FAIRNESS THROUGH THE LENS OF COOPERATIVE GAME THEORY: AN EXPERIMENTAL APPROACH

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Fairness through the Lens of Cooperative Game Theory: An Experimental Approach*

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Abstract

This paper experimentally investigates cooperative game theory from a normative perspective. Subjects designated as Decision Makers express their view on what is fair for others, by recommending a payoff allocation for three subjects (Recipients) whose substitutabilities and complementarities are captured by a characteristic function. We show that axioms and solution concepts from cooperative game theory provide valuable insights into the data. Axiomatic and regression analysis suggest that Decision Makers' choices can be (noisily) described as a convex combination of the Shapley value and equal split solution. A mixture model analysis, examining the distribution of *Just Deserts* indices describing how far one goes in the direction of the Shapley value, reveals heterogeneity across characteristic functions. Aggregating opinions by averaging, however, shows that the societal view of what is fair remains remarkably consistent across problems.

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1   Introduction

Since its origin, game theory has developed on two main fronts, with non-cooperative games on the one hand, and cooperative games on the other. Much effort has been devoted to testing solution concepts and strategic thinking in non-cooperative games.[1]

There is a smaller experimental literature devoted to testing cooperative game theory, which uses the *characteristic function* to describe the worth of each coalition. By allowing subjects to bargain with each other, authors have tested whether observed outcomes match the predictions of cooperative solution concepts such as the core and Shapley value. Kalisch, Milnor, Nash and Nering (1954), one of the earliest papers in the field of experimental economics, inform subjects of their role in a characteristic function and let them interact informally to reach an agreement. Other experimental papers impose a formal bargaining protocol, in addition to specifying a characteristic function, to concentrate on a particular question of interest. For instance, Murnighan and Roth (1977) consider the effect of messages during negotiation, and the announcement of payoff decisions, on the resulting allocations. Bolton, Chatterjee, and McGinn (2003) study the impact of communication constraints in a three-person bargaining game in characteristic-function form. Nash, Nagel, Ockenfels and Selten (2012) introduce tension between short-term incentives to distribute unequally, and long-term incentives to maintain cooperation, in a three-person repeated game of coalition formation, where the stage-game corresponds to a characteristic function.

Many solution concepts in cooperative game theory also have a normative interpretation, capturing what one might consider a fair outcome.[2] That is, how should one distribute the generated resource in a setting with complementarities and substitutabilities among individuals? The *core*, for instance, selects those payoff allocations that give each group of individuals no less than

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[1] See, for example, the survey of Crawford, Costa-Gomes and Iriberri (2013).
[2] See, for example, Moulin (2003) for a textbook introduction to cooperative game theory from this perspective.

their worth; the *Shapley value*, on the other hand, pays people in relation to their marginal contributions to coalitions. These two prominent solution concepts each capture a different way of giving people their "just deserts."

This paper experimentally tests axiomatic principles and cooperative game solutions from this normative perspective. To do this, we designate three subjects in each experimental session to be *Recipients*, with the remaining subjects designated as *Decision Makers*. The role of each Decision Maker is to recommend how to distribute money among the three Recipients, given the worths of different groups of Recipients. At the end of each session, one of the Decision Makers' recommended distributions is randomly chosen to be implemented. Importantly, our experimental design ensures that Decision Makers are impartial observers, that is, their monetary payoffs are independent of their recommendation (in contrast to dictator and ultimatum games). Moreover, the design eliminates any strategic channels that might affect recommendations (in contrast to ultimatum games, or settings where reciprocity may be a concern). These features allow us to identify concerns for fairness, and test whether Decision Makers' choices are guided by principles proposed in the theory of cooperative games.

To better understand Decision Makers' choices, we test the validity of normative properties, or axioms, that may be more fundamental (and, in particular, shared by many solutions). We first test axioms as they apply directly to the characteristic functions we tested, both at the individual and aggregate levels. We then consider a linear regression analysis to study how coalitions' worths impact the amount of money allocated to a Recipient, and show how the axioms translate to coefficient restrictions in the regression. This allows us to extrapolate whether these axioms would be satisfied in yet-untested characteristic functions. Through these approaches, we find suggestive evidence that Decision Makers respect *symmetry*, *desirability*, *monotonicity*, and *additivity*; but, especially at the aggregate level, they appear to violate the *dummy player* property, whereby a Recipient who adds no value to any coalition should get a zero payoff. At a theoretical level, we prove that satisfying symmetry and additivity (along with efficiency, which must be satisfied in our experiment) means

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that Decision Makers' choices should be characterized as a linear combination of the Shapley value and equal split solutions.[3] This suggests a regression model with much fewer parameters, imposing relationships across coefficients for different coalitions' worths and requiring them to be the same across Recipients. Regression analysis shows that despite its simplicity, this model does a nice job of explaining observed choices.

In each of the seven characteristic functions tested, a significant fraction of observed payoff allocations do indeed fall on, or near, the line joining the Shapley value to equal split. The weight on the Shapley value in this linear combination can be interpreted as an index of "just deserts," describing how far the solution departs from equal split in order to reward individuals for their marginal contributions. We perform a Gaussian mixture analysis to infer the relative prevalence of Just Deserts indices in our subject pool. This reveals a diversity of opinions regarding what is fair in each characteristic function, with the distribution of fairness ideals varying across characteristic functions. We then examine the data from a different perspective, by averaging suggested allocations in each characteristic function. Since averaging cancels noise, such aggregation of opinions can also shed light on whether the heterogeneity across characteristic functions arises from noise or systematic shifts in opinion. We find that the average suggested allocation in each characteristic function can be explained nearly perfectly (an R-squared of over 98%) as a linear combination of the Shapley value and equal split, with a Just Deserts index of about 38%. With the caveat that averaging choices results in a small dataset of only 14 observations, it appears that even though individuals are not always consistent, the societal view of what is fair appears to be remarkably so.

In addition to the aforementioned literature on cooperative games, our study also relates to two other literatures. The first is the literature on other-regarding preferences, including Fehr and Schmidt (1999), Bolton and Ockenfels (2000), Andreoni and Miller (2002), Charness and Rabin (2002), Karni and Safra (2002), and Fisman, Kariv and Markovits (2007), among others.[4]

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[3]This result holds for the set of 3-player characteristic functions studied here.

[4]For interesting discussions of this literature, see the book by Camerer (2003) and the

If one were to apply this literature to our problem, then choices for others should be independent of the worths of sub-coalitions (and in many cases, should be an equal split). Our results provide a more complex picture of what people see as fair for others, as a significant fraction of Decision Makers do take the worths of sub-coalitions into account when choosing how to allocate money. While Decision Makers pick an equal split when the characteristic function is fully symmetric, they often opt for an unequal split when facing other characteristic functions; and their behavior seems to accord with some basic principles of cooperative game theory.

Our paper also contributes to the literature testing theories of distributive justice, which is discussed in Konow (2003). Many contributions to that literature involve opinion surveys asking participants to choose between different norms or outcomes given vignettes describing hypothetical situations (see e.g. Yaari and Bar-Hillel (1984) and Kahneman, Knetsch and Thaler (1986) for early contributions). These papers also employ an impartial observer approach. To the extent that preferences over others' payoffs can be inferred from self-serving decisions, experiments on dictator and ultimatum games can also offer a positive evaluation of normative principles. For instance, Cappelen, Hole, Sørensen, and Tungodden (2007) add an investment phase to the classic two-person dictator game to discuss the pluralism of fairness ideals when the income to be shared between the two people is endogenously determined.[5] Parametrizing an individual's utility function by a weighted sum of his own payoff and the distance of the payoff allocation to that individual's fairness ideal, Cappelen et al. use a mixture model to study a horse-race between three classic fairness ideals (strict egalitarianism, libertarianism and liberal egalitarianism). The experimental design for our study circumvents the problem of self-serving bias, to focus directly on what people view as fair for others.

The paper is organized as follows. Section 2 describes our experimental

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survey by Sobel (2005).

[5]The income in their paper is the sum of the two individuals' independent investment outcomes. Given this additivity and the fact that there are only two people involved, their setting is quite different from our own characteristic-function based setting, where groups differ in value and different complementarities and substitutabilities arise.

design and procedure. Section 3 describes the axioms and solution concepts being tested, and their implications for the characteristic functions we study. Section 4 offers a preliminary understanding of subjects' choices through scatterplots and summary statistics. Section 5 delves into further detail, testing axioms and applying econometric techniques to provide a fuller understanding of subjects' opinions on what is fair. Section 6 offers concluding remarks.

2   Experimental Design and Procedure

2.1   Design

This experiment tests what monetary payments individuals (henceforth called Decision Makers) deem appropriate for three designated subjects (henceforth called Recipients) in view of how much different coalitions of Recipients would be worth. That is, Decision Makers' information is in the form of a *characteristic function*. The design of the experiment is simple. At the start of a session, three randomly chosen subjects are designated Recipients 1, 2 and 3, respectively. Recipients stay in that role for the duration of the experiment. All other subjects are designated Decision Makers. A session has seven rounds.

At the start of each round, each Recipient receives an empty electronic "basket." By answering trivia questions correctly, a Recipient earns assorted objects (e.g., two left shoes, a bicycle frame, one bicycle wheel) for his or her basket. Combinations of objects that form a "match" have monetary value. For instance, in a given round a complete pair of shoes – left and right – may be worth $15, while a bicycle frame with two wheels may be worth $40. The objects available to each Recipient in a round have been selected so that only combinations of two or three Recipients' baskets may have positive worth. We momentarily defer discussion of our control over the possible worths of different basket combinations, in order to describe the key role of Decision Makers.

For each round, once the content of the Recipients' baskets has been determined, Decision Makers are informed of the value of different basket combinations. We discuss later in this section the choices behind the presentation

of this information. The Decision Maker is permitted to allocate, as he or she deems fit, the monetary proceeds of the three-basket combination among the Recipients. We require monetary allocations to be efficient and nonnegative, and allow the Decision Maker to opt out of any given round without making a decision.

At minimum, all subjects receive a five-dollar show up fee. Decision Makers receive one additional dollar for each round in which they participate. At the end of the session, one round and one Decision Maker (who participated in that round) are randomly chosen. Recipients receive the monetary payoffs determined by the chosen Decision Maker in the chosen round (in addition to their show up fee). Subjects are informed only of their own payoff, and do not learn which roles other subjects played during the experiment.

The experiment was designed with the following considerations in mind. First, for characteristic functions to be meaningful to Decision Makers, the coalitions' worths should be somehow "earned" by Recipients. This is achieved here by letting Recipients earn objects by answering quiz questions correctly.

Second, to permit specific tests of solution concepts and axioms (as discussed in Section 3), we want to maintain control over the set of characteristic functions faced by Decision Makers. Subjects were told that Recipients would be earning objects in each round, but were not given information regarding how those objects and their values would be selected. In fact, for each round, we pre-selected the objects available as well as the values of different object combinations. Assuming that Recipients earn all the objects available to them in a round, one of the seven characteristic functions in Table 1 is generated.[6] We henceforth use the numbering scheme in this table to identify characteristic functions. The session-dependent mapping between rounds and characteristic functions is detailed further below.

Third, the above points relate to a more general concern: we want the characteristic functions to be earned while mitigating the possibility that in-

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[6]Precisely to reduce the probability that some other characteristic functions would be generated, Recipients were afforded multiple opportunities to earn available objects. Incidentally, any superadditive characteristic function can be generated through this process.

	R1+R2	R1+R3	R2+R3	R1+R2+R3
CF1	60	0	0	60
CF2	40	40	0	40
CF3	40	40	20	50
CF4	80	60	40	90
CF5	30	15	15	30
CF6	40	40	0	70
CF7	40	40	40	60

TABLE 1: The seven characteristic functions (CF) studied are described in the rows. The numerical values in the last four columns are the dollar amounts generated by combining the baskets of the Recipients listed, where Recipient i is denoted Ri.

formation extraneous to the monetary values of basket combinations affects Decision Makers' choices. For this reason, subjects remain in separate roles throughout the experiment, so that Decision Makers cannot consider their own experience as a Recipient when determining payoff allocations. Moreover, a Decision Maker's chosen payoff allocation need not reflect strategic concerns, both because it cannot influence his or her own payoff, and because Recipients play no further strategic role. Decision Makers are presented *only* with the computed values of different basket combinations. They do not learn which objects are in the Recipients' baskets or the values of different object combinations. Similarly, Decision Makers do not see the quiz questions Recipients faced, or how well the Recipients performed.[7] Finally, they do not learn the outcomes of other Decision Makers' choices, and cannot communicate with other subjects. In addition to maintaining the purity of the characteristic function, this allows our setting to remain as close as possible to standard split-the-pie problems. The above features have the added benefit of simplifying the Decision Maker's problem from a computational standpoint.

Fourth, Decision Makers are informed in the instructions that the Recipient numbers they see on their screen in each round are randomly generated aliases

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[7]Notice in passing that keeping such background information from Decision Makers is not unrealistic outside of the lab, in the sense that one does not necessarily know precisely whether other peoples' successes are due to luck, hard work, nepotism, etc.

for the true Recipients.[8] That is, the Recipient whose alias is Ri ($i = 1, 2, 3$) on the Decision Maker's screen in a given round is equally likely to be given the alias R1, R2 or R3 in the next round. Such randomization helps rule out the possibility that a Decision Maker's payoff allocation for a Recipient is influenced by his or her choice for that Recipient in a previous round.

Finally, we run six different sessions to help wash out potential effects[9] arising from the order in which the characteristic functions are presented, employing a Latin square design for characteristic functions one through six. Table 2 details the session-dependent mapping between rounds and characteristic functions. The seventh characteristic function is fully symmetric and all standard solution concepts prescribe an equal split. Consequently, this characteristic function is left as a consistency check in the final round of all sessions, where it cannot affect subsequent behavior.

	Round						
	1	**2**	**3**	**4**	**5**	**6**	**7**
Session 1	1	6	2	5	3	4	7
Session 2	2	1	3	6	4	5	7
Session 3	3	2	4	1	5	6	7
Session 4	4	3	5	2	6	1	7
Session 5	5	4	6	3	1	2	7
Session 6	6	5	1	4	2	3	7

TABLE 2: The ordering of characteristic functions in the six sessions. Round entries identify the characteristic function using the scheme from Table 1. The Latin square design means each possible pair from CF1-CF6 is adjacent in some session.

2.2 Procedure

The six experimental sessions were conducted in April and May 2013. All sessions were held at a computer lab at Brown University, with subjects par-

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[8]The characteristic function the Decision Maker sees is permuted accordingly.

[9]We will not return to this point later, as we do not find evidence of order effects in our data. For instance, the probability of splitting equally in a characteristic function is independent of the previously tested characteristic function.

ticipating anonymously through their computer terminal.[10] Subjects were recruited via the BUSSEL (Brown University Social Science Experimental Laboratory) website,[11] and were allowed to participate in only one of the six sessions.

Sessions lasted approximately thirty to forty minutes. At the start of each session, the supervisor read aloud the experimental instructions, which were simultaneously available on each subject's computer screen. The onscreen instructions contained a practice screen for inputting Recipients' payoffs, to get accustomed to the interface. These instructions are available in Appendix D. The session supervisor then summarized the instructions using a presentation projected onto a screen. That presentation is available in Appendix E. Subjects learned their role as Recipient or Decision Maker only after going through all of the instructions.

A total of 107 subjects participated in the experiment, for an average of nearly eighteen subjects per session. With three subjects selected to be Recipients in each session, a total of 89 subjects acted as Decision Makers. Nearly all Decision Makers chose to actively participate in each round.[12] All subjects received payment in cash at the end of the session.

After completing all seven rounds but before learning their payoff, subjects

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[10]The interface for the experiment was programmed by Possible Worlds Ltd. to run through a web browser.

[11]This site, available at bussel.brown.edu, offers an interface to register in the system and sign up for economic experiments. To do so, the information requested from subjects is their name and email address and, if applicable, their school and student ID number. The vast majority of subjects registered through the site are Brown University and RISD graduate and undergraduate students, but participation is open to all interested individuals of at least 18 years of age without discrimination regarding gender, race, religious beliefs, sexual orientation or any other personal characteristics.

[12]In the first couple of sessions, after everyone except one or two Decision Makers had completed all seven rounds, a connectivity issue with the server prevented the remaining Decision Makers from entering their choice in the final one or two rounds. Of course, the last round was always CF7. Since it was through no fault of their own, those few subjects were paid $1 for each of those missing decisions. This did not affect any of the remaining payment process. The connectivity problem was ultimately identified and corrected. Aside from this, two Decision Makers voluntarily opted out of one round, and one opted out of three rounds. Letting n_i be the number of responses for CFi, we have $n_1 = 88$, $n_2 = 89$, $n_3 = 88$, $n_4 = 88$, $n_5 = 86$, $n_6 = 87$, $n_7 = 84$.

in each session were presented with an optional exit survey via the computer interface. This survey collected basic demographic information (major, gender, age and number of siblings) and allowed subjects to describe how they made their choices as Decision Makers, if applicable.

3   Theoretical Benchmark

3.1   Solution Concepts

Let I be a set of n individuals. A *coalition* is any subset of I. Following von Neumann and Morgenstern (1944), a *characteristic function v* associates to each coalition S a worth $v(S)$.[13] The amount $v(S)$ represents how much members of S can share should they cooperate. Assuming that the grand coalition forms (that is, all players cooperate), how should $v(I)$ be split among individuals? This is the central question of cooperative game theory.

The *equal-split* solution simply divides $v(I)$ equally among all individuals. By contrast, cooperative game theory provides a variety of solution concepts that account for the worths of sub-coalitions, each capturing a distinct notion of fairness. Prominent solution concepts are the *Shapley value* (Shapley, 1953), the *core* (Gillies, 1959), and the *nucleolus* (Schmeidler, 1969).

The Shapley value. Consider building up the grand coalition by adding individuals one at a time, giving each their *marginal contribution* $v(S \cup \{i\}) - v(S)$ to the set S of individuals preceding i. The Shapley value achieves a notion of fairness by averaging these payoffs over all possible ways to build up the grand coalition. That is, the Shapley value is computed as

$$Sh_i(v) = \sum_{S \subseteq \in I \setminus \{i\}} p_i(S)[v(S \cup \{i\}) - v(S)],$$

where $p_i(S) = \frac{|S|!(n-|S|-1)!}{n!}$ is the fraction of possible orderings where the set of individuals preceding i is exactly S. This formula also has an axiomatic foun-

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[13]With the convention that $v(\emptyset) = 0$.

dation. The Shapley value is the only single-valued solution that is efficient, symmetric, additive and satisfies the dummy player axiom. Many alternative axiomatic characterizations have been proposed since then. Axioms are defined formally and discussed below, as we explain the rationale behind our selection of characteristic functions for the experiment. We will also test their validity experimentally.

The core. The core looks for payoff vectors $x \in R^I$ with the feature that there is no coalition whose members would be better off by cooperating on their own; that is, the core requires that $\sum_{i \in S} x_i \geq v(S)$ for each coalition S, with $\sum_{i \in I} x_i = v(I)$ for the grand coalition. While often interpreted from a positive standpoint, the core is also normatively appealing as it respects property rights for individuals and groups: picking a payoff vector outside the core means robbing some individuals from what they deserve.

The nucleolus. Like the Shapley value, the nucleolus prescribes a unique solution in all cases. Given a payoff vector x, the *excess surplus* of a coalition S is the amount it receives net of what it could obtain on its own, that is, $\sum_{i \in S} x_i - v(S)$. The nucleolus interprets excess surplus as a welfare criterion for a coalition, and chooses the payoff vector that lexicographically maximizes all coalitions' excess surpluses, starting from the coalition with the lowest excess surplus and moving up. By contrast, the core simply requires each coalition's excess surplus to be nonnegative. Hence, whenever the core is nonempty, it must contain the nucleolus.

3.2 Normative Principles

We now turn our attention to some normative properties (or axioms) which may guide Decision Makers' choices, even if they do not follow one of the above solution concepts. A significant part of cooperative game theory precisely aims at defining such principles, and understanding which combinations characterize solution concepts. Some properties are satisfied by multiple reasonable solution concepts, and may thus appear, at least on a theoretical level, to be

more universal and fundamental. Others are satisfied by a narrower class of solution concepts, and thus sharply capture the essence of what distinguishes some solutions from others. Testing the axioms, in addition to examining the explanatory power and the relative prevalence of a handful of solution concepts, offers a fuller picture of what people view as fair.

Individual i is a *dummy player* if $v(S) = v(S \setminus \{i\})$, for any coalition S containing i. In order to test this property, we included in our study a characteristic function with a dummy player, namely Recipient 3 in CF1. The dummy player axiom requires that such individuals receive a zero payoff. It is satisfied by the Shapley value, the core, and thus any selection of it as well (such as the nucleolus for instance). The equal split solution, on the other hand, violates the dummy axiom. Hence characteristic functions with a dummy player offer a stark test of the difference between equal split and most standard solutions from cooperative game theory.

Suppose that for any (non-singleton) coalition containing individual j but not i, replacing j with i strictly increases profit. In this case, we say that individual i is *more desirable* than j. If replacing j with i never makes a difference, we say that i and j are *symmetric.* A payoff vector *respects symmetry* if it allocates the same amount to symmetric individuals. It *respects desirability* if it allocates a strictly larger amount to i than to j when i is more desirable than j.[14] The Shapley value respects both symmetry and desirability. The core always contains payoff vectors that respect both symmetry and desirability, but may contain additional payoff vectors. The equal split solution respects symmetry, but by definition, systematically violates desirability.

The properties introduced so far apply pointwise: that is, for given characteristic functions. The following properties relate payoff vectors across characteristic functions.

Suppose that one selects a payoff vector x for a characteristic function v, and a payoff vector $\hat{x}$ for a characteristic function $\hat{v}$. Suppose further that the only difference between v and $\hat{v}$ is that the worth of coalition S has increased.

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[14]Comparisons of payoffs in terms of the individuals' relative desirability were first suggested by Maschler and Peleg (1966).

Then the payoff vectors x and $\hat{x}$ *respect monotonicity* if the payoff of each member of S increases, that is, $\hat{x}_i > x_i$ for all $i \in S$. The Shapley value selects payoff vectors that systematically respect this property. Young (1985) provides an example of two characteristic functions with singleton cores that violate monotonicity. However, one can show that the core does admit a single-valued selection (e.g. the nucleolus) that respects monotonicity for games with only three individuals, as in our experiment. Of course, the equal split solution violates monotonicity since it overlooks the worth of coalitions other than the grand coalition.

A cornerstone of Shapley's (1953) characterization of his value is the additivity axiom. Given two characteristic functions v and $\hat{v}$, the sum $v + \hat{v}$ is the characteristic function where the worth of each coalition is the sum of its worth in v and in $\hat{v}$. Suppose that one selects the payoff vector x for characteristic function v, and the payoff vector $\hat{x}$ for characteristic function $\hat{v}$. To *respect additivity*, one's choice for the characteristic function $v + \hat{v}$ must be the payoff vector $x + \hat{x}$. *Linearity* is a strengthening of the additivity axiom: if one selects the payoff vector x for v, and $\hat{x}$ for $\hat{v}$, linearity requires that one's choice for the characteristic function $\alpha v + \beta \hat{v}$ is $\alpha x + \beta \hat{x}$.

3.3 Theoretical Implications and Motivations for CF 1-7

In our setting, the underlying set of individuals I is simply the three subjects who have been selected to be Recipients. To ensure that subjects acting as Decision Makers are not overwhelmed by numbers, we tested only characteristic functions for which the monetary payoff of singleton coalitions is zero.

As noted earlier, the fully symmetric CF7 serves as a consistency check, since all standard solution concepts, including the core and the Shapley value, would prescribe an equal division there. On the other hand, CF1-CF6 allow us to distinguish between some different solution concepts (see Table 3 for the payoff allocations selected in those characteristic functions).

Since the Shapley value need not belong to the core, it is possible to test the relative prevalence of these two competing norms. To make this comparison

	CF1	CF2	CF3	CF4	CF5	CF6
Shapley	$(30,30,0)$	$(\frac{80}{3}, \frac{40}{6}, \frac{40}{6})$	$(\frac{70}{3}, \frac{40}{3}, \frac{40}{3})$	$(40,30,20)$	$(\frac{25}{2}, \frac{25}{2}, 5)$	$(\frac{110}{3}, \frac{50}{3}, \frac{50}{3})$
Core	P_1	$\{(40,0,0)\}$	$\{(30,10,10)\}$	$\{(50,30,10)\}$	$\{(15,15,0)\}$	P_2
Nucleolus	$(30,30,0)$	$(40,0,0)$	$(30,10,10)$	$(50,30,10)$	$(15,15,0)$	$(40,15,15)$

TABLE 3: The Shapley value, the core, and the nucleolus in CF1-CF6, where $P_1 = \{(x, 60 - x, 0) \mid x \in [0,60]\}$ and $P_2 = \{(70 - x - y, x, y) \mid x, y \in [0,30]\}$.

most meaningful, we include some characteristic functions whose core is single-valued (CF2-CF5). For characteristic functions with only three individuals and singleton coalitions that generate zero profit, the core is single-valued if and only if $v(\{1,2\}) + v(\{1,3\}) + v(\{2,3\}) = 2v(\{1,2,3\})$. Under this condition, the Shapley value is exactly halfway between the equal-split solution and the single payoff vector in the core (since the core is single-valued, it also coincides with the nucleolus).

We also include two characteristic functions with multi-valued cores (CF1, CF6). These allow us to test further axioms, in addition to alleviating collinearity from those cases with a single-valued core. The dummy player axiom can be tested in CF1 (where Recipient 3 plays the dummy role). The monotonicity axiom can be tested by comparing the choices in CF2 with those in CF3 and CF6. Under the reasonable assumption that Decision Makers would choose an equal split in a hypothetical characteristic function where only the grand coalition has positive worth (equal to \$30), the additivity axiom can be examined using Decision Makers' choices in both CF2 and CF6. Moreover, the linearity axiom can be tested directly using the fact that CF3 is the average of CF2 and CF7.

In each of CF1-7, every pair of Recipients can be ranked in terms of either symmetry or desirability. In particular, Recipient i is more desirable than (symmetric to) Recipient j if and only if $v(\{i,k\}) > v(\{j,k\})$ (resp., $v(\{i,k\}) = v(\{j,k\})$). Table 4 shows the ranking of Recipients in each of our seven characteristic functions. Both symmetry and desirability can be tested within each characteristic function, with the exceptions of CF4 (which is fully asymmetric) and CF7 (which is fully symmetric). Notice that Recipient 1 is

always more desirable than, or symmetric to, Recipient 2; and in turn, Recipient 2 is always more desirable than, or symmetric to, Recipient 3. This was only for the purpose of normalization when designing the characteristic functions: as discussed in Section 2.1, Recipients' true identities (as R1, R2 or R3) are masked by a randomly generated alias in each round (with the characteristic function permuted accordingly), so that Decision Makers cannot identify a pattern.

	CF 1 and 5	CF 2, 3 and 6	CF4	CF7
Rankings	$R1 \sim R2 \succ R3$	$R1 \succ R2 \sim R3$	$R1 \succ R2 \succ R3$	$R1 \sim R2 \sim R3$

TABLE 4: The ranking of Recipients in each of the seven characteristic functions, where $Ri \succ Rj$ ($Ri \sim Rj$) means that Ri is more desirable than (symmetric to) Rj.

The simplex representations in Figure 1 (discussed further in Section 4) visualize the payoff allocations corresponding to the different solution concepts in CF1-7. Since R1 is either symmetric to, or more desirable than R2, most solution concepts would require that R1's payoff is at least as high as that of R2. In Figure 1, this corresponds to a payoff allocation in the "left" half of each triangle (that is, left of the vertical line which bisects the bottom edge). Similarly, since R2 is either symmetric to, or more desirable than R3, this corresponds to a payoff allocation in the "bottom" half of each triangle (that is, below the diagonal line which bisects the right edge). Given our normalization of Recipient rankings, the solution concepts in our setting thus prescribe choosing an allocation in the "bottom-left" subtriangle of the simplex. Even through the simplices in Figure 1 represent different total monetary amounts, allocations can be compared even *across* simplices as describing the percentages allotted to different Recipients. As can be seen from Figure 1, the locations of the core allocations (or nucledolus when the core is multi-valued) in CF1-CF7 loosely grid up the bottom-left subtriangle. This variation in the percentages allocated to different Recipients in CF1-CF7 allows us to perform a fuller test of Decision Makers' view of fairness.

Finally, we introduced variation in the worth of the grand coalition across

different characteristic functions in order to identify its effect. However, in both CF1 and CF7 the grand coalition is worth \$60, since it is interesting to see whether the choices in these two cases differ. We also included some characteristic functions where the worth of the grand coalition is not divisible by three, as we conjecture that this may motivate Decision Makers to take another look at the worths of sub-coalitions.

4 Description of the Data

A total of 107 subjects participated in the experiment, with 89 serving as Decision Makers and 18 serving as Recipients; demographic details are provided in Appendix B.[15] Before analyzing the choices made by the Decision Makers, we must first ascertain that Recipients answered sufficiently many quiz questions in each round to generate the desired characteristic functions. The data confirms that this was indeed the case in every session.

By depicting a Decision Maker's allocation for the three Recipients in the simplex (as standard in the cooperative games literature), Figure 1 provides a visualization of all Decision Makers' choices for each characteristic function (where a ball's radius is proportional to the fraction of Decision Makers who picked its center). The payoff allocations in each simplex are read as follows: R3's payoff is read off the tick marks on the vertical axis, R2's payoff is given by the diagonal indifference curves (whose levels are noted by the tick marks on the horizontal axis), and R1's payoff is given by what remains from the total value. In other words, the top (bottom right, bottom left) corner of the simplex corresponds to giving everything to R3 (R2, R1).

The Shapley value, the core, and the nucleolus (when the core is multivalued) are marked in the figure; these are best viewed in color. Recall that in

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[15]While we collected demographic data, the number of subjects per demographic category may too be small to draw conclusive inferences. We do find that economics majors are about 10% less likely to choose an equal split in any given characteristic function. Age and gender seem to have no substantial effect on behavior. Neither does the number of siblings, with the possible exception of being an only child, which also decreases the probability of splitting equally (this should be taken with a grain of salt: only 6 Decision Makers were only children).
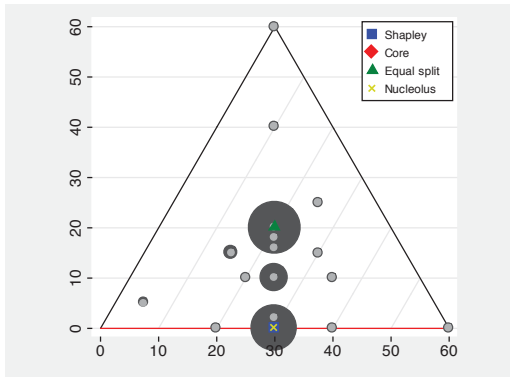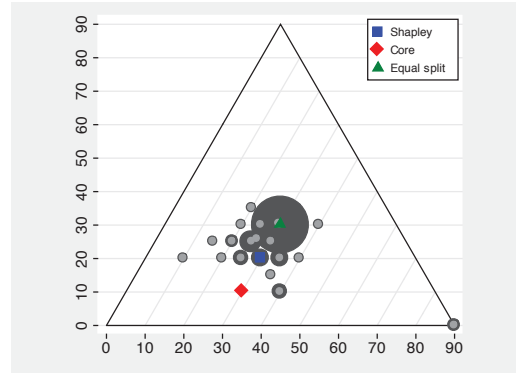
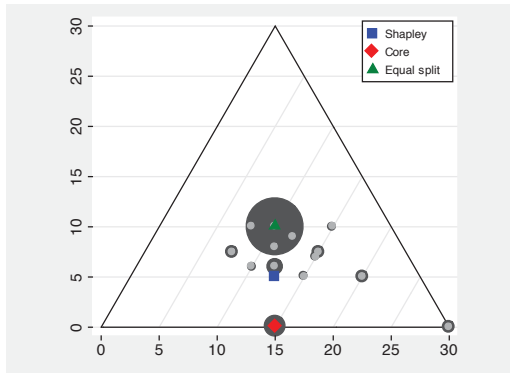(A) Characteristic function 1
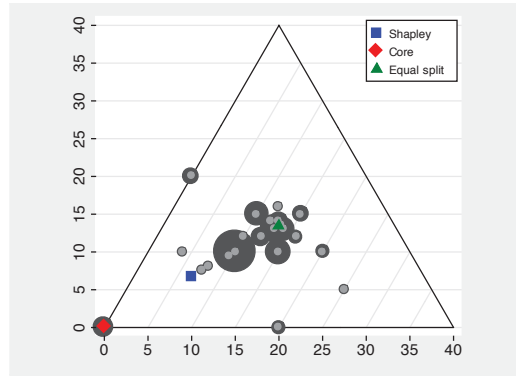
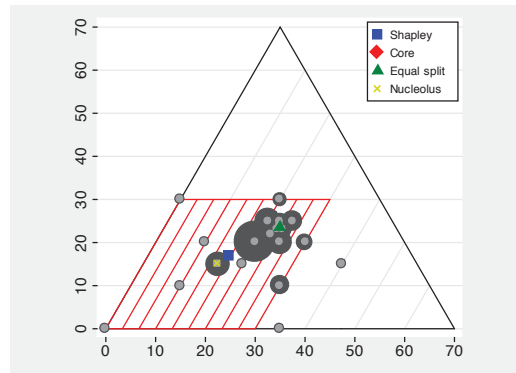(B) Characteristic function 2
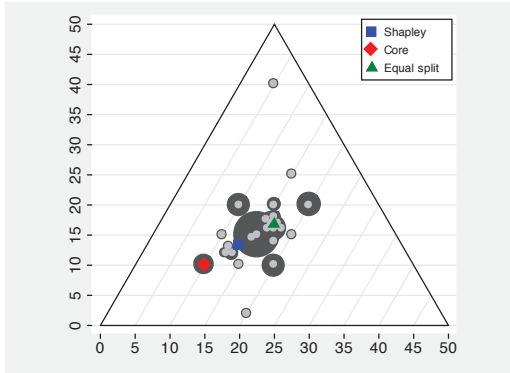
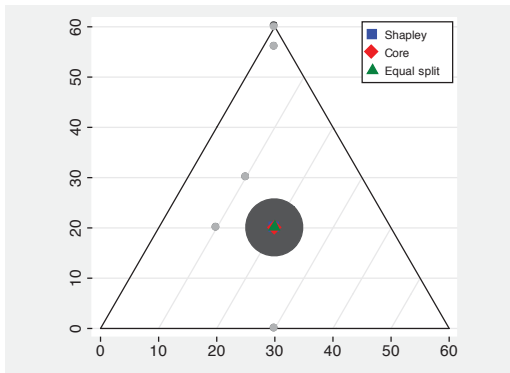
(C) Characteristic function 3

(D) Characteristic function 4

(E) Characteristic function 5

(F) Characteristic function 6

(G) Characteristic function 7

FIGURE 1: Frequency-weighted scatterplots of Decision Makers' allocations (with outliers). Vertical ticks give R3's payoff; R2's payoff is read through the diagonal indifference curves; R1's payoff is what remains. Thus, the top (bottom left, bottom right) corner represents giving the entire sum to R3 (resp. R1, R2).

the fully symmetric characteristic function, CF7, the only choice consistent with standard solution concepts is to split proceeds equally among the Recipients. As seen in Figure 1G, nearly all the Decision Makers participating in CF7 did, in fact, opt for an equal split; the 5 subjects who chose an unequal allocation in CF7 were also outliers in other characteristic functions.[16] Since their choices do not conform to any standard principles, we have dropped these five subjects from all ensuing analysis, leaving 84 Decision Makers. Table 5 below summarizes the remaining data more succinctly, giving the mean payoffs chosen by Decision Makers for each characteristic function.

	CF1	CF2	CF3	CF4	CF5	CF6	CF7
Recipient 1	$24.3 (0.73)	$17.7 (0.73)	$19.1 (0.52)	$34.0 (0.70)	$10.5 (0.23)	$27.7 (0.70)	$20 (0)
Recipient 2	$24.4 (0.75)	$11.4 (0.51)	$15.2 (0.33)	$29.0 (0.52)	$11.1 (0.26)	$21.6 (0.57)	$20 (0)
Recipient 3	$11.3 (1.05)	$10.9 (0.45)	$15.7 (0.47)	$26.9 (0.54)	$ 8.4 (0.33)	$20.7 (0.53)	$20 (0)

TABLE 5: Average amounts allocated to Recipients per characteristic function (after dropping the five outliers), with standard errors in parentheses.

Inspection of Table 5 suggests a clear departure from the equal split norm. This means a significant fraction of the data cannot be explained by a model with purely egalitarian preferences, or more generally, with preferences defined over only individuals' final payoffs (such as in the seminal models of Fehr and Schmidt (1999) and Bolton and Ockenfels (2000)). Even though all subjects in Table 5 choose to split equally in CF7, a substantial number depart from equal split when the characteristic function becomes asymmetric. For instance, the mean payoffs in CF1 and CF7 are quite different even though $60 is shared in both cases.

Table 6 shows that a significant fraction of decisions depart from equal split in each of CF1-CF6. In CF2, CF3 and CF6, the worth of the grand coalition is

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[16]Some of their survey responses suggest a lack of understanding of basket worths or of the setting, or that they were intentionally allocating payoffs in an arbitrary manner; e.g., in describing how they made their choices in the exit survey, one of these five outliers wrote "Pretty arbitrary", and another explained that "i gave one person all of the money because i thought it would increase the recipients average earnings" (sic).

	CF1	CF2	CF3	CF4	CF5	CF6	CF7
Equal splits	41%	23.8%	18.1%	57.8%	65.4%	20.7%	100%

TABLE 6: Percent of payoff allocations that are "equal splits" as defined by choosing payoffs for Recipients that differ by at most one dollar.

not divisible by three. Decision Makers can input numbers with decimal places, but may find payments in whole dollars to be simpler. Throughout the paper, we will thus count a Decision Maker's chosen allocation as an *equal split* if payoffs across Recipients differ by at most one dollar. For those characteristic functions where the total worth is divisible by three (CF1, CF4, CF5 and CF7), everyone who satisfies our equal split criterion is in fact splitting exactly equally. There are 27 Decision Makers who split the money exactly equally in *all* four characteristic functions where the total worth is divisible by three. Even allowing for differences of a dollar, the proportion of equal splits is lower in CF2, CF3 and CF6, where the total worth is not perfectly divisible. There are at least two possible theories to explain this. On the one hand, there may be a fraction of Decision Makers who wish to opt for an "equal split," but who round in multiples of five instead of singles, and don't discriminate regarding which Recipients get more. On the other hand, imperfect divisibility might motivate Decision Makers to think further about the problem, and take a closer look at the worths of sub-coalitions. If one also counts Decision Makers in CF2, CF3, and CF6 who select payoff allocations for the Recipients that differ by (at most) five dollars, the percentages would be more in line with those for CF1, CF4 and CF5 in Table 6. However, this may count too many people: in CF3, for instance, among Decision Makers satisfying the five-dollar criterion but not the one-dollar criterion, 73.7% choose the allocation ($20, $15, $15), which is compatible with rewarding the most desirable Recipient (R1) and treating the symmetric Recipients (R2 and R3) equally.

Our ensuing analysis provides evidence that subjects' choices are not arbitrary, but are guided by basic normative principles. Moreover, we show that solutions from cooperative game theory provide insight into their behavior.

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5 Data Analysis

To better understand Decision Makers' choices, we begin in Section 5.1 by testing whether they satisfy the axioms introduced in Section 3.2 (as they apply to the characteristic functions studied here), both at the individual and aggregate levels.

In Section 5.2, we consider all Decision Makers' choices across CF1-7 to perform a regression analysis, testing how the amounts allocated to Recipients depend on the characteristic function. As we show in Proposition 1, axioms translate to coefficient restrictions in the regression, which allows us to extrapolate from Decision Makers' choices whether they would satisfy these axioms in yet untested characteristic functions. We find that Decision Makers' choices are well-described as a convex combination of the Shapley value and equal split solution. This accords with our theoretical result, Proposition 2, characterizing the solution concepts which are consistent with those axioms that Decision Makers seem to satisfy. The weight on the Shapley value (which we find to be about 38% in our data) can be interpreted as a Just Deserts index, capturing how much Decision Makers reward a Recipient's position in the characteristic function.

In Section 5.3, we examine each characteristic function in isolation. An individual level analysis shows that a significant fraction of Decision Makers choose a payoff allocation which is consistent with a linear combination of the Shapley value and equal split solution. However, there is heterogeneity in the Just Deserts index across subjects. We then perform a Gaussian mixture model analysis to identify the most likely norms of fairness. This raises the question of whether the distribution of Decision Makers' Just Deserts indices is consistent across different characteristic functions. Our analysis shows that this is not the case in general, with heterogeneity in fairness ideals across different characteristic functions.

In Section 5.4, we seek additional insight by studying the Decision Makers' average choice across different characteristic functions. Averaging offers a way of canceling individual noise and aggregating conflicting opinions into a societal

choice. In contrast to the heterogeneity of the Just Deserts index distribution across characteristic functions, we find that average behavior is remarkably consistent across characteristic functions.

5.1 Testing Axioms

Dummy player. CF1 is the only characteristic function among those we tested which has a dummy player (Recipient 3). There is a substantial fraction (34.9%) of subjects satisfying the dummy player property, as well as a substantial fraction (41%) of subjects who violate it because they split equally. We also observe that 15.7% of Decision Makers who violated it by picking an allocation that is a convex combination of the equal split solution and the Shapley value, with the vast majority of these allocating $10 to Recipient 3 (which is halfway between the two solutions). Hence most subjects' choices can be categorized into one of the three above norms. There are many reasons why one may see few norms here; for instance, the Shapley value is an element of the core, and coincides with the nucleolus. A more complex picture arises in some of the other characteristic functions.

Remember, of course, that all of the above Decision Makers split the $60 in CF7 (the same amount which is available in CF1) equally among Recipients. A substantial proportion of subjects thus respond to elements of the characteristic function other than the total amount to be distributed.

Symmetry and desirability. The average payoff allocations in Table 5 suggest that symmetry and desirability comparisons are respected at the aggregate level, with symmetric Recipients allocated approximately equal average payoffs, and more desirable Recipients allocated seemingly higher average payoffs. This is confirmed statistically. For each characteristic function and each applicable desirability comparison Ri≻Rj, the null hypothesis that the payoffs of Ri and Rj are equal is rejected by both a paired t-test and a Wilcoxon signed rank sum test at all conventional levels of significance ($p \leq .001$), with the exception of only a 5% significance level for respecting the comparison R2≻R3 in CF4 (the p-value is 0.0387). Similarly, for each characteristic function and

each applicable symmetry comparison Ri~Rj, the null hypothesis that the payoffs of Ri and Rj are equal cannot be rejected by either a paired t-test or a Wilcoxon signed rank sum test. Appendix C.1 provides full details.

More information can be gleaned by examining the data at the individual level. Decision Makers opting for an equal split clearly respect all symmetry comparisons, but violate all desirability comparisons. Among Decision Makers opting for an unequal split in a given characteristic function, Table 7 shows that a substantial portion respect all applicable symmetry and desirability comparisons. In CF4, no two players are symmetric. It appears that this feature may have complicated the problem, adding some noise. However, among non-equal splits in CF4, 100% respect at least one of the three applicable desirability comparisons; that is, no one selects strictly higher payoffs for R3 than for R2 and strictly higher payoffs for R2 than R1. Moreover, 94.3% respect at least two of the desirability comparisons, and 62.9% choose either R1=R2>R3 or R1>R2=R3.

	CF1	CF2	CF3	CF4	CF5	CF6
Respect rankings	85.7%	56.3%	63.2%	31.4%	67.9%	55.4%

TABLE 7: For each characteristic function, the percentage of chosen allocations (among non-equal splits) respecting symmetry and desirability comparisons.

Monotonicity Among those characteristic functions tested here, monotonicity has implications only when moving from CF2 to either CF3 or CF6. In the former case, Recipient 2 and 3's payoffs should increase because the worths of both the grand coalition and {2, 3} increase. In the latter case, all three recipients' payoffs should increase because the worth of the grand coalition increases. The average payoff allocations in Table 5 appear to confirm these comparisons, which are verified statistically. The null hypotheses that the relevant payoffs are unchanged are rejected at all conventional levels of significance, using both paired t-tests and Wilcoxon signed rank sum tests. This continues to hold true even when testing only those choices which are not equal splits.

At the individual level, 85.5% of Decision Makers allocated weakly more money to both Recipients 2 and 3 when moving from CF2 to CF3 (with 65.1% allocating strictly more to both). Similarly, 93.9% of Decision Makers allocated weakly more money to all three Recipients when moving from CF2 to CF6 (with 80.5% allocating strictly more to all three).

Additivity   No two characteristic functions among those we tested add up to one of the others. Notice, however, that CF6 can be written as the sum of CF2 and the characteristic function given by $v(\{1, 2, 3\}) = 30$ and $v(S) = 0$ for all other coalitions S. It is safe to assume that Decision Makers would opt to allocate $10 to each Recipient in v. Hence, under this assumption, additivity can be tested by checking whether each Recipient is allocated an extra $10 when moving from CF2 to CF6.

Again, the average payoff allocations in Table 5 strongly suggest that this relationship holds. To confirm this, we test the null hypotheses that each Recipient's payoff in CF6 is exactly ten dollars larger than that in CF2. The null cannot be rejected for any of the Recipients using both paired t-tests and Wilcoxon signed rank sum tests; this continues to hold true even when testing only those choices which are not equal splits (p-values are reported in Appendix C.2). At the individual level, subjects who opt for equal split will automatically satisfy the property. When the worth of the grand coalition is not divisible by three, as is the case for CF2 and CF6, additivity will be closely satisfied by those equal splitters who round to a dollar. Even among those who do not split equally in CF2 and CF6, we observe 17 Decision Makers who satisfy the additivity axiom with exact equality for all three Recipients.

A strengthening of the additivity axiom is to require full linearity of the solution. Among the characteristic functions we tested, note that CF3 is the average of CF2 and CF7. Again, the average payoff allocations in Table 5 strongly suggest that Decision Makers' decisions respect linearity. To confirm this, we test the null hypotheses that each Recipient's payoff in CF3 is exactly the average of those in CF2 and CF7. The null cannot be rejected for any of the Recipients using both paired t-tests and Wilcoxon signed rank sum tests.

Equal splitters satisfy linearity by definition; however, the above conclusions hold true even when testing these hypotheses when taking out Decision Makers whose choices count as equal splits in all three characteristic functions (p-values are reported in Appendix C.2). At the individual level, even among the latter category of non-equal splitters, there are 8 Decision Makers who satisfy linearity with exact equality.

5.2 Regression Analysis

To understand how the amount of money m^i allocated to Recipient i depends on coalitions' worths in the characteristic function, let us examine all[17] Decision Makers' choices (across all characteristic functions) using the following linear regression model:

$$m^i = \alpha_0^i + \alpha_{ij}^i v(\{i,j\}) + \alpha_{ik}^i v(\{i,k\}) + \alpha_{jk}^i v(\{j,k\}) + \alpha_{ijk}^i v(\{i,j,k\}) + \epsilon^i, \ (1)$$

for each Recipient $i = 1, 2, 3$, where $\alpha_0^i, \alpha_{ij}^i, \alpha_{ik}^i, \alpha_{jk}^i, \alpha_{ijk}^i$ are the parameters to estimate, and ϵ^i captures noise. That is, for each Recipient, an observation is the amount allocated to that Recipient, yielding up to seven decisions for each Decision Maker (one per characteristic function). Such a linear specification provides a first-order understanding of the relationship between the Decision Makers' chosen allocations and the characteristic function. In addition, notice that two main solution concepts, the Shapley value and equal split, are in fact linear functions of groups' worths.

Overview of regression results. Table 8 provides results from three regressions, one per Recipient, using robust standard errors. By efficiency, any Recipient's payoff is fixed once one knows the other two Recipients' payoffs. The results are computed by performing the regressions for each Recipient separately and then applying seemingly unrelated estimation, allowing errors

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[17]Running a separate regression for each Decision Maker would be of theoretical interest as well, but such an analysis is not practical for this particular experiment given that each Decision Maker made only seven choices, while the linear model below has five parameters.

	Recipient 1	Recipient 2	Recipient 3
$v(\{1,2,3\})$	0.337***	0.348***	0.315***
	(0.0190)	(0.0187)	(0.0192)
$v(\{1,2\})$	0.0790***	0.0404	-0.119***
	(0.0223)	(0.0210)	(0.0210)
$v(\{1,3\})$	0.0449*	-0.133***	0.0884***
	(0.0195)	(0.0192)	(0.0187)
$v(\{2,3\})$	-0.112***	0.0395***	0.0727***
	(0.0165)	(0.0116)	(0.0102)
Constant	-0.742	1.016*	-0.274
	(0.538)	(0.447)	(0.411)
Observations	575	575	575
R^2	0.637	0.673	0.590
Adjusted R^2	0.635	0.670	0.587

Robust standard errors in parentheses, clustered at the DM level
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

TABLE 8: Regression of each Recipient's allocation on coalitions' worths.

to be both correlated across Recipients and clustered at the level of the Decision Maker.[18] Roughly speaking, the results show that each Recipient starts with 33 cents on the dollar for the worth of the grand coalition, loses about 12 cents on the dollar for the worth of the pairwise coalition that does not include him, and finally gains (on average) 6 cents on the dollar for the worth of each pairwise coalition that includes him. As an example, if the characteristic function were $v(\{1, 2, 3\}) = v(\{1, 2\}) = v(\{1, 3\}) = 60$ and $v(\{2, 3\}) = 0$, a back-of-the-envelope calculation estimates that Recipient 1 would receive $27.2, and Recipients 2 and 3 would receive $16.4 each.[19] While we did not test this particular characteristic function, notice that CF2 amounts to scaling

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[18]This technique uses a Huber-White sandwich covariance estimator generalized to allow for clustering (Rogers, 1993). We use this same approach for testing coefficients within and across these equations. Also, dropping one characteristic function at a time shows no substantial effects on estimated values of coefficients, suggesting that collinearity does not pose a significant problem here.

[19]The fitted values from the regression are actually $27.184, $16.297, and $16.791, resp.

it by 2/3; and indeed, scaling the above estimates by that factor yields \$18.1 for R1 and \$10.9 for R2 and R3, which are not statistically different from the average observed choices in CF2 (\$17.7 for R1, \$11.4 for R2, and \$10.9 for R3). The R^2 values, which are on the order of sixty-percent, suggest that the linear model does a reasonably good job of fitting the data.

For Recipients 1 and 3, all of the coefficients on coalitions' worths are significant at all conventional levels (with p-values ≤ 0.001), with the exception of α_{13}^1 whose p-value is 0.021. The intercepts for these Recipients are also not significantly different from zero (with p-values of 0.168 and 0.506, respectively). Similar results hold for Recipient 2, with the exceptions of α_{12}^1 (whose p-value is 0.054, and is thus different from zero only at more permissive levels of statistical significance) and the intercept, which is significantly different from zero (p-value of 0.023) and amounts to a transfer of a dollar to Recipient 2. However, this intercept is not significant when running the regression for Recipient 2 separately (p-value of 0.109).

Testing axioms through coefficient restrictions. Regression analysis gives us the opportunity to study the data from a different perspective. Provided that a linear model adequately describes behavior, this approach encompasses more information: we can extrapolate whether an axiom holds (at least at the aggregate level), even when it does not directly apply to the particular characteristic functions tested here. For instance, we can test whether Decision Makers would allocate a zero payoff to Recipient 2 should he be a dummy player, even though we did not include a characteristic function to test that directly. Similarly, we can test whether both Recipients 1 and 2's payoffs would increase should the worth of coalitions $\{1, 2\}$ increase, even though we did not gather data for such cases. Indeed, the axioms discussed in Section 3.2 translate into coefficient restrictions.[20]

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[20]The only property from Section 3.2 that cannot be translated into coefficient restrictions in (1) is additivity, since the equation subsumes it. We did test for omitted variables after regressing for each Recipient separately, and there is no evidence that terms in addition to the linear ones are needed (all three p-values are larger than 0.5).

Proposition 1. *Suppose that allocated payoffs are determined by the model underlying the linear regression, that is (1) without the noise ϵ^i; and that they sum up to the worth of the grand coalition, as required in the experiment. Then:*

(i) *The dummy player property is satisfied if and only if $\alpha_0^i = 0$ and $\alpha_{ijk}^i = -\alpha_{jk}^i$, for each i, for all distinct $i, j, k \in \{1, 2, 3\}$;*

(ii) *Symmetry is satisfied if and only if $\alpha_{ij}^i = \alpha_{ik}^i$, $\alpha_{ij}^i = \alpha_{ij}^j$, $\alpha_{jk}^i = -(\alpha_{ij}^i + \alpha_{ik}^i)$, $\alpha_0^i = 0$, and $\alpha_{ijk}^i = 1/3$, for all distinct $i, j, k \in \{1, 2, 3\}$;*

(iii) *Monotonicity is satisfied if and only if for all Recipients i and all coalitions S, we have $\alpha_S^i > 0$ when $i \in S$ and $\alpha_S^i < 0$ when $i \notin S$;*

(iv) *Desirability holds if symmetry and monotonicity are satisfied.*

We can safely conclude that the dummy player property does not hold, as the coefficient restrictions $\alpha_{ijk}^i = -\alpha_{jk}^i$, are rejected for each i at all conventional levels of significance ($p = 0.000$). As for monotonicity, Table 8 shows that each of the twelve coefficients (four for each Recipient) have the appropriate sign. Verifying the axiom then amounts to checking that these coefficients are significantly different from zero. Only the coefficient of $v(\{1, 2\})$ for Recipient 2 is not significantly different from zero at the 5% level, as seen in Table 8. However, that coefficient is very close to the threshold for significance, as the p-value is 0.054. The data thus strongly suggests that Monotonicity holds. To verify symmetry, we consider each of the corresponding fifteen equalities given in Proposition 1 as a different null hypothesis, using a chi-squared test after seemingly unrelated estimation with clustering at the level of the Decision Maker. With the exception of the null hypotheses that $\alpha_0^2 = 0$ and $\alpha_{23}^2 = \alpha_{23}^3$, which are rejected with p-values around 0.02, none of the other thirteen possible equalities can be rejected at a 5% level of significance (see the p-values reported in Appendix C.4). The data thus suggests that, for the most part, symmetry holds. The evidence in favor of Monotonicity and Symmetry also suggests that the data respects desirability.

A More Parsimonious Model. The coefficient restrictions arising from symmetry in Proposition 1(*ii*) suggest a more parsimonious model than that

underlying (1). Imposing these restrictions, one finds that $m^i = v(\{1,2,3\})/3 + \alpha(v(\{i,j\}) + v(\{i,k\}) - 2v(\{j,k\}))$, where $\alpha = \alpha_{ij}^i$. This can be further rearranged as $m^i = \delta Sh^i(v) + (1-\delta)ES^i(v)$, simply by taking $\delta = 6\alpha$. More basically, it is natural to ask which class of solution concepts emerges if one drops dummy player from the classic characterization of the Shapley value; that is, keeping additivity, efficiency, and symmetry. A clean characterization emerges for the domain V of three-player characteristic functions for which the worth of each coalition is a rational number, and singleton coalitions are worth nothing. Naturally, V contains all seven characteristic functions we tested.

Proposition 2. *A single-valued solution concept $\sigma : V \to \mathbb{R}^3$ is additive, symmetric, and efficient if and only if σ is a linear combination of the Shapley value and the equal split solution, that is, $\sigma = \delta Sh + (1-\delta)ES$. Moreover, δ can be identified by knowing $\sigma^i(v)$ for any one characteristic function v and any one Recipient i such that $Sh^i(v) \neq ES^i(v)$, as $\delta = \frac{\sigma^i(v) - ES^i(v)}{Sh^i(v) - ES^i(v)}$.*

In view of this result, we investigate how the amounts that Decision Makers allocate to Recipients relate to their payments according to the Shapley value and the equal split solution. We achieve this through a regression where we pool Decision Makers choices for two out of the three Recipients (since the sum of payoffs for all three is fixed):

$$m^i = \beta_0 + \beta_{Sh}Sh^i(v) + \beta_{ES}ES^i(v) + \epsilon, \tag{2}$$

for Recipients $i = 1, 2$ (or $i = 1, 3$, or $i = 2, 3$, depending on which Recipient one excludes from the analysis), where $\beta_0, \beta_{Sh}, \beta_{ES}$ are the parameters to estimate, and ϵ captures noise. This regression treats the two Recipients symmetrically, meaning that there are up to 14 observations for each Decision Maker (two per characteristic function).

By definition, regression (2) cannot capture the data quite as well as (1), since it incorporates the numerous coefficient restrictions from Proposition 1(*ii*), a couple of which were rejected in a statistical sense. On the other hand, to the extent that all the coefficient restrictions appear to be satisfied, or not far from being satisfied, the simpler model still captures Decision Makers' choices

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	Dropping R1	Dropping R2	Dropping R3
Shapley	0.401***	0.379***	0.356***
	(0.0484)	(0.0388)	(0.0423)
Equal Split	0.586***	0.643***	0.641***
	(0.0481)	(0.0391)	(0.0465)
Constant	0.429	-0.477*	0.118
	(0.264)	(0.211)	(0.212)
Observations	1150	1150	1150
R^2	0.637	0.653	0.660
Adjusted R^2	0.636	0.652	0.660

Robust standard errors in parentheses, clustered at the DM level
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

TABLE 9: Regression of Recipients' allocations on Shapley and equal split.

well. Its decisive comparative advantage is providing a good understanding of choices with much fewer parameters.

As can be seen in the Table, the estimated weights on the Shapley value and equal split sum up to approximately one, as expected.[21] Also, the intercepts in two out of the three cases are not significantly different from zero (the p-values are 0.108 and 0.579); the intercept when dropping Recipient 2 is different from zero at the 5% level (p-value 0.026) but at -$0.47 its magnitude is small.

The equal split solution overlooks complementarity and substitutability among Recipients, as captured by the worth of pairwise coalitions. By contrast, the Shapley value rewards Recipients for their role in the creation of the surplus. The weight placed on the Shapley value in the above model can be interpreted as a measure of "just deserts," capturing how much one wishes to reward Recipients according to their roles. While the above linear model is quite simple, one could imagine other, nonlinear solution concepts to capture a Decision Maker's departure from equal split; for instance, the nucleolus, which refines the core whenever it is nonempty. However, the estimated coefficient on

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[21]The p-values when testing the null hypothesis that the sum of these coefficients equals one are 0.389 when dropping R1, 0.084 when dropping R2, and 0.835 when dropping R3.

the nucleolus, if it is added as an explanatory variable in the above regression, turns out to be nonsignificant (p-value 0.308).[22]

5.3 Taxonomy of Decision Makers' Notions of Fairness

The purpose of this section is to better understand the data at the level of individual Decision Makers and individual characteristic functions. One can get a first impression of the distribution of fairness ideals from the weighted scatterplots in Figure 1. To systematically categorize choices while allowing for noise, we perform a Gaussian mixture model analysis per characteristic function. Suppose that for each characteristic function, Decision Makers belong to different populations characterized by a fairness ideal and some covariance matrix to capture noise according to a normal distribution. Observed choices then come from a mixture of normal distributions, and the maximum likelihood criterion allows us to estimate the weights on each population.

The regression analysis from the previous subsection suggests that Decision Makers' choices as a whole can be described, up to some error term, as a linear combination of the Shapley value and the equal split solution. In each characteristic function, the line joining the payoff allocations prescribed by these two solutions seems to play a central role at the level of individual decisions as well. Indeed, Table 10 shows that in each of CF1-CF6, a significant fraction of Decision Makers' choices fall almost perfectly[23] on this line. Our mixture model analysis is guided by these results, trying to categorize all Decision Makers' fairness ideals in each characteristic function in terms of a *Just Deserts index* that describes the weight placed on the Shapley value.

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[22]In CF1-CF7, the nucleolus falls on the same line segment as the Shapley value and equal split. However, these three solutions are not collinear, since the ratio of the distances from the nucleolus to equal split and from the Shapley value to equal split, is not constant for R1 and R2 (it happens to be constant for R3). Consequently the modified regression, where we add the nucleolus, is performed by dropping R3.

[23]"Almost perfectly" allows for small rounding errors in case of indivisibilities, and is defined as follows. We compute a given Decision Maker's δ using their choice for R1, and construct the hypothetical choices they would make, using this δ, for R2 and R3; we then check whether the actual choices for R2 and R3 are each at most one dollar away from the hypothetical ones.

CF1	CF2	CF3	CF4	CF5	CF6
89.2%	69.0%	73.5%	66.3%	85.2%	67.1%

TABLE 10: Percent of payoff allocations falling "almost perfectly" on the line passing through the Shapley value and the equal split solution, allowing for rounding error.

For each characteristic function, we consider a mixture model with pre-specified means, which are given by those monetary allocations corresponding to Just Deserts indices ranging from 0 to 2, in increments of one-quarter.[24] Note that each such Just Deserts index identifies a unique payoff allocation on the line passing through the equal split and Shapley value, which gives the fairness ideal of that population. Indices less than zero contradict desirability rankings and are likely to be "mistakes." An index of two corresponds to the core in characteristic functions where it is single-valued. Larger indices are more extreme and not supported by the data. Given that Recipients' payoffs sum up to a fixed amount, Decision Makers' choices are fully described by the payoff they assign to any two Recipients. We conduct the mixture model analysis in the space of monetary payoffs for Recipients 1 and 3 (we do not include Recipient 2 because in CF4, the Shapley value and equal split happen to allocate the same amount to him). Errors within each mixture are captured by a bivariate Gaussian distribution centered around that mixture's prespecified mean. The estimated mixture weights and covariance matrices, computed through the *EM* algorithm (Dempster, Laird and Rubin, 1977), are given in Appendix C.5. The estimated mixture weights, with mixture means represented in terms of Just Deserts indices, are shown in Figure 2 for each characteristic function.

The mixture model analysis confirms what we already knew for CF1, with three underlying fairness ideals associated to the Just Deserts indices $\delta = 0$ (equal split), $\delta = 1$ (Shapley value and nucleolus), and $\delta = 1/2$ (halfway

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[24]The only exception is CF1, where we restrict attention to indices less than or equal to 1, because larger indices correspond to giving a negative payoff to R3.
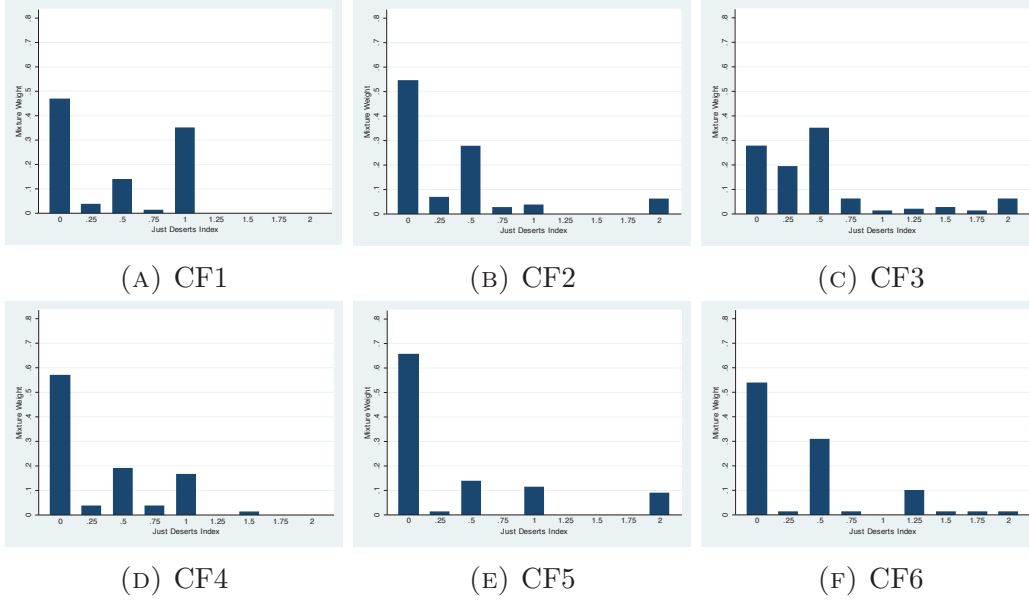
FIGURE 2: Estimated mixture weights for each of CF1-CF6, using a bivariate Gaussian mixture model with prespecified means.

between the other two). Among CF1-CF6, this characteristic function is the one where the Shapley value is most prevalent, perhaps because it belongs to the core, and further coincides with the nucleolus. In those characteristic functions where the core is single-valued (CF2-CF5), the corresponding index ($\delta = 2$) often appears as an isolated peak of the distribution, although small (6% in both CF2 and CF3, and 8.6% in CF5). The index 0 corresponding to equal split plays an important role in all of the characteristic functions. The index $\delta = 1/2$ also plays an important role, serving as a local peak and garnering a share of at least 10% in all the characteristic functions. The Shapley value itself, which is important in CF1, is less important in other characteristic functions; however, it is a local peak in CF2, CF4, and CF5, with a share of at least 10% in CF4-CF5. In the case of CF6, the local peak at the index $\delta = 5/4$ corresponds to the nucleolus. Aside from these commonalities, the distributions across characteristic functions are rather different.

5.4 Aggregating Opinions: A Thought Exercise

We have argued that for each characteristic function, Decision Makers' choices appear overall consistent with a fairness ideal that falls on the line joining the Equal Split solution to the Shapley value. On the other hand, the weight placed on the Shapley value may vary with individuals and characteristic functions. Individuals may simply have different opinions regarding what is fair for others in each characteristic function. Of course, observed choices may also be noisy around an individual's fairness ideal. The fact that the fairness ideal may vary across characteristic functions raises some interesting questions. Do different characteristic functions make certain fairness ideals more salient? Alternatively, do Decision Makers simply lack consistency? In the latter case, Decision Makers might correct their behavior when alerted of its inconsistency with axioms they find desirable. In this sense, axiomatic work may be helpful for thinking through the logical implications of one's principles.[25]

As a thought exercise, we consider the *average* payoff allocation selected for Recipients in each characteristic function, and study how this average varies across characteristic functions.[26] Such averaging is of interest for several reasons. First, averaging cancels noise. While the context and method of aggregation are quite different, the possibility of getting closer to the "truth" through aggregation is reminiscent of the Condorcet jury theorem. Second, average payoffs can be interpreted as what the society *as a whole* should view as fair, given a variety of opinions among its citizens. There can be many ways to aggregate opinions. The simple average is perhaps the most natural, and has some theoretical reasons in its favor. Rubinstein and Fishburn (1986) show the simple average is the *only* aggregator that picks the common opinion when all Decision Makers agree, that is efficient, and for which a Recipient's

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[25]This is reminiscent of the anecdote whereby Savage himself made Allais-type choices when confronted with that paradox for the first time; instead of rejecting the theory, he corrected his choices because of his desire to be consistent with the system of axioms that he felt appropriate. The work of Dal Bó and Dal Bó (2010), though in a quite different setting, suggests a possible avenue for further investigating the moral suasion of normative properties (such as those encapsulated in axioms).

[26]Taking this average is purely an ex-post theoretical exercise. Recipients were not paid according to such averages, and such averages were never mentioned to subjects.

payoff depends only on the amounts Decision Makers' allocated to him.[27]

Suppose that each Decision Maker j (noisily) follows the underlying linear model which allocates to Recipient i the monetary amount $m_j^i = \delta_j Sh^i + (1 - \delta_j)ES^i$. Then, the average monetary allocation $\bar{m}^i$ for Recipient i should have the same structure, with $\delta = \sum_{j=1}^{n} \delta_j/n$. Pooling the data on the average payoffs of Recipients 1 and 3, our dataset yields 14 observations (two average payoffs from each CF1 through CF7). While this is a relatively small number of observations, the linear relationship apparent in this data is quite striking. A regression using the model $\bar{m}^i = \alpha_0 + \alpha_{ES}ES^i + \alpha_{Sh}Sh^i + \epsilon^i$ yields an adjusted R-squared value of 0.995. Moreover, one cannot reject the null hypothesis that the coefficients on equal split and Shapley value sum to one (p-value 0.378) and the intercept is zero (p-value 0.329).

Hence, despite some noisiness in individual observations, there is surprising coherency in the data when aggregating opinions. To get a clearer picture of this highly linear aggregate relationship, it is helpful to plot the data in two dimensions. If average payoffs are given by $\bar{m}^i = \delta Sh^i + (1 - \delta)ES^i$, then subtracting ES^i from both sides implies that $\bar{m}^i - ES^i$ (a Recipients average payoff net of equal split) is simply given by $\delta(Sh^i - ES^i)$. Figure 3 depicts the average payoff data under these transformations, as well as the fitted line from the corresponding regression (which yields an adjusted R-squared of 0.980).
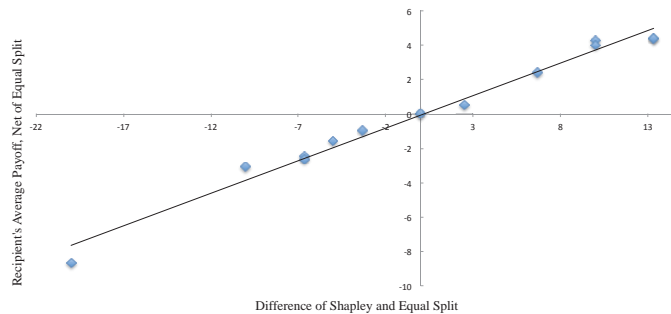
FIGURE 3: Average payoffs of R1 and R3 net of the equal split solution for the given characteristic function, plotted against the difference between the Shapley value and equal split for that Recipient. The slope of the fitted regression line is about 0.38.

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[27]As an example, the aggregation method giving each Recipient the median payoff chosen for him by Decision Makers would satisfy the first and last properties, but violate the second.

6 Concluding Remarks

Our experimental results illustrate how people's view on what is fair for others may be informed by more than the mere comparison of final payoffs: Decision Makers' choices regarding how to split the worth of the grand coalition vary with the worth of sub-coalitions. Looking back at the weighted scatterplots of observed choices in Figure 1, and Recipients' average payoffs in Table 5, the distribution of choices appears to vary across characteristic functions, even when normalizing by the worth of the grand coalition. The theory of cooperative games sheds light on the commonality behind choices. A first step towards understanding observed choices is to think about them in relation to the axis passing through the equal split and the Shapley value, rather than in terms of the share of the total worth allocated to each Recipient. A second step in understanding choices is to express them in terms of Just Deserts indices; that is, to measure their departure from equal split against the Shapley value's departure from equal split. While the scatterplots across characteristic functions appear quite different, some commonalities emerge under this transformation: we see the importance of the indices $\delta = 0, 1/2, 1, 2$. Although the weights on these indices vary across characteristic functions, the average payoffs noted in Table 5 are well described by a Just Deserts index of about 38%.

We now discuss which conclusions may be robust to changing the setting, and two directions for further research.

Context-Specific versus Robust Conclusions. The weight placed on the Shapley value represents how much one wishes to reward Recipients for their perceived role in the creation of the surplus, and is thus likely to vary with the Decision Makers' subjective views of how justified the characteristic functions are. For instance, we anticipate that a much smaller weight would be placed on the Shapley value if characteristic functions were defined arbitrarily, instead of arising from success in a quiz.[28] One could also imagine other tasks, or

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[28]The role of luck versus earning has been documented in other contexts; see for instance Ruffle (1998) in the case of a dictator game or Durante, Putterman and van der Weele

a setting with greater transparency in the relationship between the task and characteristic functions,[29] that would result in higher weights on the Shapley value. Subjective views may also vary with the population studied, particularly with perfect information; for instance, students in the humanities might value success in a quiz about literature more than a quiz about mathematics.

At the same time, the ability to explain a large fraction of observed divisions in terms of linear combinations of equal split and Shapley payoffs should remain valid in all these variants, as this result is grounded on more basic principles such as additivity, efficiency and symmetry. Interestingly, this opens the possibility of learning about a subject pool's subjective views on just deserts in a specific context. By observing the allocations chosen for some characteristic functions, one can estimate the Just Deserts index and extrapolate what the fair division would be when a different characteristic function emerges in the same context.

Social Choice Theory Beyond Cooperative Games. Economists have made important contributions in the theory of Distributive Justive beyond the analysis of cooperative games, with the development in particular of the theory of social choice. The approach of testing normative solutions in cooperative game theory from a positive standpoint, by observing what disinterested Decision Makers pick for others, could be extended to that field as well. A key feature of the social choice theory is the formulation of ideas through axioms, which can similarly be tested in a laboratory setting.

Interplay with Distributional Preferences. While we have characterized Decision Makers' choices for others, we have not tested whether those choices arise from preference maximization over final payoff allocations. In particular, preference maximization has no testable implications in our experiment, because subjects' choices are restricted to distributing pots of money

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(2013) in an experiment with income redistribution and taxation.

[29]Notice that there is a tension, however, between i) making the source of the characteristic functions transparent, ii) having characteristic functions be earned, and iii) keeping control over the set of characteristic functions faced by Decision Makers.

(as in standard dictator and ultimatum games). Adapting the approach of Andreoni and Miller (2002) and Fisman, Kariv and Markovits (2007) to derive GARP conditions in our cooperative game context would require considering "hyperplane games" (Maschler and Owen, 1989) or more general games with "non-transferable utilities." With the exception of those Decision Makers who consistently choose to split equally, our results indicate that if a Decision Maker does maximize a preference over payoff allocations, then it must vary with the additional information captured in the characteristic function. In other words, the preference must be context-dependent.

Context-dependent preferences over final payoff allocations also feature in the literature on intentions and reciprocity (Charness and Rabin, 2002; Falk, Fehr and Fischbacher, 2003; Falk and Fischbacher 2006). We eliminate such channels in our experimental design to be able to identify normative principles. Our analysis of what is objectively fair can, however, shed light on the appropriate reference point (or benchmark) individuals consider when assessing the intentions behind a proposed allocation in settings with complementarities and substitutabilities.

Appendices

A   Proofs

Proof of Proposition 1   Items (i) and (iii) hold trivially. Item (iv) follows
from (ii) and (iii). We now prove item (ii) regarding symmetry. For the
sufficient condition, letting x denote α_{12}^1, the conditions on the coefficients α
implies that Recipient i's payoff equals $xv(\{i,j\}) + xv(\{i,k\}) - 2xv(\{j,k\}) +$
$v(\{i,j,k\})/3$. Symmetry follows at once. Conversely, if the solution satisfies
symmetry, then Proposition 2, proved below, implies that it can be written as
$\delta\sigma^{Sh} + (1-\delta)ES$, for some real number δ. The result then follows from the fact
that the set of coefficients defining the Shapley value satisfy the conditions in
the statement, as do the coefficients defining the equal split solution. $\square$

Proof of Proposition 2   The sufficient condition follows at once from the
fact that both the Shapley value and the equal split solution satisfy efficiency,
symmetry, and additivity.

We thus focus on the necessary condition. Additivity implies that, for any
characteristic function v, $\sigma(2v) = \sigma(v+v) = \sigma(v) + \sigma(v) = 2\sigma(v)$. Similarly,
$\sigma(v) = \sigma(\frac{v+v}{2}) = \sigma(\frac{v}{2}) + \sigma(\frac{v}{2}) = 2\sigma(\frac{v}{2})$, and hence $\sigma(\frac{v}{2}) = \frac{\sigma(v)}{2}$. Also, $\sigma(0) = 0$,
by symmetry and efficiency. Hence $0 = \sigma(v-v) = \sigma(v) + \sigma(-v)$, and $\sigma(-v) =$
$-\sigma(v)$. It is easy to extend such arguments to conclude that σ must be linear
with respect to the field of rational numbers: $\sigma(\alpha v + \beta v') = \alpha\sigma(v) + \beta\sigma(w)$,
for all rational numbers α and β, and all characteristic functions v and w.

For each $S = \{1,2\}$, $\{1,3\}$, $\{2,3\}$, and $\{1,2,3\}$, let v_S be the char-
acteristic function where the worth of a coalition is positive if and only if
it contains S, in which case its worth is 1. The collection of vectors $<$
$v_{\{1,2\}}, v_{\{1,3\}}, v_{\{2,3\}}, v_{\{1,2,3\}} >$ forms a basis of V (understood as vector space
over the field of rational numbers). By symmetry and efficiency, $\sigma(v_{\{1,2,3\}}) =$
$(1/3, 1/3, 1/3)$, $\sigma(v_{\{1,2\}}) = (x, x, 1-2x)$, $\sigma(v_{\{1,3\}}) = (y, 1-2y, y)$, $\sigma(v_{\{2,3\}}) =$
$(1-2z, z, z)$, for some real numbers x, y, z.

Linearity implies that $\sigma(v_{\{1,2\}} + v_{\{1,3\}}) = (x+y, 1-2y+x, 1-2x+y)$. Notice

that players 2 and 3 are symmetric in $v_{\{1,2\}} + v_{\{1,3\}}$, and hence $1 - 2y + x = 1 - 2x + y$, or $x = y$. A similar reasoning implies that $x = z$. Any game v can be rewritten as $v(\{1,2\})v_{\{1,2\}} + v(\{1,3\})v_{\{1,3\}} + v(\{2,3\})v_{\{2,3\}} + (v(\{1,2,3\}) - v(\{1,2\}) - v(\{1,3\}) - v(\{2,3\}))v_{\{1,2,3\}}$. By linearity (notice that coefficients are indeed rational numbers given that $v \in V$), we conclude that

$$\sigma(v) = (x, x, 1 - 2x)v(\{1,2\}) + (x, 1 - 2x, x)v(\{1,3\}) + (1 - 2x, x, x)v(\{2,3\}) +$$

$$(1/3, 1/3, 1/3)(v(\{1,2,3\}) - v(\{1,2\}) - v(\{1,3\}) - v(\{2,3\})).$$

It follows by simple algebra that $\sigma = \delta Sh + (1 - \delta)ES$, where $\delta = 6x - 2$. $\square$

B Demographic information across sessions

Among the 89 Decision Makers, only 82 provided demographic information in the optional exit survey.[30] The data reveals that 23.2% of Decision Makers were economics concentrators (including joint concentrations with related departments, such as applied math-economics). There was a wide variety of other concentrations reported, with biology-related concentrations a popular choice. Decision Makers ranged in age from eighteen to twenty-four, with a mean age of 20.2. The gender distribution was 40.2% male and 59.8% female.[31] In terms of siblings, 7.2% of Decision Makers were only children, 55.4% have one sibling, 24.1% have two siblings, and 13.2% have three or more siblings.

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[30]Recipients were also given the opportunity to respond to the exit survey. We do not include their information here in order to accurately represent the population whose choices we are analyzing.

[31]Enrollment at Brown University is 47% male and 53% female according to http://colleges.usnews.rankingsandreviews.com/best-colleges/brown-university-3401.

C Additional details for statistical tests

C.1 Symmetry and desirability comparisons

The following table gives the p-values arising under a paired t-test and a Wilcoxon signed rank sum test, of the null hypothesis that the payoffs of the given pair Recipients are equal in the given characteristic function. In each case, the null hypothesis of equality cannot be rejected whenever a symmetry comparison applies, but is rejected whenever a desirability comparison applies. All p-values are rounded to three decimal places.

	H_0: **R1=R2**		H_0: **R2=R3**		H_0: **R1=R3**	
	Paired t-test	Wilcoxon	Paired t-test	Wilcoxon	Paired t-test	Wilcoxon
CF1	0.954	0.755	0.000	0.000	0.000	0.000
CF2	0.000	0.000	0.375	0.389	0.000	0.000
CF3	0.000	0.000	0.447	0.743	0.001	0.000
CF4	0.000	0.000	0.011	0.039	0.000	0.000
CF5	0.116	0.230	0.000	0.000	0.000	0.000
CF6	0.000	0.000	0.321	0.942	0.000	0.000

C.2 Additivity

The following gives p-values arising under a paired t-test and a Wilcoxon signed rank sum test, of the null hypothesis that Ri's payoff in CF6 is exactly ten dollars more than Ri's payoff in CF2, for each $i = 1, 2, 3$. We repeat the tests dropping individuals classified as equal splitters in both CF2 and CF6. In each case, the null cannot be rejected at conventional significance levels.

	Paired t-test p-value		**Wilcoxon test p-value**	
	all (no outliers)	drop equal splits	all (no outliers)	drop equal splits
R1	0.993	0.943	0.573	0.633
R2	0.806	0.712	0.160	0.347
R3	0.800	0.661	0.767	0.520

The following table gives the p-values arising under a paired t-test and a Wilcoxon signed rank sum test, of the null hypothesis that the payoff of Ri in CF3 is exactly the average of the payoff of Ri in CF2 and the payoff of Ri in CF7, for each $i = 1, 2, 3$. These tests are also repeated when dropping individuals who are classified as equal splitters in all three of CF2, CF3, and CF7. In each case, the null cannot be rejected at conventional levels of significance.

	Paired t-test p-value		Wilcoxon test p-value	
	all (no outliers)	drop equal splits	all (no outliers)	drop equal splits
R1	0.770	0.888	0.164	0.591
R2	0.330	0.360	0.275	0.377
R3	0.636	0.547	0.833	0.743

C.3 Testing Coefficients in Regression (1)

H_0	$\alpha_{12}^1 = \alpha_{13}^1$	$\alpha_{12}^2 = \alpha_{23}^2$	$\alpha_{13}^3 = \alpha_{23}^3$	$\alpha_{12}^1 = \alpha_{12}^2$	$\alpha_{13}^1 = \alpha_{13}^3$	$\alpha_{23}^2 = \alpha_{23}^3$
p-value	0.224	0.963	0.418	0.309	0.189	0.021
H_0	$-\alpha_{23}^1 = \alpha_{12}^1 + \alpha_{13}^1$		$-\alpha_{13}^2 = \alpha_{12}^2 + \alpha_{23}^2$		$-\alpha_{12}^3 = \alpha_{13}^3 + \alpha_{23}^3$	
p-value	0.686		0.070		0.129	
H_0	$\alpha_0^1 = 0$	$\alpha_{12}^2 = 0$	$\alpha_{13}^3 = 0$	$\alpha_{123}^1 = 1/3$	$\alpha_{123}^2 = 1/3$	$\alpha_{123}^3 = 1/3$
p-value	0.168	0.023	0.506	0.851	0.447	0.352
H_0	$\alpha_{123}^1 = -\alpha_{23}^1$		$\alpha_{123}^2 = -\alpha_{13}^2$		$\alpha_{123}^3 = -\alpha_{12}^3$	
p-value	0.000		0.000		0.000	

C.4 Testing Coefficients in Regression (2)

H_0	$\beta_{ES}^1 + \beta_{Sh}^1 = 1$	$\beta_{ES}^2 + \beta_{Sh}^2 = 1$	$\beta_{ES}^3 + \beta_{Sh}^3 = 1$
p-value	0.197	0.080	0.275

C.5 Mixture Model Analysis

The analysis is in terms of monetary payoffs. An index δ corresponds to a mean allocation (m^1, m^3); covariance matrices are in dollars squared.

	CF1	**CF2**	**CF3**	**CF4**	**CF5**	**CF6**
$\delta = 0$	0.465	0.542	0.274	0.566	0.654	0.537
Cov	5.820, −7.616	3.016, −0.726	21.619, −12.561	0, 0	0, 0	3.746, −1.718
	−7.616, 13.150	−0.726, 3.122	−12.561, 15.577	0, 0	0, 0	−1.718, 5.141
$\delta = 1/4$	0.036	0.065	0.192	0.036	0.012	0.012
Cov	56.250, 0	13.608, 19.034	2.089, −1.095	156.250, −31.250	31.641, −7.031	136.111, 77.778
	0, 0	19.034, 34.602	−1.095, 0.870	−31.250, 6.250	−7.031, 1.563	77.778, 44.444
$\delta = 1/2$	0.137	0.274	0.349	0.187	0.136	0.305
Cov	57.010, 21.973	0.043, −0.022	0.034, −0.017	13.841, −0.268	14.517, 2.761	0, 0
	21.973, 8.789	−0.022, 0.011	−0.017, 0.009	−0.268, 11.242	2.761, 2.705	0, 12
$\delta = 3/4$	0.012	0.024	0.058	0.036	0	0.012
Cov	56.250, 37.500	11.111, 27.778	5.238, −4.308	6.250, −31.250	6.086, −6.654	44.444, 77.778
	37.500, 25	27.778, 69.444	−4.308, 23.198	−31.250, 156.250	−6.654, 13.353	77.778, 136.111
$\delta = 1$	0.349	0.036	0.012	0.162	0.111	0
Cov	17.277, 3.380	3.445, −2.389	336.111, −488.889	28.880, 3.502	1.361, −2.722	11.116, 44.440
	3.380, 1.000	−2.389, 4.528	−488.889, 711.111	3.502, 5.699	−2.722, 5.444	44.440, 177.771
$\delta = 5/4$	N/A	0	0.018	0	0	0.098
Cov		177.696, −80.901	5.965, −20.876	54.835, −48.919	10.827, −12.649	0, 0
		−80.901, 43.616	−20.876, 73.151	−48.919, 67.039	−12.649, 24.024	0, 3.125
$\delta = 6/4$	N/A	0	0.024	0.012	0	0.012
Cov		274.194, −128.339	2.778, −0.556	225, 75	14.110, −20.329	69.445, 111.111
		−128.339, 67.173	−0.556, 0.111	75, 25	−20.329, 38.930	111.111, 177.778
$\delta = 7/4$	N/A	0	0.012	0	0	0.012
Cov		392.018, −186.388	5.444, −2.722	174.230, −171.932	18.535, −29.174	136.111, −38.889
		−186.388, 94.904	−2.722, 1.361	−171.932, 193.218	−29.174, 55.805	−38.889, 11.111
$\delta = 2$	N/A	0.060	0.060	0	0.086	0.012
Cov		0, 0	0, 0	249.836, −249.842	0, 0	0, 0
		0, 0	0, 0	−249.842, 273.351	0, 0	0, 0

D   Instructions via computer interface

The following are successive screenshots of the experimental instructions subjects received through the experimental interface, which was accessible at each computer terminal.

Decision-making experiment

What happens if you are chosen to be a Recipient?

There are 7 rounds. You begin each round with an empty basket. You can earn (fictitious) objects that accumulate in your basket by answering quiz questions. As an example, you may earn a 'left shoe' if you correctly answer a question, a 'right glove' if you correctly answer another, etc.

Here is an example of a question a Recipient could face in a given round:

> **Question**
> Who was the 2012 U.S. vice-presidential Republican nominee?
> ○ Mitt Romney
> ○ Ron Paul
> ○ Joe Biden
> ○ Paul Ryan

All quiz questions are multiple choice. Quiz questions could also be logic puzzles, math questions, etc. You will be informed whether your answer is correct. You may have multiple opportunities within a round to earn objects.

At the end of each round, each Recipient sees his own basket and the two other Recipients' baskets. They also learn the cash amounts that can be generated by combining baskets. Here is an example (note that both the objects and their values may differ from round to round).

> **In this round, Recipients earned the following baskets of objects:**
> Recipient 1: two left shoes, one left glove, and one leg of a tripod.
> Recipient 2: one right shoe, two right gloves, and one leg of a tripod.
> Recipient 3: one right shoe and one leg of a tripod.
>
> **The redemption value of two or more baskets is calculated by summing up the value of different object combinations. These values are:**
> Each matching pair (left and right) of shoes is worth $15.
> Each matching pair (left and right) of gloves is worth $10.
> Three legs of a tripod are together worth $5.
> No other combination of objects has any value.
>
> **Therefore, the values of all the possible basket combinations are:**
> Recipients 1 and 2's baskets together have a value of $25.
> Recipients 1 and 3's baskets together have a value of $15.
> Recipients 2 and 3's baskets together have a value of $0.
> Recipients 1, 2 and 3's baskets together have a value of $45.

CONTINUE

Decision-making experiment

What if you are a Decision Maker?

At the end of each round, each Recipient has a basket of objects which they earned by answering quiz questions. These baskets must be redeemed in combination for cash. As a Decision Maker, you have the opportunity to decide how to split the proceeds among those Recipients when all three baskets are combined. You are paid **$1** for each round in which you choose to fill in the corresponding payoffs for the Recipients.

So that you do not have to calculate the redemption value of baskets, we present you those values immediately. The next screen gives an example of what you will see in a given round, and how you input your decision.

To ensure anonymity, neither your identity nor the Recipients' identities are revealed. In addition, Recipients are only identified in each round by a randomly drawn number (1, 2, or 3). That number is re-drawn in each round.

CONTINUE

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Decision-making experiment

Example screen for the Decision Maker

Given the objects each Recipient earned by answering quiz questions correctly, we have the following results:

- If only Recipients 1 and 2's baskets are combined, then Recipients 1 and 2 could share $25.
- If only Recipients 1 and 3's baskets are combined, then Recipients 1 and 3 could share $15.
- If only Recipients 2 and 3's baskets are combined, then Recipients 2 and 3 could share $0.
- If all three baskets are combined, then Recipients 1, 2 and 3 could share $45.

We request that all three baskets be combined since doing so generates the most money to be shared. However, you can decide how to split the proceeds as you see fit.

Your options will appear in a moment.

◯ I do not wish to participate in this round, and understand that I will not receive $1 for this round.

◯ I wish to participate in this round, and earn $1 for my decision. All three baskets must be redeemed together, and all proceeds must be distributed.

CONTINUE

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In the screen above, the value of different object combinations appear sequentially, with a small delay between lines. The options seen in the screen below appear only when selecting the second radio button. The input boxes for payoffs can be accessed in any order. Once two payoff boxes are filled out, the third is automatically filled with the remaining amount. Entries are required to be nonnegative.
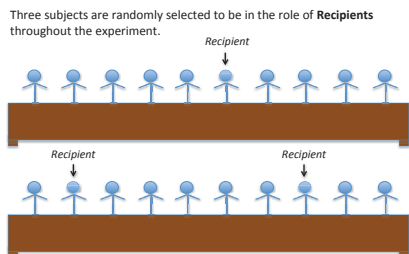
E   Instructions projected on screen

After subjects go through the instructions on the computer screen, the session supervisor gives a more graphical presentation of the instructions. Screenshots are presented below, and should be read from left to right within each row, starting from the top.

Three subjects are randomly selected to be in the role of **Recipients** throughout the experiment.

Recipient

Recipient   Recipient

The other subjects will be in the role of **Decision Makers** throughout the experiment.

There are 7 rounds. In the beginning of each round, each Recipient starts with an empty basket.

Each Recipient individually answers quiz questions. By answering questions correctly, a Recipient can earn fictitious ``objects'' for his or her personal basket.

The cash redemption value of any single basket is zero. Cash redemption value is generated when baskets of different Recipients are combined at the end of each Round.

For example, one Recipient may have earned a left shoe and a wheel-less bike frame. Another Recipient may have earned a right shoe. Another Recipient may have earned two bike wheels.

At the end of each round, after Recipients have earned objects for their individual baskets, each Decision Maker is given the opportunity to decide how to divide the cash redemption value among the Recipients when all three baskets are combined.

Decision Makers get $1 for each round they choose to participate in.

So Decision Makers don't have to compute the value of different basket combinations, the computer does this for you, saying how much different combinations of the Recipients' baskets would be worth.
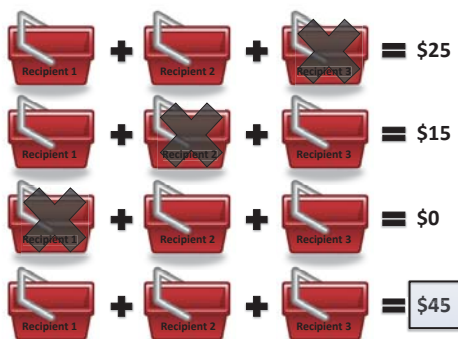
We request that you combine all three baskets, since this generates the most money. However, you can decide how to divide the money between the Recipients as you see fit.

There are **7** rounds, so each Decision Maker makes seven decisions.

Given the nature of the objects earned, baskets are not worth anything on their own. They can only have cash redemption value when combined with other basket(s).

Recipient 1 = $0

Recipient 2 = $0

Recipient 3 = $0

Recipient 1 + Recipient 2 + Recipient 3 = $25

Recipient 1 + Recipient 2 + Recipient 3 = $15

Recipient 1 + Recipient 2 + Recipient 3 = $0

Recipient 1 + Recipient 2 + Recipient 3 = $45

Two important points.

All identities remain anonymous. In fact, the DM only sees an alias for the Recipients' names (which can be either Recipient 1, 2, or 3). This alias is randomly determined in each round. So the subject who appears as Recipient 1 in a round is equally likely be given the alias of Recipient 1, 2 or 3 in the next round. The Decision Makers' identities are hidden too.

To determine everyone's payoffs from the experiment, we:
1) Randomly pick one of the seven rounds.
2) Randomly pick one of the Decision Makers who participated in that round.
3) Give the three Recipients the payoffs the chosen Decision Maker selected for the Recipients, plus the show-up fee.
4) Give all the Decision Makers $1 for each round they participated in by selecting payoffs for the Recipients, plus the show-up fee.

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