MATHEMATICAL INSTITUTIONAL ECONOMICS

By

Martin Shubik

December 2012

COWLES FOUNDATION DISCUSSION PAPER NO. 1882

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS
YALE UNIVERSITY
Box 208281
New Haven, Connecticut 06520-8281

http://cowles.econ.yale.edu/

Mathematical Institutional Economics*

Martin Shubik

December, 3, 2012

Abstract

An overview is given of the utilization of strategic market games in the development of a game theory based theory of money and financial institutions.

JEL Classifications: C72, C73, E44

Contents

*For the symposium in honor of the 70th birthday of Yakar Kannai.

1 Statics or Dynamics?

An adequate understanding of broad economic dynamics requires the presence of money and financial institutions. Due to the plethora of institutions in any advanced economy a synthetic approach is required. There is no magic set of equations of motion that fit all economies without considerable hand tailoring. There are many individual operations research and micro-economic problems that call for deep analytical treatment; but a useful overall synthesis of the many aspects of underlying macro-micro-economic structure can be organized utilizing **strategic market games**. A strategic market game provides a well-defined game theoretic model of a closed dynamic economy that is loosely coupled.

Presented here is the sketch of a search for an adequate basis for understanding economic dynamics and the means for control of an economy. It began in 1953 when I was finishing my PhD. thesis at Princeton on the topic of game theory as applied to the understanding of oligopolistic competition. John Nash had broadly generalized Cournot's concept of a noncooperative equilibrium (NCE) [34]. Lloyd Shapley had suggested the core as a solution applied to games in coalitional form [42], and had also developed the value solution [43]. I regarded both cooperative and noncooperative solution concepts as relevant to economic behavior and was tempted to use both. But I concluded that the Nash noncooperative equilibrium model connected more or less seamlessly with the work of Cournot, Bertrand, Zeuthen, Stackelberg and Chamberlin and the predominant viewpoint of economic theorists at that time was such that it made better sense to concentrate on the noncooperative equilibrium approach.

In a book Strategy and Market Structure (SMS) [45], based on my thesis, I utilized the noncooperative equilibrium approach to oligopolistic market structures. The original intent had been, first to study equilibrium with little concern for time, assets and explicit dynamics, and then try to develop an economic dynamics. I found that the former was relatively easy to do. One could catch the essence of limiting properties such the growth of the number of competitors in a market. However the latter was far more difficult to achieve than I had expected. The inordinate proliferation of plausible models led me to suggest that the study of the dynamics of economic systems be called **Mathematical Institutional Economics.** This title stressed the role of institutions as carriers of process. They appear as parts of rules of the game.

Obtaining a model is not enough. One has to have a solution concept and be able to derive the solution with the analysis. This is where the mathematics enters. I formulated a class of games called **Games of Economic Survival** [45], [54] where the firm and its owners are split into two separate entities.

The concept behind a GES is that is that the corporation and its owners, each may have different payoffs and may have different levels of strategic control over the actions of the firm. With this dichotomy one can model situations in which a limited liability controlling stockholder can use the firm to take risks involving great gains or bankruptcy that he would not risk himself.[1]

2 Static Solutions to Games in Coalitional Form

Although in the work noted above I had utilized primarily the NCE solution, it was, and still is my strong belief that the theory of games does not offer, or will ever offer a single general solution concept that covers all strategic situations in which a game theoretic analysis of the problem appears to be of value. The popular solution concepts in cooperative static models are the **Core** and **Value.**[2] Among games in strategic form the **maxmin solution** for two person constant sum games is the clearly favored solution. For all other games some variant of the Nash **noncooperative equilibrium** is favored. For games in extensive form there is no clear consensus about solution concepts. Modified forms of the noncooperative equilibrium are often used In many instances some form of learning and signalling is considered ([29], [20]) and the concept of a quasi-cooperative solution is often talked about, but rarely cleanly defined.

3 The Closed Economy Convergence of the Cournot Game

In December 1970 while on an extended visit to the Rand Corporation, I reviewed the two systemic problems in economic theory that were the highest on my list of problems.

3.1 A Quest for a Theory of Money

Sometime in the 1960s I visited Oskar Morgenstern in Princeton. He asked me if I had any specific problems or a research program that I wanted to carry out. I specified the

[1] For a modern mathematical institutional treatment of the firm see Tirole [60].

[2] Less popular are the nucleolus, bargaining set and stable set.

theory of money. He commented that this was possibly one of the hardest problems in economic theory

I spent 1960–1970 wasting considerable amounts of time trying to get a satisfactory model of a closed economy incorporating money. and I failed to do so. In 1970 I made a conscious decision to abandon looking for a theory of money. I argued that I was already forty four and that I was probably too old to try to generate a new idea which was needed.

3.2 The CE as a Limit for Many Solutions

I decided that I should devote all of my time on what I regarded as the second most important problem on my list. This was completing the game theory solution convergence package on market games with Lloyd Shapley. All of our results had been for games in coalitional form. The convergence of the **core, value** and the **nucleolus** [40] to the **CE** bolstered the interpretation of the CE.[3] In essence the **core** convergence illustrates that the CE is impervious to the power of groups. The convergence of the **value** illustrated that the **CE** reflects the axioms of the **value** or fair division taking into account the property claims reflected in the initial ownership distribution. The **nucleolus** can be viewed in terms of a maxmin taxation policy where a single point in the core is selected in such a way that the maximum tax that any agent is charged it minimized.[4] The CE itself may be regarded as showing that a correctly selected price system contains all the information needed to enable all individuals independently to select the appropriate allocations that resulted in a Pareto optimal outcome with all books balanced. Unfortunately the static existence proofs provided no dynamics showing the emergence of such prices. The work of Scarf [39] providing a computational way of arriving at a CE, but this algorithm makes no use of competitive conditions so critical to the folk lore of free competition.

Missing from the list above is the convergence of the **NCE.** In order to study it one needs to work with the game represented in strategic form. The strategic form provides a far finer grained model of the economy than does the coalitional or characteristic function form. Furthermore it provides a half-way house to the fully time dependent description of a game in extensive form. The existence of an **NCE,** like the **CE** offers a justification for decentralization in the sense that the equilibrium in both instances is derived from independent behavior. They differ in as much as the CE solution postulates price as given, but in the NCE solution the agents take into account their influence on price explicitly and the limiting aspect of increasing the number of agents in the market gradually attenuates the power of the individuals

[3]There are also other coalitional solution concepts such as the bargaining set. the kernel and the stable set. But the core, value and nuceolus are more condusive to economic interpretation.

[4]Shapley and Shubik worked primarily with sidepayment games (SP) where there is no problem with the uniqueness of the CE. Matters are somewhat trickier with NSP games where multiple CEs are encountered.

over price.

3.2.1 A Conceptual Sensitivity Analysis of the CE

How to submit an economic model to a sensitivity analysis always presents a challenge. There are several straightforward ways concerning structure. They involve changing the values of parameters, or changing functional forms or algorithms arguing that there are sufficient reasons to believe that in some appropriate way the new forms or algorithms are "close" to the previous ones. Another approach to sensitivity analysis of a model involves changing the behavioral assumptions made and selecting a different solution concept.

The belief in the virtues of the invisible hand and market competition has been around for some time. Its prime claim for attention by economists is that it is efficient and decentralized. Noted here is that for an exchange economy with the appropriate structure a strong case can be made out for the **CE** as being a passive solution where individual behavior is guided by a set of prices supplied be either a central computing system or by the forces of competition and in either case the individual has no personal power over price.

We have:

- **CE** illustrating efficiency obtained either from competition creating prices or by a central agency announcing them.
- The **NCE** attains the **CE** as a limit of a sequence of games with oligopolistic prices that approach the **CE** prices. The **NCE** is generically inefficient but approaches efficiency.
- The **core** shrinks to the **CE** outcome as the countervailing power of groups rule out other core points.
- The **nucleolus** lies within the core hence it converges to the **CE.**
- The **Value** moves across the Pareto Surface and eventually converges to the **CE** illustrating that the axiomatic properties defining the Value are shared by the CE.

This multiplicity of solutions which, in an exchange economy lead to the same outcome although they are conceptually highly different makes a compelling case for the **CE** in a mass market economy.[5] Left out unfortunately is any actual dynamics. Time, at best, is only implicit. They deal with the final outcomes, not how one gets there. The extensive form or other representations cast in a multiperiod or continuous time mode are required to consider actual dynamics.

[5] There are two versions to be considered of the exchange economy as a coalitonal game. They are the side-payment (SP) and the no-sidpayment games (NSP). The most import difference is that the CE is unique in the game with side-payments may be multiple for NSP, the core and value solutions are consistent with the multiple equilibra.

3.3 From NCE Convergence to Commodity Money

In looking for the model to illustrate convergence of the NCE I tried to build on both the Cournot (quantity strategy) and Bertrand (price strategy) formulations, but narrowed down my investigation to a closed variant of the Cournot open model. Utilizing the mantra of keeping matters as simple as possible to catch the phenomenon being considered.

I decided to use a model where initially there were n individuals and n commodities. All individuals have a symmetric utility function in all the n commodities Thus one should be able to build an intrinsically symmetric game. With a complete market structure the number of markets would be $n(n-1)/2$ if every pair of goods could be exchanged with each other directly. I was concerned with whether I could construct a symmetric game with a minimal amount of simple markets; where I mean by a simple market a mechanism where quantities of good i are exchanged for good j. A price would be a ratio q_i/q_j. If I wanted to select a minimal network of markets that could permit efficient trade at least one market for each good was required. This called for $n-1$ markets. However I observed that I needed n independent strategy sets, but there were only $n-1$ independent prices. If one insisted that all players were to denominate the quantity of good offered for sale in terms of a specific commodity, then the player whose commodity is selected plays a nonsymmetric role. I overcame this difficulty by considering a somewhat different game with n monopolists trading in $n+1$ commodities where each monopolist held one unit of his special good and a large enough supply of the $n+1$st good. I set the price of the $n+1$st good equal to one and had the strategies of each player i be an offer q_i for the sale of his commodity combined with bids $(b_{i1}, b_{i2}, ..., b_{in})$ to purchase all commodities. I then tried this out on a specific simple model and considered the replication of the model with k players of each type and let k become large. Much to my joy the example was tractable, I could calculate a specific noncooperative equilibrium and show that it moved toward the Pareto optimal surface as k became large.

I took my preliminary model to Shapley who constructively tore it to bits in many different ways. After each tearing up it was possible to rebuild and strengthen it. In particular Lloyd was quick in pointing out that if some commodity was not offered for sale, but the $n+1$st commodity had been bid for it, as price was defined as a ratio of two quantities, there would be a division by zero. We noted that the specialist's role on the New York exchange requires that he make an "orderly market," i.e., that he has a small inventory available for sale. This can be treated mathematically by defining an "epsilon-related" game to the game under consideration where some small amount "epsilon" is available in each market. This cuts out the singularity.

The model appeared to me to fill the gap we had in our investigation of the convergence of game solutions to the competitive equilibrium. But there was a fundamental difference that finally dawned on me. The cooperative models are at a far higher level of abstraction than this model. This model required the strategic or extensive form of the game, not the coalitional form or characteristic function. But the strategic

form is at a lower level of abstraction than the coalitional form and the strategic and the extensive forms provide enough structure for a process description of the game.

4 A Game Theoretic Approach to The Theory of Money[7]

The problem being tackled was the embedding of the strategic market game into a general equilibrium setting. I had tried to construct a model with as few markets as possible needed to include efficient outcomes. As soon as one tries to obtain a fully dynamic model one is faced with multitude or a Pandora's box of variations. The first model I looked at had n markets and $n+1$ commodities. This structure was sufficient, but not necessary to allow for efficient trade providing that there was "enough" of $n+1$ commodity present and it was well distributed (see [50], [15]).

I was so intent on finishing the convergence package with Lloyd that for several weeks I thought only of the noncooperative game convergence. After some time it dawned on me that in fact this model provided the entry into the development of a general theory of money. The $n+1$st commodity that had been introduced could be regarded as a commodity money, and the condition that individuals were required to bid using this commodity imposed on each individual optimization a set of cash flow constraints such that if each individual did not have enough of the $n+1$st commodity they could not necessarily attain the optimum outcome that could be obtained in the CE utilizing only wealth constraints. Mathematically these extra conditions said the none of the cash flow constraints had a positive shadow price. In other words no constraint was binding.

My second most important problem turned out to solve my most important problem.

The model provided a key into creating a host of strategic process models consistent with the structure of the general equilibrium economy. Heeding the warning of Kenneth Arrow that one can easily be swamped by a plethora of models in attempting to deal generally with economic dynamics I considered how to cut down on the number of potential models in a reasonable manner. I devised the concept of **minimal institution.** A minimal institution is a mechanism that is just able to perform a function or set of functions for which it has been designed. The removal of any part of the mechanism disables it from performing its function. A simple example of a minimal institution is provided by price formation mechanisms. The simplest mechanism (measured in terms of size of strategy set needed for each market) is given by the **sell-all** model.

[5]There have been many distinguished works in the formalization of monetary theory (see Kiyotaki and Wright [25]. Woodford [64] since the 1950s but with the exception of Stephan Morris and a few others [12], [33], the predominant approach has not been game theoretic.

Suppose, for any set N of agents trading in a set M of commodities plus a commodity money an agent i has an initial endowment of $(q_1^i, q_2^i, ..., q_m^i, q_{m+1}^i)$ where $q_{m+1}^i \geq 0$ is agent i's holding of commodity money.

All individuals are required to offer all of their commodities for sale (a tax collector's dream). A strategy of an individual i is a vector of m dimensions $(b_1^i, b_2^i, ..., b_m^i)$ where b_j^i is the bid in money by individual i for good j. The sum of all bids $\sum_{j=1}^m b_j^i \leq q_{m+1}^i$, the amount of money she has on hand. The price of good j is determined by the amount of money bid for the goods offered

$$p_j = \frac{\sum_{i=1}^n b_j^i}{\sum_{i=1}^n q_j^i}$$

This is a cash only economy with no credit indicated. The price formation explicitly spells out many of the assumptions that are implicit in "cash-in-advance" models that there is a finite period under consideration where the market meets only once and the earnings from sales are not available for use until next period.

4.1 Necessary Functions and Sufficient Institutions

The underlying theme in the portrayal of economic process models calls for the specification of functional need such as the need to be able to borrow, with the construction of the simplest institutional form, such as a money market that provides for the function.

Elementary observations indicate the functions; economy and parsimony justify and limit the mechanisms and institutional observation match them with the economic reality for which they provide abstractions. Thus it is shown in detail elsewhere [50] that there are three basically simple price formation mechanisms, "sell-all" described above is the simplest, with one move per player; buy-sell the next with two moves and bid offer or the double auction mechanism with four moves for each player [13]. The double auction mechanism is close to that of many stock markets. Multiperiod versions have been analyzed and simulated by many individuals [17], [19].

It is straightforward to observe that the general exchange model for $m+1$ commodities calls for $m(m+1)/2$ markets where bids and offers are aggregated and prices formed. Even casual empiricism indicates that that there are costs associated with the running of institutions. Instead of using $m(m+1)/2$ markets if one commodity is designated as a money and it is in sufficient supply so that all "can pay cash" one needs only m markets. But observation confirms that often even the rich do not have enough cash on hand to meet their immediate payments. The economy needs to make sure that there is enough money and that it can be lent and borrowed, hence some form of credit market is called for. It is easy to show that in general if the economy is to be flexible it must take into account error and uncertainty, but this may lead to states where an individual cannot repay the debt owed. This calls for the invention of bankruptcy and reorganization laws [55], [14], [65].

One can take any basic function such as insurance or controlling the money supply and in a similar manner consider the minimal institutions required for these functions. In essence, there are minimal representations of the basic functions of a financial system including the role of markets as price forming mass decision aggregating and disaggregating devices and information processing mechanisms. These provide the links between financial institutions and basic micro-economic theory.

4.2 Dynamics and a Loosely a Coupled System

The key distinction between a strategic market game and a conventional GE market economy appears in the different stresses on statics and dynamics and is directly displayed in the mathematics. The mathematics of general equilibrium is without equations of motion. It provides existence proofs of equilibrium overwhelmingly in terms of equations and interior solutions. As Koopmans phrased it, it is pre-institutional. In contrast strategic market games, are process models and cannot avoid institutions. The institutions are the **carriers of process** they arise naturally in the definition of the rules of the game. Although one might wish to study equilibrium market prices, the manner of price formation must be made sufficiently explicit that the model could be utilized as a **playable experimental game.** In order to meet this criterion the mathematics must be able to supply how every position in the state space including all boundaries can be obtained regardless of the existence of an equilibrium.

4.3 Fiat Money or Gold?

A comment on "enough money" was made above. history tells us that gold has, for many years played an important role in providing a monetary commodity. Yet as population has grown and the economies grew in size and complexity, the lack of flexibility in the supply of gold, its costs and sources are such as to rule it out as a convenient money for the nation state. With the growth of the state and its role in enforcing commercial, contract, taxation and other laws gold was replaced by fiat money. The key feature in contrasting fiat money and gold is to understand the locus of the supply, the flexibility of removing money from the economy and the key to its maintenance of value. Smith and Shubik [57] provide an analysis contrasting the financial role and flexibility of the gold industry and the role of a government central bank producing fiat money. Even without invoking dynamics the recouping of the expenses of the governmental infrastructure required to support the issue of fiat are sufficient to justify an equilibrium with a fiat money with positive worth.

4.4 Debt, Bankruptcy and Reorganization Accommodate Disequilibrium

A viable dynamic structure must be able to absorb divergence from the position of equilibrium. In a dynamic market economy with money and debt and independent decision-makers it is easy to show that some combination of the functions of bankruptcy, reorganization or renegotiation is a logical necessity to resolve inconsistent outcomes.

Bankruptcy and reorganization provide a means to reconcile claims that cannot be satisfied by the market mechanisms. They provide for a reallocation of assets as part of the rules of the game. Under these rules all real assets and fiat money are conserved and redistributed. Credit is not conserved.[6]

The existence of credit and bankruptcy and reorganization are the key factors in being able to construct an economy as a loosely coupled system.

5 The Change in Paradigm

The general equilibrium system being pre-institutional rightly abstracts away from public goods and government because they are not relevant to the questions it asks and answers. Those questions deal primarily with individually owned easily transferable goods.[7] In such an economy it is implicitly assumed that the government and laws of society are a free good and do not require an explicit treatment.

As noted above a translation of an intrinsically timeless institution-free economic model into a process model requires elementary institutions as **the carriers of process.** The combination of considerations of economy, parsimony, efficiency and control all conspire to create financial control processes to facilitate production and exchange. The legal, enforcement and government monetary mechanisms appear as a public good of sufficient size and power that even without enlarging the economic model to include other public goods such as defense, transportation, health and education the government must be added as an agent of considerable size. Technically the economy together with its control processes need to be modeled as a game with one atomic player and a measure of small players.

5.1 The Paper Trail

As the economy increases in complexity so do the "paper" (or computer cipher) instruments increase and the derivative industry may build pyramids of paper on

[6] Although it is true that real assets are not destroyed, many are highly specialized for their institutions. Many off-balance sheet item are destroyed with the dissolving of a firm. These include items such as reputatuon and the knowledge of routines that are features of the information structure of living institutions.

[7] Primarily the goods are fungible chattels.

paper. But along with complexity, indivisibility, joint ownership, immobility and other physical or societal properties real assets tend to have their counterpart in financial instruments. Finance is to real assets as a matching virtual world. The trade is less in the real assets than in the financial assets representing claims to the ownership of rights with respect to real assets.

When one considers the trade among physical and financial assets and their transformation over time it is reasonable to ask if there are a basic set of fundamental goods and services and financial assets that serve as the building blocks of more complex instruments and also provide us with a transformation matrix over time [49], [53].

5.2 Real and Other Legal Persons

Already in the general equilibrium analysis an implicit distinction has been made between preference maximizing or utility seeking natural persons and profit maximizing firms. The distinction called for is given in the law as between natural and other legal persons including various forms of corporations. A key open question is what are the goals of the corporation? General equilibrium with its extreme abstraction of the firm and complete markets proves that an efficient equilibrium is attained with the firms acting as profit maximizers flowing through all profits to the natural persons who are their ultimate owners [10]. Even with the GE abstraction Arrow [3] heads towards institutions by noting a role for the stockmarket.

Biological considerations have the expected life of the individual as finite; but the law may permit the legal persons it has created to live indefinitely. Thus for many relevant economic questions economic dynamics must deal with the life cycles of both individuals and institutions.

The development of the discipline of finance has been based primarily on individuals and institutions maximizing monetary profits. Much useful analysis has been based on the concept of the profit maximizing institution; but context counts and there are many instances where profit maximization is a poor surrogate for a far more nuanced set of goals by large and small corporations [6], [46], [62]. The size of the firm's bureaucracy and concern for its employees may dominate concern for stockholders.

6 Time and Uncertainty

The initial development of the general equilibrium was based on an ingenious abstraction of both time and uncertainty from the economic structure [10]. Technically this can be done for finite partitions of time and uncertainty by multiplying the dimensions of the state space. For any finite period of time the overall economy may be presented as a game in strategic form.

Although this technique has been of considerable value in answering questions about a world with the availability of complete markets and perfectly informed in-

dividuals applied finance in reaction to observations on the existence of incomplete markets, transaction costs and individuals with asymmetric markets has developed usefully using a partial equilibrium setting introducing explicitly many empirically observed imperfections.

Since the late 1970s a body of literature has sprung up in macroeconomics utilizing dynamic programming methods for producing highly aggregated closed stochastic models of the economy. This has been led primarily by Robert Lucas [27], [28]. More or less at the same time Karatzas, Shubik, Sudderth and others [23], [55], [24] considered mathematically similar models that, however differed in several basic ways. The former were aimed simultaneously at macroeconomic policy and with the aim to put a sound mathematical microeconomic foundation under macroeconomics, in contrast with the ad hoc models of the Keynesians and others; the latter were concerned with building logically completely defined strategic market game dynamic models consistent with general equilibrium that illustrated basic properties of the financial system such as goods, markets, money markets, bankruptcy structures, an elementary central bank and other minimal institutions. Specific instances of these games can be constructed as playable experimental games.

An instance of the type of game being analyzed is given by the value function [23]

$$V(s;p) = \sup_{0\leq b\leq s}\left[u\left(\frac{b}{p}\right) + \beta EV(s-b+pY;p)\right]$$

where the state is described by the initial amount of money s held by an individual and the initial market price p. b is the bid and Y is a random variable. Under more or less standard micreconomic assumptions it is shown that such a game with a fixed amount of money has an equilibrium wealth distribution that maps into itself. We were unable to prove convergence from non-equilibrium positions, however simulations by Miller and Shubik [32] indicated that it is probably true. However when we introduced banking into the model the simulations showed considerable fluctuations.

6.1 Overlapping Generations or Dynasties

Operations research, much of microeconomics and finance deal with phenomena that are more or less local both in space and time, thus it is usually reasonable to leave demographic variables out of consideration. this is not so with those concerned with macroeconomic questions.

The phenomenal example of 114 pages more or less hand computed in the work of Maurice Allais [1] opened up the importance of overlapping generation (OLG) models in macroeconomics.

OLG also removes the controversial "natural discount model" from providing the mathematical convenience of summable infinite series for wealth calculations and linking the "natural rate" of interest in a simple manner with this discount. It adds to highlighting the potential of the money rate of interest as a control variable.

6.2 The Utility of Wealth?

Preference and utility theory have been among the gems of microeconomics [22], [18], in spite of the considerable work many problems on the interfaces among micro-economic theory, macro-economics and finance illustrate the difficulties in producing a unified structure for economic theory. Earning and spending wealth are contextually two highly different occupations. The context of buying goods and services subject to a wealth (or income?) constraint is usually considerably different from the context of the activities of earning a living or maximizing wealth. The individual evidently has a utility for wealth and this is at the heart of many financial models. Since the time of Bernoulli [31], [8] [30] the shape of the utility of monetary wealth has been considered.

The financial theorists implicitly accept that the utility function for money is what the rich are optimizing. If you are a billionaire in mergers and acquisitions you have a utility for money where the money summarizes your chips in the grand game of buying and selling ownership claims to production assets and financial instruments. This is unlike the laborer making $30,000 a year where this income constraint is directly relevant to the choices of payments for food clothing and shelter. The theory of consumer choice does better with those in the range from 0–$150,000 a year where the money is still mor or less "eating money." Much of the income of the rich is utilized as "investing money," blue chips devoted to investing in ownership claims. The study of the consumption function for the rich calls for scholars in marketing, behavioral economics and social psychology.

6.2.1 Fiduciary Behavior?

It is often overlooked in much economic theory and in applying game theory to economics that most economic decisions are made by fiduciaries using **other people's money**.

The law calls for some form of prudent man rule where a trustee is expected to act with due prudence as if the assets were his own, In essence this topic is a miasma of law and custom where the economic theorist enters like Alexander the Great with his sword to cut the Gordian Knot and assume that for some purposes the head of a corporation or a trustee maximizes the ultimate owner's welfare.

The applied game theorist brings Agency Theory and asymmetric information and the institutionalists bring the essay and historical study of the corporation [63].

6.2.2 Utility, Bankruptcy and Default

Bankruptcy has been noted above. Yet another aspect of bankruptcy and default [8] that has to be accounted for in economic theory is how one accounts for bankruptcy

[8]The term bankruptcy is an art form in the law, but economists often utilize bankruptcy and default interchangeably. Depending on the application more or less precision is called for.

penalties and the role that money and other features play. The magnitude of bankruptcy is denominated in money. The penalties associated with bankruptcy involve a mixture of law, custom, social, political and economic factors. The penalties are by no means only economic. But under any utilitarian measure they enter into any utility function. Given that the penalty is related to a monetary loss it is methodologically sound that the value of wealth should be defined, not merely in the positive orthant but also in the negative.

7 Money, Credit, and Conservation

Given that there is a distinction between government money and credit the mathematical economist attempting to model economies containing both a government money and credit must be able to make the distinctions between them. In doing so how these instruments enter and leave the economy must be noted. This amounts to being able to specify the rules of conservation of these instruments and how and when conservation is violated.

A useful way to approach this problem is as that of a naive physicist. Consider that government money consists only of special blue Poker chips. At least in theory, one should be able to trace how and when these chips, or aggregates of chips enter the economy, where they are at any time and how they exit the economy. Tracking the history of a specific dollar bill is not unlike try to track a single particle. One might devise electronic means. In the United States there is a hobbyist group: Where's George?(www:wheresgeorge.com/) devoted to tracking dollar bills. We could consider all the other forms of near-money as different colored chips and try to trace each of them. While logically feasible this is technologically impractical unless one has an important specific question that justifies expensive, limited special data gathering.

Both in theory and fact anyone can create credit and many individuals do, even in the form of going into a store where one is known , having forgotten one's wallet; one may make the purchase against the promise to drop in soon and bring the money.

A question raised in finance has been: Could everyone be a banker, issuing their own notes that are accepted as a means of payment? Fisher Black [7] discussed this and Sahi and Yao [37] and Sorin [59] provided formal game models showing this possibility. Angerer, Huber, Shubik and Sunder [2] provided experimental evidence on its feasibility and discussed the institutional conditions that make this possibility highly unlikely.

In the United States The Federal Reserve has defined three monetary aggregates M1, M2, and M3. M1, has the transaction deposits of banks and cash in circulation. M2 includes savings accounts, small time deposits at banks, and retail money market funds; and M3 includes large time deposits, repurchase agreements, Eurodollars, and institutional money market funds. This gives us 9 types of chips. Tracking the quantities and velocities of all of these would require an 18 dimensional phase space.

Fortunately for some (but by no means all) important macro- and micro-economic questions limiting models to two types of money, government and bank money (M1) is often useful and the statistics are plentiful, while the velocity statistics still are both hard to measure and raise conceptual problems.

8 Where is Schumpeter?

Even with the addition of government and public goods that immediately helped to provide a game theoretic link between micro- and macro-economics a fundamental fact of economic life is still missing in the discussion up to this point. That is the role of innovation and **the breaking the circular flow of money in an innovating economy.** Schumpeter put forth this observation first in his thesis then in his book [41]. Although it over a hundred years since Schumpeter argued that innovation is essentially a disequilibrium phenomenon and that it is critical to capitalistic competition it has defied adequate mathematization. Essentially the reasons for this delay are that even at the highest level of abstraction a model must have a clear description of cash flows and the money supply; how to model innovation and what is operationally meant by breaking the circular flow. At any point of time all resources are given. Thus in a closed system an innovation must involve a reallocation of existing resources . The idea for an innovation and the implementation of the innovation are separate, The genesis of the idea may require relatively few resources, it involves primarily the perception of the existence of a new algorithm or formula for a projected new product and possibly some preliminary development. This may lead to a feasibility investigation followed by the commitment of financing to divert the resources from their current use.

Dynamic programming models of innovation both by Robinson Crusoe in a one person non-monetary economy [52] and by a firm in a monetary economy [53] can be constructed as strategic market games. There is a fundamental contrast in coordination and control. In the former, Crusoe is in control of his own fate and money is irrelevant. In the latter the real good sector of the economy must accommodate with the financial sector. The process involves perception, evaluation, coordination and control.

The basic aspects of breaking the circular flow of money require only a single random event. It appears that with random occurrences every period the type of stochastic increasing returns to scale phenomena indicated by Brian Arthur [4] may be present.

9 What is a Theory?

What is a theory? Different disciplines utilize the word theory differently. Furthermore model and theory appear on occasion to be used interchangeably. Several

suggestions are noted below.

A conventional way to approach this question might be to go to the dictionary for aid, and it is useful; to do so providing that one recognizes the weaknesses of dictionary construction. Another way is to seek currently institutionalized scientific authorities. The National Academy of Sciences of the United States suggests:

> A plausible or scientifically acceptable, well-substantiated explanation of some aspect of the natural world; an organized system of accepted knowledge that applies in a variety of circumstances to explain a specific set of phenomena and predict the characteristics of as yet unobserved phenomena.

It also defines a fact as:

> In science, a "fact" typically refers to an observation, measurement, or other form of evidence that can be expected to occur the same way under similar circumstances. However, scientists also use the term "fact" to refer to a scientific explanation that has been tested and confirmed so many times that there is no longer a compelling reason to keep testing it or looking for additional examples.

Aristotle contrasted theory to "practice." Praxis, the Greek term for "doing," is concerned with application, while; pure theory is not concerned with immediate application. An often used example comes from medicine. Medical research may be concerned with attempting to understand the causes for a disease without being immediately concerned with practise. In contrast good practitioners are more concerned with curing patients of a disease, and if they find a cure but not a deep explanation they are reasonably content. Central bankers may have the same view of the financial system. Unfortunately their level of success if far from those in medicine.

A mathematical view of a theory is deductive. A theory's (possibly full sensory or empirical) content is given by basic axioms and a formal logic develops the theory. The logical consequence of the axioms are presented as theorems.

A semantic view of theories, is as models providing a logical framework connected with some aspect of observation. The are abstractions or simplifications of some aspects of the real world.

In economics there are many subdivisions that tend to intermix theory and practise. Possibly the major rift is between micro- and macro-economics

There are many subdivisions of microeconomics (including the often not recognized applied field of operations research) where practitioners and theorists are highly intermixed.

A distinction often made to sort out the pure theorists as contrasted with those highly tainted with empirical concerns is between those devoted primarily to normative concerns "what should be" as contrasted with those more inclined to positive

economics stressing "what is." Recently in finance and macro-economics the term "engineer" has been used to indicate those involved with problems at hand. This may even include retreaded PhD. physicists or top probability theorists devising complex derivatives or algorithms to take advance of local correlations in time series in stock trading.

Small "purist" areas are the bastion of some game theorists and general equilibrium theorists devoted to exploring classical logical positivist axiomatic and mathematical methods where the axioms that might cause worry to the empiricists are not where the attention is lavished. These include fair division, bargaining and the mathematics of preference theory.

9.1 Let a Thousand Specializations Blossom!

Beyond the major divisions of micro- and macro-economics not only are there many economic theories with adjectives attached such as international economics, welfare economics, labor economics, health economics and so forth; there are also divisions such as behavioral economics where the assumptions of economy, including individual optimization and the standard models of utilitarian economic agents are replaced. Experimental gaming such as that of Gode and Sunder [21] has indicated that the double auction market is reasonably efficient even when operated by agents with limited intelligence. Vernon Smith has shown the worth of studies in equilibrium [58], and Plott [35] and Roth [36] have dealt with institution design and the economist as an engineer.

The main thrust of macroeconomics is clearly operational. It deals with the dynamics of the whole economy encompassing features such as inflation, economic cycles and growth, unemployment, and monetary and fiscal policy. An honorable employment for the macroeconomist is to give operational quantitative and qualitative advice to governments.

The political economists, economic historians and historians of economic thought still provide broad insights utilizing the essay form as their way to deal with the imponderables.

Especially in application the closely related disciplines of finance, accounting, and law intersect with many of economic specializations.. The disciplines of sociology social psychology and psychology serve to challenge the axioms underlying many economic models. And recently the disciplines of Physics, Ecology and Biology have been considered as potential contributors to economic understanding.

9.2 On Knowing Your Business

As anyone who has tried to deal with microeconomic dynamics of any variety knows, one is buried in a multitude of plausible choices in modeling. These reflect nonsymmetric information, set up costs, time lags, unit size and a myriad of other micro-

economic or other details that may appear to be sufficiently relevant to the question at hand that they cannot be left out. Taking this into account one could despair that there is no meaningful theory of economic process. All items appear to fall under Edgeworth's prescient remark at his inaugural address. He commented:

> It is worth while to consider why the path of applied economics is so slippery; and how it is possible to combine an enthusiastic admiration of theory with the coldest hesitation in practice. The explanation may be partially given in the words of a distinguished logician who has well and quaintly said, that if a malign spirit sought to annihilate to whole fabric of useful knowledge with the least effort and change, it would by no means be necessary that he should abrogate the laws of nature. The links of the chain of causation need not be corroded. Like effects shall still follow like causes; only like causes shall no longer occur in collocation. Every case is to be singular; every species, like the fabled Phoenix, to be unique. Now most of our practical problems have this character of singularity; every burning question is a Phoenix in the sense of being sui generis.
>
> — F.Y. Edgeworth, 1891 [16].

One might regard Edgeworth's comments as an observation of extreme pessimism however it is suggested here that it contains a basic nub of truth when directed at application. When dealing with application there is no substitute for knowing your business. The basic reason why more or less applied economics is split into so many fields is that in application, in each of these specializations relevant details must be added.

10 Why Unify Macro- and Micro-economics?

A good theory is capable of bringing out the commonality that underlies the many models that can be grouped under that theory. The nature of the questions to be asked determines much of the detail of the model constructed to answer the question at hand. The questions at hand may be aimed at problems requiring direct immediately applied advice in running an economy or they may be devoted purely to theory; asking of the postulated theory questions such as "is it logically possible that under the basic axioms of this theory could an equilibrium exist with a constant flow of bankruptcies?"

Utilizing strategic market games and experimental gaming together with consideration of the methods of physics concerning scaling, conservation and dimensional analysis we [51] have endeavored to construct a methodology aimed at building or joining models and theories in the broad subject of political economy with its many branches and ramifications. As matters stand today there is a broad collection of

specialized theories many of which share at least the common assumptions of the presence of economic agents.

For most questions asked of the many sub-theories and models that abound there is no need to unify the theories utilized in the sub-disciplines. The sub-disciplines supply the special substructure and detail needed to answer pertinent questions.

There are however, questions that arise the may fall between two or among several of the subdivisions in political economy. For these one has to stitch together essentially diverse bodies of work and to try to reconcile different gestalts.

My concrete concern has been with a viable theory of money that involves at least three large sub-disciplines of political economy they are general equilibrium microeconomics; macroeconomics and finance; as well as three allied disciplines game theory, econophysics and accounting and less directly economic history; oligopoly theory and experimental gaming. There are many questions in trying to construct a viable theory of money and financial institutions that cannot be answered adequately without taking into account inputs from these diverse sources. A good example is the old chestnut going back to Adam Smith, the doctrine of "Bills only" that has been a source of controversy for nearly 250 years with verbal comments and debates by many individuals such as Bagehot [5] and more recently Lucas [28] and Sargent [38]. This topic cannot be resolved without paying attention to the specific details required to construct a playable experimental game with strict care on the different time scales assumed for the interactions of all parties and instruments as well as the rules preserving or breaking conservation; and the common knowledge and information conditions assumed.

11 From Statics to Dynamics

In spite of the many successful applications of mathematics to equilibrium economics, the development of economics as a science has a considerable way to go. In particular, as is well known, there is no generally acceptable theory of dynamics. Yet the whole basis of macroeconomics requires economic dynamics.

In the broad sweep of the development of both pure and applied economics, techniques come and go and fashions change. It has been fashionable for microeconomic theorists to view macroeconomics as ad hoc and "unscientific," while macroeconomists return the compliment by branding topics such as general equilibrium (GE) theory as irrelevant.

GE theory provided great and elegant insights into the potential role of price in providing optimal allocations under the appropriate conditions. But it did not solve the key problem as to how coordination was to come about. That was relegated to the unspecified dynamics, and led to a clash between those who advocated competitive price formation and those who conceived of a centralized agency announcing prices. This clash could not be resolved by a mathematics that did not deal explicitly with

price formation.[9]

The strategic market game construction adds onto the GE a building kit to develop economic models that meet the strict standards of strategic and extensive form game theory while utilizing the physical considerations of scaling, dimension checking, conservation, closure and consistency to help make sure that the verbal description of the institutions and instruments included and their mathematical representations are consistent and complete. It provides a half way house between GE and dynamics inasmuch as the process model can be solved for their equilibria rather than their full dynamics

Items such as the definition of the periodically taxable profit of the firm in a dynamic incomplete markets economy do not coincide with the formally correct but sterile definitions of long term profit. In any model that tries to reconcile theory and practise the accounting and the model must be reconciled.

The prolific work of Lucas [28] and his colleagues, has, in part been offered as a formal critique of ad hoc macroeconomics where the careful dynamic programming methods he employs are more consistent with a correctly closed general dynamic equilibrium model. Unfortunately there are many problems with the representative agent and as the Lucas critique appears to not deal in mere theory but also is aimed at commenting on policy it appears to be woefully constrained by the representative agent and the curse of having to utilize few dimensions. Even with just one commodity and a money, the individual agent model yields a wealth distribution [23].

The Lucas critique is rightly immediately concerned with linking theory and its empirical validation. Rather than fitting macroeconomic data from a high dimensional world, a different empirical approach is to be able to formulate the dynamic models as playable games and test them in the gaming laboratory. This is the approach suggested here. Experience to date indicates that experimental subjects may conform to predicted behavior when the dimensions are low and many well defined counterfactual but logically coherent rules or laws are imposed on the game such as the existence of perfect enforcement of contract and instantaneous cost free clearing of accounts (see: [2], on everyone a banker).

12 Mathematical Institutional Economics: A Reprise

When I first coined the phrase "Mathematical Institutional Economics" I had only a vague idea of why I named it such. I knew that von Neumann and Morgenstern [61] had warned that it was not clear as to what form economic dynamics would take. It took me many years to appreciate the fact that the dynamics of a nation state or world could not be studied fruitfully without considering the monetary and financial control system that was in place to control and coordinate the system

[9] Although the debate on Marshallian versus Walrasian stability considering aggregated supply and demand functions could be regarded as an attempt to attack this problem.

The opening up of general equilibrium meant that the economic theorist is overwhelmed with a myriad of ways to complete and well define any dynamic model rising from being required to attach equations of motion to the **static mathematical abstraction.** It has to be clothed with some set of **institutions that are the carriers of process.** In picking the institutions the economic modeler has to face up to two problems. They are the empirical validity of the assumptions and the desire for abstraction in order to help analysis. This leads to the concept of a **minimal institution**,the selection of which requires both institutional knowledge and the ability to abstract.

The proliferation of alternative sets of sufficiently good institutions in some contexts signals the approach of economics to ecology and biology. Especially when one considers innovation, as noted in Section 7. above.

It is probably premature to speculate how the inevitable development of connections between economics, ecology and biology will develop. It is already fairly clear that the stress on Pareto optimal is disappearing to be replaced by concepts of viability within a context.

Two types of experimental and operational gaming are important for the current work and work ahead. Rigid form experimental gaming forces the formal economic modeler to "beta -test" the models in order to check for closure and consistency. Free-form operational gaming as is used in military war games offers the opportunity to better explore the softer sciences interlinkage to economic model building [9] [11] in these games area experts have the opportunity in midgame to halt the game to debate and challenge the axioms and context of the game. For, example this approach could be applied to economic policy. One could have an institute devoted to operational gaming of new tax laws. This is probably not politically feasible because there may be political opposition to having most of the hidden loopholes removed.

As an admirer of formal theories with clean axioms, interesting theorems and proofs and concerned with invariant properties I suggest that what the approach suggested melding the modeling of context and institutions with analysis takes needed steps towards a mathematical institutional economics suitable to provide better insight in understanding the financial control of the overall economy in an evolutionary sociopolitical environment in which all economies must function.

References

[1] Allais, M. 1947. *Économie et intérêt.* Paris: Imprimerie Nationale.

[2] Angerer, Martin Huber, J., M. Shubik and S. Sunder, 2010. "An economy with personal currency: Theory and experimental evidence." *Annals of Finance* 6(4): 475–509.

[3] Arrow, K. J. 1964. "The role of securities in the optimal allocation of risk bearing," *Review of Economic Studies* 31: 91–96.

[4] Arthur, W. B. 1994. *Increasing Returns and Path Dependence in the Economy,* Ann Arbor, University of Michigan Press.

[5] Bagehot, W. [1873] 1962. *Lombard Street.* Reprint, with a new introduction by Frank C. Genovese. Homewood, IL.

[6] Baumol, W. J. 1959. *Business Behavior, Value and Growth.* New York: Macmillan.

[7] Black, F. 1970. "Banking and Interest rates in a World without Money," *Journal of Banking Research* 1, 8–20.

[8] Bernouilli, D. 1954. "Exposition of a new theory on the measurement of risk" (translated from the Latin original 1738) *Econometrica* 22: 23–36.

[9] Brewer G and M. Shubik 1979. *The War Game.* Cambridge: Harvard University Press.

[10] Debreu, G. 1959. *Theory of Value.* New York: Wiley

[11] DeLeon, P. 1973. "Scenario design: An overview," RAND R-1218-ARPA

[12] Diamond D., and P. Dybvig P 1983. "Bank runs, deposit insurance, and liquidity," *Journal of Political Economy* 91(3): 401–419.

[13] Dubey P. 1982. "Price-quantity strategic market games," *Econometrica* 50.

[14] Dubey, P., J.Geanakoplos and M. Shubik J. 2005. "Default and punishment in general equilibrium," *Econometrica* 73(1): 1-37.

[15] Dubey, P., and M. Shubik. 1978. "The noncooperative equilibria of a closed trading economy with market supply and bidding strategies," *Journal of Economic Theory* 17(1):1–20.

[16] Edgeworth, F. Y. 1891. "An introductory lecture on political economy," *Economic Journal* 1(4): 625–634.

[17] Farmer, J. D., L. Gillemot, G. Iori, S. Krishnamurthy, D. E. Smith, and M. G. Daniels. 2005. "A random order placement model of price formation in the continuous double auction," in L. Blume and S. Durlauf, eds., *The Economy as an Evolving Complex System, III*. New York: Oxford University Press, pp. 133–173.

[18] Fishburn, P. C. 1970. *Utility Theory for Decisionmaking.* New York: Wiley

[19] Friedman, D., and J. Rust, eds. 1993. "The double auction market," *Institutions, Theories, and Evidence: Proceedings of the Workshop on Double Auction Markets*, Held June, 1991 in Santa Fe, New Mexico.

[20] Fudenberg, D., and D.K. Levine. 1998. *The Theory of Learning in Games.* MIT Press: Cambridge.

[21] Gode, D. K., and S. Sunder. 1993. 1"Allocative efficiency of markets with zero intelligence traders: Market as a partial substitute for individual rationality," *Journal of Political Economy* 101(1): 119–137.

[22] Hicks, J. R. [1939] 1946. *Value and Capital*, 2nd ed. Oxford: Clarendon.

[23] Karatzas, I., M. Shubik, and W. Sudderth. 1994. "Construction of stationary Markov equilibria in a strategic market game," *Journal of Mathematical Operations Research* 19(4): 975–1006.

[24] Karatzas, I., M. Shubik and W. Sudderth. 2002. "A stochastic overlapping generations economy with inheritance," *Journal of Economics* 77(3): 207–240.

[25] Kiyotaki, N., and R. Wright. 1989. "On money as a medium of exchange," *Journal of Political Economy* 97: 927–993.

[26] Leijonhufvud, A. 1984. "Hicks on time and money," in D. Collard et al., eds., *Economic Theory and Hicksian Themes.* Oxford: Oxford University Press, pp. 26–46.

[27] Lucas, R. E. 1980. "Equilibrium in a pure currency economy," *Economic Economic Inquiry* 18(2): 203–220.

[28] Lucas, R. E. 1996. "Nobel lecture: Monetary neutrality," *Journal of Political Economy* 104 (4): 661–682.

[29] Mailath G., and L. Sameulson, 2006. *Repeated Games and Reputation.* Oxford University Press: Oxford.

[30] Markowitz, H. M. 1952. "The utility of wealth," *Journal of Political Economy* 60(2): 151–158.

[31] Menger K. 1967. "The role of uncertainty in economics" (translated from the German 1934), in M. Shubik, ed., *Essays in Mathematical Economics in Honor of Oskar Morgenstern.* Princeton: Princeton University Press.

[32] Miller, J., and M. Shubik. 1994. "Some dynamics of a strategic market hame with a large number of agents," *Journal of Economics*, 60(1): 1–28.

[33] Morris, S., and H. S. Shin. 1998. "Unique equilibrium in a model of self-fulfilling attacks," *American Economic Review* 88: 587-597;

[34] Nash, J. F. 1951. "Noncooperative games," *Annals of Mathematics* 54 (2): 286–295.

[35] Plott, C. R. 2001. *Public Economics, Political Processes and PolicyApplications. Collected Papers on the Experimental Foundations of Economics and Political Science*, Vol. 1. Cheltenham, UK: Edward Elgar Publishing.

[36] Roth, A. E. 2001. "The economist as engineer: Game theory, experimentation, and computation as tools for design economics," Fisher-Schultz Lecture, *Econometrica*, (July) 70(4): 1341-01378.

[37] Sahi, S., and S. Yao. 1989. "The noncooperative equilibria of a trading economy with complete markets and consistent prices," *Journal of Mathematical Economics* 18 (4): 325–346.

[38] Sargent, T. J. 2001. "Where to draw lines: Stability versus efficiency," *Economica* 78(6): 197–214.

[39] Scarf, H. E. 1973. *The Computation of Economic Equilibria* (with Terje Hansen). New Haven: Yale University Press.

[40] Schmeidler, D. 1969. "The nucleolus of a characteristic function game," *SIAM Journal of Applied Mathematics* 17: 1163-1170.

[41] Schumpeter, J. A. 1934. *The Theory of Economic Development.* Cambridge: Harvard University Press.(1911 in German)

[42] Shapley, L. S. 1952. "Notes on the n-person game, III: Some variants of the von Neumann-Morgenstern definition of solution," RAND Publication RM-817.

[43] Shapley, L. S. (1953). "A value for n-person games," in A. W. Tucker and R. D. Luce, eds., *Contributions to the Theory of Games II.* Princeton: Princeton University Press, pp. 307–317.

[44] Shapley, L. S., and M. Shubik. 1977. "Trade using one commodity as a means of payment," *Journal of Political Economy* 85 (5).

[45] Shubik, M. 1959. *Strategy and Market Structure*. Wiley: New York.

[46] Shubik, M. 1961. "Objective functions and models of corporate optimization," *Quarterly Journal of Economics* 75 (3): 345–375.

[47] Shubik, M. ed. 1967. *Essays in Mathematical Economics in Honor of Oskar Morgenstern*. Princeton: Princeton University Press.

[48] Shubik, M. 1972. "A theory of money and financial institutions: Fiat money and noncooperative equilibrium in a closed economy," *International Journal of Game Theory* 1(4): 243–268.

[49] Shubik, M. 1975. "On the eight basic units of a dynamic economy controlled by financial institutions," *Review of Income and Wealth* 21(2): 183–120.

[50] Shubik, M. 1999. *The Theory of Money and Financial Institutions, Volume 1.* Cambridge: MIT Press.

[51] Shubik, M., and D. E. Smith. 2013. "The control of a monetary economy," Unpublished manuscript in process.

[52] Shubik, M., and W. Sudderth. 2011, "Cost Innovation: Schumpeter and Equilibrium. Part 1. Robinson Crusoe," CFDP 1786, Cowles Foundation, New Haven, CT.

[53] Shubik, M, and W. Sudderth. 2011, "Cost innovation: Schumpeter and equilibrium. Part 2, Innovation and the money Supply," in process.

[54] Shubik, M., and G. Thompson. 1959. "Games of economic survival," *Naval Logistics Research Quarterly* 6: 111–123.

[55] Shubik, M., and W. Whitt. 1973. "Fiat money in an economy with one nondurable good and no credit (A noncooperative sequential game)," in A. Blaquiere, ed., *Topics in differential games*. Amsterdam: North- Holland, pp. 401-448.

[56] Shubik, M., and C. Wilson. 1977. "The optimal bankruptcy rule in a trading economy using fiat money," *Zeitschrift für Nationalökonomie* 37(3–4): 333–354.

[57] Smith, D. E., and M. Shubik. 2011. "Endogenizing the provision of money: Costs of commodity and fiat monies in relation to the value of trade," *Journal of Mathematical Economics* 47: 508–530.

[58] Smith, V. 1979. *Research in Experimental Economics*, Vol. 1. Greenwich, CT: JAI Press, p. 49.

[59] Sorin, S. 1996. "Strategic market games with exchange rates," *Journal of Economic Theory* 68(2): 431–446.

[60] Tirole, J. 2005. *The Theory of Corporate Control.* Princeton: Princeton Universaity Press.

[61] von Neumann, J., and O. Morgenstern. 1944. *Theory of Games and Economic Behavior*. Princeton: Princeton University Press.

[62] Williamson, O. E. 1975. *Markets and hierarchies: Analysis and Antitrust Implications*. New York: Free Press

[63] Williamson, O. E. 2002. "The theory of the firm as governance structure: From choice to contract," *Journal of Economic Perspectives* 16(3): 171–195.

[64] Woodford, M. 2003. *Interest and Prices*. Princeton: Princeton University Press

[65] Zame, W. R. 1993. "Efficiency and the role of default when security markets are incomplete," *American Economic Review* 83: 1142–1164.