# A WEB GAMING FACILITY FOR RESEARCH AND TEACHING

**By**

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# A WEB GAMING FACILITY FOR RESEARCH AND TEACHING

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[1]With the fine research assistance of Jarus Singh

## Abstract

This essay considers the potential for utilizing web games for research and teaching. It discusses a specific gaming facility that has been constructed and utilized. The gaming facility can be made available for use for those interested in utilizing it for teaching and/or research purposes. The goal is to have this facility be of use for both single play and repeated matrix games. Much of the discussion here is aimed at single play games as a desirable benchmark preliminary to the study of repeated games. Properties of the one stage games are discussed and instructions for the use of the system are supplied. Extensions to multistage games and incomplete information are noted.



**JEL Classification**: C7, C9, D03

**Key words**: matrix games, experimental, teaching and computerized games,


## 1. On Matrix Games

It is difficult for an undergraduate in any of the social sciences to obtain his or her BA without at some point having been subjected to the paradox and the wonders of the Prisoner's Dilemma Game. This simple $2\times 2$ matrix game has been the subject of many thousands of essays and experiments and poses a deep problem in economics, social-psychology and psychology concerning individual competition and cooperation.

Two immediate features that make the $2\times 2$ game attractive are that binary choice plays a central role in the study of individual behavior and two person interactions are central to socio-psychological analysis.

Matrix games, such as the Prisoner's Dilemma Game (PD) have been used both for one-shot experiments and to study repeated play. There are various theories as to what one should do in both instances In the approaches as to how the games are played and how they should be played there has been, to some extent, a dichotomy between normative and behavioral theorizing. In much of pure game theory the solution concepts suggested can be interpreted as normative: The rational individual should play such and such. A behavioral approach is concerned with how individuals play. Furthermore the behavioral assumptions made by the game theorists by no means fully represent the views of the social-psychologists or the psychologists.

A debate goes on as to whether the Nash noncooperative equilibrium (or some of its many variants) is to be considered as a normative or behavioral solution. It is a thesis of this essay that a way to consider the links between the two approaches is via experimental gaming.

There is little question that the analysis of the playing of the Prisoner's Dilemma Game poses several deep and difficult problems but even if one restricts oneself to only $2\times 2$ games with strictly different entries (no ties) with payoffs 1, 2, 3, and 4, there are, even at that level of simplicity $4!\times 4!=576$ different games that can be constructed.[2] They merit concern along with the PD. Taking into account that changing the rows and columns of any one of these games leaves the games strategically unchanged to a game theorist (but not necessarily to a social psychologist) who can argue that there are really only 144 different games. A psychologist could say that games with rows and/or columns permuted look different to the players and one could make a case for experimenting to see if a different behavior is manifested with a change in row or column representation.

Undoubtedly one of the main topics in game theory, and in different ways in psychology and social-

[2] The game with entries 1,2,3,4 seems to be extremely special, but it serves as a jumping off point for considering games where each agent selects numbers within the interval [0,1] which enables on to consider finite games with entries of any size.

psychology is how people learn (and teach) when playing in repeated games. There is a considerable literature involving individuals such as Selten, Aumann, Kreps, Fudenberg, Samuelson, Mailath, and many others considering items such as reputation, threats and signaling in multistage games. The concern is also with lack or presence of common knowledge and initial and future information conditions.

These concerns with dynamics are undoubtedly of import, and there is a need for massive experimentation with dynamics. Nevertheless it is worth asking if there is anything left to learn about the simple one-shot play of the $2 \times 2$ matrix game where the cues are primarily structural rather than behavioral. The belief expressed here is that there are still some items worth considering and they can serve as useful benchmarks prior to our natural concern with repeated plays. Furthermore by building this apparatus first designed to experiment with the one shot games one can easily extended it to serve as a platform that can be enlarged for repeated play in some situations as is noted below.

## 2. On the Classification of the Structure of 2 x 2 Games

This essay deals primarily with the gaming system constructed and how to use it. A companion essay [Shubik 2012] discusses the question of what constitutes a solution to a game; but included here are some insights into the structure of matrix games and brief comments on two solution concepts; the noncooperative equilibrium and the cooperative game.

Over the course of the last forty years there have been several approaches to classifying $2 \times 2$ matrix games. A natural question to ask prior to discussing these attempts is why should one study the $2 \times 2$ matrix games in the first place?

An appropriate philosophical view of matrix games is that if one is going to start simple maybe one should first address the $0 \times 0$ and then the $1 \times 1$ matrix games. I leave the $0 \times 0$ game to the more philosophically inclined and observe that there are some problems with the contextual interpretation of the $1 \times 1$ game. With a choice set for each player of one strategy, there is no choice. As anyone who has utilized experimental gaming knows, even with the stark abstract representation an implicit contextual assumption has been made that there is a history such that either the players do not have the choice not to play, or that they have implicitly consented to play. Lest I be accused of nit picking I leave further interpretations of the $1 \times 1$ to the reader. The suggestion here is that the $2 \times 2$ game is inordinately suited to experimentation. An important property of the $2 \times 2$ is that it is the smallest intrinsically symmetric strategic structure where each player has an explicit real choice. However if ties are feasible we must take into account pathologies such as games with constant payoffs regardless of choice (suppose all $a_{ij} = a$; all $b_{ij} = a$).

| | 1 | 2 |
|---|---|---|
| 1 | $a_{11}, b_{11}$ | $a_{12}, b_{12}$ |
| 2 | $a_{21}, b_{21}$ | $a_{22}, b_{11}$ |

Table1

Why not experiment with $3 \times 3$ or bigger games? Much of the answer lies in the psychology, the mathematics and the modeling. In the seminal article "The Magic Number $7 \pm 2$," George Miller [Miller 1956] suggested around $5-9$ separate items are about as large a number as can be absorbed by an individual's short term span of attention. The eight payoff numbers of the $2 \times 2$ fall into this span, but the eighteen numbers associated with the $3 \times 3$ do not. For any matrix $3 \times 3$ or larger some extra structure must be imposed on the entries in the rows and columns if they are meant to represent any structure encountered in the investigation of society. For example, we have approximated a Cournot duopoly economic model with continuous payoffs by a matrix with a matrix grid up to may be $20 \times 20$ or $30 \times 30$

(Shubik and Siegel, [1963]). In experimental games with a matrix larger than an $2\times 2$ it is usually desirable to provide a context and to impose some structure on the jumble of numbers in the matrix. Even with the $2\times 2$ game there is a tendency to hang special names on some of the structures such as the Prisoner's Dilemma, The Stag Hunt, The Battle of the Sexes, although there is little evidence that individuals who do not know the name of the structures are able to associate the names and the structures [Powers and Shubik, 1982?].

Rapoport, Guyer and Gordon published a pioneering work [RGG 1976] on the $2\times 2$ games that contained many experiments and used the classification of all strongly ordinal $2\times 2$ matrix games suggested by Rapoport and Guyer [RG 1966]. Given the date of the work it was a tour de force to have not only experimented with 78 one shot games (see Appendix 1), but also reported on both cardinal and ordinal variations as well as considering repeated games. Since then other classifying schemes such as that of Peter Borm [1987] in which fifteen classes of games have been suggested.

The number 78 noted above was arrived at by reducing the complete class of the 576 $2\times 2$ matrix games observing that there are 66 games each with 8 representations obtained from interchanging rows, columns and the roles of the two players and there are 12 games that because of the symmetric roles of the players have only 4 representations; giving a total of $66\times 8+12\times 4=576$ games. Recently Robinson and Goforth [RG 2005] presented a topological classification of all of the 576 games and argued that one should consider that from the view point of game theoretic analysis one should consider the full set of 144 games obtained by just considering the row and column transpositions as this set is symmetric from the viewpoint of the players.

Baranyi, Lee and Shubik [1982] studied the structure of the payoff sets of the complete set of the 576 $2\times 2$ matrix games and showed that the payoff structures can be represented by 24 payoff sets (see Appendix 2). They did not indicate any behavioral structure in their representations. The work by Robinson and Goforth did, as is noted below.

A natural question to ask of the $2\times 2$ matrix games is what does the structure look like when there are ties Kilgore and Frazer [1988] considered the possibility of ties. We have observed that the set of all games without ties is $4!\times 4!$ , but with ties one has $4^{2^4}=65,536$ possibilities. Kilgore and Frazer were able to show that, using the classification Rapoport, et al. [1966] of one could reduce the 65,536 to 726, but this number of games is already not within easy range of experimentation. The difference between the games with and without ties can be viewed as considering selecting the payoffs from a set of numbers with replacement and without replacement.

In their classification and display of $2\times 2$ games Robinson and Goforth utilized a display that enables the strategic structure and optimum response to be shown easily on the same graph. An example for the Prisoner's Dilemma is noted below.

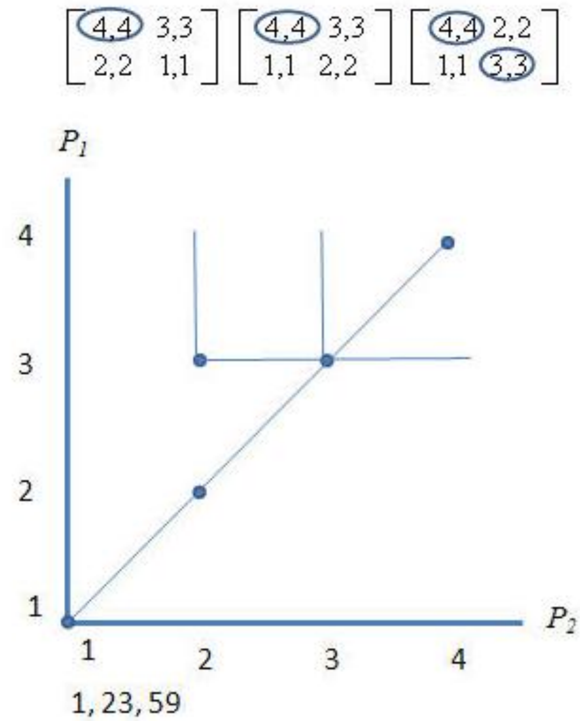


**Figure 1**

The vertex in the lower left has coordinates (1,1) and the four cells are represented by the points (3,3), (4,1), (1,4), and (2,2). The solid lines link the strategic choices for the Row Player and the dashed lines for the column player. The arrows indicate an optimal response; thus this representation skillfully mixes considerations of strategy and behavior sketched on the payoff structure (which is given by the convex hull of the four points).

Robinson and Goforth [2005], like Rapoport et al., try to find a natural taxonomy for all the games. They propose a periodic table analogy based on topological considerations, where they discuss and develop the concept of structural closeness of any two games They extend their work by taking into account ties [Robinson, Goforth and Cargill, 2007]. however they were not able to extend fully this method of classification. It is somewhat difficult to envision a topological approach offering a serviceable taxonomy for matrix games with more than two strategies each and more than two players.

In my view it is highly desirable to investigate and experiment with games with weak orderings because they can be used to reflect lack of fineness of perception. Many decision problems involve individuals with different perceptions and expertise, where in several payoff entries A cannot distinguish the payoffs yet B, may be able to distinguish between them.

Unfortunately the number of cases encountered in attempts to classify the $2\times 2$ games with ties is so large as to make it difficult to use the whole class for experimental games in the way that is feasible for the set of games with strictly ordinal payoffs. As soon as one goes to the $3\times 3$ matrix the number of strategically different matrices is $9!\times 9!/3!\times 3!$ or approximately 3.658 billion. It is fairly evident that from the viewpoint of experimental gaming one needs to specify some extra structure or set of structures to be considered. All possible worlds will not do; although one still may wish to contemplate what structure gives interesting 3 x 3 experiments. Baranyi, Lee and Shubik [1992] carried out some estimates of the limiting properties for the shape of the payoff set, the Pareto set and the individually rational set of outcomes for an arbitrarily large matrix games with more than two players and more than two strategies; but there is still considerable work to be done considering different forms of aggregation as the number of players and/or strategies become large. The imposition of some forms of aggregation and communication and encounter structure is called for when considering games larger than the $2 \times 2$.

In conforming to the idea of keeping an approach as simple as possible in the experimental apparatus discussed here I suggest that it is useful to consider a cardinal classification scheme where we consider the payoffs to be measurable and comparable, hence the operator addition may be regarded as meaningful. If we regard the ordinal entries 1, 2, 3, 4, as cardinal and comparable then we can add the payoffs in each cell and observe that the is a natural categorization of all of the games into four categories where the jointly maximal payoff adds to 8, 7, 6, or 5. We note immediately that all games that have a cell with sum 8 are all games of coordination and have that cell as not merely a joint maximum, but also as a noncooperative equilibrium point. All games with the maximum sum of 5 are constant sum games, or games of pure opposition. Table 3 shows the distribution of the joint maximum for all 78 games

| Sum | Frequency |
|---|---|
| 8 | 17 |
| 7 | 35 |
| 6 | 23 |
| 5 | 3 |
| Total | 78 |

Table 2

The mixed motive games have a maximum of 6 or 7. A further classification of these games requires a variety of extra conditions such as symmetry, strategy domination, existence of a pure strategy equilibrium point, uniqueness and individual rationality.

If players are offered a money payoff for the sum of the score from playing a set of the games each is motivated to maximize the expected score in each game.

### 3. A Web approach to Experimental Gaming and Teaching Games

In the late 1950s, Merrill Flood and I discussed the potential value of utilizing the gambling information generated in Las Vegas to provide mass crude statistics on gambling with individuals using their own time and money. After several failures to obtain the appropriate connections to realize such a scheme we abandoned the idea. We nevertheless felt and I still believe, especially given the vast improvements in computer and communication technology, that the social sciences could benefit from the gathering of mass game playing statistics. Life is lived for the most pert in more or less noisy environments. The hope is that large samples, even if gathered with less control than one might have in the laboratory will provide insights into game playing behavior such that they can serve to connect various theories of behavior in these structures with some evidence of play.

In some of the taxonomies for the one shot games use is made of optimal response considerations. On a little reflection an optimal response requires that the individuals be given information of a previous position in the outcome of the game against which an agent can respond. In essence by requiring that an initial condition be specified one has embedded the single shot game into an ongoing socio-psychological process, far richer than that called for by the single shot-in-the-dark play noted here below. When initial conditions are specified that include history the solution set is enlarged.

### 3.1. An Apparatus for Teaching and Research

The view of a web gaming facility suggested here is evolutionary. Start small and then build out. Concern oneself with one relatively simple experimental goal and one teaching goal. get the system set up to perform these modest tasks, Debug the system with one or two operational runs accepting as a given the overwhelming odds that in the first few runs something will be forgotten or something will go wrong; but as the system functions purposefully the errors will be corrected and the omissions discovered.

An example of a somewhat different website, but built in the same spirit is that of Ariel Rubinstein (http://gametheory.tau.ac.il/).

In the early days of operations research, gaming and simulation, especially in the construction of simulation languages there was an enthusiasm for the construction of grand purpose simulators. An example of such a simulator was provided by Jay Forrester in his development of Systems Dynamics that was meant to be destined to be able to solve all feedback system economic problems and influenced the thinking of the Club of Rome. Unfortunately as is now well recognized by economists studying evolutionary game theory or incomplete contracts, ecological systems have the nasty habit of depending on unaccounted for dynamics and unintended consequences from actions that should have caused no problems.

In spite of the considerable flourishing of experimental gaming in economics as evinced by the work of Siegal, Smith, Plott, Roth, Guth, Holt, Sunder, Fehr, Rubinstein, Schotter, Huber and many others there has been little work to try to mass produce data from the web.

The approach adopted here is that in a way similar to how individuals can play backgammon or chess on the web, one should be able to construct an apparatus to gather large sample size experimental data from web games where interest in the games and small monetary prizes help to motivate the players. I conjecture that the use of the games as part of a teaching program in game theory, economics or social psychology can produce considerable experimental data of use from large numbers of students motivated by the classes and possibly a small monetary prize.

Ideally I believe that we should be considering developing a set of standardized experiments that could be analyzed more or less automatically. In some sense they would be an analogue of a Tibetan prayer wheel that produces its prayers without human intervention. This collection can be made without the use of a formal laboratory and nevertheless conforming to privacy conditions of the players. In essence this already happens in the gathering of traffic statistics.

Furthermore although at this time there are many legal, technical and societal problems involved it appears to be feasible to design a set of experimental games where a not for profit experimental establishment could charge players an admission fee to play in an appropriately designed and parameterized non-constant sum game using say 97–98% of the money for prizes and cover experimental costs with the remaining 2–3%. This would have the extra benefit that the players have some of their own money at stake and experimental costs could be highly reduced or covered. There is no attempt to do this here, but it is noted as a relatively natural step in the future.

### 3.2. The Specific Apparatus for Teaching and Research

The approach adopted here was to design an easy to use game structure on the Web that could first be employed to construct two games to test the play of the complete set of all strategically different $2\times2$ (cardinalized strictly ordinal) games by visitors to a dedicated website (http://www.museumofmoney.org) These visitors could be from the general public, students in a class or professionals such as attendees at a conference on game theory or social psychology.

It is aimed also for use in the classroom motivated by a class exercise and/or by a small monetary prize. I believe that the latter should be pedagogically preferred. Reasonably simple games in the classroom appear to be effective teaching devices and can yield useful data to students and teachers Shubik [1978]

### 3.3. The apparatus and the Program for One-shot Games Research

The first goal has been to set up conditions to experiment with one-shot play of all 144 strategically different $2\times2$ matrix games. In **Appendix 2** we give a listing of all 144 games. The matrices are presented in string form, the first two numbers are the contents of the upper left entry; the next two upper right; the next two lower left and the final two lower right. Robinson and Goforth provide a handy set of all of the optimal response diagrams classified into four sets. We utilize other representations as we comment on both ordinal and cardinal constructions. In **Appendix 1 some of** the results of the plays of the Rapoport et al., 78 games are shown. Underneath each pair of numbers in the payoff matrices appears the percentage of the player pairs making this selection; thus for example, Game 3 is The Stag Hunt and 62% chose the Pareto optimal noncooperative equilibrium, while 4% chose the risk dominant noncooperative equilibrium. Other results concerning this run are discussed in a companion paper. Here the major purpose in displaying the output is to show that one obtains considerable experimental information directly from the display. To the right and below each matrix appears the information on the number of Row and Column players who selected left or right or up or down. A color coding indicates at a glance the density of responses.

Our web game experimental apparatus can be utilized for sequential games with a live individual against an artificial player, or (with some modifications for pairs of live individuals in a classroom) however the first concern here has been with the one shot games where the influence of structure should be at its highest. In a related paper [Shubik, 2012] a number of different solutions for the one shot game are discussed.

#### 3.3.1. A problem with timing

As is well known in experimental gaming in general the distortions caused by time pressures may be of considerable importance. Here where we are dealing with purely abstract games, in as sterile and barren a context as is feasible, problems with time are manifested in both the possibilities of boredom and the pressures of being overwhelmed by too many decisions. An immediate goal has been to obtain some insights into as to how individuals would play the 144 games that constitute the full closed set of all strategic variations in the $2 \times 2$ game. But it is fairly evident that to have an individual decide on how to play 144 games in a single session is too much. Even half that number may be a burden. But as many of this set of matrix games have dominant strategies, preliminary testing suggested that for almost all individuals the playing of 78 matrix games would require no less than 30 minutes and no more than two hours, with most of the players within the range of around 50–70 minutes. One eminent game theorist declined to participate in the game because he estimated that for him to think sufficiently carefully about how he should play 78 matrix games might require of the order of a week's work. This is illustrative of the difference between a normative and behavioristic approach to simple matrix games.

The number of 78 (=66 + 12) games was selected because one can construct two sets of games where in the first set there are 66 games where the score from the sum of the equilibrium solutions is in favor of Row and the remaining 12 games are completely symmetric with the roles of the Row player and the Column player being identical. The second set of games has the same 12 symmetric games and the 66 transposes where to roles of Row and Column is interchanged. If the performance of two sets of players on the 12 symmetric games are not statistically different this would justify combining the data from the two runs giving a coverage of all 144 games.

In operational gaming, such as war gaming, context and timing are of considerable importance. The argument being that human decision-making does not take place in a vacuum, and thus it is important to set the stage appropriately. The games presented here are at the other extreme. They are simple abstract structures supplied with as little a context as possible. A question that merits answering is how much do these abstract structures guide the decision-making?

#### 3.3.2. Facts and rat facts

It has been suggested that there are many types of facts in empirical work depending heavily on both the context and the population studied. Experimental game theory at most is of the order of sixty to seventy years old and has utilized only modest resources. In biological research many interesting items may be found out about rats; but the most important questions often involve how well can one generalize from the rat facts to observations about humans? Can we generalize from games played by professional game theorists as players to the same games and briefings played with undergraduates or social psychologists or the man on the street? If they do we may have some extra confidence in the influence of structure in context impoverished structures. One cannot dismiss the possibility that the results from the anonymous matrix game paradigm may be regarded as grossly pathological when context is taken into account. In actual decision-making, when confronted with choices between A and B, the optimal solution may to generate and choose C.

**4. The Gaming System**

An immediate use of the gaming system has been aimed at the study of the complete set of $2\times 2$ games, but an understanding of the ease with new games can be constructed and utilized is given in **Appendix 1** where the specifics and details of use are noted.

**5 Other Uses for Teaching and Research**

The concern here has been primarily with a program for experimentation with one shot experimental games; but it is evident that many of the more interesting questions in the development and applications of game theory involve the investigation of repeated games with many time periods in order to consider items such as learning, teaching, reputation, incomplete information and lack of common knowledge.

**5.1. Multistage games and artificial players**

In our initial construction of the system we have not yet addressed the building of a facility to run multistage games in a flexible manner on the web. There are many technical problems involving timing and coordination of players, not unlike the programming problems faced in playing on-line backgammon or chess; but somewhat worse. However, there are two simple, useful extensions that may be made that do not require heavy costs. They involve adjustments for both the classroom and the web. In particular it is relatively easy to have an individual play against an artificial player. In matrix games the construction of an artificial player involves little more than specifying a formal game strategy to serve as the artificial player. The individual then plays against a sequence of the same matrix with the artificial agent updated according to the strategy specified.

**5.1.1. An inventory of artificial players**

For game structures as simple as these it is easy to write an artificial player who is nothing more than a strategy or algorithm for how to select a move at each information set. In his Prisoner's Dilemma experiment Axelrod [Axelrod 1984] asked all participants to submit a strategy which was then treated as a player to be matched against all others. As soon as a sequential game is of any length the proliferation of strategies is enormous. It is evident from experimental work that individuals tend to keep the complexity of a formal strategy highly limited. A few examples include the random or entropic player who selects a move with probability of 50:50 each period; the optimal responder, who maximizes against the previous outcome. A third artificial player (designed primarily for the Prisoner’s Dilemma played repeatedly by the same pair, or any other game for which the concept of double-cross can be made meaningful) would start with tit-for-tat, then increase the punishment as a function of the number of times double crossed.

**5.1.2. On modifications**

In a classroom it requires a relatively inexpensive adjustment to have two individuals play against each other if they are in the same place at the same time. The only extra software required is that after Row or Column has moved the other player prompted for her move. When this is forthcoming, both are informed and the game goes to the next play.

The two variants noted above that provide simple extensions have not been done as yet, in keeping with the philosophy advocated here modifications should be made in demand to use. For individuals wishing to adapt this apparatus for their own teaching and/or research requirements they can obtain. The apparatus is available to Yale faculty by contacting martin.shubik@yale.edu and samuel.cohen@yale.edu at The Center for Media and Instructional Innovation to register as a game constructor and monitor and to obtain operating information.

**5.2. Multistage Games and Incomplete Information**

The games discussed so far have considered two players facing jointly known matrix games; however instead of entering numbers into the payoffs for both agents A and B, instead they only see their own payoff they may be asked to deduce the payoffs of the other player after repeated play (see Shubik [ 1962)] for an example of such an experiment where the matrix perceived by A at the start looks like

| | 1 | 2 |
|---|---|---|
| 1 | $a_{11}$, ? | $a_{12}$, ? |
| 2 | $a_{21}$, ? | $a_{22}$, ? |

Table 3

and similarly for B.

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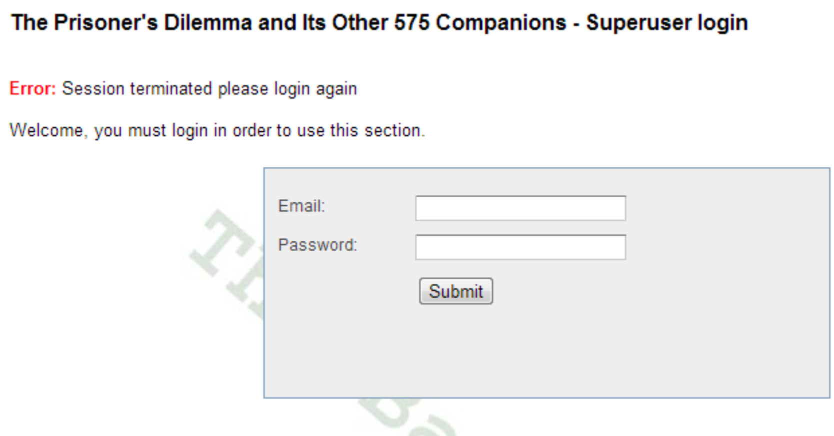
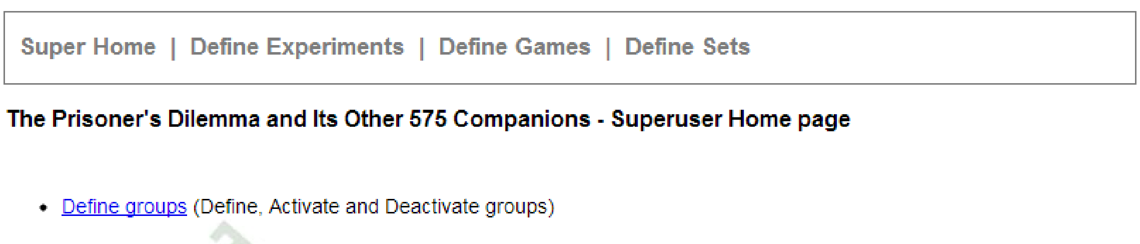
## APPENDIX 1.A: Explanation of Administrator's Gaming System

**Home page**:




This is the homepage that can be found at the URL: http://econgames.research.yale.edu/monitor/. To log in as an administrator, enter the login credentials used to register you for the website and your individual password. The error message that appears in the picture above is normal.

**Main page**:

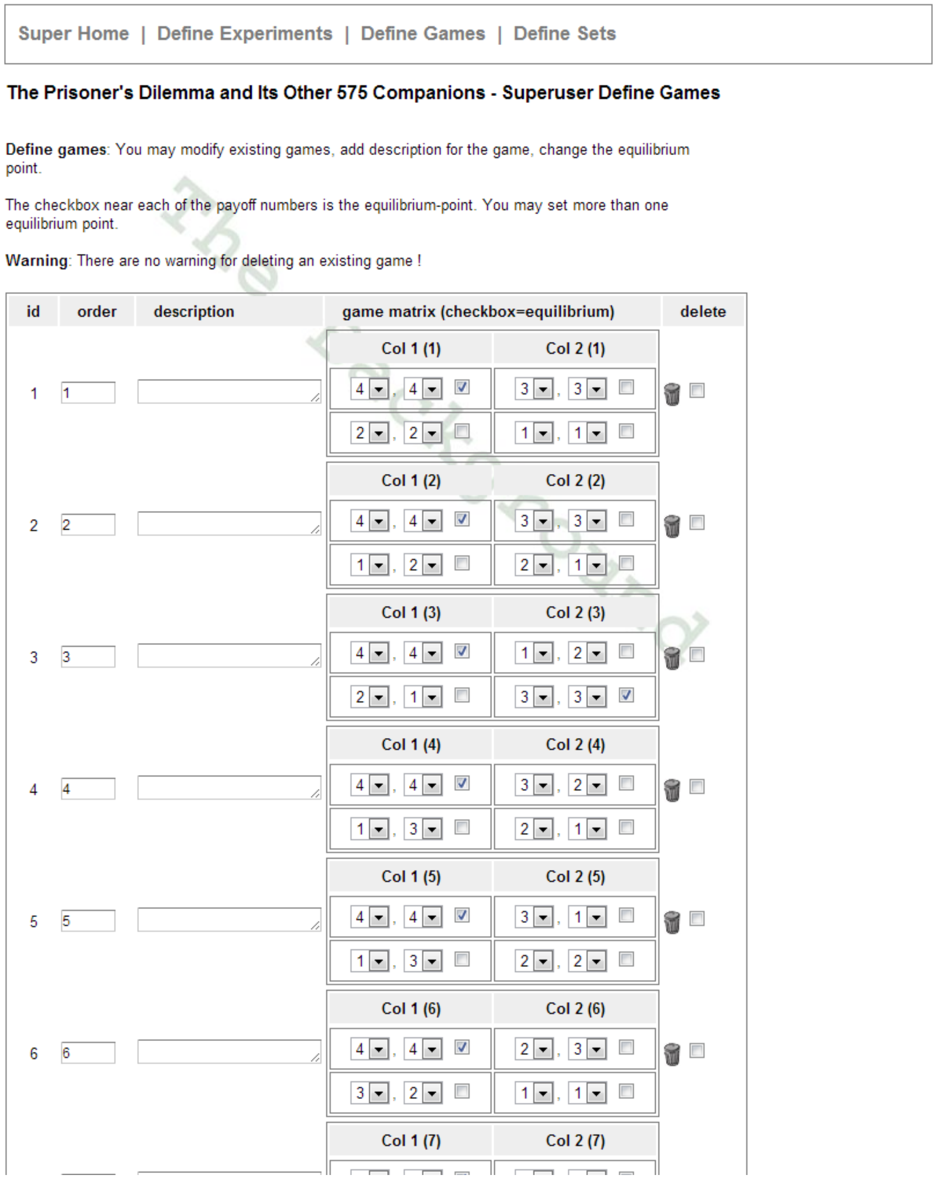



Immediately after logging into the system, this is the page that will display. This page can also be accessed by clicking "Super Home" on the bar across the top of the screen.

On this screen there are choices for where to proceed next. Each of these is explained in more detail when the particular page is displayed in this appendix.

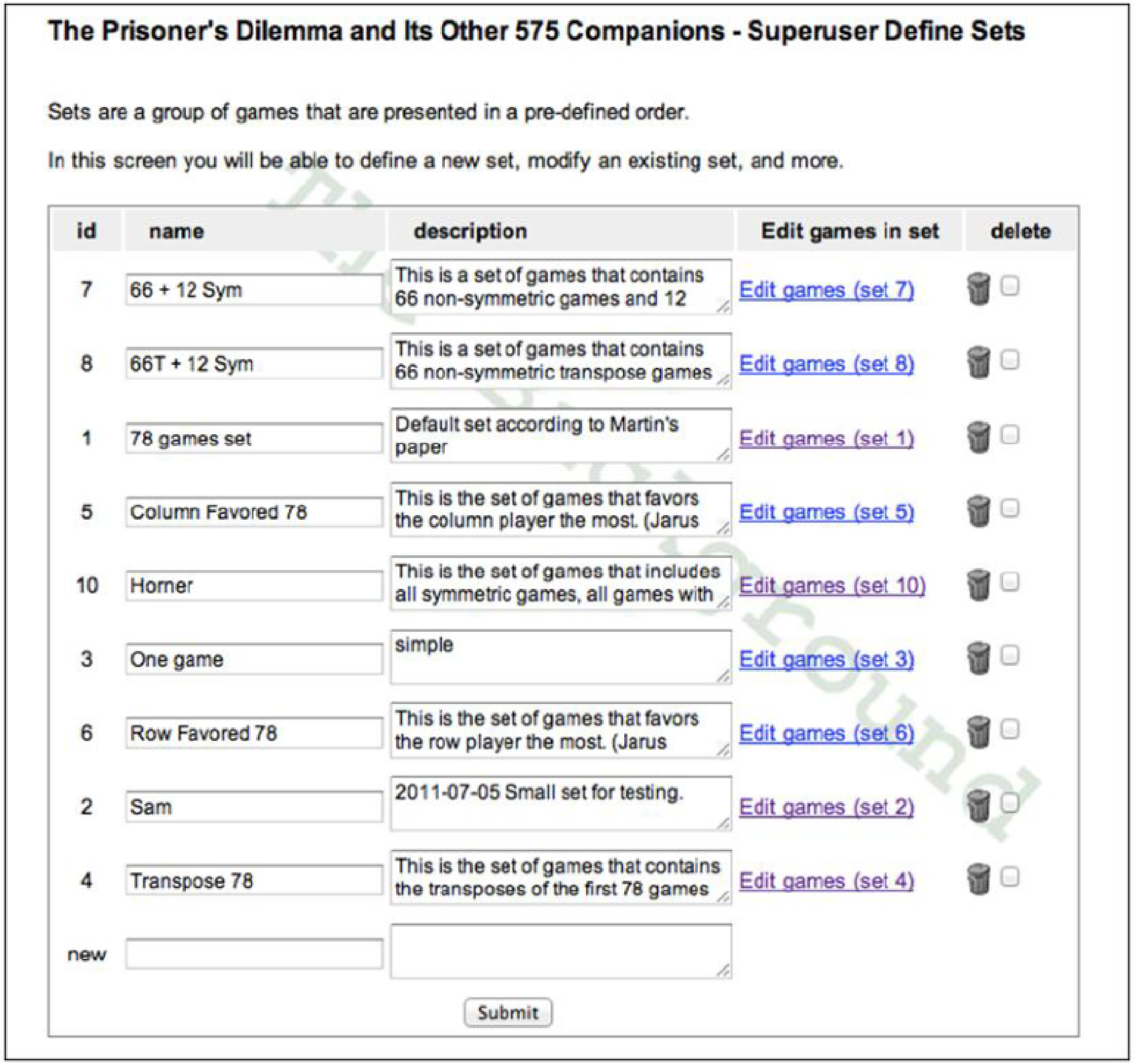
- Clicking "Define Experiments" will bring you to the page that lists all experiments that have been run using this gaming system. Clicking "Define groups" on the lower part of the page will also bring you to the same page that clicking "Define Experiments" does.
- Clicking "Define Games" will bring you to the page that lists all of the matrices that are in use in the gaming system.
- Clicking "Define Sets" will bring you to the page that lists all of the available sets of games that will be used in experiments and allows for the creation of new sets to be used in experimentation.

**Games page:**




The games page appears as above. Each game is assigned a game id sequentially based on when it is created. Descriptions are optional, but can be used to keep track of what type of game is being played or of specific games such as prisoners dilemma or stag hunt. The payoffs to each player are specified in the cells of the matrix; payoffs to the row player are specified before payoffs to the column player. Checkboxes indicate which cells of the matrix correspond to Nash equilibria of the normal form game. To create a new game, go to the bottom of the screen and specify the payoffs (1, 2, 3, or 4) to each player and (optional) a description. Then click "Submit". This will create the new game in the system and can be used later in creating sets for experimentation. Here, a superuser can also delete games. To delete a game, simply check the box next to the trash can image and click submit on the bottom of the page. Be very careful, however, since games, once deleted, cannot be recovered and there is no warning message or confirmation screen displayed before a game is deleted.

**Sets page:**



**The Prisoner's Dilemma and Its Other 575 Companions - Superuser Define Sets**

Sets are a group of games that are presented in a pre-defined order.

In this screen you will be able to define a new set, modify an existing set, and more.

| id | name | description | Edit games in set | delete |
|---|---|---|---|---|
| 7 | 66 + 12 Sym | This is a set of games that contains 66 non-symmetric games and 12 | Edit games (set 7) | |
| 8 | 66T + 12 Sym | This is a set of games that contains 66 non-symmetric transpose games | Edit games (set 8) | |
| 1 | 78 games set | Default set according to Martin's paper | Edit games (set 1) | |
| 5 | Column Favored 78 | This is the set of games that favors the column player the most. (Jarus | Edit games (set 5) | |
| 10 | Horner | This is the set of games that includes all symmetric games, all games with | Edit games (set 10) | |
| 3 | One game | simple | Edit games (set 3) | |
| 6 | Row Favored 78 | This is the set of games that favors the row player the most. (Jarus | Edit games (set 6) | |
| 2 | Sam | 2011-07-05 Small set for testing. | Edit games (set 2) | |
| 4 | Transpose 78 | This is the set of games that contains the transposes of the first 78 games | Edit games (set 4) | |
| new | | | | |

Submit

The sets page lets one edit and create new sets. One does so by entering a new set name next to "new" and providing a description. Once one clicks 'Submit', the page refreshes and a link allowing one to edit the games in the set appears in the column labeled "Edit games in set". By clicking on "Edit games" for the new set, one is taken to a page of games which looks like the following image:

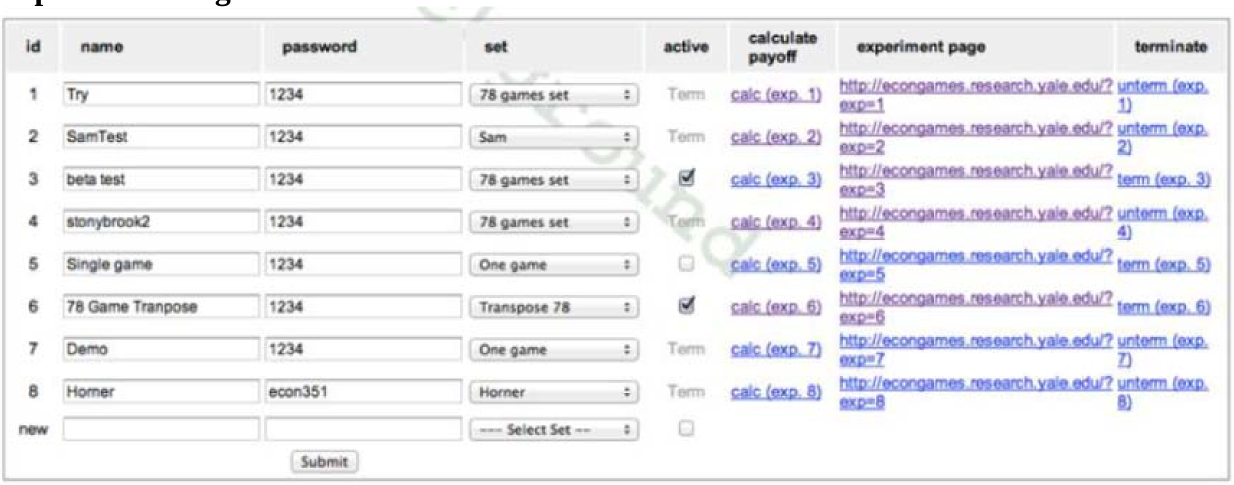
**Set details**

**Set id**: 7 **Set name**: 66 + 12 Sym
**Set description**: This is a set of games that contains 66 non-symmetric games and 12 symmetric ones. (Jarus added 10/23)

| game-id | order | description | game matrix (*=equilibrium) | ? as other player payoff | Include in the set |
|---|---|---|---|---|---|
| 1 | 1 | | 4,4* 3,3<br>2,2 1,1 | | |
| 2 | 2 | | 4,4* 3,3<br>1,2 2,1 | | |
| 3 | 3 | | 4,4* 1,2<br>2,1 3,3* | | |
| 4 | 4 | | 4,4* 3,2<br>1,3 2,1 | | |
| 5 | 5 | | 4,4* 3,1<br>1,3 2,2 | | |
| 6 | 6 | | 4,4* 2,3<br>3,2 1,1 | | |

Here, the user can rearrange the order of games. If one checks a box in the column "? as other player payoff", then the game (if included in the set) will only show each user their own payoffs when playing the game, and place question marks for the payoffs of the opposing player. When checking boxes in the last column labeled "Include in the set", the user can select which games are to be included in the set they have created. Games (from "Define Games") will be displayed to participants and included in the set if "include in the set" is checked off; furthermore, games will appear in the order specified by the numbers in the "order" column with the game with order 1 coming before the game with order 2 and so on.

**Experiments Page**

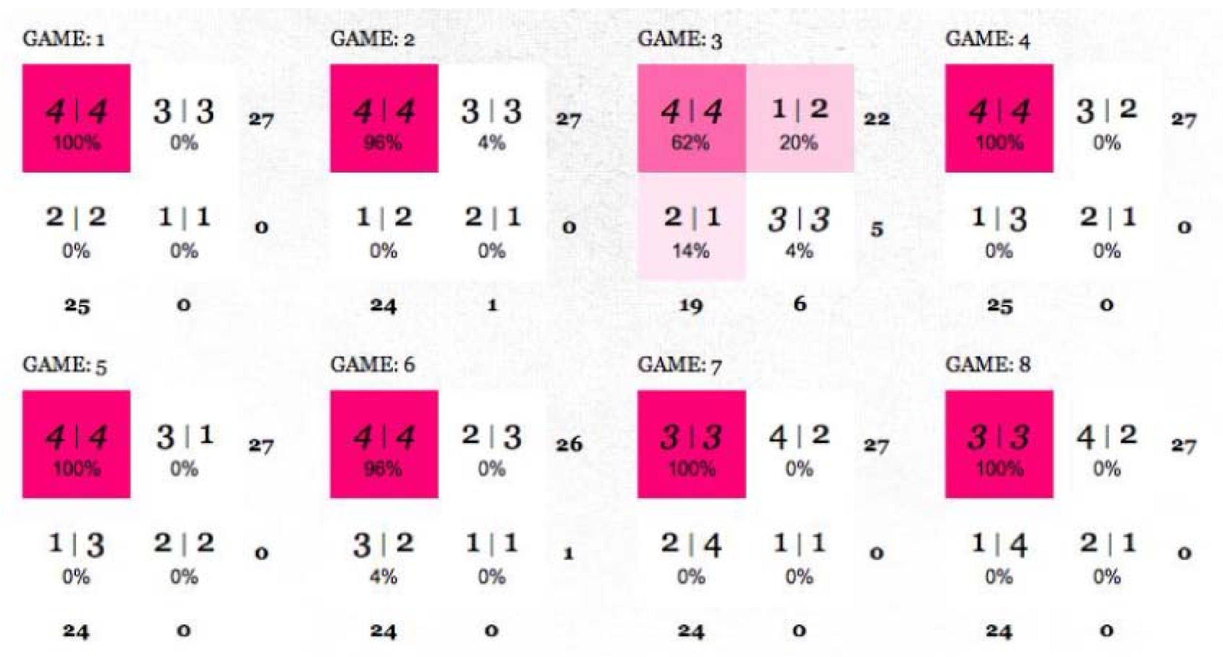


| id | name | password | set | active | calculate payoff | experiment page | terminate |
|---|---|---|---|---|---|---|---|
| 1 | Try | 1234 | 78 games set | Term | calc (exp. 1) | http://econgames.research.yale.edu/?exp=1 | unterm (exp. 1) |
| 2 | SamTest | 1234 | Sam | Term | calc (exp. 2) | http://econgames.research.yale.edu/?exp=2 | unterm (exp. 2) |
| 3 | beta test | 1234 | 78 games set | ☑ | calc (exp. 3) | http://econgames.research.yale.edu/?exp=3 | term (exp. 3) |
| 4 | stonybrook2 | 1234 | 78 games set | Term | calc (exp. 4) | http://econgames.research.yale.edu/?exp=4 | unterm (exp. 4) |
| 5 | Single game | 1234 | One game | ☐ | calc (exp. 5) | http://econgames.research.yale.edu/?exp=5 | term (exp. 5) |
| 6 | 78 Game Tranpose | 1234 | Transpose 78 | ☑ | calc (exp. 6) | http://econgames.research.yale.edu/?exp=6 | term (exp. 6) |
| 7 | Demo | 1234 | One game | Term | calc (exp. 7) | http://econgames.research.yale.edu/?exp=7 | unterm (exp. 7) |
| 8 | Homer | econ351 | Horner | Term | calc (exp. 8) | http://econgames.research.yale.edu/?exp=8 | unterm (exp. 8) |
| new | | | --- Select Set -- | ☐ | | | |

Submit

This page can be accessed by clicking “Define Experiments” in the navigation bar on the top of the website. To create a new experiment one has to enter a name for the experiment, provide a password that users will use, and then select a set of games to use (from “Define Sets”). After the user hits submit at the bottom of the page, a link to that experiment will be generated. If a user wishes to end the experiment they are running, they can hit the link in the column marked “terminate”. Players will not be able to participate in an experiment while it is terminated. Terminated experiments can be un-terminated in the same fashion. To view the results of an experiment, one clicks on the links in the “calculate payoff” column.

This produces an image similar to the following:




The games appear in the order in which they were presented to the experiment subjects. Each game is shown in normal form with payoffs to row players on the left and payoffs to column players on the right. The bold numbers below each column and to the right of each row show the number of players that selected that strategy. The percentages displayed under each box are calculated according to the following formula: (number of row players who played given strategy)(number of column players who played given strategy)(100)/(total row players*total column players). The shading of payoffs gives us a visual representation of the aggregate results. Darker colors mean the payoff pair was realized more frequently as compared to payoff pairs with lighter colored backgrounds. At the bottom of the results page, an image similar to the following appears:

⊞ 1) Student: 5 - martin.shubik@yale.edu - r player
Total Payoff: 4963

⊞ 2) Student: 34 - [redacted] - r player
Total Payoff: 5037

⊞ 3) Student: 35 - [redacted] - c player
Total Payoff: 6855

⊞ 4) Student: 38 - [redacted] - r player
Total Payoff: 5021

⊞ 5) Student: 40 - [redacted] - c player
Total Payoff: 6865

⊞ 6) Student: 41 - [redacted] - r player
Total Payoff: 2261

⊞ 7) Student: 42 - [redacted] - r player
Total Payoff: 5123

⊞ 8) Student: 44 - [redacted] - c player
Total Payoff: 6879

⊞ 9) Student: 36 - [redacted] - c player
Total Payoff: 6801

⊞ 10) Student: 48 - [redacted] - r player
Total Payoff: 5073

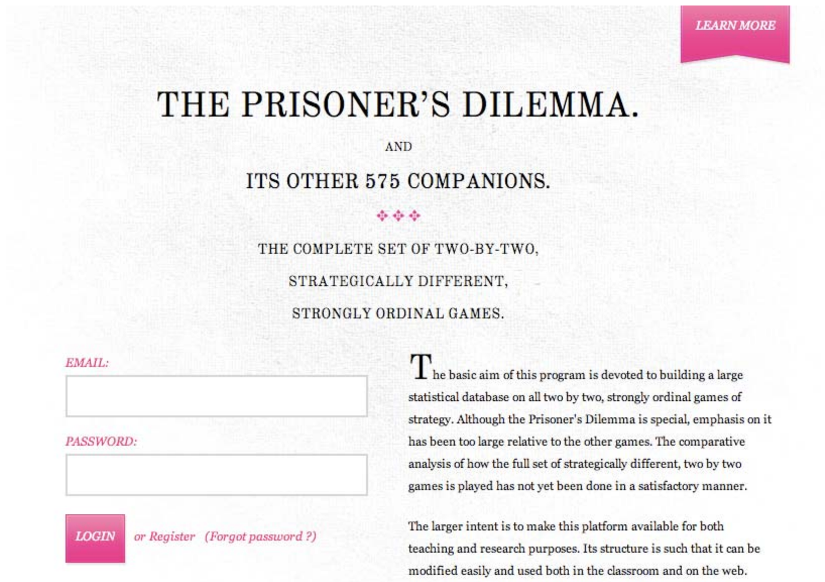
The monitor can see the e-mail addresses of each person who participated in the experiment and their scores. After the monitor has seen the data, he or she may remove the e-mail addresses before passing on any results to others, thus preserving privacy out of the classroom, if used in class.

⊟ 1) Student: 5 - martin.shubik@yale.edu - r player
Game 1 (r1) payoff (4 * 25 + 3 * 0): 100
Game 2 (r1) payoff (4 * 24 + 3 * 1): 99
Game 3 (r1) payoff (4 * 19 + 1 * 6): 82
Game 4 (r1) payoff (4 * 25 + 3 * 0): 100
Game 5 (r1) payoff (4 * 24 + 3 * 0): 96
Game 6 (r1) payoff (4 * 24 + 2 * 0): 96
Game 7 (r1) payoff (3 * 24 + 4 * 0): 72
Game 8 (r1) payoff (3 * 24 + 4 * 0): 72
Game 9 (r1) payoff (3 * 23 + 4 * 1): 73
Game 10 (r1) payoff (2 * 24 + 4 * 0): 48
Game 11 (r1) payoff (2 * 24 + 4 * 0): 48
Game 12 (r1) payoff (2 * 23 + 4 * 1): 50
Game 13 (r1) payoff (3 * 24 + 4 * 0): 72
Game 14 (r1) payoff (3 * 23 + 4 * 0): 69
Game 15 (r1) payoff (3 * 23 + 4 * 0): 69
Game 16 (r1) payoff (3 * 23 + 4 * 0): 69
Game 17 (r1) payoff (2 * 23 + 4 * 0): 46
Game 18 (r1) payoff (2 * 23 + 4 * 0): 46
Game 19 (r1) payoff (3 * 23 + 4 * 0): 69
Game 20 (r1) payoff (3 * 23 + 4 * 0): 69
Game 21 (r1) payoff (2 * 23 + 4 * 0): 46
Game 22 (r1) payoff (4 * 21 + 3 * 2): 90
Game 23 (r1) payoff (4 * 21 + 3 * 2): 90
Game 24 (r1) payoff (4 * 21 + 3 * 2): 90
Game 25 (r1) payoff (4 * 21 + 3 * 2): 90
Game 26 (r1) payoff (4 * 20 + 2 * 3): 86
Game 27 (r1) payoff (4 * 21 + 2 * 2): 88

A score is calculated for each individual matrix game by finding the sum of a player's payoffs against all of the other players playing as the opponent. For example, in the visual above, Student 5 is a row player. He played top. 24 column players played left (each giving him a payout of 4) and 1 played right (giving him a payout of 3). His score for the game is 4*24+3*1=99.

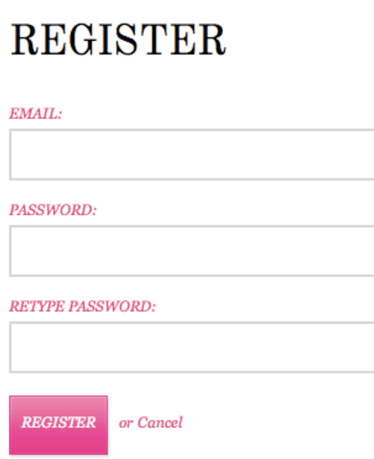
## APPENDIX 1.B: Explanation of Player's Gaming System

**Home Page:** http://econgames.research.yale.edu/?exp=xx, where xx corresponds to the number of a particular experiment.




This is how the home page appears for players. To start playing the games, they are requested to enter their email addresses and the password specific to the experiment they are participating in.

**Registration Page:**




This appears if a player needs to register (if they have not already) in order to take experiments.

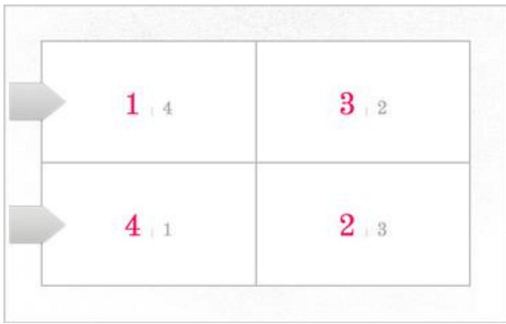
**Rules Page**:



*LOGOUT* *LEARN MORE*

THE PRISONER'S DILEMMA AND ITS OTHER 575 COMPANIONS

# GAME RULES




ONE

✣

*No prior knowledge of Game Theory is required to play, but a simple example of a single game is provided to illustrate the choices you face.*

A 2 x 2 matrix game is shown to the left. There are two players identified as the row player and the column player. A row player has a move that consists of choosing either the top row (UP) or the bottom row (DOWN). The choice of the column player is to select the left (LEFT) or right (RIGHT) column.

The table that has two entries in each cell shows the points obtained by the two players. The first number specifies the points to the row player and the second to the column player. For example if the row player chooses UP and column chooses LEFT, the row player obtains 1 and the column player obtains 4. If they choose DOWN and RIGHT respectively the payoff to row is 2 and to column is 3.


TWO

✣ ✣

Select a row by clicking anywhere on it. Once a row is selected, click the Next button to advance to the next game matrix. The image to the left is show with the first row selected.

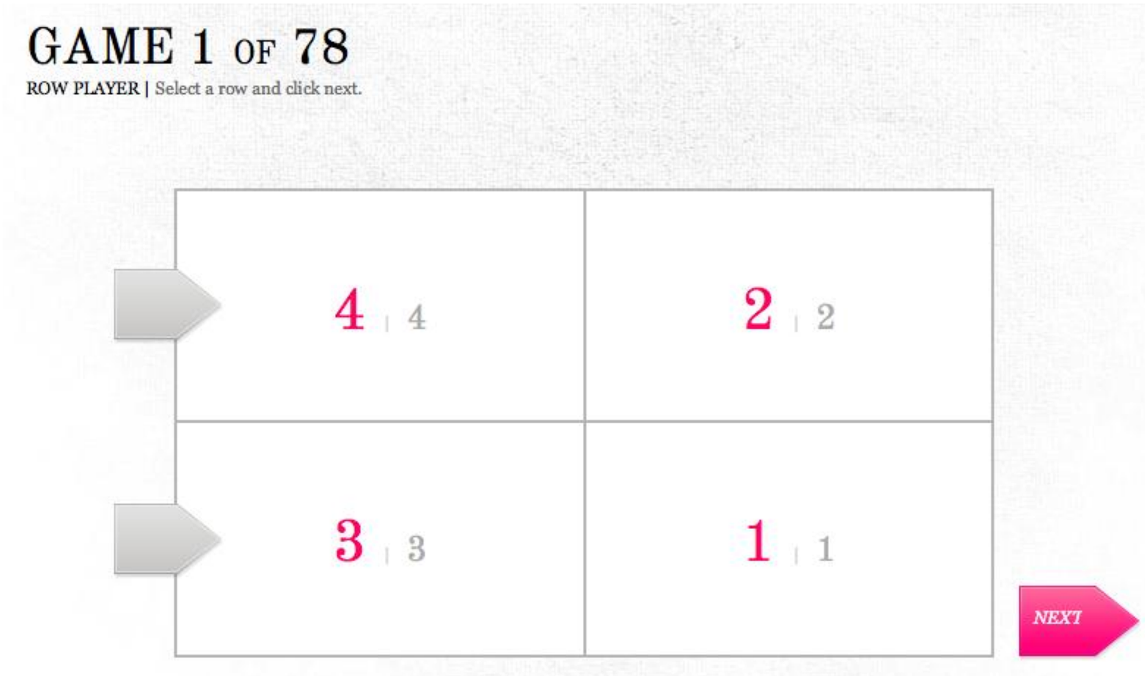
You will be matched in all of the 24 games you are playing in by all of the row players. Your final score will be calculated by adding the score you obtained in each of the 24 games against everyone of the column players and then adding together your score in each of the 24 games. A prize will of $xx will be given to the player with the highest score. In the event of ties there will be a randomization to select one winner.

You already played 78 games, you will be directed to the thank you page.



The rules page describes the rules for playing the games. It explains how to interpret the game diagrams, select moves and view results.

**Games Pages:**




These pages display the games and let players pick strategies. Row players pick a row by clicking on it. Column players click on columns to select them. A player may change their choice by clicking on the row/column they did not originally select. To finalize one's decision and continue to the next game, a player should click on the "NEXT" button.

**Thank You Page:**

This page appears after a player has completed the experiment.

## APPENDIX 2: Two Separated Game Sets

*Game #* gives the number of the game in our research set (out of 144). *Matrix* provides the payoffs of this game in matrix form. *Favors* gives information on which player the game favors. *Row PSNE* provides the payoff to the row player for the pure strategy Nash equilibrium that has the highest sum of payoffs for both players, 2.5 if there is no pure strategy Nash equilibrium. *Col* PSNE provides the payoff to the column player for the pure strategy Nash equilibrium that has the highest sum of payoffs for both players, 2.5 if there is no pure strategy Nash equilibrium. For example, if a game has two Nash equilibria, one with payouts for both players (2,2) and the other with payouts (4,4), *Row PSNE* and *Col PSNE* would both be 4. *Favors* is “column” if *Col PSNE>Row PSNE*, “row” if *Row PSNE>Col PSNE* and tie otherwise. *Type* is “M” is the game has no pure strategy Nash equilibria, and therefore has only a mixed strategy Nash equilibrium. *Type* is “S” if the game is symmetric. *Alt. Eq.* provides (*Row PSNE*, *Col PSNE*) for any other equilibria of a game that has the same sum of players’ payoffs.

**Column Favored:**

| Game # | Matrix | | Row PSNE | Col PSNE | Favors | Type | Alt. Eq. |
|---|---|---|---|---|---|---|---|
| 2 | (1, 2) | (3, 1) | 2 | 4 | column | | |
| | (2, 4) | (4, 3) | | | | | |
| 5 | (1, 3) | (3, 4) | 3 | 4 | column | | |
| | (4, 1) | (2, 2) | | | | | |
| 16 | (1, 2) | (2, 3) | 3 | 4 | column | | |
| | (3, 4) | (4, 1) | | | | | |
| 18 | (1, 2) | (2, 1) | 3 | 4 | column | | |
| | (3, 4) | (4, 3) | | | | | |
| 27 | (1, 3) | (2, 2) | 3 | 4 | column | | |
| | (4, 1) | (3, 4) | | | | | |
| 30 | (1, 2) | (2, 3) | 3 | 4 | column | | |
| | (4, 1) | (3, 4) | | | | | |
| 33 | (1, 1) | (2, 3) | 3 | 4 | column | | |
| | (4, 2) | (3, 4) | | | | | |
| 34 | (1, 1) | (2, 2) | 3 | 4 | column | | |
| | (4, 3) | (3, 4) | | | | | |
| 38 | (1, 4) | (4, 2) | 2 | 3 | column | | |
| | (2, 3) | (3, 1) | | | | | |
| 40 | (1, 4) | (4, 1) | 2 | 3 | column | | |
| | (2, 3) | (3, 2) | | | | | |
| 43 | (1, 3) | (4, 2) | 2 | 4 | column | | |
| | (2, 4) | (3, 1) | | | | | |

| | | | | |
|---|---|---|---|---|
| 45 | (1, 3) | (4, 1) | 2 | 4 column |
| | (2, 4) | (3, 2) | | |
| 49 | (1, 2) | (4, 1) | 2 | 4 column |
| | (2, 4) | (3, 3) | | |
| 63 | (1, 3) | (4, 2) | 3 | 4 column |
| | (3, 4) | (2, 1) | | |
| 66 | (1, 3) | (4, 1) | 3 | 4 column |
| | (3, 4) | (2, 2) | | |
| 71 | (1, 2) | (4, 1) | 3 | 4 column |
| | (3, 4) | (2, 3) | | |
| 76 | (1, 1) | (4, 2) | 3 | 4 column |
| | (3, 4) | (2, 3) | | |
| 79 | (1, 1) | (2, 2) | 3 | 4 column |
| | (3, 4) | (4, 3) | | |
| 80 | (1, 1) | (2, 3) | 3 | 4 column |
| | (3, 4) | (4, 2) | | |
| 86 | (1, 3) | (2, 1) | 3 | 4 column |
| | (3, 4) | (4, 2) | | |
| 87 | (1, 3) | (2, 2) | 3 | 4 column |
| | (3, 4) | (4, 1) | | |
| 94 | (1, 2) | (2, 1) | 3 | 4 column |
| | (4, 3) | (3, 4) | | |
| 96 | (1, 3) | (2, 1) | 3 | 4 column |
| | (4, 2) | (3, 4) | | |
| 101 | (1, 1) | (3, 2) | 2 | 4 column |
| | (2, 4) | (4, 3) | | |
| 103 | (1, 1) | (3, 3) | 2 | 4 column |
| | (2, 4) | (4, 2) | | |
| 105 | (1, 1) | (3, 4) | 2 | 3 column |
| | (2, 3) | (4, 2) | | |
| 108 | (1, 2) | (3, 3) | 2 | 4 column |
| | (2, 4) | (4, 1) | | |
| 110 | (1, 2) | (3, 4) | 2 | 3 column |
| | (2, 3) | (4, 1) | | |
| 112 | (1, 3) | (3, 1) | 2 | 4 column |
| | (2, 4) | (4, 2) | | |
| 114 | (1, 3) | (3, 2) | 2 | 4 column |
| | (2, 4) | (4, 1) | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 118 | (1, 4) | (3, 1) | 2 | 3 | column | | |
| | (2, 3) | (4, 2) | | | | | |
| 120 | (1, 4) | (3, 2) | 2 | 3 | column | | |
| | (2, 3) | (4, 1) | | | | | |
| 125 | (1, 1) | (3, 4) | 3 | 4 | column | | |
| | (4, 2) | (2, 3) | | | | | |
| 128 | (1, 2) | (3, 4) | 3 | 4 | column | | |
| | (4, 1) | (2, 3) | | | | | |
| 133 | (1, 3) | (3, 4) | 3 | 4 | column | | |
| | (4, 2) | (2, 1) | | | | | |
| 6 | (1, 3) | (3, 2) | 2.5 | 2.5 | tie | M | |
| | (4, 1) | (2, 4) | | | | | |
| 39 | (1, 4) | (4, 1) | 2.5 | 2.5 | tie | M | |
| | (2, 2) | (3, 3) | | | | | |
| 44 | (1, 3) | (4, 2) | 2.5 | 2.5 | tie | M | |
| | (2, 1) | (3, 4) | | | | | |
| 50 | (1, 2) | (4, 1) | 2.5 | 2.5 | tie | M | |
| | (2, 3) | (3, 4) | | | | | |
| 57 | (1, 4) | (4, 3) | 2.5 | 2.5 | **tie** | M | |
| | (3, 1) | (2, 2) | | | | | |
| 59 | (1, 4) | (4, 1) | 2.5 | 2.5 | tie | M | |
| | (3, 2) | (2, 3) | | | | | |
| 64 | (1, 3) | (4, 2) | 2.5 | 2.5 | tie | M | |
| | (3, 1) | (2, 4) | | | | | |
| 65 | (1, 3) | (4, 1) | 2.5 | 2.5 | tie | M | |
| | (3, 2) | (2, 4) | | | | | |
| 72 | (1, 2) | (4, 1) | 2.5 | 2.5 | tie | M | |
| | (3, 3) | (2, 4) | | | | | |
| 1 | (1, 4) | (3, 3) | 2 | 2 | tie | S | |
| | (2, 2) | (4, 1) | | | | | |
| 3 | (1, 1) | (3, 2) | 4 | 4 | tie | S | |
| | (2, 3) | (4, 4) | | | | | |
| 7 | (1, 2) | (3, 3) | 4 | 4 | tie | S | |
| | (4, 4) | (2, 1) | | | | | |
| 10 | (1, 1) | (3, 4) | 3 | 4 | tie | S | *(4,3) |
| | (4, 3) | (2, 2) | | | | | |
| 12 | (1, 4) | (2, 2) | 3 | 3 | tie | S | |
| | (3, 3) | (4, 1) | | | | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 19 | (1, 1) | (2, 3) | 4 | 4 | tie | S | |
| | (3, 2) | (4, 4) | | | | | |
| 26 | (1, 3) | (2, 2) | 4 | 4 | tie | S | |
| | (4, 4) | (3, 1) | | | | | |
| 35 | (1, 1) | (2, 4) | 3 | 3 | tie | S | |
| | (4, 2) | (3, 3) | | | | | |
| 41 | (1, 3) | (4, 4) | 4 | 4 | tie | S | |
| | (2, 2) | (3, 1) | | | | | |
| 53 | (1, 1) | (4, 2) | 2 | 4 | tie | S | *(4,2) |
| | (2, 4) | (3, 3) | | | | | |
| 69 | (1, 2) | (4, 4) | 4 | 4 | tie | S | |
| | (3, 3) | (2, 1) | | | | | |
| 74 | (1, 1) | (4, 3) | 3 | 4 | tie | S | *(4,3) |
| | (3, 4) | (2, 2) | | | | | |
| 8 | (1, 2) | (3, 1) | 4 | 4 | tie | | |
| | (4, 4) | (2, 3) | | | | | |
| 15 | (1, 3) | (2, 2) | 4 | 4 | tie | | |
| | (3, 1) | (4, 4) | | | | | |
| 20 | (1, 1) | (2, 2) | 4 | 4 | tie | | |
| | (3, 3) | (4, 4) | | | | | |
| 21 | (1, 1) | (2, 4) | 3 | 3 | tie | | |
| | (3, 3) | (4, 2) | | | | | |
| 24 | (1, 4) | (2, 1) | 3 | 3 | tie | | |
| | (4, 2) | (3, 3) | | | | | |
| 28 | (1, 3) | (2, 1) | 4 | 4 | tie | | |
| | (4, 4) | (3, 2) | | | | | |
| 29 | (1, 2) | (2, 3) | 4 | 4 | tie | | |
| | (4, 4) | (3, 1) | | | | | |
| 31 | (1, 2) | (2, 4) | 3 | 3 | tie | | |
| | (4, 1) | (3, 3) | | | | | |
| 32 | (1, 2) | (2, 1) | 4 | 4 | tie | | |
| | (4, 4) | (3, 3) | | | | | |
| 37 | (1, 4) | (4, 3) | 2 | 2 | tie | | |
| | (2, 2) | (3, 1) | | | | | |
| 42 | (1, 3) | (4, 4) | 4 | 4 | tie | | |
| | (2, 1) | (3, 2) | | | | | |
| 48 | (1, 2) | (4, 4) | 4 | 4 | tie | | |
| | (2, 1) | (3, 3) | | | | | |

| Game # | Matrix | | Row PSNE | Col PSNE | Favors | Type | Alt. Eq. |
|---|---|---|---|---|---|---|---|
| 54 | (1, 1) | (4, 4) | 4 | 4 | tie | | |
| | (2, 2) | (3, 3) | | | | | |
| 55 | (1, 1) | (4, 4) | 4 | 4 | tie | | |
| | (2, 3) | (3, 2) | | | | | |
| 58 | (1, 4) | (4, 2) | 3 | 3 | tie | | |
| | (3, 3) | (2, 1) | | | | | |
| 60 | (1, 4) | (4, 1) | 3 | 3 | tie | | |
| | (3, 3) | (2, 2) | | | | | |
| 61 | (1, 3) | (4, 4) | 4 | 4 | tie | | |
| | (3, 2) | (2, 1) | | | | | |
| 62 | (1, 3) | (4, 4) | 4 | 4 | tie | | |
| | (3, 1) | (2, 2) | | | | | |
| 67 | (1, 2) | (4, 3) | 3 | 4 | tie | | *(4,3) |
| | (3, 4) | (2, 1) | | | | | |
| 70 | (1, 2) | (4, 4) | 4 | 4 | tie | | |
| | (3, 1) | (2, 3) | | | | | |
| 77 | (1, 1) | (4, 4) | 4 | 4 | tie | | |
| | (3, 2) | (2, 3) | | | | | |
| 78 | (1, 1) | (4, 4) | 4 | 4 | tie | | |
| | (3, 3) | (2, 2) | | | | | |
| | | | 232.5 | 281.5 | **Total:** | | |

**Row Favored:**

| Game # | Matrix | | Row PSNE | Col PSNE | Favors | Type | Alt. Eq. |
|---|---|---|---|---|---|---|---|
| 4 | (1, 4) | (3, 3) | 4 | 2 | row | | |
| | (4, 2) | (2, 1) | | | | | |
| 9 | (1, 1) | (3, 2) | 3 | 2 | row | | |
| | (4, 3) | (2, 4) | | | | | |
| 11 | (1, 4) | (2, 3) | 3 | 2 | row | | |
| | (3, 2) | (4, 1) | | | | | |
| 13 | (1, 4) | (2, 1) | 4 | 3 | row | | |
| | (3, 2) | (4, 3) | | | | | |
| 14 | (1, 3) | (2, 4) | 4 | 2 | row | | |
| | (3, 1) | (4, 2) | | | | | |
| 17 | (1, 2) | (2, 4) | 4 | 3 | row | | |
| | (3, 1) | (4, 3) | | | | | |

| | | | | |
|---|---|---|---|---|
| 22 | (1, 4) | (2, 3) | 4 | 2 row |
| | (4, 2) | (3, 1) | | |
| 23 | (1, 4) | (2, 2) | 4 | 3 row |
| | (4, 3) | (3, 1) | | |
| 25 | (1, 3) | (2, 4) | 3 | 2 row |
| | (4, 1) | (3, 2) | | |
| 36 | (1, 1) | (2, 4) | 4 | 3 row |
| | (4, 3) | (3, 2) | | |
| 46 | (1, 2) | (4, 3) | 4 | 3 row |
| | (2, 4) | (3, 1) | | |
| 47 | (1, 2) | (4, 3) | 4 | 3 row |
| | (2, 1) | (3, 4) | | |
| 51 | (1, 1) | (4, 3) | 4 | 3 row |
| | (2, 2) | (3, 4) | | |
| 52 | (1, 1) | (4, 2) | 4 | 2 row |
| | (2, 3) | (3, 4) | | |
| 56 | (1, 4) | (4, 3) | 3 | 2 row |
| | (3, 2) | (2, 1) | | |
| 68 | (1, 2) | (4, 3) | 4 | 3 row |
| | (3, 1) | (2, 4) | | |
| 73 | (1, 1) | (4, 3) | 4 | 3 row |
| | (3, 2) | (2, 4) | | |
| 75 | (1, 1) | (4, 2) | 4 | 2 row |
| | (3, 3) | (2, 4) | | |
| 81 | (1, 1) | (2, 4) | 4 | 3 row |
| | (3, 2) | (4, 3) | | |
| 88 | (1, 3) | (2, 4) | 3 | 2 row |
| | (3, 2) | (4, 1) | | |
| 90 | (1, 4) | (2, 2) | 4 | 3 row |
| | (3, 1) | (4, 3) | | |
| 91 | (1, 4) | (2, 3) | 4 | 2 row |
| | (3, 1) | (4, 2) | | |
| 95 | (1, 2) | (2, 4) | 4 | 3 row |
| | (4, 3) | (3, 1) | | |
| 97 | (1, 3) | (2, 4) | 4 | 2 row |
| | (4, 2) | (3, 1) | | |
| 98 | (1, 4) | (2, 1) | 4 | 3 row |
| | (4, 3) | (3, 2) | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| 100 | (1, 4) | (2, 3) | 3 | 2 | row | |
| | (4, 1) | (3, 2) | | | | |
| 104 | (1, 1) | (3, 4) | 4 | 3 | row | |
| | (2, 2) | (4, 3) | | | | |
| 109 | (1, 2) | (3, 4) | 4 | 3 | row | |
| | (2, 1) | (4, 3) | | | | |
| 115 | (1, 3) | (3, 4) | 4 | 2 | row | |
| | (2, 1) | (4, 2) | | | | |
| 117 | (1, 4) | (3, 1) | 4 | 3 | row | |
| | (2, 2) | (4, 3) | | | | |
| 119 | (1, 4) | (3, 2) | 4 | 3 | row | |
| | (2, 1) | (4, 3) | | | | |
| 121 | (1, 4) | (3, 3) | 4 | 2 | row | |
| | (2, 1) | (4, 2) | | | | |
| 135 | (1, 4) | (3, 1) | 4 | 3 | row | |
| | (4, 3) | (2, 2) | | | | |
| 137 | (1, 4) | (3, 2) | 4 | 3 | row | |
| | (4, 3) | (2, 1) | | | | |
| 139 | (1, 1) | (4, 3) | 4 | 3 | row | |
| | (2, 4) | (3, 2) | | | | |
| 126 | (1, 2) | (3, 1) | 2.5 | 2.5 | tie | M |
| | (4, 3) | (2, 4) | | | | |
| 130 | (1, 3) | (3, 1) | 2.5 | 2.5 | tie | M |
| | (4, 2) | (2, 4) | | | | |
| 134 | (1, 4) | (3, 1) | 2.5 | 2.5 | tie | M |
| | (4, 2) | (2, 3) | | | | |
| 136 | (1, 4) | (3, 2) | 2.5 | 2.5 | tie | M |
| | (4, 1) | (2, 3) | | | | |
| 138 | (1, 4) | (3, 3) | 2.5 | 2.5 | **tie** | M |
| | (4, 1) | (2, 2) | | | | |
| 141 | (1, 3) | (4, 1) | 2.5 | 2.5 | tie | M |
| | (2, 2) | (3, 4) | | | | |
| 142 | (1, 4) | (4, 2) | 2.5 | 2.5 | tie | M |
| | (2, 1) | (3, 3) | | | | |
| 143 | (1, 4) | (4, 3) | 2.5 | 2.5 | tie | M |
| | (2, 1) | (3, 2) | | | | |
| 144 | (1, 4) | (4, 2) | 2.5 | 2.5 | tie | M |
| | (3, 1) | (2, 3) | | | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | (1, 4) | (3, 3) | 2 | 2 tie | S | |
| | (2, 2) | (4, 1) | | | | |
| 3 | (1, 1) | (3, 2) | 4 | 4 tie | S | |
| | (2, 3) | (4, 4) | | | | |
| 7 | (1, 2) | (3, 3) | 4 | 4 tie | S | |
| | (4, 4) | (2, 1) | | | | |
| 10 | (1, 1) | (3, 4) | 4 | 3 tie | S | *(3,4) |
| | (4, 3) | (2, 2) | | | | |
| 12 | (1, 4) | (2, 2) | 3 | 3 tie | S | |
| | (3, 3) | (4, 1) | | | | |
| 19 | (1, 1) | (2, 3) | 4 | 4 tie | S | |
| | (3, 2) | (4, 4) | | | | |
| 26 | (1, 3) | (2, 2) | 4 | 4 tie | S | |
| | (4, 4) | (3, 1) | | | | |
| 35 | (1, 1) | (2, 4) | 3 | 3 tie | S | |
| | (4, 2) | (3, 3) | | | | |
| 41 | (1, 3) | (4, 4) | 4 | 4 tie | S | |
| | (2, 2) | (3, 1) | | | | |
| 53 | (1, 1) | (4, 2) | 4 | 2 tie | S | *(2,4) |
| | (2, 4) | (3, 3) | | | | |
| 69 | (1, 2) | (4, 4) | 4 | 4 tie | S | |
| | (3, 3) | (2, 1) | | | | |
| 74 | (1, 1) | (4, 3) | 4 | 3 tie | S | *(3,4) |
| | (3, 4) | (2, 2) | | | | |
| 82 | (1, 2) | (2, 1) | 4 | 4 tie | | |
| | (3, 3) | (4, 4) | | | | |
| 83 | (1, 2) | (2, 3) | 4 | 4 tie | | |
| | (3, 1) | (4, 4) | | | | |
| 84 | (1, 2) | (2, 4) | 3 | 3 tie | | |
| | (3, 3) | (4, 1) | | | | |
| 85 | (1, 3) | (2, 1) | 4 | 4 tie | | |
| | (3, 2) | (4, 4) | | | | |
| 89 | (1, 4) | (2, 1) | 3 | 3 tie | | |
| | (3, 3) | (4, 2) | | | | |
| 92 | (1, 1) | (2, 2) | 4 | 4 tie | | |
| | (4, 4) | (3, 3) | | | | |
| 93 | (1, 1) | (2, 3) | 4 | 4 tie | | |
| | (4, 4) | (3, 2) | | | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| 99 | (1, 4) | (2, 2) | 3 | 3 | tie | |
| | (4, 1) | (3, 3) | | | | |
| 102 | (1, 1) | (3, 3) | 4 | 4 | tie | |
| | (2, 2) | (4, 4) | | | | |
| 106 | (1, 2) | (3, 1) | 4 | 4 | tie | |
| | (2, 3) | (4, 4) | | | | |
| 107 | (1, 2) | (3, 3) | 4 | 4 | tie | |
| | (2, 1) | (4, 4) | | | | |
| 111 | (1, 3) | (3, 1) | 4 | 4 | tie | |
| | (2, 2) | (4, 4) | | | | |
| 113 | (1, 3) | (3, 2) | 4 | 4 | tie | |
| | (2, 1) | (4, 4) | | | | |
| 116 | (1, 3) | (3, 4) | 2 | 2 | tie | |
| | (2, 2) | (4, 1) | | | | |
| 122 | (1, 1) | (3, 2) | 4 | 4 | tie | |
| | (4, 4) | (2, 3) | | | | |
| 123 | (1, 1) | (3, 3) | 3 | 3 | tie | |
| | (4, 2) | (2, 4) | | | | |
| 124 | (1, 1) | (3, 3) | 4 | 4 | tie | |
| | (4, 4) | (2, 2) | | | | |
| 127 | (1, 2) | (3, 3) | 3 | 3 | tie | |
| | (4, 1) | (2, 4) | | | | |
| 129 | (1, 2) | (3, 4) | 4 | 3 | tie | *(3,4) |
| | (4, 3) | (2, 1) | | | | |
| 131 | (1, 3) | (3, 1) | 4 | 4 | tie | |
| | (4, 4) | (2, 2) | | | | |
| 132 | (1, 3) | (3, 2) | 4 | 4 | tie | |
| | (4, 4) | (2, 1) | | | | |
| 140 | (1, 2) | (4, 4) | 4 | 4 | tie | |
| | (2, 3) | (3, 1) | | | | |
| | | **Total:** | 281.5 | 232.5 | | |