof of Adao, Costinot, and Donaldson

(2017) but extending to the case of labor mobility and agglomeration spillovers. We assume that the agglomeration function, the labor supply function, and the exogenous shifters are the same as for the Generalized Economy, so that we do not use superscripts for those objects.

As in the Generalized Spatial Model, each country has a representative agent with preferences for consumption and labor supply in different markets, with utility function given by

$$U_c\left(\left\{C_j^N\right\}_j, \left\{L_j^N\right\}_j\right).$$

The main difference is that we explicitly allow for preferences over goods, z:

$$C_j^N \equiv V^N\left(\left\{c_{z,ij}^N\right\}_{z,i}\right),$$

where $V^N(\cdot)$ is twice differentiable, quasi-concave, homothetic, and increasing in all arguments. Notice that $V^N(\cdot)$ allows for the possibility that goods from different origins are imperfect substitutes.

The representative household's budget constraint is

$$\sum_i \sum_z p_{z,ij}^N c_{z,ij}^N = w_j^N L_j^N.$$

There are many perfectly competitive firms supplying each good in any market. The production technology uses only labor and entails external economies of scale at the market level. In particular, the technology of producing good z in i and delivering to j is given by

$$Y_{z,ij}^N = \Psi_i\left(\left\{L_j^N\right\}_j\right) \frac{L_{z,ij}^N}{\tau_{ij}\alpha_{z,ir}^N},$$

where $\alpha_{z,ij}^N$ is good-specific productivity shifter of producing in i and delivering in j.

Equilibrium. We use the fact that $V^N(\cdot)$ is homothetic to derive the price index in market j:

$$P_j^N = P_j^N\left(\left\{p_{k,oj}^N\right\}_{k,o}\right) \equiv \min_{\left\{c_{k,oj}\right\}_{k,o}} \sum_{k,o} p_{k,oj}^N c_{k,oj}^N \quad \text{s.t.} \quad V^N\left(\left\{c_{k,oj}\right\}_{k,o}\right) \geq 1 \tag{64}$$

where the associated spending share on good z from i is

$$x_{ij,z}^N \in X_{ij,z}^N\left(\left\{p_{k,oj}^N\right\}_{k,o}\right). \tag{65}$$

Conditional on prices, the representative household solves the utility maximization problem that yields the labor supply in market j:

$$L_j^N = \Phi_j\left(\left\{\omega_i^N\right\}_i\right). \tag{66}$$

Profit maximization implies that

$$p_{z,ij}^N = \tau_{ij} p_i^N \alpha_{z,ij}^N \tag{67}$$

where

$$p_i^N = \frac{w_i^N}{\Psi_i\left(\left\{L_j^N\right\}_j\right)} \tag{68}$$

Finally, the labor market clearing condition is

$$w_i^N L_i^N = \sum_j \sum_z x_{z,ij}^N \cdot \left(w_j^N L_j^N\right). \tag{69}$$

The competitive equilibrium corresponds to $\left\{\left\{p_{z,ij}^N\right\}_{z,i}, w_j^N, L_j^N, P_j^N\right\}_j$ such that equations (64)–(69) hold. Thus, the equilibrium can be written as $\left\{p_i^N, \omega_i^N, L_i^N, P_i^N\right\}_i$ solving (12)–(17) with $\Phi_j(\cdot)$, $\Psi_j(\cdot)$, and

$$X_{ij}^N\left(\left\{\tau_{oj}p_o^N\right\}_o\right) \equiv \left\{x_{ij}^N = \sum_z x_{ij,z}^N :\ x_{ij,z}^N \in X_{ij,z}^N\left(\left\{\tau_{oj}p_o^N\alpha_{k,oj}^N\right\}_{o,k}\right)\right\}$$

such that

$$P_i^N\left(\left\{\tau_{oj}p_o^N\right\}_o\right) = P_i^N\left(\left\{\tau_{oj}p_o^N\alpha_{k,oj}^N\right\}_{o,k}\right).$$

Equivalence. We now construct an equivalent Generalized Spatial Economy. We only need to show that there exist preferences and technology that are consistent with $\left\{\left\{X_{ij}^N(.)\right\}_i, \Phi_j(.), \Psi_j(.)\right\}_j$. We also assume that the production function of the market-specific composite good in the Generalized Economy is

$$Y_{ij} = \Psi_i\left(\left\{L_j\right\}_j\right)\frac{L_{ij}}{\tau_{ij}}.$$

In addition, consider the preferences in Section 3 with

$$V_j\left(\left\{c_{ij}\right\}_i\right) \equiv \max_{\left\{c_{z,ij}\right\}_{z,i}} V^N\left(\left\{c_{z,ij}\right\}_{z,i}\right) \quad \text{s.t.} \quad \textstyle\sum_z \alpha_{z,ij}^N c_{z,ij} = c_{ij}\ . \tag{70}$$

Intuitively, the preference structure in (70) implies that, if the representative household acquires c_{ij} units of $i's$ composite good for j's consumption, then it optimally allocates the composite good into the production of different goods, given the exogenous weights $\alpha_{z,ij}^N$ that are now embedded into the representative agent's preferences. Since the relative price of goods in market i only depends on $\alpha_{z,ij}^N$, this decision yields allocations that are identical to those in the competitive equilibrium of the decentralized economy.

To see this, denote the spending shares associated with the cost minimization problem with $V_j(\cdot)$ by $x_{ij} \in X_{ij}\left(\left\{\tau_{ij}p_i\right\}_i\right)$. Thus, the equivalence follows from showing that

$$X_{ij}\left(\left\{\tau_{oj}p_o\right\}_o\right) = X_{ij}^N\left(\left\{\tau_{oj}p_o\right\}_o\right) \quad \forall \left\{\tau_{oj}p_o\right\}_o. \tag{71}$$

First, we show that $x_{ij} \in X_{ij}\left(\left\{\tau_{oj}p_o\right\}_o\right) \implies \exists x_{ij,z}^N \in X_{ij,z}^N\left(\left\{\tau_{oj}p_o\alpha_{z,oj}^N\right\}_{k,o}\right)$ with $x_{ij} = \sum_z x_{ij,z}^N$. Let $\left\{c_{z,ij}\right\}_{z,i}$ be the solution of the good allocation problem in the definition of $V_j\left(\left\{c_{ij}\right\}\right)$ in (70). We proceed by contradiction to show that $\left\{c_{z,ij}\right\}_{z,i}$ implies spending shares, $\left\{x_{z,ij}\right\}_{z,i} =$

$\left\{\tau_{oj}p_o\alpha^N_{z,ij}c_{z,ij}\right\}_{z,i}$, such that $x_{z,ij} \in X^N_{ij,z}\left(\left\{\tau_{oj}p_o\alpha^N_{z,oj}\right\}_{k,o}\right)$. Suppose there exists a feasible allocation $\left\{c^N_{z,ij}\right\}_{z,i}$ such that

$$V^N\left(\left\{c^N_{z,ij}\right\}_{z,i}\right) > V^N\left(\left\{c_{z,ij}\right\}_{z,i}\right) \quad \text{and} \quad \sum_i\sum_z \tau_{ij}p_i\alpha^N_{z,ij}c^N_{z,ij} \leq 1. \tag{72}$$

Notice that $\sum_i\sum_z \tau_{ij}p_i\alpha^N_{z,ij}c^N_{z,ij} \leq 1$, which implies that the allocation $c^N_{ij} \equiv \sum_z \alpha^N_{z,ij}c^N_{z,ij}$ is feasible in the Generalized Spatial Competitive Economy. Thus,

$$V^N\left(\left\{c_{z,ij}\right\}_{z,i}\right) = V_j\left(\left\{c_{ij}\right\}\right) \geq V_j\left(\left\{c^N_{ij}\right\}\right) \geq V^N\left(\left\{c^N_{z,ij}\right\}_{z,i}\right),$$

which is a contradiction of inequality (72).

Second, we show that $x_{ij} = \sum_z x^N_{ij,z}$ with $x^N_{ij,z} \in X^N_{ij,z}\left(\left\{\tau_{oj}p_o\alpha^N_{z,oj}\right\}_{k,o}\right)$, and $c^N_{ij} = \sum_z \alpha^N_{z,ij}c^N_{z,ij} \implies x_{ij} \in X_{ij}\left(\left\{\tau_{oj}p_o\right\}_o\right)$. We start with $c^N_{ij} = \sum_z \alpha^N_{z,ij}c^N_{z,ij}$ implied by the solution of the consumer's problem in the Neoclassical Economy. We proceed by contradiction to show that $\left\{c^N_{ij}\right\}_i$ is optimal in the Generalized Spatial Competitive Economy given prices $\left\{\tau_{ij}p_i\right\}_i$. Suppose there exists a feasible allocation $\left\{c_{ij}\right\}_i$ in the Generalized Spatial Competitive Economy such that

$$V_j\left(\left\{c_{ij}\right\}\right) > V_j\left(\left\{c^N_{ij}\right\}\right) \quad \text{and} \quad \sum_i p_{ij}c_{ij} \leq \sum_i p_{ij}c^N_{ij} = 1.$$

Let $\left\{c_{z,ij}\right\}_{z,i}$ be the be the solution of the good allocation problem in the definition of $V_j\left(\left\{c_{ij}\right\}\right)$ in (70). Thus,

$$\sum_i \tau_{ij}p_i\sum_z \alpha^N_{z,ij}c_{z,ij} = \sum_i \tau_{ij}p_ic_{ij} \leq 1$$

and, by revealed preference,

$$V_j\left(\left\{c^N_{ij}\right\}\right) \geq V^N\left(\left\{c^N_{z,ij}\right\}_{z,i}\right) \geq V^N\left(\left\{c_{z,ij}\right\}_{z,i}\right) = V_j\left(\left\{c_{ij}\right\}\right).$$

This establishes the contradiction. Since we have found preferences and technology that imply the mappings $\left(\Phi_j(.), \Psi_j(.), X^N_{ij}(.)\right)$, we have proven the equivalence.

D.2.2 New Trade Theory

Environment. The utility function is as in the Generalized Spatial Model. We assume that C_j has a nested preference structure across sectors, $C^N_j = V^N_j\left(\left\{C_{k,j}\right\}_k\right)$ with $V^N_j(\cdot)$ strictly quasi-concave and homogeneous of degree one. Sectors are divided into two groups: competitive sectors, $k \in K^{N_C}$, and monopolistic competitive sectors, $k \in K^{N_M}$.

In any competitive sector $k \in K^{N_C}$, firms in each country produce one homogeneous good with the production technology in (4). In particular, assume that technology is subject to external economies of scale with the marginal production cost given by $\zeta_{kr}\Psi^{N_C}_{kr}\left(\left\{L_j\right\}_j\right)$. Let $C^{N_C}_{k,j}$ be an aggregator of goods from different origins r, $C^{N_C}_{k,j} \equiv V^{N_C}_{k,j}\left(\left\{c_{kr,j}\right\}_r\right)$, where $V^{N_C}_{k,j}(\cdot)$ is twice differentiable, increasing, quasi-concave, and homogeneous of degree one. Notice that the utility function allows the goods produced in different regions to be perfect substitutes and, therefore, it covers homogeneous goods.

In any sector $k \in K^{N_M}$, there is large mass of potential entrants in each region that produce a differentiated good, indexed by z, and operate in monopolistic competition. We assume that all potential entrants in sector-region (k, r) have access to the same increasing returns technology where, in terms of labor, the fixed entry cost is $\mu_{kr} \Psi_{kr}^{N_E} \left(\{L_j\}_j \right)$ and the marginal production cost is $\zeta_{kr} \cdot \Psi_{kr}^{N_P} \left(\{L_j\}_j \right)$. We explicitly allow $\Psi_{kr}^{N_P}(\cdot)$ and $\Psi_{kr}^{N_E}(\cdot)$ to depend on employment, but we assume that firms perceive them as given. So, these function incorporate external agglomeration and congestion forces at the market level.

We also assume that, for $k \in K^{N_M}$, preferences are CES across the available differentiated goods with elasticity $\sigma > 1$:

$$C_{k,j}^N = \left[\int_{z \in Z_{k,j}} (c(z))^{\frac{\sigma}{\sigma-1}} \, dz \right]^{\frac{\sigma-1}{\sigma}},$$

where $Z_{k,j}$ is the set of goods in sector $k \in K^{N_M}$ available in market j.

Equilibrium. As in the Generalized Spatial Model, the representative household's problem yields the labor supply in region-sector j,

$$L_j^N \in \Phi_j \left(\{\omega_i^N\}_i \right). \tag{73}$$

Consider now a competitive sector $k \in K^{N_C}$. Cost minimization implies that

$$p_{kr,j}^N = \tau_{kr,j} \frac{w_{kr}^N}{\Psi_{kr}^{N_C} \left(\left\{ L_j^N \right\}_j \right)} \tag{74}$$

For the monopolistic competitive sector $k \in K^{N_M}$, all firms in region r choose the same price:

$$p_{kr,j}^N = \tau_{kr,j} \frac{\sigma}{\sigma-1} \frac{w_{kr}^N}{\Psi_{kr}^{N_P} \left(\left\{ L_j^N \right\}_j \right)}. \tag{75}$$

We now characterize the mass of operating firms, M_{kr}. The labor market clearing and the free entry conditions in (k, r) imply

$$M_{kr} = \frac{1}{\sigma \mu_{kr}} \cdot \frac{L_{kr}^N}{\Psi_{kr}^{N_E} \left(\left\{ L_j^N \right\}_j \right)}.$$

Thus, in the monopolistic competitive sector $k \in K^{N_M}$, we can express prices as

$$p_{kr,j}^N = \tau_{kr,j} \frac{w_{kr}^N}{\Psi_{kr}^{N_M} \left(\left\{ L_j^N \right\}_j \right)} \tag{76}$$

with

$$\Psi_{kr}^{N_M} \left\{ L_j^N \right\}_j \equiv \frac{\sigma-1}{\sigma} \Psi_{kr}^{N_P} \left(\left\{ L_j^N \right\}_j \right) \left(\frac{1}{\sigma \mu_{kr}} \cdot \frac{L_{kr}^N}{\Psi_{kr}^{N_E} \left(\left\{ L_j^N \right\}_j \right)} \right)^{\frac{1}{1-\sigma}}. \tag{77}$$

Using these expressions, it is straightforward to show that the labor market clearing condition in sector k of region r, $i=(k,r)$, is

$$w_{kr}^N L_{kr}^N = \sum_j x_{kr,j}^N w_j^N L_j^N. \tag{78}$$

Equivalence. We now construct an equivalent Generalized Spatial Model. To establish the equivalence, we need to set $\Psi_{kr}(\cdot) = \Psi_{kr}^{N_M}(\cdot)$ for $k \in K^{N_M}$ and $\Psi_{kr}(\cdot) = \Psi_{kr}^{N_C}(\cdot)$ for $k \in K^{N_C}$. We also need to specify sector-level preferences such that $V_{k,j}\left(\{c_{kr,j}\}_r\right) = V_{k,j}^{N_C}\left(\{c_{kr,j}\}_r\right)$ for $k \in K^{N_C}$ and $V_{k,j}\left(\{c_{kr,j}\}_r\right) = \left[\sum_r \left(c_{kr,j}\right)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{\sigma-1}{\sigma}}$ for $k \in K^{N_M}$. In addition, we must specify the same upper-level consumption aggregator across sectors: $V_j\left(\{c_{kr,j}\}_{k,r}\right) \equiv V_j^N\left(\left\{V_{k,j}\left(\{c_{kr,j}\}_r\right)\right\}_k\right)$.

D.2.3 New Economic Geography

Environment. For the next equivalence result we consider an economy with production structure and preference for goods identical to those in the New Trade Theory Economy of Section D.2.2. We assume that each country c is populated by a continuum of individuals with identical preferences for goods. These individuals differ in terms of mobility across markets. As in Krugman (1991), there are two groups of markets in each country, $J_c^{N_I}$ and $J_c^{N_M}$. Market $j \in J_c^{N_I}$ is populated by a subset of completely immobile individuals such that

$$L_j^N = \bar{L}_j \quad \forall j \in J_c^{N_I}, \tag{79}$$

In addition, there is a mass $\bar{L}_c$ of individuals that is completely mobile across markets $j \in J_c^{N_M}$ such that

$$\sum_{j \in J_c^{N_M}} L_j^N = \bar{L}_c. \tag{80}$$

Mobile individuals have identical preferences for being employed in any $j \in J_c^{N_M}$:

$$U_j\left(\omega_j^N, L_j^N\right) = \omega_j^N \left(L_j^N\right)^\beta.$$

where ω_j^N is the real wage in market j.

Equilibrium. We restrict attention to equilibria with positive employment in every $j \in J_c^{N_M}$, and analyze separately the cases of $\beta \neq 0$ and $\beta = 0$.

If $\beta = 0$, any employment allocation is feasible as long as $\nu_i \omega_i^N = \bar{u}$. Thus, the labor supply is

$$\left\{L_i^N\right\}_i = \Phi_c^N\left(\left\{\nu_i \omega_i^N\right\}_i\right) = \begin{cases} L_j^N = \bar{L}_c,\ L_i^N = 0 & \text{if } \ \omega_i^N > \omega_i^N \ \forall i \in J_c^{N_M} \\ \left\{L_j^N : \sum_j L_j^N = \bar{L}_c\right\} & \text{if } \ \omega_i^N = \bar{u} \ \forall i \in J_c^{N_M} \end{cases}. \tag{81}$$

If $\beta \neq 0$, in this case, any $j \in J_c^{N_M}$ with positive employment must have

$$\omega_j^N \left(L_j^N\right)^\beta = \bar{u} \quad \Longrightarrow L_j^N = \left(\frac{\bar{u}}{\omega_i^N}\right)^{\frac{1}{\beta}}$$

From equation (80),

$$L_j^N = \Phi_j^N\left(\left\{\omega_i^N\right\}_i\right) \equiv \bar{L}_c \frac{\left(\omega_j^N\right)^{-\frac{1}{\beta}}}{\sum_{i\in J_c^{N_M}}\left(\omega_i^N\right)^{-\frac{1}{\beta}}} \tag{82}$$

The equilibrium of this economy is $\left\{p_i^N, P_i^N, L_i^N, \omega_i^N\right\}$ solving (12)–(17) with $\Phi_j\left(\left\{\omega_i^N\right\}_i\right) = \bar{L}_j$ if $j \in J^{N_I}$ and $\Phi_j\left(\left\{\omega_i^N\right\}_i\right) = \Phi_j^N\left(\left\{\omega_i^N\right\}_i\right)$ if $j \in J^{N_M}$.

Equivalence. To establish equivalence, we construct preferences for the the representative household in the Generalized Spatial Model that yield the labor supply function $\Phi_j(\cdot) = \Phi_j^N(\cdot)$. Specifically, consider the following preferences:

$$U_c\left(\left\{C_j, L_j\right\}_j\right) = \begin{cases} \left[\sum_{j\in J_c^M}\left(C_j\right)\left(L_j\right)^{\beta}\right]^{\frac{1}{1+\beta}} & \text{if } \sum_{j\in J_c^M} L_j = \bar{L}_c \text{ and } L_i = \bar{L}_i \ \forall i \in J^I \\ -\infty & \text{otherwise} \end{cases}$$

Since the budget constraint implies that $C_j = \omega_j L_j$, the labor supply function is the solution of

$$\left\{\Phi_j\left(\left\{\omega_i\right\}_i\right)\right\}_j = \arg\max_{\{L_j\}}\left[\sum_{j\in J_c^M}\left(\omega_j\right)\left(L_j\right)^{1+\beta}\right]^{\frac{1}{1+\beta}} \quad \text{s.t.} \sum_{j\in J_c^M} L_j = \bar{L}_c.$$

If $\beta = 0$, it is straightforward to see that the solution of the utility maximization problem yields equation (81). If $\beta \neq 0$, the solution of the maximization problem is the same as equation (82). Since we have assumed a production structure and preferences for goods identical to those in the New Trade Theory Economy, the assumptions on technology and consumption aggregator imposed in the previous section imply that the functions $X_{ij}(\cdot)$ and $\Psi_i(\cdot)$ deliver the equivalence.

D.2.4 Spatial Assignment Models

Environment. Suppose that countries are populated by a continuum of individuals, $\iota \in I_c$, that are heterogeneous in terms of preferences and efficiency across markets (i.e, sector-region pairs). We assume individual ι has market specific preferences, $a_j(\iota)$, and market specific efficiency, $e_j(\iota)$. In particular, if employed in market j, we assume that individual ι has homothetic preferences given by

$$U_j(\iota) = a_j(\iota) + V_j^N\left(\left\{c_{ij}^N\right\}_j\right),$$

with a budget constraint given by

$$\sum_i p_{ij}^N c_{ij}^N = w_j^N e_j(\iota).$$

We further assume that individuals take independent draws of $(a_j(\iota), e_j(\iota))$ from a common distribution:

$$\left\{a_j(\iota), e_j(\iota)\right\}_j \sim F^N(\boldsymbol{a}, \boldsymbol{e}).$$

On the production side, we maintain the same structure of the Generalized Spatial Model. That is, there is a representative competitive firm in each market with the production technology in (4).

Equilibrium. We start by characterizing spending shares across markets. Conditional on choosing j, individuals choose spending shares that minimize total cost:

$$P_j^N\left(\left\{p_{oj}^N\right\}_j\right) \equiv \sum_o p_{oj}^N c_{oj}^N \text{ s.t. } V_j^N\left(\left\{c_{oj}^N\right\}_o\right) = 1, \tag{83}$$

with associated spending shares given by

$$x_{ij} \in X_{ij}^N\left(\left\{p_{oj}^N\right\}_o\right). \tag{84}$$

The solution of this problem implies that, for individual ι, the utility of being employed in j is $U_j(\iota) = a_j(\iota) + \omega_j^N e_j(\iota)$. Thus, the set of individuals choosing j is

$$I_j\left(\left\{\omega_i^N\right\}_i\right) \equiv \left\{(\boldsymbol{a}, \boldsymbol{e}):\ a_j + e_j\omega_j \geq a_i + e_i\omega_i\ \forall i\right\},$$

with the associated labor supply given by

$$L_j = \Phi_j^N\left(\left\{\omega_i^N\right\}_i\right) \equiv \int_{I_j\left(\left\{\omega_i^N\right\}_i\right)} e_j\ dF_c(\boldsymbol{a}, \boldsymbol{e}). \tag{85}$$

Notice that the function $\Phi_j^N(\cdot)$ is homogeneous of degree zero with $\frac{\partial \Phi_j^N}{\partial \omega_j} \geq 0$ and $\frac{\partial \Phi_j^N}{\partial \omega_i} \leq 0$.[46]

Profit maximization and labor market clearing are still given by (15)–(17). Thus, the equilibrium can be written as $\left\{p_i^N, P_i^N, L_i^N, \omega_i^N\right\}$ solving (12)–(17) with $\Psi_j(\cdot)$, $X_{ij}(\cdot) = X_{ij}^N(\cdot)$, and $\Phi_j(\cdot) = \Phi_j^N(\cdot)$.

Equivalence. To establish the equivalence, it is sufficient to show that there are preferences for the representative household of the Generalized Spatial Model that yield $\Phi_j(\cdot) = \Phi_j^N(\cdot)$ and $X_{ij}(\cdot) = X_{ij}^N(\cdot)$. Specifically, consider the following preferences:

$$C_j = V_j^N\left(\left\{c_{ij}\right\}_j\right),$$

and

$$U\left(\left\{C_j\right\}_j \left\{L_j\right\}_j\right) \equiv \max_{\left\{\left\{I_j(a,e)\right\}_j\right\}_{(a,e)}} \sum_j C_j + \int \sum_j a_j I_j(a,e)\ dF^N(a,e)$$

subject to

$$L_j = \int e_j I_j(a,e)\ dF^N(a,e)\ \forall j$$

$$\sum_j I_j(a,e) = 1\ \forall (a,e),$$

$$I_j(a,e) \geq 0\ \forall j,\ \forall (a,e).$$

It is straight forward to see that the first-stage problem in the Generalized Spatial Model yields

$$X_{ij}\left(\left\{p_{oj}\right\}_o\right) = X_{ij}^N\left(\left\{p_{oj}\right\}_o\right) \text{ and } P_j\left(\left\{p_{oj}\right\}_o\right) = P_j^N\left(\left\{p_{oj}\right\}_o\right).$$

[46] The homogeneity of $\Phi_j^N(\cdot)$ follows immediately from the definition of I_j. To see that $\frac{\partial \Phi_j^N}{\partial \omega_j} \geq 0$ and $\frac{\partial \Phi_j^N}{\partial \omega_i} \leq 0$, notice that $I_i\left(\tilde{\boldsymbol{\omega}}_c\right) \subset I_i\left(\boldsymbol{\omega}_c\right)$ and $I_j\left(\boldsymbol{\omega}_c\right) \subset I_j\left(\tilde{\boldsymbol{\omega}}_c\right)$ whenever $\tilde{\omega}_j > \omega_j$ and $\tilde{\omega}_i = \omega_i$.

Also, the second-stage problem in the Generalized Spatial Model yields a labor supply function that solves

$$\{\Phi_j(\{\omega_i\}_i)\}_j = \arg\max_{\{L_j\}_j} U\left(\{\omega_j L_j\}_j \{L_j\}_j\right).$$

Using the definition above, the solution of this problem is

$$\Phi_j(\{\omega_i\}_i) = \int e_j I_j^*(a,e)\ dF^N(a,e)\ \forall j$$

where

$$\left\{\{I_j^*(a,e)\}_j\right\}_{(a,e)} \equiv \arg \max_{\left\{\{I_j(a,e)\}_j\right\}_{(a,e)}} \int \sum_j (a_j + \omega_j e_j)\, I_j(a,e)\ dF^N(a,e)$$

subject to

$$\forall (a,e):\ \sum_j I_j(a,e) = 1,\ \text{and}\ I_j(a,e) \geq 0.$$

To solve this problem, we substitute the first constraint into the objective function to eliminate $I_o(a,e)$ for an arbitrary o. Then, we consider the problem's Lagrangian:

$$\max_{\{I_j(a,e)\geq 0\}_{j\neq o}} \int (a_o + \omega_o e_o)\ dF(a,e) + \int \sum_j (a_j + \omega_j e_j - a_o - \omega_o e_o)\, I_j(a,e)\ dF^N(a,e).$$

The first-order condition of this problem implies that, for all $j \neq o$, $I_j(a,e) = 0$ if $a_o + \omega_o e_o > a_j + \omega_j e_j$. Thus, $I_o(a,e) = 1$ if, and only if, $a_o + \omega_o e_o \geq a_j + \omega_j e_j$. Since o was arbitrarily chosen, we can write

$$\forall i:\ I_j^*(a,e) = 1\ \Leftrightarrow\ (a,e) \in I_j(\{\omega_i\}_i) \equiv \{(\boldsymbol{a},\boldsymbol{e}):\ a_j + \omega_j e_j \geq a_o + \omega_o e_o\ \forall o\}.$$

Thus, the system of labor supply constraints implies that

$$\Phi_j(\{\omega_i\}_i) = \int_{I_j(\{\omega_i\}_i)} e_j\ dF^N(a,e),$$

and, therefore,

$$\Phi_j(\{\omega_i\}_i) = \Phi_j^N(\{\omega_i\}_i).$$

D.2.5 Spatial Assignment Models with Other Factors in Production

Environment. Consider an economy with a representative household with the preferences in (1)–(10) subject to the budget constraint in (11). We denote an origin sector-region pair as $i \equiv (k,r)$ and a destination sector-region pair as $j \equiv (s,d)$. We impose additional restrictions on preferences to obtain the equivalence result. First, assume that individuals employed in all sectors of region r have identical preferences, $V_{sd}(\cdot) = V_d(\cdot)$, and face identical prices, $p_{kr,sd} = p_{kr,d}$. Second, assume that preferences are such that the labor supply function is invertible (up to a scalar).

In sector-region pair, there is a representative competitive firm that uses labor, L_{kr}^N, and another factor, T_{kr}^N, in production, with the following Cobb-Douglas production function:

$$Y_{kr}^N = \tilde{\zeta}_{kr} \tilde{\Psi}_{kr}^N\left(\left\{L_{sd}^N\right\}_{sd}\right)\left(L_{kr}^N\right)^{\alpha_{kr}^N}\left(T_{kr}^N\right)^{1-\alpha_{kr}^N}. \tag{86}$$

Each region r has an endowment of the other factor, $\bar{T}_r^N$. We assume that the other factor is mobile across sectors within a region, but that it is immobile across regions – like land in spatial models. Similar to Caliendo et al. (2018b), there is a national mutual fund that owns the other factor in all regions. We assume that the local government in region r owns a share κ_r of the national fund, and it transfers all dividends to local residents. In particular, we impose that the per-capita transfer rate to individuals employed in sector k of region r, ρ_{kr}^N, is inversely proportional to the share of labor in the total cost of the sector,

$$\rho_{kr}^N = \rho_r^N / \alpha_{kr}^N. \tag{87}$$

Equilibrium. To characterize the equilibrium, it is useful to work with the adjusted wage rate, $\tilde{w}_{kr}^N \equiv w_{kr}^N / \alpha_{kr}^N$. The representative household's cost minimization problem yields spending share and price indices that are given by, for all s,

$$x_{kr,sd}^N \in X_{kr,sd}\left(\left\{p_{kr,sd}^N\right\}_{kr}\right) = X_{kr,d}\left(\left\{p_{kr,d}^N\right\}_{kr}\right) \quad \text{and} \quad P_{sd}^N = P_{sd}\left(\left\{p_{kr,sd}^N\right\}_{kr}\right) = P_d\left(\left\{p_{kr,dd}^N\right\}_{kr}\right). \tag{88}$$

As in Section (3), the utility maximization problem of the representative household yields the labor supply function. Using the transfer rule in (87), the labor supply in j is

$$L_{sd} \in \Phi_{sd}\left(\left\{\rho_r^N \tilde{\omega}_{kr}^N\right\}_{kr}\right). \tag{89}$$

Thus, the optimization of consumption and labor choice is corresponds directly to the one of the Generalized Economy with transfers such that the budget constraint in market j is $\sum_{kr} c_{kr,sd} p_{kr,sd} = \rho_d^N w_{sd} L_{sd}$.

In addition, the profit maximization problem of firms implies that

$$p_{kr,sd}^N = \tau_{kr,sd} p_{kr}^N$$

where

$$p_{kr}^N = \frac{\tilde{w}_{kr}^N}{\zeta_{kr} \tilde{\Psi}_{kr}^N\left(\left\{L_{sd}^N\right\}_{sd}\right)} \cdot \left(\frac{R_{kr}^N}{\tilde{w}_{kr}^N}\right)^{1-\alpha_{kr}^N}$$

where R_{kr}^N is the price of other factor faced by the producer in sector k of region r, and, abusing notation, $\zeta_{kr} \equiv \tilde{\zeta}_{kr}(1-\alpha_{kr}^N)^{(1-\alpha_{kr}^N)}$.

To obtain the equilibrium level of R_{kr}^N, consider the market clearing condition for the other factor in region r: $\bar{T}_r^N = \sum_k T_{kr}^N = \sum_k \left(1-\alpha_{kr}^N\right) \tilde{w}_{kr}^N L_{kr}^N / R_{kr}^N$. Since the other factor is perfectly mobile across sectors, $R_{kr}^N = R_r^N$ for all k and, therefore,

$$R_r^N = \frac{\sum_k \left(1-\alpha_{kr}^N\right) \tilde{w}_{kr}^N L_{kr}^N}{\bar{T}_r^N}$$

We use this expression to eliminate R_{kr}^N in the expression of $p_{kr,kr}^N$ for sector k in region r. After some manipulation, we obtain

$$p_{kr,kr}^N = \frac{\tilde{w}_{kr}^N}{\tilde{\Psi}_{kr}^N\left(\left\{L_{sd}^N\right\}_{sd}\right)} \left(\frac{1}{\bar{T}_r^N} \sum_s \left(1-\alpha_{sr}^N\right) \frac{\rho_r^N \tilde{\omega}_{sr}^N}{\rho_r^N \tilde{\omega}_{kr}^N} L_{sr}^N\right)^{1-\alpha_{kr}^N}$$

Thus, the invertibility of the labor supply function yields

$$p_{kr}^{N} = \frac{\tilde{w}_{kr}^{N}}{\Psi_{kr}^{N}\left(\left\{L_{sd}^{N}\right\}_{sd}\right)} \tag{90}$$

with

$$\Psi_{kr}^{N}\left(\left\{L_{sd}^{N}\right\}_{sd}\right) \equiv \tilde{\Psi}_{kr}^{N}\left(\left\{L_{sd}^{N}\right\}_{sd}\right)\left(\frac{1}{\bar{T}_{r}^{N}}\sum_{s}\left(1-\alpha_{sr}\right)\Phi_{kr,sr}^{-1}\left(\left\{L_{sd}^{N}\right\}_{sd}\right)L_{sr}^{N}\right)^{\alpha_i - 1}. \tag{91}$$

where we used invertibility of labor supply up to a scalar to write

$$\frac{\rho_{r}^{N}\tilde{\omega}_{sr}^{N}}{\rho_{r}^{N}\tilde{\omega}_{kr}^{N}} = \Phi_{kr,sr}^{-1}\left(\left\{L_{sd}^{N}\right\}_{sd}\right).$$

To close the equilibrium, we consider the labor market clearing condition that can be written in terms of the revenue share accruing to labor in every sector-region pair:

$$\tilde{w}_{kr}^{N}L_{kr}^{N} = \sum_{sd} x_{kr,sd}^{N}\rho_{d}^{N}\tilde{w}_{sd}^{N}L_{sd}^{N}. \tag{92}$$

Finally, the transfer rate in region r is determined by its share in the dividend paid by the mutual fund:

$$\kappa_r \sum_{sd}(1-\alpha_{sd}^{N})\tilde{w}_{sd}^{N}L_{sd}^{N} = \sum_{k}(\rho_{kr}^{N}-1)\alpha_{kr}\tilde{w}_{kr}^{N}L_{kr}^{N} = \sum_{k}(\rho_{r}^{N}-\alpha_{kr})\tilde{w}_{kr}^{N}L_{kr}^{N}$$

$$\rho_{r}^{N} = \frac{\kappa_r \sum_{sd}(1-\alpha_{sd}^{N})\tilde{w}_{sd}^{N}L_{sd}^{N} + \sum_{k}\alpha_{kr}\tilde{w}_{kr}^{N}L_{kr}^{N}}{\sum_{k}\tilde{w}_{kr}^{N}L_{kr}^{N}} \tag{93}$$

where the left hand side is region r's total transfer payments, and the right hand side is region r's share in the total land revenue in the country.

The equilibrium of this economy is characterized by $\left\{p_i^N, P_i^N, L_i^N, \omega_i^N\right\}$ that solve equations (88)–(92), with $\left(\Phi_j(\cdot), \Psi_j^N(\cdot), X_{ij}(\cdot)\right)$, conditional on the transfer rule $\{\rho_r\}$ in (93).

Equivalence. To establish the equivalence, we consider the Generalized Spatial Model of Section 3, with $\Psi_{kr}(\cdot) = \Psi_{kr}^{N}(\cdot)$ in (91) and the transfer rule in (93). This establishes that the Generalized Spatial Model is equivalent to spatial assignment models with other factors of production that are mobile across sectors but not across regions – e.g., land and other natural resources. A similar argument yields the equivalence with models with other factors of production that are mobile across both regions and sectors. The only restriction is that the invertibility step to obtain (91) requires the same transfer rate across markets in the country, as in Caliendo et al. (2018b).

D.2.6 Special Case with Mobile Capital

Environment. Consider the simplified economy of Section 2. Assume that assume that preferences are such that the labor supply function is invertible (up to a scalar), so that we can write

$$\frac{w_j}{w_i} = \Phi_{i,j}^{-1}\left(\boldsymbol{L}\right). \tag{94}$$

We introduce capital by assuming that the production function takes the following Cobb-Douglas form:

$$Y_i = \frac{1}{\kappa_i}\tau_i \left(L_i\right)^{\alpha_i} \left(K_i\right)^{1-\alpha_i},$$

where $\kappa_i \equiv \alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i}$.

Assume that capital is fully mobile across regions, so that rent is identical in all regions: $R_i = R$ for all i. There is an exogenous capital endowment in the economy given by $\bar{K}$.

Equilibrium. The cost minimization problem of the firm and the zero profit conditions imply that, in every region i,

$$p_i = \frac{w_i}{\tau_i}\left(\frac{R}{w_i}\right)^{1-\alpha_i}. \tag{95}$$

In this economy, capital market clearing condition requires $R\bar{K} = \sum_i RK_i$. Using the fact that firms spend a share $1-\alpha_i$ of their revenue on capital, we get the following expression for the rent in equilibrium:

$$R = \frac{1}{\bar{K}}\sum_j \frac{1-\alpha_j}{\alpha_j} w_j L_j.$$

Substituting this expression into (95),

$$p_i = \frac{w_i}{\tau_i}\left(\frac{1}{\bar{K}}\sum_j \frac{1-\alpha_j}{\alpha_j}\frac{w_j}{w_i}L_j\right)^{1-\alpha_i},$$

which combined with the inverse labor supply in (94) yields

$$p_i = \frac{w_i}{\tau_i}\left(\frac{1}{\bar{K}}\sum_j \frac{1-\alpha_j}{\alpha_j}\Phi_{i,j}^{-1}\left(\boldsymbol{L}\right)L_j\right)^{1-\alpha_i}.$$

Equivalence. We establish the equivalence with the model of Section 2 by specifying

$$\Psi_i\left(\boldsymbol{L}\right) \equiv \left(\frac{1}{\bar{K}}\sum_j \frac{1-\alpha_j}{\alpha_j}\Phi_{i,j}^{-1}\left(\boldsymbol{L}\right)L_j\right)^{\alpha_i-1}. \tag{96}$$

An illustrative example. To gain intuition for the labor productivity spillovers implied by factor mobility, consider the special case of a gravity labor supply structure: $\Phi_i\left(\boldsymbol{\omega}\right) = \frac{\omega_i^{\phi}}{\sum_j \omega_j^{\phi}}$ such that $\frac{w_j}{w_i} = \left(\frac{L_j}{L_i}\right)^{\frac{1}{\phi}}$. In this case,

$$\Psi_i\left(\boldsymbol{L}\right) = \left[\frac{\bar{K}\left(L_i\right)^{\frac{1}{\phi}}}{\sum_j \frac{1-\alpha_j}{\alpha_j}\left(L_j\right)^{1+\frac{1}{\phi}}}\right]^{1-\alpha_i}$$

Thus, for $i \neq j$, the elasticity of labor productivity in market i to employment in market j is

$$\psi_{ij} \equiv \frac{\partial \log \Psi_i (\boldsymbol{L})}{\partial \log L_j} = -(1-\alpha_i) \frac{\frac{1-\alpha_j}{\alpha_j} (L_j)^{1+\frac{1}{\phi}}}{\sum_{j'} \frac{1-\alpha_{j'}}{\alpha_{j'}} (L_{j'})^{1+\frac{1}{\phi}}} \left(1 + \frac{1}{\phi}\right) < 0.$$

Intuitively, since the labor-to-capital spending ration is constant, higher employment in market j triggers an increase in the capital demand in market j, which causes rent prices to increase in the entire economy. The higher capital cost increases the production cost everywhere and, therefore, acts as a congestion on other markets.

D.3 Extensions

D.3.1 Generalized Spatial Model with Multiple Labor Types

Multiple Worker Types. Consider an extension of our model with multiple worker groups – groups are indexed by g and g'. We write the equilibrium in terms of factor-content of trade as in Adao, Costinot, and Donaldson (2017). Each market now is defined as a triple of sector-region-group. We denote origin markets as $i \equiv (k, r, g)$, and destination markets as $j \equiv (s, d, g')$. As before, the representative consumer has preferences over consumption and labor across markets (i.e., sector-region-group markers):

$$U_c \left(\{C_j\}, \{L_j\}_j \right).$$

We assume that the consumption index depends on the factor content of trade from different sectors and regions. That is, the consumption index depends directly on a composite good produced by each sector-region-group triple:

$$C_j = V_j \left(\{c_{ij}\}_i \right)$$

Finally, assume that there is a competitive firm producing the market-level composite good with production function given by

$$Y_i = \Psi_i \left(\{L_o\}_o \right) L_i.$$

All our results remain valid in this environment with spending shares in terms of factor content of trade. That is, x_{ij} is the spending share on the composite good produced in the sector-region-group triple.

Equivalent Armington Economy Multiple Worker Types. To gain intuition for this economy, we now derive preferences in terms of factor content of trade in the case of an Armington economy with multiple labor types. Assume that the representative household has preferences over the allocation of the multiple worker groups across sector-region pairs j,

$$U_c \left(\left\{ C_j^g \right\}_{j,g}, \left\{ L_j^g \right\}_{j,g} \right).$$

The consumption index is a function of the quantities consumed of goods produced in different origin sector-region pairs i:

$$C_j^g = \tilde{V}_j \left(\left\{ c_{ij}^g \right\}_i \right).$$

Assume that each sector-region pair i has a representative firm that combines labor from different worker types with a constant returns to scale technology:

$$Y_i = F_i\left(\{\Psi_i^g\left(\boldsymbol{L}\right)L_i^g\}_g\right)$$

where $F_{ij}(.)$ is homogeneous of degree one.

Thus, as in the equivalence with the Ricardian economy above, we must define preferences of the representative agent that incorporate the technology to produce final goods,

$$C_j = V_j\left(\left\{c_{ij}^g\right\}_{i,g}\right) \equiv \tilde{V}_j\left(\left\{F_i\left(\left\{c_{ij}^g\right\}_g\right)\right\}_i\right),$$

where c_{ij}^g is the amount of "effective" labor of group g in market i used in the production of goods shipped to market j.

let the production technology of "effective" labor of group g in market i be

$$c_i^g = \Psi_i^g\left(\boldsymbol{L}\right)L_i^g.$$

In equilibrium, the production cost of "effective" labor of group g in market i

$$p_i^g = \frac{w_i^g}{\Psi_i^g\left(\boldsymbol{L}\right)}$$

In this case, the spending share on factor g in sector-region pair i is simply

$$x_{ij}^g = \alpha_i^g\left(\{p_i^g\}_g\right)x_{ij}$$

where $\alpha_i^g\left(\{p_i^g\}_g\right)$ is the share of factor g in the production cost of sector-region pair i, and x_{ij} is the spending share on goods from sector-region pair i.

D.3.2 Generalized Spatial Model with Intermediate Goods in Production

We now derive the decomposition between direct and indirect effects in a model with input-output linkages.

Preferences. On the consumption side, we maintain the same structure of Section 3, in which the representative household preferences yield a market-level price index of

$$P_j^C = P_j^C\left(\boldsymbol{p}_j\right) \equiv \min_{\boldsymbol{c}_j}\sum_o p_{oj}c_{oj} \quad \text{s.t.} \quad V_j\left(\boldsymbol{c}_j\right) = 1, \tag{97}$$

with the associated final spending share on goods from origin i given by

$$x_{ij}^C = X_{ij}^C\left(\boldsymbol{p}_j\right). \tag{98}$$

Also, the utility maximization problem of the representative agent yields the labor supply in any market j:

$$L_j = \Phi_j\left(\boldsymbol{\omega}\right). \tag{99}$$

We also maintain the assumption of iceberg trade costs such that

$$p_{ij} = \tau_{ij}p_i \tag{100}$$

Production. The main change is on the production function, which we assume to take the following Cobb-Douglas form between labor and an intermediate input aggregator:

$$Y_i = \frac{1}{\kappa_i} \Psi_i (\boldsymbol{L}) (L_i)^{\varpi_i} (M_i)^{1-\varpi_i} ,$$

where $\kappa_i = \varpi_i^{\varpi_i} (1-\varpi_i)^{1-\varpi_i}$, and M_i is an index of intermediate inputs used in production:

$$M_i = F_i \left(\{M_{ji}\}_j \right).$$

In this case, the cost minimization problem of the representative firm implies that the zero profit condition is

$$p_i = \frac{(w_i)^{\varpi_i} \left(P_i^M\right)^{1-\varpi_i}}{\Psi_i (\boldsymbol{L})} \tag{101}$$

where

$$P_i^M = P_i^M (\boldsymbol{p}_i) \equiv \min_{\boldsymbol{M}_i} \sum_o p_{ji} M_{ji} \quad \text{s.t.} \quad F_i \left(\{M_{ji}\}_j \right) = 1 \tag{102}$$

with associated input spending shares given by

$$x_{ji}^M = X_{ji}^M \left(\{p_{ji}\}_j \right) \equiv \frac{\partial \ln P_i^M}{\partial \ln p_{ji}}. \tag{103}$$

Market clearing. To close the model, consider the market clearing condition for labor in each market. The total revenue of market i from sales in market j is

$$X_{ij} = x_{ij}^C w_j L_j + x_{ij}^M (1-\varpi_j) \sum_d X_{jd}$$

$$X_{ij} = x_{ij}^C w_j L_j + x_{ij}^M \frac{1-\varpi_j}{\varpi_j} w_j L_j$$

$$X_{ij} = \left(x_{ij}^C + x_{ij}^M \frac{1-\varpi_j}{\varpi_j} \right) w_j L_j$$

Thus,

$$\frac{1}{\varpi_i} w_i L_i = \sum_j \left(\varpi_j x_{ij}^C + x_{ij}^M (1-\varpi_j) \right) \frac{1}{\varpi_j} w_j L_j. \tag{104}$$

Equilibrium. The equilibrium entails $\{w_i, P_i, L_i, p_i\}$ that satisfy (97)–(104) given $p_m \equiv 1$ for a reference market.

There are two points that are worth mentioning. The equilibrium requires knowledge of the labor share, ϖ_i, and the cost function, $F_i(.)$ (which determines the producer price index $P_i^M(\cdot)$ and the intermediate spending shares $\pi_{ij}(\cdot)$). Second, this environment is a generalization of the model in Caliendo and Parro (2014) that imposes a gravity structure on the demand for final products, $X_{ij}(\cdot)$, and intermediate products, $X_{ij}^M(\cdot)$. In particular, their model assumes that final and intermediate consumption is identical within each sector, but have different sector-level spending shares.

To write the labor and the trade modules, we combine first equations (99) and (101):

$$\log \omega_i = \frac{1}{\varpi_i} \log Q_i + \frac{1}{\varpi_i} \log \Psi_i (\boldsymbol{\Phi}(\boldsymbol{\omega})), \tag{105}$$

where

$$Q_i = \frac{p_i}{\left(P_i^C\right)^{\varpi_i}\left(P_i^M\right)^{1-\varpi_i}} \tag{106}$$

with P_i^C given by (97) and P_i^M given by (102).

The trade module follows from the combination of (101) and (104):

$$\left[\frac{p_i \Psi_i\left(\boldsymbol{\Phi}\left(\boldsymbol{\omega}\right)\right)}{\left(P_i^M\right)^{1-\varpi_i}}\right]^{\frac{1}{\varpi_i}} \frac{\Phi_i(\boldsymbol{\omega})}{\varpi_i} = \sum_j X_{ij}\left(\{p_{oj}\}_o\right)\left[\frac{p_j \Psi_{ij}\left(\boldsymbol{\Phi}\left(\boldsymbol{\omega}\right)\right)}{\left(P_j^M\right)^{1-\varpi_j}}\right]^{\frac{1}{\varpi_i}} \frac{\Phi_j(\boldsymbol{\omega})}{\varpi_j} \tag{107}$$

with P_i^C given by (97), P_i^M given by (102), and

$$X_{ij}\left(\{p_{oj}\}_o\right) \equiv X_{ij}^C\left(\{p_{oj}\}_o\right)\varpi_j + X_{ij}^M\left(\{p_{oj}\}_o\right)(1-\varpi_j)$$

Decomposition of direct and indirect effects. In terms of the modified competitiveness measure, we have the same labor market module equation:

$$\log\hat{\boldsymbol{\omega}} = \bar{\boldsymbol{\beta}}\log\hat{\boldsymbol{Q}} \tag{108}$$

with $\bar{\boldsymbol{\beta}} = \left(\bar{\boldsymbol{\varpi}} - \bar{\boldsymbol{\psi}}\bar{\boldsymbol{\phi}}\right)^{-1}$ and $\bar{\boldsymbol{\varpi}}$ is a diagonal matrix with entries ϖ_i.

We also have that

$$\log\hat{\boldsymbol{Q}} = \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\varpi}}\bar{\boldsymbol{x}}^C - \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\varpi}}\right)\bar{\boldsymbol{x}}^M\right)\log\hat{\boldsymbol{p}} - \bar{\boldsymbol{\varpi}}\log\boldsymbol{\eta}^C(\hat{\boldsymbol{\tau}}) - \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\varpi}}\right)\log\boldsymbol{\eta}^M(\hat{\boldsymbol{\tau}}) \tag{109}$$

where the consumption and the production cost exposure are given by

$$\log\hat{\eta}_i^C(\hat{\boldsymbol{\tau}}) \equiv \sum_j x_{ji}^C \log\hat{\tau}_{ji}, \tag{110}$$

$$\log\hat{\eta}_i^M(\hat{\boldsymbol{\tau}}) \equiv \sum_j x_{ji}^M \log\hat{\tau}_{ji}. \tag{111}$$

Notice that, if the production and the consumption shares are the same $x_{ji} = x_{ji}^M = x_{ji}^C$, then $\log\hat{\eta}_i^C(\hat{\boldsymbol{\tau}}) = \log\hat{\eta}_i^M(\hat{\boldsymbol{\tau}})$.

Define $\chi_{oij} \equiv \frac{\partial\log X_{ij}(\{p_{oj}\}_o)}{\partial\log p_{oj}}$, with the associate matrix $\bar{\boldsymbol{\chi}} \equiv [\sum_d y_{id}\chi_{jid}]_{i,j}$. As before, we define the revenue exposure as

$$\log\hat{\eta}_i^R(\hat{\boldsymbol{\tau}}) \equiv \sum_j\sum_o y_{ij}^0 \chi_{oij}\log\hat{\tau}_{oj}. \tag{112}$$

Thus, the trade module yields

$$\left[\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}} - \bar{\boldsymbol{\chi}}\bar{\boldsymbol{\varpi}}\right]\bar{\boldsymbol{\varpi}}^{-1}\log\hat{\boldsymbol{p}} + \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}}\right)\left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{\varpi}}^{-1}\bar{\boldsymbol{\psi}}\right)\bar{\boldsymbol{\phi}}\log\hat{\boldsymbol{\omega}} = \log\boldsymbol{\eta}^R(\hat{\boldsymbol{\tau}}) + \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}}\right)\bar{\boldsymbol{\varpi}}^{-1}\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\varpi}}\right)\log\hat{\boldsymbol{P}}^M$$

Let us define

$$\bar{\boldsymbol{x}} \equiv \bar{\boldsymbol{\varpi}}\bar{\boldsymbol{x}}^C + \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\varpi}}\right)\bar{\boldsymbol{x}}^M$$

$$\bar{\boldsymbol{\mu}}^M \equiv \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}}\right)\bar{\boldsymbol{\varpi}}^{-1}$$

$$\bar{\boldsymbol{\mu}} \equiv \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}}\right)\left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{\varpi}}^{-1}\bar{\boldsymbol{\psi}}\right)\bar{\boldsymbol{\phi}}$$

$$\bar{\boldsymbol{\gamma}} \equiv \left[\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}} - \bar{\boldsymbol{\chi}}\bar{\boldsymbol{\varpi}} + \bar{\boldsymbol{\mu}}\bar{\boldsymbol{\beta}}\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}\right)\bar{\boldsymbol{\varpi}} - \bar{\boldsymbol{\mu}}^M\bar{\boldsymbol{x}}^M\right]\bar{\boldsymbol{\varpi}}^{-1}$$

By substituting (108) and (109) into the expression above, we obtain

$$\bar{\boldsymbol{\gamma}}\log\hat{\boldsymbol{p}} = \log\boldsymbol{\eta}^R(\hat{\boldsymbol{\tau}}) + \bar{\boldsymbol{\mu}}\bar{\boldsymbol{\beta}}\bar{\boldsymbol{\varpi}}\log\boldsymbol{\eta}^C(\hat{\boldsymbol{\tau}}) + \left(\bar{\boldsymbol{\mu}}^M + \bar{\boldsymbol{\mu}}\bar{\boldsymbol{\beta}}\right)\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\varpi}}\right)\log\boldsymbol{\eta}^M(\hat{\boldsymbol{\tau}}).$$

Applying this expression into (109),

$$\log\hat{\boldsymbol{Q}} = \bar{\boldsymbol{\alpha}}^R\log\boldsymbol{\eta}^R(\hat{\boldsymbol{\tau}}) - \bar{\boldsymbol{\alpha}}^C\bar{\boldsymbol{\varpi}}\log\boldsymbol{\eta}^C(\hat{\boldsymbol{\tau}}) - \bar{\boldsymbol{\alpha}}^M\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\varpi}}\right)\log\boldsymbol{\eta}^M(\hat{\boldsymbol{\tau}}) \tag{113}$$

where $\bar{\boldsymbol{\alpha}} \equiv \bar{\boldsymbol{M}}'\left(\bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}'\right)^{-1}\bar{\boldsymbol{M}}$, $\bar{\boldsymbol{\alpha}}^R \equiv \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}\right)\bar{\boldsymbol{\alpha}}$, $\bar{\boldsymbol{\alpha}}^C \equiv \bar{\boldsymbol{I}} - \bar{\boldsymbol{\alpha}}^R\bar{\boldsymbol{\mu}}\bar{\boldsymbol{\beta}}$, and $\bar{\boldsymbol{\alpha}}^M \equiv \bar{\boldsymbol{\alpha}}^C - \bar{\boldsymbol{\alpha}}^R\bar{\boldsymbol{\mu}}^M$.

Thus, equations (108) and (113) yield

$$\log\hat{\boldsymbol{\omega}} = \bar{\boldsymbol{\beta}}\left[\bar{\boldsymbol{\alpha}}^R\log\boldsymbol{\eta}^R(\hat{\boldsymbol{\tau}}) - \bar{\boldsymbol{\alpha}}^C\bar{\boldsymbol{\varpi}}\log\boldsymbol{\eta}^C(\hat{\boldsymbol{\tau}},\hat{\boldsymbol{\zeta}}) - \bar{\boldsymbol{\alpha}}^M\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\varpi}}\right)\log\boldsymbol{\eta}^M(\hat{\boldsymbol{\tau}})\right] \tag{114}$$

Notice that if $x_{ji} = x_{ji}^M = x_{ji}^C$ then $\log\hat{\eta}_i^C(\hat{\boldsymbol{\tau}}) = \log\hat{\eta}_i^M(\hat{\boldsymbol{\tau}})$, as discussed above, so that then the relationship can be written as

$$\log\hat{\boldsymbol{\omega}} = \bar{\boldsymbol{\beta}}\left[\bar{\boldsymbol{\alpha}}^R\log\boldsymbol{\eta}^R(\hat{\boldsymbol{\tau}}) - \left[\bar{\boldsymbol{\alpha}}^C\bar{\boldsymbol{\varpi}} + \bar{\boldsymbol{\alpha}}^M\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\varpi}}\right)\right]\log\boldsymbol{\eta}^C(\hat{\boldsymbol{\tau}})\right].$$

Thus, under the assumption of $x_{ji} = x_{ji}^M = x_{ji}^C$, the model with intermediate inputs can generate the same counterfactuals as a model without intermediate inputs, as long as the elasticities of the models with and without the intermediates are set to be the same.

An illustrative example. To see this point more clearly, we consider a simple example that draws on the model of Section 2. In particular, we assume the presence of a single homogeneous good as in Section 2 such that the production function with intermediate goods is

$$Y_i = \tau_i\Psi_i\left(\boldsymbol{L}\right)\left(L_i\right)^{\varpi}\left(M_i\right)^{1-\varpi}.$$

The derivations above yield the following expression for the labor market module:

$$\log\boldsymbol{\omega} - \varpi^{-1}\log\boldsymbol{\Psi}\left(\boldsymbol{\Phi}\left(\boldsymbol{\omega}\right)\right) = \varpi^{-1}\log\boldsymbol{\tau}.$$

Consider the case of log-linear functions of agglomeration and labor supply: $\Phi_i\left(\boldsymbol{\omega}\right) = \omega_i^{\phi}$ and $\Psi_i(\boldsymbol{L}) = L_i^{\psi}$. Thus,

$$\log\omega_i = \frac{\varpi^{-1}}{1 - \varpi^{-1}\psi\phi}\log\tau_i = \frac{1}{\varpi - \psi\phi}\log\tau_i.$$

The interpretation of this condition is that, for given elasticities ϕ and ψ, a lower value of ϖ (higher share of intermediates) means a stronger response of labor outcomes to economic shocks. However, the response of real wages to shocks in τ_i is going to be the same if the aggregate elasticity $(\varpi - \psi\phi)^{-1}$ is set to be the same across models. This is a similar point to the one made by Allen, Arkolakis, and Takahashi (2018) in that, for certain counterfactuals, the predictions of a spatial model with respect to fundamentals may be the same with intermediate inputs or not as long as some aggregate elasticities are set to be invariant across models.

D.3.3 Generalized Spatial Model with Commuting

We now define a Generalized Spatial Competitive Economy with Commuting between markets.

Preferences. We assume that the representative household has preferences over consumption and labor for individuals residing in market j and commuting to market d:

$$U\left(\{C_{jd}\}_{j,d}, \{L_{jd}\}_{j,d}\right)$$

where L_{jd} is the mass of workers residing in market j and working on market d, and C_{jd} denoting the associated consumption index of these workers.

We assume that individuals consume in their market of residence. For labor in market j commuting to d, the homothetic consumption index is

$$C_{jd} = V_j\left(\{c_{ijd}\}_i\right)$$

and the budget constraint is

$$\sum_i p_{ij} c_{ijd} = w_d L_{jd}.$$

As in the baseline model, the first-stage problem yields the price index and the spending shares,

$$P_j\left(\{p_{ij}\}_i\right) \quad \text{and} \quad X_{ij}\left(\{p_{oj}\}_o\right). \tag{115}$$

Notice that, because $V_j(.)$ does not vary with the commuting destination, the price index and the spending shares do not vary with the commuting destination. This implies that

$$\sum_i p_{ij} c_{ijd} = P_j C_{jd} \quad \Rightarrow \quad C_{jd} = \frac{w_d}{P_j} L_{jd} = \omega_{jd} L_{jd}$$

where $\omega_{jd} = w_d/P_j$ is the real wage of working in market d and residing in market j.

Thus, the second-stage problem is

$$\max_{\{L_{jd}\}_{jd}} U\left(\{(\omega_{jd}) L_{jd}\}_{j,d}, \{L_{jd}\}_{j,d}\right)$$

which yields the labor supply mapping,

$$L_{jd} \in \Phi_{jd}(\boldsymbol{\omega}, \boldsymbol{P}) \equiv \Phi_{jd}\left(\left\{\omega_i \frac{P_i}{P_o}\right\}_{oi}\right). \tag{116}$$

Production. As in the baseline model, we consider the profit maximization problem of firms in market i yields the same equilibrium conditions

$$p_{ij} = \tau_{ij} p_i, \tag{117}$$

$$p_i = \frac{w_i}{\Psi_i\left(\{L_{jd}\}_{j,d}\right)}. \tag{118}$$

Notice that we all agglomeration to depend on the entire vector of commuting flows, $\{L_{ij}\}_{i,j}$. This general formulation covers two possible specifications of agglomeration forces. When agglomeration depends only on employment in each market, $\Psi_i\left(\{L_{jd}\}_{j,d}\right) = \Psi_i\left(\left\{\sum_j L_{jd}\right\}_d\right)$. Alternatively, when agglomeration depends only on residence population in each market, we have that $\Psi_i\left(\{L_{jd}\}_{j,d}\right) = \Psi_i\left(\{\sum_d L_{jd}\}_j\right)$.

Market clearing. To close the model, we consider the labor market clearing condition: total labor payments to labor in market i equals total revenue of market i from selling to all other markets in the world economy. That is,

$$\sum_o w_i L_{oi} = \sum_j x_{ij}\left(\sum_d w_d L_{jd}\right). \tag{119}$$

Equilibrium. The competitive equilibrium in this economy corresponds to $\{p_i, w_i, P_j, L_{ij}\}$ such that conditions (115)–(119) hold. In this case, we need to extend the notion of labor supply to capture commuting flows across markets. In other words, counterfactual predictions require knowledge of the extended labor supply mapping with between-market worker commuting flows, $L_{jd} \in \Phi_{jd}\left(\{\omega_{oi}\}_{oi}\right)$.

Let bold variable with a tilde denote the $N^2 \times 1$ vector of stacked market-to-market vector, with $\tilde{\boldsymbol{L}} \equiv \{L_{jd}\}_{jd}$ and $\tilde{\boldsymbol{\omega}} \equiv \{\omega_{jd}\}_{jd}$.

Using this notation, the combination of equations (116) and (118) yields the labor market module

$$\omega_i = \frac{p_i}{P_i}\Psi_i\left(\{\Phi_{jd}\left(\boldsymbol{\omega}, \boldsymbol{P}\right)\}_{jd}\right) \tag{120}$$

The combination of (118) and (119) yields the trade module:

$$p_i \Psi_i\left(\tilde{\boldsymbol{\Phi}}\left(\tilde{\boldsymbol{\omega}}\right)\right)\sum_o \Phi_{oi}\left(\tilde{\boldsymbol{\omega}}\right) = \sum_j x_{ij}\left(\sum_d p_d \Psi_d\left(\tilde{\boldsymbol{\Phi}}\left(\tilde{\boldsymbol{\omega}}\right)\right)\Phi_{jd}\left(\tilde{\boldsymbol{\omega}}\right)\right), \tag{121}$$

where the price index and the spending shares are given by (115).

Decomposition of direct and indirect effects. We now log-linearize the system to obtain the decomposition into direct and indirect spillover effects. The labor market module in (116) implies that

$$\log\hat{\omega}_i = \log\hat{p}_i - \log\hat{P}_i + \sum_{jd}\psi_{i,jd}\sum_l\sum_o \phi_{jd,ol}\left(\log\hat{\omega}_l + \log\hat{P}_l - \log\hat{P}_o\right)$$

$$\log\hat{\omega}_i - \sum_{jd}\psi_{i,jd}\sum_l\left(\sum_o \phi_{jd,ol}\right)\log\hat{\omega}_l = \log\hat{p}_i - \log\hat{P}_i + \sum_{jd}\psi_{i,jd}\sum_l\left(\sum_o\left(\phi_{jd,ol} - \phi_{jd,lo}\right)\right)\log\hat{P}_l$$

In matrix form, we write

$$\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\psi}}\bar{\boldsymbol{\phi}}^{\omega}\right)\log\hat{\boldsymbol{\omega}} = \log\hat{\boldsymbol{p}} - \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\psi}}\bar{\boldsymbol{\phi}}^{P}\right)\log\hat{\boldsymbol{P}}$$

where

$$\bar{\boldsymbol{\psi}} = [\psi_{i,jd}]_{i,jd} \quad \bar{\boldsymbol{\phi}}^{\omega} \equiv [\sum_o \phi_{jd,ol}]_{jd,l} \quad \bar{\boldsymbol{\phi}}^{P} \equiv [\sum_o(\phi_{jd,ol} - \phi_{jd,lo})]_{jd,l}.$$

We now have two elasticity matrices of commuting flows: $\bar{\boldsymbol{\phi}}^{\omega}$ and $\bar{\boldsymbol{\phi}}^{P}$. First, a change in the real wage of market l affects the payoff of all commuting flows with destination l and, therefore, has a total effect on the flow in jd of $\phi^{\omega}_{jd,l} \equiv \sum_o \phi_{jd,ol}$. Second, conditional on real wages, a change in the price index of market l has an effect on the payoff of all pairs with an origin effect in l, generating a total response in the jd flow of $\phi^{P}_{jd,l} \equiv \sum_o (\phi_{jd,ol} - \phi_{jd,lo})$.

Recalling that $\log \hat{\boldsymbol{P}} = \log \hat{\boldsymbol{\eta}}^C + \bar{\boldsymbol{x}} \log \hat{\boldsymbol{p}}$, we get that

$$\log \hat{\boldsymbol{\omega}} = \bar{\boldsymbol{\beta}} \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\pi}} \bar{\boldsymbol{x}} \right) \log \hat{\boldsymbol{p}} - \bar{\boldsymbol{\beta}} \bar{\boldsymbol{\pi}} \log \hat{\boldsymbol{\eta}}^C \tag{122}$$

where we define

$$\bar{\boldsymbol{\beta}} \equiv \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\psi}} \bar{\boldsymbol{\phi}}^{\omega} \right)^{-1} \quad \text{and} \quad \bar{\boldsymbol{\pi}} \equiv \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\psi}} \bar{\boldsymbol{\phi}}^{P} \right).$$

From the trade module in

$$\begin{aligned} \log \hat{p}_i + \textstyle\sum_{jd} \psi_{i,jd} \phi^{\omega}_{jd,l} \log \hat{\omega}_l + \sum_{jd} \psi_{i,jd} \phi^{P}_{jd,l} \log \hat{P}_l + \sum_o \frac{L_{oi}}{\sum_l L_{li}} \left(\phi^{\omega}_{oi,l} \log \hat{\omega}_l + \phi^{P}_{oi,l} \log \hat{P}_l \right) = \\ \log \eta^R_i + \textstyle\sum_o \left(\sum_j y_{ij} \chi_{oij} \right) \log \hat{p}_o + \log \hat{p}_d + \sum_j y_{ij} \sum_d \frac{L_{jd}}{\sum_o L_{jo}} \left(\sum_{ko} \psi_{d,ko} \phi^{\omega}_{ko,l} \log \hat{\omega}_l + \sum_{ko} \psi_{d,ko} \phi^{P}_{ko,l} \log \hat{P}_l \right) \\ + \textstyle\sum_j y_{ij} \sum_d \frac{L_{jd}}{\sum_o L_{jo}} \left(\phi^{\omega}_{jd,l} \log \hat{\omega}_l + \phi^{P}_{jd,l} \log \hat{P}_l \right) \end{aligned}$$

Thus,

$$\begin{aligned} \log \hat{\boldsymbol{p}} + \left(\bar{\boldsymbol{\psi}} + \bar{\boldsymbol{L}}^E \right) \left(\bar{\boldsymbol{\phi}}^{\omega} \log \hat{\boldsymbol{\omega}} + \bar{\boldsymbol{\phi}}^{P} \log \hat{\boldsymbol{P}} \right) = \log \hat{\boldsymbol{\eta}}^R + \bar{\boldsymbol{\chi}} \log \hat{\boldsymbol{p}} \\ + \bar{\boldsymbol{y}} \bar{\boldsymbol{L}} \left(\log \hat{\boldsymbol{p}} + \bar{\boldsymbol{\psi}} \left(\bar{\boldsymbol{\phi}}^{\omega} \log \hat{\boldsymbol{\omega}} + \bar{\boldsymbol{\phi}}^{P} \log \hat{\boldsymbol{P}} \right) \right) + \bar{\boldsymbol{y}} \bar{\boldsymbol{L}}^R \left(\bar{\boldsymbol{\phi}}^{\omega} \log \hat{\boldsymbol{\omega}} + \bar{\boldsymbol{\phi}}^{P} \log \hat{\boldsymbol{P}} \right) \end{aligned}$$

where $\bar{\boldsymbol{L}} = [L_{ij} / \sum_o L_{io}]_{i,j}$, $\bar{\boldsymbol{L}}^R = [L^R_{i,jd}]_{i,jd}$ with $L^R_{j,od} = L_{od} / \sum_i L_{ji} 1[j = o]$, and $\bar{\boldsymbol{L}}^E = [L^E_{i,jd}]_{i,jd}$ with $L^E_{i,jd} = L_{jd} / \sum_o L_{oi} 1[i = d]$.

Rearranging the expression above,

$$\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\chi}} - \bar{\boldsymbol{y}} \bar{\boldsymbol{L}} \right) \log \hat{\boldsymbol{p}} = \log \hat{\boldsymbol{\eta}}^R - \bar{\boldsymbol{\mu}} \left(\bar{\boldsymbol{\phi}}^{\omega} \log \hat{\boldsymbol{\omega}} + \bar{\boldsymbol{\phi}}^{P} \log \hat{\boldsymbol{P}} \right)$$

with $\bar{\boldsymbol{\mu}} \equiv \bar{\boldsymbol{\psi}} + \bar{\boldsymbol{L}}^E - \bar{\boldsymbol{y}} \left(\bar{\boldsymbol{L}} \bar{\boldsymbol{\psi}} + \bar{\boldsymbol{L}}^R \right)$.

Using (122),

$$\bar{\boldsymbol{\gamma}} \log \hat{\boldsymbol{p}} = \log \hat{\boldsymbol{\eta}}^R + \bar{\boldsymbol{\mu}} \left(\bar{\boldsymbol{\phi}}^{\omega} \bar{\boldsymbol{\beta}} \bar{\boldsymbol{\pi}} - \bar{\boldsymbol{\phi}}^{P} \right) \log \hat{\boldsymbol{\eta}}^C$$

with $\bar{\boldsymbol{\gamma}} \equiv \bar{\boldsymbol{I}} - \bar{\boldsymbol{\chi}} - \bar{\boldsymbol{y}} \bar{\boldsymbol{L}} + \bar{\boldsymbol{\mu}} \left(\bar{\boldsymbol{\phi}}^{\omega} \bar{\boldsymbol{\beta}} \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\pi}} \bar{\boldsymbol{x}} \right) + \bar{\boldsymbol{\phi}}^{P} \bar{\boldsymbol{x}} \right)$.

By combining this expression and (122),

$$\log \hat{\boldsymbol{\omega}} = \bar{\boldsymbol{\beta}} \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\pi}} \bar{\boldsymbol{x}} \right) \left(\bar{\boldsymbol{\alpha}} \log \hat{\boldsymbol{\eta}}^R + \bar{\boldsymbol{\alpha}} \bar{\boldsymbol{\mu}} \left(\bar{\boldsymbol{\phi}}^{\omega} \bar{\boldsymbol{\beta}} \bar{\boldsymbol{\pi}} - \bar{\boldsymbol{\phi}}^{P} \right) \log \hat{\boldsymbol{\eta}}^C \right) - \bar{\boldsymbol{\beta}} \bar{\boldsymbol{\pi}} \log \hat{\boldsymbol{\eta}}^C,$$

which implies that

$$\log \hat{\boldsymbol{\omega}} = \bar{\boldsymbol{\beta}} \left(\bar{\boldsymbol{\alpha}}^R \log \hat{\boldsymbol{\eta}}^R - \bar{\boldsymbol{\alpha}}^C \log \hat{\boldsymbol{\eta}}^C \right) \tag{123}$$

where $\bar{\boldsymbol{\alpha}} \equiv \bar{\boldsymbol{M}}' \left(\bar{\boldsymbol{M}} \bar{\boldsymbol{\gamma}} \bar{\boldsymbol{M}}' \right)^{-1} \bar{\boldsymbol{M}}$, $\bar{\boldsymbol{\alpha}}^R \equiv \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\pi}} \bar{\boldsymbol{x}} \right) \bar{\boldsymbol{\alpha}}$, $\bar{\boldsymbol{\alpha}}^C \equiv \bar{\boldsymbol{\pi}} - \bar{\boldsymbol{\alpha}}^R \bar{\boldsymbol{\mu}} \left(\bar{\boldsymbol{\phi}}^{\omega} \bar{\boldsymbol{\beta}} \bar{\boldsymbol{\pi}} - \bar{\boldsymbol{\phi}}^{P} \right)$.