DECOUPLING MARKETS AND INDIVIDUALS: RATIONAL EXPECTATIONS EQUILIBRIUM OUTCOMES FROM INFORMATION DISSEMINATION AMONG BOUNDEDLY-RATIONAL TRADERS

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Abstract

Attainment of rational expectations equilibria in asset markets calls for the price system to disseminate traders' private information to others. It is known that markets populated by asymmetrically-informed profit-motivated human traders can converge to rational expectations equilibria. This paper reports comparable market outcomes when human traders are replaced by boundedly-rational algorithmic agents who use a simple means-end heuristic. These algorithmic agents lack the capability to optimize; yet outcomes of markets populated by them converge near the equilibrium derived from optimization assumptions. These findings suggest that market structure is an important determinant of efficient aggregate level outcomes, and that care is necessary not to overstate the importance of human cognition and conscious optimization in such contexts

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Our knowledge of the very narrow limits of human rationality must dispose us to doubt that business firms, investors or consumers possess either the knowledge or computational ability that would be required to carry out the rational expectations strategy.

Herbert Simon (1969)

The claim that the market can be trusted to correct the effect of individual irrationalities cannot be made without supporting evidence, and the burden of specifying a plausible corrective mechanism should rest on those who make this claim.

Tversky and Kahneman, 1986, p. S275).

The principal findings of experimental economics are that impersonal exchange in markets converges in repeated interaction to the equilibrium states implied by economic theory, under information conditions far weaker than specified in the theory.

Vernon Smith (2008)

1. Introduction

A central feature of economic theory is derivation of equilibrium in economies populated by agents who maximize some well-ordered function such as profit or utility. Although it is recognized that actions of economic agents are subject to institutional constraints and feedback (North 1990), exploration of the extent to which equilibrium arises from characteristics of the institutional environment, as opposed to the behavior of individuals, has been limited; Becker (1962) being a notable exception. The normal modeling technique is to ascribe sophisticated computational abilities to a representative agent to solve for equilibrium (Muth 1961).

In Sciences of the Artificial, Simon (1969, Chapter 3) questioned the plausibility of human agents, with their limited cognitive abilities, forming rational expectations by intuition.

Accumulated observational evidence on these cognitive limits of individuals shifted the burden of proof, and led to calls for evidence that markets can overcome such behavioral limitations (Tversky and Kahneman 1986 quoted above; Thaler 1986).

Laboratory studies of markets populated by asymmetrically-informed profit-motivated human subjects have revealed that their aggregate level outcomes tend to converge near the predictions of rational expectations theory (Forsythe, Palfrey and Plott 1982; Plott and Sunder 1982—henceforth PS, 1988; and Forsythe and Lundholm 1990). However, since complex patterns of human behavior can only be inferred from actions, not observed directly, it is difficult to know from such experiments which elements of trader behavior and faculties are necessary or sufficient for various kinds of markets to attain their theoretical equilibria. This difficulty has led some to claim that inability of human beings to optimize by intuition implies that economic theories based on optimization assumptions are prima facie invalid (Tversky and Kahneman 1986).

Such doubts about the achievability of mathematically derived equilibria, when individual agents are not able to perform complex optimization calculations, are understandable. From a constructivist point of view (Smith 2008), rational expectations equilibria place heavy demands on individual cognition to learn others' preferences or strategies, and to arrive at unbiased estimates of underlying parameters of the economy by observing market variables. In theory, disseminating and detecting information in markets calls for bootstrapping—rational assessments are necessary to arrive in equilibrium and such assessments require observation of equilibrium outcomes. Cognitive and computational demands on individuals to arrive at economic equilibria, especially rational expectations equilibria, are quite high, generating questions about the plausibility of equilibrium models (Simon 1969).

Replacing humans by simple algorithms can allow us to decompose the complexity of trader behavior into simpler elements, and establish causal links between specific characteristics of trader behavior and market outcomes. Using the Gode and Sunder (1993) approach, we find and report that in markets with uncertainty and asymmetric information, simple zero-intelligence adaptive algorithmic traders are able to attain outcomes approximating rational expectations equilibria. Since the statistical distribution of these outcomes is centered near the PS observations of markets with human traders, the convergence of their outcomes to equilibrium can be attributed to the minimal levels of intelligence with which the algorithms are endowed. Since this level of intelligence is far less than what is assumed in deriving equilibria, it is reasonable to infer that the convergence of markets to rational expectations equilibria emerge mainly from the properties of the market and simple and plausible decision heuristics, rather than from complex and sophisticated optimization (Becker 1962, Gigerenzer et al., 1999, and Smith 2008).

1.1 Background

Economic theory is commonly understood to require individual agents to have sophisticated information processing capabilities and maximization objectives. However an alternative conceptualization of how equilibria are attained is that market structure generates constraints which guide human behavior without making extensive computational demands on individual problem solving. This conceptualization of structure builds on the work of Becker (1962), Smith (1962) and Gode and Sunder (1993). Becker (1962) showed that the downward slope of demand and the upward slope of supply functions arise from individuals having to act within their budget constraints, even if they choose randomly from their opportunity sets. Smith

(1962) reported that classroom double auction markets populated by a mere handful of profit-motivated student traders with minimal information arrive in close proximity of Walrasian equilibrium. Moreover, Smith's auction markets had little resemblance to the tâtonnement story used to motivate theoretical derivations.

Gode and Sunder (1993) put Becker's constrained random choice together with Smith's double auctions and reported the results of computer simulations of simple double auctions populated by ZI (for "zero intelligence") algorithmic traders who bid or ask randomly within their budget constraints (i.e., buyers do not bid above their private values and sellers do not ask below their private costs). Although these traders do not remember, optimize, seek higher profits, or learn, simulated markets populated by such traders also reach the proximity of their theoretical equilibria, especially in their allocative efficiency. In simple double auctions without uncertainty or information asymmetry, theoretical equilibria are attainable with individuals endowed with only minimal levels of intelligence (not trading at a loss). Jamal and Sunder (1996) extended the results to markets with shared uncertainty with algorithmic agents using means-end heuristic (henceforth M-E,) developed by Newell and Simon (1972).

PS found that markets with uncertainty and asymmetrically distributed information (with two or three states of the world) disseminate information and converge near rational expectations equilibria when populated with profit-motivated human traders. The present paper examines whether the PS results can also be achieved without profit maximization on the part of traders. Do minimally intelligent traders using the M-E heuristic also yield market outcomes predicted by rational expectations?

Substitution of human subjects of traditional laboratory markets by algorithmic agents using M-E heuristic has the advantage of helping us gain precise control of traders' information processing and decision making (i.e., “cognitive”) abilities and process. This control allows us to hold trader “cognition” constant at a specified level and explore the outcome properties of market structures and environment. In contrast, we can neither observe nor hold invariant the cognitive processes used by human traders (also, see Angerer et al. 2011; and Huber et al. 2010, 2011). Moreover, use of algorithmic traders enables us to run longer computational experiments, randomize parameters in the experimental setting, and conduct replications without significant additional cost in time or money.

The paper is organized in four sections. The second section describes a simple M-E heuristic used by minimally-intelligent algorithmic traders in a double auction market. In the third section, we implement this heuristic in a market where some traders have perfect insider information (while others have no information) and compare the simulation results with data from the profit-motivated human experiments reported by PS. The fourth section presents the implications of the findings and some concluding remarks.

2. Means-End Heuristic

Simon (1955) proposed bounded rationality as a process model to understand and explain how humans, with their limited knowledge and computational capacity behave in complex settings. He postulated that humans develop and use simple heuristics to seek and attain merely satisfactory, not optimal, outcomes. To understand human problem-solving Newell, Shaw and Simon (1957) developed General Problem Solver (GPS). Newell and Simon (1972) adduced a large body of data which show that GPS is a robust model of human problem-solving in a wide variety of task environments. The key heuristic used by GPS is means-ends analysis (M-E or the heuristic of reducing differences). Gigerenzer et al. (1999) have focused on the usefulness and effectiveness of fast and frugal heuristics like M-E in human life, whereas Tversky and

Kahneman (1974) have documented a similar heuristic which they relabeled as "anchor and adjust." GPS recognizes knowledge states, differences between knowledge states, operators, goals, sub-goals and problem solving heuristics as entities. GPS starts with an initial (or current) knowledge state, and a goal or desired knowledge state. GPS then selects and applies operators that reduce the difference between the current state and the goal state. The M-E heuristic for carrying out this procedure can be summarized in four steps: (i) compare the current knowledge state a with a goal state b to identify difference d between them; (ii) find an operator o that will reduce the difference d in the next step; (iii) apply the operator o to the current knowledge state a to produce a new current knowledge state a* that is closer to b than a ; and (iv) repeat this process until the current knowledge state a* is acceptably close to the goal state b. Knowledge states of traders can be represented as aspiration levels (Simon 1956) that adjust in response to experience. The M-E heuristic for a trader thus requires a mechanism for setting an initial aspiration level, and a method for adjusting these levels in light of experience (e.g., Jamal and Sunder 1996).

2.1 Market Environment

Market environment is defined by four elements: (1) uncertainty, (2) distribution of information, (3) security payoffs, and (4) rules of the market. Following PS we examine markets with either two (X and Y) or three (X, Y, and Z) states of the world, where each state si occurs with a known probability πi. One half of the traders in the markets are informed about the realized state of the world before the trading starts each period, while the other half are uninformed. At the beginning of each period, each trader is endowed with two identical securities which pay a single state contingent dividend DXj at the end of the trading period. There are three types of traders and each trader type gets a different dividend in a given state (e.g., state x - see Table 1). The rules of the double auction are as follows: after a bid or ask is generated (see section 2.3 for details on bid/ask generation), the highest bid price is compared to the lowest ask price. If the bid price is equal to or greater than the ask price a trade occurs. The recorded transaction price is set to be equal to the midpoint between the bid and ask prices.

Insert Table 1 about Here

2.2 Implementing M-E Heuristic

We implement the M-E heuristic in two steps. First, each agent's initial knowledge state (aspiration level) is set equal to the expected value of the payoff based on its private information. The second step implements the idea that subjects without perfect information make gradual adjustments by applying weight γ ($0 \leq \gamma \leq 1$) to newest observed price P_t, and weight ($1- \gamma$) to the past Current Aspiration Level (CAL_t). This process can be represented as a first order adaptive process:

$$CAL_{t+1} = (1 - \gamma)\ CAL_t \ + \gamma\ P_t. \qquad (1)$$

If CAL_0 is the initial value of CAL_t, by substitution,

$$CAL_{t+1} = (1 - \gamma)^{t+1}\ CAL_0 \ + \gamma\ ((1 - \gamma)^t P_1 + (1 - \gamma)^{t-1}\ P_2 + \ldots + (1 - \gamma)\ P_{t-1} + P_t). \qquad (2)$$

In the context of double auction markets, these two elements of the M-E heuristic—setting an initial aspiration level and gradually adapting it in light of observed transaction prices—can be interpreted in appropriate ways. We describe the structure of each market, the implementation of the heuristic in that market, followed by an examination of the simulation outcomes, and a comparison of these outcomes with the previously reported results obtained in laboratory experiments with profit-motivated human subjects.

2.3 Minimally Intelligent Algorithmic Agents

Algorithmic agents deployed in these markets use an M-E heuristic to estimate a "current aspiration level" (*CAL*), and use the *CAL* to implement a Zero-Intelligence (ZI) strategy after Gode and Sunder (1993) consisting of bidding randomly below and asking above their aspiration levels. Traders draw a uniformly distributed random number between 0 and an upper limit of 1. If number drawn is less than or equal to 0.5, the trader will generate a bid. If the number drawn is greater than 0.5, an ask will be generated. If the action is a bid, then the amount of the bid is determined by drawing a second randomly generated number between a lower bound of 0 and an upper bound of the individual trader's *CAL*. This bid is then compared to the highest bid that currently exists in the market. If the new bid is higher than the existing highest bid, the new bid becomes the new highest bid in the market. Correspondingly, if the action is an ask, then the amount of the ask is determined by generating a second random number in the range having a lower bound of the traders *CAL* and an upper bound of 1. This newly generated ask is then compared to the existing lowest ask in the market. If the new ask is less than the existing ask, then the new ask becomes the new lowest ask in the market. Bids and asks are generated randomly, distributed independently, identically, and uniformly over these ranges (see Figure 1). These algorithmic agents are myopic, making no attempt to anticipate, backward induct, or theorize about the behavior of other traders. They simply use the knowledge of observable past market events (transaction prices) to estimate their opportunity sets, and choose randomly from these sets.

Insert figure 1 about here

These markets are populated in equal numbers by traders of each payoff type who are, and are not, informed about the realized state of world. As shown in Figure 2, the informed algorithmic traders begin by setting their initial *CAL* using the perfect signal they have about the realized state of the world for any given trader type j:

$$\text{If realized state} = X,\ CAL_X = D_{Xj}$$

$$\text{If realized state} = Y,\ CAL_Y = D_{Yj} \qquad (3)^1$$

The uninformed traders of type j use their unconditional expected dividend value to set their initial *CAL* using the prior state probabilities:

$$CAL_j = \Pr(X) * (D_{Xj}) + \Pr(Y) * (D_{Yj}) \qquad (4)^2$$

Since they know the state with certainty, informed traders do not update their *CAL*s in response to observed transactions; they learn nothing about the state of the world from transaction prices.[3] Uninformed traders of every dividend type, however, update their *CALs* after each transaction using the M-E heuristic (i.e., first order adaptive process):

$$CAL_{t+1} = (1-\gamma)(CAL_t) + (\gamma P_t), \qquad (5)$$

with a given randomly chosen value of the adaptive parameter γ for the simulation (see Section 2.4 below). Submission of bids and asks continues with the updated *CALs* serving as constraints on the opportunity sets of traders until the next transaction occurs, and this process is repeated for 10,000 cycles to the end of the period. At the end of each period the realized state is revealed to all traders, dividends are paid to their accounts, and each trader's security endowment is refreshed for the following period. The uninformed algorithmic traders carry their end-of-period *CAL* forward and use it in the following period.[4]

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[1] For 3-state markets, if realized state = Z, $CAL_Y = D_{Zj}$.

[2] For 3-state markets, $CAL_j = \Pr(X)*(D_{Xj}) + \Pr(Y)*(D_{Yj}) + \Pr(X)*(D_{Zj})$.

[3] The informed traders could, for example, learn that in some states market prices are higher than their own dividend in that state, and thus raised their *CAL* to that higher level. Human traders, presumably, make this adjustment but our algorithmic traders do not. We should not, therefore, expect the markets with these minimally-intelligent agents to behave identically to the human markets.

[4] At this stage, it would have been possible for the agents to keep track of the prices associated with each realized state and use this information in subsequent periods. In the spirit of minimal intelligence, our agents do not do so,

In the following period, informed traders again get a perfect signal about the state and set their $CAL = D_{Xj}$ (or D_{Yj}) depending on whether the signal received is *X or Y*. The uninformed traders use their end-of-period *CAL* from the preceding period as CAL_0 to trade and to generate CAL_1 after the first transaction, and so on. This process is illustrated in Figure 2.

Insert Figure 2 about here

2.4 Experimental Design

We use the market design parameters from the PS human experiments for our simulations (see Table 1). We ran 50 replications of four markets numbered 2, 3, 4 and 5 as reported by PS's human experiments (three states in Market 5, and two in the other three markets).[5] The participants were freshly endowed with two securities every period. For each of the 50 replications, the adjustment parameter γ was randomly and independently drawn from a uniform distribution $U(0.05, 0.5)$. In each market, there are 12 traders who traded single period securities. A random state of nature—*X*, *Y*, (or *Z* in case of 3-states)—was drawn at the start of each period to match the actual realizations observed in the PS markets. Except for a few initial periods (when no trader was informed), and in some final periods (when all traders were informed), six of these twelve traders had perfect inside information and the other six were uninformed. For consistency and ease of reference we identify these markets using the same numbers as used by PS.[6]

and uninformed agents simply carry forward their *CAL* from the end of one period to the beginning of the next period. The *CAL* of informed agents responds to a perfect signal about the state realized in each period and is not dependent on experience in previous periods.

[5] PS found that the information structure of their Market 1 was too complex for it to reach rational expectations equilibrium in less than a dozen periods. Accordingly, we have not tried to replicate that market in these simulations.

[6] In this paper we only report periods where one half of the traders in the market are informed and the other half are uninformed. We have also simulated periods where all traders were informed, or all were uninformed. The results are not qualitatively different from human participants reported in PS. Full simulation results, including all periods with informed/uninformed traders are available at http://www.zitraders.com. This website also gives an outline of

In addition to the design parameters, Table 1 also shows the respective equilibrium predictions of prices and allocations of the four markets. These markets were designed by PS such that the price predictions of both the rational expectations and the Walrasian or prior information (PI) equilibria in the high price states (state X in Markets 2, 3, and 4; states X and Y in Market 5) are identical. However, in the low-price state (Y in Markets 2, 3, and 4; Z in Market 5), the price predictions of the two equilibrium models are distinct. Also, as shown in Table 1, the allocation predictions overlap partially in the high-price state(s) but are distinct in the low price state. These differences in equilibrium predictions allowed PS to conclude that behavior of their human participants was better explained by the RE equilibria than the competing PI model.

3. Experimental Results – Markets with Asymmetric Insider Information

Figure 3, Panel A shows the time chart of prices observed in five asymmetric information periods of a market populated with profit-motivated human traders reported in PS against the background of rational expectations (RE - solid horizontal line) and Walrasian (PI - broken horizontal line) predictions for respective periods . Panel B shows a time chart of prices from 50 replications and their median (in solid black line) of the same market with M-E heuristic algorithmic traders. The adaptive parameter γ is randomly and independently drawn each period from a uniform distribution $U(0.05, 0.5)$ and the same value is applicable to all traders. Six of the twelve traders have perfect inside information and the other six are uninformed. Allocative efficiency and trading volume is shown numerically for each period in Panel A of Table 2.

Panels A and B in Figure 3 indicate: (1) In state X (with low RE price of 0.24), transaction prices of both human traders (Panel A) and algorithmic traders (Panel B) approach the RE equilibrium level from above. (2) In state Y (with higher RE price of 0.35), transaction

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the code, and allows visitors to rerun simulations with their own random seeds, and see the charts of market behavior dynamically, as well as obtain data files for further analysis.

prices of both human traders (Panel A) and algorithmic traders (Panel B) generally approach and get close to the equilibrium level from below. (3) As shown in Table 2, in State *X* (low RE price) periods, average trading volume for human traders across the five periods is 19.5 while the average volume for algorithmic traders is 17.5. The allocative efficiency of human trader markets across the five periods is 64%, while efficiency of the simulated markets is 81%. Note that allocative efficiency occurs from having the appropriate number of trades and the securities being acquired by the appropriate type of trader. These efficiency levels (bellow 100) suggest the wrong type of trader is accumulating securities in some periods. In State *Y* (low RE price) periods, human traders' average volume is 19.3 (vs. 23.7 for algorithmic traders) and human trader efficiency is 100%, while algorithmic traders achieve efficiency levels of 99%. This means the appropriate number of trades is taking place, and securities are being held by the appropriate trader type.

There are also important differences between the two panels: (1) convergence of prices to RE predictions in Panel A (with human traders) is tighter and progressively faster in later periods, as compared to Panel B where there is little change from early to later realizations of the same state (*X* or *Y*). Efficiency results also show human subjects improving over time (when State is X), whereas markets populated with algorithmic traders show less improvement over time.

Insert Figure 3 and Table 2 about Here

Replication of the 2-state markets with different parameters (Figures 4 and 5 and Table 2) show essentially the same pattern of convergence except that in State *Y* (low RE price) human traders have a tendency to converge quickly to the RE price, especially in later periods (not coming from above or below) whereas the paths with algorithmic traders depend on history in the previous period (because the *CAL* from the uninformed is carried forward from previous periods). If the previous period is State *X* (high RE price) the simulation converges from above, and if the previous period is State *Y* (low RE price), the simulation converges from below the RE price. As expected, algorithmic traders adjust slowly and learn myopically without any global awareness of equilibrium prices.

Insert Figures 4 and 5 about Here

Figure 6, Panel A displays data for a three-state market reported by PS with human traders, and identical market replicated for this paper with algorithmic traders. Figure 6, Panel B shows a chart of median prices from 50 replications of the same market with minimally intelligent M-E algorithmic traders. In both panels, the solid horizontal line indicates the rational expectations (dashed for PI) equilibrium price for the respective periods. Allocative efficiency and trading volume are shown numerically for each period in Table 2 Panel D, the average efficiency and volume (across 50 replications) are shown numerically.

Figure 6 indicates: (1) In state *Z* (with high RE price of 0.32), for both human (Panel A) and algorithmic traders (Panel B) transaction prices approach and get close to the RE equilibrium level from below. (2) In state *Y* (with medium RE price of 0.245), transaction prices also generally approach and get close to the equilibrium level from below in both panels. The only exception occurs in Period 11 when the market converges from above in both human and simulated markets. It appears that moving from a high equilibrium price state to a lower price state may cause convergences from above. Otherwise, both humans and our simulated traders tend to be conservative and approach the equilibrium price from below. ((3) Trading volume in all three states is generally greater than the predicted volume of 16 trades. For human traders volume tends to range from 15-23 trades, whereas algorithmic traders volume ranges from 14-24 trades. (5) In all periods of State *Z* (high RE price), allocative efficiency for human traders is 100% whereas algorithmic traders achieve 99% efficiency. In State *Y* (medium RE price) periods, allocative efficiency of human traders averages 97% (100% efficiency in all periods except the first realization of State *Y*) whereas algorithmic traders achieve 95% efficiency and do not achieve 100% efficiency in any individual period. In State *X* (low RE price) periods, allocative efficiency of human traders averages 88% whereas algorithmic traders achieve 92% efficiency.

Insert Figure 6 about Here

Table 2 shows volume and efficiency numerically. Again, it is clear that, outcomes of markets with profit-motivated human and minimally intelligent algorithmic traders exhibit the same central tendencies of convergence towards the predictions of rational expectations models. Apparently, the structural constraints of the market rules, and simple means-end heuristics proposed by Newell and Simon (1972) are sufficient to yield this result even as the number of states in the market increases from 2 states to 3.

3.1 Statistical Analysis of Price Changes, Volume and Efficiency

To assess price convergence to the rational expectations equilibrium, we report results of a procedure used by Gode and Sunder (1993) who regressed the root mean squared deviation between transaction and RE equilibrium prices on the natural logarithm of the transaction sequence number within a period. If prices move towards RE levels over time, the slope coefficient of this regression should be less than zero. Four panels of Figure 7 show the behavior of this root mean square deviation over time for the four human and simulated market pairs. Results of ordinary least squares regressions of MSD on log of transaction sequence number in human and simulated markets are shown in two triplets in each panel (slope, p-value, and R^2)

respectively. Three of the four human (with the exception of Market 2), as well as all four simulated markets exhibit significant convergence to RE equilibrium, and the zero-slope hypothesis is rejected in favor of negative slope alternative at $p < 0.000$ for the seven markets. Explanatory power is generally high with 80% of the reduction in the deviation from RE equilibria being explained by log of transaction sequence number. Figure 7 shows that root mean squared deviation of transaction from RE equilibrium prices tends towards 0.

Figure 7 about Here

Across all 32 periods of the four markets, the difference between the trading volume (Figure 8) and efficiency (Figure 9) of human and simulated markets is not statistically different (average volume of simulated market is about one trade greater than for human markets with t-statistic of 1.35 and the average efficiency of simulated markets is 1.6% lower than that of markets with human traders (t-statistic of -1.08). There is no significant difference between the volumes and efficiency of markets with human traders as opposed to algorithmic traders. The inference is not that these simple algorithms capture all or even most of the behavior of the humans; that is not true. However, when seen through the perspective of aggregate market outcomes—prices, allocations, trading volume, and efficiency—these differences get attenuated to a point of statistical insignificance.

Figure 8 and 9 about Here

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4. Discussion and Concluding Remarks

We have presented evidence that individual behavior, modeled by simple means-end heuristics and zero-intelligence, yields near-equilibrium market outcomes whose formal derivation requires strong assumptions about optimization by individual agents. Even if this key assumption of theory is descriptively invalid, it does not necessarily undermine the validity and predictive value of the theory at the aggregate level. Our findings are consistent with Gigerenzer et al. (1999) who built on Simon's bounded rationality paradigm by proposing that individuals use "fast and frugal" heuristics to successfully accomplish complex tasks.

The computational or other "cognitive" abilities of our algorithmic traders do not exceed, indeed are far weaker than, the documented faculties of human cognition. Yet, these simulated markets with insider trading based on asymmetric access to information converge to the close proximity of rational expectations equilibria and attain high allocative efficiency. Contrary to claims made in behavioral economics literature (e.g., Tversky and Kahneman 1974, Thaler 1986), we find that individuals using a simple means-end heuristic (analogous to Tversky and Kahneman's 1974 anchor and adjust heuristic) in a market setting generate outcomes close to the rational expectations equilibrium. We interpret the results to suggest that, even in these relatively complex market environments (compared to Gode and Sunder 1993, 1997, and Jamal and Sunder 1996), allocative efficiency of markets remains largely a function of their structure, not intelligence or optimizing behavior of agents. Stress on understanding the role of market structure, rather than human cognition, may help advance our understanding of links between economic theory and market outcomes.

5. References

Angerer, Martin, Juergen Huber, Martin Shubik and Shyam Sunder. "An Economy with Personal Currency: Theory and Experimental Evidence." *Annals of Finance*, Volume 6, Number 4, October 2010, 475-509.

Becker, Gary, "Irrational Behavior and Economic Theory," Journal of Political Economy. Vol 70, (1962), 1-13.

Forsythe, Robert, and Russell Lundholm. 1990. "Information Aggregation in an Experimental market," Econometrica 58:2 (March), 309-347.

Forsythe, Robert, Thomas Palfrey and Charles Plott. "Asset Valuation in an Experimental Market." Econometrica, Volume 50, Number 3, May 1982, 537-567.

Friedman, Milton. "The Methodology of Positive Economics," in Essays in Positive Economics. Chicago: University of Chicago Press, 1953.

Gigerenzer, Gerd, P. Todd and the ABC Research Group. 1999. *Simple Heuristics That Make us Smart*. Oxford University Press, New York..

Gode, Dhananjay, and Shyam Sunder, "Allocative Efficiency of Markets with Zero Intelligence Traders: Market as a Partial Substitute for Individual Rationality," The Journal of Political Economy. Vol. 101 No. 1 (February 1993), 119-137.

Gode, Dhananjay, and Shyam Sunder, "What Makes Markets Allocationally Efficient?" The Quarterly Journal of Economics. (May 1997).

Huber, Juergen, Martin Shubik and Shyam Sunder."Three Minimal Market Institutions: Theory and Experimental Evidence." *Games and Economic Behavior* 70 (2010) 403–424.

Huber, Juergen, Martin Shubik and Shyam Sunder. "Sufficiency of an Outside Bank and a Default Penalty to Support the Value of Fiat Money: Experimental Evidence." Cowles Foundation Working Paper, June 12, 2011 (http://faculty.som.yale.edu/shyamsunder/Research/Experimental%20Economics%20and%20Finance/Presentations%20and%20Working%20Papers/Shubik-Huber-Sunder/SufficiencyPaperJune122011%20%282%29.pdf).

Jamal, Karim, and Shyam Sunder, "Bayesian Equilibrium in Double Auctions Populated by Biased Heuristic Traders," Journal of Economic Behavior and Organizations, Vol 31 (1996), 273 - 291.

Muth, J.F. (1961). Rational expectations and The Theory of Price Movements. *Econometrica*, 29:315-335.

Newell, Allan, J.C. Shaw and Herbert A. Simon. Preliminary description of the General Problem Solving Program I (GPS I). CIP Working Paper No.7, (December 1957).

Newell, Allan, and Herbert A. Simon, Human Problem Solving. Prentice Hall, 1972: Englewood Cliffs, N.J.

North, Douglas C., Instititions, Institutional Change and Economic Performance. Cambridge University Press,1990: New York, NY.

Plott, Charles R. and Shyam Sunder, "Efficiency of Controlled Security Markets with Insider Information: An Application of Rational Expectation Models," Journal of Political Economy, Vol. 90, No. 4 (August 1982), 663-698.

Plott, Charles R. and Shyam Sunder, "Rational Expectations and the Aggregation of Diverse Information in Laboratory Security Markets." Econometrica, Vol. 56, No. 5 (September, 1988), 1085-1118.

Simon, Herbert A. "A Behavioral Model of Rational Choice," Quarterly Journal of Economics, Vol. 69, (1955), 99-118.

Simon, Herbert. A. “Rational Choice and the Structure of the Environment,” Psychological Review, Vol. 63, (1956), 129-138.
Simon, Herbert A. 1969. The Sciences of the Artificial. Cambridge, Mass.: MIT Press.
Smith, Vernon L. 1962. “An Experimental Study of Competitive Market Behavior.” The Journal of Political Economy, Vol. 70 No. 2, April 1962, 111-137.
Smith, Vernon L. 2008. *Rationality in Economics: Constructivist and Ecological Forms.* Cambridge University Press.
Sunder, Shyam, "Determinants of Economic Interaction: Behavior or Structure." Journal of Economic Interaction and Coordination. Vol. 1 No. 1 (2006); pp. 21-32.
Thaler, Richard. 1986. “The Psychology and Economics Conference Handbook: Comments on Simon, on Einhorn and Hogarth, and on Tversky and Kahneman.” The Journal of Business. Vol. 59 No. 4, Part 2, October 1986, S279-S284.
Tversky, Amos. and Daniel Kahneman, “Judgment under Uncertainty: Heuristics and Biases,” Science, (1974), Vol. 185, 1124-1131.
Tversky, Amos. and Daniel Kahneman, “Rational Choice and the Framing of Decisions.” The Journal of Business. Vol. 59 No. 4, Part 2, October 1986, S251-S278.

Table 1 – Simulation Parameters

Market	Corresponding Market	State	Probability	Dividends For Each Trader Type			RE Predictions Price	PI Predictions Price
				Type I	Type II	Type III	(Allocation to)*	(Allocation to)*
2	Plott and Sunder 1982 Market 2	X	0.333	0.1	0.2	0.24	0.24(III)	0.266(I_u)
		Y	0.667	0.35	0.3	0.175	0.35(I)	0.35(I_i)
3	Plott and Sunder 1982 Market 3	X	0.4	0.4	0.3	0.125	0.4(I)	0.4(I_i)
		Y	0.6	0.1	0.15	0.175	0.175(III)	0.22(I_u)
4	Plott and Sunder 1982 Market 4	X	0.4	0.375	0.275	0.1	0.375(I)	0.375(I_i)
		Y	0.6	0.1	0.15	0.175	0.175(III)	0.21(I_u)
5	Plott and Sunder 1982 Market 5	X	0.35	0.12	0.155	0.18	0.18(III)	0.212(I_u)
		Y	0.25	0.17	0.245	0.1	0.245(II)	0.245(II_i)
		Z	0.4	0.32	0.135	0.16	0.32(I)	0.32(I_i)

Plott and Sunder (1982) conducted an experiment with profit oriented human traders to ascertain whether they traded at prices (and quantities) predicted by rational expectations models. Table 1 shows the parameters used in the experiment and the predictions about price and which trader type should hold securities in these markets. Our simulation uses the same parameters as those used in the PS experiment.

*Allocation code: I, II, and III for all traders of types I, II, and III respectively. I_i for informed traders of type I, I_u for uninformed traders of type I, and similarly for informed and uninformed traders of types II and III.

Table 2: Transaction and Efficiency Data

Panel A - Number of Transactions (Efficiency Levels) of Market 2

Period (State)	7 (X)	8 (Y)	9 (X)	10 (Y)	11 (Y)	Avg.
Human Data	22 (57%)	19 (100%)	17 (70%)	19 (100%)	20 (100%)	19.4 (85.4%)
Simulation (50 Reps)	19 (78%)	25 (99%)	16 (83%)	25 (99%)	21 (98%)	21.2 (91.4%)

Panel B – Number of Transactions (Efficiency Levels) of Market 3

Period (State)	3 (Y)	4 (X)	5 (Y)	6 (Y)	7 (X)	8 (Y)	9 (X)	10 (Y)	Avg.
Human Data	15 (79%)	19 (100%)	15 (88%)	14 (89%)	19 (100%)	14 (98%)	15 (100%)	15 (99%)	15.8 (94.1%)
Simulation (50 Reps)	14 (87%)	25 (100%)	12 (81%)	14 (87%)	25 (100%)	12 (81%)	25 (100%)	12 (80%)	17.4 (89.5%)

Panel C – Number of Transactions (Efficiency Levels) of Market 4

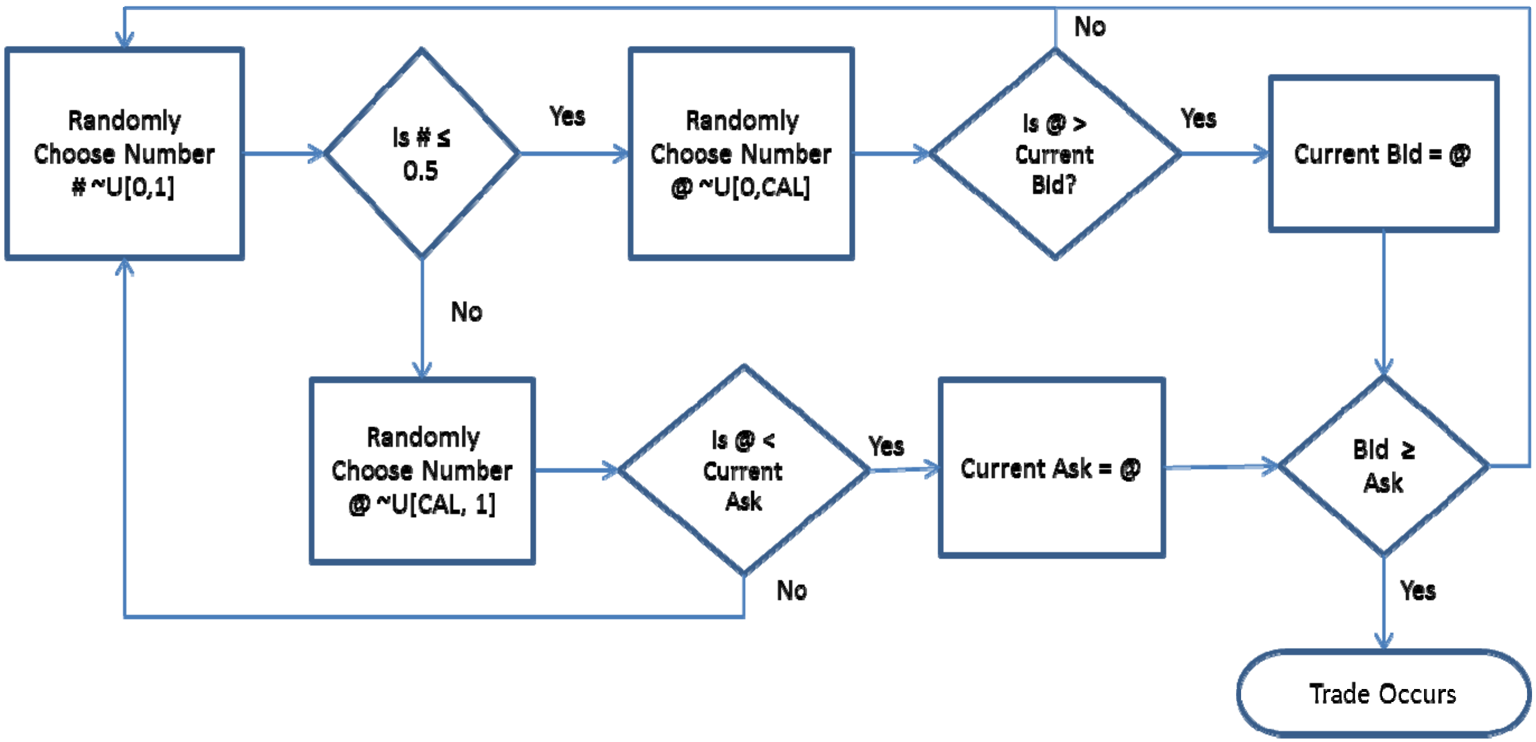
Period (State)	5 (Y)	6 (X)	7 (Y)	8 (Y)	9 (X)	10 (Y)	11 (X)	12 (Y)	13 (X)	Avg.
Human Data	17 (92%)	23 (100%)	17 (95%)	12 (93%)	20 (100%)	14 (94%)	21 (100%)	18 (94%)	21 (100%)	18.1 (96.4%)
Simulation (50 Reps)	14 (90%)	25 (100%)	12 (81%)	14 (88%)	25 (100%)	12 (80%)	24 (100%)	12 (81%)	24 (100%)	18.0 (94.1%)

Panel D – Number of Transactions (Efficiency Levels) of Market 5

Period (State)	4 (X)	5 (X)	6 (Y)	7 (Z)	8 (Z)	9 (Y)	10 (Y)	11 (X)	12 (Y)	13 (Z)	Avg.
Human Data	15 (82%)	16 (94%)	17 (87%)	20 (100%)	23 (100%)	21 (100%)	20 (100%)	18 (87%)	18 (100%)	16 (100%)	18.4 (95%)
Simulation (50 Reps)	14 (93%)	16 (95%)	22 (99%)	23 (99%)	24 (98%)	16 (87%)	21 (97%)	13 (87%)	22 (99%)	23 (99%)	19.4 (95.3%)

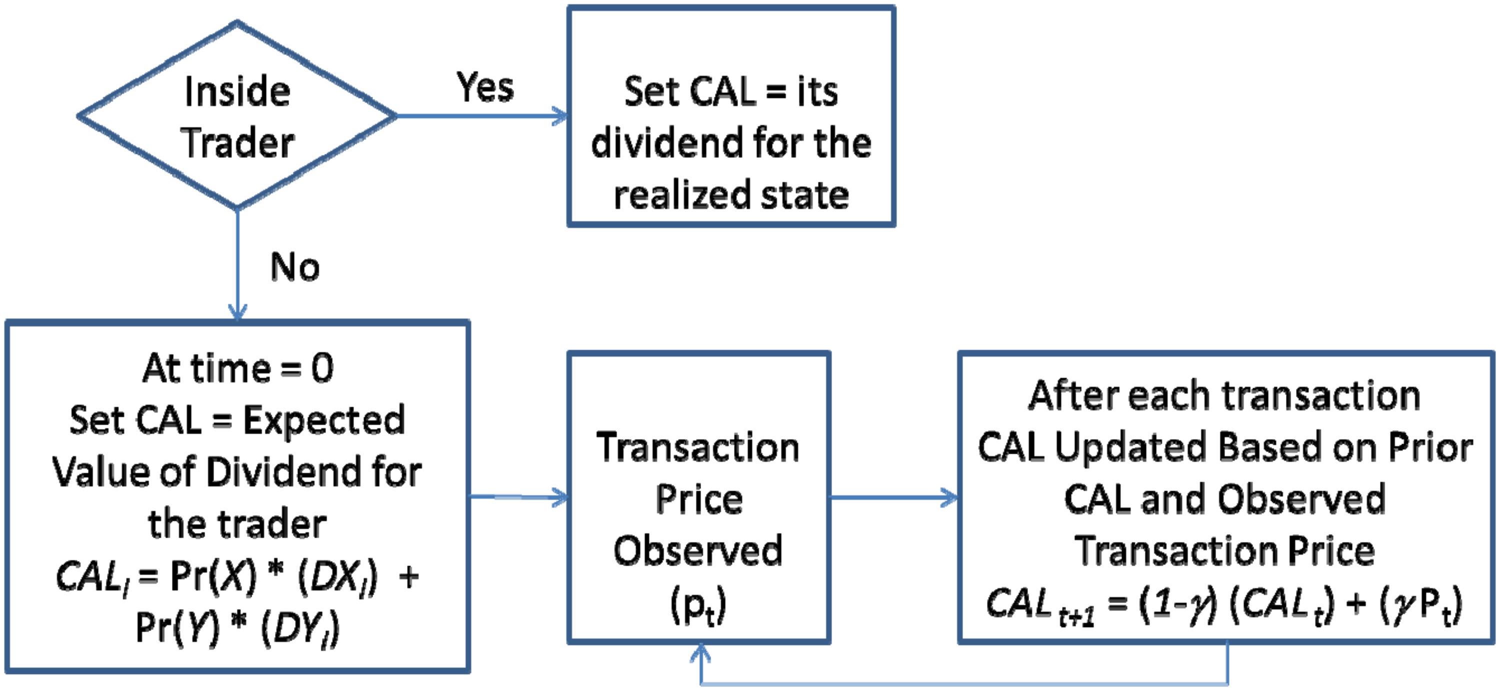
Plott and Sunder (1982) conducted an experiment with profit oriented human traders to ascertain whether they traded at prices (and quantities) predicted by rational expectations models. Table 2 shows the number of transactions and efficiency levels attained by human traders, as well as simulated algorithmic traders who use a simple linear heuristic to update aspiration levels. The behavior of simulated traders is qualitatively similar to that of human traders with respect to both number of transactions and efficiency levels in all markets simulated, and across all state realizations.

Figure 1: Bid, Ask, and Transactions Algorithm



The algorithmic traders in our simulated markets send a message by first drawing a random number from a uniform distribution bounded by 0 and 1. If the number is less than or equal (greater than) to 0.5, the message is a bid (ask). If the message is a bid, the trader draws a second random number from a uniform distribution bounded by 0 and the current aspiration level (*CAL*). If the message is an ask, the second random number is drawn from a uniform distribution between *CAL* and 1. If the trader's bid is more than the highest current bid in the market, the former becomes the current bid. If the trader's ask is less than the current (lowest) ask, then the former becomes the current ask in the market. When the market bid is equal to (or exceeds) the market ask, a trade occurs at the mid-point of the bid and ask. Visit www.zitraders.com for outline of the code.

Figure 2: Algorithm for Setting Current Aspiration Levels (CAL's)*



*There are two types of traders in our simulated markets--informed and uniformed. They set a current aspiration level (*CAL*) to generate bids and asks. The informed (insider) traders get a perfect signal about the realized state and set their *CAL* = their dividend in that state. The uninformed traders set their *CAL*= expected value, and then update it using a simple linear updating rule after each transaction. The end-of-period *CAL* of uninformed traders is carried forward to the next period. Informed traders do not carry forward *CAL* from previous periods. Visit www.zitraders.com for outline of the code.

Figure 3 - Market 2 of Plott and Sunder (1982)

RE (PI) Eq. Prices are State X = 0.24 (0.266), State Y = 0.35 (0.35)

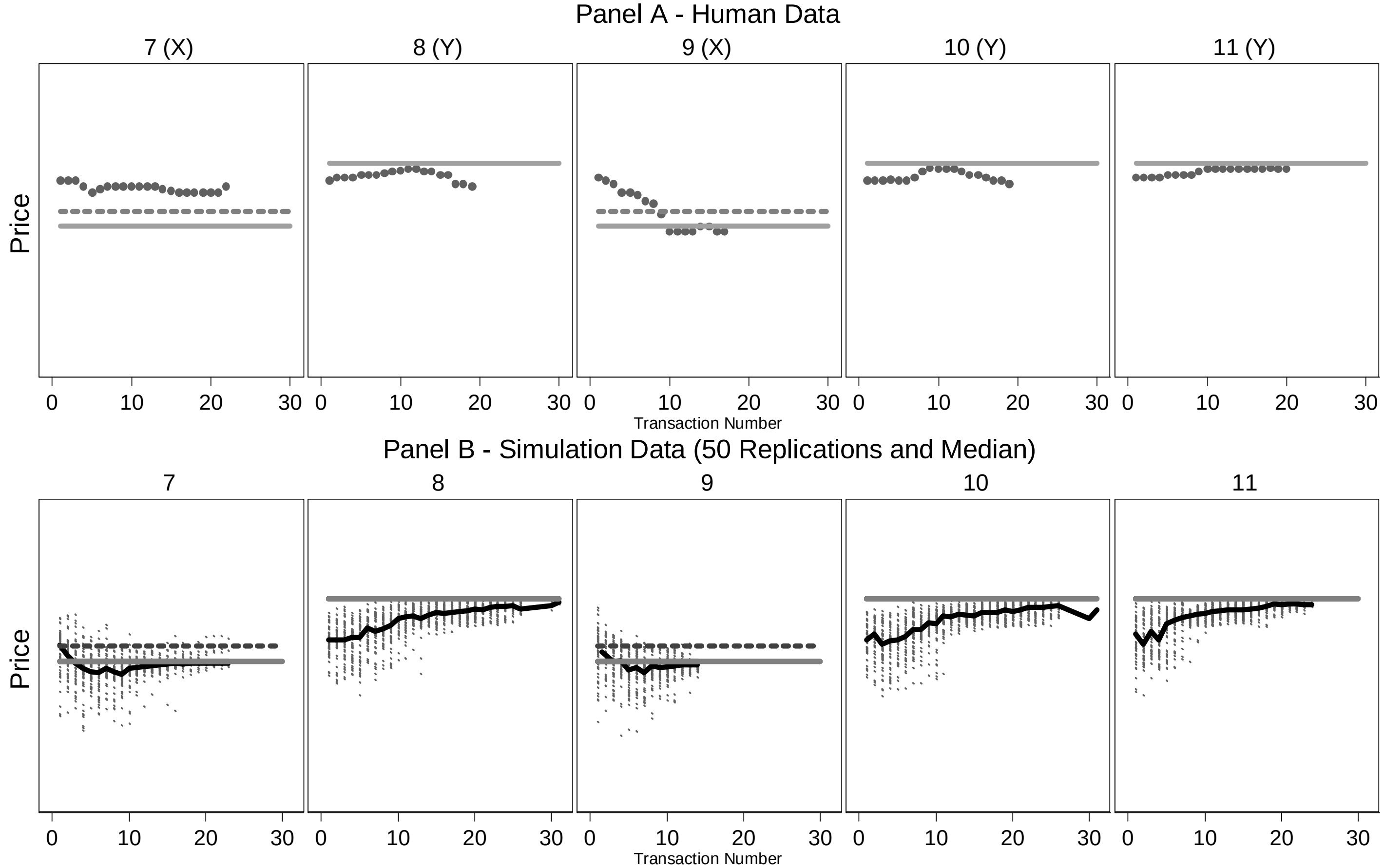


Figure 3 plots the price paths for periods 7-11 of Market 2 in Plott and Sunder (1982) where participants have different information sets. Panel A provides the price history observed using human participants in PS. Panel B provides data from 50 replications of a computer simulation using algorithmic traders. In both panels the solid grey line is the rational expectations (RE) predicted price while the dashed line is the prior information (PI) predicted price. Each dot in Panel B is an observed price and the dark line is the median price observed over 50 replications. The price cloud (of dots) shows the range of prices and how it decreases over time.

Figure 4 - Market 3 of Plott and Sunder (1982)

RE (PI) Eq. Prices are State X = 0.4 (0.4), State Y = 0.175 (0.22)

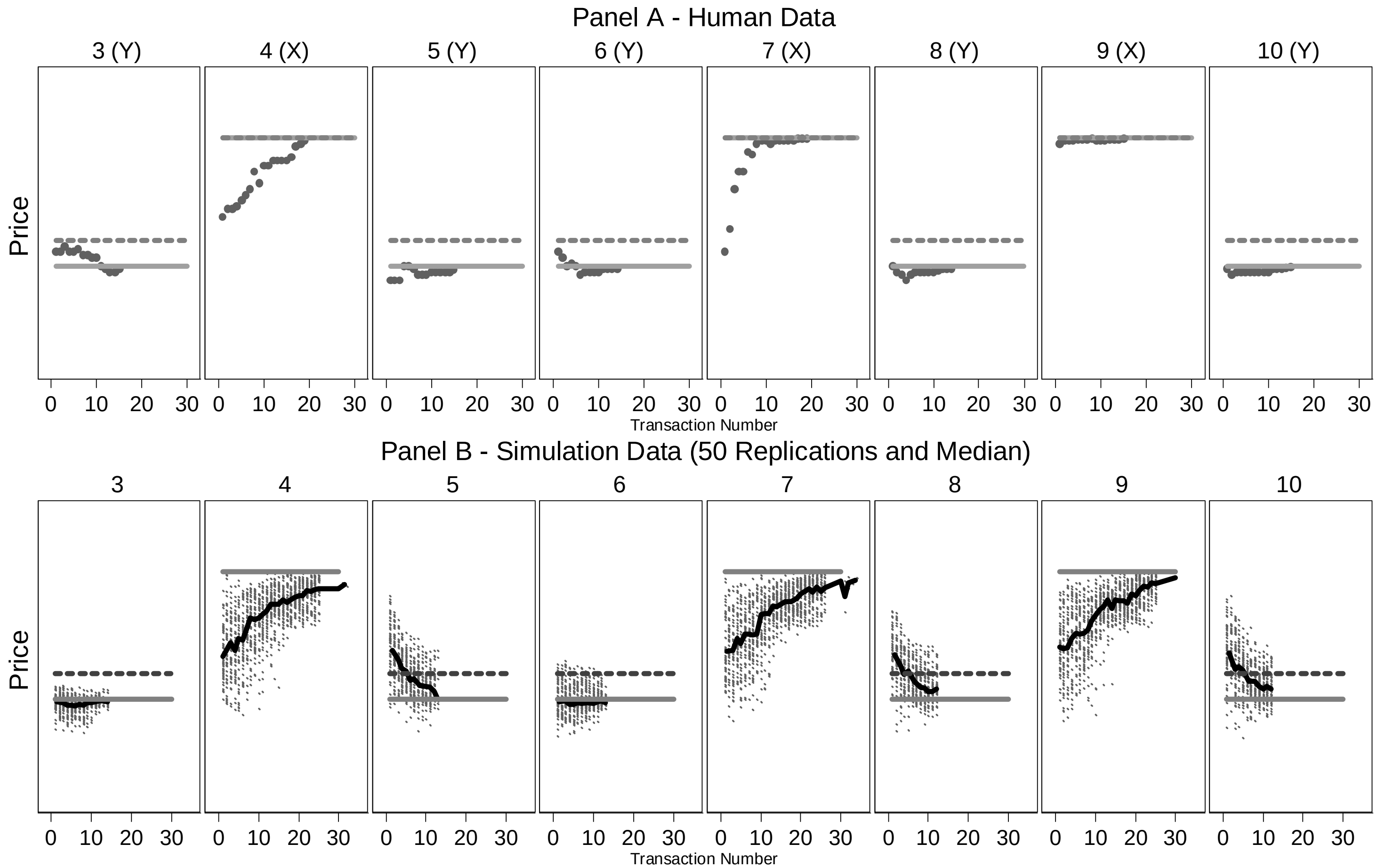


Figure 4 plots the price paths for periods 3-10 of Market 3 in Plott and Sunder (1982) where participants have different information sets. Panel A provides the price history observed using human participants in PS. Panel B provides data from 50 replications of a computer simulation using algorithmic traders. In both panels the solid grey line is the rational expectations (RE) predicted price while the dashed line is the prior information (PI) predicted price. Each dot in Panel B is an observed price and the dark line is the median price observed over 50 replications. The price cloud (of dots) shows the range of prices and how it decreases over time.

Figure 5 - Market 4 of Plott and Sunder (1982)

RE (PI) Eq. Prices are State X = 0.375 (0.375), State Y = 0.175 (0.21)

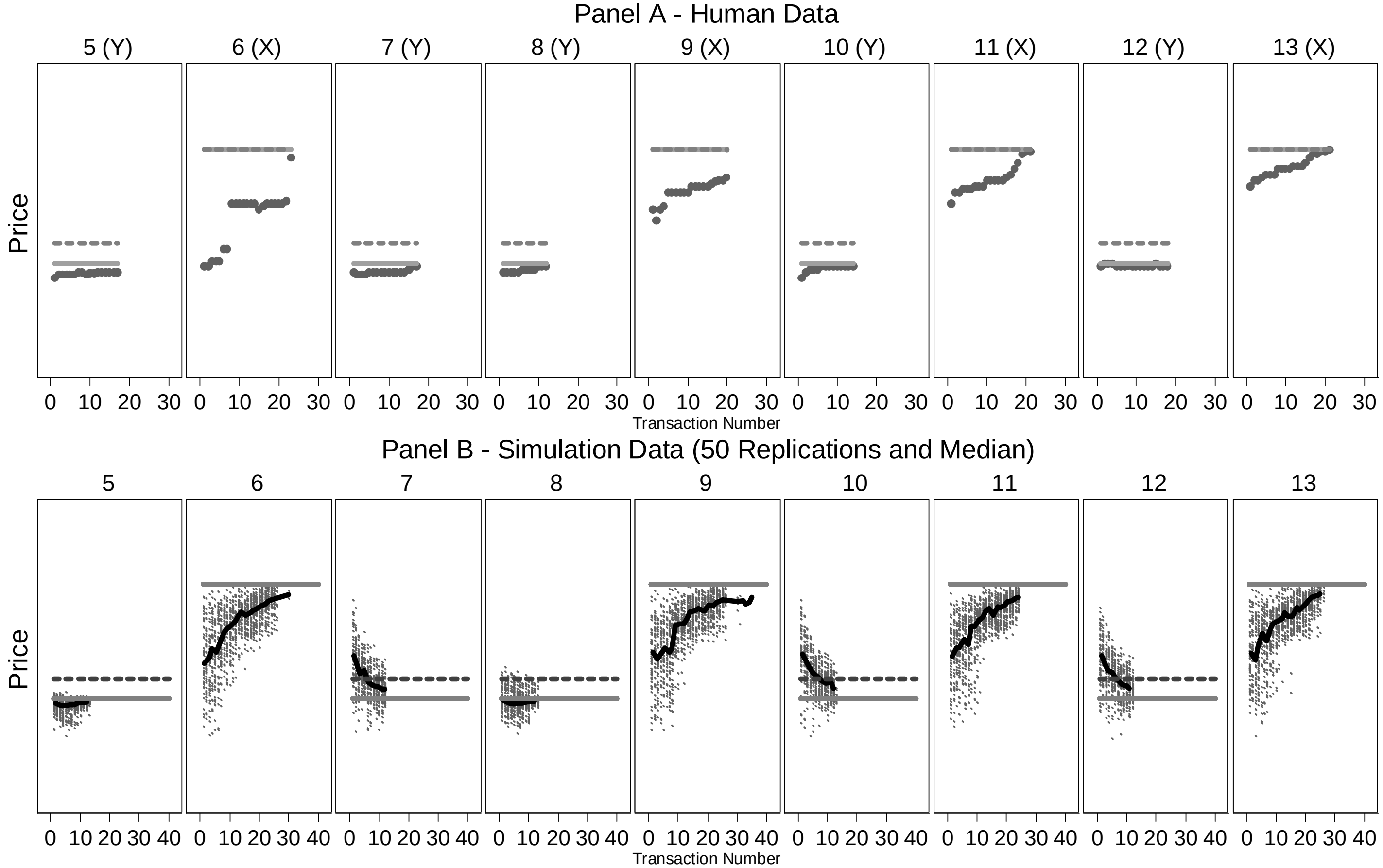


Figure 5 plots the price paths for periods 5-13 of Market 4 in Plott and Sunder (1982) where participants have different information sets. Panel A provides the price history observed using human participants in PS. Panel B provides data from 50 replications of a computer simulation using algorithmic traders. In both panels the solid grey line is the rational expectations (RE) predicted price while the dashed line is the prior information (PI) predicted price. Each dot in Panel B is an observed price and the dark line is the median price observed over 50 replications. The price cloud (of dots) shows the range of prices and how it decreases over time.

Figure 6 - Market 5 of Plott and Sunder (1982)

RE (PI) Eq. Prices are State X = 0.18 (0.212), State Y = 0.245 (0.245), State Z = 0.32 (0.32)

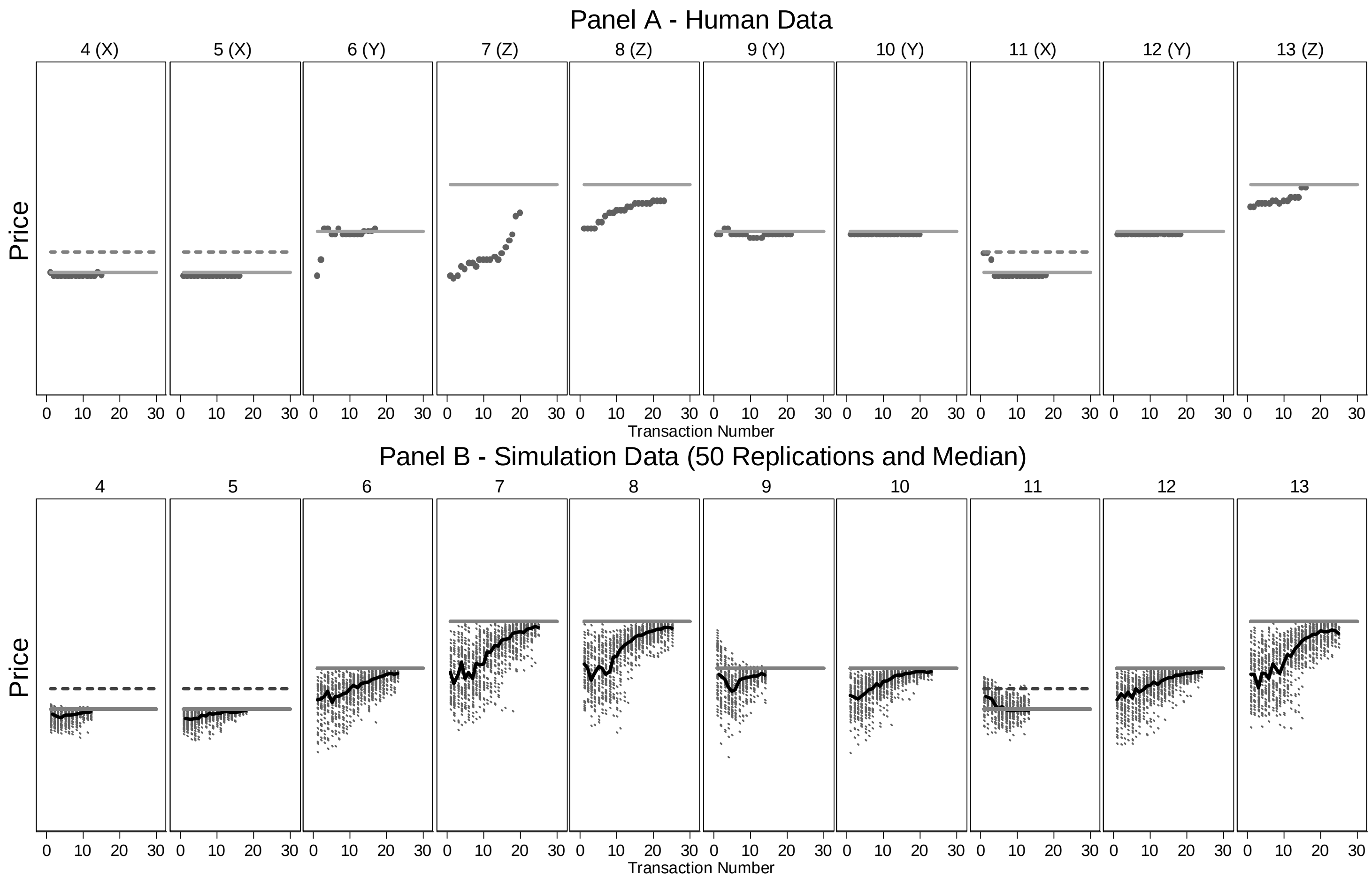
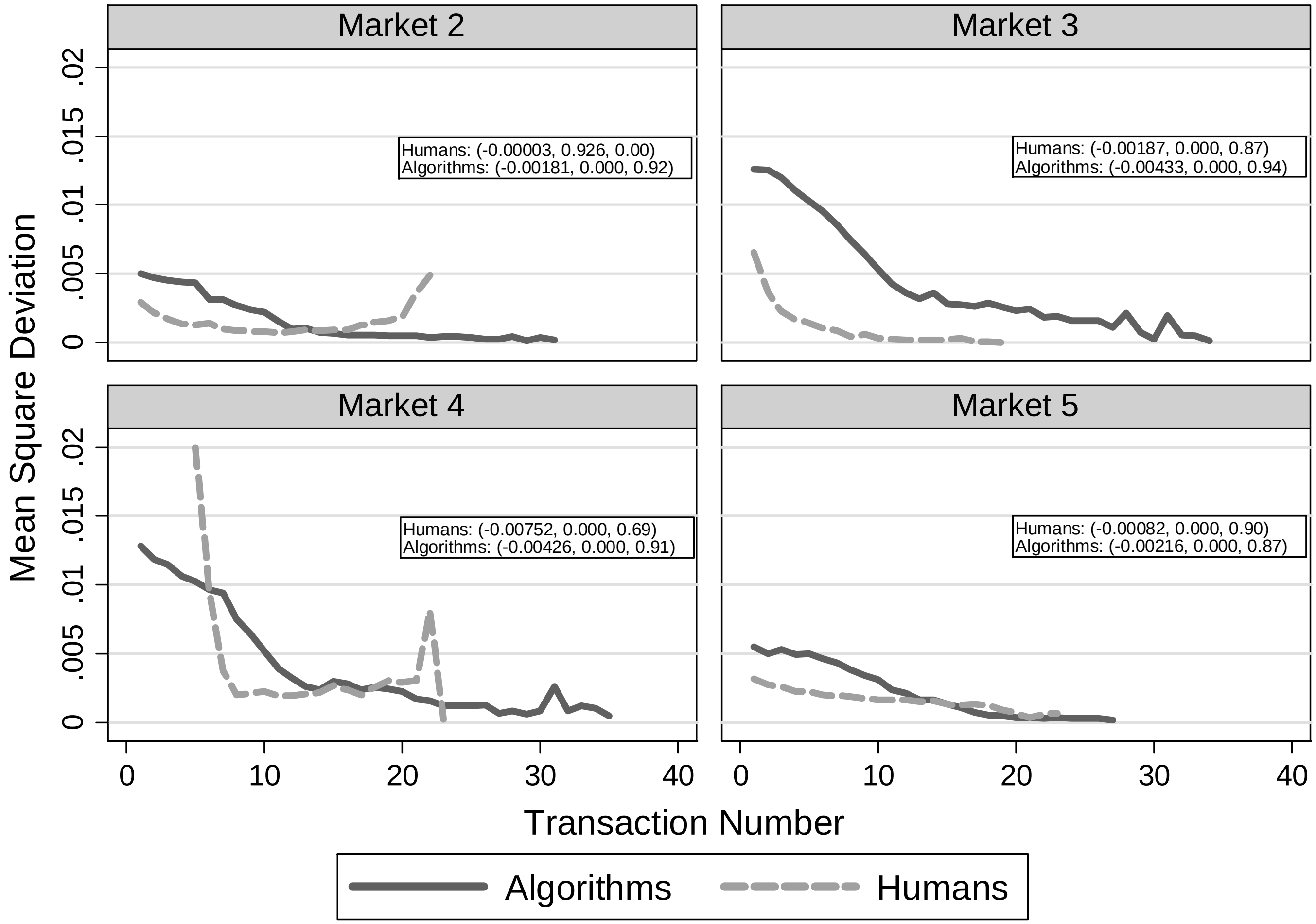


Figure 6 plots the price paths for periods 4-13 of Market 5 in Plott and Sunder (1982) where participants have different information sets. Panel A provides the price history observed using human participants in PS. Panel B provides data from 50 replications of a computer simulation using algorithmic traders. In both panels the solid grey line is the rational expectations (RE) predicted price while the dashed line is the prior information (PI) predicted price. Each dot in Panel B is an observed price and the dark line is the median price observed over 50 replications. The price cloud (of dots) shows the range of prices and how it decreases over time.

Figure 7: Root Mean Squared Deviation of Prices from Equilibrium vs. Transaction Number



To assess price convergence over time towards the rational expectations price, Figure 7 plots the root mean square deviation of prices from the rational expectations equilibrium by transaction number for both the Plott and Sunder (1982) human traders and the algorithmic traders in our simulation. For both human and simulated traders the root mean square price deviation tends towards zero. For human Market 4, the first five root mean squared deviation observations are more than 0.02 for a maximum of 0.145 for transaction 3, and do not appear in the chart. The y-scale is chosen to highlight the relationships for the remaining data. OLS regression (*MSD = α + β ln transaction No.*) estimates of *β*, *p*-value and R^2 for human and algorithmic markets are shown numerically inside each chart. For example, the human participants in market 5 have the following values: β = -0.00082, p-value = 0.000 and R^2 = 0.90. These results show that observed prices are converging towards the RE equilibrium price in all markets except for human participants in Market 2.

Figure 8: Comparison of Transaction Volume by Market and Period

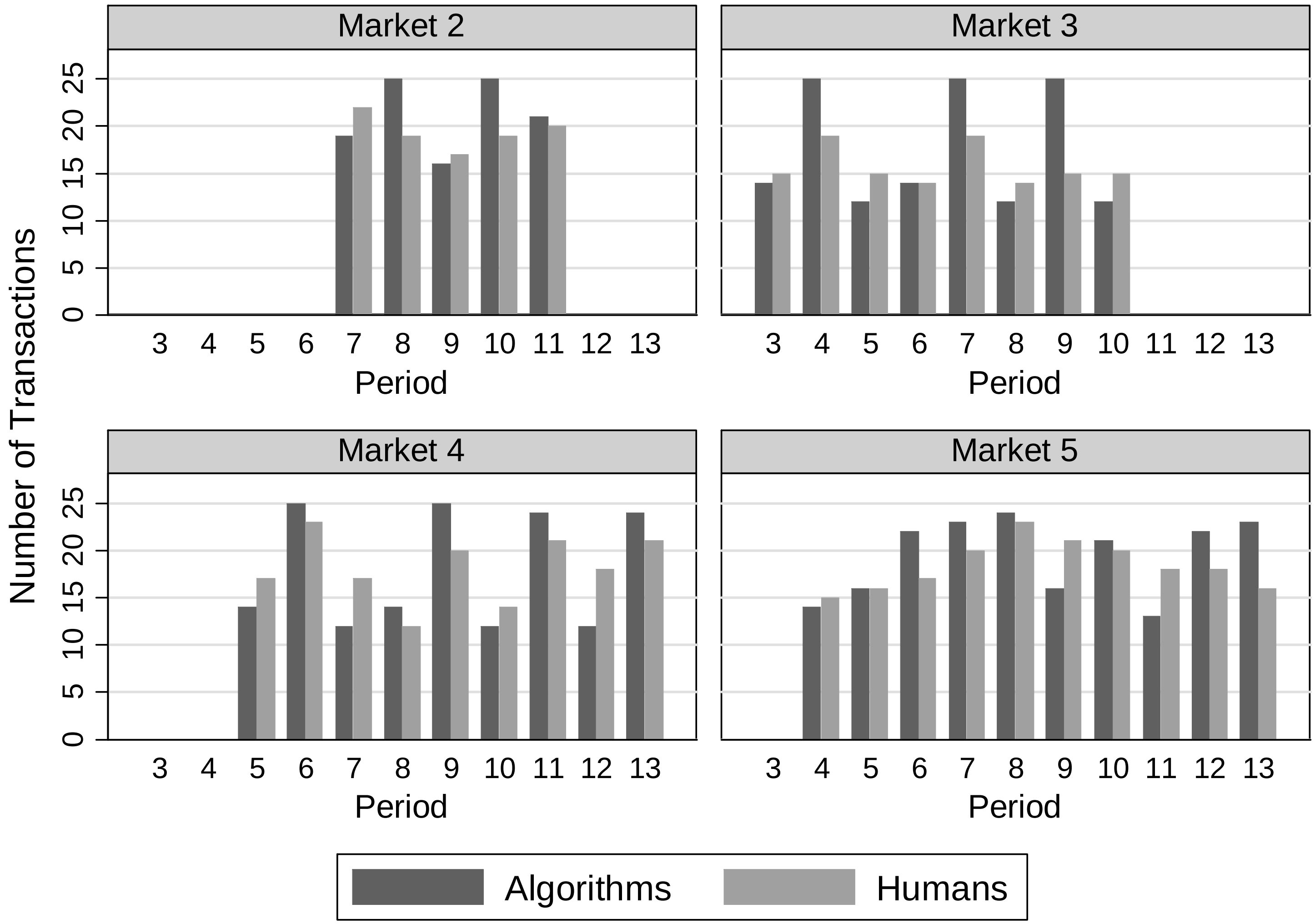


Figure 8 provides a comparison of the number of transactions for both the Plott and Sunder (1982) human traders and our algorithm traders. The mean volume for human traders is 17.81 over 32 periods. The mean volume for the algorithmic traders is 18.78 over 32 periods. Overall the mean difference between the human and algorithmic traders is 0.97 (t-statistic 1.35 is not significant at 5% level) across all 32 periods.

Figure 9: Comparison of Efficiency by Market and Period

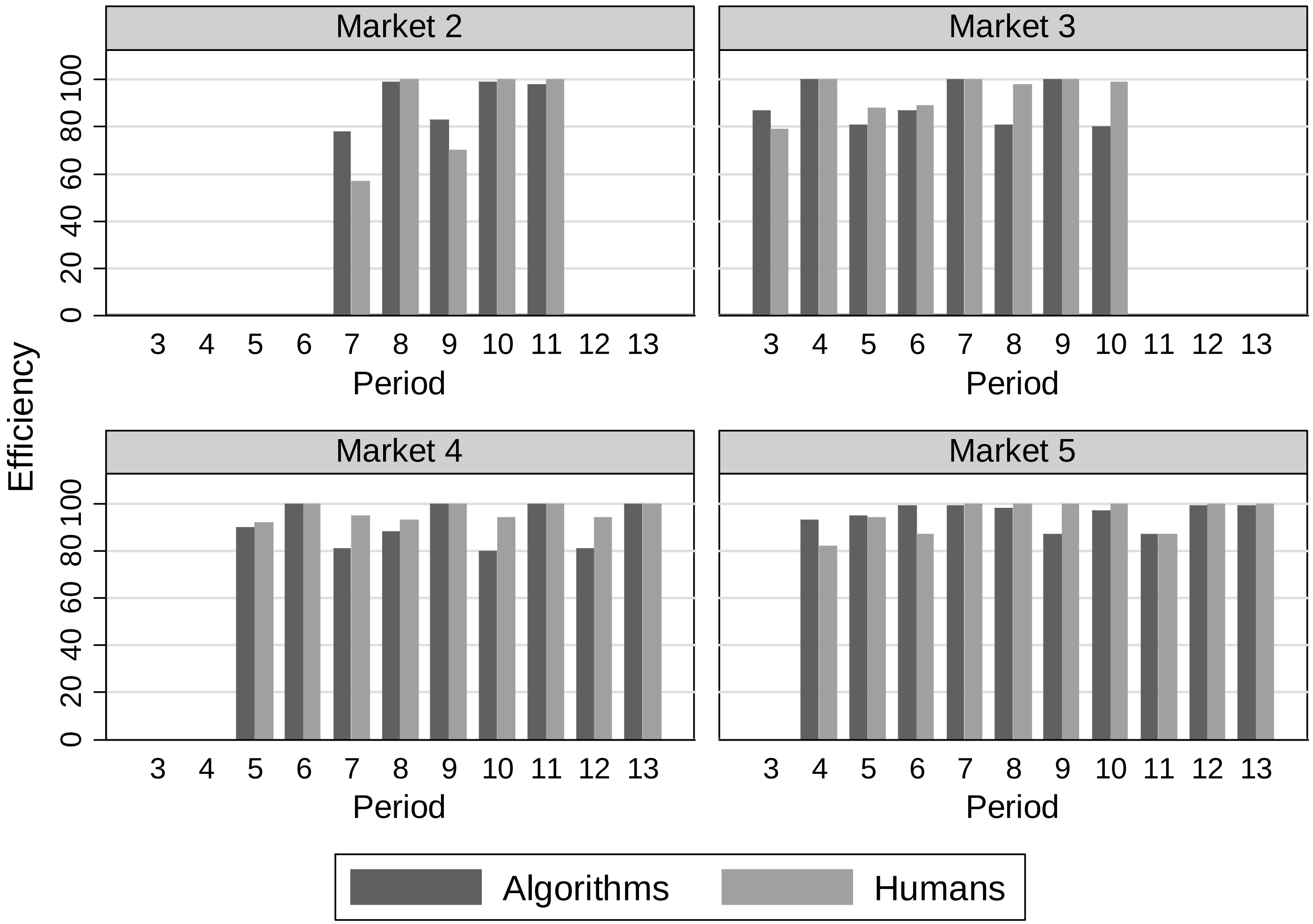


Figure 9 provides a comparison of market efficiency for both the Plott and Sunder (1982) human traders and our algorithm traders. The mean efficiency for human traders is 93.69% over 32 periods. The mean efficiency for the algorithmic traders is 92.06%. Overall the mean difference between the human and algorithmic traders is -1.625% (t-statistic -1.08 is not significant at 5% level) across all 32 periods.