INFORMATION AND MARKET POWER

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August 2015

COWLES FOUNDATION DISCUSSION PAPER NO. 2017

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS
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http://cowles.yale.edu/

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August 3, 2015

Abstract

We analyze demand function competition with a finite number of agents and private information. We show that the nature of the private information determines the market power of the agents and thus price and volume of equilibrium trade.

We establish our results by providing a characterization of the set of all joint distributions over demands and payoff states that can arise in equilibrium under any information structure. In demand function competition, the agents condition their demand on the endogenous information contained in the price.

We compare the set of feasible outcomes under demand function to the feasible outcomes under Cournot competition. We find that the first and second moments of the equilibrium distribution respond very differently to the private information of the agents under these two market structures. The first moment of the equilibrium demand, the average demand, is more sensitive to the nature of the private information in demand function competition, reflecting the strategic impact of private information. By contrast, the second moments are less sensitive to the private information, reflecting the common conditioning on the price among the agents.

JEL CLASSIFICATION: C72, C73, D43, D83, G12.

KEYWORDS: Demand Function Competition, Supply Function Competition, Price Impact, Market Power, Incomplete Information, Bayes Correlated Equilibrium, Volatility, Moments Restrictions, Linear Best Responses, Quadratic Payoffs.

*We gratefully acknowledge financial support from NSF ICES 1215808. We would like to thank Alessandro Pavan, Marzena Rostek, Jean Tirole, Xavier Vives, and Marek Weretka as well as many seminar participants, for informative discussions.

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1 Introduction

1.1 Motivation

Consider a market where traders submit demand functions, specifying their demand at any given price, and the price is then chosen to clear the market. A trader exercises *market power* if, by reducing demand at any given price, he can strategically influence the market price. The standard measure of market power is *price impact*: how much do changes in the trader's demand shift market prices? Market power leads to inefficiency, as the marginal value exceeds the market price. Market power is linked to the number of firms in the market. As the number of firms increases, market power decreases, and we have a competitive economy in the limit.

This standard understanding of market power is based on symmetric information. But if there is asymmetric information, so that the level of market clearing prices conveys payoff relevant information to consumers, then there is an additional channel of price impact. The slope of the demand curves that consumers submit will then reflect not only market power - i.e., their ability to strategically control prices - but also their desire to reflect the information contained in market prices in their demand. These two aspects of the market - strategic manipulation of prices and learning from prices - are intrinsically linked and interact in subtle ways.

We study how asymmetric information affects the outcome of the demand function competition. We consider a model with a finite number of agents which trade a divisible good with an exogenous linear supply. Agents' have quadratic utility over their holding of the good. The utility of an agent is subject to payoff shocks, which are correlated across agents. More precisely, the payoff state of each agent is the sum of the realized idiosyncratic and the aggregate payoff shock. The agents have only incomplete information about the realized payoff states. We restrict attention to Gaussian information structures. In our benchmark model we focus on demand function competition, although we later compare this market structures with other market structures, such as quantity (Cournot) competition. Throughout the paper we focus on symmetric environments and symmetric equilibria.

We begin by studying a key element of demand function competition which is price impact. In demand function competition the quantity bought by agent i is contingent on the realized price. This implies that agent j, by changing the demand schedule he submits, will change the price and the quantity bought by agent i. This in turn implies that the change in price caused by a reduction in the demand of agent j is not trivially calculated by using the slope of the exogenous supply curve. Importantly, the price impact of agent j depends on the *slope* of the demand schedules submitted by all agents other than agent j. The change in price caused by the change in quantity bought by agent j is called the *price impact* of agent j. In particular, the price impact of an agent in demand function competition can be larger or smaller than the one given by the slope of the exogenous supply curve.

We begin by studying how the nature of the information structure changes the price impact of each agent. The payoff state of each agent is the sum of the idiosyncratic and the aggregate payoff shock. We study a particular class of information structures which we call *noise free* information structures. In a noise free information structure, an agent receives a one dimensional signal which is a linear combination of the common and idiosyncratic component of his payoff shock. Importantly, the linear combination in the signal may not have the same weights on the two components, and hence the signal realization, even though it does not contain any noise, might not allow an agent to perfectly infer the realization of his payoff shock. We show that the price impact of an agent in the Bayes Nash equilibrium in which agents receive noise free signals can be any number between $-1/2$ and $+\infty$. Hence, independent of the number of agents or the specification of the payoff environment (but depending on the information structure), there can be a market shutdown (if price impact is $+\infty$), agents can behave competitively (if price impact is 0), or agents can trade more than optimal (if price impact is negative).

The key variable in the interaction of strategic manipulation and learning is what consumers learn from prices. Consider a noise-free information structure in which the private signal of an agent substantially overweights the realization of the aggregate shock. Then an agent by relying on his private signal can almost perfectly infer the size of the aggregate shock, and hence almost perfectly anticipate what the realized price will be. As the agent can to a large extent infer the realized price from his private signal, he submits almost perfectly elastic demands at the expected price. If the realization of the price is lower than expected by agent i, then agent i must interpret this as a very high realization of his idiosyncratic shock, and hence he would want to buy a very large amount. Similarly, if the realized price is higher than expected by agent i, then agent i must interpret this as a very low realization of his idiosyncratic shock, and hence he would want to buy a very small amount. This behavior is consistent with submitting a very elastic demand function, as small price changes cause a large change in the quantity bought by the agent. By symmetry, every agent submits a very elastic demand, and thus individual agent cannot change prices by reducing the demand. Hence, agents behave (almost) competitively as they have (almost) no price impact.

Conversely, if the noise free signal were to substantially overweight the idiosyncratic component, then every individual agent has very little information on the realized aggregate shock. Hence, a higher realization of the equilibrium price will be interpreted as a higher realization of the aggregate shock and thus his expected valuation for the good increases. In consequence, an agent submits a demand function that is very inelastic (and possibly even with a positive slope), as an increase in price does not make the agent want to buy any less quantity of the good. An increase in price is interpreted as a higher realization of the common shock, which increases the demand of all agents. Hence, an individual agent will have a very large price impact. Thus we provide a sharp account of how price impact depends on the information structure.

We proceed to characterize all distributions over quantities and payoff shocks which are consistent with being the outcomes of the demand function competition game for some given payoff uncertainty. But rather than fixing a parameterized class of information structures and solving for the Bayes Nash equilibria, we show that we can characterize the set of outcomes that can arise without explicit reference to the underlying information structure. We refer to set of resulting outcome distribution as Bayes correlated equilibria. We provide a complete characterization of the Bayes correlated equilibria in terms of three variables: (i) the correlation between aggregate payoff shock and aggregate demand, (ii) the correlation between idiosyncratic payoff shock and idiosyncratic demand and (iii) price impact. The price impact parametrizes the set of distributions by increasing or decreasing the quantity bought by each agent. A low price impact is reflected in a higher average quantity bought and a higher variance in the quantity bought by agents. Interestingly, the set of feasible correlations and price impact are independent of the payoff environment, and hence are determined solely by the information structure. These three parameters, plus the specification on the payoff environment, fully characterize the distribution over outcomes in the demand competition game. The restrictions on the correlations and price impact are as follows: the correlations must be positive and the price impact can be any number between $-1/2$ and $+\infty$.

The Bayes correlated equilibrium is defined in terms of the distribution of quantities bought by each agent and realization of payoff shocks. Importantly, we do not need to specify the strategies submitted by agents to induce a particular distribution. This makes the solution concept particularly useful to compare outcomes across different market structure. In particular, we compare the set of distributions over outcomes that are consistent with demand function competition with the set of distributions over outcomes that are consistent with Cournot competition. After all, the outcome of both market competitions is a quantity bought by each agent, a price and the realization of the payoff shocks. Hence, although from a game theoretic perspective the strategies used by agents in each game are not comparable, both games are comparable in terms of outcomes.

With quantity competition, each agent submits a single demanded quantity that is independent of the price. As a consequence the price impact is constant across information structures, and so is the average demand. On the other hand, in Cournot competition the set of feasible distributions over outcomes is characterized by three correlation coefficient whereas these in demand function competition these are restricted to a two-dimensional subspace. The intuition for the reduction in the dimensionality of the feasible correlations is as follows. In demand function competition the quantity bought by an agent is a function of the information contained in his private signal *and* the information in prices. This implies that the response to aggregate and idiosyncratic payoff shocks are disentangled. In particular, the variance of the average quantity bought by agents is a function on the information agents have on the aggregate shock, which is determined by a unique correlation. Similarly, the variance of the idiosyncratic quantity bought by agents is a function on the information agents have on the idiosyncratic shock, which is also determined by a unique correlation. The correlation in the quantity bought by agents is a function of the variance of the average quantity bought by agents and the variance of the idiosyncratic quantity bought by agents. Hence, the correlation in agents' actions is a function of the two correlations which measure the information agents have about the realization of the aggregate shock and idiosyncratic shock. Thus, while the feasible distribution over outcomes are less restricted under demand function competition with respect to the first moment, the Cournot competition imposes less restrictions on the second moments.

The difference in the way these markets structures are sensitive to private information is best illustrated by comparing the behavior under noise free signals. In demand function competition confounding of aggregate and idiosyncratic shock in the private signals of the agents leads to a change in the price impact. Price impact changes the aggregate and idiosyncratic variance of the quantity bought by an agent in the same direction (that is, with the same sign). Importantly, agents can disentangle their response to aggregate and idiosyncratic payoff shock using the realized price, although this implies agents have an endogenous price impact. In the Cournot competition agents cannot disentangle their response to aggregate and idiosyncratic payoff shock, as they cannot condition in prices. This implies that, in the Cournot competition confounding of aggregate and idiosyncratic shock in the private signals always leads to, either an increase of aggregate variance and decrease of idiosyncratic variance of the quantity bought by agents, or vice versa. As a matter of fact, in the Cournot competition the maximum variance in the average quantity bought by agents can grow without bounds as the size of the idiosyncratic payoff shocks grow without bounds By contrast, in the demand function competition the maximum variance in the average quantity bought by agents is bounded by the size of the aggregate payoff shocks.

Finally, as an extension, we show how the same techniques can be applied to other market structures. In particular, we solve for a particular class of market structures in which the agents can condition their demand on a noisy realization of prices. Hence, allowing us to provide a model that smoothly connects demand function competition and Cournot competition. We also solve for a static model of trading as in Kyle (1985). Although we provide only a brief analysis of these alternative market structures, we believe a more exhaustive comparison is material for future work.

We have pursued the strategy of characterizing the set of outcomes that can arise without explicit reference to the underlying information structure in other recent work (most closely related is Bergemann, Heumann, and Morris (2015)). In this paper, we pursue this strategy in a game which provides strategic foundations for competitive equilibria. Thus we view the results in this paper as providing a benchmark model for studying the role of information in markets, and present a number of novel questions that can be asked and answered using this approach.

First, in studying a substantive economic question, such as the interaction of market power and information, we can identify which features of the information structure drive results, rather than solving within a low dimensional parameterized class of information structures. As we look at the joint distribution of quantities and prices that can arise in equilibrium, it turns out that extremal distributions, arise when agents observe "noise-free" information structures, where each agent observes perfectly a linear combination of the common component and his idiosyncratic component. Any other outcome that could arise in any other (perhaps multidimensional) information structure could also arise in an information structure where agents observed one dimensional confounding signals with noise. The impact of the noise is merely to reduce variation in the outcome. Thus our transparent analysis under noise free information structures frames and bounds what could happen all information structures.

Second, in understanding possible outcomes in a market setting, we can abstract from the consumers' information structures and strategies (i.e., the demand curves that they submit) and identify equilibrium conditions on the joint distribution of quantities (and thus prices) condition on the distribution of valuations.

Third, we can compare the equilibrium outcomes under demand function competition with other uniform price mechanism that match demand and supply.

1.2 Related Literature

The market setting is that analyzed in the seminal work of Klemperer and Meyer (1989) on *supply function competition*, where market participants submitted supply curves when there is uncertainty about demand and symmetric information. Because of our interest in strategic uncertainty in financial markets, we focus on a model of *demand function competition.* With appropriate re-labelling and sign changes, the model is identical to supply function competition. We follow an important paper of Vives (2011a) in introducing asymmetric information into this setting, and we also follow Vives (2011a) in assuming symmetric consumers with normally distributed common and idiosyncratic components of their values, which allows a tractable and transparent analysis of closed form linear equilibria.

Vives (2011a) also highlighted the interaction of market power with asymmetric information. We depart from Vives (2011a) in studying what can happen in his model, in all (linear) equilibria, for all possible (symmetric and normal) information structures. While he assumes that consumers observe conditionally independent normal signals of their true valuations (which reflect common and idiosyncratic components), we break the link between the degree of confounding in the information structure (the degree to which common and idiosyncratic signals can be distinguished) and the accuracy of signals. This allows us to offer a sharper account of the how market power and asymmetric information interact.

Weretka (2011) studies a model of equilibrium in economies in which agents are not price takers. Importantly, agents do not have incomplete information and agents can have any price impact, which they take as exogenous. An interesting aspect of the equilibrium trade in any noise free information structure is that agents have *ex-post* complete information. That is, an agent can perfectly infer the state of the world using the information contained in their private signal and the realized price. Hence, the price impact is unrelated to the amount of ex-post uncertainty agents have on the state of the world. In this sense, our work provides an information based foundation to the approach of Weretka (2011), as we show that any price impact is consistent with the Bayes Nash equilibrium of the demand function competition game, even when agents have ex-post complete information.

We provide a sharp comparison between the set of feasible outcomes under demand function competition and Cournot competition. We use the results found in Bergemann, Heumann, and Morris (2015) to provide the comparison with Cournot competition. The approach of characterizing the set of outcomes for all information structures follows the work of Bergemann and Morris (2015).

The remained of the paper is organized as follows. Section 2 describes the model and the payoff environment. Section 3 describes the Bayes Nash equilibrium in a small class of information structures that we refer to as noise free information structures. Section 4 introduces a second solution concept, Bayes correlated equilibrium, and with it, we can describe the equilibrium behavior for all possible information structures. Section 5 compares the feasible equilibrium outcomes under demand competition with the ones arising in quantity competition. Section 6 concludes.

2 Model

Payoffs We consider an economy with finite number of agents (buyers), indexed by $i \in N = \{1, ..., N\}$. There is a divisible good which is purchased by the agents. The realized utility of a trader who buys an amount a_i of the asset at price p is given by:

$$u_i(\theta_i, a_i, p) \triangleq \theta_i a_i - \frac{1}{2}a_i^2 - a_i p, \tag{1}$$

where θ_i is the (marginal) willingness to pay, the payoff state, of trader i. The aggregate demand of the buyers is denoted by A with

$$A \triangleq \sum_{i=1}^{N} a_i.$$

The asset is supplied by a competitive market of producers represented by an aggregate supply function:

$$p = c_0 + cA.$$

The aggregate supply function could be the results of a competitive supply with a quadratic aggregate cost function given by:

$$c(A) \triangleq c_0 A + \frac{1}{2}cA^2.$$

We assume that the willingness to pay, θ_i, is symmetrically and normally distributed across agents. Thus, for any pair of agents $i, j \in N$ their willingness to pay is distributed according to:

$$\begin{pmatrix} \theta_i \\ \theta_j \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_\theta \\ \mu_\theta \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \rho_{\theta\theta}\sigma_\theta^2 \\ \rho_{\theta\theta}\sigma_\theta^2 & \sigma_\theta^2 \end{pmatrix} \right). \tag{2}$$

The expected willingness to pay is given by the mean $\mu_\theta \in \mathbb{R}_+$ and the variance is denoted by σ_θ^2. The correlation across agents is given by the correlation coefficient $\rho_{\theta\theta}$. By symmetry, and for notational convenience we omit the subscripts in description of the moments (thus μ_θ instead of

μ_{θ_i}, σ_θ instead of σ_{θ_i}, and $\rho_{\theta\theta}$ instead of $\rho_{\theta_i\theta_j}$). For the symmetrically distributed random variables $\{\theta_i\}_{i=1}^N$ to form a feasible multivariate normal distribution it has to be that $\rho_{\theta\theta} \in [-\frac{1}{N-1}, 1]$.

With the symmetry of the payoff states across agents, a useful and alternative representation of the environment is obtained by decomposing the random variable into a common and an idiosyncratic component. Thus for a given profile of realized payoff states $(\theta_1, ..., \theta_N)$, we define the average payoff state:

$$\bar{\theta} \triangleq \frac{1}{N}\sum_{i\in N}\theta_i, \tag{3}$$

and, correspondingly, we define the idiosyncratic component of agent i payoff state:

$$\Delta\theta_i \triangleq \theta_i - \bar{\theta}. \tag{4}$$

Now, we can describe the payoff environment in terms of the common and idiosyncratic component $\theta_i = \bar{\theta} + \Delta\theta_i$ with

$$\begin{pmatrix} \bar{\theta} \\ \Delta\theta_i \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_\theta \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{\bar{\theta}}^2 & 0 \\ 0 & \sigma_{\Delta\theta}^2 \end{pmatrix}\right), \tag{5}$$

where the variance of common component is given by

$$\sigma_{\bar{\theta}}^2 = \rho_{\theta\theta}\sigma_\theta^2$$

and the variance of the idiosyncratic component is the residual

$$\sigma_{\Delta\theta}^2 = (1 - \rho_{\theta\theta})\,\sigma_\theta^2.$$

We shall use the decomposition of the individual variable into a common and an (orthogonal) idiosyncratic component later also for the action and the signal variables. Henceforth we use a bar above a variable to denote the average over all agents, as in $\overline{\theta}$ and a Δ to denote the idiosyncratic component relative to the average, as in $\Delta\theta_i$.

Signals Each agent receives a signal, possibly noisy, possibly multi-dimensional about his payoff state and the payoff state of all the other agents. We shall restrict attention throughout the paper to symmetric and normally distributed signals. The signal that agent i receives, $s_i = \left(s_i^1, ..., s_i^K\right)$, is a K-dimensional vector for some finite K. The joint distribution of the signals (types) and the payoff states of the agents can therefore be described for any pair of agents i and j as a multivariate normal distribution:

$$\begin{pmatrix} \theta_i \\ \theta_j \\ s_i \\ s_j \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_\theta \\ \mu_\theta \\ \mu_s \\ \mu_s \end{pmatrix}, \begin{pmatrix} \Sigma_{\theta\theta} & \Sigma_{\theta s} \\ \Sigma_{\theta s} & \Sigma_{ss} \end{pmatrix} \right), \tag{6}$$

where μ_s is the mean realization of the signal vector s_i and the submatrices $\Sigma_{\theta\theta}$, $\Sigma_{\theta s}$ and Σ_{ss} together form the variance-covariance matrix of the joint distribution. The only restriction that we impose on the joint distribution is that the entire matrix describing the variance covariance matrix in (6) is positive semi-definite (this is a necessary and sufficient condition for a matrix to be a valid variance covariance matrix). With minor abuse of language, we refer to joint distribution of states and signals as an information structure $\mathcal{I}$.

Strategies The agents simultaneously submit demand functions $x_i(s_i, p)$. The demand function $x_i(s_i, p)$ represents the demand of agent i at price p given the private information conveyed by the signal s_i:

$$x_i : \mathbb{R}^K \times \mathbb{R} \to \mathbb{R}. \tag{7}$$

The equilibrium price p^* is determined by the submitted demand functions and the market clearing condition:

$$p^* = c_0 + c \sum_{i \in N} x_i(s_i, p^*).$$

We analyze the symmetric Bayes Nash equilibrium in demand function competition. Given the market clearing condition, the equilibrium demand function $x^*(s_i, p)$ solves for each agent i and each signal realization s_i the following maximization problem:

$$x(s_i, p) \in \underset{x_i(\cdot) \in C(\mathbb{R})}{\arg\max}\ \mathbb{E}[u_i(\theta_i, x_i(p^*), p^*)|s_i]. \tag{8}$$

subject to the market clearing condition:

$$p^* = c_0 + c \sum_{j \neq i} x(s_j, p^*) + x_i(p^*). \tag{9}$$

For the moment, we shall merely require that for every signal s_i the submitted demand function $x_i(p)$ is a continuous function defined the real line $\mathbb{R}$, thus $x_i(\cdot) \in C(\mathbb{R})$. In the subsequent equilibrium analysis, we find that given the linear quadratic payoff environment, and the normality of the signal and payoff environment, the resulting equilibrium demand function is a linear function in the signal vector s_i and the price p.

Definition 1 (Symmetric Bayes Nash Equilibrium)
The demand functions $\{x(s_i, p)\}_{i=1}^N$ *constitute a symmetric Bayes Nash equilibrium if for every agent* i *and every signal realization* s_i*, the best response condition (8) and the market clearing condition (9) are satisfied.*

3 Noise Free Information Structures

We begin our analysis with the conventional approach. Namely, we fix the information structure of the agents and then determine the structure of the equilibrium demand function. The novel aspect in the analysis is the nature of the private information that we refer to as noise free information structure. Namely, the signal of each agent is a linear combination of the idiosyncratic and the common component of the payoff state. In turn, the resulting structure of the equilibrium demand functions will suggest a different and novel approach that describes the equilibrium directly in terms of the outcomes of the game, namely the quantities, price, and payoff states in a way that is independent of the specific information structure.

3.1 Noise Free Information Structure

We begin the equilibrium analysis with a class of one dimensional signals. For now, we shall consider the class of information structures in which the one dimensional signal of each agent i is given by:

$$s_i = \Delta\theta_i + \lambda \cdot \bar{\theta}. \tag{10}$$

where $\lambda \in \mathbb{R}$ is the weight that the common component of the payoff state receives in the signal that the agent i receives. As the idiosyncratic and the common component, $\Delta\theta_i$ and $\bar{\theta}$ respectively of the signal are normal distributed, the signal s_i is also normally distributed. We call the signal s_i noise-free as it is generated by the component of the payoff state, and no extraneous noise enters the signal. However to the extent that the composition of the idiosyncratic and common component in the signal differ from its composition in the payoff state, that is as long as $\lambda \neq 1$, agent i faces residual uncertainty about his willingness to pay since the signal s_i confounds the idiosyncratic and the common component. Given the multivariate normal distribution, the conditional expectation of agent i about his payoff state θ_i given by

$$\mathbb{E}\left[\theta_i \left| s_i \right.\right] = \mu_\theta + \frac{\lambda\rho_{\theta\theta}\sigma_\theta^2 + \left(1-\rho_{\theta\theta}\right)\sigma_\theta^2}{\lambda^2\rho_{\theta\theta}\sigma_\theta^2 + \left(1-\rho_{\theta\theta}\right)\sigma_\theta^2}\left(s_i - \lambda\mu_\theta\right),$$

and the conditional expectation about the common component, or for that matter the payoff state θ_j of any other agent j is given by

$$\mathbb{E}\left[\overline{\theta}\left|s_i\right.\right]=\mathbb{E}\left[\theta_j\left|s_i\right.\right]=\mu_\theta+\frac{\lambda\rho_{\theta\theta}\sigma_\theta^2}{\lambda^2\rho_{\theta\theta}\sigma_\theta^2+\left(1-\rho_{\theta\theta}\right)\sigma_\theta^2}\left(s_i-\lambda\mu_\theta\right),$$

where the later conditional expectation is less responsive to the signal s_i as the idiosyncratic component of agent i payoff state drops out in the updating rule. In the above expressions of the conditional expectations we can cancel the total variance σ_θ^2 of the payoff state, and what matters is only the relative contribution of each component of the shock, 1 and λ, respectively.

In the important special case of $\lambda=1$, we can verify that $\mathbb{E}\left[\theta_i\left|s_i\right.\right]=\theta_i$. By contrast, if $\lambda>1$, then the signal overweighs the common component relative to its payoff relevance, and if $\lambda<1$, then the signal underweights the common component relative to its payoff relevance. If $\lambda=0$, then the signal s_i only conveys information about the idiosyncratic component, if $\lambda\to\pm\infty$, then the signal s_i only conveys information about the common component.

3.2 Price Impact

Each agent submits a demand function that describes his demand for the asset at a given market price. Thus the demand of each agent is conditioned on the private signal s_i and the equilibrium price p. Heuristically, each agent i therefore solves a pointwise maximization problem given the conditioning information (p,s_i):

$$\max_{a_i}\mathbb{E}\left[\theta_i a_i-\frac{1}{2}a_i^2-pa_i\middle|p,s_i\right].\tag{11}$$

The resulting first order condition is

$$\mathbb{E}[\theta_i|s_i,p]-p-a_i-\frac{\partial p}{\partial a_i}a_i=0$$

and thus determines the demand of agent i:

$$a_i=\frac{\mathbb{E}[\theta_i|s_i,p]-p}{1+\frac{\partial p}{\partial a_i}}.\tag{12}$$

Importantly, with a finite number of agents, an increase in the demand by agent i affects the equilibrium price for all the traders, which is, still heuristically, captured by the partial derivative $\partial p/\partial a_i$. We shall refer to this derivative, provided that it exists, as the *price impact* of agent i. The price impact represents the "market power" of agent i in the sense that it indicates how strongly the trader can influence the equilibrium price by changing his own demanded quantity. In the absence of market power, we have $\partial p/\partial a_i = 0$, and the resulting demanded quantity by agent i always equals the difference between his marginal willingness to pay and the price of the asset:

$$a_i = \mathbb{E}[\theta_i|s_i, p] - p.$$

In the presence of market power, $\partial p/\partial a_i > 0$, each agent i lowers his demand, and hence we will observe a strategic demand reduction.

3.3 Demand Function Equilibrium

Next, we derive the equilibrium demand functions of the traders. Given the linear quadratic payoff environment, and the normality of the state and signal environment we will find that in equilibrium the traders submit symmetric linear demands:

$$x_i\left(s_i, p\right) \triangleq \beta_0 + \beta_s s_i + \beta_p p.$$

The demand function of each trader i consists of a stochastic intercept of his demand function, $\beta_0 + \beta_s s_i$, and the price sensitivity $\beta_p p$, where the slope of the individual demand function β_p is typically negative.[1]

Given the candidate equilibrium demand functions, the market clearing condition can be written as

$$p = c_0 + c\sum_{i=1}^{N}\left(\beta_0 + \beta_s s_i + \beta_p p\right). \tag{13}$$

For a moment let us consider the market clearing condition from the point of view of a specific trader k. Given the candidate equilibrium strategies of the other agent, trader k can anticipate that

[1] The demand function competition, plotted as $(q\left(p\right), p)$ therefore can be viewed as intermediate structure between Cournot competition and Bertrand competition. Cournot competition generates a vertical demand function in which the buyer fixed demand q and the price responds whereas Betrand competition generates a horizontal demand function in which the buyer fixes the price p and realized demand q adjusts. With symmetric information, there exists a continuum of symmetric price equilibria in which the supply is shared among the buyers (where the continuum can be maintained due to the discontinuity in the supply as a function of the price). With asymmetric information, the bidding game is a generalization of the first price auction with variable supply which appears to be an open problem in the auction theory. The resulting equilibrium is likely to allocate all units to the winning bidder, and zero units to the losing bidders, and hence lead to a very inefficient allocation relative to a discriminatory price auction in which bidders submit bids for every marginal unit. Ausubel, Cramton, Pycia, Rostek, and Weretka (2014) analyze a corresponding model of *fixed* supply under uniform price *and* pay-as-you-bid auction rules.

a change in his demanded quantity a_k will impact the equilibrium price:

$$p = c_0 + c \sum_{i \neq k} \left(\beta_0 + \beta_s s_i + \beta_p p \right) + c a_k .$$

In other words, from the point of view of trader k, the market clearing condition is represented by a residual supply function for trader k :

$$p = c_0 + c \sum_{i \neq k} \left(\beta_0 + \beta_s s_i \right) + c \left(N - 1 \right) \beta_p p + c a_k ,$$

and after collecting the terms involving the market clearing price p on the lhs:

$$p = \frac{c}{1 - \left(N - 1 \right) c \beta_p} \left(\frac{c_0}{c} + \sum_{i \neq k} \left(\beta_0 + \beta_s s_i \right) + a_k \right) .$$

The residual supply function that trader k is facing thus has a random intercept determined by the signal realizations $s_{-k} = (s_1, ..., s_{k-1}, s_{k+1}, ...s_N)$ of the other traders and a constant slope that is given by the responsiveness of the other traders to the price. Thus, we observe that under the hypothesis of a symmetric linear demand function equilibrium, the price impact of trader k, $\partial p / \partial a_k$ is going to be a constant as well, and we denote it by m :

$$m \triangleq \frac{\partial p}{\partial a_k} = \frac{c}{1 - \left(N - 1 \right) c \beta_p} . \tag{14}$$

Thus, the price impact of trader k is determined in equilibrium by the price sensitivity β_p of all the other traders. Typically, the demand function of the buyers is downward sloping, or $\beta_p < 0$. Thus, an increase in the (absolute) price sensitivity $\left| \beta_p \right|$ of the other traders decreases the price impact m, the market power, of trader k.

We now return to the market clearing condition (13) and express it in terms of the signals received. Thus, after rearranging the market clearing condition, we find that the equilibrium price is informative about the average signal $\overline{s}$ received by the agents:

$$\frac{1}{N} \sum_{i=1}^{N} s_i = \frac{\left(1 - N c \beta_p \right) p - c_0 - N c \beta_0}{N c \beta_s} . \tag{15}$$

The average signal $\overline{s}$ (following the same notation as for the payoff state in (3)) is perfectly informative of the average state $\overline{\theta}$

$$\overline{s} = \frac{1}{N} \sum_{i} s_i = \frac{1}{N} \sum_{i} \left(\Delta \theta_i + \lambda \overline{\theta} \right) = \lambda \overline{\theta} .$$

Thus, in equilibrium, the conditional expectation of each agent regarding his expected payoff state $\mathbb{E}[\theta_i|s_i,p]$ equals $\mathbb{E}[\theta_i|s_i,\bar{s}]$:

$$\mathbb{E}[\theta_i|s_i,p] = \mathbb{E}[\theta_i|s_i,\bar{s}] = s_i + \left(\frac{1-\lambda}{\lambda}\right)\bar{s} = \theta_i, \tag{16}$$

which in turn allows each agent to infer his payoff state θ_i perfectly.

We can now find the equilibrium demand function by requiring that the quantity demanded satisfies the best response condition (12) for every signal realization s_i and every price p, or equivalently every average signal realization $\bar{s}$:

$$\beta_0 + \beta_s s_i + \beta_p p = \frac{\mathbb{E}[\theta_i|s_i,p] - p}{1+m} = \frac{\mathbb{E}[\theta_i|s_i,\bar{s}] - p}{1+m},$$

using the fact the price impact $\partial p/\partial a_i$ equals a constant m introduced earlier in (14).

Thus, using the resulting conditional expectation in terms of $(s_i, \bar{s})$, or more explicitly the market clearing condition expressed in terms of the average signal $\bar{s}$, we can find the equilibrium demand function by matching the three coefficients $\left(\beta_0, \beta_s, \beta_p\right)$ and the price impact m:

$$\begin{aligned}\beta_0 + \beta_s s_i + \beta_p p &= \frac{s_i + \frac{1-\lambda}{\lambda}\bar{s} - p}{1 + \frac{\partial p}{\partial a_i}} \\ &= \frac{s_i + \frac{1-\lambda}{\lambda}\left(\frac{(1-Ncc_p)p - P_0 - Ncc_0}{Ncc_s}\right) - p}{1+m}.\end{aligned}$$

The results are summarized in the following proposition that describes the linear and symmetric equilibrium demand function.

Proposition 1 (Demand Function Equilibrium with Noise Free Signals)
For every noise free information structure λ, there exists a unique symmetric linear Bayes Nash equilibrium. The coefficients of the linear demand function are given by

$$\beta_0 = -\frac{(1-\lambda)\,c_0}{Nc}, \quad \beta_s = \frac{1}{1+m}, \quad \beta_p = \frac{1-\lambda}{Nc} - \frac{\lambda}{1+m}, \tag{17}$$

and

$$m = \frac{1}{2}\left(-Nc\frac{(N-1)\lambda - 1}{(N-1)\lambda+1} + \sqrt{(Nc\frac{(N-1)\lambda-1}{(N-1)\lambda+1})^2 + 2Nc + 1} - 1\right). \tag{18}$$

The coefficients of the demand function, $\beta_0, \beta_s, \beta_p$ are described in terms of the primitives of the model and the price impact m, which in turn is a function of the primitives c, N, and the information structure λ. Importantly, even when we hold the payoff environment as represented by c and N (and the distribution of payoff states fixed), the price impact varies greatly with the information structure and we develop the impact that the information structure has on the equilibrium strategy of the agents in more detail in the next subsection. For now we observe that from the equilibrium strategies we can immediately infer the realized demands in equilibrium which have a very transparent structure.

Corollary 1 (Realized Demand)
In the unique symmetric linear Bayes Nash equilibrium, the realized demand a_i of each agent is given by: $a_i = \Delta a_i + \bar{a}$ with:

$$\Delta a_i = \frac{\Delta\theta_i}{1+m} \text{ and } \bar{a} = \frac{\bar{\theta}}{1+c+m}. \tag{19}$$

The description of the equilibrium trades in (19) indicate that the individual trades are measurable with respect to idiosyncratic as well as the common component of the payoff shock of agent i, $\Delta\theta_i$ and $\bar{\theta}$ respectively. Thus the demand functions remain equilibrium strategies even under complete information, and hence form an ex post equilibrium, and not merely a Bayes Nash equilibrium. Moreover, as the equilibrium strategy does not depend on the conditional expectation, the above characterization of the equilibrium is valid beyond the multivariate normal distribution. As long as we maintain the quadratic payoff environment, any symmetric and continuous joint distribution of the payoff state $(\theta_1, ..., \theta_N)$ would lead to the above characterization of the ex post equilibrium in demand functions.

We observe that the level of realized demand depends on the payoff state and the equilibrium price impact. Thus we find that an increase in the market power leads to an uniform reduction of the realized demand for all realizations of the payoff state. Notably, the socially efficient level of trade would be realized at $m = 0$, and an increase in market power leads to a dampening of the quantity traded (both on average across agents and the idiosyncratic quantity by an individual agents). The idiosyncratic component Δa_i in the realized demand is not affected by the supply condition c, as the sum of the idiosyncratic terms sum to zero, by definition of the idiosyncratic trade. However, even the level of idiosyncratic demand is affected by the price impact as it influences the response of all the other traders.

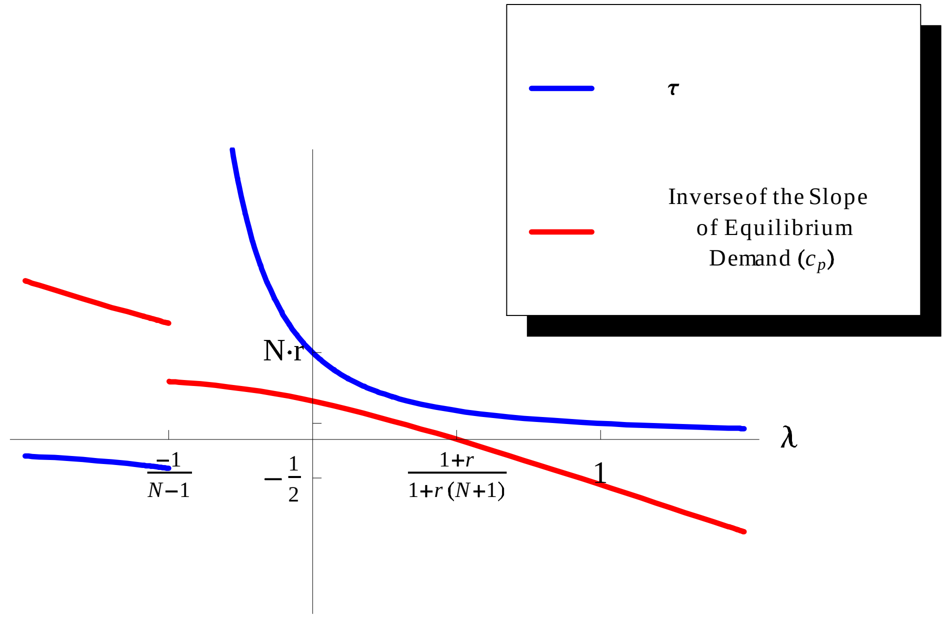

Figure 1: Equilibrium Demand and Price Impact with Noise Free Information

3.4 Information Structure and Price Impact

We derived the equilibrium demand for a given information structure λ in Proposition 1. But the above analysis also lays the groundwork to understand how the information structure impacts the demand function and consequently the price impact and the market power of each each agent.

Figure 1 graphically summarizes the price impact m and the price sensitivity of the equilibrium demand function β_p as a function of the information structure λ as derived earlier in Proposition 1.

Earlier we noticed that the price impact of a given trader is supported by the price sensitivity of the other traders, see (14). Thus the central aspect to understand is how each trader responds to the new information contained in the price relative to his private signal s_i. From (16), we can write the conditional expectation of the agent about his payoff state given his signal s_i and the signals of all the other agents s_{-i} as:

$$\mathbb{E}[\theta_i|s_i, s_{-i}] = \left(\frac{N-1}{N} + \frac{1}{\lambda N}\right) s_i + \left(\frac{1-\lambda}{\lambda}\right) \frac{1}{N} \sum_{j \neq i} s_j, \tag{20}$$

where the conditional expectation follows from the fact that the signals are noise free and that the equilibrium is informative. Noticeably, the expectation does not refer to either the nature of the supply function or the moments of the common prior. Thus the conditional expectation of agent i is a linear combination of his own signal and the signal of the other agents. Importantly, as the weight λ on the common component turns negative there is a critical value

$$\lambda = -\frac{1}{N-1}, \tag{21}$$

at which the conditional expectation of agent i is independent of his own signal s_i, and equal to the sum of the signals of the other agents:

$$\mathbb{E}[\theta_i | s_i, s_{-i}] = -\sum_{j \neq i} s_j.$$

We can thus expect the demand behavior to change dramatically around the critical information structure λ in which each agent learns nothing from his own signal in the presence of all the other signals. Proposition 2 and 3 therefore describe separately the qualitative behavior of the equilibrium behavior for $\lambda \geq -1/(N-1)$ and $\lambda < -1/(N-1)$.

Indeed for the moment, let us suppose that the signal overweighs the common shock relative to its payoff importance, that is $\lambda > 1$. Each trader submits a demand function given his private information, and has to decide how to respond to the price, that is how to set β_p, negative or positive, small or large ? We saw that the equilibrium price p will be perfectly informative about the average signal $\overline{s}$ and thus about the average state $\overline{\theta}$. In particular, a higher price p reflects a higher average state $\overline{\theta}$. So how should the demand of agent i respond to a market clearing price p that is higher (or lower) than expected on the basis of the private signal ? Thus suppose the equilibrium price p reflects a higher common state $\overline{\theta}$ than agent i would have expected on the basis of his conditional expectation $\mathbb{E}\left[\overline{\theta}\,|s_i\right]$, or $\overline{\theta} > \mathbb{E}\left[\overline{\theta}\,|s_i\right]$. As the signal is noise free, and given by the weighted linear combination $s_i = \Delta\theta_i + \lambda\overline{\theta}$, agent i will have to revise his expectation $\mathbb{E}\left[\Delta\theta_i\left|\overline{\theta}, s_i\right.\right]$ about the idiosyncratic component downwards, after all the weighted sum of the realization $\Delta\theta_i$ and $\overline{\theta}$ still have to add up to s_i. Also, given s_i and the realization of the common component $\overline{\theta}$,, the conditional expectation equals the realized idiosyncratic component, $\mathbb{E}\left[\Delta\theta_i\left|\overline{\theta}, s_i\right.\right] = \Delta\theta_i$, and thus

$$\mathbb{E}\left[\Delta\theta_i\left|\overline{\theta}, s_i\right.\right] - \mathbb{E}\left[\Delta\theta_i\,|s_i\right] = \Delta\theta_i - \mathbb{E}\left[\Delta\theta_i\,|s_i\right] = -\lambda\left(\overline{\theta} - \mathbb{E}\left[\overline{\theta}\,|s_i\right]\right), \tag{22}$$

to balance the upward revision of the common component. Moreover, since the signal overweighs the common component, the downward revision of the idiosyncratic component occurs at the multiple λ of the upwards revision, thus leading to a decrease in the expectation of the payoff state θ_i :

$$\mathbb{E}\left[\theta_i\left|\overline{\theta}, s_i\right.\right] - \mathbb{E}\left[\theta_i\,|s_i\right] = \theta_i - \mathbb{E}\left[\theta_i\,|s_i\right] = -(\lambda - 1)\left(\overline{\theta} - \mathbb{E}\left[\overline{\theta}\,|s_i\right]\right) < 0. \tag{23}$$

We can therefore conclude that in equilibrium the realized price p and the realized willingness to pay θ_i move in the opposite direction, thus suggesting that $\beta_p < 0$. Moreover as the weight λ on the common component in the signal increases, the downward revision of the expectation becomes more pronounced, and hence induces each agent to lower his demand more aggressively in response to higher prices, thus depressing β_p further. And we observed earlier, see (14), that from the point of view of agent i, the sensitivity to price by all other agents implies that the price impact of agent i is decreasing. Together this leads to the comparative static result of price sensitivity and price impact with respect to the information structure parametrized by the weight λ, as stated below in Proposition 2.1 and 3.1. As λ is increasing, the price sensitivity decreases and the resulting price impact m converges to 0 as $\lambda \to \infty$.

Proposition 2 (Information Structure and Price Impact)
For $\lambda \geq -1/(N-1)$:

1. *the price impact $m \in (0, \infty)$ and the price sensitivity $\beta_p \in \left(-\infty, \frac{1}{c(N-1)}\right)$ are decreasing in λ;*
2. *as $\lambda \to -1/(N-1)$, we have:*
$$\lim_{\lambda \downarrow -1/(N-1)} m = +\infty, \quad \lim_{\lambda \downarrow -1/(N-1)} \beta_p = \frac{1}{c(N-1)};$$
3. *as $\lambda \to \infty$, we have:*
$$\lim_{\lambda \to \infty} m = 0, \quad \lim_{\lambda \to \infty} \beta_p = -\infty.$$

Thus we find that the information structure has a profound effect on the responsiveness of the agents to the price p, and through the equilibrium price sensitivity of the agents, it affects the market power of each agent as captured by the price impact.

Evidently, there are two forces which determine the price impact. We can understand the price impact by looking at the response of the other agents if agent i decides to submit a higher demand than expected. First, if the agents observe higher prices, then they interpret this as a higher realization of $\bar{\theta}$ than they would have originally estimated. This results in an increase in the submitted demand functions. On the other hand, an unexpected increase in the price also results in the agents interpreting this as a lower realization of $\Delta\theta_i$ than they would have originally estimated. Thus, they tend to reduce their demand. It is particularly easy to explain which of these forces dominate by inspecting the two limit cases of $\lambda = 0$ and $\lambda = \infty$.

In the case $\lambda \to \infty$, the agents can anticipate the equilibrium price based only on their own signals, and thus they submit a perfectly elastic demand. As $\lambda \to \infty$, the private signal of each agent is a very good predictor of the average signal. Thus, they can correctly anticipate what will be the equilibrium price. In consequence, if an agent faces a higher than anticipated equilibrium price, he attributes this to a negative shock on the value of $\Delta\theta_i$. Therefore, he reduces his equilibrium demand which reduces the price which in turn implies a very elastic demand.

In the case of $\lambda \to 0$, each signal conveys only purely idiosyncratic information and by construction they idiosyncratic signals sum up to 0. Therefore, agents have zero information on what the equilibrium price should be based on the signal they receive individually. The equilibrium in this case consists of individual demand functions that are perfectly collinear with the supply function, and with prices adjusting to the average signal. This equilibrium suffers from the classic Grossman-Stiglitz paradox that prices are not measurable with respect to the information agent have. Yet, in the limit we get the same intuitions without suffering from the paradox. As $\lambda \to 0$, all the information that the agents have are about their idiosyncratic shocks. Thus, they forecast the average payoff state from the equilibrium price they observe. If an agent decreases the quantity he submits, the response of the other agents is to forecast a lower average state and thus they also decreasing the quantity they demand. A decrease in demand by an agent is thus reinforced by the best response of the other agents. Similarly, if an agent increases the price by increasing the demand he submits, he expects that all other agents should do the same. Therefore, in the limit case as $\lambda \to 0$, we obtain a collusive price level.

For negative values of λ we have that both forces reinforce each other. Depending on the value of λ, agents interpret a high price as a lower shock to $\bar{\theta}$ and $\Delta\theta$, in which case the price impact gets to be above Nc. The other situation is that agents interpret a high price as a higher shock to $\bar{\theta}$ and $\Delta\theta$, in which case the price impact gets to be below 0, as agents decrease the demand of other agents by increasing the price level. The ensuing comparative static result holds locally for all $\lambda \in \mathbb{R}$ except for the critical value of $\lambda = -1/(N-1)$, where we observe a discontinuity in the price sensitivity and the price impact of the agents.

Even before we reach the critical value $\lambda = -1/(N-1)$, the interaction between the equilibrium price and the equilibrium update on the willingness to pay becomes more subtle if the common shock $\bar{\theta}$ receives a smaller weight in the signal than it receives in the payoff state of agent i. A higher than expected price still implies a higher than expected common shock, but because $\lambda < 1$, the resulting downward revision of the idiosyncratic component $\Delta\theta_i$ is smaller, and in consequence the resulting revision on the payoff state θ_i balances in favor of the payoff state. Thus a higher price now indicates a higher expected payoff state θ_i, and if λ falls sufficiently below 1, then the price sensitivity β_p even turns positive, in fact it hits zero at $\lambda = (1+c)/(1+c(N+1)) < 1$.

Proposition 3 (Information Structure and Negative Price Impact)
For $\lambda < -1/(N-1)$:

1. *the price impact $m \in \left(-\frac{1}{2}, 0\right)$ and the price sensitivity $\beta_p \in \left(\frac{1+2c}{c(N-1)}, \infty\right)$ are decreasing in λ;*
2. *as $\lambda \to -1/(N-1)$, we have:*
$$\lim_{\lambda \uparrow -1/(N-1)} m = -\frac{1}{2}, \quad \lim_{\lambda \uparrow -1/(N-1)} \beta_p = \frac{1+2c}{c(N-1)};$$
3. *as $\lambda \to -\infty$, we have:*
$$\lim_{\lambda \to \infty} m = 0, \quad \lim_{\lambda \to \infty} \beta_p = +\infty.$$

It remains to understand the source of the discontinuity in the price sensitivity and the price impact at the critical value of $\lambda = -1/(N-1)$. We observed that as the information structure λ approaches the critical value, the impact that the agent i's signal has on his expectation of θ_i converges to 0 as

$$\mathbb{E}[\theta_i | s_i, s_{-i}] = \left(\frac{N-1}{N} + \frac{1}{\lambda N}\right) s_i + \left(\frac{1-\lambda}{\lambda}\right) \frac{1}{N} \sum_{j \neq i} s_j. \tag{24}$$

This implies that the total weight that an agent puts on his own signal goes to 0 as he is only learning from the stochastic intercept of the residual supply that he faces.[2] Importantly, as λ approaches $-1/(N-1)$ from the right, a high signal implies a low payoff type θ_i for agent i who receives the signal, but also for all other agents as $\lambda \in (-1/(N-1), 0)$ implies that

$$\frac{\partial \bar{\theta}}{\partial s_i}, \frac{\partial \theta_i}{\partial s_i} \leq 0. \tag{25}$$

Now, as λ is close to $-1/(N-1)$ a small upward shift in the quantity bought by any agent i is interpreted as a large upward shift in the equilibrium expectation of agent i as he is only putting a small weight on his own signal. But importantly, while a smaller signal realization s_i leads agent i to revise his expectation about θ_i moderately upwards, by all other agents $j \neq i$, it will correctly

[2] Note that the weight an agent puts on his own signal is not only the coefficient β_s but also depends on the coefficient β_p

by interpreted as a much larger increase in their expectation of θ_j and thus lead to a huge upward shift in the amount bought by all other agents. In the limit as $\lambda \to -1/(N-1)$, an arbitrarily small increase in the amount bought by agent i leads to an arbitrarily large increase in the equilibrium expectation of θ_j for all other agents which implies an arbitrarily high increase in the quantity bought by other agents, which implies an arbitrarily large increase in price. Thus, a small change in the quantity bought by agent i implies an arbitrarily high positive impact on prices, which leads to arbitrarily large market power as stated in Proposition 2.1.

Now, the discontinuity at $\lambda = -1/(N-1)$ arises as when λ approaches $-1/(N-1)$ from the left, the sign in the updating rule for the payoff state of agent i and all the other agents differ, namely evaluating (24) at $\lambda \leq -1/(N-1)$ gives us:

$$\frac{\partial \theta_i}{\partial s_i} \geq 0 \text{ and } \frac{\partial \bar{\theta}}{\partial s_i} \leq 0. \tag{26}$$

In turn, an small increase in the payoff state of agent i caused by the signal s_i implies a large decrease in the common component $\bar{\theta}$, and hence a large decrease in the payoff state θ_j of the all other agents, $j \neq i$. When we translate this into demand behavior, we find that a small increase in quantity demand by agent i implies a large decrease in the demand by all the other agents. Thus, an increase in the quantity bought by agent i reduces the price at which he buys, as other agents are decreasing the quantity they buy. In fact, this means that the price impact m turns from positive to negative when $\lambda < -1/(N-1)$. In fact if we consider the objective function of agent i :

$$\mathbb{E}[\theta_i | s_i] a_i - \frac{1}{2} a_i^2 - p a_i, \tag{27}$$

and notice that $\partial p / \partial a_i = m$, so that

$$\mathbb{E}[\theta_i | s_i] a_i - \left(\frac{1}{2} + m \right) a_i^2,$$

then with < 0, it is as if the objective function of agent is becoming less concave as m decreases. Thus, as λ approaches $-1/(N-1)$ from the left, even though an agent i's signal s_i is almost non-informative of his type, as the agent is almost risk neutral, his demand response to his own signal remains bounded away from 0, which also bounds the response of other agents to any shift in the amount bought by an individual agent, explaining the finite limit from the left.

It has been prominently noted that demand function competition under complete information typically has many, often a continuum, of equilibrium outcomes. In a seminal contribution, Klemperer and Meyer (1989) showed that only one of these outcomes survives if the game is perturbed with a small amount of imperfect information. The equilibrium market power under this perturbation is the same outcome as when each agent receives a noise free signal with $\lambda = 1$.

The set of outcomes described by noise free signals coincide with the outcomes described by slope takers equilibrium, as described by Weretka (2011). We established in Proposition 1 that any feasible market power can be decentralized as an ex-post complete information equilibrium.

It might be helpful to complete the discussion with the limit case in which there is no exogenous supply of the asset, that is as $c \to \infty$, and we require that the average net supply $\overline{a} = 0$ for all realization of payoff states and signals.

Proposition 4 (Equilibrium with Zero Net Supply)

1. *If $\lambda \in [-1/(N-1), 1/(N-1)]$, then there does not exist an equilibrium in linear symmetric demand functions;*

2. *If $\lambda \notin [-1/(N-1), 1/(N-1)]$, the coefficients of the linear equilibrium demand function are:*
$$\beta_s = \frac{1}{1+m},\ \beta_p = \frac{-\lambda}{1+m},$$
and the market power is given by:
$$m = \frac{1}{\lambda(N-1)-1}.$$

We could alternatively consider the case with elastic supply and then let $c \to \infty$. We then find that the expression for m would show that as $c \to \infty$, we have $m \to \infty$ for all $\lambda \in [-1/(N-1), 1/(N-1)]$. By contrast, for $\lambda \notin [-1/(N-1), 1/(N-1)]$, if $c \to \infty$, then $m \to 1/\left(\lambda(N-1)-1\right)$.

The non-existence of equilibrium for the case of zero net supply is known in the literature. It is usually attributed to the presence of only two agent. The above result shows that for every finite number of agents there exist information structures for which there are equilibria and information structure for which there are no equilibria. The range of value of λ for which there is no equilibrium decreases with the number of agents. Finally, we note that as the literature focused exclusively on the case of noisy, but non-confounding information structure, and hence $\lambda = 1$, the non-existence of equilibrium arises only for $N = 2$.

4 Equilibrium Behavior for All Information Structures

Until now we have analyzed the demand function competition for a small and special class of information structures. We now extend the analysis to the equilibrium behavior under all possible (symmetric and normally distributed) information structures. To do so, we introduce a solution concept that we refer to as Bayes correlated equilibrium. This solution concept will describe the equilibrium behavior in terms of a distribution of outcomes, namely action and states, and the price impact.

4.1 Definition of Bayes Correlated Equilibrium

The notion of Bayes correlated equilibria will be defined independently of any specific information structure of the agents. We shall merely require that the joint distribution of outcomes, namely prices, quantities, and payoff states form a joint distribution such that there exists a price impact m under which the best response conditions of the agents is satisfied and the market clears.

The equilibrium object is therefore a joint distribution, denoted by μ, over prices, individual and average quantity, and individual and average payoff state, $(p, a_i, \bar{a}, \theta_i, \bar{\theta})$. As we continue to restrict the analysis to symmetric normal distribution, such a joint distribution is a multivariate distribution given by:

$$\begin{pmatrix} p \\ a_i \\ \bar{a} \\ \theta_i \\ \bar{\theta} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_p \\ \mu_a \\ \mu_a \\ \mu_\theta \\ \mu_\theta \end{pmatrix}, \begin{pmatrix} \sigma_p^2 & \rho_{ap}\sigma_a\sigma_p & \rho_{p\bar{a}}\sigma_{\bar{a}}\sigma_p & \rho_{\theta p}\sigma_\theta\sigma_p & \rho_{p\bar{\theta}}\sigma_{\bar{\theta}}\sigma_p \\ \rho_{ap}\sigma_a\sigma_p & \sigma_a^2 & \rho_{a\bar{a}}\sigma_{\bar{a}}\sigma_a & \rho_{a\theta}\sigma_\theta\sigma_a & \rho_{a\bar{\theta}}\sigma_{\bar{\theta}}\sigma_a \\ \rho_{\bar{a}p}\sigma_{\bar{a}}\sigma_P & \rho_{a\bar{a}}\sigma_{\bar{a}}\sigma_a\sigma_{\bar{a}} & \sigma_{\bar{a}}^2 & \rho_{\theta\bar{a}}\sigma_\theta\sigma_{\bar{a}} & \rho_{\bar{a}\bar{\theta}}\sigma_{\bar{\theta}}\sigma_{\bar{a}} \\ \rho_{\theta p}\sigma_\theta\sigma_p & \rho_{a\theta}\sigma_\theta\sigma_a & \rho_{\theta\bar{a}}\sigma_\theta\sigma_{\bar{a}} & \sigma_\theta^2 & \rho_{\theta\bar{\theta}}\sigma_\theta\sigma_{\bar{\theta}} \\ \rho_{p\bar{\theta}}\sigma_{\bar{\theta}}\sigma_p & \rho_{a\bar{\theta}}\sigma_{\bar{\theta}}\sigma_a & \rho_{\bar{a}\bar{\theta}}\sigma_{\bar{\theta}}\sigma_{\bar{a}} & \rho_{\theta\bar{\theta}}\sigma_\theta\sigma_{\bar{\theta}} & \sigma_{\bar{\theta}}^2 \end{pmatrix} \right). \tag{28}$$

With a symmetric distribution across agents, the covariance between any two agents, say in the demand a_i and a_j is identical to the correlation between the demand a_i and the average demand $\bar{a}$. Thus it suffices to track the outcome of an individual agent i and the average outcome, where we use the notation introduced earlier in (3) and (4) to denote the common and idiosyncratic component of a_i, namely $\overline{a}$ and Δa_i.

Definition 2 (Bayes Correlated Equilibrium)

A Bayes correlated equilibrium is a joint (normal) distribution of $(p, a_i, \bar{a}, \theta_i, \bar{\theta})$ (as given by (28)) and a price impact $m \in (-1/2, \infty)$ such that best response condition holds for all i, a_i, p:

$$\mathbb{E}[\theta_i | a_i, p] - p - a_i - m a_i = 0; \tag{29}$$

and the market clears:

$$p = c_0 + cN\bar{a}. \tag{30}$$

The best response condition (29) reflects the fact that in the demand function competition each trader can condition his demand, and hence his expectation about the payoff state on the equilibrium price. Given the price p and the price impact m, the equilibrium quantity a_i must then be optimal for agent i. In addition the market clearing condition determines the price as a function of the realized demand, $N\bar{a}$, which by definition is equal to $\Sigma_i a_i$. Since the price p is perfectly collinear with the aggregate demand $N\bar{a}$, we will frequently refer to a Bayes correlated equilibrium in terms of the variables $(a_i, \bar{a}, \theta_i, \bar{\theta})'$, without making an explicit reference to the price.

We introduced the notion of a Bayes correlated equilibrium in Bergemann and Morris (2013) in a linear best response model with normally distributed uncertainty, and subsequently defined it for a canonical finite games, with a finite number of actions, states and players in Bergemann and Morris (2015). The present definition adapts the notion of Bayes correlated equilibrium to the demand function competition in two ways. First, it accounts for the fact that the agents can condition on the price in the conditional expectation $\mathbb{E}[\theta_i|a_i, p]$ of the best response condition. Second, it allows the price impact m to be an equilibrium object and thus determined in equilibrium.

In Section 5 we shall compare in detail the set of equilibrium outcomes under demand function competition and under quantity (Cournot) competition. Here it might be informative to simply contrast the Bayes correlated equilibrium conditions. Under quantity competition, each agent has to chose a quantity independent of the price, and hence the price impact $\partial p/\partial a_i$ is determined by the market clearing condition, and hence $\partial p/\partial a_i = c$ for all i and a_i. Likewise, the conditional expectation of each agent can only take into account the choice of the quantity a_i, but cannot condition on the price. Thus the best response condition under quantity competition can now be stated as:

$$\mathbb{E}[\theta_i|a_i] - \mathbb{E}[p|a_i] - a_i - ca_i = 0. \tag{31}$$

By contrast, the market clearing condition and the specification of the equilibrium distribution remain unchanged relative to above definition under demand function competition. Thus, already at this point, we observe that the quantity competition looses one degree of freedom as the price impact is fixed to c by the exogenous supply function, but it gains one degree of freedom as agents do not condition their best response on price.

We will now provide two different characterizations of the set of feasible outcomes as Bayes correlated equilibrium. The first characterization is purely in statistical terms, namely the moments of the equilibrium distribution. We provide a sharp characterization of what are the feasible outcomes under any information structure. This characterization in particular will allow us to understand how the conditioning on the prices, which are a source of endogenous information, restrict the set of outcomes with respect to an economy where demand decision have to be made in expectation of the realized equilibrium price, such as in the quantity competition. We then provide an equivalence result that formally connects the solution concept of Bayes correlated equilibria to the Bayes Nash equilibrium under all possible (normal symmetric) information structure. In turn, the equivalence result suggest a second characterization in terms of a canonical information structure that allow us to decentralize all outcomes as Bayes Nash equilibrium supported by the specific class of canonical information structures. This characterization provides a link between information structure as exogenous data and the equilibrium outcome. As such it is suitable to provide additional intuition behind the driving mechanisms of the equilibrium price impact.

4.2 Statistical Characterization

We provide a statistical characterization of the set of feasible distributions of quantities and prices in any Bayes correlated equilibrium. We begin with an auxiliary lemma that allows us to reduce the number of variables we need to describe the equilibrium outcome. The reduction in moments that we need to track arises purely from the symmetry property (across agents) of the outcome distribution rather than the equilibrium properties. Under symmetry across agents, we can represent the moments of the individual variables, θ_i and a_i, in terms of the moments of the common component of these variables, namely $\overline{\theta}$ and $\overline{a}$, and the idiosyncratic components, $\Delta\theta_i$ and Δa_i. By construction, the idiosyncratic and the common component are orthogonal to each other, and hence exactly half of the covariance terms are going to be equal to zero. In particular, we only need to follow the covariance between the idiosyncratic components, $\Delta\theta_i$ and Δa_i, and the covariance between the common components, $\overline{\theta}$ and $\overline{a}$, and we define their correlation coefficients:

$$\rho_{\Delta\Delta} \triangleq \text{corr}(\Delta a_i, \Delta\theta_i) \text{ and } \rho_{\bar{a}\bar{\theta}} \triangleq \text{corr}(\bar{a}, \bar{\theta}). \tag{32}$$

With this we can represent the outcome distribution as follows.

Lemma 1 (Symmetric Outcome Distribution)
If the random variables $(a_1, ..., a_N, \theta_1, ..., \theta_N)$ are symmetric normally distributed, then the random variables $(\Delta a_i, \bar{a}, \Delta\theta_i, \bar{\theta},)$ satisfy:

$$\mu_{\bar{a}} = \mu_a,\ \mu_{\bar{\theta}} = \mu_\theta,\ \mu_{\Delta a} = \mu_{\Delta\theta} = 0,$$

and their joint distribution of variables can be expressed as:

$$\begin{pmatrix} \Delta a_i \\ \bar{a} \\ \Delta\theta_i \\ \bar{\theta} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ \mu_a \\ 0 \\ \mu_\theta \end{pmatrix}, \begin{pmatrix} \frac{(N-1)\sigma_a^2(1-\rho_{aa})}{N} & 0 & \rho_{\Delta\Delta}\sigma_{\Delta a}\sigma_{\Delta\theta} & 0 \\ 0 & \frac{\sigma_a^2((N-1)\rho_{aa}+1)}{N} & 0 & \rho_{\bar{a}\bar{\theta}}\sigma_{\bar{a}}\sigma_{\bar{\theta}} \\ \rho_{\Delta\Delta}\sigma_{\Delta a}\sigma_{\Delta\theta} & 0 & \frac{(N-1)\sigma_\theta^2(1-\rho_{\theta\theta})}{N} & 0 \\ 0 & \rho_{\bar{a}\bar{\theta}}\sigma_{\bar{a}}\sigma_{\bar{\theta}} & 0 & \frac{\sigma_\theta^2((N-1)\rho_{\theta\theta}+1)}{N} \end{pmatrix}\right), \tag{33}$$

We can now characterize the set of Bayes correlated equilibrium.

Proposition 5 (Moment Restrictions of Bayes Correlated Equilibrium)
The normal random variables $(\theta_i, \bar{\theta}, a_i, \bar{a})$ and the price impact $m \in (-1/2, \infty)$ form a symmetric Bayes correlated equilibrium if and only if:

1. *the joint distribution of variables $(\theta_i, \bar{\theta}, a_i, \bar{a})$ is given by (33);*

2. *the mean individual action is:*
$$\mu_a = \frac{\mu_\theta - c_0}{1 + cN + m}; \tag{34}$$

3. *the variance of the common and the idiosyncratic components of the individual action are:*
$$\sigma_{\bar{a}} = \frac{\rho_{\bar{a}\bar{\theta}}\sigma_\theta}{1 + m + c}, \; \sigma_{\Delta a} = \frac{\rho_{\Delta\Delta}\sigma_\theta}{1 + m}; \tag{35}$$

4. *the correlation coefficients are:*
$$\rho_{\Delta\Delta}, \rho_{\bar{a}\bar{\theta}} \in [0, 1] \text{ and } \rho_{aa} = \frac{\sigma_{\bar{a}}^2}{\sigma_{\bar{a}}^2 + \sigma_{\Delta a}^2}. \tag{36}$$

The above proposition leads us conclude that the equilibrium outcome is jointly determined by the payoff environment in form of the mean and variance of the valuation, μ_θ and σ_θ, the supply function, c and c_0, and three endogenous equilibrium variables $(m, \rho_{\bar{a}\bar{\theta}}, \rho_{\Delta\Delta})$. Moreover, these three endogenous variables are unrestricted by the payoff environment of the game. The payoff relevant exogenous variables of the game only enter into the determination of the mean and variance of the equilibrium actions next to endogenous variables $(m, \rho_{\bar{a}\bar{\theta}}, \rho_{\Delta\Delta})$.

Proposition 5 informs us that the first moment, μ_a, depends only on of the equilibrium variables, namely the price impact m. By contrast, the variance of the average action depends on the correlation $\rho_{\bar{a}\bar{\theta}}$ between average action and average payoff state, and analogously the variance of the idiosyncratic component depends on the correlation $\rho_{\Delta\Delta}$ between idiosyncratic component of action and payoff state. As the first and second moment change with price impact, but only the second moment changes with the correlations between quantities and payoff shock, it is interesting to illustrate the set of feasible first and second moments.

In Figure 2 we illustrate the set of feasible normalized variance of the aggregate quantity traded ($\sigma_{\bar{a}}/\sigma_{\bar{\theta}}$) and normalized expected quantity traded $\mu_a/(\mu_\theta - c_0)$. We can see that there is a linear relation between the maximal normalized aggregate that can be achieved for any given expected quantity traded. The interpretation is as follows. Consider a analyst that know the expected quantity traded by each agent. If this quantity is very small compared with respect to the optimal one, then the analyst must infer that price impact is very large. This in turn implies that the aggregate variance is very small as well, independent on the amount of ex-post information agents have on the aggregate shock (measured by $\rho_{\bar{a}\bar{\theta}}$). On the other hand, if the analyst observes that the expected quantity traded by an agent is close to the optimal one, then he must infer that price impact is low. Nevertheless, this does not allow the analyst to infer that the variance of the aggregate quantity traded is high. This comes from the fact that the aggregate variance depends on the amount of ex-post information agents have on the aggregate shock. This implies that, even if price impact is low, the variance of the aggregate quantity traded is low. Nevertheless, if the ex-post information agents have is high (that is, $\rho_{\bar{a}\bar{\theta}}$ close to 1) and the expected quantity traded is close to one, then the variance of the aggregate quantity traded will be high.

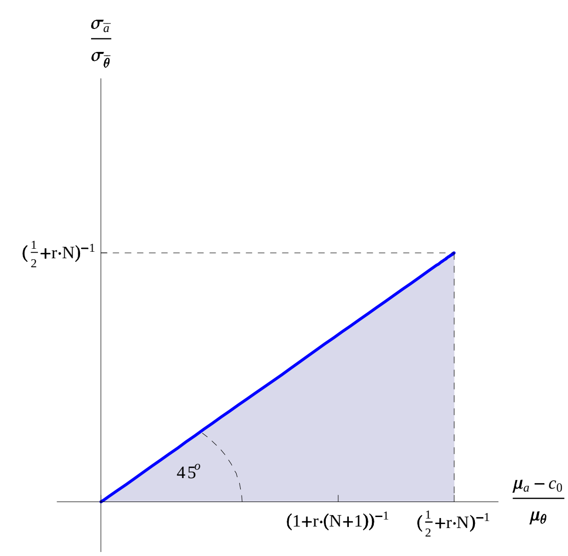

Figure 2: First and Second Moments of Average Demand

In Figure 3 we illustrate the set of feasible normalized variance of the individual quantity traded (σ_a/σ_θ) and normalized expected quantity traded $\mu_a/(\mu_\theta - c_0)$, for $\rho_{\theta\theta} = 1/2$. The basic qualitative description is similar to the one we did for variance of the aggregate quantity traded. Nevertheless, in this case the upper boundary is convex. To understand the source of the convexity it is convenient to think of the case in which c is very large. In this case, independent of the information structure the expected quantity traded by an agent will be very low (this can be seen from (34)). This will also imply that the variance of the aggregate quantity traded will be small, but this will impose no restriction on the variance of the idiosyncratic quantity traded (which can be seen from (35)). Hence, small differences in the expected quantity traded imply very big differences in the price impact (after all, the expected quantity traded will be small for all price impact). Importantly, if average quantity traded is "too low", this implies a price impact that is "too high", and hence small changes in the expected quantity traded still imply price impact that is "too high". Hence, if price impact is "too high" the variance of the idiosyncratic quantity traded remains close to 0. As the expected quantity traded increases, the price impact is not "too high"', and hence a small increment in the expected quantity traded implies very large changes in price impact. As the price impact is not "too high" in the first place, this in turns implies very large changes in the variance of the idiosyncratic quantity traded. Hence, the relation between the expected quantity traded and the variance of the idiosyncratic quantity traded is convex. This implies that the relation between the expected quantity traded and the variance of the individual quantity traded is also convex. The convexity is accentuated by an increase in c, and becomes linear as $c = 0$.

In Section 3, we analyzed the Bayes Nash equilibria under a specific class of noise free information structures. There, we established in Corollary 1 the realized demands. In particular we showed that the idiosyncratic and the common component of the demand, Δa_i and $\overline{a}$, were linear functions of the idiosyncratic and common component of the payoff shocks, $\Delta\theta_i$ and $\overline{\theta}$, respectively. It follows that the Bayes Nash equilibrium outcomes under the noise free information structure displayed maximal correlation of $\rho_{\overline{a}\overline{\theta}}$ and $\rho_{\Delta\Delta}$, namely $\rho_{\overline{a}\overline{\theta}} = \rho_{\Delta\Delta} = 1$. This is of course consistent with agents having ex-post complete information, as their action is perfectly measurable with respect to the payoff relevant shocks. Thus, the class of noise-free signals allows us to decentralize all feasible price impacts $m \in (-1/2, \infty)$ with maximal correlation between the action and payoff states. Hence, the upper boundary of the moments in Figure 2 and 3 - the dark blue line - is generated by the noise free information structures. With maximal price impact, that is as $\lambda \to -1/N$ and hence $m \to \infty$, there is zero trade in equilibrium, and the mean (and realized trade is zero). As the price impact

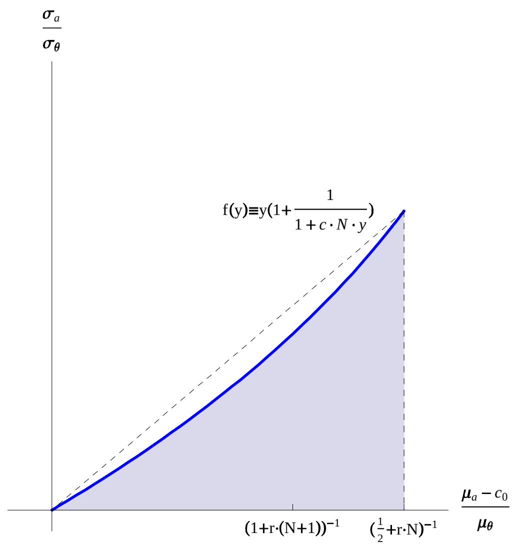

Figure 3: First and Second Moments of Individual Demand

weakens, and more weight is placed on the common payoff state in the noise-free signal, that is as $\lambda \to \infty$ and $m \to 0$, the expected quantity traded by an agent increases , and so is the maximally feasible variance of average trade. the maximum is achieved by taking limit $\lambda \to -1/N$ from the left, which yields in the limit a price impact of $-1/2$. This implies that there is more trade than optimal.

4.3 Equivalence

We now provide a result that formally connects the solution concept of Bayes correlated equilibria to the Bayes Nash equilibrium under all possible (normal symmetric) information structure. We establish that the set of Bayes Nash equilibrium in demand functions for all (normal) information structure can be equivalently be described by the set of Bayes correlated equilibria. In the Bayes Nash equilibrium, we take as given exogenous data the information structure, the type space, that the agents have, and then derive the resulting equilibrium strategies. In turn, the equilibrium strategies generate a particular joint distribution of realized quantities, prices and payoff states. But instead of describing an equilibrium outcome through the process of finding the Bayes Nash equilibrium for every possible information structure, one can simply analyze which joint distributions of realized traded quantities, prices and payoff states, can be reconciled with the equilibrium conditions of best response and market clearing for some given equilibrium price impact. This latter description of an equilibrium exactly constitutes the notion of Bayes correlated equilibrium.

Proposition 6 (Equivalence)
A set of demand functions $\{x_i(s_i, p)\}_i$ and an information structures $\mathcal{I}$ form a Bayes Nash equilibrium if and only if the resulting equilibrium distribution of $(p, a_i, \bar{a}, \theta_i, \bar{\theta})$ together with a price impact $m \in (-1/2, \infty)$ form a Bayes correlated equilibrium.

We see that Proposition 6 allow us to connect both solution concepts, and show that they describe the same set of outcomes. The Bayes Nash equilibrium in demand functions predicts not only an outcome, but also the demand function submitted by agents. In many cases this demand function can be observed in the data, so it is also empirically relevant. Yet, a Bayes correlated equilibrium also contains the information pertaining to the demand functions agents submit. Instead of describing the slope of the demand function an agent submits, a Bayes correlated equilibrium specifies the price impact each agent has. Yet, there is a bijection between price impact and the slope of the demand an agent submits in equilibrium. Thus, by specifying a Bayes correlated equilibrium we are not only specifying outcomes in terms of quantities traded, prices and types, but also the slope of the demands agents must have submitted in equilibrium.

4.4 Canonical Information Structures

With the equivalence result of Proposition 6, we know that the set of equilibrium outcomes as described by the Bayes correlated equilibrium in Proposition 5 is complete and exhaustive with respect to the set of all Bayes Nash equilibria. Yet, the equivalence does not tell us how complex the information structures have to be order in order to generate all possible information structures. We now provide an explicit characterization of the set of all equilibria by means of a class of one dimensional signal. This class of information structures is sufficiently rich to informationally decentralize all possible Bayes correlated equilibrium outcomes as Bayes Nash equilibria in demand functions. In this sense, this class can be seen as a canonical class of information structures.

The class of one-dimensional information structures that we consider are simply the noisy augmentation of the noise-free information structures that we considered earlier. Namely, let the one-dimensional signal be of the form:

$$s_i \triangleq \Delta\theta_i + \lambda\bar{\theta} + \varepsilon_i, \tag{37}$$

where, as before, $\lambda \in \mathbb{R}$ is the weight on the common component of the payoff state, and ε_i represents a normally distributed noise term with mean zero, variance σ_ε^2, and correlation coefficient $\rho_{\varepsilon\varepsilon}$

between the agents. As before, we shall use the decomposition into a common and an idiosyncratic component of the error term in the description of the equilibrium, and thus

$$\sigma_{\varepsilon}^2 \triangleq \sigma_{\bar{\varepsilon}}^2 + \sigma_{\Delta\varepsilon}^2,$$

which decomposes the variance of the error term into a common variance term $\sigma_{\bar{\varepsilon}}^2$ and an idiosyncratic variance term $\sigma_{\Delta\varepsilon}^2$. Thus, we now allow for noisy signals and the error terms are allowed to be correlated across the agents. This class of one-dimensional information structures is thus complete described by the triple $(\lambda, \sigma_{\bar{\varepsilon}}^2, \sigma_{\Delta\varepsilon}^2) \in \mathbb{R} \times \mathbb{R}_+ \times \mathbb{R}_+$.

Proposition 7 (Demand Function Equilibrium with Noisy Signals)
For every one-dimensional information structure (37), the unique linear Bayes Nash equilibrium has:

1. *price impact*

$$m = \frac{1}{2}\left(-Nc\frac{(N-1)\Lambda - 1}{(N-1)\Lambda + 1} + \sqrt{(Nc\frac{(N-1)\Lambda - 1}{(N-1)\Lambda + 1})^2 + 2Nc + 1} - 1\right) \tag{38}$$

   with weights:

$$\Lambda \triangleq \lambda\frac{b}{B}, \quad b \triangleq \frac{\sigma_{\Delta\theta}^2}{\sigma_{\Delta\theta}^2 + \sigma_{\Delta\varepsilon}^2}, \qquad B \triangleq \frac{\sigma_{\bar{\theta}}^2\lambda^2}{\sigma_{\bar{\theta}}^2\lambda^2 + \sigma_{\bar{\varepsilon}}^2}; \tag{39}$$

2. *and realized demands*

$$\bar{a} = \frac{\mathbb{E}[\bar{\theta}|\bar{s}]}{1 + m + Nc}, \qquad \Delta a_i = \frac{\mathbb{E}[\Delta\theta_i|\Delta s_i]}{1+m}, \tag{40}$$

   with conditional expectations:

$$\mathbb{E}[\bar{\theta}|\bar{s}] = B\frac{\bar{s}}{\lambda} + (1-B)\mu_\theta, \qquad \mathbb{E}[\Delta\theta_i|\Delta s_i] = b\Delta s_i. \tag{41}$$

We should highlight how closely the equilibrium in this large class of noisy information structures tracks the corresponding results for the noise free information structure. The equilibrium price impact m with noisy signals, (38), corresponds precisely to the expression in the noise-free equilibrium, see Proposition 1, after we replace λ by Λ. Moreover, Λ simply adjusts λ by the informativeness of the idiosyncratic component of the signal s_i relative to the common component of the signal s_i, the ratio b/B. In turn, each term $b, B \in [0, 1]$, represents the signal to noise ratio of the idiosyncratic and the common component in the signal s. Thus if the noise in the idiosyncratic component is large, then the effective weight Λ attached to the common component is lowered, and similarly if the noise in the common component is large, then the effective weight Λ attached to the common component increases. Similarly, the realized demand under the noisy signals, (40), corresponds exactly to the demands under the noise free signals, see Corollary 1, after we replace the realized payoff state with the conditional expectation of the payoff state.

The large class of noisy one-dimensional signals allows to relate the equilibrium description with earlier results in the literature on demand function competition. For example, we can recover the analysis of the (robust) equilibrium with complete information, as studied by Klemperer and Meyer (1989). After all, each agent has complete information about his payoff state if there is no confounding and no noise in the observation, that is $\lambda = b = B = 1$. In this case we have that

$$\Lambda_{KM} \triangleq 1,$$

and it follows that the price impact in the model studied by Klemperer and Meyer (1989) is given by:

$$m_{KM} = \frac{1}{2}\left(-Nc\frac{N-2}{N} + \sqrt{(Nc\frac{N-2}{N})^2 + 2Nc + 1} - 1\right).$$

Vives (2011b) analyzes the demand function competition with private information. He considers a class of noisy information structures in which there is no confounding between the idiosyncratic and the common component, and in which the noise in the signal of each agent is purely idiosyncratic. Thus, within the class of one-dimensional information structures defined by (37), this corresponds to imposing the restrictions that $\lambda = 1$ and $\rho_{\varepsilon\varepsilon} = 0$. The resulting weight Λ_V in the price impact is easily computed to be:

$$\Lambda_V \triangleq \frac{(1-\rho_{\theta\theta})((N\rho_{\theta\theta} + (1-\rho_{\theta\theta} + \frac{\sigma_\varepsilon^2}{\sigma_\theta^2}))}{(1+(N-1)\rho_{\theta\theta})(1-\rho_{\theta\theta} + \frac{\sigma_\varepsilon^2}{\sigma_\theta^2})}.$$

Thus, in Vives (2011b), the class of one-dimensional information structures is completely described by the one-dimensional parameter of the variance σ_ε^2 of the idiosyncratic noise. Thus, with $\sigma_\varepsilon^2 \in [0, \infty)$, it follows the possible values that the weight Λ can take on within the class of information structures considered by Vives (2011b) is given by

$$\Lambda_V \in [\frac{1-\rho_{\theta\theta}}{(1-\rho_{\theta\theta}) + N\rho_{\theta\theta}}, 1] \subset [0,1].$$

In turn this implies that the price impact in Vives (2011b) is given by the bounds that can be generated by the values of Λ restricted to be in the unit interval. By contrast, we observed earlier that in the class of the one-dimensional noisy information structures give by (37), the weight Λ could take on any value on the real line.

In Section 3 we already provided an intuition of how the confounding information as represented by λ affects the price impact and the equilibrium. It thus remains to understand what are the effects on the price impact of having noisy information, and thus allowing for $b, B \neq 1$. Clearly, having noise in the signals has the effect that it adds residual uncertainty to the information of each agents about his payoff state thus dampens, or attenuates, the response of each agent to his signals. Surprisingly, this is in fact the only effect that noise has on the equilibrium behavior of the agents. We can verify this momentarily, if we re-define for each agent his payoff state in terms of the conditional expectation. Thus, define the new payoff state to be the conditional expectation as follows:

$$\varphi_i \triangleq \mathbb{E}[\Delta\theta_i|\Delta s_i] + \mathbb{E}[\bar{\theta}|\bar{s}] = b\Delta s_i + B\bar{s} + (1-B)\mu_\theta,$$

where the second inequality simply refers to the weights b and B that emerge from the updating formula of the normally distributed signals. We can then rewrite the true signal s_i that agent i receives as follows:

$$s_i = \frac{1}{b}(b\Delta s_i + \frac{b}{B}B\bar{s}) = \frac{1}{b}(\Delta\varphi_i + \Lambda\bar{\varphi} - (1-B)\mu_\theta).$$

In turn, the signal s_i is informationally equivalent to the following signal s_i' that we obtain after multiplying s_i by the constant b:

$$s_i' = \Delta\varphi_i + \lambda\frac{b}{B}\bar{\varphi}.$$

Thus, it is easy to see that we can repeat the previous analysis but using the definition already made $\Lambda = \lambda\frac{b}{B}$. Therefore, the errors in the signals affect the price impact by dampening the response to the average and idiosyncratic part of the signal, which serves as a reweighing of the informational content of the signal.

We can obtain the *entire set* of Bayes correlated equilibria with this class of noisy one-dimensional information structures defined by (37).

Proposition 8 (Sufficiency of Canonical Signals)
For every multivariate normal distribution of random variables $(\theta_i, \bar{\theta}, a_i, \bar{a})$ and every price impact $m \in (-1/2, \infty)$ that jointly form a Bayes correlated equilibrium, there exists a canonical signal structure that decentralizes the random variables and price impact as a Bayes Nash equilibrium outcome.

We can thus generate the entire set of Bayes correlated equilibria with the class of noisy one-dimensional information structures given by (37). However, the outcome of any given Bayes correlated equilibrium could be generated by a multi-dimensional information structure. Proposition 6 and 8 jointly establish that in terms of equilibrium outcomes the equilibrium behavior under any arbitrary (normal) multi-dimensional signal structures would nonetheless lead to behavior that can be completely described by a class of one-dimensional signals.

The statistical characterization of the equilibrium outcomes in Proposition 5 informed us that the equilibrium set is completely described by the three equilibrium variables $\rho_{\bar{a}\bar{\theta}}, \rho_{\Delta\Delta}$ and m. The class of noisy one-dimensional information structures is described by a different triple, namely $\sigma^2_{\bar{\varepsilon}}, \sigma^2_{\Delta\varepsilon}$, and λ. And while there is not a one-to-one mapping from one of the variables to the other, a comparison of Proposition 5 and 7 indicates that they perform similar tasks in terms of controlling the mean and variance-covariance of the equilibrium distribution.

Vives (2011b) considers an environment in which each agent receives a one-dimensional signal such that $\lambda = 1$ and $\rho_{\varepsilon\varepsilon} = 0$. Thus the class of information structures of the form given by (37) have two additional dimensions.

Canonical Information Structures The set of one-dimensional information structures that we consider here:

$$s_i \triangleq \Delta\theta_i + \lambda\bar{\theta} + \varepsilon_i, \tag{42}$$

is completely defined by three parameters: (i) $\lambda \in \mathbb{R}$ *confounds* the idiosyncratic and the common component, (ii) $\sigma^2_{\Delta\varepsilon} \in \mathbb{R}_+$ adds ex-post uncertainty on the idiosyncratic component of the payoff shock, and (iii) $\sigma^2_{\bar{\varepsilon}} \in \mathbb{R}_+$ adds ex-post uncertainty on the common component of the payoff shock. The resulting set of information structures is canonical in the present context in the strict sense that it is necessary and sufficient to generate the behavior for all symmetric normal information structures, including all multi-dimensional information structures.

These three elements of the information structures arise commonly, though frequently only as subsets in strategic bidding and trading environments. For example, Reny and Perry (2006) consider a double auction market with a finite number of traders and a limit model with a continuum of buyers and sellers. Each trader, buyer or seller, wants to buy or sell at most one unit, and the utility function of agent i is defined by (x_i, ω) where x_i is a private component and ω is a common component. Each trader only observes x_i but not ω, though x_i is correlated with ω, and hence contains information about ω. Thus, there is confounding and ex-post uncertainty on the common component of the payoff shock, but there is no ex-post uncertainty on the idiosyncratic component of the payoff shock.

In Vives (2011a), as discussed earlier, there is only attenuation, but neither confounding nor ex-post uncertainty on the common component is present. Similarly, in the strategic trading models of Kyle (1985), (1989) and recent generalization such as Rostek and Weretka (2012) and Lambert, Ostrovksy, and Panov (2014), there is ex-post uncertainty on common and idiosyncratic component of the payoff shock, but no element of confounding. By contrast, in the analysis of the first price auction of Bergemann, Brooks, and Morris (2015), there is only ex-post uncertainty on the common component of the payoff shock, but neither confounding nor ex-post uncertainty of the idiosyncratic component of the payoff shock. Thus one important insight of the equilibrium analysis through the lens of Bayes correlated equilibrium is that all of the three above elements ought to be part of the analysis of the trading environment in order to capture informationally rich environments.

5 Comparing Market Mechanisms

The competition in demand functions provides a market mechanism that balances demand and supply with a *uniform price* across traders. The preceding analysis established that the market allocation is sensitive to the information structure that the agents possess. Moreover, we have provided a sharp characterization on how the outcome of demand function competition is sensitive to the information structure. As the competition in demand function is only one of many mechanism that match demand and supply on the basis of a uniform price, it is natural to compare the outcome under demand function competition with other uniform price market mechanisms.

An immediately relevant mechanism is competition in quantities, the Cournot oligopoly. In fact, earlier we already observed that the best response condition under quantity competition differs from the demand function competition in two important respects (see (31)). First, in demand function competition the agents can make their trade contingent on the equilibrium price in demand function competition, whereas in quantity competition the demand has to be stated unconditional in the quantity competition. Second, in demand function competition the price impact of each agent depends on the submitted demand function of all other agents whereas in quantity competition the price impact is constant and simply given by the supply conditions in the quantity competition. We show how this induces important differences in the set of possible outcomes under both forms of market competition, even when we compare across all possible information structure.

The stark difference in the ability to condition on the exact equilibrium price suggest the analysis of larger class of market mechanism, one in which the traders can only condition on an imperfect signal of the equilibrium price. We formalize such an approach to a class of market mechanism in Section 5.3. We conclude the comparison in Section 5.4 by extending the analysis to the strategic trading mechanism of Kyle (1985), (1989), where the market maker determines the price under a zero profit condition. Thus, the market maker matches the demand of the agents with his own supply until the price of the asset is equal to the conditional expectation given the information conveyed by the market.

5.1 Cournot Competition

We maintain the same payoff and information environment as in the demand function competition. The only change that arises with the quantity competition is that each agent i submits a demanded quantity q_i. As before, the market clearing prices is given by balancing demand and supply:

$$p = c_0 + c\sum_{i\in N} q_i.$$

In the present section, we denoted the demand variable from by q_i (rather than a_i) to emphasize that the market mechanism that we are considering has changed. The strategy of each trader is therefore a mapping from the private signal s_i into the demanded quantity, thus

$$q_i : \mathbb{R}^K \to \mathbb{R}.$$

The best response of agent i is:

$$q_i = \frac{1}{1+c}\mathbb{E}[\theta_i - (c_0 + cN\bar{q})|s_i]. \tag{43}$$

Definition 3 (Symmetric Bayes Nash Equilibrium with Cournot Competition) *The random variables $\{q_i\}_{i\in N}$ form a symmetric normal Bayes Nash equilibrium under competition in quantities if for all i and s_i the best response condition (43) holds.*

The market outcome under Cournot competition can also be analyzed for all information structures at once. In other words, we can define the Bayes correlated equilibrium in quantity competition and then offer a characterization of the equilibrium moments for all information structures.

Definition 4 (Bayes Correlated Equilibrium with Cournot Competition)
A Bayes correlated equilibrium is a joint (normal) distribution of $(p, a_i, \bar{a}, \theta_i, \bar{\theta})$ *(as given by (28)) such that best response condition holds for all* i, a_i*:*

$$\mathbb{E}[\theta_i - p|a_i] - a_i - ca_i = 0; \tag{44}$$

and the market clears:

$$p = c_0 + cN\bar{a}. \tag{45}$$

The equivalence between the Bayes correlated equilibrium and the Bayes Nash equilibrium for all information structures remains valid in the present game, see Bergemann and Morris (2015) for a canonical argument. Before we provide a statistical characterization of the Bayes correlated equilibrium in terms of the moments of the equilibrium distribution we define some additional statistical variables. Although we could describe the Bayes correlated equilibrium in terms of the distribution of variables as defined by (33), the change of variables has a meaningful economical interpretation. We first provide the definitions and proposition, and then we explain why different market competition require different statistical variables. We define:

$$\rho_{q\theta} \triangleq corr(q_i, \theta_i) \text{ and } rho_{q\phi} \triangleq corr(q_i, \theta_j),$$

and note that:

$$cov(\bar{q}, \bar{\theta}) = cov(q_i, \theta_j) = corr(q_i, \theta_j)\sigma_q\sigma_\theta \ ;$$

$$cov(\Delta q_i, \Delta\theta_i) = cov(q_i, \theta_i) - cov(\bar{q}, \bar{\theta}) = (\rho_{q\theta} - \rho_{q\phi})\sigma_q\sigma_\theta.$$

Hence, we can describe the distribution over variables $(\Delta a_i, \bar{a}, \Delta\theta_i, \bar{\theta})$ as follows:

$$\begin{pmatrix} \Delta q_i \\ \bar{q} \\ \Delta\theta_i \\ \bar{\theta} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ \mu_q \\ 0 \\ \mu_\theta \end{pmatrix}, \begin{pmatrix} \frac{(N-1)\sigma_q^2\left(1-\rho_{qq}\right)}{N} & 0 & (\rho_{q\theta} - \rho_{q\phi})\sigma_q\sigma_\theta & 0 \\ 0 & \frac{\sigma_q^2\left((N-1)\rho_{qq}+1\right)}{N} & 0 & \rho_{q\phi}\sigma_q\sigma_\theta \\ (\rho_{q\theta} - \rho_{q\phi})\sigma_q\sigma_\theta & 0 & \frac{(N-1)\sigma_\theta^2(1-\rho_{\theta\theta})}{N} & 0 \\ 0 & \rho_{q\phi}\sigma_q\sigma_\theta & 0 & \frac{\sigma_\theta^2((N-1)\rho_{\theta\theta}+1)}{N} \end{pmatrix}\right), \tag{46}$$

We should highlight that (33) and (33) are equivalent ways to describe the same distribution of variables. We can now characterize the Bayes correlated equilibrium of Cournot competition.

Proposition 9 (Moments of Cournot Competition)
The jointly normal random variables $(\theta_i, \bar{\theta}, q_i, \bar{q})$ *given by (46) form a Bayes correlated equilibrium if and only if:*

1. *the mean of the individual action is*

$$\mu_q = \frac{\mu_\theta - c_0}{1 + (N+1)c}; \tag{47}$$

2. *the variance of the individual action is*

$$\sigma_q = \frac{\rho_{q\theta}\sigma_\theta}{1 + c - Nc\rho_{qq}}; \tag{48}$$

3. *the correlations satisfy the following inequalities:*

$$\left(\rho_{q\theta} - \rho_{q\phi}\right)^2 \leq \frac{(N-1)^2}{N^2}\left(1 - \rho_{qq}\right)\left(1 - \rho_{\theta\theta}\right), \quad \rho_{q\phi}^2 \leq \frac{\left((N-1)\rho_{qq} + 1\right)\left((N-1)\rho_{\theta\theta} + 1\right)}{N^2}, \tag{49}$$

and

$$\rho_{qq} \in [-\frac{1}{N-1}, 1]. \tag{50}$$

We thus find that with Cournot competition, the mean demand of each agent is constant across Bayes correlated equilibrium, and thus constant across all information structures. The second moment of the distribution is characterized by the correlations $(\rho_{qq}, \rho_{q\theta}, \rho_{q\phi})$, while σ_q is determined by these correlations. Interestingly, the correlations $(\rho_{qq}, \rho_{q\theta}, \rho_{q\phi})$ are only restricted to be statistically feasible. That is, (49) and (50) are equivalent to requiring that the variance covariance matrix of (46) to be statistically feasible (this is a necessary and sufficient condition for a matrix to be a valid variance covariance matrix).

We characterized the Bayes correlated equilibrium of the demand function competition and the Cournot competition in terms of the moments of the random variables $(q_i, \bar{q}, \theta_i, \bar{\theta})$. Although we are interested in characterizing the same object, we have chosen different variables to do describe the second moments. In the case of demand function competition we chose the variables $(\rho_{\Delta\Delta}, \rho_{\bar{a}\bar{\theta}}, \sigma_{\Delta a}, \sigma_{\bar{a}})$, while in the case of the Cournot competition we chose the variables $(\rho_{q\theta}, \rho_{q\phi}, \rho_{qq}, \sigma_q)$. To a large extent the choice of variables is irrelevant beyond the algebraic tractability as there is a bijection between the two sets of variables. To make this explicit:

$$\rho_{qq} = \frac{\sigma_{\bar{q}}^2}{\sigma_{\bar{q}}^2 + \sigma_{\Delta q}^2};$$

$$\rho_{q\theta} = \operatorname{corr}(q_i, \theta_i) = \frac{\rho_{\Delta\Delta}\sigma_{\Delta q}\sigma_{\Delta\theta} + \rho_{\bar{q}\bar{\theta}}\sigma_{\bar{q}}\sigma_{\bar{\theta}}}{\sigma_\theta\sqrt{\sigma_{\Delta q}^2 + \sigma_{\bar{q}}^2}};$$

$$\rho_{q\phi} = \text{corr}(q_i, \theta_i) = \frac{\rho_{\bar{q}\bar{\theta}}\sigma_{\bar{q}}\sigma_{\bar{\theta}}}{\sigma_\theta\sqrt{\sigma_{\Delta q}^2 + \sigma_{\bar{q}}^2}}.$$

Yet there is an interesting economic justification for the choice of variables in each case.

A feature that is common to the Cournot competition and the demand function competition is that all feasible outcomes can be decentralized as a Bayes Nash equilibrium with the canonical signal structure. Interestingly, in Cournot competition, a given canonical signal structure determines the set of equilibrium correlations $(\rho_{qq}, \rho_{q\theta}, \rho_{q\phi})$ *independently* of the strategic property of the game, in particular the level of complementarity or substitutability of the game as represented by c. Moreover, the set of feasible correlations $(\rho_{qq}, \rho_{q\theta}, \rho_{q\phi})$ are independent of c, and we only require that they are statistically feasible. By contrast, in the demand function competition we have that the canonical signal structure determines $(\rho_{\Delta\Delta}, \rho_{\bar{a}\bar{\theta}}, m)$, independent of the strategic structure of the game, again represented by c. In the demand function competition, the set of feasible variables $(\rho_{\Delta\Delta}, \rho_{\bar{q}\bar{\theta}}, m)$ are also independent of the payoff relevant environment, and are directly determined by the chosen canonical signal structure.

But if we were to insist to say, characterize the set of Bayes correlated equilibria in the Cournot model in terms of the variables $(\rho_{\Delta\Delta}, \rho_{\bar{q}\bar{\theta}}, \sigma_{\Delta q}, \sigma_{\bar{q}})$ of the demand function competition, then we would have that $\rho_{\Delta\Delta}, \rho_{\bar{q}\bar{\theta}} \in [-1, 1]$. That is, the conditions on these two correlations will be relaxed. Nevertheless, we still need to satisfy the condition that $\rho_{q\theta} \geq 0$, hence we will at least need one of the two correlations to be positive. Moreover, the parameter ρ_{qq} is only constrained by the statistical restriction that (46) must be statistically feasible. Additionally, the parameters $(\rho_{\Delta\Delta}, \rho_{\bar{q}\bar{\theta}}, \sigma_{\Delta q}, \sigma_{\bar{q}})$ would have to satisfy the first order condition of the Cournot competition which would now be stated by:

$$\sigma_q \triangleq \sqrt{\sigma_{\bar{q}}^2 + \sigma_{\Delta q}^2} = \frac{\rho_{\Delta\Delta}\sigma_{\Delta q}\sigma_{\Delta\theta} + \rho_{\bar{q}\bar{\theta}}\sigma_{\bar{q}}\sigma_{\bar{\theta}}}{\sigma_\theta\sqrt{\sigma_{\Delta q}^2 + \sigma_{\bar{q}}^2}} \frac{\sigma_\theta}{1 + c(\frac{\sigma_{\bar{q}}^2}{\sigma_{\bar{q}}^2 + \sigma_{\Delta q}^2})} \triangleq \frac{\rho_{q\theta}\sigma_\theta}{1 + c\rho_{qq}}. \tag{51}$$

Hence, $\sigma_{\Delta q}$ and $\sigma_{\bar{q}}$ are defined by ρ_{qq} and (51). An alternative formulation of the same change of variables is that in demand function competition the following identity must always be satisfied:

$$\frac{N\rho_{a\phi}}{(1 + m + cN)(1 + \rho_{aa}(N - 1))} = \frac{\rho_{a\theta}}{1 + m + c(1 + (N - 1)\rho_{aa})} \tag{52}$$

Hence, in demand function competition the correlations $(\rho_{qq}, \rho_{q\theta}, \rho_{q\phi})$ live in a two dimensional space.

We can see that the algebra becomes more difficult if we express a Bayes correlated equilibrium of Cournot competition in terms of the variables $(\rho_{\Delta\Delta}, \rho_{\bar{a}\bar{\theta}}, \sigma_{\Delta q}, \sigma_{\bar{q}})$. The interesting point to highlight is how this depends on the difference between the economics of both models, rather than only on a algebraic coincidence.

5.2 Comparing Equilibrium Outcomes

The lack of conditioning information under quantity competition might be overcome by giving each agent additional information relative to the information provided in the competition with demand function. On the other hand, the price impact in the quantity competition is constant and equal to c, and so if we would like to replicate the outcome of the demand function competition, it would appear that one would have to adjust the response of the demand conditions as represented by concavity of the utility function.

Proposition 10 (Demand Function and Quantity Competition)
Let $(a_i, \bar{a}, \theta_i, \bar{\theta})$ be the outcome of the Bayes Nash equilibrium in demand functions with one dimensional signals $\{s_i\}_{i\in N}$, then $(a_i, \bar{a}, \theta_i, \bar{\theta})$ is the outcome of the Bayes Nash Equilibrium under quantity competition if each agent i receives the two-dimensional signal $(\Delta s_i, \bar{s})$ and agent's i preferences are given by:

$$\tilde{u}(\theta_i, a_i, p) \triangleq \theta_i a_i - \frac{1}{2}a_i^2 - a_i p - \frac{1}{2}(m-c)q_i^2,$$

where m is given by (38).

Thus, the outcome of any Bayes Nash equilibrium in demand function competition can be described as the equilibrium outcome of a competition in quantities after changing two distinct elements in the decision problem of each agent.

First, each agent requires more information about the payoff state. The necessary additional information is attained by splitting the original signal s_i into two signals, namely the common and the idiosyncratic component of the signal that each agent received in the demand function competition. The additional information conveyed by the two components allows the agents to improve their estimate about the demands by the others through $\bar{s}$, and yet adjust the individual demand through the knowledge of the idiosyncratic signal component Δs_i. It is perhaps worth noting that the set of signals of $\mathcal{I}_i = (\Delta s_i, \bar{s})$ is equivalent to the informational environment in which each agent receives a private signal s_i, and then agents pooled and shared their signals to obtain $\bar{s}$ before they submit their individual demand. It is then easy to see that the resulting Bayes Nash equilibrium is *privately revealing* to each agent, as defined by Vives (2011b).

Second, each agent's marginal willingness to pay needs to be modified relative to the demand function competition. This is achieved by changing the concavity in the utility function of each agent. Namely, instead of $1/2$, the quadratic in the payoff function of the trader is required to be $1/2\,(1+m-c)$. This takes into account the fact that in quantity competition price impact is constant and equal to c, while in demand function it is endogenous and equal to m.

Earlier, we represented the first and second moments under demand function competition. With Proposition 9, we can now contrast the equilibrium moments across the market mechanisms. In Figure 4 we illustrate the set of feasible first and second moment under demand function competition and quantity competition, where we describe the set of equilibrium moments under quantity in red. The most immediate contrast is the first moment. Under Cournot competition, the information structure has no influence on the price impact, and hence the nature of the best response function does not vary across information structures. In consequence, the law of iterated expectation pins down the mean of the individual demand, and of the average demand across all information structure to a unique value. However, the variance and the covariance of the demand is strongly influenced by the information structure in the Cournot competition. Moreover, as the traders cannot condition their demand on the realized prices, they lack in an important instrument to synchronize their demand. In consequence the variance of the aggregate demand can be much larger in Cournot competition than in the demand function competition as displayed in Figure 4. In other words, conditioning on prices in the demand function competition imposes constraints on the responses of the agents to their private information that in turn imposes constraints both on the individual variance, but more importantly on the variance of the aggregate demand.

Interestingly, the set of the feasible first and second (normalized) moments for the aggregate demand under demand function competition (blue shaded area) does not depend on the correlations of types. In Cournot competition this is not the case. As we take $\rho_{\theta\theta} \to 0$, we know that the maximum aggregate demand variance remains positive and bounded away from 0. Thus, the ratio between the variance of the aggregate action to the aggregate shock goes to infinity as $\rho_{\theta\theta} \to 0$. With Cournot competition each agent acts on a signal that is confounding the idiosyncratic and the aggregate component of the payoff state. By contrast, in the demand function competition, in equilibrium there is always a separation between the aggregate and idiosyncratic payoff state revealed through the price, and thus the set of feasible first and second (normalized) moments of the aggregate demand do not depend on the correlation of the shocks.

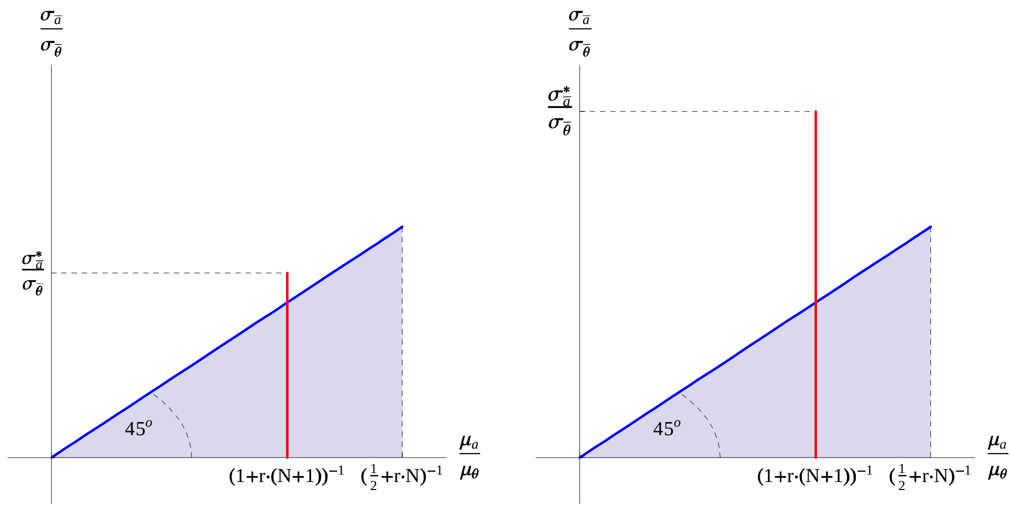

Figure 4: First and Second Equilibrium Moments of Aggregate Demand in Demand Function and Cournot Competition

The equilibrium price is a linear function of the aggregate demand. The volatility of the equilibrium price therefore follows the volatility of the aggregate demand. In particular, as the observability of the equilibrium price limits the variance of the aggregate demand, it also limits the volatility of the price. By contrast, in the Cournot competition, the demand by the agents is less synchronized, and there are information structure that decentralizes a Bayes Nash equilibrium with arbitrary large price volatility. As the determination of the individual demands are made prior to the determination of the equilibrium price, idiosyncratic uncertainty can lead to large aggregate volatility.

Let σ^*_p. be the maximum price volatility across any equilibrium of demand function competition. Similarly, let σ^{**}_p be the maximum price volatility across any equilibrium of Cournot competition. The comparison of the maximum price volatility across both forms of competition is as follows:

Proposition 11 (Price Volatility)

1. *The price volatility with demand function competition is bounded by the variance of the aggregate shock:*
$$\sigma^*_p \leq \frac{Nr \cdot \sigma_{\bar{\theta}}}{\frac{1}{2} + Nr};$$
2. *in Cournot competition the price volatility increases linearly with the size of the idiosyncratic shocks:*
$$\sigma^{**}_p \geq \frac{r\sigma_{\Delta\theta}}{2\sqrt{1+r}};$$

3. as the distribution of payoff shocks approaches purely idiosyncratic values, the maximum price volatility under Cournot competition is larger than under demand function competition:

$$\lim_{\rho_{\theta\theta}\to 0} \frac{\sigma_p^{**}}{\sigma_p^{*}} = \infty;$$

4. as the distribution of payoff shocks approaches purely common values, the maximum price volatility under demand function competition is larger than under Cournot competition:

$$\lim_{\rho_{\theta\theta}\to 1} \frac{\sigma_p^{**}}{\sigma_p^{*}} < 1.$$

Thus with demand function competition, the equilibrium price volatility is bounded by the volatility of aggregate shocks. By contrast, with Cournot competition, the volatility of the price can grow without bounds for given aggregate shock as long as the variance of the idiosyncratic payoff shock increases. The volatility of the price in the absence of aggregate uncertainty is closely related the recent work that relates idiosyncratic uncertainty to aggregate volatility. For example, Angeletos and La'O (2013) provide a model of an economy in which there is no aggregate uncertainty, but there may be aggregate fluctuations. One of the key aspects is that in the economy the production decisions are done prior to the exchange phase, and thus there are no endogenous information through prices. The present analysis with demand function competition therefore gives us an understanding how the aggregate volatility may be dampened by the presence of endogenous market information as provided by the equilibrium price in demand function competition.

Finally, we discuss how the information contained impacts and in particular shrinks the set of feasible correlations. To simplify the expressions, we will do this in a model with a continuum of agents and we take the following limits:

$$\lim_{N\to\infty} Nc = \overline{c} \text{ and } \tau = 0.$$

In the limit, we can then write (52) as follows:

$$\rho_{a\phi} = \frac{\rho_{aa}(\overline{c}+1)\rho_{a\theta}}{1+\overline{c}\cdot\rho_{aa}}. \tag{53}$$

In Cournot competition the correlations are only constrained by the statistical conditions given by:

$$\rho_{aa}\rho_{\theta\theta} \geq \rho_{a\phi}^2 \text{ and } (1-\rho_{aa})(1-\rho_{\theta\theta}) \geq (\rho_{a\theta}-\rho_{a\phi})^2.$$

By contrast, in demand function competition we need to satisfy the additional constraint (53). Interestingly, the set of feasible correlations in the demand function competition depends on the

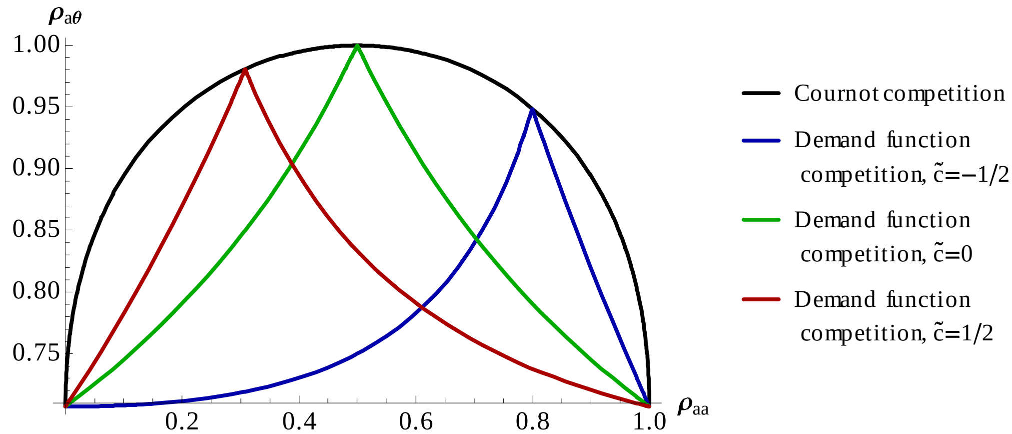

Figure 5: Feasible correlations for $\rho_{\theta\theta} = 1/2$.

level of strategic complementarity $\bar{c}$. Most strikingly, the set of feasible correlations in demand function competition is a two dimensional object whereas it is a three dimensional object with Cournot competition. The set of correlations $(\rho_{aa}, \rho_{a\theta})$ that are feasible under Cournot are given by:

$$\rho_{a\theta} \leq \sqrt{\rho_{aa}\rho_{\theta\theta}} + \sqrt{(1-\rho_{aa})(1-\rho_{\theta\theta})},$$

whereas with demand function competition we get that the set of correlations $(\rho_{aa}, \rho_{a\theta})$ are:

$$\rho_{a\theta} \leq \min\left\{\sqrt{\rho_{aa}\rho_{\theta\theta}}, \frac{1+\bar{c}\cdot\rho_{aa}}{\rho_{aa}(\bar{c}+1)}, \sqrt{(1-\rho_{aa})(1-\rho_{\theta\theta})}\frac{1+\bar{c}\cdot\rho_{aa}}{1-\rho_{aa}}\right\}.$$

We illustrate the set of feasible correlations under both forms of competition in the space of correlation coefficient $(\rho_{aa}, \rho_{a\theta})$ in Figure 5 and Figure 6.

5.3 Noisy Price Signals

We consider a market mechanism in which each agent conditions his trades on a noisy signal $p_{\bar{\eta}}$ of the market clearing price p:

$$p_{\bar{\eta}} = p + \bar{\eta} = c_0 + cN\bar{a} + \bar{\eta},$$

where $\bar{\eta}$ is a normally distributed random variable, independent of all other random variables with mean zero and variance $\sigma_{\bar{\eta}}$, and common across all agents. The variance of the noise term $\bar{\eta}$ links the model of demand function competition with the Cournot competition. On the one extreme if $\sigma_{\bar{\eta}} = 0$, then the price is observed without any noise, and we are in the demand function competition, on the the other extreme as $\sigma_{\bar{\eta}} = \infty$, there is no information in the noise price signal $p_{\bar{\eta}}$, and we are in

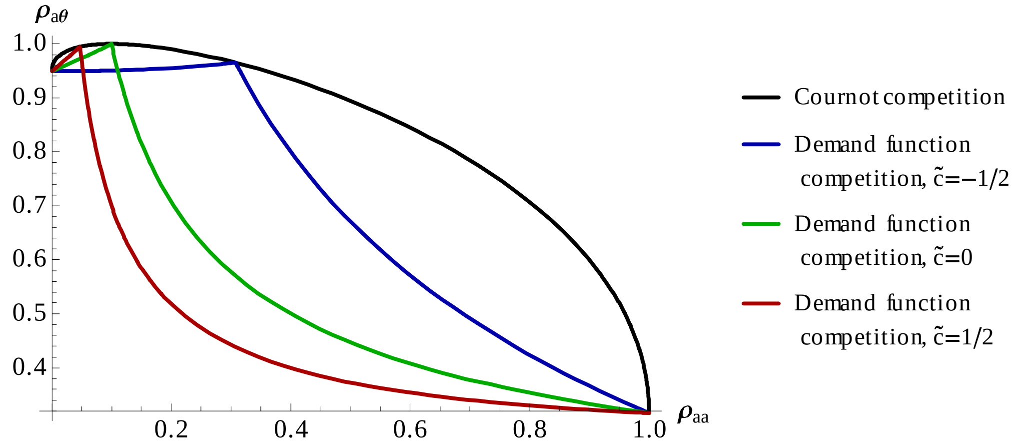

Figure 6: Feasible correlations for $\rho_{\theta\theta} = 1/10$.

the model of Cournot competition. We are interested in analyzing how the quality of the common conditioning information affects the set of market outcomes.

In an alternative, but equivalent interpretation of the current model, the agents submit demand curves to the auctioneer, but the auctioneer observes a noisy version of these demands. Thus it is as if the received demand functions are shifted by the common term $\bar{\eta}$.

In the corresponding Bayes Nash equilibrium of the game, each agents submits a demand:

$$x_i(s_i, p_{\bar{\eta}}) = \beta_0 + \beta_s s_i + \beta_p p_{\bar{\eta}}, \tag{54}$$

and the Walrasian auctioneer chooses a price p^* that clears the market:

$$c_0 + c\sum_{i\in N} x_i(s_i, p^* + \eta) = p^*. \tag{55}$$

Here, we will directly analyze the Bayes correlated equilibrium of the game. The best response condition for each agent is as in the model with demand function competition except that the conditioning event is now the noisy observation of the price:

$$a_i = \frac{1}{1+c}(\mathbb{E}[\theta_i - (c_0 + cN\bar{a})\,|a_i, p_{\bar{\eta}}]).$$

The definition of the Bayes correlation equilibrium is augmented by the noisy price signal $p_{\bar{\eta}}$.

Definition 5 (Bayes Correlated Equilibrium with Noisy Prices)
A joint distribution of variables $(\Delta a_i, \bar{a}, \Delta\theta_i, \bar{\theta}, \bar{\eta})$ and a market power m forms a Bayes correlated equilibrium if:

1. *The random variables* $(\Delta a_i, \bar{a}, \Delta\theta_i, \bar{\theta}, \bar{\eta})$ *are normally distributed.*

2. *The first order condition of agents is satisfied:*

$$a_i = \frac{1}{1+m}\mathbb{E}[\theta_i + cN\bar{a}|p_{\bar{\eta}}, a_i],$$

3. *The market power is given by:*

$$m = \frac{(c(\sigma_{\bar{\eta}} + Nc\rho_{\bar{a}\bar{\eta}}\sigma_{\bar{a}}))}{(\sigma_{\bar{\eta}} + c\rho_{\bar{a}\bar{\eta}}\sigma_{\bar{a}})}. \tag{56}$$

The new element in the above definition is the additional random variable given by $\bar{\eta}$ and we link the price impact m with the correlation between the average demand and the noisy signal through the correlation coefficient $\rho_{\bar{a}\bar{\eta}}$.

The link between market power m with the correlation between the average action and the noise the signal $\rho_{\bar{a}\bar{\eta}}$ can be understood as follows. Since the agents share the public information about the shock $\bar{\eta}$, the covariance between the aggregate demand and the shock $\bar{\eta}$ allows us to calculate the weight that agents put on the price in their demand function:

$$cov(\bar{a}, \bar{\eta}) = \rho_{\bar{a}\bar{\eta}}\sigma_{\bar{a}}\sigma_{\bar{\eta}} = \frac{\beta_p}{(1 - c\beta_p N)}\sigma^2_{\bar{\eta}}.$$

and hence,

$$\beta_p = \frac{\rho_{\bar{a}\bar{\eta}}\frac{\sigma_{\bar{a}}}{\sigma_{\bar{\eta}}}}{1 + Nc\rho_{\bar{a}\bar{\eta}}\frac{\sigma_{\bar{a}}}{\sigma_{\bar{\eta}}}}. \tag{57}$$

We can then use the same relation between β_p and m used for supply function competition without noise:

$$m = \frac{c}{1 - c(N-1)\beta_p}. \tag{58}$$

Using (57) and (58) we get (56).

Proposition 12 (BCE with Noisy Prices)

A joint normal distribution of variables $(\Delta a_i, \bar{a}, \Delta\theta_i, \bar{\theta}, \bar{\eta})$ *and a market power* m *forms a Bayes correlated equilibrium if and only if:*

1. *the first moments satisfy:*

$$\mu_a = \frac{1}{1+m}(\mu_\theta + cN\mu_a);$$

2. the second moments satisfy:

$$\sigma_{\bar{a}} = \frac{1}{1+m+cN}\left(\rho_{\bar{a}\bar{\theta}}\sigma_{\bar{\theta}}\sigma_{\bar{a}} + \rho_{\bar{a}\bar{\eta}}\sigma_{\bar{\eta}}\sigma_{\bar{a}} - \frac{1+m}{cN}\rho_{\bar{a}\bar{\eta}}\sigma_{\bar{\eta}}\sigma_{\bar{a}}\right) \geq 0, \tag{59}$$

$$\sigma_{\Delta a} = \frac{1}{1+m}\left(\rho_{\Delta\Delta}\sigma_{\Delta\theta}\sigma_{\Delta a} + \frac{1+m}{cN}\rho_{\bar{a}\bar{\eta}}\sigma_{\bar{\eta}}\sigma_{\bar{a}} - \rho_{\bar{a}\bar{\eta}}\sigma_{\bar{\eta}}\sigma_{\bar{a}}\right) \geq 0; \tag{60}$$

3. the correlations satisfy:

$$1 - \rho_{\Delta\Delta}^2 \geq 0 \ ; \ 1 - \rho_{\bar{a}\bar{\theta}}^2 - \rho_{\bar{a}\bar{\eta}}^2 \geq 0; \tag{61}$$

4. and the price impact is given by:

$$m = \frac{(c(\sigma_{\bar{\eta}} + Nc\rho_{\bar{a}\bar{\eta}}\sigma_{\bar{a}}))}{(\sigma_{\bar{\eta}} + c\rho_{\bar{a}\bar{\eta}}\sigma_{\bar{a}})} \geq -\frac{1}{2}. \tag{62}$$

As before (59), (60) and (62) form a system of equations to determine $\sigma_{\bar{a}}, \sigma_{\Delta a}$ and m simultaneously. We briefly argue how this model converges to a model of Cournot competition as $\sigma_{\bar{\eta}} \to \infty$ and a model of demand function competition as $\sigma_{\bar{\eta}} \to 0$. As $\sigma_{\bar{\eta}} \to \infty$, we have that from nonnegative values of, $\sigma_{\bar{a}}, \sigma_{\Delta a} \geq 0$, the restriction on (59) and (60)) lead to $\rho_{\bar{a}\bar{\eta}} \to 0$ As the price signal becomes very noisy, the agents put less weight on the signal and hence this reduces the correlation of the average demand with the noise. We then infer from (62) that this implies that $m \to c$, which corresponds to the price impact in the Cournot model. Finally note that adding up (59) and (60) we get:

$$(1+m+cN)\sigma_{\bar{a}}^2 + (1+m)\sigma_{\Delta a}^2 = \rho_{\bar{a}\bar{\theta}}\sigma_{\bar{\theta}}\sigma_{\bar{a}} + \rho_{\Delta\Delta}\sigma_{\Delta\theta}\sigma_{\Delta a}. \tag{63}$$

Additionally, we can use the additional information from (62) that $m \to c$, and after recalling that:

$$\sigma_{\bar{a}}^2 = \rho_{aa}\sigma_a^2, \ \sigma_{\Delta a}^2 = (1-\rho_{aa})\sigma_a^2,$$

and

$$\rho_{\bar{a}\bar{\theta}}\sigma_{\bar{\theta}}\sigma_{\bar{a}} + \rho_{\Delta\Delta}\sigma_{\Delta\theta}\sigma_{\Delta a} = \rho_{a\theta}\sigma_a\sigma_\theta = cov(a_i, \theta_i)$$

we can rewrite (63) as follows:

$$\sigma_a = \frac{\rho_{a\theta}\sigma_\theta}{1+c+Nc\rho_{aa}},$$

which is precisely the condition (48) that we obtained earlier for the Cournot competition.

We now consider the case $\sigma_{\bar{\eta}} \to 0$. In this case, (59) and (60)) can be written as follows:

$$\sigma_{\bar{a}}^2 \to \frac{1}{1+m+cN}\rho_{\bar{a}\bar{\theta}}\sigma_{\bar{\theta}}\sigma_{\bar{a}}, \quad \sigma_{\Delta a}^2 \to \frac{1}{1+m}\rho_{\Delta\Delta}\sigma_{\Delta\theta}\sigma_{\Delta a}, \tag{64}$$

which corresponds to the conditions that we had on aggregate and idiosyncratic volatility for demand function competition. Additionally, as $\sigma_{\bar{\eta}} \to 0$, we have that small changes in $\rho_{\bar{a}\bar{\eta}}$ around 0 can cause large changes in m (which is given by (62)). Hence, we will have that $m \in (-1/2, \infty)$. Note that, as $\sigma_{\bar{\eta}} \to 0$ we have a more slack in the bounds on m, as $\sigma_{\bar{a}\bar{\theta}}, \sigma_{\Delta a \Delta \theta} \geq 0$ becomes less stringent.

We assumed that agents observe a noisy signal of the realized price, and hence the agents face uncertainty about the realized price. A different, but strategically related model, would be one in which the agents can condition perfectly on the realized price, yet the price only represents a noisy signal of the realized average action:

$$p = r(N\bar{a} + \bar{\eta}), \tag{65}$$

as analyzed, for example in Vives (2014), in a model of stochastic supply. We could now link the demand function competition that would arise with $\sigma_{\bar{\eta}} = 0$ to the market outcomes with positive noise, $\sigma_{\bar{\eta}} > 0$. Interestingly, in the limit as the noise becomes arbitrarily large, the equilibrium price impact converges to a demand function competition under symmetric but imperfect competition as analyzed by Klemperer and Meyer (1989) and not the price impact of the Cournot competition. Moreover, as if the price contains a large noise term of the form by (65), then it is as if we add a fundamental (supply) shock to the economy. By contrast, in the current analysis, even as the noise grows large, we have finite and positive limits for the volatility of realized prices.

Finally, it is worth observing that the analysis of noisy signals of aggregate action has appeared elsewhere in the literature. For example, Benhabib, Wang, and Wen (2013) analyze an economy with a continuum of agents and each agent is assumed to observe the aggregate price with an idiosyncratic noise as a version of a noisy rational expectations equilibrium. Clearly, we could make the relationship more precise by taking limits of the Bayes correlated equilibrium as the number of traders become large and then obtain the restriction for economies with a continuum of agents.

5.4 Kyle Model

The final trading mechanism that we analyze is a static version of the trading model of Kyle (1985). In the spirit of the current model, we consider an environment that is more general than studied by Kyle (1985) in the sense that we allow the willingness to pay to be composed of an idiosyncratic and common component, whereas Kyle (1985) and much of the recent literature, see Lambert, Ostrovksy, and Panov (2014) consider a pure common value environment. Additionally, even though both of the aforementioned papers consider linear utilities, we keep the quadratic preferences as in (1) (this is only to avoid changing the payoff environment, it will obviously play no role).

Thus we maintain the payoff environment and only consider the following change in the trading mechanism. The agents submit quantities $a_i \in \mathbb{R}$ to a market maker. The market maker observes the total quantity demanded, which is the amount demanded by the agents *and* a demand shock η, which represent the noise traders:

$$A \triangleq \sum_{i \in N} a_i + \eta.$$

In addition, the market maker observes a signal s_m. The market maker sets the price price equal to his expected value of the average type $\bar{\theta}$ conditional on all the information he has available:

$$p = \mathbb{E}[\bar{\theta}|A, s_m].$$

As usual, we will assume that the information structure and the payoff shocks are jointly normally distributed.

Definition 6 (BCE of Kyle Model)
A jointly normal distribution of variables $(a_i, \theta_i, \bar{a}, \bar{\theta}, p, \eta)$ forms a Bayes correlated equilibrium of the Kyle model if

1. *each agents best responds:*
$$a = \frac{1}{1+m}\mathbb{E}[\theta_i - p|a_i];$$
2. *the market maker sets a fair price:*
$$p(A, s_m) = \mathbb{E}[\bar{\theta}|s_m, A];$$
3. *agents correctly anticipate the price function and hence the price impact m:*
$$m = \frac{\partial p(A, s_m)}{\partial A}.$$

We now characterize the set of outcomes under all possible information structures by means of the Bayes correlated equilibrium. For this, we will characterize the set of all possible distributions that are feasible for a fixed variance σ_η^2 of the noise trader:

$$\begin{pmatrix} \Delta a_i \\ \bar{a} \\ \Delta\theta_i \\ \bar{\theta} \\ p \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ \mu_a \\ 0 \\ \mu_\theta \\ \mu_p \end{pmatrix}, \begin{pmatrix} \frac{(N-1)\sigma_a^2(1-\rho_{aa})}{N} & 0 & \rho_{\Delta\Delta}\sigma_{\Delta a}\sigma_{\Delta\theta} & 0 & 0 \\ 0 & \frac{\sigma_a^2((N-1)\rho_{aa}+1)}{N} & 0 & \sigma_{\bar{a}}\sigma_{\bar{\theta}}\rho_{\bar{a}\bar{\theta}} & \rho_{ap}\sigma_a\sigma_p \\ \rho_{\Delta\Delta}\sigma_{\Delta a}\sigma_{\Delta\theta} & 0 & \frac{(N-1)\sigma_\theta^2(1-\rho_{\theta\theta})}{N} & 0 & 0 \\ 0 & \sigma_{\bar{a}}\sigma_{\bar{\theta}}\rho_{\bar{a}\bar{\theta}} & 0 & \frac{\sigma_\theta^2((N-1)\rho_{\theta\theta}+1)}{N} & \rho_{\theta p}\sigma_p\sigma_\theta \\ 0 & \rho_{ap}\sigma_a\sigma_p & 0 & \rho_{\theta p}\sigma_p\sigma_\theta & \sigma_p^2 \end{pmatrix}\right). \tag{66}$$

Proposition 13 (Characterization of BCE of Kyle model)
A jointly normal distribution of variables $(a_i, \theta_i, \bar{a}, \bar{\theta}, p, \eta)$ forms a Bayes correlated equilibrium of the Kyle model if and only if:

1. *the first moments are given by:*
$$\mu_a = 0 \text{ and } \mu_p = \mu_\theta;$$
2. *the second moment are given by:*
$$\sigma_a = \frac{\rho_{a\theta}\sigma_\theta}{1 + m + \rho_{ap}\sigma_p} \text{ and } \sigma_p = \rho_{\bar{\theta}p}\sigma_{\bar{\theta}};$$
3. *the correlations satisfy the following inequality:*
$$\rho_{\bar{a}\bar{\theta}}^2 - \rho_{\bar{a}p}^2 - \rho_{\bar{\theta}p}^2 \leq 1 + 2\rho_{\bar{a}\bar{\theta}}\rho_{\bar{a}p}\rho_{\bar{\theta}p} \ ; \ \rho_{\Delta\Delta}^2 \leq 1,$$
4. *and price impact $m \in (-1/2, \infty)$.*

We observe that the characterization of the equilibrium does not refer to the variance of the noise traders given by σ_η^2. As the market maker may have private information about the quantity of noise trade η, it does not appear as a restriction on the equilibrium outcome. Interestingly, the restrictions on the joint distribution of demands, states and prices $(a_i, \bar{a}, \theta_i, \bar{\theta}, p)$ is a combination of those appearing separately in the demand function competition and the Cournot competition. The restrictions on the second moments, the correlation coefficients of demands and payoff state are equivalent to those in the Cournot competition in the sense that there are no restriction beyond the purely statistical ones. By contrasts, the restrictions on the first moments reflect aspects of the demand function competition in the sense that the mean demand depends on the price impact. Similarly, as the market maker may have private information about the noise traders there is no restriction on the set of feasible price impacts.

The case in which the market maker does not have any additional information beyond the one that comes through the aggregate demand yields additional restrictions as then
$$p = \mathbb{E}[\theta|A] = \frac{\text{cov}(A, \bar{\theta})}{\text{var}(A)}A.$$
and we would have that the set of outcomes as characterized by Proposition 13, but imposing the additional constraint that:
$$p = \frac{N\rho_{\bar{a}\bar{\theta}}\sigma_{\bar{a}}\sigma_{\bar{\theta}}}{N^2\sigma_{\bar{a}}^2 + \sigma_\eta^2}A.$$

Now the price impact depends on the variance of the noise term σ_η. As the market maker does not have any additional information about the quantity of noise trade, there is a bound on the informativeness of the aggregate demand A, which in turn determines the price impact.

6 Conclusion

We studied how the information structure of agents affects the Bayes Nash equilibrium of a game in which agents compete in demand functions. We have shown that price impact strongly depends on the nature of the private information agents. The analysis provided a very clear understanding on how the information in prices affects the set of feasible outcomes. This allowed us to provide a sharp distinction between the set of feasible outcomes that can be achieved under demand function competition and under quantity competition.

7 Appendix

Proof of Proposition 6. (Only if) We first consider a price impact constant m and joint distribution of variables $(p, a, \bar{a}, \theta_i, \bar{\theta})$ that constitute a symmetric Bayes correlated equilibrium, and show there exists normal signals $\{s_i\}_{i \in N}$ and demand function $x(s_i, p)$ that constitute a symmetric Bayes Nash equilibrium in demand functions such that,

$$p = \hat{p} \text{ and } a_i = x_i(s_i, \hat{p}),$$

where $\hat{p}$ is the equilibrium price in the demand functions equilibrium.

Define a constant β as follows,

$$\beta \triangleq \frac{c - m}{mc(N-1)},$$

and suppose players receive signals $s_i = a_i + \beta p$. We will show that the demand functions

$$x(s_i, p) = s_i - \beta p \tag{67}$$

constitute a symmetric Bayes Nash equilibrium in linear demand functions. If all players submit demand functions as previously defined, then each player will face a residual demand given by,

$$p_i = \frac{1}{1 + c(N-1)\beta}(P_i + ca), \tag{68}$$

where

$$P_i \triangleq c_0 + c \sum_{j \neq N} \beta s_i.$$

Note that by definition, if $a = a_i$, then $p_i = p$.

We now consider the following fictitious game for player i. We assume all players different than i submit demand functions given by (67) first. Then player i observes P_i and decides how much quantity he wants to buy assuming the market clearing price will be given by (68). If we keep the demand functions of players different than i fixed, this fictitious game will obviously yield weakly better profits for agent i than the original game in which he submits demand functions simultaneously with the rest of the players.

In the fictitious game player i solves the following maximization problem:

$$\max_a \mathbb{E}[\theta_i a - \frac{1}{2}a^2 - p_i a | s_i, P_i].$$

The first order condition is given by (where a^* denotes the optimal demand),

$$\mathbb{E}[\theta_i|s_i, P] - a^* - p_i - \frac{\partial p_i}{\partial a} a^* = 0.$$

We can rewrite the first order condition as follows,

$$a^* = \frac{\mathbb{E}[\theta_i|s_i, P_i] - p_i}{1 + \frac{\partial p_i}{\partial a}}.$$

Also, note that,

$$\frac{\partial p_i}{\partial a} = \frac{c}{1 + c(N-1)\beta} = m.$$

Moreover, remember that if $a = a_i$ then $p_i = p$. This also implies that P_i is informationally equivalent to p. Thus, we have that if $a^* = a_i$ the first order condition is satisfied. Thus, $a_i^* = a_i$ is a solution to the optimization problem.

Finally, if agent i submits the demand function $x(s_i, p) = s_i - \beta p = a_i$ he would play in the original game in the same way as in the fictitious game. Thus, he will be playing optimally as well. Thus, the demand function $x(s_i, p)$ is a optimal response given that all other players submit the same demand. Thus, this constitutes a Bayes Nash equilibrium in demand functions.

(If) We now consider some information structure $\{\mathcal{I}_i\}_{i \in N}$ and some symmetric linear Bayes Nash equilibrium in demand functions given by $x(\mathcal{I}_i, p)$. We first note that we can always find a set of one dimensional signals $\{s_i\}_{i \in N}$ such that there exists demand functions, denoted by $x'(s_i, p)$, that constitute a Bayes Nash equilibrium and that are outcome equivalent to the Bayes Nash equilibrium given by $x(\mathcal{I}_i, p)$. For this, just define signal s_i as follows,

$$s_i \triangleq x(\mathcal{I}_i, p) - \beta_p p \text{ where } \beta_p \triangleq \frac{\partial x(\mathcal{I}_i, p)}{\partial p}.$$

We now define,

$$x'(s_i, p) \triangleq s_i + \beta_p p = x(\mathcal{I}_i, p).$$

By definition $x'(s_i, p)$ is measurable with respect to (s_i, p). We now check $x'(s_i, p)$ constitutes a Bayes Nash equilibrium. By definition, if all players $j \neq i$ submit demand functions $x'(s_j, p)$, then player i faces exactly the same problem as in the Bayes Nash equilibrium when players submit demand functions given by $x(\mathcal{I}_i, p)$, except he has information s_i instead of $\mathcal{I}_i$. From the way s_i is defined, it is clear that $\mathcal{I}_i$ is weakly more informative than s_i. Thus, if $x(\mathcal{I}_i, p)$ is a best response when player has information $\mathcal{I}_i$, then $x(\mathcal{I}_i, p)$ would also be a best response when player i has information s_i. Yet, if player submits demand function $x'(s_i, p)$ he will be submitting the same demand function as $x(\mathcal{I}_i, p)$, thus this is a best response. Thus, $x'(s_i, p)$ constitutes a Bayes Nash equilibrium that is outcome equivalent to $x(\mathcal{I}_i, p)$.

We now consider some one dimensional signals $\{s_i\}_{i\in N}$ and some symmetric linear Bayes Nash equilibrium in demand functions given by $x(s_i, p)$ that constitute a Bayes Nash equilibrium in demand functions and show that there exists a Bayes correlated equilibrium that is outcome equivalent. We know that we can write $x(s_i, p)$ as follows,

$$x(s_i, p) = \beta_0 + \beta_s s_i + \beta_p p$$

where $\beta_0, \beta_s, \beta_p$ are constant. In the Bayes Nash equilibrium in demand functions player i faces a residual demand given by,

$$p = P_i + \frac{c}{1-(N-1)\beta_p} a_i,$$

where,

$$P_i = c_0 + c(N-1)\beta_0 + c\sum_{j\neq i} \beta_s s_j.$$

In the Bayes Nash equilibrium in demand functions player i cannot do better than if he knew what was the residual demand he was facing and he responded to this. In such a case, we would solve,

$$\max_a \mathbb{E}[\theta_i a - \frac{1}{2}a_i^2 - a_i p | P_i, s_i].$$

The best response to the previous maximization problem is given by:

$$\mathbb{E}[\theta_i | P_i, s_i] - a_i^* - (P_i + \frac{c}{1(N-1)\beta_p} a_i^*) - \frac{\partial p}{\partial a_i} = 0.$$

Note that conditioning on the intercept of the residual demand that agent faces is equivalent to conditioning on the equilibrium price

$$p = P_i + \frac{c}{1-(N-1)\beta_p} a_i^*.$$

Thus, the first order condition can be written as follows,

$$\mathbb{E}[\theta_i | p, s_i] - a_i^* - p - \frac{\partial p}{\partial a_i} = 0.$$

But, note that agent i can get exactly the same outcome by submitting the demand function,

$$x(s_i, p) = \frac{\mathbb{E}[\theta_i | p, s_i] - p}{1 + \frac{\partial p}{\partial a_i}},$$

thus this must be the submitted demand function in equilibrium. Thus, in any Bayes Nash equilibrium the equilibrium realized quantities satisfy the following conditions,

$$\mathbb{E}[\theta_i|p,s_i] - a_i^* - p - \frac{\partial p}{\partial a_i} = 0.$$

Besides the market clearing condition $p = c_0 + cN\bar{a}$ is also obviously satisfied. Since in equilibrium all quantities are normally distributed, we have that $(p, \bar{a}, \Delta a_i, \bar{\theta}, \Delta\theta_i)$ form a Bayes correlated equilibrium. ■

Proof of Proposition 7. We assume agents receive a one dimensional signal of the form:

$$s_i = \Delta\theta_i + \Delta\varepsilon_i + \bar{\varepsilon} + \lambda\bar{\theta}.$$

We find explicitly the equilibrium in demand functions. We conjecture that agents submit demand functions of the form:

$$x(s_i, p) = \beta_0 + \beta_s s_i + \beta_p p. \tag{69}$$

Note that:

$$\frac{1}{N}\sum_{i\in N} x(s_i,p) = \beta_0 + \beta_s\bar{s} + \beta_p p,$$

and thus in equilibrium:

$$\hat{p} = c_0 + c\sum_{i\in N} x(s_i,\hat{p}) = c_0 + Nc(\beta_0 + \beta_s\bar{s} + \beta_p\hat{p}),$$

which leads to

$$\hat{p} = \frac{1}{1 - Nc\beta_p}(c_0 + Nc(\beta_0 + \beta_s\bar{s})).$$

Thus,

$$\bar{s} = \frac{(1 - Nc\beta_p)\hat{p} - c_0 - Nc\beta_0}{Nc\beta_s}.$$

Also, note that if all agents submit demand functions of the form (69), then agent $i \in N$ will face a residual demand with a slope given by,

$$\frac{\partial p_i}{\partial a} = m = \frac{c}{1 - c(N-1)\beta_p}.$$

As before, we use the variable p_i for the residual supply that agent i faces. We now note that:

$$\mathbb{E}[\theta_i|s_i,\hat{p}] = \mathbb{E}[\theta_i|\Delta s_i,\bar{s}] = \mathbb{E}[\Delta\theta_i|\Delta s_i] + \mathbb{E}[\bar{\theta}|\bar{s}].$$

Calculating each of the terms,

$$\mathbb{E}[\Delta\theta_i|\Delta s_i] = \frac{\sigma^2_{\Delta\theta}}{\sigma^2_{\Delta\theta}+\sigma^2_{\Delta\varepsilon}}\Delta s_i = \frac{\sigma^2_{\Delta\theta}}{\sigma^2_{\Delta\theta}+\sigma^2_{\Delta\varepsilon}}(s_i - \frac{(1-Nc\beta_p)\hat{p} - c_0 - Nc\beta_0}{Nc\beta_s});$$

$$\mathbb{E}[\bar{\theta}|\bar{s}] = \frac{\sigma^2_{\bar{\theta}}}{\sigma^2_{\bar{\theta}}+\sigma^2_{\bar{\varepsilon}}/\lambda^2}\frac{\bar{s}}{\lambda} + \frac{\sigma^2_{\bar{\varepsilon}}}{\sigma^2_{\bar{\theta}}+\sigma^2_{\bar{\varepsilon}}/\lambda^2}\mu_\theta = \frac{\sigma^2_{\bar{\theta}}}{\sigma^2_{\bar{\theta}}+\sigma^2_{\bar{\varepsilon}}/\lambda^2}\frac{(1-Nc\beta_p)\hat{p} - c_0 - Nc\beta_0}{\lambda Nc\beta_s} + \frac{\sigma^2_{\bar{\varepsilon}}}{\sigma^2_{\bar{\theta}}+\sigma^2_{\bar{\varepsilon}}/\lambda^2}\mu_\theta.$$

It is convenient to define,

$$\begin{aligned} b &\triangleq \frac{\sigma^2_{\Delta\theta}}{\sigma^2_{\Delta\theta}+\sigma^2_{\Delta\varepsilon}} = \frac{(1-\rho_{\theta\theta})\sigma^2_\theta}{(1-\rho_{\theta\theta})\sigma^2_\theta + (1-\rho_{\varepsilon\varepsilon})\sigma^2_\varepsilon}\ ; \\ B &\triangleq \frac{\sigma^2_{\bar{\theta}}}{\sigma^2_{\bar{\theta}}+\sigma^2_{\bar{\varepsilon}}/\lambda^2} = \frac{(1+(N-1)\rho_{\theta\theta})\sigma^2_\theta}{(1+(N-1)\rho_{\theta\theta})\sigma^2_\theta + (1+(N-1)\rho_{\varepsilon\varepsilon})\sigma^2_\varepsilon/\lambda^2}. \end{aligned}$$

We conjecture that the following demand functions form an equilibrium:

$$\begin{aligned} x(s_i,p) &= \frac{\mathbb{E}[\theta_i|s,p]-p}{1+m} \\ &= \frac{b(s_i - \frac{(1-Nc\beta_p)p-c_0-Nc\beta_0}{Nc\beta_s}) + B\frac{(1-Nc\beta_p)p-c_0-Nc\beta_0}{\lambda Nc\beta_s} + (1-B)\mu_\theta - p}{1+m} \end{aligned}$$

We can express the solution from matching the coefficients:

$$\begin{aligned} \beta_s &= \frac{b\left(\kappa+\sqrt{\kappa^2+2nc+1}-1\right)}{\kappa+nc} \\ m &= \frac{1}{2}\left(-\kappa+\sqrt{\kappa^2+2nc+1}-1\right) \\ \beta_0 &= \frac{(B-1)c\mu_\theta\left(\kappa+\sqrt{\kappa^2+2nc+1}-1\right)+c_0((n-2)c-\kappa)}{c\left(-\kappa+n^2c-n(\kappa+3c)\right)} \\ \beta_p &= -\frac{\kappa+c\left(\kappa+\sqrt{\kappa^2+2nc+1}-n+1\right)}{(n-1)c(nc-\kappa)}, \end{aligned}$$

with

$$\kappa \triangleq nc\frac{b(n-1)\lambda - B}{b(n-1)\lambda + B}.$$

Note that the second root of the quadratic problem would lead us to $m \leq -1/2$ and thus this does not constitute a valid equilibrium. On the other hand, the first root delivers $m \geq -1/2$, and thus this constitutes a valid equilibrium. By rewriting the terms and using the definition of ι we get the result. ■

Proof of Lemma 1. The conditions of the first moments are direct from the symmetry assumption. To be more specific, let

$$\bar{\theta} = \frac{1}{N}\sum_{i\in N}\mathbb{E}[\theta_i].$$

Taking expectations of the previous equation:

$$\mu_{\bar{\theta}} = \mathbb{E}[\bar{\theta}] = \frac{1}{N}\sum_{i\in N}\mathbb{E}[\theta_i] = \frac{1}{N}\sum_{i\in n}\mu_{\theta_i} = \mu_\theta.$$

The same obviously holds for $\mu_{\bar{a}} = \mu_a$. To prove the results on the second moments, we first prove that,

$$\sum_{i\in N}\Delta\theta_i = 0$$

For this just note that:

$$\sum_{i\in N}\Delta\theta_i = \sum_{i\in N}(\theta_i - \bar{\theta}) = \sum_{i\in N}\theta_i - N\bar{\theta} = \sum_{i\in N}\theta_i - N(\frac{1}{N}\sum_{i\in}\theta_i) = 0$$

In any symmetric equilibrium, we must have that for all $i, j \in N$, $\text{cov}(\theta_i\bar{\theta}) = \text{cov}(\theta_j, \bar{\theta})$. Thus, we have that:

$$\text{cov}(\bar{\theta},\theta_i) = \frac{1}{N}\text{cov}(\bar{\theta},\sum_{i\in N}\theta_i) = \frac{1}{N}\text{cov}(\bar{\theta},\sum_{i\in N}\bar{\theta}+\Delta\theta_i) = \text{var}(\bar{\theta}) + \frac{1}{N}\text{cov}(\bar{\theta},\underbrace{\sum_{i\in N}\Delta\theta_i}_{=0}) = \text{var}(\bar{\theta}).$$

For the rest of the moments we obviously just proceed the same way. ■

Proof of Proposition 5. (Only if) We first prove that if normal random variables $(\theta_i, \bar{\theta}, a_i, \bar{a})$ and the price impact parameter m form a Bayes correlated equilibrium then conditions 1-4 hold. Condition 1 is trivial from the fact that the definition of Bayes correlated equilibrium imposes normality. If the normal random variables are normally distributed, then their variance/covariance must be positive-semidefinite. But this is equivalent to imposing that the variance-covariance matrix of the random variables is positive semi-definite. Yet, this directly implies condition 4.

If normal random variables $(\theta_i, \bar{\theta}, a_i, \bar{a})$ and the price impact parameter m form a Bayes correlated equilibrium then we have that,

$$\mathbb{E}[\theta_i|\bar{a}, a_i] - a_i - p - ma_i = 0,$$

where we use that p and $\bar{a}$ are informationally equivalent. Taking expectations of the previous equality and using the Law of Iterated Expectations we get condition (34). If we multiply the previous equation by a_i we get:

$$\mathbb{E}[a_i\theta_i|\bar{a}, a_i] - a_i^2 - a_i(c_0 + Nc\bar{a}) - ma_i^2 - \mu_a \underbrace{(\mu_\theta - c_0 - \mu_a(1 + m + Nc))}_{=0} = 0.$$

Grouping up terms, we get:

$$\operatorname{cov}(a_i\theta_i) - \operatorname{var}(a_i) - Nc \operatorname{cov}(a_i, \bar{a}) - m \operatorname{var}(a_i) = 0.$$

But, just by rewriting the value of the variances and covariances, the previous equality can be written as follows:

$$\sigma_a = \frac{\rho_{a\theta}\sigma_\theta}{1 + m + c((N-1)\rho_{aa} + 1)}.$$

Thus, we get (35). If we repeat the same as before but multiply by $\bar{a}$ instead of a_i we get:

$$\operatorname{cov}(\bar{a}, \theta_i) - \operatorname{cov}(a_i, \bar{a}) - Nc \operatorname{var}(\bar{a}) - m \operatorname{cov}(\bar{a}, a_i) = 0.$$

As before, by rewriting the value of the variances and covariances, the previous equality can be written as follows:

$$\sigma_a = \frac{N\rho_{a\phi}\sigma_\theta}{(1 + m + cN)((1 - \rho_{aa})(N-1) + 1)}.$$

Using (35) we get (35).

(If) We now consider normal random variables $(\theta_i, \bar{\theta}, a_i, \bar{a})$ such that conditions 1-4 are satisfied. First, note that condition 4 guarantees that the variance/covariance matrix is positive-semidefinite, and thus a well defined variance/covariance matrix. Moreover, if condition 1 is satisfied, we can obviously relabel the terms such that we can rewrite the distribution as in **??**. We just need to prove that restrictions (44) and (45) of the definition of Bayes correlated equilibrium are also satisfied. We will show that the following restriction holds,

$$\mathbb{E}[\theta_i|a_i, p] - (c_0 + cN\bar{a}) - a_i - ma_i = 0. \tag{70}$$

Then obviously restriction (45) is just the determination of the price in terms of the average quantity $\bar{a}$ and it will be evidently satisfied by defining the price in this way.

We now show that conditions 2 and 3 imply that equation (70) is satisfied. We define the random variable

$$z \triangleq \mathbb{E}[\theta_i|a_i, p] - (c_0 + cN\bar{a}) - a_i - ma_i.$$

Since $(\theta_i, \bar{\theta}, a_i, \bar{a})$ are jointly normal, we have that z is normally distributed. If we calculate the expected value of z we get:

$$\mathbb{E}[z] = \mu_\theta - (c_0 + cN\mu_a) - \mu_a - m\mu_a = 0,$$

where the second equality is from condition 2. If we calculate the variance of z we get,

$$\begin{aligned}\mathrm{var}(z) &= \mathrm{var}(\mathbb{E}[\theta_i|a_i, p] - (c_0 + cN\bar{a}) - a_i - ma_i) \\ &= \mathrm{cov}(z, \mathbb{E}[\theta_i|a_i, p] - (c_0 + cN\bar{a}) - a_i - ma_i) \\ &= \mathrm{cov}(z, \mathbb{E}[\theta_i|a_i, p]) - (1+m)\,\mathrm{cov}(z, a_i) - cN\,\mathrm{cov}(z, \bar{a}) - c_0\,\mathrm{cov}(z, 1).\end{aligned}$$

Note that $\mathrm{cov}(z, 1) = \mathbb{E}[z] = 0$ by condition 2. On the other hand, it is direct that $\mathrm{cov}(z, a_i) = 0$ by (35) and $\mathrm{cov}(z, \bar{a}) = 0$ by (35). On the other hand, we can find constants $\alpha, \beta, \gamma \in R$ such that,

$$\mathbb{E}[\theta_i|a_i, p] = \mathbb{E}[\theta_i|a_i, \bar{a}] = \alpha a_i + \beta\bar{a} + \gamma.$$

Thus, we have that,

$$\mathrm{cov}(z, \mathbb{E}[\theta_i|a_i, p]) = \alpha\,\mathrm{cov}(z, a_i) + \beta\,\mathrm{cov}(z, \bar{a}) + \gamma\,\mathrm{cov}(z, 1) = 0,$$

by the same argument as before. Thus, we have that $\mathbb{E}[z] = \mathrm{var}(z) = 0$. Since z is normally distributed, this implies that $z = 0$. Thus , (70) is satisfied. Thus, by adequately defining p we have that restrictions (44) and (45) are satisfied. Hence, we get the result. ■

Proof of Proposition 9. See Bergemann, Heumann, and Morris (2015). ■

Proof of Proposition 10. First, note that $\mathrm{cov}(\bar{s}, \Delta\theta_i) = \mathrm{cov}(\bar{\theta}, \Delta s_i) = 0$. Thus,

$$\mathbb{E}[\theta_i|\bar{s}, \Delta s_i] = \mathbb{E}[\bar{\theta}|\bar{s}] + \mathbb{E}[\Delta\theta_i|\Delta s_i].$$

By definition $\sum_{i\in N} \Delta s_i = 0$, thus in equilibrium,

$$\mathbb{E}[\bar{a}|\bar{s}, \Delta s_i] = \mathbb{E}[\bar{a}|\bar{s}].$$

Thus, it is easy to see that the equilibrium actions will be given by,

$$q_i = \frac{\mathbb{E}[\Delta\theta_i|\Delta s_i]}{1+m} + \frac{\mathbb{E}[\bar{\theta}|\bar{s}]}{1+c+m}.$$

Yet, this is exactly the characterization provided in Proposition 7. ■

Profit Maximizing Price Impact To the extent that the information structure can influence the equilibrium quantities and prices, it can also influence the profits of the agents. A natural question therefore what is the nature of the profit maximizing information structure. Thus, subject to the market clearing condition and the individual best response functions, we ask which information structure maximizes the profit of the representative trader. An upper bound on the profit is obtained by maximizing the joint profit of the traders for every realization of the payoff vector $\theta = (\theta_1, ..., \theta_N)$:

$$\{a_1^*, ..., a_N^*\} = \underset{\{a_1,...,a_N\}\in\mathbb{R}^N}{\arg\max} \left\{ \sum_{i\in N} \theta_i a_i - \frac{1}{2}a_i^2 - a_i p \right\}$$

subject to

$$p = c_0 + c\sum_{i\in N} a_i.$$

The pointwise solution gives us the following profit-optimal or *collusive demands*:

$$\bar{a}^* = \frac{\bar{\theta}}{1+2cN} \ ; \ \Delta a_i^* = \Delta\theta_i. \tag{71}$$

We can compare the collusive demands with the equilibrium demands, see Proposition 7. We find that for any noise-free information structure the equilibrium profits are always strictly below the collusive demands given by (71). Interestingly, if $m = Nc$, then the equilibrium price is equal to the collusive price, and thus agents get the maximum profits from the variations in $\bar{\theta}$. Yet, in this case the trade between the agents is too low, and thus the profits are lower than the upper bound. On the other hand, as $m \to 0$ the trade between the agents approaches the collusive level, but in this case the average price is too responsive to the average payoff state, and thus the profit is too low. In fact, the maximal equilibrium profit arises at a price impact between $m = 0$ and $m = Nc$.

Proposition 14 (Profit Maximizing Price Impact)
The expected profit $\mathbb{E}[\pi]$ has a unique maximum $m^* \in [0, Nc]$.

Proof. Without loss of generality we can restrict attention to the noise free information structures. The profits of each agent can then be written in terms of the price impact as follows:

$$\mathbb{E}[\pi] = (1/2+m)\left(\frac{(\mu_\theta - c_0)^2 + \sigma_{\bar{\theta}}^2}{(1+m+Nc)^2} + \frac{\sigma_{\Delta\theta}^2}{(1+m)^2}\right) \tag{72}$$

If we maximize (72) with respect to m, we get the profit maximizing price impact which we denote m^*. We first prove that $\mathbb{E}[\pi]$ has a unique maximum in m, and that the maximum is in $(0, Nc)$.

For this, first note that the function $(1/2+x)/(1+\beta+x)^2$ is quasi-concave in x, with a unique maximum at $x=\beta$. Second, note that the function $(1/2+x)/(1+\beta+x)^2$ is strictly concave in x for $x<1/2+\beta$. Since the sum of concave functions is concave, it is easy to see that $\mathbb{E}[\pi]$ is strictly concave for $m \leq 1/2+N\cdot c$. Moreover, we have that:

$$0=\arg\max_{m}\ (1/2+m)\frac{\sigma^2_{\Delta\theta}}{(1+m)^2}\ ;\ Nc=\arg\max_{m}\ (1/2+m)\frac{(\mu_\theta-c_0)^2+\sigma^2_{\bar{\theta}}}{(1+m+Nc)^2}.$$

Thus, it is easy to see that $\mathbb{E}[\pi]$ is decreasing for $m \geq 1/2+N\cdot c$ (which is the part we cannot check it is concave) and has a unique maximum in $[0,Nc]$.

Using the previous result we can also check the m^* is monotonic increasing in $\rho_{\theta\theta}$. We note that:

$$\frac{\partial}{\partial m}\left((1/2+m)\frac{\sigma^2_{\Delta\theta}}{(1+m)^2}\right)\bigg|_{m=m^*}<0\ ;\ \frac{\partial}{\partial m}\left((1/2+m)\frac{(\mu_\theta-c_0)^2+\sigma^2_{\bar{\theta}}}{(1+m+Nc)^2}\right)\bigg|_{m=m^*}>0.$$

Using the fact that $\sigma^2_{\Delta\theta}=\frac{N-1}{N}(1-\rho)\sigma_\theta$ and $\sigma^2_{\bar{\theta}}=\frac{(N-1)\rho+1}{N}\sigma^2_\theta$ we have that m^* is increasing in $\rho_{\theta\theta}$.
■

We can understand the bounds provided in Proposition 14 by considering the solution in some special cases. If $\sigma_{\Delta\theta}=0$, then it is optimal to impose $m^*=Nc$. As there are no gains from trade between the agents, it is best to impose the optimal price level. By contrast, if $(\mu_\theta-c_0)^2+\sigma^2_{\bar{\theta}}=0$, then it is optimal to maximize trade between agents and impose $m^*=0$.

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