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THE ROLE OF INVENTORY CONTROLS

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OLIGOPOLY PRICE DISCRIMINATION: THE ROLE OF INVENTORY CONTROLS

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Abstract

Inventory controls, used most notably by airlines, are sales limits assigned to individual prices. While typically viewed as a tool to manage demand uncertainty, we argue that inventory controls also facilitate intertemporal price discrimination. In our model, competing firms first choose quantity and then choose prices in a series of advance-purchase markets. When demand becomes more inelastic over time, as in the airline and hotel markets, a monopolist can easily price discriminate; however we show that oligopoly firms generally cannot. Inventory controls let firms set increasing prices regardless of whether or not demand is uncertain.

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1 Introduction

Seminal research by Kreps and Scheinkman (1983) and Davidson and Deneckere (1986) analyzes sequential oligopoly games in which firms first choose quantity and then price. These papers are important because they describe when the Cournot model is a valid prediction of a sequential quantity-price game. A limitation of these works is that they only consider one pricing period, yet, the most widely cited examples of sequential quantity-price games include the airline, hotel, sport and entertainment industries–all industries in which firms adjust their prices over time. Airline markets in particular are known for sharp price increases in the final weeks before departure.[1] While this pattern of prices is consistent with theoretical models of demand uncertainty (Prescott 1975, Eden 1990, Dana 1999), recent empirical work on airline pricing finds that intertemporal price discrimination is the dominant reason for increasing prices (Puller, Sengupta, and Wiggins 2012).

We explore whether intertemporal price discrimination arises in an oligopoly model of sequential quantity-price games with multiple sales periods and demand becoming more inelastic over time. Our main contribution is to show that strong competitive forces prevent oligopoly firms from utilizing intertemporal price discrimination, even when consumers who arrive closer to departure are less price-sensitive and when a monopolist would charge these consumers higher prices. Once quantity is fixed, firms have very strong incentives to raise their prices in early, more-elastic periods in order to reserve their output for the later, less-elastic periods. This results in equilibrium prices that are flat over time, which is inconsistent with observed pricing patterns in airline and hotel markets. We reconcile this inconsistency by showing that if competing firms commit to use inventory controls (limits on unit sales assigned to prices), such as those actually used by airlines and hotels, they can profitably engage in intertemporal price discrimination. While inventory

[1]A report produced for Expedia, Airlines Reporting Corporation (2015), suggests that airline fares are lowest 57 days before departure and increase dramatically within the last 21 days.

controls have been studied extensively in the context of demand uncertainty (Talluri and Van Ryzin 2006, McGill and Van Ryzin 1999), we argue that they can also be used to facilitate oligopoly price discrimination, regardless of whether or not there is uncertainty about demand.

In our baseline model, firms sell a homogeneous good and have no private information. We focus on the case in which demand becomes more inelastic over time–consumers assigned to later periods have less-elastic demands–because this pattern is consistent with the empirical literature on airlines (Lazarev 2013, Williams 2018). At the deadline, no further sales can take place. We assume that there is a continuum of consumers who are assigned to one of the sequential markets, though this can easily be generalized.

For tractability, we analyze a model with two advance-purchase sales periods, although we also discuss extending the analysis to any finite number of periods. The challenge in solving our game, and the games studied by Kreps and Scheinkman (1983) and Davidson and Deneckere (1986) is that quantity-constrained price competition generates mixed-strategy equilibrium. Solving our game is even more challenging because we consider more than one sales period. We simplify the analysis by focusing on high costs of capacity; this focus eliminates mixed strategies on and off the equilibrium path.

Our main result is that there exist strong competitive forces that prevent intertemporal price discrimination. That is, under mild conditions, equilibrium prices are flat over time, even though consumers who arrive later have higher willingness to pay. This occurs because an individual airline has an incentive to raise price in the early period, shifting demand to competitors, in order to sell more in the later period, when consumers are less price-sensitive and the equilibrium price will be higher. Other firms have similar incentives, which results in uniform prices arising as the unique equilibrium outcome.

We extend the analysis by solving the model when unit-sales limits, or inventory controls, are used in conjunction with price setting. We show that setting inventory controls provides the commitment necessary for firms to increase prices over time as they limit firms' ability to shift demand to competitors in the early, less desirable, period. This result holds even in a simple extension of the model that incorporates demand uncertainty. Thus, while inventory controls have largely been studied in the context of demand uncertainty, we show that they also facilitate intertemporal price discrimination in oligopoly markets.

We also discuss a version of the model with product differentiation. When products are differentiated, prices are no longer uniform across time as firms benefit from the inability to shift all of the demand using very small price changes. However, the strategic incentives explored in this paper are still present, suggesting products must be sufficiently differentiated for prices to increase substantially across periods. For this reason, we postulate that inventory controls, pioneered by airlines, are particularly valuable in the airline industry because products are typically close substitutes and inventory controls allow firms to set higher prices to their less price-sensitive customers.

1.1 Related Literature

This paper contributes to three strands of the economics literature. First, we analyze a model of price competition with capacity constraints (Levitan and Shubik 1972, Allen and Hellwig 1986, Osborne and Pitchik 1986, Klemperer and Meyer 1986). As in Kreps and Scheinkman (1983) and Davidson and Deneckere (1986), our firms choose capacity and then price, but unlike earlier research, we consider more than one pricing period.

Second, we analyze intertemporal price discrimination. Stokey (1979) is a seminal paper that shows that monopoly intertemporal price discrimination is not always feasible. Several more-recent papers find that price adjustments over time are profitable in environments with deadlines and limited capacity (Gallego and van Ryzin 1994, Su 2007, Board and Skrzypacz 2016, Dilme and Li 2017). This is particularly true when consumers learn their preferences over time, as in Akan, Ata, and Dana (2015) and Ata and Dana (2015).

Important empirical contributions to the literature on intertemporal price discrimination, and more generally price adjustments over time, include Nair (2007) on video games, Sweeting (2012) on stadium seats, and Hendel and Nevo (2013) on storable goods. Our focus in on prices that increase as a deadline draws closer (e.g., event or departure time), a pattern that has been found in Broadway Theater (Leslie 2004) and in several airline studies (Lazarev 2013, McAfee and te Velde 2006, Williams 2018). We argue that this pattern is unlikely to exist in competitive markets unless firms use inventory controls (airlines), or their products are significantly differentiated (Broadway Theater).

Finally, our work is related to the literature on inventory controls (see Littlewood (1972), Belobaba (1987), Belobaba (1989) and Weatherford and Bodily (1992), and surveys by Talluri and Van Ryzin (2006), McGill and Van Ryzin (1999) and Stole (2003)). While prior research views inventory controls as a tool for managing aggregate demand uncertainty, our paper shows that they also facilitate intertemporal price discrimination in oligopoly markets.

2 The Model

Consider an oligopoly with n firms selling a homogeneous good to a continuum of consumers in a series of advance-purchase sales markets. For simplicity, we consider just two selling periods, $t = 1, 2$. Even though firms may charge different prices, the market demand in each period is given by continuous functions of a single price, $D_1(p)$ and $D_2(p)$. Let $D_{Tot}(p) = D_1(p) + D_2(p)$ denote the total demand at a uniform price p, and let $p_1(q)$, $p_2(q)$ and $p_{Tot}(q)$ denote the inverses of $D_1(p)$, $D_2(p)$ and $D_{Tot}(p)$, respectively. The price elasticity of demand is given by $\eta_t(p) = D_t'(p)p/D_t(p)$. We assume that the revenue function, $p_t(x)x$, is concave in each period.

Though not always essential for our results, we assume that consumers are less price-sensitive over time (Assumption 1). That is, we consider an environment in which a monopolist would set increasing prices. This assumption seems particularly relevant for our airline and hotel examples.

Assumption 1. *Demand becomes more inelastic over time, so $|\eta_2(p)| < |\eta_1(p)|$ for all p.*

Another reason to consider this restriction is that the ability to price discriminate is constrained when demand becomes more elastic over time: consumers can arbitrage price differences by delaying their purchases (see Ata and Dana (2015), who show that price discrimination is feasible when reservation values increase over time but not when they decrease over time).

We analyze a three-stage game. In the first stage (stage zero), firms simultaneously choose their capacities, $K^i, \forall i = 1, \ldots, n$. The cost per unit of capacity is c for all firms. In stages one and two, firms sell their capacity in two sequential advance-purchase sales periods. Sales in both of these periods are from a common capacity constraint (e.g., seats on the same flight). For simplicity, we assume that the marginal cost of each sale is zero (the cost of putting a passenger in an otherwise empty airline seat is zero). This is a game of complete information, so capacities, prices and sales are all observable in all periods.

We assume that consumers cannot delay their purchasing decisions but, instead, are exogenously assigned to purchase in either the first or second sales period. In our setting, this assumption can easily be relaxed by allowing consumers to learn their preferences over time and to choose when to purchase, as consumers will purchase as early as possible to avoid price increases.[2]

Products are homogeneous, so consumers purchase at the lowest price available, as long as their valuation exceeds the price. If the firms set different prices, then a firm with a higher price can have positive sales only after all of the firms with lower prices have sold all of their capacity. If two or more firms charge the same price, then we assume that firms divide the sales equally, subject to their capacity constraints. How much the firm with the higher price sells–that is, the firm's residual demand–depends on the rationing

[2]In this case, some consumers prefer to purchase in the second period because they do not know their demand until the second period. Other consumers prefer to purchase in the first period, even with the option to wait, because they know their demands early and because they rationally anticipate that the firms' prices will be higher if they wait (see, for example, Dana (1998) and Akan, Ata, and Dana (2015)).

rule. The residual demand function is $RD_t\left(p; \mathbf{p}^{-i}, \mathbf{K}^{-i}\right)$, where the arguments are firm i's own price, p, and vectors of all of the other firms' prices and capacities. Our results hold for both the efficient rationing rule and the proportional rationing rule. Recall that the residual demand for the efficient rationing rule is

$$RD_t(p; \mathbf{p}^{-i}, \mathbf{q}^{-i}) = D_t(p) - \sum_{j \neq i; p^j < p} q^j, t = 1, 2, \tag{1}$$

and the residual demand for the proportional rationing rule is

$$RD_t(p; \mathbf{p}^{-i}, \mathbf{q}^{-i}) = D_t(p) \left[1 - \sum_{j \neq i; p^j < p} \frac{q^j}{D_t(p^j)} \right], t = 1, 2, \tag{2}$$

if no other firm charges p, and the demand is shared if any of firm i's rivals is charging p.

If there were just one pricing period, then we know from Kreps and Scheinkman (1983), who analyze efficient rationing, and Davidson and Deneckere (1986), who analyze proportional rationing, that the pricing subgame would have a unique Nash equilibrium. These papers characterize profits for all capacity levels and show that the price game has a mixed-strategy equilibrium when capacities are sufficiently large. Because we have two pricing periods, characterizing the equilibrium profits is considerably more challenging.

By focusing on large capacity costs, we avoid this challenge and ensure that the pricing game has pure-strategy equilibria. Similar results may hold for low capacity costs, but we do not solve the mixed-strategy equilibria needed to prove that the results generalize. The assumption that capacity costs are large seems particularly reasonable in our airline and hotel applications since more than 75% of airlines' costs do not vary with the number of passengers served, and the remainder (reservations and sales expense and passenger service) includes mainly labor costs that do not with the number of passengers served.[3]

Our assumption on capacity costs allow us to consider both efficient and proportional

[3]Calculations based on DOT Form 41 filings and reported in ICAO, Airline Operating Costs and Productivity, February 2017, https://www.icao.int/.

rationing. Recall that in both Kreps and Scheinkman (1983) and Davidson and Deneckere (1986), when capacity is small (for example, smaller than a monopolist's output if capacity were free), the price is equal to the market-clearing price and does not depend on the rationing rule. This is because marginal revenue is positive in the pricing stage even when firms collude, which implies that marginal revenue is positive for every firm. Thus, firms can never do better by setting a price at which they do not sell all of their capacity.

We make a similar, but slightly stronger, assumption. Our assumption is stronger because it guarantees that marginal revenue is positive in the second pricing period, even if firms choose not to sell any of their capacity in the first period.

Assumption 2. *The competitive output, $D_{Tot}(c)$, is smaller than the monopoly output, $q_2^m(0)$, produced when the monopolist has zero capacity costs.*

Assumption 2 is clearly satisfied if the cost of capacity c is sufficiently large.

We also assume that the firms' total capacity does not exceed the capacity that would be produced if the market were perfectly competitive (Assumption 3).

Assumption 3. *Firms' capacities are less than the capacities in a perfectly competitive market, or $\sum_i K^i \leq D_{Tot}(c)$.*

On the one hand, Assumption 3 is reasonable because holding capacity in excess of the perfectly competitive market has no value to firms in our model. In this game, it is reasonable to expect industry profits to be negative for any capacities not satisfying Assumption 3. This differs from some dynamic games in which unused capacity might be an effective off-the-equilibrium-path instrument for punishment. Note that if industry profits are negative, some firms can increase profits by reducing capacity. On the other hand, Assumption 3 is atypical – it is usually easy to show that such capacities are never optimal. Together with Assumption 2, Assumption 3 implies that every on- or off-the-equilibrium-path pricing subgame has a pure-strategy equilibrium, which simplifies our proofs.

3 Equilibrium Characterization

We now solve the full model as described in Section 2. We solve the three-stage game (capacity in stage 0, price in stage 1, and price in stage 2) by backwards induction. All proofs, except for our main result, Proposition 2, appear in the Appendix.

3.1 The Pricing Subgames

We begin by characterizing prices in the second pricing period. Lemma 1 states that in the second period, firms set prices to clear the market. This result is well known from Kreps and Scheinkman (1983) for efficient rationing and from Davidson and Deneckere (1986) for proportional rationing.

Lemma 1. *Under either the efficient or the proportional rationing rule, if Assumptions 2 and 3 hold, then in any subgame perfect equilibrium (SPE) of the three-stage game, the price in the second selling period clears the market.*

Lemma 1 allows us to easily characterize all of the subgame perfect equilibria of the pricing subgame. We say that the equilibrium is unique when all the equilibria of the game have the same outcomes and payoffs for all players.

First, no equilibrium exists in the pricing subgame in which $p_1 \neq p_2$. If all firms charged prices p_1 and p_2, and $p_1 < p_2$, then a firm could deviate to a slightly higher price in period 1. The firm's period 1 sales would fall discretely (perhaps to zero); its period 2 sales would rise discretely; and prices would change by, at most, an arbitrarily small amount. Its profits would be strictly higher. And if all firms charged prices p_1 and p_2, and $p_1 > p_2$, then a firm could deviate to a slightly lower price in period 1. The firm's period 1 sales would rise discretely (perhaps to its capacity); its period 2 sales would fall discretely; and prices would change by, at most, an arbitrarily small amount. Its profits would again be strictly higher.

Proposition 1, below, shows that there are two types of subgame perfect equilibria in the pricing subgame. First, in uniform-price equilibria, all firms set the same price in the first and second periods. Since the market clears in the second period (Lemma 1), any uniform-price equilibrium must satisfy $D_1(p^*) + D_2(p^*) = \sum_i K^i$, so the uniform price is unique. Second, in asymmetric-price equilibria, a single firm sells in the first period; the first-period price is lower than the second-period price; and all other firms sell only in the second period.

Asymmetric-price equilibria have a particular form. Only one firm, firm i, sells in the first period – let p_1^i and q_1^i denote its first-period price and quantity, where

$$p_1^i = \underset{p \in [p_1(K^i), \infty]}{\operatorname{argmax}}\; pD_1(p) + p_2\left(\sum_i K^i - D_1(p)\right)\left(K^i - D_1(p)\right), \tag{3}$$

or, equivalently,

$$q_1^i = \underset{q \in [0, K^i]}{\operatorname{argmax}}\; p_1(q)q + p_2\left(\sum_i K^i - q\right)\left(K^i - q\right). \tag{4}$$

In both expressions, the firm's output is constrained so that first-period sales do not exceed K^i. The second-period price is larger than p_1^i and is given by

$$p_2 = p_2\left(\sum_i K^i - D_1(p_1^i)\right). \tag{5}$$

Proposition 1. *Under either the efficient or the proportional rationing rule, if Assumptions 2 and 3 hold, then every pure-strategy SPE of the pricing subgame is either a uniform-price equilibrium or an asymmetric-price equilibrium satisfying Equations (3), (4) and (5). Also,*

1) *when a uniform-price equilibrium exists, it is the unique pure-strategy SPE;*

2) *when an asymmetric-price equilibrium exists, no uniform-price equilibrium exists; and*

3) *there are, at most, n asymmetric-price equilibria.*

Intuitively, asymmetric-price equilibria exist because a lower price in the first pricing period increases sales in the first period, leading to a higher second-period price. But a firm can increase its profit in this way only if it has sufficient capacity to meet all of the demand in first period and has additional capacity to sell at the higher second-period price. Other firms free ride and sell only in the second pricing period at the higher price.

Asymmetric-price equilibria are more likely to exist when one firm has more capacity than its rivals. The incentive to deviate to a lower price is increasing in the deviating firm's capacity, decreasing in the rival firms' capacity, increasing in the elasticity of first-period demand, and decreasing in the size of first-period demand.

While asymmetric-price equilibria exist in some instances, Assumption 4 implies that only a uniform-price equilibrium exists. This relatively weak condition implies that no firm has enough capacity to profitably deviate from the symmetric uniform-price equilibrium.

Assumption 4 requires that demand in period 2 not be too much less elastic than demand in period 1. That is, at equal prices, the demand in period 2 can be less elastic, but not so much less elastic that unilaterally cutting price in the first period in order to drive up price in the second period would be profitable.

Assumption 4. *The elasticities of demand and capacities satisfy*

$$\frac{\eta_2(p)}{\eta_1(p)} > \frac{K^i}{\sum_{j=1}^n K^j}, \forall p, i. \tag{6}$$

The next proposition shows that under Assumption 4, the unique equilbrium of the two-period pricing subgame is the uniform-price equilibrium.

Proposition 2. *If Assumptions 1, 2, and 3, and 4 hold, then the unique subgame perfect Nash equilibrium of the pricing subgame is a uniform-price equilibrium.*

Proof. Let K^i denote each firm's capacity, and let $\tilde{p}$ denote the unique uniform price defined by $D_{tot}(\tilde{p}) = D_1(\tilde{p}) + D_2(\tilde{p}) = \sum_{i=1}^n K^i$.

Consider a deviation to a lower price. If $D_1(\tilde{p}) \geq \max_i K^i$, then a deviation to a lower price is not profitable, because any firm that cuts its price in period 1 will sell all of its capacity at that lower price and earn strictly lower profits.

If $D_1(\tilde{p}) < \max_i K^i$, then for any firm i such that $K^i \leq D_1(\tilde{p})$, a deviation to a lower price is not profitable by the same argument. If $K^i > D_1(\tilde{p})$, then a deviation could be profitable. But if the firm chooses a price at which $D_1(p) \geq K^i$, then profits are strictly lower. So we can think of the firm as choosing a price p, or equivalently, a quantity $q = D_1(p)$ to maximize

$$\hat{\pi}^i(q;\tilde{p},\mathbf{K}) = qp_1(q) + p_2\left(\sum_{i=1}^{n} K^i - q\right)\left(K^i - q\right). \tag{7}$$

The first-order condition is

$$\frac{d\hat{\pi}(q;\tilde{p},\mathbf{K})}{dq} = p_1(q) + qp_1'(q) - p_2\left(\sum_{i=1}^{n} K^i - q\right) - p_2'\left(\sum_{i=1}^{n} K^i - q\right)(K^i - q) = 0, \tag{8}$$

or

$$\begin{aligned}\frac{d\hat{\pi}(q;\tilde{p},\mathbf{K})}{dq} &= p_1(q)\left(1 + \frac{1}{\eta_1(p_1(q))}\right) \\ &\quad - p_2\left(\sum_{i=1}^{n} K^i - q\right)\left(1 + \frac{1}{\eta_2\left(p_2\left(\sum_{i=1}^{n} K^i - q\right)\right)}\frac{K^i - q}{\sum_{i=1}^{n} K^i - q}\right) = 0.\end{aligned} \tag{9}$$

Clearly, (7) is concave, so a deviation to a lower price cannot be profitable if the solution to (9) is less than $D_1(\tilde{p})$ or, equivalently, if $\frac{d\hat{\pi}(D_1(\tilde{p});\tilde{p},\mathbf{K})}{dq} < 0$. But

$$\begin{aligned}\frac{d\hat{\pi}(D_1(\tilde{p});\tilde{p},\mathbf{K})}{dq} &< p_1(D_1(\tilde{p}))\left(1 + \frac{1}{\eta_1(p_1(D_1(\tilde{p})))}\right) \\ &\quad - p_2\left(\sum_{i=1}^{n} K^i - D_1(\tilde{p})\right)\left(1 + \frac{1}{\eta_2\left(p_2\left(\sum_{i=1}^{n} K^i - D_1(\tilde{p})\right)\right)}\frac{K^i}{\sum_{i=1}^{n} K^i}\right)\end{aligned}$$

because $\frac{K^i - q}{(\sum_{i=1}^{n} K^i - q)} < \frac{K^i}{\sum_{i=1}^{n} K^i}$. Since $p_1(D_1(\tilde{p})) = p_2\left(\sum_{i=1}^{n} K^i - D_1(\tilde{p})\right) = \tilde{p}$, a deviation to a lower price is not profitable if

$$\frac{1}{\eta_1(p_1(D_1(\tilde{p})))} - \frac{1}{\eta_2\left(p_2\left(\sum_{i=1}^n K^i - D_1(\tilde{p})\right)\right)} \frac{K^i}{\sum_{i=1}^n K^i} < 0 \iff \frac{\eta_2(\tilde{p})}{\eta_1(\tilde{p})} > \frac{K^i}{\sum_{i=1}^n K^i}, \tag{10}$$

or, equivalently, if Assumption 4 holds.

Now consider a deviation to a higher price. If $D(\tilde{p}) \leq \sum_{j \neq i} K^j$, then a firm that deviates to a higher price has zero sales in period 1 and still sells all of its output at the uniform price, and profits are the same. If $D(\tilde{p}) > \sum_{j \neq i} K^j$, then the deviating firm's rivals sell all of their output at a price $\tilde{p}$, so its rivals' profits are unchanged. The deviating firm acts like a residual monopolist, but under Assumption 1, even a monopolist cannot increase its profits by raising its price in period 1, so the deviating firm's profits must be lower. ■

Intuitively, deviating from a uniform price is profitable for a monopolist if it raises the second-period profit by more than it lowers the first-period profit. However, since rivals free ride and sell only in period 2, an oligopoly firm that deviates from the uniform price, by lowering its first-period price earns, at most, $1/n$th of the second-period industry profits. The oligopoly firm that deviates cannot increase its profit unless it can increase the second-period industry profits by at least n times the decrease in its first-period profit. For such a deviation to be profitable, the first-period demand must be at least n times more elastic than the second-period demand. Assumption 4 guarantees that such a deviation is not profitable.

When Assumption 1 does not hold, a uniform-price equilibrium may not exist. A firm may be able to profitably deviate to a higher price in period 1, particularly under proportional rationing and when the first-period demand is large relative to second-period demand. If a uniform price equilibrium does not exist, then no pure-strategy equilibrium of the pricing subgame exists.

3.2 The Initial Capacity Choice

In many respects, Proposition 2 is the most interesting result of the paper. It specifies that for any allocation of initial capacity satisfying Assumption 4, oligopoly firms cannot price discriminate when a monopolist clearly would.

We now ask what happens when firms choose their initial capacity optimally. We replace Assumption 4, which is a restriction on capacities, with Assumption 5, which is a restriction on the elasticities. Assumption 5 is weaker. It is equivalent to Assumption 4 when the firms' capacities are symmetric. Proposition 3 establishes that Assumption 5 is sufficient to guarantee that when the firms choose capacity (the full game), the unique subgame perfect equilibrium is a uniform-price equilibrium.

Assumption 5. *The elasticity of demand satisfies*

$$\frac{\eta_2(p)}{\eta_1(p)} > \frac{1}{n}.$$

Proposition 3. *Under Assumptions 1, 2, 3, and 5, the unique symmetric pure-strategy subgame perfect Nash equilibrium of the full game is a uniform-price equilibrium.*

Proposition 3 implies that we should not expect to see intertemporal price discrimination in oligopoly markets. Unless the decrease in the elasticity of demand is very large, firms will choose symmetric capacities, and prices will be uniform over time. However, prices typically rise as the departure time approaches in the airline industry, and empirical work suggests that these increases are related to changes in the elasticity of demand, which is inconsistent with Proposition 3. The next section of the paper suggests that inventory controls are a way to reconcile this inconsistency.

If, instead, the elasticity increases over time, then competition is still a barrier to symmetric equilibria with declines in prices since firms can cut price and take all of their sales in the first period, but a uniform-price equilibrium may not exist.

4 Inventory Controls

In this section, we add inventory controls to the model. The game with inventory controls is a game in which firms first choose capacity and then, in each of the two subsequent periods, simultaneously choose price and an upper bound on quantity sold.[4]

In the previous section, we showed that prices are uniform because, by increasing their current price, firms can divert their rivals' sales from the future to the current period and divert their own sales from the present to the future period. However, firms can prevent rival firms' price changes from impacting their sales by using inventory controls. So, for example, a firm can limit the number of units available at p_1 to exactly the number of units it expects to sell in period 1. If a rival firm deviates to a higher price in period 1, the firm's own sales will be unchanged. However, there is a natural asymmetry. Inventory controls can cap sales, but they cannot prevent sales from decreasing. Thus, inventory controls do not prevent a rival from increasing its share of current sales by deviating to a lower price.

In this section, we consider a game in which firms first choose their capacity, and then choose their price and inventory controls in each selling period. To distinguish the firm's inventory controls from the firm's actual sales, we denote the inventory control for firm i in period t by k_t^i.

Proposition 4. *Under Assumptions 1, 2, 3, and 5, and under either the efficient or the proportional rationing rule, a subgame perfect Nash equilibrium of the model with inventory controls exists in which all firms set the Cournot price and quantity in each selling period.*

The capacity-then-price model with inventory controls has other equilibria. In particular, the symmetric capacity, uniform-price equilibrium characterized in Proposition 3 may still be a subgame perfect equilibrium of the inventory control game. This is because

[4]This is different from a model in which firms choose output each period, subject to a capacity constraint. If firms could choose capacity and let price clear the market each period they would produce the Cournot output each period (see Van den Berg, Bos, Herings, and Peters (2012), however we also differ from that paper because we assume that capacity costs are high).

firms may need to collectively employ inventory controls in order to benefit from using them.[5]

Proposition 4 establishes that inventory controls facilitate intertemporal price discrimination. Other equilibria exist, but Proposition 4 – as compared to Proposition 3 – establishes that inventory controls may increase profits for all firms.

5 Model Extensions

5.1 Product Differentiation

Our first extension is to consider product differentiation. Differentiation does not alter firms' incentive to attempt to shift demand to competitors in the early period. However, product differentiation makes it more costly to shift demand. With undifferentiated products, a small price change shifts all of the demand. With differentiated products, the firm's price increase must be larger, and have a first-order effect on its profits, in order to have a significant impact on a rival's sales.

Product differentiation also introduces increased complexity, so we focus our attention on two firms in a symmetric environment, and give intuition instead of analyzing the equilibrium of the model. We also maintain the assumption that capacity is sufficiently small so that firms always set market-clearing prices in the second period.

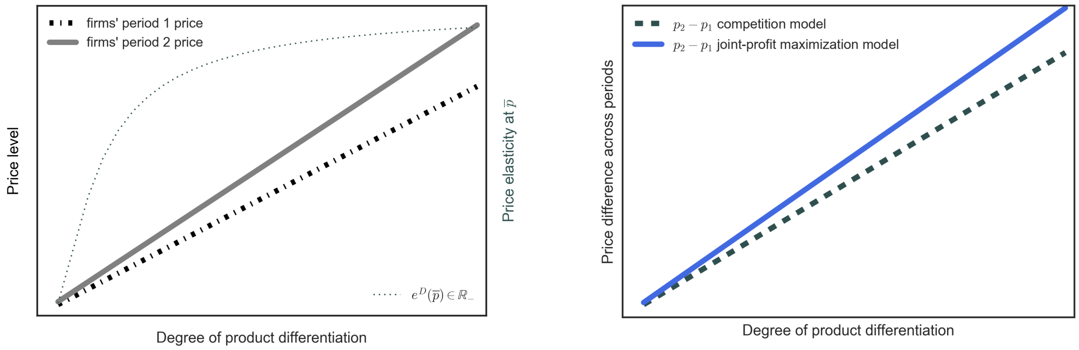
Product differentiation results in equilibrium subgame prices that are no longer uniform over time; however, prices are flatter – as a function of the degree of product differentiation – than joint-profit-maximizing prices (see Figure 1 for an example, where the left plot shows increasing differences in prices across periods as product differentiation in-

[5] If firms could announce and commit to their inventory controls each period before any firm sets price, then Proposition 4 still holds. In this case, if each firm set an inventory control equal to the Cournot output, this would result in the Cournot prices, and no unilateral inventory control deviation would effect the subsequent prices. But this timing may also eliminate uniform price equilibrium. In a duopoly model a unilateral inventory control would curtail the rival's incentives to raise price and cause the rival to equate marginal revenue across the two periods, even when the rival hadn't set an inventory control itself.

Figure 1: Intertemporal Price Discrimination as a Function of Product Differentiation

(a) Prices Across Periods

(b) Competition vs. Joint-Profit Maximization



Notes: Example constructed using a random utility model (logit) with two firms and two periods. Product differentiation is increasing towards the right of the plots. (a) The light dashed line corresponds to the own-price elasticity for a constant price offered by both firms. As products become increasingly differentiated, the difference between p_1 and p_2 increases. (b) Shows the change in price ($p_2 - p_1$) of competition model versus the joint-profit maximization model. Prices are flatter in the competition model, as the gap between the two models grows with the degree of differentiation.

creases). To see this, consider two firms, A and B, and let the inverse demand functions be $P_1^A(q_1^A, q_1^B)$, $P_1^B(q_1^A, q_1^B)$, $P_2^A(q_2^A, q_2^B)$, and $P_2^B(q_2^A, q_2^B)$. Joint-profit-maximizing firms would set marginal revenue equal to the shadow cost of capacity in each of the four product markets, so $\frac{\partial p_i^j(q_t^j, q_t^{-j})}{\partial q_t^j} q_t^j + p_t^j(q_t^j, q_t^{-j}) = \lambda, \forall t = 1, 2; j = A, B$. Suppose that the joint-profit-maximizing prices are increasing over time.

Contrast these prices with the prices that would be set by two competing firms given the same initial capacity. If Firm A sets a higher price than the joint-profit-maximizing firm, it will sell less in the first period and, hence, more in the second period. Sales for Firm B are higher in the first period, and it has less to sell in the second period; thus, in the second period, its price is higher and Firm A's demand is higher. Because it ignores the loss for Firm B, Firm A has an incentive to set a higher first-period price than the joint-profit-maximizing monopolist. Firm B has a similar incentive, and, in equilibrium, both firms' prices will be flatter relative to joint-profit-maximizing prices (see the right panel in Figure 1). It is also worth noting that prices might still be perfectly flat if sufficiently many consumers were indifferent between the firms – a symmetric increasing price equilibrium does not exist because either firm could strictly increase profits with an arbitrarily small price increase.

5.2 Aggregate Demand Uncertainty

Inventory controls are generally described as a tool for managing demand uncertainty, so it is important to describe how the model can be extended to include such uncertainty. To generate intuition, we describe an extension in which just first-period demand is uncertain. A monopolist sets the first-period price before learning the first-period demand and sets a second-period price to clear the market.

A simple way to add uncertainty to the model is assume realized demand can be high or low in the first period, but is known to be high in the second period. In this case a monopolist choosing capacity optimally would set a lower price (based on expected demand) in the first period.

However, the monopoly prices are not an equilibrium with competing firms, even if the firms have the same capacity as the monopolist. Because the monopoly prices increase in expectation, competing firms prefer to sell more of their capacity in the second period, when the expected price is higher. And any firm can shift a discrete amount of its first-period sales to its rival through an arbitrarily small price increase in period 1. Thus, expected prices must be equal in the two periods in any symmetric pure-strategy equilibrium.

5.3 Many Periods

An obvious limitation of the paper is that we consider only two pricing periods. The challenge to extending Propositions 2 and 3 to many periods is that it is more difficult to describe assumptions under which firms play pure strategies for all histories of the game, and it is difficult to analytically bound profits in subgames with mixed-strategy equilibria.

However, it is easy to see that the a symmetric increasing price equilibrium still will not exist. Obviously, on the equilibrium path of the final two periods of the many-periods game is equivalent to our study above, so prices must be equal in the final two periods. And the intuition that firms can profit from shifting lower-price sales to the rivals still holds.

Also, with more than two periods, firms will not only consider large price cuts in order to increase future prices, but may also consider smaller price cuts in order to induce rivals to make a larger price cut in the future. Taking sales away from rivals makes their capacity share large in the next period, which could make it profitable for them to make a large price cut.

6 Conclusion

While a monopolist can charge higher prices to last-minute consumers, we show that oligopoly firms generally cannot. We consider an advance-purchase, sequential-pricing model with complete information and show that capacity-constrained firms will compete on price until prices are equalized across the selling periods. High prices for late consumers are not sustainable because firms have an incentive to raise their early prices in order to claim a larger share of the late consumer market. However, the inclusion of inventory controls – which are common in the airline industry, but are typically characterized as a tool to manage demand uncertainty – facilitates intertemporal price discrimination in oligopoly markets.

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A Appendix

Proof of Lemma 1:

Proof. Suppose not. Then either some firm is charging a price below the market-clearing price or a price above the market clearing price.

Clearly, a firm charging below the market-clearing price must be selling all of its output and increasing price without reducing sales to raise profit. If a firm charges a price above the market-clearing price, then either it does not sell all of its output or some higher-priced firm does not sell all of its output, but there must exist some firm charging a price p greater than the market-clearing price that does not sell all of its output.

Suppose that the firm is not the only firm charging p. Then, the firm can decrease price arbitrarily, and its sales will increase discretely, so its profits increase.

If no other firm is charging the price p; then the firm's profit is $pRD_2(p; \mathbf{p}^{-i}, \mathbf{q}^{-i})$, where $\mathbf{p}^{-i}$ and $\mathbf{q}^{-i}$ are the other firms' prices and remaining capacities.

Under the efficient rationing rule, the derivative of profit with respect to price is $RD_2(p; \mathbf{p}^{-i}, \mathbf{q}^{-i}) + pD_2'(p)$, which is negative because $RD_2(p; \mathbf{p}^{-i}, \mathbf{q}^{-i}) < D_2(p)$ and because $pD_2'(p) + D_2(p) < 0$. This is true because $D_2(p)$ is less than the zero-cost monopoly output. So, lowering price increases profit.

Under proportional rationing, the derivative of profit with respect to price is $RD_2(p) + pRD_2'(p) = \left(pD_2'(p) + D_2(p)\right)\left[1 - \sum_{j \neq i} \frac{q^j}{D_2(p^j)}\right]$, which is negative because $pD_2'(p) + D_2(p) < 0$. This is true because the $D_2(p)$ is less than the zero-cost monopoly output. So lowering price increases profit. ■

Proof of Proposition 1:

Define $p_L = \min_i p_1^i$ to be the lowest equilibrium price offered in period 1.

And note that, given that $\sum_i K^i < D_{tot}(c)$, and given Assumption 2, it follows that, regardless of the history of the game in the first period, all firms with positive remaining capacity charge the market-clearing price in the second-period subgame.

The proof of the proposition proceeds as a series of eight claims.

1) In any equilibrium of the pricing subgame, $p_L \leq p_2$.

Suppose not, so $p_L > p_2$. Suppose also that some firm has zero sales in period 1. Since $p_L > p_2$, the firm with zero sales would be strictly better off setting a first-period price just below p_L. By deviating to $\hat{p}$, this firm increases its sales in period 1 and decreases its sales in period 2 by the same amount. The firm's profits are strictly higher because the price is higher in period 1, and because its deviation may also increase the second-period price. This is a contradiction.

Now, suppose, instead, that $p_L > p_2$ but that every firm has positive sales in period 1. It follows that every firm must be charging p_L. Otherwise, some firm j has positive sales and is charging a price $p^j > p_L$. Let $\hat{K} = \sum_{i|p_i=p_L} K^i$ denote the total capacity at price p_L. Clearly, $D_1(p_L) > \hat{K}$, because firm j's residual demand is positive. But this implies that there exists a strictly positive ϵ such that a firm i charging p_L can deviate to a higher price, $p_L + \epsilon$, and still sell all of its capacity, which is a contradiction.

If $p_L > p_2$ and all firms are charging p_L in period 1, then any firm that has excess capacity in period 1 could strictly increase its profit by deviating to a first-period price of $p_L - \epsilon$, for sufficiently small ϵ. At this price, the deviating firm sells strictly more in period 1, and strictly less in period 2. The deviating firm sells more at a first-period price that is arbitrarily close to p_L and sells less at a second-period price that is arbitrarily close to p_2, and $p_L > p_2$, so profits are higher. So, $p_L \leq p_2$.

2) In any equilibrium of the pricing subgame, when p_L is offered by two or more firms in period 1, then $p_L = p_2$.

Suppose not. So, $p_L < p_2$, and p_L is offered by two or more firms offering p_L. Suppose that firm i is one of those firms. Then, firm i's profit can be written as $p_L x_i + p_2 \left(K^i - x_i\right)$, where $x_i = \min\left\{RD_1\left(p_L; p_L, \sum_{j\neq i|p^j=p_L} K^j\right), K^i\right\}$ is firm i's sales at p_L.

At the slightly higher price $p_L + \epsilon$, the firm i's profit is

$$(p_L + \epsilon) \min\left\{RD_1\left(p_L + \epsilon; p_L, \sum_{j\neq i|p^j=p_L} K^j\right), K^i\right\} + \hat{p}_2(\cdot) \max\left\{K^i - RD_1\left(p_L + \epsilon; p_L, \sum_{j\neq i|p^j=p_L} K^j\right), 0\right\}, \quad (11)$$

which is clearly greater than the firm i's profit at p_L when $x^i = K^i$, since $p_L + \epsilon > p_L$ and $\hat{p}_2(\cdot) > p_L$. Thus, all of firm i's sales are at a higher price, and its sales volume doesn't change.

If, on the other hand, $RD_1(p_L; p_L, \sum_{j\neq i|p^j=p_L} K^j) < K^i$, so that $x^i < K^i$, then the deviation is still profitable for firm i because

$$\lim_{\epsilon\to 0} RD_1^i\left(p_L + \epsilon; p_L, \sum_{j\neq i|p^j=p_L} K^j\right) \leq RD_1^i(p_L; p_L, \sum_{j\neq i|p^j=p_L} K^j) < K^i,$$

since RD is decreasing (for either rationing rule), and the limit of (11) as ϵ goes to 0 is

$$p_L \lim_{p \downarrow p_L} RD_1^i(p; p_L, \sum_{j \neq i | p^j = p_L} K^j) + p_2 \left(K^i - \lim_{p \downarrow p_L} RD_1^i(p; p_L, \sum_{j \neq i | p^j = p_L} K^j) \right),$$

so for sufficiently small ϵ, it again follows that profits are higher because the firm sells more units at p_2 and fewer units at (or near) p_L and $p_2 > p_L$. So a deviation is profitable, which is a contradiction, so either $p_L = p_2$, or only one firm charges p_L.

3) If $p_L = p_2$, then the pricing equilibrium is a uniform-price equilibrium.

Suppose that some firm j sets a price $p^j > p_L = p_2$ in period 1 and has strictly positive sales. The residual demand at p^j is strictly positive, which implies that the residual demand in a neighborhood of p_L must also be strictly positive. Therefore, if a firm, say firm i, deviated from p_L to any price $p_L + \epsilon$, a price in a neighborhood of p_L (but below any higher-priced firm's price), it would be able to sell all of its capacity at that price.

This is because when firm i removes its capacity K^i at p_L, it increases the residual demand in a neighborhood of p_L by K^i. This is clearly true for either rationing rule. So any firm charging p_L could strictly increase its profits by increasing its price since its total sales would not be affected.

4) There exists, at most, one uniform-price equilibrium of the pricing subgame (the total sales and the transaction prices in each period are unique).

Given the capacity, the sales and volume of sales in a uniform-price equilibrium are uniquely defined, because only one price satifies $D_1(p) + D_2(p) = \sum_i K^i$.

5) A uniform-price equilibrium exists as long no firm wants to deviate to a lower price in period 1.

Consider any equilibrium in which $p_L = p_2$, and no firm has positive sales at any period 1 price other than p_L. The equilibrium price level is clearly unique.

Deviating to a higher price is never profitable. Setting a higher price in the first period lowers industry profit – given the industry capacity and Assumption 4, industry profit is clearly lower when the first-period price is higher than the second-period price – and the deviator's share of first-period revenue falls, and its share of second-period revenue rises, so the change in revenue for the deviator must be smaller than for other firms, and the deviator's profit must fall.

Deviating to a lower price might be profitable. Clearly, though, such a deviation cannot be profitable unless $K^i > D_1(p)$ so that the firm has positive sales in period 2. Otherwise, all of the deviating firm's sales would be at a lower price. And since $D(p)$ is decreasing, this implies that $K^i > D_1(p_L)$ is a necessary condition for a deviation to a lower price to be profitable, and $K^i < D_1(p_L)$ for all i is a sufficient condition for a uniform-price equilibrium to be the unique equilibrium of the pricing subgame.

6) When a uniform-price equilibrium of the pricing subgame does not exist, then an asymmetric-price equilibrium exists in which exactly one firm offers $p_L < p_2$ and all other firms have zero sales in period 1.

Suppose that a uniform-price equilibrium does not exist. Then, a deviation is profitable for some firm, and, clearly, it must be profitable for the firm with the largest capacity. For any deviation, that firm loses the same profit in period one from the price decrease, but gains more from the associated price increase in period 2.

Let firm i denote the firm with the largest capacity and p_1^i denote the firm's profit-maximizing deviation and $\hat{p}_2$ the resulting second-period price. That is, let p_1^i denote the prices that maximizes the profit function

Then, p_1^i and $\hat{p}_2$ clearly represent an asymmetric-price equilibrium. All firms except firm i sell only in period 2. Firm i sells in both periods. And no firm wants to undercut firm i in period 1 because it would sell more at the low price and less at the high price in period 2. And if it could increase the price and its profits by charging less than p_1^i, then so could firm i, in which case p_1^i is not firm i's profit-maximizing price, which is a contradiction.

7) A uniform-price equilibrium exists if and only if an asymmetric-price equilibrium does not exist.

Recall that (3) has a unique maximum and is concave. Consider the unique candidate uniform-price equilibrium with a price equal to p^*. This equilibrium exists unless some firm i wants to deviate to a lower price in the first period.

Suppose that firm i wants to deviate. Firm i's profit, (3), is maximized at the same price p_1^i. If $p_1^i < p^*$, then the deviation is profitable for firm i, and the uniform-price equilibrium does not exist. Moreover, an asymmetric-price equilibrium clearly exists, because (3) and (5) define an asymmetric-price equilibrium. Firm i cannot increase its profits by changing its price, and all of the other firms are strictly better off free riding and selling at $\hat{p}_2 > p_1^i$ rather than deviating to a first-period price below p_1^i. So, an asymmetric-price equilibrium exists and no uniform-price equilibrium exists.

Now suppose that firm i does not want to deviate. Then a uniform-price equilibrium exists. Now consider any asymmetric-price equilibrium in which firm i charges p. Clearly, $p < \hat{p}_2$ by definition of an asymmetric-price equilibrium, which implies that p is less than p^*, the uniform-price equilibrium price. Firm i's profits in the asymmetric-price equilibrium are given by equation (3).

Notice that (3) is concave. Clearly, (3) is concave if (4) is concave, and the second derivative of (4) with respect to q_1 is

$$p_1''(q_1)q_1 + 2p_1'(q_1) + p_2''\left(\sum_i K^i - q_1\right)\left(K^i - q_1\right) + 2p_2'\left(\sum_i K^i - q_1\right), \tag{12}$$

which is clearly negative because $K^i - q_1 < \sum_i K^i - q_1$ and because because both revenue functions, $p_j(x)x$, are concave.

Now, if $p > p_1^i$, then firm i can profitably deviate to p_1^i, and the price then maximizes (3). And, if $p < p_1^i$, then because (3) is concave and maximized at p_1^i it follows that firm i is strictly better off increasing its price. So, no asymmetric-price equilibrium exists.

8) There are, at most, n asymmetric-price equilibria.

We show that there exists, at most, one asymmetric-price equilibrium in which firm i is the low-priced firm in period one (or, more strictly speaking, such equilibria differ only in the prices of firms with zero sales).

In an asymmetric-price equilibrium, if firm i is the low-price firm, then it is the only firm with positive sales in period 1. Let p denote firm i's price.

If $p > p_1^i$, then firm i can profitably deviate to p_1^i. If $p < p_1^i$, then because $\pi(p)$ is concave and maximized at p_1^i, it follows that firm i is strictly better off increasing its price. So, p does not describe a situation in which an asymmetric-price equilibrium exists.

Therefore, the only asymmetric-price equilibrium that exists in which firm i is the low-price firm in the first period is given by (3) and (5).

Proof of Proposition 3:

Under Assumptions 2, 3 and 5, if a subgame perfect equilibrium exists in which every firm chooses K^* units of capacity, then, by Proposition 2, the unique subgame perfect equilibrium of the pricing subgame is a uniform-price equilibrium. Moreover, for all firm capacities in a neighborhood of K^*, Assumption 5 and Proposition 2 imply that the unique subgame perfect equilibrium of the pricing subgame is a uniform-price equilibrium, so the first-stage profit function for firm i can be written as

$$\Pi^u(K^i; K^{-i}) = \left(p_{tot}\left(\sum_j K^j\right) - c\right)K^i, \tag{13}$$

where K^{-i} is the capacity of the other firms.

Firm i's capacity, K^i, maximizes firm i's profits only if $K^i = K^*$ is the solution to

$$\frac{\partial \Pi^u(K^i; K^*)}{\partial K^i} = p_{tot}((n-1)K^* + K^i) - c + p_{tot}'((n-1)K^* + K^i)K^i = 0, \tag{14}$$

which is concave and has a unique solution, $K^i(K^*)$, which is decreasing in K^*. So, (14) uniquely defines a symmetric solution K^*, and it is easy to see that K^* must be exactly equal to the Cournot quantity associated with n firms, production cost c, and demand $D_{tot}(p)$.

We have shown that $K^i = K^*$ is local best response. Next, we show that $K^i = K^*$ is the global best response when rival firms choose K^*.

Suppose that $K^i < K^*$. If a uniform price equilibrium exists when firm i chooses K^i and other firms choose K^*, then firm i's profits are given by (13), and so firm i's profits at K^i are strictly lower than at K^*.

If a uniform-price equilibrium does not exist, then an asymmetric-price equilibrium must exist. Under Assumption 5, firm i cannot profit by deviating from the uniform-price equilibrium even if its capacity is K^*, so firm i is not the low-priced firm in the first period. The only asymmetric-price equilibrium that can exist is one in which one of firm i's rivals is the firm that sells at the low price in the first period. There are $n-1$ such equilibria because any of the $n-1$ firms with capacity K^* could set the low price in the first period.

Firm i's profit in all of these asymmetric-price equilibria is

$$\Pi^a(K^i;K^*) = \left[p_2\left((n-1)K^* + K^i - D_1(p_1)\right) - c\right]K^i, \tag{15}$$

where p_1 is the price charged in the first period, and so p_1 maximizes

$$D_1(p_1)p_1 + p_2\left((n-1)K^* + K^i - D_1(p_1)\right)\left(K^* - D_1(p_1)\right). \tag{16}$$

Firm i's first order-condition is

$$p_2'\left((n-1)K^* + K^i - D_1(p_1)\right)\left(1 - D_1'(p_1)\frac{dp_1}{dK^i}\right) + p_2\left((n-1)K^* + K^i - D_1(p_1)\right) - c = 0. \tag{17}$$

Because $p_1 < p_2$, $D(p_1)$ is greater than first-period sales at the uniform price. This implies that $n-1$ firms are each selling less than $K^* - D(\tilde{p})/n$ in period 2, where $\tilde{p}$ is the uniform price. In this case, ignoring the impact of K^i on p_1, firm i's best response is greater than $K^* - D(\tilde{p})/n$, which implies that $K^i > K^*$, which is a contradiction. And, as K^i increases, the optimal first-period price falls ($dp_1/dK^i < 0$). Thus, ignoring the impact of K^i on p_1 does not alter the result. This is still a contradiction.

Now suppose that $K^i > K^*$. Again, the equilibrum of the pricing subgame may be an asymmetric-price equilibrium or a uniform-price equilibrium. If it is a uniform-price equilibrium, then by the same argument, profits are strictly lower, which is a contradiction.

If it is an asymmetric-price equilibrium, then it must be an asymmetric-price equilibrium in which firm i sets a low price in the first period. This is because an asymmetric-price equilibrium exists only if a firm wants to deviate from the uniform-price equilibrium, and equation (10) tells us that a firm wants to deviate only if $\eta_2(p)/\eta_1(p)$ exceeds its share of capacity. But by Assumption 5, this happens only if the capacity share exceeds $1/n$ and only firm i's share of capacity exceeds $1/n$.

So, if firm i deviates to $K^i > K^*$, then its profit must be

$$\max_{p_1} D_1(p_1)\, p_1 + p_2\left((n-1)K^* + K^i - D_1(p_1)\right)\left(K^i - D_1(p_1)\right).$$

Rewriting this as a function of quantity yields

$$\max_{q_1} p_1(q_1)q_1 + p_2\left((n-1)K^* + K^i - q_1\right)\left(K^i - q_1\right). \tag{18}$$

Thus, the firm's profit in stage 1 is

$$\max_{q_1} p_1(q_1)q_1 + p_2\left((n-1)K^* + K^i - q_1\right)\left(K^i - q_1\right) - cK^i, \tag{19}$$

and its maximized stage 1 profit is

$$\max_{q_1,K_1} p_1(q_1)q_1 + p_2\left((n-1)K^* + K^i - q_1\right)\left(K^i - q_1\right) - cK^i, \tag{20}$$

which we can rewrite using a change of variables ($q_2 = K^i - q_1$) as

$$\max_{q_1,q_2} p_1(q_1)q_1 - cq_1 + p_2\left((n-1)K^* + q_2\right)q_2 - cq_2. \tag{21}$$

Therefore, q_1 is the first-period monopoly output. and q_2 is the second-period best response to $(n-1)K^*$. But this is not an equilibrium unless $p_1 < p_2$, or equivalently the Lerner index in the first period is smaller than the Lerner index in period 2, or

$$\frac{p_1'(q_1)q_1}{p_1(q_1)} < \frac{p_2'\left((n-1)K^* + q_2\right)q_2}{p_2\left((n-1)K^* + q_2\right)} \tag{22}$$

$$\frac{1}{\eta_1(p_1)} < \frac{1}{\eta_2(p_2)}\frac{q_2}{((n-1)K^* + q_2} \tag{23}$$

or

$$\frac{\eta_2(p_2)}{\eta_1(p_1)} < \frac{q_2}{((n-1)K^* + q_2}, \tag{24}$$

which violates Assumption 4 because $q_2 < K^*$. So, this is a contradiction and no global deviation is profitable.

Proof of Proposition 4:

Proof. Consider an equilibrium in which, on the equilibrium path, firms choose capacity equal to the sum of the Cournot capacity in each period, $q_1^C + q_2^C$, and then set the Cournot price, p_t^C, and set inventory controls equal to the Cournot output in each selling period,

$k_1^i = q_1^C$. Off of the equilibrium path, they set the market-clearing price in the last period and, they use price and inventory controls to allocate their output across the two selling periods in order to equalize the marginal revenue given the initial capacities in the first selling period.

Clearly, no deviation is profitable in the final period. Lemma 1 holds, and the second-period prices for all firms are equal to the market-clearing prices (the presence of inventory controls does not change this result).

Consider the first selling period, and suppose that firm i deviates to a lower price. Assumption 1 implies that $p_1^C < p_2^C$, so a small decrease in price discontinuously increases firm i's first-period sales, decreases firm i's second-period sales, and decreases firm i's profits. More importantly, profits cannot be higher than profits in a uniform-price equilibrium because an asymmetric-price equilibrium does not exist by Proposition 2, and profits in the candidate equilibrium are even higher than profits in the uniform price equilibrium. So, no deviation to a lower price is profitable.

Suppose, instead, that firm i deviates to a higher price in the first selling period. Under the efficient rationing rule, the residual demand function facing the deviating firm is $RD_1^i(p; p_1^C, q_1^C) = D_1(p) - (n-1)q_1^C$. Note that the rival firms' quantities are no longer their capacities, but, instead, are their inventory controls.

Since the shadow cost of capacity is c on the equilibrium path (and more generally is the same in both periods), firm i's first-period profit function is $(D_1(p) - (n-1)q_1^C)(p - c)$ or, equivalently, $(p_1((n-1)q_1^C + q) - c)q$. Thus, the optimal price deviation is given by the first-order condition, which is

$$p_1'\left((n-1)q_1^C + q\right)q + p_1\left((n-1)q_1^C + q\right) = c.$$

But this implies that $q = q_1^C$ and that the optimal price and quantity is the first-period Cournot output (or. more generally. the output that equalizes the marginal revenue across the two periods), so no deviation to a higher price is profitable.

Under proportional rationing, the residual demand function facing the deviating firm is $RD_1^i(p; p_1^C, q_1^C) = D_1(p)\left[1 - \frac{(n-1)q_1^C}{D_1(p_1^C)}\right] = \frac{1}{n}D_1(p)$ since $D_1(p_1^C) = nq_1^C$. The shadow cost of capacity is c on the equilibrium path (but, more generally, is equalized in the two periods), so firm i's first-period profit function is $\frac{1}{n}D_1(p)(p - c)$, or equivalently, $p_1(nq) - c)q$. The first-order condition is $p_1(nq) + p_1'(nq)\,q = c$, which implies that $q = q_1^C$, so no deviation to a higher price is profitable.

In stage 0, firms choose capacity expecting to equalize marginal revenue across periods 1 and 2. It is easy to see that $K^i = q_1^C + q_2^C$ is a best response to $K^j = q_1^C + q_2^C$ for all $j \neq i$. ■