WHAT IS A SOLUTION TO A MATRIX GAME

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These notes are provided to describe many of the problems encountered concerning both structure and behavior in specifying what is meant by the solution to a game of strategy in matrix or strategic form. In the short term in particular, it is often reasonable for the individual to accept as given, both the context in which decisions are being made and the formal structure of the rules of the game. A solution is usually considered as a complete set of equations of motion that when applied to the game at hand selects a final outcome. There are many different theories and conjectures about how games of strategy are, or should be played. Several of them are noted below. They are especially relevant to the experimental gaming facility noted in the companion paper.
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Keywords: Matrix games, Solution concepts, Experimental gaming

[1] I wish to thank my research assistants Ian Hoffman, Aaron Manilow and Jarus Singh for their fine assistance.

1. Preliminary Remarks

There are still considerable questions as to what constitutes a solution to an n-person game in strategic form. Without delving into the sometimes important modeling problems involving broad context, psychology, social psychology, sociology and many other factors, we try to define an abstract model trading off analysis against nuance. For simplicity and clarity our remarks are confined primarily to one-period two-person $k \times k$ matrix games, with an emphasis on the 2×2 games first with ordinal, then with cardinal, entries.

In the $k \times k$ matrix game we denote the most desired outcome to each player by k^2 and the least desired by 1. In 2×2 matrix games the entries are 4, 3, 2, and 1.[2]

In human affairs, if the game analogy is to be used at all, it is best to consider it as a game within a game. Although in our modeling we make a formal distinction between structure and behavior, in an evolutionary process there is an interaction between the two, where in the short run, structure constrains behavior, however in the long run behavior modifies structure. In essence, custom precedes law. Having observed this we limit our investigation considerably to highly-formal models in our discussion of structure, intent and behavior in short-run scenarios.

2. What Is a Solution?

Harsanyi and Selten [1988] embarked on a project to find conditions that would plausibly select a unique noncooperative equilibrium point to be the solution to any matrix game. Central to this approach was the acceptance of the Nash's [Nash 1953] formal concept of a noncooperative equilibrium as the central necessary property of any solution.

The viewpoint espoused here is that the search for a unique noncooperative equilibrium solution to all games poses many interesting philosophical problems in an abstract world inhabited by abstract von Neumann game players with unlimited intelligence and perception and no passions or personality traits. These players act in an institution free world where context is implicitly accounted for in the matrix game or the extensive form of the game. Unfortunately, as a portrayal of human decision-making it fails to appreciate the fundamental limitations in attempting to portray an open evolving system where the dynamics are context dependent and the institutions of any society are the carriers of process.

At the highest level of abstraction, if one concedes that it is a least theoretically possible to portray a game in extensive form, a solution is nothing more than a path down the game tree to some terminal point. A more specialized solution contains rules for selecting specialized paths. The noncooperative equilibrium concept stresses mutually consistent expectations, but this is only one among several desirable properties that may be ascribed to a special solution.

2.1. Players, Strategies and Outcomes

A game in strategic form has three basic components, a set of players N, a set of strategies S associated with the players, and a set of outcomes associated with every n-tuple of strategies, i.e., with every selection of a strategy by each of the n players.

In the limited context proposed here we can separate out structure and behavior, thereby leaving out the evolutionary aspect of interaction between the two. In order to keep the structure simple we primarily limit our concern to the set of all 2×2 matrix games with ordinal payoffs and possible ties. We return later to a brief discussion of who the players are, how the outcomes are calculated and what the strategies might be. Here we consider the anatomy of the set of all games.

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[2]It is fairly evident that for many purposes it is desirable to generalize to any size matrix with different strategies for each agent and with entries of any size; but for expository purposes and in concert with the first set of experiments considered, the heavy restrictions on structure are left in.

The domain of the set of ordinal 2×2 matrix games consists of $4^4 \times 4^4 = 65{,}536$ different games. If symmetry conditions are taken into account, this can be reduced to 726 strategically different games as indicated in Table 1.

	4321	3321	3221	3211	2221	2211	2111	1111	
4321	78	72	72	72	24	36	24	6	384
3321		21	36	36	12	18	12	3	138
3221			21	36	12	18	12	3	102
3211				21	12	18	12	3	66
2221					3	6	4	1	14
2211						8	6	3	17
2111							3	1	4
1111								1	1
Total									726

Table 1
Ordinal 2×2 Games with Ties

Even limiting ourselves to this class of games it is fairly obvious that to experiment with all 726 games would be a Herculean task. Going to the 3×3 matrix the number of strategically different matrices is around $9! \times 9!/3! \times 3!$ or 3.658 billion.

As is noted below the only complete set of matrix games susceptible to exhaustive investigation is the set of strictly ordinal 2×2 games.

2.1.1. A Caveat on the 2 x 2 Games

The 2×2 game is attractive for both experimental and didactic reasons, as has been noted above. In particular, much of decision-making involves binary choice and much of human interaction is dyadic. Yet it is important to keep in mind the open question as to whether the two-person, two choice paradigm, though simple and experimentally attractive is central or is misleading in the sense that it directs attention away from more basic problems in strategic analysis and gives us a false impression of techniques and insights that do not generalize to larger strategic settings.

When contrasting the 2×2 game with larger matrices one must note that for the 2×2 games pure strategy equilibria exist except for a few games; in contrast as the matrices grow large, the chance of the existence of a pure strategy equilibrium approaches $1 - 1/e$ and the mixed strategies, although few for the 2×2 game, proliferate with more strategies and players (von Stengl [1999]).

2.2. Structure and Behavior

How should, or how will individuals play the complete set of 2×2 matrix games? Utilizing a closed complete set of one shot matrix games Rappaport, Guyer and Gordon [1976] addressed this problem for the 2×2 game with strictly ordinal preferences (no ties considered). There are $4! \times 4! = 546$ games in this class. Removing the symmetries, they reduced them to 78 and they ran experiments on all 78 games. Given the state of computer assistance at that time even this was a daunting task. Their book remains to this day a somewhat unappreciated classic in experimental gaming. They had an ambitious program aimed at studying behavior in all of these structures for both one shot and repeated plays.

Prior to discussing behavior, some further observations are made on structure.

2.2.1. Some Notes on Structure

RGG argued that their set of 78 games is the simplest non-trivial closed set of all different strategic structures, as has been noted in the companion essay [Shubik 2012)]. RGG constructed taxonomy of these games based on behavioral considerations. It is possible to construct several reasonably natural structural taxonomies of these games. Possibly the least controversial is to break them into 3 categories. For the unreduced 576 games the first category consists of 144 games of coordination, where one cell has the joint optimum; the second category consists of 24 games of pure opposition and the third category has 408 games where the coincidence of joint interest or opposition of interest is mixed.

	L	*R*
L	4,4	a_1, a_2
R	b_1, b_2	c_1, c_2

Table 2
Games of coordination

where a_i, b_i, c_i are any permutation of 1, 2, 3.

	L	*R*
L	4,1	2,3
R	3,2	1,4

Table 3
A constant sum game

Because the class of 2×2 matrix games is the only non-trivial class of $n \times n$ games that can be explored exhaustively[3] Barany, Lee and Shubik [BLS 1992] considered, as a simple example, the four pairs of payoffs as points in a two dimensional space. Connecting the payoffs with arcs enabled them to illustrate all 24 convex hulls of these games and they noted that these can be reduced by rotation and reflection to 7 basic shapes. If the payoffs are interpreted as cardinal, these hulls indicate the payoff sets attainable by the employment of mixed strategies.

Of the 24 structures there are:

6 structures with a 1-point Pareto set,
13 structures with a 2-point Pareto set,
4 structures with a 3-point Pareto set,
1 structure with a 4-point Pareto set.

Table 4

Appendix 1 shows the 24 structures in detail. There are only 7 different shapes that generate the others by rotation. Three of the structures are illustrated below in order to illustrate a further link between side-payment and no-side-payment game structures and to show extremes as well as how considerations of individual rationality limit the acceptable segment of the payoff sets.

The structurally most optimal games. Figure 1 shows the one-dimensional payoff set for the 6 structurally most optimal games. The Pareto set consists of the single point (4,4). The payoff set is a line from (1,1) to (4,4).

[3]The 1×1 game does not have much to analyze. Its existence merits consideration. It is formally defined; however the meaning of the choice of no choice poses some problems.

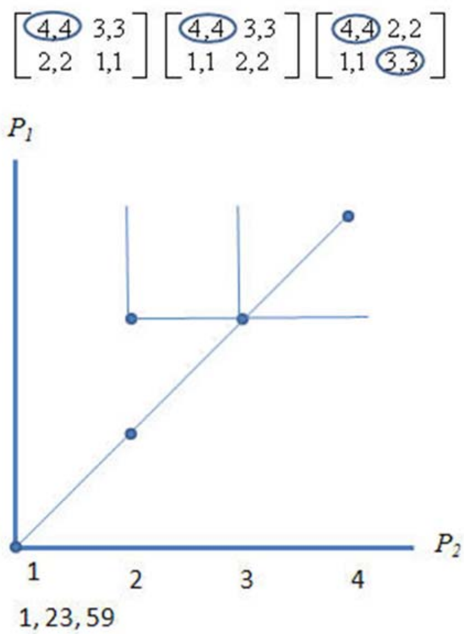

Figure 1
Some coordination with IR limits

We include in Figure 1 the limitation on the payoff set placed by individual rationality. This is obtained from calculating the maxmin for each player and plotting the joint maxmin point. This has been done for the 3 games shown (there are 6 in total). Games 1 and 23 (RGG numbering) have the *IR* limit at (3,2), while the Game 59 has the *IR* point at (2,2).

The Prisoner's Dilemma and permutations. The Prisoner's Dilemma game belongs to the payoff structure shown in Figure 2. It is the two dimensional set of points in the area denoted by *ABCD* with the maxmin at (2,2) which is also the unique NE for these games with ordinal payoffs. Thus the individually rational payoff set is bounded by *AKDL*.

A link between games in strategic form and side-payment games in coalitional or cooperative form can be forged if we were to consider the payoffs as directly transferrable. The individually rational payoff set is enlarged from the area enclosed by *AKDL* to that enclosed by *AIDJ*. The cooperative game with side payments would have a solution on the line *IDJ*; without side payments, its outcome would be restricted to *KDL*.

The Prisoner's Dilemma game lies in the set of mixed motive games. The sets of outcomes from the games of pure opposition (Figure 3) can be represented in Figure 2 by points M, N and P on the line BC. There are no side payments feasible in a game of pure opposition because the individually rational segment of the Pareto set is always a single point (*M*, *N* or *P*).

The Pareto set for the games of coordination (Figure 1) with side payments is given by the line through *GH*. The individually rational segment of this line is determined by the maxmin conditions of the specific game. The cooperative solutions, the core and the value can be easily calculated. The core for a two-person game coincides with the individually rational segment of the optimal surface which is *GH*.

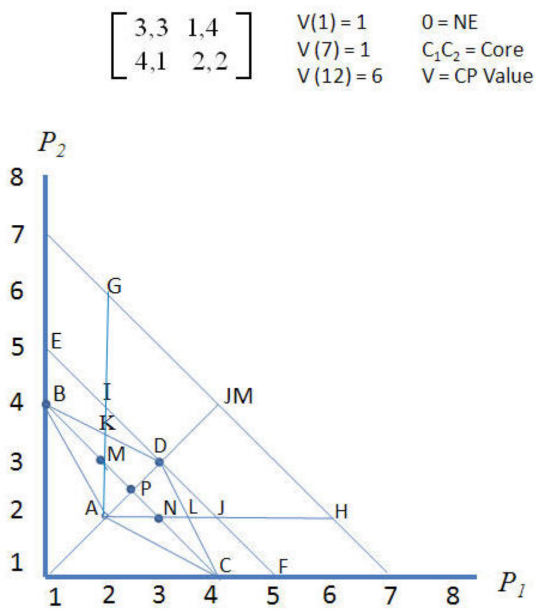

Figure 2
The P.D.: Non-cooperative and cooperative structure

The constant sum games: All is optimal. The constant sum games also have a one-dimensional strategy set as is indicated in Figure 2. Structurally they may be regarded as the most cooperative in the sense that all outcomes are Pareto optimal. There is no gain from cooperative behavior. In Game 11 the unique solution is selected by the minmax conditions at (2,3) and in Game 45 at (3,2), while Game 75 requires mixed strategies with expected payoff of (2.5, 2.5).

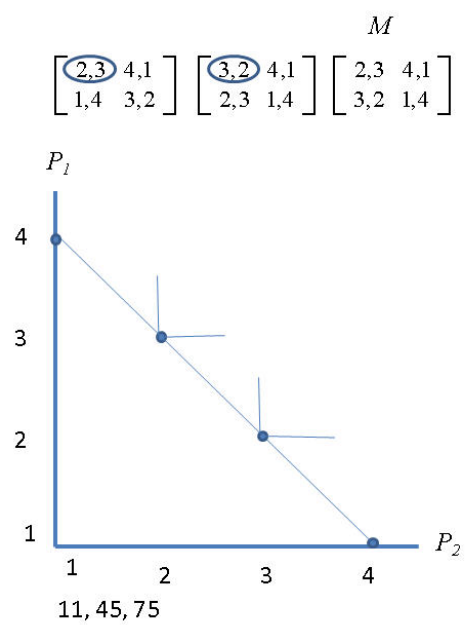

Figure 3
Games of pure opposition

The social psychologist may argue that there is no such thing as a game of pure opposition. At least both sides must consent to playing the game. This, itself, indicates that a considerable context, such as being[4] part of an experiment, has been accepted.

2.3. 78 or 144 Games?

The original work of RGG reduced the 576 games to 78. However Robinson and Goforth [2005] noted that the reduction to 144 games was more appropriate. The computation of 576 games as 576 = 66 × 8 + 12 × 4 shows that although the permutation of rows and columns reduces the games to 144, the interchange of the row and column player only yields the strategically identical game for 12 out of the 144. 144 games is a rather large number to have a single individual play, but 78, though rather large, is feasible. One can split the 144 games into two sets of 78 games. In the first set, there are 66 games favoring the column player and 12 symmetric games.[5] In the other set, there are 66 games favoring the row player (the transposes of those favoring the column player) and 12 symmetric games.

If we consider that the payoffs are comparable and additive, we may obtain upper and lower bounds on the sum of the payoffs for any solution concept utilized. In particular, we may obtain upper bounds for the sum of all noncooperative equilibrium outcomes by observing that all of the mixed strategy games yield (2.5,2,5) and assigning the NE that assigns the largest joint payoff to the players when there are two NEs, and the smallest for the lower bound. Table 5 illustrates the bounds on the scores attainable from playing 78 games together with the upper and lower feasible scores.

Joint Maximum	538
78a Row NCE	514
78a Column NCE	512
78a Row IR (ps)	364
78a Column IR	364
78b Row NCE	512
78b Column NCE	514
78b Row IR (ps)	364
78b Column IR	364
Joint Minimum	242

Table 5
Scores from various behaviors

Out of the 78 games there are 21 with a maximum score of 8 (the coordination games); 31 with 7; 23 with 6 and 3 with 5 (the strict opposition games); thus the joint maximum is 534. The individually rational (IR) bound for any individual is given by the maxmin calculation. The noncooperative equilibrium scores presented favor the row players as they are obtained by selecting the best joint score NE whenever there are two equilibrium points. All mixed strategy equilibria are evaluated at (2.5,2.5).

The joint minimum is noted with the word “joint” in it to denote that it takes considerable coordination of efforts to score as badly as this. There are 21 games with a joint minimum of 2, 31 with 3, 23 with 4, and 3 with 5.

Table 6 shows the distribution of the 80 pure strategy noncooperative equilibria over all 78 games.

[4]A reasonable analogy is that for game theory. It is a “Flatland” (see Abbot [1884]).

[5]These two sets of games are specified in Appendix 2 of the associated paper, Shubik [2012].

	Number of games	Pareto optimal	Not optimal
0 PSNE	9		
1 PSNE	58	57	1
2 PSNE	11	16	6

Table 6
Equilibrium Point Distribution

Nine of the games call for mixed strategies. Only 7 out of 80 pure strategy NEs are not optimal.[6] The special attraction of the Prisoner's Dilemma is that it is the only 2×2 matrix game where the unique NE is not Pareto optimal. As soon as we go to larger matrices or more players the proportions change considerably and one must be cautious in generalizing from the 2×2 matrix game to more complex structures without specific justification. The noncooperative equilibria are generically not Pareto optimal (see Dubey and Rogawski [1990]).

A reasonable question to consider is how badly suboptimal they are. This requires setting up a measure. Table 5 suggests that a reasonable measure is to consider the distance between the individually rational outcome and the net maximum to be 100% and then to measure by how large a percentage the expected value for the NE fails.

2.4. Behavior Related With Structure?

The set of games we are considering offer a complete closed set of different strategic environments. They present a closed domain in which to examine behavior under all possible structures in a highly limited world. By considering games played only once, we have ruled out the evolutionary interaction between behavior and structure. But given the class of simple mathematical structures we can ask, and hopefully answer, some questions about how they might be played or suggest some normative features about how they should be played. We first address the normative aspects of how these games should be played.

2.4.1. A General Solution Concept

A completely general solution concept for a game in strategic form is that it is some subset of strategy pairs. A strategy pair $(s_i, \bar{s}_j)$ where *i* and *j* lie in the strategy sets *S* and $\bar{S}$ of the two players, will select an entry in the payoff matrix[7]. Without further qualification this suggests that all outcomes in the bimatrix are in some solution set. A more useful solution concept should be one that selects a subset of the outcomes. Thus we need to specify operators on the payoffs of the individuals in the matrix game that cut down on the set of outcomes. Ideally this might cut down the outcomes in some solution to a single point.

The approach here for the normative solutions is not directly on the selection of strategies, but on conditions imposed on outcomes that limit the set of outcomes.

2.4.2. Normative Criteria

A listing of normative criteria for the properties that we might like a solution to have includes: 1) Existence over the whole domain of games; 2) Uniqueness of the selection for any game; 3) Symmetry; 4) Individual rationality (IR); and 5) Row or column domination. These items may or may not be regarded

[6]This table shows only the pure strategy equilibria. 20 of the games have a mixed strategy noncooperative equilibrium; however only in the games of pure opposition is the mixed strategy Nash equilibrium always Pareto optimal.

[7]It is reiterated that this general definition of solution for a game in extensive form is useful for existence proofs, but is so general as to appear to be of little operational use, except for small games. A solution is a set of paths down the game tree from the origin to a set of terminal points. This permits any set of completed plays to be selected as a solution.

as solutions by themselves, but as properties that can be present individually or severally in defining a solution.

Some comments are noted on these conditions.

Individual rationality: In any matrix game individual rationality can always be satisfied. This normative condition requires that the outcome of the game be restricted to the subset of individually rational outcomes.

Symmetry: Social symmetry requires that an interchange of names of players should not influence the outcome from any acceptable solution. For example, in front of the judge the same crime will receive the same penalty regardless whether the names of the individuals charged are switched.

Structural symmetry is a property of the game structure. All individuals have equivalent strategy sets. In the reduced set of all the 2×2 games studied here only 12 games have the structural symmetry property.

Existence: Does the solution proposed always exist over the selected domain of games? For example, if the solution concept selected is that of a pure strategy non-cooperative equilibrium (PSNE) only 69 out of the 78 games have PSNE.

Uniqueness is clearly an extremely strong condition, highly desirable for planning and prediction. 58 out of the 78 games have a unique PSNE.

Row and column domination and independence from irrelevant alternatives amount to the same for the matrix game. If some row or column of payoffs is dominated by another row or column there is no motivation to employ it.

Ideally it would be highly desirable to have a solution that has all of these properties. We can construct a weaker solution that satisfies only some of the desiderata. Five criteria have been suggested, thus we can consider the various combinations of the 5 properties.

Then there are two well-defined conditions that separate the schools of thought leaning towards cooperative solutions and noncooperative solutions.

1) Group rationality or Pareto optimality (PO).
2) Consistency of prior expectations, or the NCE property, or rational expectations (NCE).

Pareto optimality is a normative criterion of considerable attractiveness. It implies that nothing should go to waste; it should not be feasible for any individual in the society to improve her payoff without decreasing the payoff to someone else. The non-cooperative equilibrium solution to the Prisoner's Dilemma game does not have this property.

A basic critique of those supporting Pareto optimality is that it is a static criterion that gives no indication as to how it is to be achieved. Neither the costs of the resources consumed in the dynamics of process, nor the coordination often needed, is accounted for.

Should one regard the non-cooperative equilibrium solution as normative or as a behavioristic observation? The formal mathematics requires that one assumes that all players are equally capable individualistic optimizers. However, one could interpret this non-cooperative individualistic behavior as a norm. The study of mass markets can reinterpret the many person game as a two-person game of the individual playing a single aggregate anonymous player called “the aggregated others.” It is here that questions of aggregation and low information enter fairly naturally. The topics of asymmetric information and aggregation are critical to both experimentation and the development of the theory of the playing of multistage games, but are not covered in these two expository papers.

Appendix 2 illustrates some properties of the 144 games. The games are shown with an indication after the display of the matrix of: 1) the shape of the strategy set; 2) the joint maximum; 3) the number of pure strategy non-cooperative equilibria; 4) whether the game is symmetric; 5) and 6) the row and column scores from the non-cooperative equilibria; 7) the number of dominated rows and columns; 8) the number of Pareto optimal outcomes; and 9) the number of the associated transposed game.

More sociological criteria. Less clearly defined, but relevant to more sociologically slanted concepts of solution are desiderata such as: 1) "fairness," 2) envy-free outcomes, and 3) decentralized structures.

Beyond these conditions there are many more considerations such as the role of language, the distinction between face-to-face and anonymous communication, the role of gesture, and the role of outside enforcement. We limit our considerations here only to the conditions noted above beginning with individual rationality.

2.4.3. Preference Conditions

The assumptions concerning preferences appear to be more positive or empirical than normative. Even in this limited instance we could consider: 1) independent, ordinal preferences where the preference ranking depends only on the outcome to the individual; 2) ordinal preferences where the preference ranking depends on the outcomes to both players; 3) independent cardinal utility with no comparability; and 4) independent cardinal utility with comparability.

There is a vast and sophisticated literature in both individual and social preference theory (see for example Fishburn [1970], Hammond et al. [2004, 2007]) covering items such the independence of irrelevant alternatives, partial orderings, and experimental attempts to measure utility all of which we bypass here. In much experimental gaming, a draconian simplification is made. It is assumed that as a reasonably crude approximation individuals may be risk averse, but may have linear utility for money in the small range covered by many experiments.

2.4.4. Mixed, Correlated or Pure Strategies?

As was indicated in von Neumann and Morgenstern [1944] an axiom extending individual preferences over gambling is sufficient to justify a cardinal measure of utility for an individual with completely ordered preferences. The presence of a cardinal utility measure under uncertainty provides the means to guarantee the existence of a noncooperative equilibrium in all matrix games. Whether individuals utilize mixed strategies in any or all contexts is an open empirical question.

One can go a step further and also consider under what circumstances a correlated equilibrium would be reasonable. A simple example both illustrates and raises questions about the potential for a correlated equilibrium. Table 7 shows a version of the Prisoner's Dilemma game where the incentive for the players to abandon the unique pure strategy equilibrium and replace it by playing a correlated mixed strategy with the probability of (.5,.5) on the strategy pairs (*U*,*R*) and (*D*,*L*) gives expected payoff of (50.5,50.5) to the players, in contrast with the payoff of (2,2) at the PSNE. Unfortunately this does not have the self-policing properties of an NE. If column departs from the agreement and merely plays *R* then he will obtain an expected payoff of 51, slightly better than 50.5. This contrasts with the game shown in Table 8, where if the two players assign a correlated mix of (.5,.5) to (*U*,*L*) and (*D*,*R*) they obtain an expectation of 30 each. But now if one player deviates she obtains an expectation of 25. Hence the correlated strategy pair is a correlated non-cooperative equilibrium in the sense that no individual has an incentive to violate her commitment to adhering to the correlated strategy. Left out of this discussion is how the players go about agreeing to play a correlated strategy. What mechanism or institution is called for?

	L	*R*
U	3,3	1,100
D	100,1	2,2

Table 7
A Prisoner's Dilemma Game Variant

	L	*R*
U	50,10	0,0
D	0,0	10,50

Table 8
A Battle of the Sexes Game Variant

There are two attractive features to correlated strategies. The first is purely mathematical: they are far easier to locate and calculate than mixed strategies (see Gilboa, and Zemel [1984]). The second is that they may arise quite naturally in repeated play situations (see Hart and Mascolell [2005]).

At the level of simplicity presented here there are 9 games without pure strategy equilibria. If we limit the non-cooperative equilibrium to pure strategy solutions, these games have no equilibrium. If we permit mixed strategies then we should be able to view these games as having equilibria based on cardinal utilities. In the preliminary experimental games, a chi square test rejected the hypothesis that the noncooperative strategies were being played (see Appendix 6).

Appendices 3 and 4 show the set of symmetric games and the set of mixed strategy games respectively. The full lines with an arrow on them indicate the optimal response of the row player from his current position if he had historical information on the previous play. The dashed lines with an arrow show the same for the column player. Appendix 5 covers the remaining matrices in the set of 144.

3. Norms, Laws and Individual Behavior

Many normative considerations have been noted. They each provide different limitations to behavior if they are to be realized. Some sets of the constraints may be inconsistent.

A way of considering the linkage between the norms, laws and customs of the society and individual or group selection of strategies is, that, to a reasonable first order approximation, they represent the longer term context in which short term behavior takes place.

3.1. Behavior and Decentralization

A favorite norm in economic theory is decentralization. The reason why competitive individualistic markets appear to be so attractive is that they have the special property that any submarket of a market is a market. The essential structure remains the same at any size. Furthermore, under reasonable conditions in classes of games such as the strategic market games, the noncooperative equilibria approach efficiency as the market sizes increase.

3.2. The NCE and "The Price of Anarchy"

In computer science there is interest in a problem colorfully entitled "The Price of Anarchy" (see Halpern [2003)], Rothblum [2007], Roughgarden [2009]). This problem, in game theoretic terms, explores the question as to how inefficient the noncooperative equilibrium is in comparison with the joint maximum in various problems. One of the reasons for including the consideration of the joint maximum that could be achieved if the matrix games considered here were played as cooperative games with side-payments, is to connect this approach with the anarchy literature and to explore how relatively inefficient are the noncooperative equilibria in comparison with the highest fruits of cooperation with a single one-dimensional numerical measure that can be interpreted as a monetary worth. This may be regarded as an observable approximation that serves as a proxy for utility maximization in much of economic activity. It is easy enough to produce situations where this simplifying approximation may be deemed insufficient; but it is proposed here that it is worthwhile in the exploration of these abstract games to try to construct the simplest possible index of inefficiency of the NE prior to seeking more complex measures.

3.3. Measures of Inefficiency

Even with the simplification of a one-dimensional measure there are several choices to be made. Three are noted. Let JM stand for joint maximum, SumNE stand for the sum of the expected payoffs at a non-cooperative equilibrium, and IR stand for the sum of the two minimal individually rational payoffs. We may consider these three indices:

Index 1: $I(1)$ = SumNE/JM expressed as a percentage.

Index 2: $I(2)$ = (JM-SumNE)/JM expressed as a percentage.

Index 3: $I(3)$ = (SumNE-IR)/(JM-IR).

The first index expresses the payoffs at the NE as a percentage of the joint maximum. The second is essentially the complement of the first. It indicates by what percentage efficiency is lost. The third index is possibly more preferable inasmuch that it is concerned with the range between the lowest individually rational outcomes and the joint max[8]. Using criteria 1 and 3 in application to the Prisoner's Dilemma game (table 9), $I(1) = 4/6 = 66.7\%$ whereas $I(3) = (4 - 4)/(6 - 4) = 0$ It is completely inefficient inasmuch as it offers no gain over purely introspective maxmin behavior. For constant sum games $I(3)$ yields 0/0, but in this instance it is reasonable to define 0/0=1.

	L	*R*
U	3,3	1,4
D	4,1	2,2

Table 9
A Prisoner's Dilemma Game

3.4. An Added Incentive for a Measure

The measure of the inefficiency of the noncooperative equilibrium with respect to the joint maximum serves to provide a crude estimate of the amount of resources a society could afford to spend on a coordinating device, be it interpreted as a government, rule-maker, coordinator and enforcer or a referee. In much of microeconomic theory, for example general equilibrium theory, government is left out. In macroeconomic theory it is always in. In any reconciliation of the two, the abstract game model requires not merely a large number of consumers and producers, but at the very least a large atomic player, the government, its agencies and other institutions, to provide and enforce the laws and other means for coordination.

4. Behavioral Solutions

In the development of game theory, three forms are suggested as basic abstractions for a game: the extensive form, the strategic form and the coalitional, or cooperative, form. The third is essentially static and given over to the exploration of normative theorizing dealing with the constraints imposed by properties such as fairness, individual rationality, group rationality, and optimality. The other two representations deal with playing the game, not just negotiating over how the proceeds from the play should be split. The selection of strategies by all individuals can be regarded as behavioral even though the outcome may satisfy some normative properties.

[8] It is independent of affine transformations.

4.1. Some Popular Players

For the simple 2×2 matrix game played once it is possible to more or less list all the ways the game can be played, given some weak conditions on context. The listing below was compiled from unpublished joint work with Michael Schapira.

This includes among the most well-known player types: 1) the random or entropic player; 2) the optimal response player; and 3) the risk minimizing or maxmin player.

The random, or entropic player at most needs to know the size of his strategy set, but nothing else.

The optimal response player requires information on the initial conditions, the knowledge of her preferences and the ability to look across a row or column.

The risk minimizing or maxmin player must be able to view only his payoffs in the bimatrix as a whole and compute the maxmin over all rows or columns. In constructing simple automata one might wish to deny the ability to look across rows or columns and restrict scope to neighbors in the matrix.

We could consider that the game is played by two different behavioral types. Thus, there are 9 pairs of players to be considered.

4.2. A Listing of Many Player Types

Before listing the players, we may note that it is possible and meaningful even in a one play game to make the decision as to whether to supply any history in the initial conditions. We have the opportunity to merely supply the matrix with no previous history given whatsoever, or we may brief the players accordingly: "your predecessors played the strategy pair $(s_i, \bar{s}_j)$ and you both have been informed of this." A knowledge of history could be given for many periods back, but in keeping it simple we note at most, only one.

1) **The Constant Player (2):** A constant player is a player that selects the same strategy regardless of the specific game being played, or the history of the play. We may denote by C_L the player who always selects L and by C_R the player who always plays R. The number in the parenthesis indicates the number of player types in this category.

2) **The Entropic Player (1):** With careful institutional design the entropic player does not even need to know the bounds on his strategy set. They may be forced on him. For example in a double auction market, on the up side, any bid above his credit line may be rejected; on the downside there is a natural minimum at zero.

3) **The Maxmax Player (1)**: The maxmax player is the ultimate cooperator. She always chooses the row or column with the joint maximum in it.

4) **The Maxmin Player (2):** She is the ultimate pessimist. We may distinguish two types differentiated by some sophistication. One employs only pure strategies; the other may employ mixed strategies.

5) **The Maxmin the Difference Player (2):** She is the ultimate opponent. We may distinguish two types differentiated by some sophistication. One employs only pure strategies, the other may employ mixed strategies. (Maxmin the difference manifests itself in a well-known military context of optimizing the damage exchange rate. It can be called with some reason, "illfare economics" and has a natural use in establishing threats.)

6) **The Minmax Player (1):** The minmax player may be regarded by some as irrational in the sense that the action to damage the other may incur damage to the self larger than the minimum payoff

she could guarantee herself. Such behavior falls into the domain of suicide bombers and may require that the meaning and dimensionality of the payoffs be considered in a different light.

7) **The Regression Player** (∞)**:** We note, but do not consider further, the infinite number of introspective players whose behavior is based on considering regresses such as "I believe that the other player believes that I believe … etc…."

8) **The Best Response Player (1):** She always selects the strategy that is the best response to the current profile. We note that there is a considerable difference between simultaneous and sequential optimal response. The coordination problem is far more difficult with the first (see Quint,Shubik and Yan [1995]).

9) **The Markovian Player (16):** The Markovian player is a player whose selection depends solely on his knowledge of the last strategy profile employed. For the 2×2 games considered here the number of different players is $2^4 = 16$ which is large but manageable. For even a 3×3 matrix the number would be unwieldy.

For the 2×2 game the Markovian strategies can be enumerated fairly simply. All of the players (who may be regarded as strategies) are of the form:

If the previous state were (*L*,*L*), I play *i* (where i= 1,2);
If the previous state were (*L*,*R*), I play *j* (where *j* = 1,2);
If the previous state were (*R*,*L*), I play *k* (where *k* = 1,2);
If the previous state were (*R*,*R*), I play *l* (where *l* = 1,2).

The actions of any two players selected from the sixteen sets suggested above are sufficient to determine an outcome to the game. A single outcome may be regarded as a solution by itself. After all, it provides a complete set of instructions as to how to get to a final payoff. We may, however wish to regard a solution as consisting of a set of outcomes having certain properties in common.

4.3. An Aside on Evolutionary Game Theory

In this paper, as the main concern is with the behavior of humans, often in dyadic relationships and in relatively small populations as compared with insects or smaller biological organisms, the discussion of evolutionary game theory is omitted beyond noting that both it and many of the problems with the non-cooperative equilibrium as applied to humans and other biological entities have been covered by books such as Weibull [1995] and Samuelson [1997] and thoughtfully surveyed by Hammerstein and Selten [1994] and by George Mailath [1998]. It is my belief that the analogies between relatively low intelligence organisms acting in natural environments and humans acting within institutions erected by society as the carriers of process lead to highly different dynamics. The generalities that exist tend to be in the statics in environment poor models such as matrix games[9].

5. Behavioral Solutions with Normative Properties

Before selecting among player types, we note a few solution possibilities where each agent is expected to play a type similar to him or herself.

1. Non-cooperative equilibrium
2. Joint Maxmin
3. Maxmin vs minmax

[9]Sergiu Hart [1999] utilizing a simple example of a game in extensive form contrasts the selection and mutation aspects of evolutionary dynamics with the backward induction approach of perfect equilibria.

4. Minmax vs maxmin
5. Joint minmax
6. Maxmax NSP
7. Maxmax SP
8. MinMax difference
9. SP Value based on 7 and 8
10. Optimal response (simultaneous)
11. Optimal response (sequential)
12. Core

The Non-cooperative equilibrium: needs no further comment beyond the observation that there is the assumption that all agents are optimizers of the same type and that this is common knowledge.

Joint Maxmin: if both agents are extremely cautious, in some instances, their actions may jointly maximize as in Table 10 below. Over all games, their overall scores would be 432 as contrasted with 472 for the noncooperative players.

	L	*R*
U	4,4	3, 1
D	1, 3	2, 2

Table 10
JM and *IR* Coincide

Maxmin vs. minmax: Here one is implicitly assuming a cautious player versus an aggressive or hostile player. Over all games, their overall scores would be 360 and 357 as contrasted with 472 for the noncooperative players.

Joint minmax describes two aggressive players. Over all games, their overall scores would be 288 as contrasted with 472 for the noncooperative players.

The four solutions immediately above are purely individualistic. They and solutions 5 and 8 require no coordination or communication except to resolve non-uniqueness of the non-cooperative equilibrium. Solutions 7, 9, and 12 require communication leading up to a common agreement "to make the pie as large as possible, but argue over how to slice it."

The individual trained in economic theory will immediately reject many of the players described above in Sections 4.1 and 5 because some will not satisfy individual rationality and others appear to change the meaning of the payoffs and still others appear to have little intelligence, not unlike the automata that inhabit many models in evolutionary game theory.

Given the welter of types and solutions suggested we note that there are more solutions suggested than there are outcomes in a 2×2 matrix. Given one short game played without face-to-face communication, we limit our considerations to two types of agents, the individual maximizers and the more cautious maxmin agents. We also consider the influence of the normative conditions suggested in section 2.4.2 above.

5.1. Normative Limits

Prior to turning to notes on the preliminary run of 78 games, some limits imposed on the 2×2 games by normative considerations are noted.

Group rationality or Pareto optimality (PO). This limits the solution set to somewhere from 1 to k outcomes from k^2. **Existence** is satisfied for all games

PO and uniqueness holds for 3 out of the 78 games. **Existence with both properties** is not satisfied for the remaining games.

PO and Pure Strategy Non-cooperative Equilibrium (PSNE) holds for 57 out of 78 games. **Existence** is not satisfied for the remaining games (this percentage of PO equilibria decreases considerably as the matrix size increases, to the point that generalizing from the 2×2 is misleading).

NCE limits the solution set to somewhere between 0 to 2 pure strategy NEs for the cardinal games with at least 1 mixed strategy NE for any game with 0 pure strategy NEs. For an ordinal game, a mixed strategy is not defined.

NCE and Uniqueness holds for 58 games. **Existence** is not satisfied for the remaining games.

NCE, Uniqueness, PO and Existence. For the cardinal utility games the 3 constant sum games satisfy all criteria, as do 15 of the 21 games of coordination. If the games have ordinal preferences we must reduce the constant sum games to those that have a pure strategy solution.

In general, for a $k \times k$ ordinal matrix game with ties the percentage of constant sum games is diminishing at the rate of $1/k^{k^2}$ thus the percentage of all the closed strategic set of 2×2 games that satisfies all desiderata is 6.25%. For a 3×3 matrix this has dropped to .0005%.

The only two sets of games that fill a reasonable list of desirable properties are precisely the vanishingly small subset of constant sum games for which von Neumann noted that one could extend the concept of individual rationality and the games of coordination.

It is easy to note that properties such as Pareto optimality become vanishingly small when compared to all outcomes but much of human endeavor is anti-entropic. When one considers "all possible worlds" life is lived on a set of measure zero and what we call good solutions have desirable properties for the appropriately selected relevant set.

5.2. Some Tentative Experimental Observations

The initial run of a set of 78 games was at a game theory conference at Stony Brook in July 2011. The players were all professional game theorists. There were several difficulties and error entries into one matrix and an accidental replication of another. Furthermore, several participants dropped out before completing all games and the data on some of the latter games reflects the fewer numbers. Thus the run may be more appropriately regarded as a debugging run of the experimental set up rather than a clean experiment. Appendix 7 provides a synopsis of the results.

The calculation of final score has each row player matched against each column player for all of the games. Each player is assigned the payoff achieved in each play of all games in which he participates. Thus, for example if there were 10 row and 10 column players the sample size for each game would be 100. The final score for a player is the sum of his scores over all games in which he is matched.

The only formal statistical analysis performed on the pilot study (beyond the display itself) was to check for a frequency interpretation of mixed strategy equilibria. The results were uniformly negative. The games of coordination indicated the selection of the joint maximum for the most part with a significant decrease for the Stag Hunt game with nevertheless the joint maximum (4,4) obtaining 62% versus the risk dominant NE of (3,3) obtaining 4% and lack of coordination accounting for 35%. The games of pure opposition, when mixed strategies were not involved gave the saddle point results. The Prisoner's Dilemma (Game 12) has 96% choose the unique NE which is also the Maxmin solution. The key feature in the rapid choice play appeared to be the guidance that the presence of one or two dominated strategies offered.

Possibly the two surprises in the run were the poorness of the evidence for mixed strategies and the disinterest in risk dominance.

6. Why 1, 2, 3, 4?

A natural skeptical observation of the game theorist to make is "Why confine yourself to the payoff matrices with 1, 2, 3, 4 as entries? Why not use 1, 2, 75, 963 against 6, 4, 371 and 427, or any other eight numbers?" The experimentalist's answer is that he is keeping it simple. Unless one can show that the simplest cases are not worth checking, check them first. Once there are results on the simplest cases, then consider more complex experiments, such as RGG did in varying the size of the entries in the matrix.

Ideally from the viewpoint of theory it would be handy to have a general measure of inefficiency of the NCEs for matrix games. One approach to such a measure would be to consider all 2×2 games generated by selecting the entries randomly. We can consider a drawing 8 i.i.d. random variables from the interval [0,1] solving each game generated for its inefficiency and taking the average over a large sample generated by the randomization.

With linear utility, using the measure suggested in Section 3.3, the efficiency of all Prisoner's Dilemma games is 0. The efficiency of the largest NE in the Stag Hunt is 1 and the efficiency of the risk dominant equilibrium is (8 – 4)/16 – 4) = 2/3. With colleagues this work is being extended to the 8-dimesional hypercube to consider 2×2 games with payoffs of any size (within a finite bound).The loss of efficiency of the best noncooperative equilibrium for the 2×2 matrix game studied here is essentially 15%; it should be larger for the larger class of games.

The discussion in this essay has been devoted to games with complete knowledge. A natural extension is to consider economic agents knowing only their own payoffs together with a common prior over the others. This possibility, together with an efficiency measure, is considered elsewhere DSS?

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Appendix 1: Payoff Sets

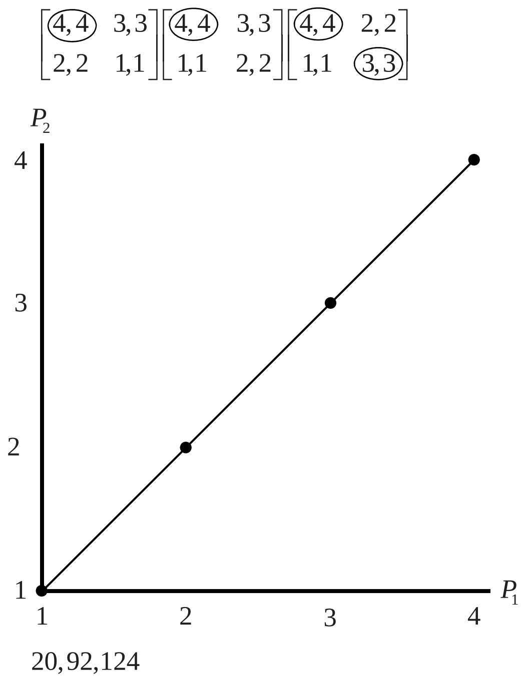


FIGURE 1

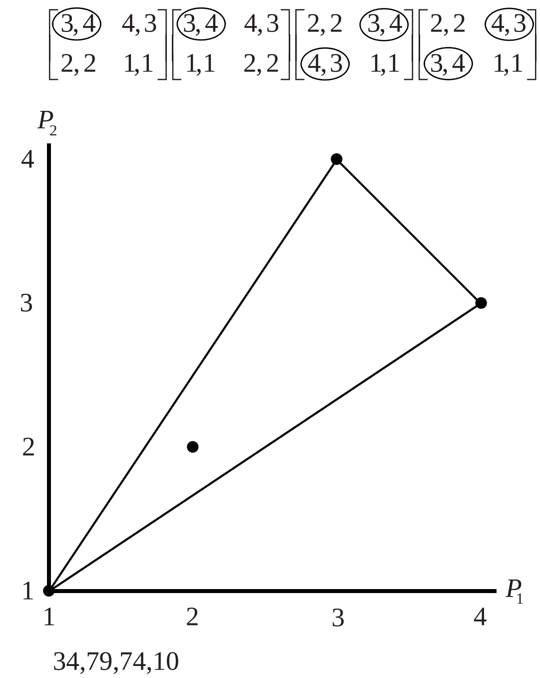


FIGURE 2

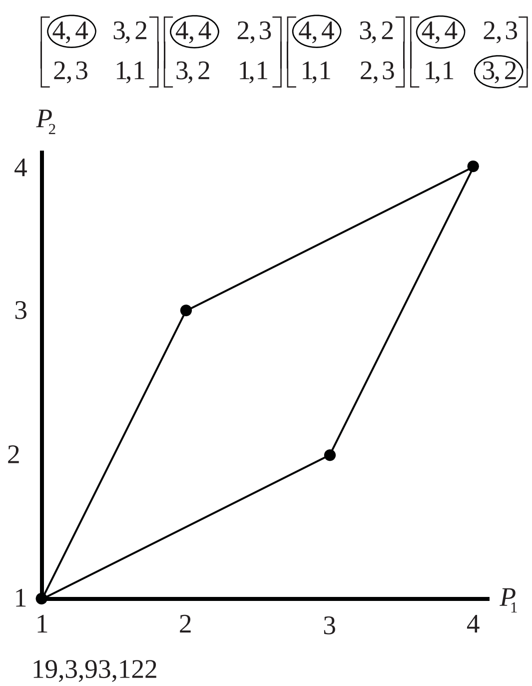


FIGURE 3

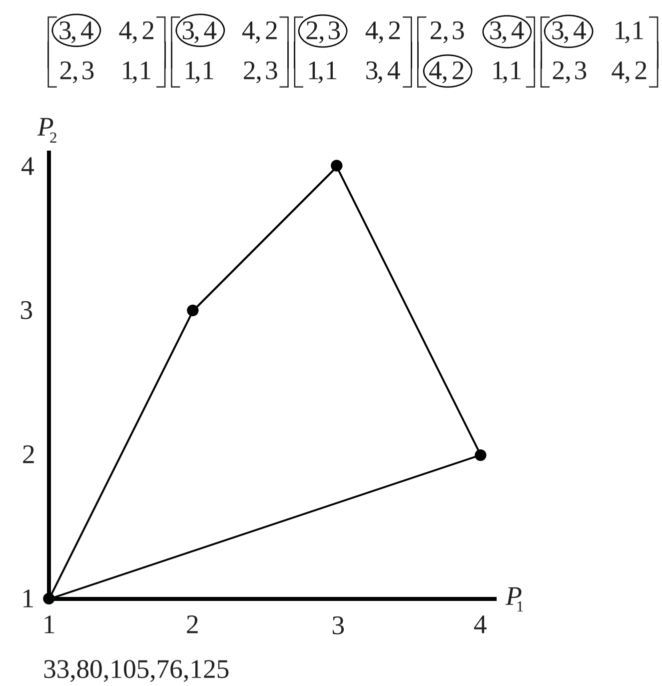
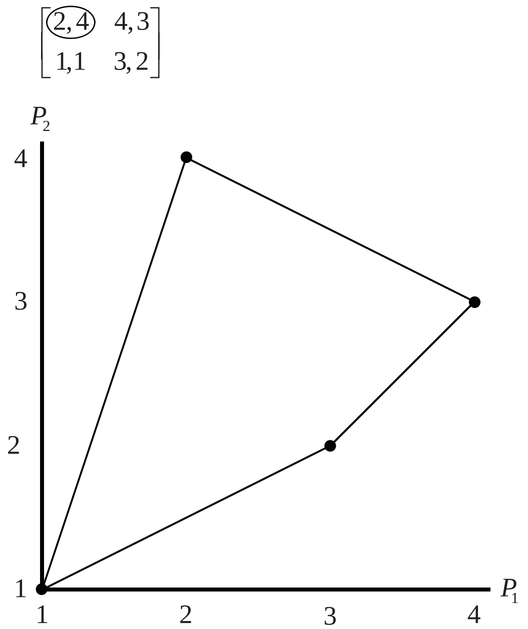
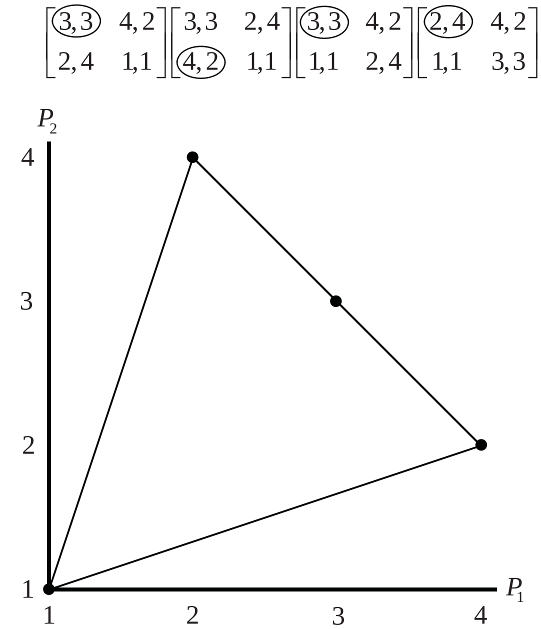


FIGURE 4



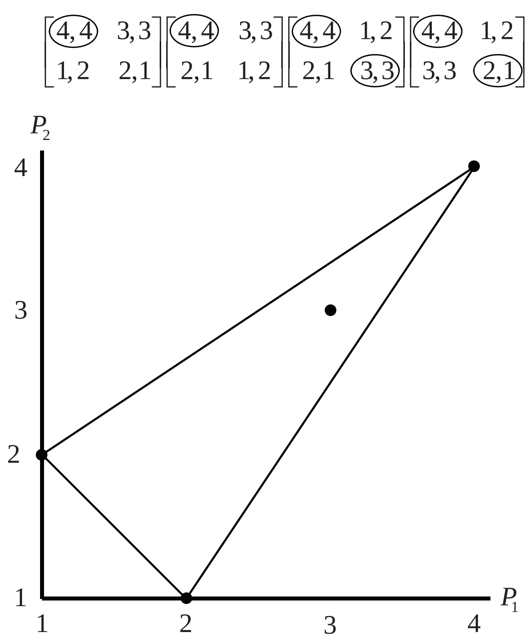
101

FIGURE 5



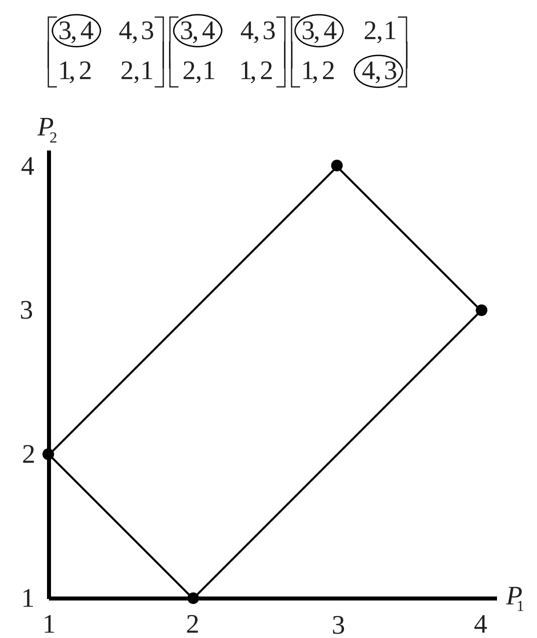
35,53,21,103

FIGURE 6



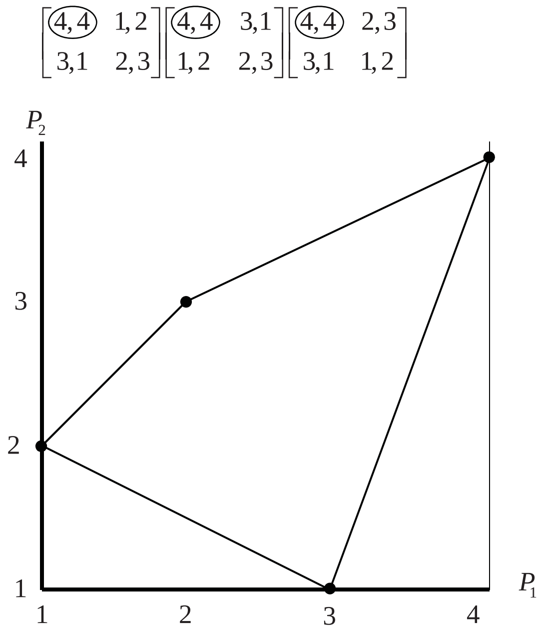
32,82,69,48

FIGURE 7



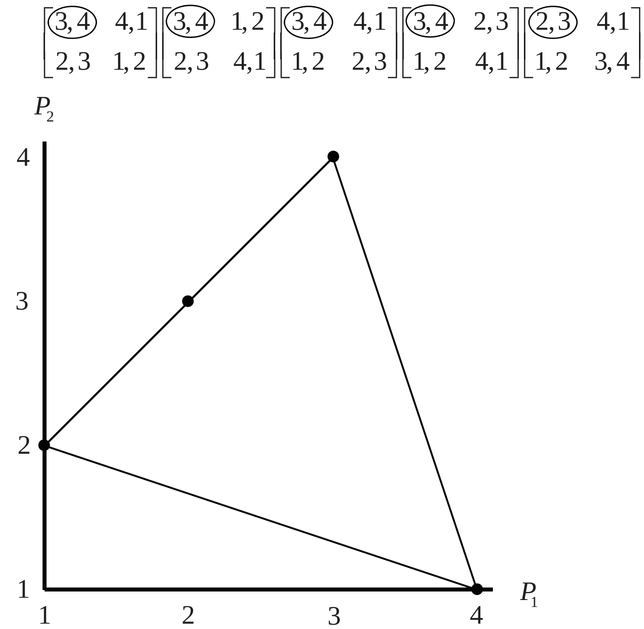
18,94,67

FIGURE 8



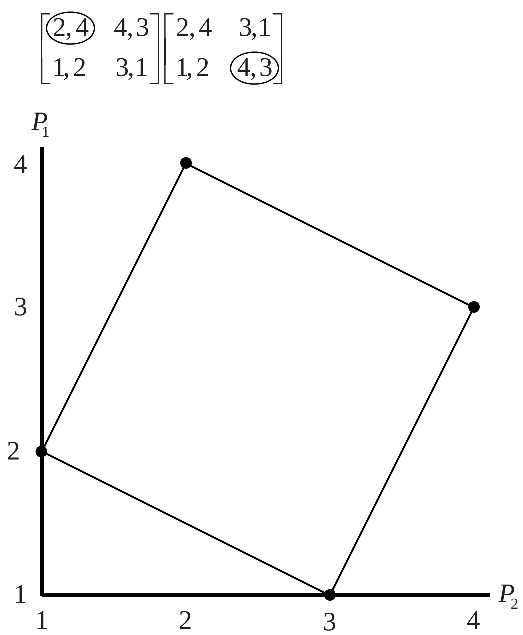
140,29,106

FIGURE 9



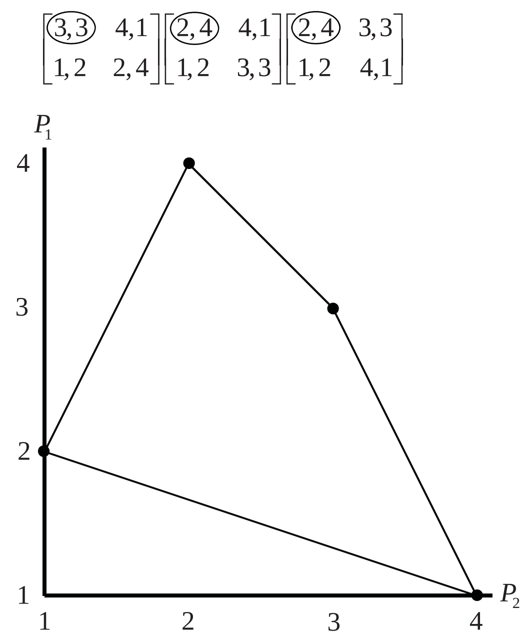
30,128,16,71,110

FIGURE 10



2,46

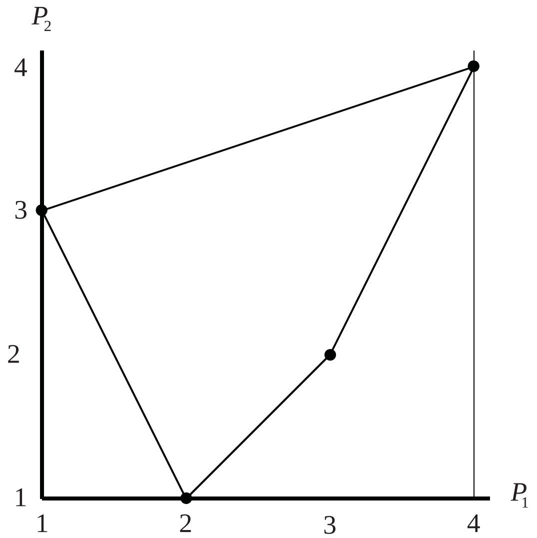
FIGURE 11



84,108,41

FIGURE 12

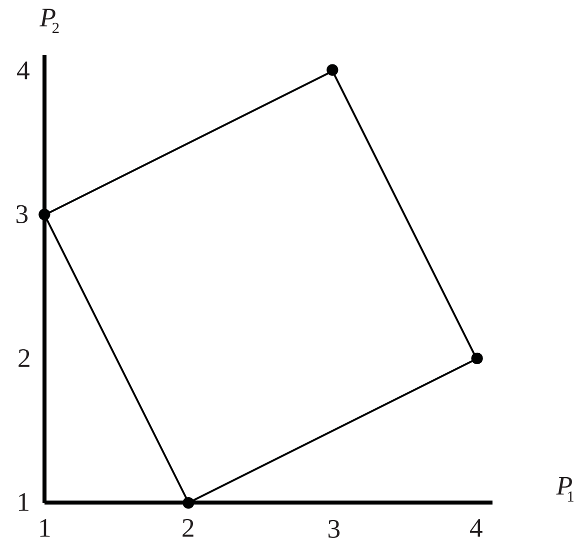
$$\begin{bmatrix} \enclose{circle}{4,4} & 3,2 \\ 1,3 & 2,1 \end{bmatrix}\begin{bmatrix} \enclose{circle}{4,4} & 3,2 \\ 2,1 & 1,3 \end{bmatrix}\begin{bmatrix} \enclose{circle}{4,4} & 2,1 \\ 3,2 & 1,3 \end{bmatrix}\begin{bmatrix} \enclose{circle}{4,4} & 2,1 \\ 1,3 & 3,2 \end{bmatrix}$$



28,85,113,132

FIGURE 13

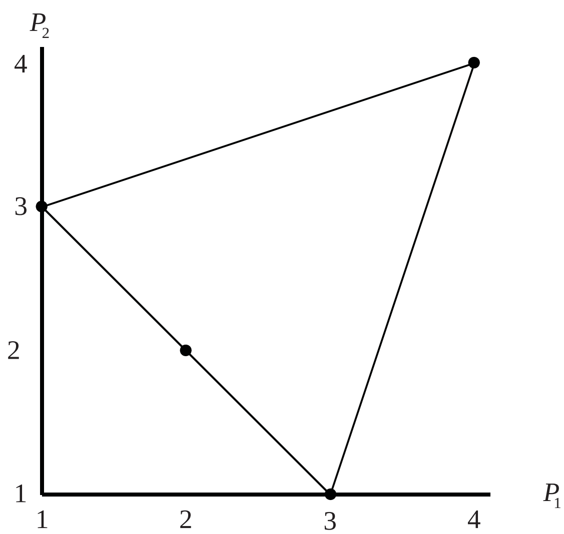
$$\begin{bmatrix} \enclose{circle}{3,4} & 4,2 \\ 1,3 & 2,1 \end{bmatrix}\begin{bmatrix} \enclose{circle}{3,4} & 2,1 \\ 1,3 & 4,2 \end{bmatrix}\begin{bmatrix} \enclose{circle}{3,4} & 4,2 \\ 2,1 & 1,3 \end{bmatrix}\begin{bmatrix} \enclose{circle}{3,4} & 2,1 \\ 4,2 & 1,3 \end{bmatrix}$$



86,63,96,44

FIGURE 14

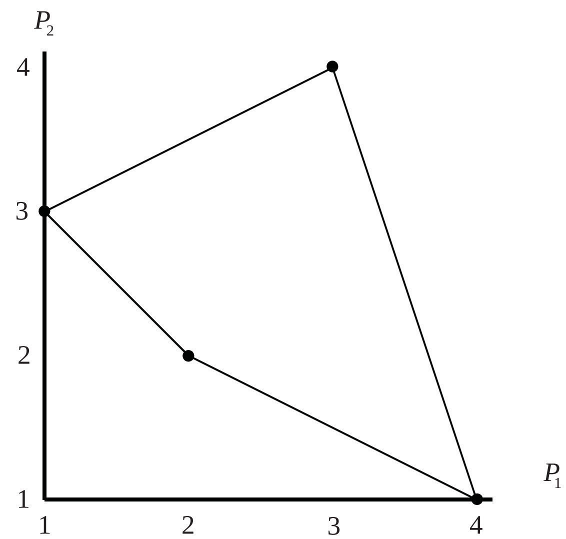
$$\begin{bmatrix} \enclose{circle}{4,4} & 3,1 \\ 1,3 & 2,2 \end{bmatrix}\begin{bmatrix} \enclose{circle}{4,4} & 3,1 \\ 2,2 & 1,3 \end{bmatrix}\begin{bmatrix} \enclose{circle}{4,4} & 1,3 \\ 3,1 & \enclose{circle}{2,2} \end{bmatrix}$$



26,15,41

FIGURE 15

$$\begin{bmatrix} \enclose{circle}{3,4} & 4,1 \\ 1,3 & 2,2 \end{bmatrix}\begin{bmatrix} \enclose{circle}{3,4} & 2,2 \\ 1,3 & 4,1 \end{bmatrix}\begin{bmatrix} \enclose{circle}{3,4} & 1,3 \\ 2,2 & 4,1 \end{bmatrix}\begin{bmatrix} \enclose{circle}{3,4} & 4,1 \\ 2,2 & 1,3 \end{bmatrix}\begin{bmatrix} \enclose{circle}{2,2} & 4,1 \\ 1,3 & \enclose{circle}{3,4} \end{bmatrix}$$

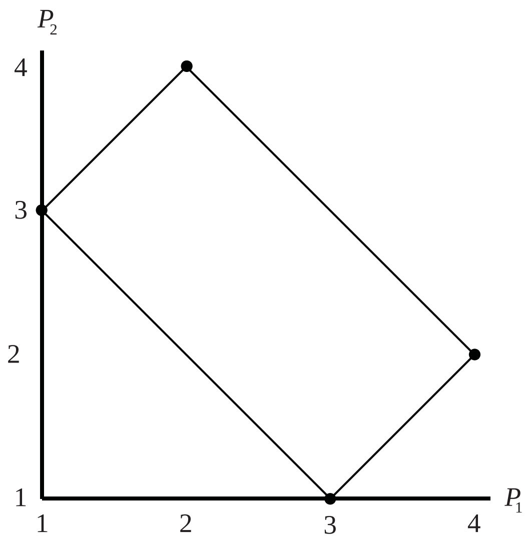


87,66,5,27,116

FIGURE 16

$$M$$

$$\begin{bmatrix} \textcircled{2,4} & 3,1 \\ 1,3 & 4,2 \end{bmatrix}\begin{bmatrix} 2,4 & 3,1 \\ 4,2 & 1,3 \end{bmatrix}\begin{bmatrix} \textcircled{2,4} & 4,2 \\ 1,3 & 3,1 \end{bmatrix}$$

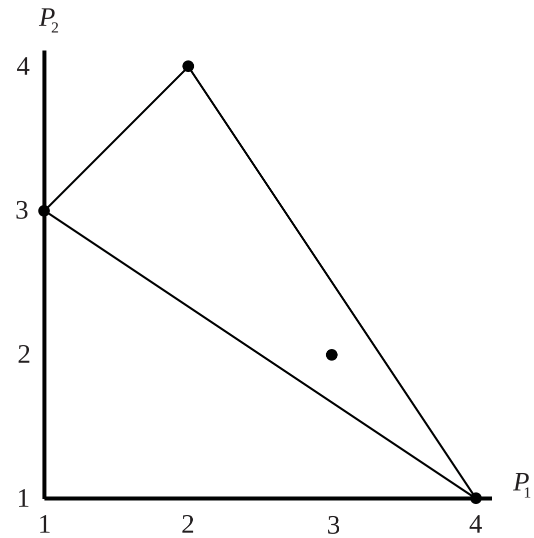


43,64,112

FIGURE 17

$$M$$

$$\begin{bmatrix} \textcircled{2,4} & 4,1 \\ 1,3 & 3,2 \end{bmatrix}\begin{bmatrix} \textcircled{2,4} & 3,2 \\ 1,3 & 4,1 \end{bmatrix}\begin{bmatrix} 2,4 & 4,1 \\ 3,2 & 1,3 \end{bmatrix}\begin{bmatrix} \textcircled{3,2} & 4,1 \\ 1,3 & \textcircled{2,4} \end{bmatrix}$$

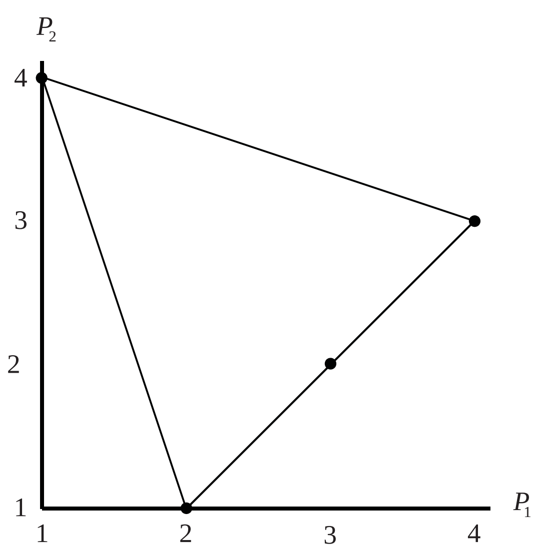


114,45,6,88

FIGURE 18

$$M$$

$$\begin{bmatrix} 3,2 & 2,1 \\ 4,3 & 1,4 \end{bmatrix}$$

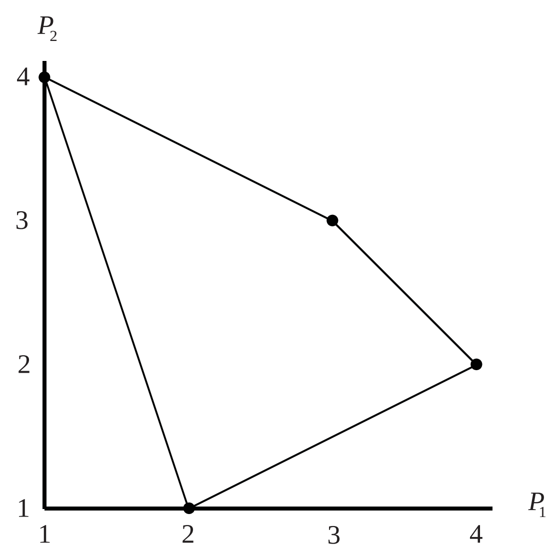


143

FIGURE 19

$$M$$

$$\begin{bmatrix} \textcircled{3,3} & 4,2 \\ 1,4 & 2,1 \end{bmatrix}\begin{bmatrix} \textcircled{3,3} & 4,2 \\ 2,1 & 1,4 \end{bmatrix}\begin{bmatrix} 3,3 & 2,1 \\ 4,2 & 1,4 \end{bmatrix}$$

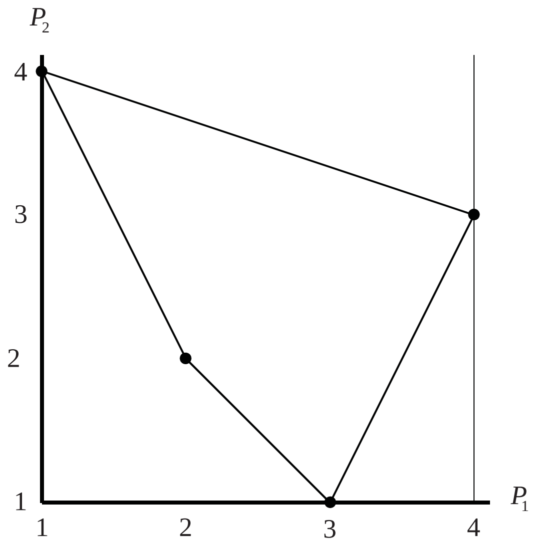


89,24,142

FIGURE 20

M

$$\begin{bmatrix} 2,2 & 3,1 \\ 4,3 & 1,4 \end{bmatrix}$$

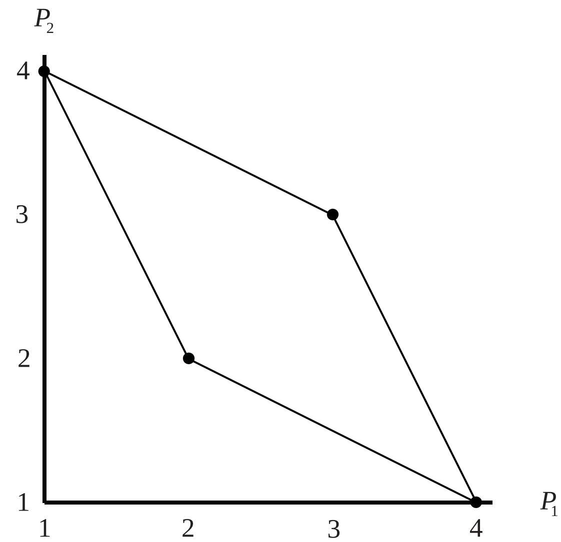


57

FIGURE 21

M

$$\begin{bmatrix} \enclose{circle}{3,3} & 4,1 \\ 1,4 & 2,2 \end{bmatrix}\begin{bmatrix} \enclose{circle}{3,3} & 4,1 \\ 2,2 & 1,4 \end{bmatrix}\begin{bmatrix} 2,2 & 4,1 \\ 3,3 & 1,4 \end{bmatrix}\begin{bmatrix} \enclose{circle}{2,2} & 4,1 \\ 1,4 & 3,3 \end{bmatrix}$$

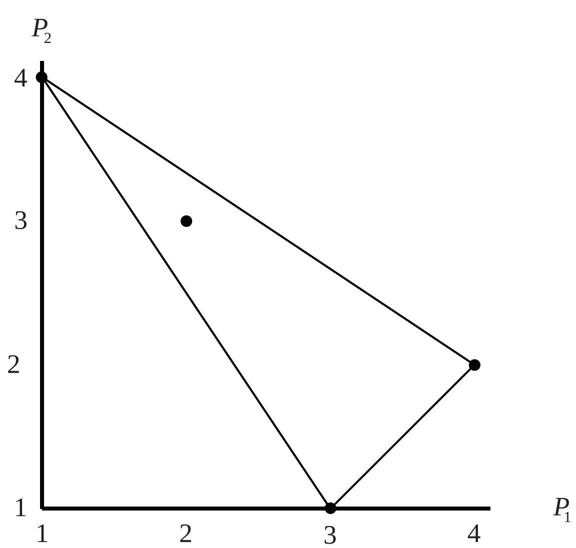


12,99,138,1

FIGURE 22

M

$$\begin{bmatrix} \enclose{circle}{2,3} & 4,2 \\ 1,4 & 3,1 \end{bmatrix}\begin{bmatrix} 2,3 & 3,1 \\ 4,2 & 1,4 \end{bmatrix}$$



118,144

FIGURE 23

M

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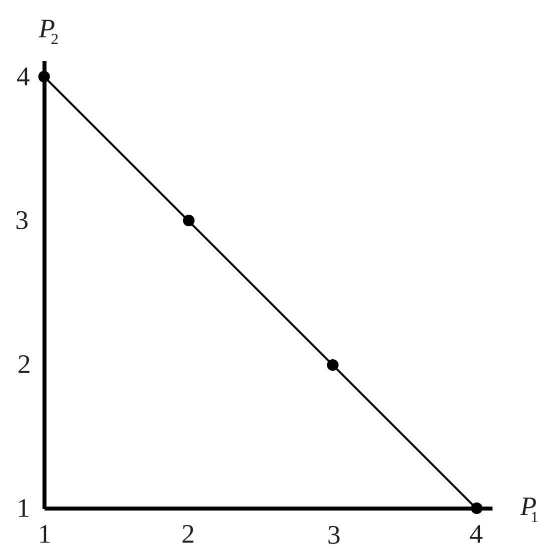


120,100,136

FIGURE 24

Game #	Payoff Matrix	Shape	Joint Max	PSNEs	Symmetric	Nash Payoff		Dom.	Pareto Optima	Transpose
						Row	Col.			
1	(1,4) (3,3) (2,2) (4,1)	22	6	1	Sym	2	2	2	3	NA
2	(1,2) (3,1) (2,4) (4,3)	11	7	1		2	4	2	2	115
3	(1,1) (3,2) (2,3) (4,4)	3	8	1	Sym	4	4	2	1	NA
4	(1,4) (3,3) (4,2) (2,1)	20	6	1		4	2	1	3	108
5	(1,3) (3,4) (4,1) (2,2)	16	7	1		3	4	1	2	117
6	(1,3) (3,2) (4,1) (2,4)	18	6	0		2.5	2.5	0	3	134
7	(1,2) (3,3) (4,4) (2,1)	7	8	2	Sym	4 3	4 3	0	1	NA
8	(1,2) (3,1) (4,4) (2,3)	9	8	1		4	4	1	1	113
9	(1,1) (3,2) (4,3) (2,4)	5	7	1		3	2	1	2	105
10	(1,1) (3,4) (4,3) (2,2)	2	7	2	Sym	3 4	4 3	0	2	NA
11	(1,4) (2,3) (3,2) (4,1)	24	5	1		3	2	2	4	120
12	(1,4) (2,2) (3,3) (4,1)	22	6	1	Sym	3	3	2	3	NA
13	(1,4) (2,1) (3,2) (4,3)	19	7	1		4	3	1	2	128
14	(1,3) (2,4) (3,1) (4,2)	17	6	1		4	2	2	2	112
15	(1,3) (2,2) (3,1) (4,4)	15	8	1		4	4	1	1	131
16	(1,2) (2,3) (3,4) (4,1)	10	7	1		3	4	1	2	137
17	(1,2) (2,4) (3,1) (4,3)	11	7	1		4	3	2	2	86
18	(1,2) (2,1) (3,4) (4,3)	8	7	1		3	4	2	2	109
19	(1,1) (2,3) (3,2) (4,4)	3	8	1	Sym	4	4	2	1	NA
20	(1,1) (2,2) (3,3) (4,4)	1	8	1		4	4	2	1	102
21	(1,1) (2,4) (3,3) (4,2)	6	6	1		3	3	1	3	123
22	(1,4) (2,3) (4,2) (3,1)	23	6	1		4	2	2	3	114

Appendix 2

Game #	Payoff Matrix	Shape	Joint Max	PSNEs	Symmetric	Nash Payoff Row	Nash Payoff Col.	Dom.	Pareto Optima	Transpose
23	(1,4) (2,2) (4,3) (3,1)	21	7	1		4	3	2	2	87
24	(1,4) (2,1) (4,2) (3,3)	20	6	1		3	3	1	3	127
25	(1,3) (2,4) (4,1) (3,2)	18	6	1		3	2	2	3	118
26	(1,3) (2,2) (4,4) (3,1)	15	8	1	Sym	4	4	2	1	NA
27	(1,3) (2,2) (4,1) (3,4)	16	7	1		3	4	1	2	135
28	(1,3) (2,1) (4,4) (3,2)	13	8	1		4	4	2	1	83
29	(1,2) (2,3) (4,4) (3,1)	9	8	1		4	4	1	1	132
30	(1,2) (2,3) (4,1) (3,4)	10	7	1		3	4	2	2	98
31	(1,2) (2,4) (4,1) (3,3)	12	6	1		3	3	2	3	89
32	(1,2) (2,1) (4,4) (3,3)	7	8	1		4	4	2	1	107
33	(1,1) (2,3) (4,2) (3,4)	3	7	1		3	4	2	2	81
34	(1,1) (2,2) (4,3) (3,4)	2	7	1		3	4	2	2	104
35	(1,1) (2,4) (4,2) (3,3)	6	6	1	Sym	3	3	2	3	NA
36	(1,1) (2,4) (4,3) (3,2)	5	7	1		4	3	1	2	125
37	(1,4) (4,3) (2,2) (3,1)	21	7	1		2	2	1	2	116
38	(1,4) (4,2) (2,3) (3,1)	23	6	1		2	3	1	3	88
39	(1,4) (4,1) (2,2) (3,3)	22	6	0		2.5	2.5	0	3	138
40	(1,4) (4,1) (2,3) (3,2)	24	5	1		2	3	1	4	100
41	(1,3) (4,4) (2,2) (3,1)	15	8	2	Sym	4 2	4 2	0	1	NA
42	(1,3) (4,4) (2,1) (3,2)	13	8	1		4	4	1	1	106
43	(1,3) (4,2) (2,4) (3,1)	17	6	1		2	4	1	2	97
44	(1,3) (4,2) (2,1) (3,4)	14	7	0		2.5	2.5	0	2	126

Game #	Payoff Matrix	Shape	Joint Max	PSNEs	Symmetric	Nash Payoff Row	Nash Payoff Col.	Dom.	Pareto Optima	Transpose
45	(1,3) (4,1) (2,4) (3,2)	18	6	1		2	4	1	3	91
46	(1,2) (4,3) (2,4) (3,1)	11	7	2		4 2	3 4	0	2	133
47	(1,2) (4,3) (2,1) (3,4)	8	7	1		4	3	1	2	94
48	(1,2) (4,4) (2,1) (3,3)	7	8	1		4	4	1	1	82
49	(1,2) (4,1) (2,4) (3,3)	12	6	1		2	4	1	3	121
50	(1,2) (4,1) (2,3) (3,4)	10	7	0		2.5	2.5	0	2	143
51	(1,1) (4,3) (2,2) (3,4)	2	7	1		4	3	1	2	79
52	(1,1) (4,2) (2,3) (3,4)	4	7	1		4	2	1	2	101
53	(1,1) (4,2) (2,4) (3,3)	6	6	2	Sym	4 2	2 4	0	3	NA
54	(1,1) (4,4) (2,2) (3,3)	1	8	1		4	4	1	1	92
55	(1,1) (4,4) (2,3) (3,2)	3	8	2		4 2	4 3	0	1	122
56	(1,4) (4,3) (3,2) (2,1)	19	7	1		3	2	1	2	110
57	(1,4) (4,3) (3,1) (2,2)	21	7	0		2.5	2.5	0	2	141
58	(1,4) (4,2) (3,3) (2,1)	20	6	1		3	3	1	3	84
59	(1,4) (4,1) (3,2) (2,3)	24	5	0		2.5	2.5	0	4	136
60	(1,4) (4,1) (3,3) (2,2)	22	6	1		3	3	1	3	99
61	(1,3) (4,4) (3,2) (2,1)	13	8	2		4 3	4 2	0	1	140
62	(1,3) (4,4) (3,1) (2,2)	15	8	1		4	4	1	1	111
63	(1,3) (4,2) (3,4) (2,1)	14	7	1		3	4	1	2	95
64	(1,3) (4,2) (3,1) (2,4)	17	6	0		2.5	2.5	0	2	130
65	(1,3) (4,1) (3,2) (2,4)	18	6	0		2.5	2.5	0	3	144
66	(1,3) (4,1) (3,4) (2,2)	16	7	1		3	4	1	2	90

Appendix 2

Game #	Payoff Matrix	Shape	Joint Max	PSNEs	Symmetric	Nash Payoff Row	Nash Payoff Col.	Dom.	Pareto Optima	Transpose
67	(1,2) (4,3) (3,4) (2,1)	8	7	2		4 3	3 4	0	2	129
68	(1,2) (4,3) (3,1) (2,4)	11	7	1		4	3	1	2	96
69	(1,2) (4,4) (3,3) (2,1)	7	8	2	Sym	4 3	4 3	0	1	NA
70	(1,2) (4,4) (3,1) (2,3)	9	8	1		4	4	1	1	85
71	(1,2) (4,1) (3,4) (2,3)	10	7	1		3	4	1	2	119
72	(1,2) (4,1) (3,3) (2,4)	12	6	0		2.5	2.5	0	3	142
73	(1,1) (4,3) (3,2) (2,4)	5	7	1		4	3	1	2	80
74	(1,1) (4,3) (3,4) (2,2)	2	7	2	Sym	4 3	3 4	0	2	NA
75	(1,1) (4,2) (3,3) (2,4)	6	6	1		4	2	1	3	103
76	(1,1) (4,2) (3,4) (2,3)	4	7	2		3 4	4 2	0	2	139
77	(1,1) (4,4) (3,2) (2,3)	3	8	1		4	4	1	1	93
78	(1,1) (4,4) (3,3) (2,2)	1	8	2		4 3	4 3	0	1	124
79	(1,1) (2,2) (3,4) (4,3)	2	7	1		3	4	1	2	51
80	(1,1) (2,3) (3,4) (4,2)	4	7	1		3	4	1	2	73
81	(1,1) (2,4) (3,2) (4,3)	5	7	1		4	3	2	2	33
82	(1,2) (2,1) (3,3) (4,4)	7	8	1		4	4	1	1	48
83	(1,2) (2,3) (3,1) (4,4)	9	8	1		4	4	2	1	28
84	(1,2) (2,4) (3,3) (4,1)	12	6	1		3	3	1	3	58
85	(1,3) (2,1) (3,2) (4,4)	13	8	1		4	4	1	1	70
86	(1,3) (2,1) (3,4) (4,2)	14	7	1		3	4	2	2	17
87	(1,3) (2,2) (3,4) (4,1)	16	7	1		3	4	2	2	23
88	(1,3) (2,4) (3,2) (4,1)	18	6	1		3	2	1	3	38

Appendix 2

Game #	Payoff Matrix	Shape	Joint Max	PSNEs	Symmetric	Nash Payoff		Dom.	Pareto Optima	Transpose
						Row	Col.			
89	(1,4) (2,1) (3,3) (4,2)	20	6	1		3	3	2	3	31
90	(1,4) (2,2) (3,1) (4,3)	21	7	1		4	3	1	2	66
91	(1,4) (2,3) (3,1) (4,2)	23	6	1		4	2	1	3	45
92	(1,1) (2,2) (4,4) (3,3)	1	8	1		4	4	1	1	54
93	(1,1) (2,3) (4,4) (3,2)	3	8	1		4	4	1	1	77
94	(1,2) (2,1) (4,3) (3,4)	6	7	1		3	4	1	2	47
95	(1,2) (2,4) (4,3) (3,1)	11	7	1		4	3	1	2	63
96	(1,3) (2,1) (4,2) (3,4)	14	7	1		3	4	1	2	68
97	(1,3) (2,4) (4,2) (3,1)	17	6	1		4	2	1	2	43
98	(1,4) (2,1) (4,3) (3,2)	19	7	1		4	3	2	2	30
99	(1,4) (2,2) (4,1) (3,3)	22	6	1		3	3	1	3	60
100	(1,4) (2,3) (4,1) (3,2)	24	5	1		3	2	1	4	40
101	(1,1) (3,2) (2,4) (4,3)	5	7	1		2	4	1	2	52
102	(1,1) (3,3) (2,2) (4,4)	1	8	1		4	4	2	1	20
103	(1,1) (3,3) (2,4) (4,2)	6	6	1		2	4	1	3	75
104	(1,1) (3,4) (2,2) (4,3)	2	7	1		4	3	2	2	34
105	(1,1) (3,4) (2,3) (4,2)	4	7	1		2	3	1	2	9
106	(1,2) (3,1) (2,3) (4,4)	9	8	1		4	4	1	1	42
107	(1,2) (3,3) (2,1) (4,4)	7	8	1		4	4	2	1	32
108	(1,2) (3,3) (2,4) (4,1)	12	6	1		2	4	1	3	4
109	(1,2) (3,4) (2,1) (4,3)	8	7	1		4	3	2	2	18
110	(1,2) (3,4) (2,3) (4,1)	10	7	1		2	3	1	2	56

Appendix 2

Game #	Payoff Matrix	Shape	Joint Max	PSNEs	Symmetric	Nash Payoff Row	Nash Payoff Col.	Dom.	Pareto Optima	Transpose
111	(1,3) (3,1) (2,2) (4,4)	15	8	1		4	4	1	1	62
112	(1,3) (3,1) (2,4) (4,2)	17	6	1		2	4	2	2	14
113	(1,3) (3,2) (2,1) (4,4)	13	8	1		4	4	1	1	8
114	(1,3) (3,2) (2,4) (4,1)	18	6	1		2	4	2	3	22
115	(1,3) (3,4) (2,1) (4,2)	14	7	1		4	2	2	2	2
116	(1,3) (3,4) (2,2) (4,1)	16	7	1		2	2	1	2	37
117	(1,4) (3,1) (2,2) (4,3)	21	7	1		4	3	1	2	5
118	(1,4) (3,1) (2,3) (4,2)	23	6	1		2	3	2	3	25
119	(1,4) (3,2) (2,1) (4,3)	19	7	1		4	3	1	2	71
120	(1,4) (3,2) (2,3) (4,1)	24	5	1		2	3	2	4	11
121	(1,4) (3,3) (2,1) (4,2)	20	6	1		4	2	1	3	49
122	(1,1) (3,2) (4,4) (2,3)	3	8	2		4 3	4 2	0	1	55
123	(1,1) (3,3) (4,2) (2,4)	6	6	1		3	3	1	3	21
124	(1,1) (3,3) (4,4) (2,2)	1	8	2		4 3	4 3	0	1	78
125	(1,1) (3,4) (4,2) (2,3)	4	7	1		3	4	1	2	36
126	(1,2) (3,1) (4,3) (2,4)	11	7	0		2.5	2.5	0	2	44
127	(1,2) (3,3) (4,1) (2,4)	12	6	1		3	3	1	3	24
128	(1,2) (3,4) (4,1) (2,3)	10	7	1		3	4	1	2	13
129	(1,2) (3,4) (4,3) (2,1)	8	7	2		4 3	3 4	0	2	67
130	(1,3) (3,1) (4,2) (2,4)	17	6	0		2.5	2.5	0	2	64
131	(1,3) (3,1) (4,4) (2,2)	15	8	1		4	4	1	1	15
132	(1,3) (3,2) (4,4) (2,1)	13	8	1		4	4	1	1	29

Appendix 2

Game #	Payoff Matrix		Shape	Joint Max	PSNEs	Symmetric	Nash Payoff Row	Nash Payoff Col.	Dom.	Pareto Optima	Transpose
133	(1,3)	(3,4)	14	7	2		3	4	0	2	46
	(4,2)	(2,1)					4	2			
134	(1,4)	(3,1)	23	6	0		2.5	2.5	0	3	6
	(4,2)	(2,3)									
135	(1,4)	(3,1)	21	7	1		4	3	1	2	27
	(4,3)	(2,2)									
136	(1,4)	(3,2)	24	5	0		2.5	2.5	0	4	59
	(4,1)	(2,3)									
137	(1,4)	(3,2)	19	7	1		4	3	1	2	16
	(4,3)	(2,1)									
138	(1,4)	(3,3)	22	6	0		2.5	2.5	0	3	39
	(4,1)	(2,2)									
139	(1,1)	(4,3)	5	7	2		4	3	0	2	76
	(2,4)	(3,2)					2	4			
140	(1,2)	(4,4)	9	8	2		4	4	0	1	61
	(2,3)	(3,1)					2	3			
141	(1,3)	(4,1)	16	7	0		2.5	2.5	0	2	57
	(2,2)	(3,4)									
142	(1,4)	(4,2)	20	6	0		2.5	2.5	0	3	72
	(2,1)	(3,3)									
143	(1,4)	(4,3)	19	7	0		2.5	2.5	0	2	50
	(2,1)	(3,2)									
144	(1,4)	(4,2)	23	6	0		2.5	2.5	0	3	65
	(3,1)	(2,3)									

Game # corresponds to the numbering system established in the companion paper

Payoff Matrix gives the normal form of each game with payoffs listed as (row payoff, column payoff)

Shape corresponds to the shape of the payoff set's convex hull as shown in Appendix 1

Joint Max gives the highest possible combined payoff for the two players

Symmetric is marked "Sym" if the game is symmetric, otherwise it is left blank

Nash Payoff lists the payoffs of the noncooperative equilibrium

if there are two equilibria with different payoff sums, the one with the highest sum is listed first

Dom. specifies the number of row and column strategies that are strictly dominated

Pareto Optima gives the number of payoff pairs that are Pareto optimal

Transpose lists the game number corresponding to the transpose of the game shown

Appendix 3: Symmetric Games

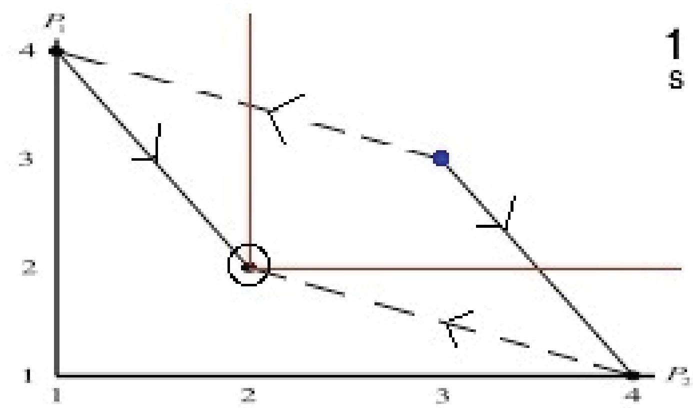
The following is a list of all the symmetric games in the set of 144. Each entry includes a matrix representation of the game, the maxmin payoffs, the jointmax payoffs and their sums and a game diagram.

In the game diagrams, solid lines indicate the change in payoffs when the row player changes his strategy. Dashed lines indicate the change in payoffs when the column player changes his strategy. The arrows show the best responses for each player. A circled payoff is a non-cooperative equilibrium. If an 's' is printed under the game number, the game is symmetric, if an 'm' is printed under the game number, the game has no equilibrium in pure strategies. Any blue payoff is a jointmax payoff. The red lines indicate the maxmin payoffs for both players.

Games4321_4321(:,:,1) =

1, 4	3, 3
2, 2	4, 1

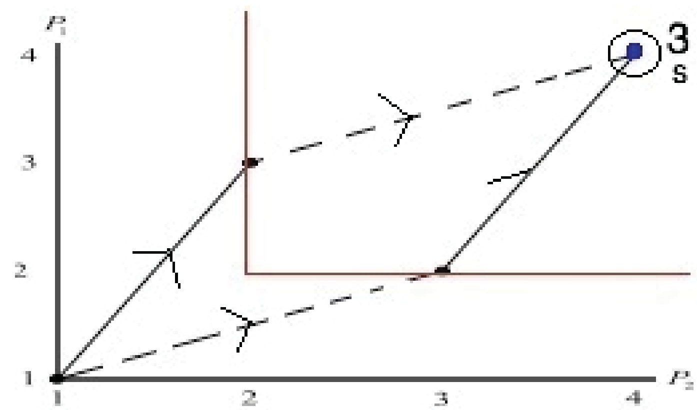
maxmin P1: 2
maxmin P2: 2
joint max P1: 3
joint max P2: 3
joint max sum: 6



Games4321_4321(:,:,3) =

1, 1	3, 2
2, 3	4, 4

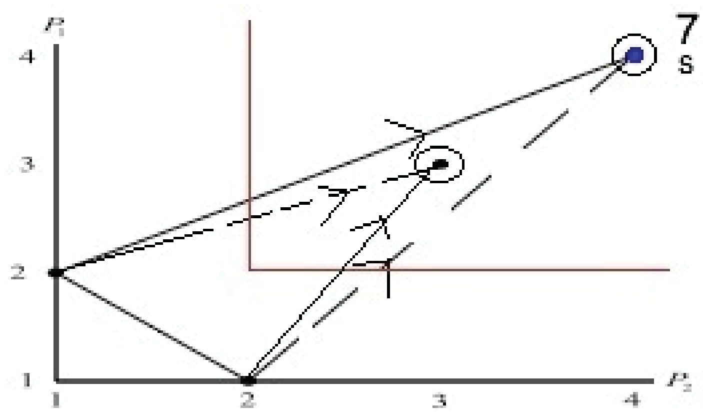
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,7) =

1, 2	3, 3
4, 4	2, 1

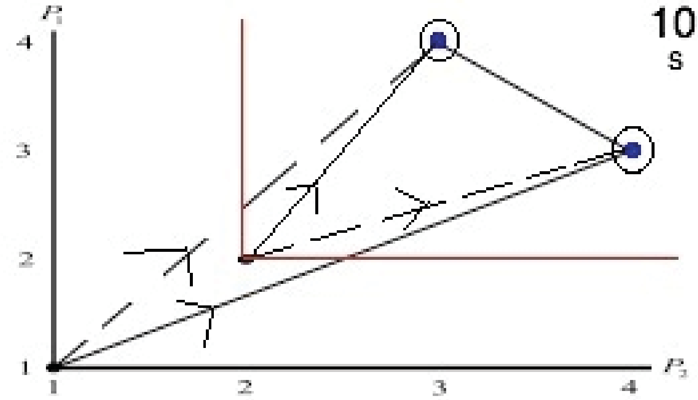
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,10) =

1, 1	3, 4
4, 3	2, 2

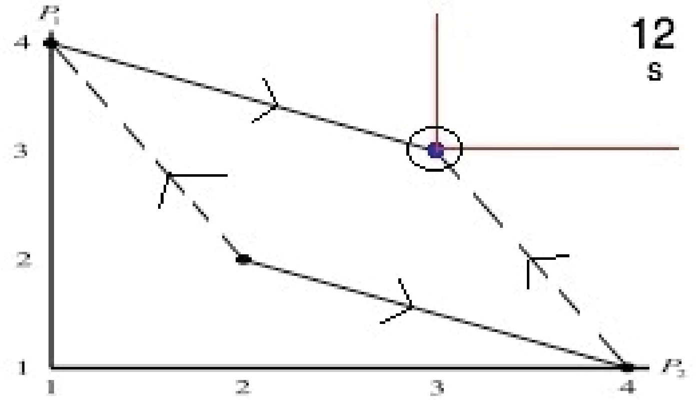
maxmin P1: 2
maxmin P2: 2
joint max P1: 3, 4
joint max P2: 4, 3
joint max sum: 7



Games4321_4321(:,:,12) =

1, 4	2, 2
3, 3	4, 1

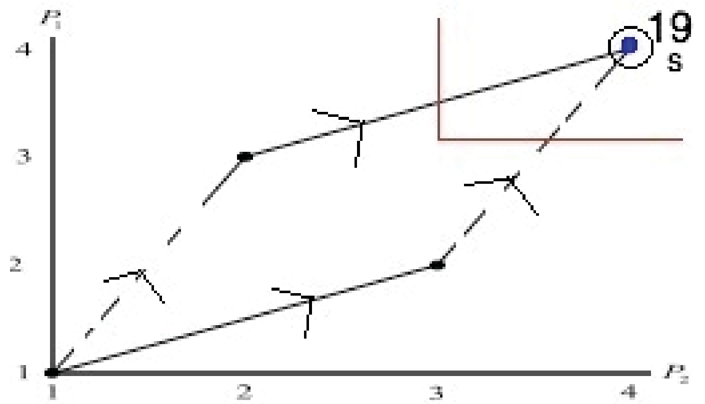
maxmin P1: 3
maxmin P2: 3
joint max P1: 3
joint max P2: 3
joint max sum: 6



Games4321_4321(:,:,19) =

1, 1	2, 3
3, 2	4, 4

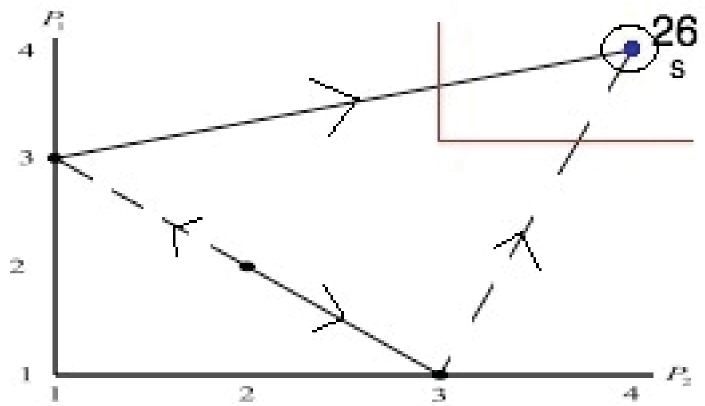
maxmin P1: 3
maxmin P2: 3
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,26) =

1, 3	2, 2
4, 4	3, 1

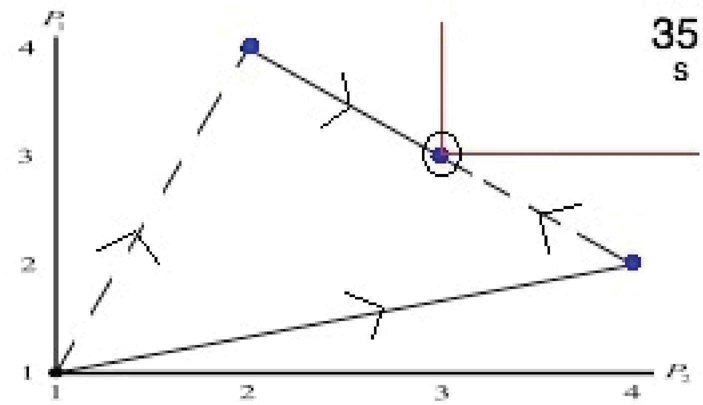
maxmin P1: 3
maxmin P2: 3
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,35) =

1, 1	2, 4
4, 2	3, 3

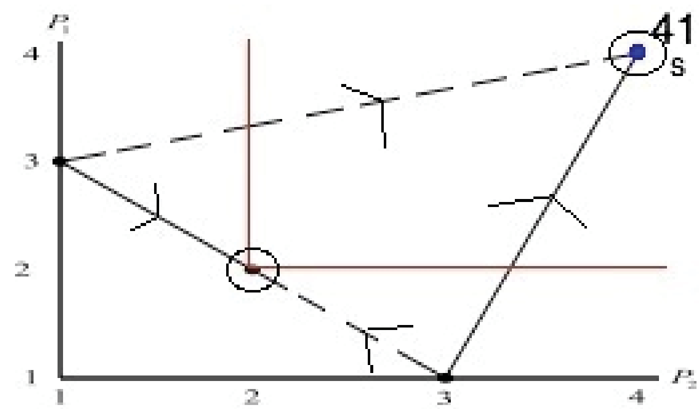
maxmin P1: 3
maxmin P2: 3
joint max P1: 2, 3, 4
joint max P2: 4, 3, 2
joint max sum: 7



Games4321_4321(:,:,41) =

1, 3	4, 4
2, 2	3, 1

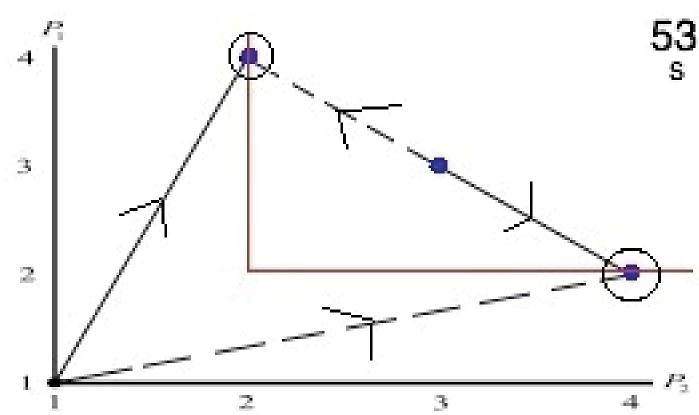
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,53) =

1, 1	4, 2
2, 4	3, 3

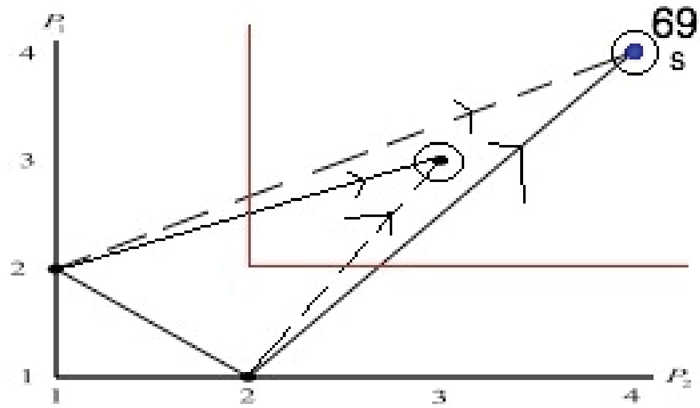
maxmin P1: 2
maxmin P2: 2
joint max P1: 2, 3, 4
joint max P2: 4, 3, 2
joint max sum: 6



Games4321_4321(:,:,69) =

1, 2	4, 4
3, 3	2, 1

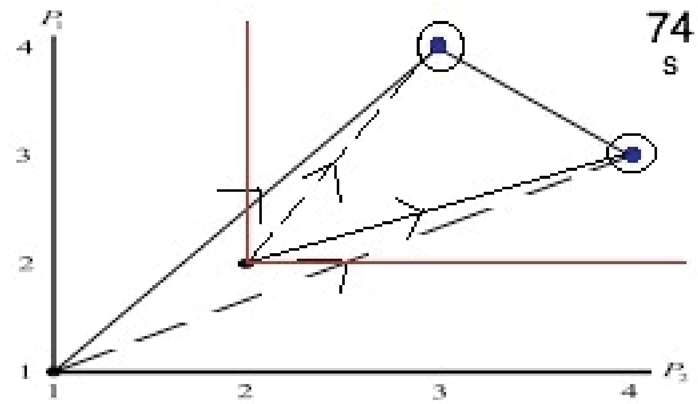
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,74) =

1, 1	4, 3
3, 4	2, 2

maxmin P1: 2
maxmin P2: 2
joint max P1: 3, 4
joint max P2: 4, 3
joint max sum: 7



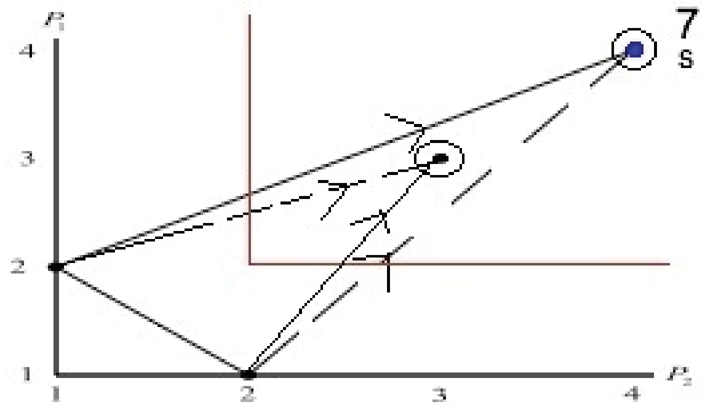
Appendix 4: Games with 2 Non-Cooperative Equilibria

The following is a list of all the games with 2 non-cooperative equilibria in pure strategies in the set of 144. Each entry includes a matrix representation of the game, the maxmin payoffs, the jointmax payoffs and their sums and a game diagram.

Games4321_4321(:,:,7) =

1, 2	3, 3
4, 4	2, 1

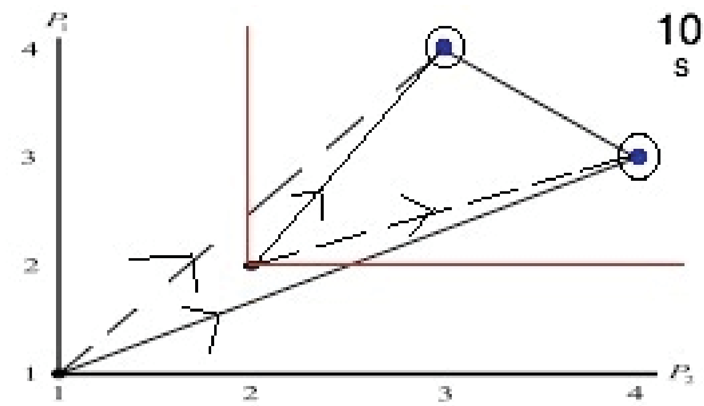
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,10) =

1, 1	3, 4
4, 3	2, 2

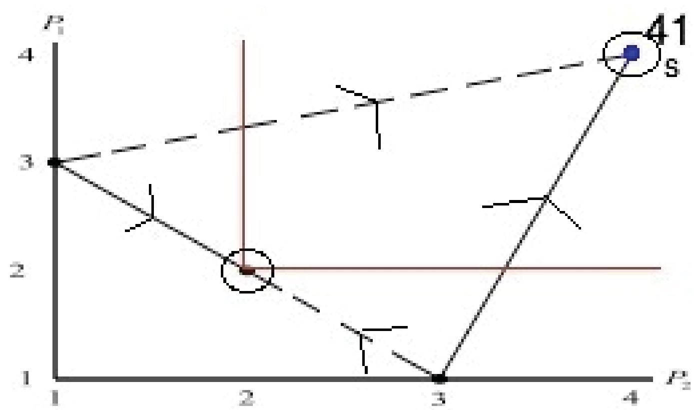
maxmin P1: 2
maxmin P2: 2
joint max P1: 3, 4
joint max P2: 4, 3
joint max sum: 7



Games4321_4321(:,:,41) =

1, 3	4, 4
2, 2	3, 1

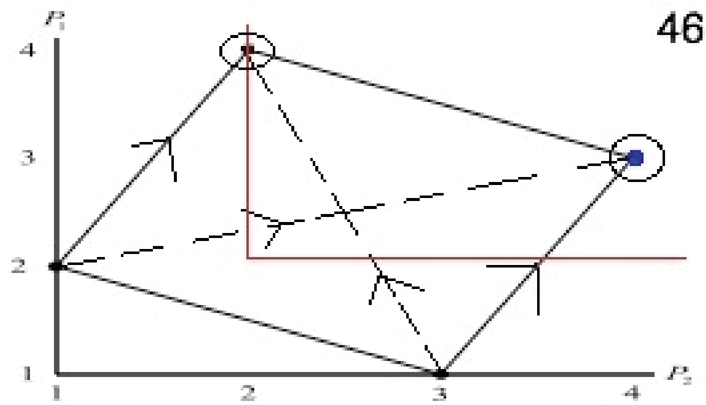
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,46) =

1, 2	4, 3
2, 4	3, 1

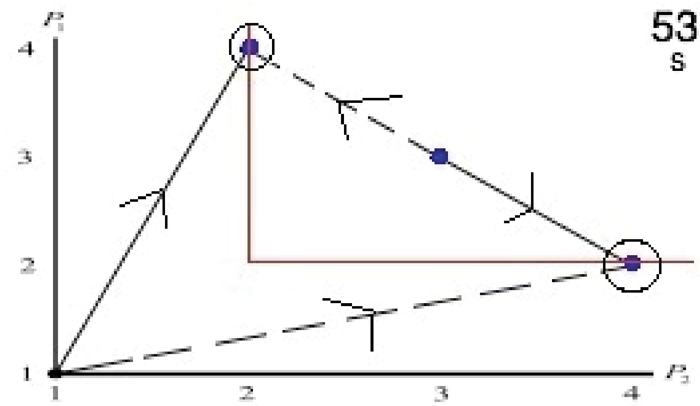
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,53) =

1, 1	4, 2
2, 4	3, 3

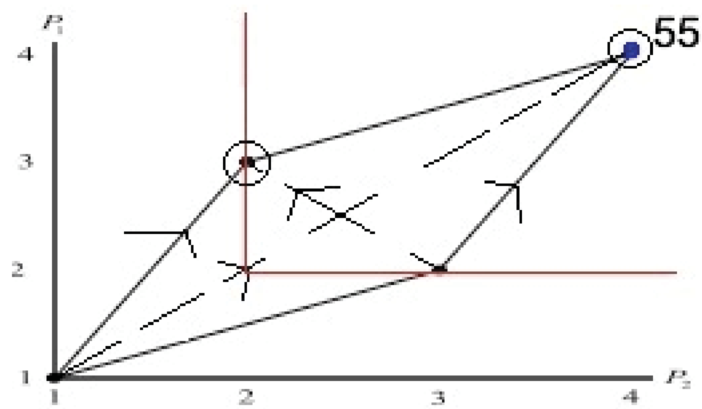
maxmin P1: 2
maxmin P2: 2
joint max P1: 2, 3, 4
joint max P2: 4, 3, 2
joint max sum: 6



Games4321_4321(:,:,55) =

1, 1	4, 4
2, 3	3, 2

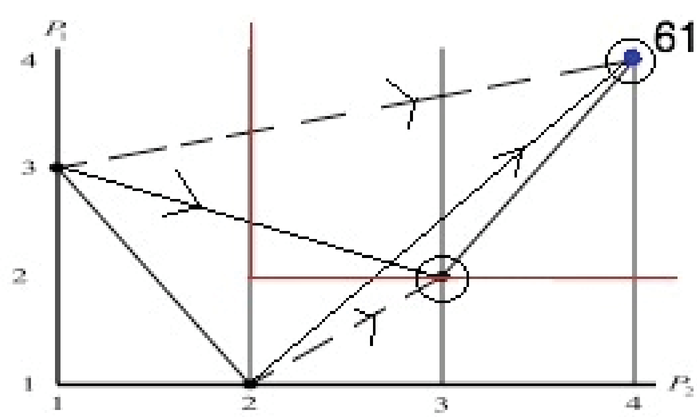
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,61) =

1, 3	4, 4
3, 2	2, 1

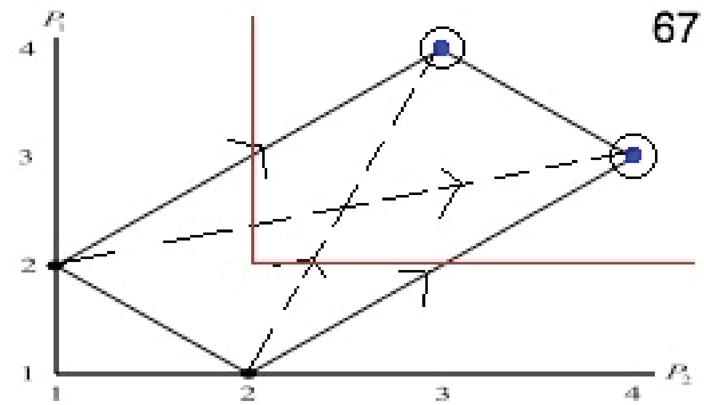
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,67) =

1, 2	4, 3
3, 4	2, 1

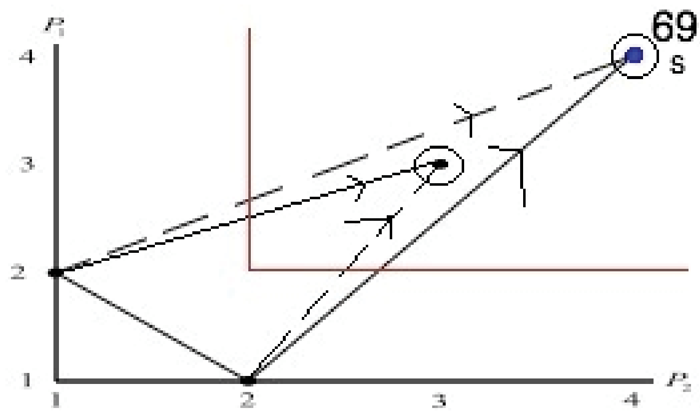
maxmin P1: 2
maxmin P2: 2
joint max P1: 3, 4
joint max P2: 4, 3
joint max sum: 7



Games4321_4321(:,:,69) =

1, 2	4, 4
3, 3	2, 1

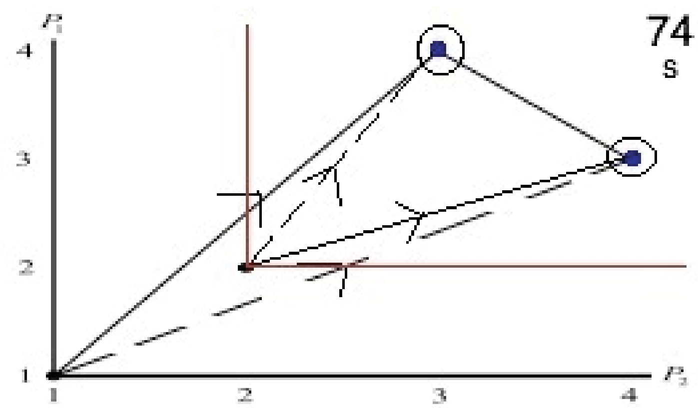
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,74) =

1, 1	4, 3
3, 4	2, 2

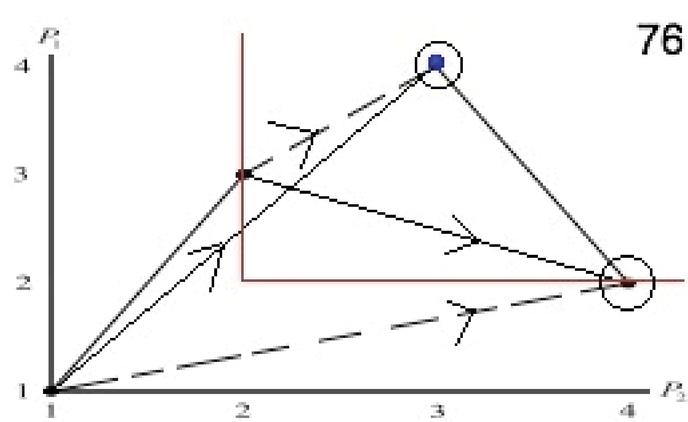
maxmin P1: 2
maxmin P2: 2
joint max P1: 3, 4
joint max P2: 4, 3
joint max sum: 7



Games4321_4321(:,:,76) =

1, 1	4, 2
3, 4	2, 3

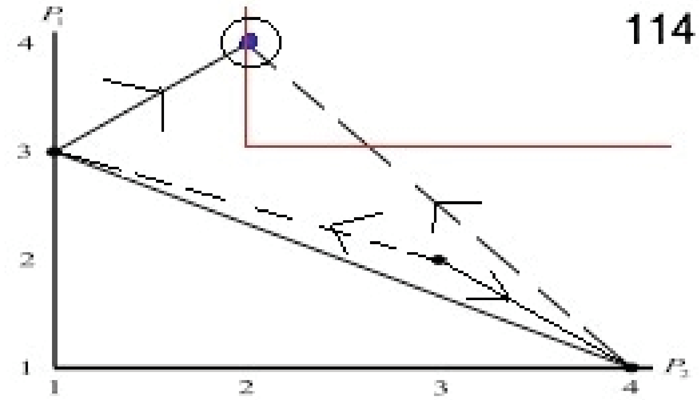
maxmin P1: 2
maxmin P2: 2
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,114) =

1, 3	3, 2
2, 4	4, 1

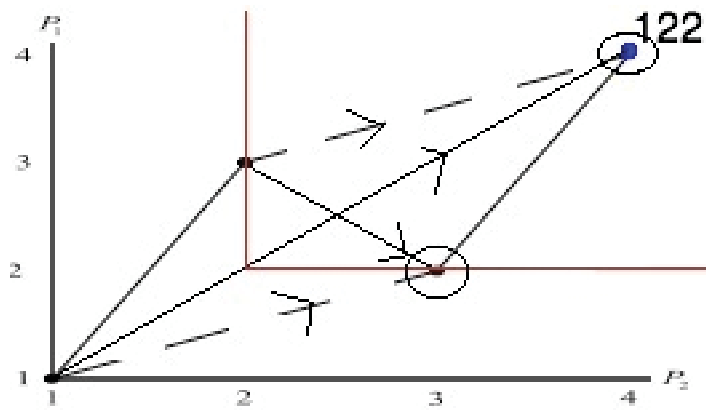
maxmin P1: 2
maxmin P2: 3
joint max P1: 2
joint max P2: 4
joint max sum: 6



Games4321_4321(:,:,122) =

1, 1	3, 2
4, 4	2, 3

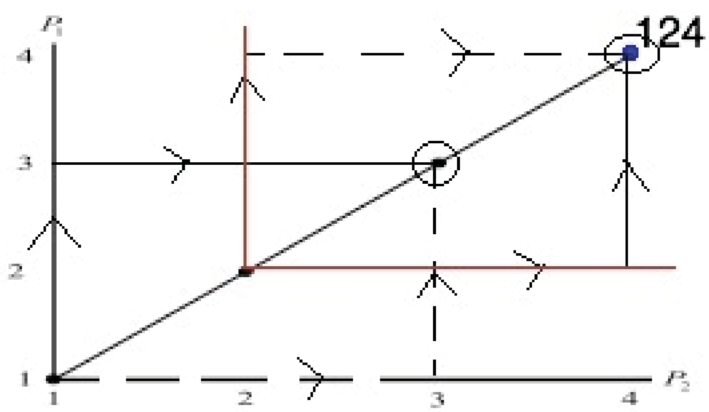
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,124) =

1, 1	3, 3
4, 4	2, 2

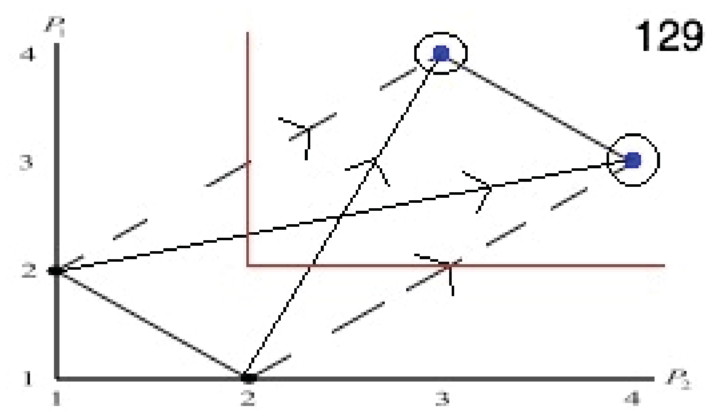
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,129) =

1, 2	3, 4
4, 3	2, 1

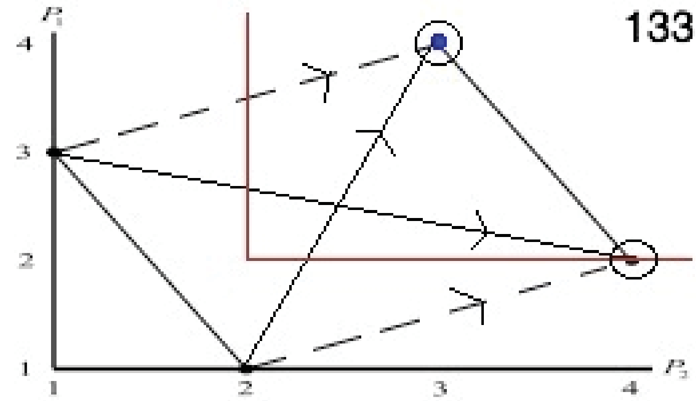
maxmin P1: 2
maxmin P2: 2
joint max P1: 3, 4
joint max P2: 4, 3
joint max sum: 7



Games4321_4321(:,:,133) =

1, 3	3, 4
4, 2	2, 1

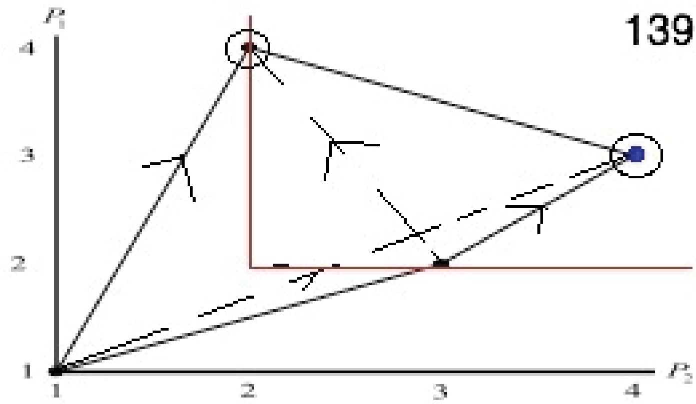
maxmin P1: 2
maxmin P2: 2
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,139) =

1, 1	4, 3
2, 4	3, 2

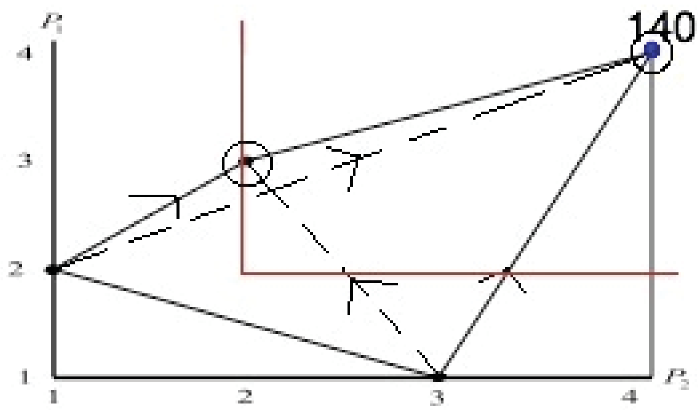
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,140) =

1, 2	4, 4
2, 3	3, 1

maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



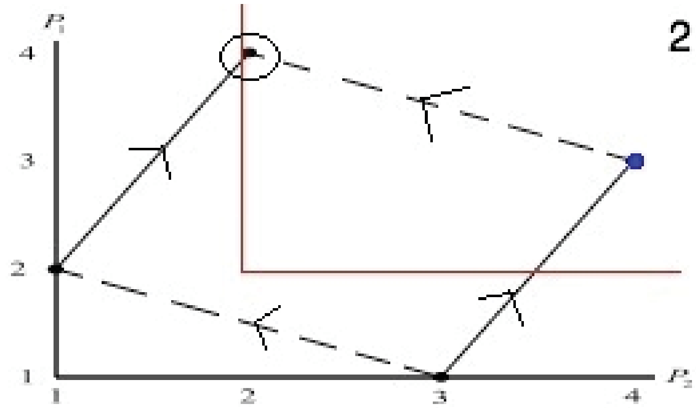
Appendix 5: All Other Game Diagrams

The following is a list of all other games in the set of 144. Each entry includes a matrix representation of the game, the maxmin payoffs, the jointmax payoffs and their sums and a game diagram.

Games4321_4321(:,:,2) =

1, 2	3, 1
2, 4	4, 3

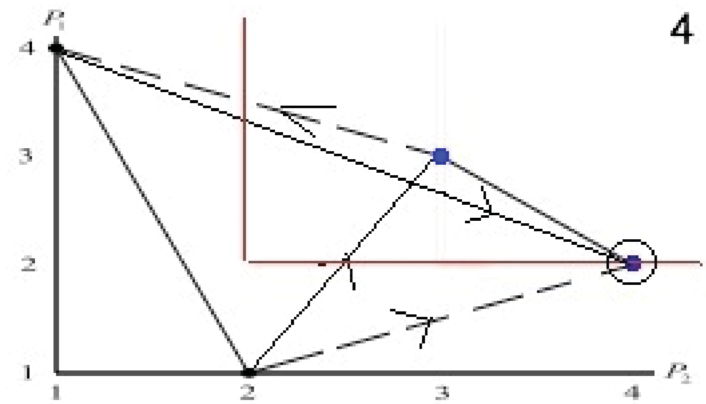
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,4) =

1, 4	3, 3
4, 2	2, 1

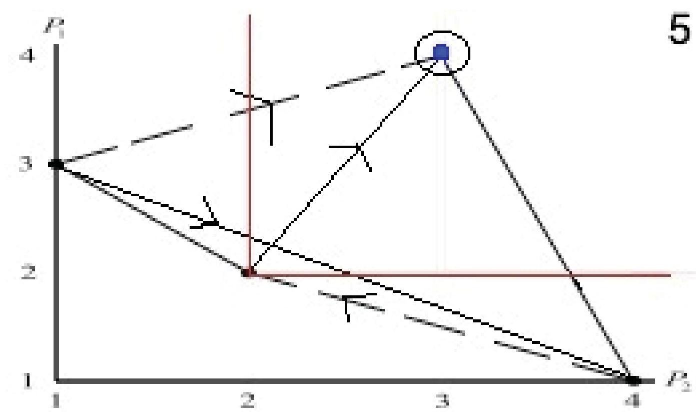
maxmin P1: 2
maxmin P2: 2
joint max P1: 3, 4
joint max P2: 3, 2
joint max sum: 6



Games4321_4321(:,:,5) =

1, 3	3, 4
4, 1	2, 2

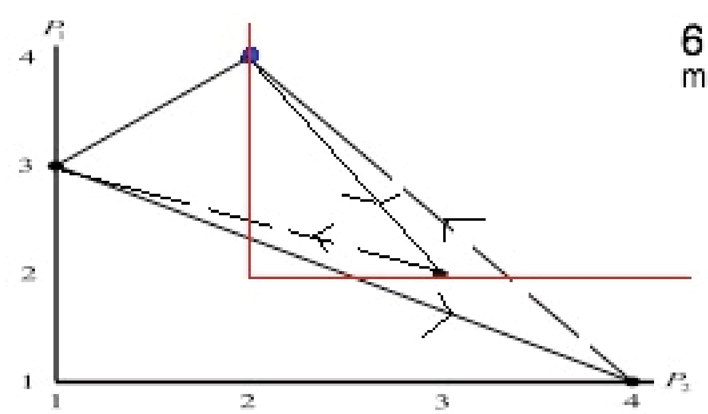
maxmin P1: 2
maxmin P2: 2
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,6) =

1, 3	3, 2
4, 1	2, 4

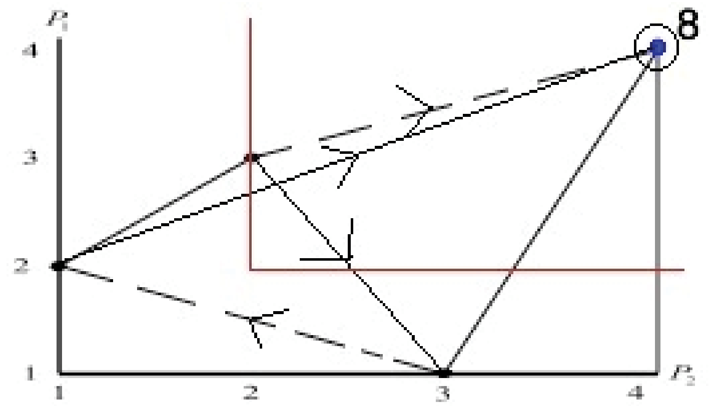
maxmin P1: 2
maxmin P2: 2
joint max P1: 2
joint max P2: 4
joint max sum: 6



Games4321_4321(:,:,8) =

1, 2	3, 1
4, 4	2, 3

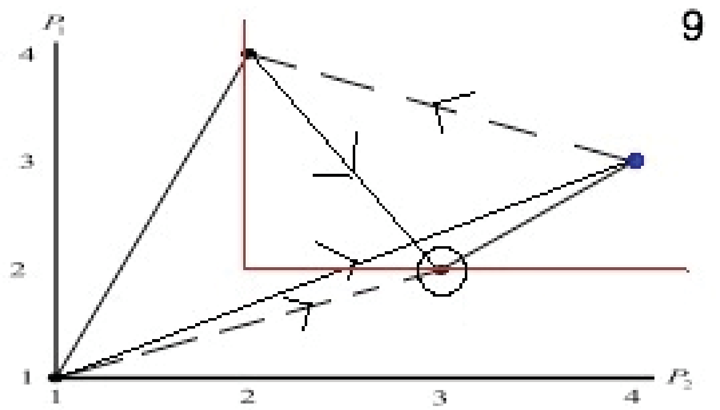
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,9) =

1, 1	3, 2
4, 3	2, 4

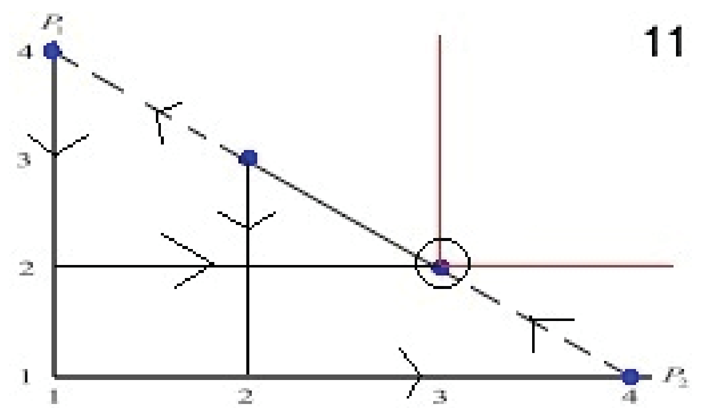
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,11) =

1, 4	2, 3
3, 2	4, 1

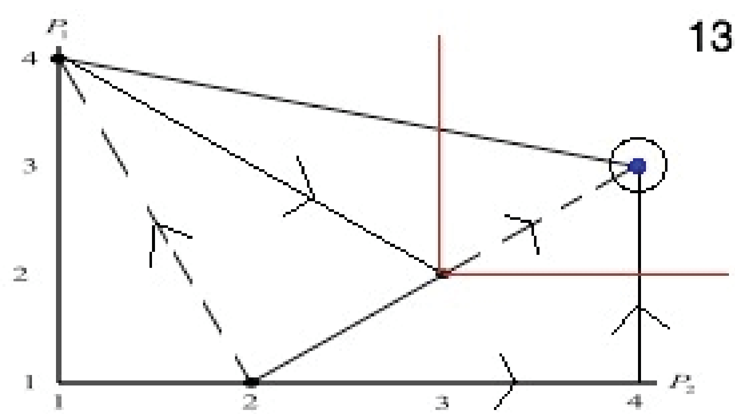
maxmin P1: 3
maxmin P2: 2
joint max P1: 1, 2, 3, 4
joint max P2: 4, 3, 2, 1
joint max sum: 5



Games4321_4321(:,:,13) =

1, 4	2, 1
3, 2	4, 3

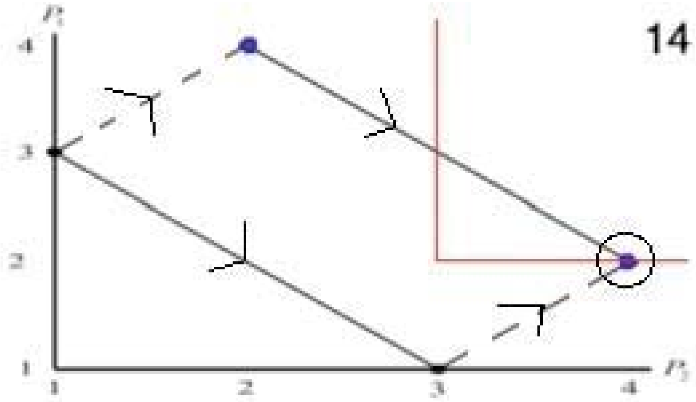
maxmin P1: 3
maxmin P2: 2
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,14) =

1, 3	2, 4
3, 1	4, 2

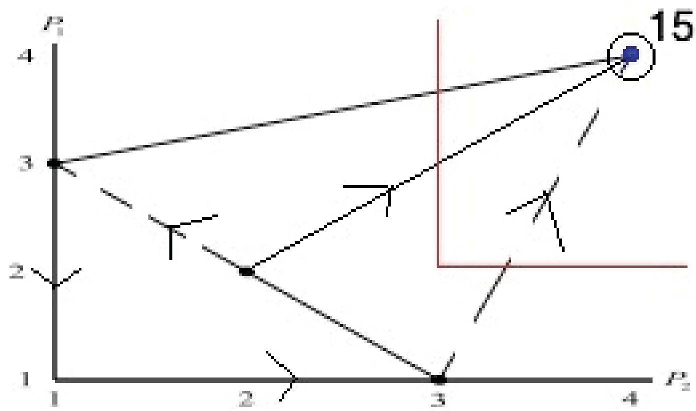
maxmin P1: 3
maxmin P2: 2
joint max P1: 2, 4
joint max P2: 4, 2
joint max sum: 6



Games4321_4321(:,:,15) =

1, 3	2, 2
3, 1	4, 4

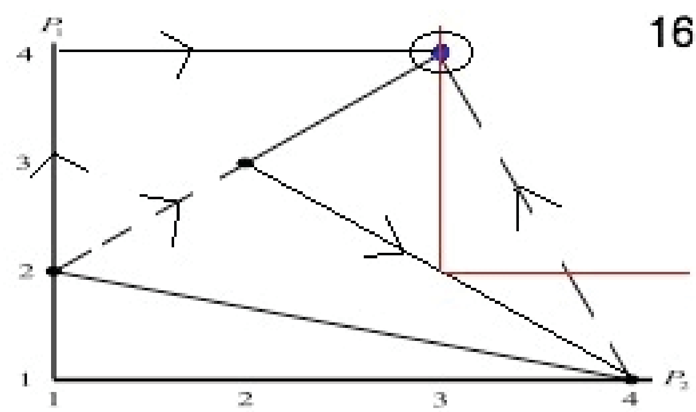
maxmin P1: 3
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,16) =

1, 2	2, 3
3, 4	4, 1

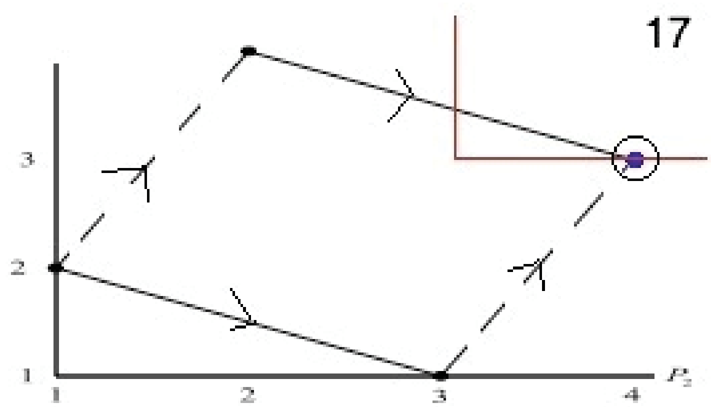
maxmin P1: 3
maxmin P2: 2
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,17) =

1, 2	2, 4
3, 1	4, 3

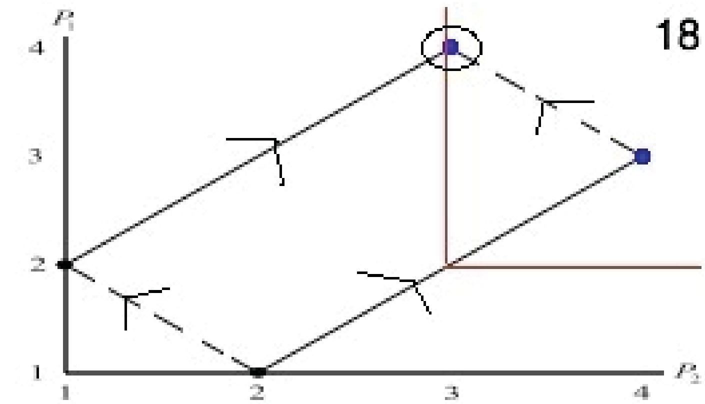
maxmin P1: 3
maxmin P2: 3
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,18) =

1, 2	2, 1
3, 4	4, 3

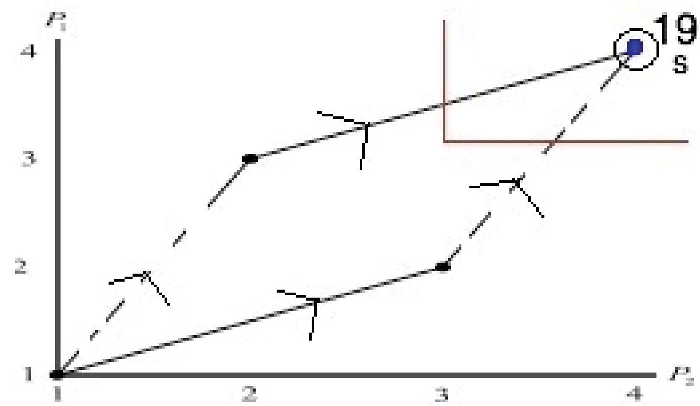
maxmin P1: 3
maxmin P2: 2
joint max P1: 3, 4
joint max P2: 4, 3
joint max sum: 7



Games4321_4321(:,:,19) =

1, 1	2, 3
3, 2	4, 4

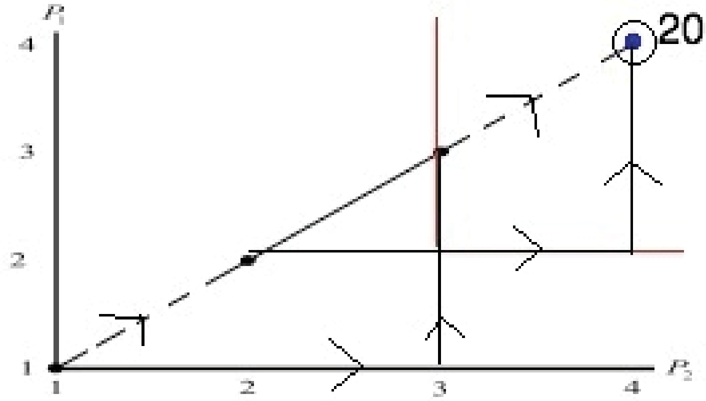
maxmin P1: 3
maxmin P2: 3
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,20) =

1, 1	2, 2
3, 3	4, 4

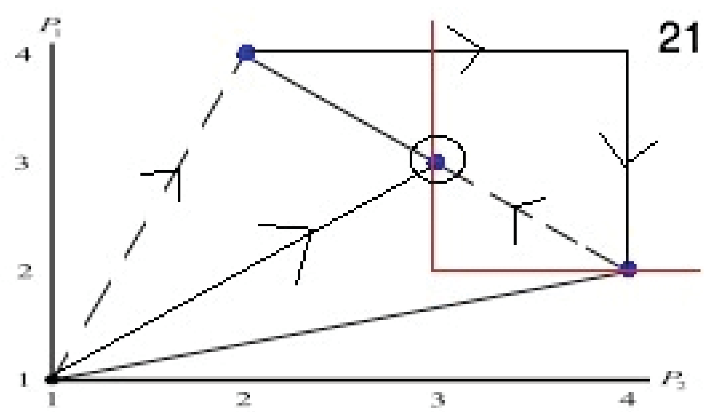
maxmin P1: 3
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,21) =

1, 1	2, 4
3, 3	4, 2

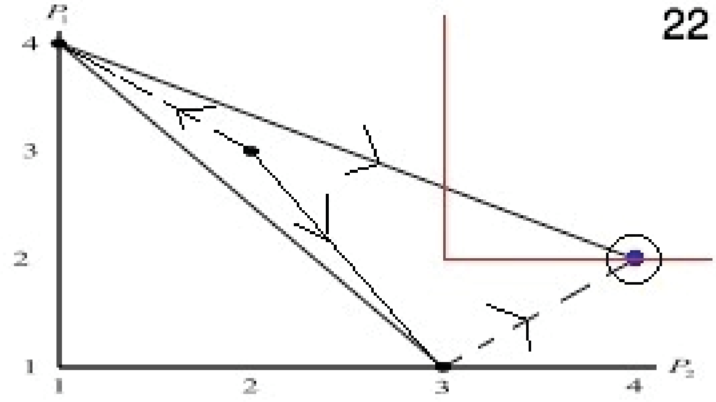
maxmin P1: 3
maxmin P2: 2
joint max P1: 2, 3, 4
joint max P2: 4, 3, 2
joint max sum: 6



Games4321_4321(:,:,22) =

1, 4	2, 3
4, 2	3, 1

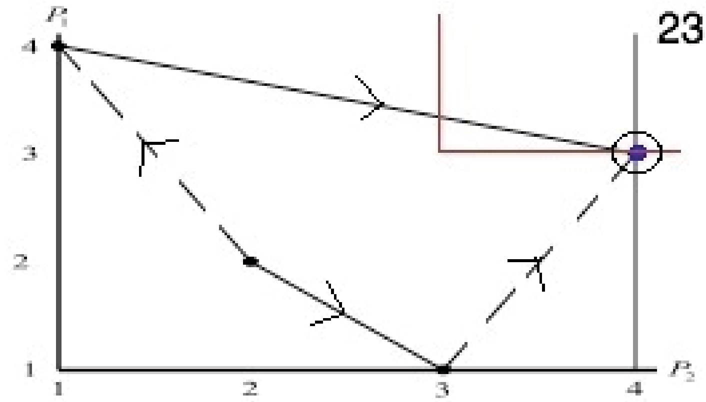
maxmin P1: 3
maxmin P2: 2
joint max P1: 4
joint max P2: 2
joint max sum: 6



Games4321_4321(:,:,23) =

1, 4	2, 2
4, 3	3, 1

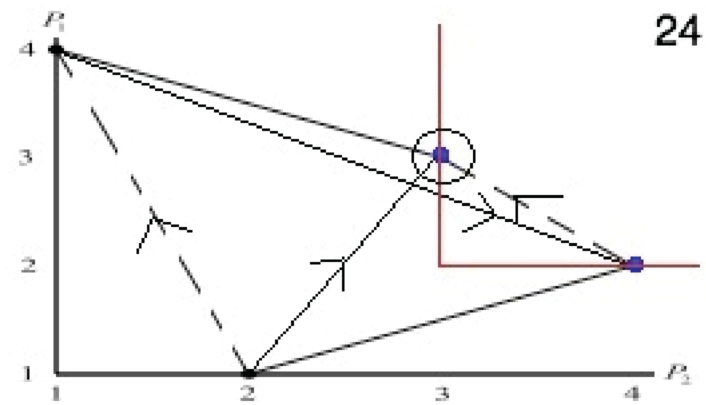
maxmin P1: 3
maxmin P2: 3
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,24) =

1, 4	2, 1
4, 2	3, 3

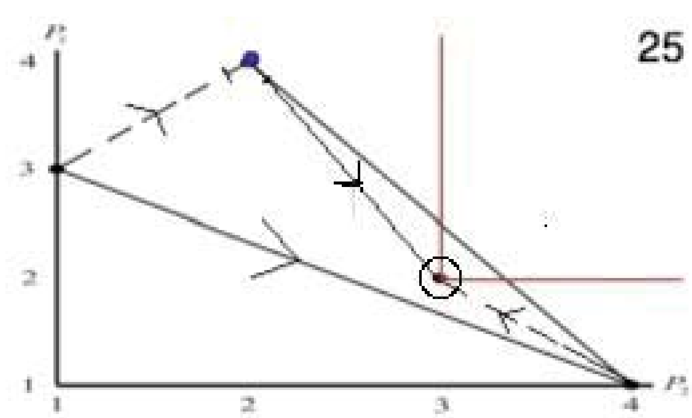
maxmin P1: 3
maxmin P2: 2
joint max P1: 3, 4
joint max P2: 3, 2
joint max sum: 6



Games4321_4321(:,:,25) =

1, 3	2, 4
4, 1	3, 2

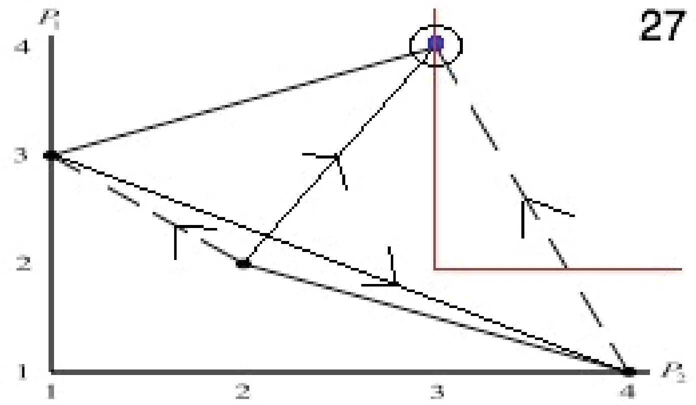
maxmin P1: 3
maxmin P2: 2
joint max P1: 2
joint max P2: 4
joint max sum: 6



Games4321_4321(:,:,27) =

1, 3	2, 2
4, 1	3, 4

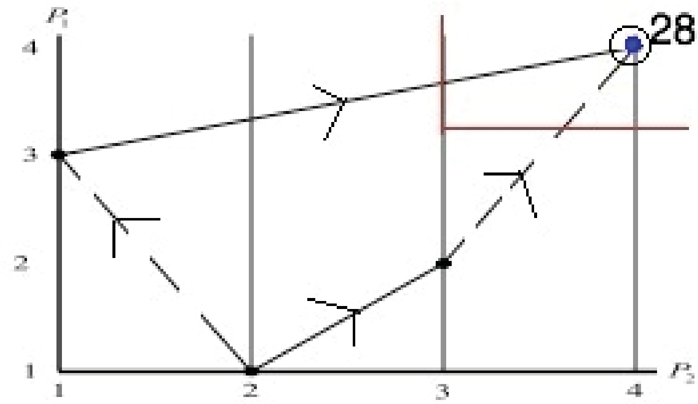
maxmin P1: 3
maxmin P2: 2
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,28) =

1, 3	2, 1
4, 4	3, 2

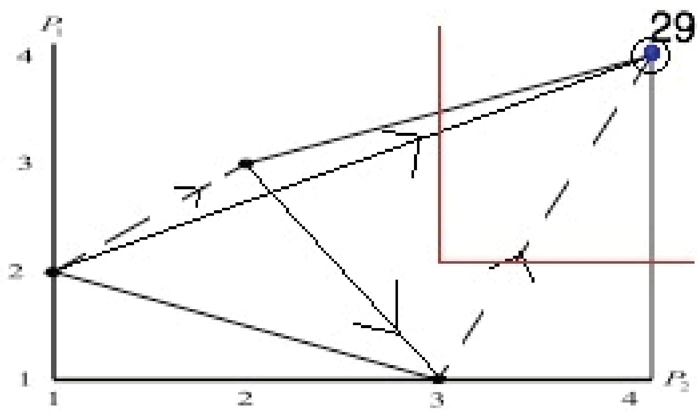
maxmin P1: 3
maxmin P2: 3
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,29) =

1, 2	2, 3
4, 4	3, 1

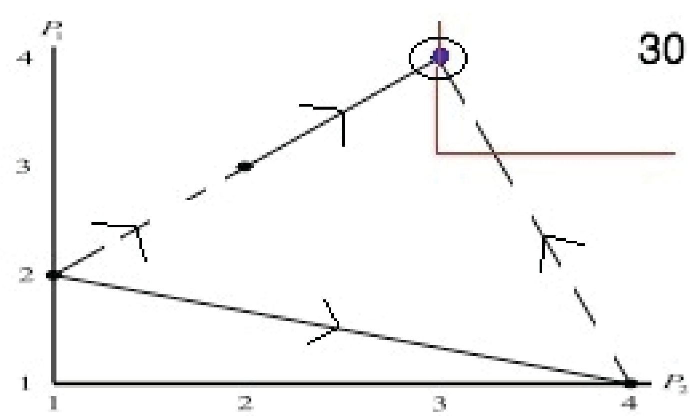
maxmin P1: 3
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,30) =

1, 2	2, 3
4, 1	3, 4

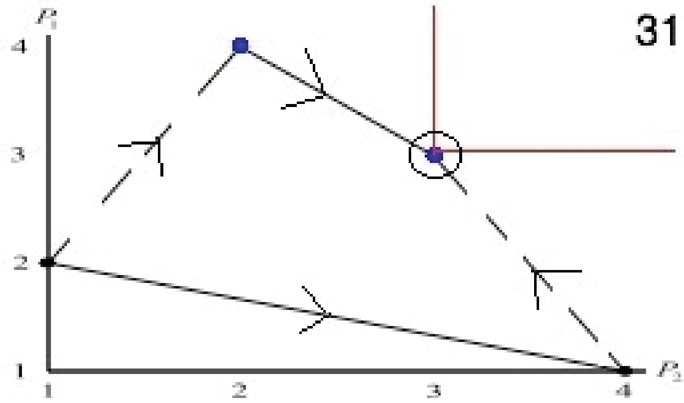
maxmin P1: 3
maxmin P2: 3
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,31) =

1, 2	2, 4
4, 1	3, 3

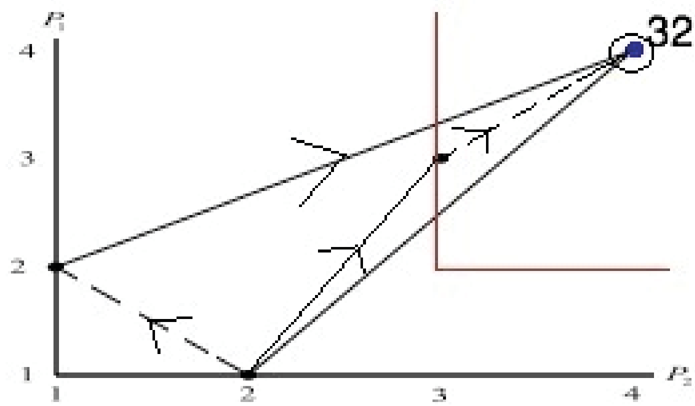
maxmin P1: 3
maxmin P2: 3
joint max P1: 2, 3
joint max P2: 4, 3
joint max sum: 6



Games4321_4321(:,:,32) =

1, 2	2, 1
4, 4	3, 3

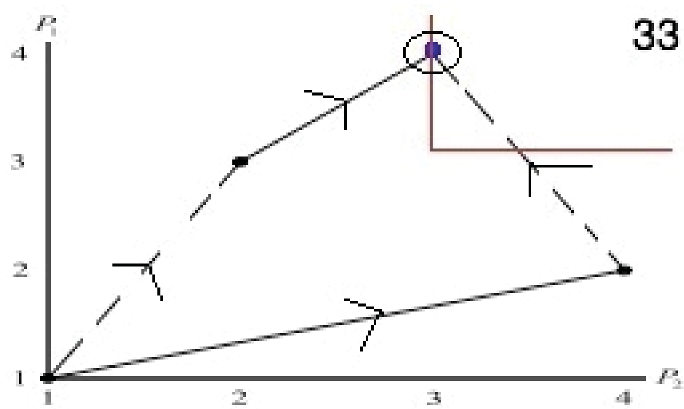
maxmin P1: 3
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,33) =

1, 1	2, 3
4, 2	3, 4

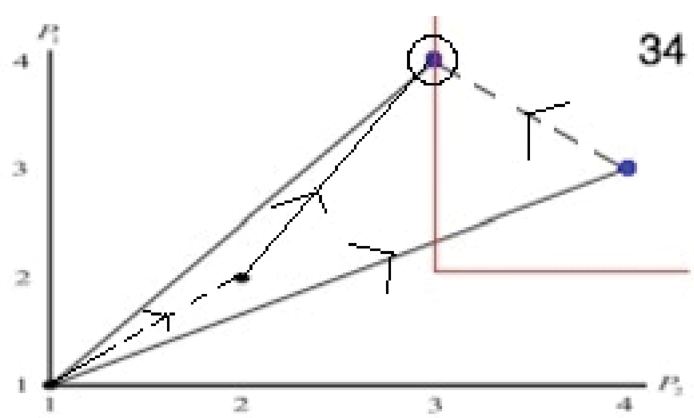
maxmin P1: 3
maxmin P2: 3
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,34) =

1, 1	2, 2
4, 3	3, 4

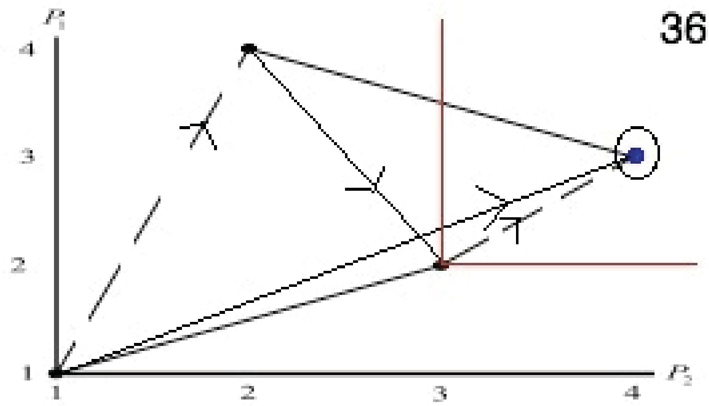
maxmin P1: 3
maxmin P2: 2
joint max P1: 3, 4
joint max P2: 4, 3
joint max sum: 7



Games4321_4321(:,:,36) =

1, 1	2, 4
4, 3	3, 2

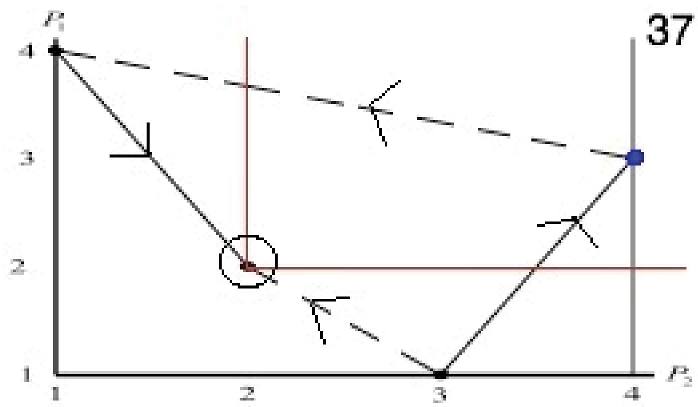
maxmin P1: 3
maxmin P2: 2
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,37) =

1, 4	4, 3
2, 2	3, 1

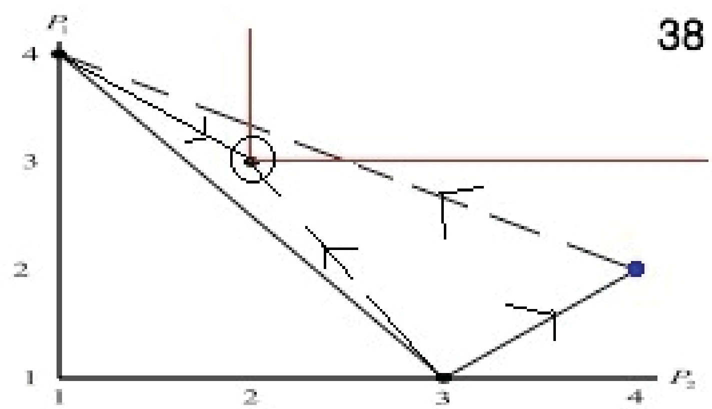
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,38) =

1, 4	4, 2
2, 3	3, 1

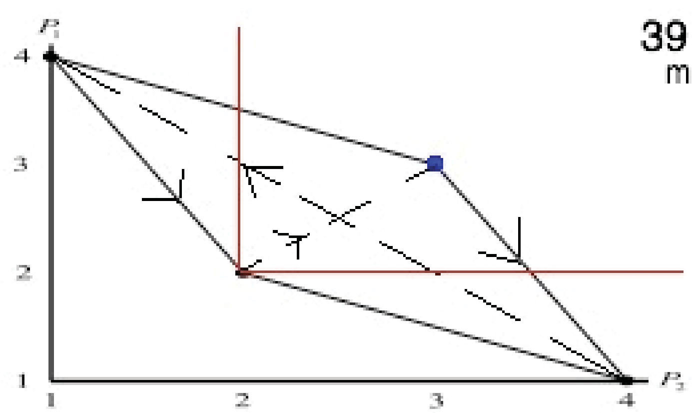
maxmin P1: 2
maxmin P2: 3
joint max P1: 4
joint max P2: 2
joint max sum: 6



Games4321_4321(:,:,39) =

1, 4	4, 1
2, 2	3, 3

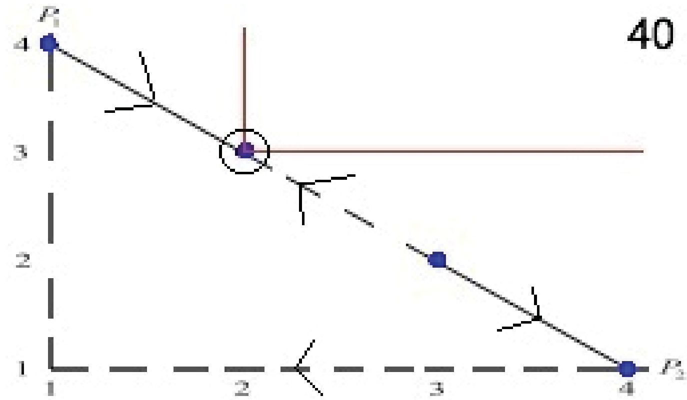
maxmin P1: 2
maxmin P2: 2
joint max P1: 3
joint max P2: 3
joint max sum: 6



Games4321_4321(:,:,40) =

1, 4	4, 1
2, 3	3, 2

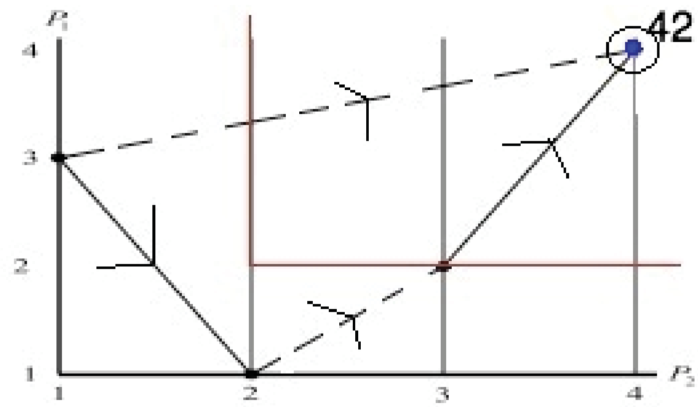
maxmin P1: 2
maxmin P2: 3
joint max P1: 1, 2, 3, 4
joint max P2: 4, 3, 2, 1
joint max sum: 5



Games4321_4321(:,:,42) =

1, 3	4, 4
2, 1	3, 2

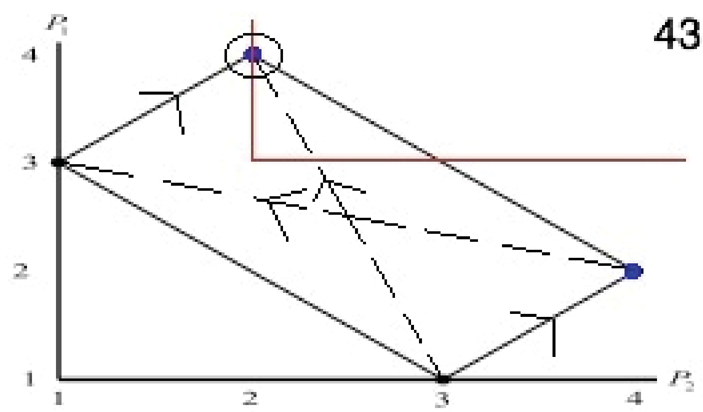
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,43) =

1, 3	4, 2
2, 4	3, 1

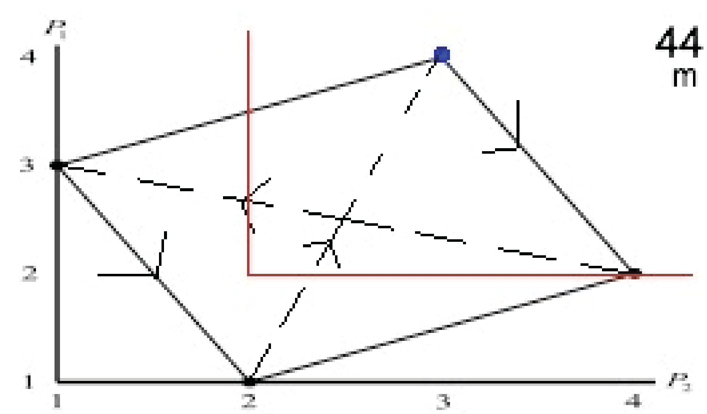
maxmin P1: 2
maxmin P2: 3
joint max P1: 2, 4
joint max P2: 4, 2
joint max sum: 6



Games4321_4321(:,:,44) =

1, 3	4, 2
2, 1	3, 4

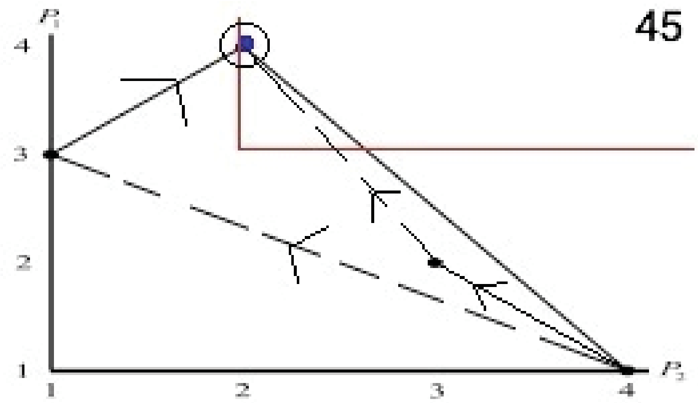
maxmin P1: 2
maxmin P2: 2
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,45) =

1, 3	4, 1
2, 4	3, 2

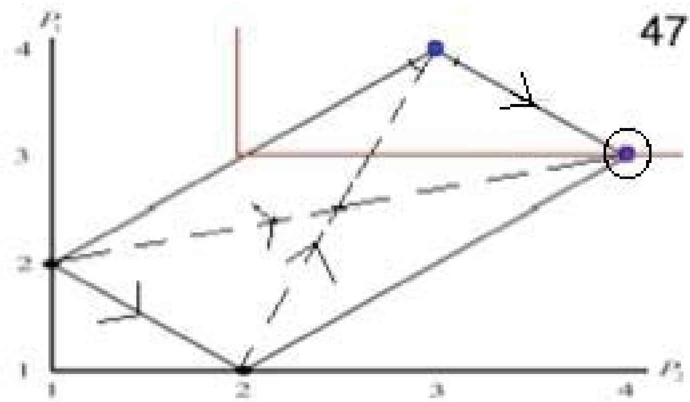
maxmin P1: 2
maxmin P2: 3
joint max P1: 2
joint max P2: 4
joint max sum: 6



Games4321_4321(:,:,47) =

1, 2	4, 3
2, 1	3, 4

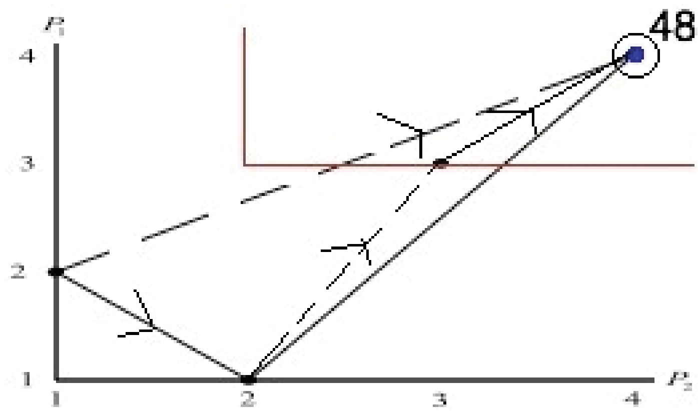
maxmin P1: 2
maxmin P2: 3
joint max P1: 3, 4
joint max P2: 4, 3
joint max sum: 7



Games4321_4321(:,:,48) =

1, 2	4, 4
2, 1	3, 3

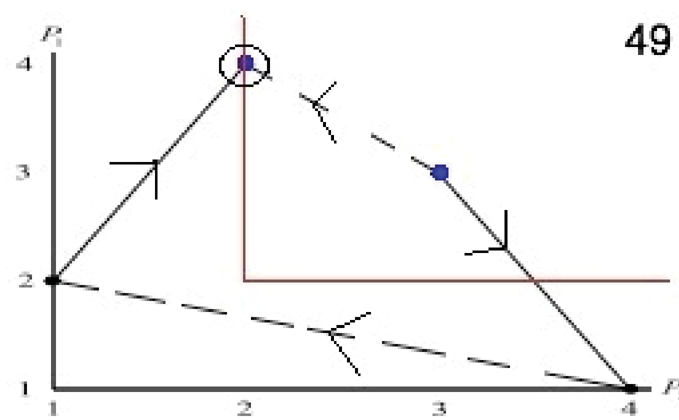
maxmin P1: 2
maxmin P2: 3
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,49) =

1, 2	4, 1
2, 4	3, 3

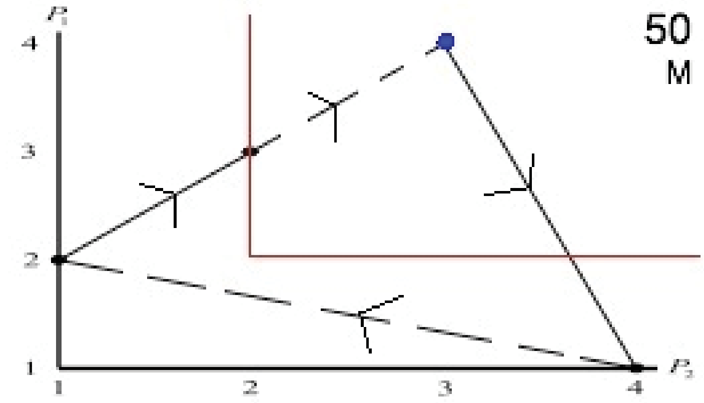
maxmin P1: 2
maxmin P2: 2
joint max P1: 2, 3
joint max P2: 4, 3
joint max sum: 6



Games4321_4321(:,:,50) =

1, 2	4, 1
2, 3	3, 4

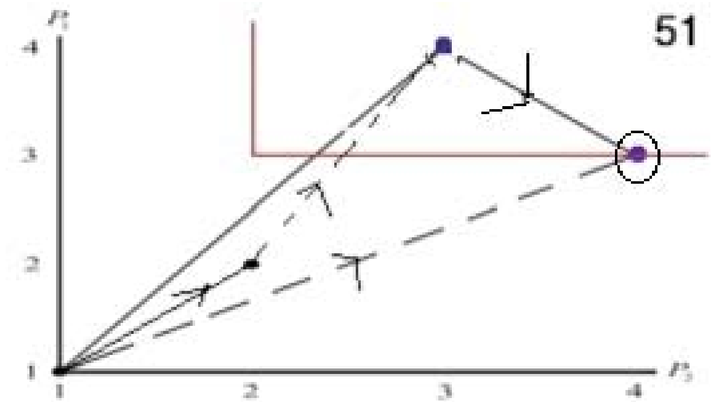
maxmin P1: 2
maxmin P2: 2
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,51) =

1, 1	4, 3
2, 2	3, 4

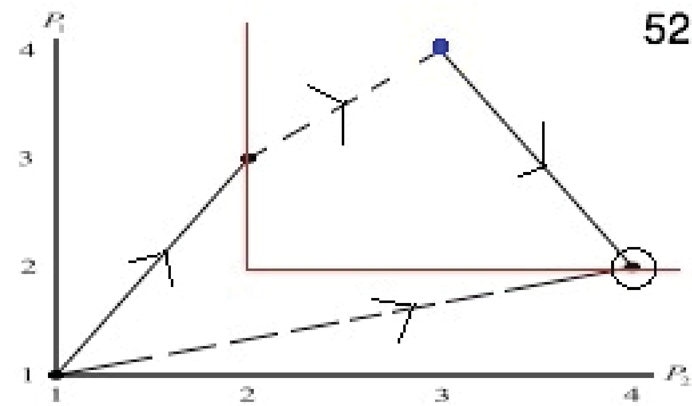
maxmin P1: 2
maxmin P2: 3
joint max P1: 3, 4
joint max P2: 4, 3
joint max sum: 7



Games4321_4321(:,:,52) =

1, 1	4, 2
2, 3	3, 4

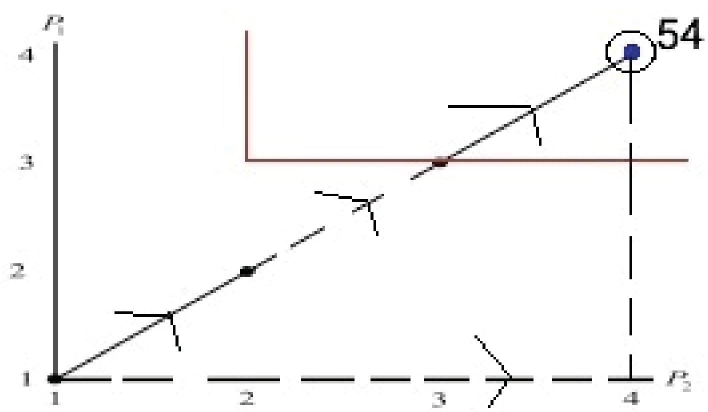
maxmin P1: 2
maxmin P2: 2
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,54) =

1, 1	4, 4
2, 2	3, 3

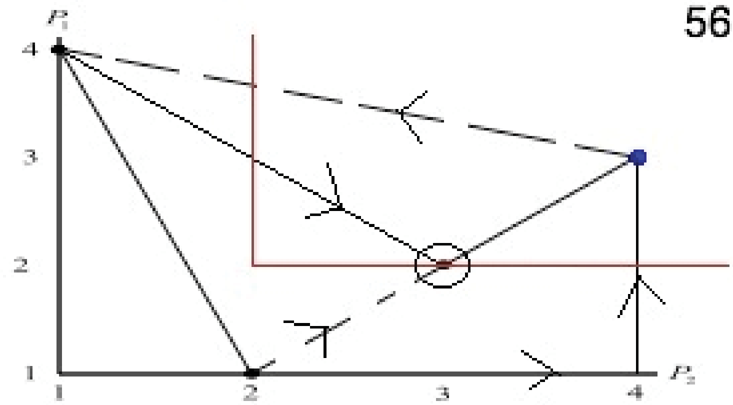
maxmin P1: 2
maxmin P2: 3
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,56) =

1, 4	4, 3
3, 2	2, 1

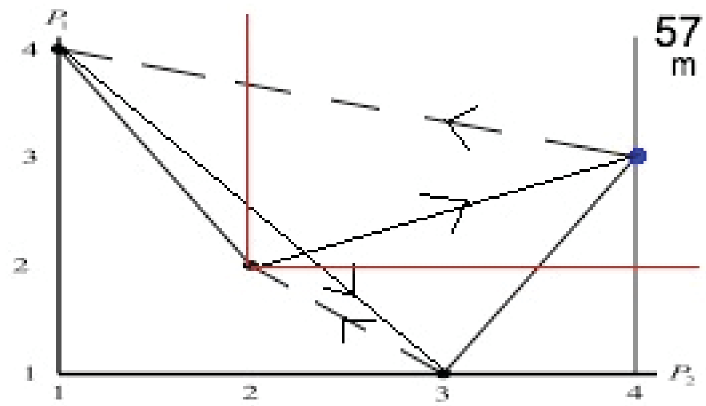
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,57) =

1, 4	4, 3
3, 1	2, 2

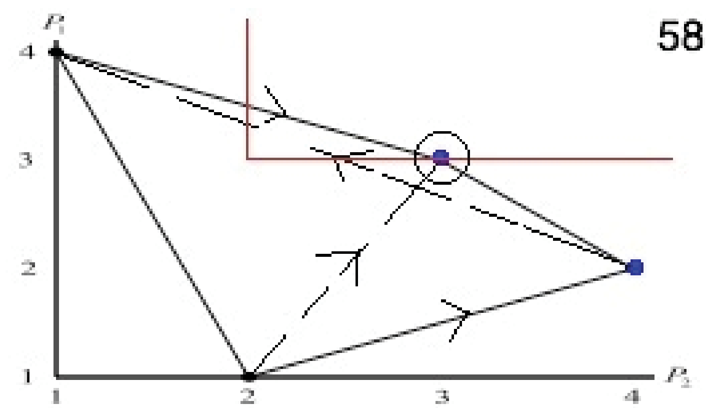
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,58) =

1, 4	4, 2
3, 3	2, 1

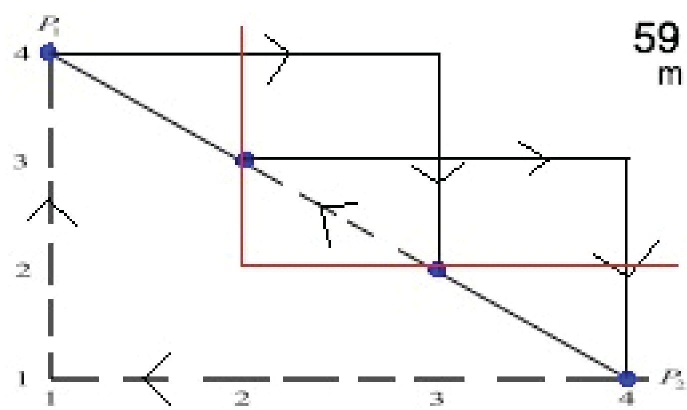
maxmin P1: 2
maxmin P2: 3
joint max P1: 3, 4
joint max P2: 3, 2
joint max sum: 6



Games4321_4321(:,:,59) =

1, 4	4, 1
3, 2	2, 3

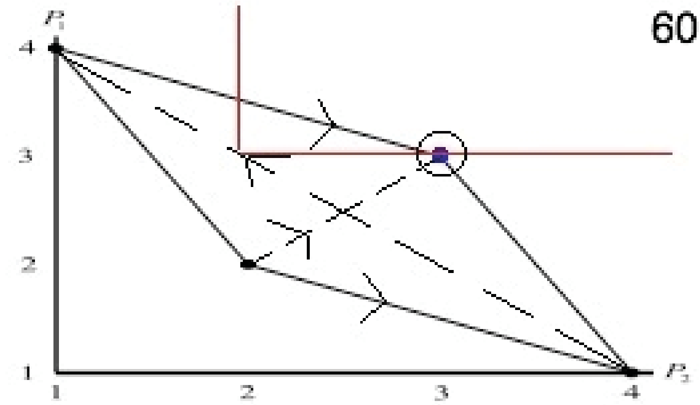
maxmin P1: 2
maxmin P2: 2
joint max P1: 1, 2, 3, 4
joint max P2: 4, 3, 2, 1
joint max sum: 5



Games4321_4321(:,:,60) =

1, 4	4, 1
3, 3	2, 2

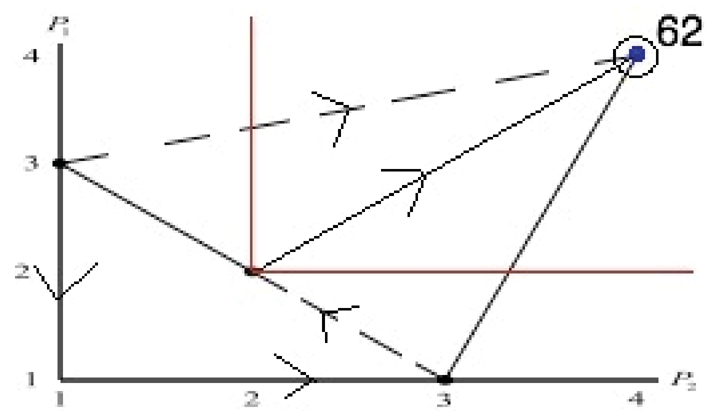
maxmin P1: 2
maxmin P2: 3
joint max P1: 3
joint max P2: 3
joint max sum: 6



Games4321_4321(:,:,62) =

1, 3	4, 4
3, 1	2, 2

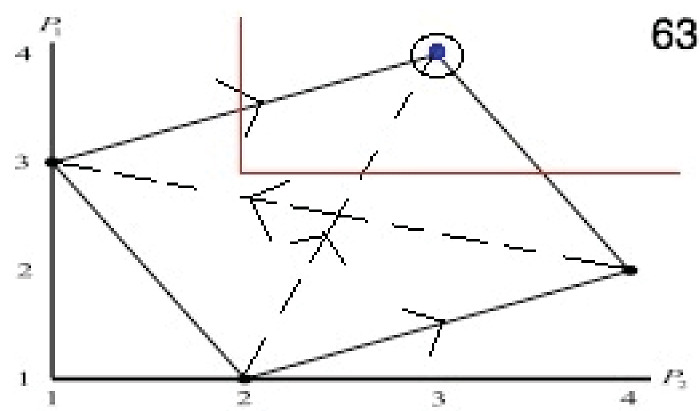
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,63) =

1, 3	4, 2
3, 4	2, 1

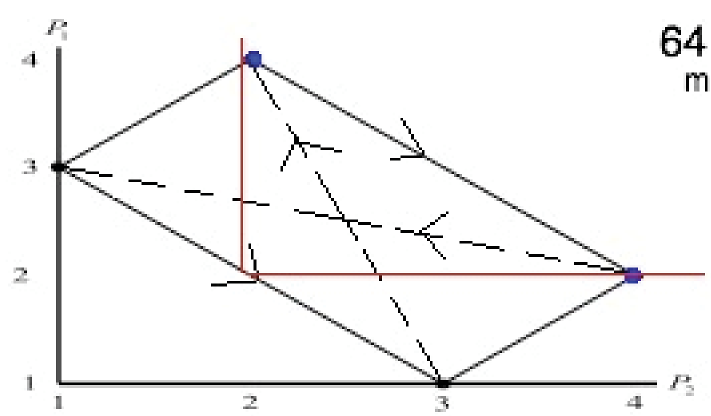
maxmin P1: 2
maxmin P2: 3
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,64) =

1, 3	4, 2
3, 1	2, 4

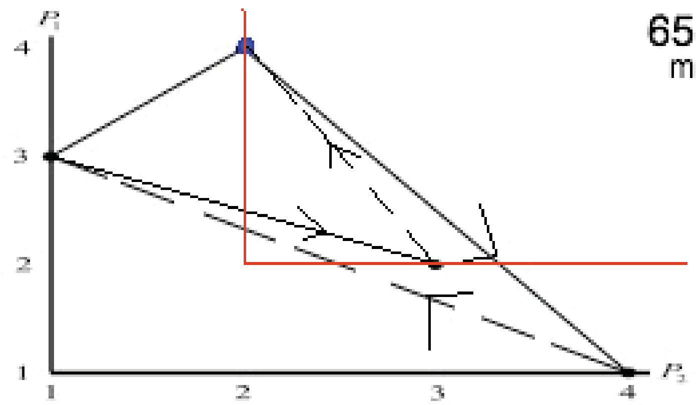
maxmin P1: 2
maxmin P2: 2
joint max P1: 2, 4
joint max P2: 4, 2
joint max sum: 6



Games4321_4321(:,:,65) =

1, 3	4, 1
3, 2	2, 4

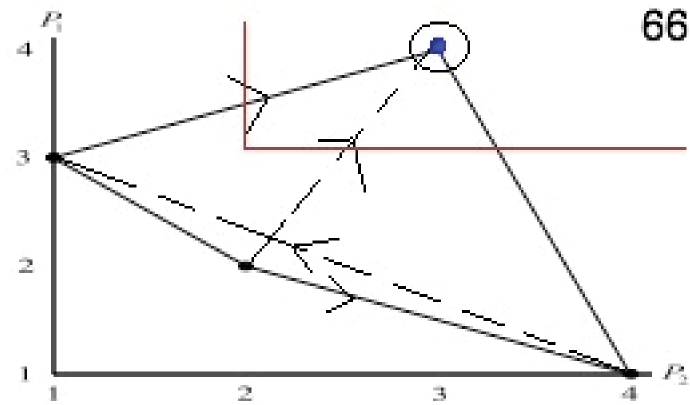
maxmin P1: 2
maxmin P2: 2
joint max P1: 2
joint max P2: 4
joint max sum: 6



Games4321_4321(:,:,66) =

1, 3	4, 1
3, 4	2, 2

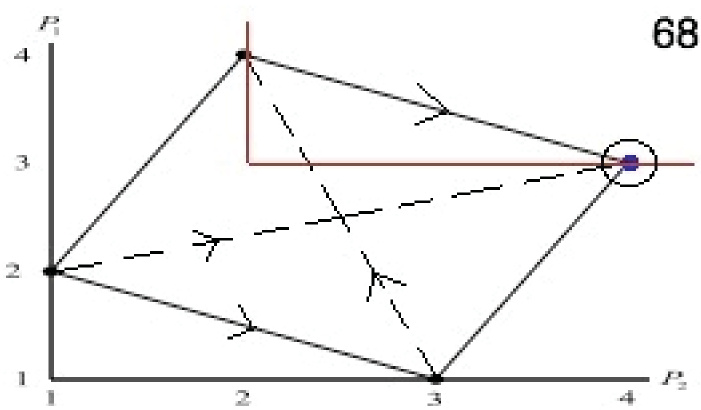
maxmin P1: 2
maxmin P2: 3
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,68) =

1, 2	4, 3
3, 1	2, 4

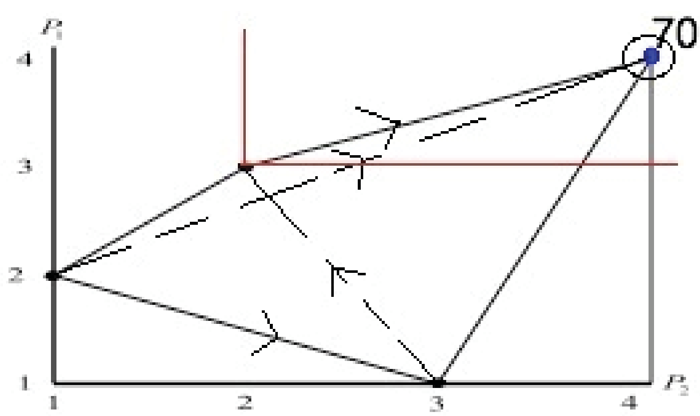
maxmin P1: 2
maxmin P2: 3
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,70) =

1, 2	4, 4
3, 1	2, 3

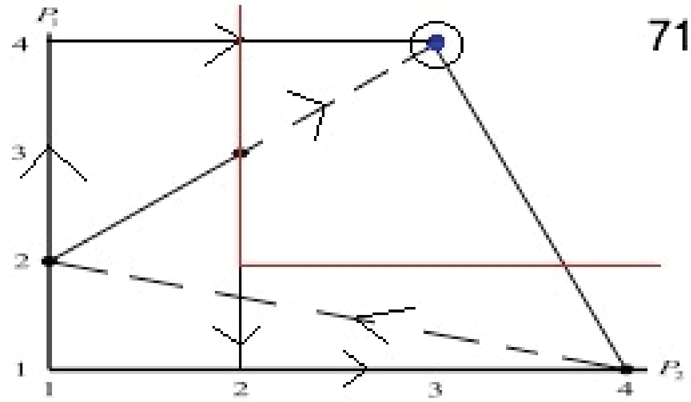
maxmin P1: 2
maxmin P2: 3
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,71) =

1, 2	4, 1
3, 4	2, 3

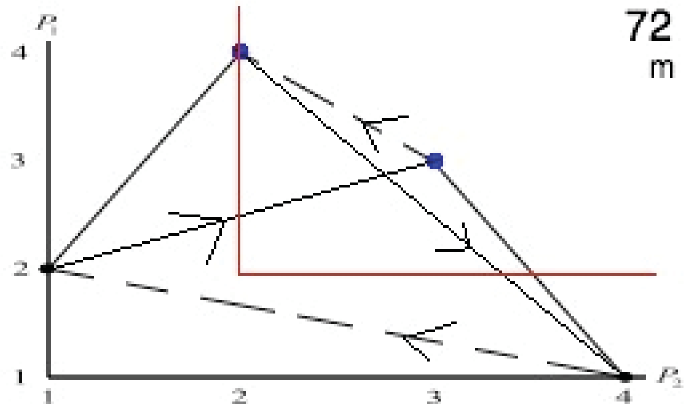
maxmin P1: 2
maxmin P2: 2
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,72) =

1, 2	4, 1
3, 3	2, 4

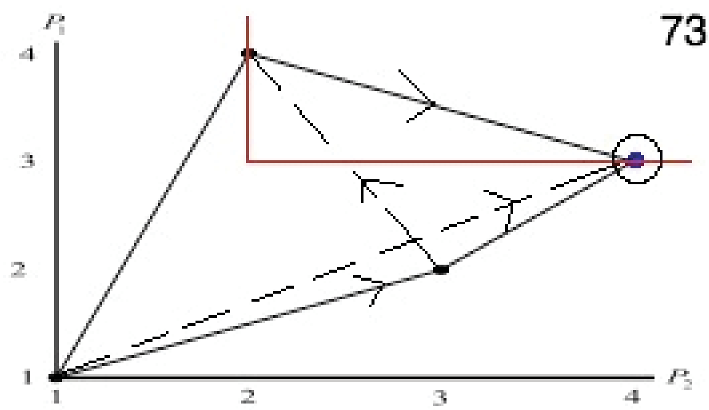
maxmin P1: 2
maxmin P2: 2
joint max P1: 2, 3
joint max P2: 4, 3
joint max sum: 6



Games4321_4321(:,:,73) =

1, 1	4, 3
3, 2	2, 4

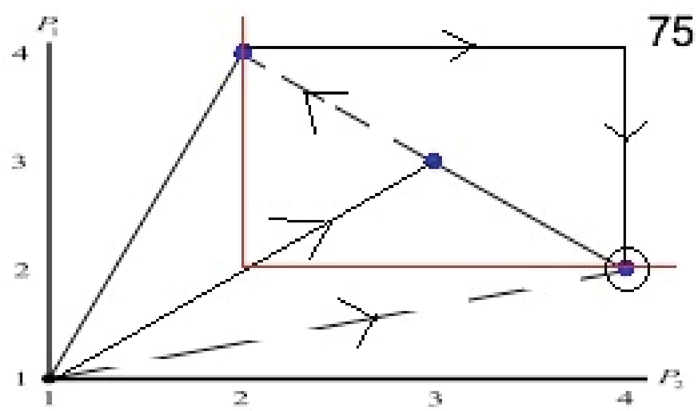
maxmin P1: 2
maxmin P2: 3
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,75) =

1, 1	4, 2
3, 3	2, 4

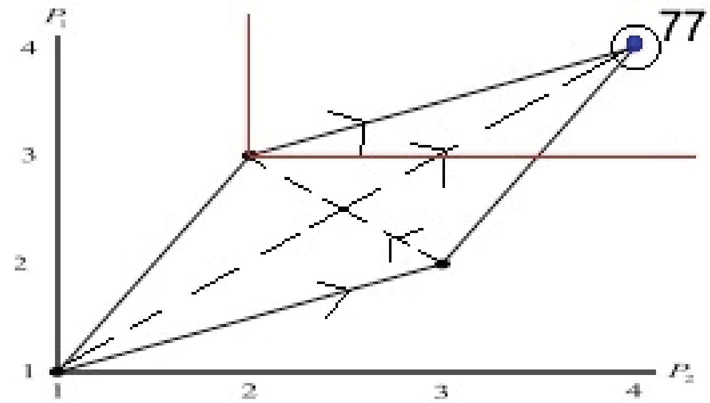
maxmin P1: 2
maxmin P2: 2
joint max P1: 2, 3, 4
joint max P2: 4, 3, 2
joint max sum: 6



Games4321_4321(:,:,77) =

1, 1	4, 4
3, 2	2, 3

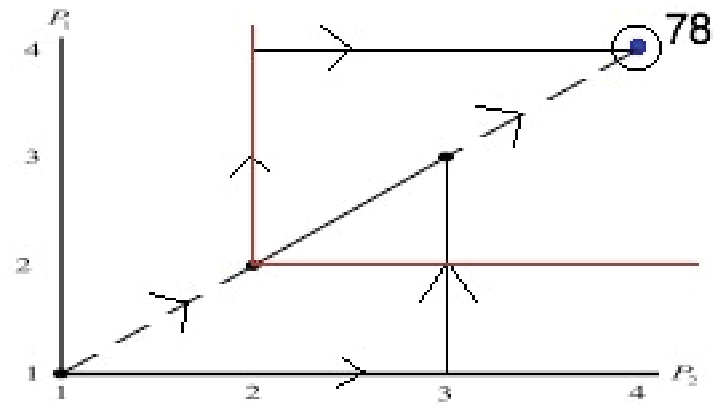
maxmin P1: 2
maxmin P2: 3
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,78) =

1, 1	4, 4
3, 3	2, 2

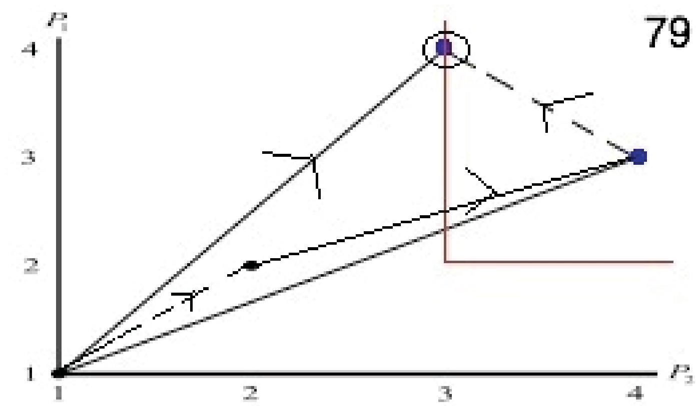
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,79) =

1, 1	2, 2
3, 4	4, 3

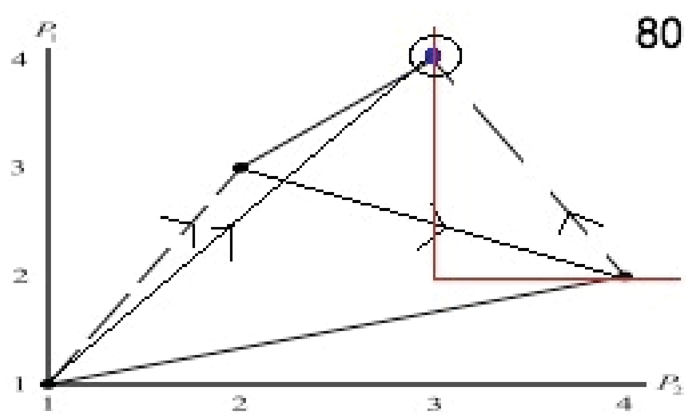
maxmin P1: 3
maxmin P2: 2
joint max P1: 3, 4
joint max P2: 4, 3
joint max sum: 7



Games4321_4321(:,:,80) =

1, 1	2, 3
3, 4	4, 2

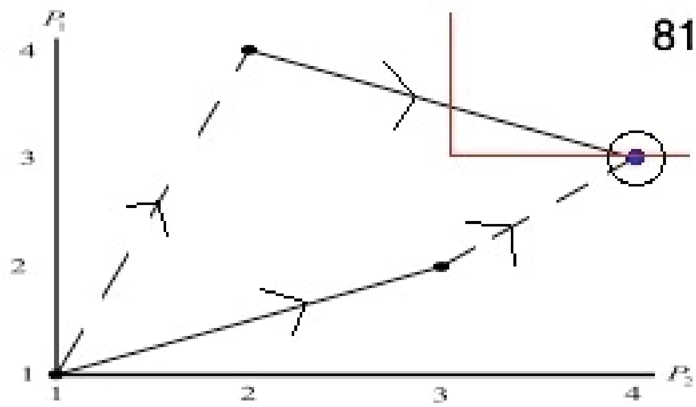
maxmin P1: 3
maxmin P2: 2
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,81) =

1, 1	2, 4
3, 2	4, 3

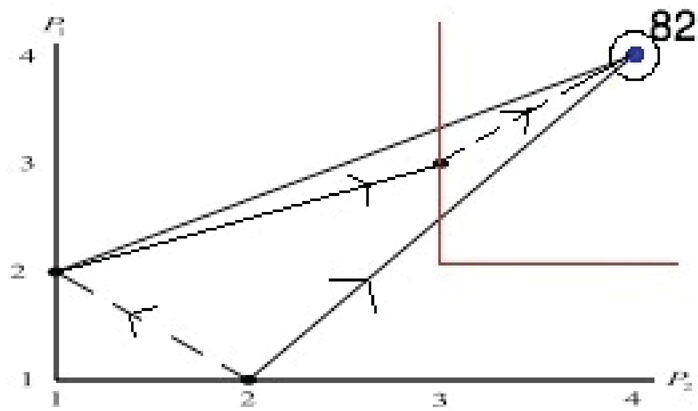
maxmin P1: 3
maxmin P2: 3
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,82) =

1, 2	2, 1
3, 3	4, 4

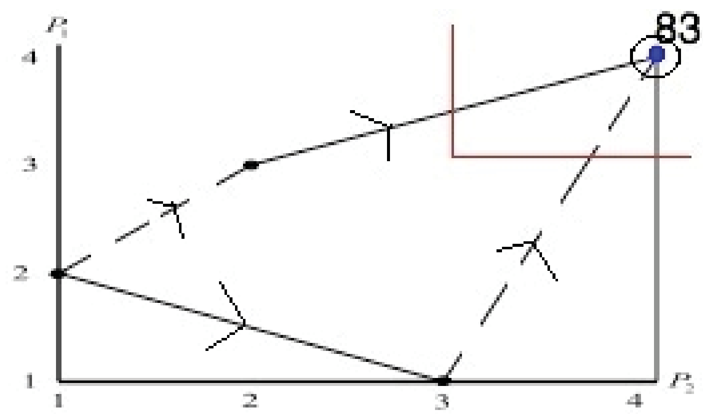
maxmin P1: 3
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,83) =

1, 2	2, 3
3, 1	4, 4

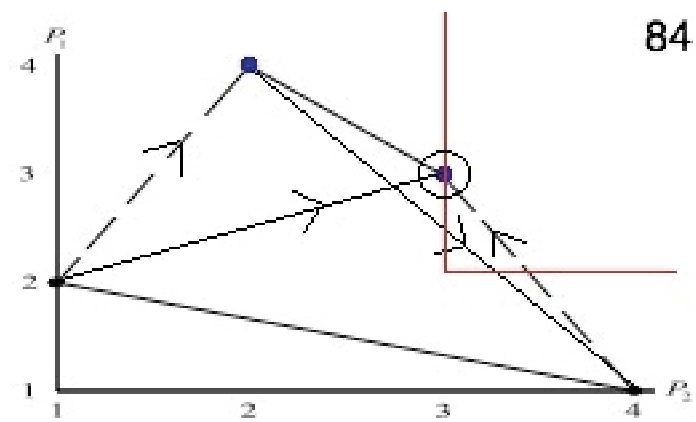
maxmin P1: 3
maxmin P2: 3
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,84) =

1, 2	2, 4
3, 3	4, 1

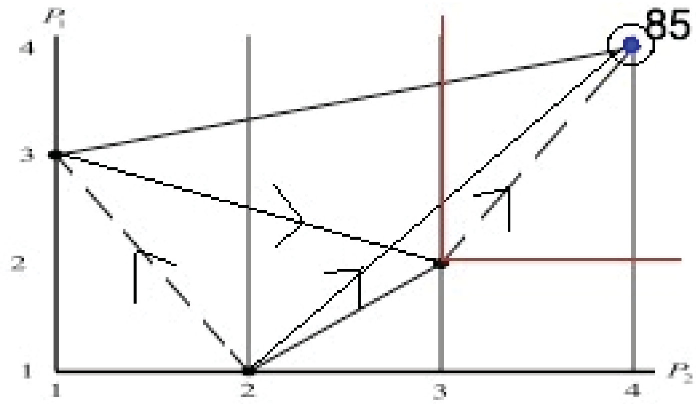
maxmin P1: 3
maxmin P2: 2
joint max P1: 2, 3
joint max P2: 4, 3
joint max sum: 6



Games4321_4321(:,:,85) =

1, 3	2, 1
3, 2	4, 4

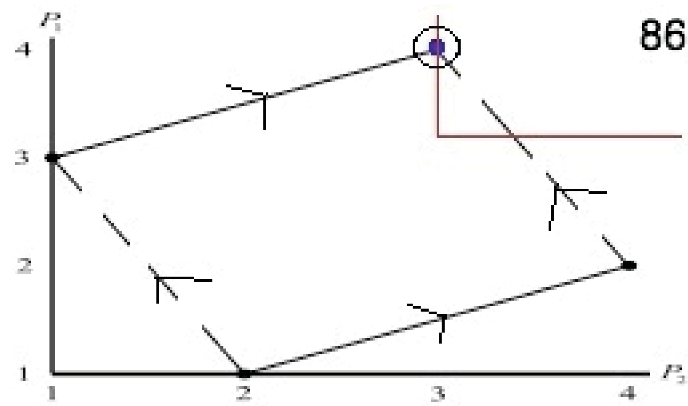
maxmin P1: 3
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,86) =

1, 3	2, 1
3, 4	4, 2

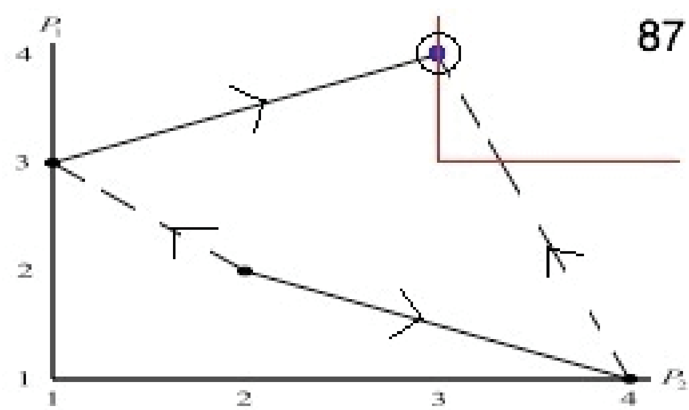
maxmin P1: 3
maxmin P2: 3
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,87) =

1, 3	2, 2
3, 4	4, 1

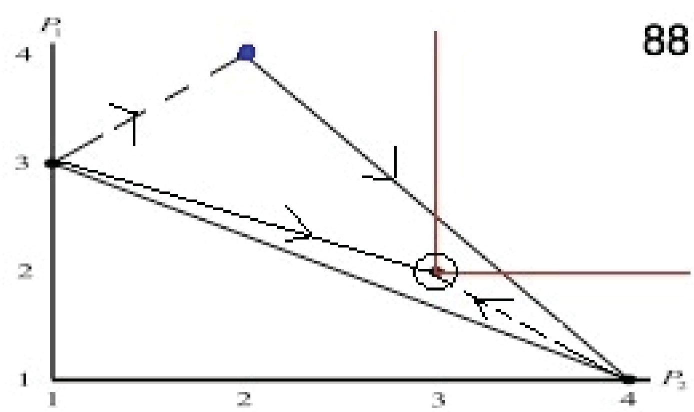
maxmin P1: 3
maxmin P2: 3
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,88) =

1, 3	2, 4
3, 2	4, 1

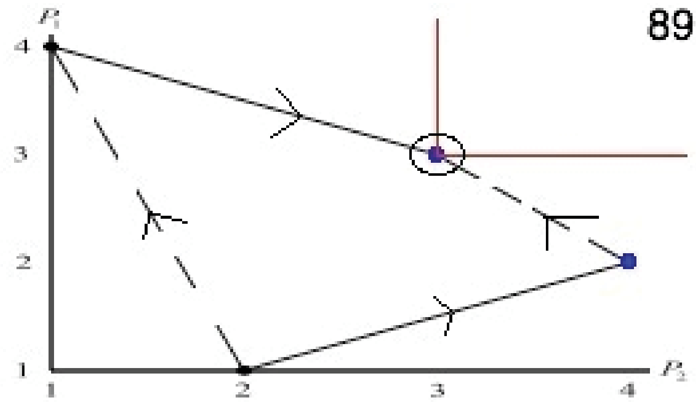
maxmin P1: 3
maxmin P2: 2
joint max P1: 2
joint max P2: 4
joint max sum: 6



Games4321_4321(:,:,89) =

1, 4	2, 1
3, 3	4, 2

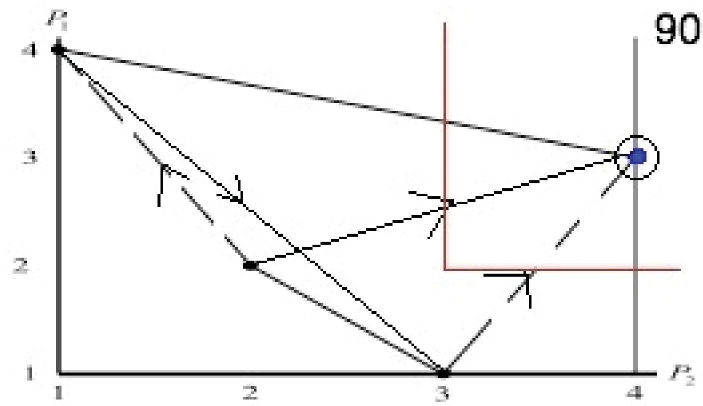
maxmin P1: 3
maxmin P2: 3
joint max P1: 3, 4
joint max P2: 3, 2
joint max sum: 6



Games4321_4321(:,:,90) =

1, 4	2, 2
3, 1	4, 3

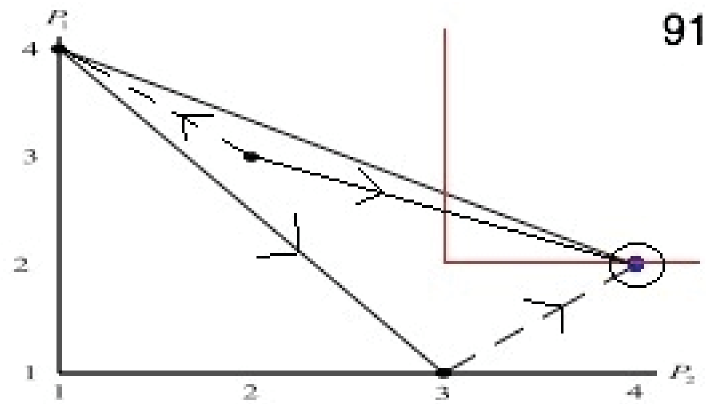
maxmin P1: 3
maxmin P2: 2
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,91) =

1, 4	2, 3
3, 1	4, 2

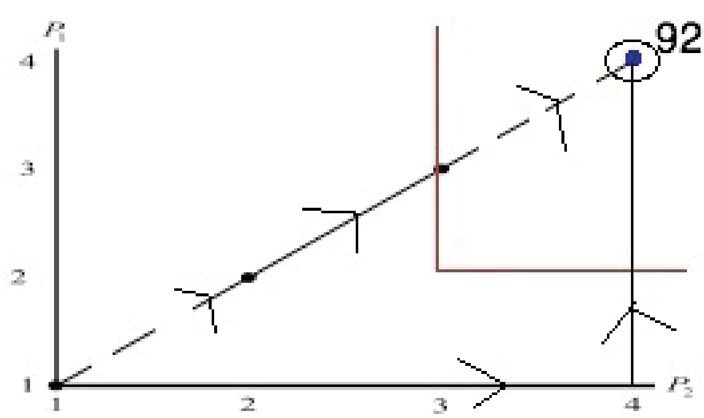
maxmin P1: 3
maxmin P2: 2
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,92) =

1, 1	2, 2
4, 4	3, 3

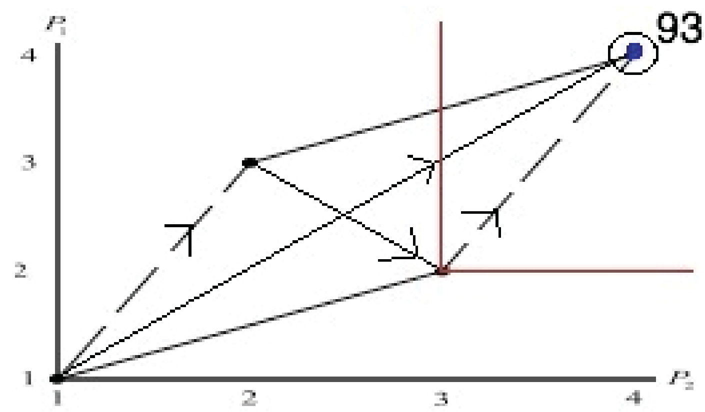
maxmin P1: 3
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,93) =

1, 1	2, 3
4, 4	3, 2

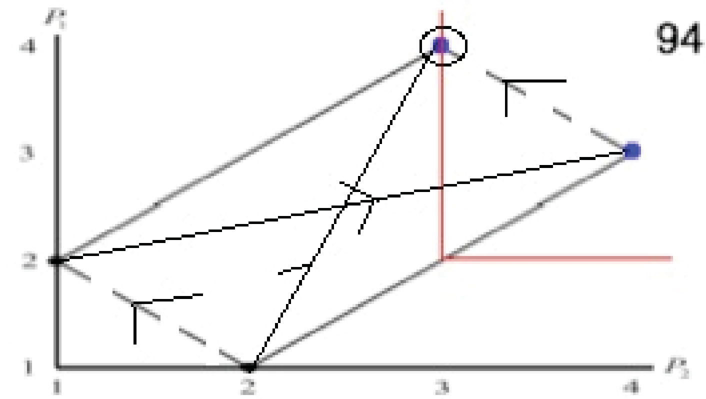
maxmin P1: 3
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,94) =

1, 2	2, 1
4, 3	3, 4

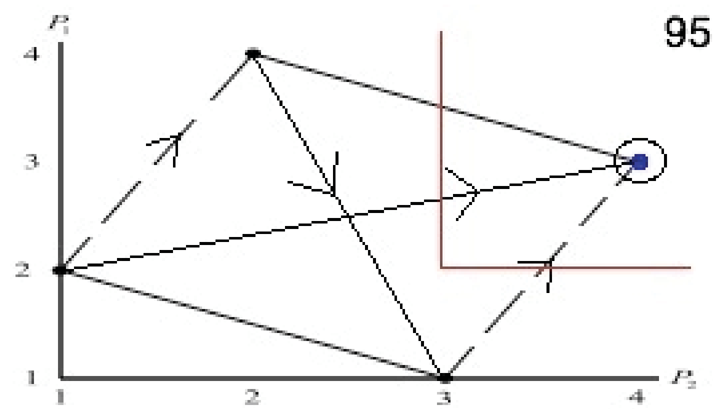
maxmin P1: 3
maxmin P2: 2
joint max P1: 3, 4
joint max P2: 4, 3
joint max sum: 7



Games4321_4321(:,:,95) =

1, 2	2, 4
4, 3	3, 1

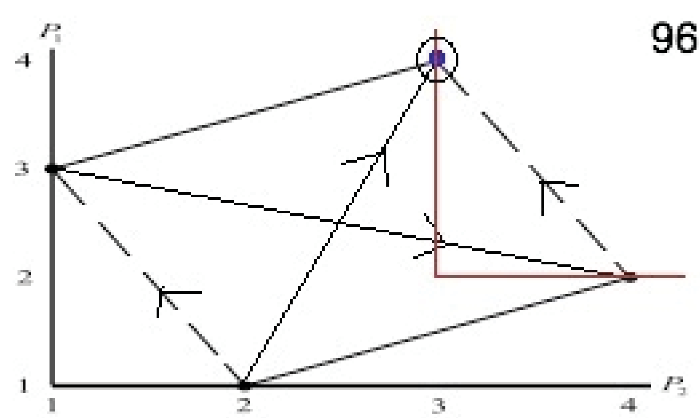
maxmin P1: 3
maxmin P2: 2
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,96) =

1, 3	2, 1
4, 2	3, 4

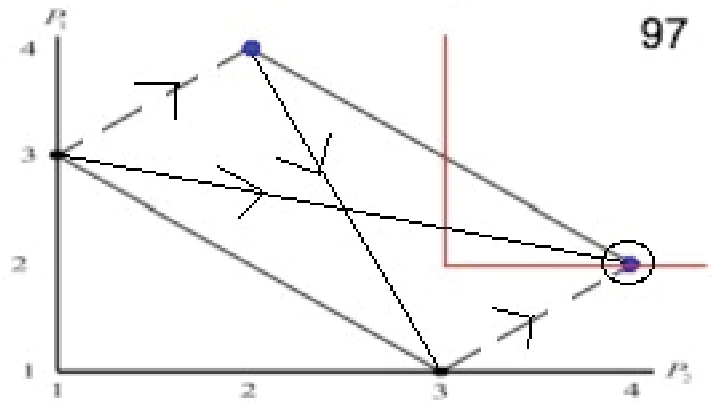
maxmin P1: 3
maxmin P2: 2
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,97) =

1, 3	2, 4
4, 2	3, 1

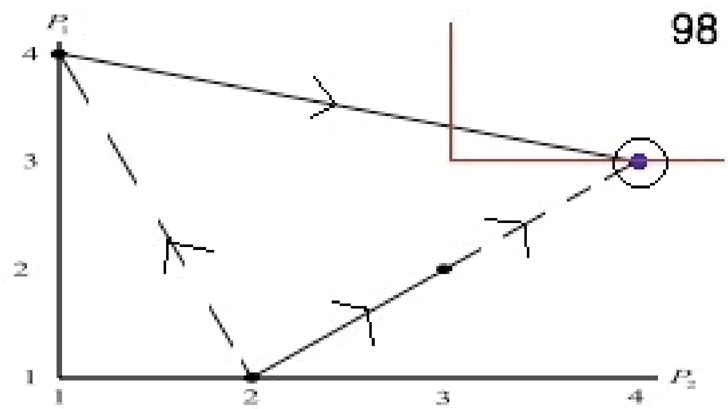
maxmin P1: 3
maxmin P2: 2
joint max P1: 2, 4
joint max P2: 4, 2
joint max sum: 6



Games4321_4321(:,:,98) =

1, 4	2, 1
4, 3	3, 2

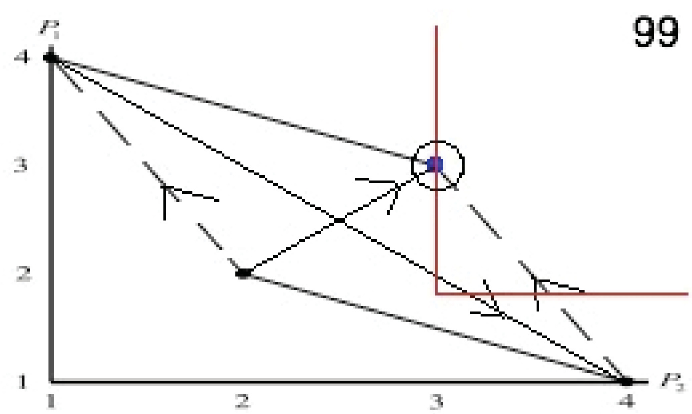
maxmin P1: 3
maxmin P2: 3
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,99) =

1, 4	2, 2
4, 1	3, 3

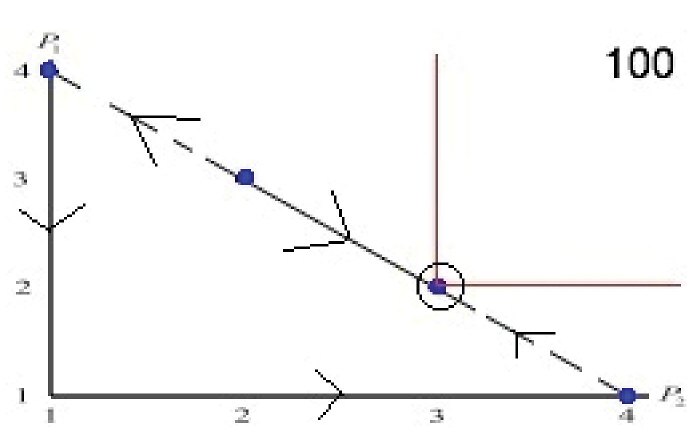
maxmin P1: 3
maxmin P2: 2
joint max P1: 3
joint max P2: 3
joint max sum: 6



Games4321_4321(:,:,100) =

1, 4	2, 3
4, 1	3, 2

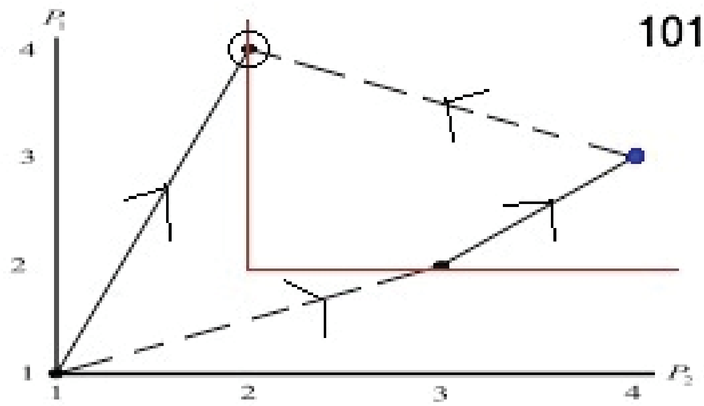
maxmin P1: 3
maxmin P2: 2
joint max P1: 1, 2, 3, 4
joint max P2: 4, 3, 2, 1
joint max sum: 5



Games4321_4321(:,:,101) =

1, 1	3, 2
2, 4	4, 3

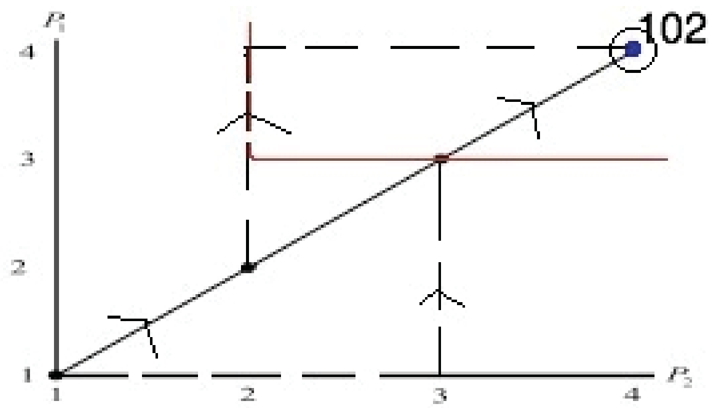
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,102) =

1, 1	3, 3
2, 2	4, 4

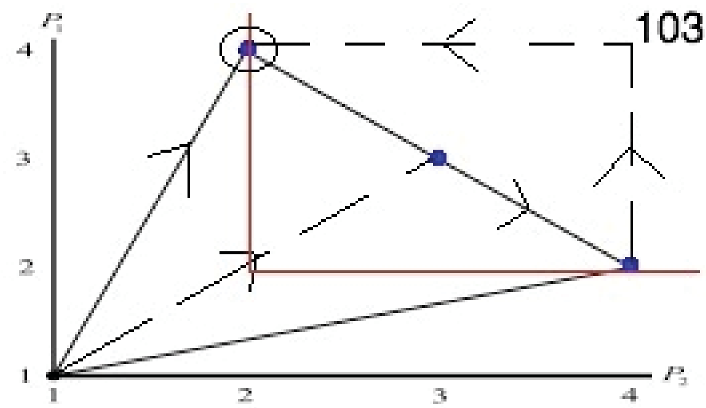
maxmin P1: 2
maxmin P2: 3
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,103) =

1, 1	3, 3
2, 4	4, 2

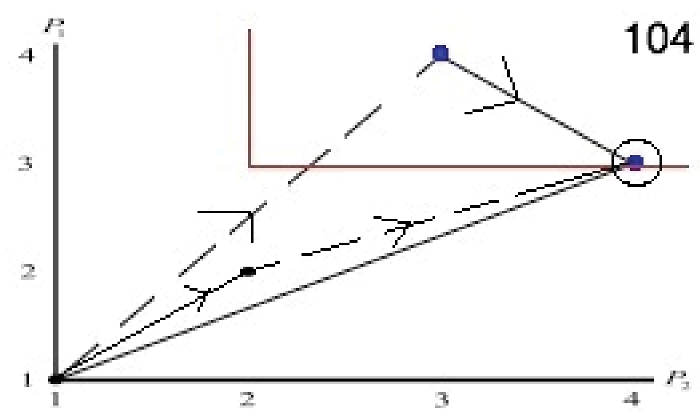
maxmin P1: 2
maxmin P2: 2
joint max P1: 2, 3, 4
joint max P2: 4, 3, 2
joint max sum: 6



Games4321_4321(:,:,104) =

1, 1	3, 4
2, 2	4, 3

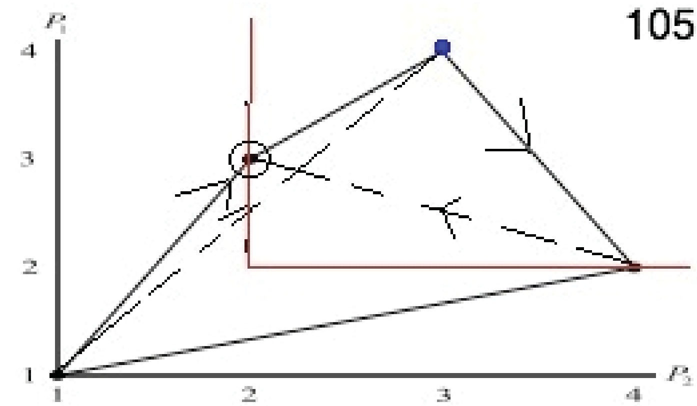
maxmin P1: 2
maxmin P2: 3
joint max P1: 3, 4
joint max P2: 4, 3
joint max sum: 7



Games4321_4321(:,:,105) =

1, 1	3, 4
2, 3	4, 2

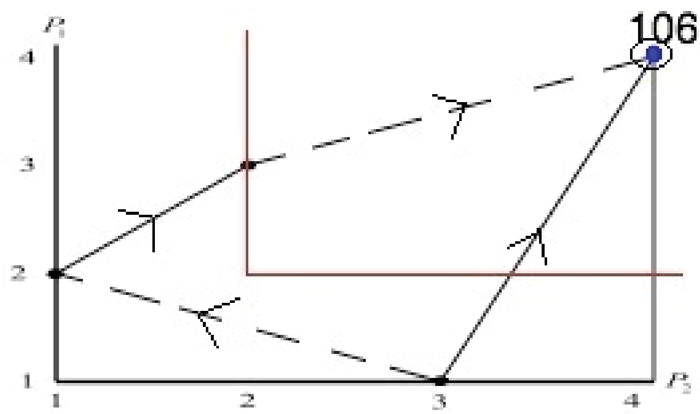
maxmin P1: 2
maxmin P2: 2
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,106) =

1, 2	3, 1
2, 3	4, 4

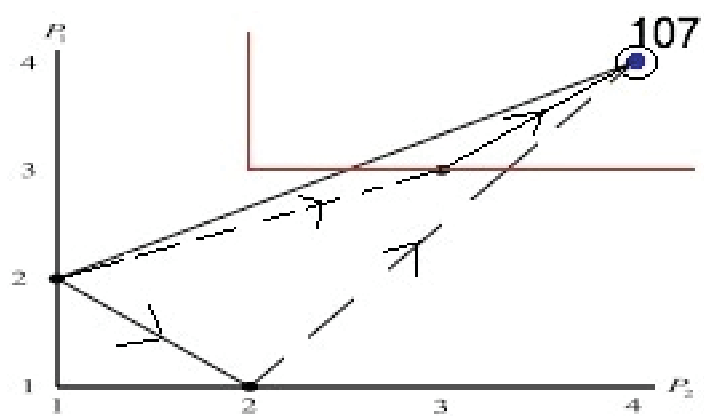
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,107) =

1, 2	3, 3
2, 1	4, 4

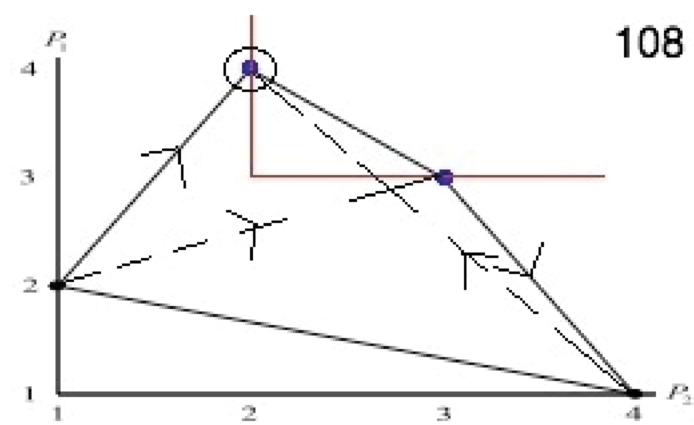
maxmin P1: 2
maxmin P2: 3
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,108) =

1, 2	3, 3
2, 4	4, 1

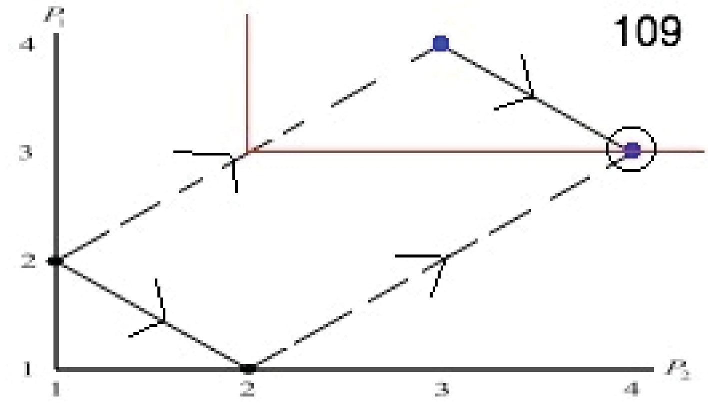
maxmin P1: 2
maxmin P2: 2
joint max P1: 2, 3
joint max P2: 4, 3
joint max sum: 6



Games4321_4321(:,:,109) =

1, 2	3, 4
2, 1	4, 3

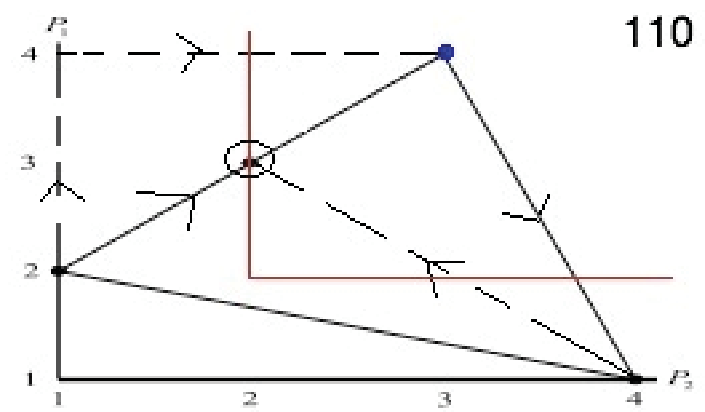
maxmin P1: 2
maxmin P2: 3
joint max P1: 3, 4
joint max P2: 4, 3
joint max sum: 7



Games4321_4321(:,:,110) =

1, 2	3, 4
2, 3	4, 1

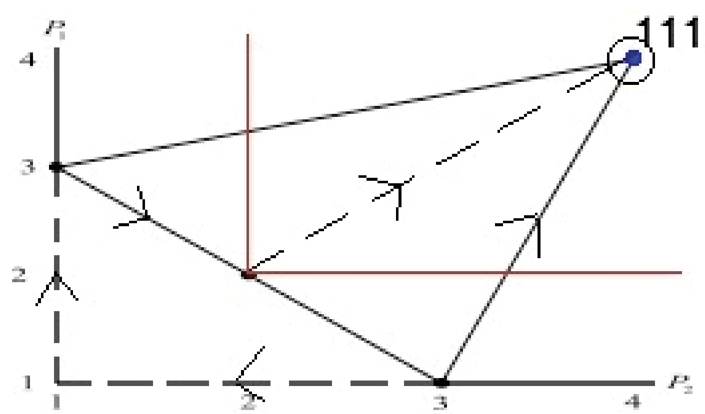
maxmin P1: 2
maxmin P2: 2
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,111) =

1, 3	3, 1
2, 2	4, 4

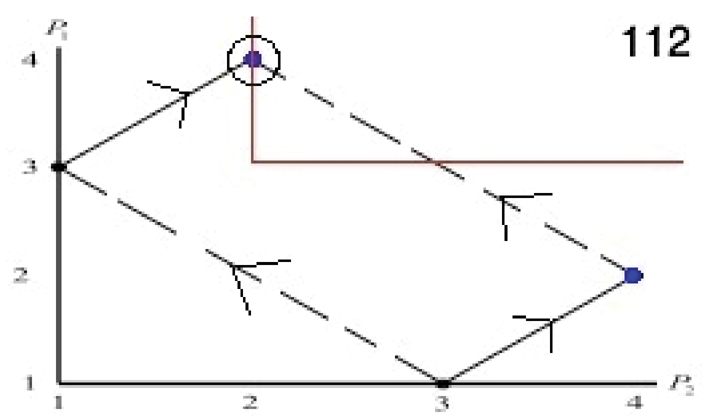
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,112) =

1, 3	3, 1
2, 4	4, 2

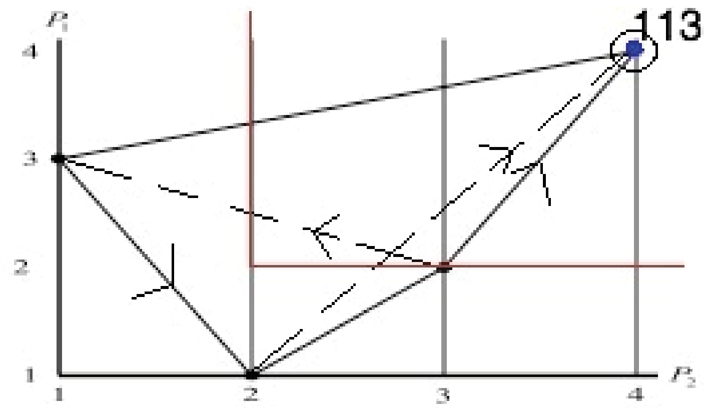
maxmin P1: 2
maxmin P2: 3
joint max P1: 2, 4
joint max P2: 4, 2
joint max sum: 6



Games4321_4321(:,:,113) =

1, 3	3, 2
2, 1	4, 4

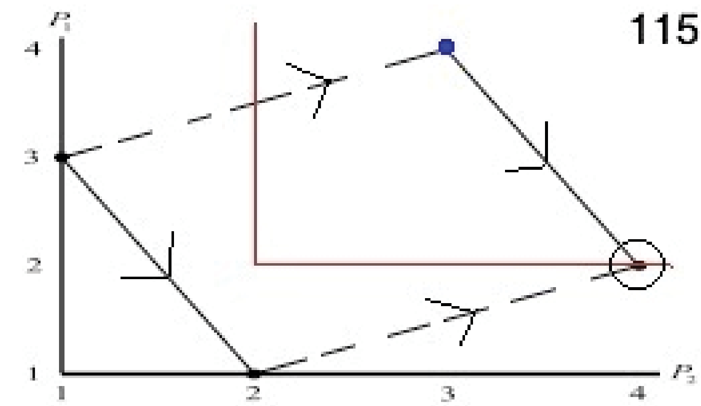
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,115) =

1, 3	3, 4
2, 1	4, 2

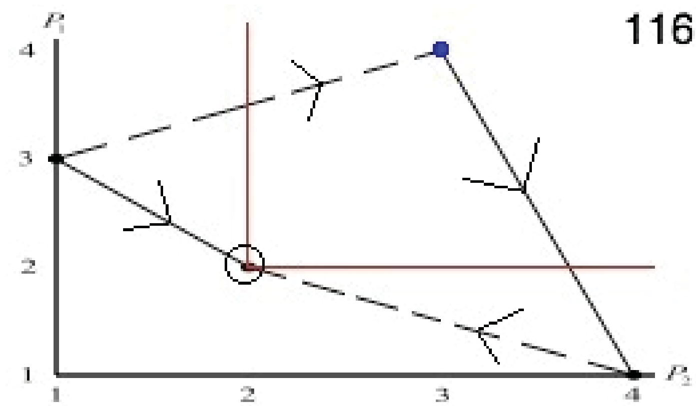
maxmin P1: 2
maxmin P2: 2
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,116) =

1, 3	3, 4
2, 2	4, 1

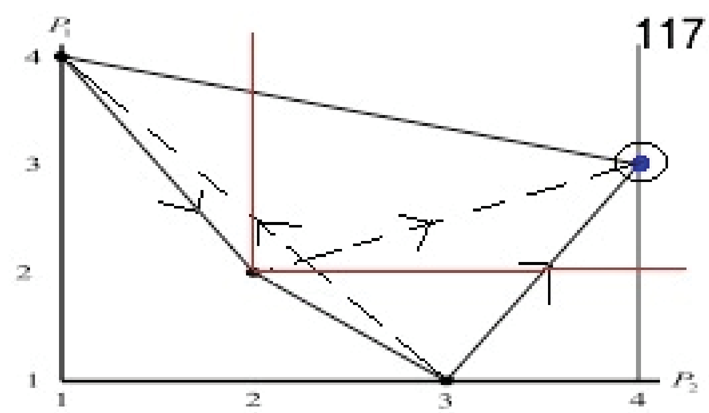
maxmin P1: 2
maxmin P2: 2
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,117) =

1, 4	3, 1
2, 2	4, 3

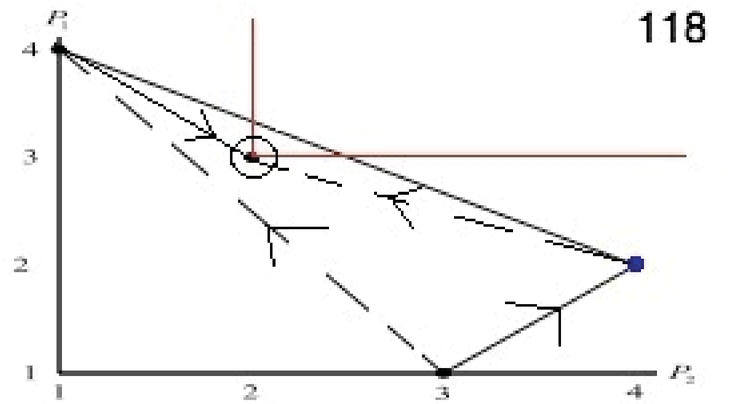
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,118) =

1, 4	3, 1
2, 3	4, 2

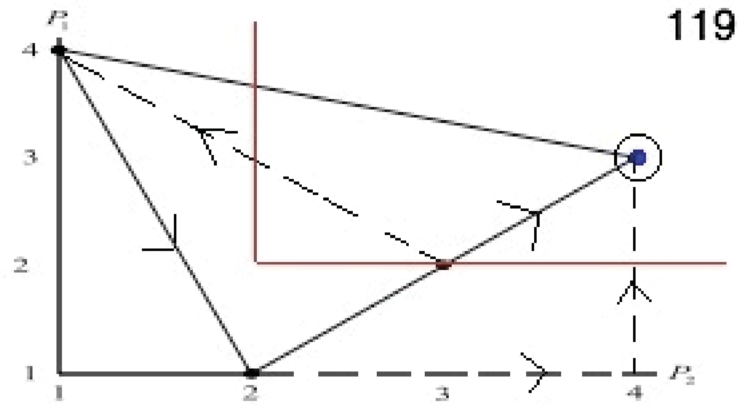
maxmin P1: 2
maxmin P2: 3
joint max P1: 4
joint max P2: 2
joint max sum: 6



Games4321_4321(:,:,119) =

1, 4	3, 2
2, 1	4, 3

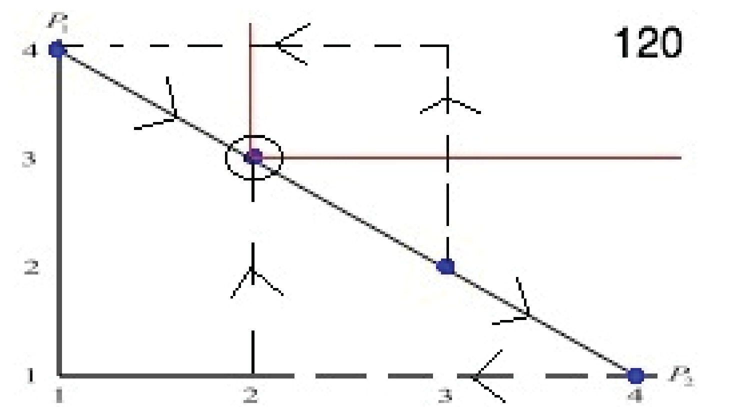
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,120) =

1, 4	3, 2
2, 3	4, 1

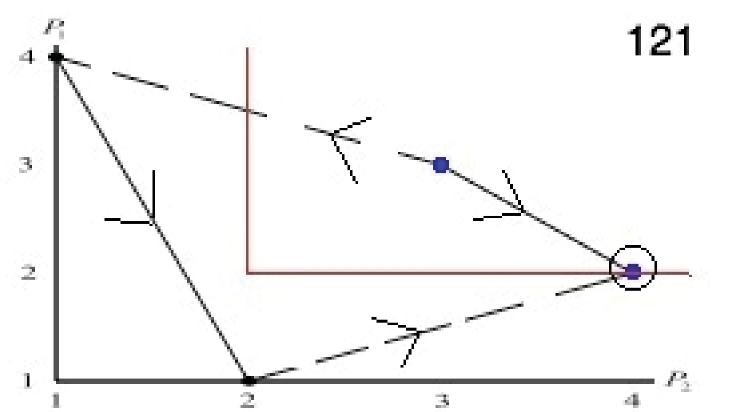
maxmin P1: 2
maxmin P2: 3
joint max P1: 1, 2, 3, 4
joint max P2: 4, 3, 2, 1
joint max sum: 5



Games4321_4321(:,:,121) =

1, 4	3, 3
2, 1	4, 2

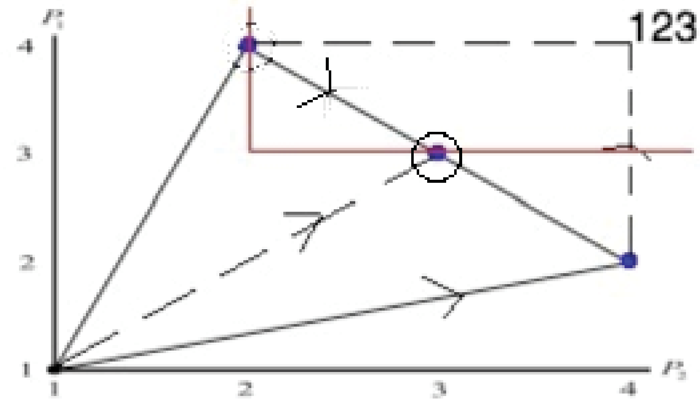
maxmin P1: 2
maxmin P2: 2
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,123) =

1, 1	3, 3
4, 2	2, 4

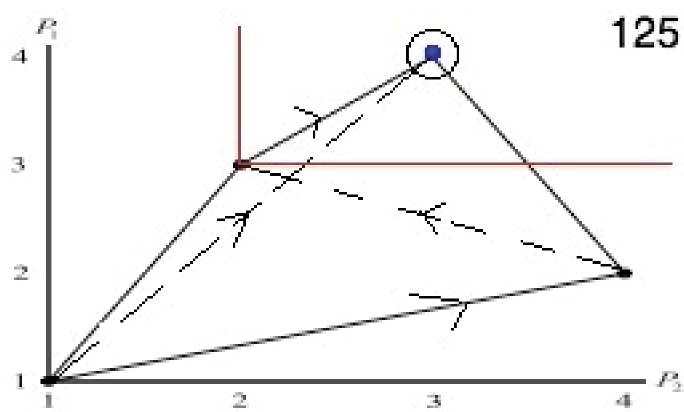
maxmin P1: 2
maxmin P2: 3
joint max P1: 2, 3, 4
joint max P2: 4, 3, 2
joint max sum: 6



Games4321_4321(:,:,125) =

1, 1	3, 4
4, 2	2, 3

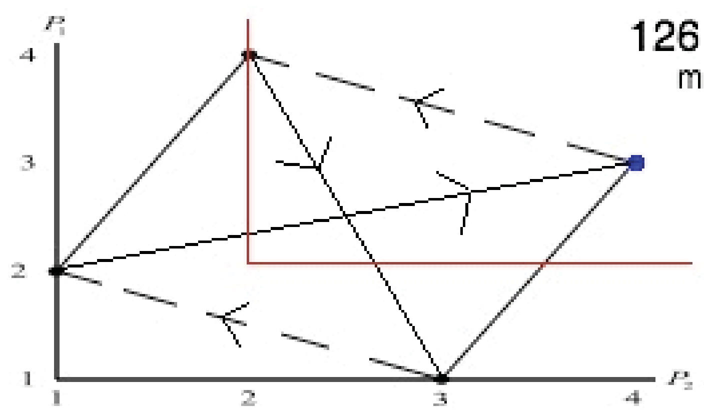
maxmin P1: 2
maxmin P2: 3
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,126) =

1, 2	3, 1
4, 3	2, 4

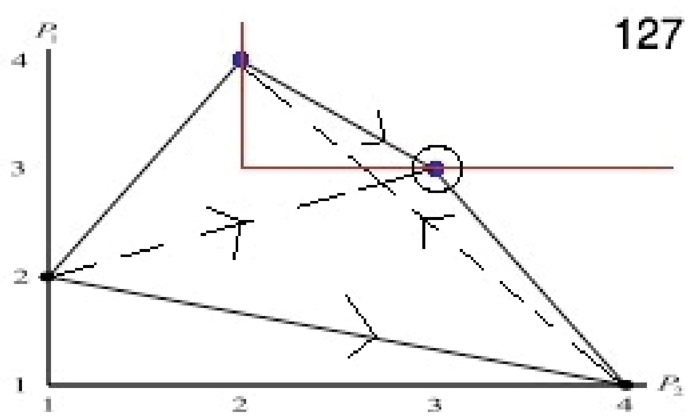
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,127) =

1, 2	3, 3
4, 1	2, 4

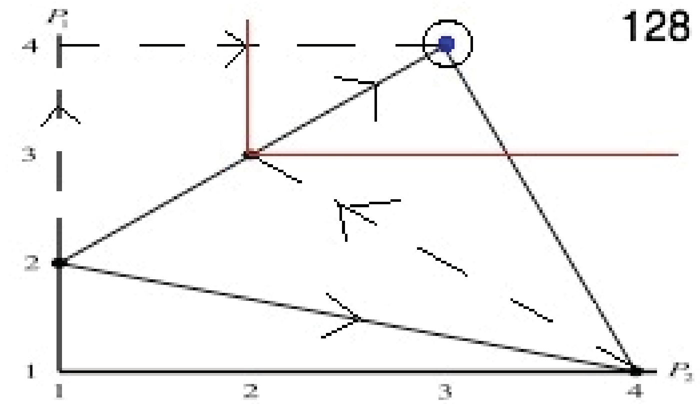
maxmin P1: 2
maxmin P2: 3
joint max P1: 3, 4
joint max P2: 3, 2
joint max sum: 6



Games4321_4321(:,:,128) =

1, 2	3, 4
4, 1	2, 3

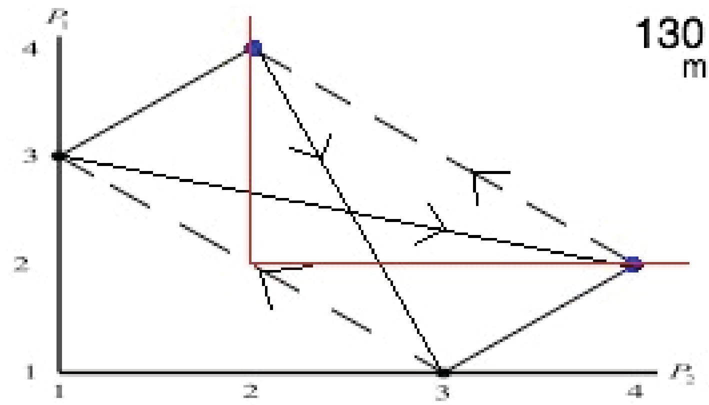
maxmin P1: 2
maxmin P2: 3
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,130) =

1, 3	3, 1
4, 2	2, 4

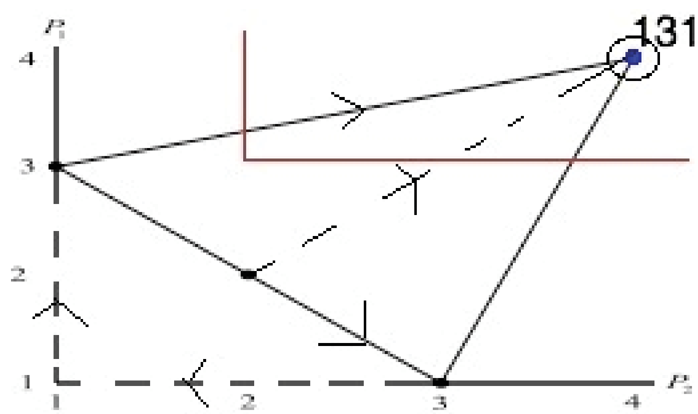
maxmin P1: 2
maxmin P2: 2
joint max P1: 2, 4
joint max P2: 4, 2
joint max sum: 6



Games4321_4321(:,:,131) =

1, 3	3, 1
4, 4	2, 2

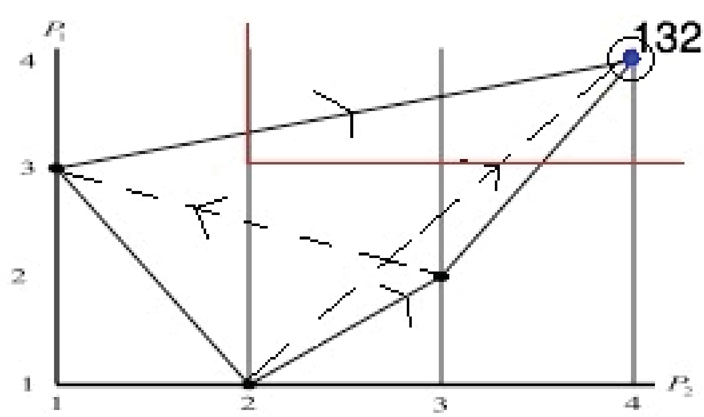
maxmin P1: 2
maxmin P2: 3
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,132) =

1, 3	3, 2
4, 4	2, 1

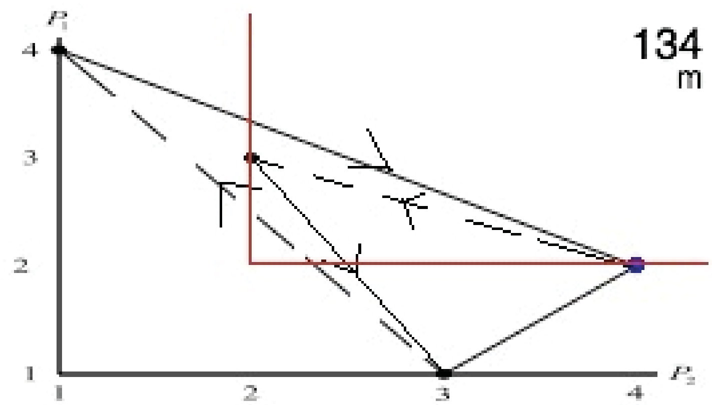
maxmin P1: 2
maxmin P2: 3
joint max P1: 4
joint max P2: 4
joint max sum: 8



Games4321_4321(:,:,134) =

1, 4	3, 1
4, 2	2, 3

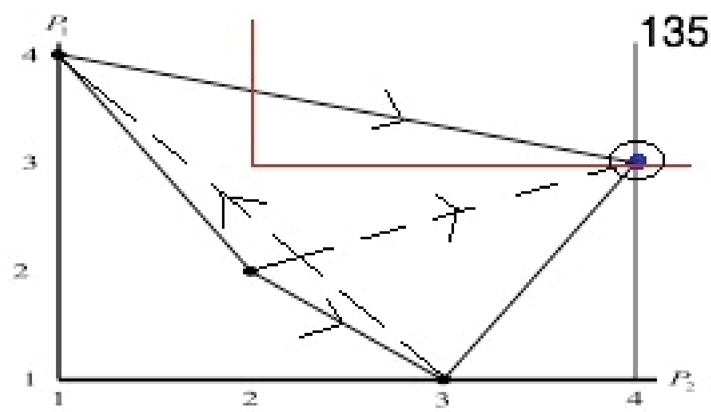
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 2
joint max sum: 6



Games4321_4321(:,:,135) =

1, 4	3, 1
4, 3	2, 2

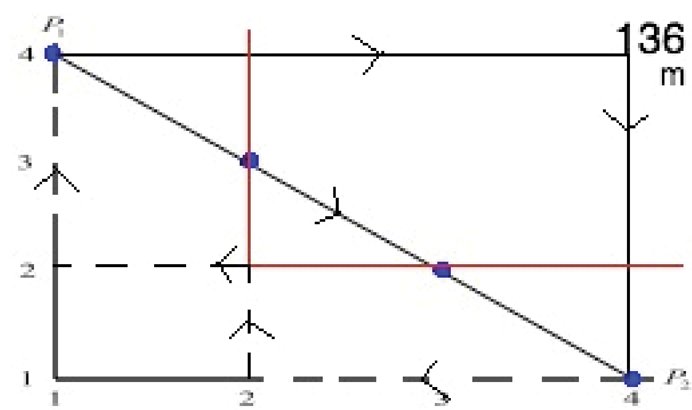
maxmin P1: 2
maxmin P2: 3
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,136) =

1, 4	3, 2
4, 1	2, 3

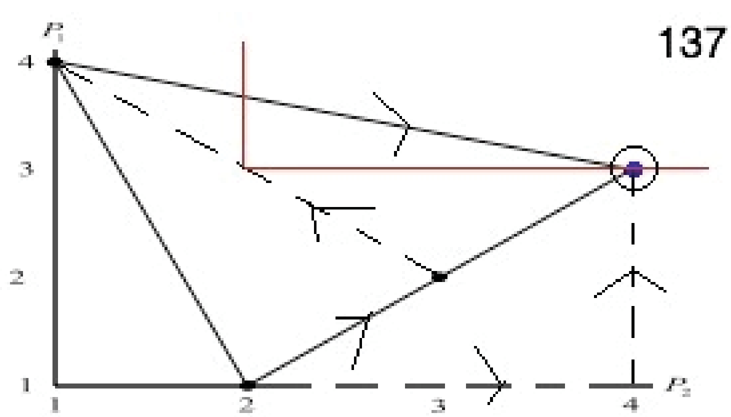
maxmin P1: 2
maxmin P2: 2
joint max P1: 1, 2, 3, 4
joint max P2: 4, 3, 2, 1
joint max sum: 5



Games4321_4321(:,:,137) =

1, 4	3, 2
4, 3	2, 1

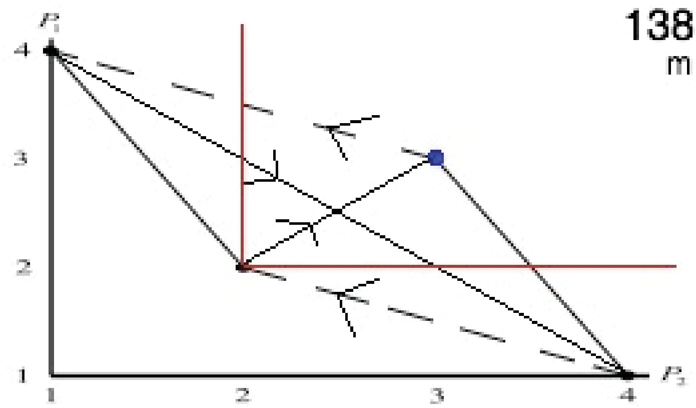
maxmin P1: 2
maxmin P2: 3
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,138) =

1, 4	3, 3
4, 1	2, 2

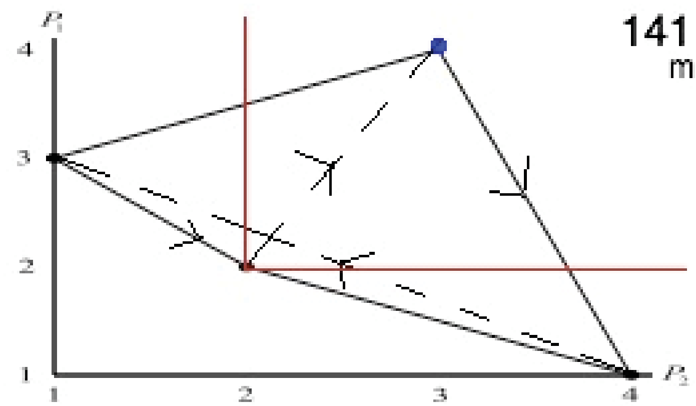
maxmin P1: 2
maxmin P2: 2
joint max P1: 3
joint max P2: 3
joint max sum: 6



Games4321_4321(:,:,141) =

1, 3	4, 1
2, 2	3, 4

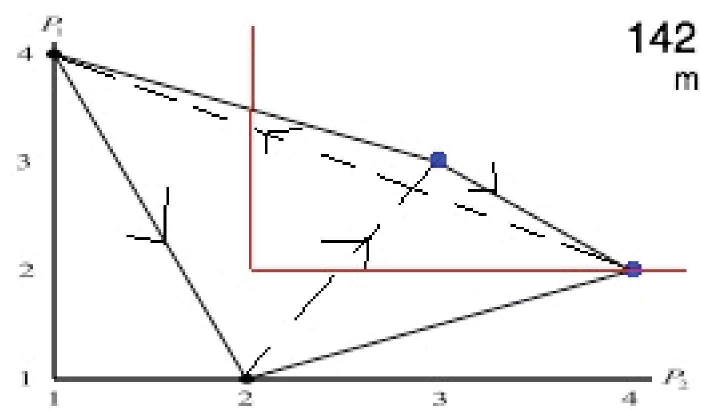
maxmin P1: 2
maxmin P2: 2
joint max P1: 3
joint max P2: 4
joint max sum: 7



Games4321_4321(:,:,142) =

1, 4	4, 2
2, 1	3, 3

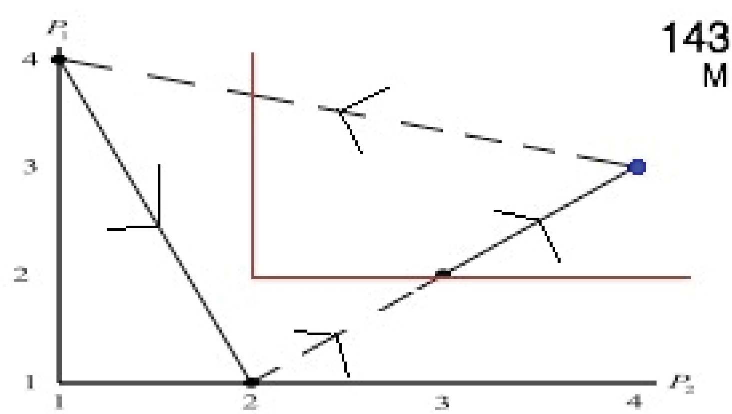
maxmin P1: 2
maxmin P2: 2
joint max P1: 3, 4
joint max P2: 3, 2
joint max sum: 6



Games4321_4321(:,:,143) =

1, 4	4, 3
2, 1	3, 2

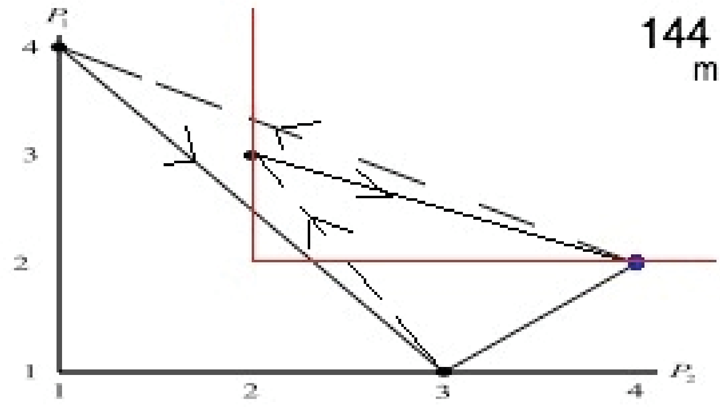
maxmin P1: 2
maxmin P2: 2
joint max P1: 4
joint max P2: 3
joint max sum: 7



Games4321_4321(:,:,144) =

1, 4	4, 2
3, 1	2, 3

maxmin P1: 2
maxmin P2: 2
joint max P1: 2, 4
joint max P2: 4, 2
joint max sum: 6



Appendix 6: Chi-Square Results for Mixed Strategies

The following list provides chi-square tests on whether the population of players plays mixed strategies on average. The chi-square tests were done using an online resource[1]. In each case there are 3 degrees of freedom.

Game 6:
chi-square value = 425.87
P = <.0001

Game 44:
chi-square value = 1471.21
P = <.0001

Game 57:
chi-square value = 126
P = <.0001

Game 64:
chi-square value = 245.33
P = <.0001

Game 136:
chi-square value = 325.07
P = <.0001

Game 138:
chi-square value = 154.23
P = <.0001

Game 141:
chi-square value = 238.51
P = <.0001

Game 142:
chi-square value = 303.41
P = <.0001

Game 143:
chi-square value = 207.2
P = <.0001

Game 144:
chi-square value = 246.19
P = <.0001

[1] http://faculty.vassar.edu/lowry/csfit.html

Appendix 7: Results from Stonybrook Experiment

How to Read the Results:

The games are numbered in the order in which they were presented to the experiment subjects. Each game is shown in matrix form with payoffs to row players on the left and column players on the right. The bold numbers below each column and to the right of each row show the number of players that selected that strategy. The percents displayed under each box are calculated according to the following formula: (number of row players who played given strategy)(number of column players who played given strategy)(100)/(total row players*total column players). The shading of payoffs gives us a visual representation of the aggregate results.

Errors:

The 78-game Stonybrook set was supposed to be made up of all 12 symmetric games in our set and 66 games with transposes. No two games in the set were supposed to be transposes of each other so that the 78 game set would be representative of all 2x2 games with ordinal payoffs. The list of errors in this set is as follows:

- Game 3, 60 and 63 are identical
- Game 29 and 30 are transposes
- Game 40 and 78 are transposes

As a result, neither the following games nor their transposes are in the set:

(1, 2)	(3, 3)
(4, 4)	(2, 1)

(1, 3)	(2, 4)
(3, 1)	(4, 2)

(1, 1)	(2, 3)
(3, 2)	(4, 4)

(1, 3)	(2, 2)
(4, 1)	(3, 4)

(1, 3)	(4, 4)
(3, 1)	(2, 2)

(1, 3)	(3, 1)
(2, 2)	(4, 4)

(1, 3)	(3, 1)
(2, 4)	(4, 2)

(1, 4)	(3, 1)
(4, 3)	(2, 2)