INTERTEMPORAL PRICE DISCRIMINATION IN SEQUENTIAL QUANTITY-PRICE GAMES

By

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Intertemporal Price Discrimination in Sequential Quantity-Price Games

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Abstract

This paper develops an oligopoly model in which firms first choose capacity and then compete in prices in a series of advance-purchase markets. We show the existence of multiple sales opportunities creates strong competitive forces that prevent firms from utilizing intertemporal price discrimination. We then show that intertemporal price discrimination is possible, but only when firms adopt inventory controls (sales limit restrictions) and demand becomes more inelastic over time. Therefore, in addition to being useful to manage demand uncertainty, we show that inventory controls are also a tool to soften price competition. We discuss model extensions, including product differentiation, aggregate demand uncertainty, and longer sales horizons.

JEL Classification: D21, D43, L13

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1 Introduction

In many market settings, firms commit to capacities and then sell their inventories to consumers over time. Classic examples include selling seats on a plane or train, or selling rooms at a hotel or on a cruise ship. In these industries, capacity is fixed, and firms frequently adjust their prices over time both in response to demand shocks that affect scarcity and to changes in the overall price sensitivity of arriving consumers. Sophisticated pricing systems, commonly referred to as revenue management, automate this process. Although there exists a substantial literature in economics, marketing, and operations research on the design, implementation, and consequences of using these pricing systems, much of this existing research focuses on a monopolist facing uncertain demand. It is unclear how these insights carry over to the common setting where firms compete for sales and have the incentive to undercut each others' ability to price discriminate before the sales deadline.

In this paper, we extend the seminal research by Kreps and Scheinkman (1983) and Davidson and Deneckere (1986) on sequential quantity-price games to multiple sales periods. We show that when firms first choose capacity and then compete for customers over time, strong competitive forces exist that prevent firms from utilizing intertemporal price discrimination. This occurs even in situations where this form of price discrimination would clearly increase industry profits. We highlight two main contributions. First, we show that equilibrium prices are flat or uniform over time, unless firms make additional commitments. This occurs because the existence of multiple sales periods creates a costless arbitrage opportunity in which a capacity-constrained firm can increase its profits by shifting sales in periods with price sensitive consumers to its capacity-constrained rivals. This strategy allows the firm to increase its market share in periods with price insensitive consumers. Other firms have similar strategic incentives. In the absence of additional commitments, these strong competitive forces lead to intense price competition that prevent firms from setting increasing prices when demand becomes less price sensitive over time. It also prevents firms from setting decreasing prices when demand becomes more price sensitive over time. Second, we show that intertemporal price discrimination is possible when firms adopt inventory controls, but only if demand becomes more inelastic over time. Inventory controls are sales limits that prevent firms from selling *too much* capacity at a given price. These controls are a key feature in the revenue management systems that have been adopted by airlines, cruises, hotels, trains, and entertainment or sports events. Therefore, in addition to being a beneficial tool to manage remaining inventory when demand is uncertain, we identify another benefit of using inventory controls—they can increase industry profits by softening price competition. We discuss extensions of our baseline model that reflect market characteristics in the aforementioned examples, including product differentiation, aggregate demand uncertainty, discounting, and longer sales horizons.

The baseline model in Section 2 considers an oligopoly setting where firms sell a homogeneous good. Firms first choose capacity—an output constraint that is common across selling periods—and then compete in prices in a series of sequential markets. In each period, firms' remaining capacities are observed, and then firms simultaneously choose prices, and consumers make their purchase decisions. After the final period, unsold inventory is scrapped with zero value, e.g., tickets for a concert that has already happened no longer hold any value. For tractability reasons, the baseline model considers two advance-purchase sales periods. We assume that there are a continuum of consumers, some of whom arrive in each one of two sequential markets. We assume that consumers assigned to the early market can wait and purchase in the later market. We also allow the elasticity of demand to change over time. We emphasize the case in which demand becomes more inelastic for two reasons. First, it is clear that a monopolist would set increasing prices in this case, and second, prices tend to rise in several industries in which firms have fixed inventory, adjust prices over time, and face competition, e.g., see Puller, Sengupta, and Wiggins (2012) and Siegert and Ulbricht (2020) for descriptive analyses on airline pricing.

Because firms are free to sell their capacity in any time period, our result may not seem surprising. However, if firms choose output each period as opposed to prices (and prices clear the market each period), then firms would equalize marginal revenues across time and not equalize prices. One of our contributions is showing that our sequential quantity-price game is different from the sequential capacity-output model, even though the two models always coincide under our assumptions when there is only one sales period. An additional contribution is characterizing sufficient conditions under which uniform prices arise as the unique pure-strategy equilibrium outcome. Although we find that uniform pricing arises whether the elasticity of demand is increasing or decreasing over time, we also show there exist important asymmetries in the sufficient conditions that guarantee uniform pricing in these two scenarios.

We then enrich the model by allowing firms to implement inventory controls each period in conjunction with price setting. In this model, firms choose an initial overall capacity limit and then simultaneously choose sales (or quantity) limits and prices in each period. We show there exist equilibria in which prices are increasing when demand becomes more inelastic over time. Price discrimination is possible because inventory controls protect a firm from rivals who want to exploit the arbitrage opportunity by increasing its market share in early periods with more elastic demand and decreasing its market share in late periods with less elastic demand. Because deviating to a higher price in the early period is not profitable with inventory controls, firms instead choose to sell at Cournot prices in each period, thereby facilitating intertemporal price discrimination and increasing profits. On the other hand, we also show that inventory controls are ineffective at facilitating price discrimination when demand becomes more elastic over time. This is for two reasons. First, a rival can increase its market share in early, more profitable periods by cutting price—inventory controls do not protect the firm because they only create an upper bound on sales (they cap total sales in early, low-price periods, but do not put a floor on sales in early, high-price periods). Second, forward-looking consumers constrain firms' early-period pricing because they can choose to purchase in the later period.

Our results provide an additional rationale for the widespread use of inventory controls in the aforementioned industries. As detailed in Hortaçsu, Natan, Parsley, Schwieg, and Williams (2021), firms in capacity-constrained industries dynamically release inventory over time, thus preventing them from selling all remaining inventory at a single price. For example, customers shopping for airline tickets may observe a warning that there are "5 seats left" at a particular price.[1] This occurs because prices have corresponding sales-quantity limits. If a firm attempted to shift price-sensitive consumers to its rivals, inventory controls will cause rival firm prices to rise as units are sold, thereby decreasing the effectiveness of the strategic force explored in this paper. In the previous airline example, at most five

[1]Examples can be viewed at https://travelupdate.com/5-seats-left-booking-flights/, accessed 10/5/2021.

seats would be sold at the current price instead of potentially the entire plane. Our analysis suggests it is profitable for firms to adopt inventory controls because they soften price competition. Indeed, managers at American Airlines describe inventory controls as a "strong competitive tool" that determines the "revenue-mix", or *who purchases and at what prices* (Smith, Leimkuhler, and Darrow, 1992).

Finally, we discuss model extensions. First, we consider product differentiation. We argue that prices are no longer uniform across time because firms are unable to shift all of their sales to rivals using very small price changes. However, the strategic incentives explored in our baseline model are still present. We use an example to show that products must be highly differentiated in order for prices to increase substantially over time absent firms using inventory controls. Nonetheless, our analysis suggests that the adoption of inventory controls is most valuable when products are close substitutes. We also show how our results can generalize to many periods with additional assumptions. Finally, we discuss discounting, models with aggregate demand uncertainty, and specify alternative ways to model inventory controls.

1.1 Related Literature

This paper contributes to several strands of literature in economics, management, and marketing. First, we contribute to a large literature on price competition with capacity constraints (Levitan and Shubik, 1972; Allen and Hellwig, 1986; Osborne and Pitchik, 1986; Klemperer and Meyer, 1986; Acemoglu, Bimpikis, and Ozdaglar, 2009). Our work complements Van den Berg, Bos, Herings, and Peters (2012), who consider a similar model setup but do not allow firms to shift low-priced sales to rivals.[2]

Second, we analyze intertemporal price discrimination. Stokey (1979) is a seminal paper that shows that intertemporal price discrimination is not always feasible in the monopoly setting.[3] In recent theoretical work, including Öry (2016) and Dilmé and Li (2019), forward-looking buyers who can postpone their purchases constrain the a monopolist's ability to discriminate. These so called Coasian forces are also present in our model when demand becomes more elastic over time. When demand becomes more inelastic over time, consumers cannot make themselves better off by postponing their purchases because they face increasing prices. Here, competition is a key constraint on price discrimination—the temptation to arbitrage differences in rivals' prices can be so strong that it prevents intertemporal price discrimination.[4]

Another reason that prices may adjust over time is that consumers may learn more about their preferences as the time of consumption approaches, and firms may set price accordingly. When consumers learn over time, monopolists may be able to perfectly price discriminate (Akan, Ata, and Dana, 2015; Ata and Dana, 2015). In competitive markets, learning may result in prices that increase over time Dana (1998). Consumers may also learn about their preferences through repeat purchase, and learning through repeat purchase also interacts in important ways with competition, particularly when consumers are forward looking (Villas-Boas, 2004). However, in this paper we assume that consumers know their preferences upon arrival and we abstract from repeat purchases. Therefore, these effects are

[2]See Benassy (1989) and Reynolds and Wilson (2000) for related pricing games, Aguirre (2017) for a related quantity games, and De Frutos and Fabra (2011) for a related price and capacity game.

[3]Nair (2007) and Williams (2021) conduct empirical studies on intertemporal price discrimination where prices rise (fall) over time.

[4]See also Champsaur and Rochet (1989).

not present.

The marketing literature has also emphasized strategies for shifting demand when capacity is fixed, particularly service industries (Shugan, 2002). While our model does not have have multiple consumption periods, this issue is relevant for the hotels as well as other industries where capacity is constrained. Firms often have strong financial incentives to use price and non-price strategies to smooth demand variation, particularly when it is costly to shift supply, and including when the demand variation is unpredictable (Shugan, 2002; Dana, 1999). Non-price strategies may also be available, including advertising (Horstmann and Moorthy, 2003) and the timing of product introductions (Radas and Shugan, 1998).

Finally, we analyze inventory controls. Research in operations management has shown that inventory controls can be an effective tool to manage demand uncertainty, but this research abstracts from their ability to increase profits with known demand in the presence of competition.[5]

2 The Model

Consider an oligopoly with n firms selling a homogeneous good to a continuum of consumers in a series of advance-purchase sales markets. For tractability, we consider just two selling periods, or stages. Some consumers arrive in Stage 1 while others arrive in Stage 2. We assume that consumers who arrive earlier can purchase in either Stage 1 or Stage 2, while consumers who arrive later can only purchase in Stage 2. Consumption takes place afterwards, in Stage 3.

We assume that consumers know their valuations for the good when they arrive. The valuations of consumers who arrive early do not change if they wait

[5]See Talluri and Van Ryzin (2006) and McGill and Van Ryzin (1999) for overviews of this work.

to purchase later.[6] Although we do not explicitly consider discounting, all of our results generalize since we can interpret all prices as prices in the units of Stage 3 dollars. That is, we treat all payments as if they are made at the time of consumption.

We represent preferences using market demand functions, denoted by $D_1(p)$ and $D_2(p)$, respectively. We assume these functions are strictly decreasing and twice differentiable. We let $P_1(q)$ and $P_2(q)$ denote the inverse demands associated with $D_1(p)$ and $D_2(p)$, respectively, and we assume that $P_t''(q)q+2P_t'(q) < 0, \forall t = 1, 2$. Throughout the paper, we use p_t^i to denote Firm i's price; we use $\mathbf{p}_t$ to denote the vector of all firms' prices; and we use p_t to denote the Stage t price when all Stage t transactions occur at this price. We use $-i$ to denote firms other than i. We let $D^{tot}(p) = D_1(p) + D_2(p)$ denote the total demand when prices are the same in both stages and $P^{tot}(q)$ denote the associated inverse total demand when q units of total output are sold at a uniform price.

The cost per unit of capacity for all firms is c. We make the simplifying assumption that the marginal cost of production for each unit sold is zero (all the costs of production are associated capacity, not sales). We let $\eta_t(p) = D_t'(p)p/D_t(p)$ denote the price elasticity of demand in Stage t.

Each firm's strategy consist of three choices, capacity and two prices. The game proceeds in three stages (see Figure 1). First, in Stage 0, firms simultaneously choose their capacities, denoted by $K^i \geq 0$ for firm i or by the vector $\mathbf{K}$. Then, in Stage 1, firms simultaneously choose prices ($\mathbf{p}_1$), and consumers who arrive in Stage 1 then make their purchase decisions. Sales, $\mathbf{q}_1 \geq \mathbf{0}$, are constrained only

[6]Alternatively, following Dana (1998) and Akan, Ata, and Dana (2015), we could have assumed that some consumers do not learn their demands until Stage 2 and then make additional mild assumptions that imply that these consumers would never want to purchase in Stage 1 even if they were able to.

by the firms' initial capacities, $\mathbf{K}_1 = \mathbf{K}$ in Stage 1. Sales in Stage 2 are constrained by firms' residual capacities, $\mathbf{K}_2 = \mathbf{K} - \mathbf{q}_1 \geq \mathbf{0}$. That is, the capacity constraint is common across stages. Note that we are making the natural, but empirically strong, assumption that the firm cannot refuse sales at its Stage 1 price in order to reserve more of its inventory for Stage 2. This is important because of the strategic uncertainty about rival firms' prices. We relax this assumption in Section 5 where we introduce inventory controls. In Stage 2, firms simultaneously choose prices ($\mathbf{p}_2$), and consumers who arrive in Stage 2 (or waited) make their purchase decisions. Capacity not used in Stage 2, $\mathbf{K}_2 - \mathbf{q}_2$, has zero value (it is scrapped at no cost). We ignore discounting.

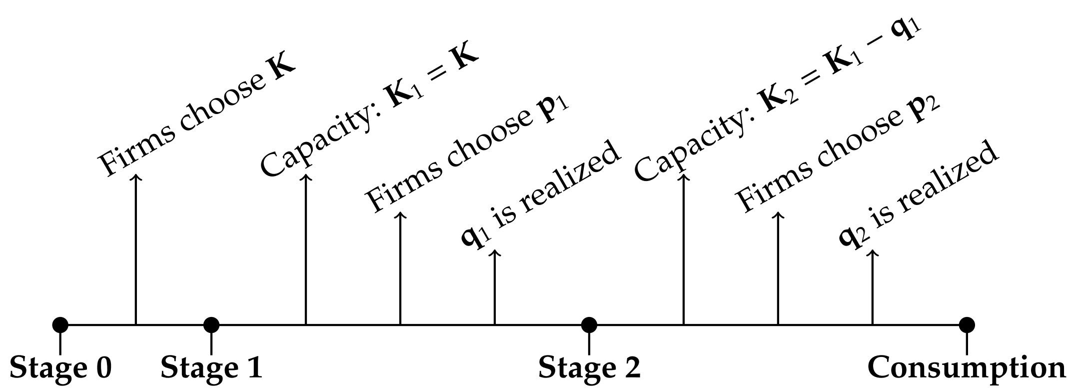


Figure 1: Timing of the Game

2.1 Pure Strategies and Residual Demand

Quantity-price games, including Kreps and Scheinkman (1983) and Davidson and Deneckere (1986), are known to have mixed strategy equilibrium off of the equilibrium path, which makes them difficult to solve. Because we have multiple pricing periods, characterizing the equilibria of the pricing subgame is considerably more challenging. To simplify our analysis, we assume that capacity costs are sufficiently large so that all of the pricing subgames have pure-strategy equilibria.

In the Appendix, we derive a lower bound on capacity costs that implies that in every equilibrium, total industry capacity is less than $\text{argmax}_q P_2(q)q$. In other words, no firm can profit by holding back some of its capacity in the final period of the game. Let c_L denote this lower bound. We make the following assumption:

Assumption 1. *The cost of capacity satisfies $c \geq c_L$.*

This assumption reflects industries such as airlines and hotels, where capacity costs are high. Note that Assumption 1 is only a sufficient condition. It may be possible that smaller capacity costs generate pure strategy equilibria. Our results may hold even when some of the off-the-equilibrium-path pricing subgames do not have pure strategy equilibria. Alternative assumptions also give the same results, such as assuming demand is isoelastic (see Madden 1998).

In addition to capacity costs, we must define the how capacity is rationed, particularly when firms set different prices. Throughout the paper, we accommodate multiple rationing rules, stated below.

Assumption 2. *Rationing is either efficient or proportional.*

We define residual demand for firm i in stage t as $RD_t\left(p_t^i; \mathbf{p}_t^{-i}, \mathbf{K}_t^{-i}\right)$, which can be written explicitly as

$$RD_t(p_t^i; \mathbf{p}_t^{-i}, \mathbf{K}_t^{-i}) = D_t(p_t^i) - \sum_{j:\ p_t^j < p_t^i} K_t^j, \forall t = 1, 2, \tag{1}$$

under the efficient rationing rule, and as

$$RD_t(p_t^i; \mathbf{p}_t^{-i}, \mathbf{K}_t^{-i}) = D_t(p_t^i)\left[1 - \sum_{j:p^j<p^i} \frac{K_t^j}{D_t(p_t^j)}\right], \forall t = 1, 2, \tag{2}$$

under the proportional rationing rule.

3 A Benchmark Result

Before characterizing the equilibrium of our game, we consider a useful benchmark result. Suppose that firms are constrained to set the same price in Stage 1 and Stage 2—that is, $p_1^i = p_2^i, \forall i$. Then, in equilibrium, $\mathbf{K}$ must be the symmetric Cournot output, or the Cournot output when demand is $D_1(p) + D_2(p)$. This is because Assumptions 1 and 2 imply that the equilibrium price in the pricing subgame is always equal to the market clearing price, so the Stage 0 capacity game reduces to a standard Cournot model. We formalize this idea in the following lemma. All proofs, except the proof of Lemma 3, are in the Appendix.

Lemma 1. *When firms are constrained to choose the same price in Stage 1 and Stage 2, if Assumptions 1 and 2 hold, then the equilibrium price in every Stage 1 and Stage 2 pricing subgame is the market-clearing price defined implicitly by $D_1(p) + D_2(p) = \sum_i K^i$. The equilibrium capacities chosen in Stage 0 are the Cournot capacities associated with demand $D^{tot}(p) = D_1(p) + D_2(p)$.*

Note that as the number of firms goes to infinity, the price converges to the cost of capacity, c. Therefore, we refer to c as the competitive price.

4 Equilibrium Characterization

We now solve for the subgame perfect equilibrium of the full model described in Section 2, starting with Stage 2 and working backwards to Stage 0.

4.1 The Final Pricing Period

We begin by characterizing equilibrium prices in the final pricing period (Stage 2). Lemma 2 states that in every Stage 2 subgame, firms set prices to clear the market.

Lemma 2. *Under Assumptions 1 and 2, in any subgame perfect equilibrium (SPE) of the three-stage game, the price in the second selling period clears the market.*

Assumption 1, which implies Lemma 2, is important because it allows us to easily characterize all of the pure-strategy subgame-perfect equilibria of the pricing subgame.

4.2 No Intertemporal Price Discrimination in Symmetric Equilibria

We define a uniform-price equilibrium to be an equilibrium in which all transactions occur at the same price. That is, either prices are equal across time (so transactions are necessarily at the same price), or prices decline over time, but all consumers purchase in Stage 2 at the same price. We say that an equilibrium is *symmetric* as long as the transactions prices in each stage are the same for all firms. We say that the equilibrium is *unique* if the prices that consumers pay, and the capacities that firms choose, are uniquely defined.

Lemma 3. *Under Assumptions 1 and 2, any symmetric pure-strategy equilibrium of the pricing subgame is a uniform-price equilibrium.*

Proof. Suppose that a subgame perfect equilibrium of the pricing subgame exists in which $p_1 < p_2$. If firm i deviates to a price $\hat{p} > p_1$, but arbitrarily close to p_1, in Stage 1, then its sales would be the larger of 0 and $RD_1(\hat{p}; \mathbf{p_1}, \mathbf{K^{-i}})$. If the residual demand is zero then Stage 1 sales are unchanged, equal to $D_1(p_1)$, and the Stage 2 market clearing price is unchanged by Lemma 2. This implies that firm i's profits are strictly higher, which is a contradiction. If the residual demand is positive, then $RD_1(\hat{p}; \mathbf{p_1}, \mathbf{K_1^{-i}}) + \sum_{j \neq i} K_1^j$ is arbitrarily close to $D(p_1)$ for both rationing rules because $\hat{p}$ is arbitrarily close to p_1, so the market clearing price in Stage 2 is arbitrarily close to p_2. This implies that firm i's profits are strictly higher following its deviation, which is a contradiction.

Suppose that a subgame perfect equilibrium of the pricing subgame exists in which $p_1 > p_2$. In this case, since consumers can choose to wait, there are no transactions in Stage 1, and all transactions take place in Stage 2 at a price p_2. This implies that all transactions prices are the same. That is, there must also exist a payoff-equivalent equilibrium in which the Stage 2 prices are unchanged, but $p_1^i = p_2$ for all i.[7] ■

Lemma 3 demonstrates the strong competitive forces in the model. If prices changed over time (in a symmetric equilibrium), individual firms could change their prices in order to increase their sales in the higher-priced period. Prices cannot rise over time because firms can raise their Stage 1 price to shift sales to

[7] Note that even if consumers could not wait, which we think is unrealistic, any firm with strictly positive sales in Stage 2 could deviate to a price $\hat{p} < p_1$ that is arbitrarily close to p_1. Total Stage 1 sales would be arbitrarily close to $D(p_1)$ under either rationing rule because $\hat{p}$ is arbitrarily close to p_1. Then the market clearing price in Stage 2 is arbitrarily close to p_2, which implies the deviation is profitable. So, the proof does not depend on our assumption that consumers can wait.

rivals in Stage 1 and thereby sell more in Stage 2. Similarly, prices cannot fall over time for the same reason. However, recall that even a monopolist cannot benefit from declining prices because we assumed that consumers can wait until prices are lower to make their purchases. Therefore, there are two reasons that prevent prices from declining over time.

Although symmetric equilibria must have uniform prices, asymmetric equilibria may also exist. In the following subsections, we characterize reasonable conditions under which only symmetric equilibria exist.

4.3 Decreasing Elasticity of Demand

4.3.1 The Pricing Subgame

In Proposition 1, below, we show that there are two types of pure-strategy subgame perfect equilibria in the pricing subgame when demand becomes more inelastic over time. Since the market clears in Stage 2 by Lemma 2, any uniform-price equilibrium must satisfy $D_1(p^*) + D_2(p^*) = \sum_i K^i$. The uniform price is unique by Lemma 1, though consumption can take place in both periods or just in Stage 2.

In an asymmetric-price equilibrium, a single firm sells in Stage 1. The Stage 1 price is lower than the Stage 2 price, and all other firms sell only in Stage 2. Intuitively, the firm that sells in Stage 1 is pushing up the price in Stage 2 by limiting Stage 2 capacity. So in a sense the firm is providing a public good. It follows that only one firm sets a low Stage 1 price and the others free ride.

Let Firm i be the firm that sells in the Stage 1, and let p_1^i and q_1^i denote its first-period price and quantity, where

$$p_1^i = \operatorname*{argmax}_{p \in [P_1(K^i), \infty]} pD_1(p) + P_2\left(\sum_i K^i - D_1(p)\right)\left(K^i - D_1(p)\right), \tag{3}$$

or, equivalently,

$$q_1^i = \operatorname*{argmax}_{q \in [0, K^i]} P_1(q)q + P_2\left(\sum_i K^i - q\right)\left(K^i - q\right). \tag{4}$$

Firm i's first-period sales do not exceed K^i, and the second-period price is higher than p_1^i and is given by

$$P_2\left(\sum_i K^i - D_1(p_1^i)\right). \tag{5}$$

Note that Proposition 1 holds regardless of whether or not the elasticity is decreasing.

Proposition 1. *Under Assumptions 1 and 2, every pure-strategy subgame-perfect equilibrium of the pricing subgame is either a uniform-price equilibrium or an asymmetric-price equilibrium satisfying Equations (3), (4) and (5). When a uniform-price equilibrium exists, it is the unique pure-strategy subgame-perfect equilibrium. When a uniform-price equilibrium does not exist, then at least one, and at most n, asymmetric-price equilibria exist.*

Intuitively, asymmetric-price equilibria exist because a lower price in Stage 1 increases sales in Stage 1, leading to less output sold and a higher price in Stage 2. A firm can increase its profit in this way only if the elasticity is decreasing (so increasing prices is desirable) and only if it has sufficient capacity to meet all of the demand in Stage 1 plus enough additional capacity to profit from selling at the higher price in Stage 2. Other firms free ride and sell only in Stage 2 at the higher price.

Asymmetric-price equilibria are more likely to exist than uniform-price equilibria when one firm chooses significantly more capacity than its rivals in Stage 0. The incentive to deviate to a lower price is increasing in the deviating firm's capacity, decreasing in the rival firms' capacities, increasing in the relative elasticity of Stage 1 demand, and decreasing in the relative magnitude of Stage 1 demand.

Like Lemma 2, Proposition 1 highlights the pressure on competing firms to equalize prices across periods. Unless one firm is sufficiently large and can unilaterally implement an asymmetric-price equilibrium, the equilibrium is a uniform-price equilibrium. Although Proposition 1 shows that asymmetric-price equilibria of the pricing subgame may exist, we now show that under relatively mild additional assumptions, the uniform-price equilibrium is unique even when the elasticity of demand is decreasing.

Importantly, we show in the next section that when firms choose their capacities optimally, they will choose symmetric capacities which results in a uniform-price equilibrium in the pricing sub-game.

4.3.2 Initial Capacity Choice

We now consider the full game, which includes Stage 0. We make an additional assumption (Assumption 3).

Assumption 3. *The elasticity of demand satisfies*

$$\frac{\eta_2(p)}{\eta_1(p)} > \frac{1}{n}, \forall p > 0.$$

This assumption implies that the elasticity of demand is not decreasing too quickly. Intuitively, we need this assumption in order to guarantee that a firm does not want to choose so much capacity that it can supply all of demand at a lower price in period 1 and still have sufficient remaining capacity to make profits selling at higher price in period 2 (even though its rivals are selling all of their capacity in period 2). Assumption 3 is a sufficient condition that implies such deviations are not profitable.

Using this additional assumption, we now show in Proposition 2 below that intertemporal price discrimination is impossible in oligopoly markets when demand becomes more inelastic over time.

Proposition 2. *If demand becomes more inleastic over time, then under Assumptions 1, 2 and 3, the unique pure-strategy subgame-perfect Nash equilibrium of the full game is a uniform-price equilibrium. The equilibrium capacity and profits are equal to the Cournot capacity and profits given demand* $D_1(p) + D_2(p)$.

4.4 Increasing Elasticity of Demand

We now establish results under the case in which demand becomes more elastic over time. In this case, consumers have an incentive to wait to purchase. These Coasian forces can prevent even a monopolist from using intertemporal price discrimination.

Proposition 3 establishes that prices are always uniform in the pricing subgame when demand becomes more elastic over time.

Proposition 3. *When demand is constant or becomes more elastic over time, then under Assumptions 1 and 2, the uniform-price equilibrium is the unique pure-strategy subgame-perfect equilibrium of the pricing subgame.*

This result holds for two reasons. First, the same competitive forces that constrain firms when the elasticity of demand is increasing constrain firms when the elasticity of demand is decreasing. That is, firms want to shift lower priced sales onto their rivals. Second, price discrimination is also constrained by the fact that consumers can wait and purchase in Stage 2 if prices decline over time.

4.4.1 Initial Capacity Choice

We now consider the full game, including the initial capacity choice.

Proposition 4. *When demand is constant or becomes more elastic over time, then under Assumptions 1 and 2, the unique pure-strategy subgame-perfect Nash equilibrium of the full game is a uniform-price equilibrium. Equilibrium capacity and profits are equal to the Cournot capacity and profits given demand* $D_1(p) + D_2(p)$.

Proposition 4 follows immediately from previous results. When demand becomes more elastic over time, Assumption 3 is always satisfied, so the Cournot model is even more robust to breaking up demand into multiple pricing periods. However, this is largely because consumers have the option to wait.[8]

5 Inventory Controls

In the previous section, we showed that firms choose capacity equal to the Cournot output and set the same price in both pricing periods. They set the one-shot Cournot price and quantity, even when profits would be higher with intertemporal price discrimination.

We now show that inventory controls allow firms to price discriminate and earn higher profits, but only if demand becomes more inelastic over time. We

[8] In an earlier version of the paper, we assumed that consumers did not have the option to wait and showed that a uniform-price equilibrium may not always exist under increasing elasticity of demand.

model inventory controls as an upper bound on the quantity sold each pricing period, and we allow firms to set inventory controls when they set their price. So firms first choose their initial capacity, and then, in each of the subsequent pricing periods, simultaneously choose both their price and an inventory control. For two pricing periods the timing is shown in Figure 2.

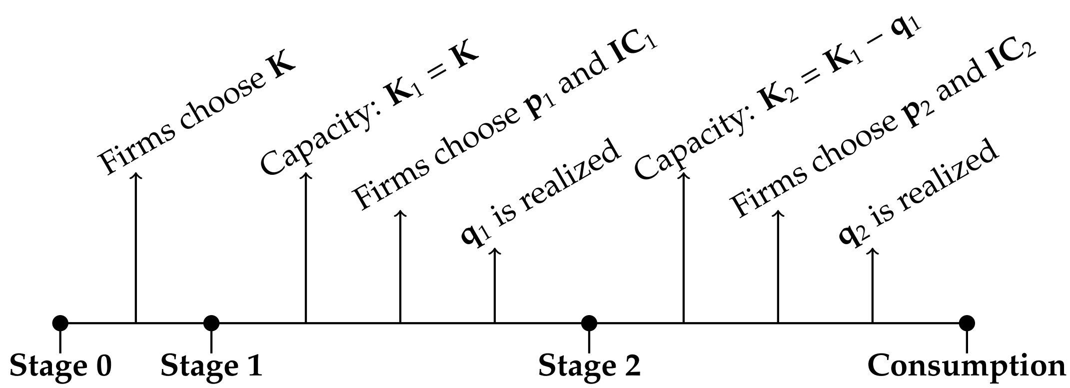


Figure 2: Timing of the Game with Inventory Controls

Inventory controls guarantee that if a rival deviates to a higher price in Stage 1, then its own sales will not increase. Inventory controls place a cap on sales but not a floor. Hence, inventory controls highlight another natural asymmetry between increasing and decreasing elasticity of demand: Inventory controls can prevent a rival from increasing a firm's sales by deviating to a higher price, but they cannot prevent a rival from lowering a firm's sales by deviating to a lower price.

Proposition 5. *If demand becomes more inelastic over time, then under Assumptions 1, 2 and 3, a subgame-perfect Nash equilibrium of the model with inventory controls exists in which all firms set the Cournot price and set inventory controls equal to the Cournot quantity in each selling period. Profits are strictly higher in this equilibrium than in the uniform-price equilibrium.*

In the equilibrium described in Proposition 5, firms commit to inventory controls that are equal to each firm's equilibrium sales in each stage. Inventory controls do not restrict output on the equilibrium path, but they do act as a strategic commitment device because they constrain the firm's off-the-equilibrium-path output. In equilibrium, firms sell the Cournot output associated with each stage, and so prices rise over time because demand becomes more inelastic.[9] This is contrast to the model without inventory controls in which firms prices are constant and firms sell the Cournot quantity associated with the aggregate demand, $D_1(p) + D_2(p)$.

The model with inventory controls does have other equilibria. In particular, the symmetric capacity, uniform-price equilibrium characterized in Proposition 2 may still be a subgame perfect equilibrium of the inventory control game. There are many different increasing price paths that can be supported with inventory controls. We think that it is natural for firms to coordinate on the Cournot quantities.

6 Extensions

6.1 Product Differentiation

Introducing product differentiation does not alter the firms' incentives to attempt to shift demand to their competitors in Stage 1 when demand becomes more inelastic over time. However, product differentiation does make shifting demand

[9] We could have assumed that firms commit to observable inventory controls before setting their price. In this case inventory controls would serve the same purpose, but also place an observable limit on the firm's own sales which reduces the firm's return from price cutting. While this means that the set of subgame perfect Nash equilibria is undoubtedly different, Proposition 5 still holds – if each firm sets an inventory control equal to the Cournot output, then firms would clearly set Cournot prices, and no unilateral inventory control deviation could increase profits.

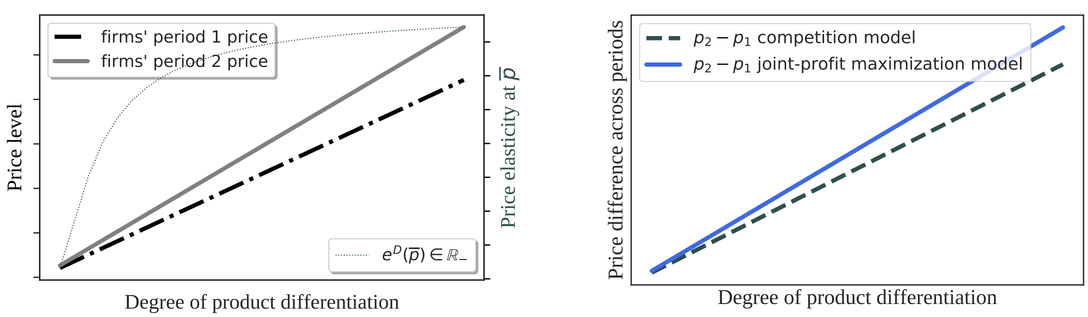
more costly. When products are homogeneous, a small price increase shifts every consumer to the rivals. With differentiated products, any price increase will shift fewer consumers to the rivals, and the profit increase in Stage 2 will be smaller.

Product differentiation also introduces increased complexity, so to illustrate its impact, we focus our attention on two firms in a symmetric environment. We provide intuition instead of analyzing the equilibrium of the model. We maintain the assumption that capacity is sufficiently small so that firms always set market-clearing prices in Stage 2.

Figure 3: Intertemporal Price Discrimination as a Function of Product Differentiation

(a) Prices Across Periods

(b) Competition vs. Joint-Profit Maximization



Notes: Example constructed using a random utility model (logit) with two firms and two periods. Product differentiation is increasing towards the right of the plots. (a) The light dashed line corresponds to the own-price elasticity for a constant price offered by both firms. As products become increasingly differentiated, the difference between p_1 and p_2 increases. (b) Shows the change in price ($p_2 - p_1$) of competition model versus the joint-profit maximization model. Prices are flatter in the competition model, as the gap between the two models grows with the degree of differentiation.

Product differentiation results in equilibrium subgame prices that are no longer uniform over time; however, prices are flatter for any amount of product differentiation than the joint-profit-maximizing prices (see Figure 3 for an example, where the left plot shows increasing differences in prices across periods as product differentiation increases). To see this, consider two firms, A and B, and let the inverse demand functions be $P_1^A\left(q_1^A, q_1^B\right)$, $P_1^B\left(q_1^A, q_1^B\right)$, $P_2^A\left(q_2^A, q_2^B\right)$, and $P_2^B\left(q_2^A, q_2^B\right)$. Joint-profit-maximizing firms would set marginal revenue equal to the shadow cost of capacity in each of the four product markets, so $\frac{\partial P_i^j\left(q_t^j, q_t^{-j}\right)}{\partial q_t^j} q_t^j + P_t^j\left(q_t^j, q_t^{-j}\right) = \lambda, \forall t = 1, 2; j = A, B$. Suppose that the joint-profit-maximizing prices are increasing over time.

Contrast these prices with the prices that would be set by two competing firms given the same initial capacity. If Firm A sets a higher price than the joint-profit-maximizing firm, it will sell less in Stage 1 and hence, more in Stage 2. Sales for Firm B are higher in Stage 1, and it has less to sell in Stage 2; thus, in Stage 2, its price is higher, and Firm A's demand is higher. Because it ignores the loss for Firm B, Firm A has an incentive to set a higher first-period price than the joint-profit-maximizing monopolist. Firm B has a similar incentive. In equilibrium, both firms' prices will be flatter relative to joint-profit-maximizing prices (see the right panel in Figure 3). It is also worth noting that prices might still be perfectly flat if sufficiently many consumers were indifferent between the firms—a symmetric increasing price equilibrium does not exist because either firm could strictly increase profits with an arbitrarily small price increase.

6.2 Many Periods

An obvious limitation of the paper is that we consider only two pricing periods. Extending the model without inventory controls to more than two periods is difficult because stronger assumptions are required in order to ensure firms play pure strategies on and off the equilibrium path. In addition, it is difficult to find sufficient conditions that rule out asymmetric equilibria. However, we use an example to show that Proposition 5 can be generalized to many periods.

Instead of strengthening Assumption 1, we assume isoelastic demand, i.e.,

$p(q) = q^{1/\epsilon}$, because, with isoelastic demand, even the monopolist's marginal revenue is always positive. That is, $p(q) + p'(q)q = (1 + 1/\epsilon)q^{1/\epsilon} > 0$, if $\epsilon > 1$. In any equilibrium, firms must sell all of their capacity. Consider any vector of capacities $\mathbf{K}$ and a sequence of isoelastic demands satisfying $|\epsilon_t|$ strictly increasing in t, for $t = 1, \ldots, T$. If the game has only one pricing period, demand is equal to $p(q) = q^{1/\epsilon_T}$, and firms set the marketing clearing price for all $\mathbf{K}$.

Now proceed by induction.[10] Suppose that for all $\mathbf{K}$, there exists an equilibrium of the s-period pricing game (the final s periods) in which firms sell all of their capacity and equalize their marginal revenue across periods. This clearly holds for $s = 1$. We now show that it follows that for all $\mathbf{K}$, there exists an equilibrium of the $s + 1$-period pricing game in which firms sell all of their capacity and set prices and inventory controls that equalize their marginal revenue across periods.

First, there clearly exists a unique vector of inventory controls for period 1 that equalizes marginal revenue between period 1 and the remaining s periods for all firms. That is, letting k_t^i denote firm i's inventory control and sales in period t, the inventory controls are uniquely defined by $k_t^i p'(\sum_j k_t^j) + p(\sum_j k_t^j) = k_\tau^i p'(\sum_j k_\tau^j) + p(\sum_j k_\tau^j), \forall t, \tau, i$.

Second, if each firm chooses these inventory controls and sets the market-clearing price in period 1 then no deviation is profitable. No deviation in the inventory control is profitable. In addition, no price decrease is profitable because the firm would sell more at the low price in period 1 and sell less in every period that has a higher price; so what remains is to show that no price increase is profitable. We consider both rationing rules.

[10] As in the two-period model, total demand grows as each period is added in the inductive proof, but we could also have held total demand fixed and divided demand into more discrete periods.

Under the proportional rationing rule, the residual demand on the inverse demand curve is $p_1\left(\frac{q^i}{Z}\right)$ where $Z = 1 - \sum_{j \neq i} \frac{k_1^j}{D_1(p_1^j)}$. At equal prices, firm i's marginal revenue on this demand curve is the same as firm i's marginal revenue $p_1(q)$, so deviating to a higher price implies that the period 1 marginal revenue is higher than marginal revenue in every other period. Therefore, profits are lower and no deviation to a higher price is profitable under proportional rationing.

Under the efficient rationing rule, the residual demand on the inverse demand curve is $p_1\left(q^i + \sum_{j \neq i} k_1^j\right)$. At equal prices, firm i's marginal revenue on this demand curve is strictly higher than on $p_1(q)$, so deviating to a higher price implies that the period 1 marginal revenue is higher than the marginal revenue in every other period. Therefore, profits are lower and no deviation to a higher price is profitable under efficient rationing.

6.3 Aggregate Demand Uncertainty

Inventory controls are generally described as a tool for managing demand uncertainty, so it is useful to describe how the model could be extended to include such uncertainty. We describe how our results may generalize when firms must set price before demand is realized.

To generate intuition, we begin by describing a potential extension in which aggregate demand is uncertain only in Stage 1. That is, firms set the Stage 1 prices before learning the Stage 1 demand, and then firms set prices in Stage 2 that clear the market. This sort of (slow) updating has been documented in the airline industry (Hortaçsu, Natan, Parsley, Schwieg, and Williams, 2021).

Suppose that the number of consumers in Stage 1 can be either high or low, and that the consumers in Stage 1 are known to have more elastic demand than consumers in Stage 2. A monopolist choosing prices and capacity optimally would like to set a lower price in Stage 1 than in Stage 2, but it would also like to limit sales in Stage 1 to reserve sufficient capacity for Stage 2 in the event that Stage 1 demand is unexpectedly high. This is why inventory controls are useful for a monopolist—they protect the firm from unwanted excess sales due high demand shocks when prices are relatively low.

Clearly the monopoly prices are not an equilibrium with competing firms—even if the competing firms have the same capacity as the monopolist—because the monopoly prices increase over time, and competing firms prefer to sell more of their capacity in Stage 2 when the expected price is relatively higher. Any firm can shift a discrete amount of its Stage 1 sales to its rival through an arbitrarily small price increase in its Stage 1 price. Therefore, the expected price in Stage 2 must be equal to the price charged in Stage 1 in any symmetric pure-strategy equilibrium.

In this stylized setting, a monopolist benefits from inventory controls because aggregate demand is uncertain. In the oligopoly setting, firms benefit from inventory controls both because they facilitate intertemporal price discrimination and because aggregate demand is uncertain.

Next, suppose demand is uncertain in both the first and second periods and that prices are set each period before demand is observed. We assume that Stage 2 demand is either high or low and consider two scenarios for optimal Stage 2 pricing. First, suppose that a monopolist would choose a Stage 2 price at which the market clears when demand is low. There is no excess capacity when demand is low, but excess demand when demand is high. In so doing, the firm forgoes some potentially higher-priced sales to reduce the risk of unsold capacity. With competition, the strategic considerations explored in this paper are still present in this scenario—competing firms will want to shift at least some early-period sales to rivals to increase their later-period sales at higher prices. Therefore, our main results generalize. Second, suppose that a monopolist would choose a Stage 2 price that results in excess capacity in Stage 2 when demand is low. In so doing, the firm forgoes sales when demand is low in order to increase margins when demand is high. In this case, second-period competition is difficult to model and it is unclear whether or not our main insights still hold. In particular, payoffs will depend on the rationing rule, and competing firms will adopt mixed strategies in Stage 2 since otherwise a small price decrease would strictly increases a firm's sales when demand is low. This is a promising area for future research.

6.4 Discounting

In our baseline model, we abstract from the effects of discounting by assuming that all consumers pay at the time of consumption. It is worthwhile considering how Lemma 3 would change if payments were made at the time of purchase, and both firms and consumers discount the future. First, it is easy to see that if the discount factor is the same for all players, then equilibrium prices would increase over time. Equilibrium prices would be the prices that made consumers (and firms) indifferent between purchasing in period 1 or period 2. Hence, Lemma 3 would need to be modified to state the prices are no longer uniform, but that equilibrium prices make consumers indifferent between purchasing in Stage 1 and Stage 2. However, this implies that firms are not engaging in intertemporal price discrimination.

Note that a uniform-price equilibrium may exist in which all consumers postpone their purchases until Stage 2. In such an equilibrium, all sales take place at the same price (and Stage 1 prices are not uniquely defined). The conditions under which this equilibrium exists would be essentially identical to Assumption 3.

If the firms' discount factor is higher than the consumers' discount factor, then the only symmetric equilibrium would be a uniform-price equilibrium in which all purchases take place in Stage 2, as this is Pareto improving. Again, firms' Stage 1 prices are not uniquely defined, but transacted prices are uniform. This equilibrium requires Assumption 3.

7 Conclusion

We establish that inventory controls can facilitate intertemporal price discrimination in an oligopoly. When a single firm serves the market, and demand becomes more inelastic over time, then the firm can clearly charge higher prices to late-arriving consumers. However, in our oligopoly model, strong competitive forces arise that prevent increasing prices over time. Individually, firms have an incentive to move their capacity to the period with a highest price. Consequently, firms will compete until prices are equalized over time, even though each firm has market power, and firms would collectively earn higher profits if prices were increasing.

In order to coordinate on increasing prices when late-arriving consumers have higher willingness to pay, firms must shield themselves from these strong competitive forces. We show that firms commit to caps on their sales each period by adopting inventory controls. This strategy increases profits and allows prices to rise over time. We hypothesize that capacity-constrained firms have adopted inventory controls in order to soften price competition and engage in intertemporal price discrimination.

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A Appendix

Derivation of c_L in Assumption 1:

Formally, we implicitly define $K^{max}(c)$ by $\Pi^m(K^{max}(c), c) = 0$, where Π^m denotes Stage 0 profits for a monopolist as a function of its capacity choice, or

$$\Pi^m(K, c) = \max_{q_1, q_2; q_1+q_2 \leq K} \left[P_1(q_1)q_1 + P_2(q_2)q_2 - cK\right]. \tag{6}$$

By the implicit function theorem and the generalized envelope theorem,

$$\frac{dK^{max}(c)}{dc} = -\frac{\partial\Pi^m(K^{max}(c), c)/\partial c}{\partial\Pi^m(K^{max}(c), c)/\partial K} = -\frac{K^{max}(c)}{c + \lambda}, \tag{7}$$

where λ is the Lagrangian multiplier on the constraint $q_1 + q_2 \leq K$. Therefore, K^{max} is a continuous, decreasing function. It is also true that

$$\lim_{c \to 0} K^{max}(c) > \operatorname*{argmax}_{q_1} P_1(q_1)q_1 + \operatorname*{argmax}_{q_2} P_2(q_2)q_2, \tag{8}$$

and

$$\lim_{c \to \infty} K^{max}(c) = 0. \tag{9}$$

Therefore, there exists a unique, strictly positive capacity cost, c_L, satisfying

$$\operatorname*{argmax}_{q} P_2(q)q = K^{max}(c_L). \tag{10}$$

Since the monopolist's optimal capacity choice must be less than $K^{max}(c_L)$ when $c \geq c_L$, it follows that if $c \geq c_L$, then the monopolist sells all of its remaining capacity in Stage 2, regardless of how much of its capacity it sells in Stage 1.

Proof of Lemma 1:

Proof. Suppose not, then so some firm with positive capacity is charging a price not equal to the market-clearing price. Clearly, trade must take place at that price since otherwise profits are negative.

Suppose that some firm sets a price strictly below the market-clearing price with strictly positive probability. Let p_L be the lowest such price. Clearly, a firm setting a price equal to p_L sells all of its capacity since p_L is below the market-clearing price. Then, $D_1(p_L) + D_2(p_L)$ exceeds the combined capacity of every firm because p_L is below the market clearing price, and the market-clearing price is defined by $D_1(p) + D_2(p) = \sum_i K^i$, and both demand functions are strictly decrease in p. Therefore, $D_1(p_L) + D_2(p_L)$ exceeds the capacity of the firm or firms setting a price equal to p_L. But, this implies that there exists a price strictly higher than p_L at which a firm setting a price equal to p_L would also sell all of its capacity, which is a contradiction. So all firms are charging a price greater than or equal to the market clearing price.

Now suppose that some firm sets a price strictly greater than the market-clearing price with strictly positive probability. Let p_H be the highest such price. Because industry capacity is equal to demand at the market-clearing price, and all firms are charging a price greater than or equal to the market-clearing price, it follows that at least one firm charging p_H does not sell all of its capacity. If two or more firms set a price equal to p_H with strictly positive probability, then a firm that does not sell all of its capacity can strictly increase profits by decreasing its price to $p_H - \epsilon$, which is a contradiction.

If only one firm charges p_H with strictly positive probability, and that firm has positive sales in Stage 1, then that firm's revenue (all costs are sunk) is equal to

$p_H RD_1(p_H; \mathbf{p}^{-i}, \mathbf{K}^{-i}) + p_H D_2(p_H)$ and all of the other firms' sales are in Stage 1 only. Alternatively, if only one firm sets a price p_H with strictly positive probability and its sales are zero in Stage 1, then its revenue is equal to $p_H RD_2(p_H; \mathbf{p}^{-i}, \mathbf{K}_2^{-i})$, where $\mathbf{K}_2^{-i}$ is the other firms' remaining capacity at the start of Stage 2.

Clearly the firm charging p_H will not sell all of its capacity in either case, because p_H exceeds the market-clearing price, and the other firms are all setting prices at or above the market-clearing price, so total consumption must be less than available capacity.

Assume that the firm's rivals are playing pure strategies. Under the efficient rationing rule, if the firm has positive sales in Stage 1, then the derivative of its revenue with respect to its price is $RD_1(p_H; \mathbf{p}^{-i}, \mathbf{K}^{-i}) + p_H D_1'(p_H) + D_2(p_H) + p_H D_2'(p_H)$, which is negative because 1) $RD_1(p; \mathbf{p}^{-i}, \mathbf{K}^{-i}) < D_1(p), \forall p$; 2) $p_H D_1'(p_H) + D_1(p_H) < 0$; and 3) $p_H D_2'(p_H) + D_2(p_H) < 0$. The second and third statements are true because, by Assumption 1, $D_1(p_H) + D_2(p_H)$ is less than the revenue-maximizing output (marginal revenue is positive). So, lowering price below p_H increases profit, which is a contradiction. Under the efficient rationing rule, if the firm charging p_H has zero sales in Stage 1, then the derivative of profit with respect to price is $D_2(p_H) + p_H D_2'(p_H)$, which is negative because, by Assumption 1, $D_2(p_H)$ is less than the revenue-maximizing output (marginal revenue is positive). So, lowering price below p_H increases profit, which is a contradiction.

Under the proportional rationing rule, if the firm charging p_H has positive sales in Stage 1, then the derivative of profit with respect to Firm i's price is

$$\begin{aligned} RD_1(p;\mathbf{p}^{-i},\mathbf{K}^{-i}) + p_H RD_1'(p_H;\mathbf{p}^{-i},\mathbf{K}^{-i}) + p_H D_2(p_H) + D_2'(p_H) = \\ \Big(p_H D_1'(p_H) + D_1(p_h)\Big)\left[1 - \sum_{j\neq i}\frac{K^j}{D_2(p^j)}\right] + \left(p_H D_2'(p_H) + D_2(p_H)\right), \quad (11) \end{aligned}$$

which is negative because $p_H D_1'(p_H) + D_1(p_H) < 0$, and $p_H D_2'(p_H) + D_2(p_H) < 0$. These are both true because, by Assumption 1, $D_1(p_H) + D_2(p_H)$ is less than the revenue-maximizing output. So lowering price below p_H increases profit, which is a contradiction. Under the proportional rationing rule, if the firm charging p_H has zero sales in Stage 1, then the derivative of profit with respect to Firm i's price is

$$\begin{aligned} RD_2(p;\mathbf{p}^{-i},\mathbf{K}_2^{-i}) + p_H RD_2'(p_H;\mathbf{p}^{-i},\mathbf{K}_2^{-i}) = \\ \Big(p_H D_1'(p_H) + D_1(p_h)\Big)\left[1 - \sum_{j\neq i}\frac{K_2^j}{D_2(p^j)}\right] + \left(p_H D_2'(p_H) + D_2(p_H)\right), \quad (12) \end{aligned}$$

which is negative because $p_H D_1'(p_H) + D_1(p_H) < 0$ and $p_H D_2'(p_H) + D_2(p_H) < 0$. This is true because $D_1(p_H) + D_2(p_H)$ is less than the revenue-maximizing output. So lowering price below p_H increases profit, which is a contradiction.

Under either rationing rule, if rivals are playing mixed strategies then the firm's expected profit is a weighted average of the above pure-strategy profit functions, all of which are strictly decreasing at the price p_H, so we have a contradiction. ■

Proof of Lemma 2:

Proof. Suppose not. First, suppose that some firm is charging a price strictly below the market-clearing price with positive probability. Let p_L be the lowest such price. Clearly any firm charging p_L sells all of its capacity (because p_L is below the market clearing price), but then there must exist a strictly higher price at which the same firm sells all of its capacity and earns strictly higher profits, which is a contradiction.

Now suppose instead that some firm charges a price strictly above the market-clearing price with positive probability. Let p_H be the highest such price offered. Clearly at least one firm offering to sell at price p_H does not sell all of its capacity, because p_H is above the market clearing price. If two or more firms charge p_H with strictly positive probability, then at least one of the firms does not sell all of its capacity, and that firm can strictly increase its sales and profits by decreasing its price to $p_H - \epsilon$, which is a contradiction. If only one firm is charging the price p_H with strictly positive probability, and if other firms are playing pure strategies, then a firm charging p_H earns revenues (or continuation profits) equal to $p_H RD_2(p_H; \mathbf{p}^{-i}, \mathbf{q}^{-i})$, where $\mathbf{p}^{-i}$ and $\mathbf{q}^{-i}$ are the other firms' prices and remaining capacities.

Under the efficient rationing rule, the derivative of profit with respect to the continuation price is $RD_2(p; \mathbf{p}^{-i}, \mathbf{q}^{-i}) + pD_2'(p)$, which is stictly negative at $p = p_H$ because $RD_2(p_H; \mathbf{p}^{-i}, \mathbf{q}^{-i}) < D_2(p_H)$ and $p_H D_2'(p_H) + D_2(p_H) < 0$. The latter is true because $D_2(p_H)$ is less than the remaining industry capacity (p_H is above the market clearing price) and the initial industry capacity, so, by Assumption 1, $D_2(p_H)$ is also less than the revenue-maximizing output. So, lowering price below p_H strictly increases profit, which is a contradiction.

Under the proportional rationing rule, the derivative of profit with respect to Firm i's price is $RD_2(p;\mathbf{p}^{-i},\mathbf{q}^{-i})+pRD_2'(p;\mathbf{p}^{-i},\mathbf{q}^{-i}) = \left(pD_2'(p)+D_2(p)\right)\left[1-\sum_{j\neq i}\frac{q^j}{D_2(p^j)}\right]$, which is strictly negative at $p = p_H$ because $p_H D_2'(p_H) + D_2(p_H) < 0$. This is true because $D_2(p_H)$ is less than the remaining industry capacity (p_H is above the market clearing price) and the initial industry capacity, so, by Assumption 1, $D_2(p_H)$ is less than the revenue-maximizing output. Lowering price below p_H strictly increases profit, which is a contradiction.

Finally, since a deviation is profitable regardless of what prices the rivals charge, it follows that a deviation is profitable even when rivals' pricing strategies are mixed. ■

Proof of Proposition 1:

Let $p_L = \min_i p_1^i$ denote the lowest equilibrium price offered in Stage 1. Recall that by Lemma 2 and under Assumption 1, all firms with positive remaining capacity in the Stage 2 charge the market-clearing price. The proof of the proposition proceeds as a series of six claims.

1) In any pure strategy equilibrium of the pricing subgame that has positive sales in both stages, $p_L \leq p_2$.

If a pure strategy equilibrium exists in which $p_L > p_2$, then all consumers who arrive in Stage 1 must be waiting to purchase until Stage 2. So, sales are zero at p_L, which is a contradiction.

2) In any pure strategy equilibrium of the pricing subgame, if p_L is offered by two or more firms in Stage 1, and if sales at p_L are strictly positive, then $p_L = p_2$.

Suppose not. Then it follows that $p_L < p_2$, by Claim 1 above. Since p_L is offered by two or more firms, let Firm i be one of these firms. Then Firm i's continuation profit can be written as $p_L x_i + P_2\left(K^i - x_i\right)$, where $x_i = \min\left\{RD_1\left(p_L; p_L, \sum_{j\neq i|p^j=p_L} K^j\right), K^i\right\}$ is Firm i's sales at p_L.

If Firm i deviates to a slightly higher price $p_L + \epsilon$, its profit is

$$(p_L+\epsilon)\min\left\{RD_1\left(p_L+\epsilon; p_L, \sum_{j\neq i|p^j=p_L} K^j\right), K^i\right\} \\ + \hat{P}_2(\cdot)\max\left\{K^i - RD_1\left(p_L+\epsilon; p_L, \sum_{j\neq i|p^j=p_L} K^j\right), 0\right\}, \quad (13)$$

where $\hat{P}_2(\cdot)$ is the market clearing price in period 2, which is a continuous and decreasing function of the total capacity remaining after Stage 1.

If $x^i = K^i$, then Firm i's profit is clearly higher since $p_L + \epsilon > p_L$ and $\hat{P}_2(\cdot) > p_L$, so all of Firm i's sales are at a higher price and its sales volume does not change.

If, on the other hand, $x^i < K^i$ and $RD_1(p_L; p_L, \sum_{j\neq i|p^j=p_L} K^j) < K^i$, then the same deviation is still profitable for Firm i because

$$\lim_{\epsilon\to 0} RD_1^i\left(p_L+\epsilon; p_L, \sum_{j\neq i|p^j=p_L} K^j\right) \leq RD_1^i\left(p_L; p_L, \sum_{j\neq i|p^j=p_L} K^j\right) < K^i,$$

since RD is decreasing in price (for either rationing rule), and so the limit of (13) as ϵ goes to 0 is

$$p_L \lim_{p \downarrow p_L} RD_1^i \left(p; p_L, \sum_{j \neq i | p^j = p_L} K^j \right) + P_2 \left(K^i - \lim_{p \downarrow p_L} RD_1^i (p; p_L, \sum_{j \neq i | p^j = p_L} K^j) \right).$$

Profits are higher because the firm sells more units at p_2 and fewer units at p_L and $p_2 > p_L$. A deviation is profitable, which is a contradiction.

3) If $p_L = p_2$, then the equilibrium is a uniform-price equilibrium.

Suppose not, so some Firm j sets a price $p^j > p_L = p_2$ in Stage 1. Because consumers can wait, it follows that Firm j's sales are zero, so the equilibrium is a uniform-price equilibrium.

4) There exists at most one uniform-price equilibrium of the pricing subgame (the total sales and the transaction price is unique).

Given capacities, the price and volume of sales in a uniform-price equilibrium are uniquely defined because only one price satisfies $D_1(p) + D_2(p) = \sum_i K^i$.

5) Any pure-strategy subgame-perfect equilibrium is either a uniform-price equilibrium or an asymmetric price equilibrium. Either a uniform-price equilibrium exists or one or more asymmetric price equilibria exists, but not both.

As above, consider the unique candidate uniform-price equilibrium. Suppose this equilibrium does not exist. Then it must be that deviating in Stage 1 is profitable. But deviating to a higher price in Stage 1 is never profitable. Consumers prefer to wait and buy at the market clearing price in Stage 2. So deviating to a lower price must be profitable.

If deviating from the uniform-price to a lower price in Stage 1 is profitable for some firm, then it is clearly also profitable for the firm that has the largest capacity.

Let i denote the firm with the largest capacity; let p_1^i denote the firm's profit-maximizing deviation in Stage 1; and let $\hat{p}_2$ denote the resulting second-period market-clearing price.

Then it follows that p_1^i and $\hat{p}_2$ must define an asymmetric-price equilibrium. Firm i sells in both periods (otherwise the deviation isn't profitable) so all other firms must sell only in Stage 2. Clearly Firm i has no incentive to deviate since by construction p_1^i is its best response to the other firms' strategies. And if any other firm could increase its profits by charging a price less than p_1^i, then it follows that Firm i could also increase its profit by deviating to that same price (because Firm i has more capacity), in which case p_1^i is not Firm i's profit-maximizing price, which is a contradiction.

Similarly, if an asymmetric price equilibrium exists, then p_1^i must be the best response for Firm i to other firms' prices, even if they were all charging p_2 in Stage 1. So a uniform-price equilibrium does not exist.

6) There exist at most n asymmetric-price equilibria.

We show that there exists, at most, one asymmetric-price equilibrium in which Firm i is the low-priced firm in period one (or, more strictly speaking, such equilibria differ only in the prices of firms with zero sales).

In an asymmetric-price equilibrium, if Firm i is the low-price firm, then it is the only firm with positive sales in Stage 1. Let p denote Firm i's equilibrium price. As in Claim 5 let p_1^i denote Firm i's best response when rival firm's are charging the unique uniform-price equilibrium price, which is the same as its optimal price when rivals are setting the market clearing price in Stage 2.

However, if $p > p_1^i$, then Firm i can profitably deviate to p_1^i because regardless of what price it sets, its rivals are selling at the market clearing price in Stage 2.

And, if $p < p_1^i$, then because $\pi(p)$ is concave and maximized at p_1^i, it follows that Firm i is strictly better off increasing its price. So, p cannot be an asymmetric-price equilibrium price unless $p = p_1^i$.

Therefore, the only one asymmetric-price equilibrium that can exist in which Firm i is the low-price firm in the first period and that equilibrium is given by (3) and (5). Since there are n firms there are at most n asymmetric-price equilibria.

Proof of Proposition 2:

Proof. The proof proceeds in two steps. We first prove Claim 1 below, which shows the pricing subgame has a uniform-price equilibrium for at least some values of first-stage capacity, and then we prove that the capacities chosen in equilibrium fall within that range.

Claim 1. *If demand becomes more inleastic over time, then under Assumptions 1, 2 and if capacities satisfy*

$$\frac{\eta_2(p)}{\eta_1(p)} > \frac{K^i}{\sum_{j=1}^n K^j}, \forall p > 0, i = 1, ..., n. \tag{14}$$

then the unique subgame-perfect pure-strategy equilibrium of the Stage 1 and Stage 2 pricing subgame is a uniform-price equilibrium.

Proof of Claim 1:

Proof. Let K^i denote each firm's capacity, and let $\tilde{p}$ denote the unique uniform price defined by $D_{tot}(\tilde{p}) = D_1(\tilde{p}) + D_2(\tilde{p}) = \sum_i K^i$. By Assumption 1 and Proposition 1, the uniform-price equilibrium is unique if it exists, or no deviation is profitable.

Suppose that $D_1(\tilde{p}) \geq \max_i K^i$. Then a deviation to a lower price is not profitable, because any firm that cuts its price in Stage 1 will sell all of its capacity at the lower deviation price and hence earn strictly lower profits.

Now suppose that $D_1(\tilde{p}) < \max_i K^i$. Then for any Firm i such that $K^i \leq D_1(\tilde{p})$, a deviation to a lower price is not profitable by the same argument. When $K^i > D_1(\tilde{p})$, then a deviation to a lower price could increase the market-clearing price in period 2, and could increase the firm's profits, but only if demand is becoming less elastic over time so the firms jointly prefer to set prices that increase over time.

Let Firm i be the deviating firm, and let $p_2(\cdot)$ denote the second-period market-clearing price as a function of remaining capacity. Firm i's problem is to choose a price $p^i < \tilde{p}$, or equivalently, a quantity $q^i = D_1(p^i)$ to maximize its continuation profit,

$$\hat{\pi}^i(q^i; \tilde{p}, \mathbf{K}) = q^i p_1(q^i) + P_2\left(\sum_{i=1}^{n} K^i - q^i\right)\left(K^i - q^i\right), \tag{15}$$

subject to $q^i \in \left(D_1(\tilde{p}), K^i\right]$ – higher output levels are not feasible, and lower output levels are inconsistent with a lower first period price. The first-order condition is

$$\frac{d\hat{\pi}(q^i; \tilde{p}, \mathbf{K})}{dq} = P_1(q^i) + q^i P_1'(q^i) - P_2\left(\sum_{i=1}^{n} K^i - q^i\right) - P_2'\left(\sum_{i=1}^{n} K^i - q^i\right)(K^i - q^i) = 0, \tag{16}$$

or

$$\begin{aligned} \frac{d\hat{\pi}(q^i; \tilde{p}, \mathbf{K})}{dq} &= P_1(q^i)\left(1 + \frac{1}{\eta_1(P_1(q^i))}\right) \\ &\quad - P_2\left(\sum_{i=1}^{n} K^i - q^i\right)\left(1 + \frac{1}{\eta_2\left(P_2\left(\sum_{i=1}^{n} K^i - q^i\right)\right)} \frac{K^i - q^i}{\sum_{i=1}^{n} K^i - q^i}\right) = 0. \end{aligned} \tag{17}$$

Clearly, the objective function, equation (15), is concave, so (17) implies that a deviation to a lower price is profitable if and only if $\lim_{q \downarrow D_1(\tilde{p})} \frac{d\hat{\pi}(q; \tilde{p}, \mathbf{K})}{dq} > 0$, or equivalently, $\lim_{p\uparrow\tilde{p}} \frac{d\hat{\pi}(D_1(p);\tilde{p},\mathbf{K})}{dq} > 0$. But clearly

$$\begin{aligned}\lim_{p\uparrow\tilde{p}} \frac{d\hat{\pi}(D_1(p);\tilde{p},\mathbf{K})}{dq} &< P_1(D_1(\tilde{p}))\left(1+\frac{1}{\eta_1(P_1(D_1(\tilde{p})))}\right)\\ &\quad - P_2\left(\sum_{i=1}^{n} K^i - D_1(\tilde{p})\right)\left(1+\frac{1}{\eta_2\left(P_2\left(\sum_{i=1}^{n} K^i - D_1(\tilde{p})\right)\right)}\frac{K^i}{\sum_{i=1}^{n} K^i}\right)\end{aligned}$$

because $\frac{K^i - q}{\left(\sum_{i=1}^{n} K^i - q\right)} < \frac{K^i}{\sum_{i=1}^{n} K^i}$. Since $P_1(D_1(\tilde{p})) = P_2\left(\sum_{i=1}^{n} K^i - D_1(\tilde{p})\right) = \tilde{p}$, it follows that a deviation to a lower price is not profitable if

$$\frac{1}{\eta_1(P_1(D_1(\tilde{p})))} - \frac{1}{\eta_2\left(P_2\left(\sum_{i=1}^{n} K^i - D_1(\tilde{p})\right)\right)}\frac{K^i}{\sum_{i=1}^{n} K^i} < 0 \iff \frac{\eta_2(\tilde{p})}{\eta_1(\tilde{p})} > \frac{K^i}{\sum_{i=1}^{n} K^i}, \tag{18}$$

or equivalently, if Equation (14) holds.

Finally, consider a deviation to a higher price. If $D_1(\tilde{p}) < \sum_{j\neq i} K^j$, for all i, then no such deviation can have any effect on first or second period sales. The firms that do not deviate can meet all of the demand at the price $\tilde{p}$. If, on the other hand, $D_1(\tilde{p}) > \sum_{j\neq i} K^j$, for some i, then a firm can deviate to a higher price and have positive sales. However even a monopolist would not find such a deviation profitable when demand is becoming less elastic over time, so no firm will deviate to a higher price. ■

We now continue the proof of Proposition 2 by showing that the Stage 0 equilibrium capacities satisfy Claim 1.

Under Assumptions 1 and 3, if a subgame perfect equilibrium exists in which every firm chooses K^* units of capacity, then, by Claim 1, the unique subgame perfect equilibrium of the pricing subgame is a uniform-price equilibrium. Similarly, if all firm capacities in a neighborhood of K^*, then the unique subgame perfect equilibrium of the pricing subgame is a uniform-price equilibrium, so the first-stage profit function for Firm i can be written as

$$\Pi^u(K^i; K^{-i}) = \left(P_{tot}\left(\sum_j K^j\right) - c\right)K^i, \tag{19}$$

where K^{-i} is the capacity of the other firms.

Firm i's capacity, K^i, maximizes Firm i's profits only if $K^i = K^*$ is the solution to

$$\frac{\partial \Pi^u(K^i; K^*)}{\partial K^i} = P_{tot}((n-1)K^* + K^i) - c + P'_{tot}((n-1)K^* + K^i)K^i = 0, \tag{20}$$

which is concave and therefore has a unique solution, $K^i(K^*)$. Clearly $K^i(K^*)$ is decreasing in K^*, so (20) uniquely defines a symmetric solution K^*, and it is easy to see that K^* must be exactly equal to the Cournot quantity associated with n firms, production cost c, and demand $D_{tot}(p)$. So we have shown that $K^i = K^*$ is a local best response. Next, we show that $K^i = K^*$ is the global best response when rival firms choose K^*.

Suppose that $K^i < K^*$. If a uniform price equilibrium exists when Firm i chooses K^i and other firms choose K^*, then Firm i's profits are given by (19), and so Firm i's profits at K^i are strictly lower than at K^*.

If, on the other hand, a uniform-price equilibrium does not exist when Firm i chooses K^i and other firms choose K^*, then by Proposition 1 an asymmetric-price equilibrium must exist. Under Assumption 3, Firm i cannot profit by deviating from the uniform-price equilibrium even if its capacity is K^*, so Firm i is not the low-priced firm in the first period. The only asymmetric-price equilibrium that can exist is one in which one of Firm i's rivals is the firm that sells at the low price in the first period. There are $n-1$ such equilibria because any of the $n-1$ firms with capacity K^* could set the low price in the first period.

Firm i's first-stage profit in all of these asymmetric-price equilibria is

$$\Pi^a(K^i; K^*) = \left[P_2\left((n-1)K^* + K^i - D_1(p_1)\right) - c\right] K^i, \tag{21}$$

where p_1 is the price charged in the first period, and so p_1 maximizes

$$D_1(p_1)p_1 + P_2\left((n-1)K^* + K^i - D_1(p_1)\right)(K^* - D_1(p_1)). \tag{22}$$

Firm i's first order-condition is

$$P_2'\left((n-1)K^* + K^i - D_1(p_1)\right)\left(1 - D_1'(p_1)\frac{dp_1}{dK^i}\right) + P_2\left((n-1)K^* + K^i - D_1(p_1)\right) - c = 0. \tag{23}$$

Because $p_1 < p_2$, $D(p_1)$ is greater than first-period sales at the uniform price. This implies that $n-1$ firms are each selling less than $K^* - D(\tilde{p})/n$ in period 2, where $\tilde{p}$ is the uniform price. In this case, ignoring the impact of K^i on p_1, Firm i's best response is greater than $K^* - D(\tilde{p})/n$, which implies that $K^i > K^*$, which is a contradiction. And, as K^i increases, the optimal first-period price falls ($dp_1/dK^i < 0$). Thus, ignoring the impact of K^i on p_1 does not alter the result. Deviating to a lower K^i is still not profitable.

Now suppose that $K^i > K^*$. Again, the equilibrum of the pricing subgame may be an asymmetric-price equilibrium or a uniform-price equilibrium. If it is a uniform-price equilibrium, then by the same argument, profits are strictly lower.

If it is an asymmetric-price equilibrium, then it must be an asymmetric-price equilibrium in which Firm i sets a low price in the first period. This is because an asymmetric-price equilibrium exists only if a firm wants to deviate from the uniform-price equilibrium, and (18) tells us that a firm wants to deviate only if

$\eta_2(p)/\eta_1(p)$ exceeds its share of capacity. But by Assumption 3, this happens only if the capacity share exceeds $1/n$ and only Firm i's share of capacity exceeds $1/n$.

So, if Firm i deviates to $K^i > K^*$, then its profit must be

$$\max_{p_1} D_1(p_1)\,p_1 + P_2\left((n-1)K^* + K^i - D_1(p_1)\right)\left(K^i - D_1(p_1)\right).$$

Rewriting this as a function of quantity yields

$$\max_{q_1} P_1(q_1)q_1 + P_2\left((n-1)K^* + K^i - q_1\right)\left(K^i - q_1\right). \tag{24}$$

Thus, the firm's profit in stage one is

$$\max_{q_1} P_1(q_1)q_1 + P_2\left((n-1)K^* + K^i - q_1\right)\left(K^i - q_1\right) - cK^i, \tag{25}$$

and its maximized profit in stage one is

$$\max_{q_1,K_1} P_1(q_1)q_1 + P_2\left((n-1)K^* + K^i - q_1\right)\left(K^i - q_1\right) - cK^i, \tag{26}$$

which we can rewrite using a change of variables ($q_2 = K^i - q_1$) as

$$\max_{q_1,q_2} P_1(q_1)q_1 - cq_1 + P_2\left((n-1)K^* + q_2\right)q_2 - cq_2. \tag{27}$$

Therefore, q_1 is the first-period monopoly output, and q_2 is the second-period best response to $(n-1)K^*$. But this is not a profitable deviation for firm i unless $p_1 < p_2$ (otherwise both prices are lower than the uniform price), or equivalently the Lerner index in the first period is smaller than the Lerner index in period 2, or

$$\frac{P_1'(q_1)q_1}{P_1(q_1)} < \frac{P_2'\left((n-1)K^* + q_2\right)q_2}{P_2\left((n-1)K^* + q_2\right)} \tag{28}$$

$$\frac{1}{|\eta_1(p_1)|} < \frac{1}{|\eta_2(p_2)|}\frac{q_2}{(n-1)K^* + q_2} \tag{29}$$

or

$$\frac{\eta_2(p_2)}{\eta_1(p_1)} < \frac{q_2}{(n-1)K^* + q_2}, \tag{30}$$

which violates Assumption 3 because $q_2 < K^*$. So, this is a contradiction. Hence there exists no profitable deviation for any firm. ■

Proof of Proposition 3:

Proof. This follows immediately from Proposition 1, which shows that a pure strategy equilibrium exists and that any pure strategy equilibrium must be a uniform-price equilibrium or an asymmetric price equilibrium in which the Stage 1 price is strictly lower than the Stage 2 price. But if the elasticity of demand is increasing, an asymmetric price equilibrium cannot exist. The firm selling in Stage 1 prefers to sell all of its capacity at the market clearing price in Stage 2. ■

Proof of Proposition 4:

Proof. By Proposition 3 all transactions take place at the same price, and by Lemma 1 firms set the Cournot capacities as if there were one combined sales period. ■

Proof of Proposition 5:

Proof. Let k_t^i denote the inventory control for Firm i in period t. Let q_t^{iC} denote the output of firm i in period t when firms play a sequential Cournot game.

Consider an equilibrium of the inventory control game in which, on the equilibrium path, firms choose capacity equal to the sum of the Cournot capacity in each period, $K^i = q_1^{iC} + q_2^{iC}$, set the Cournot price, p_t^C in each period, and then set inventory controls equal to the Cournot output in each period, i.e., $k_t^i = q_t^{iC}$.

Clearly no deviation is profitable in the final period. That is, in every Stage 2 subgame firms set the market clearing price and set a non-binding inventory control. This is because Lemma 2 holds, so any second-period price not equal to the market-clearing price is less profitable. Introducing inventory controls does not change this result.

Next, consider a deviation by Firm i to a lower price in the first selling period. Decreasing demand elasticity implies that $p_1^C < p_2^C$, so a small decrease in its first-period price discontinuously increases Firm i's first-period sales, decreases Firm i's second-period sales, and decreases Firm i's profits. More generally, if Firm i had a profitable deviation to a lower price in period one, then that price would define an asymmetric price equilibrium, but by Proposition 1 an asymmetric-price equilibrium does not exist. So deviating to a lower price is not profitable.

Now consider a deviation by Firm i to a higher price in the first period. Under the efficient rationing rule, the residual demand function facing the deviating firm is $RD_1^i(p^i; p-i_1, q-i_1) = D_1(p)-(n-1)q_1^C$. This is because of the rival firms' inventory controls, $k_1^j = q_1^C$ (if any firm deviates in stage zero, then k_1^j equals the Cournot output given the new capacity constraint).

Since the shadow cost of capacity is c on the equilibrium path (and, more generally, is equalized across periods), Firm i's first-period profit function is $(D_1(p^i) - (n-1)q_1^C)(p^i - c)$ or, equivalently, $(p_1((n-1)q_1^C + q^i) - c)q^i$ where p_1 is the first period inverse demand function. Thus, the optimal price deviation is given by the first-order condition, which is

$$P_1'\left((n-1)q_1^C + q\right)q + P_1\left((n-1)q_1^C + q\right) = c.$$

But this implies that $q = q_1^C$ and that the optimal price and quantity is the first-period Cournot output (or, more generally, is the output that equalizes the marginal revenue across the two periods), so no deviation to a higher price is profitable.

Under the proportional rationing rule, the deviating firm's residual demand function is

$$RD_1^i(p^i; p_1^C, q_1^C) = D_1(p^i)\left[1 - \frac{(n-1)q_1^C}{D_1(p_1^C)}\right] = \frac{1}{n}D_1(p^i),$$

since $D_1(p_1^C) = nq_1^C$. The shadow cost of capacity is c on the equilibrium path (and, more generally, is equalized across the two periods), so Firm i's first-period profit function is $\frac{1}{n}D_1(p)(p-c)$, or equivalently, $p_1(nq) - cq$. The first-order condition is $P_1(nq) + P_1'(nq)\,q = c$, which implies that $q = q_1^C$, so no deviation to a higher price is profitable.

In Stage zero, firms choose capacity expecting to equalize marginal revenue across periods, so $K^i = q_1^C + q_2^C$ is a best response to $K^j = q_1^C + q_2^C$ for all $j \neq i$. ■