FINANCING OF PUBLIC GOODS THROUGH TAXATION IN
A GENERAL EQUILIBRIUM ECONOMY: EXPERIMENTAL EVIDENCE

By

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Abstract

We use a laboratory experiment to compare general equilibrium economies in which agents individually allocate their private goods among consumption, investment in production, and replenishing/ refurbishing a depreciating public facility in a dynamic game with long-term investment opportunities. The public facility is financed either by voluntary anonymous contributions (VAC) or taxes. We find that rates of taxation chosen by majority vote remain at an intermediate level (far from zero or 100%), and the experimental economies sustain public goods at levels between the finite- and infinite-horizon optima. This contrasts with a rapid decline of public goods under VAC. Both the payoff efficiency and production of private goods are higher when taxes are set endogenously instead of being fixed at the optimum level externally. When subjects choose between VAC and taxation, 23 out of 24 majority votes favor taxation.

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1. Introduction

Public goods are crucial for the functioning of a society, and the problem of financing their production has attracted much interest.[1] Game theoretic models suggest that egoistic individuals have little reason to finance production or maintenance of public goods through individual voluntary anonymous contributions (VACs). Laboratory public good experiments tend initially to yield average contributions around 50 percent of the collective optimum, gradually declining towards a 5-20 percent range.

However, there is little reason for society to confine its search for efficient solutions for the pervasive problem of financing the provision of public goods (PGs) and common pool resources (CPRs) to only VACs. Institutions may evolve to address various problems of economizing through socio-political and economic processes of adjustment, experimentation, and feedback over rules and conventions. It is reasonable to conjecture that the scope of such social evolution includes the provision of PGs and CPRs. In modern democratic societies taxes, set by an elected government, are the most common way to finance such goods. We therefore explore how efficient the provision of PGs is in a system with taxes set externally or by subjects through a vote.[2] In an additional treatment subjects can also vote on whether they want to implement a system with taxes or with VACs.

Walker et al. (2000) seem to have been the first to consider the efficiency implications of a combined common-property-with-voting allocation scheme in the laboratory. They reported that voting on the use of a CPR is more efficient than appropriation of the resource by individual members of the group. In most cases proposals adopted by vote were socially optimal, indicating that groups can coordinate on the efficient use of a CPR. Earlier, Ostrom et al. (1992) showed that communication in a CPR-game significantly increased average net yield. Magreiter et al. (2005) studied asymmetric endowments and found that homogeneity made efficient agreements more likely. However, the

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[1] For surveys of the substantial pre-1995 literature on experimental gaming with public goods see Ledyard (1994) and Bergstom et al. (1986). From considerable literature since then, we mention only a few. Fehr and Gächter (2000) consider public goods experiments without punishment for free riding; Brandts and Schram (2001) consider voluntary contribution mechanisms for public goods; Palfrey and Prisbrey (1997) consider public goods provision where the individuals have different marginal values for their private goods; Ahn et al. (2009) present an experiments on endogenous group sizes; Hatzipanayotou and Michael (2001) deal with public goods, tax policies and unemployment in less developed countries. Modeling in the last of these papers is closer to the spirit of our own emphasis on the importance of institutional structures in the economy.

[2] Agranov and Palfrey (2015) examine the "positive questions about the political-economic equilibrium that determines tax policy, rather than the normative concerns about optimal tax rates." (fn. 2) Their concerns about inequality and distribution are absent in the present study.

common-property setting of these three studies is quite distinct from the public good we explore here. Kroll et al. (2007) employed a more familiar public-goods setting and reported that voting by itself did not promote cooperation; but the ability of voters to punish defectors did. With perfect enforcement they observed 100 percent contribution rates in most periods. While these results are useful, contributions or a tax of 100 percent are neither realistic nor desirable in practice. We explore an economy with private and public goods where voting is used to determine the tax rate in a setting where the optimum rates of consumption and taxation lie at an intermediate level (68.3 percent for consumption and 21.5 percent for taxation).

Prior experimental studies (see Carpenter 2000, for an overview) have observed cooperation where conventional theory predicts its absence. We also build on studies of experimental production economies with taxation, e.g., Riedl and van Winden (2007). The endogenous taxation through voting is similar to the work of Sutter and Weck-Hannemann (2003, 2004). In our process-oriented strategic market game the maintenance of an existing public good is financed through a tax on private income. The unique equilibrium solution for any given tax rate yields an optimal consumption/investment policy for each individual. General dynamic programming analysis of our basic model enables us to solve for an optimal rate of taxation for society as a whole.[3]

We set up and examine a model economy dynamic game with long-term investment opportunities in the laboratory with a 2x2 +3 design of four main and three robustness treatments. Our two variables for the 2x2 design are taxes (set exogenously or through vote) and the initial stock of public good (starting at 100 or 50 percent of the optimum). Treatment 1 features an exogenously given tax rate and the starting stock of the public good is at the infinite horizon GE optimum, contrasted with starting at 50 percent of the optimum in Treatment 2. The taxation is fixed at the theoretical optimal level to maintain the optimal stock of the public good facility in both cases.[4] In Treatments 3 and 4, the tax rate is set endogenously through subjects' vote (following Black 1958 at the median choice) once every five periods, starting with optimal and suboptimal public facility levels in Treatments 3 and 4, respectively. As a comparison and bridge to existing literature, we also examine the performance of otherwise identical economies in which (a) taxation is replaced by individual VACs, labeled Treatment

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[3]   Appendix   A   presents   an   EXCEL-sheet   (the   infinite-horizon   model   can   be   found   at http://www.uibk.ac.at/ibf/mitarbeiter/huberj/model_infinite_online-material.xlsx, while the finite-horizon model is at http://www.uibk.ac.at/ibf/mitarbeiter/huberj/model_finite_online-material.xlsx) which allows one to manipulate different input variables of the model and see the charts of respective changes in equilibrium levels of utility and other variables.
[4] For the basic model of tax-financed public goods, see Karatzas et al. 2006. Basics on exchange economies and money are provided in Lucas (1978, 1980) and Lucas and Stokey (1983, 1987).

0, (b) tax rate is set exogenously at the finite horizon optimum, labeled Treatment T5, and (c) subjects choose between VACs and taxation systems by majority vote, labeled Treatment T6.

We find that the four main treatments with finite horizon experimental economies sustain public goods between the finite- and infinite-horizon optima, and exhibit 85 to 97 percent efficiency (actual/general equilibrium payoff to subjects). Efficiency and production of private goods are significantly higher when the rate of taxation is determined by voting compared to being fixed at the GE optimum. In the two voting treatments taxes remain at an intermediate level, converging neither to zero nor to 100%. Irrespective of whether we start at 50% or 100% of the optimum, the stock of the public good converges near the same level between the infinite- and finite-horizon optima by the end of the experiment. This holds also in supplemental treatment T6 in which the majority chose taxation regime over VACs 23 out of 24 times. In T5, with an ever lower tax rate (set externally at the finite-horizon optimum), the stock of the public good falls to a level lower than in T1 to T4 and T6, but higher than in T0 (VAC). In all except the VAC treatments, the ending stock of public goods exceeds the finite horizon optimum. We also find that the share of total earnings derived from the public good is higher in the taxation-treatments. By contrast, in VAC treatments most earnings come from direct consumption of the private good. Total production of private goods is not reduced by taxes, and is highest when subjects can choose the tax rate endogenously (compared to VACs or externally fixed tax rate).

These results from a general equilibrium laboratory economy suggest that taxation is an efficient social institution to address the problem of under-production of public goods through voluntary contributions. The model and experimental design are presented in Section 2, followed by results, and a discussion in the subsequent sections of the paper.

2. A Dynamic General Equilibrium Model of an Economy with a Public Good with Laboratory Implementation

We consider a version of Samuelson's (1954) pure public good embedded in a parallel dynamic control process that has been solved for its type-symmetric non-cooperative (rational expectations) equilibrium for any tax rate (see Karatzas et al. 2011).[5] The dynamic structure of the game also includes a government and voting.

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[5] Formally, with a continuum of agents Karatzas et al. (2011) solve for any tax level; then after solving this set with taxation level as a parameter they solve for the optimum from the point of view of a benevolent central government. The

The basic model involves the maintenance/refurbishment of a depreciating public good facility such as a transportation or sewage system (see Karatzas et al. 2006 for a description).[6] The game has a government and n individual agents. At the outset, the government is endowed with G units of the public good and M units of money; each agent has a units of private good, m units of money, and a private good production function. The government has the right to collect as tax a fraction θ of individuals' income from the sale of private goods, and a production function that transforms the private goods bought from tax revenues into the public good.[7]

Figure 1 gives a time line of events within one period. A period begins with government in possession of taxes gathered in the preceding period in the form of money (M in period 1), the n agents carrying their after-tax money balances from the previous period (m in period 1), and the units of the private good they produced at the end of the previous period (a in period 1). We use a sell-all market mechanism, in which individuals' entire balance of private goods is automatically offered for sale in a market (see Huber et al. 2010 for properties of the sell-all mechanism). In the experiment each individual automatically bids his total money balance b to buy the private good from the market. The government also bids all its money balance $\bar{b}$ for the private good. A price p is computed as the ratio of the total money bid (by agents and the government) divided by the total number of units of private goods available.

(Insert Figure 1 about here)

The fixed money supply in the economy in conjunction with the sell-all market game imposes a good deal of regularity on price dynamics: If aggregate production increases (decreases), then the price must fall (rise). We see this as a virtue as it promotes order in an environment with a lot of moving parts (money, private goods, public goods, taxation, production, and consumption), and permits sharper focus on the question of efficient provision of public goods under taxation.

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theory approximates equilibrium as though the number of agents is large enough that they have no influence on the price. Use of $n = 10$ in the laboratory experiment ignores the presence of a small oligopolistic influence.

[6] It also could be a wage-supported bureaucracy that provides a self-policing system for the economy. Although bureaucracy could be one of the most important and earliest of costs of public goods, it is rarely mentioned in discussions of public goods. Also, capital cost of creating a new public good facility is typically considerably higher than the cost to operate and replenish an existing facility. The two may also differ in their political feasibility. In the present model and experiment, we confine ourselves to consideration of financing the operation and replenishing of an existing public good facility in absence of uncertainty.

[7] Even at this level of simplicity, given that production takes time, there are accounting questions to be considered in the definition of periodic income and profits. In a stationary equilibrium the timing differences disappear.

The quantity of private goods the government and individual agents get equals the money they bid divided by the price of the private good ($k_i = b_i/p$ units for individual i; $k = \bar{b}/p$ units for the government). Each agent, being a producer as well as a consumer, divides the units bought between consumption and production.[8] In addition, each agent receives the price multiplied by the number of units sold as his income in units of money. This money income is taxed at a uniform tax rate, either pre-set to the equilibrium rate $\theta = 21.5\%$ (in Treatments 1 and 2) or set endogenously through a vote by subjects (in Treatments 3 and 4), where all subjects pay the median of the tax rates proposed by individuals.

Each of the n producer/consumer agents has a concave private good production function $f(k) = 80*k^{0.25}$ with a one period production time lag, and a payoff function of the form $u(c, G) = (c + G/4)$, with c being the consumption of private goods and G being the stock of public goods. We calibrated the game so that in equilibrium approximately half of the expected earnings come from the public good and the other half from private consumption.

Before the end of the period the stock of the public good is depreciated by 10%. The government then uses all k units of private good it buys to produce $F(k) = 2*k^{0.5}$ units of the public good which is added to replenish the stock of the public good at the beginning of the next period. The government carries the tax collected as its money balance to buy private goods in the following period. In infinite horizon equilibrium the production of public goods precisely covers depreciation; otherwise the amount of the stock of the public good changes. This describes one full period of the game. Holdings of the goods (public and private) and cash are carried over from one period to the next in all treatments.

In implementing an experimental game with a finite termination we are faced with the question of how to value the stock of public good and money holdings at the end of the game. With zero valuation, we expect that the maintenance of the public good facility will tend to drop off towards the end of experimental sessions. We set up an Excel worksheet to numerically solve the finite horizon dynamic program when the value of the stock of public good is zero at the end of the session (see footnote 4 and Appendix A). The terminal or "salvage value" of left over money, private and the

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[8] In this respect our experiment is similar to Lei and Noussair's (2002) growth experiment; the same subjects simultaneously play the roles of both the firm and the consumer.

public good are all set to zero. Subjects are instructed that the session will end with $1/6^{th}$ chance each after period 25, 26, 27, 28, 29, or 30.[9]

Instructions given to subjects are included as Appendix B. Instructions supplemented by trial rounds allowed subjects to gain a reasonable understanding of the decisions they had to make, the opportunity sets from which various decisions had to be chosen, and how their own and others' decisions were linked to their payoffs. It is unlikely, and almost impossible, for any subjects to have fully understood the mathematical structure and properties of the model economy in this experiment (or for that matter, in most experiments where the mathematical structure is nontrivial, and optimal strategy is far from obvious). It is not the purpose of the experiment to assess the cognitive capacities of subjects to intuitively arrive at optimal solutions to stochastic dynamic programs; that would be outside the scope of this paper. Our aim in this explorative study is to find out how production, endogenously set tax rates and the stock of the public good evolve in these economies populated with agents having abilities and incentives of ordinary people. While implementing a complex model in the laboratory is a challenge, we took care to explain the instructions to the subjects and ensure that they correctly understood their information, opportunity sets, and how their payoffs depended on their own and others' actions.

If the future has little value and the maintenance of a public good is costly, one might as well forego maintenance in favor of immediate consumption. In our experiment this was avoided by selecting no discount on the future.[10] A further experimental difficulty occurs as the experiment time horizon is finite. We expect and empirically find some drop off near the end of the play as the remaining public goods are of no further value once the last period is over.

Implementation of the experiment

The experiment consists of variations on the regime to finance a public good (exogenous fixed or varying tax rate, endogenous tax rate, and VAC) and the initial stock of the public good the economy is endowed with (optimum, half of optimum) in a 2x2 + 3 design (see Table 1).

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[9] For the equilibrium calculations presented in the results section we used $1/6^{th}$ chance of ending the game after each of the periods 25 to 30. In the experiment we ran the first session with a random termination, which happened to be after period 26. For better comparability and ease of exposition in figures all other sessions were also ended after this period.

[10] With zero discount rate the payoffs of a dynamic program may become unbounded; in our experiment this can be handled by maximizing the average payoff per period.

The variations in the regime are:

- Control treatment T0 in which subjects make voluntary anonymous contributions (VACs) for production of the public good. This treatment serves as a benchmark for comparison with the results from experimental literature on VAC partial equilibrium economies.

- Exogenously fixed tax rate (at infinite horizon equilibrium level of 21.5%) in regimes T1 and T2. In T1 (and T3), the starting level of the stock of public good is at its steady state (i.e., infinite horizon) equilibrium of 427. In order to assess the dynamic ability of the system to adjust when the starting point is not at the optimum, we use Treatment T2 (and T4) in which the starting level is 50% of the optimum.

- Whether governments have the ability or incentives to set the rate of taxation at the optimal level is controversial. We therefore contrast the results of equilibrium exogenous tax rate economies (T1 and T2) against economies with an endogenously determined tax rate (median of individual proposals solicited once every five periods) in regimes T3 and T4; and

- Taxes fixed each period at the finite-horizon optimum (dropping from above 25 percent in the first period to almost zero in the last) in T5; and

- Institutional evolution through voting between VAC and taxation regimes in T6.

In T3 and T4, each subject proposes a tax rate and the median of the ten proposals (mean of the fifth and sixth highest proposals) is applied to all subjects for five periods, until the next vote is taken. Subjects therefore have the collective freedom to change the provision for public goods.[11]

In T6 subjects first experience five periods with VAC (same as in T0) and then five periods with taxes (setup of T3). After these ten periods, subjects collectively decide by majority vote whether they want to implement VAC or taxes for the next five periods.[12] The selected mechanism is implemented for five periods, until the next vote is taken.

Table 2 gives the values of the parameters' equilibria of the design. In stationary (i.e., infinite horizon) equilibrium price is $p = 27.67$; each individual should buy 170 units of the private good and consume 68.27%, i.e., 116 units, while the remaining 54 units are put into production to produce $80*54^{0.25} = 217$ units for the next period. The tax is at 21.5 percent and generates 12.940 tax income

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[11] These treatments are close to Robbett (2014), where subjects choose their tax rates, and interior optima existed.
[12] There was one tie vote of 5:5 and one institution was picked randomly.

which the government uses to buy 467.7 units of the private good to produce $2*467.7^{0.5} = 43.25$ units of the public good which is, just enough to offset the periodic depreciation (10% of 427 units) of the equilibrium stock of the public good.

(Table 2 about here)

The experiment consists of a total of 32 independent runs, each with a different cohort of 10 subjects for a total of 320 subjects. All subjects were BA or MA students in management or economics at the University of Innsbruck, Austria. All sessions were carried out using a program written in z-Tree (Fischbacher, 2007) and recruitment was done with ORSEE (Greiner, 2015). Average duration of a session was approximately 60 minutes and average earnings were 15 euros.

3. Hypotheses and Results

This laboratory experiment explores several related questions: (1) how VAC and taxation regimes affect the provision of a public good; (2) whether the tax rates determined by popular vote tend towards zero over time; (3) whether the steady state stock of public good depends on the initial conditions; and (4) whether the efficiency of the system is affected by the regime for financing the provision of a public good. Based on the literature discussed in the introductory section, we set up these questions in the form of null hypotheses of "no difference". Most of the tests (except on the stock of public good) are conducted on data from the final five periods of each run.[13] We use the data for the final period for the stock of the public good; being a cumulative stock magnitude, it is not susceptible to large period-to-period variations.

The hypotheses and results are summarized in Table 3 and results are presented in Figures 2, 3 and 4. Figure 2 charts the evolution of tax rates in treatments T3, T4, and T6 as well as VAC rates in supplemental treatment T0 (chain-dotted lines). Additional lines in black and green show general equilibrium predictions for infinite and finite horizon economies as theoretical benchmarks for comparison.[14]

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[13] This is done as a compromise among three considerations: (1) data from these late periods reflect most of the learning that takes place during a run; (2) use of five periods mitigates excessive dependence on data from a single final period; and (3) uncertainty about the end of the run mitigates against the data from these periods being unduly influenced by any end-of-the-session effects.

[14] Since these experimental economies are known to subjects to last for a finite number of periods, strictly speaking, the finite horizon equilibrium is the appropriate benchmark for comparing the empirical data. However, we add the infinite horizon equilibrium as an additional benchmark in case subjects ignore the impending end of the economy until close to the

(Insert Table 3 here)

3.1. Endogenously set tax rates (Figure 2):

> *Null (Alternative) Hypothesis Ia: Endogenously determined tax rates stabilize near (below) the optimal level.*

> *Null (Alternative) Hypothesis Ib: Endogenously determined tax rates are equal to (more than) zero.*

Paying low or no taxes leaves more for private consumption initially, but ends up hurting everyone by depleting the stock of the public good. An economy that attains general equilibrium will generate tax rates near 21.5 percent under null hypothesis Ia and lower ones under the alternative. Null hypothesis Ib for the extreme tax rate is zero. These are tested on data from Treatments T3 and T4.[15]

The top left panel of Figure 2, displaying taxes in Treatment 3 (with the initial stock of public good at the steady state level of 427 units), shows the endogenously determined tax rate usually remained below 21.5 percent and declined from a range of 17.5-22 (average 19.8) in the first vote to 5-22.5 percent (average 14.3) in the sixth and final vote. Note that the finite horizon optimal tax rate (broken green line) declines from 25% to near zero, because the terminal conditions assign zero value to the stock of public good at the end of the session. In Treatment 4 (top right panel) the endogenously determined tax rates also declined slightly from 11-23 percent (average 18.3) in the first vote to 8-23 (average 16.3 percent) in the sixth and final vote. The changes in tax rates are not statistically significant ($p = 0.1720$ for T3 and $p = 0.5346$ for T4 in a Two-sample Wilcoxon rank-sum test).

(Insert Figure 2 about here)

In all 12 endogenous taxation economies agents voted to pay taxes higher than the finite horizon optimum in the second half of the sessions, and taxes clearly did not approach the extreme values of zero or 100 percent, as in earlier public goods literature.[16] Null hypothesis Ia, on the tax rates stabilizing near the optimal (versus declining towards zero) is rejected when all tax rates are compared to the optimum of 21.5 percent. However, when testing each of the twelve votes (six for each of T3 and T4) separately, the null is rejected seven times and not rejected five times. Hence, while the null of

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end. We also add as benchmarks the theoretical minimum (where no subject contributes anything to the public good) and maximum levels of production and public goods.

[15] In T6 23 out of 24 votes resulted in taxes rather than VAC being implemented and the resulting tax votes resulted in comparable tax rates as in T3 and T4.

[16] As for proposals of extremes (zero or 100% taxes): these happen quite infrequently. In total only 31 out of a total of 720 votes (4.3%) across T3 and T4 were for zero or 100% taxes. Subjects did of course only learn the actual tax rate, not all ten proposals in their economy.

Hypothesis Ia is rejected overall, tax rates were close to the infinite-horizon optimum in almost half of the individual votes. As for Hypothesis Ib, not a single vote yielded a tax rate of zero. An enforced tax that is equal for all does not lead to a breakdown, as is usually observed in VAC public goods experiments.

3.2 Public Good Provisioning under VAC and Endogenous Taxation

> *Null (Alternative) Hypothesis II: Provision for public goods is equal (higher) under taxation than under VAC.*

This hypothesis is tested on data from Treatments T1 to T4 vs. T0. Most VAC literature reports low contributions to public goods. If endogenously determined tax rates are an effective solution to the problem, we should expect to reject the null hypothesis in favor of the alternative.

The chain-dotted lines in each of the panels of Figure 2 show the realized VACs as a percentage of individual income in the two sessions of Treatment 0. Contributions dropped steadily over time, asymptotically approaching zero, and remained less than the finite- as well as infinite-horizon optima throughout. This is consistent with the results of prior laboratory experiments with voluntary anonymous contributions for public goods. Null hypothesis II, stating that treatments with taxes lead to the same average contributions than the treatment with VACs is clearly rejected in favor of the alternative with data from all periods, as well as with data from only the final five periods (Wilcoxon-signed ranks tests, *p*-values <0.01).[17]

3.3. Dependence of public good provision on initial conditions

> *Null (Alternative) Hypothesis IIIa: The final level of the public good does not depend on initial endowment of the public good (is higher with higher initial endowment).*

> *Null (Alternative) Hypothesis IIIb: The final level of the public good does not depend on whether it is financed by fixed or endogenously determined tax rates (is higher with fixed tax rates).*

> *Null (Alternative) Hypothesis IIIc: The final level of the public good does not depend on whether it is financed by taxes or VAC (is higher when financed by taxes).*

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[17] Mann-Whitney U-test comparing average contributions for entire runs confirm this, as all p-values are below 0.05 for four individual tests comparing each of T1, T2, T3, and T4 to T0.

The three sub-hypotheses are tested on the stock of the public good in the last period. We use data from (a) Treatment T1 vs. T2 and T3 vs. T4; (b) Treatment T1 vs. T3 and T2 vs. T4; and (c) four comparisons of T0 against each of T1 to T4, respectively.

The top left panel of Figure 3 shows the development over all periods of the stock of the public good in all seven treatments plus four benchmarks (finite and infinite horizon optima as well as theoretically possible maximum and minimum developments).[18] The same conventions are used to show data in the other three panels of Figure 3.

(Insert Figure 3 about here)

Starting from the optimal level in all but T2 and T4, the stock of public good declined to the neighborhood of 370 to 390 units, irrespective of whether the tax rate was fixed (T1) or determined by vote by participants (T3 and T6). Starting from the suboptimal level, the stock of public goods rose gradually to the neighborhood of 360 irrespective of whether the tax rate was fixed (T2) or determined by participants' vote (T4). In T5 does the stock of the public good decline substantially in the last few periods, as tax rates (lowered each period) become insufficient to sustain the stock of PG.

Hypotheses IIIa compares the final stock of the PG between treatments where the stock started at the optimum vs. half of the optimum. The two Mann-Whitney U-tests yield *p*-values of 0.248 and 0.423, for T1 vs. T2, and T3 vs. T4, respectively. Hence null Hypothesis IIIa is not rejected.

Hypothesis IIIb compares the final stock of the PG between treatments where the tax rate is fixed at the optimum or is set endogenously. There are strong theoretical arguments why subjects should be expected to vote for rather low tax rates (below GE level), but on the other hand earlier experimental evidence (e.g., Kroll et al., 2007) suggests high tax rates when taxes are enforceable (as they are in our case). Hence we have no clear expectation on this hypothesis. The two Mann-Whitney U-tests yield *p*-values of 0.286 and 0.831, for T1 vs. T3, and T2 vs. T4, respectively. Hence null Hypothesis IIIb of no difference in the final stocks of public goods under two tax policies is not rejected. It seems reasonable to infer, on the basis of these 20 independent sessions of experimental economies, that the stocks of public goods tend towards the range midway between the infinite-horizon

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[18] In an unconstrained environment, one would expect the finite horizon equilibrium stock of the public good to be exhausted to zero at the end of the session. Since the stock of public good depreciates at a constant rate of 10% per period, exhaustion close to zero at the end would require lower investment in early periods. The lower payoff in those periods prevents the optimal level of public good from being driven to exhaustion at the end even in a finite-horizon economy.

and finite-horizon optima and are not significantly different in different taxation regimes, as the tax rates determined by vote are sufficiently high to sustain a high stock of PGs.

Finally, the dotted line in Figure 3 depicts the time path for T0 in which taxation was replaced by individual VACs. In these two sessions, the stock of public goods declined steadily and sharply to an average of 159 units at the end of period 25. This is much lower than levels observed in any period of any of the 20 economies with taxation. Null hypothesis IIIc of equality of the final stock of PG between VAC treatment T0 and each of T1-T4 is rejected. The *p*-values of the Mann-Whitney U-tests are 0.046 (N = 8) for T3 and T4, and 0.064 (N = 6) for T1 and T2. The data confirm that the final stock of the PG is lower in T0 with VAC than in any other treatment. These results are consistent with those obtained in voluminous experimental literature on partial equilibrium economies in which public goods are financed by VACs.

Figure 4, depicting in the top panel the average stock of public goods across treatments (over all periods on the left; last five periods on the right), confirms these observations: especially on the top right panel we observe the very low remaining stock of PGs in T0, and the comparatively high levels in T1, T3 and T6. T5, where the tax is fixed at the finite-horizon optimum ends very close to the finite-horizon optimum of the stock of PGs.

(Insert Figure 4 about here)

Summing up on the stock of PG: Failure to reject hypothesis IIIa suggests that this economy tends to sustain a high level of PGs. Failure to reject null hypothesis IIIb suggests that taxes do not need to be fixed at the optimum, but that endogenous choice through vote can lead to equally good results. Rejection of null hypothesis IIIc suggests that the financing regimes for public goods matter for its steady state level, as any tax regime produced higher final levels of the PG than VACs did.

3.4. Efficiency:

We measure efficiency of the economy as the total points earned by all participating subjects in a period as a percentage of the number of points they would have earned in that period if the economy had achieved the infinite horizon (steady state) general equilibrium. Here we use average efficiency across the last five periods for the statistical tests. In addition, results on the full data are provided in Figures 3 and 4. Note that points earned by each individual are the private goods consumed plus the

stock of the public good divided by four.[19] Efficiency is defined as these earnings divided by the GE earnings.

Null(Alternative) Hypothesis IVa: Efficiency is the same irrespective of the initial endowment of the public good (lower with suboptimal endowment).

Null (Alternative) Hypothesis IVb: Efficiency is the same irrespective of fixed or endogenous tax rates (lower with endogenously sets tax rates).

Null (Alternative) Hypothesis IVc: Efficiency is the same irrespective of public good financing by taxation or VAC (lower with VAC financing).

Hypothesis IVa is tested by comparing data from treatments T1 vs. T2 and T3 vs. T4. Hypothesis IVb is tested by comparing data from two pairs of treatments T1 vs. T3 and T2 vs. T4. Hypothesis IVc is tested by comparing data from four pairs of treatments T0 vs. T1, T2, T3, and T4, respectively. The development of efficiency in each treatment is shown in the bottom left panel of Figure 3, while average efficiency across all and the last five periods (along with four benchmarks) is shown in the second row of panels of Figure 4.[20] In Figure 3 the third bar (T2) is lower than the second (T1) in both panels, and the fifth bar (T4) is lower than the fourth (T3) in both panels, favoring the alternative hypothesis IVa (suboptimal initial endowment generates lower efficiency). Statistically, Mann-Whitney U-tests (N = 10) reject the null hypothesis for the last five periods of T3 vs. T4 (but not for T1 vs. T2).

Hypothesis IVb, comparing efficiency in treatments with the tax rate fixed at the infinite horizon GE optimum of 21.5 percent versus endogenous choice of taxes reveals differences, as efficiency is significantly higher in T3 than in T1 ($p = 0.011$ for the last five periods) and also marginally higher in T4 than in T2 ($p = 0.088$ for the last five periods). Hence the endogenous choice of taxes resulted in higher efficiency levels (through higher production of private goods; see next section) than with the tax rate set exogenously at the optimum.

To examine hypothesis IVc, VAC yields higher efficiency initially (the first bar), driven by high consumption and low investment in the production of PGs. However, this profligacy catches up with the economy and, in the last five periods, every alternative yields higher average efficiencies,

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[19] Note that this efficiency measure is only an approximation because it ignores the stock of public good left for the future at the end of the laboratory sessions. For individual periods, efficiencies can exceed 100% when agents consume unsustainable amounts by cutting back on investments.

[20] Appendix C Figure A1 (enclosed, and to be made available online), shows the period-by-period efficiency for various treatments.

even ignoring the poor shape in which VAC leaves the stock of public goods. Compared to T3, T0 has lower efficiency in last five periods (Mann-Whitney U-test, $p = 0.046$, $N = 6$).

3.5. Production of the private good

> *Null (Alternative) Hypothesis Va: Production of the private good is the same (different) irrespective of whether taxes are fixed or set endogenously by vote.*

> *Null (Alternative) Hypothesis Vb: Production of the private good is the same (different) irrespective of the public good being financed by taxation or VAC.*

These two null hypotheses are tested two-tailed, because there is no relevant basis for assuming the deviations from the null to be in either direction. The third row of panels in Figure 4 shows the average total production of private goods in the sessions of each treatment, while the top right panel of Figure 3 provides the respective development over the 25 periods.[21] Production tended to decline over the 25 periods of all sessions from near optimal (2170) to the neighborhood of 1,500.[22] A reason for this could be the choice of a concave production function ($80*k^{0.25}$) in which the extra output from positive deviations from optimal input (54 units of the private good) is much smaller than the loss of output from comparable negative deviations. Thus, while the average input is close to the optimum (average of 53.2 in the first ten periods; 44.8 overall), average output is lower due to dispersion of inputs across individual subjects. In addition, optimal production would fall sharply in periods 26 to 30 in the finite-horizon benchmark. Thus, the decline observed in the experiments is also justified by this benchmark. Of the four main treatments, the total production is the highest in T3 and T4 where taxes are set endogenously, and it also falls less in these treatments.

We test for differences in average production across all (rather than only the last five) periods, as total production is relevant in each period and as the initial stock of PG should be irrelevant for production. To test Hypothesis Va we run pairwise Mann-Whitney U-tests between treatments T1 vs. T3 and T2 vs. T4. T3, with a high initial stock of the PG and endogenously set tax stands out as the treatment with the highest average production, which is significantly higher than production in T1 (but also higher than in T2 and T4, each difference significant with $p < 0.02$). The other treatment with endogenously set taxes, T4, had the second highest average production, which was significantly higher than in T2 ($p = 0.011$). We conclude that production was higher in treatments where subjects had

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[21] Figure A2 in Appendix C included here and available online shows period-by-period details of individual runs.
[22] Note that production is also at 2170 in the finite-horizon-benchmark in periods 1-25, as subjects need to produce units of the private good in order to earn money and be able to consume and produce units for the next period. As the rules specify that there will certainly be at least 25 periods, production is at the long-term optimum of 2170 throughout periods 1-25. After period 25 production quickly and steadily drops.

control over the taxes they pay compared to those treatments where taxes were set exogenously. This points to the possibility that subjects are more committed to and more ready to invest in an environment where they felt they had more control over the economy they acted in.

To test Hypothesis Vb we run pairwise Mann-Whitney U-tests between T0 and each of T1 to T4. We find no significant differences for T1 and T4, but significantly different production in T2 (where it is lower than in T0) and T3 (where it is higher than in T0; p-values 0.064 and 0.046, respectively). Hence, we conclude that in our setting taxes did not deter production, when compared to a VAC-regime. When comparing tax-treatments subjects produced more when they had control over the tax rate they had to pay than when it was set externally.

3.6. Decomposition of earnings from public goods and private consumption

> *Null (Alternative) Hypothesis VIa: Initial endowment makes no difference to the percentage of earnings from the public good (greater proportion of earnings from higher endowment).*

> *Null (Alternative) Hypothesis VIb: The method of determining tax rate, endogenously or fixed, makes no difference to the percentage of earnings from the public good (lower proportion of earnings from endogenous tax rate).*

> *Null (Alternative) Hypothesis VIc: Financing of public goods by taxation or VAC makes no difference to the percentage of earnings from the public good (lower proportion of earnings from VAC financing).*

We set the parameters of the game so that in equilibrium roughly one half of points are earned from the public good and the other half from the consumption of the private good. The fourth row of panels in Figure 4 shows the percentage of points actually earned from the public good (with the remainder earned from consumption of private goods), and the bottom right panel of Figure 3 gives the respective development over time.

In T1 and T3, where the stock of the public good started at the optimum, the share of points earned from PGs remained close to the GE level of 50 percent throughout. In T2 and T4, by contrast, the stock of PG started at half of optimum, and the share of points earned from the public good was initially below one third. However, through high-enough taxes the stock of the public good grew over time and its contribution to total points earned rose to roughly 50 percent in the second half of the experiment in both T2 and T4.

Testing Hypotheses VIa and VIb on the averages of the last five periods we find that among the tax treatments T1 has a significantly higher ratio from the PG than T2 (rejecting null VIa) and also

than T3 (rejecting null VIb). The higher ratio in T1 is not so much due to a higher stock of PG, but due to lower production of the private good in this treatment especially compared to T3. We do, however, find no differences between T3 vs. T4 and T2 vs. T4.

As for Hypothesis VIc: The dotted line in Figure 3 shows the development in the VAC treatment. It illustrates nicely what happened in this treatment: as the stock of the public good drew down due to low contributions (see top left panel of Figure 3), the share of points earned from the public good fell to 22 percent in the last period. Differences between this treatment and the other four are significant (*p*-values of 0.046 in T0 vs. each of T3 and T4, respectively, and *p*=0.064 in T0 vs. T1 and T2).

3.7. Democratic Choice of Financing Regime

> *Null (Alternative) Hypothesis VII: Citizens have no preference between financing the public good by VAC or taxation when given a chance to decide by popular vote (prefer taxation).*

The alternative hypothesis is consistent with the findings of Gürerk et al. (2006) who introduced "voting by feet" dynamics in a traditional public goods setting. Two institutions ran simultaneously in their experiment. Both institutions had VACs, but punishment (sanctioning) was possible in only one of them. They found that contributions in the sanctioning institution converged towards 100%, and to 0% in the sanction-free environment. While initially some 70% of subjects chose to be in the sanction-free institutions, they gradually switched until 90% chose the sanctioning institution in the last few periods of the session, where high contributions and high earnings prevailed. With high contributions, sanctioning itself was rarely needed.

Real societies can, through vote or revolution, choose their institutions. We capture part of this process in Treatment T6 where subjects decided every five periods by majority vote whether to finance the public good through VACs or taxes. Subjects first experienced five periods with each of the two institutions T0 and T3. Then the initial endowments were reinitialized and one of the two institutions was chosen by a majority vote. The vote was repeated every five periods. We conducted four runs of this treatment for a total of 24 votes on choosing the institution.

In 23 out of 24 majority votes subjects chose taxes over VACs.[23] Most of voting decisions were not close, with on average 7.6 of 10 votes for taxation, and had a slight upward trend over time. Only

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[23] This is nicely in line with e.g. Robbett (2014), who showed that when allowed to vote on taxes subjects in an experiment converged towards their respective optimum level.

one decision (the third vote in run 3) favored VACs by 6:4 vote. One other vote in run 3 was a 5-5 tie (resolved randomly by computer in favor of taxes). We infer that with some experience and given the choice subjects choose a system with perfectly enforced taxes that makes them better off. The results for T6 are also given in Figures 2 (bottom panel), 3 and 4.

As seen in the top panels of Figure 4, the average stock of public goods in all as well as the last five periods of the T6 sessions as high or higher than any other treatment. The same is true of the volume of production in the T6 sessions (third row of panels in Figure 4). Efficiency of these sessions was among the highest, which is especially impressive given the high stock of public goods at the end of these sessions (the third row of panels). Finally the percentage of payoff from public goods was also comparatively high. In this treatment, where subjects arguably had more control over the environment they acted in (deciding on whether to implement taxes or VACs and then deciding on the tax rate) they seemed very committed to sustain an economy with high production levels and a high stock of PGs.

5. Discussion and Concluding Remarks

Public goods decisions are made in rich institutional settings. States evolved over centuries by enforcing weights and measures, commercial codes, accounting rules, law and order, and tax collection. In this study we take it as a given that the structure of government is able to serve these functions.

We reported on a novel laboratory experiment to explore the suitability of setting taxes through democratic voting to pay for public goods in a general equilibrium economy. We found that the four main treatments with finite horizon experimental economies sustained public goods between the finite- and infinite-horizon optima, and at 85 to 97 percent efficiency. Both efficiency and the production of private goods were higher when the rate of taxation was determined by vote instead of being fixed at the GE optimum. Production of private goods was also not harmed by taxation. In the two treatments with voting, taxes remained at an intermediate level, converging neither to zero nor to 100%. Irrespective of whether we started at 50% or 100% of the optimum, the stock of the public good converged to the same level between the finite- and infinite-horizon optima. This held also in robustness treatment T6 in which 23 out of 24 times subjects chose taxation over a voluntary contribution (VAC) regime by a majority vote. In all treatments except the one with VACs the ending stock of public goods exceeded the finite horizon optimum.

Our results suggest that the important social problem of financing public goods can be addressed, fairly and efficiently, by societies through taxes set by democratic vote. We also found evidence that production was higher the more control subjects had over the economy they acted in. Dependence on voluntary contributions among large groups may be too unreliable a basis for providing services essential to their productivity, social cohesion, even survival. In the experiment the level of VACs is significantly lower than the level of tax contribution in any given period. While we know voluntary contributions to public goods rapidly deteriorate in many designs, it is important to establish the result in this design, particularly given the concavity of public goods production function.

Still, voluntary contribution mechanisms have the inherent appeal of being decentralized, and thus insulated from tyranny. Taxation necessitates centralized power and a centralized enforcement mechanism, and has historical associations with oppression. Democratic government and taxation based on popular voting attempt to balance the consequences of centralization by fairness by broad acceptace. Our experimental results suggest that such a reasonable balance is achievable for financing of public goods and services through democratic mechanisms. We find that the majority of subjects voted 23 times out of 24 to favor a system with taxes over VACs.

Subjects cut the tax rates marginally as the sessions progressed towards the end when the remaining stock of public good became worthless. They made up for lower taxation by saving more of their private goods, so that tax proceeds remained about the same regardless of whether taxes were set exogenously at the ex ante optimal level or set endogenously by a vote; and increased efficiency by doing so. This powerful result raises interesting questions for future research; e.g., is it the tax level itself or exogenous tax policy that induces suboptimal dis-saving?

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Table 1: Design of the Experiment	Initial Level of Public Good	
Regimes for Public Good Provision	100 percent of Optimal	50 percent of Optimal
Voluntary anonymous contributions	**Treatment 0: 2 sessions***	
Taxation	Rate fixed at 21.5% (infinite-horizon opt.)	**Treatment 1: 4 sessions**	**Treatment 2: 4 sessions**
	Rate set by vote	**Treatment 3: 6 sessions**	**Treatment 4: 6 sessions**
	Rate fixed at finite-horizon optimum	**Treatment 5: 6 sessions**	
Vote on system**	**Treatment 6: 4 sessions**	

*Voluntary contributions specified in units of money in one session and in percent of wealth in the other.

** subjects decide by majority vote whether they implement a system with voluntary anonymous contributions or with taxes.

Table 2: Experimental Parameters and Design		
Parameters		
Number of Agents	n	10
Initial money endowment of agents	m	4,700
Initial pvt. good endowment of agents	a	217
Agents' pvt. Good production function	$f(k)$	$80*k^{0.25}$
Single period agent payoff	$u(x, G)$	$x + G/4$
Session agent payoff		Sum of period-wise payoffs
Initial government public good endow.	G	427 (T1, T3) or 213.5 (T2, T4)
Initial government money endowment	M	13,000
Government's public good prod. function	$F(k)$	$2*k^{0.5}$
Natural rate of discount	β	1
Depreciation rate (per period)	η	0.1
Terminal value of public good		0
Session termination		Announced: random btw. periods 25 and 30 Actual: always ended after vote in period 26
Equilibrium Outcomes		
Price of private goods	p	27.67
Per capita production of pvt. good		217
Per capita purchase of pvt. good		170
Per capita consumption of pvt. good		116 (68.27% of 170)
Per capita pvt. Good into production		54 (31.73% of 170)
Production of public good		42.7

Table 3: Summary of Hypotheses and Tests

Hypothesis	Variable	Null Alternative	T1 vs. T0	T2 vs. T0	T3 vs. T0	T4 vs. T0	T1 vs. T2	T3 vs. T4	T1 vs. T3	T2 vs. T4	T3	T4	T6
Ia	Tax rates	ETR = Equil. ETR < Equil.									Reject. p<0.01	Reject. p<0.01	
Ib	Tax rates	ETR = 0 ETR > 0									Reject. p<0.01	Reject. p<0.01	
II	Provision for PG	ETR = VAC ETR > VAC	Reject. p<0.01	Reject. p<0.01	Reject. p<0.01	Reject. p<0.01							
IIIa	Final Level of PG	HIE =LIE					Not rej. p=0.25	Not rej. p=0.43					
IIIb	Final Level of PG	ETR = FTR							Not rej. p=0.29	Not rej. p=0.83			
IIIc	Final Level of PG	ETR = VAC ETR > VAC	Reject. p=0.06	Reject. p=0.06	Reject. p=0.05	Reject. p=0.05							
IVa	Efficiency	HIE =LIE HIE > LIE					Not rej. p=1.00	Reject. p=0.02					
IVb	Efficiency	ETR = FTR ETR < FTR							Reject. p=0.01	Reject. p=0.09			
IVc	Efficiency	Tax = VAC Tax > VAC	Not rej. p=0.64		Reject. p=0.05								
Va	Pvt. Good Production	ETR = FTR ETR <> FTR							Reject. p<0.01	Reject. p=0.02			
Vb	Pvt. Good Production	Tax = VAC Tax <> VAC	Not rej. p=0.36	Reject. p=0.06	Reject. p=0.05	Not rej. p=0.18							
VIa	% Earn from PG	HIE =LIE HIE > LIE					Reject. p=0.02	Not rej. p=0.87					
VIb	% Earn from PG	ETR = FTR ETR < FTR							Reject. p=0.01	Not rej. p=0.39			
VIc	% Earn from PG	Tax = VAC Tax > VAC	Reject. p=0.06	Reject. p=0.06	Reject. p=0.05	Reject. p=0.05							
VII	VAC or Tax by vote	Tax = VAC Tax > VAC											Rej. at p<0.01

ETR = Endogenously determined tax rate; VAC =Voluntary anonymous contributions; HIE/LIE = High/Low initial endowment of public good; FTR = fixed (at optimum level) tax rate; Var Level of PG = Variation of final level of public good across sessions of the same treatment.

Figure 1: Time line of a period in the experiment

a   b   c   d

Start of Period t Start of Period $t+1$
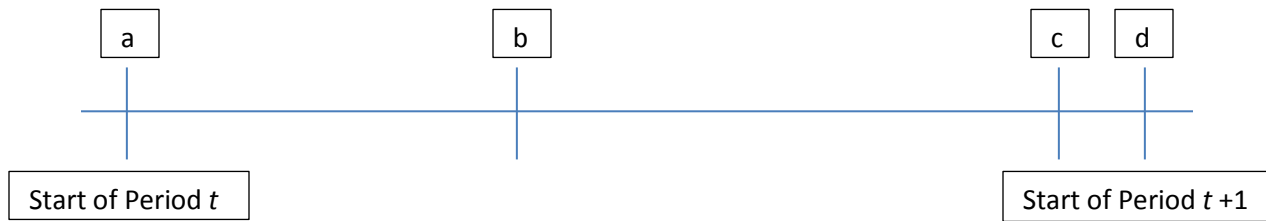
a. Endowment:

– Government has money M_t and public good G_t carried over from prior period ($M_t = 13{,}000$ and $G_t = 427$ or 213.5 for $t = 1$)

– Each subject i has money m_{it} and private good v_{it} ($m_{it} = 4{,}700$ and $v_{it} = 217$ for $t = 1$)

b. Call Auction:

– Government bids all its money balance M_t

– Each subject i bids all its money balance m_{it} and offers all its units of private good v_{it}

– Price of private good is calculated: $p_t = [M_t + \text{sum}(m_{it})]/\text{sum}(v_{it})$

– Each subject i receives m_{it}/p_t units of the private good, and $(1\text{-}T)*(v_{it})*(p_t)$ units of money after paying taxes at rate T on money received in auction

– Government receives M_t/p_t units of the private good, and $T*(p_t)*\text{sum}(v_{it})$ units of tax money

c. Investment:

– Each subject chooses to invest a part or whole of its post-auction private good, x_{it} /Min [0, m_{it}/p_t], into production of private good for the following period and consumes the balance to earn points). Total consumption (points) for a subject in the period is equal to these uninvested units of the private good, $(m_{it})/(p_t)\text{-}x_{it}$, plus one-fourth of the quantity of the public good held by the government, G_t

d. Production:

– Production for subjects takes place. This production endowment of the private good is carried to the following period $t+1$: $v_{i,t+1} = 80*(x_{it})^{\wedge}(1/4)$.

– The public good stock depreciates by 10%.

– The Government invests all units of its private good into production of the public good. If G_t is the initial stock of public good in period t and $x_t = M_t/(p_t)$ is it's quantity of the private good purchased by the Government in the period t auction, then the initial stock in $t+1$ is $G_{t+1} = 0.9*(G_t) + 2*(x_t)^{\wedge}(1/2)$.
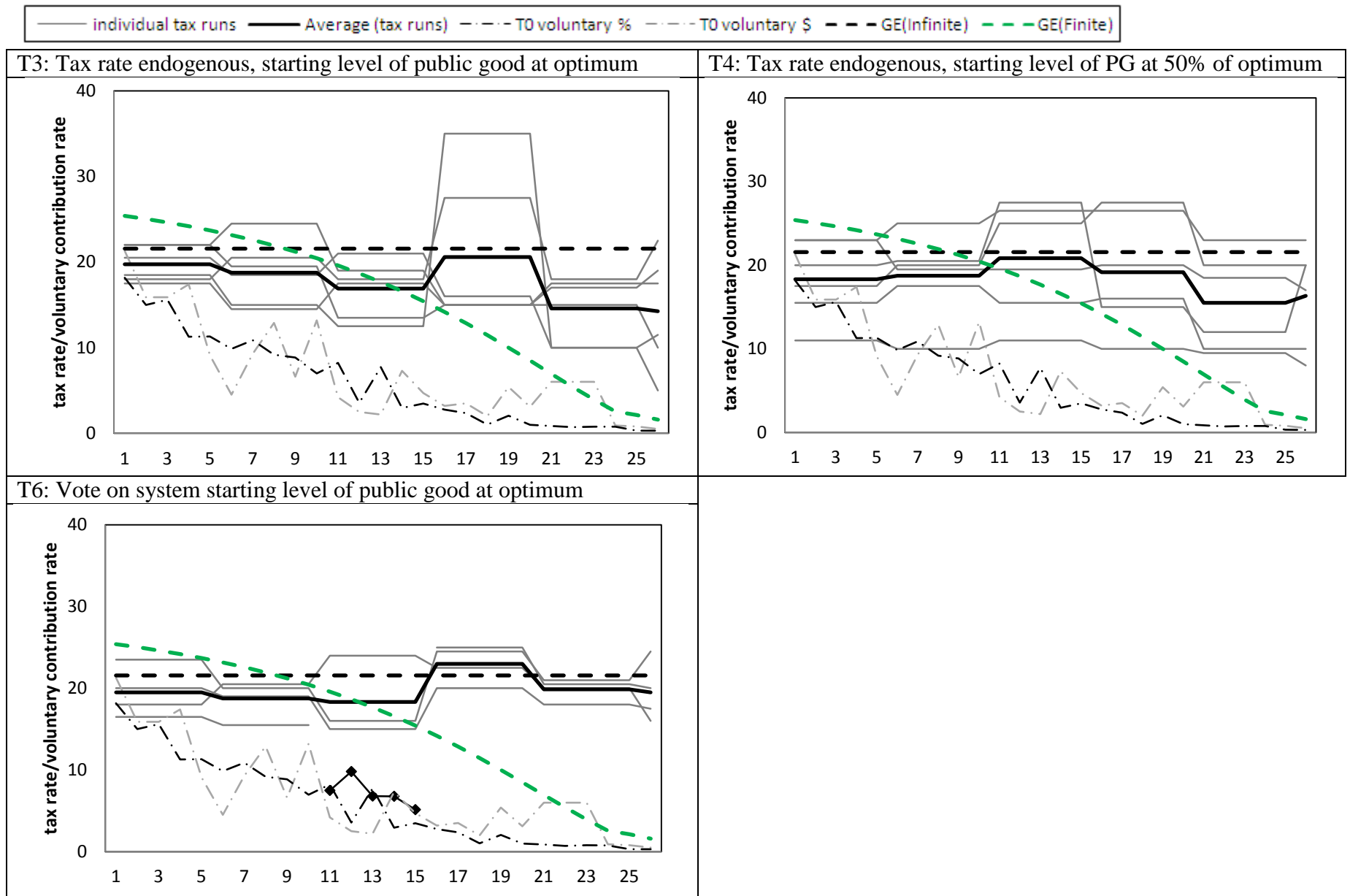
Figure 2: Evolution of tax rates over time in the three treatments with endogenous choice of tax rates: T3 on top left, T4 on top right, and T6 (vote on which system to implement) on the bottom left.
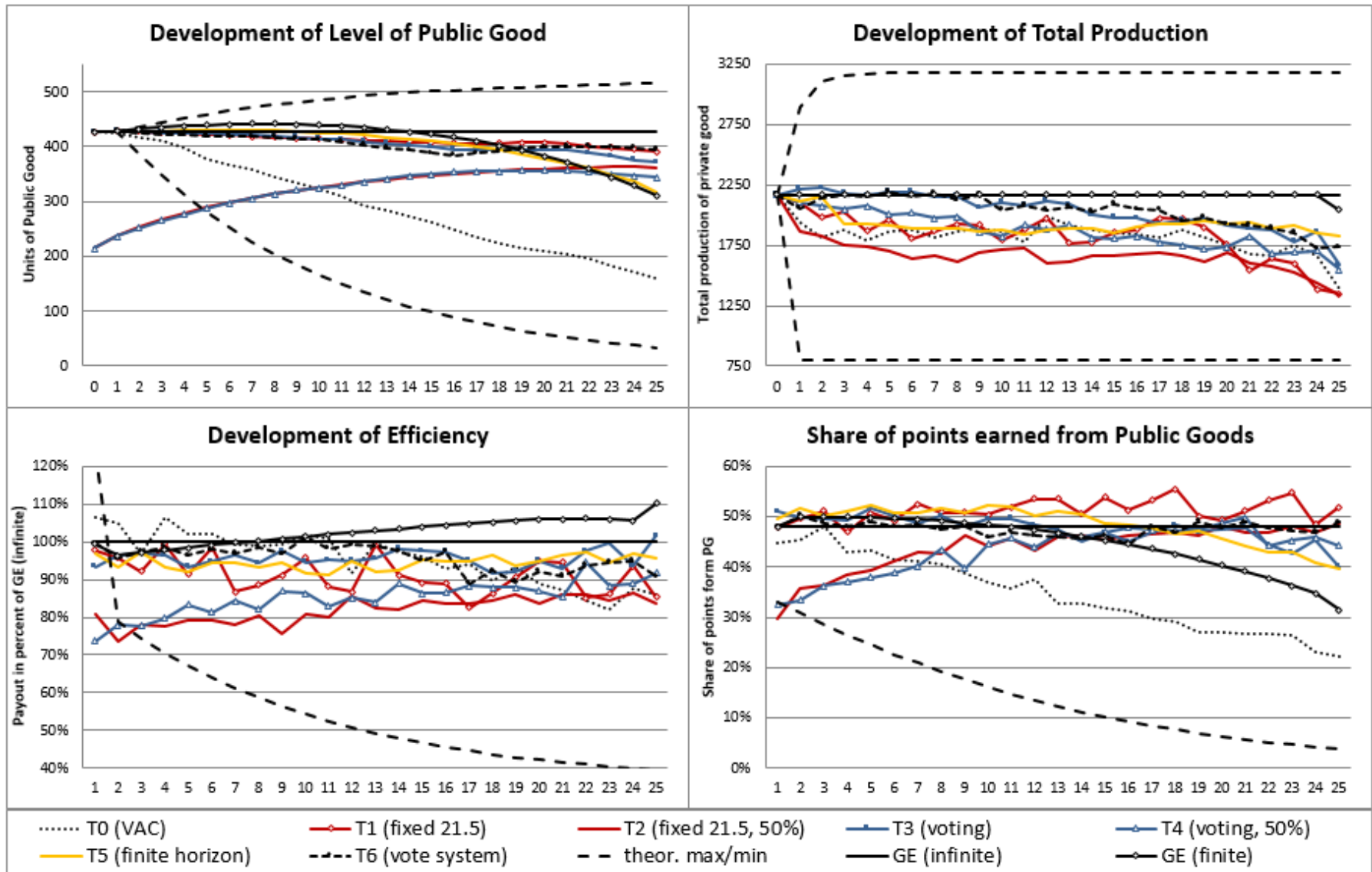
Figure 3: Development over 25 periods of the stock of public goods (top left panel), total production (top right panel), efficiency (bottom left panel), and share of points earned from the public good (bottom right panel). Theoretical maxima are not visible in the two bottom panels (where it is at 100% and overlaps with general equilibrium (infinite) on the left and is at 100% and not shown on the right).
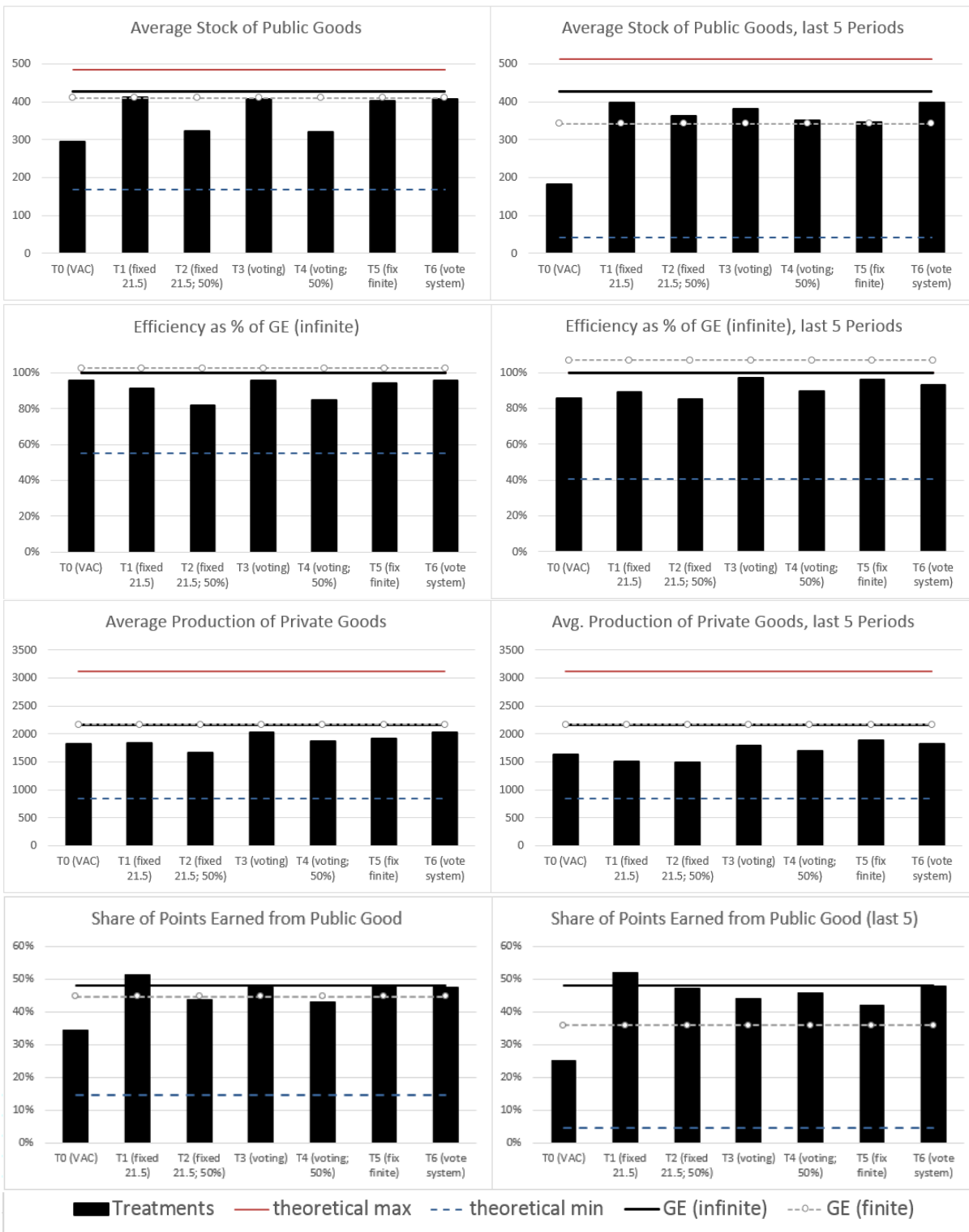
Figure 4: Key results for stock of public goods (top panel), efficiency (second panel), total production (third panel), and share of points earned from the public good (bottom panel). Averages of all periods shown on the left and averages for the last five periods on the right.

APPENDIX A – explanation of online material

As supporting material for this paper we provide two MS EXCEL worksheets, one for infinite horizon, one for a finite horizon of 30 periods. In both worksheets all relevant input variables can be varied in cells E2 to E17. The respective notation can be found in cells A2 to A17. Especially noteworthy in the infinite horizon setting are the tax rate (E7) and the consumption rate (E12) as these are the two variables for which we optimized by use of the solver function of MS EXCEL.

In rows 19 to 24 (22 to 28 in the finite setting) the sell-all market is modeled, with period 1 in column E, and subsequent periods to the right, up to period 20 in the infinite setting and period 30 in the finite setting. Right below, are the productions of private and public goods, again from period 1 (column E) to period 20 (30 in the finite setting).

Several graphs from Columns H to AD illustrate the results and their sensitivity to variations in the input variables. Figures 7 and 8 give screenshots of part of the respective excel sheets, which would be continued in further rows down and further columns to the right.

Figure A1: MS EXCEL screenshot for model with infinite horizon. The Graph in the top rows of columns I to N shows total utility as a function of consumption rate (E12) and tax rate (E7).
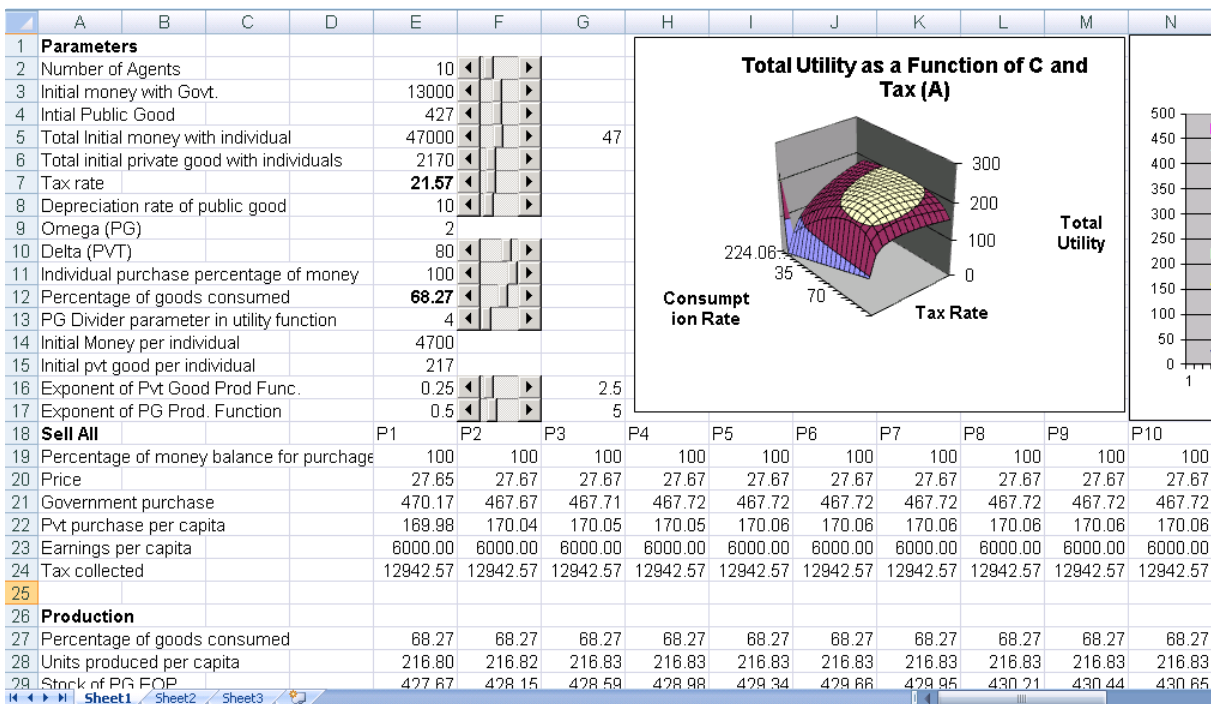
Figure A2: MS EXCEL screenshot for model with finite horizon of 30 periods. Here the tax rate (E7) and consumption rate (E12) are no longer fixed for several periods, but instead change from period to period. The respective values are displayed in rows 19 and 20.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	**Parameters**														
2	Number of Agents				10										
3	Initial money with Govt.				13000										
4	Intial Public Good				427										
5	Total Initial money with individual				47000		47								
6	Total initial private good with individuals				2170										
7	Tax rate				**see row 22**										
8	Depreciation rate of public good				10										
9	Omega (PG)				2										
10	Delta (PVT)				80										
11	Individual purchase percentage of money				100										
12	Percentage of goods consumed				**see row 23**										
13	PG Divider parameter in utility function				4										
14	Initial Money per individual				4700										
15	Initial pvt good per individual				217										
16	Exponent of Pvt Good Prod Func.				0.25		2.5								
17	Exponent of PG Prod. Function				0.5		5								
18															
19	Tax rate				25.35	25.02	24.62	24.18	23.69	23.15	22.56	21.91	21.20	20.43	19.58
20	percentage of goods consumed				68.05	66.94	67.14	67.36	67.60	67.86	68.14	68.43	68.75	69.09	69.45
21															
22	**Sell All**				P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11
23	Percentage of money balance for purchage				100	100	100	100	100	100	100	100	100	100	100
24	Price				27.65	27.72	27.83	27.95	28.09	28.24	28.41	28.59	28.80	29.02	29.26
25	Government purchase				470.17	548.72	541.12	532.28	522.48	511.67	499.80	486.78	472.52	456.93	439.91
26	Pvt purchase per capita				169.98	162.30	163.04	163.94	164.91	165.99	167.18	168.48	169.91	171.47	173.17
27	Earnings per capita				6000.00	6020.00	6044.05	6070.72	6100.37	6133.15	6169.36	6209.30	6253.34	6301.82	6355.12
28	Tax collected				15210.73	15060.78	14879.47	14677.26	14451.02	14199.10	13918.69	13606.78	13259.78	12874.01	12445.35
29															

Time Series of Variables

Legend:
- Price
- Government
- Pvt purchase
- Percentage consumed
- Units produced capita
- Stock of PG
- Utility per cap

Sheet1 / Sheet2 / Sheet3

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APPENDIX B: Instructions

Dear participant: Welcome to the experiment. Please do not talk to any other subject for the duration of the experiment.

You are one of ten subjects populating a small economy with money and two kinds of goods: one private and one public good. As subjects, you will produce, sell, buy, and consume the private good. The government (played by the experimenter) will tax the income of subjects (from sale of the private good) and use the proceeds to buy some of the private good, to be used to produce the public good. The tax rate will be either fixed, or determined by the vote of the ten subjects once every five periods. Your earnings for each period depend on the quantity of private good you consume, and the quantity of the public good provided by the government for benefit of all in that period.

Money and Goods

There is money and two kinds of goods in the economy:

- A private good produced, sold, bought and consumed by the participating subjects; some the private good is also bought by the government and used to produce the public good.
- The public good (e.g., a public facility) which depreciates at the rate of 10 percent per round. The government uses tax collected from subjects to replenish the depreciating stock of public good.

In round 1 each subject starts with 4,700 units of money and 217 units of the private good. The government starts with 13,000 units of money and 427 (*213.5 in half of the runs*) units of the public good.

At the beginning of each round, all private good produced in (and carried over from) the preceding round) is sold in a market. Thus, the initial private good endowment of 217 units in the hands of each subject (for a total of 2,170) is sold at the start of round 1.

Money serves only a means of exchange in this economy, but it has no role in savings, etc. An amount of money is given to you at the beginning of the session, and any balance left over at the end of the session has no value to you. Each round all money you have (either initial endowment or earned from sale of goods the round before) is spent for the purchase of goods at the start of each round. No borrowing is possible.

At the start of a period all money held by the government and individuals is tendered to buy units of the private good. In the first period 2,170 units are sold for a total of 60,000 units of money.

Total agent and government bids in money = 60,000/2,170 (total number of units of private good) = 27.65. These numbers will change in subsequent rounds.

Each individual buys 170 units and earns 217*27.65=6,000 units of money. Your first decision is how many of these 170 units you invest into production for the next period, with the remainder being consumed this period. Your money income (6,000 in the first period) is taxed by the government at a rate set by all subjects through a vote (see details below).

On the left side of the Screen 2 you learn the total money bid for private good, the resulting price, the units bought by the government, and government's tax revenue (all of which is spent to buy private goods in the following round). On the right side of Screen 2 you see how many units you bought, your spending, income, tax, and the initial and final money balances (the latter to be carried over to the following round).

Screen 2

Out of the units of private good you bought, you have to decide on how many you wish to consume, and how many you wish to invest to produce private goods to be sold during the next round. The following equation and chart show the relationship between the units you invest and the units produced:

UNITS OF THE PRIVATE GOOD PRODUCED = $80*(\text{UNITS INVESTED})^{0.25}$.

Note, for example, that investing 1 unit produces 80 units; investing 40 units produces 201.19 units.

Public Good

The government starts with a stock of 427 units of the public good. This stock depreciates by 10% each round, like, for example, roads deteriorate. To maintain or upgrade the public good the government taxes the subjects' income (from sales of goods) at the selected rate. All tax receipts are used to buy the private good and all private goods are used to produce new units of the public good according to the following function:

UNITS OF THE PUBLIC GOOD PRODUCED = $2*$(UNITS OF PRIVATE GOOD INVESTED)$^{0.5}$.

Taxes

All individual income (proceeds from sale of private good) will be taxed at a flat tax rate (which is either fixed by the experimenter in advance, or is set by the vote of ten subjects). In the latter case, every five rounds (i.e., at the beginnings of rounds 1, 6, 11, 16, etc.) each subject is asked to submit his/her suggested percent rate of taxation to be applicable to all ten subjects. You are free to suggest any integer number between zero (no tax) and 100 (everything taken by the government) as the percent tax rate. The computer collects the suggested tax rates from the ten subjects, sorts them from highest to lowest, and sets the median (average of the 5th and the 6th suggested rates) as the tax rate for all subjects. The selected tax rate is announced, and it remains in effect for five rounds until the next tax rate is determined though another vote. (*In half of the treatments the tax rate was fixed at 21.5. percent and no vote was carried out*)

Points earned

The points you earn in each round are calculated as:

POINTS = CONSUMPTION OF PRIVATE GOOD + PUBLIC GOOD/4.

For example, if you consume 60 units of private good and the government provides 200 units of public good, you earn 60 + 200/4 = 110 points in that period. Both higher private good consumption as well as higher stock of the public good increase your earnings. Chart 1 and Table 1 show the number of points resulting from various combinations of private good consumption and public good provision by government.

(Insert Chart 1)

History screen:

After all subjects have entered their consumption/investment decisions, computer carries out all the calculations, and a history screen provides a round-by-round overview of the results (the accounting of public goods on the left, your consumption and production of goods in the middle, the points you earn during the round on the right, and the summary of the round at the bottom.

History Screen

Period		Time remaining [sec]: 9
1		

Units of public goods at beginning of period	375.0		Units of private goods you bought	112.5		
Depreciation rate	0.10		Units you consume	87.5		
Units of public goods after depreciation	337.5		Units you invest for production	25.0		
					Points you earn this period	
Units of private good the government bought	375		Units of private good you produce	150.0		
Units of public good produced	37.5		(will be sold at the start of next period)		Points earned:	51.6
New level of public good	375.0		Total production by all ten subjects	1500.0		

Period	goods sold	price	money end	tax rate (%)	goods bought	consumed	in production	goods produced	public goods	points this period	Total points
1	150.0	40.00	4500	25.0	112.5	87.5	25.0	150.0	375.0	52	52

Final payment:

There is 1/6 chance that the experiment will last for 25, 26, 27, 28, 29, or 30 rounds. The actual number of rounds in the session will be determined randomly before we start, but will not be announced to you until the session ends.

The points earned during all rounds are added up (column "Total points" in the History Screen). Your take-home payment in euro is TOTAL POINTS / 200. For example, if the experiment ends in round 28 and you earned a total of 3,000 points during these 28 rounds, your take-home payment is 3,000/200 = 15 Euros.

Appendix C: Supplemental analysis for Referees and to be available online

1. Figures C1-C5
2. Regression results Table C1
3. A PDF file for editors (the source Karatzas et al. 2006 referenced in Section 2 of the paper and included in the list of references).

Figure C1: Stock of Public Good in Economies Grouped by Four Types of Sessions
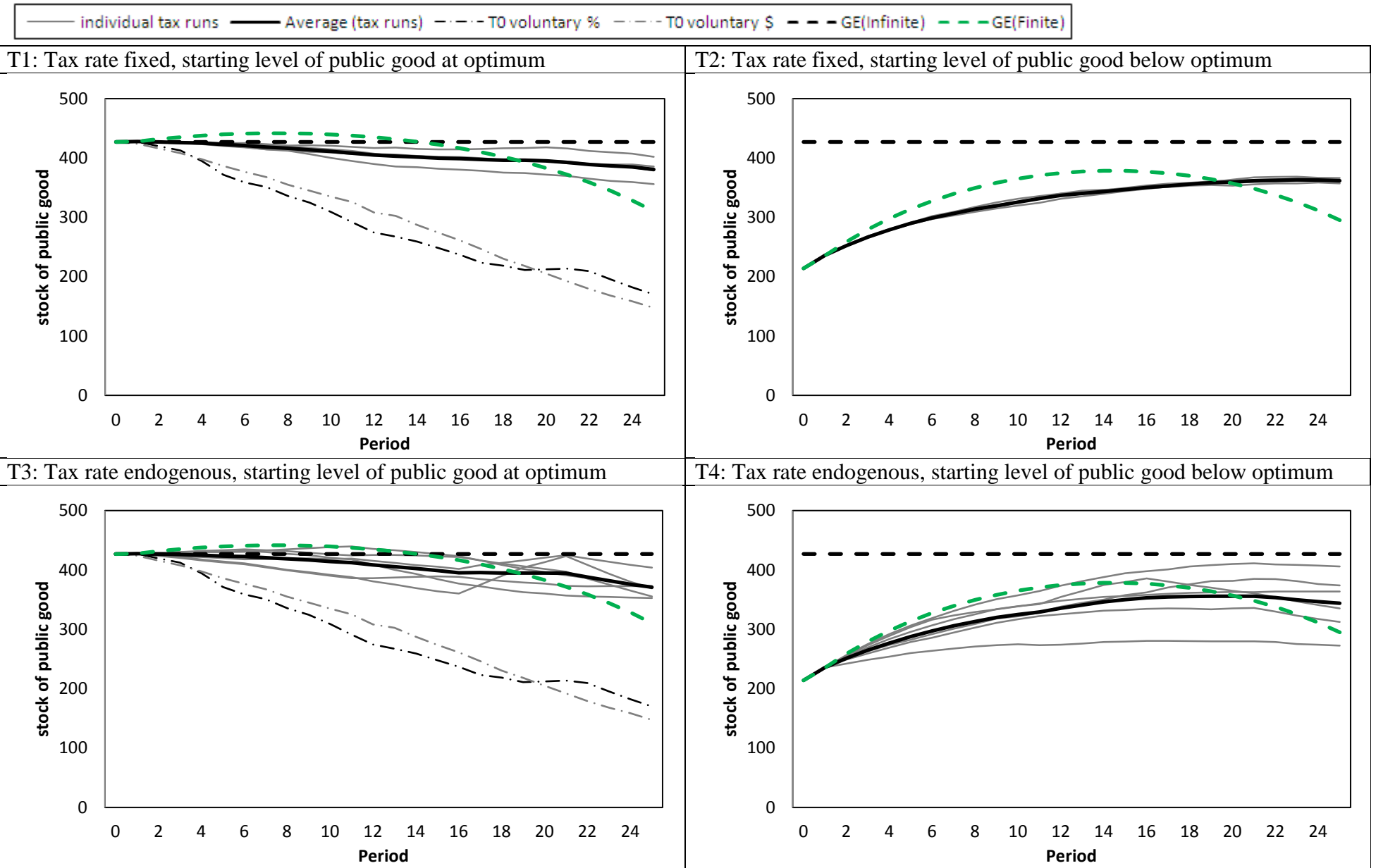
Figure C2: Efficiency of the Economies with GE(Infinite) being the Benchmark
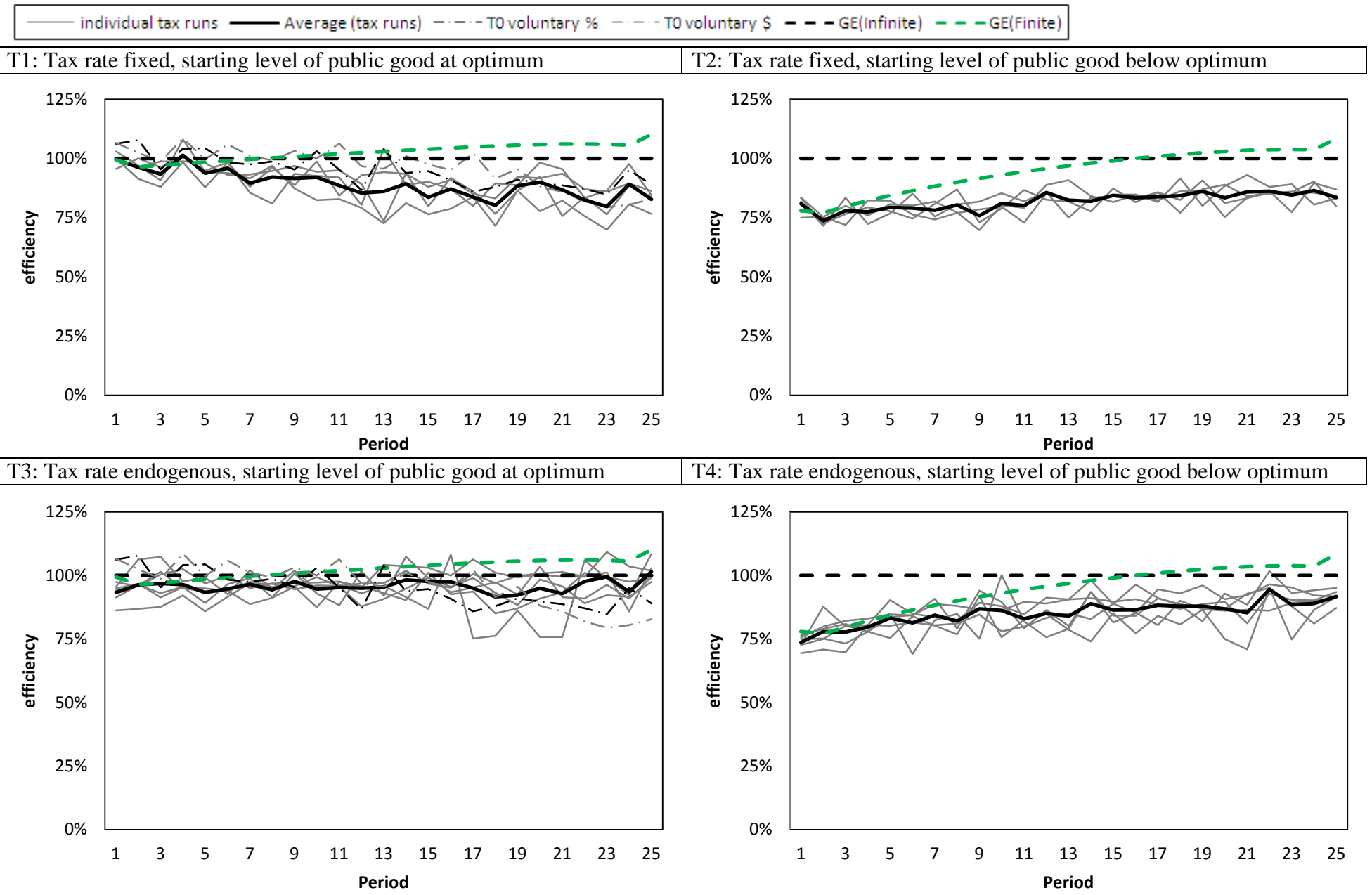
Figure C3: Total Production of Private Goods in the Economies

Figure C4: Percentage of Utility Earned from the Public Good (rather than from Consumption of Private Goods)

Legend: individual tax runs — Average (tax runs) — · — T0 voluntary % — — T0 voluntary $ — — GE(Infinite) — — GE(Finite)
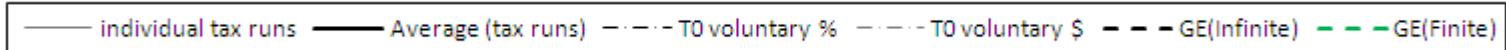
T1: Tax rate fixed, starting level of public good at optimum
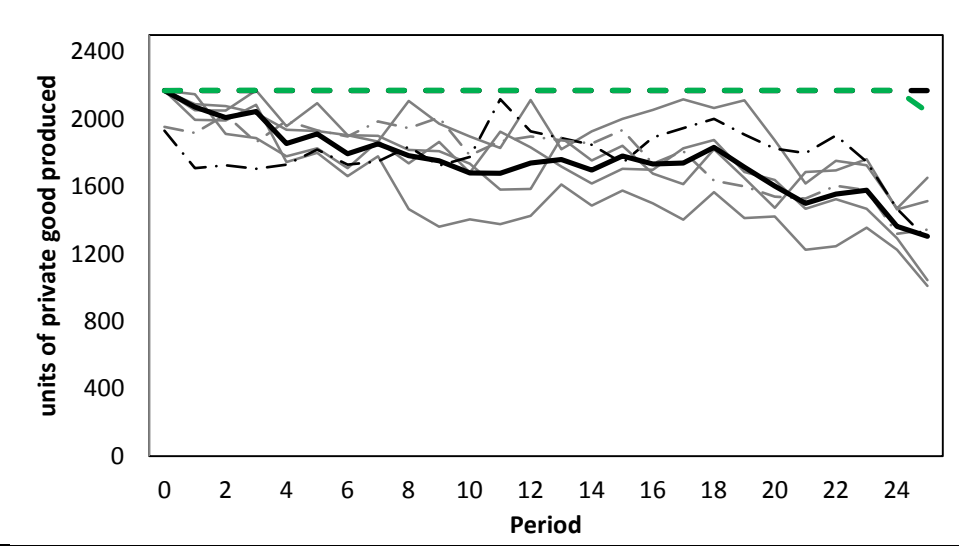
T2: Tax rate fixed, starting level of public good below optimum
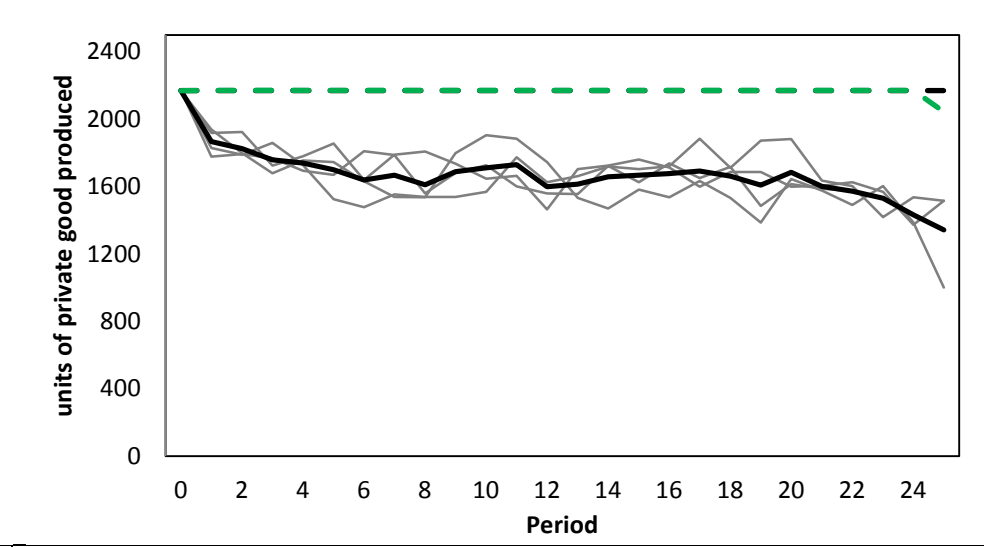
T3: Tax rate endogenous, starting level of public good at optimum
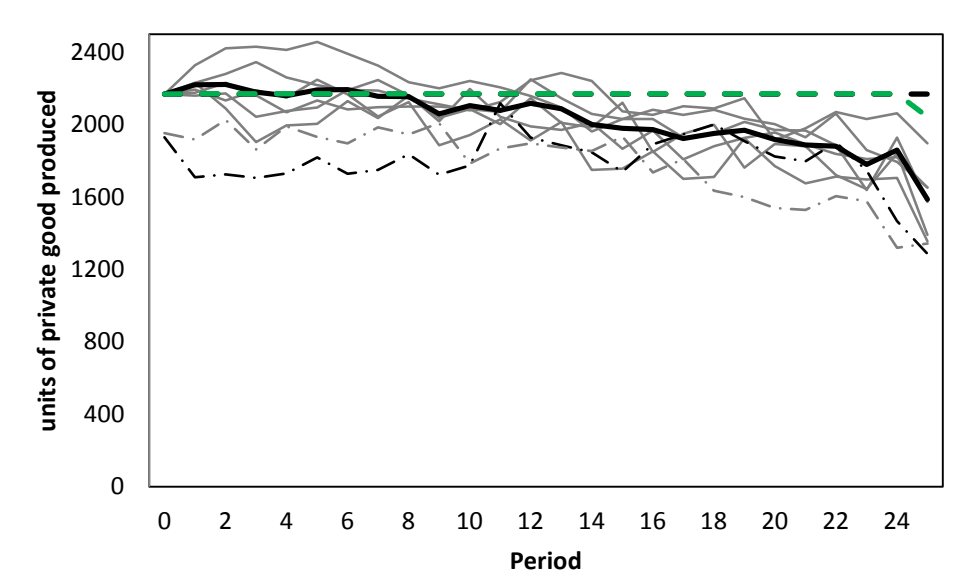
T4: Tax rate endogenous, starting level of public good below optimum
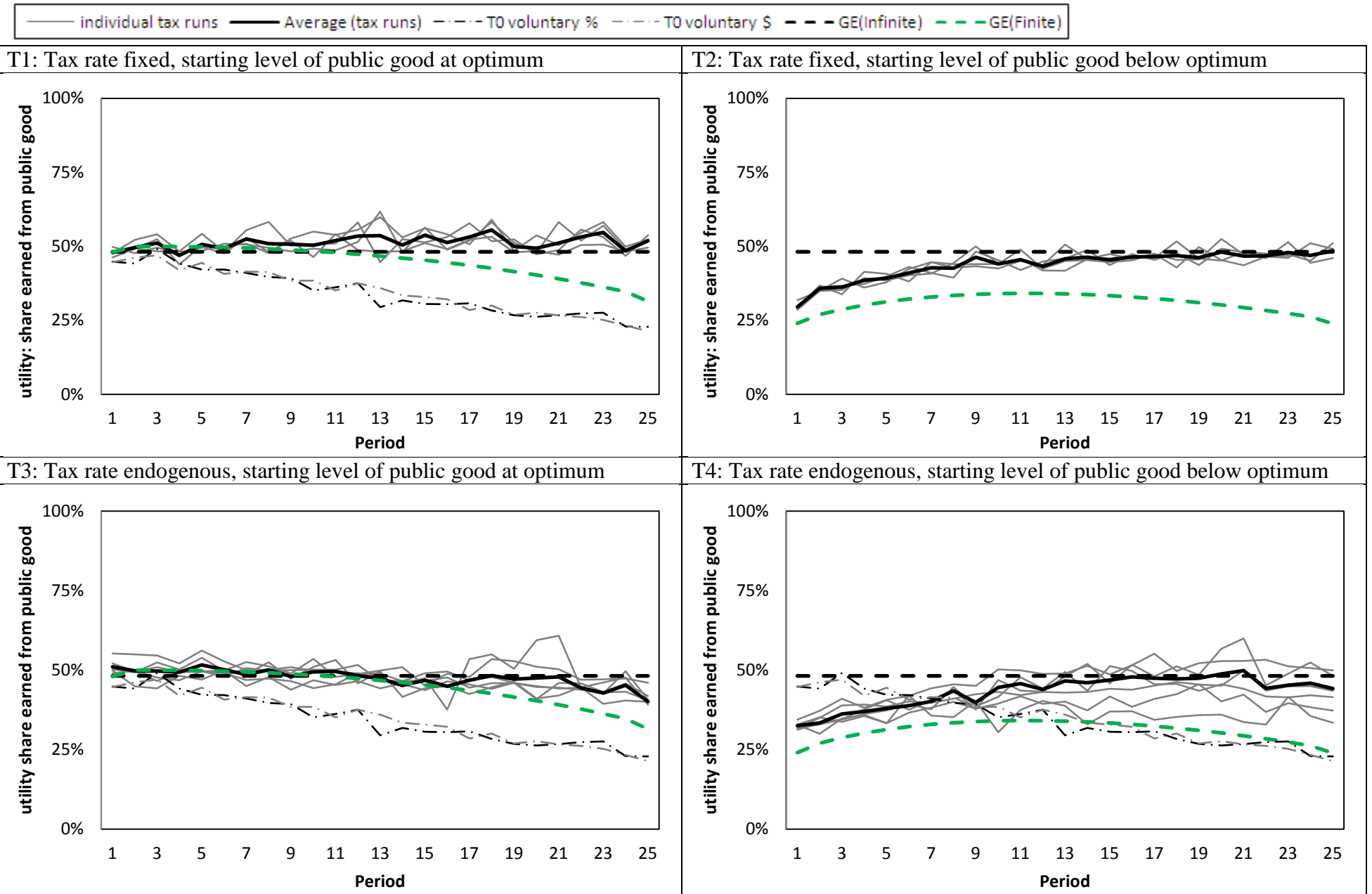
Figure C5: Data for T6 where subjects voted on the system to be implemented. Notation follows that of Figures 2 to 6.
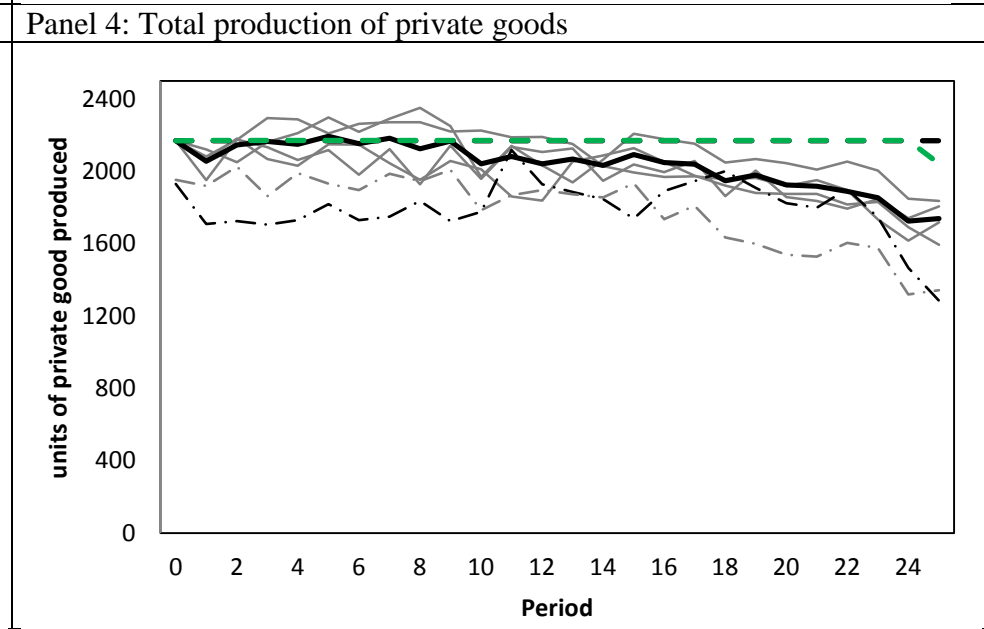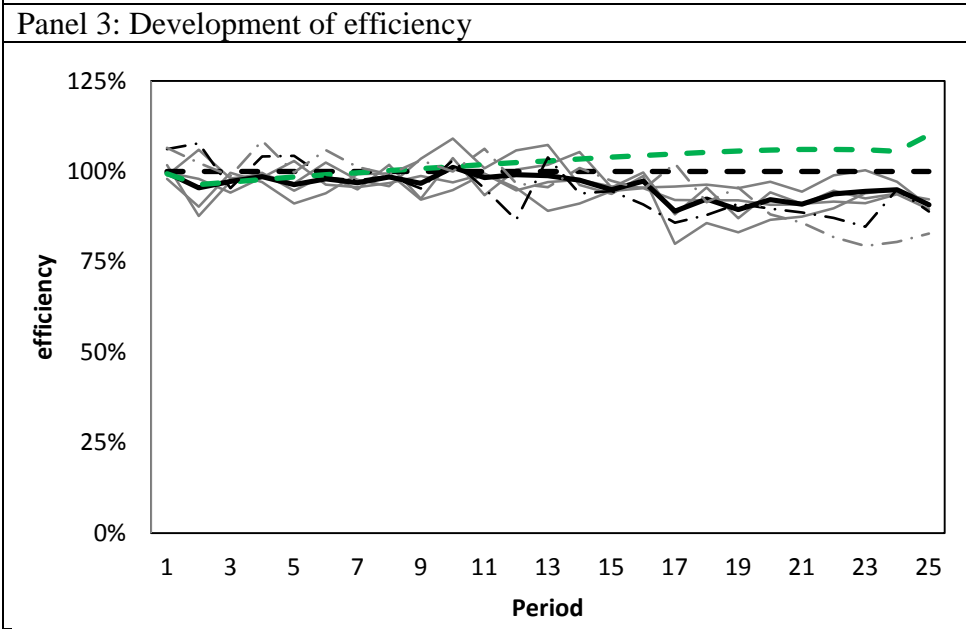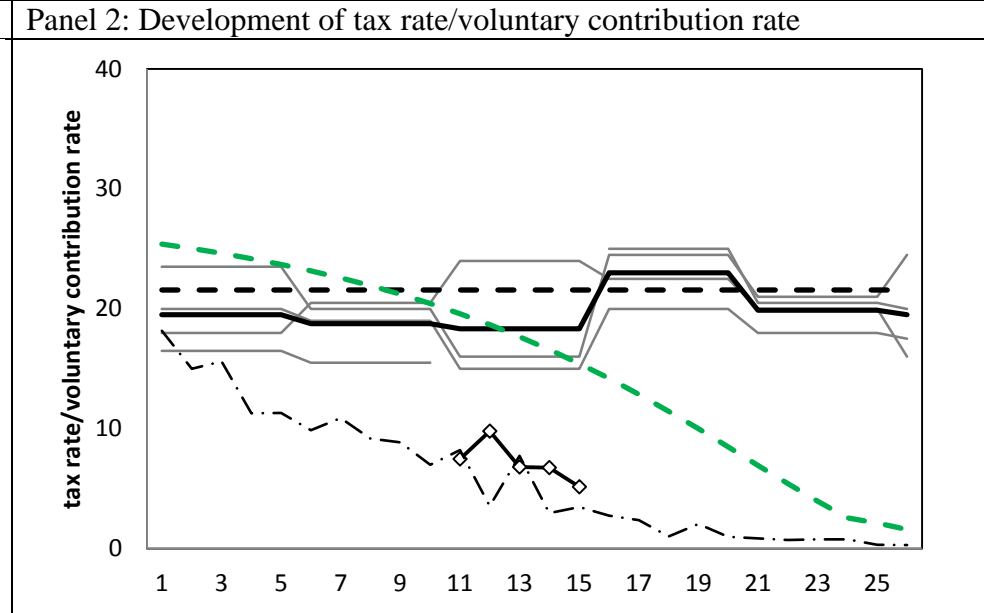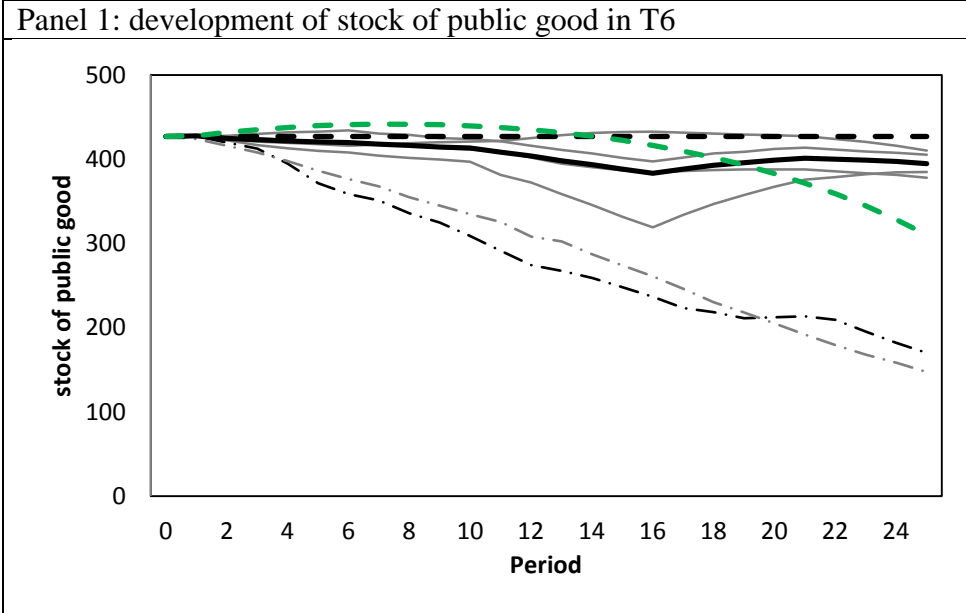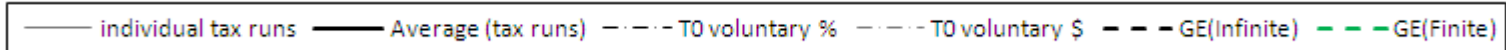
Appendix C: Regression Analysis of Data (Not for publication)

In the following four sets of GLS-regressions we use six models to examine each of the four dependent variables (stock of public goods, total production of private goods, efficiency, and share of points earned from public goods). The six models are (1) using one dummy per treatment; (2) adding 'Period' (for period sequence number) to capture time trends; and (3) adding interaction terms between treatment and period. In Models 1, 2 and 3 this is applied to data from all periods; in Models 4, 5 and 6 the same models are estimated with data from only the last five periods of each session. In all regressions, Treatment 1 (tax fixed at GE optimum of 21.5% and stock of public goods starting at optimum of 427) is the benchmark.

The results are presented in the same order they are presented throughout the paper and e.g., in Figures 3 and 4. The first dependent variable is the stock of public goods. Model 1, confirming results from or pairwise tests and from Figures 3 and 4, shows that overall the stock of PG was significantly lower (as compared to T1) under T0 (VAC), in T2 and T4 (where it started at only half the initial levels as in T1), and slightly lower in T5. Model 2 shows that overall there is no time trend in PG, as the increase of PGs in T2 and T4 counters the respective drop in most other treatments. Model 3 confirms this by delivering significantly positive interaction terms for T2 and T4 with period, but significantly negative values for T0 and T5 with period. Models 4, 5 and 6, using only data from the last five periods, confirm these results, mostly with the same levels of significance; sometimes with more extreme differences (especially for T0 and T5 where the stock of PG drops most), and sometimes with smaller differences.

Table C1: Random-effects GLS regression with clustered standard errors at the run level

Dependent variable: Public goods; T1 being the benchmark

VARIABLES	(1) Model 1	(2) Model 2	(3) Model 3	(4) Model 4	(5) Model 5	(6) Model 6
T0 (VAC)	-121.4***	-121.4***	4.314	-212.9***	-212.9***	-11.11
	(5.457)	(5.460)	(8.888)	(12.68)	(12.72)	(33.12)
T2 (fixed 21.5, 50%)	-76.33***	-76.33***	-164.0***	-21.52**	-21.52**	-82.92***
	(4.943)	(4.946)	(1.728)	(9.061)	(9.091)	(14.58)
T3 (voting)	-0.714	-0.714	4.031	-7.303	-7.303	55.27
	(6.546)	(6.550)	(6.442)	(11.39)	(11.43)	(50.19)
T4 (voting, 50%)	-79.79***	-79.79***	-160.2***	-36.67*	-36.67*	-37.05
	(12.99)	(13.00)	(4.864)	(20.11)	(20.18)	(27.91)
T5 (finite horizon)	-14.92***	-14.92***	33.56***	-74.05***	-74.05***	233.6***
	(5.749)	(5.753)	(2.161)	(9.392)	(9.422)	(17.54)
T6 (vote system)	0.580	0.580	-6.744	13.14	13.14	-18.26
	(9.615)	(9.621)	(4.451)	(11.79)	(11.82)	(36.91)
Period		-1.303	-1.979***		-6.212***	-3.281***
		(0.814)	(0.419)		(1.077)	(0.560)
T0 (VAC) & Period			-9.312***			-8.408***
			(0.632)			(1.023)
T2 (fixed 21.5, 50%) & Period			6.495***			2.558***
			(0.434)			(0.583)
T3 (voting) & Period			-0.351			-2.607
			(0.614)			(1.998)

VARIABLES		
T4 (voting, 50%) & Period	5.959***	0.0160
	(0.832)	(1.047)
T5 (finite horizon) & Period	-3.591***	-12.82***
	(0.438)	(0.636)
T6 (vote system) & Period	0.543	1.308
	(0.620)	(1.313)

	(1)	(2)	(3)	(4)	(5)	(6)
Constant	404.6***	422.2***	431.3***	383.5***	532.6***	462.2***
	(4.881)	(11.74)	(0.866)	(8.807)	(27.18)	(14.27)
Observations	832	832	832	160	160	160
R^2	0.497	0.525	0.851	0.820	0.842	0.858
Number of runs	32	32	32	32	32	32
Periods	all	all	all	last 5	last 5	last 5

Robust standard errors in parentheses
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

In the same six regressions for the second dependent variable, Model 1 reports significantly higher production in T3, T4, T5 and T6 than in T1, with the difference being highly significant for T3 and T6. These two treatments are comparable with T1, as they all start with the same level of PG. The possibility to decide the tax rate endogenously (always in T3 and chosen in 23 out of 24 cases in T6) obviously motivated subjects to invest more in production, probably because they felt to have more control over the whole economy they are acting in. All the results hold and mostly become even stronger in Models 2 to 6. All models with a 'Period' variable also clearly show that production usually decreased over time.

Tabel C2: Random-effects GLS regression with clustered standard errors at the run level
Dependent variable: total production of private goods; T1 is benchmark

VARIABLES	(1) Model 1	(2) Model 2	(3) Model 3	(4) Model 4	(5) Model 5	(6) Model 6
T0 (VAC)	74.83	74.83	-60.99	137.4	137.4	798.5
	(73.68)	(73.72)	(99.93)	(108.2)	(108.5)	(1,104)
T2 (fixed 21.5, 50%)	-78.17	-78.17	-216.1***	21.44	21.44	-239.3
	(75.25)	(75.30)	(52.36)	(99.05)	(99.37)	(984.4)
T3 (voting)	291.9***	291.9***	268.0***	317.0***	317.0***	358.7
	(80.70)	(80.75)	(58.34)	(104.4)	(104.8)	(801.8)
T4 (voting, 50%)	133.6*	133.6*	81.99*	216.3**	216.3**	-737.0
	(79.37)	(79.42)	(48.83)	(98.41)	(98.73)	(777.9)
T5 (finite horizon)	158.2*	158.2*	-91.44	351.5***	351.5***	258.0
	(83.82)	(83.87)	(78.62)	(96.98)	(97.30)	(863.2)
T6 (vote system)	297.5***	297.5***	218.9***	342.8***	342.8***	95.94
	(78.59)	(78.64)	(58.15)	(99.01)	(99.33)	(790.4)
Period		-16.13***	-23.28***		-69.59***	-78.37**
		(1.573)	(2.703)		(7.141)	(32.57)
T0 (VAC) & Period			10.06			-27.55
			(7.577)			(44.40)
T2 (fixed 21.5, 50%) & Period			10.22**			10.86
			(4.149)			(42.31)
T3 (voting) & Period			1.771			-1.736
			(3.928)			(33.63)

T4 (voting, 50%) & Period			3.825			39.72
			(3.008)			(32.68)
T5 (finite horizon) & Period			18.49***			3.899
			(3.938)			(35.85)
T6 (vote system) & Period			5.816			10.29
			(3.564)			(33.07)
Constant	1,717***	1,935***	2,032***	1,421***	3,091***	3,302***
	(73.57)	(72.30)	(40.24)	(89.69)	(187.8)	(775.3)
Observations	832	832	832	160	160	160
R^2	0.300	0.571	0.613	0.370	0.566	0.578
Number of runs	32	32	32	32	32	32
Periods	all	all	all	last 5	last 5	last 5

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

For the third dependent variable, efficiency, i.e., the points earned as a percentage of the GE-level, T0 has higher efficiency than T1 in Models 1 to 3, using all data, but not in Models 4 to 6 using only the last five periods. This is attributable to the fact that in T0 there is high private consumption (leading to many points earned), but almost no investment in the public good (leading to lower earnings in later periods). In Models 1 to 3 efficiency is lower than in T1 in T2 and T4, which is clearly attributable to the lower initial level of PGs in these two treatments. In T3, T5, and T6, however, efficiency is higher than in T1, both for all data (Models 1 and 2) as well as for the last five periods (Models 4 and 5)

Table C3: Random-effects GLS regression with clustered standard errors at the run level

Dependent variable: Efficiency; T1 being the benchmark

	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
T0 (VAC)	0.0598***	0.0598***	0.104***	0.000529	0.000529	0.368
	(0.0144)	(0.0144)	(0.0236)	(0.0406)	(0.0407)	(0.237)
T2 (fixed 21.5, 50%)	-0.0703***	-0.0703***	-0.201***	0.0140	0.0140	0.283
	(0.0142)	(0.0142)	(0.0145)	(0.0287)	(0.0288)	(0.247)
T3 (voting)	0.0670***	0.0670***	-0.0191	0.139***	0.139***	0.365
	(0.0158)	(0.0158)	(0.0222)	(0.0320)	(0.0321)	(0.273)
T4 (voting, 50%)	-0.0372**	-0.0372**	-0.187***	0.0698**	0.0698**	0.396
	(0.0181)	(0.0181)	(0.0170)	(0.0318)	(0.0319)	(0.242)
T5 (finite horizon)	0.0611***	0.0611***	-0.0322**	0.135***	0.135***	0.0242
	(0.0170)	(0.0170)	(0.0147)	(0.0302)	(0.0303)	(0.262)
T6 (vote system)	0.0688***	0.0688***	0.0215	0.109***	0.109***	0.114
	(0.0152)	(0.0152)	(0.0197)	(0.0294)	(0.0295)	(0.271)
Period		-0.000244	-0.00626***		-0.00714***	-0.00130
		(0.000847)	(0.00176)		(0.00245)	(0.0101)
T0 (VAC) & Period			-0.00326			-0.0153
			(0.00212)			(0.0106)
T2 (fixed 21.5, 50%) & Period			0.00965***			-0.0112
			(0.00179)			(0.0107)
T3 (voting) & Period			0.00637***			-0.00944
			(0.00222)			(0.0116)
T4 (voting, 50%) & Period			0.0111***			-0.0136

VARIABLES	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
			(0.00185)			(0.0105)
T5 (finite horizon) & Period			0.00691***			0.00460
			(0.00185)			(0.0111)
T6 (vote system) & Period			0.00350*			-0.000202
			(0.00199)			(0.0115)
Constant	0.888***	0.891***	0.973***	0.829***	1.000***	0.860***
	(0.0141)	(0.0161)	(0.0142)	(0.0280)	(0.0599)	(0.233)
Observations	832	832	832	160	160	160
R^2	0.417	0.418	0.563	0.485	0.501	0.517
Number of runs	32	32	32	32	32	32
Periods	all	all	all	last 5	last 5	last 5

Robust standard errors in parentheses
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

The fourth dependent variable is the share of points earned for the public good rather than from consuming the private good. The experiment was calibrated to have almost equal earnings from both these sources in GE. We see T1 was the one resulting in the highest share of points earned from the PG among all seven treatments. This is a result of the setup, as T1 started with the optimum stock of PG and the tax rate was fixed at the optimum of 21.5%. Hence the stock of PG was kept high and earnings are thus high as well. In all other treatment the share is significantly lower in Models 1, 2, 4 and 5 and the share is falling in all models that have a period variable included. Among the treatments T0 stands out with a 19.6 percentage points (Models 1 and 2) and 31.4 percentage points (models 4 and 5) lower share of points earned from PGs than in T1. This is due to the very low investment in PGs in T0. Also note that the share of points earned from PGs increases in T2 and T4 because these two treatments started at only 50% of the GE-level of PGs and the stock increases over time in these treatments.

Table C4: Random-effects GLS regression with clustered standard errors at the run level

Dependent variable: Share of points from PG; T1 being the benchmark

	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
T0 (VAC)	-0.196***	-0.196***	-0.0208	-0.314***	-0.314***	-0.199
	(0.00875)	(0.00875)	(0.0141)	(0.0186)	(0.0187)	(0.174)
T2 (fixed 21.5, 50%)	-0.0936***	-0.0936***	-0.129***	-0.0721***	-0.0721***	-0.428**
	(0.00894)	(0.00895)	(0.0144)	(0.0189)	(0.0190)	(0.178)
T3 (voting)	-0.0562***	-0.0562***	0.0145	-0.110***	-0.110***	-0.230
	(0.0102)	(0.0102)	(0.0223)	(0.0195)	(0.0196)	(0.180)
T4 (voting, 50%)	-0.111***	-0.111***	-0.125***	-0.119***	-0.119***	-0.332*
	(0.0168)	(0.0168)	(0.0171)	(0.0288)	(0.0289)	(0.173)
T5 (finite horizon)	-0.0693***	-0.0693***	0.0437***	-0.174***	-0.174***	0.116
	(0.0117)	(0.0117)	(0.0147)	(0.0203)	(0.0203)	(0.177)
T6 (vote system)	-0.0675***	-0.0675***	-0.0280	-0.0832***	-0.0832***	-0.229
	(0.0125)	(0.0125)	(0.0175)	(0.0195)	(0.0196)	(0.177)
Period		-0.000589	0.00262**		-0.00339*	-0.00605
		(0.000893)	(0.00122)		(0.00198)	(0.00695)
T0 (VAC) & Period			-0.0130***			-0.00480
			(0.00122)			(0.00714)

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T2 (fixed 21.5, 50%) & Period			0.00262**			0.0149**
			(0.00123)			(0.00743)
T3 (voting) & Period			-0.00523***			0.00500
			(0.00171)			(0.00749)
T4 (voting, 50%) & Period			0.00104			0.00887
			(0.00166)			(0.00719)
T5 (finite horizon) & Period			-0.00837***			-0.0121*
			(0.00128)			(0.00725)
T6 (vote system) & Period			-0.00293**			0.00609
			(0.00128)			(0.00725)
Constant	0.550***	0.558***	0.514***	0.568***	0.649***	0.713***
	(0.00870)	(0.0142)	(0.0141)	(0.0179)	(0.0508)	(0.168)
Observations	832	832	832	160	160	160
R^2	0.411	0.415	0.655	0.764	0.767	0.789
Number of runs	32	32	32	32	32	32
Periods	all	all	all	last 5	last 5	last 5

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Appendix C is included in this submission for the purpose of review by the editors and the referees. It need not be published, since the material in this appendix has been taken from a published paper (Karatzas et al. 2006) which has been appropriately cited in the paper so the interested readers can look it up.

In the formal model in [1] there is a continuum of producer-consumer agents $\alpha \in I$ who hold cash $m_n^\alpha \geq 0$ and goods $q_n^\alpha \geq 0$ at the start of each period n.

The government provides a quantity $G_n \geq 0$ of a generic public good in every period n. We assume that the government spends all its tax revenues on the production and maintenance of the public good.

Each agent α makes a cash bid b_n^α to purchase the sole producer-consumeer good for sale in period n where

$$b_n^\alpha \in \left[0, m_n^\alpha + \frac{\bar\theta \hat p_n q_n^\alpha}{1+\rho}\right], \tag{1}$$

The second term in the expression above is a lending limit imposed by the central bank, if the agetns choose to borrow. The $\hat p_n$ is the bank's estimate of the price p_n of the private good, and, in a rational expectations equilibrium, $\hat p_n = p_n$, thus in equilibrium the individual will be able to pay back her loan. This appears as a a somewhat "secured" loan. In actuality banks handle this feature by adding a "haircut". Although default is possible out of equilibrium, we do not model it explicitly.

Concerning the possiblity that agents wish to deposit and earn interest this is implicitly covered in the selection of b_n^α, it is as though there is a sweep program that gathers any spare cash of the individuals. In the theory below and experiments above with $1 + \rho = 1/\beta$ there is no need for borrowing or depositing..

In period n each agent α sells her goods q_n^α in a market at price p_n and pays taxes in the amount $\theta p_n q_n^\alpha$. The government spends its total tax revenues

$$B_n^G = \int_I \theta p_n q_n^\alpha \, d\alpha = \theta p_n \int_I q_n^\alpha \, d\alpha = \theta p_n Q_n \tag{2}$$

in the private goods market. Thus the price p_n is formed as

$$p_n = \frac{B_n + B_n^G}{Q_n}, \tag{3}$$

where

$$B_n = \int_I b_n^\alpha \, d\alpha$$

is the total bid by the agents. Each agent α purchases the quantity b_n^α/p_n of goods and inputs $k_n^\alpha \in [0, b_n^\alpha/p_n]$ for production.

The government acquires the quantity

$$k_n^G = \frac{B_n^G}{p_n}$$

of the private good all of which is used for the production or maintenance of the public good.

Suppose $G_n \geq 0$ is the quantity of the public good available at the beginning of period n and that $\eta \in (0, 1]$ is the depreciation rate. Then the amount of the public good in the next period is

$$G_{n+1} = (1 - \eta)G_n + F(k_n^G), \tag{4}$$

where $F(\cdot)$ is the government's production function for the public good.

The utility of an agent α in period n is

$$u(x_n^\alpha, G_n),$$

a concave increasing function of the agent's private consumption

$$x_n^\alpha = \frac{b_n^\alpha}{p_n} - k_n^\alpha$$

and the public good G_n provided by the government. For fixed values of G, we assume that $u(\cdot, G)$ is concave, increasing, and differentiable. As before each agent α seeks to maximize her total discounted utility, where $0 \leq \beta < 1$.

$$\sum_{n=1}^\infty \beta^{n-1} u(x_n^\alpha, G_n).$$

The government is regarded as a controller who moves first in the game. Its actions are specified by its selection of its control variables, the interest rate ρ and the tax rate θ. The tax rate determines the income level of the government and thereby also determines its production of the public good. Furthermore the goal or the objective function of the government is assumed known to the agents. For example, the government may wish to supply some level of the public good subject to some condition on inflation.

An equilibrium for the Model

A type-symmetric equilibrium is constructed in which money and prices may inflate, but consumption and production remain constant.

Suppose that all agents begin with the same amount of cash $m^\alpha = M$ and goods $q^\alpha = Q = f(k) + y$. Assume each agent makes the same bid $b^\alpha = b = aM$ and inputs the same amount $k^\alpha = k$ for production. Since the government bids its total income, namely $\theta p Q$, the price of the private good is given by

$$p = \frac{\int_I b^\alpha \, d\alpha + \theta p Q}{Q} = \frac{aM}{Q} + \theta p$$

and hence

$$p = \frac{aM}{\bar{\theta}Q}.$$

The amount of private good consumed by each agent in the period is

$$x^\alpha = \frac{b^\alpha}{p} - k^\alpha = \frac{aM}{p} - k = \bar{\theta}Q - k. \tag{5}$$

The government inputs

$$k^G = \frac{\theta p Q}{p} = \theta Q$$

for production of the public good and thus produces the quantity $F(\theta Q)$. Assume that the government holds the quantity G_n of the public good in each period n equal to a constant G. Then

$$G = (1 - \eta)G + F(\theta Q),$$

and thus

$$G = \frac{1}{\eta} F(\theta Q) \tag{6}$$

is the amount of the public good provided by the government in every period. The utility received by each agent in every period is

$$u(x^\alpha, G) = u(\bar{\theta}Q - k, \frac{1}{\eta}F(\theta Q)). \tag{7}$$

Suppose that every agent $\alpha \in I$ begins with cash $m^\alpha = M > 0$ and goods $q^\alpha = Q = f(k_1) + y$, where $f'(k_1) = (1 + \rho)/(\beta\bar{\theta})$. Assume also that the government initially provides the quantity $G = F(\theta Q)/\eta$ of the public good. Then there is an equilibrium in which all agents bid the proportion

$$a = \frac{(1 + \rho)(1 - \beta)}{\rho} \tag{8}$$

of their cash and input k_1 for production in every period. In this equilibrium, the government inputs $k^G = \theta Q$ for production of the public good in every period thereby holding $G_n = G$ for all n.

The proof given in [1].

In this equilibrium money and prices inflate at the rate $\tau = \beta(1+\rho)$ as they did in the equilibrium of Theorem 1. To see this, suppose that all the agents begin at $m^\alpha = M$ and $q^\alpha = Q$ and play the strategy of the theorem. Then at the next stage, each agent has cash

$$\tilde{M} = \tilde{m}^\alpha = (1+\rho)(m^\alpha - am^\alpha) + \bar{\theta}pq^\alpha$$

$$= (1+\rho)(M - aM) + \bar{\theta} \cdot \frac{aM}{\bar{\theta}Q} \cdot Q$$

$$= \tau M.$$

The price at the next stage is

$$\tilde{p} = \frac{a\tilde{M}}{\bar{\theta}Q} = \tau \frac{aM}{\bar{\theta}Q} = \tau p.$$

A control problem for the government

A benevolent but controlling government will try to maximize the welfare of the agents through its choice of the values of its control variables, the interest rate ρ and the tax rate θ. Observe that, in the equilibrium the total discounted utility of every agent is

$$\sum_{n=1}^{\infty} \beta^{n-1} u(x_n^\alpha, G_n) = \frac{1}{1-\beta} \cdot u(\bar{\theta}Q - k_1, \frac{1}{\eta}F(\theta Q)),$$

where

$$k_1 = (f')^{-1}\left(\frac{1+\rho}{\bar{\theta}\rho}\right) \qquad \text{and} \qquad Q = f(k_1).$$

Thus the utility of an agent can be written as a function $\varphi(\theta, \rho)$ of the the government's control variables θ and ρ.

We illustrate the government's optimization with a simple example.

Example 1 *Suppose* $f(k) = 2\sqrt{k}, F(k) = k, \beta = 1/2, y = 0, \eta = 1,$

$u(x, G) = \log(xG)$. Then

$$f'(k_1) = \frac{1}{\sqrt{k_1}} = \frac{1+\rho}{\beta\bar{\theta}} = \frac{2(1+\rho)}{\bar{\theta}}$$

so the equilibrium values are

$$k_1 = \frac{\bar{\theta}^2}{4(1+\rho)^2}, \qquad Q = f(k_1) = \frac{\bar{\theta}}{1+\rho},$$

and

$$G = F(k^G) = k^G = \theta Q = \frac{\theta\bar{\theta}}{1+\rho}.$$

The utility to maximize is $\varphi(\theta, \rho)$, where

$$\varphi(\theta, \rho) = \frac{1}{1-\beta} \cdot u(\bar{\theta}Q - k_1, \theta Q) =$$

$$\frac{1}{1-\beta} \cdot u\left(\frac{\bar{\theta}^2}{1+\rho} - \frac{\bar{\theta}^2}{4(1+\rho)^2}, \frac{\bar{\theta}\theta}{1+\rho}\right).$$

Since $u(x, G) = \log(xG)$, simple algebra shows that

$$\varphi(\theta, \rho) = 3\log\bar{\theta} + \log\theta + \log(3 + 4\rho) - 3\log(1 + \rho) + C,$$

where C does not depend on θ or ρ. So

$$\frac{\partial\varphi}{\partial\theta}(\theta, \rho) = \frac{-3}{1-\theta} + \frac{1}{\theta} = 0$$

if $\theta = 1/4$. This gives the optimal tax rate at 25%. It's also easy to see that

$$\frac{\partial\varphi}{\partial\rho}(\theta, \rho) < 0$$

for all $\rho > 0$. Hence small values of ρ are better for the agents.[1]

The example is fairly typical in that there is an optimal tax rate that is an interior point of (0,1). This is intuitively obvious since a tax rate of 0 results in no production of the public good and a rate of 1 means that the agents have no incentive to produce the private good.

References

[1] Karatzas, I., Shubik, M. and Sudderth, W. 2011. The Control of a Competitive Economy with a Public Good without Randomness. Journal of Public Economic Theory, 14 (4): 503-537